數位訊號處理概論
Introduction to Digital Signal Processing: HW5, Quiz5 TA Review

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Remind – Final Exam

- time: **2022/06/15 13:20-15:10**
- scope: everything we cover this semester
- 採實體考試與線上考試並行
- 需要線上考試的同學請在6/12 22:00 前在eeclass 的問卷中登記, 如要實體考試則不需額外登記, 如沒有特殊原因不能臨時改
- 詳細規則於6/11公告
- 1 A4 cheat sheet is allowed, and printed from everywhere is not accepted.
- **Don’t cheat, also we won’t allow any cheating behaviors. Don’t try to challenge.**

Quiz5 & HW5

- We will upload the hw5 scores on Sunday
- If you have other problems:
  - Please email **both TAs**
    - 簡婉軒: wschien@gap.nthu.edu.tw
    - Shreya: shreya@gapp.nthu.edu.tw
  - Please explain in English for HW q5-q8 and Quiz q13-q25
  - Quiz5: Due time: **2022/06/10 17:20**
  - HW5: Due time: **2022/06/13 17:20**

HW5-1

1. Please answer the following questions: (25%)
   (a) If a system function has zeros at the origin of the z-plane then the system function is a proper rational function. Why?
   (b) Can you obtain the z-transform of $u[n]$ at $z = 0$? If you can, what is its value? If you cannot, why not?
   (c) What is the preferred method for obtaining the inverse z-transform of rational functions? Describe this method.
   (d) Explain why a one-sided z-transform is able to determine response to a LCCDE with initial conditions while the two-sided z-transform cannot.
   (e) Can any system with a rational system function be decomposed into a product of an allpass system and a minimum-phase system? If yes, then explain how?

Please refer to professor’s slides and textbook

HW5-2

2. Determine the impulse response of the system for all possible regions of convergence. (5%)

$$y[n] - \frac{5}{2}y[n-1] + y[n-2] = x[n-1]$$

Solution:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-1}}{1 - \frac{5}{2}z^{-1} + z^{-2}} = \frac{\frac{2}{3}}{1 - 2z^{-1}} + \frac{-\frac{2}{3}}{1 - \frac{1}{2}z^{-1}}$$

ROC: $|z| > 2$

$$h[n] = \frac{2}{3} \cdot 2^n u[n] - \frac{2}{3} \cdot \left(\frac{1}{2}\right)^n u[n]$$

ROC: $|z| < \frac{1}{2}$

$$h[n] = -\frac{2}{3} \cdot 2^n u[-n-1] + \frac{2}{3} \cdot \left(\frac{1}{2}\right)^n u[-n-1]$$

ROC: $\frac{1}{2} < |z| < 2$

$$h[n] = -\frac{2}{3} \cdot 2^n u[-n-1] - \frac{2}{3} \cdot \left(\frac{1}{2}\right)^n u[n]$$

HW5-3

3. Consider the finite length sequence $x[n] = u[n] - u[n-N]$ (15%)
   (a) Determine the z-transform $X(z)$ of the sequence $x[n]$.
   (b) Determine and plot the sequence $y[n] = x[n] * x[n]$.
   (c) Determine the z-transform $Y(z)$ of the sequence $y[n]$.

(a) Solution:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n} = \sum_{n=0}^{N-1} z^{-n} = \frac{1-z^{-N}}{1-z^{-1}}, \quad \text{ROC: } |z| > 0$$

(b) Solution:

$$y[n] = \sum_{m=-\infty}^{\infty} x[m]x[n-m] = \begin{cases} n+1, & 0 \le n \le N-1 \\ 2N-1-n, & N-1 \le n \le 2N-1-n \\ 0, & \text{elsewhere} \end{cases}$$

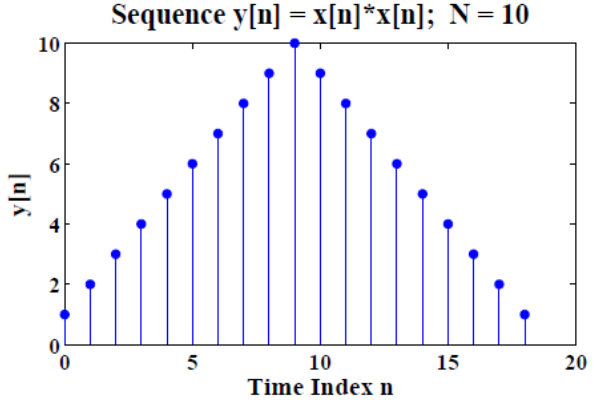

FIGURE 3.31: Sequence $y[n] = x[n] * x[n]$.

