Part I. Paper Assignment

1. Determine the system function, magnitude response, and phase response of the following systems and use the pole-zero pattern to explain the shape of their magnitude response:

(a) $y[n] = \frac{1}{4}(x[n] + x[n-1]) - \frac{1}{4}(x[n-2] + x[n-3])$

System function: $H(z) = \frac{Y(z)}{X(z)} = \frac{1}{4}(1 + z^{-1} - z^{-2} - z^{-3}) = \frac{1}{4}(1 - z^{-1})(1 + z^{-1})^2$

⇨ Zero: 1, -1; Pole: 0

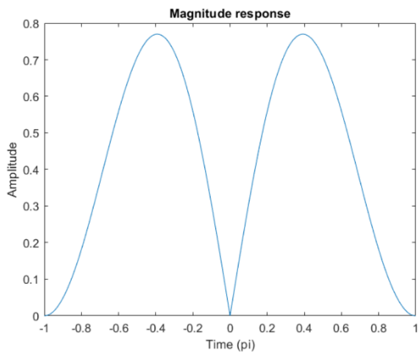
$z = e^{j\omega} = \cos(\omega) + j\sin(\omega)$

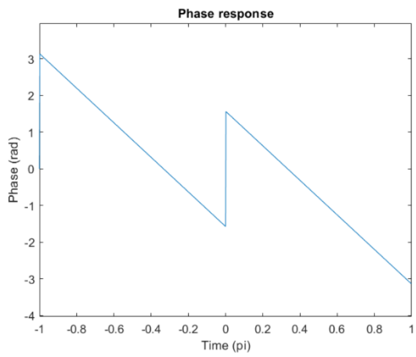
⇨ $H(e^{j\omega}) = 0.25[(1 + \cos(\omega) - \cos(2\omega) - \cos(3\omega)) + j(-\sin(\omega) + \sin(2\omega) + \sin(3\omega))]$

- Magnitude response: $|H(e^{j\omega})|$

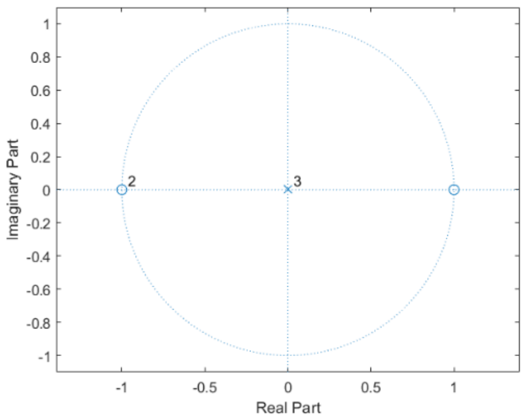
$$|H(e^{j\omega})| = 0.25\sqrt{(1 + \cos(\omega) - \cos(2\omega) - \cos(3\omega))^2 + (-\sin(\omega) + \sin(2\omega) + \sin(3\omega))^2}$$

- Phase response: $\angle H(e^{j\omega}) = \frac{-\sin(\omega)+\sin(2\omega)+\sin(3\omega)}{1+\cos(\omega)-\cos(2\omega)-\cos(3\omega)}$





- Pole-zero pattern:



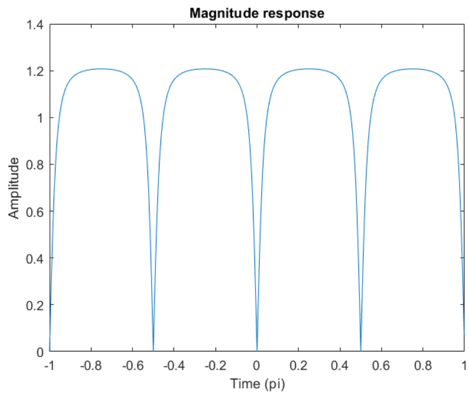
由 pole-zero pattern 我們可以推測 magnitude response 在 ±π 處會是 0，因為 zero 的緣故。

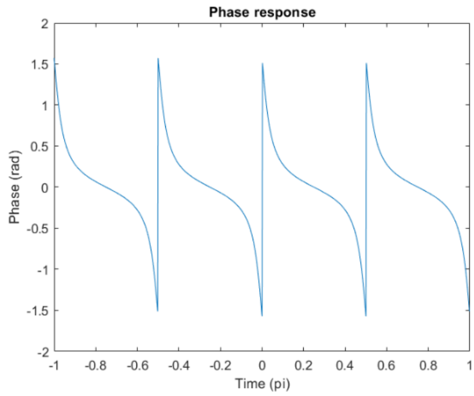
(b) $y[n] = x[n] - x[n-4] + 0.6561y[n-4]$

System function: $H(z) = \frac{1-z^{-4}}{1-0.6561z^{-4}}$ ➔ Zero: $\pm1, \pm j$; Pole: $\pm0.9, \pm0.9j$

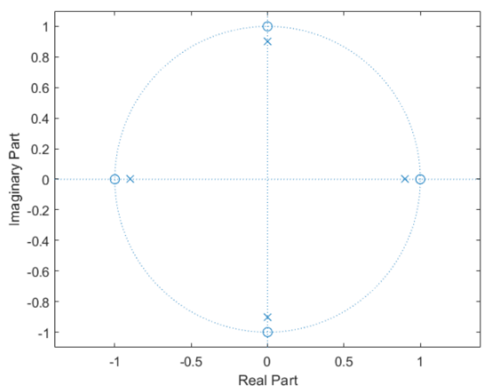
$$H(e^{j\omega}) = \frac{(1-\cos(4\omega))-j\sin(4\omega)}{(1-0.6561\cos(4\omega))-0.6561j\sin(4\omega)}$$

Magnitude response: $|H(e^{j\omega})|$ ； Phase response: $\angle H(e^{j\omega})$





Pole-zero pattern:



由 pole-zero pattern 我們可以推測 magnitude response 在 $\pm1, \pm j$ 處會是 0，因為 zero 的緣故。再加上在 zero 附近皆存在一個 pole，因此 magnitude response 的 rising time 和 falling time 會比 1(a)來的小。

2. Consider a periodic signal

$$x[n] = \sin(0.1\pi n) + \frac{1}{3}\sin(0.3\pi n) + \frac{1}{5}\sin(0.5\pi n).$$

For each of the following systems, determine if the system imparts (i) no distortion, (ii) magnitude distortion, and/or (iii) phase (or delay) distortion.

(a) $h[n] = \{1_{n=0}, -2, 3, -4, 0, 4, -3, 2, -1\}$

$H(z) = \sum_{n=-\infty}^{+\infty} h[n]z^{-n} = 1 - 2z^{-1} + 3z^{-2} - 4z^{-3} + 4z^{-5} - 3z^{-6} + 2z^{-7} - z^{-8}$

$\Rightarrow |H(e^{j\omega})| \neq \text{constant}, \angle H(e^{j\omega}) \neq -\omega n_d$

⇒ This system not only imparts magnitude but also phase distortion.

(b) $y[n] = 10x[n-10]$

$H(e^{j\omega}) = 10e^{-j10\omega} \Rightarrow |H(e^{j\omega})| = 10 = \text{constant}, \angle H(e^{j\omega}) = -10\omega$

⇒ This system imparts no distortion.

3. An economical way to compensate for the droop distortion in S/H DAC is to use an appropriate digital compensation filter prior to DAC.

(a) Determine the frequency response of such an ideal digital filter $H_r(e^{j\omega})$ that will perform an equivalent filtering given by following $H_r(j\Omega)$

$$H_r(j\Omega) = \begin{cases} \frac{\Omega T/2}{\sin(\Omega T/2)} e^{j\Omega T/2}, |\Omega| < \pi/T \\ 0, \text{otherwise} \end{cases}$$

$$X_c(j\Omega) = \begin{cases} T\,X(e^{j\Omega T}), |\Omega| < \pi/T \\ 0, \text{otherwise} \end{cases} \leftrightarrow X(e^{j\Omega T}) = \frac{1}{T}\sum_{-\infty}^{+\infty} X_c[j(\Omega - \frac{2\pi}{T}k)]$$

$$H_r(e^{j\Omega T}) = \frac{1}{T}\frac{\Omega T/2}{\sin(\Omega T/2)} e^{j\Omega T/2}, |\Omega| < \frac{\pi}{T}, \omega = \Omega T$$

$$H_r(e^{j\omega}) = \frac{1}{T}\frac{\omega/2}{\sin(\omega/2)} e^{j\omega/2}, |\omega| < \pi$$

(b) One low-order FIR filter suggested in Jackson (1996) is

$$H_{FIR}(z) = -\frac{1}{16} + \frac{9}{8}z^{-1} - \frac{1}{16}z^{-2}$$

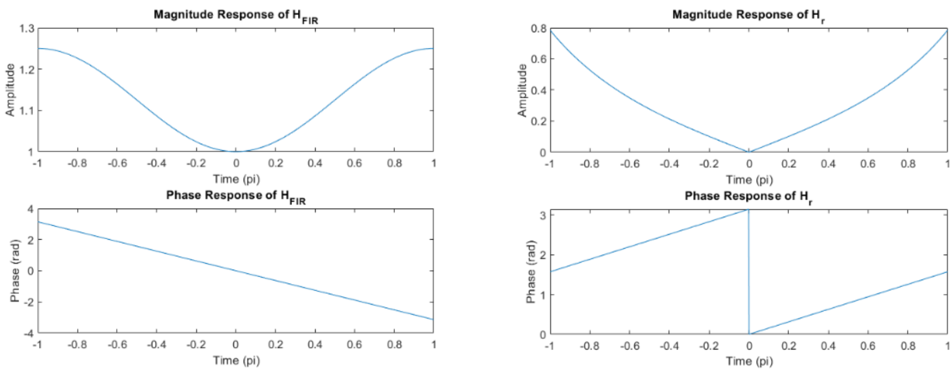
Compare the magnitude response of $H_{FIR}(e^{j\omega})$ with that of $H_r(e^{j\omega})$ above.

$z = e^{j\omega} = \cos(\omega) + j\sin(\omega)$

$$H_{FIR}(e^{j\omega}) = \frac{1}{16}[(-1 + 18\cos(\omega) - \cos(2\omega)) + j(-18\sin(\omega) + \sin(2\omega))]$$

$$|H_{FIR}(e^{j\omega})| = \frac{1}{16}\sqrt{(-1 + 18\cos(\omega) - \cos(2\omega))^2 + (-18\sin(\omega) + \sin(2\omega))^2}$$

$$|H_r(e^{j\omega})| = \frac{1}{T}\frac{\omega/2}{\sin(\omega/2)}, |\omega| < \pi$$



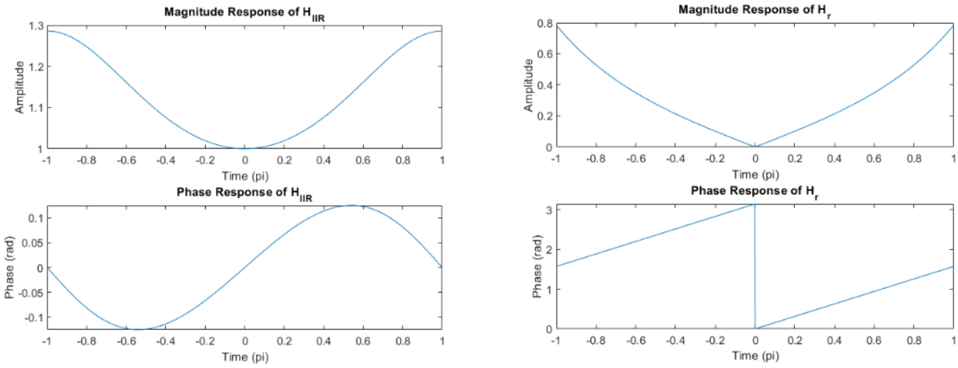
We can the curve of magnitude response of $H_{FIR}(e^{j\omega})$ is smoother and larger than magnitude response of $H_r(e^{j\omega})$.

(c) Another low-order IIR filter suggested in Jackson (1996) is

$$H_{IIR}(z) = \frac{9}{8 + z^{-1}}$$

Compare the magnitude response of $H_{IIR}(e^{j\omega})$ with that of $H_r(e^{j\omega})$ above.

$H_{IIR}(e^{j\omega}) = \frac{9\times(8+\cos\omega+j\sin\omega)}{65+16\cos\omega} \Rightarrow |H_{IIR}(e^{j\omega})| = \frac{9}{65+16\cos\omega}\sqrt{65+16\cos\omega}$



We can the curve of magnitude response of $H_{IIR}(e^{j\omega})$ is smoother and larger than magnitude response of $H_r(e^{j\omega})$.

4. Consider the following continuous-time system

$$H(s) = \frac{s^4 - 6s^3 + 10s^2 + 2s - 15}{s^5 + 15s^4 + 100s^3 + 370s^2 + 744s + 720}$$

(a) Show that the system $H(s)$ is a nonminimum phase system.

Minimum phase system: All poles and zeros are inside unit circle.

$s^4 - 6s^3 + 10s^2 + 2s - 15 = (s+1)(s-3)(s^2-4s+5)$

$s^5 + 15s^4 + 100s^3 + 370s^2 + 744s + 720 = (s+5)(s^2+4s+8)(s^2+6s+18)$

⇨ Zeros: -1, 3, $2\pm j$ & Poles: -5, $-3\pm 3j$, $-2\pm 2j$

⇨ Only one zero is inside unit circle, so this is a nonminimum phase system.

