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1. linear, time-invariant, causal, and stable

(a) $y[n] = x[-n]$: linear, not time-invariant, not causal, stable #

(i) linear

Let $x_1[n] \to y_1[n] = x_1[-n]$ $\quad ax_1[n]+bx_2[n] \Rightarrow ax_1[-n]+bx_2[-n] = ay_1[n]+by_2[n]$

$x_2[n] \to y_2[n] = x_2[-n]$

(ii) not time-invariant

Let $y_1[n] = x_1[-n]$ $\quad y_1[n-n_0] = x_1[-n+n_0]$

$x_2[n] = x_2[n-n_0]$, $y_2[n] = x_2[-n] = x_2[-n-n_0] \neq y_1[n-n_0]$

(iii) not causal

Output depends on input at future when $n<0$

(iv) Stable

If $|x[n]| < B \ \forall n$, then $|y[n]| = |x[-n]| < B$

(b) $y[n] = \cos(\pi n)x[n]$: linear, time-invariant, causal, stable #

(i) linear

Let $x_1[n] \to y_1[n] = \cos(\pi n)x_1[n]$ $\quad ax_1[n]+bx_2[n] \to \cos(\pi n)ax_1[n] + \cos(\pi n)bx_2[n]$

$x_2[n] \to y_2[n] = \cos(\pi n)x_2[n]$ $\quad = ay_1[n]+by_2[n]$

(ii) not time-invariant

Let $y_1[n] = \cos(\pi n)x_1[n]$ $\quad y_1[n-n_0] = \cos(\pi(n-n_0))x_1[n-n_0]$

$x_2[n] = x_1[n-n_0]$, $y_2[n] = \cos(\pi n)x_2[n] = \cos(\pi n)x_2[n-n_0] \neq y_1[n-n_0]$

(iii) Causal

(iv) stable : If $|x[n]| < B$, then $|y[n]| = |\cos(\pi n)x[n]| < |x[n]| < B$

(c) $y[n] = \sum_{k=n-1}^{\infty} x[k]$: linear, time-invariant, not causal, unstable #

(i) linear

Let $x_1[n] \to y_1[n] = \sum_{k=n-1}^{\infty} x_1[k]$ $\quad ax_1[n]+bx_2[n] \to ay_1[n]+by_2[n]$

$x_2[n] \to y_2[n] = \sum_{k=n-1}^{\infty} x_2[k]$

(ii) time-invariant

Let $y_1[n] = \sum_{k=n-1}^{\infty} x_1[k]$ $\quad y_1[n-n_0] = \sum_{k=n-1-n_0}^{\infty} x_1[k]$

$x_2[n] = x_1[n-n_0]$, $y_2[n] = \sum_{k=n-1}^{\infty} x_2[k] = \sum_{k=n-1}^{\infty} x_1[k-n_0] = \sum_{k=n-n_0-1}^{\infty} x_1[k] = y_1[n-n_0]$

(iii) not causal

(iv) unstable : If $|x[n]| < B$, then $|y[n]| = \left|\sum_{k=n-1}^{\infty} x[k]\right| = \infty$

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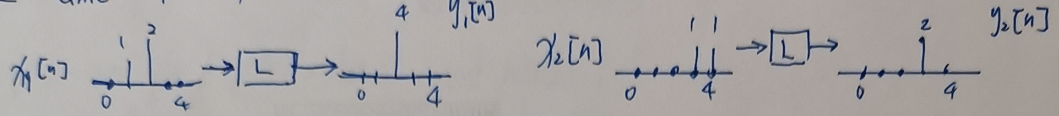
(linear)

2. L time-invariant ?



- $x_1[n] = a_1 + 2a_2$ $\quad y_1[n] = 4a_2$, $x_2[n] = a_3 + a_4$ $\quad y_2[n] = 2a_3$
- If L is time-invariant, $x_1[n-2] = a_3 + 2a_4 \to y_1[n-2] = 4a_4$

  Due to L is a linear system, $\underbrace{x_1[n-2] - x_2[n]}_{x_3[n]} = a_4 \to \underbrace{y_1[n-2] - y_2[n]}_{y_3[n]} = 4a_4 - 2a_3$

