



# Image Deblurring

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# Problem definition

Blurred image



Latent image



Blur kernel

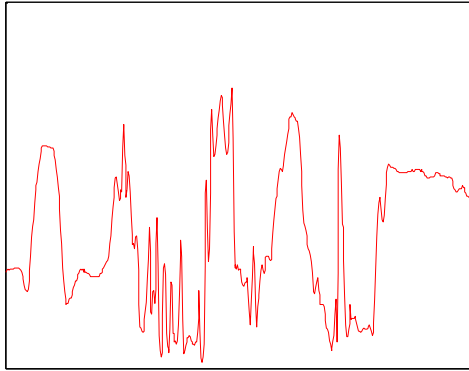


$$B = I \otimes K + n$$

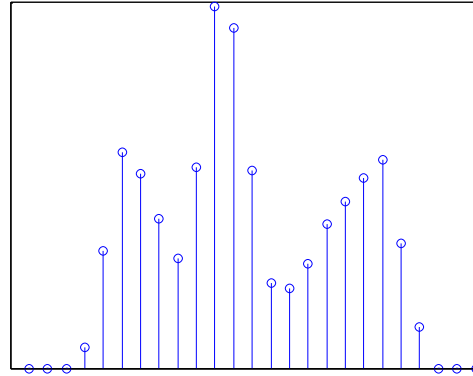
**Deblurring:** Inverse problem to solve latent  $I$  from given observed noisy  $B$  with (*non-blind*) or without (*blind*) the kernel  $K$

# Simple example

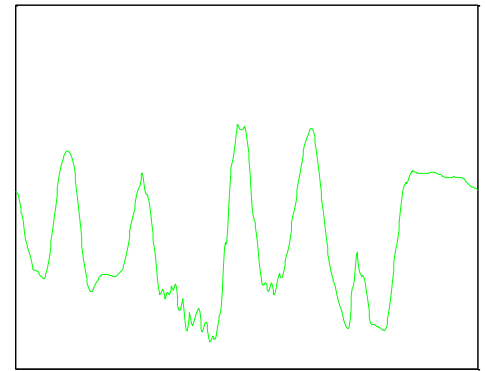
Spatial domain



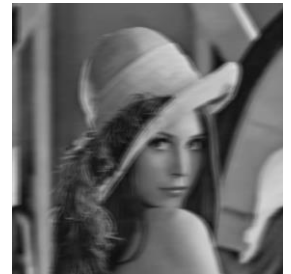
*I*



*K*

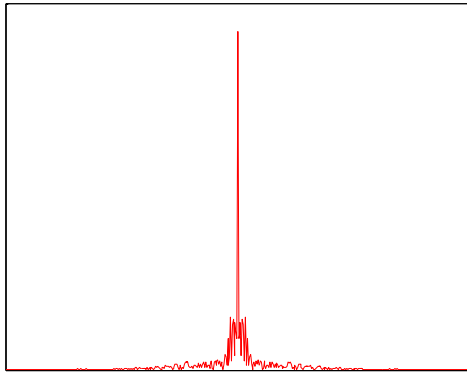


*B*

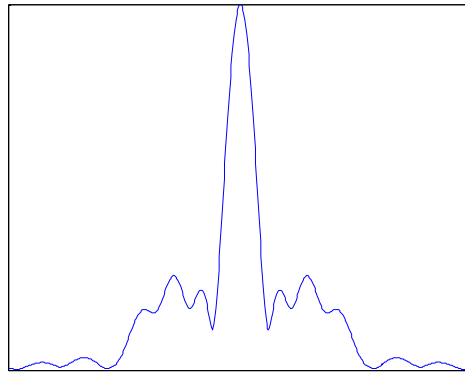


# Simple example

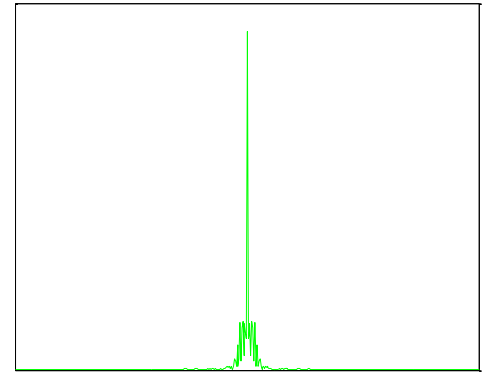
DFT domain (magnitude)



