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1.

$$A_s = -20 \cdot \log_{10} \delta_s = -20 \times (-4) = 80 \quad \#$$

$$W_c = \frac{W_p + W_s}{2} = 0.4\pi \quad \#$$

$$\Delta W = 0.5\pi - 0.3\pi = 0.2\pi$$

2.

(a)

$$\cos(\tfrac{2\pi}{M}n) \rightarrow \pi\delta(\omega - \tfrac{2\pi}{M}) + \pi\delta(\omega + \tfrac{2\pi}{M}) \quad ; \quad x[n]\,y[n] \xrightarrow{DTFT} \frac{1}{2\pi}\int_0^{2\pi} X(e^{j\theta})\,Y(e^{j(\omega-\theta)})\,d\theta$$

$$W(e^{j\omega}) = 0.5W_R(e^{j\omega}) - \frac{1}{4}W_R(e^{j(\omega - \frac{2\pi}{M})}) - \frac{1}{4}W_R(e^{j(\omega + \frac{2\pi}{M})})$$

(b) $\sigma_t^2 \triangleq \int_{-\infty}^{\infty} t^2 |x_c(t)|^2 dt$, which is called "duration"

Duration of Hann window is less than rectangular window

$\Rightarrow$ sidelobes are lower compared to rectangular window

According to uncertainty principle $\sigma_t \sigma_\Omega \geq \frac{1}{2}$

$\Rightarrow$ mainlobe of Hann window would be less than that of rectangular window

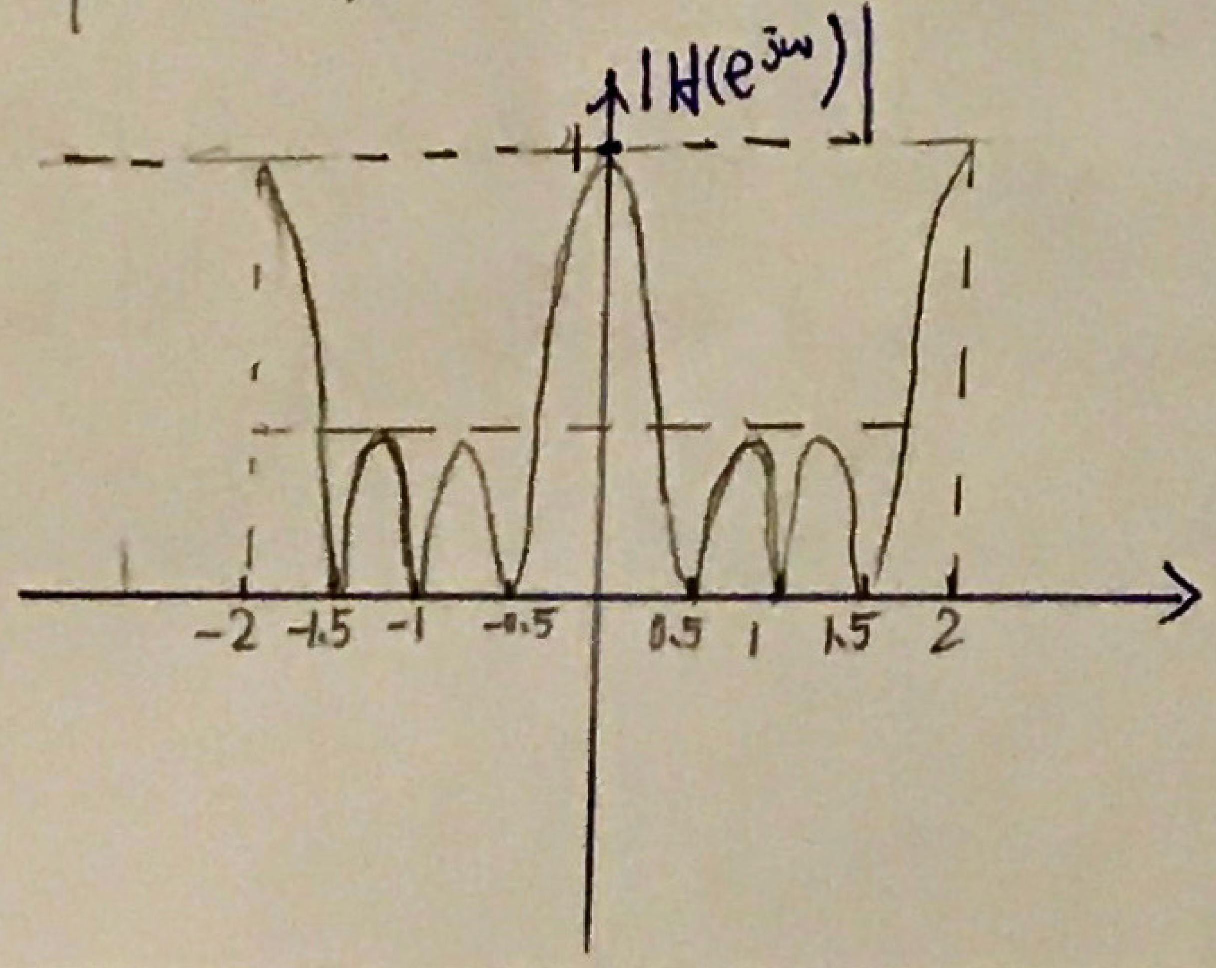
3.

