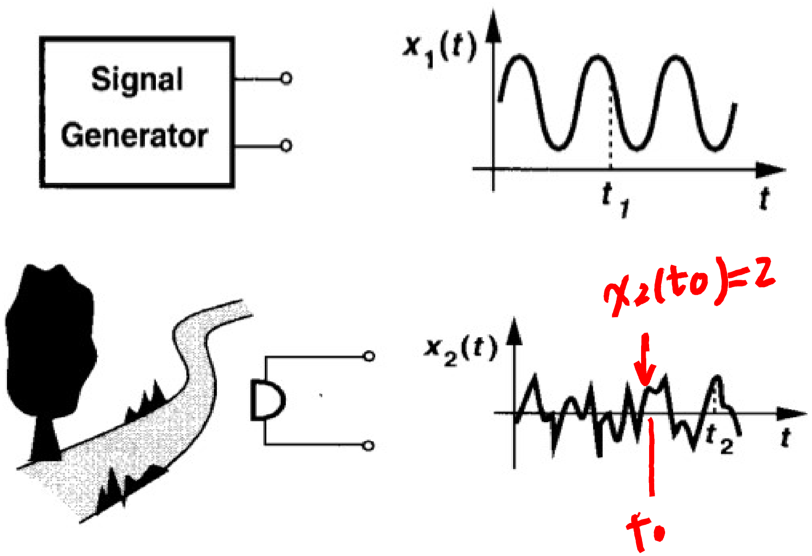
EE4280 Lecture 9: Noise

Ping-Hsuan Hsieh (謝秉璇)
Delta Building R908
EXT 42590
phsieh@ee.nthu.edu.tw

What is Noise?

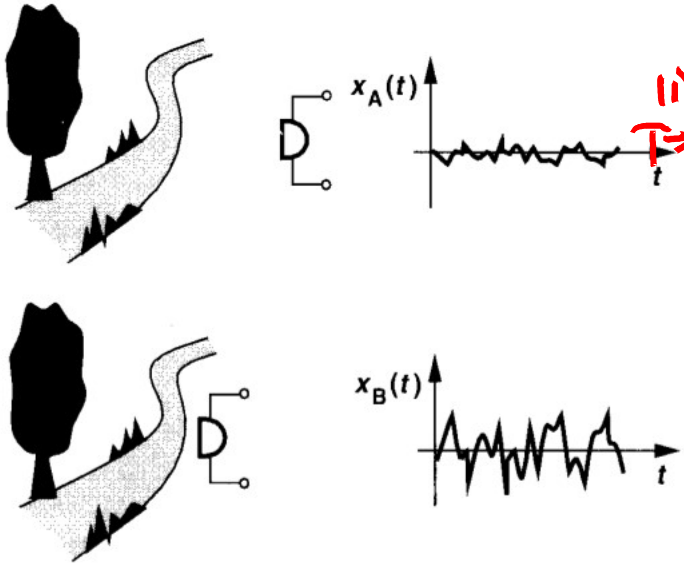
Noise is a random process. We consider a phenomenon random because we do not know everything about it, or simply because we do not <u>need</u> to know everything about it.



Since the instantaneous noise amplitude is not known, we resort to 'statistical' models, i.e., some properties that can be predicted.

Statistical Characterization

1. Mean and Average Power

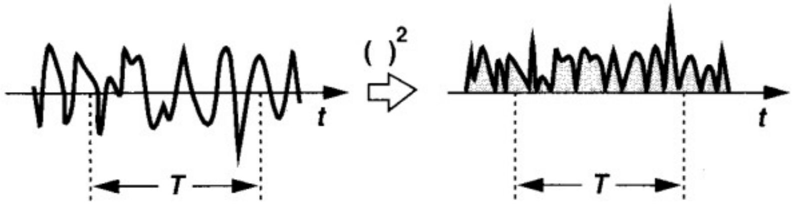


$$\lim_{T\to\infty}\frac{1}{T}\int_T x_A(t)\,dt = \overline{x_A} = 0$$

$$\lim_{T\to\infty}\frac{1}{T}\int_T x_A^2(t)\,dt = \overline{x_A^2} = 25\,\mu V^2 = (5\,mV_{rms})^2$$