$\times$



$=$

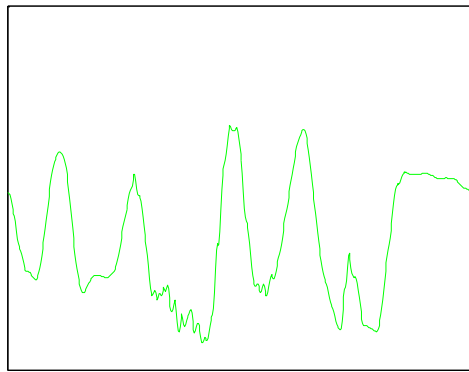
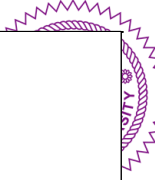


*I*

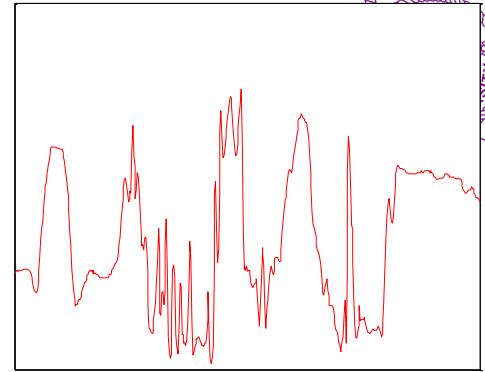
*K*

*B*

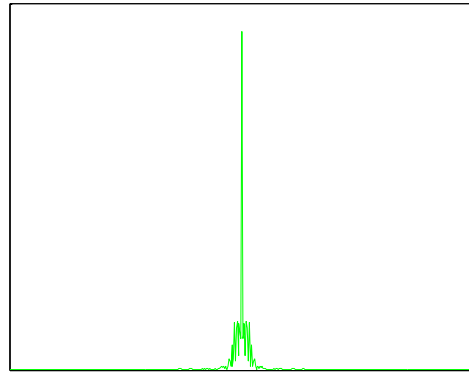




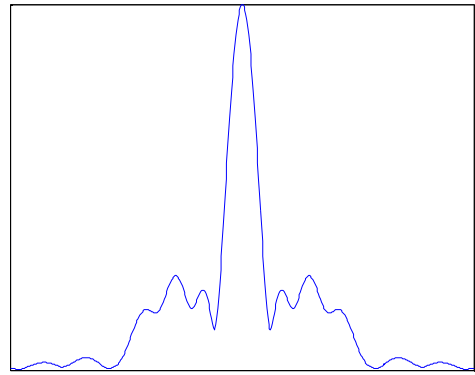
# Simple solution?



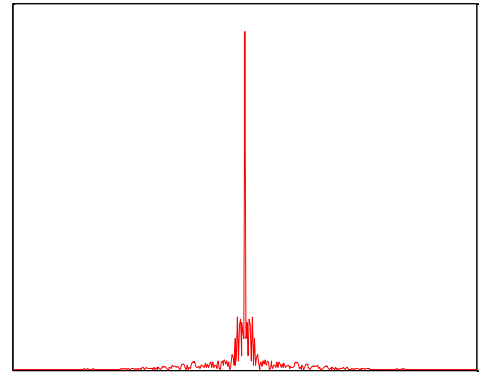
DFT domain (magnitude)



÷



=



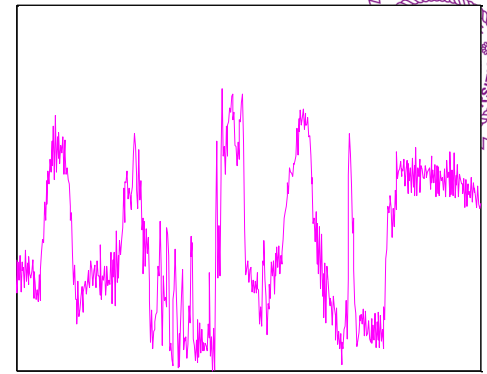
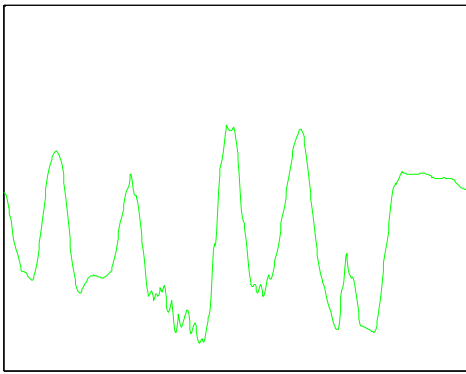
***B***

