# emd

Empirical mode decomposition

# **Syntax**

```
[imf,residual] = emd(X)[imf,residual,info] = emd(X)
[ ___ ] = emd( ___ ,Name,Value)
```
[emd\(](#page-0-4) **\_\_\_** )

# **Description**

<span id="page-0-2"></span><span id="page-0-1"></span>

# <span id="page-0-4"></span><span id="page-0-3"></span>**Examples**

# <span id="page-0-0"></span>**Perform Empirical Mode Decomposition and Visualize Hilbert Spectrum of Signal**

```
Load and visualize a nonstationary continuous signal composed of sinusoidal waves
with a distinct change in frequency. The vibration of a jackhammer and the sound of
fireworks are examples of nonstationary continuous signals. The signal is sampled at a
rate fs.
```

```
load('sinusoidalSignalExampleData.mat','X','fs')
t = (0:\text{length}(X)-1)/fs;plot(t,X)
xlabel('Time(s)')
```


The mixed signal contains sinusoidal waves with different amplitude and frequency values.

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[View MATLAB Command](matlab:openExample()

To create the Hilbert spectrum plot, you need the intrinsic mode functions (IMFs) of the signal. Perform empirical mode decomposition to compute the IMFs and residuals of the signal. Since the signal is not smooth, specify 'pchip' as the interpolation method.

```
[imf,residual,info] = emd(X,'Interpolation','pchip');
```
The table generated in the command window indicates the number of sift iterations, the relative tolerance, and the sift stop criterion for each generated IMF. This information is also contained in info. You can hide the table by adding the 'Display',0 name value pair.

Create the Hilbert spectrum plot using the imf components obtained using empirical mode decomposition.



The frequency versus time plot is a sparse plot with a vertical color bar indicating the instantaneous energy at each point in the IMF. The plot represents the instantaneous frequency spectrum of each component decomposed from the original mixed signal. Three IMFs appear in the plot with a distinct change in frequency at 1 second.

### <span id="page-1-0"></span>**Zero Crossings and Extrema in Intrinsic Mode Function of Sinusoid**

This trigonometric identity presents two different views of the same physical signal:

nis trigonometric identity presents two different views of the same physical signal:  
\n
$$
\cos 2 \pi f_1 t + \frac{1}{4} (\cos 2 \pi (f_1 + f_2)t + \cos 2 \pi (f_1 - f_2)t) = (2 + \cos^2 \pi f_2 t) \cos 2 \pi f_1 t.
$$

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Generate two sinusoids, s and z, such that s is the sum of three sine waves and z is a single sine wave with a modulated amplitude. Verify that the two signals are equal by calculating the infinity norm of their difference.

 $t = 0:1e-3:10;$ omega1 = 2\*pi\*100; omega2 = 2\*pi\*20;  $s = 0.25*cos((omegaega1-omegae)^*t) + 2.5*cos(omegaega1*t) + 0.25*cos((omegaega1+omegaega2)*t);$  $z = (2 + cos(omega2/2*t).2).*cos(omega24*t);$ norm(s-z,Inf)

ans = 3.2729e-13

5  $\overline{2}$ 

Plot the sinusoids and select a 1-second interval starting at 2 seconds.

```
plot(t,[s' z'])
xlim([2 3])
xlabel('Time (s)')
ylabel('Signal')
```


Obtain the spectrogram of the signal. The spectrogram shows three distinct sinusoidal components. Fourier analysis sees the signals as a superposition of sine waves.



Use emd to compute the intrinsic mode functions (IMFs) of the signal and additional diagnostic information. The function by default outputs a table that indicates the number of sifting iterations, the relative tolerance, and the sifting stop criterion for each IMF. Empirical mode decomposition sees the signal as z.

 $[imf,-,info] = end(s);$ 

The number of zero crossings and local extrema differ by at most one. This satisfies the necessary condition for the signal to be an IMF.

info.NumZerocrossing - info.NumExtrema

ans  $= 1$ 

Plot the IMF and select a 0.5-second interval starting at 2 seconds. The IMF is an AM signal because emd views the signal as amplitude modulated.



