Part I. Paper Assignment

1. This problem examines conversions between various filter specifications. Given the absolute specifications $\delta_s = 0.0001$ and $\omega_p = 0.3\pi$, $\omega_s = 0.5\pi$, determine the relative specifications A_s and ω_c, $\Delta\omega$.

$$A_S = -20\log_{10}\delta_s = 80\ \text{dB},\ \ \omega_c = \frac{\omega_P+\omega_s}{2} = 0.4\pi,\ \ \Delta\omega = \omega_s - \omega_P = 0.2\pi$$

2. The Hann window function can be written as $w[n] = [0.5 - 0.5\cos(2\pi n/M)]w_R[n]$ where $w_R[n]$ is the rectangular window of length M + 1.
   (a) Express the DTFT of w[n] in terms of the DTFT of $w_R[n]$.
   (b) Explain why the Hann window has the wider mainlobe but lower sidelobes than the rectangular window of the same length.

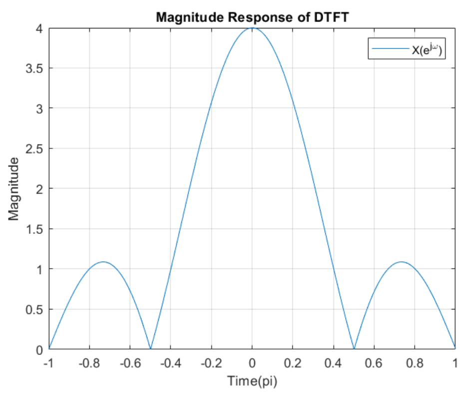
(a) $x[n]\cos(\omega_c n) \leftrightarrow \frac{1}{2}X\left(e^{j(\omega+\omega_c)}\right) + \frac{1}{2}X\left(e^{j(\omega-\omega_c)}\right)$

$$\Rightarrow W\left(e^{j\omega}\right) = \frac{1}{2}W_R\left(e^{j\omega}\right) - \frac{1}{4}\left(W_R\left(e^{j\left(\omega+\frac{2\pi}{M}\right)}\right) + W_R\left(e^{j\left(\omega-\frac{2\pi}{M}\right)}\right)\right)$$

(b) Hann window 用原本一半的 Rectangular window 再減掉 1/4 往左右平移 $e^{j2\pi/M}$ 的 Rectangular 訊號，也因此 Hann window 會有比較寬的 mainlobe 和比較小的 sidelobes。

3. Consider a FIR filter with impulse response h[n] = u[n] − u[n − 4].
   (a) Determine and sketch the magnitude response $|H(e^{j\omega})|$.
   (b) Determine and sketch the amplitude response $A(e^{j\omega})$. Compare this sketch with that in (a) and comment on the difference.
   (c) Determine and sketch the phase response $\angle H(e^{j\omega})$.
   (d) Determine and sketch the angle response $\Psi(e^{j\omega})$. Compare this sketch with that in (c) and comment on the difference.

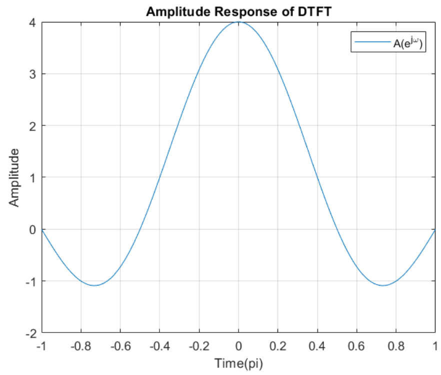
(a) $H\left(e^{j\omega}\right) = \frac{1-e^{-j4\omega}}{1-e^{j\omega}} \Rightarrow \left|H\left(e^{j\omega}\right)\right| = \left|\frac{1-\cos 4\omega + j\sin 4\omega}{1-\cos\omega - j\sin\omega}\right|$



(b) Odd order M; even tap L = M + 1, symmetric → Type II

$$H(e^{j\omega}) = h[0] + h[1]e^{-j\omega} + h[2]e^{-j2\omega} + h[3]e^{-j3\omega} = \left(2h[1]\cos\left(\frac{\omega}{2}\right) + 2h[0]\cos\left(\frac{3\omega}{2}\right)\right)e^{-j\left(\frac{3}{2}\right)\omega} = A(e^{j\omega})e^{-j\left(\frac{3}{2}\right)\omega}$$