***K***

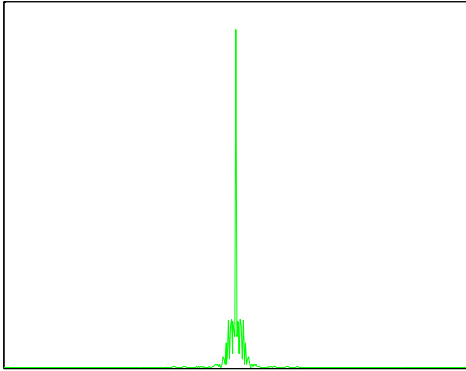
***$\hat{I}$***



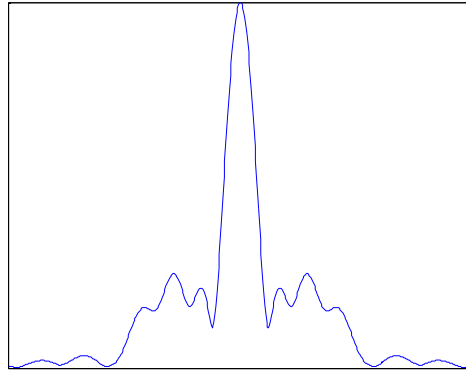
# Instability



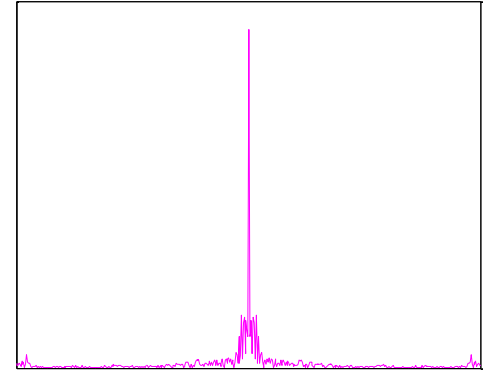
DFT domain (magnitude)



$\div$



$=$



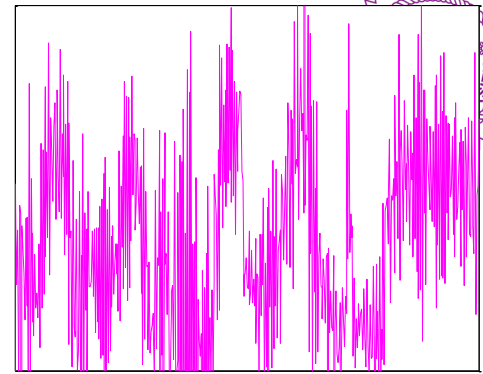
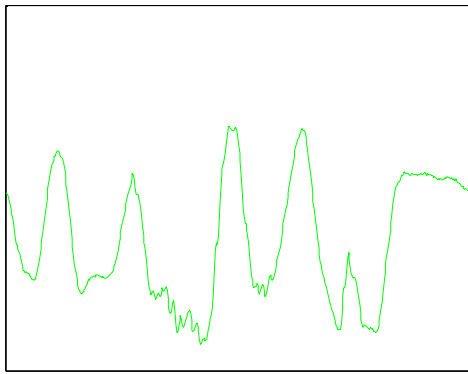
$B$  (+AWGN  $\sigma = 0.1$ )

