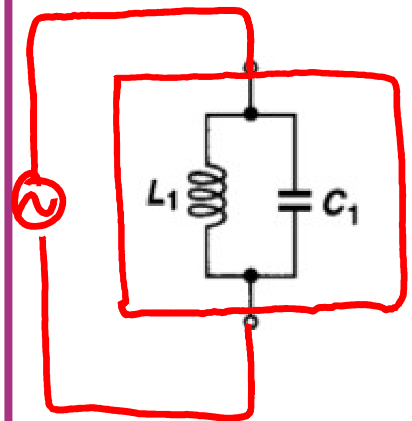
EE4280 Lecture 5:
LC Oscillators

Ping-Hsuan Hsieh (謝秉璇)
Delta Building R908
EXT 42590
phsieh@ee.nthu.edu.tw

Starting from LC Tank …

- **At resonant frequency of** $\omega_{res} = 1/\sqrt{L_1 C_1}$
  - The inductor and the capacitor impedance are equal and opposite
- **Ideally without any loss, the impedance goes to infinity → infinite *Q***
  - Inductive when $\omega < \omega_{res}$, voltage leads current by 90°
  - Capacitive when $\omega > \omega_{res}$, current leads voltage by 90°

$$Z_{tank} = Z_L // Z_C = \frac{sL \cdot \frac{1}{sC}}{sL + \frac{1}{sC}} = \frac{sL}{1+s^2LC} = \frac{j\omega L}{1-\omega^2 LC}$$

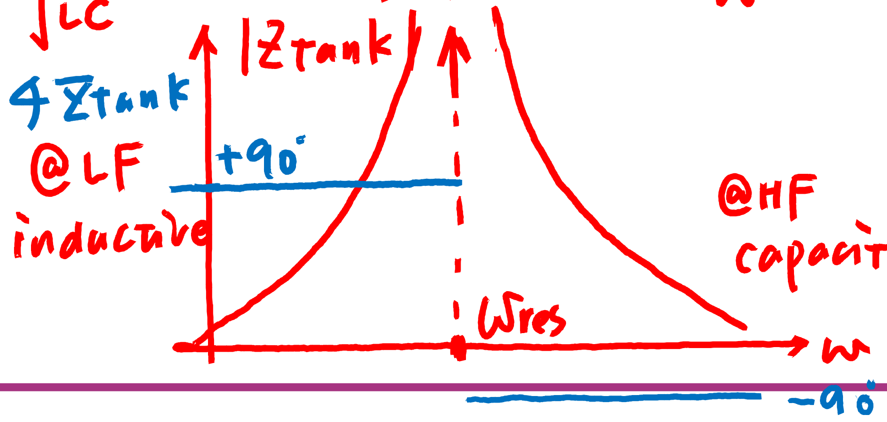


$V\sin\left(2\pi \frac{1}{\sqrt{LC}} t\right)$

@LF $Z_{tank} \sim j\omega L$

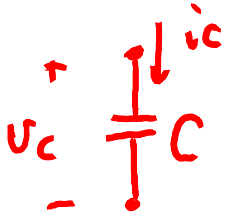
@HF $Z_{tank} \sim 1/sC$

@ $\omega_{res} = \frac{1}{\sqrt{LC}}$ when $|Z_L| = |Z_C|$ $\omega L = \frac{1}{\omega C}$ $Z_{tank} \to \infty$ (open)

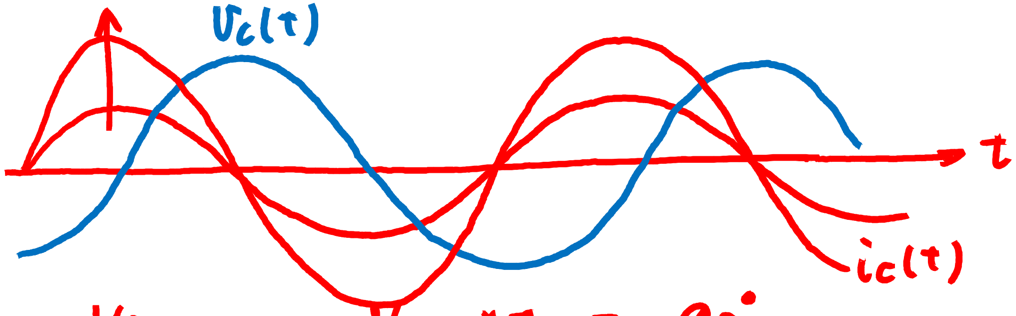


Capacitor and Inductor

$|I| \uparrow$ with $\omega \uparrow$, $C \uparrow$



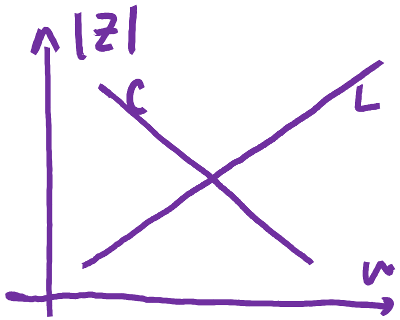
$$i_c = C \cdot \frac{dv_c}{dt}$$



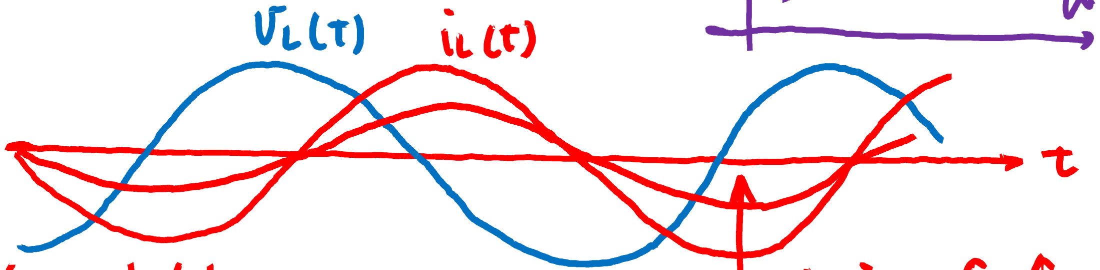
$$Z_C = \frac{1}{sC} = \frac{1}{j\omega C} = \frac{V_c}{I_c}$$

