HW3 Program Assignment

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P8

Compute and plot the phase response using the functions freqz, angle, phasez, unwrap, and phasedelay for the following systems:

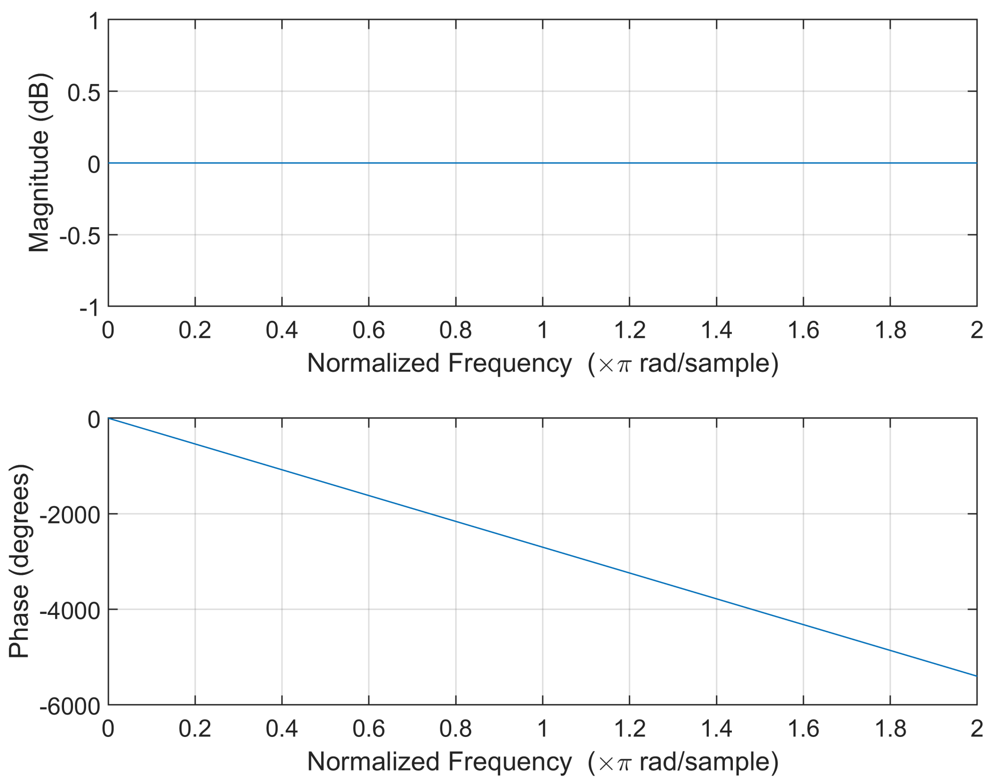
(a) $y[n] = x[n - 15]$

```
close all; clear;
fprintf('8(a)\n');
```

```
8(a)
```

```
om = linspace(0, 2*pi, 1000);
b = zeros(1, 16); b(16) = 1; a = [1];
fprintf('By "freqz"\n'); freqz(b, a, om)
```

```
By "freqz"
```

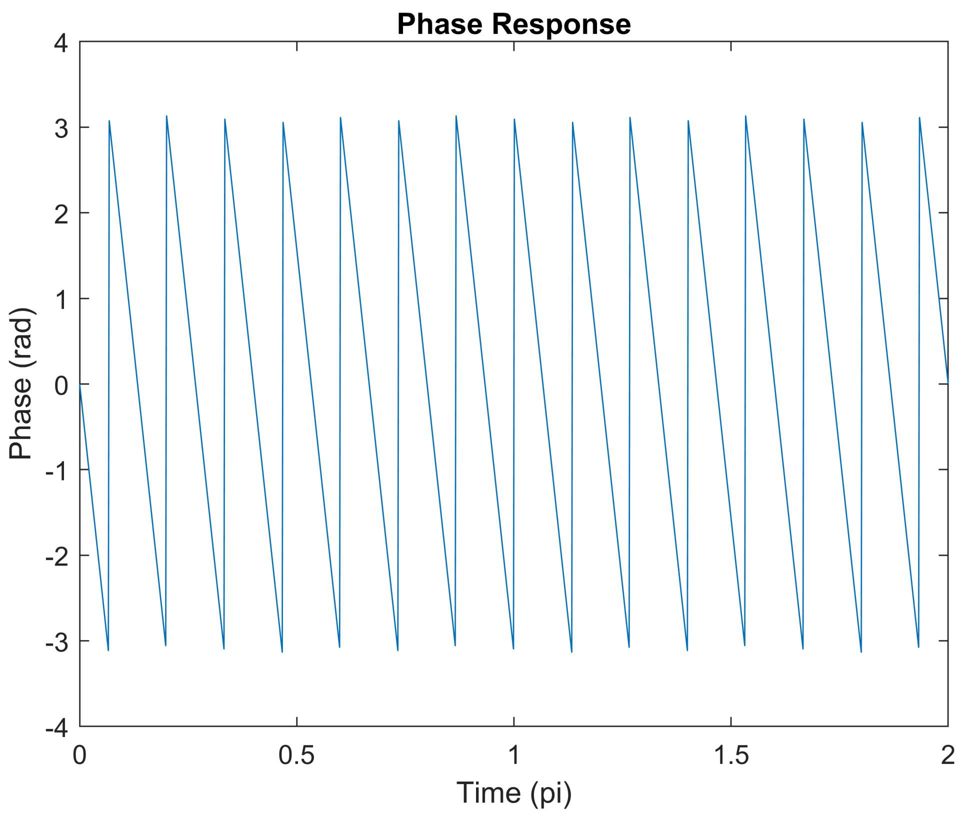



```
H1 = freqz(b, a, om);
```

```
figure;
fprintf('By "angle"\n');
```

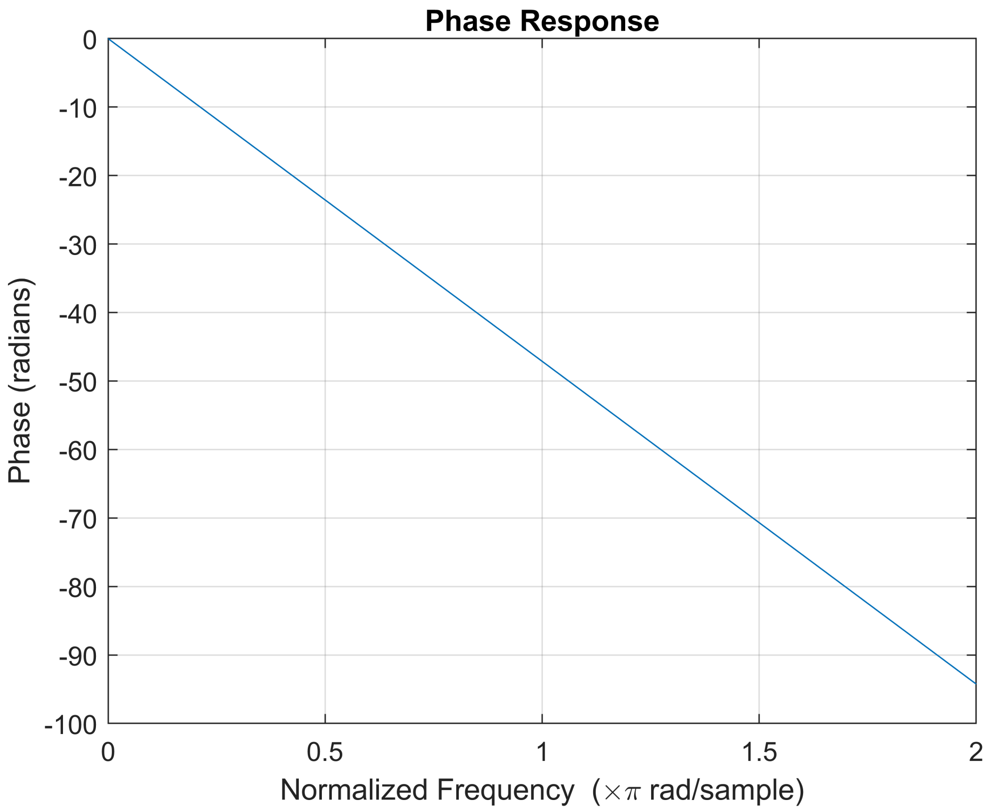
```
By "angle"
```

```
plot(om/pi, angle(H1));
title('Phase Response');
ylabel('Phase (rad)');
xlabel('Time (pi)');
```




```
fprintf('By "phasez"\n'); phasez(b, a, om)
```

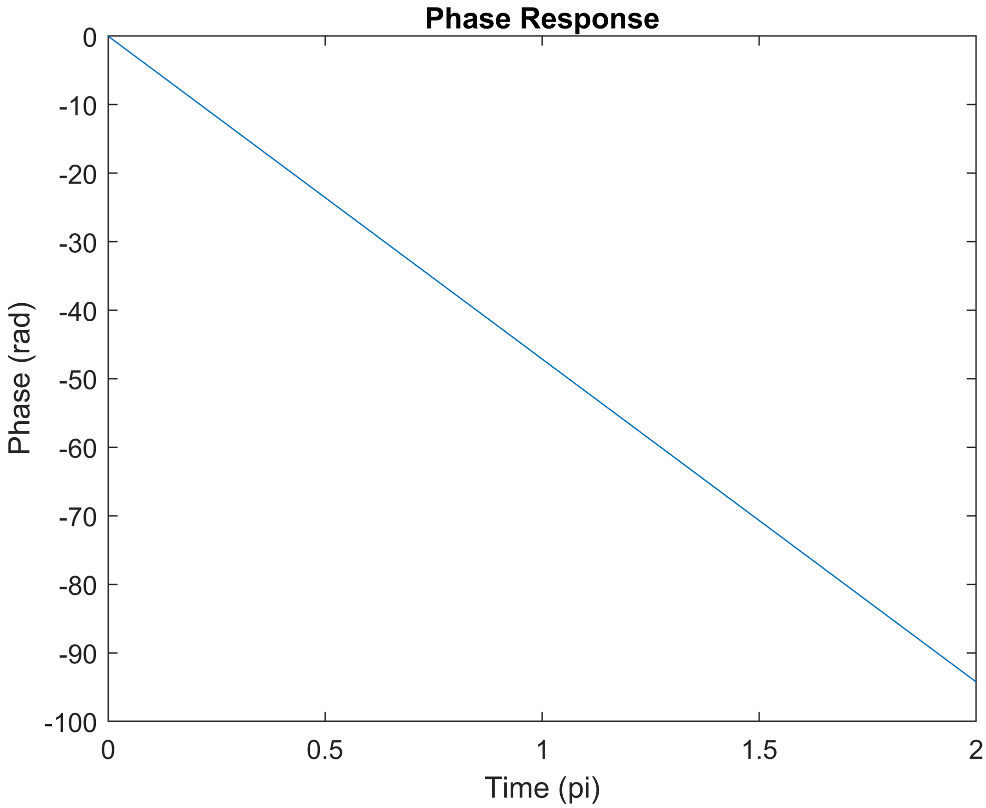
```
By "phasez"
```




```
figure;
fprintf('By "unwrap"\n');
```

