11020EE 366000 Introduction to Digital Signal Processing

Quiz #2

March 30, 2022

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1. **(10 pts.)**

   (a) Prove Parseval's theorem using the multiplication and conjugation properties. [5 pt]
   Hint: Parseval's theorem:

$$\sum_{n=-\infty}^{\infty} x_1[n]\, x_2^*[n] = \frac{1}{2\pi}\int_{2\pi} X_1(e^{j\omega})X_2^*(e^{j\omega})\, d\omega.$$

   (b) Consider $f(t) = t$ if $0 \le t < 2\pi$, use Parseval's theorem to show that: [5 pt]

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

   Hint: Parseval's theorem: **(clearly write down the left-side and right-side computing)**

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} |f(t)|^2\, dt = \sum_{n=-\infty}^{\infty} |c_n|^2$$

2. **(20 pts.)** Consider an LTI system with impulse response $h[n] = a^n u[n]$ with $|a| < 1$.

   (a) Compute the deterministic autocorrelation function $\varphi_{hh}[m]$ for this impulse response. (Hint: $\varphi_{hh}[m] = h[m] * h[-m]$) [5 pt]

   (b) Determine the magnitude-squared function $|H(e^{j\omega})|^2$ for this impulse response. [7 pt]

   (c) Use Parseval's theorem to evaluate the integral for this system. [8 pt]

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} |H(e^{j\omega})|^2\, d\omega$$

3. **(10 pts.)**

   (a) Given a sequence $x[n]$ with Fourier transform $X(e^{j\omega})$, determine the Fourier transform of the following sequence in terms of $X(e^{j\omega})$: [5 pt]

$$x_1[n] = (x[n] - jx^*[-n])/2$$

   (b) If given a sequence with Fourier transform $X(e^{j\omega}) = \frac{1}{(1+0.8-j\omega)}$, determine the Fourier transform of the following sequence in terms of $X(e^{j\omega})$: [5 pt]

$$x_2[n] = e^{j\pi\frac{n}{2}} x[n + 2]$$

4. **(20 pts.)** Consider the discrete-time system shown in Figure q4-1. Where

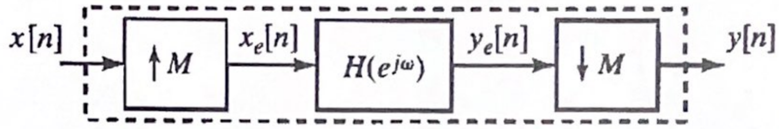

Figure q4-1

(i) M is a n integer.

(ii) $x_e[n] = \begin{cases} x[n/M] & n = kM, \ k \\ 0 & \text{otherwisw.} \end{cases}$ is any integer

(iii) $y[n] = y_e[nM]$.

(iv) $H(e^{j\omega}) = \begin{cases} M & |\omega| \le \frac{\pi}{4} \\ 0 & \frac{\pi}{4} < |\omega| \le \pi. \end{cases}$

(a) Assume that $M = 2$ and $X(e^{j\omega})$, the DTFT of $x[n]$, is real and is as shown in Figure q4-2. Make an appropriately labeled sketch of $X_e(e^{j\omega})$, $Y_e(e^{j\omega})$ and $Y(e^{j\omega})$, the DTFTs of $x_e[n]$, $y_e[n]$, and $y[n]$, respectively. Be sure to clearly label salient amplitudes and frequencies. [6 pt]

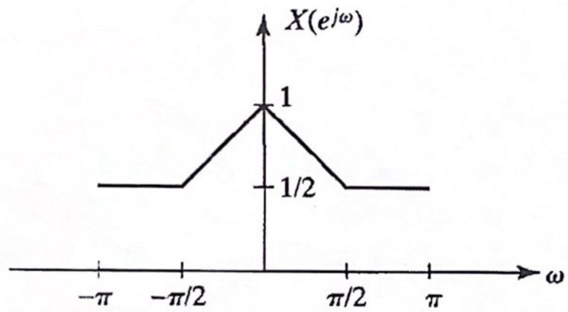

Figure q4-2

(b) For $M = 2$ and $X(e^{j\omega})$ as given in Figure 4-2, find the value of [7 pt]

$$\varepsilon = \sum_{n=-\infty}^{\infty} |x[n] - y[n]|^2.$$

(c) For $M = 2$, the overall system is LTI. Determine and sketch the magnitude of the frequency response of the overall system $|H_{eff}(e^{j\omega})|$. [7 pt]

5. **(10 pts.)** Determine sequences corresponding to each of the following Fourier transforms:

(a) $X_1(e^{j\omega}) = [1 + 5cos(2\omega) + 8cos(4\omega)]e^{-j3\omega}$. [5 pt]

(b) $X_2(e^{j\omega}) = \omega e^{j(\pi/2-5\omega)}$. ***[write final value of*** $x_2[n]$***]*** [5 pt]

6. **(10 pts.)** Determine whether or not each of the following signals is periodic. If periodic, find its fundamental frequency.

(a) $x_1(t) = sin(\sqrt{2}t) + cos(2\sqrt{2}t)$. [5 pt]

(b) $x_2(t) = \frac{1}{3}\left\{sin\left(\frac{t}{11}\right) + cos\left(\frac{t}{79}\right) + sin\left(\frac{t}{31}\right)\right\}$. [5 pt]

7. **(20 pts.)** $X(e^{j\omega})$ denotes the Fourier transform of the complex-valued signal $x[n]$, where the real and imaginary parts of $x[n]$ are given in Figure q7. (Note: The sequence is zero outside the interval shown.)

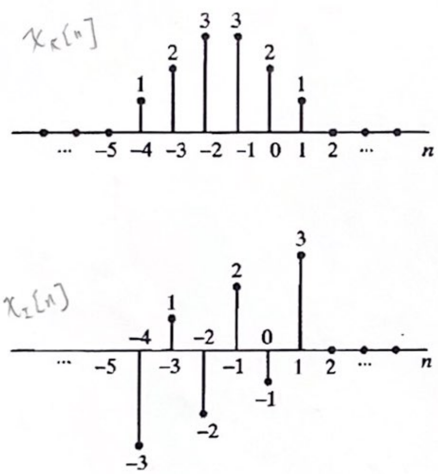

Figure q7

Perform the following calculations without explicitly evaluating $X(e^{j\omega})$.

(a) Evaluate $X(e^{j\omega})\big|_{\omega=0}$. [2 pt]

(b) Evaluate $X(e^{j\omega})\big|_{\omega=\pi}$. [3 pt]

(c) Evaluate $\int_{-\pi}^{\pi} X(e^{j\omega})d\omega$. (*Hint:* can use Inverse Fourier Transform to evaluate this) [5 pt]

(d) Determine and sketch the signal (in the time domain) whose Fourier transform is $X(e^{-j\omega})$ ***[separate plot for real and imaginary]*** [5 pt]

(e) Determine and sketch the signal (in the time domain) whose Fourier transform is $j\text{Im}\{X(e^{j\omega})\}$ ***[separate plot for real and imaginary]*** [5 pt]