$$\angle V_c - \angle I_c = -90^\circ$$

$$\frac{|V_c|}{|I_c|} = \frac{1}{\omega C}$$



$$v_L = L \cdot \frac{di_L}{dt}$$



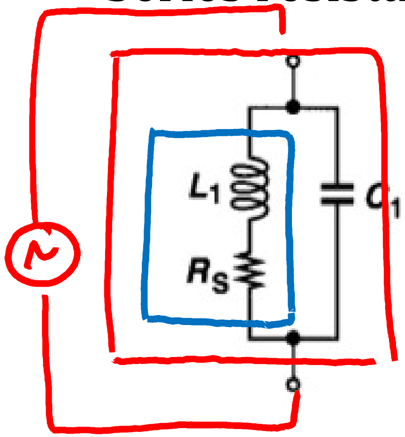
$|I|$ with $\omega \uparrow$ $L \uparrow$

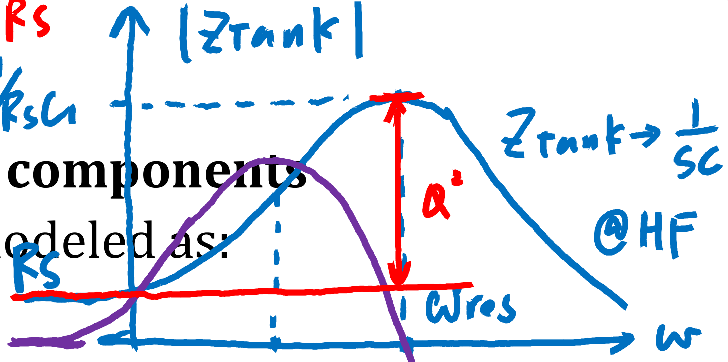
$$Z_L = sL = j\omega L = \frac{V_L}{I_L} \qquad \frac{|V_L|}{|I_L|} = \omega L$$

Consider Loss in the Tank

$R_p = \frac{L_1}{R_S C_1} = Q^2 R_S$

- **In practice, devices suffer from resistive components**
  - Series resistance of the metal wire can be modeled as:





$= (sL + R_S) // \frac{1}{sC}$

$$Z_{eq}(s) = \frac{R_S + L_1 s}{1 + L_1 C_1 s^2 + R_S C_1 s}$$

$$|Z_{eq}(s = j\omega)|^2 = \frac{R_S^2 + L_1^2\omega^2}{(1 - L_1 C_1 \omega^2)^2 + R_S^2 C_1^2 \omega^2}$$

$\cong \frac{sL_1}{sRC_1} = \frac{L_1}{RC_1}$ @ $\omega = \frac{1}{\sqrt{LC}}$

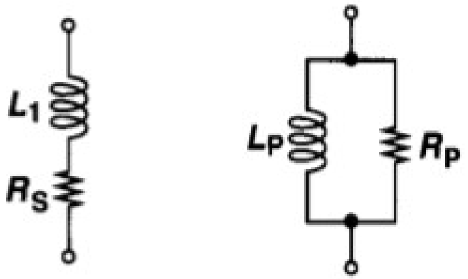
with $\omega L \gg R_S$ sufficient Q

Q of the inductor $= \omega L_1 / R_S$

  - The impedance does not go to infinity → this circuit has a finite Q

→ The impedance peaked in the vicinity of $\omega = 1/\sqrt{L_1 C_1}$

  - For easy analysis and to provide intuition:



$$L_1 s + R_S = \frac{R_P L_P s}{R_P + L_P s}$$

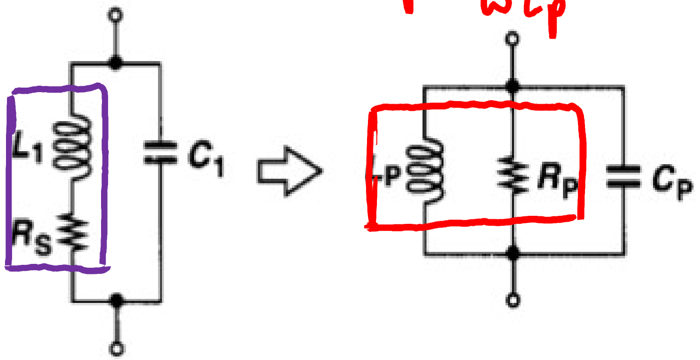
for $Q \gg 1$

$$\rightarrow\ L_P = L_1\left(1 + \frac{R_S^2}{L_1^2\omega^2}\right) = L_1\left(1 + \frac{1}{Q^2}\right) \cong L_1$$

RLC Tank

- **As the operating frequency is high enough and** $Q_L = \dfrac{\omega L_1}{R_S} >> 1$

