HW6 Program Assignment

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P1

Consider again the inverse DFT given in (8.2).

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]W_N^{-\text{kn}},\ n = 0, 1, \ldots, N-1 \quad (8.2)$$

(a) Replace k by $\langle -k \rangle_N$ in (8.2) and show that the resulting summation is a DFT expression, that is, $\text{IDFT}\left\{X\left[k\right]\right\} = \frac{1}{N}\text{DFT}\{X[\langle -k \rangle_N]\}$.

Ans.

$$\text{IDFT}\left\{X\left[k\right]\right\} = x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]W_N^{-\text{kn}}$$

$$\text{DFT}\{X[\langle -k \rangle_N]\} = \sum_{k=0}^{N-1} X[\langle -k \rangle_N]W_N^{\text{kn}} = \sum_{k=0}^{N-1} X[k]W_N^{-\text{kn}}$$

$$\Longrightarrow \text{IDFT}\left\{X\left[k\right]\right\} = \frac{1}{N}\sum_{k=0}^{N-1} X[k]W_N^{-\text{kn}} = \frac{1}{N}\text{DFT}\{X[\langle -k \rangle_N]\}$$

(b) Develop a MATLAB function x = IDFT(X,N) using the fft function that uses the above approach. Verify your function on signal x[n] = {1, 2, 3, 4, 5, 6, 7, 8}.

```
close all; clear;
fprintf('1(b)\n');
```

```
1(b)
```

```
open IDFT.m;

x = [1 2 3 4 5 6 7 8]
```

```
x = 1×8
     1     2     3     4     5     6     7     8
```

```
X = fft(x); N = length(X);
x_verify = IDFT(X, N)
```

```
x_verify = 1×8 complex
```

```
1.0000 + 0.0000i   2.0000 + 0.0000i   3.0000 + 0.0000i   4.0000 + 0.0000i ...
```

(Comment)

由上面的結果顯示，從自己設計的 IDFT 把經由 fft 轉換得到的 X 轉回去 time domain 的 x_verify 與原本的 x 相同。

P2

In this problem we will investigate differences in the speeds of DFT and FFT algorithms when stored twiddle factors are used.

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \\ X[4] \\ X[5] \\ X[6] \\ X[7] \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & W_8 & W_8^2 & W_8^3 & W_8^4 & W_8^5 & W_8^6 & W_8^7 \\ 1 & W_8^2 & W_8^4 & W_8^6 & 1 & W_8^2 & W_8^4 & W_8^6 \\ 1 & W_8^3 & W_8^6 & W_8^1 & W_8^4 & W_8^7 & W_8^2 & W_8^5 \\ 1 & W_8^4 & 1 & W_8^4 & 1 & W_8^4 & 1 & W_8^4 \\ 1 & W_8^5 & W_8^2 & W_8^7 & W_8^4 & W_8 & W_8^6 & W_8^3 \\ 1 & W_8^6 & W_8^4 & W_8^2 & 1 & W_8^6 & W_8^4 & W_8^2 \\ 1 & W_8^7 & W_8^6 & W_8^5 & W_8^4 & W_8^3 & W_8^2 & W_8 \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \\ x[4] \\ x[5] \\ x[6] \\ x[7] \end{bmatrix} \quad (8.8)$$

(a) Write a function W = dft_matrix(N) that computes the DFT matrix W_N given in (8.8).

Ans.

$$W_N = e^{-j\frac{2\pi}{N}},\ (W_N)_{(n+1)\times(m+1)} = \left(e^{-j\frac{2\pi}{N}}\right)^{nm},\ n, m = 0, 1, \ldots, N-1$$

```
close all; clear;
fprintf('2(a)\n');
```

```
2(a)
```

```
open dft_matrix.m;
```

(b) Write a function X = dftdirect_m(x,W) that modifies the dftdirect function using the matrix W from (a). Using the tic and toc functions compare computation times for the dftdirect and dftdirect_m function for N = 128, 256, 512, and 1024. For this purpose generate an N-point complex-valued signal as x = randn(1,N) + 1j*randn(1,N). (verify your code with fft first)

```
close all; clear;
fprintf('2(b)\n');
```

```
2(b)
```

```
open dftdirect_m.m;

N_set = [128 256 512 1024];
for i = 1:1:length(N_set)
    fprintf('N = %d\n', N_set(i));
    x = randn(1,N_set(i)) + 1j*randn(1,N_set(i));

    fprintf('  by dftdirect_m\n');
    tic;
    W = dft_matrix(N_set(i));
    X1 = dftdirect_m(x, W)
    toc;

    fprintf('  by dftdirect\n');
    tic;
    X2 = dftdirect(x)
    toc;
end
```

```
N = 128
  by dftdirect_m
X1 = 1×128 complex
  14.8600 -16.9911i   9.9053 -18.4707i  12.4703 - 8.8463i -11.5983 -11.2830i ⋯
Elapsed time is 0.036558 seconds.
  by dftdirect
X2 = 1×128 complex
  14.8600 -16.9911i   9.9053 -18.4707i  12.4703 - 8.8463i -11.5983 -11.2830i ⋯
Elapsed time is 0.105756 seconds.
N = 256
  by dftdirect_m
X1 = 1×256 complex
  -9.3494 -10.8823i  27.1163 + 7.5972i  12.2987 - 2.1780i  13.4434 +26.4978i ⋯
Elapsed time is 0.057577 seconds.
  by dftdirect
X2 = 1×256 complex
  -9.3494 -10.8823i  27.1163 + 7.5972i  12.2987 - 2.1780i  13.4434 +26.4978i ⋯
Elapsed time is 0.094122 seconds.
N = 512
  by dftdirect_m
X1 = 1×512 complex
  26.0656 - 3.7932i -19.1526 +43.1327i  41.4818 +33.4331i  28.1913 -13.2635i ⋯
Elapsed time is 0.215067 seconds.
  by dftdirect
X2 = 1×512 complex
  26.0656 - 3.7932i -19.1526 +43.1327i  41.4818 +33.4331i  28.1913 -13.2635i ⋯
Elapsed time is 0.328506 seconds.
N = 1024
  by dftdirect_m
X1 = 1×1024 complex
10^2 ×
   0.3573 + 0.3994i   0.1048 - 0.2627i  -0.6048 + 0.2213i   0.1171 + 0.3169i ⋯
Elapsed time is 0.719672 seconds.
  by dftdirect
```

```
X2 = 1×1024 complex
10^2 ×
    0.3573 + 0.3994i   0.1048 - 0.2627i  -0.6048 + 0.2213i   0.1171 + 0.3169i ···
Elapsed time is 1.553048 seconds.
```