$K$

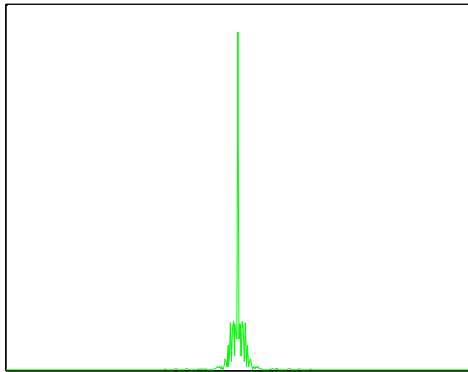
$\hat{I}$



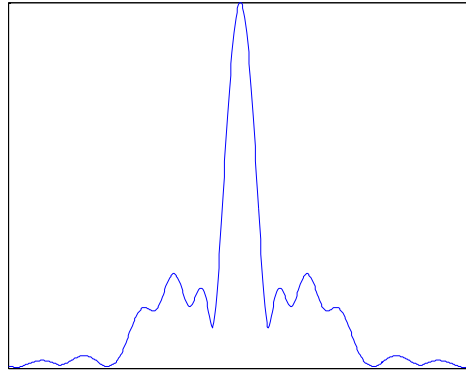
# Instability



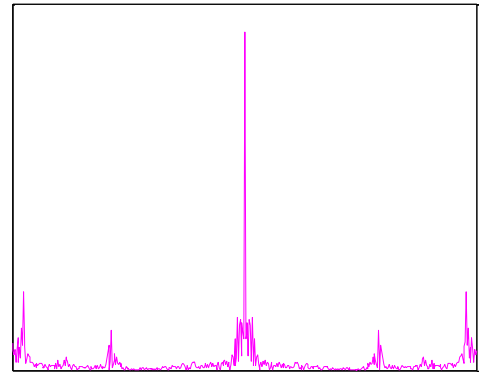
DFT domain (magnitude)



$\div$



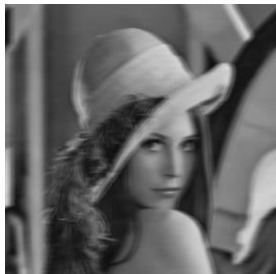
$=$



$B$  (+AWGN  $\sigma = 0.5$ )

$K$

$\hat{I}$



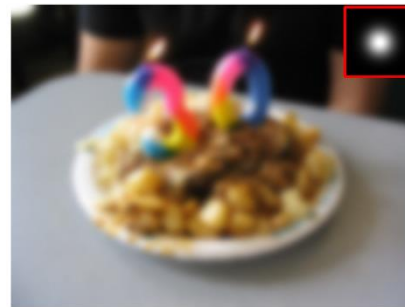
Similar inputs result in different outputs!





# Difficulty of deblurring (1/3)

- For non-blind deconvolution (given kernel  $\mathbf{K}$ )
  - $\mathbf{K}$  may have zeros in magnitude response
    - Some information of  $I$  is gone and can't be recovered
    - $FFT\{\mathbf{B}\}/FFT\{\mathbf{K}\}$  will amplify noise at frequencies with zeros
  - Deblurring quality is kernel-dependent
    - Good quality for sharp kernel (e.g. **camera shake**)
    - Blurred result for lowpass kernel (e.g. defocus blur)





## Difficulty of deblurring (2/3)

- For blind deconvolution (find the kernel  $\mathbf{K}$ )
  - Multiple solutions of  $\mathbf{K}$  are possible
    - Addressed by prior knowledge ( $\mathbf{K}$  is sharp)
  - Convolution doesn't hold at saturation regions
  - no information for  $\mathbf{K}$  in smooth texture regions



# Difficulty of deblurring (3/3)

- Algorithm limit
  - To sum up, deblurring tries to restore the image as sharp as possible, not to recover it exactly
  - Shift-invariant camera shake is the major target here
    - Other blur kernels are more difficult: shift-variant kernel, motion blur, ...

# Wiener deconvolution

- Wiener filter for each frequency component

$$\hat{I} = B \cdot \frac{1}{K} \quad \Rightarrow \quad \hat{I}(f) = B(f) \cdot \frac{1}{K(f)} \left[ \frac{|K(f)|^2}{|K(f)|^2 + \frac{1}{\text{SNR}(f)}} \right]$$