$Q_P = \dfrac{R_P}{\omega L_P}$



$$L_P \approx L_1$$

$$R_P \approx \frac{L_1^2\omega^2}{R_S} = \left(\frac{\omega L_1}{R_S}\right)^2 \cdot R_S$$

$$\approx Q^2 R_S$$

- **At** $\omega = 1/\sqrt{L_1C_1}$**, the tank reduces to a simple resistor**

$$Z_{tank} = \frac{L_1}{R_S \cdot C_1} = Q^2 \cdot R_S$$

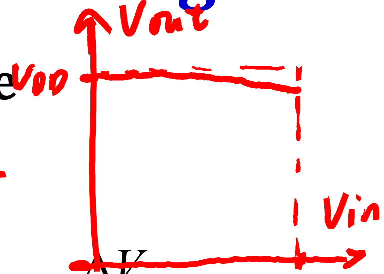
Common-Source Stage with LC Tank Load (I)

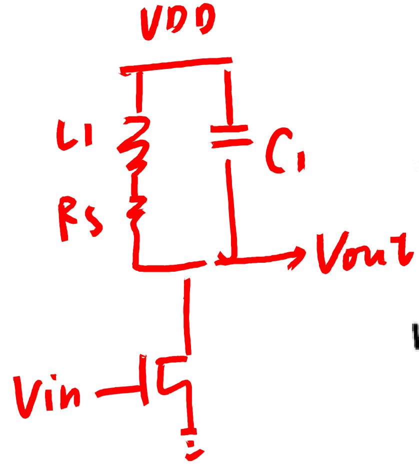
- **With a single stage**
  - At low-frequencies

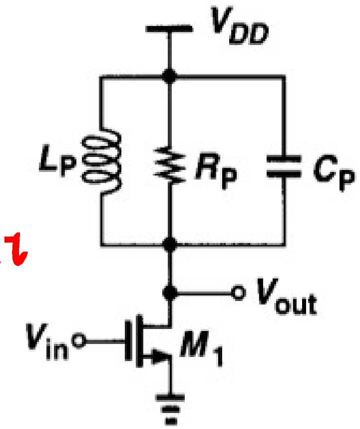
    @DC:

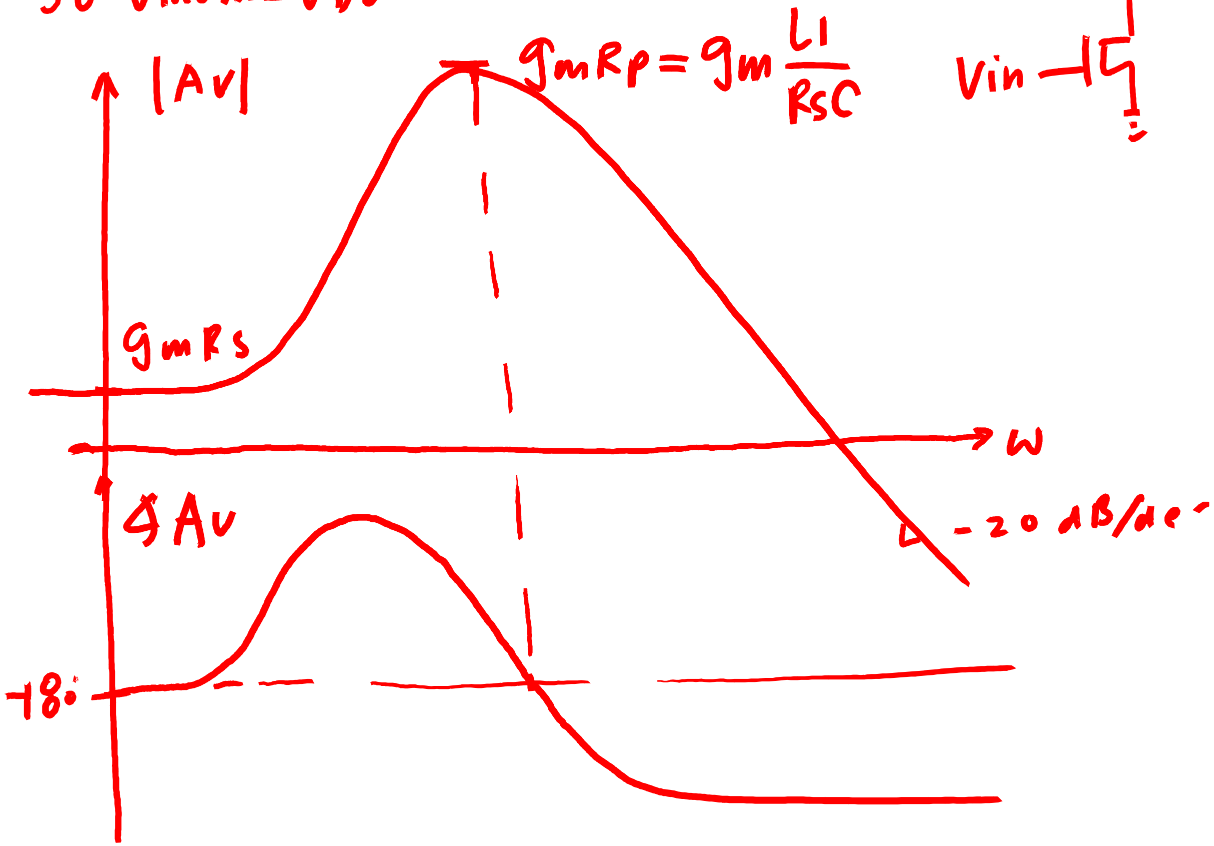
  - At resonant frequency $\dfrac{\Delta V_{out}}{\Delta V_{in}} = -g_{m1} R_P$

