Chap2
Discrete-time signals and systems

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Signals

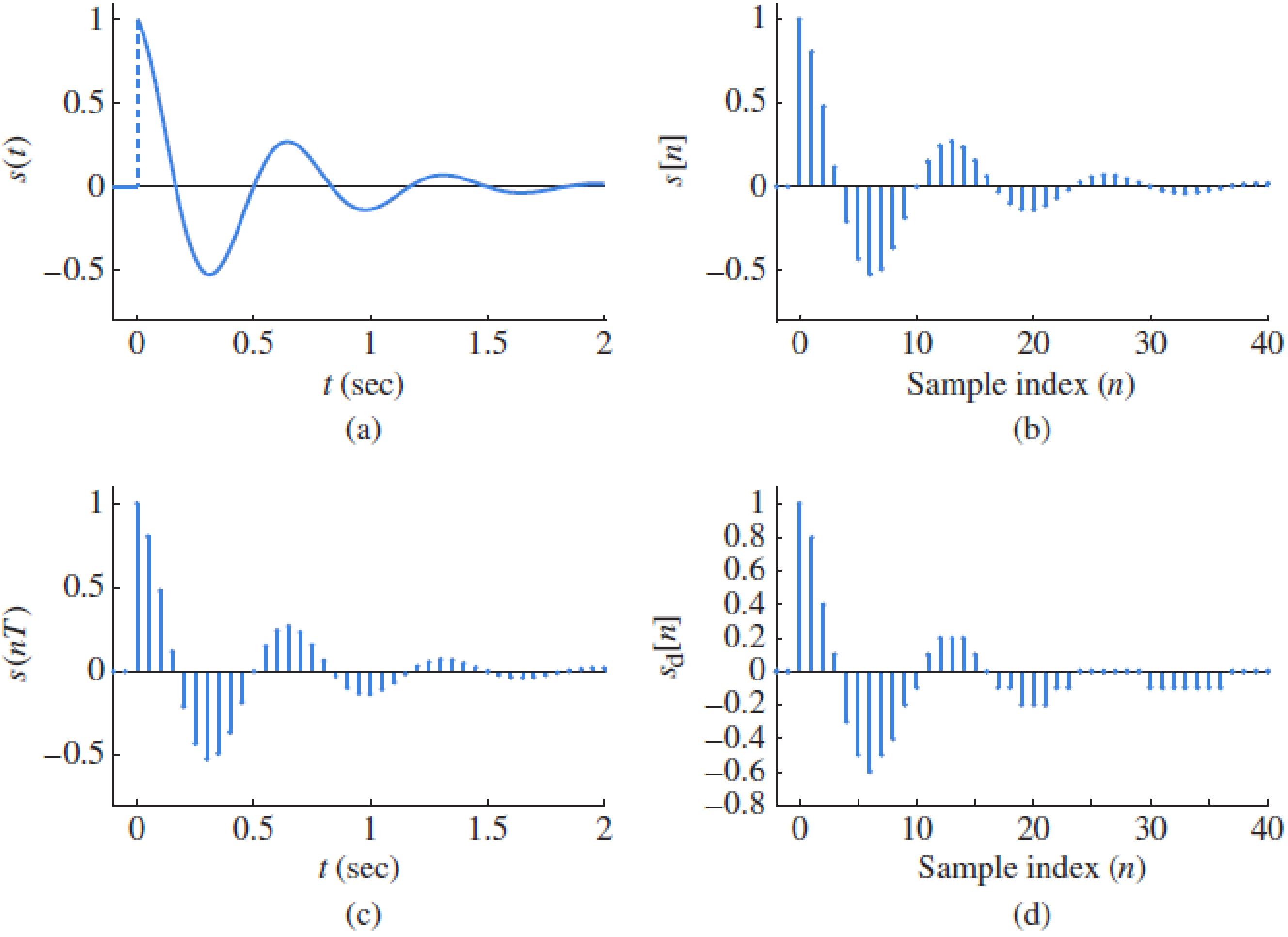


Figure 1.5 Plots illustrating the graphical representation of continuous-time signals (a), discrete-time signals (b) and (c), and digital signals (d).

Analog-to-digital conversion

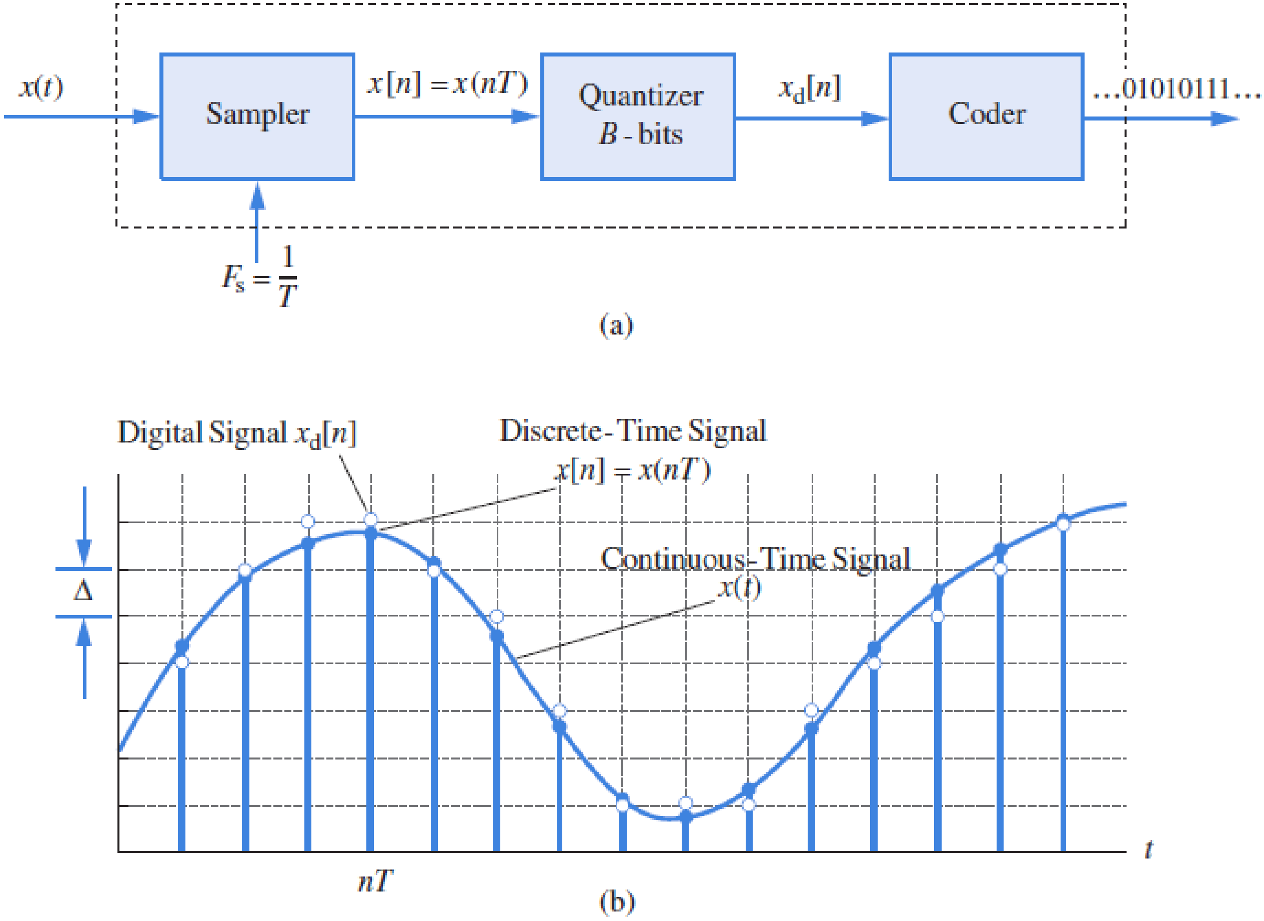


Figure 1.8 (a) Block diagram representation of the analog-to-digital conversion process. (b) Examples of the signals $x(t)$, $x[n]$, and $x_d[n]$ involved in the process. The amplitude of $x[n]$ is known with infinite precision, whereas the amplitude of $x_d[n]$ is known with finite precision Δ (quantization step or resolution).