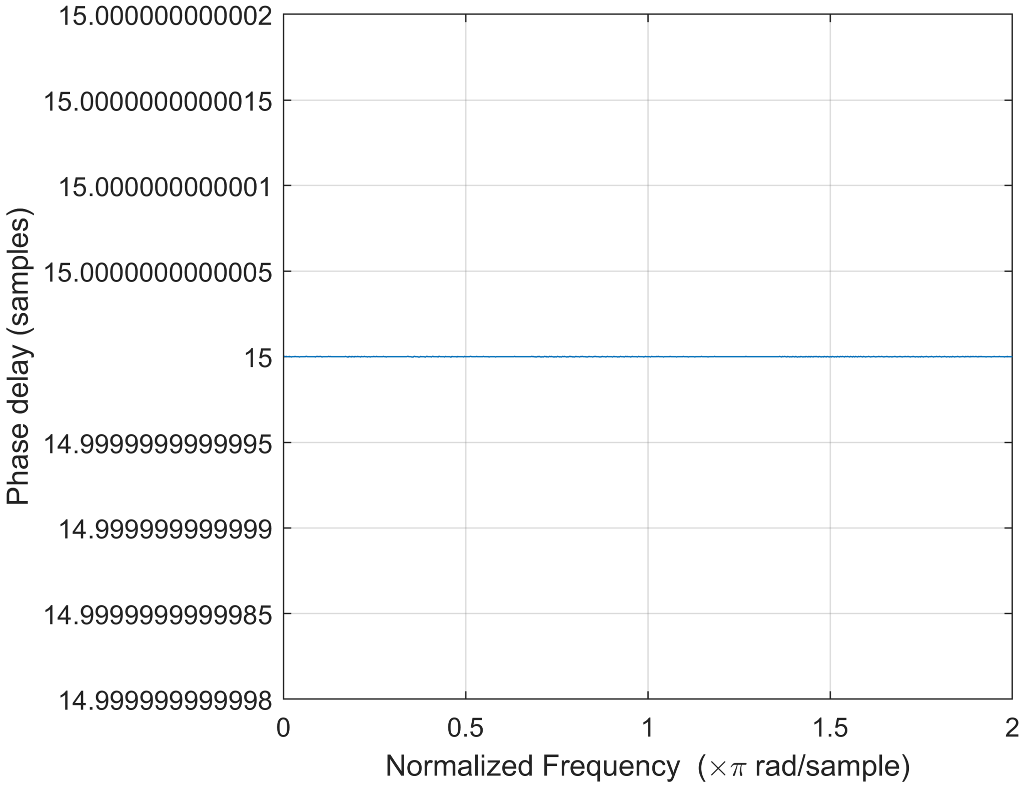
```
By "unwrap"
```

```
plot(om/pi, unwrap(angle(H1)));
title('Phase Response');
ylabel('Phase (rad)');
xlabel('Time (pi)');
```




```
fprintf('By "phasedelay"\n'); phasedelay(b, a, om)
```

```
By "phasedelay"
```



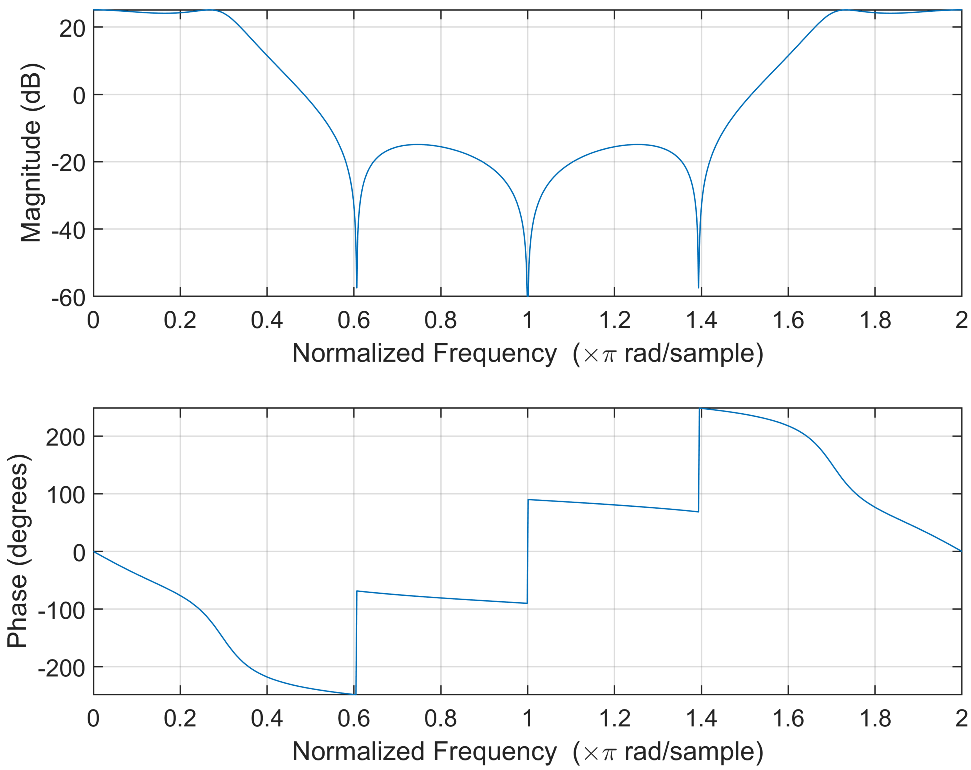
(b) $H(z) = \dfrac{1 + 1.655z^{-1} + 1.655z^{-2} + z^{-3}}{1 - 1.57z^{-1} + 1.264z^{-2} - 0.4z^{-3}}$

```
fprintf('8(b)\n');
```

```
8(b)
```

```
om = linspace(0, 2*pi, 1000);
b = [1 1.655 1.655 1]; a = [1 -1.57 1.264 -0.4];
fprintf('By "freqz"\n'); freqz(b, a, om)
```

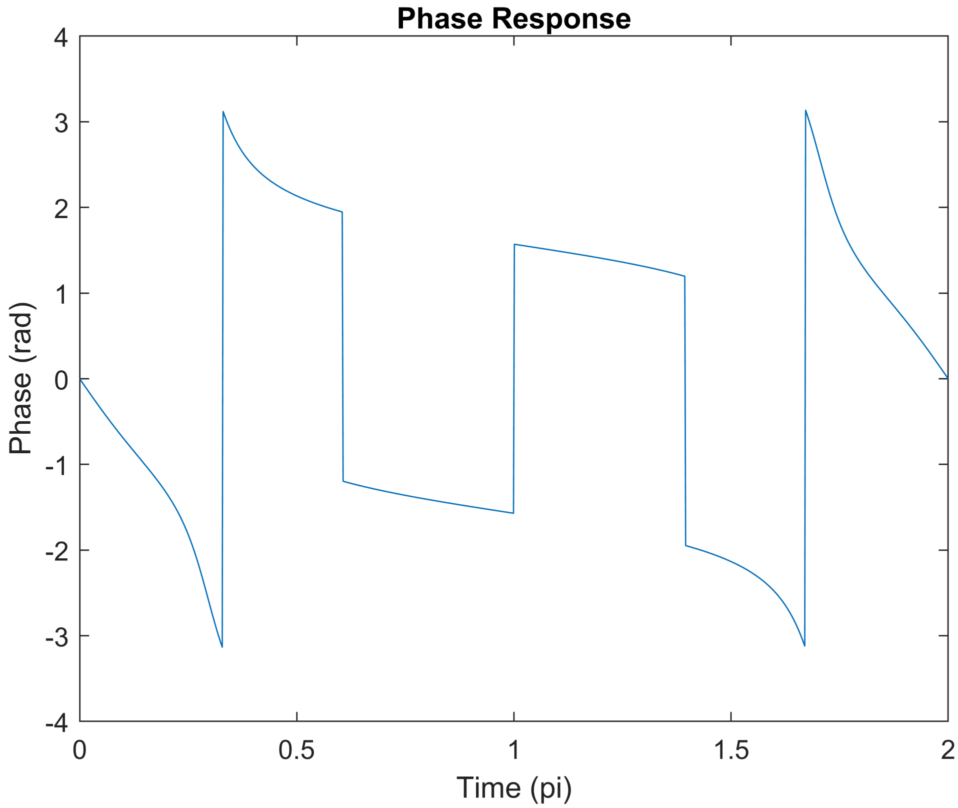
```
By "freqz"
```




```
H1 = freqz(b, a, om);
figure;
fprintf('By "angle"\n');
```

