National Tsing Hua University
Department of Electrical Engineering
EE3660 Intro. to Digital Signal Processing, Spring 2020

Homework Assignment #5: Chap. 10

Due: May 7, 2020

I Paper Assignment (30%)

1. (4%) This problem examines conversions between various filter specifications.

   Given the absolute specifications $\delta s = 0.0001$ and $\omega_p = 0.3\pi$, $\omega_s = 0.5\pi$, determine the relative specifications A_s and ω_c, $\Delta\omega$.

2. (6%) The Hann window function can be written as

$$w[\mathrm{n}] = [0.5 - 0.5\cos(2\pi \mathrm{n}/M)]w_R[\mathrm{n}].$$

   where $w_R[\mathrm{n}]$ is the rectangular window of length $M+1$.

   (a) Express the DTFT of w[n] in terms of the DTFT of $w_R[\mathrm{n}]$.

   (b) Explain why the Hann window has the wider mainlobe but lower sidelobes than the rectangular window of the same length.

3. (12%) Consider an FIR filter with impulse response $h[n] = u[n] - u[n-4]$.

   (a) Determine and sketch the magnitude response $|H(e^{j\omega})|$

   (b) Determine and sketch the amplitude response $A(e^{j\omega})$. Compare this sketch with that in (a) and comment on the difference.

   (c) Determine and sketch the phase response $\angle H(e^{j\omega})$.

   (d) Determine and sketch the angle response $\Psi(e^{j\omega})$. Compare this sketch with that in (c) and comment on the difference.

4. (8%) Consider the type-IV linear-phase FIR filter characterized by antisymmetric impulse response and odd-M.

   (a) Show that the amplitude response $A(e^{j\omega})$ is given by (10.38) with coefficients $d[k]$ given in (10.39).

(b) Show that the amplitude response $A(e^{j\omega})$ can be further expressed as (10.40) with coefficients $\hat{d}\,[k]$ given in (10.41)

II Program Assignment (70%)

5. (10%) A lowpass FIR filter is given by the specifications: $\omega_p = 0.3\pi$, $\omega_s = 0.5\pi$, and $A_s = 50$ dB.

   Use the fir2 function to obtain a minimum length linearphase filter. Use the appropriate window function in the fir2 function. Provide a plot similar to Figure 10.12.

6. (12%) Design a highpass FIR filter to satisfy the specifications: $\omega_s = 0.3\pi$, $\omega_p = 0.5\pi$, and $A_s = 50$ dB.

   (a) Use Kaiser window to obtain a minimum length linear-phase filter. Provide a plot similar to Figure 10.12.

   (b) Repeat (a) using the fir1 function.

7. (12%) In this problem we reproduce Figures 10.4 and 10.5. For each of the following linear-phase FIR filters described by h[n], obtain impulse response, amplitude response, magnitude response, and pole-zero plots in one figure window. For frequency response plots use the interval $-2\pi \le \omega \le 2\pi$.

   (a) Type-I filter: h[n] = {1, 2, 3, −2, 5, −2, 3, 2, 1}.

   (b) Type-II filter: h[n] = {1, 2, 3, −2, −2, 3, 2, 1}.

   (c) Type-III filter: h[n] = {1, 2, 3, −2, 0, 2, −3, −2, −1}.

   (d) Type-IV filter: h[n] = {1, 2, 3, −2, 2, −3, −2, −1 }.

8. (18%) Consider a Blackman window of length L = 21.

   (a) Compute and plot the log-magnitude response in dB over $-\pi \le \omega \le \pi$. In the plot measure and show the value of the peak of the first sidelobes.

   (b) Compute and plot the accumulated amplitude response in dB using the cumsum function. In the plot measure and show the value of the peak of the first sidelobe.

   (c) Repeat (a) and (b) for L = 41.

9. (18%) An ideal lowpass filter has a cutoff frequency of $\omega_c = 0.4\pi$. We want to obtain a length L

= 40 linear-phase FIR filter using the frequency-sampling method.

(a) Let the sample at ω_c be equal to 0.5. Obtain the resulting impulse response h[n]. Plot the log-magnitude response in dB and determine the minimum stopband attenuation.

