Part I. Paper Assignment

1. Determine the system function, magnitude response, and phase response of the following systems and use the pole-zero pattern to explain the shape of their magnitude response:

(a) $y[n] = \frac{1}{4}(x[n] + x[n-1]) - \frac{1}{4}(x[n-2] + x[n-3])$

System function: $H(z) = \frac{Y(z)}{X(z)} = \frac{1}{4}(1 + z^{-1} - z^{-2} - z^{-3}) = \frac{1}{4}(1 - z^{-1})(1 + z^{-1})^2$

⇨ Zero: 1, -1; Pole: 0

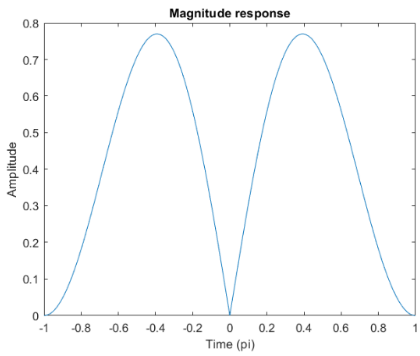
$z = e^{j\omega} = \cos(\omega) + j\sin(\omega)$

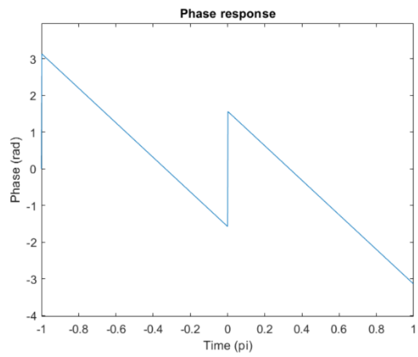
⇨ $H(e^{j\omega}) = 0.25[(1 + \cos(\omega) - \cos(2\omega) - \cos(3\omega)) + j(-\sin(\omega) + \sin(2\omega) + \sin(3\omega))]$

- Magnitude response: $|H(e^{j\omega})|$

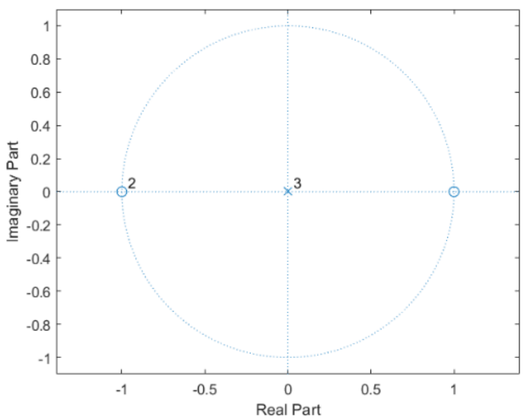
$$|H(e^{j\omega})| = 0.25\sqrt{(1 + \cos(\omega) - \cos(2\omega) - \cos(3\omega))^2 + (-\sin(\omega) + \sin(2\omega) + \sin(3\omega))^2}$$

- Phase response: $\angle H(e^{j\omega}) = \frac{-\sin(\omega) + \sin(2\omega) + \sin(3\omega)}{1 + \cos(\omega) - \cos(2\omega) - \cos(3\omega)}$





- Pole-zero pattern:



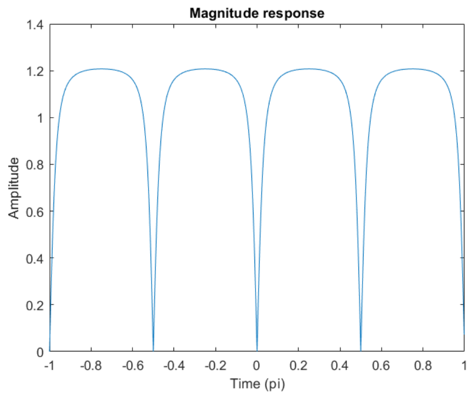
由 pole-zero pattern 我們可以推測 magnitude response 在 ±π 處會是 0，因為 zero 的緣故。