(c) Solution:

$$Y(z) = X(z)X(z) = \left(\frac{1-z^{-N}}{1-z^{-1}}\right)^2, \quad \text{ROC: } |z| > 0$$

HW5-4

4. Consider a stable system with input $x[n]$ and output $y[n]$. Determine its impulse response $h[n]$ if we are given that: (5%)

$x[n] = \left(\frac{1}{3}\right)^{|n|}$ and $y[n] = 2\left(\frac{1}{3}\right)^{n} u[n] - 2^{n+2}u[-n]$

$$X(z) = \frac{1}{1-\frac{1}{3}z^{-1}} + \frac{-1}{1-3z^{-1}} = \frac{-\frac{8}{3}z^{-1}}{1-\frac{10}{3}z^{-1}+z^{-2}}, \quad \text{ROC: } \frac{1}{3} < |z| < 3$$

$$Y(z) = \frac{2}{1-\frac{1}{3}z^{-1}} - 4 + \frac{4}{1-4z^{-1}}$$
$$= \frac{2+8z^{-1}-\frac{16}{3}z^{-2}}{(1-\frac{1}{3}z^{-1})(1-4z^{-1})}, \quad \text{ROC: } \frac{1}{3} < |z| < 4$$

$$H(z) = \frac{Y(z)}{X(z)} = -\frac{3}{4}z - \frac{19}{8} + \frac{3}{2}z^{-1} + \frac{-\frac{5}{4}}{1-4z^{-1}}, \quad \text{ROC: } |z| < 4$$

$$h[n] = -\frac{3}{4}\delta[n+1] - \frac{19}{8}\delta[n] + \frac{3}{2}\delta[n-1] + \left(\frac{5}{4}\right)\cdot 4^n u[-n-1]$$

HW5-5

5. Show that the group delay of an LTI system with frequency response function $H(e^{j\omega}) = H_R(\omega) + jH_I(\omega)$ can be expressed as: (10%)

$$\tau(\omega) = \frac{H_R(\omega)G_R(\omega) + H_I(\omega)G_I(\omega)}{|H(e^{j\omega})|^2}.$$

Where $G(e^{j\omega}) = G_R(\omega) + jG_I(\omega)$ is the DTFT of $nh[n]$.

Proof:

$$\tau_{gd}(\omega) = -\frac{d\Psi(\omega)}{d\omega}$$

$$\Psi(\omega) = \tan^{-1}\frac{H_I(\omega)}{H_R(\omega)} + 2k\pi, \quad \Psi(\omega) \text{ is continuous}$$

$$-\frac{d\Psi(\omega)}{d\omega} = -\frac{d}{d\omega}\left(\tan^{-1}\frac{H_I(\omega)}{H_R(\omega)} + 2k\pi\right) = \frac{\left(\frac{dH_I(\omega)}{d\omega}\right)H_R(\omega) - \left(\frac{dH_R(\omega)}{d\omega}\right)H_I(\omega)}{H_R^2(\omega) + H_I^2(\omega)}$$

$$nh[n] \xrightarrow{\text{DTFT}} \mathrm{j}\cdot\frac{d\left[H_R(\omega) + \mathrm{j}H_I(\omega)\right]}{d\omega} = \mathrm{j}\frac{dH_R(\omega)}{d\omega} - \frac{dH_I(\omega)}{d\omega}$$

Hence, we have

$$\frac{dH_R(\omega)}{d\omega} = G_I(\omega); \quad \frac{dH_I(\omega)}{d\omega} = -G_R(\omega)$$

$$H_R^2(\omega) + H_I^2(\omega) = |H(e^{j\omega})|^2$$

Thus, we proved that

$$\tau_{gd}(\omega) = -\frac{d\Psi(\omega)}{d\omega} = \frac{H_R(\omega)G_R(\omega) + H_I(\omega)G_I(\omega)}{|H(e^{j\omega})|^2}$$

HW5-6

6. Determine location of poles and zeros in following conditions: (20%)
   (a) We want to design a second-order IIR filter using pole-zero placement that satisfies the following requirements: (1) The magnitude response is 0 at $\omega_1 = 0$ and $\omega_3 = \pi$; (2) The maximum magnitude is 1 at $\omega_{2,4} = \pm\pi/4$; and (3) the magnitude response is approximately $\frac{1}{\sqrt{2}}$ at frequencies $\omega_{2,4} \pm 0.05$. Determine the location of the two poles and zeros of the required filter and then compute its system function H(z).
   (b) Now consider the second-order notch IIR filter that satisfies the following requirements: (1) The magnitude response has notches at $\omega_{1,2} = \pm 2\pi/3$; (2) The maximum magnitude response is 1; (3) the magnitude response is approximately $\frac{1}{\sqrt{2}}$ at frequencies $\omega_{1,2} \pm 0.01$.

