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1.

(a) $N=8$

$$\tilde{x}[n] = 2\cos\left(\frac{\pi}{4}n\right)$$

$$= e^{j\frac{\pi}{4}n} + e^{-j\frac{\pi}{4}n} = e^{j\cdot\frac{2\pi}{8}\cdot n} + e^{-j\frac{2\pi}{8}n}$$

$$\tilde{c}_k = \begin{cases} 1, & k = 1+8m \text{ or } k = -1+8m, \quad m \in \mathbb{Z} \\ 0, & \text{otherwise} \end{cases} \Rightarrow X[k] = N\tilde{c}_k = \begin{cases} 8, & k=1+8m \text{ or } k=-1+8m, \ m \in \mathbb{Z} \\ 0, & \text{otherwise} \end{cases}$$

#

(b) $N=8$

$$\tilde{x}[n] = \frac{3}{2j}\left(e^{j\frac{\pi}{4}n} - e^{-j\frac{\pi}{4}n}\right) + 2\left(e^{j\frac{3}{4}\pi} + e^{j\frac{-3}{4}\pi}\right)$$

$$= \frac{3}{2j}\left(e^{j\frac{2\pi}{8}n} - e^{-j\frac{2\pi}{8}n}\right) + 2\left(e^{j\frac{2\pi}{8}\cdot 3} + e^{j\frac{2\pi}{8}(-3)}\right)$$

$$\tilde{c}_k = \begin{cases} \frac{3}{2j}, & k = 1+8m \\ -\frac{3}{2j}, & k = -1+8m \\ 2, & k = 3+8m,\ 3-8m \\ 0, & \text{otherwise} \end{cases}, \quad m \in \mathbb{Z} \Rightarrow X[k] = N\tilde{c}_k = \begin{cases} \frac{12}{j}, & k=1+8m \\ \frac{-12}{j}, & k=-1+8m \\ 16, & k=3+8m \text{ or } 3-8m \\ 0, & \text{otherwise} \end{cases}, m \in \mathbb{Z}$$

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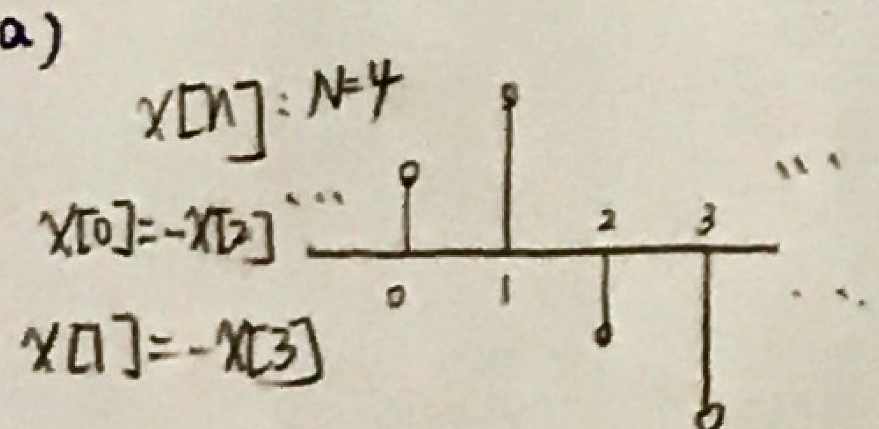
2.

(a)



$$X[k] = \sum_{n=0}^{N-1} x[n] e^{j\frac{2\pi}{N}kn}$$

$$= \sum_{n=0}^{\frac{N-2}{2}} x[n] e^{-j\frac{2\pi}{N}kn} - \sum_{n=0}^{\frac{N-2}{2}} x[n] \underbrace{e^{-j\frac{2\pi}{N}k\left(n+\frac{N}{2}\right)}}$$

$$e^{-j\frac{2\pi}{N}k\left(n+\frac{N}{2}\right)} = e^{-j\frac{2\pi}{N}kn} \cdot e^{-j\frac{2\pi}{N}k\cdot\frac{N}{2}}$$

$$\underset{m\in\mathbb{Z}}{\overset{k=2m\text{代入}}{=\!=}} e^{-j\frac{2\pi}{N}kn} \cdot e^{-j\frac{2\pi}{N}2m\cdot\frac{N}{2}}$$