  Then $x_3[n] + x_3[n+1] = a_3 + a_4 = x_2[n] \to y_3[n] + y_3[n+1] = 4a_4 - 2a_3 + 4a_3 - 2a_3 \neq y_2[n]$
- Therefore, L isn't a time-invariant system #

3. $x[n] \leftrightarrow X(z) = \frac{1}{1-2z^{-1}}$ w/ ROC: $|z|<2$

(a) $y[n] = \left(\frac{1}{3}\right)^n x[n]$ $\qquad x[n] = -2^n u[-n-1]$

$\Rightarrow Y(z) = \frac{1}{1-\frac{2}{3}z^{-1}}$ with ROC: $|z| < \frac{2}{3}$ #

(b) $y[n] = x[n] * x[-n]$

$\Rightarrow Y(z) = X_1(z)X_2(z)$, ROC $\supset (R_1 \cap R_2)$, $X_1(z) = \frac{1}{1-2z^{-1}}$ w/ ROC: $|z|<2$

$x[-n] \to \overset{"}{X_1}(z^{-1})$ w/ ROC: $|z| > \frac{1}{2}$, $X_2(z) = \frac{1}{1-2z}$

$\Rightarrow Y(z) = \frac{1}{1-2z^{-1}-2z+4} = \frac{z}{-2z^2+5z-2}$ with ROC: $\frac{1}{2} < |z| < 2$ #

(c) $y[n] = n\,x[n]$

$\Rightarrow Y(z) = -z\,\frac{d(1-2z^{-1})^{-1}}{dz} = -z \times -(1-2z^{-1})^{-2} \times 2z^{-2} = \frac{2z}{(z-2)^2}$ with ROC: $|z|<2$ #

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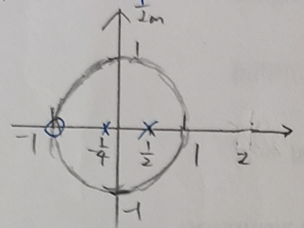
4. Causal impulse response $h[n]$, $H(z)=\dfrac{1+z^{-1}}{(1-0.5z^{-1})(1+0.25z^{-1})}$

(a) pole-zero plot (b) stable? (c) $h[n]=$?

(a)



zero : -1

pole : $\frac{1}{2}, -\frac{1}{4}$

Causal $\to h[n]=a^n u[n]$

$\Rightarrow \left(|z|>\frac{1}{2}\right)\cap\left(|z|>\frac{1}{4}\right)$

$\Rightarrow$ ROC : $|z|>\frac{1}{2}$ #

(b) This is a stable system because R.O.C. include $|z|=1$ #

(c) $H(z)=\dfrac{2}{1-\frac{1}{2}z^{-1}}+\dfrac{-1}{1+\frac{1}{4}z^{-1}} \Rightarrow h[n]=2\left(\frac{1}{2}\right)^n u[n]-\left(-\frac{1}{4}\right)^n u[n]$ #

5. (a) $X(z)=\dfrac{z}{z^3+2z^2+\frac{5}{4}z+\frac{1}{4}}$, $|z|>1$

$z^3+2z^2+\frac{5}{4}z+\frac{1}{4}=z(z+1)^2+\frac{1}{4}(z+1)=(z+1)\left(\frac{1}{4}+z^2+z\right)=(z+1)\left(z+\frac{1}{2}\right)^2$

$\hookrightarrow X(z)=\dfrac{4z}{z+1}+\dfrac{-4z}{z+\frac{1}{2}}+\dfrac{-4\times-\frac{1}{2}z}{\left(z+\frac{1}{2}\right)^2} \to x[n]=4(-1)^n u[n]+(-4)\left(-\frac{1}{2}\right)^n u[n]+(-4)\left(-\frac{1}{2}\right)^n n\,u[n]$ #

(b) $X(z)=\dfrac{z}{\left(z^2-\frac{1}{3}\right)^2}$, $|z|<0.5$ $\sqrt{\frac{1}{3}}=0.577$ $\left(z^2-\frac{1}{3}\right)^2=\left(z-\frac{1}{\sqrt{3}}\right)^2\left(z+\frac{1}{\sqrt{3}}\right)^2$

