數位訊號處理概論
Introduction to Digital Signal Processing: HW4, Quiz4, Topic5 TA Review

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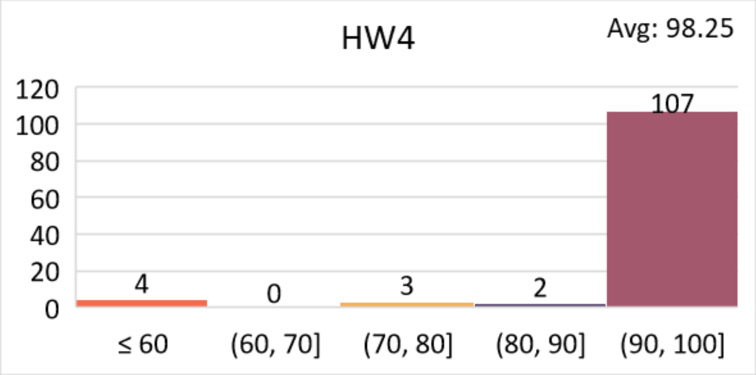
Remind

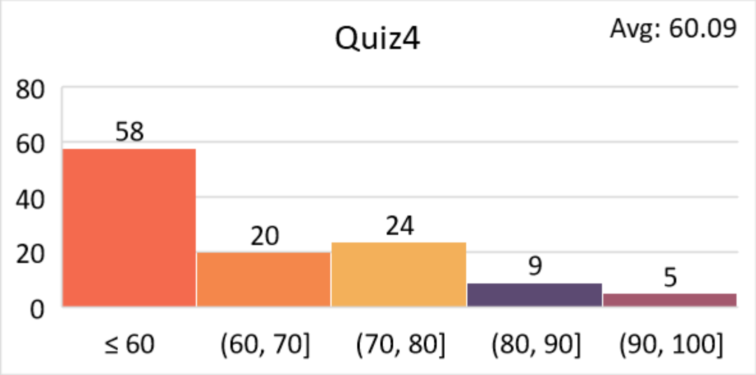
- HW 5:
  - due time: **2022/06/08 at 13:20**
  - Submit format: 108060001_王小名.pdf
  - Please provide detailed answers or explanations in English

- Quiz 5:
  - time: **2022/06/08 13:20-15:10**
  - scope: everything we cover in topic 5
  - Online in eeclass [Exam]
  - Restrict switching to other windows (Answers will be submitted automatically)
  - 1 A4 cheat sheet and 1 A4 double-sided blank sheet are allowed, and printed from everywhere is not accepted.

Quiz4 & HW 4

- Quiz4有另外上傳調整後的成績，請確認自訂欄的Quiz4成績
- If you have other problems:
  - Please email **both TAs**
    - 簡婉軒: wschien@gap.nthu.edu.tw
    - Shreya: shreya@gapp.nthu.edu.tw
  - Please explain in English for HW q5-q7 and Quiz q11-q25
  - Due time: **2022/06/03 17:20**





HW4-1

1. Please answer the following questions: (20%)
   (a) Explain clearly the spectral effect of windowing a sinusoidal signal using a rectangular window.
   (b) Describe the uncertainty principle in signal processing. Which signal satisfies it with equality?
   (c) What is a spectrogram and when is its use required in practice?
   (d) It is said that the “zero-padding” operation in DFT provides a high-density (visually smooth) spectrum but not a high-resolution (quality) spectrum. Do you agree or disagree? Explain.

(a) ref to May11-2022 p24-25
(b) ref to May11-2022 p26-28
(c) ref to May11-2022 p36
(d) ref to May4-2022 p30

HW4-2

2. Consider a real-valued N-point sequence x[n], where X[k] is its N-point DFT. (20%)
   (a) Show that $X[0]$ is real-valued.
   (b) Show that $X[\langle N-k\rangle_N] = X^*[k]$.

(a) Proof:

$$X[0] = \sum_{n=0}^{N-1} x[n] W_N^{n\cdot 0} = \sum_{n=0}^{N-1} x[n]$$

which is real-valued since $x[n]$ is real-valued.

