

Homework Assignment #5: Chap. 10

Due: May 7, 2020

I Paper Assignment (30%)

1. (4%) This problem examines conversions between various filter specifications.
Given the absolute specifications $\delta_s = 0.0001$ and $\omega_p = 0.3\pi$, $\omega_s = 0.5\pi$, determine the relative specifications A_s and ω_c , $\Delta\omega$.

2. (6%) The Hann window function can be written as

$$w[n] = [0.5 - 0.5 \cos(2\pi n/M)]w_R[n].$$

where $w_R[n]$ is the rectangular window of length $M + 1$.

- (a) Express the DTFT of $w[n]$ in terms of the DTFT of $w_R[n]$.
 - (b) Explain why the Hann window has the wider mainlobe but lower sidelobes than the rectangular window of the same length.
3. (12%) Consider an FIR filter with impulse response $h[n] = u[n] - u[n - 4]$.
 - (a) Determine and sketch the magnitude response $|H(e^{j\omega})|$
 - (b) Determine and sketch the amplitude response $A(e^{j\omega})$. Compare this sketch with that in (a) and comment on the difference.
 - (c) Determine and sketch the phase response $\angle H(e^{j\omega})$.
 - (d) Determine and sketch the angle response $\Psi(e^{j\omega})$. Compare this sketch with that in (c) and comment on the difference.
 4. (8%) Consider the type-IV linear-phase FIR filter characterized by antisymmetric impulse response and odd- M .
 - (a) Show that the amplitude response $A(e^{j\omega})$ is given by (10.38) with coefficients $d[k]$ given

in (10.39).

- (b) Show that the amplitude response $A(e^{j\omega})$ can be further expressed as (10.40) with coefficients $\hat{d}[k]$ given in (10.41)

II Program Assignment (70%)

5. (10%) A lowpass FIR filter is given by the specifications: $\omega_p = 0.3\pi$, $\omega_s = 0.5\pi$, and $A_s = 50$ dB.
- Use the `fir2` function to obtain a minimum length linearphase filter. Use the appropriate window function in the `fir2` function. Provide a plot similar to Figure 10.12.
6. (12%) Design a highpass FIR filter to satisfy the specifications: $\omega_s = 0.3\pi$, $\omega_p = 0.5\pi$, and $A_s = 50$ dB.
- (a) Use Kaiser window to obtain a minimum length linear-phase filter. Provide a plot similar to Figure 10.12.
- (b) Repeat (a) using the `fir1` function.
7. (12%) In this problem we reproduce Figures 10.4 and 10.5. For each of the following linear-phase FIR filters described by $h[n]$, obtain impulse response, amplitude response, magnitude response, and pole-zero plots in one figure window. For frequency response plots use the interval $-2\pi \leq \omega \leq 2\pi$.
- (a) Type-I filter: $h[n] = \{1, 2, 3, -2, 5, -2, 3, 2, 1\}$.
- (b) Type-II filter: $h[n] = \{1, 2, 3, -2, -2, 3, 2, 1\}$.
- (c) Type-III filter: $h[n] = \{1, 2, 3, -2, 0, 2, -3, -2, -1\}$.
- (d) Type-IV filter: $h[n] = \{1, 2, 3, -2, 2, -3, -2, -1\}$.
8. (18%) Consider a Blackman window of length $L = 21$.
- (a) Compute and plot the log-magnitude response in dB over $-\pi \leq \omega \leq \pi$. In the plot measure and show the value of the peak of the first sidelobes.
- (b) Compute and plot the accumulated amplitude response in dB using the `cumsum` function. In the plot measure and show the value of the peak of the first sidelobe.
- (c) Repeat (a) and (b) for $L = 41$.
9. (18%) An ideal lowpass filter has a cutoff frequency of $\omega_c = 0.4\pi$. We want to obtain a length L

= 40 linear-phase FIR filter using the frequency-sampling method.

- (a) Let the sample at ω_c be equal to 0.5. Obtain the resulting impulse response $h[n]$. Plot the log-magnitude response in dB and determine the minimum stopband attenuation.
- (b) Now vary the value of the sample at ω_c and find the largest minimum stopband attenuation. Obtain the resulting impulse response $h[n]$ and plot the log-magnitude response in dB in the plot window of (a).
- (c) Compare your results with those obtained using the `fir2` function (choose hamming window).