Systems

DSP enables

- Complex algorithm
- Reliable computing

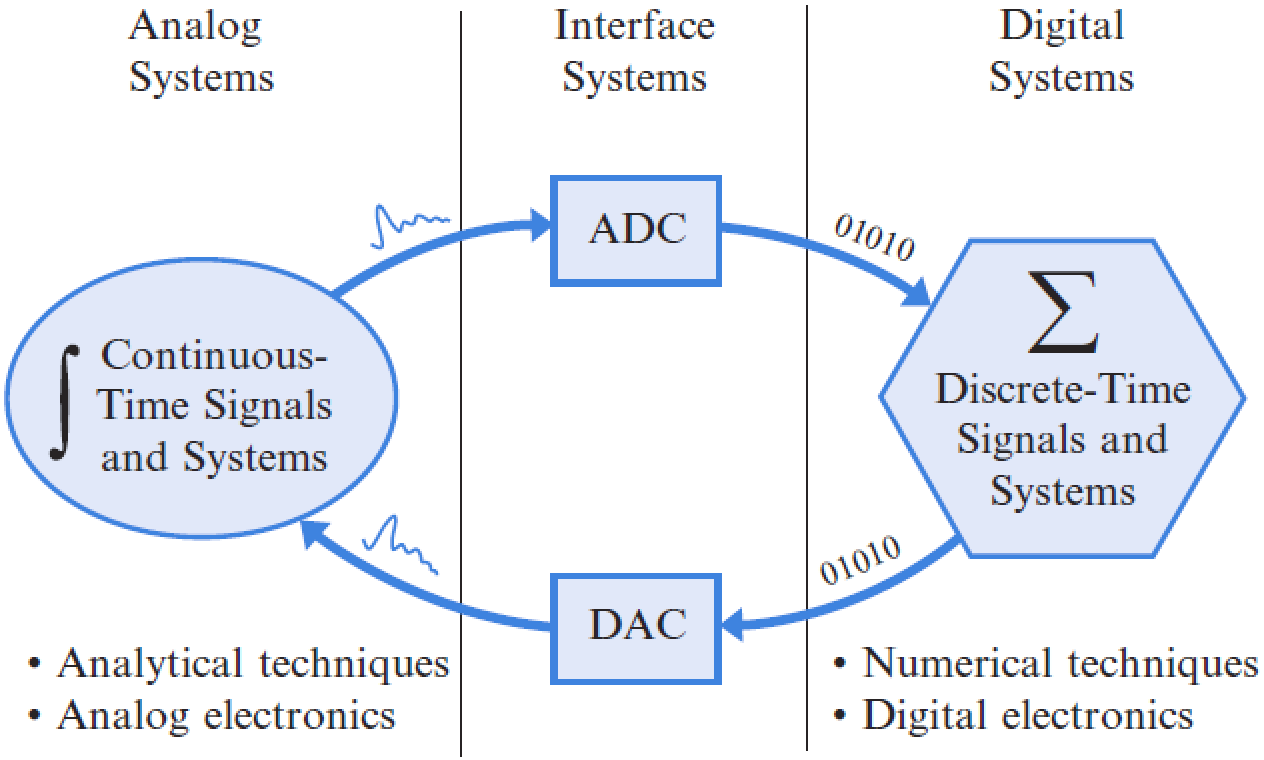


Figure 1.11 The three classes of system: analog systems, digital systems, and interface systems from analog-to-digital and digital-to-analog.

Application example

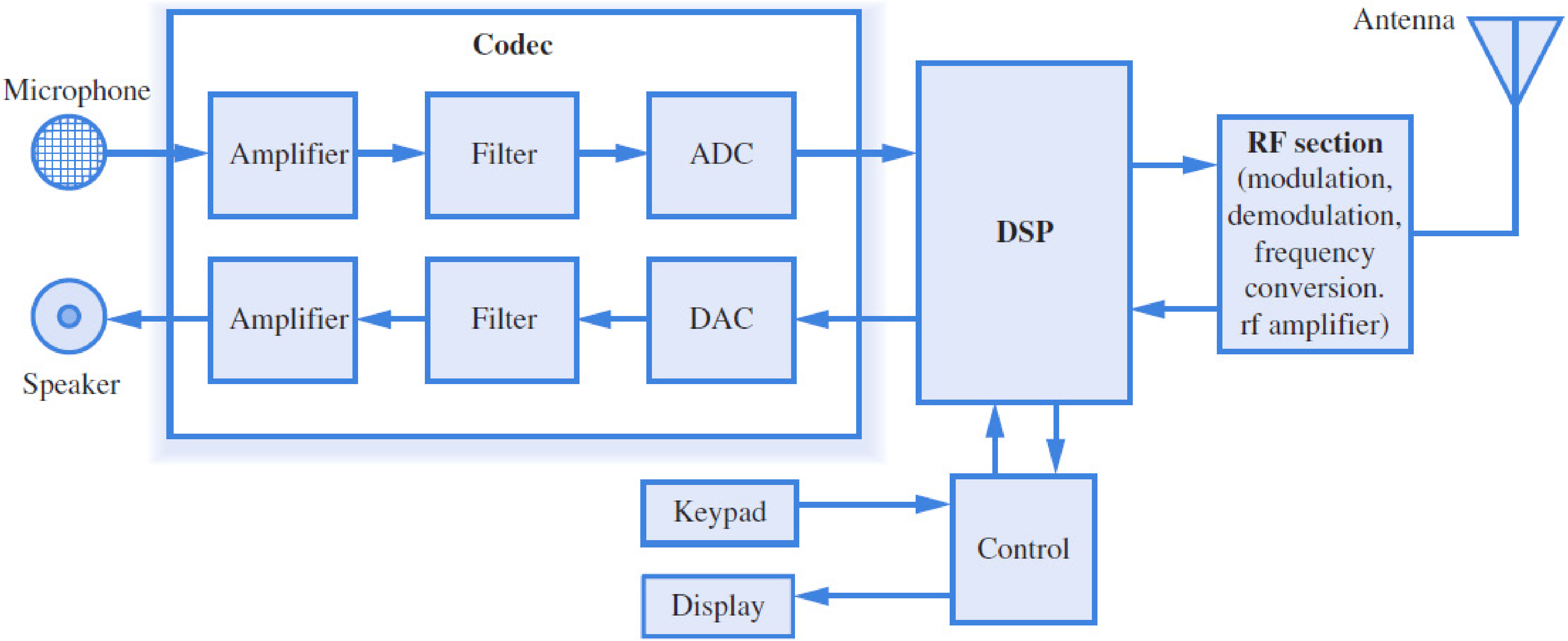


Figure 1.14 Simplified block diagram of a digital cellular phone.