(b) Proof:

If $k = 0$, since $x[0]$ is real, we have

$$X[0] = X^*[0]$$

If $1 \leq k \leq N-1$, we have

$$\begin{aligned} X[\langle N-k\rangle_N] &= \sum_{n=0}^{N-1} x[n] W_N^{n\langle N-k\rangle_N} = \sum_{n=0}^{N-1} x[n] W_N^{n(N-k)} \\ &= \sum_{n=0}^{N-1} x[n] W_N^{-nk} = \left(\sum_{n=0}^{N-1} x[n] W_N^{nk}\right)^* \\ &= X^*[k] \end{aligned}$$

Hence, we proved $X[\langle N-k\rangle_N] = X^*[k]$ for every k.

HW4-3

3. Determine the N-point DFTs of the following sequence defined over $0 \leq n < N$ (5%)

$$x[n] = 4sin(0.2\pi n), N = 10$$

Solution:
The DFT of $x[n]$ is:

$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn} = \sum_{n=0}^{9} \frac{2}{\mathrm{j}} \left(\mathrm{e}^{\mathrm{j}\frac{2\pi}{10}n} - \mathrm{e}^{-\mathrm{j}\frac{2\pi}{10}n} \right) \mathrm{e}^{-\mathrm{j}\frac{2\pi}{10}nk}$$

$$= \frac{2}{\mathrm{j}} \sum_{n=0}^{9} \mathrm{e}^{\mathrm{j}\frac{2\pi}{10}n} \cdot \mathrm{e}^{-\mathrm{j}\frac{2\pi}{10}nk} - \frac{2}{\mathrm{j}} \sum_{n=0}^{9} \mathrm{e}^{-\mathrm{j}\frac{2\pi}{10}n} \cdot \mathrm{e}^{-\mathrm{j}\frac{2\pi}{10}nk}$$

$$= -20\mathrm{j}\delta[k-1] + 20\mathrm{j}\delta[k-9]$$

HW4-4

4. Determine DFS coefficients of the following periodic sequence: (5%)

$$\tilde{x}[n] = 2cos(0.25\pi n)$$

Solution:

The DFT of $\tilde{x}[n] = 2\cos(\pi n/4)$ is:

$$X[k] = \sum_{n=0}^{7} \left(e^{j\frac{2\pi}{8}n2} + e^{-j\frac{2\pi}{8}n2} \right) e^{-j\frac{2\pi}{8}nk}$$
$$= 8\delta[k-2] + 8\delta[k-6]$$

The DFS of $\tilde{x}[n] = 2\cos(\pi n/4)$ is:

$$\tilde{X}[k] = 8\delta[\langle k \rangle_8 - 2] + 8\delta[\langle k \rangle_8 - 6]$$

Or -1 -7

HW4-5

5. Show that the N- point DFT of the circularly folded sequence $X[\langle -n\rangle_N]$ is given by $X[\langle -k\rangle_N]$. (10%)

- Mistakenly we updated the HW file twice, so some people solve the previous Q5 ($X[\langle N-k\rangle_N]$) also, so here we considered both the questions.

$$\sum_{n=0}^{N-1} x[\langle -n\rangle_N]e^{-j\frac{2\pi}{N}nk} = x[0] + \sum_{n=1}^{N-1} x[N-n]e^{-j\frac{2\pi}{N}nk}$$

$$= x[0] + \sum_{n=1}^{N-1} x[n]e^{-j\frac{2\pi}{N}(N-n)k} = x[0] + \sum_{n=1}^{N-1} x[n]e^{j\frac{2\pi}{N}nk}$$

$$= \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}n(-k)} = \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}n\langle -k\rangle_N}$$

$$= X[\langle -k\rangle_N]$$

HW4-6

6. Let $x_c(t)$ be a continuous-time signal with CTFT $X_c(j\Omega)$.(20%)
   a) Define time-duration ΔT_1 and bandwidth ΔF_1 as

$$\Delta T_1 \triangleq \frac{\int_{-\infty}^{\infty} x_c(t)dt}{x_c(0)}, \qquad \Delta F_1 \triangleq \frac{\int_{-\infty}^{\infty} x_c(j2\pi F)dF}{x_c(0)}.$$

   Show that $\Delta T_1 \Delta F_1 = 1$.

   b) Let $x_{c_1}(t) = u(t+1) - u(t-1)$ and $x_{c_2}(t) = \cos(\pi t)[u(t+1) - u(t-1)]$. Evaluate ΔT_1, ΔF_1 and their product for these two waveforms and explain for which waveform the definition of time-duration and bandwidth in (a) is reasonable.