III Reference

$$H(e^{j\omega}) = \sum_{k=1}^{\frac{M+1}{2}} \left(d[k] \sin \left[\omega \left(k - \frac{1}{2} \right) \right] \right) j e^{-\frac{j\omega M}{2}}$$

$$\triangleq jA(e^{j\omega}) e^{-j\omega M/2}. \quad (10.38)$$

$$d[k] = 2h \left[\frac{M+1}{2} - k \right]. \quad k = 1, 2, \dots, \frac{M+1}{2} \quad (10.39)$$

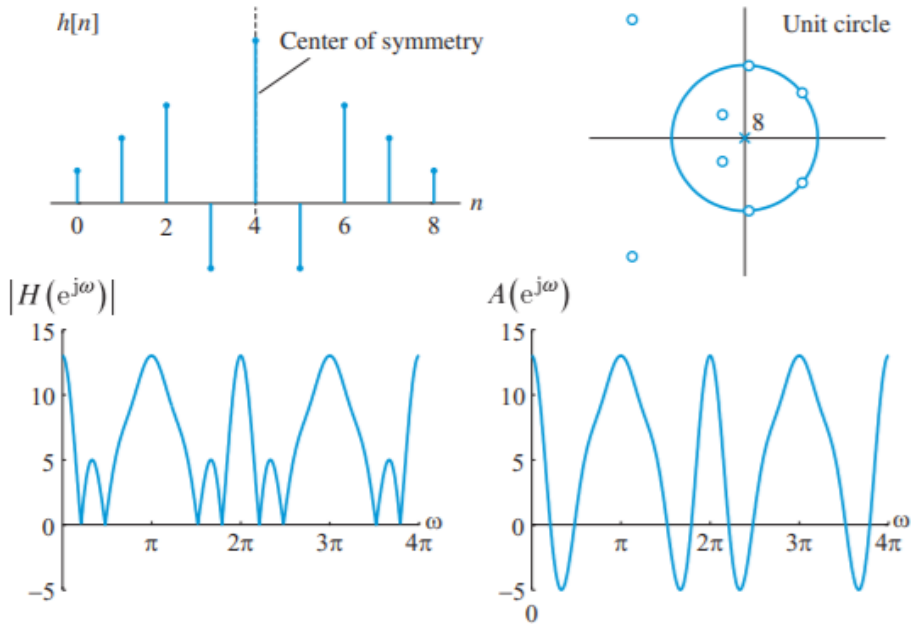
$$A(e^{j\omega}) = \sin\left(\frac{\omega}{2}\right) \sum_{k=0}^{(M-1)/2} \hat{d}[k] \cos \omega k. \quad (10.40)$$

$$d[k] = \begin{cases} \frac{1}{2}(2\hat{d}[0] - \hat{d}[1]), & k = 1 \\ \frac{1}{2}(2\hat{d}[k-1] - \hat{d}[k]), & 2 \leq k \leq (M-1)/2. \\ \frac{1}{2}(2\hat{d}[(M-1)/2]), & k = (M+1)/2 \end{cases} \quad (10.41)$$

$$H(e^{j\omega}) = \sum_{N=0}^M h[n] e^{j\omega n} \triangleq A(e^{j\omega}) e^{j\Psi(e^{j\omega n})}. \quad (10.42)$$