   Using the pole-zero placement approach determine locations of two poles and two zeros of the required filter and then compute its system function H(z).

(a) Solution:

$$\text{zeros:} \quad z_1 = e^{j0} = 1, \quad z_2 = e^{j\pi} = -1$$

$$\text{poles:} \quad p_1 = re^{j\frac{\pi}{4}}, \quad p_2 = re^{-j\frac{\pi}{4}}, \quad r \in (0,1)$$

The system function is:

$$H(z) = b_0 \frac{(1 - z^{-1})(1 + z^{-1})}{(1 - re^{j\frac{\pi}{4}} z^{-1})(1 - re^{-j\frac{\pi}{4}} z^{-1})}$$

The frequency response is:

$$H(e^{j\omega}) = b_0 \frac{(1 - e^{-j\omega})(1 + e^{-j\omega})}{(1 - re^{j\frac{\pi}{4}} e^{-j\omega})(1 - re^{-j\frac{\pi}{4}} e^{-j\omega})}$$

Constrain $|H(e^{j\omega})|_{\max} = 1$, we have

$$|b_0| = \frac{(1-r)\sqrt{1+r^2}}{\sqrt{2}}$$

Choose $r = 0.95$.

(b) Solution:

$$\text{zeros:} \quad z_1 = e^{j\frac{2\pi}{3}}, \quad z_2 = e^{-j\frac{2\pi}{3}}$$

$$\text{poles:} \quad p_1 = re^{j(\frac{2\pi}{3}+\phi)}, \quad p_2 = re^{-j(\frac{2\pi}{3}+\phi)}, \quad r \in (0,1)$$

The system function is:

$$H(z) = b_0 \frac{(1 - e^{j\frac{2\pi}{3}} z^{-1})(1 - e^{-j\frac{2\pi}{3}} z^{-1})}{(1 - re^{j(\frac{2\pi}{3}+\phi)} z^{-1})(1 - re^{-j(\frac{2\pi}{3}+\phi)} z^{-1})}$$

The frequency response is:

$$H(e^{j\omega}) = b_0 \frac{(1 - e^{j\frac{2\pi}{3}} e^{-j\omega})(1 - e^{-j\frac{2\pi}{3}} e^{-j\omega})}{(1 - re^{j(\frac{2\pi}{3}+\phi)} e^{-j\omega})(1 - re^{-j(\frac{2\pi}{3}+\phi)} e^{-j\omega})}$$

Constrain $|H(e^{j\omega})|_{\max} = 1$, we have

$$|b_0| = \frac{(1-r)|1 - re^{-2j(\frac{2\pi}{3}+\phi)}|}{|1 - e^{-j\phi}||1 - e^{-j(\frac{4\pi}{3}+\phi)}|}$$

Choose $r = 0.9, \phi = 0.01$.

HW5-7

7. Derive (eq-q7.1) formula for the impulse response of an ideal bandpass filter by (10%)

$$h_{bp}[n] = 2\frac{\sin \omega_c(n-n_d)}{\pi(n-n_d)} cos\omega_0 n \qquad \text{eq-q7.1}$$

Using impulse response (eq-q7.2) of the ideal low pass filter and the modulation property of DTFT.

$$h_{lp}[n] = \frac{\sin \omega_c(n-n_d)}{\pi(n-n_d)} \qquad \text{eq-q7.2}$$

Solution:

$$h_{bp}[n] = 2\frac{\sin \omega_c(n-n_d)}{\pi(n-n_d)} \cos \omega_0 n \qquad (5.72)$$

$$h_{lp}[n] = \frac{\sin \omega_c(n-n_d)}{\pi(n-n_d)} \qquad (5.70)$$

Modulation Property:

$$x[n]\cos\omega_0 n = \frac{1}{2}X(e^{j(\omega+\omega_0)}) + \frac{1}{2}X(e^{j(\omega-\omega_0)})$$

$$H_{bp}(e^{j\omega}) = \begin{cases} e^{-j\omega n_d}, & \omega_\ell \le |\omega| \le \omega_h \\ 0, & \text{otherwise} \end{cases}$$

$$H_{lp}(e^{j\omega}) = \begin{cases} e^{-j\omega n_d}, & |\omega| \le \omega_c \\ 0, & \omega_c < |\omega| \le \pi \end{cases}$$

$$H_{bp}(e^{j\omega}) = H_{lp}(e^{j(\omega-\omega_0)}) + H_{lp}(e^{j(\omega+\omega_0)})$$

where $\omega_0 = \frac{\omega_\ell+\omega_h}{2}$ and $\omega_c = \frac{\omega_h-\omega_\ell}{2}$.