(b) Decompose $H(s)$ into the product of minimum phase component $H_{min}(s)$ and an all pass component $H_{ap}(s)$.

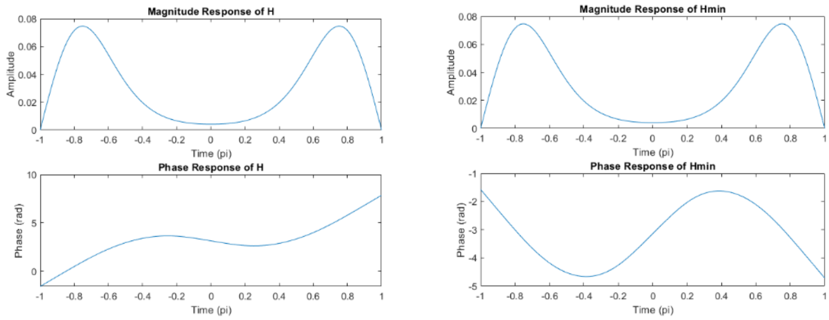
$$H(s) = \frac{(s+1)(1-3s)(5s^2-4s+1)}{(1+5s)(8s^2+4s+1)(18s^2+6s+1)} \times \frac{(s-3)(s^2-4s+5)(1+5s)(8s^2+4s+1)(18s^2+6s+1)}{(s+5)(s^2+4s+8)(s^2+6s+18)(1-3s)(5s^2-4s+1)}$$

$$= H_{min}(s) \times H_{ap}(s)$$

⇨ $H_{min}(s) = \frac{(s+1)(1-3s)(5s^2-4s+1)}{(1+5s)(8s^2+4s+1)(18s^2+6s+1)}$

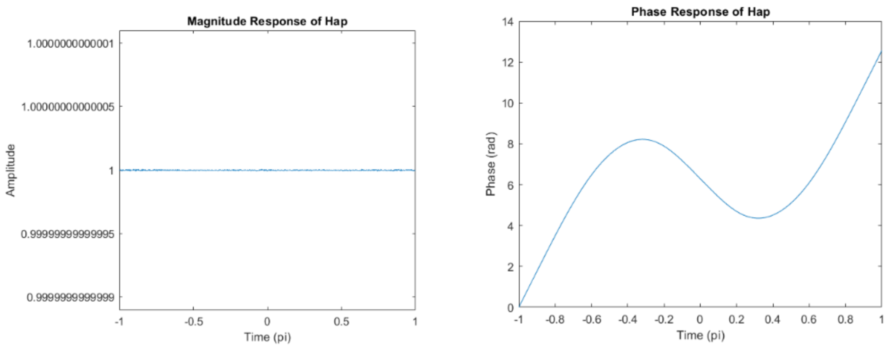
⇨ $H_{ap}(s) = \frac{(s-3)(s^2-4s+5)(1+5s)(8s^2+4s+1)(18s^2+6s+1)}{(s+5)(s^2+4s+8)(s^2+6s+18)(1-3s)(5s^2-4s+1)}$

(c) Briefly plot the magnitude and phase responses of $H(s)$ and $H_{min}(s)$ and explain your plots.



We can find that $|H_{min}(s)| = |H(s)|$ and $\angle H_{min}(s) < \angle H(s)$

(d) Briefly plot the magnitude and phase responses of $H_{ap}(s)$.

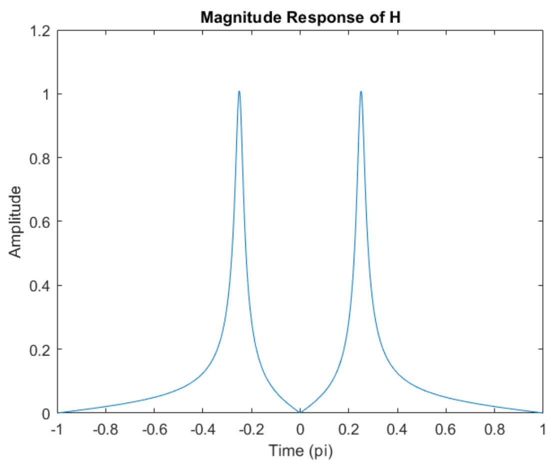


We can find that $|H_{ap}(s)| = 1$ and $\angle H_{ap}(s) + \angle H_{min}(s) = \angle H(s)$

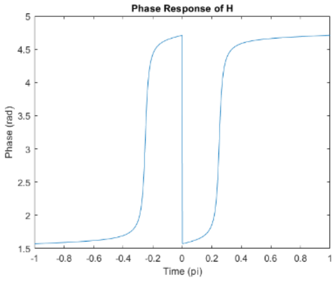
5. We want to design a second order IIR filter using pole-zero placement that satisfies the following requirements: (1) the magnitude response is 0 at $\omega_1 = 0$ and $\omega_3 = \pi$ (2) The maximum magnitude is 1 at $\omega_{2,4} = \pm\pi/4$ and (3) the magnitude response is approximately $1/\sqrt{2}$ at frequencies $\omega_{2,4} \pm 0.05$. Determine locations of two poles and two zeros of the required filter and then compute its system function H(z).

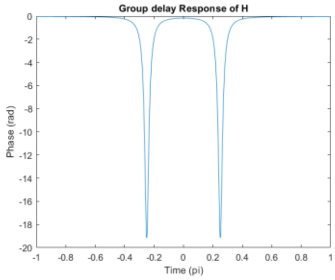
Zero: ± 1; Pole: $0.74 \pm 0.74j \Longrightarrow H(z) = \frac{1}{18}\frac{1-Z^{-2}}{1-1.49z^{-1}+1.11z^{-2}}$

(a) Briefly graph the magnitude response of the filter.



(b) Briefly graph phase and group-delay responses.





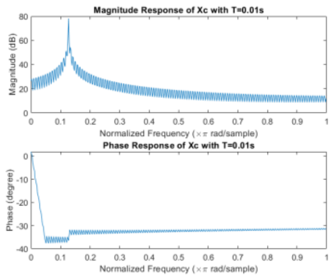
6. The following signals $x_c(t)$ is sampled periodically to obtain the discrete-time signal $x[n]$. For each of the given sampling rates in F_S Hz or in T period, (i) determine the spectrum $X(e^{j\omega})$ of $x[n]$; (ii) plot its magnitude and phase as a function of ω in rad/sam and as a function of F in Hz; and (iii) explain whether $x_c(t)$ can be recovered from $x[n]$.

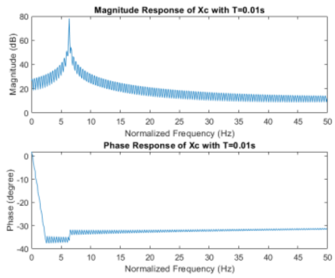
(a) $x_c(t) = 5e^{j40t} + 3e^{-j70t}$, with sampling period $T = 0.01, 0.04, 0.1$

$$e^{j\omega_0 n} \leftrightarrow 2\pi\delta(\omega-\omega_0), \sin(\omega_0 n) = \frac{\pi}{j}[\delta(\omega-\omega_0) - \delta(\omega+\omega_0)]$$

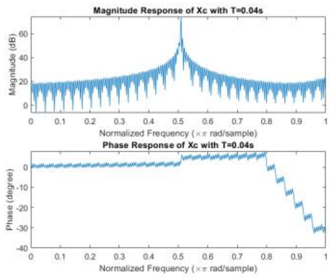
$$x[n] = x_c(nT) \longrightarrow X(e^{j\omega}) = 2\pi[5\delta(\omega - 40\pi T) + 3\delta(\omega + 70\pi T)]$$

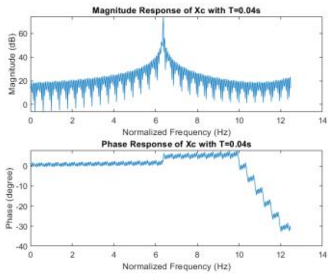
i. For T = 0.01s



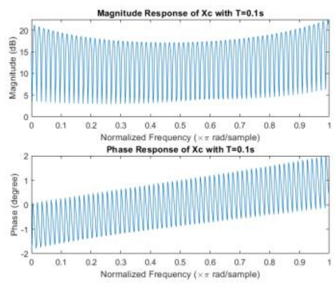


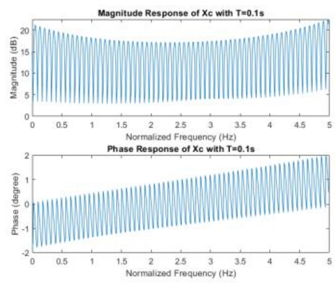
ii. For T = 0.04s





iii. For T = 0.1s





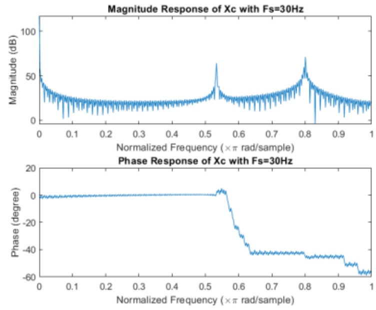
The sampling theorem shows that sampling signal can be recovered if $T_H \geq 2 \times T$, which T_H is $\pi/35$ s, which equal to 0.089s, in the signal $x_c(t)$.
Therefore, the sampling signals with $T = 0.01s$ and $0.04s$ can be recovered.

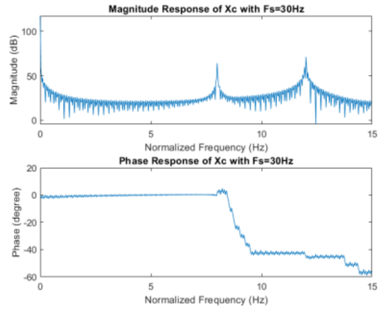
(b) $x_c(t) = 3 + 2\sin(16\pi t) + 10\cos(24\pi t)$, with sampling rate $F_S = 30, 20, 15$ Hz.

$x[n] = x_c(nT), T = F_S^{-1}$

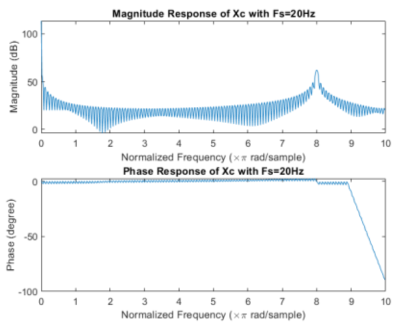
$$\Rightarrow X(e^{j\omega}) = 6\pi\delta(\omega) + \frac{5\pi}{j}\left[\delta\left(\omega - \frac{16\pi}{F_S}\right) - \delta\left(\omega + \frac{16\pi}{F_S}\right)\right] + 10\pi\left[\delta\left(\omega - \frac{24\pi}{F_S}\right) + \delta\left(\omega + \frac{24\pi}{F_S}\right)\right]$$

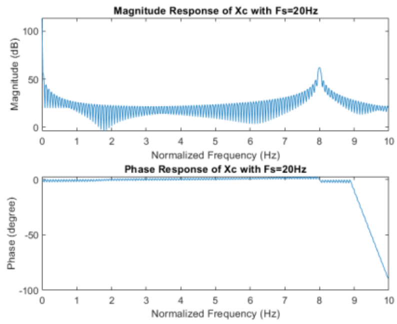
i. For $F_S = 30\text{Hz}$



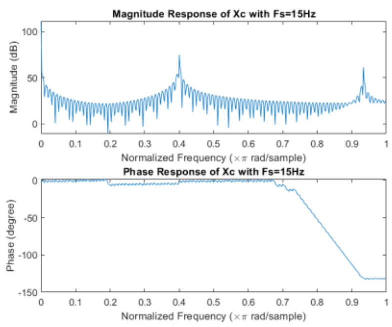


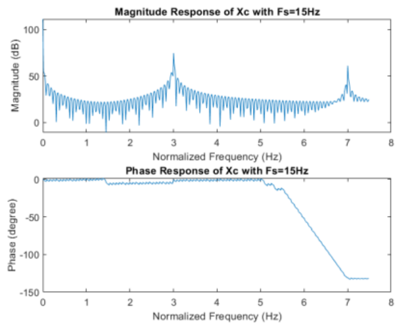
ii. For $F_S = 20\text{Hz}$





iii. For $F_S = 15\text{Hz}$





The sampling theorem shows that sampling signal can be recovered if $F_S \geq 2 \times F_H$, which F_H is 12Hz in the signal $x_c(t)$.
Therefore, only the sampling signal with $F_S = 30$ Hz can be recovered.

7. An 8-bit ADC has an input analog range of ±5 volts. The analog input signal is

$$x_c(t) = 2\cos(200\pi t) + 3\sin(500\pi t)$$

The converter supplies data to a computer at a rate of 2048 bits/s. The computer, without processing, supplies these data to an ideal DAC to form the reconstructed signal $y_c(t)$. Determine:

(a) The quantizer resolution (or step).

$B = 8$, $\Delta = \left(5V - (-5V)\right)/2^8 = 39m$

The quantizer step (Δ) is $39mV$.

(b) The SQNR in dB.