### <span id="page-3-0"></span>**Compute Intrinsic Mode Functions of Vibration Signal**

Simulate a vibration signal from a damaged bearing. Perform empirical mode decomposition to visualize the IMFs of the signal and look for defects.

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A bearing with a pitch diameter of 12 cm has eight rolling elements. Each rolling element has a diameter of 2 cm. The outer race remains stationary as the inner race is driven at 25 cycles per second. An accelerometer samples the bearing vibrations at 10 kHz.

 $fs = 10000$ ;  $f0 = 25;$  $n = 8$ ;  $d = 0.02$ ;  $p = 0.12;$ 





The vibration signal from the healthy bearing includes several orders of the driving frequency.

 $t = 0:1/fs:10-1/fs;$ yHealthy = [1 0.5 0.2 0.1 0.05]\*sin(2\*pi\*f0\*[1 2 3 4 5]'.\*t)/5;

A resonance is excited in the bearing vibration halfway through the measurement process.

yHealthy =  $(1+1./(1+1)$ inspace(-10,10,length(yHealthy)).^4)).\*yHealthy;

The resonance introduces a defect in the outer race of the bearing that results in progressive wear. The defect causes a series of impacts that recur at the ball pass frequency outer race (BPFO) of the bearing:<br>BPFO =  $\frac$ impacts that recur at the ball pass frequency outer race (BPFO) of the bearing:

$$
BPPO = \frac{1}{2}nf_0 \left[1 - \frac{d}{p}\cos\theta\right],
$$

where  $f_0$  is the driving rate,  $\,n$  is the number of rolling elements,  $d$  is the diameter of the rolling elements,  $\,p$  is the pitch diameter of the bearing, and  $\theta$  is the bearing contact angle. Assume a contact angle of 15° and compute the BPFO.

```
ca = 15;
bpfo = n*f0/2*(1-d/p*cosd(ca));
```
Use the [pulstran](https://www.mathworks.com/help/signal/ref/pulstran.html) function to model the impacts as a periodic train of 5-millisecond sinusoids. Each 3 kHz sinusoid is windowed by a flat top window. Use a power law to introduce progressive wear in the bearing vibration signal.

```
fImpack = 3000;tImpack = 0:1/fs:5e-3-1/fs;wImpact = flattopwin(length(tImpact))'/10;
xImpact = sin(2*pi*fImpact*tImpact).*wImpact;
tx = 0:1/bpfo:t(end);tx = [tx; 1.3.^{\text{dx}}-2];nWear = 49000;
nSamples = 100000;
yImpact = pulstran(t,tx',xImpact,fs)/5;
yImpact = [zeros(1,nWear) yImpact(1,(nWear+1):nSamples)];
```
Generate the BPFO vibration signal by adding the impacts to the healthy signal. Plot the signal and select a 0.3-second interval starting at 5.0 seconds.

```
yBPFO = yImpact + yHealthy;
xLimLeft = 5.0;
xLimRight = 5.3;
yMin = -0.6;yMax = 0.6;
```

```
plot(t,yBPFO)
hold on
[limLeft,limRight] = meshgrid([xLimLeft xLimRight],[yMin yMax]);
plot(limLeft,limRight,'--')
hold off
```


Zoom in on the selected interval to visualize the effect of the impacts.



Add white Gaussian noise to the signals. Specify a noise variance of  $\rm\,1/150^2$  .

rn = 150; yGood = yHealthy + randn(size(yHealthy))/rn; yBad = yBPFO + randn(size(yHealthy))/rn;



Use emd to perform an empirical mode decomposition of the healthy bearing signal. Compute the first five intrinsic mode functions (IMFs). Use the 'Display' name-value pair to show a table with the number of sifting iterations, the relative tolerance, and the sifting stop criterion for each IMF.

```
imfGood = emd(yGood,'MaxNumIMF',5,'Display',1);
Current IMF | #Sift Iter | Relative Tol | Stop Criterion Hit 
     1 | 3 | 0.015551 | SiftMaxRelativeTolerance
     2 | 3 | 0.078816 | SiftMaxRelativeTolerance
     3 | 8 | 0.085954 | SiftMaxRelativeTolerance
     4 | 1 | 0.0043681 | SiftMaxRelativeTolerance
     5 | 2 | 0.010567 | SiftMaxRelativeTolerance
```
The decomposition stopped because 'MaxNumIMF' was reached. Use emd without output arguments to visualize the first three modes and the residual.