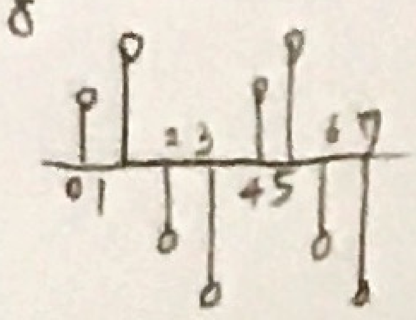
$$= e^{-j\frac{2\pi}{N}kn}$$

$\therefore X[k]=0$ for k is even #

2.

(b)

$N=8$



$$X[K] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}$$

$$= \sum_{n=0}^{\frac{N}{4}-1} x[n] e^{-j\frac{2\pi}{N}kn} - \sum_{n=0}^{\frac{N}{4}-1} x[n] e^{-j\frac{2\pi}{N}k(n+\frac{N}{4})} + \sum_{n=\frac{N}{2}}^{\frac{3}{4}N-1} x[n] e^{-j\frac{2\pi}{N}kn} - \sum_{n=\frac{N}{2}}^{\frac{3}{4}N-1} x[n] e^{-j\frac{2\pi}{N}k(n+\frac{N}{4})}$$

$$e^{-j\frac{2\pi}{N}k(n+\frac{N}{4})} = e^{-j\frac{2\pi}{N}kn} \cdot e^{-j\frac{2\pi}{N}k\cdot\frac{N}{4}}$$

$$e^{-j\frac{2\pi}{N}k\cdot\frac{N}{4}} = e^{-j2\pi\frac{k}{4}}$$

$$\overset{k=4l}{=\!=} \quad e^{-j2\pi l} = 1 \quad \because \ 0 \le l \le \frac{N}{4}-1$$

$\therefore \ X[K] = 0 \quad$ for $k = 4l, \quad 0 \le l \le \frac{N}{4}-1$

3.

(a) $x_3[n] = x_1[n] * x_2[n] \longleftrightarrow X_3(e^{j\omega}) = X_1(e^{j\omega}) \cdot X_2(e^{j\omega})$

$$X_3[K] = X_3\left(e^{j(\frac{2\pi K}{N})}\right), \quad 0 \le K \le N-1$$

$$= X_1\left(e^{j\frac{2\pi K}{N}}\right) \cdot X_2\left(e^{j(\frac{2\pi K}{N})}\right), \quad 0 \le K \le N-1$$

$$= X_1[K]\, X_2[K]$$

From definition of the circular convolution, $X_4[K] = X_1[K]\,X_2[K] = X_3[K]$

But $x_3[n]$ has length of $N_1 + N_2 - 1$. In order to calculate the N-point DFT of $y[n]$, we first form a periodic sequence of period N,

$$\tilde{x}_3[n] = \sum_{l=-\infty}^{+\infty} x_3[n + lN]$$

Thus, $x_4[n] = \sum_{l=-\infty}^{+\infty} x_3[n + lN]$ 井

3.

(b)

case 1: $N \geq L(N_1+N_2-1)$

$x_3[n]$ has the length of N_1+N_3-1

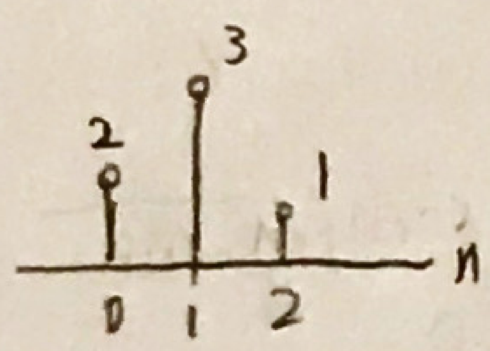
and $x_4[n] = \sum_{l=-\infty}^{+\infty} x_3[n+lN]$ 時 $x_3[n]$ 不會彼此 overlap