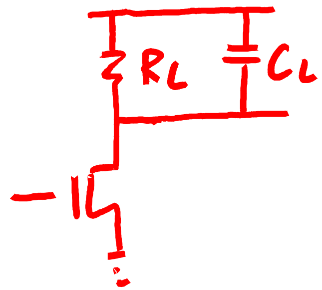
So $V_{incm} \hat{=} V_{DD}$



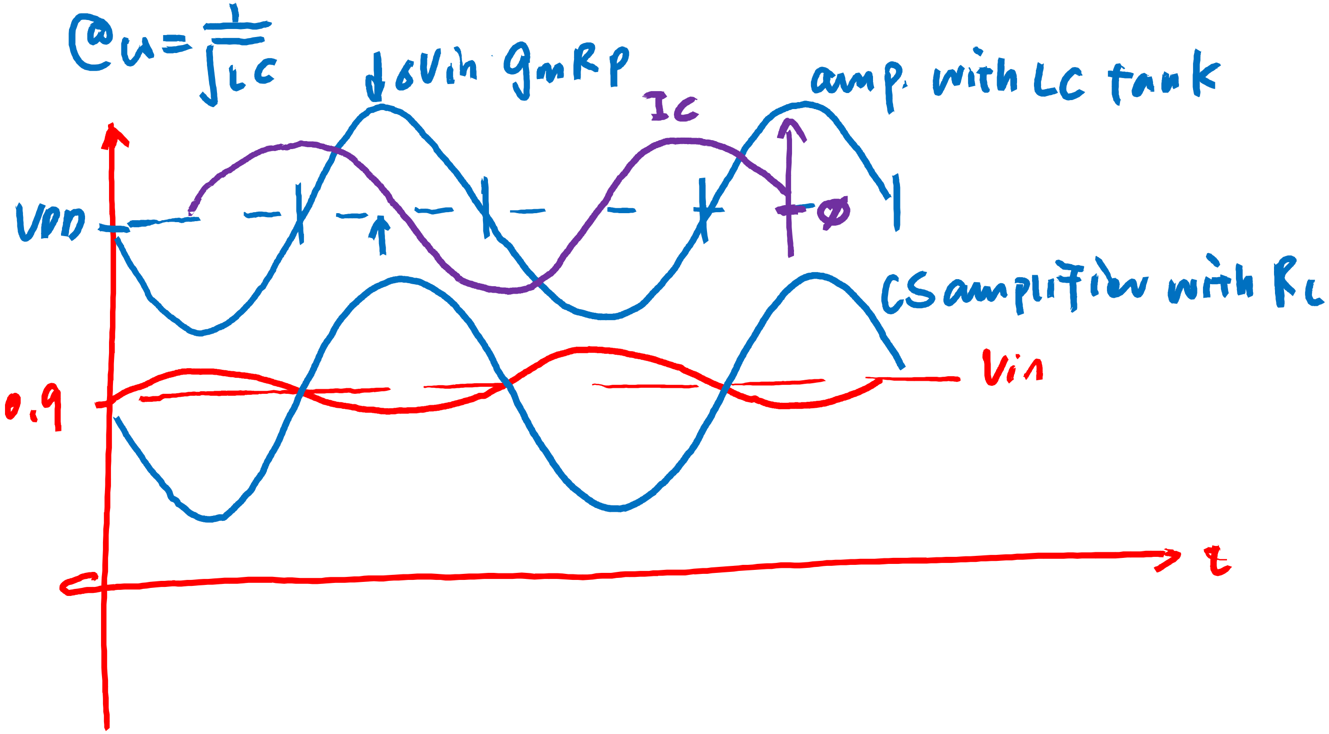


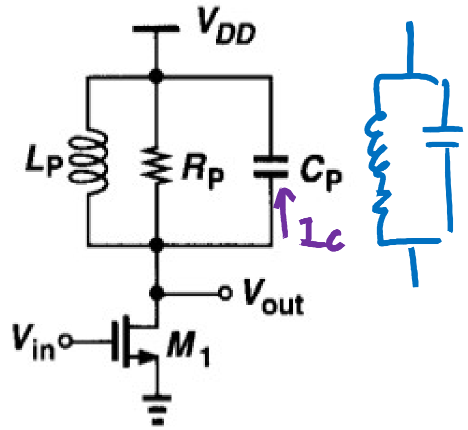


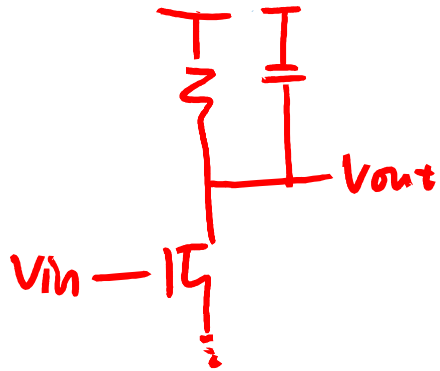




Common-Source Stage with LC Tank Load (II)