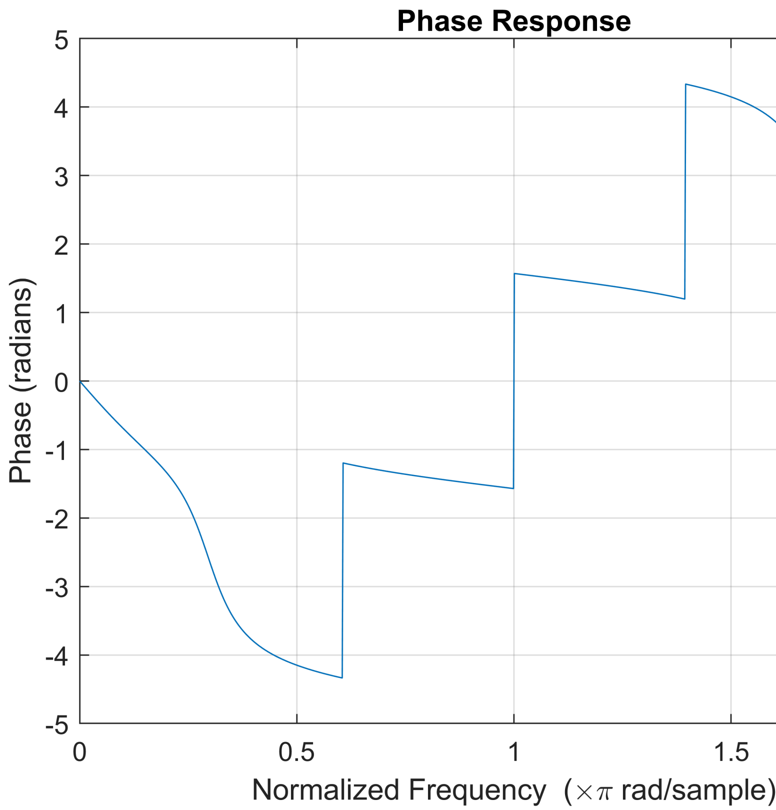
```
By "angle"
```

```
plot(om/pi, angle(H1));
title('Phase Response');
ylabel('Phase (rad)');
xlabel('Time (pi)');
```




```
fprintf('By "phasez"\n'); phasez(b, a, om)
```

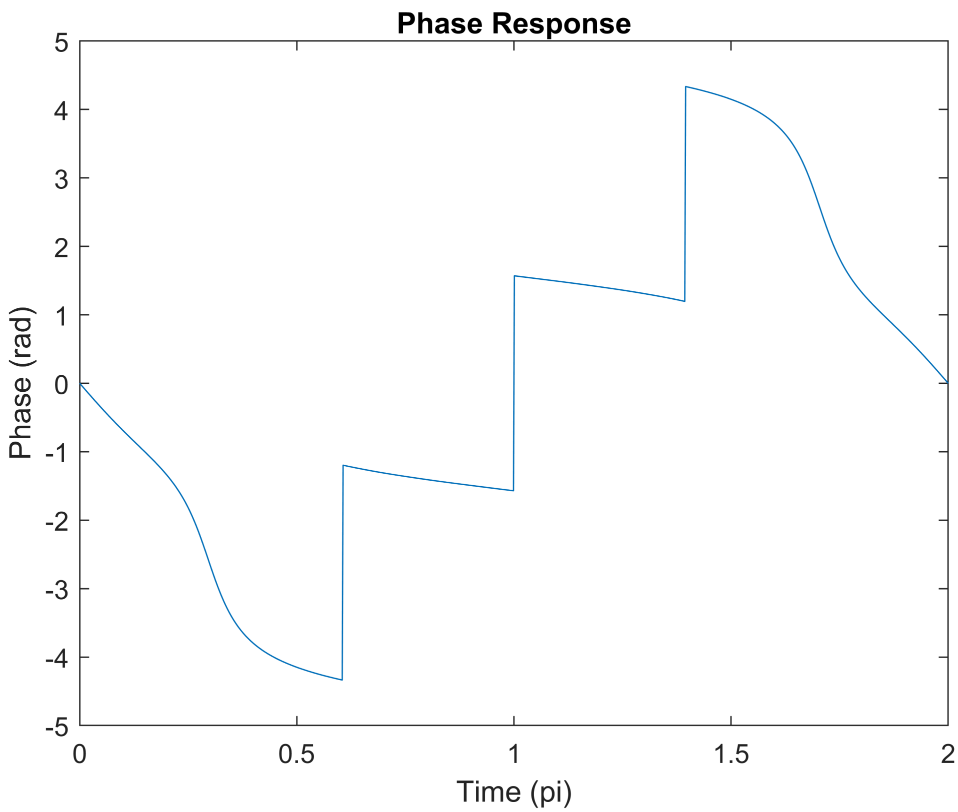
```
By "phasez"
```




```
figure;
fprintf('By "unwrap"\n');
```

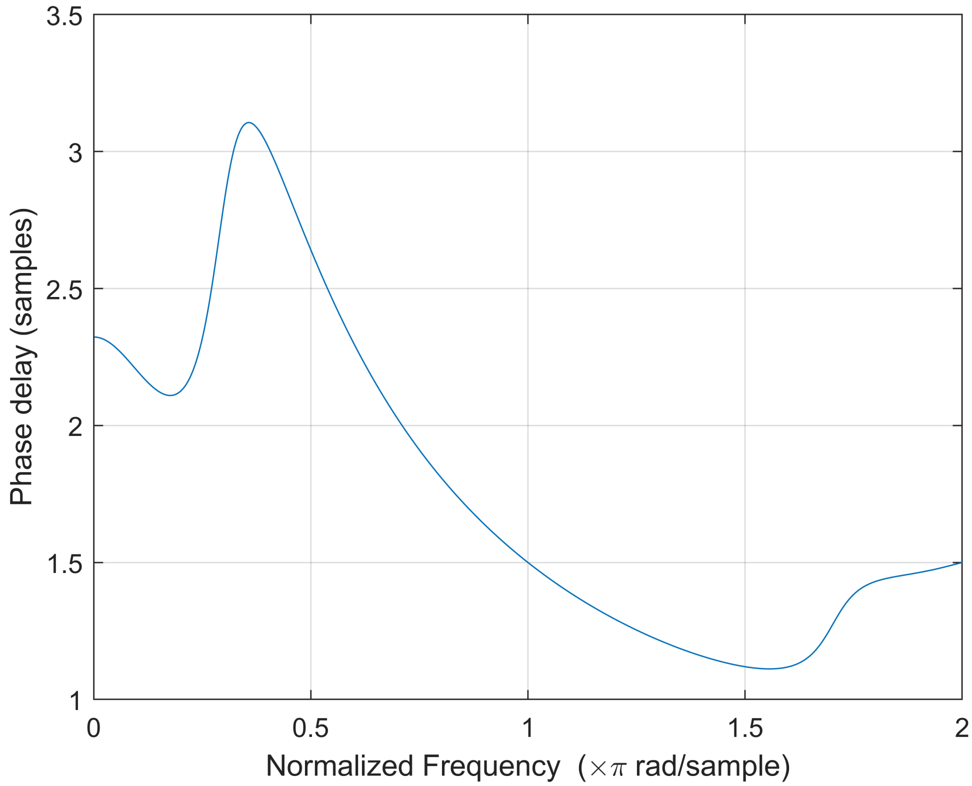
```
By "unwrap"
```

```
plot(om/pi, unwrap(angle(H1)));
title('Phase Response');
ylabel('Phase (rad)');
xlabel('Time (pi)');
```




```
fprintf('By "phasedelay"\n'); phasedelay(b, a, om)
```

```
By "phasedelay"
```



P9

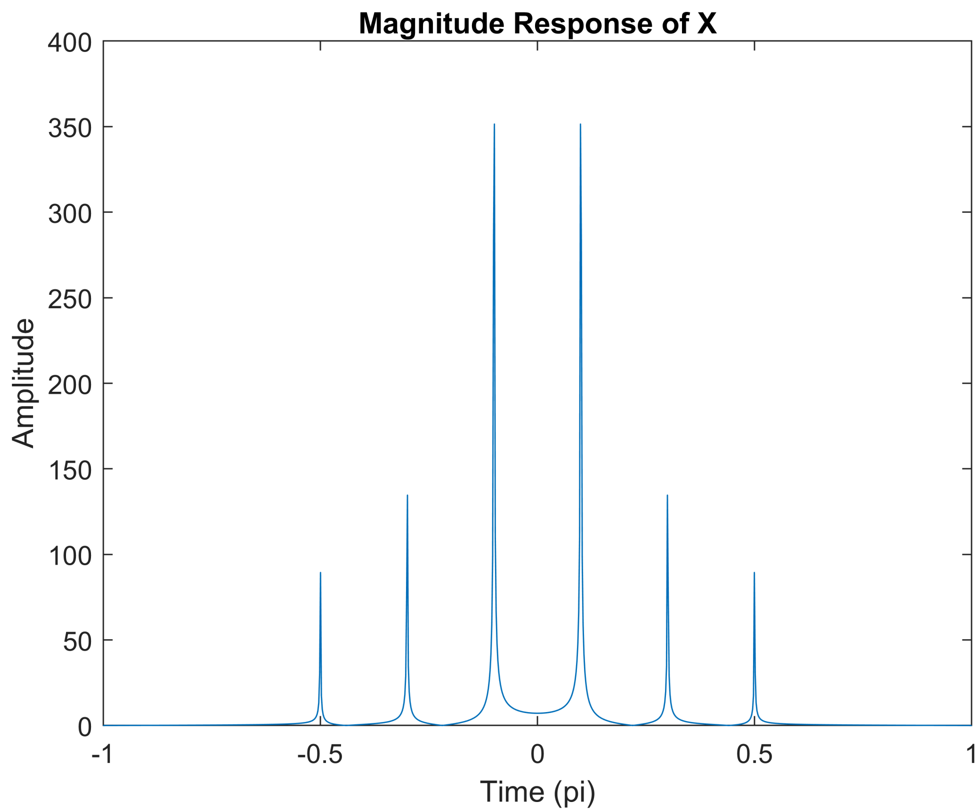
According to problem 2 in paper assignment, plot magnitude response, phase response and group-delay response for each of the systems.

```
close all; clear;
fprintf('9\n');
```