$\hookrightarrow X(z)=\dfrac{-\frac{1}{4}z}{z-\frac{1}{\sqrt{3}}}+\dfrac{\frac{1}{4}\times\frac{1}{\sqrt{3}}z}{\left(z-\frac{1}{\sqrt{3}}\right)^2}+\dfrac{\frac{1}{4}z}{z+\frac{1}{\sqrt{3}}}+\dfrac{-\frac{1}{4}\times\frac{-1}{\sqrt{3}}z}{\left(z+\frac{1}{\sqrt{3}}\right)^2} \to x[n]=\left(-\frac{1}{4}\right)\left(\frac{1}{\sqrt{3}}\right)^n u[n]+\left(\frac{1}{4}\right)\left(\frac{1}{\sqrt{3}}\right)^n n\,u[n]+$

$\left(\frac{1}{4}\right)\left(\frac{-1}{\sqrt{3}}\right)^n u[n]+\left(-\frac{1}{4}\right)\left(\frac{-1}{\sqrt{3}}\right)^n n\,u[n]$ #

6. Autocorrelation for real-value, absolutely summable sequence $x[n]$

- $\gamma_{xx}[\ell]\triangleq\sum_n x[n]x[n-\ell]$, Let $X(z)$ w/ ROC : $\alpha<|z|<\beta$

(a) Show $R_{xx}(z)=X(z)X(z^{-1})$ (b) Let $x[n]=a^n u[n]$, $|a|<1$, Determine $R_{xx}(z)$

(a) $x_1[n]*x_2[n]\xleftrightarrow{z}X_1(z)X_2(z)$ $r_{xx}[\ell]\triangleq\sum_n x[n]x[n-\ell]$; $X(z)=\sum_{n=-\infty}^{+\infty}x[n]z^{-n}$, $X(z^{-1})=\sum_n x[n]z^{n}$

$x[-n]\xleftrightarrow{z}X(z^{-1})$

- Assume $x_1[n]=x[-n] \to r_{xx}[\ell]\triangleq\sum_n x[n]x[n-\ell]=\sum_n x[n]x_1[\ell-n]=x[n]*x_1[n]$

$x_1[\ell-n]=x[n-\ell] \Rightarrow R_{xx}(z)=X(z)X_1(z)=X(z)X(z^{-1})$ #

- $X(z^{-1})$ w/ ROC : $\frac{1}{\alpha}>|z|>\frac{1}{\beta}$

- ROC of $R_{xx}(z)$: $(\alpha<|z|<\beta)\cap\left(\frac{1}{\beta}<|z|<\frac{1}{\alpha}\right) \to \max\left(\alpha,\frac{1}{\beta}\right)<|z|<\min\left(\beta,\frac{1}{\alpha}\right)$ #

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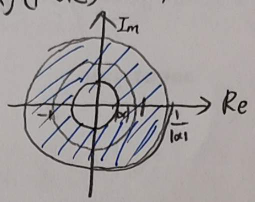
6. (b) $x[n]=\alpha^n u[n],\ |\alpha|<1 \rightarrow X(z)=\frac{z}{z-\alpha}$ with ROC: $|z|>|\alpha|$

$R_{xx}(z)=X(z)X(z^{-1})=\frac{z}{(z-\alpha)(1-\alpha z)}$ w/ zero: 0, pole: $\alpha, \frac{1}{\alpha}$ #



7. Determine DTFT

(a) $x_1[n]=(\frac{1}{4})^n\cos(\frac{\pi n}{4})u[n-2]$ Modulation: $x[n]\cos(\omega_c n)\longleftrightarrow \frac{1}{2}X(e^{j(\omega+\omega_c)})+\frac{1}{2}X(e^{j(\omega-\omega_c)})$

$a^n u[n]\longleftrightarrow \frac{1}{1-ae^{-j\omega}}$, $x[n-n_0]\longleftrightarrow X(e^{j\omega})e^{-j\omega n_0}$, $\cos(\omega_0 n)\longleftrightarrow \pi\sum_{k=-\infty}^{+\infty}\{\delta(\omega-\omega_0-2\pi k)+\delta(\omega+\omega_0-2\pi k)\}$

$x[n]=(\frac{1}{4})^n u[n-2]\longleftrightarrow X(e^{j\omega})=\frac{e^{-j2\omega}}{1-\frac{1}{4}e^{-j\omega}}$ $x_1[n]=x[n]\cos(\frac{\pi}{4}n)$