Compute and visualize the IMFs of the defective bearing signal. The first empirical mode reveals the high-frequency impacts. This high-frequency mode increases in energy as the wear progresses. The third mode shows the resonance in the vibration signal.



[The next step in the analysis is to compute the Hilbert spectrum of the extracted IMFs. For more details, see the Compute Hilbert](https://www.mathworks.com/help/signal/ref/hht.html#mw_025b9602-e0a8-4b16-9224-21b3b95eb156) Spectrum of Vibration Signal example.

### <span id="page-7-0"></span>**Visualize Residual and Intrinsic Mode Functions of Signal**

Load and visualize a nonstationary continuous signal composed of sinusoidal waves with a distinct change in frequency. The vibration of a jackhammer and the sound of fireworks are examples of nonstationary continuous signals. The signal is sampled at a rate fs.

[View MATLAB Command](matlab:openExample()

```
load('sinusoidalSignalExampleData.mat','X','fs')
t = (0:\text{length}(X)-1)/fs;plot(t,X)
xlabel('Time(s)')
```


The mixed signal contains sinusoidal waves with different amplitude and frequency values.

Perform empirical mode decomposition to plot the intrinsic mode functions and residual of the signal. Since the signal is not smooth, specify 'pchip' as the interpolation method.



emd generates an interactive plot with the original signal, the first 3 IMFs, and the residual. The table generated in the command window indicates the number of sift iterations, the relative tolerance, and the sift stop criterion for each generated IMF. You can hide the table by removing the 'Display' name-value pair or specifying it as 0.

Right-click on the white space in the plot to open the **IMF selector** window. Use **IMF selector** to selectively view the generated IMFs, the original signal, and the residual.



Select the IMFs to be displayed from the list. Choose whether to display the original signal and residual on the plot.



The selected IMFs are now displayed on the plot.



Use the plot to visualize individual components decomposed from the original signal along with the residual. Note that the residual is computed for the total number of IMFs, and does not change based on the IMFs selected in the **IMF selector** window.

# **Input Arguments** [collapse all](javascript:void(0);) **intervention of the collapse all collapse all collapse all collapse all**

# <span id="page-10-0"></span>**<sup>X</sup> — Time-domain signal** vector | timetable

Time-domain signal, specified as a real-valued vector, or a single-variable timetable with a single column. If X is a timetable, X must contain increasing, finite row times.

# <span id="page-10-1"></span>**Name-Value Pair Arguments**

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1,Value1,...,NameN,ValueN.

**Example:** 'MaxNumIMF',5



<span id="page-10-2"></span>**'SiftRelativeTolerance' — Cauchy-type convergence criterion** 0.2 (default) | positive scalar

Cauchy-type convergence criterion, specified as the comma-separated pair consisting of 'SiftRelativeTolerance' and a positive scalar. SiftRelativeTolerance is one of the sifting stop criteria, that is, sifting stops when the current relative tolerance is less than SiftRelativeTolerance. For more information, see [Sift Relative Tolerance](#page-13-0).

**'SiftMaxIterations' — Maximum number of sifting iterations** 100 (default) | positive scalar integer

Maximum number of sifting iterations, specified as the comma-separated pair consisting of 'SiftMaxIterations' and a positive scalar integer. SiftMaxIterations is one of the sifting stop criteria, that is, sifting stops when the current number of iterations is larger than SiftMaxIterations.

SiftMaxIterations can be specified using only positive whole numbers.

### **'MaxNumIMF' — Maximum number of IMFs extracted** 10 (default) | positive scalar integer

Maximum number of IMFs extracted, specified as the comma-separated pair consisting of 'MaxNumIMF' and a positive scalar integer. MaxNumIMF is one of the decomposition stop criteria, that is, decomposition stops when number of IMFs generated is equal to MaxNumTMF.

MaxNumIMF can be specified using only positive whole numbers.