(a) Proof:

$$X_c(0) = \int_{-\infty}^{\infty} x_c(t)\mathrm{d}t$$

$$x_c(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X_c(\mathrm{j}2\pi F)\mathrm{d}2\pi F$$

Hence, we can prove that

$$\Delta T_1 \Delta F_1 = \frac{\int_{-\infty}^{\infty} x_c(t)\mathrm{d}t}{x_c(0)} \cdot \frac{\int_{-\infty}^{\infty} X_c(\mathrm{j}2\pi F)\,\mathrm{d}F}{X_c(0)} = 1$$

(b) Solution:
For $x_{c_1}(t) = u(t+1) - u(t-1)$,

$$X_{c1}(\mathrm{j}\Omega) = \int_{-1}^{1} \mathrm{e}^{-\mathrm{j}\Omega t}\mathrm{d}t = \frac{2\sin\Omega}{\Omega}$$

Hence, $\Delta T_1 = 2$, and $\Delta F_1 = 1/2$. Thus, $\Delta T_1 \Delta F_1 = 1$.
For $x_{c_2}(t) = \cos(\pi t)[u(t+1) - u(t-1)]$,

$$X_{c2}(\mathrm{j}\Omega) = \frac{\sin(\pi - \Omega)}{\pi - \Omega} + \frac{\sin(\pi + \Omega)}{\pi + \Omega}$$

Thus, we have $X_{c2}(\mathrm{j}\Omega)|_{\Omega=0} = 0$.
We can conclude that the definition is reasonable for waveform like $x_{c_1}(t)$.

HW4-7

7. The DFT of the product $w[n]x[n]$ of two sequences (windowing operation) is given by the circular convolution of their respective DFTs (20%)

$$w[n]x[n] \overset{DFT}{\longleftrightarrow} \frac{1}{N} W[k] \circledast X[k]]. \qquad \text{(q7)}$$

a) Prove above expression (q7) by direct computation of the DFT of $w[n]x[n]$.

b) Prove above expression (q7) by first starting with the circular convolution of $w[n]$ and $x[n]$ and then using duality between DFT and IDFT relations.

(a) Proof:

$$w[n]x[n] \overset{\text{DFT}}{\longleftrightarrow} \frac{1}{N} W[k] \, \textcircled{N} \, X[k] \qquad (7.148)$$

$$\begin{aligned}
\sum_{n=0}^{N-1} w[n]x[n]e^{-j\frac{2\pi}{N}nk} &= \sum_{n=0}^{N-1} w[n]e^{-j\frac{2\pi}{N}nk}\left(\frac{1}{N}\sum_{m=0}^{N-1} X[m]e^{j\frac{2\pi}{N}mn}\right) \\
&= \frac{1}{N}\sum_{m=0}^{N-1} X[m]\left(\sum_{n=0}^{N-1} w[n]e^{-j\frac{2\pi}{N}n(k-m)}\right) \\
&= \frac{1}{N}\sum_{m=0}^{N-1} X[m]W[k-m] \\
&= \frac{1}{N}\sum_{m=0}^{N-1} X[m]W[\langle k-m\rangle_N] \\
&= \frac{1}{N} W[k] \, \textcircled{N} \, X[k]
\end{aligned}$$

(b) Proof:

$$\begin{aligned}
\frac{1}{N} W[k] \, \textcircled{N} \, X[k] &= \frac{1}{N}\sum_{m=0}^{N-1} X[m]W[\langle k-m\rangle_N] \\
&= \frac{1}{N}\sum_{m=0}^{N-1}\left(\sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}nm}\right) W[\langle k-m\rangle_N] \\
&= \sum_{n=0}^{N-1} x[n]\left(\frac{1}{N}\sum_{m=0}^{N-1} W[\langle k-m\rangle_N]e^{-j\frac{2\pi}{N}nm}\right) \\
&= \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}nk}\left(\frac{1}{N}\sum_{m=0}^{N-1} W[\langle k-m\rangle_N]e^{j\frac{2\pi}{N}n(k-m)}\right) \\
&= \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}nk}\left(\frac{1}{N}\sum_{m=0}^{N-1} W[\langle k-m\rangle_N]e^{j\frac{2\pi}{N}n\langle k-m\rangle_N}\right) \\
&= \sum_{n=0}^{N-1} (x[n]w[n])e^{-j\frac{2\pi}{N}nk} = \text{DFT}(x[n]w[n])
\end{aligned}$$

Quiz4

- 1-5題觀念題請直接參考老師的slides
- 6-9有再重新調整過答案，請同學再確認分數有沒有算錯

Quiz4-6

6 Let $X[k]$ be the N-point DFT of an N-point sequence $x[n]$. Which of the following assumptions are correct?

(a) If $x[n]$ satisfies the condition $x[n] = x[\langle N-1-n\rangle_N]$, then $X\left[\frac{N}{2}\right] = 0$ for N even.