(b) Now vary the value of the sample at ω_c and find the largest minimum stopband attenuation. Obtain the resulting impulse response h[n] and plot the log-magnitude response in dB in the plot window of (a).

(c) Compare your results with those obtained using the fir2 function (choose hamming window).

III Reference

$$H(e^{j\omega}) = \sum_{k=1}^{\frac{M+1}{2}} \left(d[k] sin\left[\omega\left(k-\frac{1}{2}\right)\right]\right) j\, e^{-\frac{j\omega M}{2}}$$

$$\triangleq jA(e^{j\omega})e^{-j\omega M/2}. \qquad (10.38)$$

$$d[k] = 2h\left[\frac{M+1}{2} - k\right]. \quad k = 1,2, \dots, \frac{M+1}{2} \qquad (10.39)$$

$$A(e^{j\omega}) = \sin(\tfrac{\omega}{2}) \sum_{k=0}^{(M-1)/2} \hat{d}[k] \cos \omega k. \qquad (10.40)$$

$$d[k] = \begin{cases} \frac{1}{2}(2\hat{d}[0] - \hat{d}[1]), \ k = 1 \\ \frac{1}{2}(2\hat{d}[k-1] - \hat{d}[k]), \ 2 \leq k \leq (M-1)/2. \\ \frac{1}{2}(2\hat{d}[(M-1])/2), \ k = (M+1)/2 \end{cases} \qquad (10.41)$$

$$H(e^{j\omega}) = \sum_{N=0}^{M} h[n]\, e^{j\omega n} \triangleq A(e^{j\omega})e^{j\Psi(e^{j\omega n})}. \qquad (10.42)$$