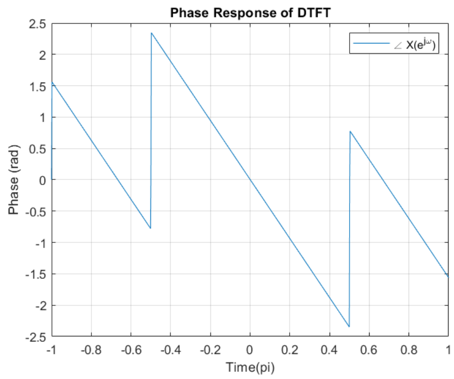
$$\Rightarrow A(e^{j\omega}) = \left(2\cos\left(\frac{\omega}{2}\right) + 2\cos\left(\frac{3\omega}{2}\right)\right)e^{-j\left(\frac{3}{2}\right)\omega}$$



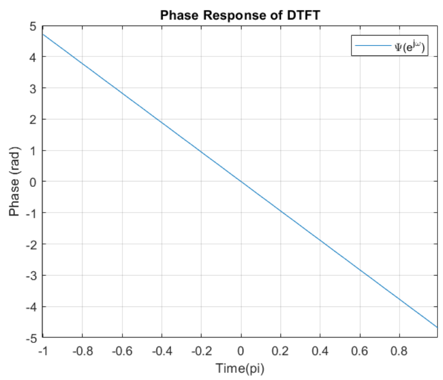
$$\Rightarrow |A(e^{j\omega})| = H(e^{j\omega})$$

(c) $H(e^{j\omega}) = \frac{1-e^{-j4\omega}}{1-e^{j\omega}} = |H(e^{j\omega})|e^{j\angle H(e^{j\omega})}$

$$\Rightarrow \angle H(e^{j\omega}) = \angle(1 - e^{-j4\omega}) - \angle(1 - e^{j\omega})$$



(d) $H(e^{j\omega}) = A(e^{j\omega})e^{-j\left(\frac{3}{2}\right)\omega} = A(e^{j\omega})e^{j\Psi(e^{j\omega})} \Rightarrow \Psi(e^{j\omega}) = -\left(\frac{3}{2}\right)\omega$



$$\Rightarrow \Psi(e^{j\omega}) = \text{unwrap}\ \angle H(e^{j\omega})$$

4. Consider the type-IV linear-phase FIR filter characterized by antisymmetric impulse response and odd-M.
   (a) Show that the amplitude response $A(e^{j\omega})$ is given by (10.38) with coefficients d[k] given in (10.39).

$$H(e^{j\omega}) = \sum_{k=1}^{\frac{M+1}{2}} \left(d[k] \sin\left[\omega\left(k - \frac{1}{2}\right)\right]\right) je^{-\frac{j\omega M}{2}} \approx jA(e^{j\omega})e^{-\frac{j\omega M}{2}} \quad (10.38)$$

$$d[k] = 2h\left[\frac{M+1}{2} - k\right] \quad (10.39)$$

   (b) Show that the amplitude response $A(e^{j\omega})$ can be further expressed as (10.40) with coefficients $\tilde{d}[k]$ given in (10.41).

$$A(e^{j\omega}) = \sin\left(\frac{\omega}{2}\right) \sum_{k=0}^{(M-1)/2} \tilde{d}[k] \cos(\omega k) \quad (10.40)$$

$$d[k] = \begin{cases} \frac{1}{2}(2\tilde{d}[0] - \tilde{d}[1]), & k = 1 \\ \frac{1}{2}(\tilde{d}[k-1] - \tilde{d}[k]), & 2 \le k \le (M-1)/2 \\ \frac{1}{2}\tilde{d}[(M-1)/2], & k = (M+1)/2 \end{cases} \quad (10.41)$$