$$\Psi(e^{j\omega n}) \triangleq -\alpha\omega + \beta. \quad (10.43)$$

Type I: Symmetric Impulse Response, Even Order $M = 8$



Type II: Symmetric Impulse Response, Odd Order $M = 7$

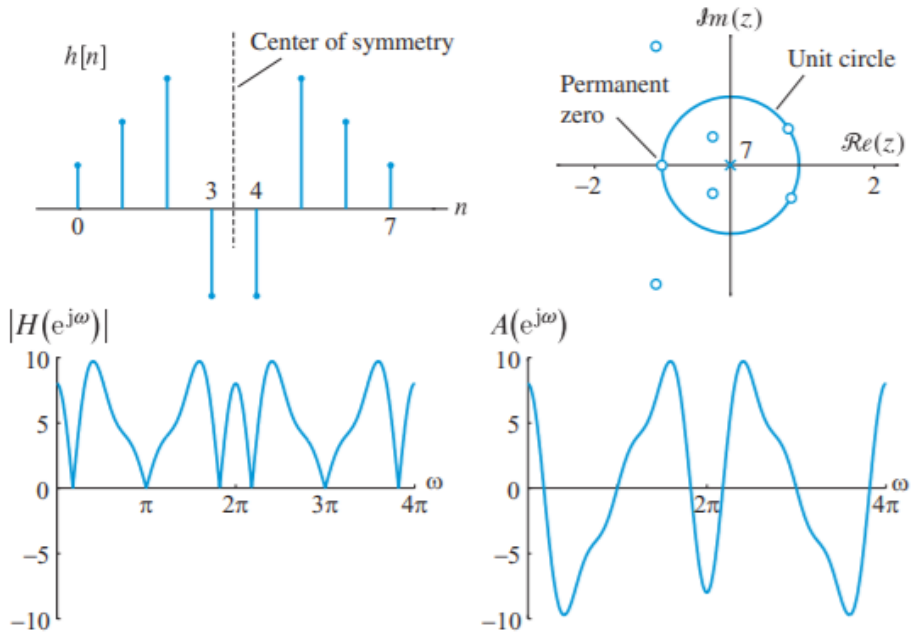
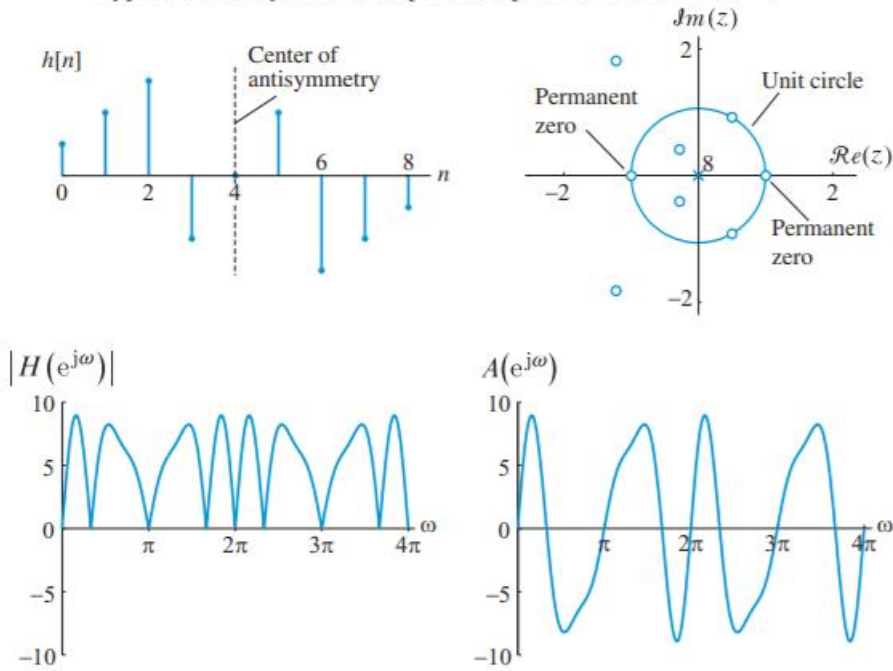


Figure 10.4 Impulse response, pole-zero pattern, magnitude response, and amplitude response