Chap 2 Discrete-time signals and systems

- 2.1 Discrete-time signals
- 2.3 Discrete-time systems
- 2.4 Convolution of LTI systems
- 2.5 Properties of LTI systems
- 2.9 FIR spatial filters
- 2.11 Continuous-time LTI systems

Discrete-time sampling

Sampling:
$x[n] = x(nT)$

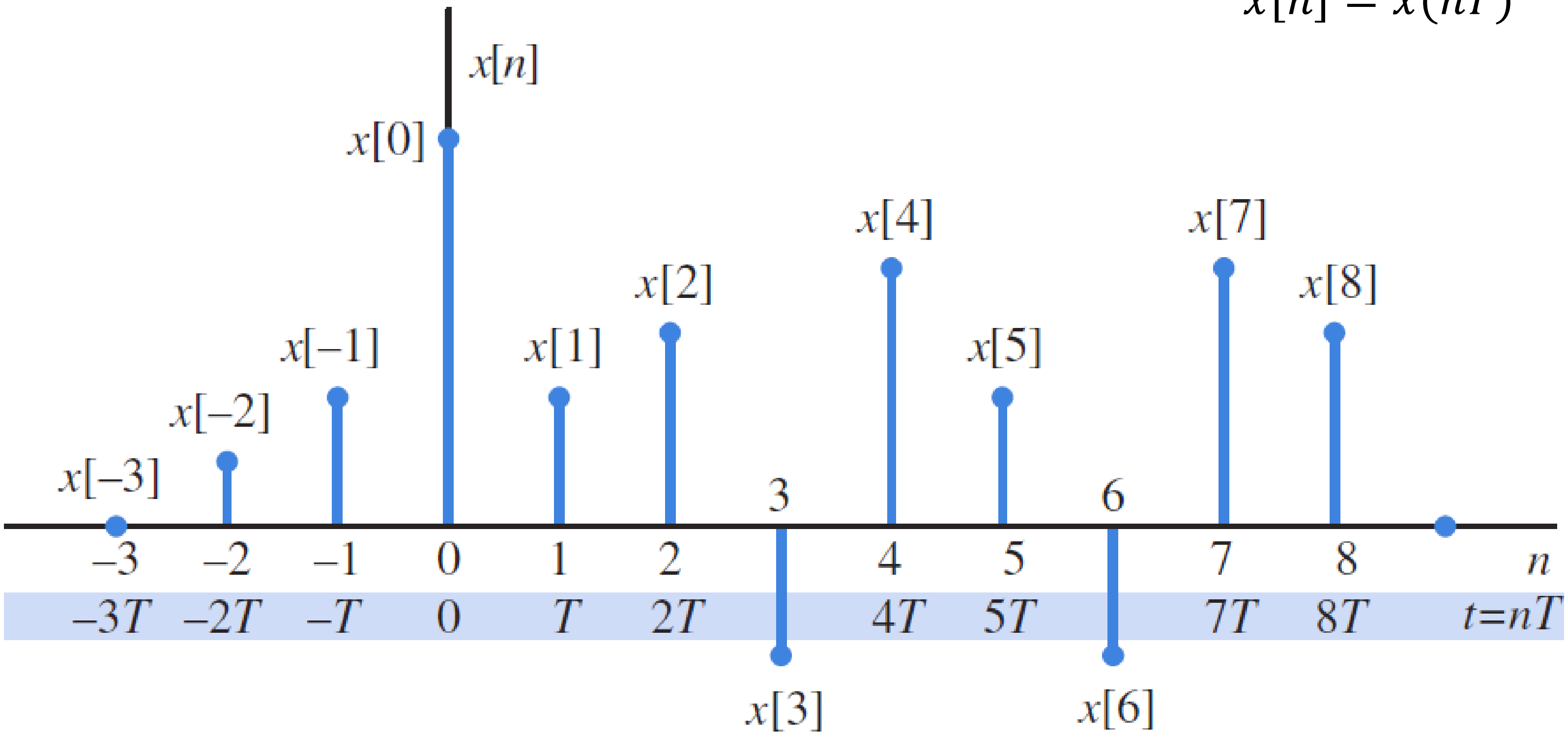
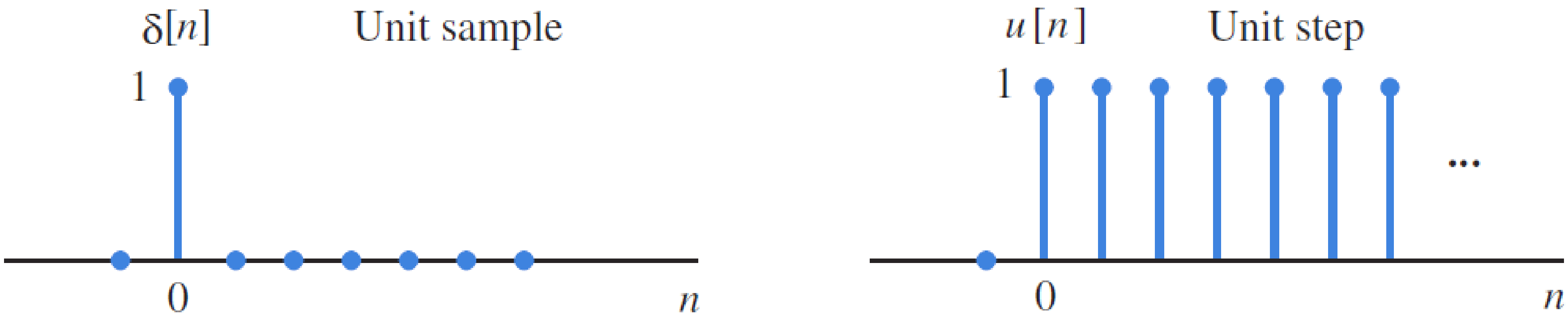


Figure 2.1 Representation of a sampled signal.