(a) $H(e^{j\omega}) = \sum_{n=0}^{M} h[n]e^{-j\omega n},\ \ h[n] = h[M-n]$

➔ $H(e^{j\omega}) = h[0] + h[1]e^{-j\omega} + \cdots + h[M-1]e^{-j(M-1)\omega} + h[M]e^{-jM\omega}$

$$= \left(h[0]e^{j\frac{M}{2}\omega} + h[1]e^{j\left(\frac{M}{2}-1\right)\omega} + \cdots + h[M-1]e^{-j\left(\frac{M}{2}-1\right)\omega} + h[M]e^{-j\frac{M}{2}\omega}\right) e^{-j\frac{M}{2}\omega}$$

$$= \left(2h[0]\sin\left(\frac{M}{2}\omega\right) + 2h[1]\sin\left(\left(\frac{M}{2}-1\right)\omega\right) + 2h[2]\sin\left(\left(\frac{M}{2}-2\right)\omega\right) + \cdots + 2h\left[\frac{M-1}{2}\right]\sin\left(\left(\frac{M}{2} - \frac{M-1}{2}\right)\omega\right)\right) je^{-j\frac{M}{2}\omega}$$

$$= \left(d[k]\sin\left(\frac{M\omega}{2}\right) + d[k-1]\sin\left(\left(\frac{M}{2}-1\right)\omega\right) + \cdots + d[1]\sin\left(\frac{\omega}{2}\right)\right) je^{-j\left(\frac{M}{2}\right)\omega}$$

➔ $H(e^{j\omega}) = \sum_{k=1}^{\frac{M+1}{2}} \left(d[k]\sin\left[\omega\left(k-\frac{1}{2}\right)\right]\right) je^{-\frac{j\omega M}{2}} = jA(e^{j\omega})e^{-\frac{j\omega M}{2}}$

➔ $d[k] = 2h\left[\frac{M+1}{2} - k\right]$

(b) $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta \ \rightarrow 2\sin\beta\cos\alpha = \sin(\alpha+\beta) - \sin(\alpha-\beta)$

$$A(e^{j\omega}) = \sum_{k=1}^{\frac{M+1}{2}} \left(d[k]\sin\left[\omega\left(k-\frac{1}{2}\right)\right]\right)$$

$$= d[1]\sin\left(\frac{\omega}{2}\right) + d[2]\sin\left(\frac{3\omega}{2}\right) + \cdots + d[k-1]\sin\left(\left(\frac{M}{2}-1\right)\omega\right) + d[k]\sin\left(\frac{M\omega}{2}\right)$$

$$= d[1]\sin\left(\frac{\omega}{2}\right) + d[2]\left(2\sin\left(\frac{\omega}{2}\right)\cos(\omega) + \sin\left(\frac{\omega}{2}\right)\right) + d[3]\left(2\sin\left(\frac{\omega}{2}\right)\cos(2\omega) + \sin\left(\frac{3\omega}{2}\right)\right) + \cdots + d[k-1]\left(2\sin\left(\frac{\omega}{2}\right)\cos\left(\frac{(M-3)\omega}{2}\right) + \sin\left(\frac{(M-4)\omega}{2}\right)\right) +$$

$$d[k]\left(2\sin\left(\frac{\omega}{2}\right)\cos\left(\frac{(M-1)\omega}{2}\right)+\sin\left(\frac{(M-2)\omega}{2}\right)\right)$$

$$=\sin\left(\frac{\omega}{2}\right)\left[\left(d[1]+\cdots+d\left[\frac{M+1}{2}\right]\right)+2\left(d[2]+\cdots+d\left[\frac{M+1}{2}\right]\right)\cos\omega+\right.$$

$$\left.2\left(d[3]+\cdots+d\left[\frac{M+1}{2}\right]\right)\cos 2\omega+\cdots+2d\left[\frac{M+1}{2}\right]\cos\left(\frac{M+1}{2}\omega\right)\right]$$

$$=\sin\left(\frac{\omega}{2}\right)\sum_{k=0}^{(M-1)/2}\tilde{d}[k]\cos(\omega k)$$

➔ $A(e^{j\omega})=\sin\left(\frac{\omega}{2}\right)\sum_{k=0}^{(M-1)/2}\tilde{d}[k]\cos(\omega k)$ with

$$d[k]=\begin{cases}\frac{1}{2}(2\tilde{d}[0]-\tilde{d}[1]),\ k=1\\ \frac{1}{2}(\tilde{d}[k-1]-\tilde{d}[k]),\ 2\le k\le (M-1)/2\\ \frac{1}{2}\tilde{d}[(M-1)/2],\ k=(M+1)/2\end{cases}$$