Hence,

$$h_{bp}[n] = 2h_{lp}\cos\omega_0 n = 2\frac{\sin \omega_c(n-n_d)}{\pi(n-n_d)}\cos\omega_0 n$$

HW5-8

8. Determine the group delay of the following systems: (10%)

(a) $y[n] = x[n] - 0.9x[n-1]$

(b) $y[n] = 0.8y[n-1] + x[n]$

(a) Solution:

The frequency response is:

$$H(e^{j\omega}) = 1 - 0.9e^{-j\omega} = 1 - 0.9\cos\omega + j0.9\sin\omega$$

The phase response is:

$$\Psi(\omega) = \tan^{-1}\frac{0.9\sin\omega}{1-0.9\cos\omega} + 2k\pi$$

The group delay is:

$$\tau_{gd}(\omega) = -\frac{d\Psi(\omega)}{d\omega} = \frac{1-0.9\cos\omega}{1+0.9^2-1.8\cos\omega}$$

(b) Solution:

The frequency response is:

$$H(e^{j\omega}) = \frac{1}{1-0.8e^{-j\omega}} = \frac{1}{1-0.8\cos\omega + j0.8\sin\omega}$$

The phase response is:

$$\Psi(\omega) = -\tan^{-1}\frac{0.8\sin\omega}{1-0.8\cos\omega} + 2k\pi$$

The group delay is:

$$\tau_{gd}(\omega) = -\frac{d\Psi(\omega)}{d\omega} = \frac{0.8\cos\omega - 0.8^2}{1+0.8^2-1.6\cos\omega}$$

Quiz5

(c) Let us an example of anti-causal sequence whose z-transform will be in the form X(z)=1+z+z2 which has a finite value at all values of ‘z’ except at z=∞. So, ROC of an anti-causal sequence is entire z-plane except at z=∞.

1 Which of the following statement is ***false*** about region of convergence (ROC)?

- A. The ROC of a causal infinite length sequence is |z|>r1.
- B. The ROC of z-transform of any signal cannot contain poles.
- C. The ROC of z-transform of finite duration anti-causal sequence contains entire z-plane, except at z=0.
- D. The ROC of z-transform of a two-sided infinite sequence is r2<|z|<r1.
- E. The ROC is always bounded by a circle.

Region of convergence (ROC) A set of values of z for which the series (3.9) converges. It is always bounded by a circle.

較多人選這個選項, 這是課本上對ROC的定義

難易度: 未設定

2 Which of the following statement is ***false*** about z-transform?

- A. Any sequence x[n] can be uniquely characterized by its z-transform.
- B. One-sided z-transform is unique for causal signal.
- C. The z-transform converts convolution equations and LCCDE into algebraic equations.
- D. The z-transform of a causal periodic signal can be determined from the knowledge of the z-transform of its first cycle
- E. Two-sided z-transform is able to determine response to a LCCDE with initial conditions.

較多人選這個選項 Ref to june1-2022.pdf p6

(e) You had proved in hw5-1

難易度: 未設定

Quiz5

(c) If the magnitudes of the poles of the response of any system is almost equal to one, then the system decays very slowly or the transient will persist for a relatively long time.

3 Which of the following statement is ***false*** about z domain system?

- A. A linear time invariant system is said to be BIBO stable and only if the ROC of the system function includes unit circle.
- B. If all the poles of H(z) are inside the unit circle, then the system is said to be BIBO stable and causal.
- C. If one or more poles are located near the unit circle, then the rate of decay of signal is rapid.
- D. If all the poles have small magnitudes, then the rate of decay of signal is rapid.
- E. If all the poles of H(z) are outside the unit circle, then the system is neither causal nor BIBO stable.

難易度: 未設定

(a) For an ideal filter, in the magnitude response plot at the stop band it should have a sudden fall which means an ideal filter should have a zero gain at stop bands.

4 Which of the following statement is ***false*** about filters?

- A. An ideal filter should have unity gain in their stop bands.
- B. A digital resonator is a special two pole bandpass filter with the pair of complex conjugate poles located near the unit circle.
- C. A comb filter is a special case of notch filter in which the nulls occur periodically across the frequency band.
- D. All ideal filters are unstable and unrealizable, but take ideal low pass filter as prototype filter.
- E. Notch filters have perfect null at certain frequencies.