Elementary discrete-time signals

$$\delta[n] \triangleq \begin{cases} 1, & n = 0 \\ 0. & n \neq 0 \end{cases}$$

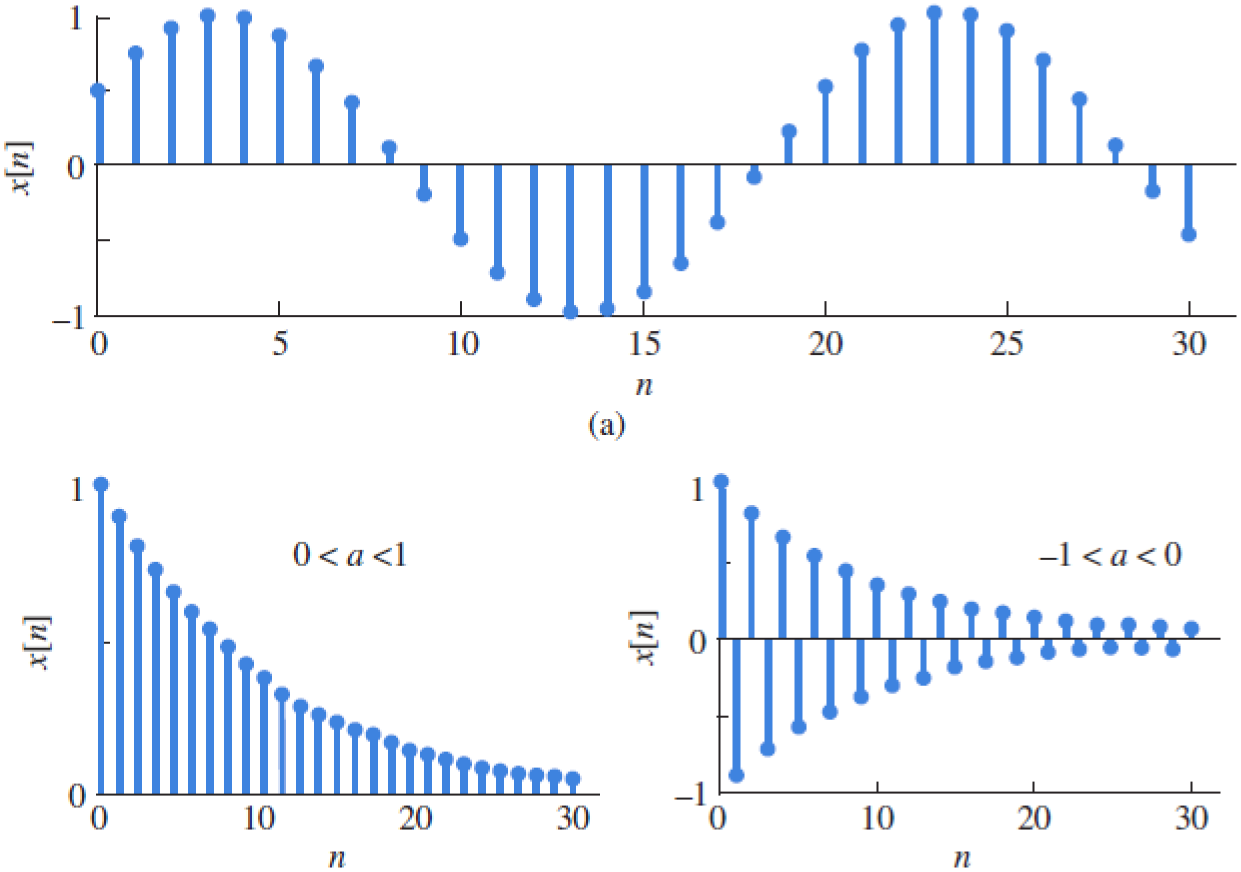
$$u[n] \triangleq \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$



(a) (b)

Figure 2.2 Some elementary discrete-time signals.

Other examples



(a)

(b)

$$x[n] = A\cos(\omega_0 n + \phi)$$

$$x[n] \triangleq Aa^n$$

Figure 2.3 Examples of a discrete-time sinusoidal signal (a), and two real exponential sequences (b).

Discrete-time systems

Operator

$$x[n] \overset{\mathcal{H}}{\longmapsto} y[n] \quad \text{or} \quad y[n] = \mathcal{H}\{x[n]\}$$

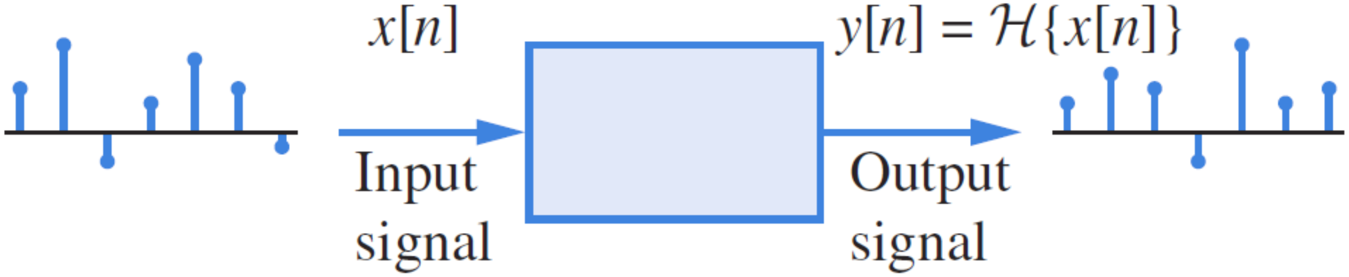


Figure 2.5 Block diagram representation of a discrete-time system.

Causality and stability

Definition 2.1 A system is called *causal* if the present value of the output does not depend on future values of the input, that is, $y[n_0]$ is determined by the values of $x[n]$ for $n \le n_0$, only.

Definition 2.2 A system is said to be *stable*, in the Bounded-Input Bounded-Output (BIBO) sense, if every bounded input signal results in a bounded output signal, that is

$$|x[n]| \le M_x < \infty \Rightarrow |y[n]| \le M_y < \infty. \tag{2.18}$$

A signal $x[n]$ is bounded if there exists a positive finite constant M_x such that $|x[n]| \le M_x$ for all n.

Linearity

Definition 2.3 A system is called *linear* if and only if for every real or complex constant a_1, a_2 and every input signal $x_1[n]$ and $x_2[n]$

$$\mathcal{H}\{a_1x_1[n] + a_2x_2[n]\} = a_1\mathcal{H}\{x_1[n]\} + a_2\mathcal{H}\{x_2[n]\}, \tag{2.19}$$

for all values of n.

Example: Is a square-law system linear or nonlinear?

$$y[n] = x^2[n]$$

Time invariance

Definition 2.4 A system is called *time-invariant* or *fixed* if and only if

$$y[n] = \mathcal{H}\{x[n]\} \Rightarrow y[n - n_0] = \mathcal{H}\{x[n - n_0]\}, \tag{2.22}$$

for every input $x[n]$ and every time shift n_0. That is, a time shift in the input results in a corresponding time shift in the output.

Example: Is a downsampler linear? Time-invariant?