Ans:

由output結果可得知，x 經過 W = dft_matrix(N) 和 X = dftdirect_m(x,W) 得到 X 的速度比直接做 DFT (X = dftdirect(x)) 快，這是因為 dftdirect 是把所有 W*x 的 term 算完後再做加總；而前者的方法是將 W 的 matrix 算完作為 input 至 dftdirect_m 得到 X，因此此方法會比較快。

(c) Write a function X = fftrecur_m(x,W) that modifies the fftrecur function given on page 439 using the matrix W from (a). Using the tic and toc functions compare computation times for the fftrecur and fftrecur_m function for N = 128, 256, 512, and 1024. For this purpose generate an N-point complex valued signal as x = randn(1,N) + 1j*randn(1,N). (verify your code with fft first)

Ans.

$$X = W_8 x = W_4 \begin{bmatrix} I & D_8 \\ I & -D_8 \end{bmatrix} \begin{bmatrix} x_E \\ x_O \end{bmatrix} = \begin{bmatrix} I & D_8 \\ I & -D_8 \end{bmatrix} \begin{bmatrix} W_4 x_E \\ W_4 x_O \end{bmatrix},$$

$$W_4 x_E = W_2 \begin{bmatrix} I & D_4 \\ I & -D_4 \end{bmatrix} \begin{bmatrix} x_{\text{EE}} \\ x_{\text{EO}} \end{bmatrix} = \begin{bmatrix} I & D_4 \\ I & -D_4 \end{bmatrix} \begin{bmatrix} W_2 x_{\text{EE}} \\ W_2 x_{\text{EO}} \end{bmatrix},$$

$$W_2 x_{\text{EE}} = W_1 \begin{bmatrix} I & D_2 \\ I & -D_2 \end{bmatrix} \begin{bmatrix} x_{\text{EEE}} \\ x_{\text{EEO}} \end{bmatrix} = \begin{bmatrix} I & D_2 \\ I & -D_2 \end{bmatrix} \begin{bmatrix} x_{\text{EEE}} \\ x_{\text{EEO}} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_{\text{EEE}} \\ x_{\text{EEO}} \end{bmatrix}$$

```
close all; clear;
fprintf('2(c)\n');
```

```
2(c)
```

```

open fftrecur_m.m;

N_set = [128 256 512 1024];
for i = 1:1:length(N_set)
    fprintf('N = %d\n', N_set(i));
    x = randn(1,N_set(i)) + 1j*randn(1,N_set(i));

    fprintf('  by fftrecur_m\n');
    tic;
    W = dft_matrix(N_set(i));
    X1 = fftrecur_m(x, W)
    toc;

    fprintf('  by fftrecur\n');
    tic;
    X2 = fftrecur(x)'
    toc;
```

```
end
```

```
N = 128
  by fftrecur_m
X1 = 128×1 complex
  -0.6522 - 5.2027i
  -2.9417 + 1.7876i
   8.5636 -18.7036i
  -6.6545 +18.0341i
  -5.6074 + 2.7283i
  -3.3954 + 1.2980i
   4.2204 +10.6196i
   5.6173 +21.1089i
  12.4469 + 3.3469i
  -3.7409 + 2.2237i
      ⋮

Elapsed time is 0.031175 seconds.
  by fftrecur
X2 = 1×128 complex
  -0.6522 + 5.2027i   4.9469 - 4.3094i  13.1789 -15.9463i -12.2957 +12.7107i ⋯
Elapsed time is 0.017077 seconds.
N = 256
  by fftrecur_m
X1 = 256×1 complex
  -2.9769 +20.1662i
   3.0187 - 7.2361i
  10.1548 - 7.3545i
   5.9500 +31.4920i
 -14.5183 -24.1378i
   2.4679 +22.1131i
  -2.7589 - 1.6438i
 -20.6436 + 3.0748i
  18.6272 + 2.7847i
  -1.1683 +33.7523i
      ⋮

Elapsed time is 0.040165 seconds.
  by fftrecur
X2 = 1×256 complex
  -2.9769 -20.1662i  -3.1185 -28.9135i  16.7614 +11.1979i  -9.5674 +50.6165i ⋯
Elapsed time is 0.062551 seconds.
N = 512
  by fftrecur_m
X1 = 512×1 complex
  22.2150 -24.7398i
   0.5101 +16.0453i
  -6.7364 - 1.8906i
 -28.1948 +13.8535i
   8.6954 -20.6527i
 -27.8050 -16.1379i
  19.3390 +19.3092i
  26.5463 + 9.4375i
  -4.6983 -36.3568i
  -5.1256 -16.3752i
      ⋮

Elapsed time is 0.126839 seconds.
  by fftrecur
X2 = 1×512 complex
```

```
  22.2150 +24.7398i  17.0689 -11.9182i   8.5055 - 3.2886i  32.0359 -27.7643i ···
Elapsed time is 0.004490 seconds.
N = 1024
  by fftrecur_m
X1 = 1024×1 complex
10^2 ×
  -0.2566 - 0.1289i
  -0.2993 + 0.4810i
  -0.0705 + 0.4556i
  -0.2974 + 0.1410i
  -0.1280 - 0.1023i
  -0.0213 + 0.3705i
   0.0716 - 0.4301i
  -0.1349 - 0.1314i
  -0.1056 + 0.1588i
  -0.0822 + 0.3717i
      ⋮
Elapsed time is 0.499928 seconds.
  by fftrecur
X2 = 1×1024 complex
10^2 ×
  -0.2566 + 0.1289i   0.1120 + 0.1124i   0.2518 + 0.5606i  -0.3665 - 0.1495i ···
Elapsed time is 0.011612 seconds.
```