$$\overline{x_B^2} = (15\,mV)^2$$

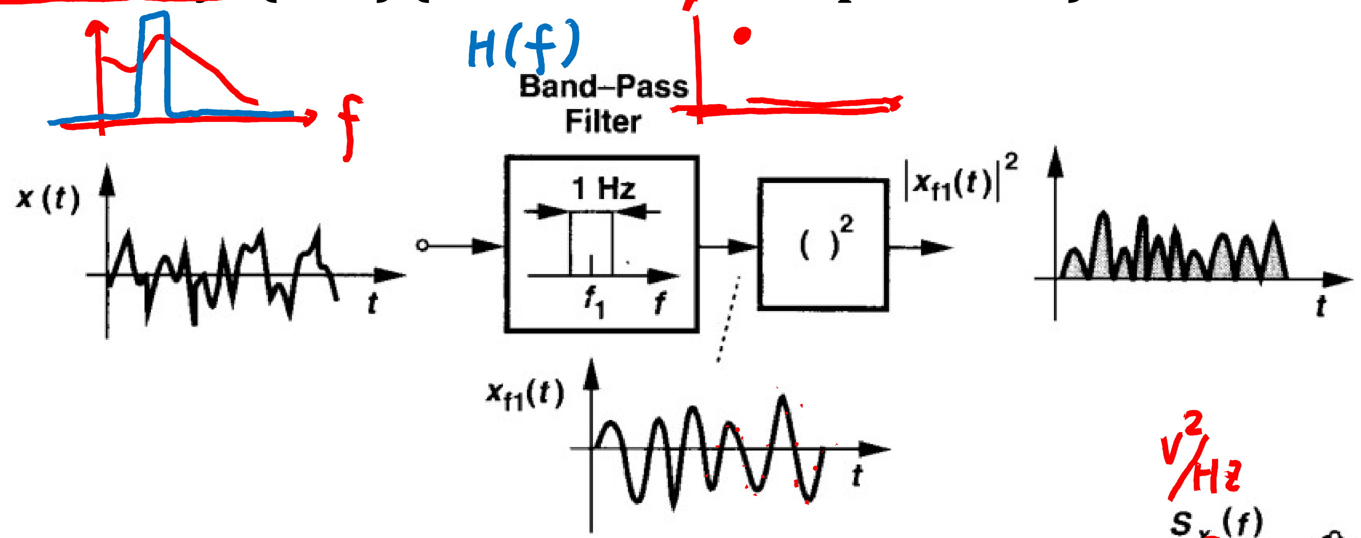
Larger fluctuations mean that the noise is ‘stronger’



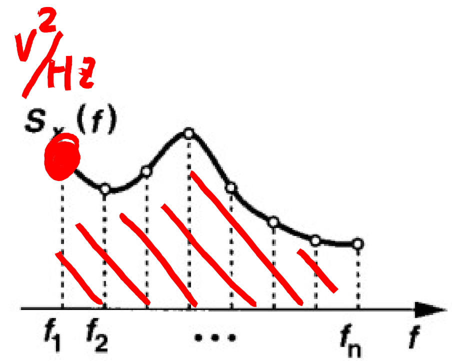
Statistical Characterization

2. Frequency Domain

For random signals, the concept of Fourier transform cannot be directly applied. But we still know that men carry less high-frequency components in their voice than women do. We define the "power spectral density" (PSD) (also called the "spectrum") as:



The PSD thus indicates how much power the signal carries in a small bandwidth around each frequency.



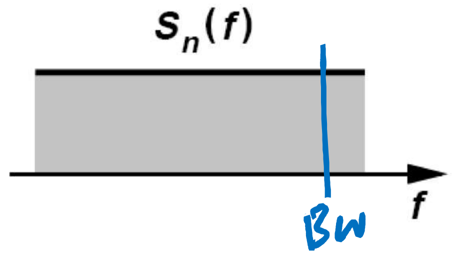
$$P_{av} = \lim_{T\to\infty} \frac{1}{T}\int_T x^2(t)\,dt = \int S_x(f)\,df$$

Statistical Characterization

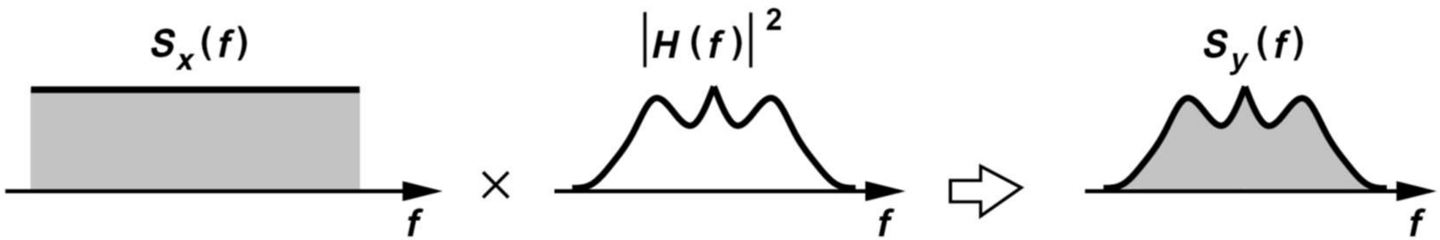
- What is the unit of $S_n(f)$?
- What is the total noise power?

A flat spectrum is called 'white'

- Is the total noise power infinite?