instable

*I*

*B*

*I-hat*





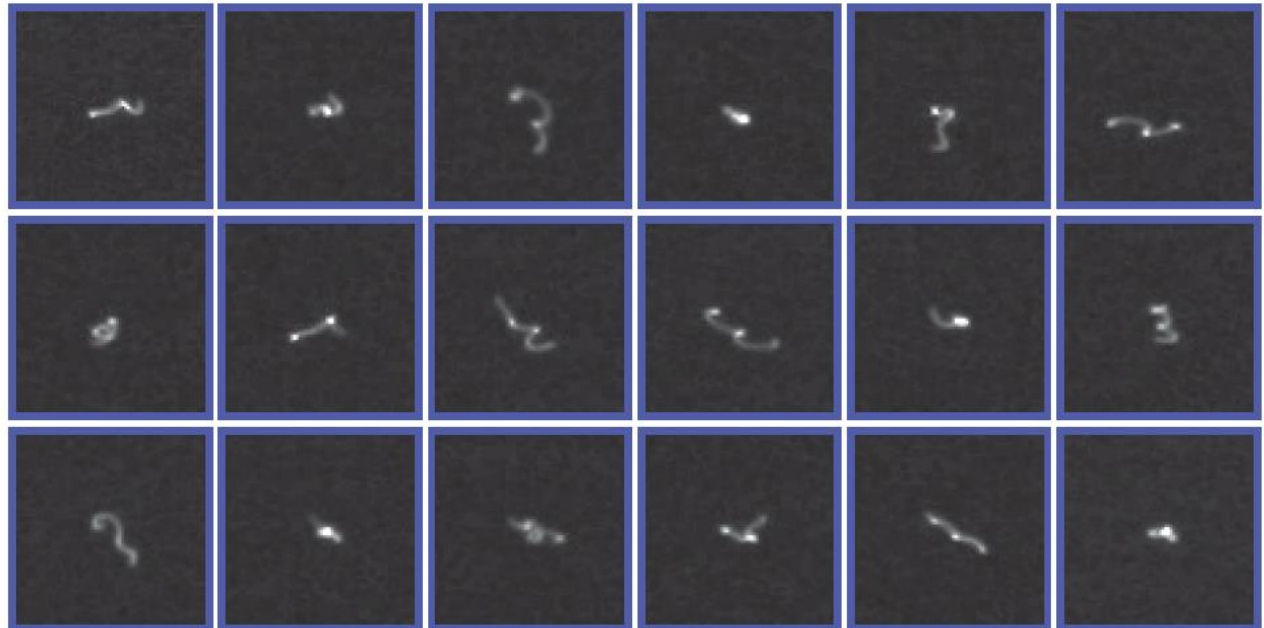
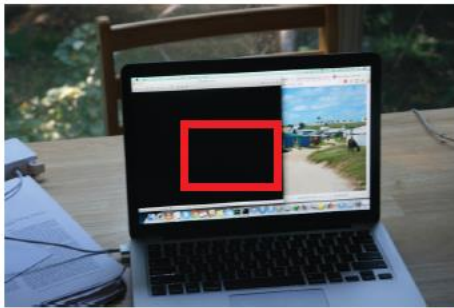
# Burst deblurring

- Accumulation of burst images in Fourier domain
  - **Benefit:** No inverse problem solving
    - e.g. ~~kernel estimation, deconvolution~~
  - **Requirement:** Burst image capturing
  - **Assumption:** Kernels in each image are *random*, and each kernel *disperses* the spectrum and preserves energy

Utilize the random behavior of burst kernels as a deterministic clue to aggregate images.

# Random nature of hand tremor

- Low correlation between successive frames for low shutter speed
  - Tremor source: arm (<5Hz), wrist (5-20Hz) and fingers (20-30Hz)

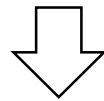


Hand tremor in successive shoots



# Blurring kernel doesn't amplify spectrum

$$k(\mathbf{x}) \geq 0 \text{ and } \int k(\mathbf{x}) = 1$$



$$|\hat{k}(\zeta)| = \left| \int k(\mathbf{x}) e^{i\mathbf{x} \cdot \zeta} d\mathbf{x} \right| \leq \int |k(\mathbf{x})| d\mathbf{x} = \int k(\mathbf{x}) d\mathbf{x} = 1$$

- The attenuation of each spectrum coefficient is random
  - Pick the largest amplitudes cross blurred images to estimate the latent one
- ⇒ Fourier burst accumulation!**



# Fourier burst accumulation (FBA)

- Perform robust weighted average for each frequency component  $\zeta$  in Fourier domain

$$u_p(\mathbf{x}) = \mathcal{F}^{-1} \left( \sum_{i=1}^M w_i(\zeta) \cdot \hat{v}_i(\zeta) \right) (\mathbf{x})$$