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難易度: 未設定

Quiz5

5 Which of the following statement is ***false*** about group delay?

- A. Group delay is useful in checking the linearity of the phase response.
- B. Group delay is the time delay that the signal component of frequency ω undergoes as it passes from the input to the output of the system.
- C. If the phase Θ(ω) of the system is linear, then the group delay of the system decreases with frequency of signal.
- D. In the IIR filter, the group delay response is used as a measure of how much dispersal a typical input will undergo when being procured by a filter.
- E. Group delay is the time delay of the amplitude envelopes of the various sinusoidal components of a signal.

(c) We know that the group delay of the system with phase Θ(ω) is defined as Tg(ω)=dΘ(ω)dω. Given the phase is linear=> the group delay of the system is constant.

Quiz5

Properties of FIR filter:

- Impulse Response is of finite duration
- It is a non-recursive system because there is no feedback path present in the FIR filter.
- They are always be designed as a linear phase.
- Stability is guaranteed is the FIR system.
- In FIR, to obtain the same frequency response as IIR large number of additions & multiplication is reviewed, so the speed is very slow.

Properties of IIR filter:

- The impulse response is of infinite duration.
- IIR system is also known as a recursive system because there is a feedback path from output to input.
- IIR system cannot be designed as a linear phase system.
- Stability cannot be guaranteed.
- In the IIR system, fewer multiplications and additions are reviewed, so processing speed is very fast.

Linear phase:

- **FIR can be easily designed to have a exact linear phase**
- No phase distortion is introduced into the signals to be filtered.
- All frequencies are shifted in time by the same amount.
- Increasing the order of the filter, but keeping all else the same, increases the sharpness of the filter roll-off
- The sharpness of the FIR and IIR filters is very different for the same order.
- Because of the recursive nature of an IIR filter, where part of the filter output is used as an input, IIR filters have sharper roll-off with the same order FIR filter.

6 Which one of the following is the advantage of FIR filter over IIR filter?

- A. FIR filter is always unstable.
- B. FIR filter can have an exact linear phase.
- C. For FIR filter, the design methods are non-linear.
- D. FIR filter cannot be realized efficiently in hardware.
- E. In FIR system, fewer multiplications and additions are revied, so processing speed is very fast.

Quiz5

(c) There are always some distortions, even in the perfect transmission line. This is due to the variation of the secondary parameters. The attenuation constant causes the frequency distortion, whereas the phase constant causes the phase distortion.

7 Which of the following statement is ***false*** about distortion?

- A. Signal distortion is the disadvantage of sampling rate conversion by converting the signal into analog signal.
- B. A distortion in the shape of the response if the phase response is not a linear function of ω.
- C. The attenuation constant causes phase distortion and the phase constant causes frequency distortion.
- D. A system introduces magnitude distortion if $|H(\omega)| \neq$ a constant.
- E. Ideal frequency-selective filters have a distortionless response over one or more frequency bands and zero response elsewhere.

難易度：未設定

8 A good filter should have a ___(a)___ ripple in the passband, ___(b)___attenuation in the stopband and very ___(c)___transition bands.

what are (a), (b), (c)?

- A. small, high, narrow
- B. small, low, wide
- C. large, high, narrow
- D. small, high, wide
- E. large, low, wide

Quiz5

9 If $X(z)$ has M finite zeros and N finite poles, then which of the following condition is true?

(a) |N-M| zeros at origin (if N > M)
(b) |N-M| zeros at origin (if N < M)
(c) |N+M| zeros at origin (if N > M)
(d) |N+M| zeros at origin (if N < M)
(e) |N-M| poles at origin (if N > M)
(f) |N-M| poles at origin (if N < M)
(g) |N+M| poles at origin (if N > M)
(h) |N+M| poles at origin (if N < M)

Partial Fraction Expansion

$$X(z) = \frac{b_0 + b_1 z^{-1} + \cdots + b_M z^{-M}}{a_0 + a_1 z^{-1} + \cdots + a_N z^{-N}} \rightarrow X(z) = \frac{z^N (b_0 z^M + \cdots + b_M)}{z^M (a_0 z^N + \cdots + a_N)}$$

Hence, it has M zeros (roots of $\sum b_k z^{M-k}$), N poles (roots of $\sum a_k z^{N-k}$), and (M-N) poles at zero if M>N (or (N-M) zeros at zero if N>M).