$\Rightarrow X_1(e^{j\omega})=\frac{1}{2}X(e^{j(\omega+\frac{\pi}{4})})+\frac{1}{2}X(e^{j(\omega-\frac{\pi}{4})})=(\frac{1}{2}e^{-j(2\omega+\frac{\pi}{2})})/(1-\frac{1}{4}e^{-j(\omega+\frac{\pi}{4})})+(\frac{1}{2}e^{-j(2\omega-\frac{\pi}{2})})/(1-\frac{1}{4}e^{-j(\omega-\frac{\pi}{4})})$ #

(b) $x_3[n]=\sin(0.1\pi n)(u[n]-u[n-10])$ $\sin(\omega_c n)=\frac{1}{2j}(e^{j\omega_c n}-e^{-j\omega_c n})$

• $x[n-n_0]\leftrightarrow X(e^{j\omega})e^{-j\omega n_0}$. • If $y[n]=\frac{1}{j}e^{j\omega_c n}x[n]\longleftrightarrow Y(e^{j\omega})=\frac{1}{j}\sum_n e^{j\omega_c n}x[n]e^{-j\omega n}=\frac{1}{j}X(e^{j(\omega-\omega_c)})$

$\Rightarrow \sin(\omega_c n)x[n]\leftrightarrow \frac{1}{2j}(X(e^{j(\omega-\omega_c)})-X(e^{j(\omega+\omega_c)}))$

• $x[n]=u[n]-u[n-10]\longleftrightarrow X(e^{j\omega})=(1-e^{-j\omega})^{-1}(1-e^{-j10\omega})$

$\Rightarrow X_3(e^{j\omega})=\frac{1}{2j}[(1-e^{-j(10\omega-\pi)})/(1-e^{-j(\omega-0.1\pi)})-(1-e^{-j(10\omega+\pi)})/(1-e^{-j(\omega+0.1\pi)})]$ #

8. $x[n]\leftrightarrow X(e^{j\omega})$, $y[n]\leftrightarrow Y(e^{j\omega})$ (a) $X(e^{j\omega})Y^*(e^{j\omega})$ (b) Show $\sum_n x[n]y^*[n]=\frac{1}{2\pi}\int_{-\pi}^{\pi}X(e^{j\omega})Y^*(e^{j\omega})d\omega$

(c) determine $\sum_n \frac{\sin(\pi n/5)}{2\pi n}\frac{\sin(\pi n/3)}{7\pi n}$

$X(e^{j\omega})=\sum_n x[n]e^{-j\omega n}$

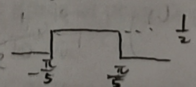
$x[n]=\frac{1}{2\pi}\int_{2\pi}X(e^{j\omega})e^{j\omega n}d\omega$ (a) $X(e^{j\omega})Y^*(e^{j\omega})\longleftrightarrow x[n]*y^*[-n]=\sum_{k=-\infty}^{+\infty}x[k]y^*[k-n]$ #

$X^*(e^{j\omega})\longleftrightarrow x^*[-n]$

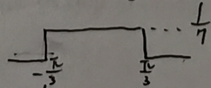
(b) $\sum_n x[n]y^*[n]=\sum_n x[n][\frac{1}{2\pi}\int_{2\pi}Y^*(e^{j\omega})e^{j\omega n}d\omega]=\frac{1}{2\pi}\int_{2\pi}[\sum_n x[n]e^{-j\omega n}]Y^*(e^{j\omega})d\omega$

$=\frac{1}{2\pi}\int_{2\pi}X(e^{j\omega})Y^*(e^{j\omega})d\omega$ #

(c) $x[n]=\frac{\sin(\pi n/5)}{2\pi n}=\frac{1}{2}\frac{\sin(\pi n/5)}{\pi n}\longleftrightarrow$



$y^*[n]=\frac{\sin(\pi n/3)}{7\pi n}=\frac{1}{7}\frac{\sin(\pi n/3)}{\pi n}\longleftrightarrow$



$\Rightarrow \frac{1}{2\pi}\int_{-\pi}^{\pi}X(e^{j\omega})Y^*(e^{j\omega})=\frac{1}{14}\times(\frac{\pi}{5}-\frac{-\pi}{5})\times\frac{1}{2\pi}=\frac{1}{70}$ #