National Tsing Hua University

Department of Electrical Engineering

EE3660 Intro. to Digital Signal Processing, Spring 2020

Homework Assignment #5: Chap10 Answer

Problem 1.

A_s =-20$*$ $\log(0.0001)$ = 80
Δω= ω_s - ω_p =0.2π
ω_c = (ω_s + ω_p)/2 =0.4π

Problem 2.

(a) Solution:

$$W(e^{j\omega}) = \frac{1}{2}W_R(e^{j\omega}) - \frac{1}{4}W_R(e^{j(\omega-\frac{2\pi}{M})}) + \frac{1}{4}W_R(e^{j(\omega+\frac{2\pi}{M})})$$

(b) Comments:
The second and third terms widen the mainlobe of Hann window and the sidelobes are lowed by the scaling factor.

Problem 3.

(a) Solution:
The DTFT of $h[n]$ is:

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h[n]e^{-j\omega n} = \sum_{n=0}^{3} e^{-j\omega n} = 1 + e^{-j\omega} + e^{-2j\omega} + e^{-3j\omega}$$
$$= (1 + \cos\omega + \cos 2\omega + \cos 3\omega) - j(\sin\omega + \sin 2\omega + \sin 3\omega)$$

Hence, the magnitude response is:

$$|H(e^{j\omega})| = \sqrt{(1 + \cos\omega + \cos 2\omega + \cos 3\omega)^2 + (\sin\omega + \sin 2\omega + \sin 3\omega)^2}$$

(b) Solution:

$$A(\mathrm{e}^{\mathrm{j}\omega}) = \sum_{k=1}^{2} b[k]\cos[\omega(k - \frac{1}{2})], \quad b[k] = 2h[2-k]$$

$$A(\mathrm{e}^{\mathrm{j}\omega}) = b[1]\cos\frac{1}{2}\omega + b[2]\cos\frac{3}{2}\omega = 2\cos\frac{1}{2}\omega + 2\cos\frac{3}{2}\omega$$

(c) Solution:

$$\angle H(\mathrm{e}^{\mathrm{j}\omega}) = -\tan^{-1}\frac{\sin\omega + \sin 2\omega + \sin 3\omega}{1 + \cos\omega + \cos 2\omega + \cos 3\omega}$$

(d) Solution:

$$\Psi(\mathrm{e}^{\mathrm{j}\omega}) = -\omega M/2 = -\frac{3}{2}\omega$$

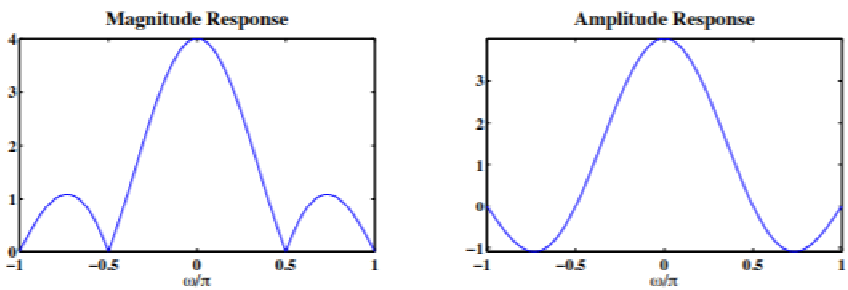

FIGURE 10.1: Plots of magnitude and amplitude responses.

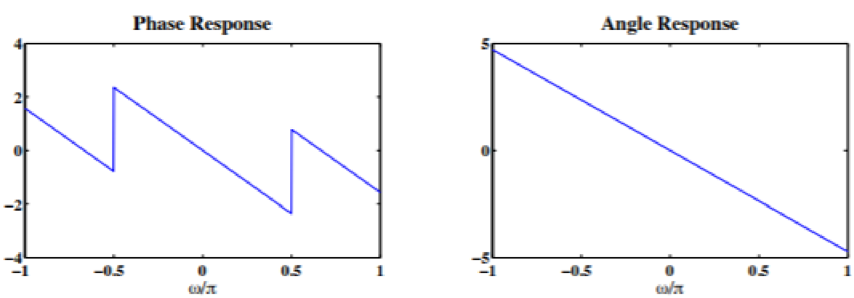

FIGURE 10.2: Plots of phase and angle responses.

Problem 4.