$\therefore \; x_4[n] = x_3[n]$

$\Rightarrow x_4[n] - x_3[n] = 0$

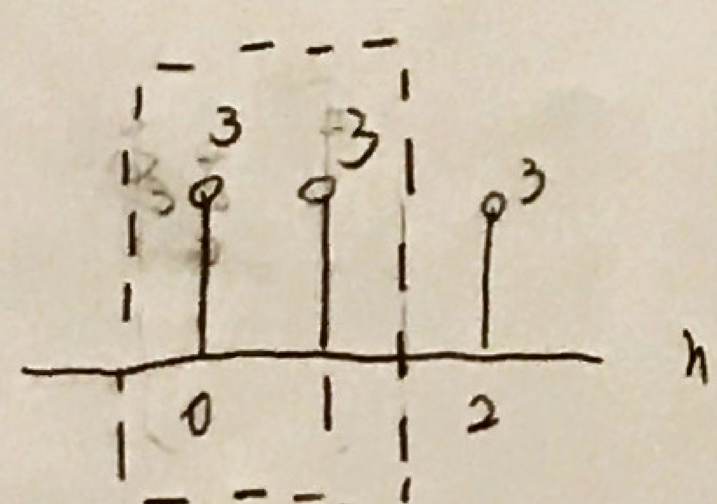
case 2: $\max(N_1, N_2) \leq N < L(N_1+N_3-1)$

Assume $x_3[n]$ $(L=3)$

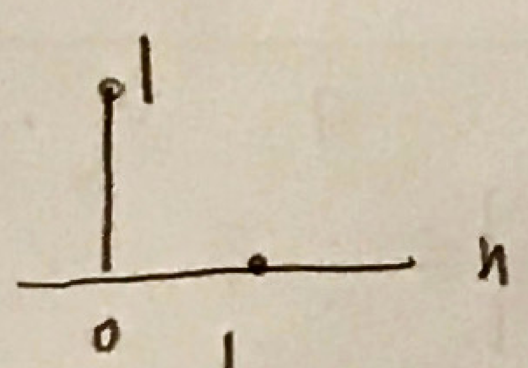


取 $N=2$

$$x_4[n] = \sum_{l=-\infty}^{+\infty} x_3[n+2l]$$



$e[n] = x_4[n] - x_3[n]$



$e[0] = x_3[N] = x_3[2] = 1$

$e[1] = x_3[1+N] = x_3[3] = 0$

$\Rightarrow e[n] = x_3[n+N]$

$$\therefore \; e[n] = \begin{cases} x_3[n+N], & \max(N_1, N_3) \leq N \leq L \\ 0, & N \geq L \end{cases}$$

3.

(c) $N_1 = 4,\ N_2 = 4 \Rightarrow L = 4+4-1 = 7$

Case I: $N = 8\ (N \geq L)$

$$x_3[n] = \{4, 11, 20, 30, 20, 11, 4\}$$

$$x_4[n] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 4 & 3 & 2 \\ 2 & 1 & 0 & 0 & 0 & 0 & 4 & 3 \\ 3 & 2 & 1 & 0 & 0 & 0 & 0 & 4 \\ 4 & 3 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 4 & 3 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 4 & 3 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 4 & 3 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 4 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 11 \\ 20 \\ 30 \\ 20 \\ 11 \\ 4 \\ 0 \end{bmatrix}$$

$$\underline{x_4[n] = \sum_{l=-\infty}^{+\infty} x_3[n + lN]}\ \#, \quad 0 \leq n \leq N-1 \quad (\text{no aliasing occured on } x_3[n])$$

And $x_4[n] = x_3[n] \Rightarrow \underline{e[n] = x_4[n] - x[3] = 0}\ \#$

Case II $N = 5\ (\max(N_1, N_2) \leq N < L)$

$$x_3[n] = \{4, 11, 20, 30, 20, 11, 4\}$$

$$x_4[n] = \begin{bmatrix} 1 & 0 & 4 & 3 & 2 \\ 2 & 1 & 0 & 4 & 3 \\ 3 & 2 & 1 & 0 & 4 \\ 4 & 3 & 2 & 1 & 0 \\ 0 & 4 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 15 \\ 15 \\ 20 \\ 30 \\ 20 \end{bmatrix}$$