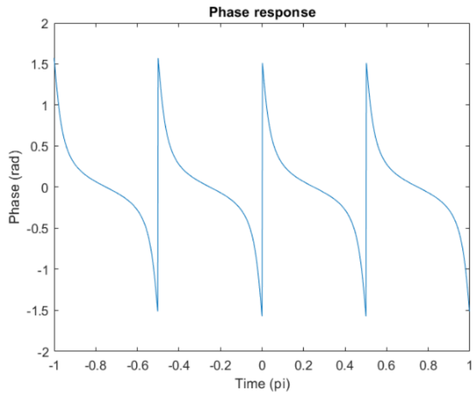
(b) $y[n] = x[n] - x[n-4] + 0.6561y[n-4]$

System function: $H(z) = \frac{1-z^{-4}}{1-0.6561z^{-4}}$ ➔ Zero: $\pm1, \pm j$; Pole: $\pm0.9, \pm0.9j$

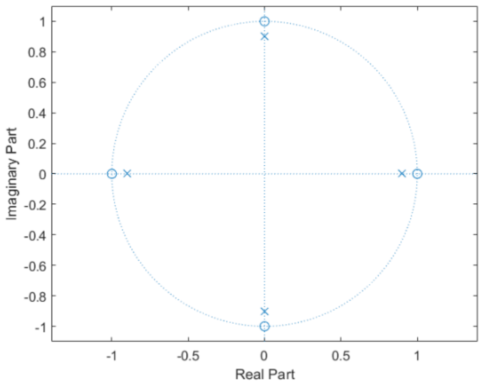
$$H(e^{j\omega}) = \frac{(1-\cos(4\omega))-j\sin(4\omega)}{(1-0.6561\cos(4\omega))-0.6561j\sin(4\omega)}$$

Magnitude response: $|H(e^{j\omega})|$ ； Phase response: $\angle H(e^{j\omega})$





Pole-zero pattern:



由 pole-zero pattern 我們可以推測 magnitude response 在 $\pm1, \pm j$ 處會是 0，因為 zero 的緣故。再加上在 zero 附近皆存在一個 pole，因此 magnitude response 的 rising time 和 falling time 會比 1(a)來的小。

2. Consider a periodic signal

$$x[n] = \sin(0.1\pi n) + \frac{1}{3}\sin(0.3\pi n) + \frac{1}{5}\sin(0.5\pi n).$$

For each of the following systems, determine if the system imparts (i) no distortion, (ii) magnitude distortion, and/or (iii) phase (or delay) distortion.

(a) $h[n] = \{1_{n=0}, -2, 3, -4, 0, 4, -3, 2, -1\}$

$H(z) = \sum_{n=-\infty}^{+\infty} h[n]z^{-n} = 1 - 2z^{-1} + 3z^{-2} - 4z^{-3} + 4z^{-5} - 3z^{-6} + 2z^{-7} - z^{-8}$

$\Rightarrow |H(e^{j\omega})| \neq \text{constant}, \angle H(e^{j\omega}) \neq -\omega n_d$

⇒ This system not only imparts magnitude but also phase distortion.

(b) $y[n] = 10x[n-10]$

$H(e^{j\omega}) = 10e^{-j10\omega} \Rightarrow |H(e^{j\omega})| = 10 = \text{constant}, \angle H(e^{j\omega}) = -10\omega$

⇒ This system imparts no distortion.

3. An economical way to compensate for the droop distortion in S/H DAC is to use an appropriate digital compensation filter prior to DAC.