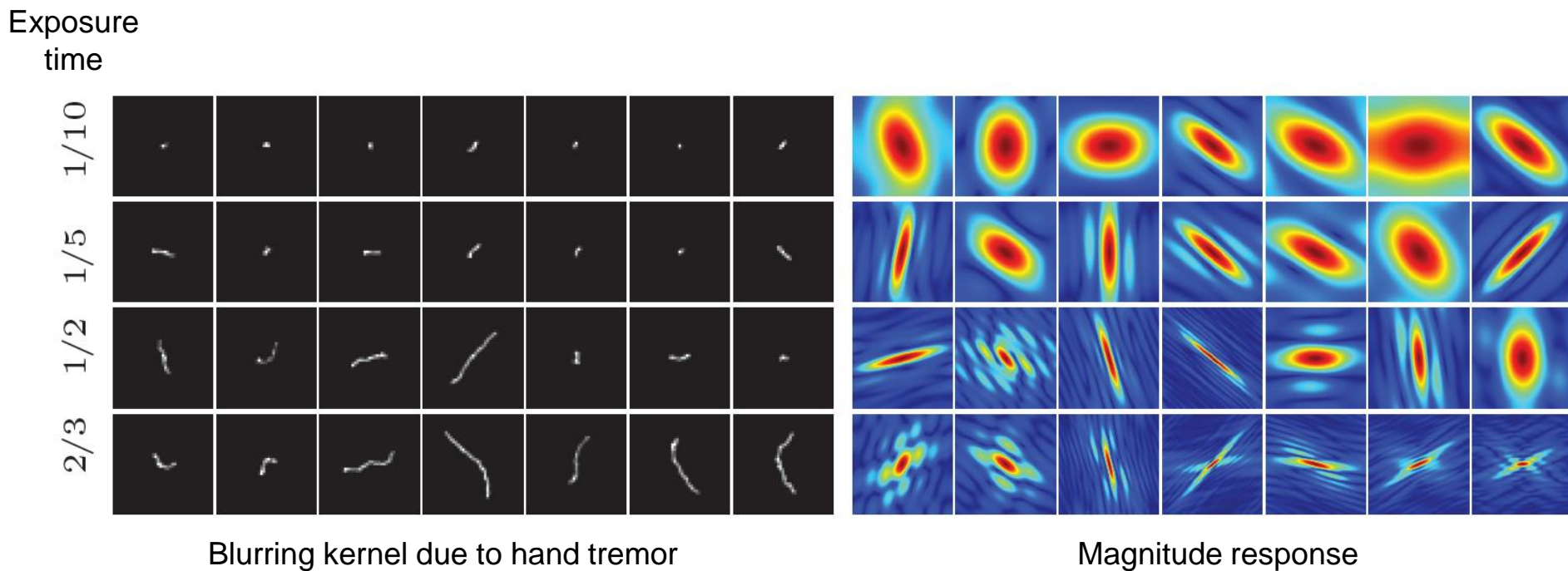
$$w_i(\zeta) = \frac{|\hat{v}_i(\zeta)|^p}{\sum_{j=1}^M |\hat{v}_j(\zeta)|^p},$$

Control the accumulation behavior:  
 $p = 0$ : simple average;  
 $p \rightarrow \infty$ : maximum pooling.



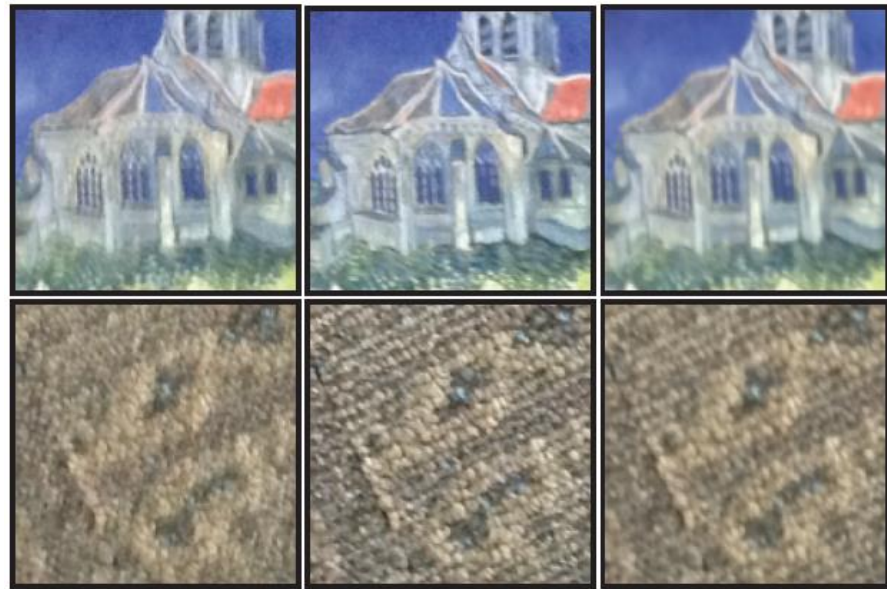
# Behavior of $\hat{k}(\zeta)$

- If hand tremor is random enough, Fourier burst accumulation can cover most of frequency components (red in the below)



# Experimental results

- 8~13 burst images
- 6~8M pixels



Typical Shot

Best Shot

Align and average



Šroubek & Milanfar [25]

Zhang *et al.* [30]

proposed method

several hours

few seconds for FBA



Typical Shot

Best Shot

Align and average

Šroubek & Milanfar [25]

Zhang *et al.* [30]

proposed method (no final sharp.)

proposed method