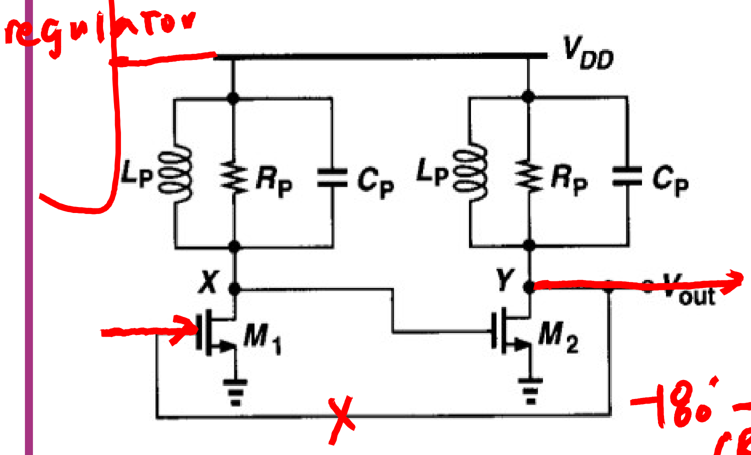


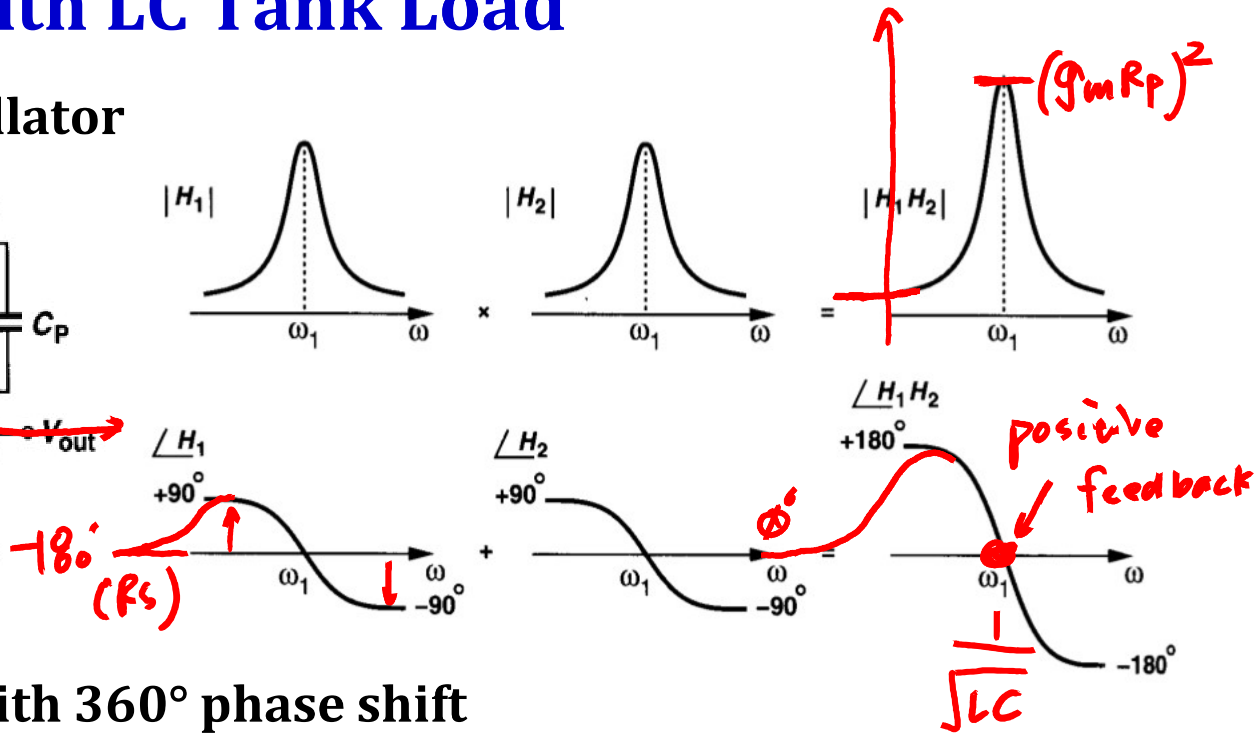
* Vout can be "momentarily" higher than VDD
* Lp // Cp behaves like open
* current of M1 flows into Rs

  there are 'reactive current' between Lp & Cp

Two-Stage CS with LC Tank Load

- **Cross-coupled oscillator**





- **Two frequencies with 360° phase shift**
  - Does not latch up at low-frequencies
  - Provide zero *additional* phase shift at resonant frequency
- **Gain requirement**
  - $\boxed{g_{m1} R_P g_{m2} R_P \geq 1}$ $g_m = xxx \frac{W}{L}(V_{DD} - V_{th})$
  - If the inductor dominates the quality factor

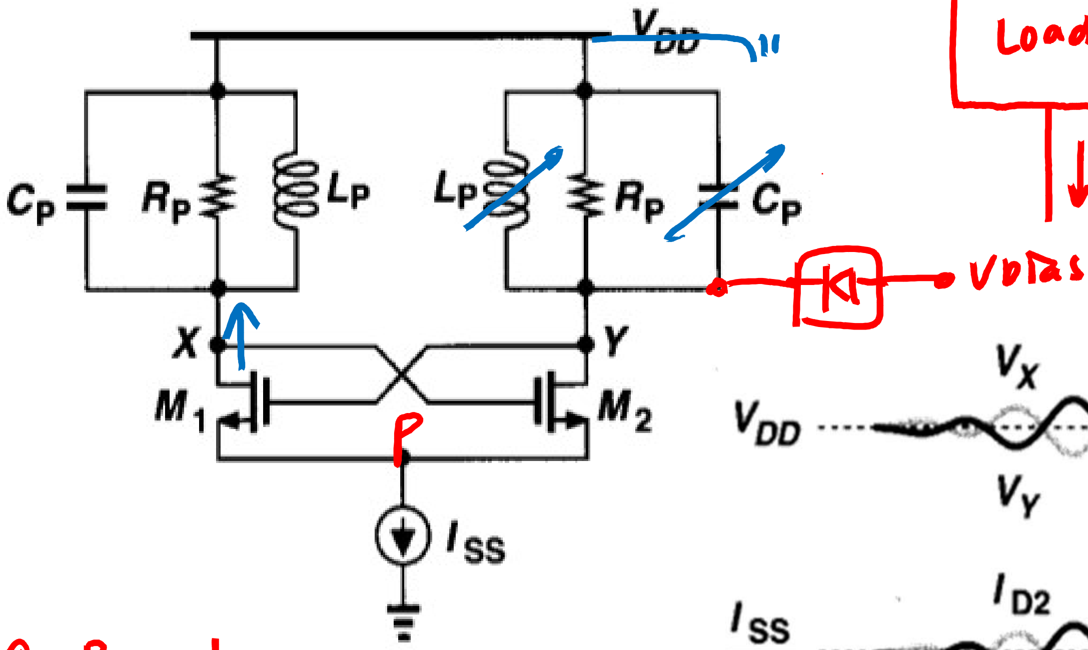
$$R_P \approx \frac{L_1^2 \omega^2}{R_S} = \frac{L_1}{C_1 R_S} \text{ at } \omega = \frac{1}{\sqrt{L_1 C_1}}$$