$\mathrm{SQNR(dB)} = 10\log_{10}\mathrm{SQNR} = 6.02B + 1.76 = 49.92\ \mathrm{dB}$

(c) The folding frequency and the Nyquist rate.

i. Folding frequency (= 0.5 Sampling Frequency)

(2048 bits/s)/(8-bit/sample) = 256 samples/s

Sampling Rate = 256-bit/sample ⇒ Folding frequency = 128 Hz

ii. Nyquist rate (= $2\Omega_H$ (rad/sec))

$F_H = \max(100\mathrm{Hz}, 250\mathrm{Hz}) = 250\mathrm{Hz}$

$\Rightarrow \Omega_H = 250 \times (2\pi) = 1570.8 \Rightarrow$ Nyquist rate $= 3141.6$ (rad/sec)

HW3 Program Assignment

By: 105060012 張育菘

P8

Compute and plot the phase response using the functions freqz, angle, phasez, unwrap, and phasedelay for the following systems:

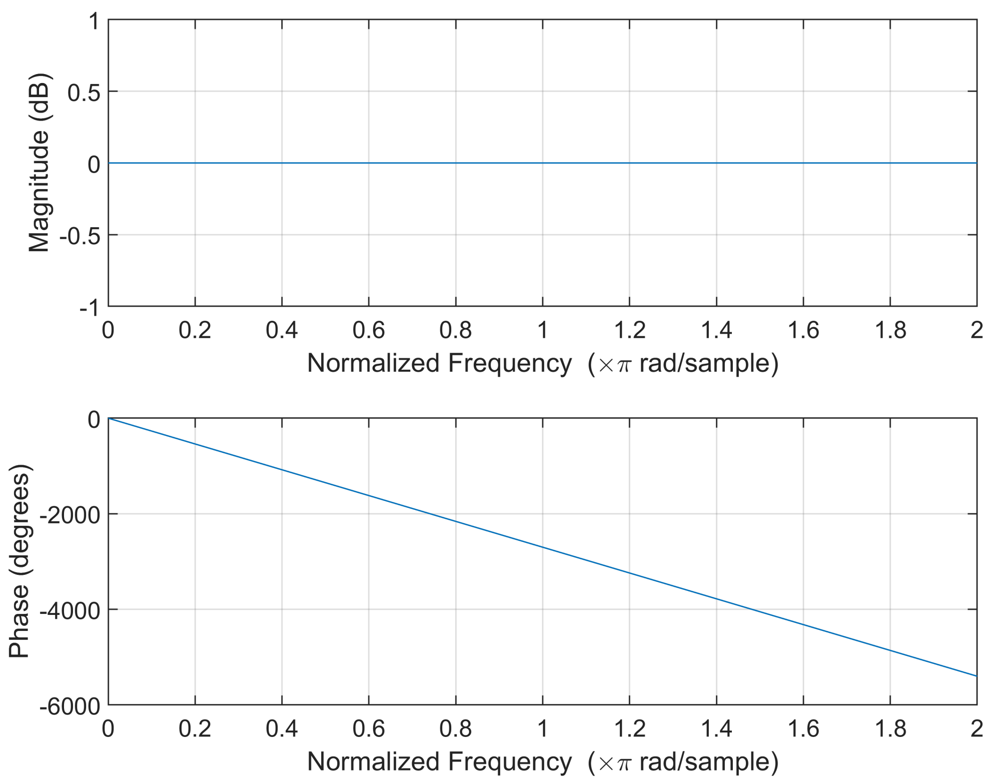
(a) $y[n] = x[n - 15]$

```
close all; clear;
fprintf('8(a)\n');
```

```
8(a)
```

```
om = linspace(0, 2*pi, 1000);
b = zeros(1, 16); b(16) = 1; a = [1];
fprintf('By "freqz"\n'); freqz(b, a, om)
```

```
By "freqz"
```

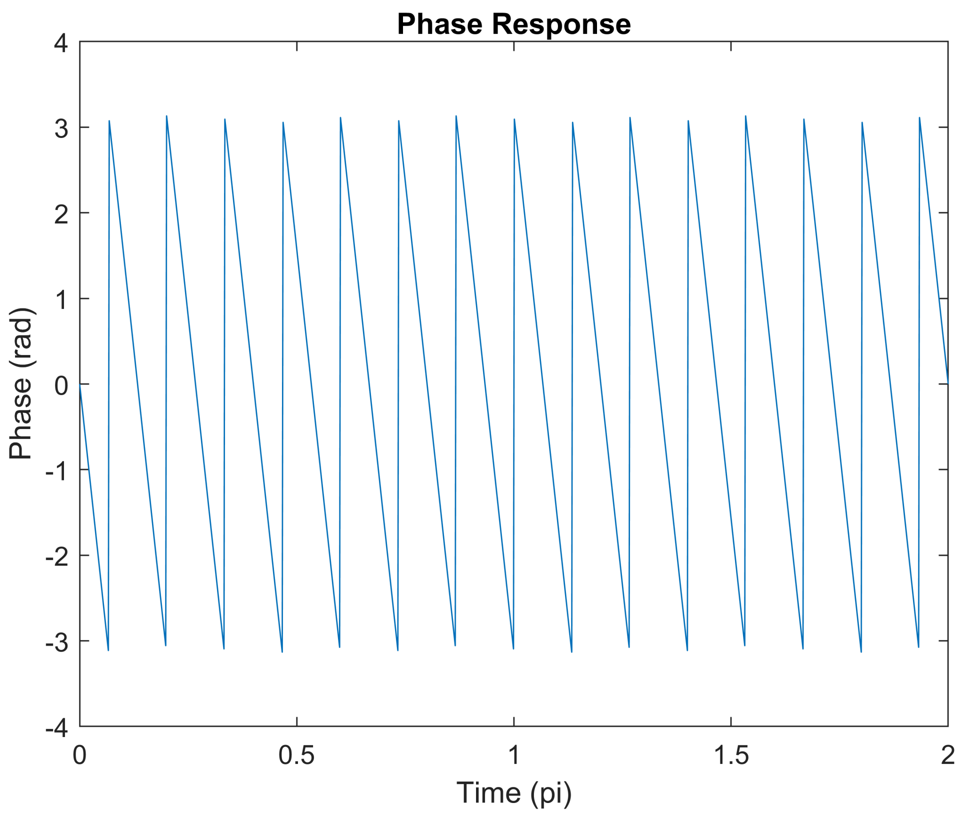



```
H1 = freqz(b, a, om);
```

```
figure;
fprintf('By "angle"\n');
```

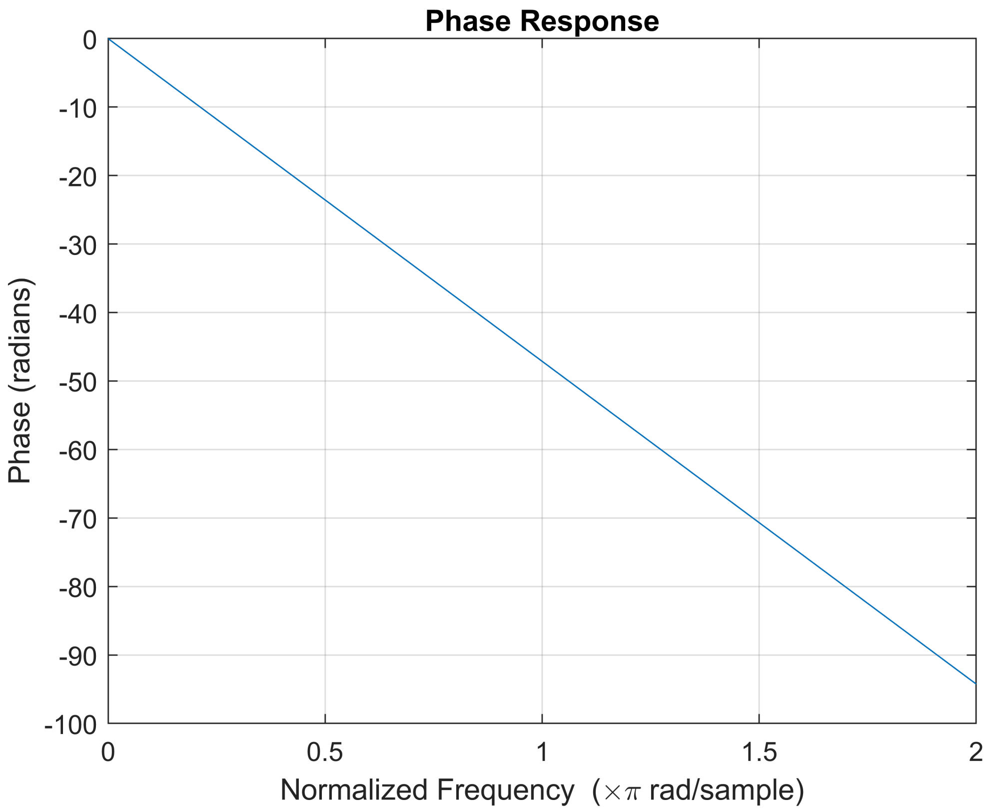
```
By "angle"
```

```
plot(om/pi, angle(H1));
title('Phase Response');
ylabel('Phase (rad)');
xlabel('Time (pi)');
```




```
fprintf('By "phasez"\n'); phasez(b, a, om)
```

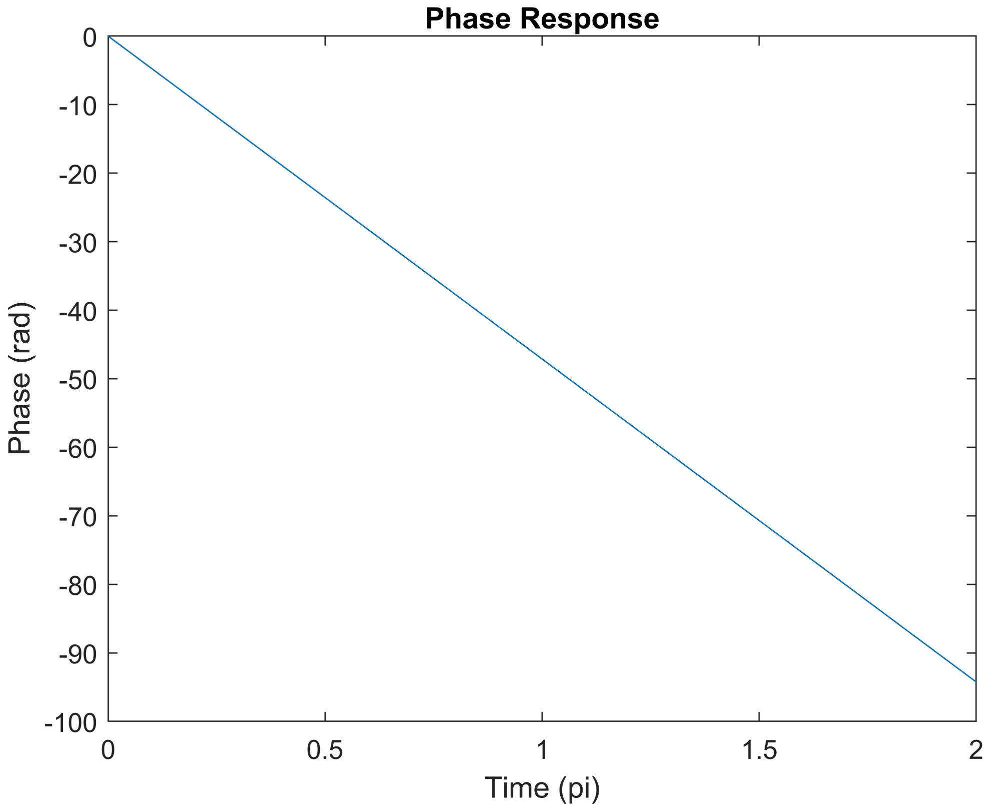
```
By "phasez"
```




```
figure;
fprintf('By "unwrap"\n');
```