(a) Determine the frequency response of such an ideal digital filter $H_r(e^{j\omega})$ that will perform an equivalent filtering given by following $H_r(j\Omega)$

$$H_r(j\Omega) = \begin{cases} \frac{\Omega T/2}{\sin(\Omega T/2)} e^{j\Omega T/2}, |\Omega| < \pi/T \\ 0, \text{otherwise} \end{cases}$$

$X_c(j\Omega) = \begin{cases} T\,X(e^{j\Omega T}), |\Omega| < \pi/T \\ 0, \text{otherwise} \end{cases} \leftrightarrow X(e^{j\Omega T}) = \frac{1}{T}\sum_{-\infty}^{+\infty} X_c[j(\Omega - \frac{2\pi}{T}k)]$

$H_r(e^{j\Omega T}) = \frac{1}{T}\frac{\Omega T/2}{\sin(\Omega T/2)} e^{j\Omega T/2}, |\Omega| < \frac{\pi}{T}, \omega = \Omega T$

$H_r(e^{j\omega}) = \frac{1}{T}\frac{\omega/2}{\sin(\omega/2)} e^{j\omega/2}, |\omega| < \pi$

(b) One low-order FIR filter suggested in Jackson (1996) is

$$H_{FIR}(z) = -\frac{1}{16} + \frac{9}{8}z^{-1} - \frac{1}{16}z^{-2}$$

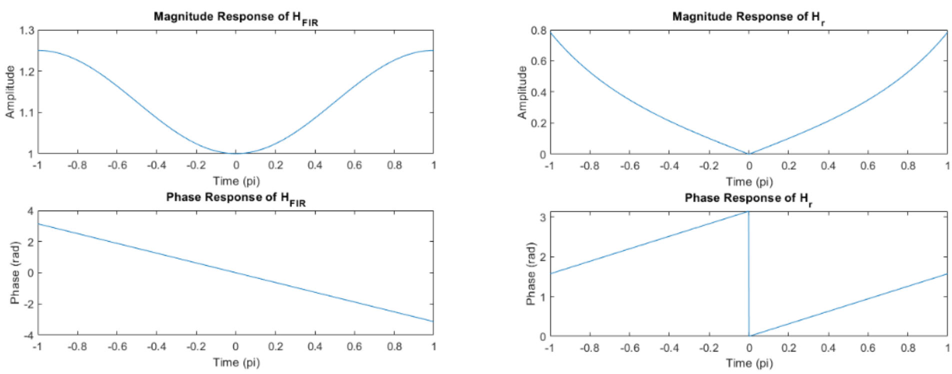
Compare the magnitude response of $H_{FIR}(e^{j\omega})$ with that of $H_r(e^{j\omega})$ above.

$z = e^{j\omega} = \cos(\omega) + j\sin(\omega)$

$H_{FIR}(e^{j\omega}) = \frac{1}{16}[(-1 + 18\cos(\omega) - \cos(2\omega)) + j(-18\sin(\omega) + \sin(2\omega))]$

$|H_{FIR}(e^{j\omega})| = \frac{1}{16}\sqrt{(-1 + 18\cos(\omega) - \cos(2\omega))^2 + (-18\sin(\omega) + \sin(2\omega))^2}$

$|H_r(e^{j\omega})| = \frac{1}{T}\frac{\omega/2}{\sin(\omega/2)}, |\omega| < \pi$



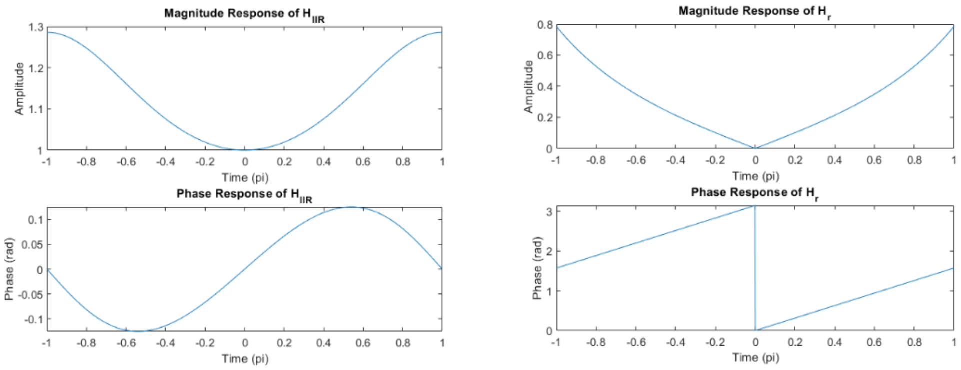
We can the curve of magnitude response of $H_{FIR}(e^{j\omega})$ is smoother and larger than magnitude response of $H_r(e^{j\omega})$.