**'MaxNumExtrema' — Maximum number of extrema in the residual signal** 1 (default) | positive scalar integer

Maximum number of extrema in the residual signal, specified as the comma-separated pair consisting of 'MaxNumExtrema' and a positive scalar integer. MaxNumExtrema is one of the decomposition stop criteria, that is, decomposition stops when number of extrema is less than MaxNumExtrema.

MaxNumExtrema can be specified using only positive whole numbers.

<span id="page-11-1"></span>**'MaxEnergyRatio' — Signal to residual energy ratio** 20 (default) | scalar

Signal to residual energy ratio, specified as the comma-separated pair consisting of 'MaxEnergyRatio' and a scalar. MaxEnergyRatio is the ratio of the energy of the signal at the beginning of sifting and the average envelope energy. MaxEnergyRatio is one of the decomposition stop criteria, that is, decomposition stops when current energy ratio is larger than MaxEnergyRatio. For more information, see [Energy Ratio](#page-13-1).

**'Interpolation' — Interpolation method for envelope construction** 'spline' (default) | 'pchip'

Interpolation method for envelope construction, specified as the comma-separated pair consisting of 'Interpolation' and either 'spline' or 'pchip'.

Specify Interpolation as:

- 'spline', if [X](#page-10-0) is a smooth signal
- 'pchip', if X is a nonsmooth signal

'spline' interpolation method uses cubic spline, while 'pchip' uses piecewise cubic Hermite interpolating polynomial method.



Toggle information display in the command window, specified as the comma-separated pair consisting of 'Display' and either 0 or 1. The table generated in the command window indicates the number of sift iterations, the relative tolerance, and the sift stop criterion for each generated IMF. Specify Display as 1 to show the table or 0 to hide the table.

# **Output Arguments** [collapse all](javascript:void(0);) and the collapse all and the collapse all and the collapse all collapse a

<span id="page-11-0"></span>**imf — Intrinsic mode function** matrix | timetable

Intrinsic mode function (IMF), returned as a matrix or timetable. Each IMF is an amplitude and frequency modulated signal with positive and slowly varying envelopes. To perform spectral analysis of a signal, you can apply the Hilbert-Huang transform to its IMFs. See [hht](https://www.mathworks.com/help/signal/ref/hht.html) and [Intrinsic Mode Functions](#page-13-2).

imf is returned as:

- A matrix whose each column is an imf, when [X](#page-10-0) is a vector
- A timetable, when X is a single data column timetable

# <span id="page-12-0"></span>**residual — Residual of the signal** column vector | single data column timetable

Residual of the signal, returned as a column vector or a single data column timetable. residual represents the portion of the original signal [X](#page-10-0) not decomposed by emd.

residual is returned as:

- A column vector, when X is a vector.
- A single data column timetable, when X is a single data column timetable.

# <span id="page-12-1"></span>**info — Additional information for diagnostics** structure

Additional information for diagnostics, returned as a structure with the following fields:

- NumIMF Number of IMFs extracted NumIMF is a vector from 1 to N, where N is the number of IMFs. If no IMFs are extracted, NumIMF is empty.
- NumExtrema — Number of extrema in each IMF

NumExtrema is a vector equal in length to the number of IMFs. The kth element of NumExtrema is the number of extrema found in the kth IMF. If no IMFs are extracted, NumExtrema is empty.

• NumZerocrossing — Number of zero crossings in each IMF

Number of zero crossings in each IMF. NumZerocrossing is a vector equal in length to the number of IMFs. The kth element of NumZerocrossing is the number of zero crossings in the kth IMF. If no IMFs are extracted, NumZerocrossing is empty.

• NumSifting — Number of sifting iterations used to extract each IMF

NumSifting is a vector equal in length to the number of IMFs. The kth element of NumSifting is the number of sifting iterations used in the extraction of the kth IMF. If no IMFs are extracted, NumSifting is empty.