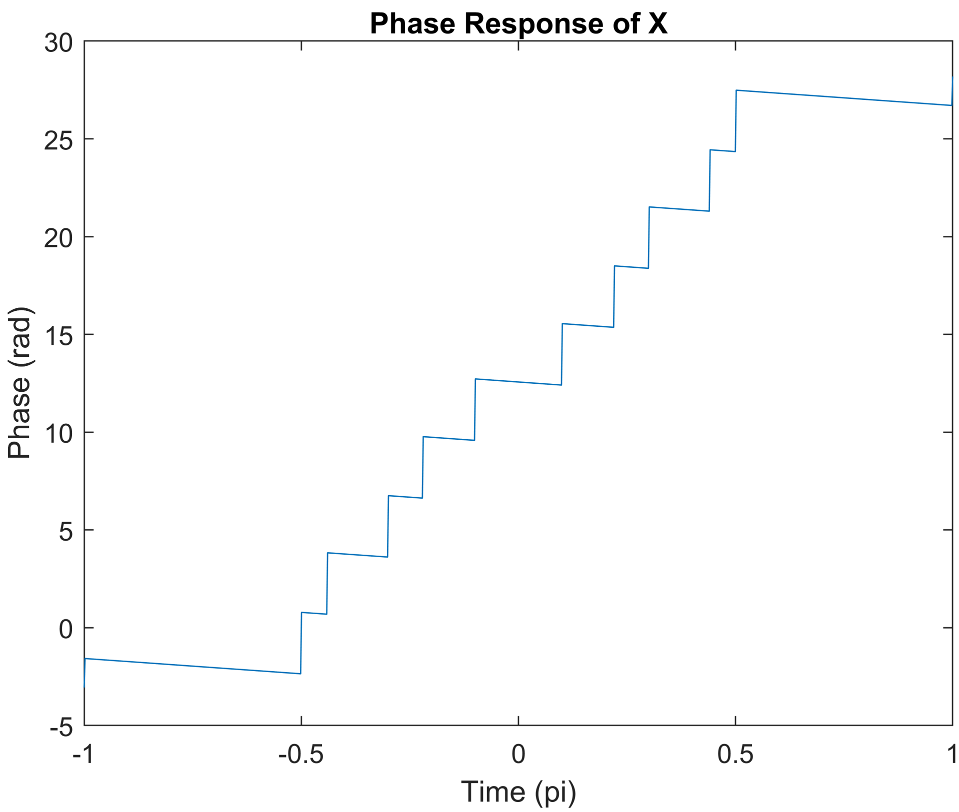
```
9
```

```
om=linspace(-pi, pi, 1000);
n = linspace(0,1000,1001);
x = sin(0.1*pi*n)+(1/3)*sin(0.3*pi*n)+0.2*sin(0.5*pi*n);
X = freqz(x,1,om);
gd = grpdelay(x,1,om);

figure; plot(om/pi, abs(X));
title('Magnitude Response of X');
ylabel('Amplitude');
xlabel('Time (pi)');
```




```
figure; plot(om/pi, unwrap(angle(X)));
title('Phase Response of X');
ylabel('Phase (rad)');
xlabel('Time (pi)');
```



(a) $h[n]=\{1_{n=0}, -2, 3, -4, 0, 4, -3, 2, -1\}$

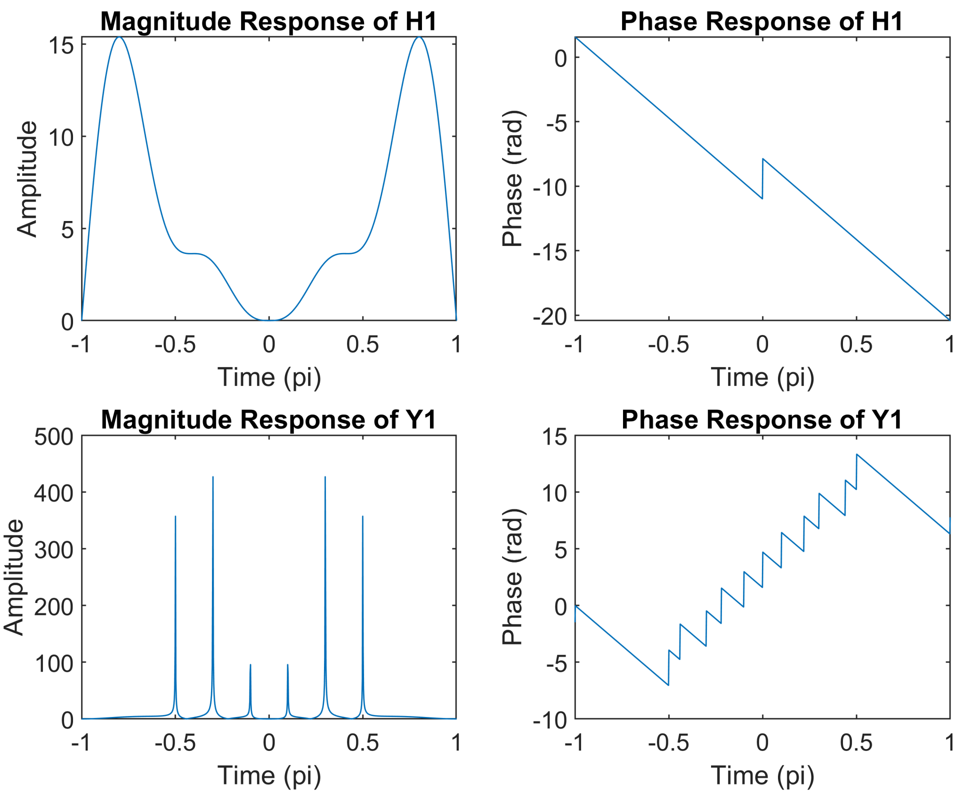
```
fprintf('9(a)\n');
```

```
9(a)
```

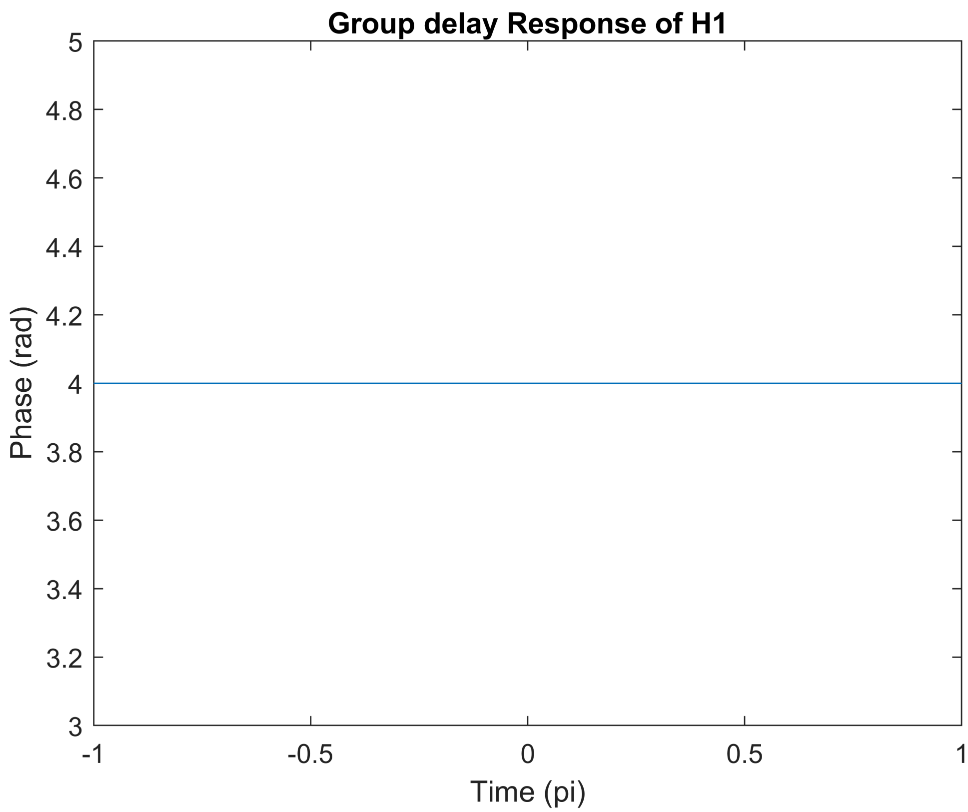
```
h1 = [1 -2 3 -4 0 4 -3 2 -1]; % start from n=0
H1 = freqz(h1,1,om); H1 = exp(-j*om*0).*H1;
gd1 = grpdelay(h1, 1, om);

figure;
subplot(2,2,1); plot(om/pi, abs(H1));
title('Magnitude Response of H1');
ylabel('Amplitude');
xlabel('Time (pi)');
subplot(2,2,2); plot(om/pi, unwrap(angle(H1)));
title('Phase Response of H1');
ylabel('Phase (rad)');
xlabel('Time (pi)');
Y1 = X.*H1;
subplot(2,2,3); plot(om/pi, abs(Y1));
title('Magnitude Response of Y1');
ylabel('Amplitude');
xlabel('Time (pi)');
```

```
subplot(2,2,4); plot(om/pi, unwrap(angle(Y1)));
title('Phase Response of Y1');
ylabel('Phase (rad)');
xlabel('Time (pi)');
```




```
figure; plot(om/pi, gd1);
title('Group delay Response of H1')
ylabel('Phase (rad)');
xlabel('Time (pi)');
```