(a) Proof:

$$H(\mathrm{e}^{\mathrm{j}\omega}) = \left(\sum_{k=1}^{(M+1)/2} d[k] \sin[\omega(k - \frac{1}{2})] \right) \cdot \mathrm{j}\mathrm{e}^{-\mathrm{j}\omega M/2} \triangleq \mathrm{j}A(\mathrm{e}^{\mathrm{j}\omega})\,\mathrm{e}^{-\mathrm{j}\omega M/2} \qquad (10.38)$$

$$d[k] = 2h[(M+1)/2 - k], \quad k = 1, 2, \ldots, (M+1)/2 \qquad (10.39)$$

$$\begin{aligned}
H(\mathrm{e}^{\mathrm{j}\omega}) &= \sum_{k=0}^{M} h[k] \cdot \mathrm{e}^{-\mathrm{j}k\omega} = \sum_{k=0}^{\frac{M-1}{2}} h[k]\mathrm{e}^{-\mathrm{j}k\omega} + \sum_{k=\frac{M+1}{2}}^{M} h[k]\mathrm{e}^{-\mathrm{j}k\omega} \\
&= \sum_{k=0}^{\frac{M-1}{2}} h[k]\mathrm{e}^{-\mathrm{j}k\omega} + \sum_{k=0}^{\frac{M-1}{2}} h[k + \frac{M+1}{2}]\mathrm{e}^{-\mathrm{j}(k+\frac{M+1}{2})\omega} \\
&= \left(\sum_{k=0}^{\frac{M-1}{2}} h[k]\mathrm{e}^{-\mathrm{j}(k-M/2)\omega} + \sum_{k=0}^{\frac{M-1}{2}} h[k + \frac{M+1}{2}]\mathrm{e}^{-\mathrm{j}(k+\frac{1}{2})\omega} \right) \cdot \mathrm{e}^{-\mathrm{j}\omega M/2} \\
&= \left(\sum_{k=0}^{\frac{M-1}{2}} h[\frac{M-1}{2} - k]\mathrm{e}^{-\mathrm{j}(\frac{M-1}{2}-k-M/2)\omega} - \sum_{k=0}^{\frac{M-1}{2}} h[M - k - \frac{M+1}{2}]\mathrm{e}^{-\mathrm{j}(k+\frac{1}{2})\omega} \right) \cdot \mathrm{e}^{-\mathrm{j}\omega} \\
&= \sum_{k=0}^{\frac{M-1}{2}} \left(h[\frac{M-1}{2} - k]\mathrm{e}^{\mathrm{j}(k+1/2)\omega} - h[\frac{M-1}{2} - k]\mathrm{e}^{-\mathrm{j}(k+1/2)\omega} \right) \cdot \mathrm{e}^{-\mathrm{j}\omega M/2} \\
&= \left(\sum_{k=0}^{\frac{M-1}{2}} 2h[\frac{M-1}{2} - k]\,\mathrm{j}\sin(k + \frac{1}{2})\omega \right) \cdot \mathrm{e}^{-\mathrm{j}\omega M/2} \\
&= \left(\sum_{k=1}^{\frac{M+1}{2}} d[k]\sin(k - \frac{1}{2})\omega \right) \cdot \mathrm{e}^{\mathrm{j}(\pi/2 - \omega M/2)}
\end{aligned}$$

(b) Proof:

$$A(\mathrm{e}^{\mathrm{j}\omega}) = \sin(\frac{\omega}{2}) \sum_{k=0}^{(M-1)/2} \tilde{d}[k] \cos k\omega \qquad (10.40)$$

$$d[k] = \begin{cases} \frac{1}{2}(2\tilde{d}[0] - \tilde{d}[1]), & k = 1 \\ \frac{1}{2}(\tilde{d}[k-1] - \tilde{d}[k]), & 2 \le k \le (M-1)/2 \\ \frac{1}{2}\tilde{d}[(M-1)/2], & k = (M+1)/2 \end{cases} \qquad (10.41)$$

$$\begin{aligned}
A(\mathrm{e}^{\mathrm{j}\omega}) &= \sin(\frac{\omega}{2}) \sum_{k=0}^{(M-1)/2} \tilde{d}[k] \cos k\omega \\
&= \frac{1}{2} \sum_{k=0}^{(M-1)/2} \tilde{d}[k][\sin(k + 1/2)\omega - \sin(k - 1/2)\omega] \\
&= (\tilde{d}[0] - \frac{1}{2}\tilde{d}[1]) \sin\frac{\omega}{2} + \sum_{k=2}^{(M-1)/2} (\tilde{d}[k-1] - \tilde{d}[k]) \sin(k - \frac{1}{2})\omega \\
&\quad + \frac{1}{2}\tilde{d}[(M-1)/2] \sin(\frac{M}{2}\omega)
\end{aligned}$$