$$\begin{array}{r|cccccccccccccc} & \cdots & & & -2 & -1 & n=0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & \cdots \\ \hline \sum_{l=-\infty}^{+\infty} x_3[n+lN] = & & & & & & 4 & 11 & 20 & 30 & 20 & 11 & 4 & & \\ & 4 & 11 & 30 & 20 & & 11 & 4 & & & & 4 & 11 & 20 & 30\ \ 20\ \ 11\ \ 4 \end{array}$$

$$= \{15, 15, 20, 30, 20\} = \underline{x_4[n]}\ \#$$

$$e[n] = x_4[n] - x_3[n]$$

$$= \{11, 4, 0, -0, 0\}$$

$$x_3[n+N] = x_3[n+5]$$

$$= \{11, 4, 0, 0, 0\}$$

$$\therefore \underline{e[n] = x_3[n+N]}$$

#

4.

$$\tilde{X}[k] = \sum_{n=0}^{N-1} \tilde{x}[n] e^{-jk\frac{2\pi}{N}n}$$

$$\tilde{X}_3[k] = \sum_{n=0}^{3N-1} \tilde{x}_3[n] e^{-jk\frac{2\pi}{3N}n}$$

$$= \sum_{n=0}^{N-1} \tilde{x}_3[n] e^{-jk\frac{2\pi}{3N}n} + \sum_{n=N}^{2N-1} \tilde{x}_3[n] e^{-jk\frac{2\pi}{3N}n} + \sum_{n=2N}^{3N-1} \tilde{x}_3[n] e^{-jk\frac{2\pi}{3N}n}$$

$$= \sum_{n=0}^{N-1} \tilde{x}_3[n] e^{-jk\frac{2\pi}{3N}n} + \sum_{n=0}^{N-1} \tilde{x}_3[n] e^{-jk\frac{2\pi}{3N}(n+N)} + \sum_{n=0}^{N-1} \tilde{x}_3[n] e^{-jk\frac{2\pi}{3N}(n+2N)}$$

$$= \sum_{n=0}^{N-1} \tilde{x}[n] e^{-jk\frac{2\pi}{3N}n} + \sum_{n=0}^{N-1} \tilde{x}[n] e^{-jk\frac{2\pi}{3N}(n+N)} + \sum_{n=0}^{N-1} \tilde{X}[n] e^{-jk\frac{2\pi}{3N}(n+2N)}$$

$$= \tilde{X}[\tfrac{1}{3}k] + \tilde{X}[\tfrac{1}{3}k] \times e^{-jk\frac{2\pi}{3}} + \tilde{X}[\tfrac{1}{3}k] \times e^{-jk\frac{4}{3}\pi}$$

$$= \left(1 + 2\cos\frac{2k}{3}\pi\right) \tilde{X}[\tfrac{1}{3}k],$$

$$\tilde{X}[\tfrac{1}{3}k] = \begin{cases} \tilde{X}[k], & k = 3m, \quad m \in Z \\ 0, & \text{otherwise} \end{cases}$$

Note that when $k = 3m$, $2\cos\frac{2\pi k}{3} = 2$.

$$\therefore \tilde{X}_3[k] = \begin{cases} \tilde{X}[\tfrac{1}{3}k] \times 3, & k = 3m, \quad m \in Z \\ 0, & \text{otherwise} \end{cases}$$

5 $X[K] = \{4,\ 2-j3,\ 3+j2,\ -4+6j,\ 8-7j,\ 8+7j,\ -4-6j,\ 3-2j,\ 2+3j\}$

$\because X[k] = X_R[k] + jX_I[k]$

$\Rightarrow \begin{cases} X_R[k] \text{ is even symmetric} \\ X_I[k] \text{ is odd symmetric} \end{cases}$ and $X[k]$ is symmetric with respect to $k=4.5$

(a)

$x[\langle n-M\rangle_N] \xrightarrow{DFT} W_N^{km} X[k]$

$W_k = e^{-j\frac{2\pi}{9}(-2)k} = e^{j\frac{4\pi}{9}k}$

$X_1[K] = \{4,\ e^{j\frac{4}{9}\pi}(2-3j),\ e^{j\frac{8}{9}\pi}(3+2j),\ e^{j\frac{4}{3}\pi}(-4+6j),\ e^{j\frac{16}{9}\pi}(8-7j),$