(b) $y[n] = 10x[n-10]$

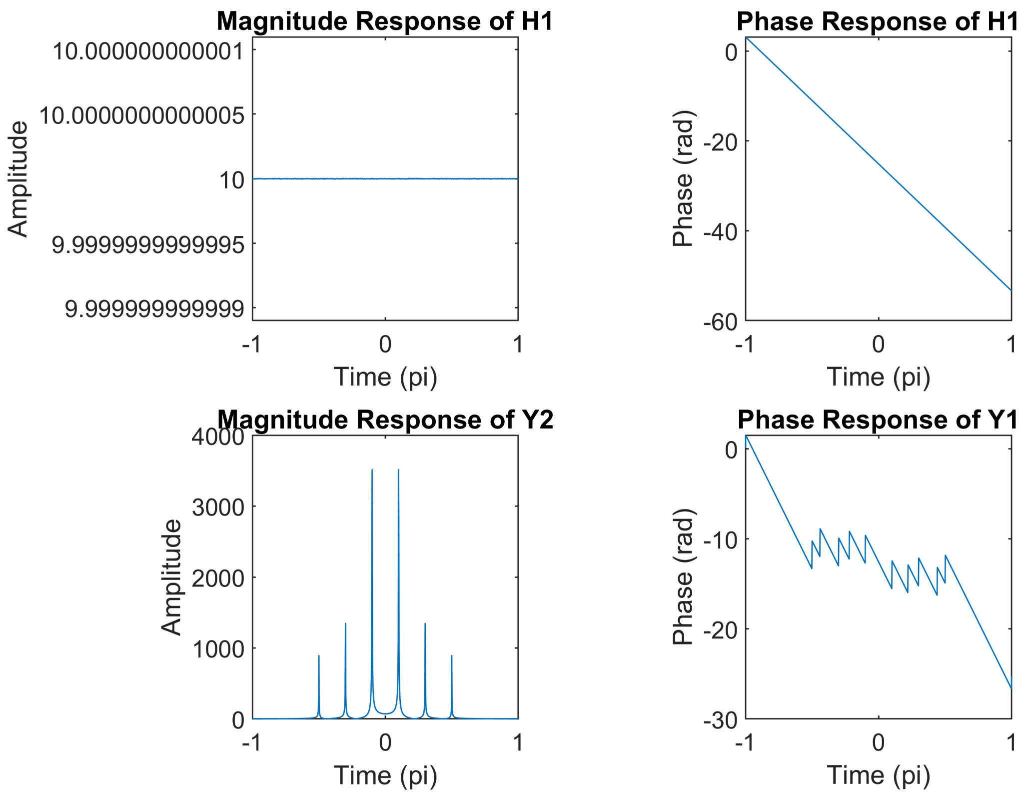
```
fprintf('9(b)\n');
```

```
9(b)
```

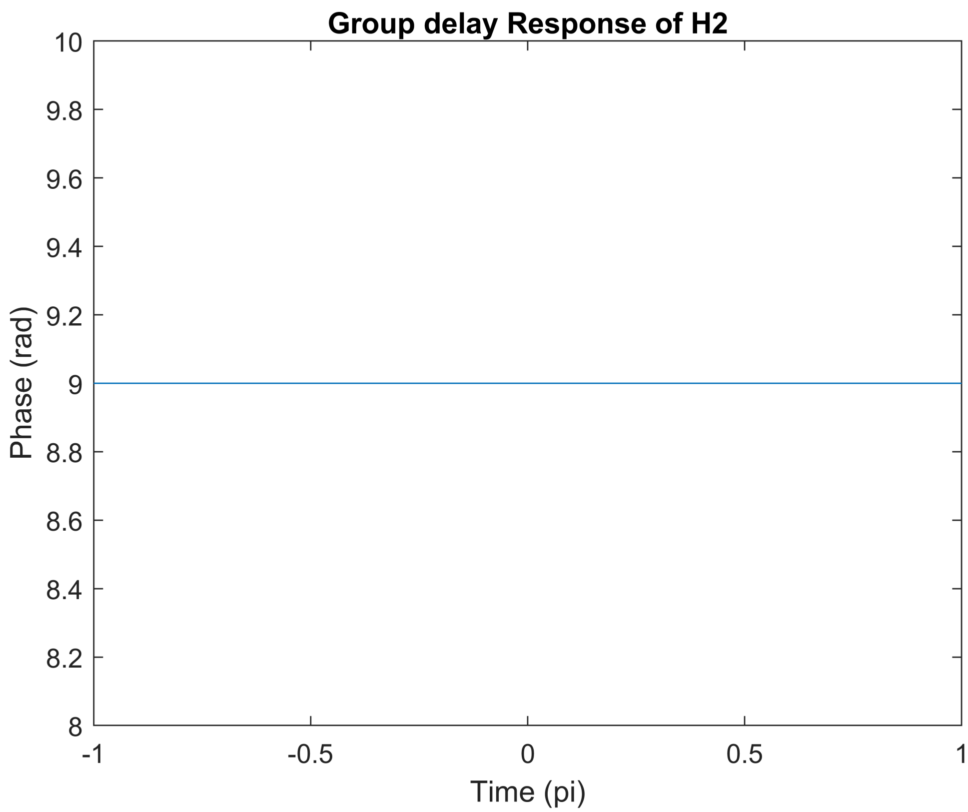
```
b = zeros(1, 10); b(10) = 10;
a = [1];
H2 = freqz(b,a,om);
gd2 = grpdelay(b, a, om);

figure;
subplot(2,2,1); plot(om/pi, abs(H2));
title('Magnitude Response of H1');
ylabel('Amplitude');
xlabel('Time (pi)');
subplot(2,2,2); plot(om/pi, unwrap(angle(H2)));
title('Phase Response of H1');
ylabel('Phase (rad)');
xlabel('Time (pi)');
Y2 = X.*H2;
subplot(2,2,3); plot(om/pi, abs(Y2));
title('Magnitude Response of Y2');
ylabel('Amplitude');
```

```
xlabel('Time (pi)');
subplot(2,2,4); plot(om/pi, unwrap(angle(Y2)));
title('Phase Response of Y1');
ylabel('Phase (rad)');
xlabel('Time (pi)');
```




```
figure; plot(om/pi, gd2);
title('Group delay Response of H2')
ylabel('Phase (rad)');
xlabel('Time (pi)');
```



P10

MATLAB provides a function called polystab that stabilizes the given polynomial with respect to the unit circle, that is, it reflects those roots which are outside the unit-circle into those that are inside the unit circle but with the same angle.

Using this function, convert the following systems into minimum-phase and maximum-phase systems. Verify your answers using a pole-zero plot for each system(plot minimum-phase and maximum-phase systems for each question).

(a) $H(z) = \dfrac{z^2 + 2z + 0.75}{z^2 - 0.5z}$

Ans:

zero = -1/2 & -3/2, pole = 1/2

For Hmin: zero = -1/2 & -2/3, pole = 1/2

$$\Longrightarrow H_{\min} = \frac{\left(1 + \frac{1}{2}z^{-1}\right)\left(1 + \frac{2}{3}z^{-1}\right)}{\left(1 - \frac{1}{2}z^{-1}\right)} = \frac{1 + \frac{7}{6}z^{-1} + \frac{1}{3}z^{-2}}{1 - \frac{1}{2}z^{-1}}$$

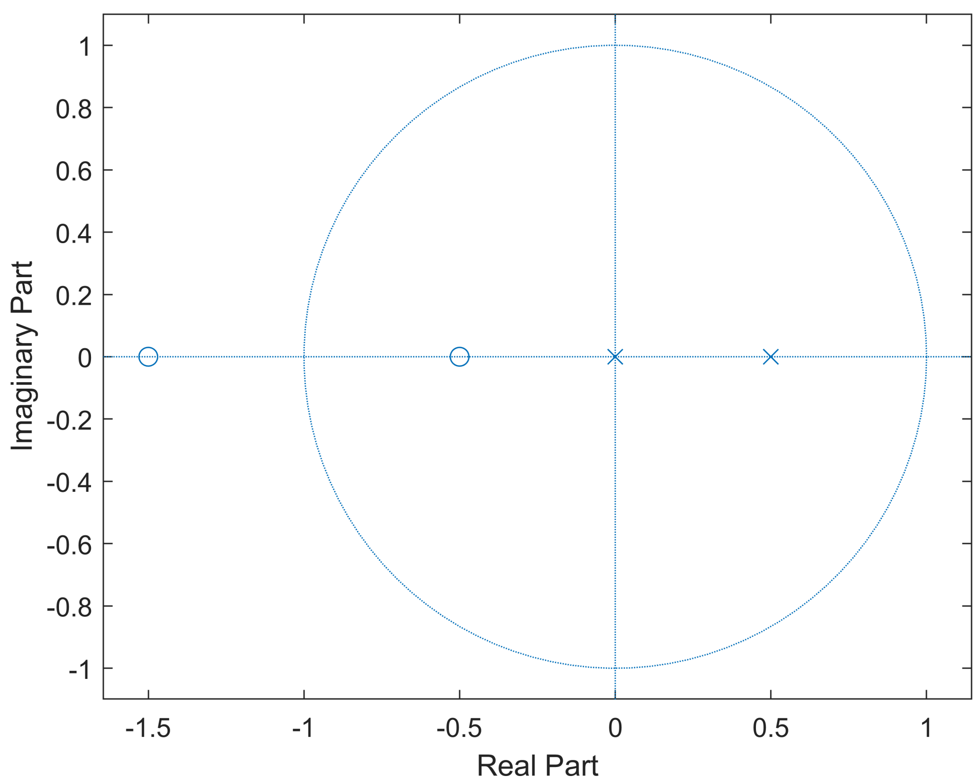
For Hmax: zero = -2 & -3/2, pole = 1/2

$$\Longrightarrow H_{\max} = \frac{\left(1+\frac{1}{2}z\right)\left(1+\frac{2}{3}z\right)}{\left(1-\frac{1}{2}z^{-1}\right)} = \frac{1+\frac{7}{6}z^{1}+\frac{1}{3}z^{2}}{1-\frac{1}{2}z^{-1}}$$