- MeanEnvelopeEnergy — Energy of the mean of the upper and lower envelopes obtained for each IMF If UE is the upper envelope and LE is the lower envelope, MeanEnvelopeEnergy is mean( $((LE+UL)/2)$ .<sup>^2</sup>). MeanEnvelopeEnergy is a vector equal in length to the number of IMFs. The kth element of MeanEnvelopeEnergy is the mean envelope energy for the kth IMF. If no IMFs are extracted, MeanEnvelopeEnergy is empty.
- RelativeTolerance — Final relative tolerance of the residual for each IMF

The relative tolerance is defined as the ratio of the squared 2-norm of the difference between the residual from the previous sifting step and the residual from the current sifting step to the squared 2-norm of the residual from the *i*th sifting step. The sifting process stops when RelativeTolerance is less than SiftRelativeTolerance. For additional information, see Sift Relative Tolerance. RelativeTolerance [is a vector equal in length to the number of IMFs. The](#page-13-0) kth element of RelativeTolerance is the final relative tolerance obtained for the kth IMF. If no IMFs are extracted, RelativeTolerance is empty.

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### **Empirical Mode Decomposition**

The empirical mode decomposition (EMD) algorithm decomposes a signal  $X(t)$  into intrinsic mode functions (IMFs) and a residual in an iterative process. The core component of the algorithm involves *sifting* a function  $X(t)$  to obtain a new function  $Y(t)$ : −

- First find the local minima and maxima of  $X(t)$ .
- Then use the local extrema to construct lower and upper envelopes  $s_-(t)$  and  $s_+(t)$ , respectively, of  $X(t)$ . Form the mean of the envelopes,  $m(t)$ .<br>Subtract the mean from  $X(t)$  to obtain the residual:  $Y(t) = X(t) - m(t)$ . envelopes,  $m(t)$ .
- •

An overview of the decomposition is as follows:

- 1. To begin, let  $r_0(t) = X(t)$ , where  $X(t)$  is the initial signal, and let  $i = 0$ .
- 2. Before sifting, check  $r_i(t)$ :
	- a. Find the total number (TN) of local extrema of  $r_i(t)$ .

b. Find the energy ratio (ER) of  $r_i(t)$  (see [Energy Ratio](#page-13-1)).

- 3. If (ER > MaxEnergyRatio) or (TN < MaxNumExtrema) or (number of IMFs > MaxNumIMF) then stop the decomposition.
- 4. Let  $r_{i, \text{Prev}}(t) = r_i(t)$ .
- 5. Sift  $r_{i,prev}(t)$  to obtain  $r_{i,Cur}(t)$ .
- 6. Check  $r_{i,\text{Cur}}(t)$

a. Find the relative tolerance (RT) of  $r_{i,\mathrm{Cur}}(t)$  (see [Sift Relative Tolerance](#page-13-0)).

- b. Get current sift iteration number (IN).
- 7. If (RT < SiftRelativeTolerance) or (IN > SiftMaxIterations) then stop sifting. An IMF has been found: IMF $_i(t) = r_{i,Cur}(t)$ . Otherwise, let  $r_{i,prev}(t) = r_{i,Cur}(t)$  and go to Step 5.
- 8. Let  $r_{i+1}(t) = r_i(t) r_{i,\text{Cur}}(t)$ .
- 9. Let  $i = i + 1$ . Return to Step 2.

For additional information, see [\[1\]](#page-13-3) and [\[3\].](#page-14-0)

### <span id="page-13-2"></span>**Intrinsic Mode Functions**

The EMD algorithm decomposes, via an iterative sifting process, a signal  $X(t)$  into IMFs  $\mathit{imfi}(t)$  and a residual  $\mathit{r}_N(t)$ :

$$
X(t) = \sum_{i=1}^{N} \text{IMF}_i(t) + r_N(t)
$$

When first introduced by Huang et al. [\[1\]](#page-13-3), an IMF was defined to be a function with two characteristics:

- The number of local extrema — the total number of local minima and local maxima — and the number of zero crossings differ by at most one.
- The mean value of the upper and lower envelopes constructed from the local extrema is zero.

.

However, as noted in [\[4\]](#page-14-1), sifting until a strict IMF is obtained can result in IMFs that have no physical significance. Specifically, sifting until the number of zero crossings and local extrema differ by at most one can result in pure-tone like IMFs, in other words, functions very similar to what would be obtained by projection on the Fourier basis. This situation is precisely what EMD strives to avoid, preferring AM-FM modulated components for their physical significance.