(c) Another low-order IIR filter suggested in Jackson (1996) is

$$H_{IIR}(z) = \frac{9}{8 + z^{-1}}$$

Compare the magnitude response of $H_{IIR}(e^{j\omega})$ with that of $H_r(e^{j\omega})$ above.

$H_{IIR}(e^{j\omega}) = \frac{9\times(8+\cos\omega+j\sin\omega)}{65+16\cos\omega} \Rightarrow |H_{IIR}(e^{j\omega})| = \frac{9}{65+16\cos\omega}\sqrt{65+16\cos\omega}$



We can the curve of magnitude response of $H_{IIR}(e^{j\omega})$ is smoother and larger than magnitude response of $H_r(e^{j\omega})$.

4. Consider the following continuous-time system

$$H(s) = \frac{s^4 - 6s^3 + 10s^2 + 2s - 15}{s^5 + 15s^4 + 100s^3 + 370s^2 + 744s + 720}$$

(a) Show that the system $H(s)$ is a nonminimum phase system.

Minimum phase system: All poles and zeros are inside unit circle.

$s^4 - 6s^3 + 10s^2 + 2s - 15 = (s+1)(s-3)(s^2 - 4s + 5)$

$s^5 + 15s^4 + 100s^3 + 370s^2 + 744s + 720 = (s+5)(s^2 + 4s + 8)(s^2 + 6s + 18)$

⇨ Zeros: -1, 3, 2±j & Poles: -5, -3±3j, -2±2j

⇨ Only one zero is inside unit circle, so this is a nonminimum phase system.

(b) Decompose $H(s)$ into the product of minimum phase component $H_{min}(s)$ and an all pass component $H_{ap}(s)$.

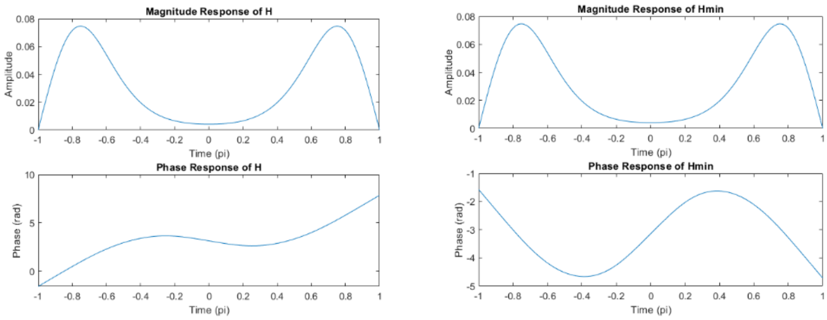
$$H(s) = \frac{(s+1)(1-3s)(5s^2-4s+1)}{(1+5s)(8s^2+4s+1)(18s^2+6s+1)} \times \frac{(s-3)(s^2-4s+5)(1+5s)(8s^2+4s+1)(18s^2+6s+1)}{(s+5)(s^2+4s+8)(s^2+6s+18)(1-3s)(5s^2-4s+1)}$$