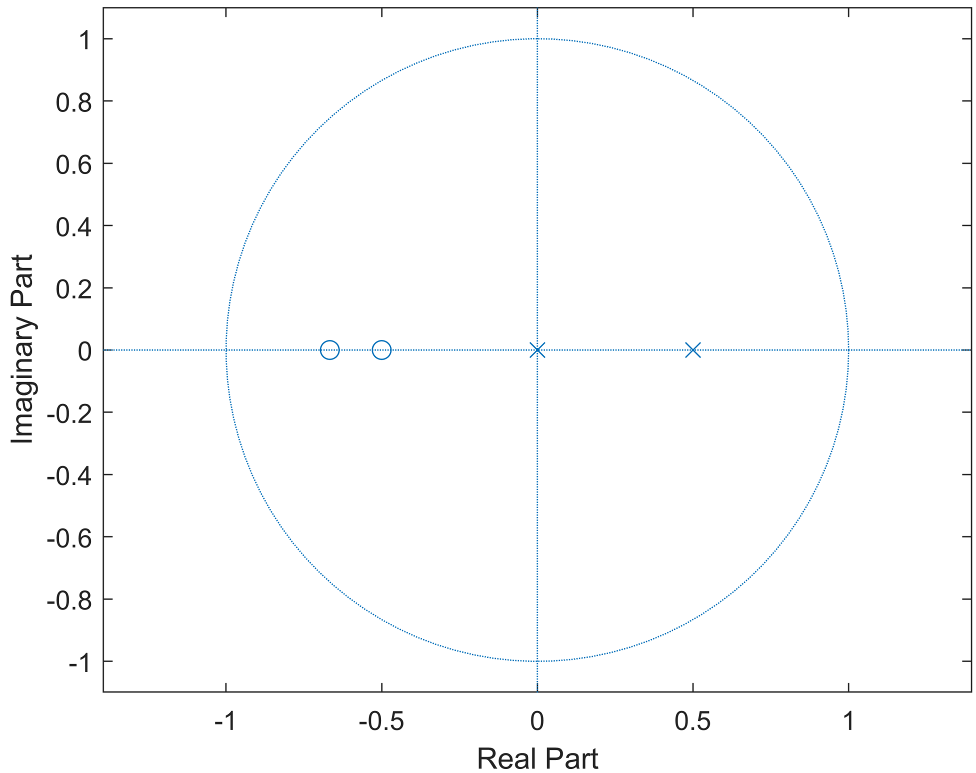
```
close all; clear;
fprintf('10(a)\n');
```

```
10(a)
```

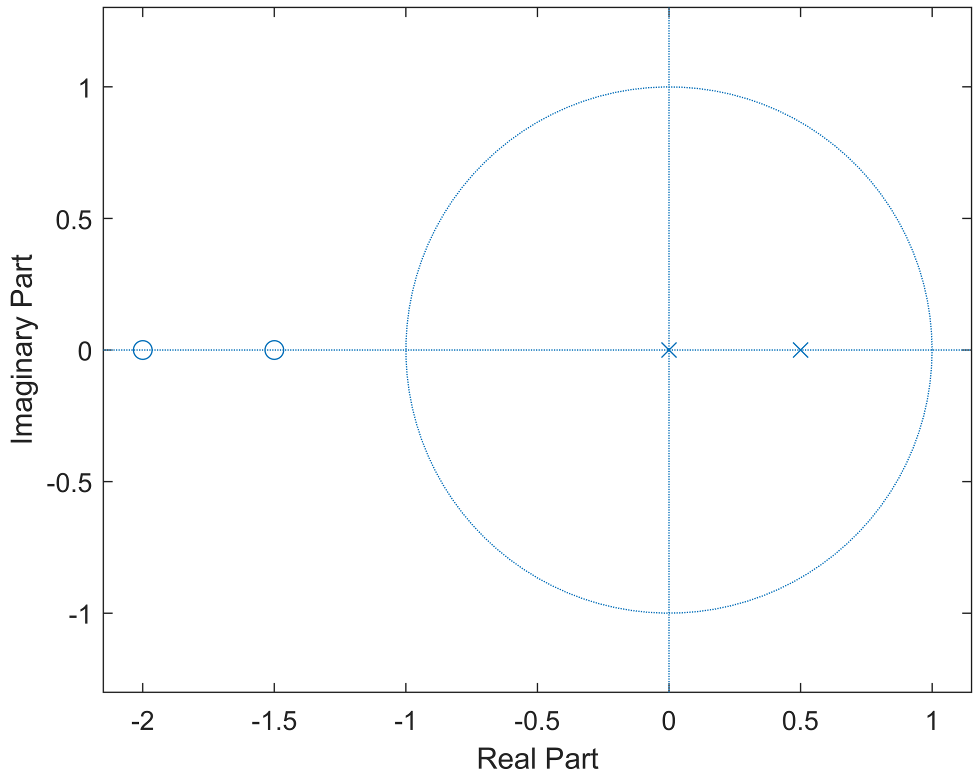
```
b = [1 2 0.75]; a = [1 -0.5];
zplane(b, a);
```



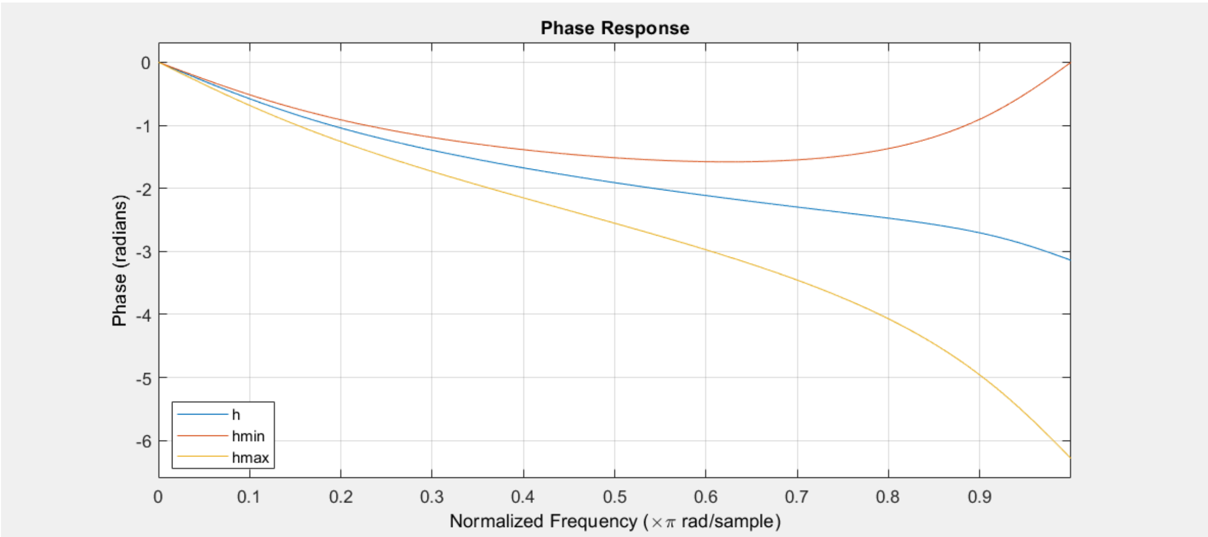

```
b_min = polystab(b); a_min = polystab(a);
zplane(b_min, a_min);
```




```
b_max = fliplr(b_min); a_max = a_min;
zplane(b_max, a_max);
```




```
hfvt = fvtool(b, a, b_min, a_min, b_max, a_max,'Analysis','phase');
legend(hfvt,'h','hmin','hmax');
```

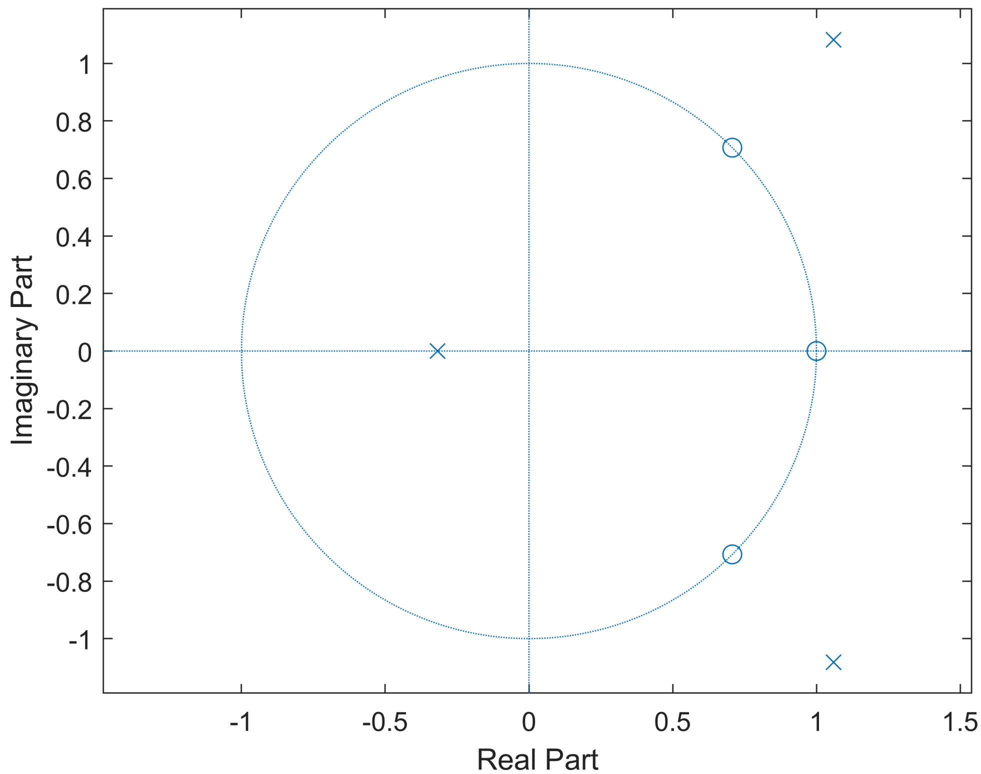


(b) $H(z) = \dfrac{1 - 2.4142z^{-1} + 2.4142z^{-2} - z^{-3}}{1 - 1.8z^{-1} + 1.62z^{-2} + 0.729z^{-3}}$