(a)

$h[n] = \{1, 1, 1, 1\}$ for $n = 0, 1, 2, 3$

$$H(e^{j\omega}) = 1 + e^{-j\omega} + e^{-j2\omega} + e^{-j3\omega}$$

$$\Rightarrow |H(e^{j\omega})| = \sqrt{(1+\cos\omega + \cos 2\omega + \cos 3\omega)^2 + (\sin\omega + \sin 2\omega + \sin 3\omega)^2} \quad \#$$



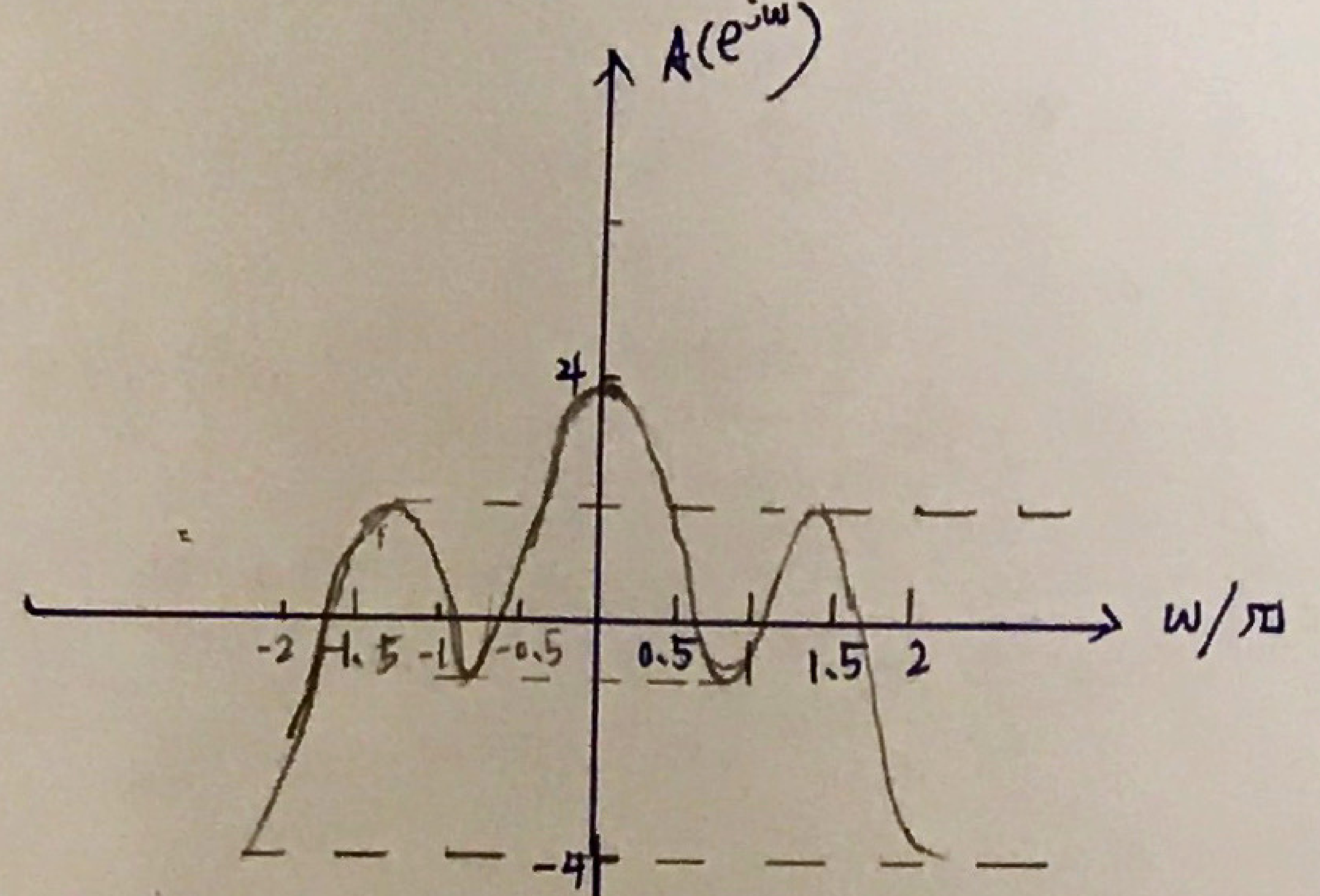
3.