$$= H_{min}(s) \times H_{ap}(s)$$

⇨ $H_{min}(s) = \frac{(s+1)(1-3s)(5s^2-4s+1)}{(1+5s)(8s^2+4s+1)(18s^2+6s+1)}$

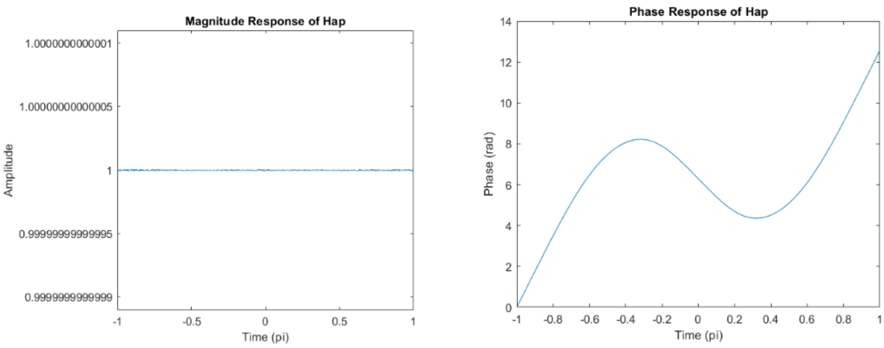
⇨ $H_{ap}(s) = \frac{(s-3)(s^2-4s+5)(1+5s)(8s^2+4s+1)(18s^2+6s+1)}{(s+5)(s^2+4s+8)(s^2+6s+18)(1-3s)(5s^2-4s+1)}$

(c) Briefly plot the magnitude and phase responses of $H(s)$ and $H_{min}(s)$ and explain your plots.



We can find that $|H_{min}(s)| = |H(s)|$ and $\angle H_{min}(s) < \angle H(s)$

(d) Briefly plot the magnitude and phase responses of $H_{ap}(s)$.

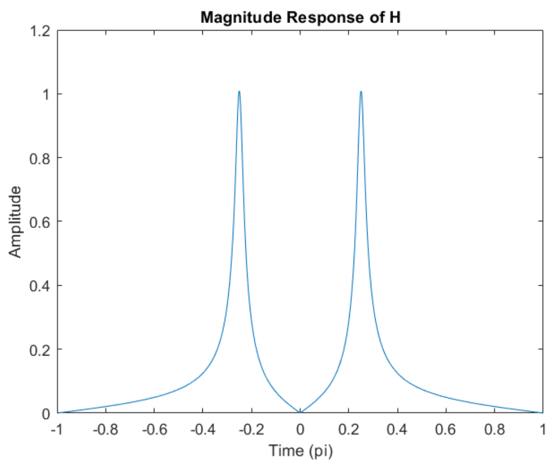


We can find that $|H_{ap}(s)| = 1$ and $\angle H_{ap}(s) + \angle H_{min}(s) = \angle H(s)$

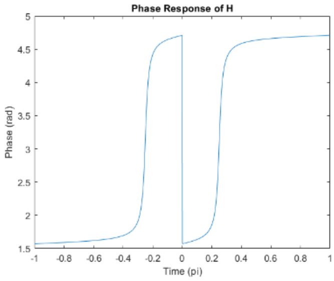
5. We want to design a second order IIR filter using pole-zero placement that satisfies the following requirements: (1) the magnitude response is 0 at $\omega_1 = 0$ and $\omega_3 = \pi$ (2) The maximum magnitude is 1 at $\omega_{2,4} = \pm\pi/4$ and (3) the magnitude response is approximately $1/\sqrt{2}$ at frequencies $\omega_{2,4} \pm 0.05$. Determine locations of two poles and two zeros of the required filter and then compute its system function H(z).

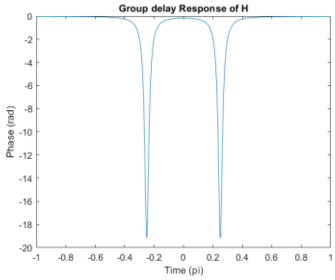
Zero: $\pm$1; Pole: 0.74$\pm$0.74j $\Longrightarrow H(z) = \frac{1}{18}\frac{1-Z^{-2}}{1-1.49z^{-1}+1.11z^{-2}}$

(a) Briefly graph the magnitude response of the filter.



(b) Briefly graph phase and group-delay responses.