Quiz5

10

Given the z-transform pair $x[n] \leftrightarrow X(z) = \frac{1}{1-\frac{1}{3}z^{-1}}$ with $ROC: |z| > \frac{1}{3}$, determine which z-transform properties can be easily applied to estimate the z-transform of $y[n] = x[n] * x[-n-1]$.

(No partial points, you have to select all correct answers.)
(多選題)

- [x] A. Convolution
- [x] B. Time shifting
- [x] C. Folding
- [ ] D. Differentiation
- [ ] E. Linearity
- [ ] F. Scaling

Solution:

Convolution, time shifting and folding:

$$Y(z) = X(z)X(1/z)z = \frac{-3}{1-3z^{-1}}\frac{1}{1-\frac{1}{3}z^{-1}} = \frac{-3}{1-\frac{10}{3}+z^{-2}}$$

ROC: $\frac{1}{3} < |z| < \frac{2}{3}$

難易度：未設定

11

Given the z-transform pair $x[n] \leftrightarrow X(z) = \frac{1}{1-\frac{1}{2}z^{-1}}$ with $ROC: |z| > \frac{1}{2}$, determine which z-transform properties can be easily applied to estimate the sequence corresponding to $Y(z) = \frac{dX(z)}{dz}$.

(No partial points, you have to select all correct answers.)
(多選題)

- [ ] A. Convolution
- [x] B. Time shifting
- [ ] C. Folding
- [x] D. Differentiation
- [ ] E. Linearity
- [ ] F. Scaling

$Y(z) = dX(z)/dz$

Differentiation and time-shifting: $y[n] = -(n-1)x[n-1]$

$$= -(n-1)\left(\frac{1}{2}\right)^{n-1}u[n-1]$$

Quiz5

12 The one-sided z-transform of $x[n] = \delta[n-k]$ is _z^(-k)_, and the one-sided z-transform of $x[n] = \delta[n+k]$ is _0_.
(忽略空白)

Answering format:

- real numbers: a or -a
- imaginary numbers: a±bj or −a±bj
- exponential numbers: a^b or a^(-b) or (-a)^b or (-a)^(-b)
- if a, b are not integers, represent by irreducible fraction, eg., 2/5, 9/4
- no spaces

Quiz5

Let the system is $y[n] = \frac{1}{2}(x[n] + x[n-1])$

13 Determine the magnitude response of the system.

- A. $|cos\omega|$
- B. $|cos\frac{\omega}{2}|$
- C. $|cos\frac{\omega}{6}|$
- D. $|cos\frac{\omega}{4}|$

Difficulty: Not specified

14 Determine the phase response of the system.

- A. $\frac{\omega}{2}$
- B. ω
- C. $-\omega$
- D. $-\frac{\omega}{2}$

Difficulty: Not specified

15 The frequency response of the system is:

- A. $\frac{1}{3}(1 + cos\omega - jsin\omega)$
- B. $\frac{1}{3}(1 + cos\omega + jsin\omega)$
- C. $\frac{1}{2}(1 + cos\omega + jsin\omega)$
- D. $\frac{1}{2}(1 + cos\omega - jsin\omega)$

Solution:

The system function is:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{2}(1 + z^{-1})$$

The frequency response is:

$$H(e^{j\omega}) = \frac{1}{2}(1 + e^{-j\omega}) = \frac{1}{2}(1 + \cos\omega - j\sin\omega)$$

The magnitude response is:

$$|H(e^{j\omega})| = \frac{1}{2}\sqrt{(1+\cos\omega)^2 + \sin^2\omega} = |\cos\frac{\omega}{2}|$$

The phase response is:

$$\angle H(e^{j\omega}) = \tan^{-1}\frac{-\sin\omega}{1+\cos\omega} = -\tan^{-1}\tan\frac{\omega}{2} = -\frac{\omega}{2}$$

Quiz5

For the following input–output pairs determine whether or not there is an LTI system producing y[n] when the input is x[n]. If such a system exists, determine if it is unique or not.

16 $x[n] = \left(\frac{1}{4}\right)^n$ all $n \mapsto y[n] = \left(\frac{1}{2}\right)^n$ all n.

- A. Exists but NOT unique
- B. Exists and unique
- C. NOT exists

Difficulty: Not specified

4 point(s)

Solution:

$$y[n] = x[n/2]$$

NOT exists. The above system is not LTI.