<u>**Important Theorem**</u>



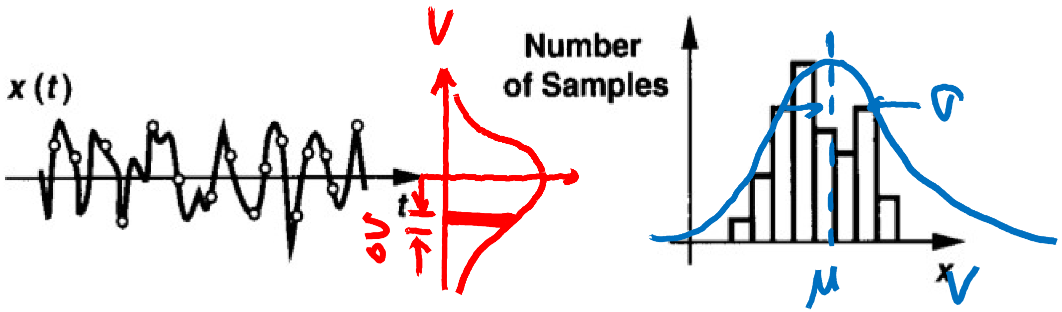
If a signal with spectrum $S_X(f)$ is applied to a linear time-invariant system with transfer function $H(s)$, the output spectrum is given by

$$S_Y(f) = S_X(f)|H(f)|^2$$

Statistical Characterization

3. Amplitude Distribution

By sampling the time-domain waveform for a long time, we can construct a "probability density function" (PDF). The PDF in essence indicates "how often" the amplitude is between certain limits.



For example, a Gaussian distribution is defined by a mean and a standard deviation. We say the noise amplitude rarely exceeds 4σ.

<u>Note:</u> Generally PDF and PSD bear no relationship.

Thermal Noise: Gaussian, white

Flicker Noise: Gaussian, not white

Correlated and Uncorrelated Sources

Can we use superposition for average noise power from a few noise components?

$$P_{av} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{+T/2} [x_1(t) + x_2(t)]^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int \left(x_1^2(t) + 2x_1(t)x_2(t) + x_2^2(t) \right) dt$$

$x(t) = x_1(t) + x_2(t)$

$\lim_{T\to\infty} \frac{1}{T} \int_T x(t)\,dt = \lim_{T\to\infty} \frac{1}{T} \int_T \left(x_1(t) + x_2(t) \right) dt$

$= P_{av1} + P_{av2}$

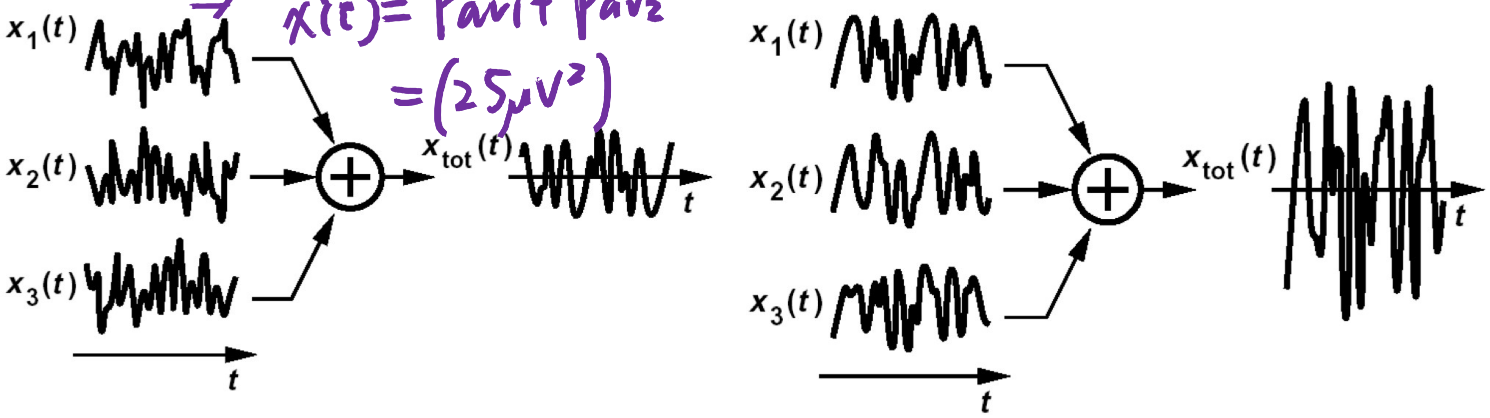
$\overline{x(t)} = \overline{x_1(t)} + \overline{x_2(t)} = \emptyset$

$\overline{x_1^2(t)} = (3\,\text{mVrms})^2$ + $\overline{2x_1(t)x_2(t)} \to \emptyset$ if $x_1(t)$ and $x_2(t)$ are independent to E.O.