(b)

$$H(e^{jw}) = e^{-j\frac{3}{2}w}\left(e^{j\frac{3}{2}w} + e^{j\frac{1}{2}w} + e^{-j\frac{1}{2}w} + e^{-j\frac{3}{2}w}\right)$$

$$= 2e^{-j\frac{3}{2}w}\left(\cos\frac{3}{2}w + \cos\frac{1}{2}w\right)$$

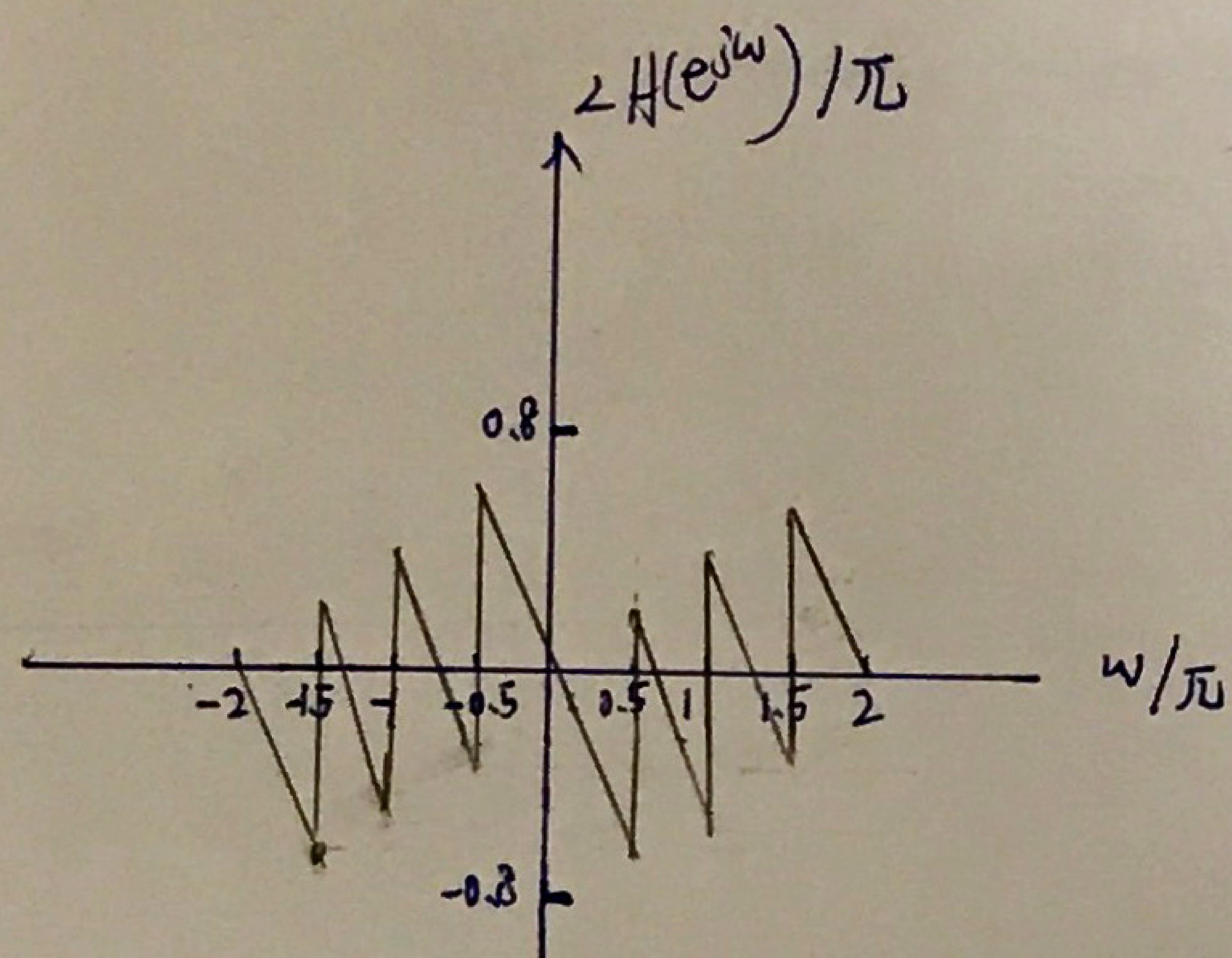
$$\Rightarrow A(e^{jw}) = 2\left(\cos\frac{3}{2}w + \cos\frac{1}{2}w\right) \#$$