(b) If $x[n]$ satisfies the condition $x[n] = x[\langle N-1-n\rangle_N]$, then $X[0] = 0$ for N even.

(c) If $x[n]$ satisfies the condition $x[n] = x[\langle n+M\rangle_N]$ where N=2M, then $X[2l+1] = 0$ for $l = 0, 1, \ldots, M-1$.

(d) If $x[n]$ satisfies the condition $x[n] = -x[\langle N-1-n]$, then $X[0] = 0$ for N even.

(e) If $x[n]$ satisfies the condition $x[n] = -x[\langle N-1-n]$, then $X[0] = 0$ for N odd.

(No partial points, you have to select all correct answers.)
(多選題)

- [x] A. (a)
- [ ] B. (b)
- [x] C. (c)
- [x] D. (d)
- [x] E. (e)

The N-point DFT of the N-pt sequence, $x[n]$ is given by

$$X[k] = \sum_{n=0}^{N-1} x[n]W_N^{kn}, \quad 0 \le k \le (N-1)$$

$$X[0] = \sum_{n=0}^{N-1} x[n]$$

(d)(e) Suppose $x[n] = -x[N-1-n]$. For N even, all elements of $x[n]$ will cancel with an antisymmetric component. For N odd, all elements have a counterpart with opposite sign. However, $x[(N-1)/2]$ must also be zero.
Therefore, for $x[n] = -x[N-1-n]$, $X[0] = 0$.

(a) Proof:

$$X[N/2] = \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}n\frac{N}{2}} = \sum_{n=0}^{N-1} x[n]\cos \pi n$$

When N is even, for any $n \in [0, N/2-1]$, there exists $x[N-1-n] = x[n]$. If we group these pairs and notice that

$$cos\pi(N-1-n) = \cos(n+1)\pi = -\cos n\pi$$

we can conclude that

$$X[N/2] = 0.$$

(b) Proof:

$$X[0] = \sum_{n=0}^{N-1} x[n]$$

When N is even, for any $n \in [0, N/2-1]$, there exists $x[N-1-n] = -x[n]$. Hence, we have

$$X[0] = \sum_{n=0}^{N-1} x[n] = X[0] = \sum_{n=0}^{N/2-1} x[n] + \sum_{n=0}^{N/2-1} x[N-1-n]$$

$$= \sum_{n=0}^{N/2-1} (x[n] - x[n]) = 0$$

(c) Proof:

$$\begin{aligned}X[2\ell+1] &= \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}n(2\ell+1)} \\ &= \sum_{n=0}^{N/2-1} x[n]e^{-j\frac{2\pi}{N}n(2\ell+1)} + \sum_{n=0}^{N/2-1} x[n+N/2]e^{-j\frac{2\pi}{N}(n+N/2)(2\ell+1)} \\ &= \sum_{n=0}^{N/2-1} x[n]e^{-j\frac{2\pi}{N}n(2\ell+1)} + \sum_{n=0}^{N/2-1} x[n]e^{-j\frac{2\pi}{N}n(2\ell+1)}e^{-j\pi(2\ell+1)} \\ &= \sum_{n=0}^{N/2-1} x[n]e^{-j\frac{2\pi}{N}n(2\ell+1)}(1-1) \\ &= 0\end{aligned}$$

Quiz4-7

7 Determine the N-point DFTs of the following sequences defined over $0 \leq n < N$
$x[n] = 6(\cos)^2(0.2\pi n),\, N = 10$

Answer:
$X[k] = A\delta[k] + B\delta[k-2] + C\delta[k+D]$

$[A, B, C, D] = [$ 30 , 15 , 15 , -8 $]$
(忽略空白)
Or -15

Solution:
The DFT of $x[n]$ is:

$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn} = \sum_{n=0}^{9} \left(3 + \frac{3}{2} e^{j\frac{2\pi}{10}n2} - \frac{3}{2} e^{-j\frac{2\pi}{10}n2}\right) e^{-j\frac{2\pi}{10}nk}$$
$$= 30\delta[k] + 15\delta[k-2] - 15\delta[k-8]$$