Type III: Anti-Symmetric Impulse Response, Even Order $M = 8$



Type IV: Anti-symmetric Impulse Response, Odd Order $M = 7$

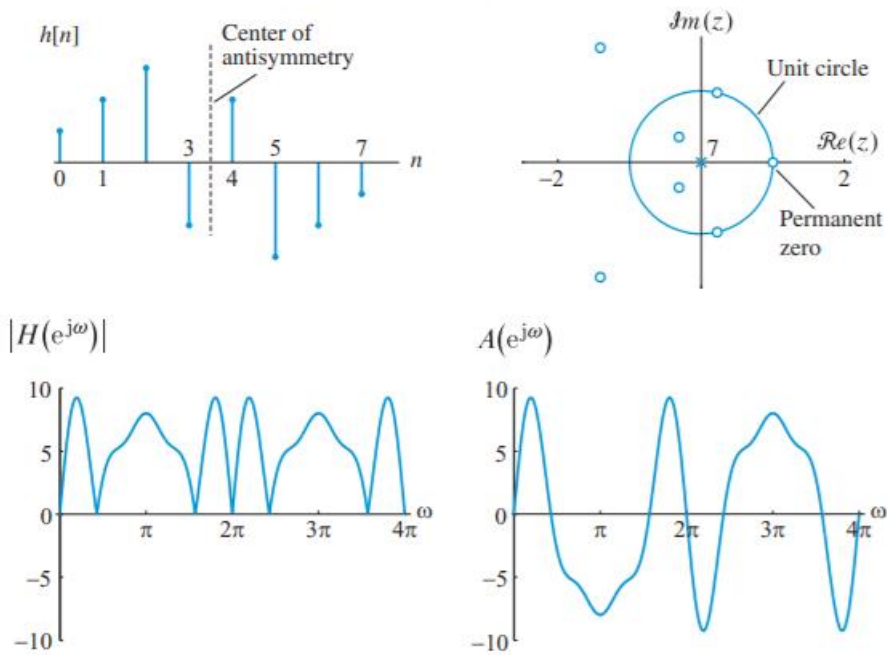


Figure 10.5 Impulse response, pole-zero pattern, magnitude response, and amplitude response

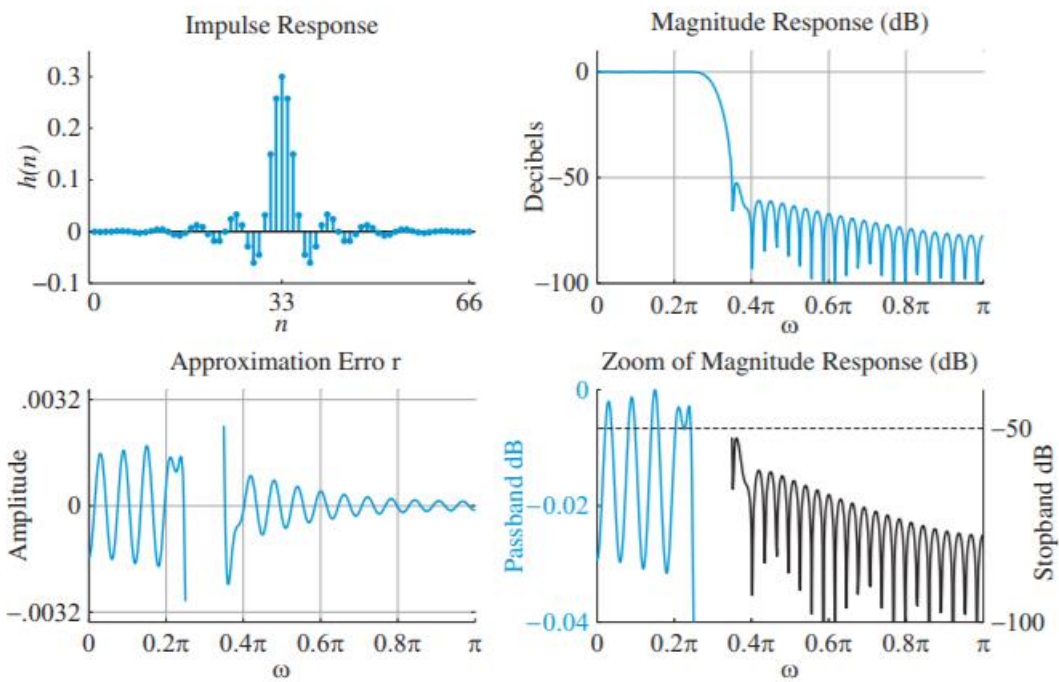


Figure 10.12 Impulse, approximation error, and magnitude response plots of the filter designed in Example 10.2 using a Hamming window to satisfy specifications: $\omega_p = 0.25\pi$, $\omega_s = 0.35\pi$, $A_p = 0.1$ dB, and $A_s = 50$ dB.