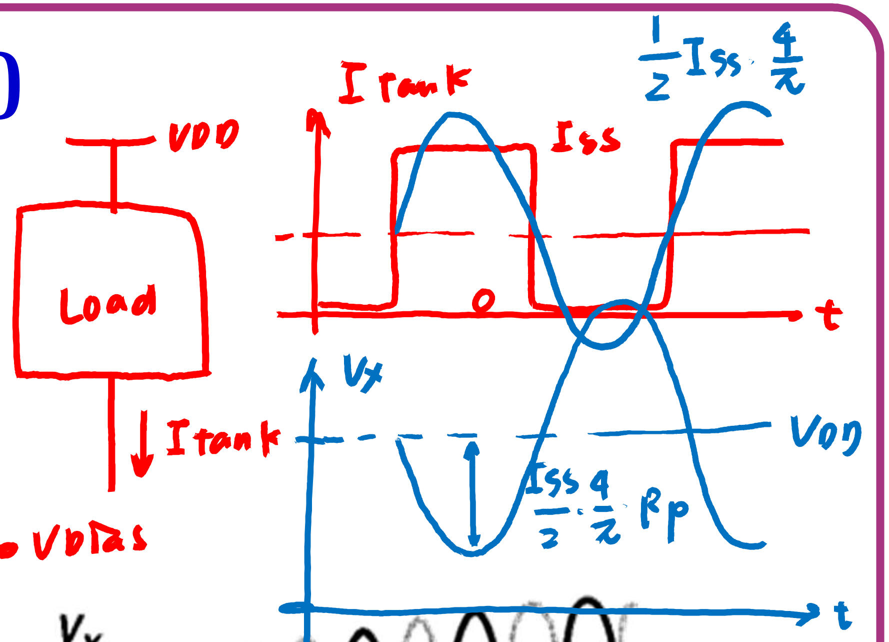
→ larger inductor is preferred to save power

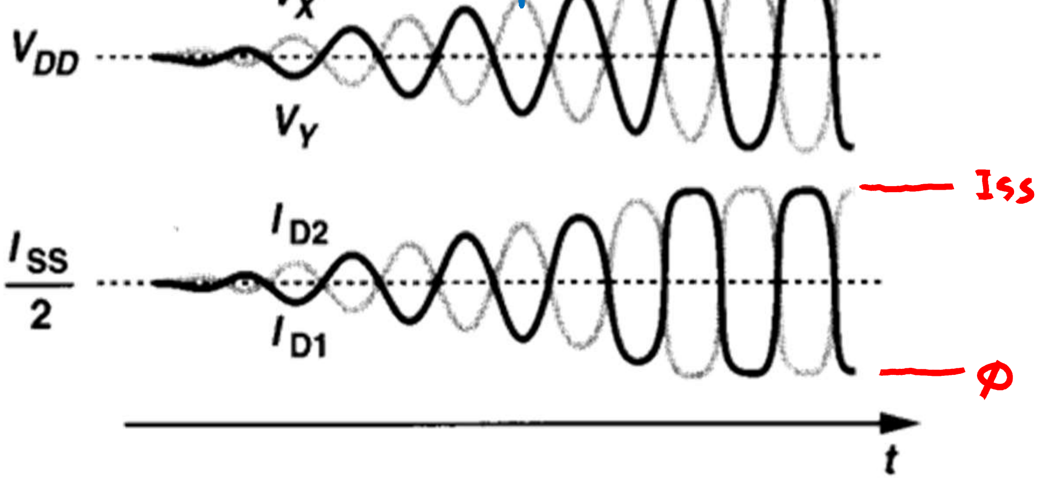
→ The drain current (bias condition) and output swing depend on V_{DD}

Cross-Coupled Oscillator (I)