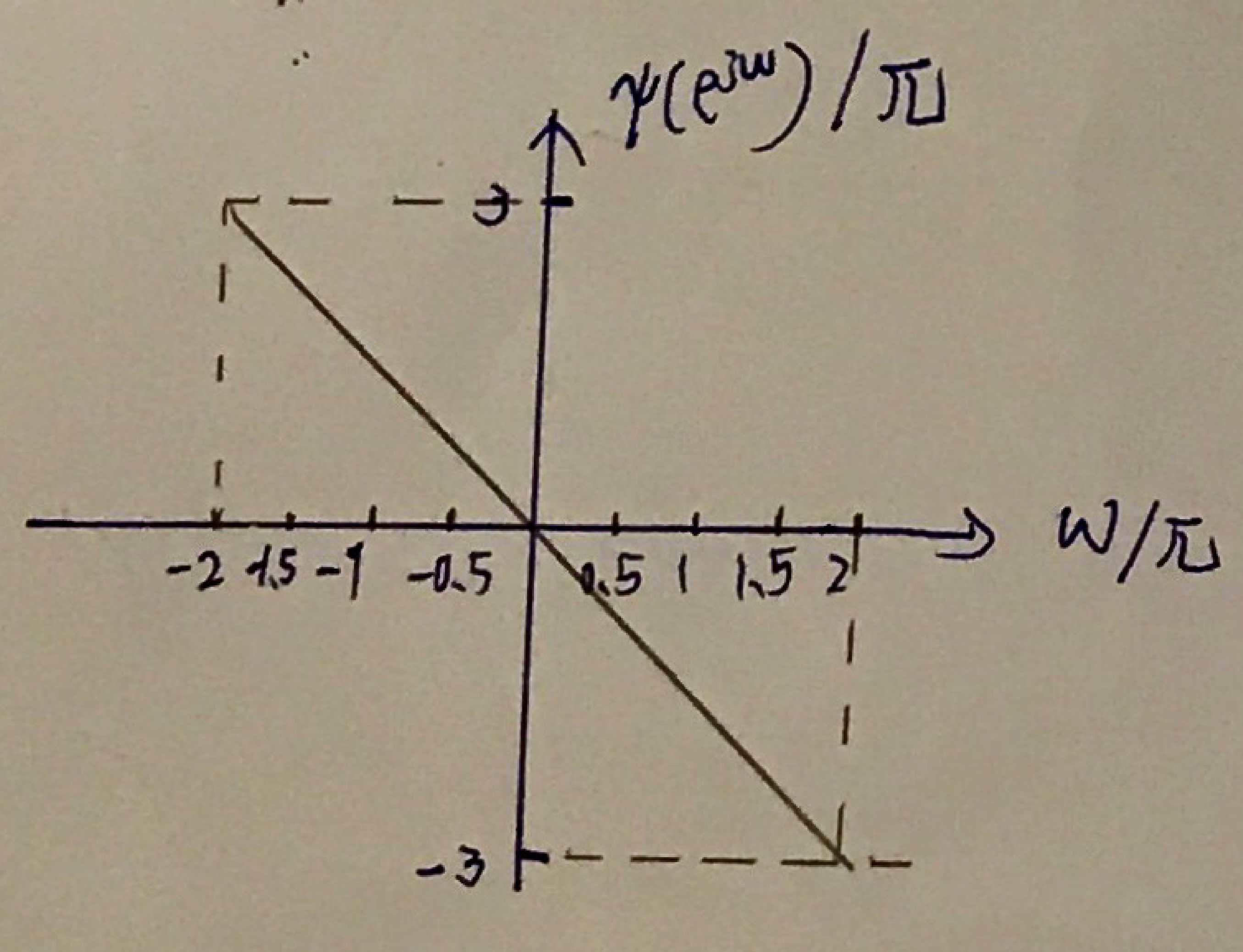
$A(e^{jw})$ have negative values, while $|H(e^{jw})|$ can' only take positive value.

And $|H(e^{jw})| = |A(e^{jw})|$

(c) $$\angle H(e^{jw}) = \tan^{-1}\left(\frac{-\sin w - \sin 2w - \sin 3w}{1+\cos w + \cos 2w + \cos 3w}\right) \#$$



(d) $$\psi(e^{jw}) = \frac{-3}{2}w \quad \# \quad \left(\because H(e^{jw}) = A(e^{jw})e^{-j\frac{3}{2}w}\right)$$



As $\angle H(e^{jw})$ can only take principle angle, which is $[-\pi, \pi]$, while $\psi(e^{jw})$ does NOT have this limit so that $\psi(e^{jw})$ is continuous and $\angle H(e^{jw})$ is not.

4.

(a.) $h[n] = -h[M-n]$