$$y[n] = \mathcal{H}\{x[n]\} = x[nM]$$

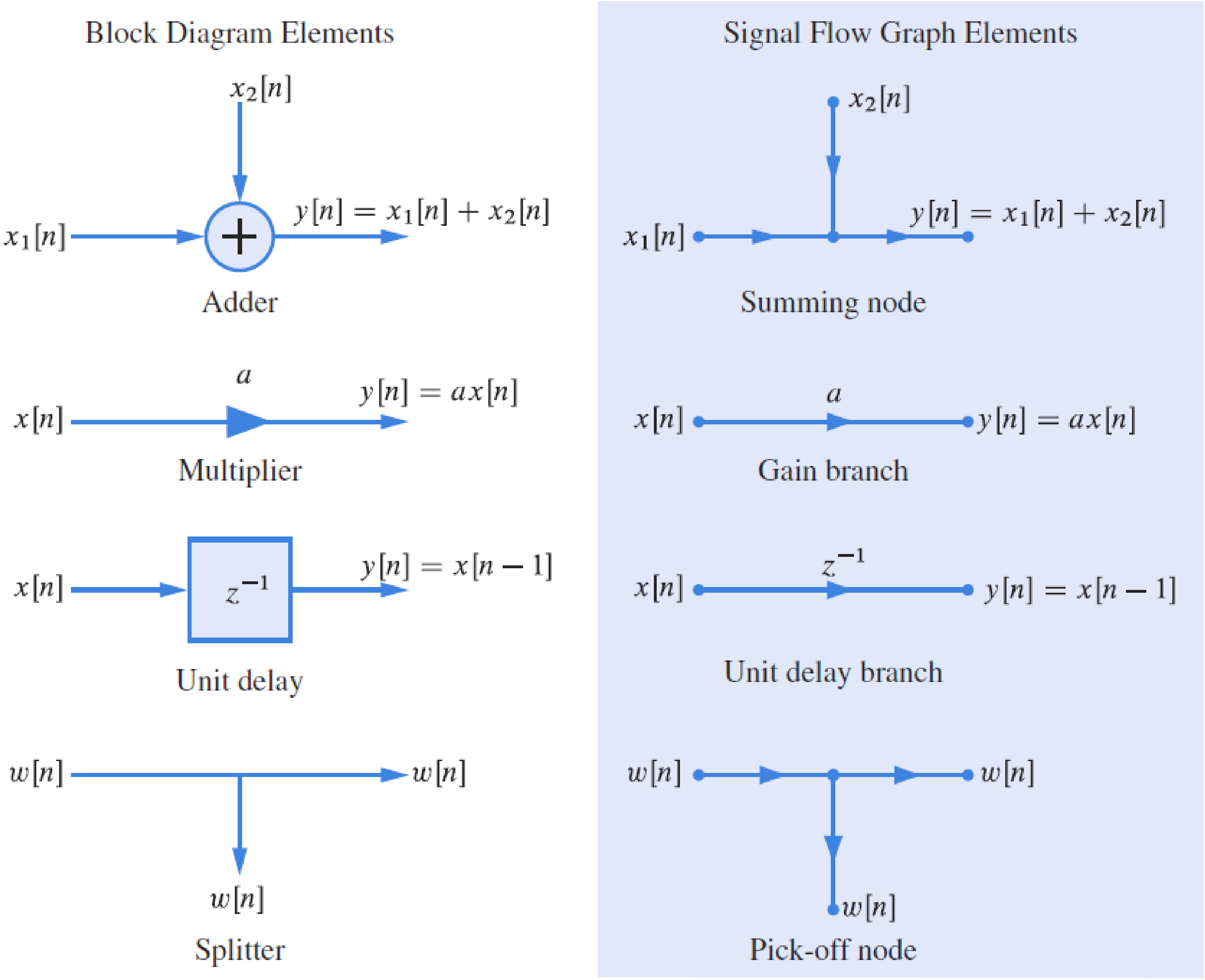


Figure 2.6 Basic building blocks and the corresponding signal flow graph elements for the implementation of discrete-time systems.

SFG example

$$w[n] = x[n] + aw[n-1], \qquad \text{(input node)}$$

$$y[n] = w[n] + bw[n-1]. \qquad \text{(output node)}$$

$$y[n] = x[n] + bx[n-1] + ay[n-1]$$

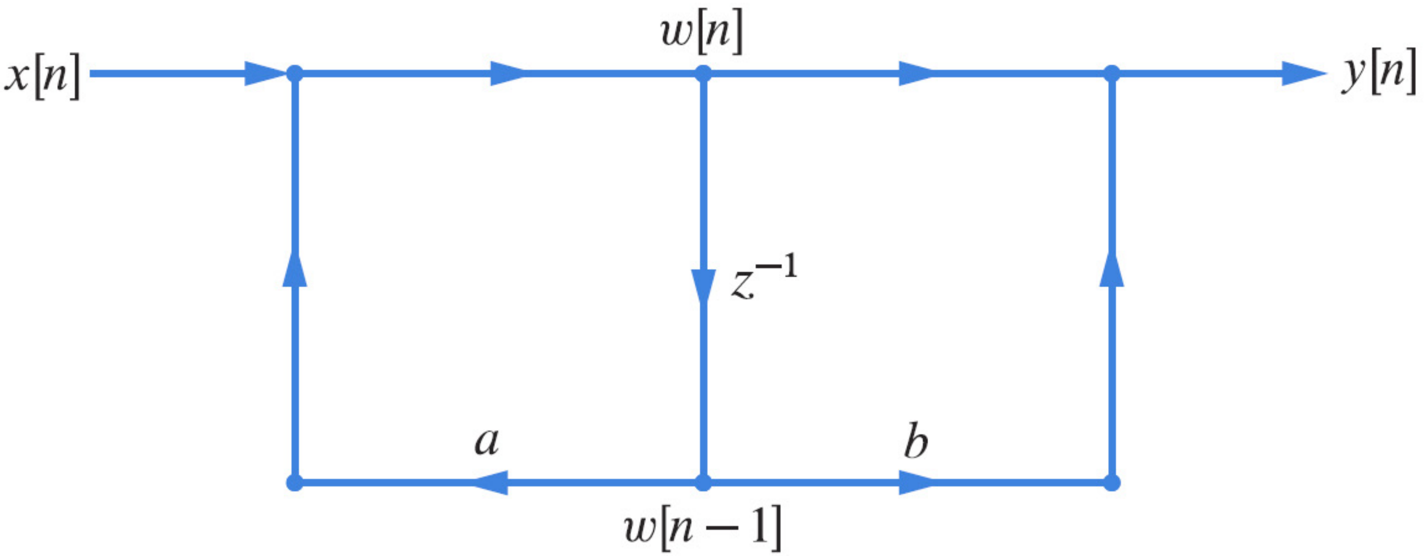


Figure 2.8 Signal flow graph of a first-order discrete-time system.

Convolution of LTI systems

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k] \xmapsto{\mathcal{H}} y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

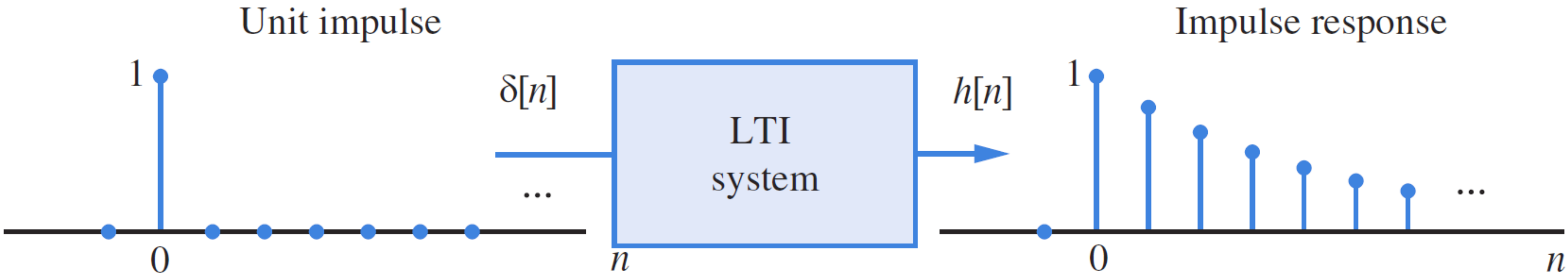


Figure 2.9 The impulse response of a linear time-invariant system.

An LTI system is represented by its impulse response.

