National Tsing Hua University
Department of Electrical Engineering
EE3660 Intro. to Digital Signal Processing, Spring 2020

Homework Assignment #3: Chap. 5-6

Due: April 16, 2020

I Paper Assignment (74%)

1. (6%) Determine the system function, magnitude response, and phase response of the following systems and use the pole-zero pattern to explain the shape of their magnitude response:

   (a) $\mathrm{y[n]} = \frac{1}{4}(x[n] + x[n-1]) - \frac{1}{4}(x[n-2] + x[n-3])$

   (b) $\mathrm{y[n]} = x[n] - x[n-4] + 0.6561y[n-4]$

2. (12%) Consider a periodic signal

$$\mathrm{x[n]} = \sin(0.1\pi n) + \frac{1}{3}\sin(0.3\pi n) + \frac{1}{5}\sin(0.5\pi n)$$

   For each of the following systems, determine if the system imparts (i) no distortion, (ii) magnitude distortion, and/or (iii) phase (or delay) distortion.

   (a) $\mathrm{h[n]} = \{1_{n=0}, -2, 3, -4, 0, 4, -3, 2, -1\}$

   (b) $\mathrm{y[n]} = 10x[n-10]$

3. (12%) An economical way to compensate for the droop distortion in S/H DAC is to use an appropriate digital compensation filter prior to DAC.

   (a) Determine the frequency response of such an ideal digital filter $H_r(e^{j\omega})$ that will perform an equivalent filtering given by following $H_r(j\Omega)$

$$H_\mathrm{r}(\mathrm{j}\Omega) = \begin{cases} \frac{\Omega T/2}{\sin(\Omega T/2)} \mathrm{e}^{\mathrm{j}\Omega T/2}, & |\Omega| < \pi/T \\ 0. & \text{otherwise} \end{cases}$$

   (b) One low-order FIR filter suggested in Jackson (1996) is

$$H_{FIR}(z) = -\frac{1}{16} + \frac{9}{8}z^{-1} - \frac{1}{16}z^{-2}$$

   Compare the magnitude response of $H_{FIR}(e^{j\omega})$ with that of $H_r(e^{j\omega})$ above.

   (c) Another low-order IIR filter suggested in Jackson (1996) is

$$H_{IIR}(z) = \frac{9}{8 + z^{-1}}$$

Compare the magnitude response of $H_{IIR}(e^{j\omega})$ with that of $H_r(e^{j\omega})$ above.

4. (12%) Consider the following continuous-time system

$$H(s) = \frac{s^4 - 6s^3 + 10s^2 + 2s - 15}{s^5 + 15s^4 + 100s^3 + 370s^2 + 744s + 720}$$

(a) Show that the system *H*(s) is a nonminimum phase system.

(b) Decompose *H*(s) into the product of minimum phase component $H_{min}(s)$ and an all pass component $H_{ap}(s)$.

(c) Briefly plot the magnitude and phase responses of *H*(s) and $H_{min}(s)$ and explain your plots.

(d) Briefly plot the magnitude and phase responses of $H_{ap}(s)$.

5. (12%) We want to design a second-order IIR filter using pole-zero placement that satisfies the following requirements: (1) the magnitude response is 0 at $\omega_1 = 0$ and $\omega_3 = \pi$ (2) The maximum magnitude is 1 at $\omega_{2,4} = \pm\frac{\pi}{4}$ and (3) the magnitude response is approximately $\frac{1}{\sqrt{2}}$ at frequencies $\omega_{2,4} \pm 0.05$

(a) Determine locations of two poles and two zeros of the required filter and then compute its system function H(z).

(b) Briefly graph the magnitude response of the filter.

(c) Briefly graph phase and group-delay responses.

6. (8%) The following signals $x_c(t)$ is sampled periodically to obtained the discrete-time signal $x[n]$. For each of the given sampling rates in F_s Hz or in T period, (i) determine the spectrum $X(e^{i\omega})$ of $x[n]$; (ii) plot its magnitude and phase as a function of ω in $\frac{rad}{sam}$ and as a function of F in Hz; and (iii) explain whether $x_c(t)$ can be recovered from $x[n]$.

(a) $x_c(t) = 5e^{i40t} + 3e^{-i70t}$, with sampling period T = 0.01, 0.04, 0.1

(b) $x_c(t) = 3 + 2\sin(16\pi t) + 10\cos(24\pi t)$, with sampling rate F_s = 30, 20, 15 Hz.

7. (12%) An 8-bit ADC has an input analog range of ±5 volts. The analog input signal is

$$x_c(t) = 2\cos(200\pi t) + 3\sin(500\pi t)$$

The converter supplies data to a computer at a rate of 2048 bits/s. The computer, without processing, supplies these data to an ideal DAC to form the reconstructed signal $y_c(t)$. Determine:

(a) the quantizer resolution (or step),

(b) the SQNR in dB,

(c) the folding frequency and the Nyquist rate.

II Program Assignment (26%)

8. (4%) Compute and plot the phase response using the functions freqz, angle, phasez, unwrap, and phasedelay for the following systems:

   (a) $y[n] = x[n-15]$

   (b) $H(z) = \frac{1+1.655z^{-1}+1.655z^{-2}+z^{-3}}{1-1.57z^{-1}+1.264z^{-2}-0.4z^{-3}}$

9. (6%) According to problem 2 in paper assignment, plot magnitude response, phase response and group-delay response for each of the systems.

10. (6%) MATLAB provides a function called polystab that stabilizes the given polynomial with respect to the unit circle, that is, it reflects those roots which are outside the unit-circle into those that are inside the unit circle but with the same angle. Using this function, convert the following systems into minimum-phase and maximum-phase systems. Verify your answers using a pole-zero plot for each system*(plot minimum-phase and maximum-phase systems for each question)*.

    (a) $H(z) = \frac{z^2+2z+0.75}{z^2-0.5z}$

    (b) $H(z) = \frac{1-2.4142z^{-1}+2.4142z^{-2}-z^{-3}}{1-1.8z^{-1}+1.62z^{-2}+0.729z^{-3}}$

11. (10%) Signal xc(t) = 5 cos(200πt + π6) + 4 sin(300πt) is sampled at a rate of Fs = 1 kHz to obtain the discrete-time signal x[n].

    (a) Determine the spectrum $X(e^{j\omega})$ of x[n] and plot its magnitude as a function of ω in $\frac{rad}{sample}$ and as a function of *F* in Hz. Explain whether the original signal xc(t) can be recovered from x[n].

    (b) Repeat part (a) for Fs = 500 Hz.

    (c) Repeat part (a) for Fs = 100

    (d) Comment on your results.