由output結果可得知，x 經過 fftrecur_m 得到 X 的速度比做 fftrecur 慢，這是因為 fftrecur 是直接算所需的 w；而前者的方法在每次迴圈都會把 W 的 matrix 作為 input 至下一層的 fftrecur_m ，等於是在每次迴圈都會傳一些不會用到的資料，因此此方法會比較慢。

P3

Consider the flow graph in Figure 8.10 which implements a DIT-FFT algorithm with both input and output in natural order. Let the nodes at each stage be labeled as sm[k],0 ≤ m ≤ 3 with s0[k] = x[k] and s3[k] = X[k], 0 ≤ k ≤ 7.

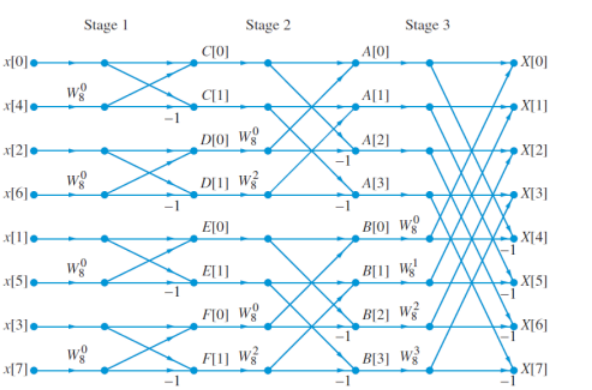


Figure 8.6 Flow graph of 8-point decimation-in-time FFT algorithm using the butterfly computation shown in Figure 8.4. The trivial twiddle factor $W_8^0 = 1$ is shown for the sake of generality.

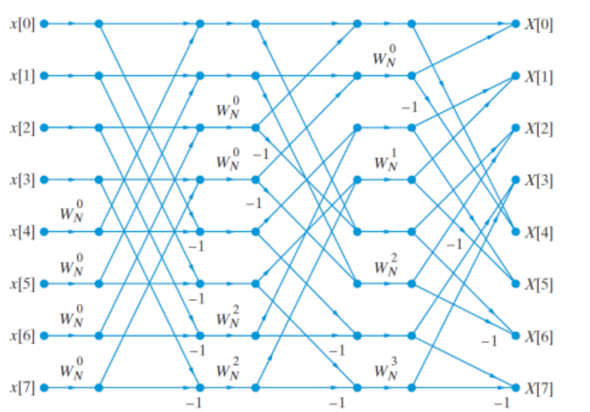


Figure 8.10 Decimation-in-time FFT algorithm with both input and output in natural order.

(a) Express $s_m[k]$ in terms of $s_{m-1}[k]$ for m = 1, 2, 3.

Ans.

$$s_1[k1] = x[k1] + x[k1+4], s_1[k1+4] = x[k1] - x[k1+4], k1 = 0,1,2,3$$

$$s_2[k2] = s_1[k2] + s_1[k2+2], s_2[k2+2] = s_1[k2+4] + W_8^2 s_1[k2+6],$$

$$s_2[k2+4] = s_1[k2] - s_1[k2+2], s_2[k2+6] = s_1[k2+4] - W_8^2 s_1[k2+6],\ k2 = 0,1$$

$$s_3[0] = s_2[0] + s_2[1], s_3[1] = s_2[2] + W_8^1 s_2[3], s_3[2] = s_2[4] + W_8^2 s_2[5], s_3[3] = s_2[6] + W_8^3 s_2[7]$$

$$s_3[4] = s_2[0] - s_2[1], s_3[5] = s_2[2] - W_8^1 s_2[3], s_3[6] = s_2[4] - W_8^2 s_2[5], s_3[7] = s_2[6] - W_8^3 s_2[7]$$

(b) Write a MATLAB function X = fftalt8(x) that computes an 8-point DFT using the equations in part (a). Verify with sequence x[n] = {0,1,2,2,3,3,3,4}.

```
close all; clear;
fprintf('3(b)\n');
```

```
3(b)
```

```
open fftalt8.m;

x = [0 1 2 2 3 3 3 4];
X_fft = fft(x,8)
```

```
X_fft = 1×8 complex
  18.0000 + 0.0000i  -3.0000 + 3.8284i  -2.0000 + 2.0000i  -3.0000 + 1.8284i ⋯
```

```
X_fftalt8 = fftalt8(x)
```

```
X_fftalt8 = 1×8 complex
  18.0000 + 0.0000i  -3.0000 + 3.8284i  -2.0000 + 2.0000i  -3.0000 + 1.8284i ⋯
```