$e^{j\frac{20}{9}\pi}(8+7j),\ e^{j\frac{8}{3}\pi}(-4-6j),\ e^{j\frac{28}{9}\pi}(3-2j),\ e^{j\frac{32}{9}\pi}(2+3j)\}$

(b)

$x[\langle -n\rangle_N] \xrightarrow{DFT} X[\langle -k\rangle_N]$

$X_2[K] = 2e^{-j\frac{4}{9}\pi k} X[\langle -k\rangle_N]$

$= \{8,\ 2e^{-j\frac{4}{9}\pi}(2+3j),\ 2e^{-j\frac{8}{9}\pi}(3-2j),\ 2e^{-j\frac{4}{3}\pi}(-4-6j),\ 2e^{-j\frac{16}{9}\pi}(8+7j),$

$2e^{-j\frac{20}{9}\pi}(8-7j),\ 2e^{-j\frac{8}{3}\pi}(-4+6j),\ 2e^{-j\frac{28}{9}\pi}(3+2j),\ 2e^{-j\frac{32}{9}\pi}(2-3j)\}$

(c)

$y[n] = h[n] \circledast_N x[n] \xrightarrow{DFT} Y[K] = H[K]X[K]$

$X[\langle -k\rangle_9] = \{4,\ 2+3j,\ 3-2j,\ -4-6j,\ 8+7j,\ 8-7j,\ -4+6j,\ 3+2j,\ 2-3j\}$

$X_3[K] = X[K]X[\langle -K\rangle_9] = \underline{\{16,\ 13,\ 13,\ 52,\ 113,\ 113,\ 52,\ 13,\ 13\}}$ #

(d)

$x[n]\cdot x[n] \xrightarrow{DFT} \frac{1}{N} X[K] \circledast_N X[k]$

$$
\frac{1}{9}\begin{bmatrix}
4 & 2+3j & 3-2j & -4-6j & 8+7j & 8-7j & -4+6j & 3+2j & 2-3j \\
2-3j & 4 & 2+3j & 3-2j & -4-6j & 8+7j & 8-7j & -4+6j & 3+2j \\
3+2j & 2-3j & 4 & 2+3j & 3-2j & -4-6j & 8+7j & 8-7j & -4+6j \\
-4+6j & 3+2j & 2-3j & 4 & 2+3j & 3-2j & -4-6j & 8+7j & 8-7j \\
8-7j & -4+6j & 3+2j & 2-3j & 4 & 2+3j & 3-2j & -4-6j & 8+7j \\
8+7j & 8-7j & -4+6j & 3+2j & 2-3j & 4 & 2+3j & 3-2j & -4-6j \\
-4-6j & 8+7j & 8-7j & -4+6j & 3+2j & 2-3j & 4 & 2+3j & 3-2j \\
3-2j & -4+6j & 8+7j & 8-7j & -4+6j & 3+2j & 2-3j & 4 & 2+3j \\
2+3j & 3-2j & -4+6j & 8+7j & 8-7j & -4+6j & 3+2j & 2-3j & 4
\end{bmatrix}
\begin{bmatrix} 4 \\ 2-3j \\ 3+2j \\ -4+6j \\ 8-7j \\ 8+7j \\ -4-6j \\ 3-2j \\ 2+3j \end{bmatrix}
= \frac{1}{9}\begin{bmatrix} 398 \\ -117+126j \\ 7-222j \\ 122+116j \\ 31+60j \\ 31-60j \\ 122-116j \\ 7+222j \\ -117-126j \end{bmatrix}
$$

#

5.

(e)

$$e^{-j\frac{2\pi}{9}\times 2\cdot n}\, x[n] \longleftrightarrow X[\langle k+2\rangle_N]$$

$$X_5[k] = X[\langle k+2\rangle_9]$$

$$= \{3+2j,\ -4+6j,\ 8-7j,\ 8+7j,\ -4-6j,\ 3-2j,\ 2+3j,\ 4,\ 2-3j\}$$ #