Quiz4-8

8 Determine DFS coefficients of the following periodic sequence:

$\tilde{x}[n] = 3\sin(0.25\pi n) + 4\cos(0.75\pi n)$

Answer:

The DFS of $\tilde{x}[n] = 3\sin(0.25\pi n) + 4\cos(0.75\pi n)$ is:

$\tilde{X}[k] = 16\delta[\langle k\rangle_8 - 3] + 16\delta[\langle k\rangle_8 + A] + 12\delta[\langle k\rangle_8 + B] - 12\delta[\langle k\rangle_8 + C]$

$[A, B, C] = [\underline{\ 3\ }, \underline{\ 1\ }, \underline{\ -1\ }]$

	A	B	C
Or	3	-1	1
	-5	-6	-2
	-5	-1	-7
	-5	-7	-1

Solution:

The DFT of $\tilde{x}[n] = 3\sin(0.25\pi n) + 4\cos(0.75\pi n)$ is:

$$X[k] = \sum_{n=0}^{7}\left[\frac{3}{2\mathrm{j}}\left(\mathrm{e}^{\mathrm{j}\frac{2\pi}{8}n2} - \mathrm{e}^{-\mathrm{j}\frac{2\pi}{8}n2}\right) + 2\left(\mathrm{e}^{\mathrm{j}\frac{2\pi}{8}n3} + \mathrm{e}^{-\mathrm{j}\frac{2\pi}{8}n3}\right)\right]\mathrm{e}^{-\mathrm{j}\frac{2\pi}{8}nk}$$

$$= -12\delta[k-2] + 12\delta[k-6] + 16\delta[k-3] + 16\delta[n-5]$$

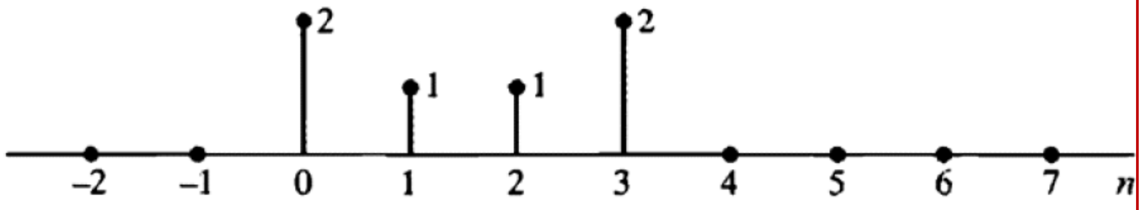
The DFS of $\tilde{x}[n] = 3\sin(0.25\pi n) + 4\cos(0.75\pi n)$ is:

$$\tilde{X}[k] = -12\delta[\langle k\rangle_8 - 2] + 12\delta[\langle k\rangle_8 - 6] + 16\delta[\langle k\rangle_8 - 3] + 16\delta[\langle k\rangle_8 - 5]$$

Quiz4-9

9 Consider the finite-length sequence $x[n]$ in Figure. The five-point DFT od $x[n]$ is denoted by $X[k]$. The sequence $y[n]$ whose DFT is $Y[k] = W_5^{-2k} X[k]$. Which statement of $y[n]$ is true?

(No partial points, you have to select all correct answers.)



(多選題)

- [ ] A. y[-1]=0
- [x] B. y[0]=1
- [ ] C. y[1]=0
- [ ] D. y[2]=1
- [ ] E. y[3]=0
- [x] F. y[4]=1
- [ ] G. y[5]=0
- [ ] H. y[6]=1
- [x] I. y[7]=0

