1.

(i) Let $u^{(\infty)}(x) = Ax + B$

$$\begin{cases} u^{(\infty)}(0) = B = 0 \\ u^{(\infty)}(L) = AL = 0 \end{cases}$$

$u^{(\infty)}(L) = 0$ #

+5

(ii) Assume $u(x,t) = X(x)T(t)$

$$\frac{X''}{X} = \frac{1}{\alpha^2}\frac{T'}{T} = -k^2$$

$$\Rightarrow \begin{cases} X(x) = C\cos kx + D\sin kx \\ T(t) = E\, e^{-\alpha^2 k^2 t} \end{cases}$$

⇐ +5

using boundary condition

$$\Rightarrow u(x,t) = \sum_{n=1}^{\infty} A_n \sin k_n x\, e^{-\alpha^2 k_n^2 t}, \quad k_n = \frac{n\pi}{L}$$

$$A_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi}{L}x\,dx$$

I.C.

⇐ +10

$$A_n = \begin{cases} \frac{160}{n^2\pi^2}, & n = 1, 5, 9, \dots \\ -\frac{160}{n^2\pi^2}, & n = 3, 7, 11, \dots \\ 0, & \text{others} \end{cases}$$

⇐ +15

(iii) 速度：

改變後的 $k' = \frac{n\pi}{2L}$ ⇒ 較慢

或寫一邊絕熱，⇒ 較慢

對一個 +2

全對 +5

new steady-state：

$$\begin{cases} u_x^{(\infty)}(0) = A = 0 \\ u^{(\infty)}(L) = B = 0 \end{cases} \Rightarrow u_x^{(\infty)} \text{ 不變}$$

(iv)

兩邊絕熱 ⇒ steady-state 為原熱能均勻分佈

$$u^{(\infty)}(x) = \frac{20 \times L}{2} \times \frac{1}{L} = 10$$

① 寫出為不為零的 constant +3

② 寫為 10，+5

2

(i)

Let $u^{(p)}(x,0) = \delta(x)$

$$\Rightarrow u^{(p)}(x,t) = \frac{1}{2\alpha\sqrt{\pi t}} e^{\frac{-x^2}{4\alpha^2 t}}$$

⇐ 有看到這個就 +3

then, $u(x,t) = f(x) * u^{(p)}(x,t)$ +5 (through x)

$$= \int_{-\infty}^{\infty} f(\tau) u^{(p)}(x-\tau, t) d\tau$$

$$u(0,t) = \int_{-\infty}^{\infty} f(\tau) u^{(p)}(-\tau, t) d\tau$$

完整寫出 ~~convolution~~ convolution +7

$$= \frac{-u_0 \alpha\sqrt{t}}{a\sqrt{\pi}} \left(e^{\frac{-L^2}{4\alpha^2 t}} - e^{\frac{-a^2}{4\alpha^2 t}} \right)$$

+10

珍惜資源・採用再生紙印製

3.(i) 有寫 $c = f\lambda$ +2

單位 +3

嘗試計算 +1

有寫

有寫 $\omega = c\pi\sqrt{(\frac{l}{3})^2+(\frac{m}{6})^2+(\frac{n}{3})^2}$ +5

代成 346 $\boxed{-1}$

算

$w=2\pi f$ rad $\boxed{-1}$

才說

60Hz → 3分

(ii) 沒化簡 $\sum_{n=1}$ +7

列對算

Ans对

沒

$\sum C_n \sin\frac{l\pi x}{3}\sin(\frac{m\pi y}{6})\sin(\frac{n\pi z}{3})$

↓

不扣，因為可被 T 吸收

x, y, z 順序錯不扣

~~不給分~~

$\sin \alpha x \sin \beta y \sin \gamma z$ (general form only) +5

x, y, z 寫成 x x x 最多 −2 −2

$\frac{c^2 U''}{U} = \frac{T''}{T} = -k^2$

+3

ADT = 0 +2

(m, n, p) = (1,1,1) +2

1(a)

2-2 給分

直

Nsym 不扣分

~~B.C.對 +5~~ 尾 ≥ −1

$u(0,t)=0$ +5 or ...