$\overline{x_2^2(t)} = (4\,\text{mVrms})^2$

We occasionally encounter correlated sources:

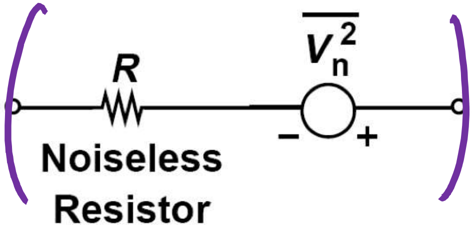
$\Rightarrow \overline{x^2(t)} = P_{av1} + P_{av2} = (25\,\mu V^2)$

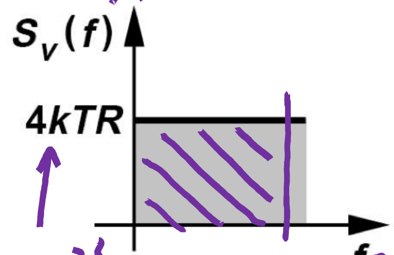


Types of Noise

1. Thermal Noise in Resistors

Random movement of charge carriers in a resistor causes fluctuations in the current. The PDF is Gaussian because there are so many carriers. The PSD is given by:



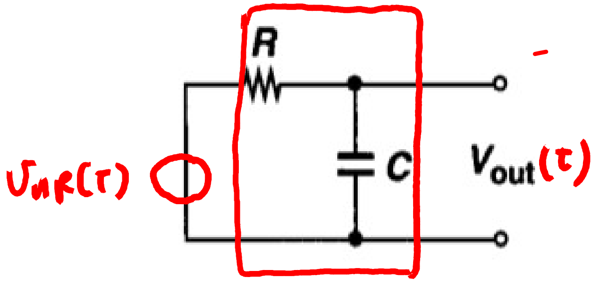


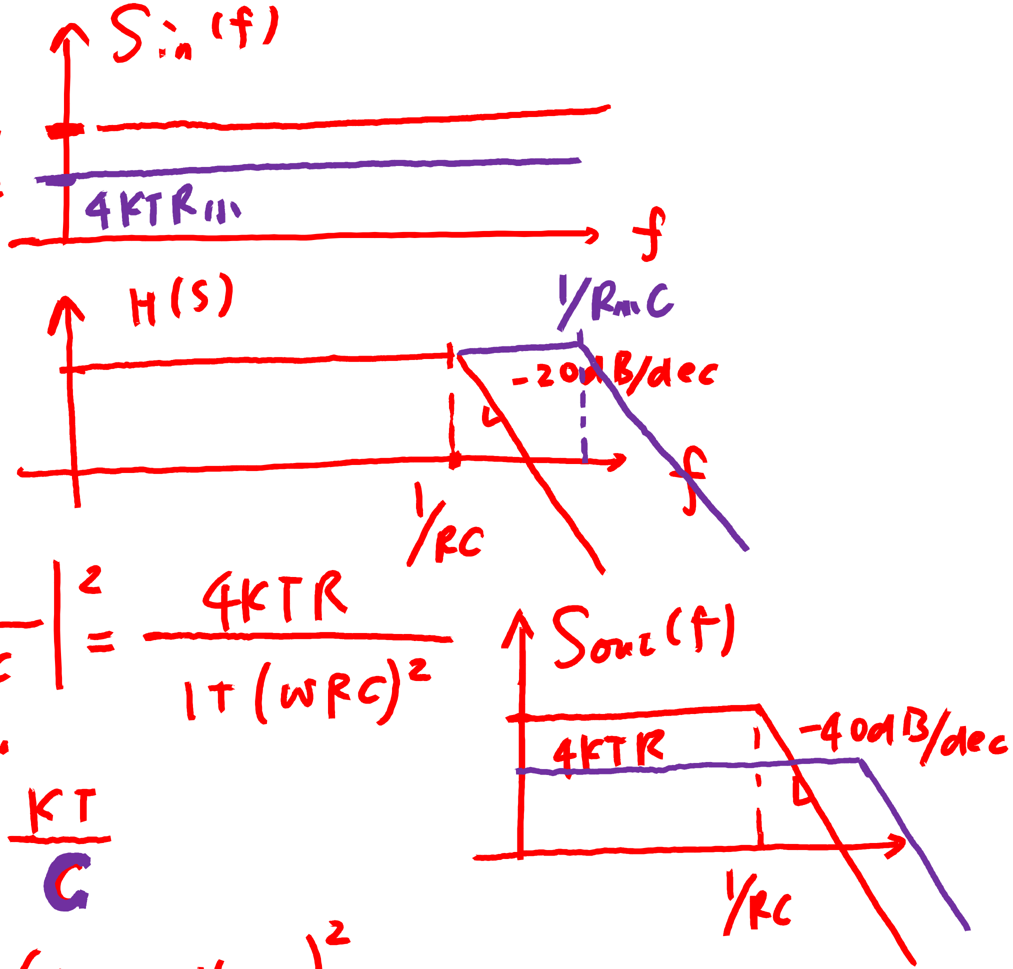
Note that the polarity of the voltage source is arbitrary.