$$\Psi(e^{j\omega n}) \triangleq -\alpha\omega + \beta. \qquad (10.43)$$

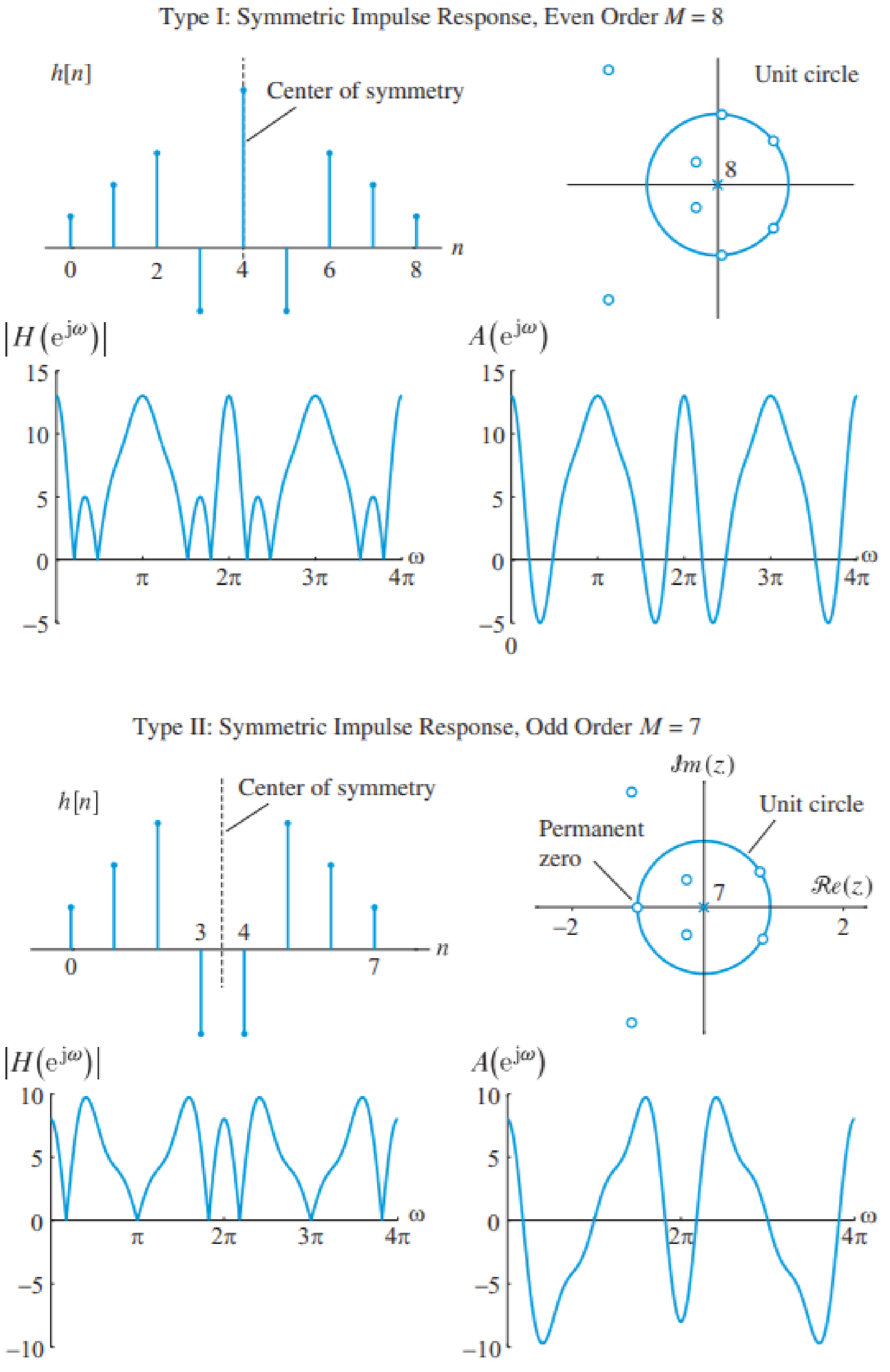

Figure 10.4 Impulse response, pole-zero pattern, magnitude response, and amplitude response