(c) Compare the coding complexity of the above function with that of MATLAB function fftditr2 shown in Figure 8.6, and comment on its usefulness.

```
close all; clear;
fprintf('3(c)\n');
```

```
3(c)
```

```
x = [0 1 2 2 3 3 3 4];
X_fftditr2 = fftditr2(x)
```

```
X_fftditr2 = 1×8 complex
  18.0000 + 0.0000i  -3.0000 + 3.8284i  -2.0000 + 2.0000i  -3.0000 + 1.8284i ⋯
```

(Comment)

就 coding complexity 而言，fftalt8 顯得複雜，因為需要將前一級算完才能算下一級，而且架構相對 fftditr2 複雜，沒辦法像 fftditr2 用三層 for 迴圈就完成此 function，因次在 coding 的複雜度方面也就比 fftditr2 複雜。

P4

Using the flow graph of Figure 8.13 and following the approach used in developing the fftditr2 function.

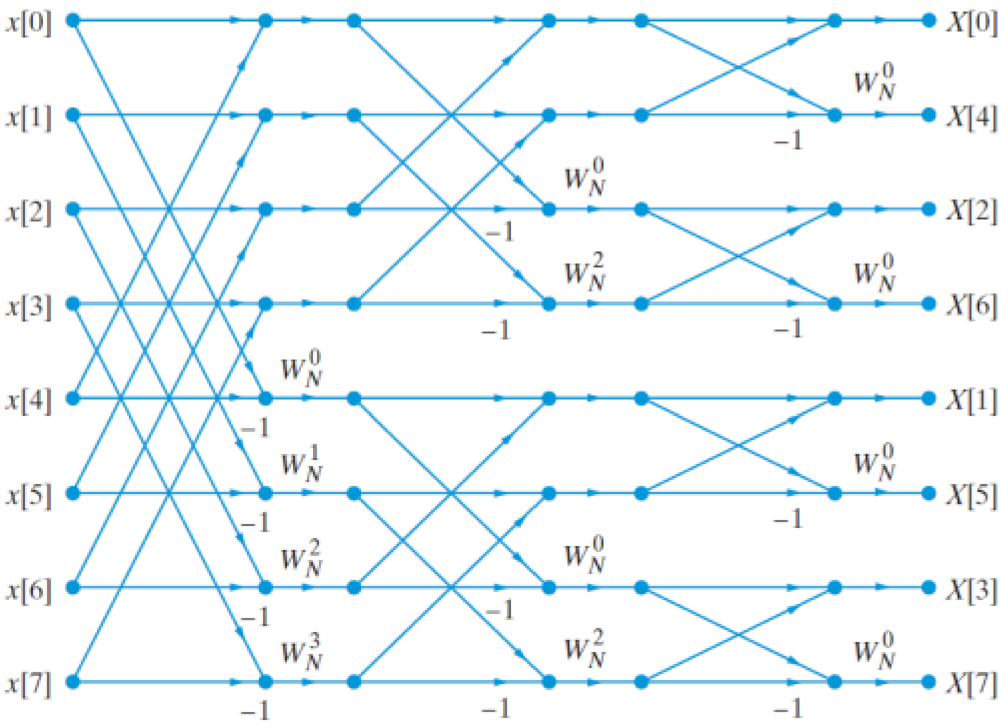


Figure 8.13 Flow graph for the decimation-in-frequency 8-point FFT algorithm. The input sequence is in natural order and the output sequence in bit-reversed order.

(a) Develop a radix-2 DIF-FFT function X = fftdifr2(x) for power-of-2 length N.

```
close all; clear;
fprintf('4(a)\n');
```

```
4(a)
```

```
open fftdifr2.m;
```

(b) Verify your function for N = 2^v, where 2 ≤ v ≤ 10. For this purpose generate an N-point complex-valued signal as x = randn(1,N) + 1j*randn(1,N).

```
fprintf('4(b)\n');
```

```
4(b)
```

```
for v = 2:10
    N = 2^v;
    x = randn(1,N) + 1j*randn(1,N);
    fprintf('for N = 2^%d\n', v);
    X_fftdifr2 = fftdifr2(x)
    X_fft = fft(x)
end
```

```
for N = 2^2
X_fftdifr2 = 1×4 complex
   0.7229 - 0.6519i   1.2888 + 2.2962i   1.1090 - 0.8002i  -1.0473 + 1.9618i
X_fft = 1×4 complex
   0.7229 - 0.6519i   1.2888 + 2.2962i   1.1090 - 0.8002i  -1.0473 + 1.9618i
for N = 2^3
X_fftdifr2 = 1×8 complex
  -0.6388 + 1.4476i  -0.6774 + 0.2921i  -0.5768 - 2.3854i  -6.3883 - 2.8715i ⋯
X_fft = 1×8 complex
  -0.6388 + 1.4476i  -0.6774 + 0.2921i  -0.5768 - 2.3854i  -6.3883 - 2.8715i ⋯
for N = 2^4
X_fftdifr2 = 1×16 complex
  -2.8699 + 0.9639i   1.1856 + 1.0836i  -8.4793 - 2.0574i   1.2718 + 5.0888i ⋯
X_fft = 1×16 complex
  -2.8699 + 0.9639i   1.1856 + 1.0836i  -8.4793 - 2.0574i   1.2718 + 5.0888i ⋯
for N = 2^5
X_fftdifr2 = 1×32 complex
  -6.8237 - 3.9361i  -0.7731 + 3.6484i  -1.3866 - 3.3637i  -0.1640 + 2.3503i ⋯
X_fft = 1×32 complex
  -6.8237 - 3.9361i  -0.7731 + 3.6484i  -1.3866 - 3.3637i  -0.1640 + 2.3503i ⋯
for N = 2^6
X_fftdifr2 = 1×64 complex
   8.4193 + 3.7853i   1.2007 + 8.2331i   6.2577 + 2.3820i  -0.9565 - 5.1766i ⋯
X_fft = 1×64 complex
   8.4193 + 3.7853i   1.2007 + 8.2331i   6.2577 + 2.3820i  -0.9565 - 5.1766i ⋯
for N = 2^7
X_fftdifr2 = 1×128 complex
  -8.2014 - 8.0684i  -0.6122 + 2.5266i -28.2804 - 9.0278i  11.8084 + 6.1246i ⋯
X_fft = 1×128 complex
  -8.2014 - 8.0684i  -0.6122 + 2.5266i -28.2804 - 9.0278i  11.8084 + 6.1246i ⋯
for N = 2^8
X_fftdifr2 = 1×256 complex
 -21.3933 -25.0174i -10.1746 +18.1729i  -7.8854 -12.4340i  -8.2055 +37.9570i ⋯
X_fft = 1×256 complex
 -21.3933 -25.0174i -10.1746 +18.1729i  -7.8854 -12.4340i  -8.2055 +37.9570i ⋯
for N = 2^9
X_fftdifr2 = 1×512 complex
  11.7374 +19.2580i  14.1321 -13.5540i   6.5551 +36.2684i   7.7291 - 1.8899i ⋯
X_fft = 1×512 complex
  11.7374 +19.2580i  14.1321 -13.5540i   6.5551 +36.2684i   7.7291 - 1.8899i ⋯
for N = 2^10
X_fftdifr2 = 1×1024 complex
10^2 ×
   0.2291 - 0.4560i   0.0540 - 0.0919i  -0.1695 - 0.2888i  -0.2992 - 0.1905i ⋯
X_fft = 1×1024 complex
10^2 ×
   0.2291 - 0.4560i   0.0540 - 0.0919i  -0.1695 - 0.2888i  -0.2992 - 0.1905i ⋯
```