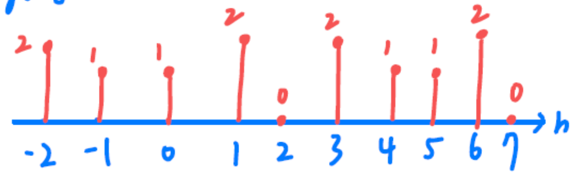
Or n=-1, 0, 4, 5, 7

如果考慮periodic

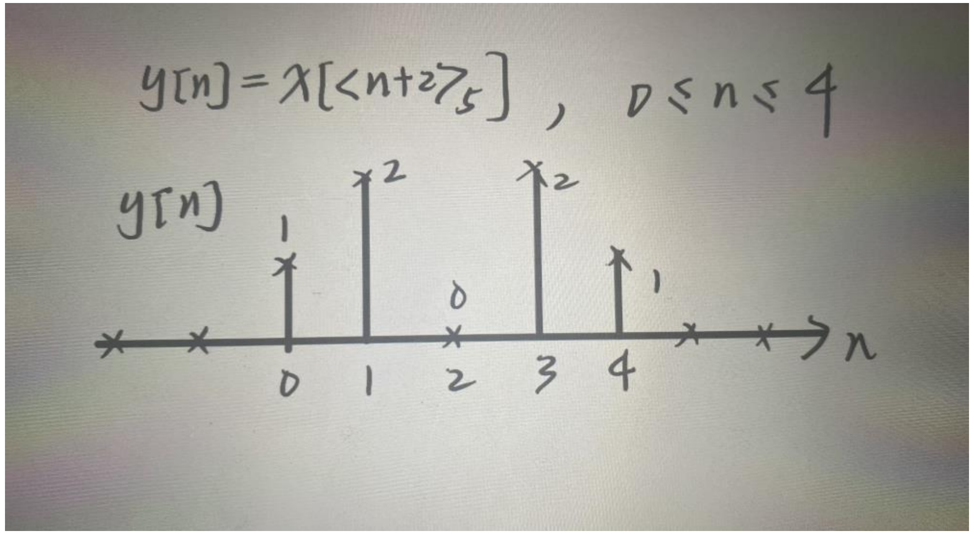
$Y[k] = W_5^{-2k} X[k] \Rightarrow y[n]$ 是 $x[n]$ 左移 2

$\Rightarrow y[n] = x[((n+2))_N]$

y[n]:



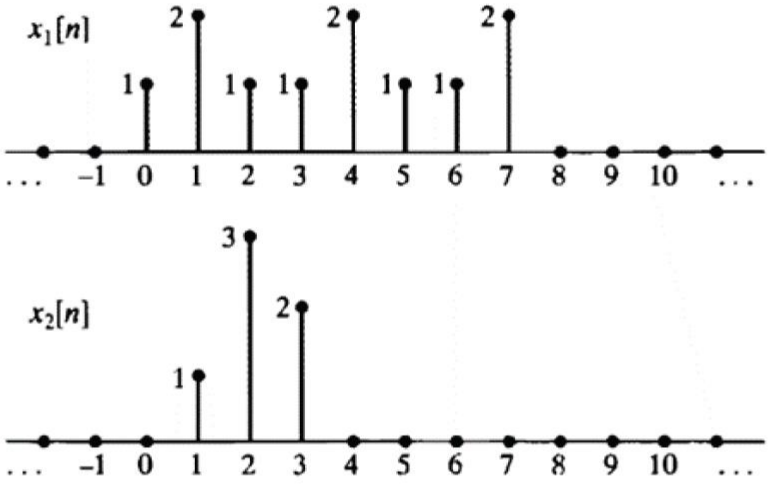
如果不考慮periodic



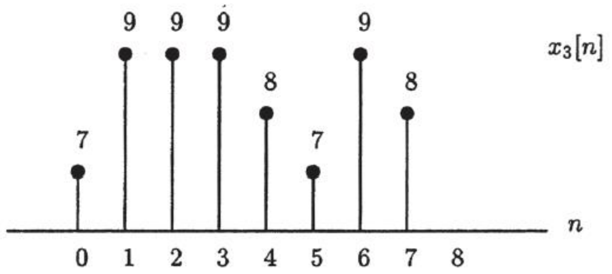
Quiz4-10

10 Two finite-length signals, $x_1[n]$ and $x_2[n]$, are sketched in Figure. Assume that $x_1[n]$ and $x_2[n]$ are zero outside of the region shown in the figure. Let $x_3[n]$ be the eight-point circular convolution of $x_1[n]$ with $x_2[n]$;
i.e., $x_3[n] = x_1[n] \circledast_8 x_2[n]$

Select all n that can make $x_3[n] = 9$.



$x_3[n]$ is the linear convolution of $x_1[n]$ and $x_2[n]$ time-aliased to $N = 8$. Carrying out the 8-point circular convolution, we get:



We thus conclude $x_3[2] = 9$.

