8 +3

$z^{-1} = \frac{-1}{2}$ $\quad \frac{-15}{2} = \frac{5}{2}b$

National Tsing Hua University
Department of Electrical Engineering
EE3660 Introduction to Digital Signal Processing, Spring 2020

Midterm Exam (100%)
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$-2 + 11z^{-1} = a(1+2z^{-1}) + b(2-z^{-1})$

$z^{-1} = 2$

$20 = 5a$

Note: Detailed derivations are required to obtain a full score for each problem.

1. (15%) Consider a system which is described by

$$2y[n] + 3y[n-1] - 2y[n-2] = -2x[n] + 11x[n-1].$$

$(2-z^{-1})(1+2z^{-1})$

(a) (5%) Determine the region of convergence for this system to be causal and also find the corresponding impulse response $h[n]$.

(b) (5%) Determine the region of convergence for this system to be stable and also find the corresponding impulse response $h[n]$.

(c) (5%) Convert this system into a minimum-phase one and determine the system function $H_{\min}(z)$.

2. (15%) Consider a sinusoidal signal $x_c(t) = \cos(2\pi F_0 t)$ with $F_0 = 11$ kHz. It is sampled at different rates of F_s and then reconstructed as $y_r(t)$ by the corresponding system

$$G(j\Omega) = \begin{cases} \frac{\sin(\Omega T/2)}{\Omega/2} e^{-j\Omega T/2}, & |\Omega| < \pi/T \\ 0, & \text{otherwise} \end{cases},$$

where $T = 1/F_s$. Note that this reconstruction system is equivalent to a sample-and-hold DAC followed by an ideal lowpass filter.

11K

(a) (5%) Determine $y_r(t)$ if $F_s = 33$ kHz.

(b) (5%) Determine $y_r(t)$ if $F_s = 6$ kHz.

(c) (5%) Consider a slightly complex scenario. Use $x_c(t)$ as a carrier signal to modulate a low-frequency signal $s(t) = \cos(2\pi F_1 t)$ where $F_1 = 44$ Hz. And sample the modulated signal $x_m(t) = s(t) \cdot x_c(t)$ at a rate of $F_s = 44$ kHz. Then process the sampled signal $x_m[n]$ by an all-pass digital filter $H(e^{j\omega}) = e^{-j4\omega^3/\pi^2}$. Finally, reconstruct $y_r(t)$ from the filtered signal using the corresponding $G(j\Omega)$. Now approximate $y_r(t)$ as accurate as you can. (Hint: use group delay to approximate the time delay of $s(t)$.)

3. (10%) The 8-point DFT of an 8-point sequence $x[n]$ is given by

$$X[k] = \{\underset{\uparrow}{1}, 2+j, 4-j, 8+j, 16, 8-j, 4-j, 2-j\}.$$

Determine the DFT of each of the following sequences.

(a) (2%) $x_1[n] = x[\langle n-2\rangle_8]$.

(b) (4%) $x_2[l] = \sum_{n=0}^{7} x[n]x^*[\langle n-l\rangle_8]$ (circular autocorrelation).

(c) (4%) Real-valued $x_3[n]$ and $x_4[n]$ such that $x[n] = x_3[n] + jx_4[n]$. You may need the symmetry properties as below.

$y[n]=x^*[n]$

$X^*[\langle k\rangle_N]$

$X^*[\langle n\rangle_N]$

$$x[n] = x_R^{ce}[n] + x_R^{co}[n] + jx_I^{ce}[n] + jx_I^{co}[n]$$

$$\updownarrow \qquad \updownarrow \qquad \qquad \updownarrow$$

$$X[k] = X_R^{ce}[k] + X_R^{co}[k] + jX_I^{ce}[k] + jX_I^{co}[k]$$

$N=3$

4. (15%) Let $X[k]$ be a real-valued N-point DFT for an N-point sequence $x[n]$.

(a) (4%) Show that $x[\frac{N-1}{2}] = x^*[\frac{N+1}{2}]$ if N is odd.