(Comment)

由上述結果我們發現 x 經過 fftdifr2 和 fft 得到的結果相同。

P5

The filterfirdf implements the FIR direct form structure.

(a) Develop a new MATLAB function y=filterfirlp(h,x) that implements the FIR linear-phase form given its impulse response in h. This function should first check if h is one of type-I through type-IV and then simulate the corresponding equations. If h does not correspond to one of the four types then the function should display an appropriate error message.

Ams.

For Type I: $y[n] = \sum_{k=0}^{\frac{M}{2}-1} h[k](x[n-k] + x[n-M+k]) + h\left[\frac{M}{2}\right]x\left[n-\frac{M}{2}\right]$

For Type II: $y[n] = \sum_{k=0}^{\frac{M-1}{2}} h[k](x[n-k] + x[n-M+k])$

For Type III: $y[n] = \sum_{k=0}^{\frac{M}{2}-1} h[k](x[n-k] - x[n-M+k])$

For Type IV: $y[n] = \sum_{k=0}^{\frac{M-1}{2}} h[k](x[n-k] - x[n-M+k])$

```
close all; clear;
fprintf('5(a)\n');
```

```
5(a)
```

```
open filterfirlp.m;
```

(b) Verify your function on each of the following FIR systems:

h1[n] = {1,2,3,2,1}, h2[n] = {1,-2,3,3,-2,1}, h3[n] = {1,-5,0,5,-1}, h4[n] = {1,-3,-4,4,3,-1}, h5[n] = {1,2,3,-2,-1},

For verification determine the first ten samples of the step responses using your function and compare them with those from the filter function.

```
fprintf('5(b)\n');
```

```
5(b)
```

```
x = ones(1, 10);
```

```
h1 = [1 2 3 2 1];
[y1, type] = filterfirlp(h1, x)
```

```
y1 = 1×10
     1     3     6     8     9     9     9     9     9     9
type = 
'Type I'
```

```
y1_verify = filterfirdf(h1, x)
```

```
y1_verify = 1×10
     1     3     6     8     9     9     9     9     9     9
```

```
h2 = [1 -2 3 3 -2 1];
[y2, type] = filterfirlp(h2, x)
```

```
y2 = 1×10
     1    -1     2     5     3     4     4     4     4     4
type = 
'Type II'
```

```
y2_verify = filterfirdf(h2, x)
```

```
y2_verify = 1×10
     1    -1     2     5     3     4     4     4     4     4
```

```
h3 = [1 -5 0 5 -1];
[y3, type] = filterfirlp(h3, x)
```

```
y3 = 1×10
     1    -4    -4     1     0     0     0     0     0     0
type = 
'Type III'
```

```
y3_verify = filterfirdf(h3, x)
```

```
y3_verify = 1×10
     1    -4    -4     1     0     0     0     0     0     0
```

```
h4 = [1 -3 -4 4 3 -1];
[y4, type] = filterfirlp(h4, x)
```

```
y4 = 1×10
     1    -2    -6    -2     1     0     0     0     0     0
type = 
'Type IV'
```

```
y4_verify = filterfirdf(h4, x)
```

```
y4_verify = 1×10
     1    -2    -6    -2     1     0     0     0     0     0
```

```
h5 = [1 2 3 -2 -1];
[y5, type] = filterfirlp(h5, x)
```

```
y5 = 1×10
     1     3     6     4     3     3     3     3     3     3
type =
'Not correspond to one of the four types.'
```

```
y5_verify = filterfirdf(h5, x)
```

```
y5_verify = 1×10
     1     3     6     4     3     3     3     3     3     3
```

P6

Consider the IIR normal direct form II structure given in Figure 9.6 and implemented by (9.18) and (9.20).