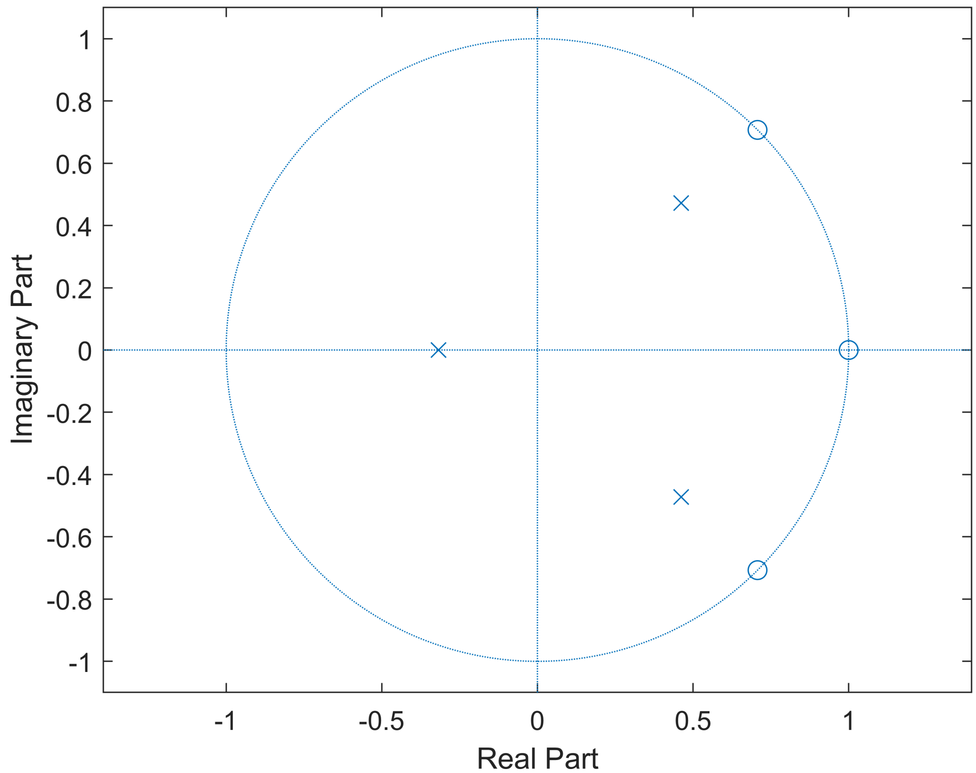
```
fprintf('10(b)\n');
```

10(b)

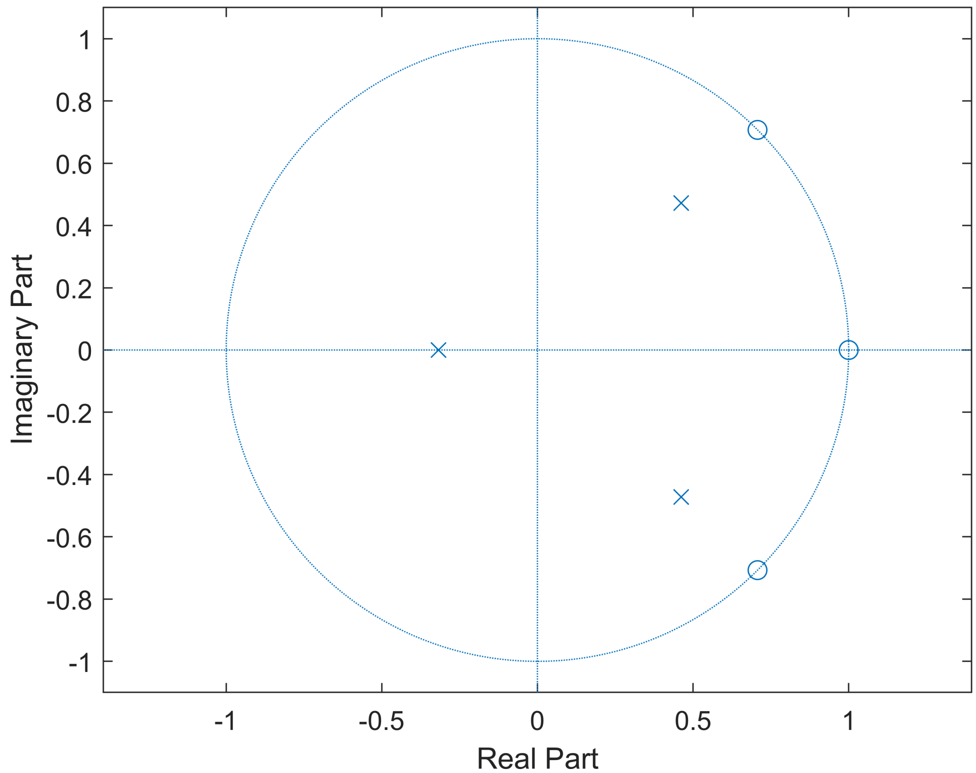
```
b = [1 -2.4142 2.4142 -1]; a = [1 -1.8 1.62 0.729];
zplane(b, a);
```



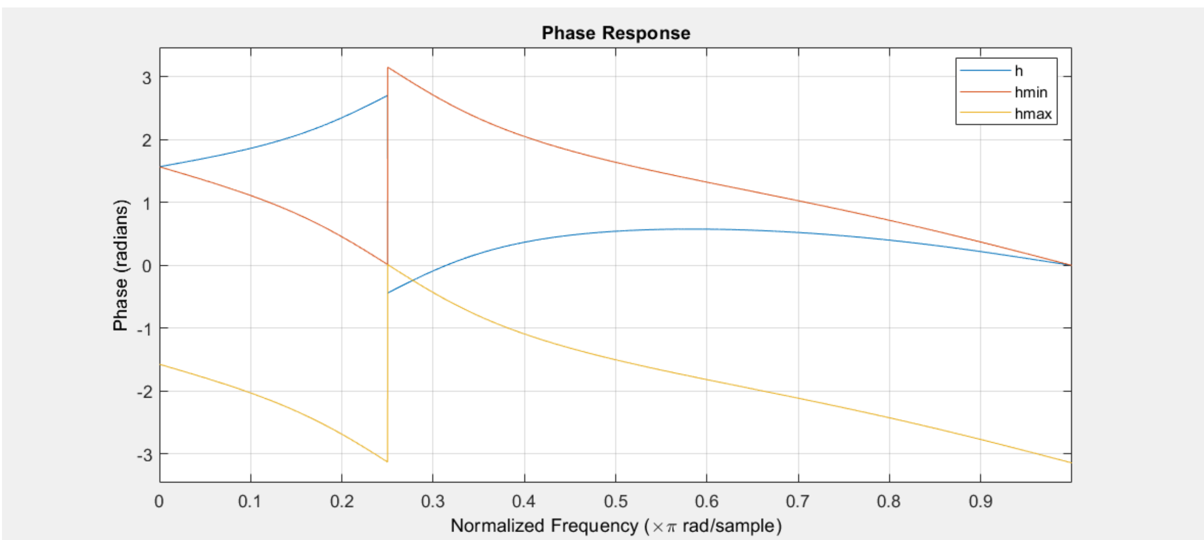

```
b_min = polystab(b); a_min = polystab(a);
zplane(b_min, a_min);
```




```
b_max = fliplr(b_min); a_max = a_min;
zplane(b_max, a_max);
```




```
hfvt = fvtool(b, a, b_min, a_min, b_max, a_max,'Analysis','phase');
legend(hfvt,'h','hmin','hmax');
```



P11

Signal $x_c(t) = 5\cos(200\pi t + \pi 6) + 4\sin(300\pi t)$ is sampled at a rate of $F_S = 1\text{kHz}$ to obtain the discrete-time signal $x[n]$.

(a) Determine the spectrum $X(e^{j\omega})$ of $x[n]$ and plot its magnitude as a function of ω in $\frac{\text{rad}}{\text{sample}}$ and as a function of F in Hz. Explain whether the original signal $x_c(t)$ can be recovered from $x[n]$.