Reference [\[4\]](#page-14-1) proposes options to obtain physically meaningful results. The emd function relaxes the original IMF definition by using [Sift Relative Tolerance](#page-13-0), a Cauchy-type stop criterion. The emd function iterates to extract natural AM-FM modes. The IMFs [generated may fail to satisfy the local extrema-zero crossings criteria. See Zero Crossings and Extrema in Intrinsic Mode Function of](#page-1-0) Sinusoid.

### <span id="page-13-0"></span>**Sift Relative Tolerance**

*Sift Relative Tolerance* is a Cauchy-type stop criterion proposed in [\[4\].](#page-14-1) Sifting stops when current relative tolerance is less than [SiftRelativeTolerance](#page-10-2). The current relative tolerance is defined as

$$
\text{Relative Tolerance} \triangleq \frac{\|r_{\text{prev}}(t) - r_{\text{cur}}(t)\|_2^2}{\|r_{\text{prev}}(t)\|_2^2}
$$

Because the Cauchy criterion does not directly count the number of zero crossings and local extrema, it is possible that the IMFs returned by the decomposition do not satisfy the strict definition of an intrinsic mode function. In those cases, you can try reducing the value of the SiftRelativeTolerance from its default value. See [\[4\]](#page-14-1) for a detailed discussion of stopping criteria. The reference also discusses the advantages and disadvantages of insisting on strictly defined IMFs in empirical mode decomposition.

### <span id="page-13-1"></span>**Energy Ratio**

Energy ratio is the ratio of the energy of the signal at the beginning of sifting and the average envelope energy [\[2\]](#page-14-2). Decomposition stops when current energy ratio is larger than [MaxEnergyRatio](#page-11-1). For the ith IMF, the energy ratio is defined as

Energy Ratio 
$$
\triangleq 10 \log_{10} \left( \frac{\|X(t)\|_2}{\|r_i(t)\|_2} \right)
$$
.

### **References**

<span id="page-13-3"></span>[1] Huang, Norden E., Zheng Shen, Steven R. Long, Manli C. Wu, Hsing H. Shih, Quanan Zheng, Nai-Chyuan Yen, Chi Chao Tung, and Henry H. Liu. "The Empirical Mode Decomposition and the Hilbert Spectrum for Nonlinear and Non-Stationary Time Series Analysis." *Proceedings of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences* 454, no. 1971 (March 8, 1998): 903–95. https://doi.org/10.1098/rspa.1998.0193.

<span id="page-14-2"></span>[2] Rato, R.T., M.D. Ortigueira, and A.G. Batista. "On the HHT, Its Problems, and Some Solutions." *Mechanical Systems and Signal Processing* 22, no. 6 (August 2008): 1374–94. https://doi.org/10.1016/j.ymssp.2007.11.028.

<span id="page-14-0"></span>[3] Rilling, Gabriel, Patrick Flandrin, and Paulo Gonçalves. "On Empirical Mode Decomposition and Its Algorithms." *IEEE-EURASIP Workshop on Nonlinear Signal and Image Processing* 2003. NSIP-03. Grado, Italy. 8–11.

<span id="page-14-1"></span>[4] Wang, Gang, Xian-Yao Chen, Fang-Li Qiao, Zhaohua Wu, and Norden E. Huang. "On Intrinsic Mode Function." *Advances in Adaptive Data Analysis* 02, no. 03 (July 2010): 277–93. https://doi.org/10.1142/S1793536910000549.

# **Extended Capabilities**

### **C/C++ Code Generation**

Generate C and C++ code using MATLAB® Coder™.

# **See Also**

# **Functions**

[hht](https://www.mathworks.com/help/signal/ref/hht.html) | [vmd](https://www.mathworks.com/help/signal/ref/vmd.html)

**Apps** [Signal Multiresolution Analyzer](https://www.mathworks.com/help/wavelet/ref/signalmultiresolutionanalyzer-app.html)

### **Topics**

[Time-Frequency Gallery](https://www.mathworks.com/help/signal/ug/time-frequency-gallery.html)

**Introduced in R2018a**