Finite impulse response (FIR) system

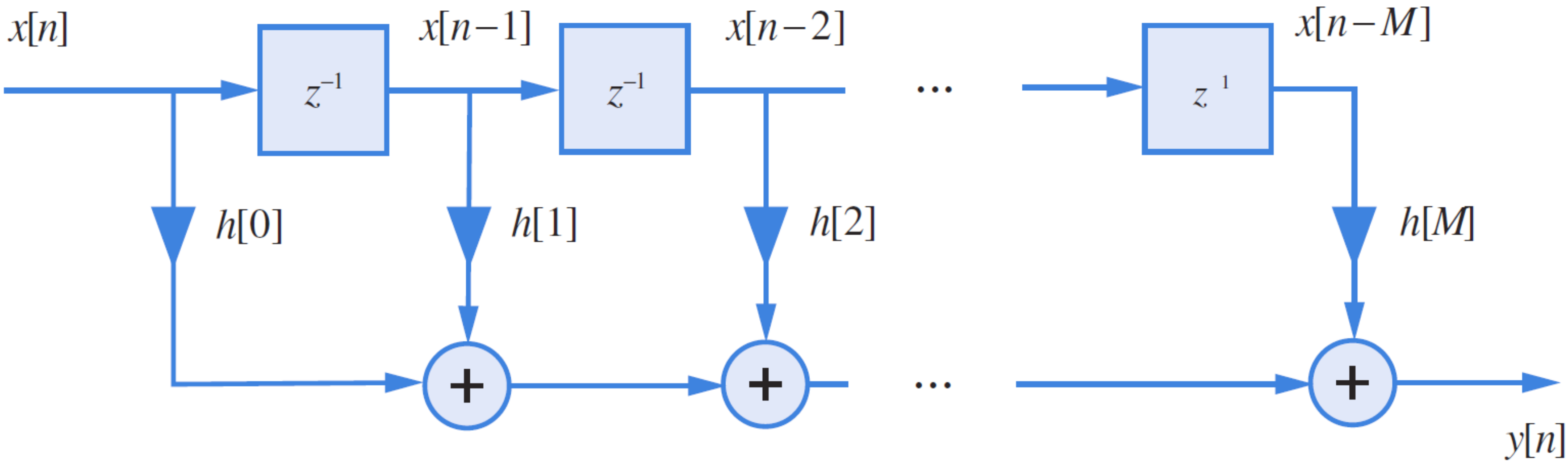


Figure 2.14 Block diagram representation of an FIR system.

Properties of convolution

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{m=-\infty}^{\infty} h[m]x[n-m] = h[n] * x[n]$$

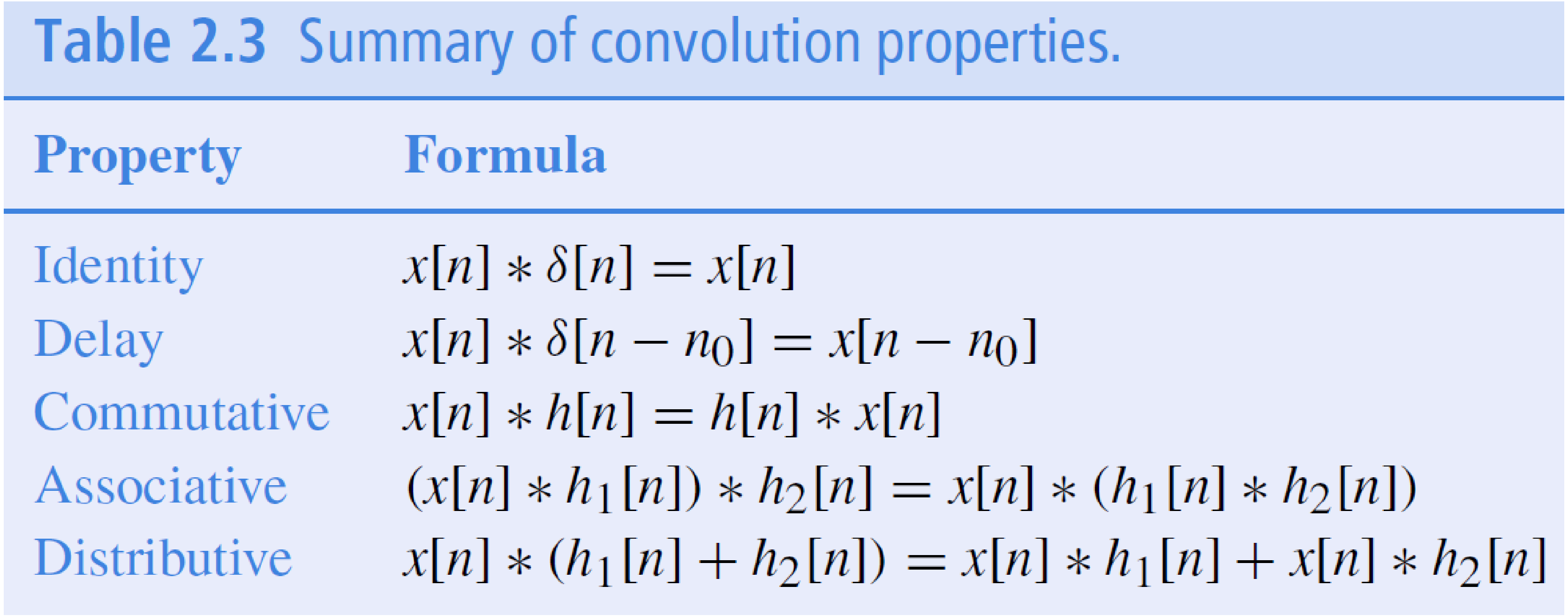


Table 2.3 Summary of convolution properties.

Property	**Formula**
Identity	$x[n] * \delta[n] = x[n]$
Delay	$x[n] * \delta[n-n_0] = x[n-n_0]$
Commutative	$x[n] * h[n] = h[n] * x[n]$
Associative	$(x[n] * h_1[n]) * h_2[n] = x[n] * (h_1[n] * h_2[n])$
Distributive	$x[n] * (h_1[n] + h_2[n]) = x[n] * h_1[n] + x[n] * h_2[n]$

Causality and stability

Result 2.5.1 A linear time-invariant system with impulse response $h[n]$ is causal if

$$h[n] = 0 \quad \text{for} \quad n < 0. \tag{2.50}$$

Result 2.5.2 A linear time-invariant system with impulse response $h[n]$ is stable, in the bounded-input bounded-output sense, if and only if the impulse response is absolutely summable, that is, if