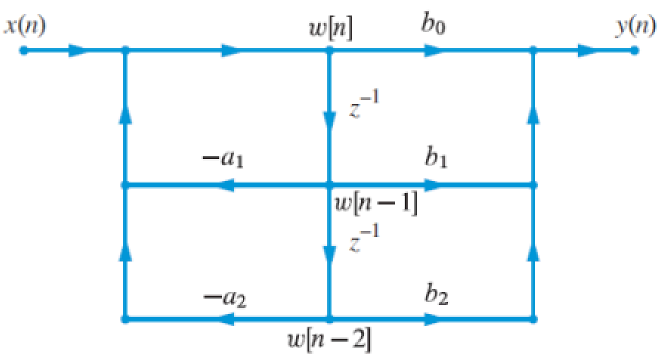


Figure 9.6 Direct form II structure for implementation of an Nth order system. For convenience, we assume that $N = M = 2$. If $N \neq M$, some of the coefficients will be zero.

$$y[n] = \sum_{k=0}^{M} b_k w[n-k]. \tag{9.20}$$

$$w[n] = -\sum_{k=1}^{N} a_k w[n-k] + x[n]. \tag{9.18}$$

(a) Using the MATLAB function filterdf1 as a guide, develop a MATLAB function y=filterdf2(b,a,x) that implements the normal direct form II structure. Assume zero initial conditions.

```
close all; clear;
fprintf('6(a)\n');
```

```
6(a)
```

```
open filterdf2.m;
```

(b) Determine y[n], 0 ≤ n ≤ 500 using your function and filterdf1 function with following inputs:

$$x[n] = \left(\frac{1}{4}\right)^n u[n], a = [1 \quad -1.5 \quad 0.5], b = 1$$

Compare your results to verify that your filterdf2 function is correctly implemented.

```
fprintf('6(b)\n');
```

```
6(b)
```

```
n = 0:1:500;
x = (1/4).^n; a = [1 -1.5 0.5]; b = [1];
y = filterdf2(b,a,x)'
```

```
y = 1×501
    1.0000    1.7500    2.1875    2.4219    2.5430    2.6045    2.6355    2.6511 ···
```

```
y_verify = filterdf1(b,a,x)'
```

```
y_verify = 1×501
    1.0000    1.7500    2.1875    2.4219    2.5430    2.6045    2.6355    2.6511 ···
```

(Comment)

由上面的結果顯示，從 filterdf1 和 filterdf2 得到的結果相同。

P7

The following numerator and denominator arrays in MATLAB represent the system function of a discrete-time system in direct form:

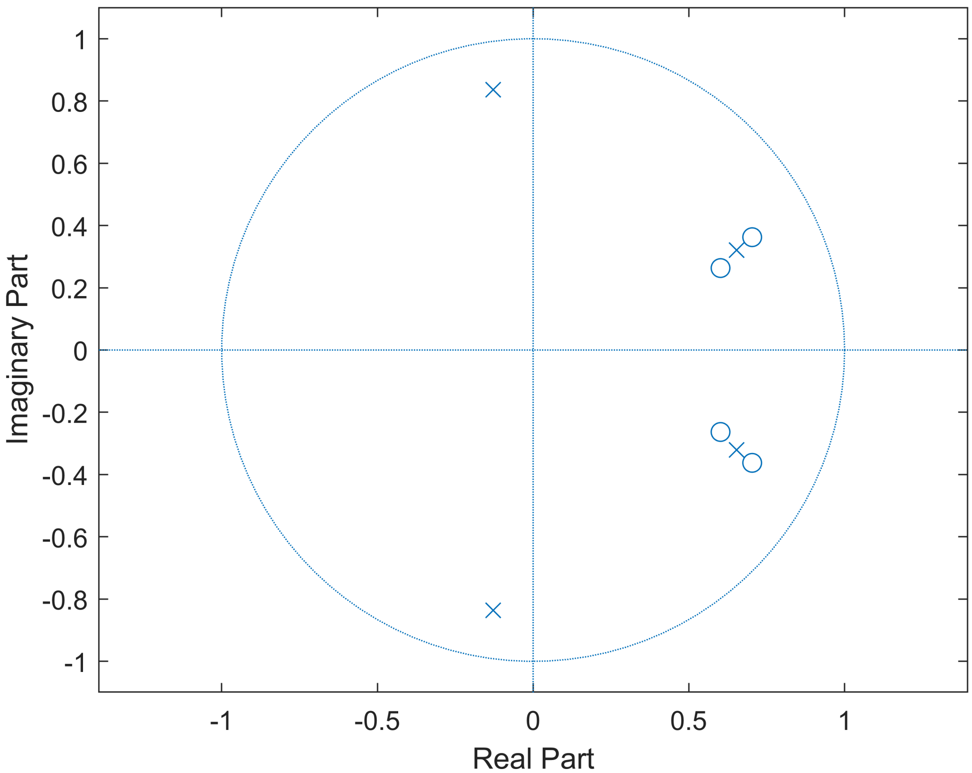
b = [1,-2.61,2.75,-1.36,0.27], a = [1,-1.05,0.91,-0.8,0.38].

