11020EE 366000 Introduction to Digital Signal Processing

Quiz #3

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1. **(25 pts.) True or False.** Write down True or False for each question, and give a short proof to the one(s) you think is true, otherwise, point out the reason why the statement is wrong or providing a counter example. [5 pt for each]
   (a) The error due to under-sampling a continuous signal can be avoided by simply amplifying the signal.
   (b) Let $x[n]$ and $y[n]$ be discrete-time signals. $X(\omega)$ and $Y(\omega)$ is their Fourier transform, respectively. If $y[n] = x[3n]$, then $Y(\omega) = X(\omega/3)$.
   (c) Let $x[n]$ and $y[n]$ be discrete-time signals. $X(\omega)$ and $Y(\omega)$ is their Fourier transform, respectively. Assume that $y[n] = x[n/2]$ when n is even and $y[n] = 0$ when n is odd. Then, $Y(\omega) = X(2\omega)$.
   (d) All practical continuous-time signals are time-limited, and cannot be strictly bandlimited.
   (e) If a signal $x_c(t) = 3 + 2sin(16\pi t) + 10cos(24\pi t)$ is sampled at a rate of $F_s = 20$ Hz to obtain the discrete-time signal $x[n]$. $x_c(t)$ can be recovered from $x[n]$.

2. **(20 pts.)** Write abbreviated final answers for following questions.
   (a) Consider a continuous-time signal

$$x_c(t) = 3cos(2\pi F_1 t + 45^{\circ}) + 3sin(2\pi F_2 t).$$

   $T = 0.001$
   $F_s = 1000$ Hz
   $\Omega_s = 2000\pi$

   It is sampled at $t = 0.001n$ to obtain $x[n]$, which is then applied to an ideal DAC to obtain another continuous-time signal $y_r(t)$. For $F_1 = 300$ Hz and $F_2 = 700$ Hz, determine $x[n]$ and determine $y_r(t)$ for the above $x[n]$ as a sinusoidal signal. [10 pt]
   (b) Consider a continuous-time signal

$$x_c(t) = 10 + 3sin(20\pi t + \pi/3) + 5cos(40\pi t).$$

   It is sampled at $t = 0.05n$ to obtain $x[n]$, which is then applied to an ideal DAC to obtain another continuous-time signal $y_r(t)$. Determine $x[n]$ and determine $y_r(t)$ as a sinusoidal signal. [10 pt]

   $T = 0.05$
   $f_s = 20$
   $\Omega_s = 40\pi$

3. **(15 pts.)** An 8-bit ADC has an input analog range of ±5 volts. The analog input signal is

$$x_c(t) = 2\,cos(200\pi t) + 3\,sin(500\pi t).$$

   The converter supplies data to a rate of 2048 bits/s. The computer, without processing, supplies these data to an ideal DAC to form the reconstructed signal $y_c(t)$. Determine:
   (a) The quantizer resolution (or step). [5 pt]
   (b) The sampling rate, and folding frequency. [5 pt]
   (c) The Nyquist rate, and the reconstructed signal $y_c(t)$. [5 pt]

4. **(10 pts.)** A complex-value continuous-time signal $x_c(t)$ has the Fourier transform shown in Figure q4, where $(\Omega_2 - \Omega_1) = \Delta\Omega$. This signal is sampled to produce the sequence $x[n] = x_c(nT)$.

   (a) Sketch the Fourier transform $X(e^{j\omega})$ of the sequence $x[n]$ for $T = \frac{\pi}{\Omega_2}$. [5 pt]

   (b) What is the lowest sampling frequency that can be used without incurring any aliasing distortion i.e., so that $x_c(t)$ can be recovered from $x[n]$? [5 pt]

$\Omega_s = \frac{2\pi}{T} = 2\Omega_2$

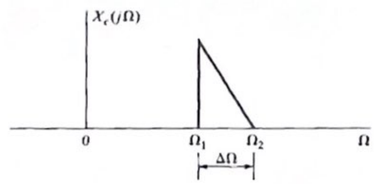

Figure q4

5. **(7 pts.)** A sinusoidal signal $s_c(x, y) = 3\cos(2.4\pi x + 2.6\pi y)$ is sampled at $\left(F_{s_x}, F_{s_y}\right)$ frequency to obtain the image $s[m,n]$. An ideal reconstruction is used on $f[m,n]$ to obtain the analog sign $s_r(x,y)$. If $F_{s_x} = 2$ sam/meter and $F_{s_y} = 3$ sam/meter, determine $s[m,n]$ and $s_r(x,y)$.

6. **(23 pts.)** Show and explain the concepts in detail.

   (a) Show that the sampler is a memoryless, linear, time-varying system. [9 pt]

   (b) What is the Nyquist rate and prove why the signal needs to satisfy the Nyquist rate while sampling (detail answer with some plot)? [7 pt]

   (c) Using Parseval's theorem, explain why the amplitude of the Fourier transform changes during downsampling but not during upsampling. [7 pt]