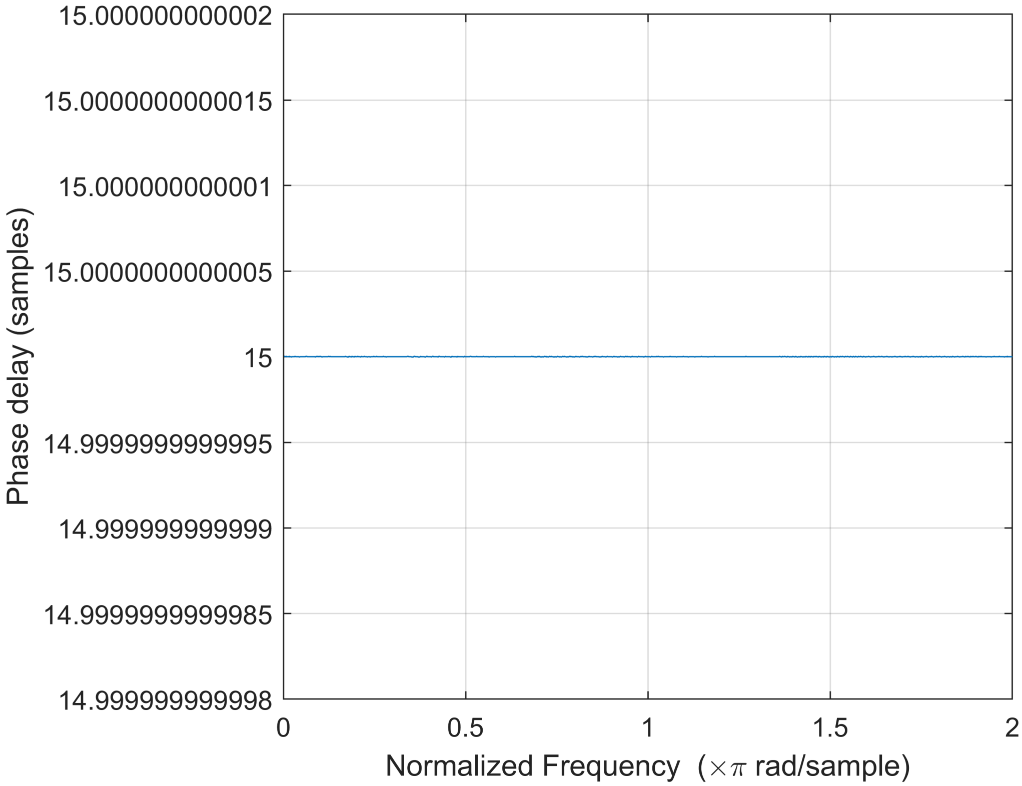
```
By "unwrap"
```

```
plot(om/pi, unwrap(angle(H1)));
title('Phase Response');
ylabel('Phase (rad)');
xlabel('Time (pi)');
```




```
fprintf('By "phasedelay"\n'); phasedelay(b, a, om)
```

```
By "phasedelay"
```



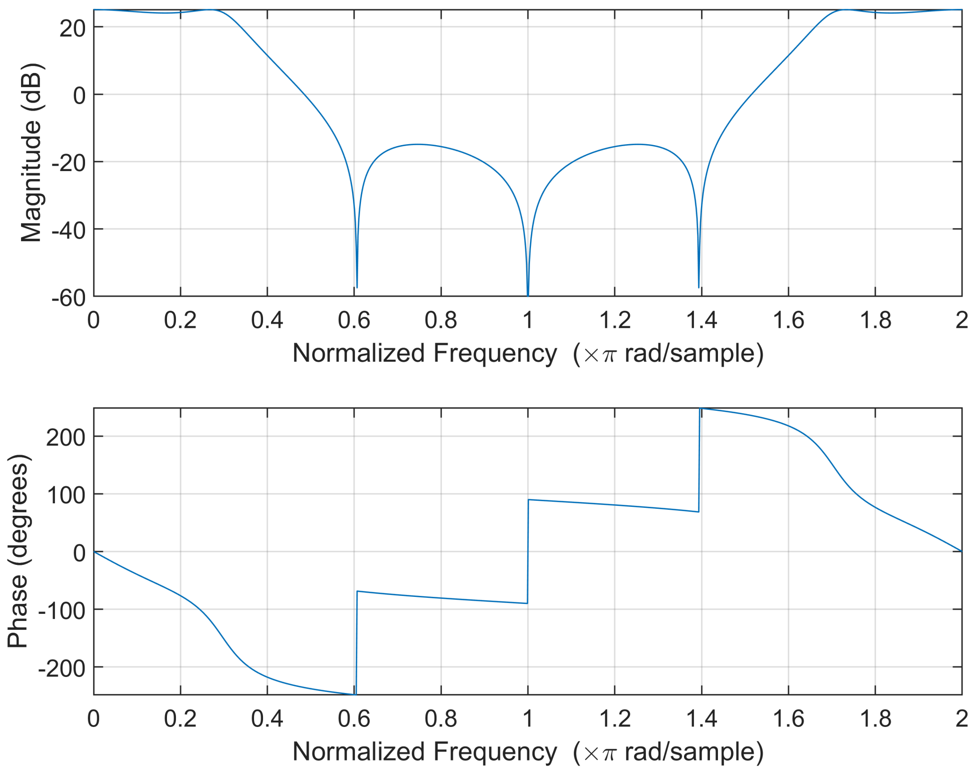
(b) $H(z) = \dfrac{1 + 1.655z^{-1} + 1.655z^{-2} + z^{-3}}{1 - 1.57z^{-1} + 1.264z^{-2} - 0.4z^{-3}}$

```
fprintf('8(b)\n');
```

```
8(b)
```

```
om = linspace(0, 2*pi, 1000);
b = [1 1.655 1.655 1]; a = [1 -1.57 1.264 -0.4];
fprintf('By "freqz"\n'); freqz(b, a, om)
```

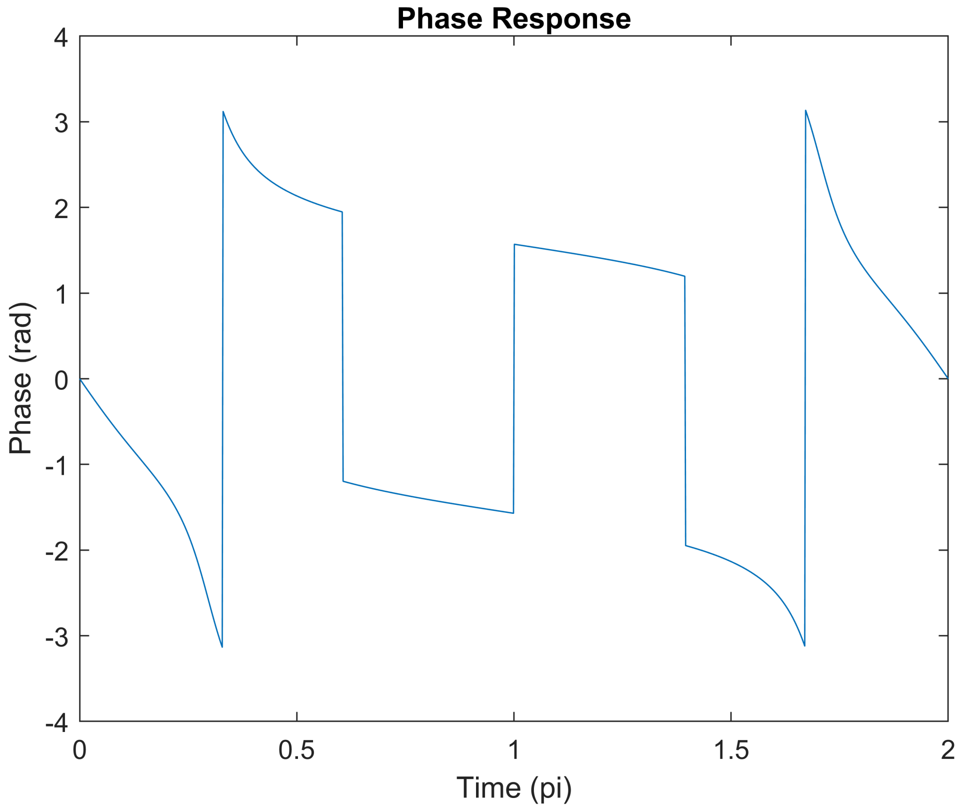
```
By "freqz"
```




```
H1 = freqz(b, a, om);
figure;
fprintf('By "angle"\n');
```

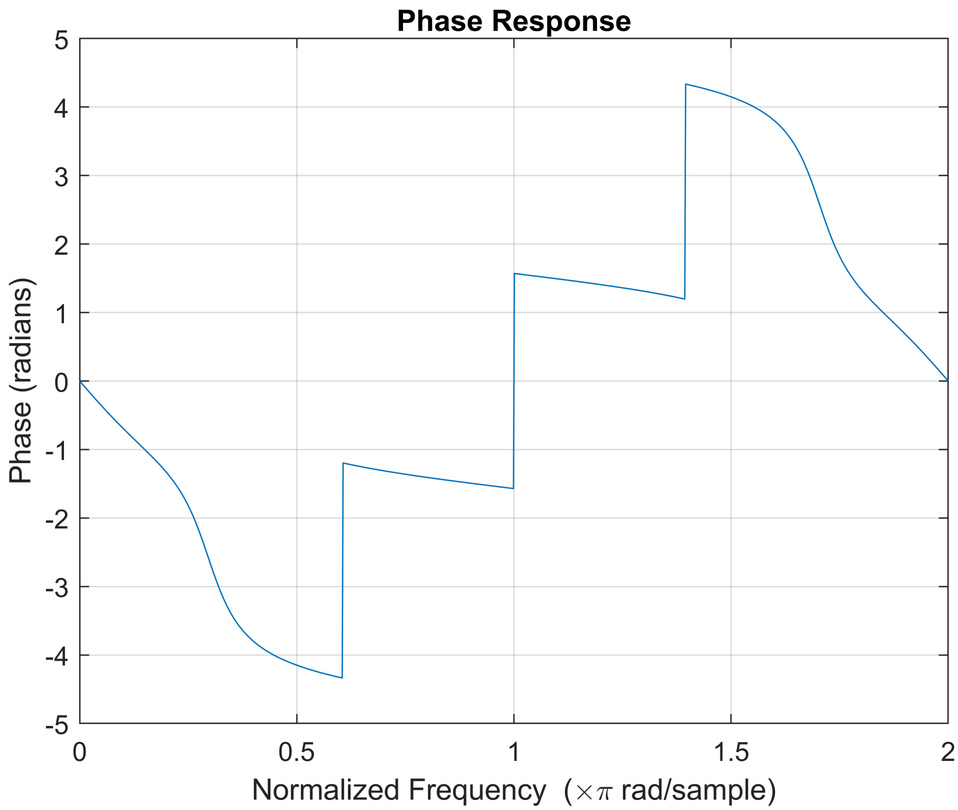
```
By "angle"
```

```
plot(om/pi, angle(H1));
title('Phase Response');
ylabel('Phase (rad)');
xlabel('Time (pi)');
```




```
fprintf('By "phasez"\n'); phasez(b, a, om)
```

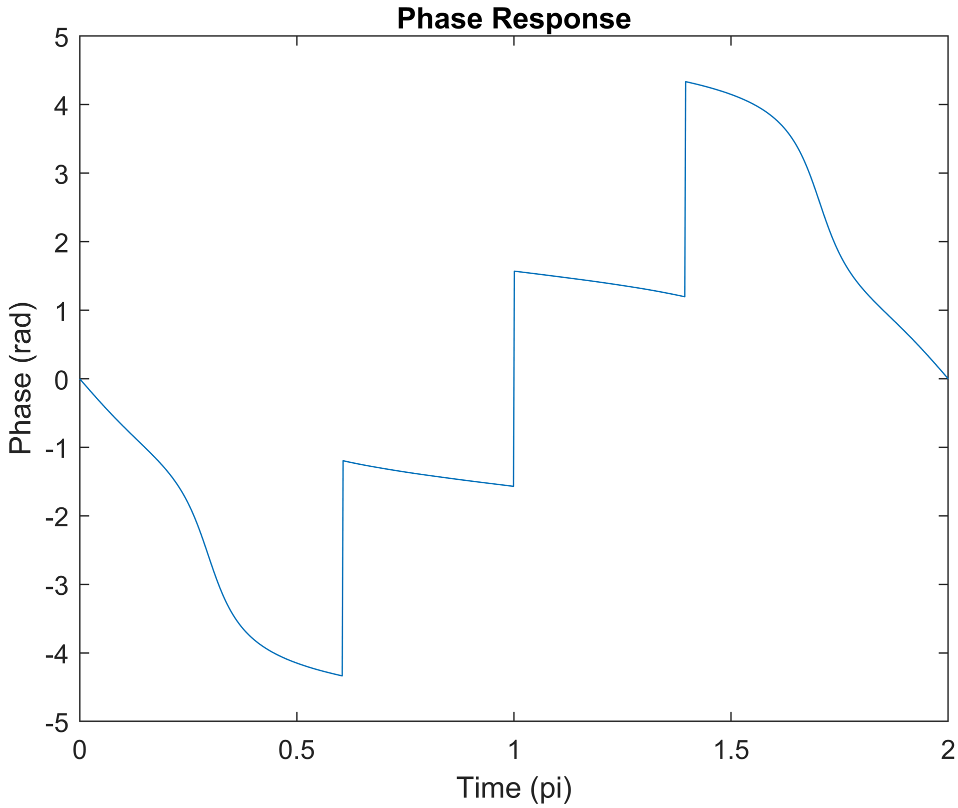
```
By "phasez"
```




```
figure;
fprintf('By "unwrap"\n');
```

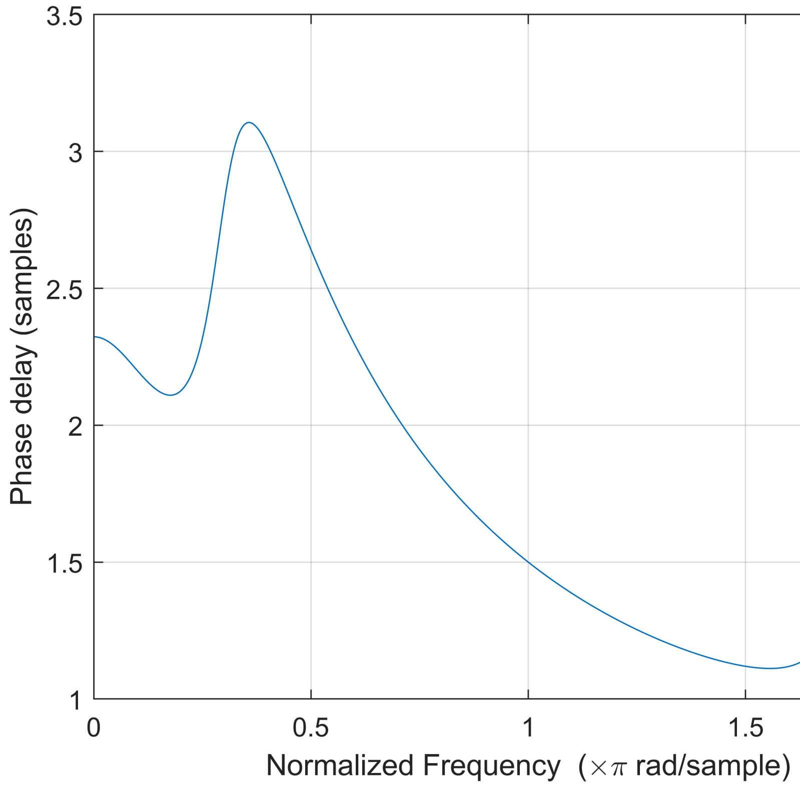
```
By "unwrap"
```

```
plot(om/pi, unwrap(angle(H1)));
title('Phase Response');
ylabel('Phase (rad)');
xlabel('Time (pi)');
```




```
fprintf('By "phasedelay"\n'); phasedelay(b, a, om)
```

```
By "phasedelay"
```



P9

According to problem 2 in paper assignment, plot magnitude response, phase response and group-delay response for each of the systems.