$u(0,0)=0=u(0,\infty)=0$ +8

+10

標錯 or 沒標 −1

有畫不扣

or

+5 −1

+8

+8

全0 +5 太sloppy 扣分

+3

4. (i) We can expand a well-behaved function to an orthogonal and complete basis. $\sin(n\frac{\pi}{L}x)$ is a complete basis, we can write $g(x,t) = \sum_{n=1}^{\infty} G_n(t)\sin(n\frac{\pi}{L}x)$

只是重述 +2

(ii)
$$\begin{cases} y(x,t) = \sum_{n=1}^{\infty} b_n(t)\sin(n\frac{\pi}{L}x) \\ c^2\frac{d^2y}{dx^2} = \frac{d^2y}{dt^2} + a\frac{dy}{dt} - g(x,t) \end{cases}$$

$$\Rightarrow -c^2\sum_{n=1}^{\infty}\left(\frac{n\pi}{L}\right)^2 b_n(t)\sin(n\frac{\pi}{L}x)$$
$$= \sum_{n=1}^{\infty} b_n''(t)\sin(n\frac{\pi}{L}x) + \sum_{n=1}^{\infty} a b_n''(t)\sin(n\frac{\pi}{L}x)$$
$$- \sum_{n=1}^{\infty} G_n(t)\sin(n\frac{\pi}{L}x)$$

3

$\Rightarrow$ Compare the coefficient of $\sin(n\frac{\pi}{L}x)$

$$c^2\left(\frac{n\pi}{L}\right)^2 b_n(t) + b_n''(t) + a b_n'(t) = G_n(t)$$

-1~2

(iii)
$$\begin{cases} y(x,0) = 0 \Rightarrow y(x,0) = \sum_{n=1}^{\infty} b_n(0)\sin(n\frac{\pi}{L}x) = 0 \\ y_t(x,0) = 0 \Rightarrow y_t(x,0) = \sum_{n=1}^{\infty} b_n'(0)\sin(n\frac{\pi}{L}x) = 0 \end{cases}$$

Imply $b_n(0) = b_n'(0) = 0$

有写出形式

+2 ~+5

From (ii)'s solution

$$b_n''(t) + a b_n'(t) + \omega_n \cancel{b_n}(t) = G_n(t)$$

尝试解

where $\omega_n = \frac{cn\pi}{L}$

in general solution

For this kind of solution may have e^{-at} term,

but $t \gg \frac{1}{a}$, it will disappear

Therefore, we just need to solve steady-state solution.

Try $b_n(t) = A e^{i\omega t}$ and $G_n(t) = g_0 \cos \omega t = g_0 \, Re[e^{i\omega t}]$

$$-A\omega^2 \cancel{e^{i\omega t}} + aAi\omega \cancel{e^{i\omega t}} + \omega_n^2 A \cancel{e^{i\omega t}} = g_0 \cancel{e^{i\omega t}}$$

$$\Rightarrow A(-\omega^2 + ia\omega + \omega_n^2) = g_0$$

$$\Rightarrow A = \frac{g_0}{(\omega_n^2 - \omega^2) + ia\omega}$$

We can express A in exponential notation $|A| e^{-i\phi}$

Laplace equation 4分

$$|A| = \frac{g_0}{\sqrt{(\omega_n^2 - \omega^2)^2 + a^2\omega^2}}$$

解出 Laplace 转换 5分

$$\tan \phi = \frac{a\omega}{\omega_n^2 - \omega^2}$$

b_n b_p +5

只看出 $t >> \frac{1}{a}$

Then we have

7分

$$b_n(t) = Re\left[|A| e^{i\phi} e^{i\omega t}\right]$$

$$= \frac{g_0}{\sqrt{(\omega_n^2 - \omega^2)^2 + a^2\omega^2}} \cos(\omega t - \phi)$$

[illegible]

8分

where $\tan \phi = \dfrac{a\omega}{\omega_n^2 - \omega^2}$

(iv) Because it has "$a y_t$" term, waves will decay.

- The shape will decrease exponentially and it can't expressed by two waves reflection in the walls, that is

$F(x+ct) + G(x-ct)$

No $\to$ 3

No + [illegible] $\to$ 5

Yes $\to$ 0

Yes + [illegible] $\to$ 1~2