National Tsing Hua University
Department of Electrical Engineering
EE3660 Intro. to Digital Signal Processing, Spring 2019

Midterm Exam (100%)
April 24, 2019

Note: Detailed derivations are required to obtain a full score for each problem.

1. (10%) Determine the impulse response $h[n]$ of the system described by

$$y[n] - y[n-1] - 2y[n-2] = 5x[n] - x[n-1]$$

for all possible regions of convergence.

2. (10%) Consider a sinusoidal signal $x_c(t) = \sin(2\pi F_0 t + \theta_0)$ with $F_0 = 50$ Hz. It is sampled at different rates of F_s and then reconstructed as $y_r(t)$ by each corresponding ideal DAC.

- **(a)** (3%) Determine $y_r(t)$ if $F_s = 120$ Hz for $\theta_0 = 0$ and $\pi/2$ respectively.
- **(b)** (3%) Determine $y_r(t)$ if $F_s = 100$ Hz in terms of θ_0.
- **(c)** (4%) Determine $y_r(t)$ if $F_s = 60$ Hz for $\theta_0 = 0$ and $\pi/2$ respectively.

3. (12%) Let $x[n]$ be a real-valued N-point sequence with N-point DFT $X[k]$.

- **(a)** (2%) Show that $X[N/2]$ is real-valued if N is even.
- **(b)** (4%) Show that $|X[k]| = |X[\langle -k \rangle_N]|$ and $\angle X[k] = -\angle X[\langle -k \rangle_N]$.
- **(c)** (6%) If $x[n]$ satisfies the condition $x[n] = x[\langle n + M \rangle_N]$ where $N = 2kM$ and k is an integer, show that $X[2kl + k] = 0$ for $l = 0, 1, ..., M-1$.

4. (13%) Let $x_1[n] = \{\underset{\uparrow}{1000}, 100, 10, 1\}$ and $x_2[n] = \{\underset{\uparrow}{8}, 4, 2, 1\}$ be four-point sequences. Let $x_3[n] = x_1[n] * x_2[n]$.

- **(a)** (3%) Determine the DTFT $X_3(e^{j\omega})$.
- **(b)** (5%) Sample frequency components as four-point DFT $X_4[k] = X_3(e^{j2\pi k/4})$ where $k = 0, 1, 2, 3$. Determine its IDFT $x_4[n]$.
- **(c)** (5%) Sample frequency components as eight-point DFT $X_5[k] = X_3(e^{j2\pi k/8})$ where $k = 0, 1, ..., 7$. Determine its IDFT $x_5[n]$.

5. (13%) Consider the discrete-time system given by

$$\sum_{k=0}^{4} \left(\frac{1}{3}\right)^k y[n-k] = \sum_{l=1}^{4} \left(\frac{1}{2}\right)^l x[n-l].$$

(a) (3%) Draw its normal direct form I structure.

(b) (3%) Draw its normal direct form II structure.

(c) (4%) Draw its transposed direct form II structure.

(d) (3%) State the benefits of the structure in (c) over those in (a) and (b), respectively.

6. (12%) Consider a lowpass linear-phase FIR filter design using the fixed windows given in the table below. The specifications are $\omega_p = 0.2\pi$, $\omega_s = 0.4\pi$, and $\delta_s = 0.025$. Determine the following design parameters to minimize the filter length: **(a)** (4%) window name, **(b)** (4%) cut-off frequency ω_c, and **(c)** (4%) window (filter) length L.

Window	Sidelobe (dB)	$\Delta\omega$	A_s (dB)
Rectangular	-13	$1.8\pi/L$	21
Barlett	-25	$6.1\pi/L$	26
Hann	-31	$6.2\pi/L$	44
Hamming	-41	$6.6\pi/L$	53
Blackman	-57	$11\pi/L$	74

7. (10%) Consider a type-II linear-phase FIR filter $y[n] = \sum_{k=0}^{M} h[k]x[n-k]$ for which $M = 5$. Implement this system using only three multiplications and draw the corresponding structures: **(a)** (5%) direct form and **(b)** (5%) transposed form.

8. (20%) Consider a MATLAB FFT function $\mathbf{myfft(x, n)}$ which performs exactly $\mathbf{n}$-point FFT of an $\mathbf{n}$-point input vector $\mathbf{x}$. It can be used for efficient computation. For example, the IDFT $\mathbf{y}$ of a given $\mathbf{512}$-point DFT $\mathbf{Y}$ (as a row vector) can be derived by the following MATLAB code.

$$\mathbf{Yflip} = [\mathbf{Y(1)}\ \mathbf{Y(512:-1:2)}];$$
$$\mathbf{y} = \mathbf{myfft(Yflip, 512)/512};$$

Write down efficient MATLAB codes for the following algorithms using only the **myfft** function and simple arithmetic/indexing/padding operations. All 1D vectors are row-wise arranged.

(a) (5%) Linear convolution. Given a 133-point **x1** and a 157-point **x2**, derive its linear convolution **x3** (289-point) through multiplication in DFT domain.

(b) (5%) Time-domain resolution scaling-up. Given a 64-point **x4**, derive its 32× scaled-up **x5** (2048-point) by zero padding, e.g. $\mathbf{zeros(1, k)}$, in DFT domain.

(c) (10%) Efficient IDFT for real-valued sequences. Given two 1024-point DFTs **X6** and **X7** of two real-valued sequences, derive their 1024-point IDFTs **x6** and **x7**, respectively, by calling $\mathbf{myfft}(..., \mathbf{1024})$ once. You may need the **real** and **imag** functions to extract the real and imaginary parts. You may also need the symmetry properties as below.

$$x[n] = x_R^{ce}[n] + x_R^{co}[n] + jx_I^{ce}[n] + jx_I^{co}[n]$$
$$X[k] = X_R^{ce}[k] + X_R^{co}[k] + jX_I^{ce}[k] + jX_I^{co}[k]$$