$$H(e^{jw}) = \sum_{n=0}^{M} h[n] e^{-jwn}$$

$$= \left(h[0] - h[0]e^{-jwM}\right) + \left(h[1]e^{-jw} - h[1]e^{-jw(M-1)}\right) + \dots$$

$$\left(h[\tfrac{M-1}{2}]e^{-jw(\frac{M-1}{2})} - h[\tfrac{M-1}{2}]e^{-jw(\frac{M+1}{2})} \right)$$

$$= 2j\,h[0]e^{-j\frac{wM}{2}}\left(\frac{e^{j\frac{wM}{2}} - e^{-j\frac{wM}{2}}}{2j}\right) + 2j\,h[1]e^{-j\frac{wM}{2}}\left(\frac{e^{jw(\frac{M}{2}-1)} - e^{-jw(\frac{M}{2}-1)}}{2j}\right)$$

$$+ \dots\ 2j\,h[\tfrac{M-1}{2}]e^{-j\frac{wM}{2}}\left(\frac{e^{jw\frac{1}{2}} - e^{-jw\frac{1}{2}}}{2j}\right)$$

$$= 2je^{-jw\frac{M}{2}} \sum_{n=0}^{\frac{M-1}{2}} h[n] \sin\left[w\left(\tfrac{M}{2} - n\right)\right]$$