<u>Example:</u> A 50-Ω resistor at room temperature exhibits

If the resistor is used in a system with 10-GHz bandwidth, then it contributes a total rms voltage of

Example: Noise Spectrum and Total Noise Power





$$\frac{V_{out}}{V_R}(s) = \frac{1}{1+sRC}$$

$$S_{out}(f) = S_{in}(f)\left|\frac{1}{1+sRC}\right|^2 = \frac{4KTR}{1+(\omega RC)^2}$$

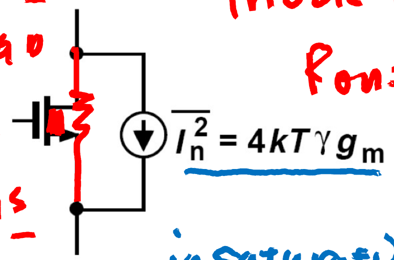
($s \rightarrow j\omega$)

$$P_{n,out} = \int S_{out}(f)\,df = \frac{KT}{C}$$

For R=50 Ω and C=1 pF: $= (6.25\,mV_{rms})^2$

Trade-offs between noise, area, speed, and power

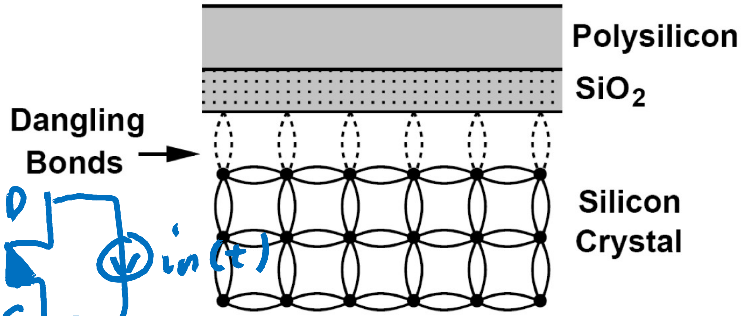
Types of Noise

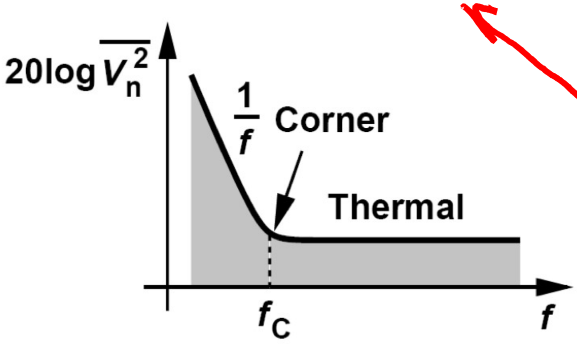


2. Thermal Noise in Transistors

3. Flicker Noise in Transistors

In MOSFETs, the extra energy states at the interface between silicon and oxide trap and release carriers randomly and at different rates. The noise in the drain current is Gaussian, but its spectrum is given by:

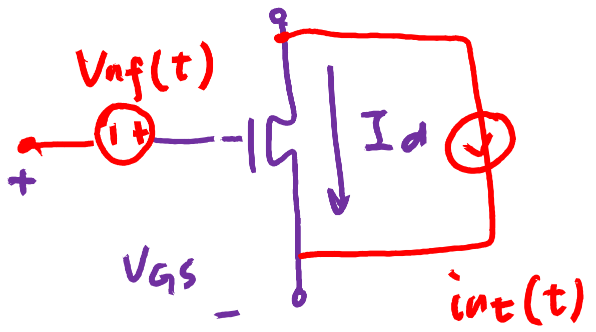


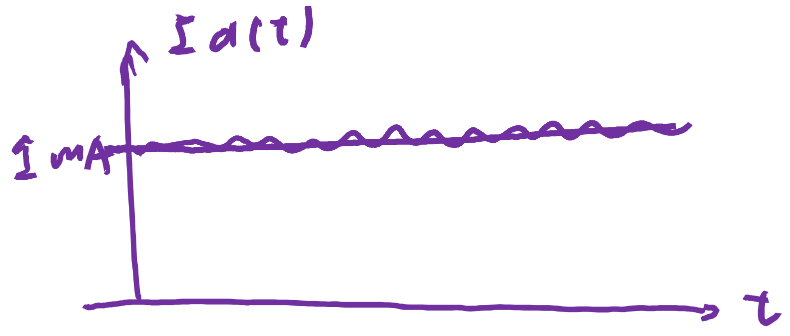


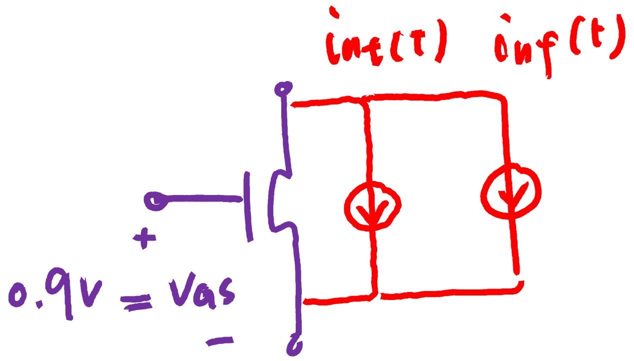
Where k is a constant and its value
vily depends on how 'clean' the
Ve often characterize the
ıoise by considering
ıuency.

Example: A Current Source

Total noise in the drain current for a band from 1 kHz to 1 MHz

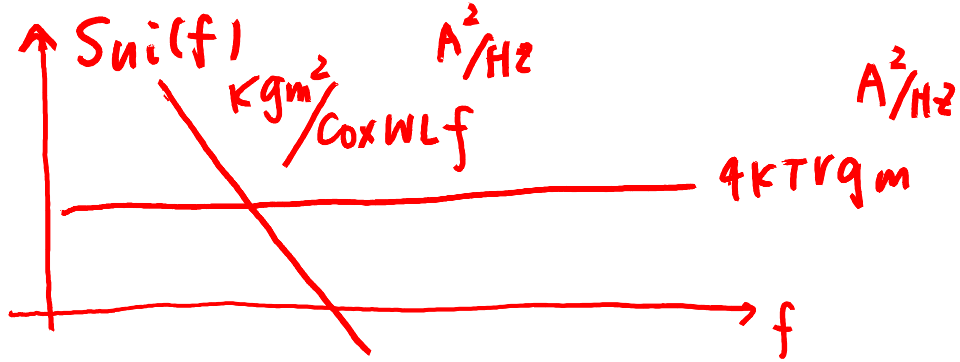




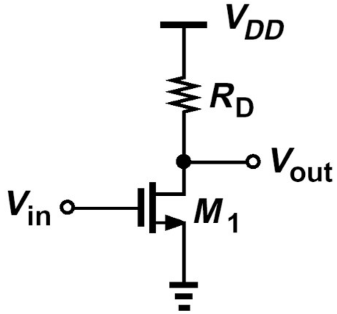


$$i_d(t) = 1\,mA + \underline{i_{n1}(t) + i_{n2}(t)}$$

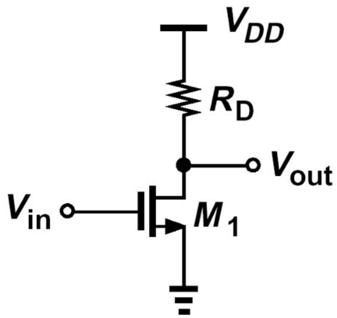
$$i_n(t) = i_{n1}(t) + i_{n2}(t)$$



Common-Source Stage

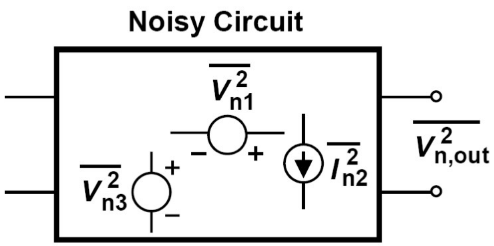


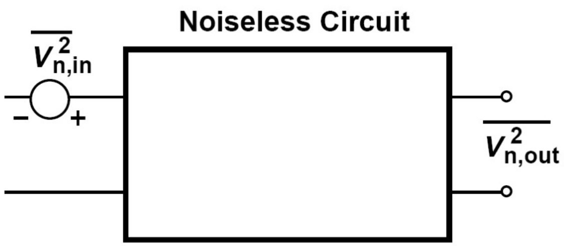
Common-Source Stage



$\overline{V^2_{n,out}} =$

<u>Input-Referred Noise</u> is the noise voltage or current that, when applied to the input of the noiseless circuit, generates the same output noise as the actual circuit does.



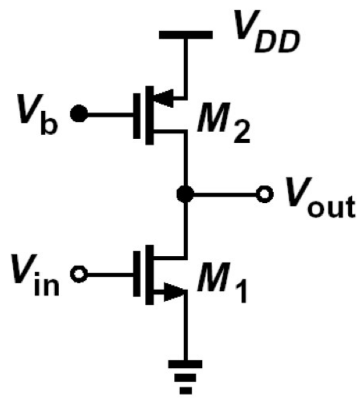


$\overline{V^2_{n,in}} =$

Common-Source Stage

$$\overline{V_{n,in}^2} = 4kT\left(\frac{2}{3g_m} + \frac{1}{g_m^2 R_D}\right) + \frac{K}{C_{ox}WL}\frac{1}{f}$$

Why does the noise decrease as R_D increases?