Determine and draw each of the following structures:

```
close all; clear;
fprintf('7\n');
```

```
7
```

```
num = [1 -2.61 2.75 -1.36 0.27];
den = [1 -1.05 0.91 -0.8 0.38];
figure; zplane(num, den);
```




```
[num,den] = eqtflength(num,den);
[zero,pole,k] = tf2zp(num,den)
```

```
zero =
   0.7033
   0.7033
   0.6017
   0.6017
pole =
  -0.1288
  -0.1288
   0.6538
   0.6538
k = 1
```

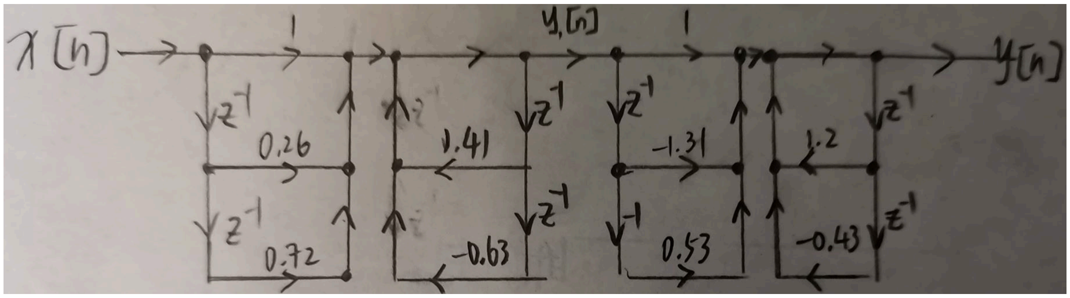
(Comment)

$$H(z) = \frac{1 - 2.61z^{-1} + 2.75z^{-2} - 1.36z^{-3} + 0.27z^{-4}}{1 - 1.05z^{-1} + 0.91z^{-2} - 0.8z^{-3} + 0.38z^{-4}} = \frac{(1 + 0.26z^{-1} + 0.72z^{-2})(1 - 1.31z^{-1} + 0.53z^{-2})}{(1 - 1.41z^{-1} + 0.63z^{-2})(1 - 1.2z^{-1} + 0.43z^{-2})}$$

(a) Cascade form with second-order sections in normal direct form I,

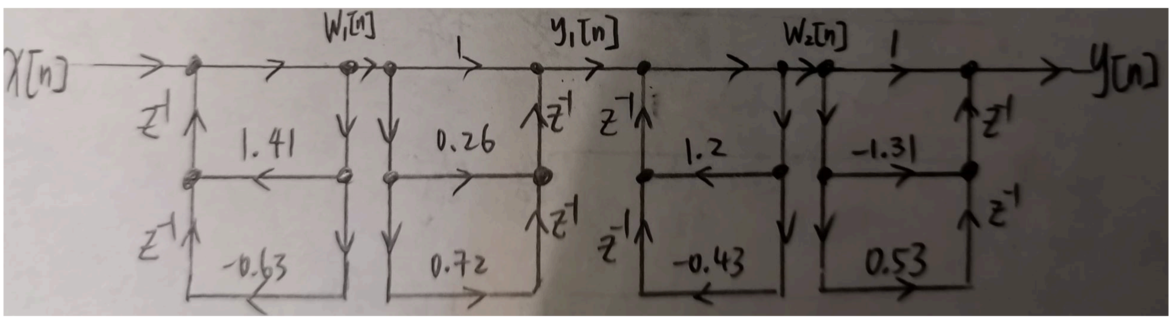
Ans.

$$y[n] = -\sum_{k=1}^{N} a_k y[n-k] + \sum_{k=0}^{M} b_k x[n-k], H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$



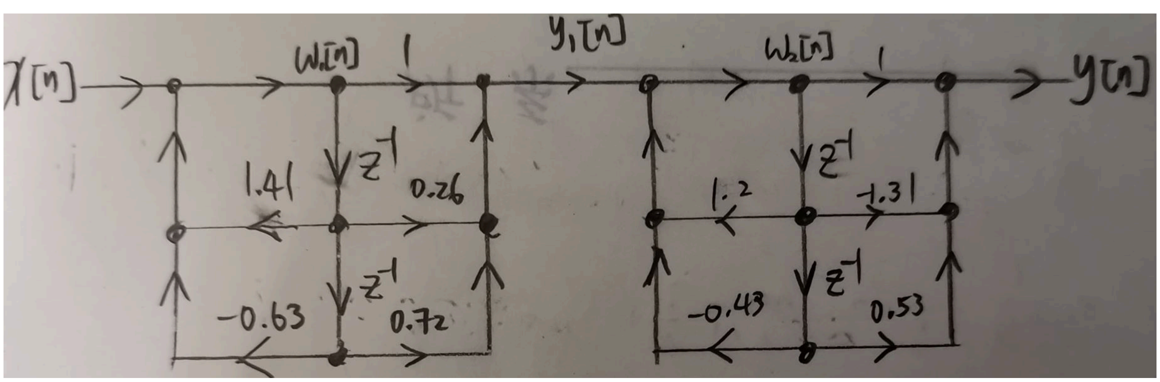
(b) Cascade form with second-order sections in transposed direct form I,