(b) (5%) Show that $|x[n]| = |x[\langle -n\rangle_N]|$.

(c) (6%) If $X[k]$ satisfies the condition $X[k] = X[\langle k+M\rangle_N]$ where $N = lM$ and l is an integer, show that $x[n] = 0$ if $l \nmid n$.

5. (15%) Complete the following example for computational Fourier analysis.

(a) (3%) For a right-sided exponential function $x_c(t) = 2^{-t/T}u(t)$, determine its CTFT $X_c(j\Omega)$ given that $e^{-at}u(t) \xrightarrow{\mathscr{F}} \frac{1}{a+j\Omega}$.

(b) (4%) Sample $x_c(t)$ with $t = nT$ to form a discrete-time signal $x[n] = x_c(nT)$ and then determine its DTFT $X(e^{j\omega})$. Also briefly state the relationship between $X(e^{j\omega})$ and $X_c(j\Omega)$.

(c) (4%) Sample the frequency components of $X(e^{j\omega})$ as four-point DFT $X_1[k] = X(e^{j2\pi k/4})$ where $k = 0, 1, ..., 3$. Determine its IDFT $x_1[n]$ which is a finite-length signal now.

(d) (4%) Now we want to have a detailed evaluation of the DTFT of $x_1[n]$. Explain how to use DFT computation to accurately derive $X_1(e^{j\omega})$ at $\omega = 2\pi k/1024$ where $k = 0, 1, ..., 1023$.

6. (10%) Consider a highpass linear-phase FIR filter design using the Kaiser window $w[n] = \frac{I_0\left(\beta\sqrt{1-[(n-\alpha)/\alpha]^2}\right)}{I_0(\beta)}$ where $0 \le n \le M$ and $I_0(x)$ is the zeroth-order modified Bessel function. The specifications are $\omega_p = 0.6\pi$, $\omega_s = 0.5\pi$, and $\delta_s = 0.01$. Determine the following design parameters to minimize the filter length: **(a)** (3%) β, **(b)** (3%) cut-off frequency ω_c, and **(c)** (4%) window (filter) length L. You may need the following empirical relations:

$$\beta = \begin{cases} 0, & A < 21 \\ 0.5842(A-21)^{0.4} + 0.07886(A-21), & 21 \le A \le 50 \\ 0.1102(A-8.7), & A > 50 \end{cases},$$

$$M = \frac{A-8}{2.285\,\Delta\omega}. \tag{1}$$

7. (20%) Consider a MATLAB FFT function **myfft512(x)** which performs exactly **512**-point DFT of a **512**-point input vector **x**. It can be used for efficient computation. For example, the IDFT **y** of a given **512**-point DFT **Y** (as a row vector) can be derived by the following MATLAB code.

$$\mathbf{Yflip}{=}\ [\mathbf{Y}(1)\ \mathbf{Y}(512:-1:2)];$$
$$\mathbf{y}{=}\ \mathbf{myfft512}(\mathbf{Yflip})/\mathbf{512};$$

Write down efficient MATLAB codes for the following algorithms using only the **myfft512** function and simple arithmetic/indexing/padding operations. All 1D vectors are row-wise arranged.

(a) (10%) Linear convolution. Given a 400-point **x1** and a 500-point **x2**, derive its linear (hard) convolution **x3** (899-point) through multiplication in DFT domain. You may consider the overlap-save or overlap-add method.

(b) (5%) Time-domain resolution scaling-up. Given a real-valued 63-point **x4**, derive its 8× scaled-up real-valued signal **x5** (504-point) by first applying 64-point DFT, then padding zeros, e.g. $\mathbf{zeros(1,k)}$, in DFT domain, and finally performing 512-point IDFT.

(c) (5%) Efficient DFT for real-valued sequences. Given two 512-point real-valued sequences **x6** and **x7**, derive their 512-point DFTs **X6** and **X7**, respectively, by calling **myfft512**(...) only once. You may need the **real** and **imag** functions to extract the real and imaginary parts. You may also need the symmetry properties in the problem **3**.