- With tail current I_{SS}







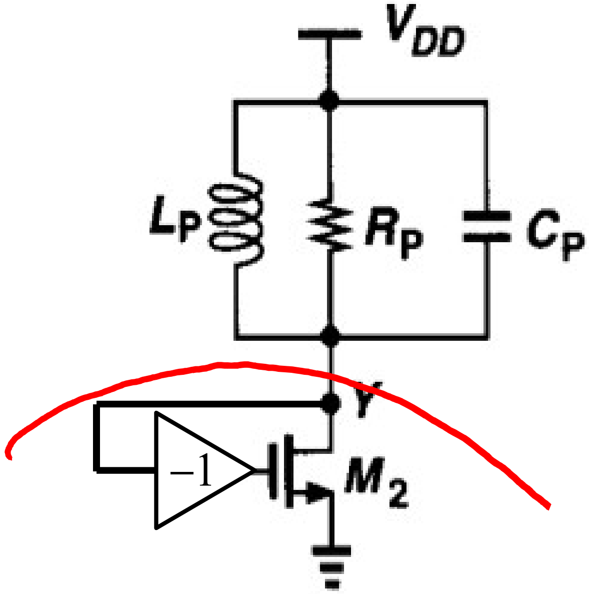
$g_m R_p \gg 1$

$g_m = \sqrt{2 \cdot xxx \cdot \frac{W}{L} I_{ss}}$

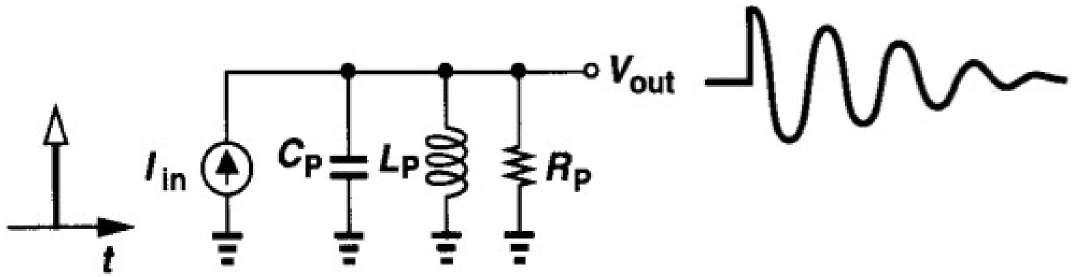
- Common mode at $\sim V_{DD}$
- I_{SS} and R_P determine oscillation amplitude

Negative Resistance (I)

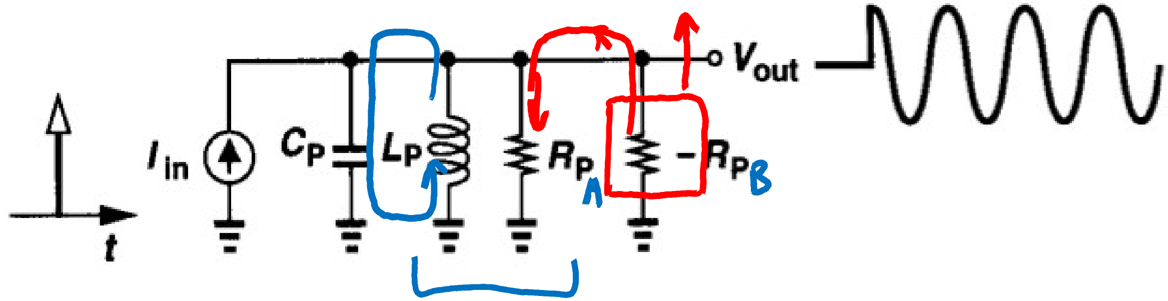
- **Half-circuit of the cross-coupled oscillator**



- Without bottom part of the circuit – only the RLC tank (lossy tank)



- Oscillation can be sustained with negative resistance



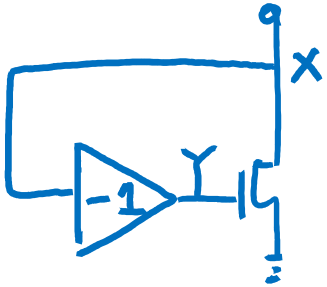
Negative Resistance (II)

- **The negative resistance has to be strong enough to sustain oscillation**

$$R_{pA} \parallel (-R_{pB}) = \frac{-R_{pA}\cdot R_{pB}}{R_{pA}-R_{pB}} < 0 \Rightarrow R_{pB} < R_{pA}$$

So that the negative res is "strong" enough

- **A positive feedback structure may create negative resistance**



open-loop $R_{out} = r_o$

closed-loop $R_{out} = \frac{r_o}{1-g_m r_o} \longrightarrow -\frac{1}{g_m}$

with $g_m r_o \gg 1$

$$\Rightarrow \frac{1}{g_m} < R_p \xrightarrow{\text{oscillation requirement}} g_m R_p > 1$$

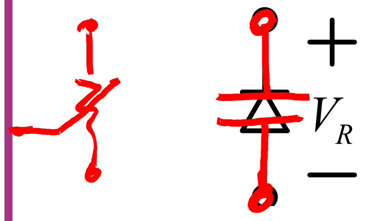
LC Voltage-Controlled Oscillator

- **The resonant frequency**

- Little dependence on bias current and transistor transconductance

→ Voltage-controlled capacitor → varactor

- For example: a reversed-biased *pn* junction



$$C_{var} = \frac{C_0}{(1 + \frac{V_R}{\phi_B})^m} \quad m \cong 0.3\text{-}0.4$$

$\phi_B \cong 0.7V$

$V_{Rmin} = 0V \qquad C_{eff} = C_0$

$V_{Rmax} = V_{DD} = 2V \qquad C_{eff} \cong 0.62\,C_0$

$f_{min} = \frac{1}{\sqrt{L C_0}}$

$f_{max} = \frac{1}{\sqrt{L \cdot 0.62 C_0}} \cong 1.27 \frac{1}{\sqrt{L \cdot C_0}}$

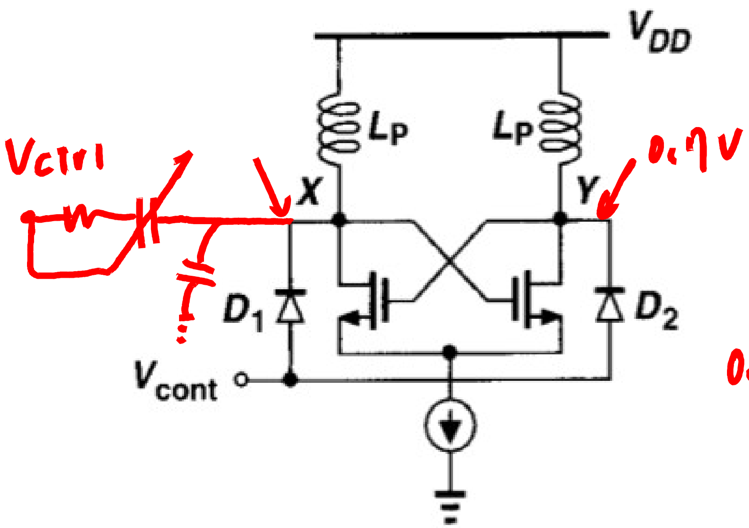
- Limited range of V_R results in limited capacitance range.
- Furthermore, to increase the operating frequency, C_0 is minimized.

