National Tsing Hua University
Department of Electrical Engineering
EE3660 Intro. to Digital Signal Processing, Spring 2020

Homework Assignment #1: Chap. 2-4

Due: March 26, 2020

I Paper Assignment (100%)

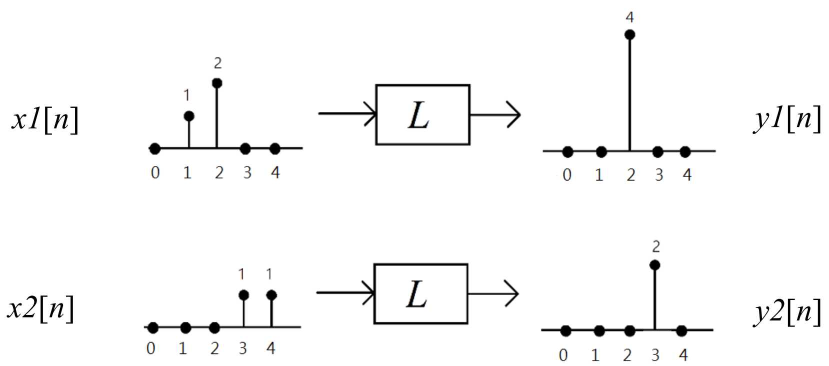
1. (12%) Explain whether the following systems are linear, time-invariant, causal, and stable.

   (a) $y[n] = x[-n]$

   (b) $y[n] = \cos(\pi n)x[n]$

   (c) $y[n] = \sum_{k=n-1}^{k=\infty} x[k]$

2. (10%) A linear system L generates output signals: $y1[n]$, $y2[n]$ in response to the input signals $x1[n]$, $x2[n]$ respectively. Explain whether the system L is time invariant.



3. (12%) Given the z-transform pair $x[n] \leftrightarrow X(z) = \frac{1}{(1-2z^{-1})}$ with ROC: $|z| < 2$, use the z-transform properties to determine the z-transform of the following sequences:

   (a) $y[n] = (\frac{1}{3})^n x[n]$

   (b) $y[n] = x[n]*x[-n]$ *(* denotes convolution)*

   (c) $y[n] = nx[n]$

4. (12%) A causal LTI system has impulse response h[n], for which the z-transform is

$$H(z) = \frac{1 + z^{-1}}{(1 - 0.5z^{-1})(1 + 0.25z^{-1})}$$

(a) Draw the pole-zero plot of H(z) and specify its ROC.

(b) Explain whether the system is stable?

(c) Find the impulse response h[n] of the system.

5. (15%) Use the method of partial fraction expansion to determine the sequences corresponding to the following z-transforms:

(a) $X(z) = \frac{z}{z^3 + 2z^2 + \frac{5}{4}z + \frac{1}{4}}$, $|z| > 1$.

(b) $X(z) = \frac{z}{(z^2 - \frac{1}{3})^2}$, $|z| < 0.5$.

6. (12%) A function called autocorrelation for a real-valued, absolutely summable sequence x[n], is defined as

$$r_{xx}[\ell] \triangleq \sum_n x[n]x[n-\ell].$$

Let X(z) be the z-transform of x[n] with ROC $\alpha < |z| < \beta$.

(a) Show that the z-transform of $r_{xx}[\ell]$ is given by $R_{xx}(z) = X(z)X(z^{-1})$.

(b) Let $x[n] = a^n u[n]$, $|a| < 1$. Determine $R_{xx}(z)$ and sketch its pole-zero plot and the ROC.

7. (12%) Determine the DTFT of following signals:

(a) $x_1[n] = \left(\frac{1}{4}\right)^n \cos\left(\frac{\pi n}{4}\right) u[n-2]$

(b) $x_3[n] = \sin(0.1\pi n)(u[n] - u[n-10])$

8. (15%) Let $x[n]$ and $y[n]$ denote complex sequences and $X(e^{j\omega})$ and $Y(e^{j\omega})$ their respective Fourier transforms

(a) Determine, in terms of x[n] and y[n], the sequence whose Fourier transform is $X(e^{j\omega})Y^*(e^{j\omega})$.

(b) Using the result in part (a), show that

$$\sum_{n=-\infty}^{\infty} x[n]y^*[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})Y^*(e^{j\omega})d\omega. \qquad \text{(eq.7b)}$$

(*eq.7b is a more general form of Parseval's theorem, as mentioned in lecture slide ch4 p17.*)

(c) Using (eq.7b), determine the value of the sum

$$\sum_{-\infty}^{\infty} \frac{sin(\pi n/5)}{2\pi n} \frac{sin(\pi n/3)}{7\pi n}$$

(*Hint: Check what is the expression of an inverse Fourier transform of a rectangular pulse mentioned in lecture slide ch4 p23.*)