17 $x[n] = e^{j2\pi\frac{n}{5}}u[n] \mapsto y[n] = 3e^{j2\pi\frac{n}{5}-j\frac{\pi}{2}}u[n]$

- A. Exists but NOT unique
- B. Exists and unique
- C. NOT exists

Difficulty: Not specified

4 point(s)

Solution:
Exists and Unique.

$$H(e^{j\omega}) = 3e^{-j\frac{\pi}{2}}$$

18 $x[n] = 2u[n] \mapsto y[n] = 3u[n] - 5u[n-5]$, for this input–output pair, the system exists and also unique, please find its frequency response function.

- A. $H(e^{j\omega}) = \frac{3}{2} - \frac{5}{2}e^{5j\omega}$
- B. $H(e^{j\omega}) = \frac{3}{2} - \frac{5}{2}e^{-5j\omega}$
- C. $H(e^{j\omega}) = \frac{3}{2} + \frac{5}{2}e^{5j\omega}$
- D. $H(e^{j\omega}) = \frac{3}{2} + \frac{5}{2}e^{-5j\omega}$

Difficulty: Not specified

4 point(s)

Exists and unique.

$$H(e^{j\omega}) = \frac{3}{2} - \frac{5}{2}e^{-5j\omega}$$

Quiz5

Consider a second-order IIR filter specification that satisfies the following requirements: (1) the magnitude response is 0 at $\omega_1 = \pi/2$ and $\omega_4 = 3\pi/2$; (2) the maximum magnitude response is 1 at $\omega_{2,3} = \pm 2\pi/3$; and (3) the magnitude response is approximately $1/\sqrt{2}$ at frequencies $\omega_{2,3} \pm 0.05$.

19 Using the pole-zero placement approach determine locations of two zeros of the required filter:

- A. $z_1 = e^{-j\frac{\pi}{2}},\ z_2 = e^{j\frac{3\pi}{2}}$
- B. $z_1 = e^{-j\frac{\pi}{2}},\ z_2 = e^{-j\frac{3\pi}{2}}$
- C. $z_1 = e^{j\frac{\pi}{2}},\ z_2 = e^{j\frac{3\pi}{2}}$
- D. $z_1 = e^{j\frac{\pi}{2}},\ z_2 = e^{-j\frac{\pi}{2}}$

Difficulty: Not specified

20 Using the pole-zero placement approach determine locations of two poles of the required filter:

- A. $p_1 = re^{j\frac{\pi}{2}},\ p_2 = re^{-j\frac{\pi}{2}}$
- B. $p_1 = re^{-j\frac{\pi}{2}},\ p_2 = re^{j\frac{2\pi}{3}}$
- C. $p_1 = re^{j\frac{\pi}{2}},\ p_2 = re^{-j\frac{2\pi}{3}}$
- D. $p_1 = re^{j\frac{2\pi}{3}},\ p_2 = re^{-j\frac{2\pi}{3}}$

Difficulty: Not specified

2 point(s)

Solution:

zeros: $z_1 = e^{j\frac{\pi}{2}}, z_2 = e^{j\frac{3\pi}{2}}$

poles: $p_1 = re^{j\frac{2\pi}{3}}, p_2 = re^{-j\frac{2\pi}{3}}$

The system function is:

$$H(z) = b_0 \frac{(1 - e^{j\frac{\pi}{2}} z^{-1})(1 - e^{j\frac{3\pi}{2}} z^{-1})}{(1 - re^{j\frac{2\pi}{3}} z^{-1})(1 - re^{-j\frac{2\pi}{3}} z^{-1})}$$

Quiz5

A continuous-time LTI system is described by the impulse response $h(t) = 4e^{-0.05t}\cos\left(10\pi t + \frac{\pi}{4}\right)u(t)$.

The input to the system is $x(t) = 4 - 3\cos\left(4\pi t + \frac{\pi}{3}\right) + 5\sin(20\pi t)$.

Answering format:

- can be real numbers (integers or fractions of integers): 6 or -6 or 6/4 or -6/4
- can be imaginary numbers: 6j or -6j or (6/4)j or -(6/4)j (remember to use bracket when there are fraction imaginary values)
- no spaces

21 Determine the system function H(s).

$$H(s) = \frac{2e^{ja\pi}}{s+b+c\pi} + \frac{2e^{jd\pi}}{s+e+f\pi}$$

fill the a, b, c, d, e, f of the H(s) shown in above format: a: 1/4 , b: 1/20 , c: -10 , d: -1/4 , e: 1/20 , f: 10

[Reminder: careful with positive and negative signs eg: 6 or -6 ; and only use fraction format for float (decimal) numbers, eg: 1/2 not 0.5]
(Not case sensitive, Ignoring space)

Difficulty: Not specified

6 point(s)