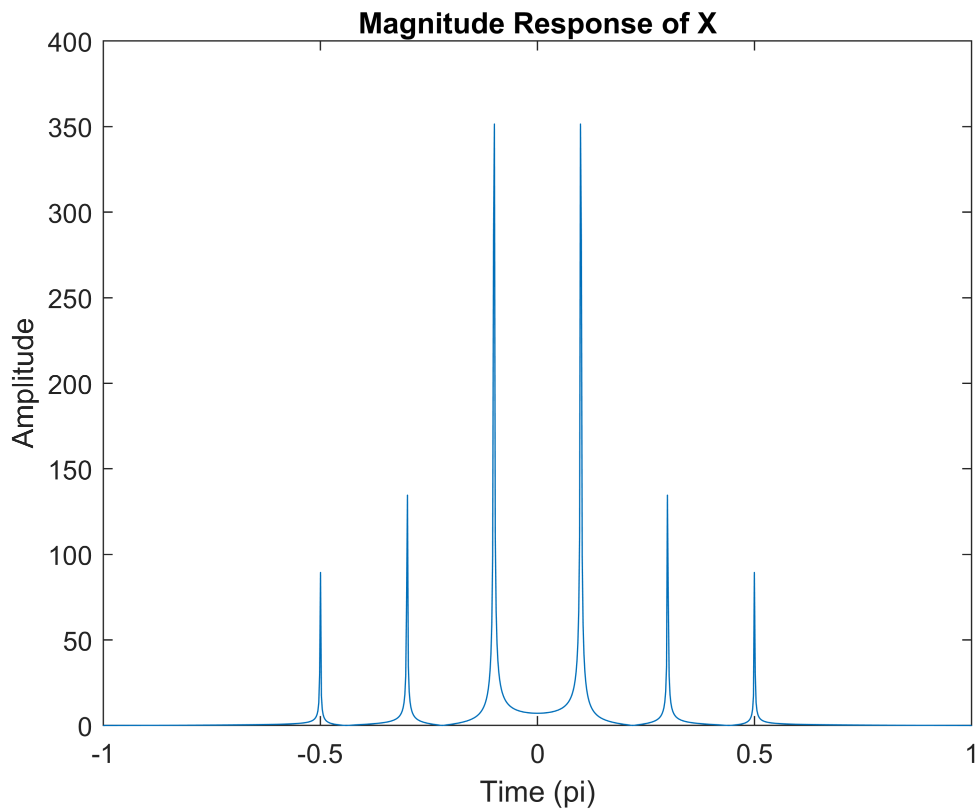
```
close all; clear;
fprintf('9\n');
```

```
9
```

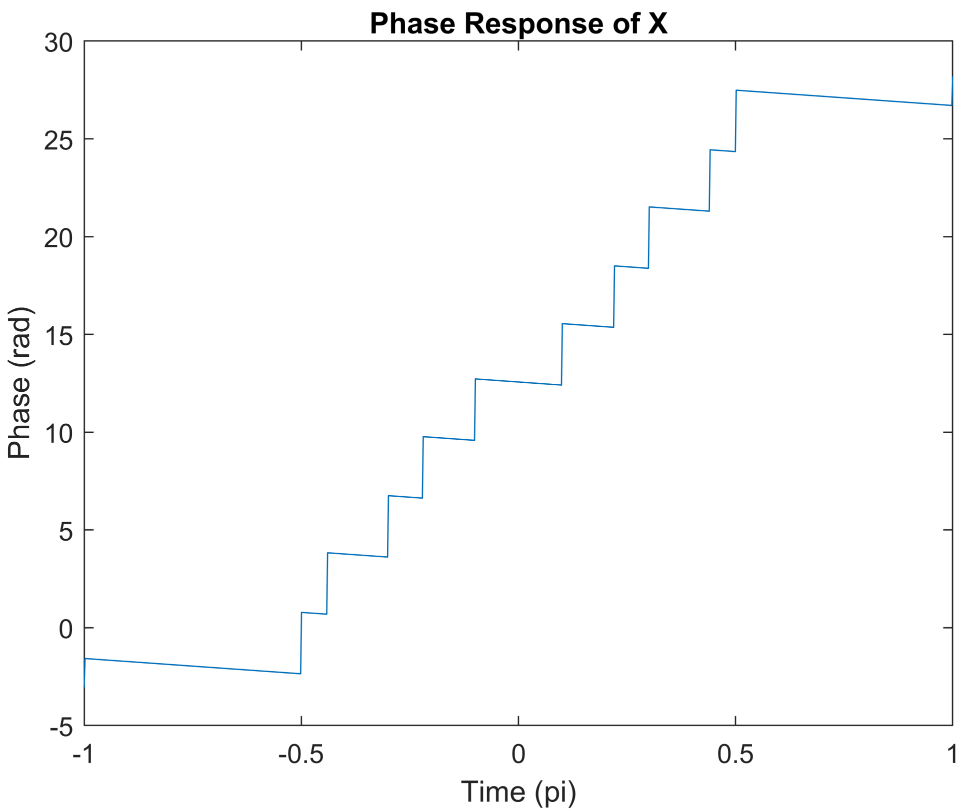
```

om=linspace(-pi, pi, 1000);
n = linspace(0,1000,1001);
x = sin(0.1*pi*n)+(1/3)*sin(0.3*pi*n)+0.2*sin(0.5*pi*n);
X = freqz(x,1,om);
gd = grpdelay(x,1,om);

figure; plot(om/pi, abs(X));
title('Magnitude Response of X');
ylabel('Amplitude');
xlabel('Time (pi)');
```




```
figure; plot(om/pi, unwrap(angle(X)));
title('Phase Response of X');
ylabel('Phase (rad)');
xlabel('Time (pi)');
```



(a) $h[n] = \{1_{n=0}, -2, 3, -4, 0, 4, -3, 2, -1\}$

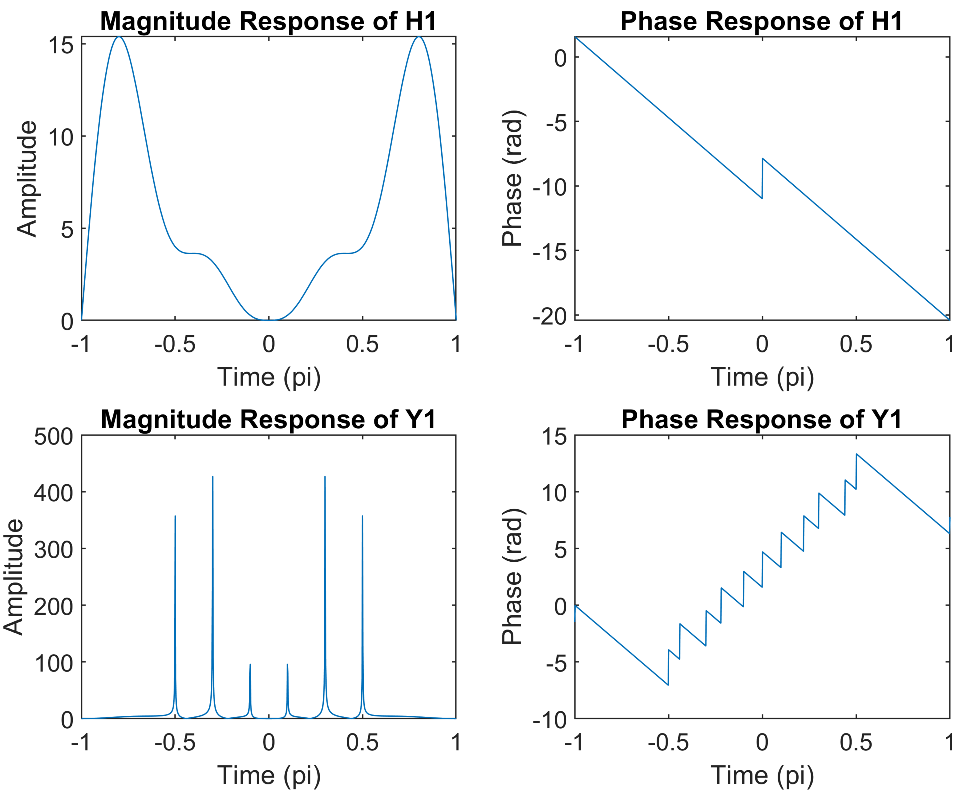
```
fprintf('9(a)\n');
```

```
9(a)
```

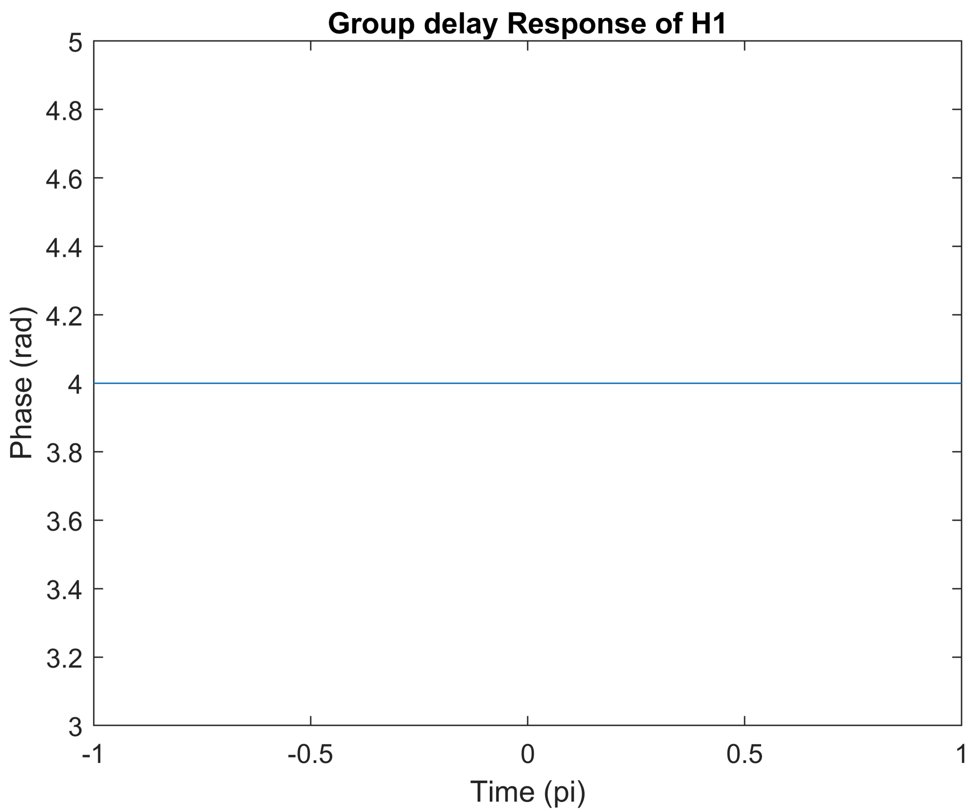
```
h1 = [1 -2 3 -4 0 4 -3 2 -1]; % start from n=0
H1 = freqz(h1,1,om); H1 = exp(-j*om*0).*H1;
gd1 = grpdelay(h1, 1, om);

figure;
subplot(2,2,1); plot(om/pi, abs(H1));
title('Magnitude Response of H1');
ylabel('Amplitude');
xlabel('Time (pi)');
subplot(2,2,2); plot(om/pi, unwrap(angle(H1)));
title('Phase Response of H1');
ylabel('Phase (rad)');
xlabel('Time (pi)');
Y1 = X.*H1;
subplot(2,2,3); plot(om/pi, abs(Y1));
title('Magnitude Response of Y1');
ylabel('Amplitude');
xlabel('Time (pi)');
```

```
subplot(2,2,4); plot(om/pi, unwrap(angle(Y1)));
title('Phase Response of Y1');
ylabel('Phase (rad)');
xlabel('Time (pi)');
```




```
figure; plot(om/pi, gd1);
title('Group delay Response of H1')
ylabel('Phase (rad)');
xlabel('Time (pi)');
```