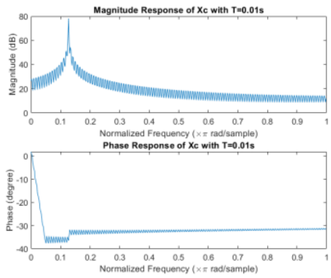
6. The following signals $x_c(t)$ is sampled periodically to obtain the discrete-time signal $x[n]$. For each of the given sampling rates in F_S Hz or in T period, (i) determine the spectrum $X(e^{j\omega})$ of $x[n]$; (ii) plot its magnitude and phase as a function of ω in rad/sam and as a function of F in Hz; and (iii) explain whether $x_c(t)$ can be recovered from $x[n]$.

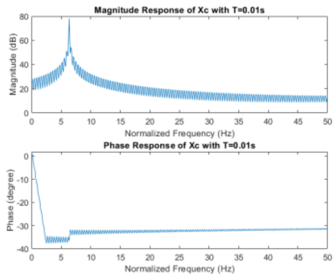
(a) $x_c(t) = 5e^{j40t} + 3e^{-j70t}$, with sampling period $T = 0.01, 0.04, 0.1$

$$e^{j\omega_0 n} \leftrightarrow 2\pi\delta(\omega - \omega_0), \sin(\omega_0 n) = \frac{\pi}{j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$$

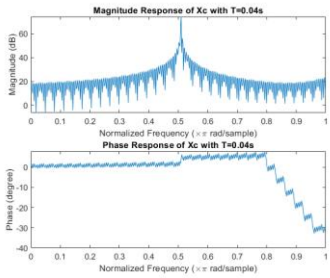
$$x[n] = x_c(nT) \longrightarrow X(e^{j\omega}) = 2\pi[5\delta(\omega - 40\pi T) + 3\delta(\omega + 70\pi T)]$$

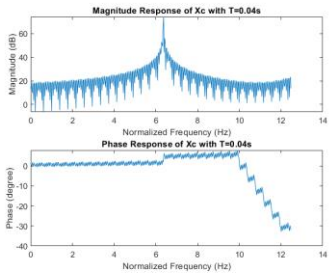
i. For T = 0.01s



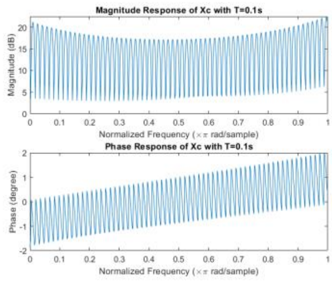


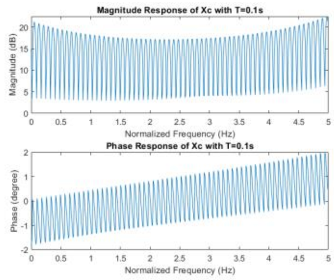
ii. For T = 0.04s





iii. For T = 0.1s





The sampling theorem shows that sampling signal can be recovered if $T_H \geq 2 \times T$, which T_H is $\pi/35$ s, which equal to 0.089s, in the signal $x_c(t)$.

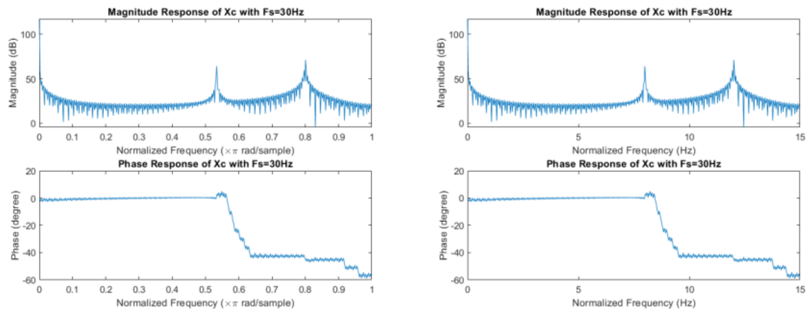
Therefore, the sampling signals with $T = 0.01\text{s}$ and 0.04s can be recovered.

(b) $x_c(t) = 3 + 2\sin(16\pi t) + 10\cos(24\pi t)$, with sampling rate $F_S = 30, 20, 15\text{ Hz}$.