11 The followig 8-point sequences are defined over $0 \leq n \leq 7$ .determine which have real-valued 8-points DFTs.
(Multiple answers question)

- [ ] A. $x_1[n] = \{0, -3, 1, -2, 0, 2, -1, 3\}$
- [x] B. $x_2[n] = \{5, 2, -9, 4, 7, 4, -9, 2\}$
- [ ] C. $x_3[n] = \{8, -3, 1, -2, 6, 2, -1, 3\}$
- [ ] D. $x_4[n] = \{0, 1, 3, -2, 5, 2, -3, 1\}$
- [x] E. $x_5[n] = \{10, 5, -7, -4, 5, -4, -7, 5\}$

12 The followig 8-point sequences are defined over $0 \leq n \leq 7$ .determine which have imaginary-valued 8-points DFTs.

(Multiple answers question)

- [x] A. $x_1[n] = \{0, -3, 1, -2, 0, 2, -1, 3\}$
- [ ] B. $x_2[n] = \{5, 2, -9, 4, 7, 4, -9, 2\}$
- [ ] C. $x_3[n] = \{8, -3, 1, -2, 6, 2, -1, 3\}$
- [ ] D. $x_4[n] = \{0, 1, 3, -2, 5, 2, -3, 1\}$
- [ ] E. $x_5[n] = \{10, 5, -7, -4, 5, -4, -7, 5\}$

13 The followig 8-point sequences are defined over $0 \leq n \leq 7$ .determine which have complex valued 8-points DFTs.

(Multiple answers question)

- [ ] A. $x_1[n] = \{0, -3, 1, -2, 0, 2, -1, 3\}$
- [ ] B. $x_2[n] = \{5, 2, -9, 4, 7, 4, -9, 2\}$
- [x] C. $x_3[n] = \{8, -3, 1, -2, 6, 2, -1, 3\}$
- [x] D. $x_4[n] = \{0, 1, 3, -2, 5, 2, -3, 1\}$
- [ ] E. $x_5[n] = \{10, 5, -7, -4, 5, -4, -7, 5\}$

Consider two finite-length sequences:

$x_1[n] = \{\underset{\uparrow}{1}, -2, 1, -3\}$, $x_2[n] = \{\underset{\uparrow}{0}, 2, -1, 0, 0, 4\}$

Answering format:

- real numbers: a or -a
- no spaces, if more than one number, separate numbers by ","
- no need to show the arrow at first number
- no partial points for some correct numbers, all numbers should be correct

Linear and circular convolution
Concepts from May11-2022.pdf

14 The linear convolution $x_1[n] \cdot x_2[n]$ is { 0,2,-5,4,-7,7,-8,4,-12 }
(Not case sensitive, Ignoring space)

Difficulty: Not specified

15 The circular convolution $x_1[n] \circledast x_2[n]$ is { -8,6,-17,4,-7,7 }
(Not case sensitive, Ignoring space)

Difficulty: Not specified

Here if u consider the N=6 or N=9 , we have given you the points.
N=9 means you should have same answer as 14

16 Determine the smallest value of N so that N-point circular convolution is equal to the linear convolution.
Answer: The smallest value of N is 9

$$\min N = 4 + 6 - 1 = 9$$

Difficulty: Not specified

{ 4, 2-j3, 3+j2, -4+j6, 8-j7} are the first five values of the 9-point DFT real-valued sequence x[n]. [Hint: to get all the 9 values use symmetry property]

17 What will be the DFT of the sequence $x_1[n] = 2x[\langle 2-n\rangle_9]$?

- A. $X_1[k] = 2W_9^{-K}X^*[k]$
- B. $X_1[k] = 2W_9^{-2K}X^*[k]$
- C. $X_1[k] = 2W_9^{-2K}X[k]$
- D. $X_1[k] = W_9^{-2K}X^*[k]$

By applying the folding and time-shifting properties, the DFT of $x_2[n]$ is:

$$X_2[k] = 2W_9^{-2k}X^*[k]$$

Difficulty: Not specified

18 The DFT of the sequence $x_2[n] = x[n]e^{-j4\pi\frac{n}{9}}$ is,

- A. $X_2[k] = X[\langle k+4\rangle_9]$
- B. $X_2[k] = X[\langle k-4\rangle_9]$
- C. $X_2[k] = X[\langle k-2\rangle_9]$
- D. $X_2[k] = X[\langle k+2\rangle_9]$

By applying the frequency-shifting property, the DFT of $x_5[n]$ is:

$$X_5[k] = X[\langle k+2\rangle_9]$$

19 Determine which DFT property can be easialy applied to estimate the DFT of the sequences $x_3[n] = x^2[n]$

- A. Time-shifting
- B. Folding and time-shifting
- C. Correlation
- D. Windowing
- E. Frequency-shifting