$$w[n] = -\sum_{k=1}^{N} a_k w[n-k] + x[n], y[n] = \sum_{k=0}^{M} b_k w[n-k]$$



(c) Cascade form with second-order sections in normal direct form II,

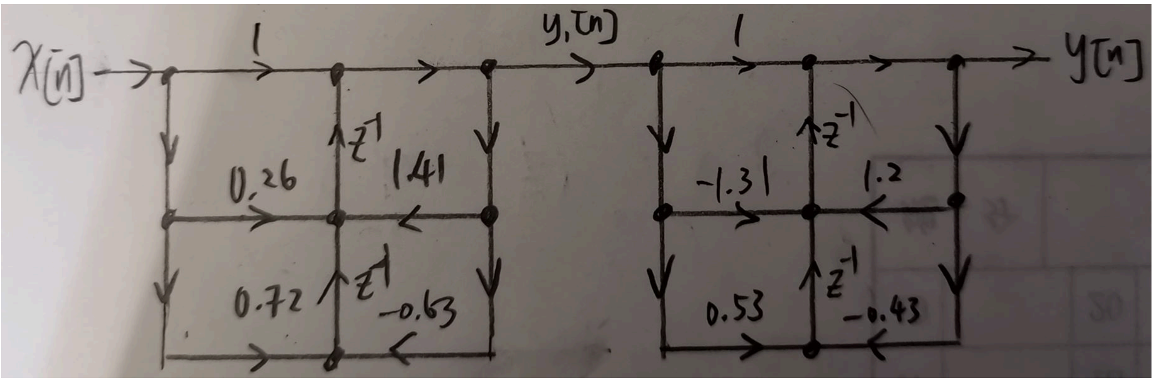
$$W(z) = \frac{1}{1 + \sum_{k=1}^{N} a_k z^{-k}}, Y(z) = \sum_{k=0}^{M} b_k z^{-k} W(z)$$



(d) Cascade form with second-order sections in transposed direct form II.

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}},$$

$$Y(z) = z^{-1}V_1(z) + b_0 X(z), V_1(z) = z^{-1}V_2(z) - a_1 Y(z) + b_1 X(z), V_2(z) = b_2 X(z) - a_2 Y(z)$$



P8

The frequency-sampling form is developed using (9.50) which uses complex arithmetic.

$$H(z) = \frac{1 - z^{-N}}{N} \sum_{k=0}^{N-1} \frac{H[k]}{1 - z^{-1} e^{j\frac{2\pi}{N}k}}, H[k] = H(z)|_{z=e^{j\frac{2\pi}{N}k}} \quad (9.50)$$

(a) Develop a MATLAB function [G,sos]=firdf2fs(h) that determines frequency sampling form parameters given in (9.51) and (9.52) given the impulse response in h. The matrix sos (second order system) should contain second-order section coefficients in the form similar to the tf2sos function while G array should contain the respective gains of second-order sections. Incorporate the coefficients for the H[0] and H[N/2] terms in sos and G arrays.

$$H(z) = \frac{1 - z^{-N}}{N} \left\{ \frac{H[0]}{1 - z^{-1}} + \frac{H\left[\frac{N}{2}\right]}{1 + z^{-1}} + \sum_{k=1}^{K} 2|H[k]| H_k(z) \right\} \quad (9.51)$$

$$H_k(z) = \frac{\cos(\angle H[k]) - z^{-1}\cos\left(\angle H[k] - \frac{2\pi k}{N}\right)}{1 - 2\cos\left(\frac{2\pi k}{N}\right) z^{-1} + z^{-2}} \quad (9.52)$$

```
close all; clear;
fprintf('8(a)\n');
```

```
8(a)
```

```
open firdf2fs.m;
```

(b) Verify your function by input h with sampled frequency response (9.53) and compare with the system function (9.54) (see example 9.6 in the textbook)

$$H[k] = H\left(e^{\frac{j2\pi k}{33}}\right) = e^{\frac{-j2\pi k}{33}} \times \begin{cases} 1 & k = 0, 1, 2, 31, 32 \\ 0.5 & k = 3, 30 \\ 0 & \text{otherwise} \end{cases} \quad (9.53)$$

$$H(z) = \frac{1 - z^{-33}}{33}\left[\frac{1}{1 - z^{-1}} + \frac{-1.99 + 1.99z^{-1}}{1 - 1.964z^{-1} + z^{-2}} + \frac{1.964 - 1.964z^{-1}}{1 - 1.857z^{-1} + z^{-2}} + \frac{-1.96 + 1.96z^{-1}}{1 - 1.683z^{-1} + z^{-2}}\right] \quad (9.54)$$

```
fprintf('8(b)\n');
```

```
8(b)
```

```
N = 33;
H = zeros(1, 33);
for k = 1:1:33
    if((k==1)||(k==2)||(k==3))
        H(k) = exp((-1*sqrt(-1)*2*pi*(k-1))/N);
    elseif((k==32)||(k==33))
        H(k) = exp((-1*sqrt(-1)*2*pi*(k-1))/N);
    elseif((k==4)||(k==31))
        H(k) = 0.5*(exp((-1*sqrt(-1)*2*pi*(k-1))/N));
    else
        H(k) = 0;
    end
end
h = ifft(H);
[G, sos] = firdf2fs(h);
sos_after = sos;
for a = 1:1:length(G)
    for b = 1:1:3
        sos_after(a,b) = sos_after(a,b)*G(a);
    end
end
sos_after % sos multiplied by the corresponding G factor
```

```
sos_after = 19×6 complex
   1.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   1.0000 + 0.0000i ···
   1.9639 + 0.0000i  -1.8567 + 0.0000i   0.0000 + 0.0000i   1.0000 + 0.0000i
   1.8567 + 0.0000i  -1.4475 + 0.0000i   0.0000 + 0.0000i   1.0000 + 0.0000i
   0.8413 + 0.0000i  -0.4154 + 0.0000i   0.0000 + 0.0000i   1.0000 + 0.0000i
  -0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   1.0000 + 0.0000i
   0.0000 + 0.0000i  -0.0000 + 0.0000i   0.0000 + 0.0000i   1.0000 + 0.0000i
   0.0000 + 0.0000i  -0.0000 + 0.0000i   0.0000 + 0.0000i   1.0000 + 0.0000i
   0.0000 + 0.0000i  -0.0000 + 0.0000i   0.0000 + 0.0000i   1.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   1.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   1.0000 + 0.0000i
      ⋮
```

由此結果我們可以發現，經由 firdf2fs 處理後的值與 (9.54) 相近，一樣只有前面 4 項有值。