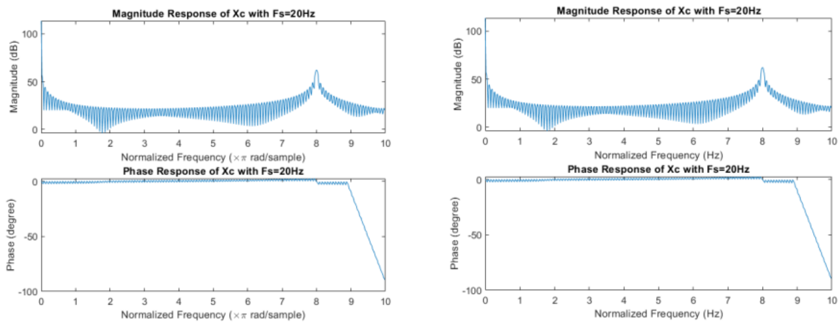
$x[n] = x_c(nT), T = F_S^{-1}$

$$\Rightarrow X(e^{j\omega}) = 6\pi\delta(\omega) + \frac{5\pi}{j}\left[\delta\left(\omega - \frac{16\pi}{F_S}\right) - \delta\left(\omega + \frac{16\pi}{F_S}\right)\right] + 10\pi[\delta\left(\omega - \frac{24\pi}{F_S}\right) + \delta\left(\omega + \frac{24\pi}{F_S}\right)]$$

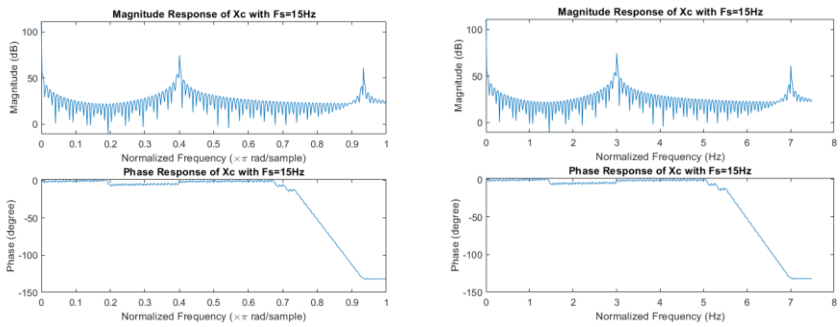
i. For $F_S = 30\text{Hz}$



ii. For $F_S = 20\text{Hz}$



iii. For $F_S = 15\text{Hz}$



The sampling theorem shows that sampling signal can be recovered if $F_S \geq 2 \times F_H$, which F_H is 12Hz in the signal $x_c(t)$.

Therefore, only the sampling signal with $F_S = 30\text{ Hz}$ can be recovered.

7. An 8-bit ADC has an input analog range of ±5 volts. The analog input signal is

$$x_c(t) = 2\cos(200\pi t) + 3\sin(500\pi t)$$

The converter supplies data to a computer at a rate of 2048 bits/s. The computer, without processing, supplies these data to an ideal DAC to form the reconstructed signal $y_c(t)$. Determine:

(a) The quantizer resolution (or step).

B = 8, $\Delta = \left(5V - (-5V)\right)/2^8 = 39m$

The quantizer step (Δ) is $39mV$.

(b) The SQNR in dB.

$SQNR(dB) = 10\log_{10} SQNR = 6.02B + 1.76 = 49.92\ dB$

(c) The folding frequency and the Nyquist rate.

i. Folding frequency (= 0.5 Sampling Frequency)

(2048 bits/s)/(8-bit/sample) = 256 samples/s

Sampling Rate = 256-bit/sample ⇒ Folding frequency = 128 Hz

ii. Nyquist rate (= $2\Omega_H$ (rad/sec))

$F_H = \max(100Hz, 250Hz) = 250Hz$

$\Rightarrow \Omega_H = 250 \times (2\pi) = 1570.8 \Rightarrow$ Nyquist rate $= 3141.6\ (rad/sec)$