11020EE 366000 Introduction to Digital Signal Processing

Quiz #1

March 9, 2022

10806107 林鈞陞

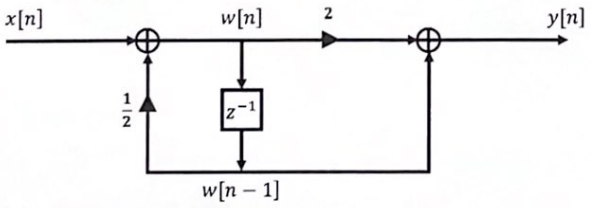
---

1. **(10 pts.)** Please brief answer the following questions:
   (a) Why we need to check whether the system is causal, stable, linear or time invariant? [4pt]
   (b) Linearity makes it possible to characterize a system in terms of the responses $h_k[n]$ to the shifted impulses $\delta[n-k]$ for all k, whereas time-invariance implies that $h_k[n] = h[n-k]$. For every system of the combination of linearity and time-invariance, (1) what is the complete and unique characterization? [2pt] (2) what are the two characteristics can its output be determined by? [4pt]

2. **(10 pts.)** A downsampler is a system,
$$y[n] = \mathcal{H}\{x[n]\} = x[nM],$$
that is used to sample a discrete-time signal $x[n]$ by a factor of M. Test the downsampler for linearity and time invariance.

3. **(20 pts.)** Consider the following difference equation:
$$y[n] - \frac{1}{4}y[n-1] - \frac{1}{8}y[n-2] = 3x[n].$$
   (a) Determine the general form of the homogeneous solution to this difference equation. [5 pt]
   (b) Both a causal and an acausal LTI system are characterized by this difference equation. Find the impulse responses of the two systems. [5 pt]
   (c) Show that the causal LTI system is stable and the acausal LTI system is unstable. [5 pt]
   (d) Find a particular solution to the difference equation when $x[n] = (1/2)^n u[n]$. [5 pt]

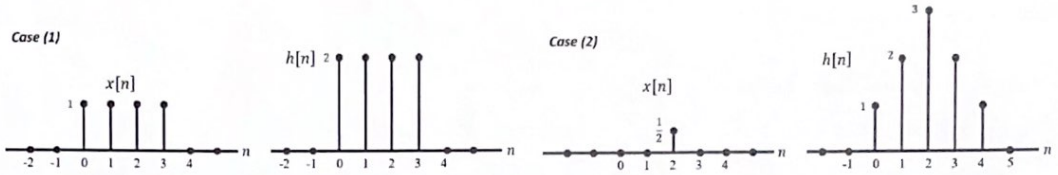
4. **(10 pts.)** Write out the input-output equation for the system.



5. **(5 pts.)** Suppose the output response of the system is $y[n]$, write the total recursive response of the system in terms of zero-input zero-state response of the system?
$$y[n] = a^{n+1}y[-1] + h[n]x[0] + h[n-1]x[1] + \cdots + h[0]x[n].$$

6. **(15 pts.)**

(a) Determine the discrete-time convolution of $x[n]$ and $h[n]$ for the following two cases. [10pt]



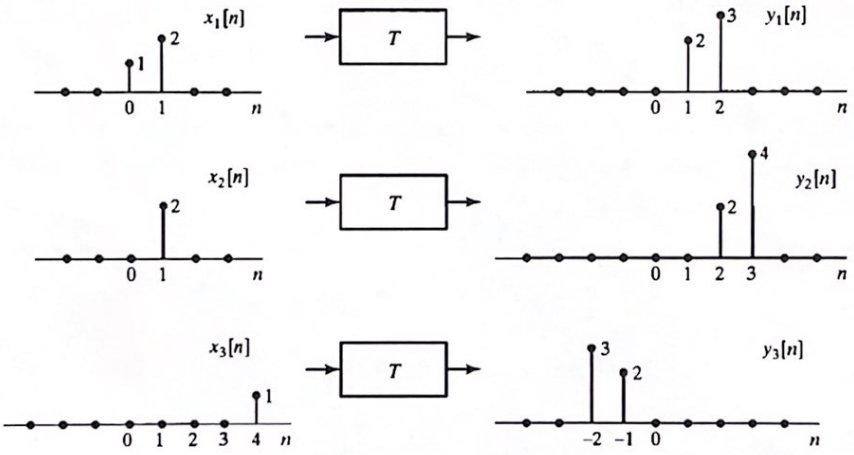
(b) Find the convolution of $x[n] = [1, 1, 1, 1, 2, 2, 2, 2]$ with $h[n] = [3, 3, 0, 0, 0, 0, 3, 3]$ by using matrix method. [5pt]

7. **(15 pts.)** The system T in below figure is known to be time-invariant. When the input to the system are $x_1[n]$, $x_2[n]$ and $x_3[n]$, the response of the system are $y_1[n]$, $y_2[n]$ and $y_3[n]$ as shown.

(a) Determine whether the system T could be linear? [5pt]

(b) If the input $x[n]$ to the system T is $\delta[n]$, what is the system response $y[n]$? [5pt]

(c) What are the possible inputs $x[n]$ for which the response of the system T can be determined from the given information alone? [5pt]



8. **(15 pts.)**

(a) State conditions when the signal is said to be periodic or aperiodic in terms of continuous time signal and discrete time signals? Are sinusoids sequence periodic sequence? [5pt]

(b) Examine whether the following signals are periodic or not? Determine the fundamental period of the signal. [10pt]

(1) $x(t) = \cos 10t - \cos(10 + \pi)\, t$

(2) $x(n) = sin\left(\frac{2\pi}{5}\right) n + cos(\frac{2\pi}{7})n$