



Do you know WHO invented Fast Fourier Transform?



Joseph Fourier (1768-1830)

had served for Napoleon army,
but was abandoned in Egypt.

Contribution: Infinite harmonic series
(e.g. Fourier series)

His presentation in **1807** to the Academy of Sciences in Paris was not well received (lack of rigor), and **refused for publication**. The work was not published until 1822, with little modification.

**Do you believe that
FFT was already invented in 1805?**



“The Prince of Mathematicians”



Carl F. Gauss (1777-1855)

the prodigy found
“ $1+..+100=101 \times 100 / 2$ ”
by himself.

“Few, but ripe.”

Lots of his works were not published but kept in his diary, which got published as his collected works after he died. Many theorems and algorithms were found very important and had been “**rediscovered**” again and again.

FFT is one of such victim.



What Gauss Invented

- Periodic functions can be interpolated by

$$f(x) = \sum_{k=0}^m a_k \cos 2\pi kx + \sum_{k=1}^m b_k \sin 2\pi kx$$

- Equivalent to Fourier series
- Coefficients a and b equivalent to DFT
- The computation for length $N=N_1N_2$ can be decomposed by N_2 sets of N_1 subsamples

$$C(k_1 + N_1k_2) = \sum_{n_2=0}^{N_2-1} \left[\sum_{n_1=0}^{N_1-1} X(N_2n_1 + n_2) W_{N_1}^{n_1k_1} W_N^{n_2k_1} \right] W_{N_2}^{n_2k_2}$$

Equivalent to the well-known **Cooley-Tukey FFT**
that was published in **1965!**

Rediscovery of FFT



Principal Discoveries of Efficient Methods of Computing the DFT

Researcher(s)	Date	Sequence Lengths	Number of DFT Values	Application
<u>C. F. Gauss [10]</u>	1805	<u>Any composite integer</u>	All	Interpolation of orbits of celestial bodies
F. Carlini [28]	1828	12	—	Harmonic analysis of barometric pressure
A. Smith [25]	1846	4, 8, 16, 32	5 or 9	Correcting deviations in compasses on ships
J. D. Everett [23]	1860	12	5	Modeling underground temperature deviations
C. Runge [7]	1903	2^nk	All	Harmonic analysis of functions
K. Stumpff [16]	1939	$2^nk, 3^nk$	All	Harmonic analysis of functions
Danielson and Lanczos [5]	1942	2^n	All	X-ray diffraction in crystals
L.H. Thomas [13]	1948	Any integer with relatively prime factors	All	Harmonic analysis of functions
I. J. Good [3]	1958	Any integer with relatively prime factors	All	Harmonic analysis of functions
<u>Cooley and Tukey [1]</u>	1965	<u>Any composite integer</u>	All	Harmonic analysis of functions
S. Winograd [14]	1976	Any integer with relatively prime factors	All	Use of complexity theory for harmonic analysis



Lesson

**Important algorithms and theorems
are timeless.**

**Effectiveness is more important than Rigor
for Engineering.**