

1. (15%)

(a) (5%) ROC  $|z| > 2$ :  $h[n] = 2 \cdot (0.5)^n u[n] - 3 \cdot (-2)^n u[n]$ .

(b) (5%) ROC  $0.5 < |z| < 2$ :  $h[n] = 2 \cdot (0.5)^n u[n] + 3 \cdot (-2)^n u[-n - 1]$ .

(c) (5%)  $\frac{11}{4} \cdot \frac{1 - \frac{2}{11} z^{-1}}{(1 - 0.5z^{-1})(1 + 0.5z^{-1})}$ .

2. (15%)

(a) (5%)  $y_r(t) = \frac{3\sqrt{3}}{2\pi} \cos(2\pi \cdot 11 \cdot 10^3 \cdot t - \pi/3)$  since  $\Omega T/2 = \pi/3$ .

(b) (5%)  $y_r(t) = \frac{3}{\pi} \cos(2\pi \cdot 10^3 \cdot t - \pi/6)$  since  $\Omega T/2 = \pi/6$ .

(c) (5%)  $y_r(t) = \frac{2\sqrt{2}}{\pi} \cos(2\pi \cdot 44 \cdot t - 7\pi/1000) \cos(2\pi \cdot 11 \cdot 10^3 \cdot t - 3\pi/4)$  since  $\Omega T/2 = \pi/4$ . The digital carrier frequency  $\omega_0 = \pi/2$ . The phase delay is  $4\omega_0^2/\pi^2 = 1$ , and the group delay is  $3 \cdot 4\omega_0^2/\pi^2 = 3$ . So, the filtered signal  $x_m[n] * h[n] = s[n - 3]x[n - 1]$ . The reconstruction further causes another 0.5 samples of delay, so we have the terms of  $\cos(2\pi \cdot 44 \cdot (t - 3.5/F_s)) \cos(2\pi \cdot 11 \cdot 10^3 \cdot (t - 1.5/F_s))$ .

3. (10%)

(a) (2%)  $e^{-jk\pi/2} X[k] = \{1, 1 - 2j, -4 + j, -1 + 8j, 16, -1 - 8j, -4 + j, 1 + 2j\}$ .

(b) (4%)  $X[k]X^*[k] = \{1, 5, 17, 65, 256, 65, 17, 5\}$ .

(c) (4%)  $X_3[k] = X_R^{ce}[k] + jX_I^{co}[k] = \{1, 2 + j, 4, 8 + j, 16, 8 - j, 4, 2 - j\}$ ;  $X_4[k] = X_I^{ce}[k] - jX_R^{co}[k] = \{0, 0, -1, 0, 0, 0, -1, 0\}$ .

4. (15%)

(a) (4%) Skipped.

(b) (5%) Skipped.

(c) (6%) Skipped.

5. (15%)

(a) (3%)  $X_c(j\Omega) = \frac{1}{\log(2/T) + j\Omega}$ .

(b) (4%)  $x[n] = 2^{-n} u[n]$  and  $X(e^{j\omega}) = \frac{1}{1 - 0.5e^{-j\omega}}$ .  $X(e^{j\omega})$  is a frequency-aliased (and scaled) version of  $X_c(j\Omega)$

(c) (4%)  $x_1[n] = \frac{1}{1 - 2^{-4}} \cdot 2^{-n}$  where  $n = 0, 1, 2, 3$ .

(d) (4%) Perform 1024-point DFT on the zero-padded  $x_1[n]$ .

6. (10%)  $A_s = 40$  dB and  $\Delta\omega = 0.1\pi$ .

(a) (3%) 3.3953.

(b) (3%)  $0.55\pi$ .

(c) (4%)  $L = 47$  (type-I);  $M = 44.5$ .

7. (20%)

(a) (10%)

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x1_0 = [x1(1 : 256) zeros(1, 256)];
x1_1 = [x1(257 : 400) zeros(1, 112)];
x2_0 = [x2(1 : 256) zeros(1, 256)];
x2_1 = [x2(257 : 500) zeros(1, 12)];
X1_0 = myfft512(x1_0);
X1_1 = myfft512(x1_1);
X2_0 = myfft512(x2_0);
X2_1 = myfft512(x2_1);
X1_0_X2_0 = X1_0 .* X2_0;
X1_1_X2_0 = X1_1 .* X2_0;
X1_0_X2_1 = X1_0 .* X2_1;
X1_1_X2_1 = X1_1 .* X2_1;
x1_0_x2_0 = myfft512([X1_0_X2_0(1) X1_0_X2_0(512 : -1 : 2)])/512;
x1_1_x2_0 = myfft512([X1_1_X2_0(1) X1_1_X2_0(512 : -1 : 2)])/512;
x1_0_x2_1 = myfft512([X1_0_X2_1(1) X1_0_X2_1(512 : -1 : 2)])/512;
x1_1_x2_1 = myfft512([X1_1_X2_1(1) X1_1_X2_1(512 : -1 : 2)])/512;
x_m = x1_1_x2_0 + x1_0_x2_1;
x3 = [x1_0_x2_0(1 : 256) x1_0_x2_0(257 : 512) + x_m(1 : 256)
      x_m(257 : 512) + x1_1_x2_1(1 : 256) x1_1_x2_1(257 : 387)];
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(1)

(b) (5%)

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x4_upsample512 = [[x4 0]; zeros(7, 64)];
X4_upsample512 = myfft512(x4_upsample512(:)');
X4 = X4_upsample512(1 : 64);
X4_pad = [X4(1 : 32) X4(33)/2 zeros(1, 7 * 64 - 1) X4(33)/2 X4(34 : 64)];
X4_padflip = [X4_pad(1) X4_pad(512 : -1 : 2)];
x5 = myfft512(X4_padflip)/64;
x5 = x5(1 : 63 * 8);
```

(c) (5%)

```
x8 = x6 + j * x7;  
X8 = myfft512(x8);  
X8_real = real(X8);  
X8_imag = imag(X8);  
X8_real_ce = (X8_real + [X8_real(1)X8_real(512 : -1 : 2)])/2;  
X8_real_co = (X8_real - [X8_real(1)X8_real(512 : -1 : 2)])/2;  
X8_imag_ce = (X8_imag + [X8_imag(1)X8_imag(512 : -1 : 2)])/2;  
X8_imag_co = (X8_imag - [X8_imag(1)X8_imag(512 : -1 : 2)])/2;  
X6 = X8_real_ce + j * X8_imag_co;  
X7 = X8_imag_ce - j * X8_real_co;
```