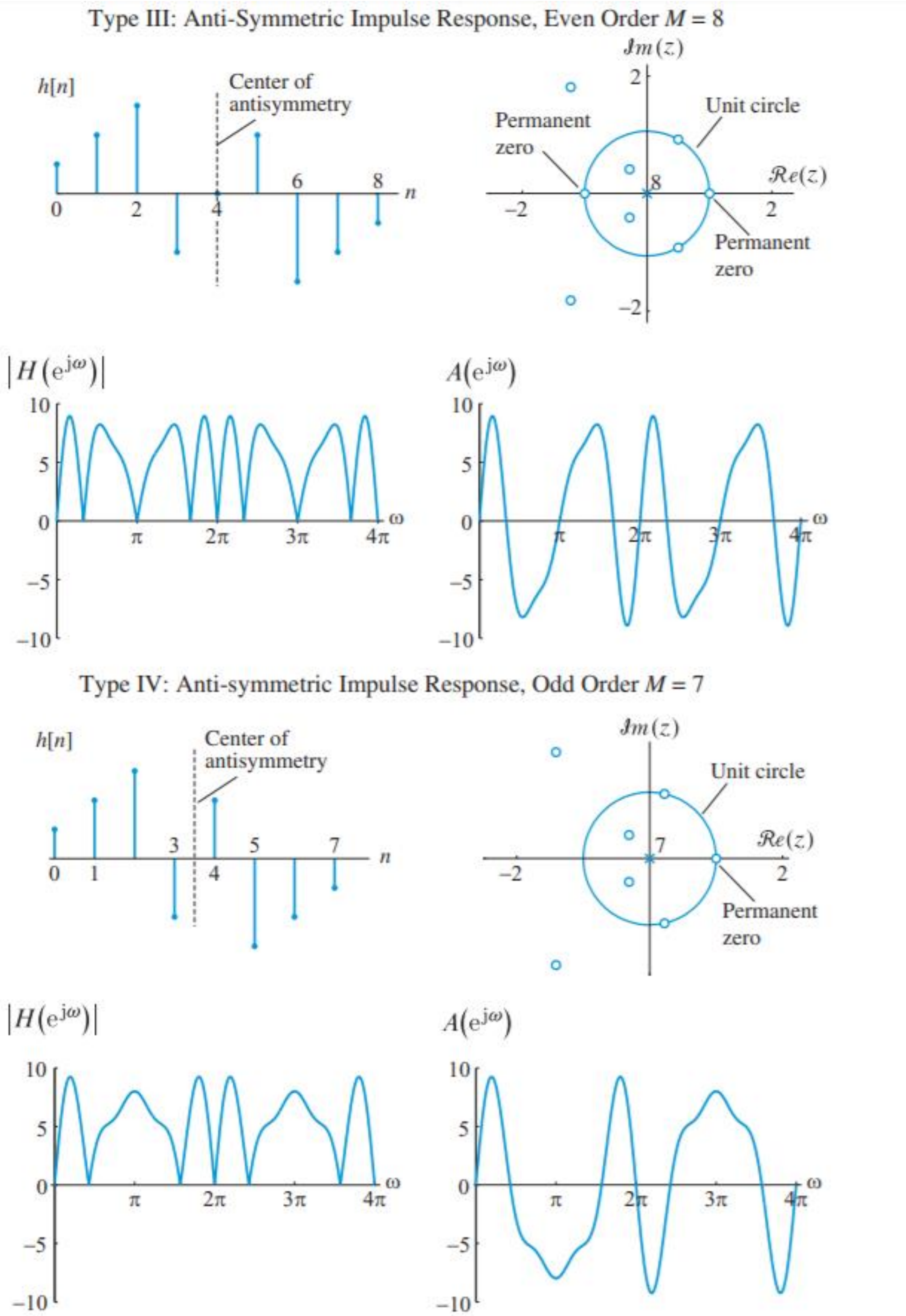

Figure 10.5 Impulse response, pole-zero pattern, magnitude response, and amplitude response

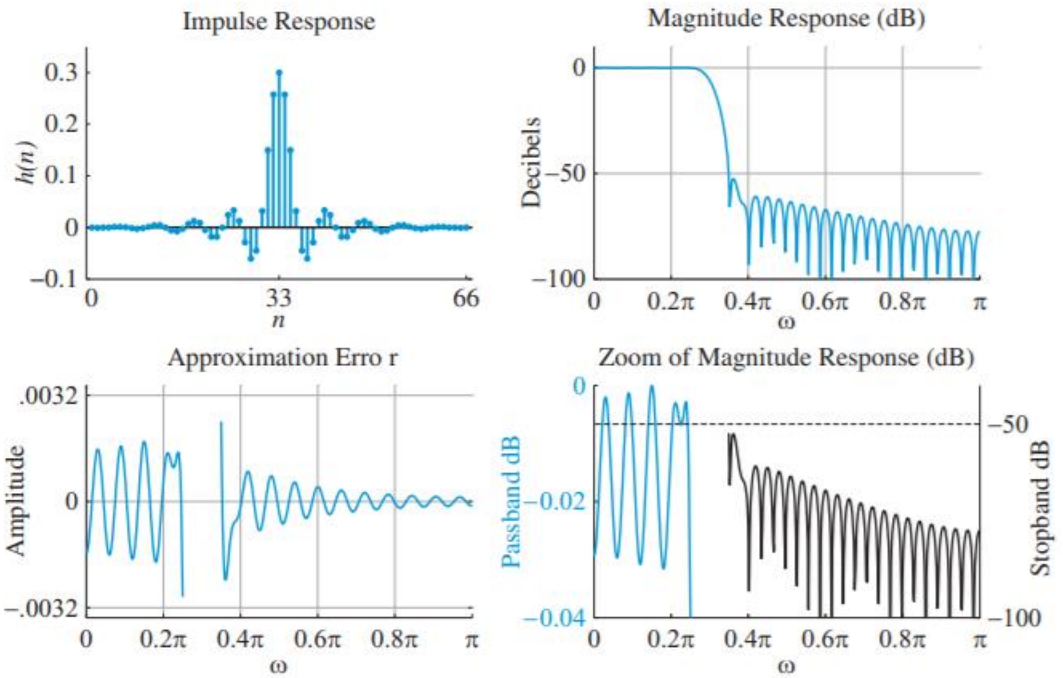

Figure 10.12 Impulse, approximation error, and magnitude response plots of the filter designed in Example 10.2 using a Hamming window to satisfy specifications: $\omega_p = 0.25\pi$, $\omega_s = 0.35\pi$, $A_p = 0.1$ dB, and $A_s = 50$ dB.