$$\sum_{n=-\infty}^{\infty} |h[n]| < \infty. \tag{2.52}$$

Examples of LTI response

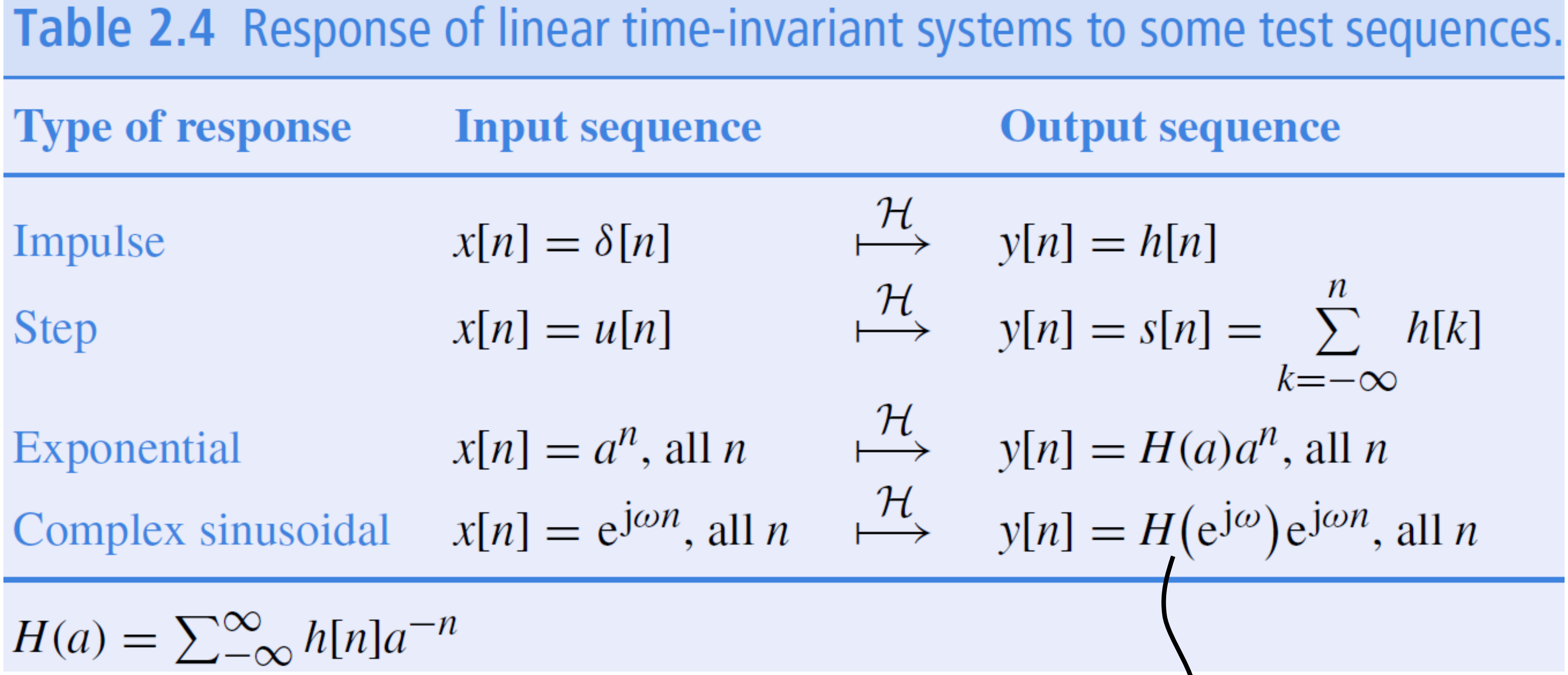


Table 2.4 Response of linear time-invariant systems to some test sequences.

Type of response	**Input sequence**		**Output sequence**
Impulse	$x[n] = \delta[n]$	$\xrightarrow{\mathcal{H}}$	$y[n] = h[n]$
Step	$x[n] = u[n]$	$\xrightarrow{\mathcal{H}}$	$y[n] = s[n] = \sum_{k=-\infty}^{n} h[k]$
Exponential	$x[n] = a^n$, all n	$\xrightarrow{\mathcal{H}}$	$y[n] = H(a)a^n$, all n
Complex sinusoidal	$x[n] = e^{j\omega n}$, all n	$\xrightarrow{\mathcal{H}}$	$y[n] = H\left(e^{j\omega}\right)e^{j\omega n}$, all n

$H(a) = \sum_{-\infty}^{\infty} h[n]a^{-n}$

Frequency response

FIR spatial (2D) filter

$$y[m,n] = \sum_{k=m-K}^{m+K} \sum_{\ell=n-L}^{n+L} x[k,\ell]h[m-k,n-\ell]$$

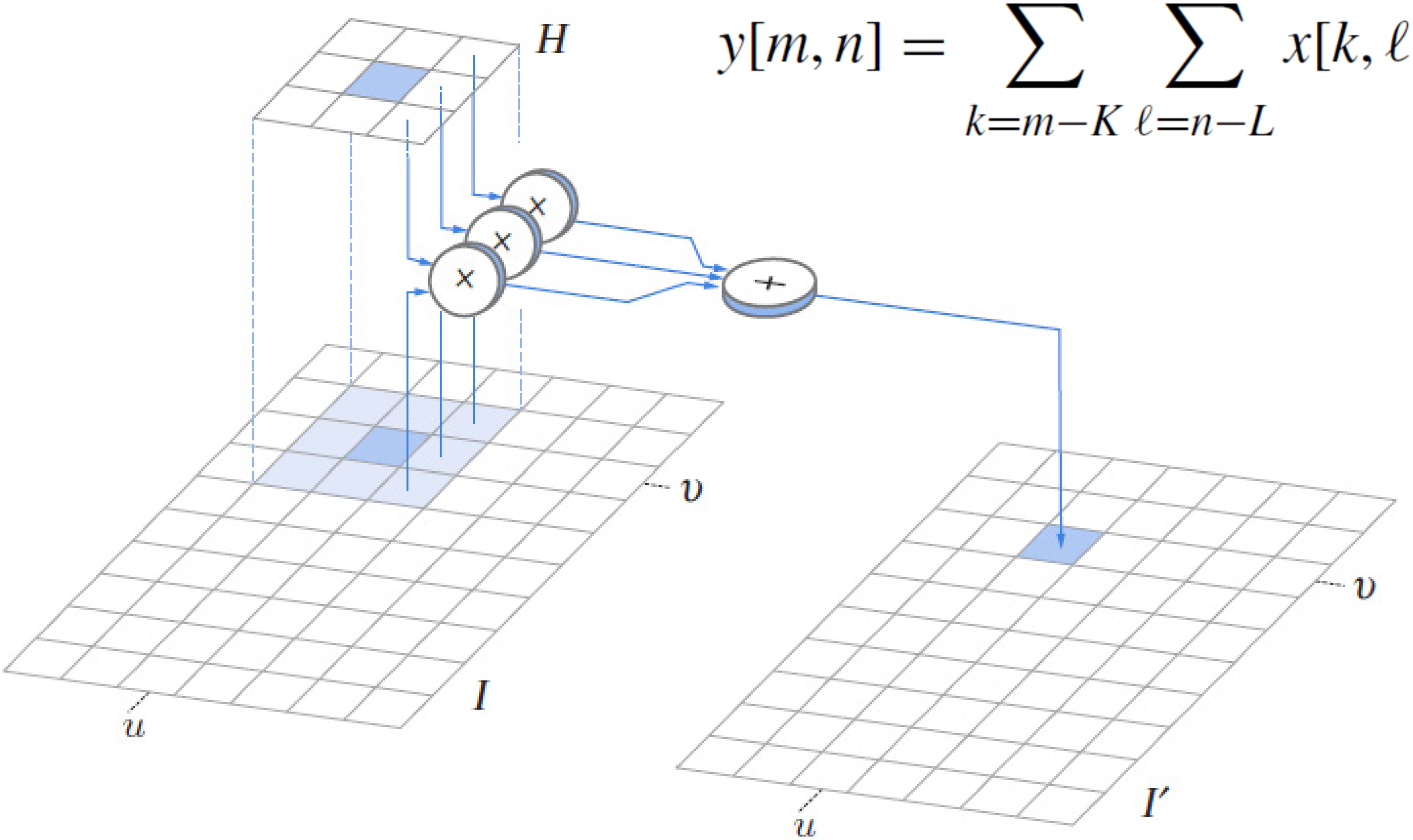


Figure 2.24 FIR spatial filter implementation.