Let $n = \frac{M+1}{2} - k \Rightarrow k = \frac{M+1}{2} - n$

$$H(e^{jw}) = 2je^{-jw\frac{M}{2}} \sum_{k=\frac{M+1}{2}}^{1} h[\tfrac{M+1}{2} - k] \sin\left[w\left(k - \tfrac{1}{2}\right)\right]$$

$$= 2je^{-jw\frac{M}{2}} \sum_{k=1}^{\frac{M+1}{2}} h[\tfrac{M+1}{2} - k] \sin\left[w\left(k - \tfrac{1}{2}\right)\right]$$

$$= j\,\underbrace{2\sum_{k=1}^{\frac{M+1}{2}} \overbrace{h[\tfrac{M+1}{2} - k]}^{d[k]} \sin\left[w\left(k - \tfrac{1}{2}\right)\right]}_{A(e^{jw})} e^{-jw\frac{M}{2}}$$

$$= jA(e^{jw})e^{-jw\frac{M}{2}} \ \#$$

$$d[k] = h[\tfrac{M+1}{2} - k] \ \#$$

4.

(b)

$$A(e^{jw}) = 2\sum_{k=1}^{\frac{M+1}{2}} h[\tfrac{M+1}{2}-k]\sin[w(k-\tfrac{1}{2})]$$

$$= 2\sum_{k=1}^{\frac{M+1}{2}} d[k]\sin(wk-\tfrac{w}{2})$$

$$2\cos\alpha\sin\beta = \sin(\alpha+\beta) - \sin(\alpha-\beta)$$

$$\sin(wk+\tfrac{w}{2}) = 2\cos wk\sin\tfrac{w}{2} + \sin(wk-\tfrac{w}{2})$$

$$d[1]\sin\tfrac{w}{2}$$

$$d[2]\sin\tfrac{3}{2}w = d[2]\sin(w\cdot 1+\tfrac{w}{2})$$

$$= d[2][2\cos w\sin\tfrac{w}{2} + \sin\tfrac{w}{2}]$$

$$d[3]\sin\tfrac{5}{2}w = d[3]\sin(w\cdot 2+\tfrac{w}{2})$$

$$= d[3][2\cos 2w\sin\tfrac{w}{2} + \sin\tfrac{3}{2}w]$$

$$= d[3][2\cos 2w\sin\tfrac{w}{2} + 2\cos w\sin\tfrac{w}{2} + \sin\tfrac{w}{2}]$$

$$\vdots$$

$$A(e^{jw}) = \sin\tfrac{w}{2}[\underbrace{(d[1]+d[2]+\dots d[\tfrac{M+1}{2}])}_{\hat{d}[0]} + 2\underbrace{(d[2]+d[3]+\dots d[\tfrac{M+1}{2}])}_{\hat{d}[1]}\cos w$$

$$+ \underbrace{2(d[3]+d[4]\dots d[\tfrac{M+1}{2}])}_{\hat{d}[2]}\cos 2w + \dots \underbrace{2d[\tfrac{M+1}{2}]}_{\hat{d}[\frac{M-1}{2}]}\cos(\tfrac{M-1}{2})w$$

$$= \sin\tfrac{w}{2}\sum_{k=0}^{\frac{M-1}{2}} \hat{d}[k]\cos wk$$

$$d[k] = \begin{cases} \tfrac{1}{2}(2\hat{d}[0]-\hat{d}[1]), & k=1 \\ \tfrac{1}{2}(\hat{d}[k-1]-\hat{d}[k]), & 2\le k\le \tfrac{M-1}{2} \\ \tfrac{1}{2}\hat{d}[\tfrac{M-1}{2}], & k=\tfrac{M+1}{2} \end{cases}$$