$$\omega = 2\pi \text{FT} = 2\pi \frac{F}{F_s}$$

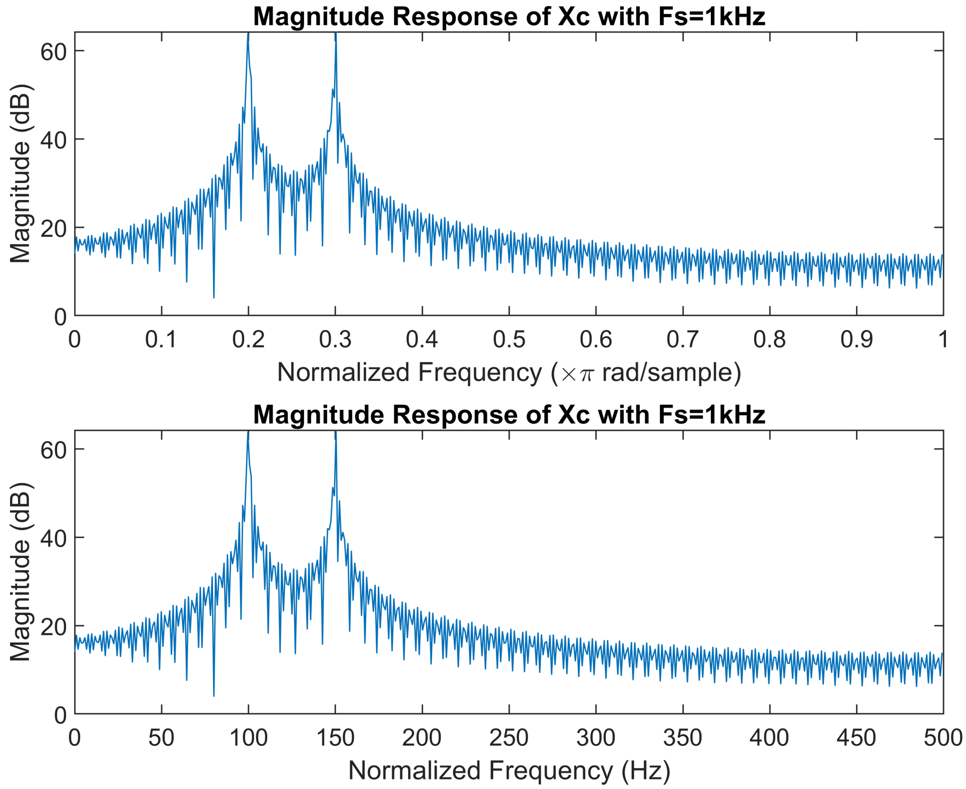
```
close all; clear;
fprintf('11(a)\n');
```

```
11(a)
```

```


t = 1000; Fs = 1000;
om = linspace(0, t, Fs*t);
x_a = 5*cos(200*pi*om + 6*pi) + 4*sin(300*pi*om);

[h_a, w_a] = freqz(x_a, 1);
figure;
subplot(2,1,1); plot(w_a/pi, 20*log10(abs(h_a)));
title('Magnitude Response of Xc with Fs=1kHz');
ylabel('Magnitude (dB)');
xlabel('Normalized Frequency (\times\pi rad/sample)');
subplot(2,1,2); plot(w_a*Fs/(2*pi), 20*log10(abs(h_a)));
title('Magnitude Response of Xc with Fs=1kHz');
ylabel('Magnitude (dB)');
xlabel('Normalized Frequency (Hz)');
```




```
% figure; stem((0:length(x_a)-1),x_a);
% title('x_c(t) with Fs=1kHz');
% ylabel('x[n]');
% xlabel('Time index (n)');
```

The frequency of signals $5\cos(200\pi t + \pi 6)$ and $4\sin(300\pi t)$ are 100Hz and 150Hz, respectively. This means that there are peaks in both 100Hz and 150Hz in the frequency domain of the original signal $x_c(t)$.

Due to sampling theorem, sampling signal can be recovered if the sampling rate (F_S) is larger than two times highest frequency in the signal(F_H), which means that $F_s \geq 2F_H$.

The original signal $x_c(t)$ can be recovered from $x[n]$ because $F_s = 1\text{kHz} \geq 2F_H = 2 \times (150)\text{Hz}$.

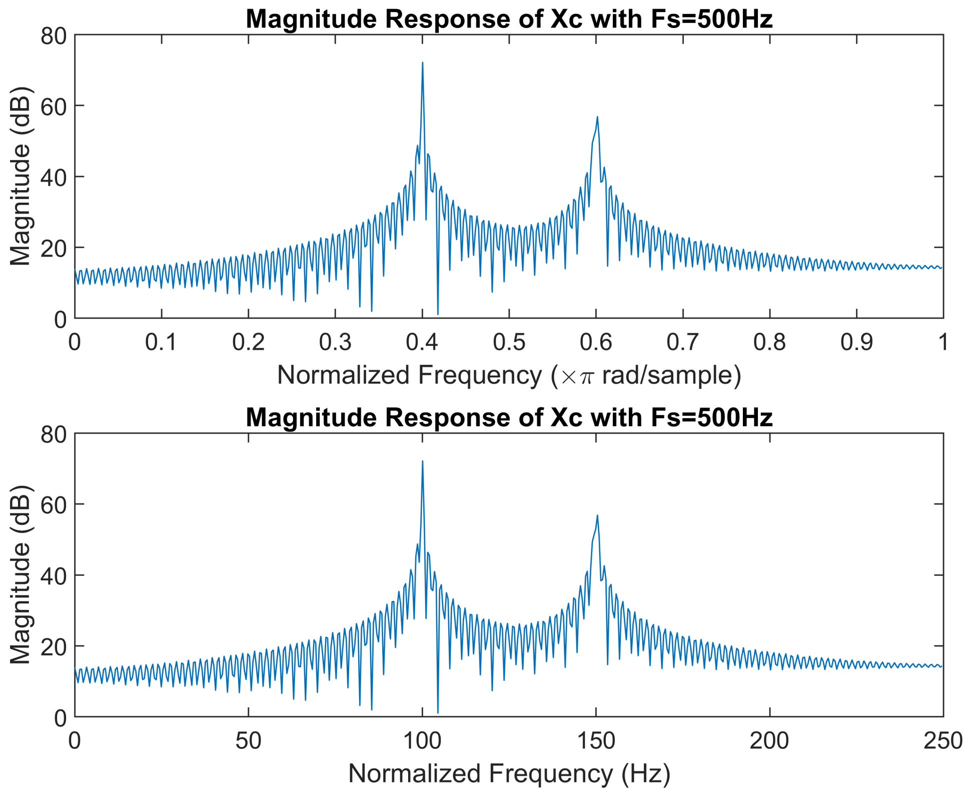
(b) Repeat part (a) for Fs = 500 Hz.

```
fprintf('11(b)\n');
```

```
11(b)
```

```
Fs = 500;
om = linspace(0, t, Fs*t);
x_b = 5*cos(200*pi*om + 6*pi) + 4*sin(300*pi*om);
```

```
[h_b, w_b] = freqz(x_b, 1);
figure;
subplot(2,1,1); plot(w_b/pi, 20*log10(abs(h_b)));
title('Magnitude Response of Xc with Fs=500Hz');
ylabel('Magnitude (dB)');
xlabel('Normalized Frequency (\times\pi rad/sample)');
subplot(2,1,2); plot(w_b*Fs/(2*pi), 20*log10(abs(h_b)));
title('Magnitude Response of Xc with Fs=500Hz');
ylabel('Magnitude (dB)');
xlabel('Normalized Frequency (Hz)')
```



The original signal $x_c(t)$ can be recovered from $x[n]$ because $F_s = 500\text{Hz} \geq 2F_H = 2 \times (150)\text{Hz}$.

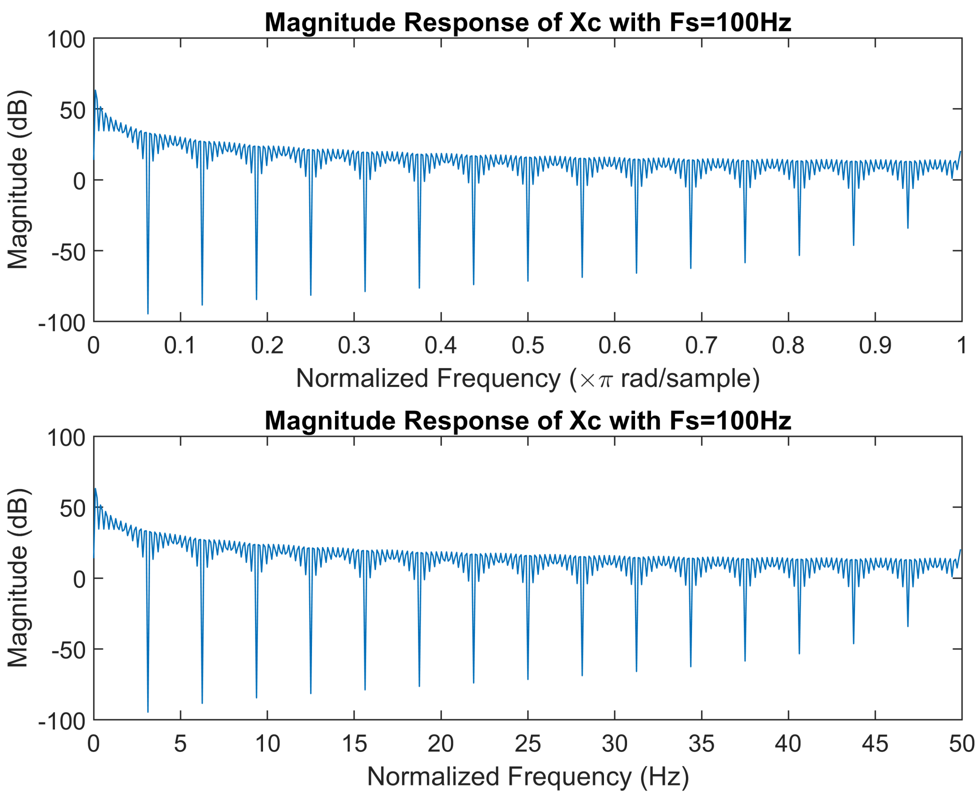
(c) Repeat part (a) for Fs = 100

```
fprintf('11(c)\n');
```

```
11(c)
```

```
Fs = 100; t = 1000;
om = linspace(0, t, Fs*t);
x_c = 5*cos(200*pi*om + 6*pi) + 4*sin(300*pi*om);
```

```
[h_c, w_c] = freqz(x_c, 1);
figure;
subplot(2,1,1); plot(w_c/pi, 20*log10(abs(h_c)));
title('Magnitude Response of Xc with Fs=100Hz');
ylabel('Magnitude (dB)');
xlabel('Normalized Frequency (\times\pi rad/sample)');
subplot(2,1,2); plot(w_c*Fs/(2*pi), 20*log10(abs(h_c)));
title('Magnitude Response of Xc with Fs=100Hz');
ylabel('Magnitude (dB)');
xlabel('Normalized Frequency (Hz)');
```



The original signal $x_c(t)$ can't be recovered from $x[n]$ because $F_s = 100\text{Hz} < 2F_H = 2\times(150)\text{Hz}$.

(d) Comment on your results.

Ans.

The sampling theorem shows that sampling signal can be recovered if $F_s \geq 2F_H$, which F_H is 150Hz in the signal $x_c(t)$. In the results above, only 11(c) doesn't meet this condition, so we can't see any peak in the plot of magnitude.