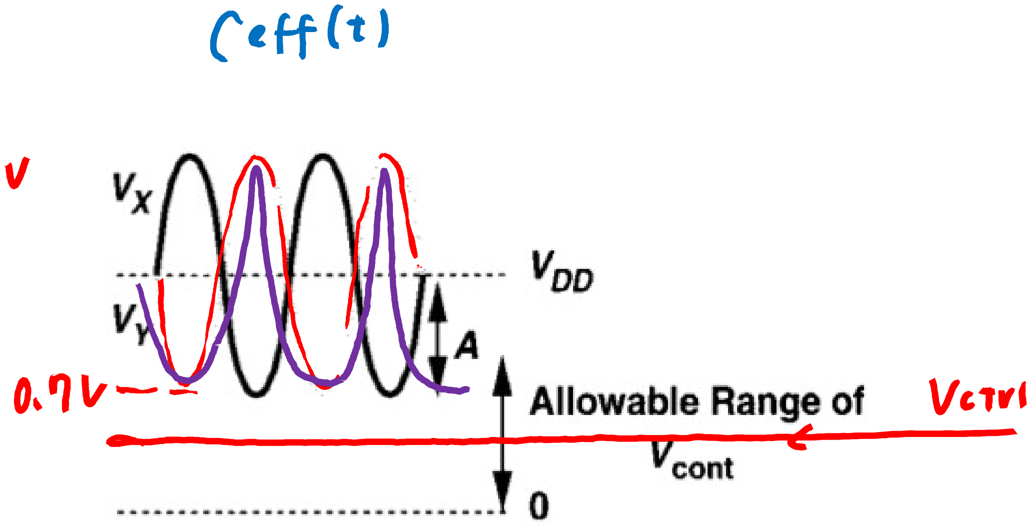
→ Trade-off between operating frequency and tuning range

Adding Varactors to Cross-Coupled Oscillator

- **To avoid forward biasing the two diodes**

→ Trade-off between signal swing and tuning range





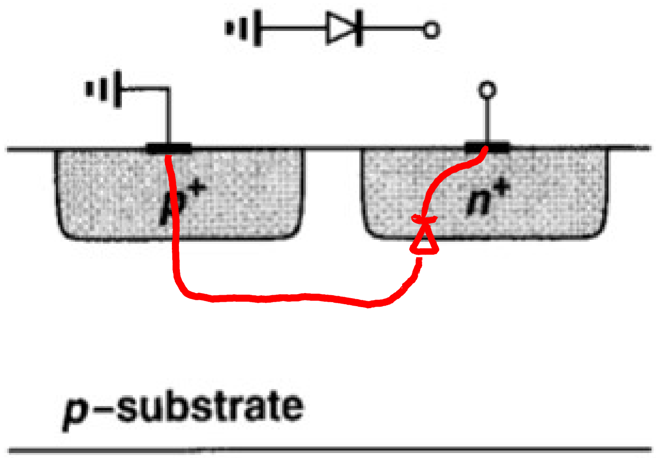
- **Capacitance depends on signal level and varies over time**

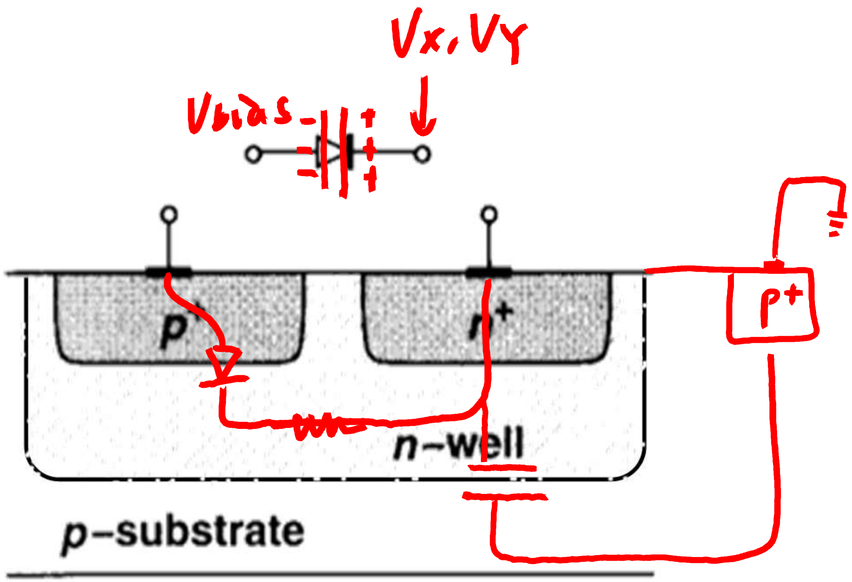
→ Average value (depending on V_{cont}) determines operating frequency

→ Oscillation waveform is distorted slightly

Varactor Diode in CMOS Technology

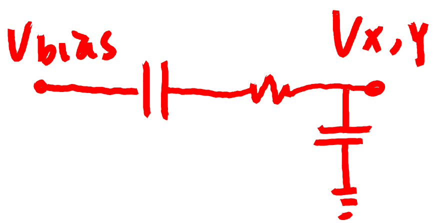
- **PN junction**





- Anode has to be grounded

→ Not tunable



- High resistivity in n-well
- High capacitance between n-well and ground

→ Fixed capacitance on signal nodes

→ Degrading tuning range

$C_{total} = C_{var} + C_{fixed}$