(b) $y[n] = 10x[n-10]$

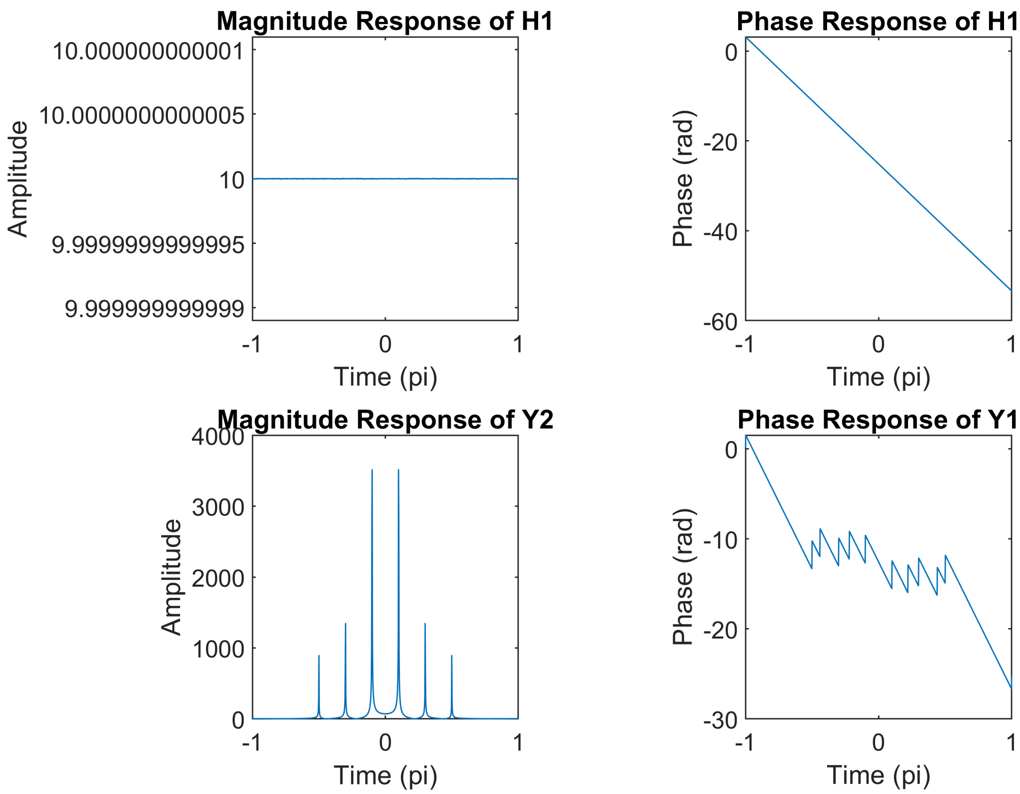
```
fprintf('9(b)\n');
```

```
9(b)
```

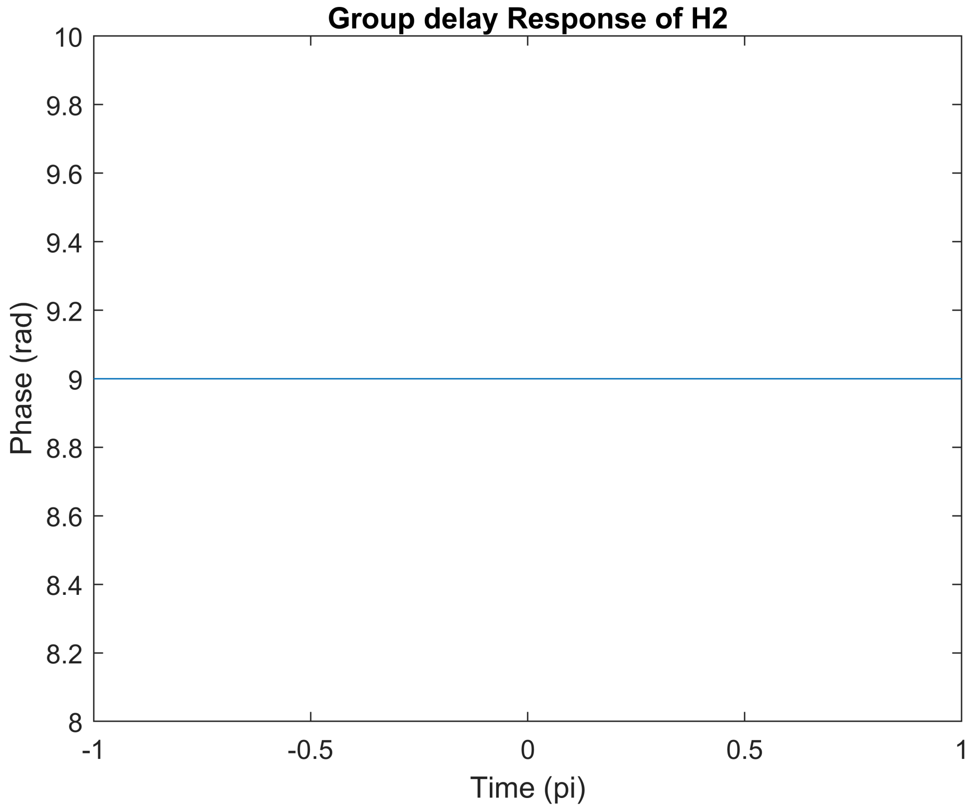
```
b = zeros(1, 10); b(10) = 10;
a = [1];
H2 = freqz(b,a,om);
gd2 = grpdelay(b, a, om);

figure;
subplot(2,2,1); plot(om/pi, abs(H2));
title('Magnitude Response of H1');
ylabel('Amplitude');
xlabel('Time (pi)');
subplot(2,2,2); plot(om/pi, unwrap(angle(H2)));
title('Phase Response of H1');
ylabel('Phase (rad)');
xlabel('Time (pi)');
Y2 = X.*H2;
subplot(2,2,3); plot(om/pi, abs(Y2));
title('Magnitude Response of Y2');
ylabel('Amplitude');
```

```
xlabel('Time (pi)');
subplot(2,2,4); plot(om/pi, unwrap(angle(Y2)));
title('Phase Response of Y1');
ylabel('Phase (rad)');
xlabel('Time (pi)');
```




```
figure; plot(om/pi, gd2);
title('Group delay Response of H2')
ylabel('Phase (rad)');
xlabel('Time (pi)');
```



P10

MATLAB provides a function called polystab that stabilizes the given polynomial with respect to the unit circle, that is, it reflects those roots which are outside the unit-circle into those that are inside the unit circle but with the same angle.

Using this function, convert the following systems into minimum-phase and maximum-phase systems. Verify your answers using a pole-zero plot for each system(plot minimum-phase and maximum-phase systems for each question).

(a) $H(z) = \dfrac{z^2 + 2z + 0.75}{z^2 - 0.5z}$

Ans:

zero = -1/2 & -3/2, pole = 1/2

For Hmin: zero = -1/2 & -2/3, pole = 1/2

$$\Longrightarrow H_{\min} = \frac{\left(1 + \frac{1}{2}z^{-1}\right)\left(1 + \frac{2}{3}z^{-1}\right)}{\left(1 - \frac{1}{2}z^{-1}\right)} = \frac{1 + \frac{7}{6}z^{-1} + \frac{1}{3}z^{-2}}{1 - \frac{1}{2}z^{-1}}$$

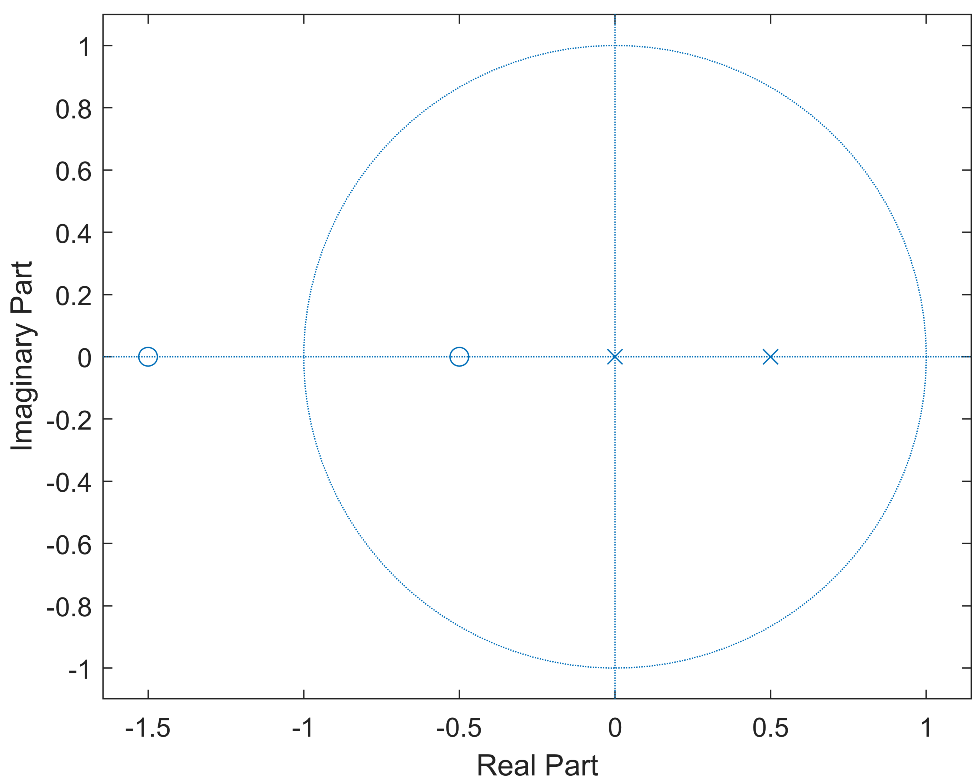
For Hmax: zero = -2 & -3/2, pole = 1/2

$$\Longrightarrow H_{\max} = \frac{\left(1+\frac{1}{2}z\right)\left(1+\frac{2}{3}z\right)}{\left(1-\frac{1}{2}z^{-1}\right)} = \frac{1+\frac{7}{6}z^{1}+\frac{1}{3}z^{2}}{1-\frac{1}{2}z^{-1}}$$

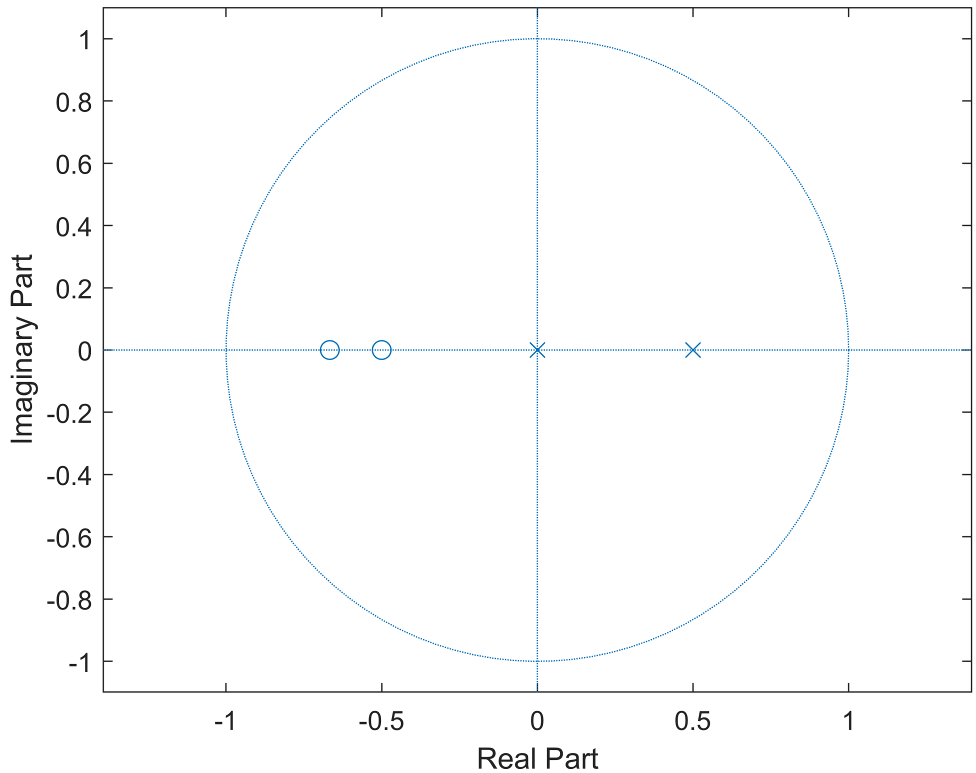
```
close all; clear;
fprintf('10(a)\n');
```

```
10(a)
```