Example of 3x3 average filter

$$y[m,n] = \sum_{k=-1}^{1} \sum_{\ell=-1}^{1} \left(\frac{1}{9}\right) x[m-k, n-\ell]$$

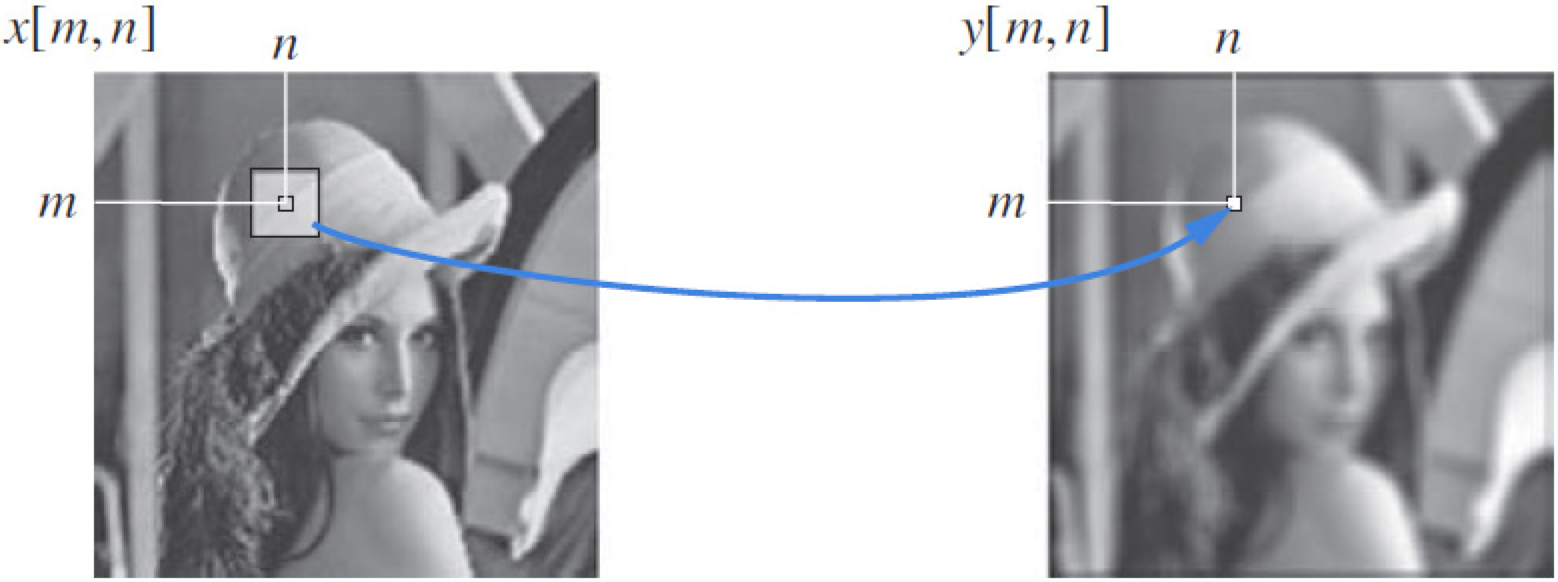


Figure 2.23 The FIR spatial filtering operation.

Continuous-time LTI systems

$$\delta(t) \xrightarrow{\mathcal{H}} h(t) \qquad \text{(Impulse Response)}$$

$$\delta(t-\tau) \xrightarrow{\mathcal{H}} h(t-\tau) \qquad \text{(Time} - \text{Invariance)}$$

$$x(\tau)\delta(t-\tau) \xrightarrow{\mathcal{H}} x(\tau)h(t-\tau) \qquad \text{(Homogeneity)}$$

$$\underbrace{\int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)\mathrm{d}\tau}_{x(t)} \xrightarrow{\mathcal{H}} \underbrace{\int_{-\infty}^{\infty} x(\tau)h(t-\tau)\mathrm{d}\tau}_{y(t)}, \qquad \text{(Additivity)}$$

Continuous-time impulse function

$$\delta_\Delta(t) = \begin{cases} 1/\Delta, & -\Delta/2 < t < \Delta/2 \\ 0, & \text{otherwise} \end{cases}$$

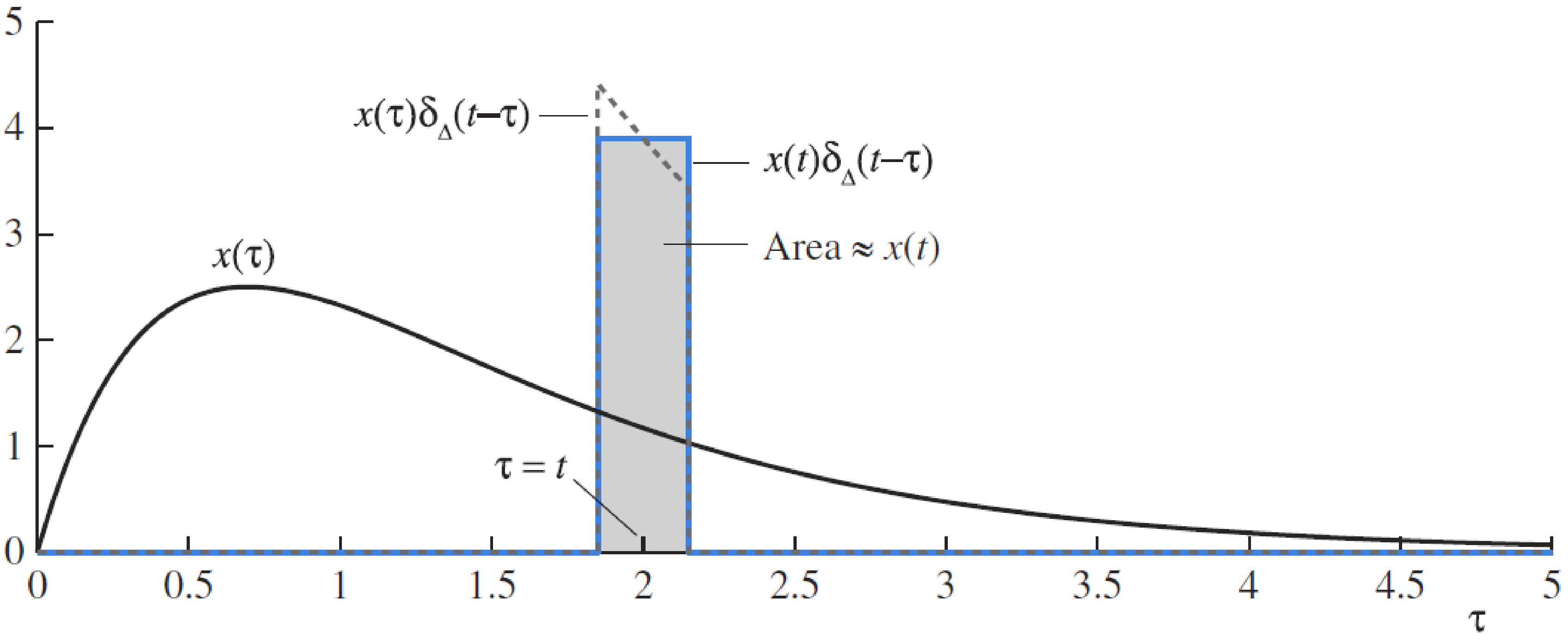


Figure 2.30 Interpretation of convolution by a narrow pulse as a scanning operation.