By applying the windowing property, the DFT of $x_4[n]$ is:

$$X_4[k] = \frac{1}{9} X[k] \circledcirc X[k]$$

Difficulty: Not specified

20 Determine which DFT property can be easily applied to find the DFT of the sequences $x_4[n] = x[n] \circledast x[\langle -n \rangle_9]$

- A. Time-shifting
- B. Folding and time-shifting
- C. Correlation
- D. Windowing
- E. Frequency-shifting

By applying the correlation property, the DFT of $x_3[n]$ is:

$$X_3[k] = X[k]X^*[k] = |X[k]|^2$$

Difficulty: Not specified

21 In the case of Hamming windowThe transition width of the main lobe is equal to _8_ π/M where M is the length of the window.

Difficulty: Not specified

22 A good window should have a __(a)__mainlobe and ___(b)_ sidelobes.

- A. a) broad, b) high
- B. a) broad, b) low
- C. a) narrow, b) low
- D. a) narrow, b) high

Difficulty: Not specified

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23 Which statements are False about windowing concepts:
a) The Hanning window has less side lobes and the leakage is less as compare to rectangular window
b) The rectangualr window has less side lobes and the leakage is less as compare to Hanning window
c) The width of the main lobe is more in Hanning window
d) In Barlett window, the triangular function resembles the tapering of rectangular window sequence parabolically from the middle to the ends.
(Multiple answers question)

- A. (a)
- B. (b)
- C. (c)
- D. (d)

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24 If {x[n]} is the signal to be analyzed, limiting the duration of the sequence to L samples, in the interval 0≤ n≤ L-1, is equivalent to multiplying {x[n]} by?

- A. Rectangular window
- B. Bartlett window
- C. Hamming window
- D. Hanning window

Difficulty: Not specified

25 Which of the following is true, If x[n] is a real sequence and X[k] is its N-point DFT,

a) X[N-k] = X[-k]
b) X[N-k] = X*[k]
c) X[-k] = X*[k]

- A. b) and c)
- B. a) and c)
- C. a) and b)
- D. All

See windowing and DFT properties concepts from May11-2022.pdf

Key Concepts

- z-transform
- Inverse z-transform
- Poles and zeros, ROC
- Minimum and all-pass system
- Sinusoidal response of real LTI systems
- Group delay
- Ideal and practical filter
- Filters

Example

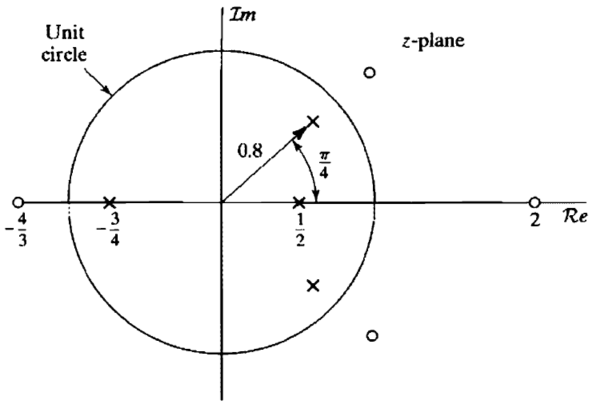
- Minimum and all-pass system

Minimum phase   All pass

$$\frac{1+0.2z^{-1}}{1+0.81z^{-1}} \times (1-9z^{-2}) = \frac{(1+0.2z^{-1})\left(1-\frac{1}{9}z^{-2}\right)}{1+0.81z^{-1}} \times \frac{(1-9z^{-2})}{\left(1-\frac{1}{9}z^{-2}\right)}$$

$$H(z) = \frac{(1+0.2z^{-1})(1-9z^{-2})}{1+0.81z^{-1}} = H_{min}(z)H_{ap}(z) = \frac{1+0.2z^{-1}}{1+0.81z^{-1}} \times (1-9z^{-2})$$

- Minimum phase system: all poles and zeros are in the unit circle(ROC).
- All pass system: poles and zeros occur in conjugate reciprocal(倒數) pairs.



$$H_k(z) = z^{-1}\frac{1-p_k^* z}{1-p_k z^{-1}} = \frac{z^{-1}-p_k^*}{1-p_k z^{-1}} \qquad (5.157)$$

Figure 5.18 Typical pole–zero plot for an all-pass system.