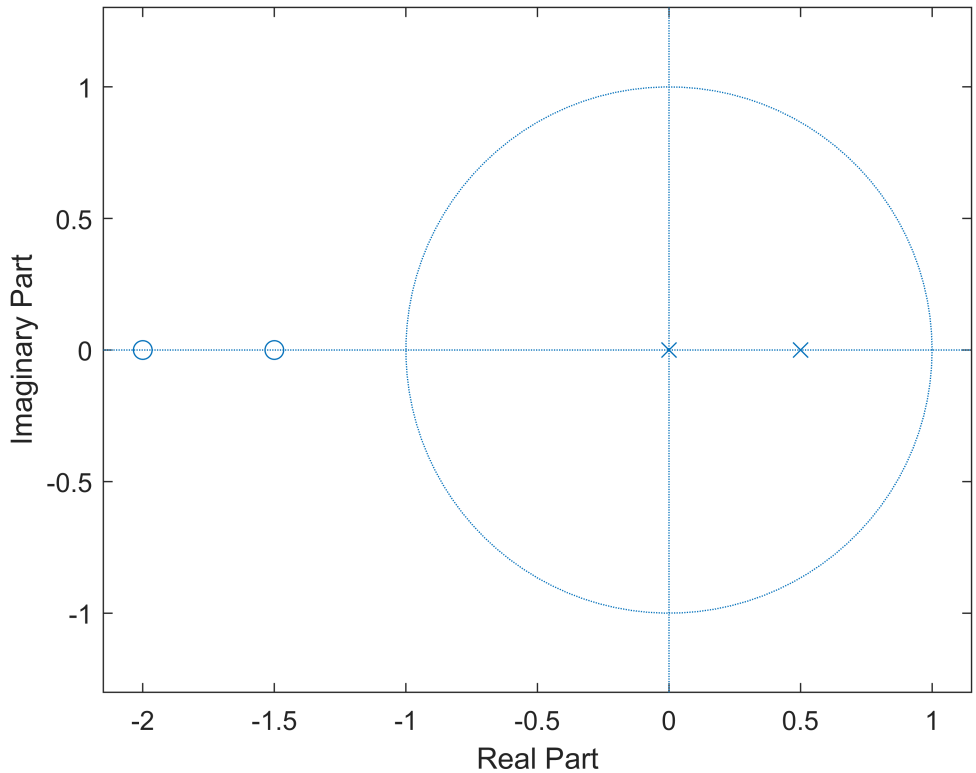
```
b = [1 2 0.75]; a = [1 -0.5];
zplane(b, a);
```



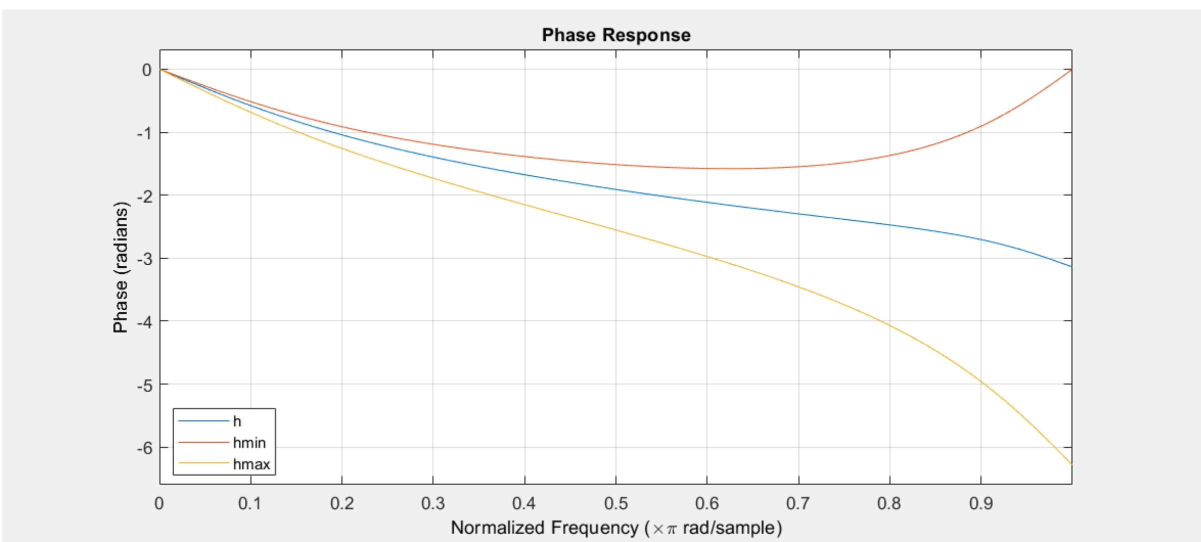

```
b_min = polystab(b); a_min = polystab(a);
zplane(b_min, a_min);
```




```
b_max = fliplr(b_min); a_max = a_min;
zplane(b_max, a_max);
```




```
hfvt = fvtool(b, a, b_min, a_min, b_max, a_max,'Analysis','phase');
legend(hfvt,'h','hmin','hmax');
```

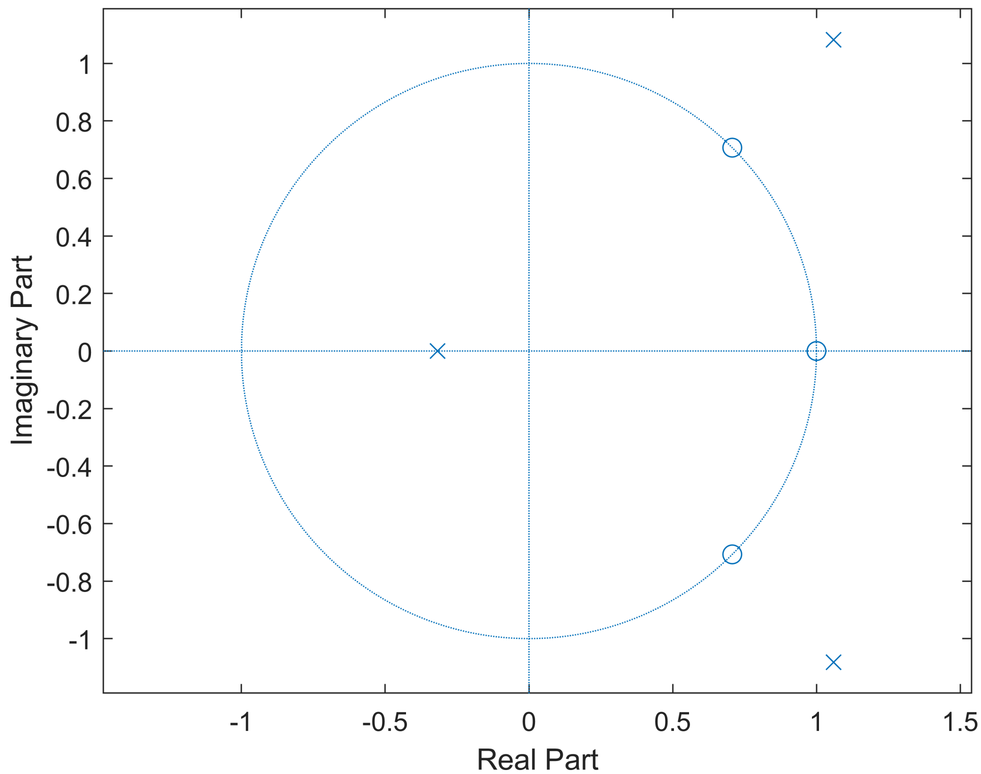


(b) $H(z) = \dfrac{1 - 2.4142z^{-1} + 2.4142z^{-2} - z^{-3}}{1 - 1.8z^{-1} + 1.62z^{-2} + 0.729z^{-3}}$

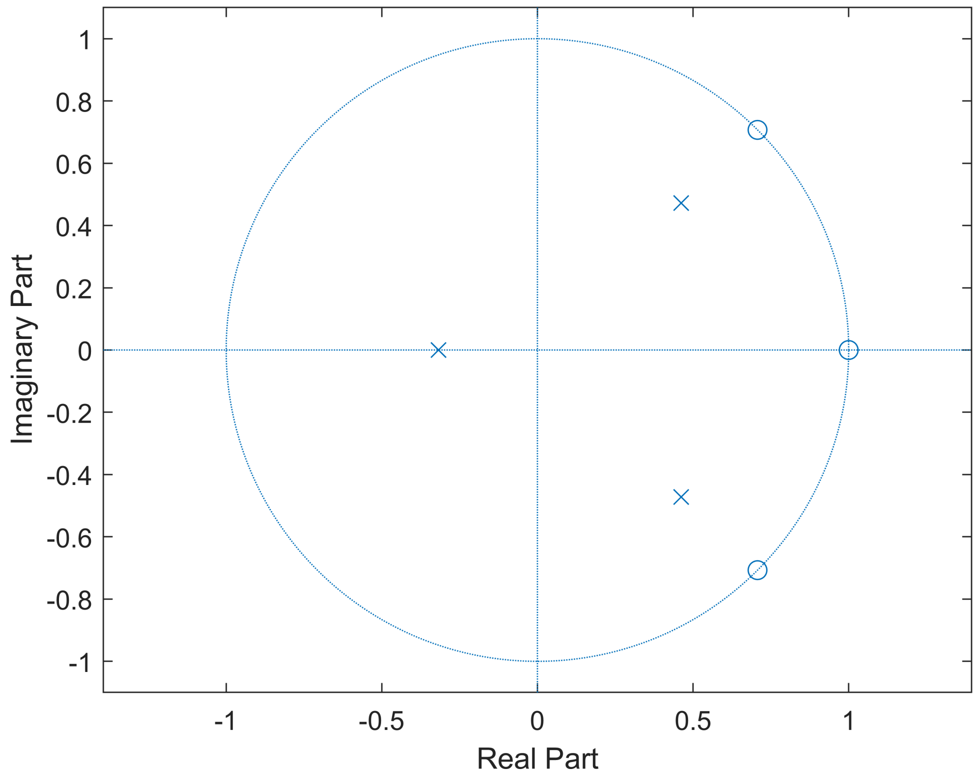
```
fprintf('10(b)\n');
```

10(b)

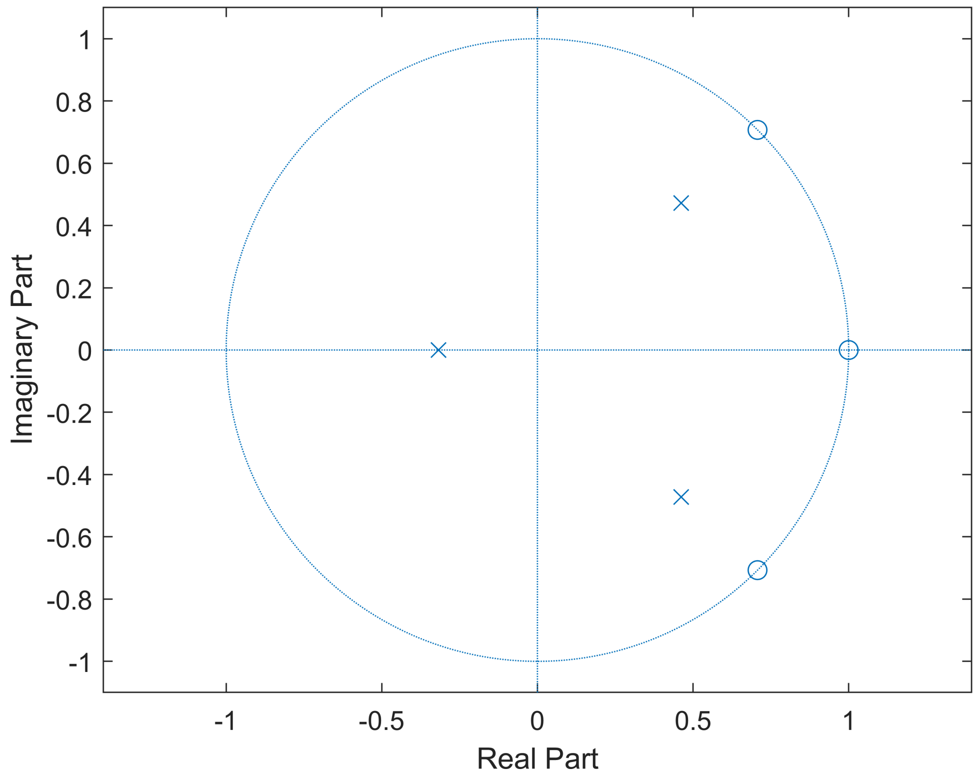
```
b = [1 -2.4142 2.4142 -1]; a = [1 -1.8 1.62 0.729];
zplane(b, a);
```



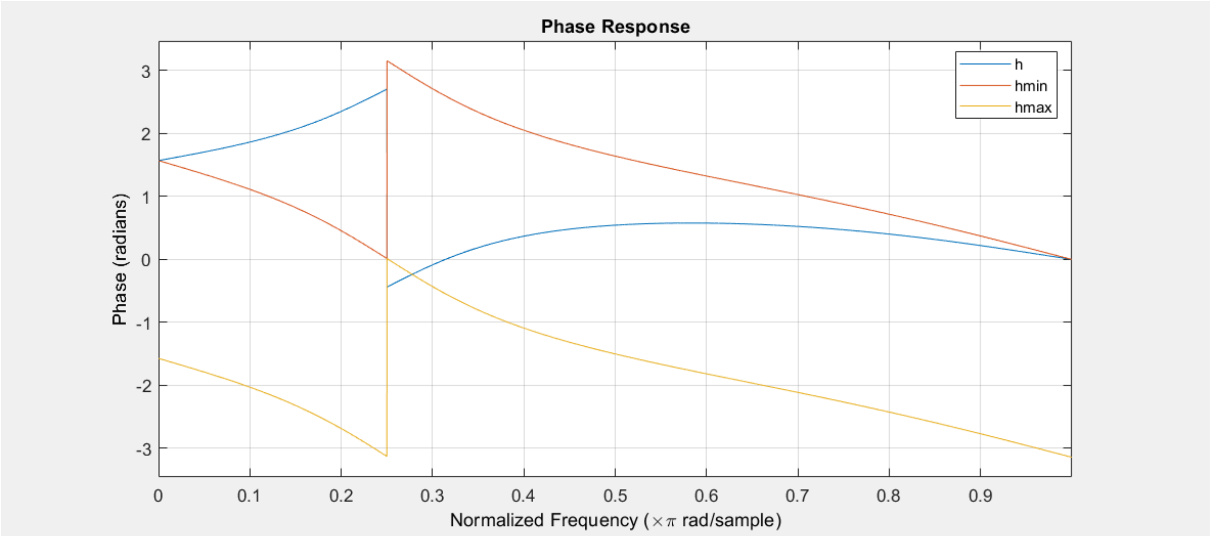

```
b_min = polystab(b); a_min = polystab(a);
zplane(b_min, a_min);
```




```
b_max = fliplr(b_min); a_max = a_min;
zplane(b_max, a_max);
```




```
hfvt = fvtool(b, a, b_min, a_min, b_max, a_max,'Analysis','phase');
legend(hfvt,'h','hmin','hmax');
```



P11

Signal $x_c(t) = 5\cos(200\pi t + \pi 6) + 4\sin(300\pi t)$ is sampled at a rate of $F_S = 1\text{kHz}$ to obtain the discrete-time signal $x[n]$.