Solution:

$$h(t) = 2e^{j\frac{\pi}{4}}e^{-0.005t}e^{j10\pi t}u(t) + 2e^{-j\frac{\pi}{4}}e^{-0.005t}e^{-j10\pi t}u(t)$$

The system function is:

$$H(s) = \frac{2e^{j\frac{\pi}{4}}}{s+0.05-10\pi} + \frac{2e^{-j\frac{\pi}{4}}}{s+0.05+10\pi}$$

22 Determine the output y(t). if y(t) is written as below, expression:

$$y(t) = C_0\,H(0) + C_1 e^{j\left(\frac{\pi}{2}\right)} e^{jV_1\pi t} H(V_1\pi) + C_2 e^{-j\left(\frac{\pi}{2}\right)} - e^{jV_2\pi t} H(-V_2\pi) + C_3\, e^{jV_3\pi t} H(V_3\pi) + C_4\, e^{-jV_4\pi t} H(-V_4\pi)$$

$$y(t) = C_0\,H(0) + C_1 e^{j\left(\frac{\pi}{2}\right)} e^{jV_1\pi t} H(V_1\pi) + C_2 e^{-j\left(\frac{\pi}{2}\right)} e^{-jV_2\pi t} H(-V_2\pi) + C_3\, e^{jV_3\pi t} H(V_3\pi) + C_4\, e^{-jV_4\pi t} H(-V_4\pi)$$

There is one typing error with "-" sign but if u know the format you should know it's a typing issue, and shouldn't affect your values or you should have asked me in the quiz time.

fill in the values for, C_0, C_1, C_2, C_3, V_1, V_2, V_3, V_4 of above y(t) output format:

C_0 : 4 , C_1: -3/2 , C_2: -3/2 , C_3 : (5/2)j , C_4:, -(5/2)j , V_1: 4 , V_2: 4 , V_3: 20 , V_4: 20

[Reminder: careful with positive and negative signs eg: 6 or -6 ; and only use fraction format for float (decimal) numbers, eg: 1/2 not 0.5 ; also the use o bracket in imaginary fractions numbers]

(Not case sensitive, Ignoring space)

Difficulty: Not specified

9 point(s)

Solution:

$$x(t) = 4 - \frac{3}{2}e^{j\frac{\pi}{3}}e^{j4\pi t} - \frac{3}{2}e^{-j\frac{\pi}{3}}e^{-j4\pi t} + \frac{5}{2j}e^{j20\pi t} - \frac{5}{2j}e^{-j20\pi t}$$

$$y(t) = 4H(0) - \frac{3}{2}e^{j\frac{\pi}{3}}e^{j4\pi t}H(4\pi) - \frac{3}{2}e^{-j\frac{\pi}{3}}e^{-j4\pi t}H(-4\pi) + \frac{5}{2j}e^{j20\pi t}H(20\pi) - \frac{5}{2j}e^{-j20\pi t}H(-20\pi)$$

Quiz5

23

The transfer function is given as :

$$H(j\omega) = \frac{1 + j\omega^2}{1 - j\omega^2}$$

the delay of the system at ω = 0 is:

- A. -2
- B. 0
- C. -4
- D. 8

Difficulty: Not specified

3 point(s)

$$H(j\omega) = \left(\frac{1+j\omega^2}{1-j\omega^2}\right)$$

$$\angle H(j\omega) = \tan^{-1}\frac{\omega^2}{1} - \tan^{-1}\frac{(-\omega^2)}{1}$$

$$= 2\tan^{-1}\omega^2$$

$$\frac{d\angle H(j\omega)}{d\omega} = -2\left[\frac{1}{1+(\omega^2)^2} \times 2\omega\right]$$

$$= -\frac{4\omega}{1+\omega^4}$$

at $\omega = 0$

$$-\frac{d\angle H(j\omega)}{d\omega} = 0$$

24

For a distortionless LTI system:

- A. group delay = phase delay
- B. group delay > phase delay
- C. group delay < phase delay
- D. group delay = phase delay/2

Difficulty: Not specified

3 point(s)

Group delay concept from June01-2022.pdf page 36-38

25

The response of a stable LTI system to a complex exponential sequence is a complex exponential sequence with the same ___(a)___, only the ___(b)___ and ___(c)___ are changed by the system. what are (a),(b),(c)?

- A. phase, frequency, amplitude
- B. frequency, amplitude, phase
- C. frequency, amplitude, group delay
- D. amplitude, phase, frequency,

Difficulty: Not specified

3 point(s)

Concepts from June01-2022.pdf page 14