(a) Determine the spectrum $X(e^{j\omega})$ of $x[n]$ and plot its magnitude as a function of ω in $\frac{\text{rad}}{\text{sample}}$ and as a function of F in Hz. Explain whether the original signal $x_c(t)$ can be recovered from $x[n]$.

$$\omega = 2\pi \text{FT} = 2\pi \frac{F}{F_s}$$

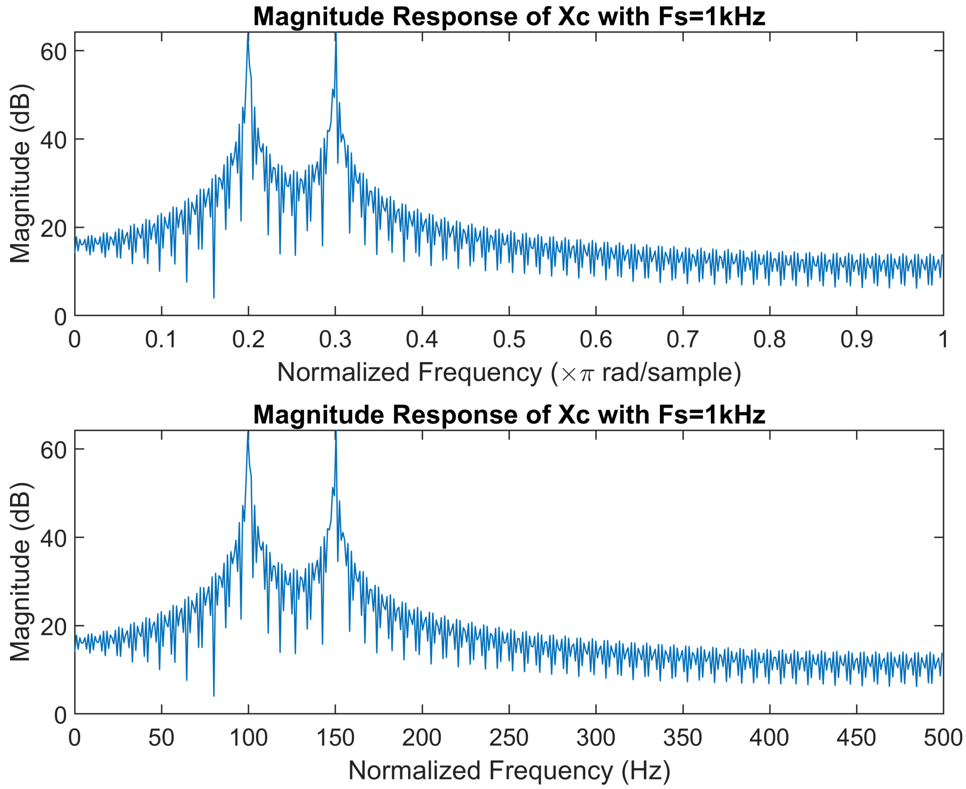
```
close all; clear;
fprintf('11(a)\n');
```

```
11(a)
```

```

t = 1000; Fs = 1000;
om = linspace(0, t, Fs*t);
x_a = 5*cos(200*pi*om + 6*pi) + 4*sin(300*pi*om);

[h_a, w_a] = freqz(x_a, 1);
figure;
subplot(2,1,1); plot(w_a/pi, 20*log10(abs(h_a)));
title('Magnitude Response of Xc with Fs=1kHz');
ylabel('Magnitude (dB)');
xlabel('Normalized Frequency (\times\pi rad/sample)');
subplot(2,1,2); plot(w_a*Fs/(2*pi), 20*log10(abs(h_a)));
title('Magnitude Response of Xc with Fs=1kHz');
ylabel('Magnitude (dB)');
xlabel('Normalized Frequency (Hz)');
```




```
% figure; stem((0:length(x_a)-1),x_a);
% title('x_c(t) with Fs=1kHz');
% ylabel('x[n]');
% xlabel('Time index (n)');
```

The frequency of signals $5\cos(200\pi t + \pi 6)$ and $4\sin(300\pi t)$ are 100Hz and 150Hz, respectively. This means that there are peaks in both 100Hz and 150Hz in the frequency domain of the original signal $x_c(t)$.

Due to sampling theorem, sampling signal can be recovered if the sampling rate (F_S) is larger than two times highest frequency in the signal(F_H), which means that $F_s \geq 2F_H$.

The original signal $x_c(t)$ can be recovered from $x[n]$ because $F_s = 1\text{kHz} \geq 2F_H = 2 \times (150)\text{Hz}$.

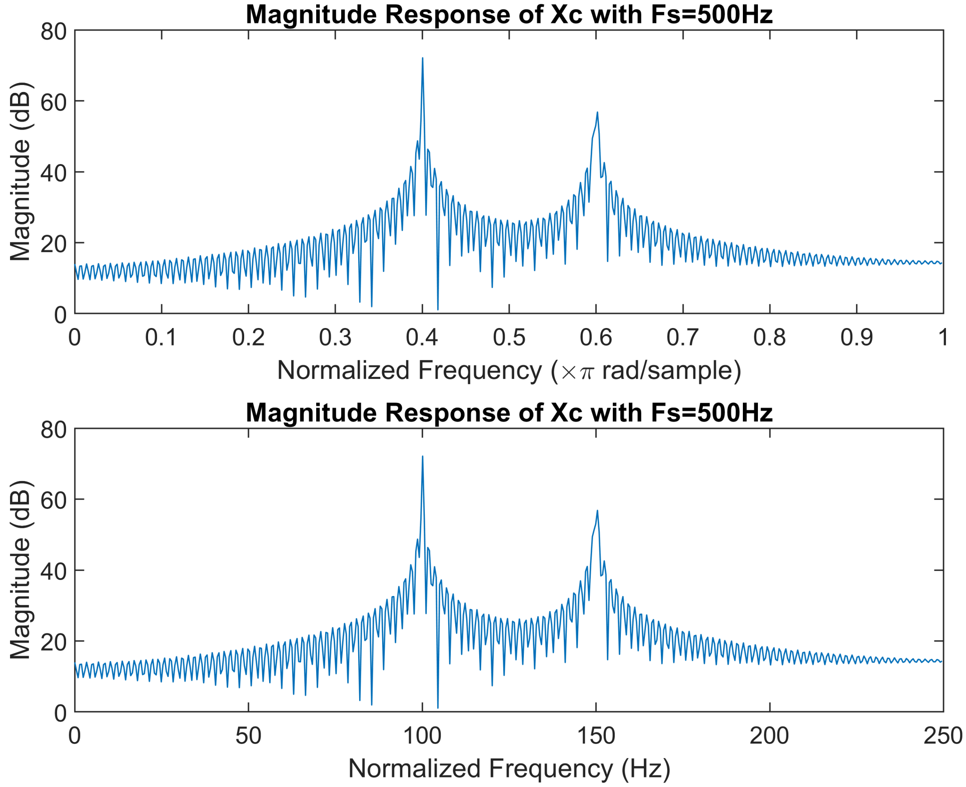
(b) Repeat part (a) for Fs = 500 Hz.

```
fprintf('11(b)\n');
```

```
11(b)
```

```
Fs = 500;
om = linspace(0, t, Fs*t);
x_b = 5*cos(200*pi*om + 6*pi) + 4*sin(300*pi*om);
```

```
[h_b, w_b] = freqz(x_b, 1);
figure;
subplot(2,1,1); plot(w_b/pi, 20*log10(abs(h_b)));
title('Magnitude Response of Xc with Fs=500Hz');
ylabel('Magnitude (dB)');
xlabel('Normalized Frequency (\times\pi rad/sample)');
subplot(2,1,2); plot(w_b*Fs/(2*pi), 20*log10(abs(h_b)));
title('Magnitude Response of Xc with Fs=500Hz');
ylabel('Magnitude (dB)');
xlabel('Normalized Frequency (Hz)')
```



The original signal $x_c(t)$ can be recovered from $x[n]$ because $F_s = 500\text{Hz} \geq 2F_H = 2 \times (150)\text{Hz}$.

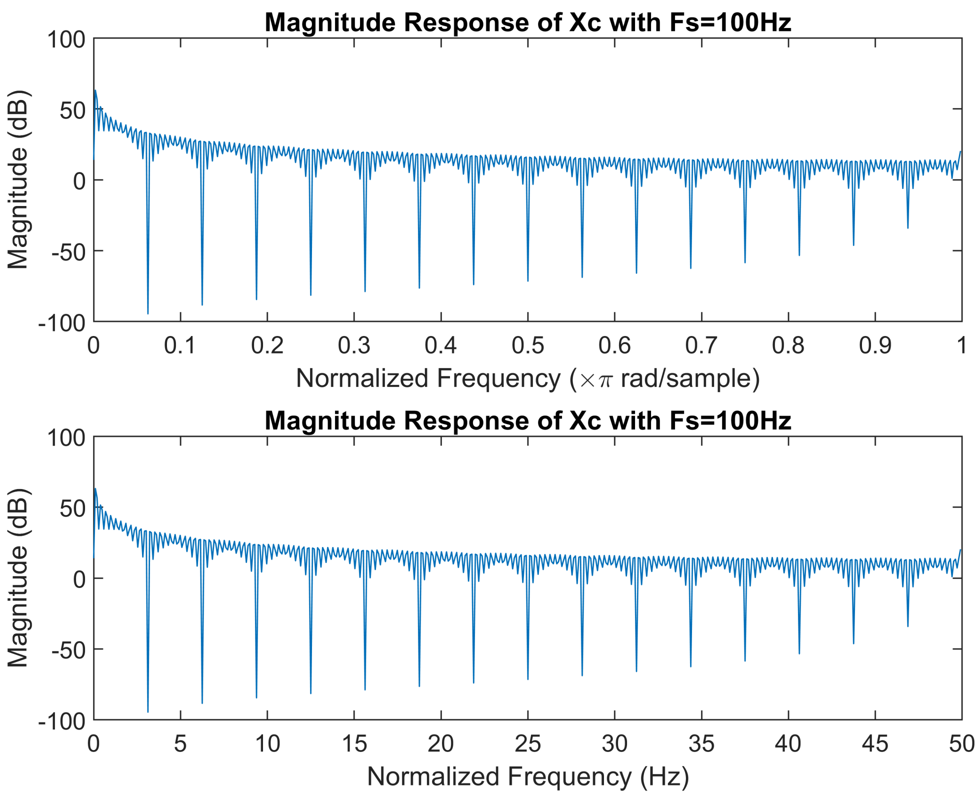
(c) Repeat part (a) for Fs = 100

```
fprintf('11(c)\n');
```

```
11(c)
```

```
Fs = 100; t = 1000;
om = linspace(0, t, Fs*t);
x_c = 5*cos(200*pi*om + 6*pi) + 4*sin(300*pi*om);
```

```
[h_c, w_c] = freqz(x_c, 1);
figure;
subplot(2,1,1); plot(w_c/pi, 20*log10(abs(h_c)));
title('Magnitude Response of Xc with Fs=100Hz');
ylabel('Magnitude (dB)');
xlabel('Normalized Frequency (\times\pi rad/sample)');
subplot(2,1,2); plot(w_c*Fs/(2*pi), 20*log10(abs(h_c)));
title('Magnitude Response of Xc with Fs=100Hz');
ylabel('Magnitude (dB)');
xlabel('Normalized Frequency (Hz)');
```



The original signal $x_c(t)$ can't be recovered from $x[n]$ because $F_s = 100\text{Hz} < 2F_H = 2\times(150)\text{Hz}$.

(d) Comment on your results.

Ans.

The sampling theorem shows that sampling signal can be recovered if $F_s \geq 2F_H$, which F_H is 150Hz in the signal $x_c(t)$. In the results above, only 11(c) doesn't meet this condition, so we can't see any peak in the plot of magnitude.