With a <u>current-source</u> load:

$$\overline{V_{n,out}^2} =$$

$$\overline{V_{n,in}^2} =$$

Consider BW limitation from C_L and a low-frequency signal V_m at input:

$$P_{n,out} =$$

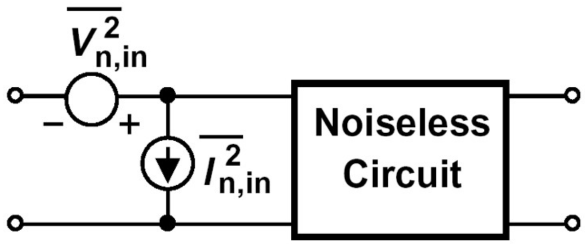
$$SNR_{out} =$$

How to reduce the noise?

Trade-offs between speed, power, and voltage headroom

Input-Referred Noise

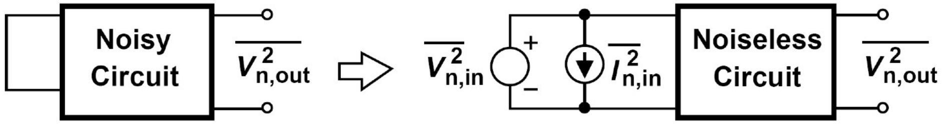
In general, we need both a voltage source and a current source at the input to model the circuit noise.



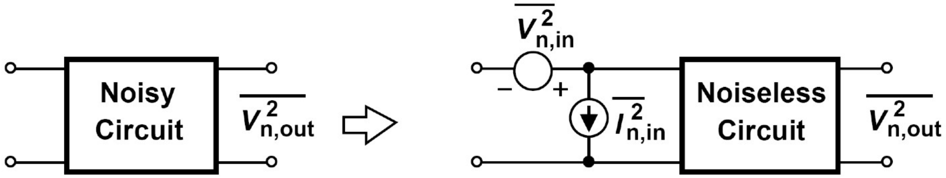
If the source impedance is high with respect to the input impedance of the circuit, then both must be considered.

How To Calculate Input-Referred Noise?

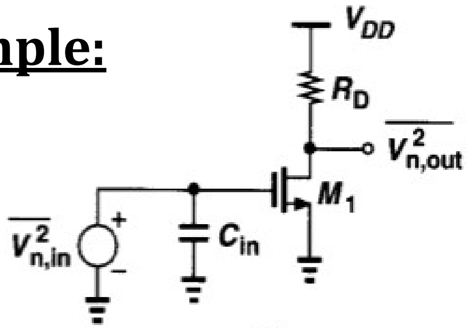
With Z_S=0:



With Z_S=∞:

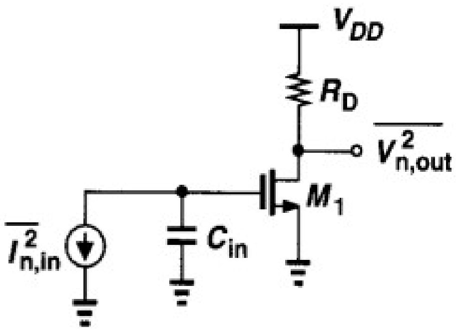


<u>Example:</u>



$$\overline{V^2_{n,in}} =$$

$$\overline{V^2_{n,out}} = \overline{I^2_{n,in}}$$



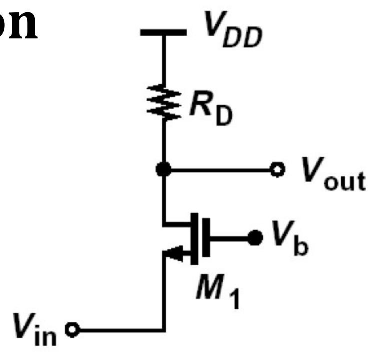
$$\overline{V^2_{n,out}} =$$

$$\overline{I^2_{n,in}} =$$

With arbitrary Z_S

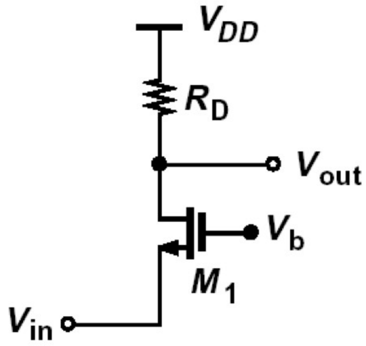
Common-Gate Stage

Basic operation



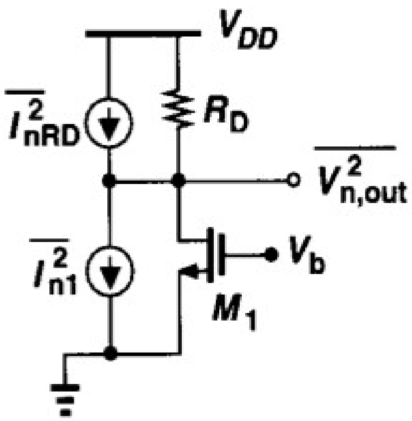
Common-Gate Stage

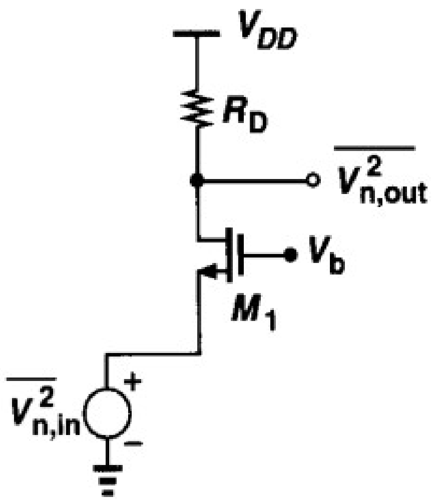
Due to the low input impedance, the input-referred noise current is not negligible even at low frequencies



Common-Gate Stage

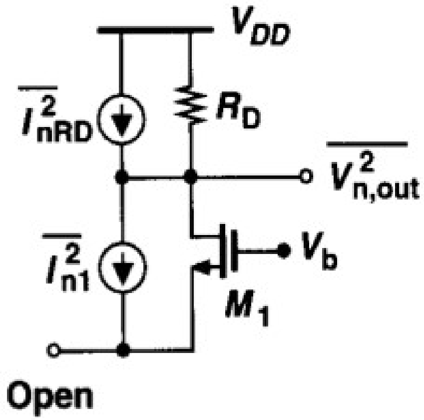
For Z_S=0

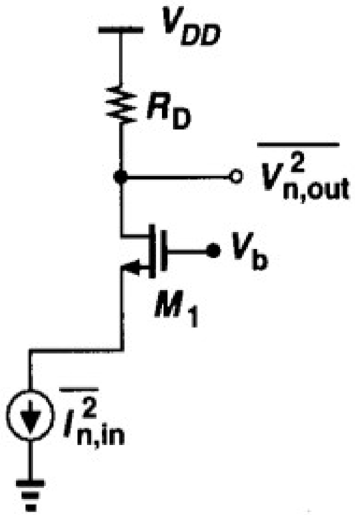




Common-Gate Stage

For Z_S=∞



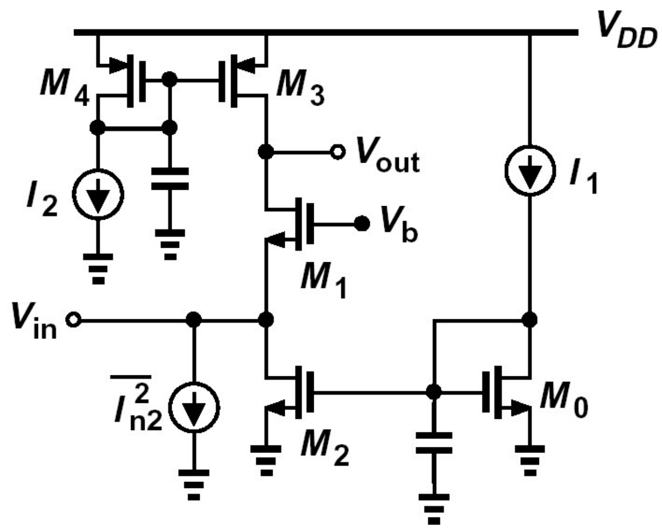


Current Source

The bias current source often contributes significant noise.

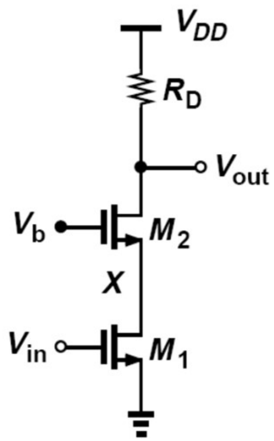
Example:

- **Capacitor C_0 shunts the noise generated by M_0 to ground.**
- **Noise from M2:**

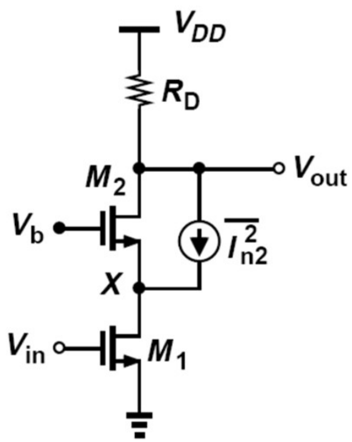


Cascode Stage

Basic operation

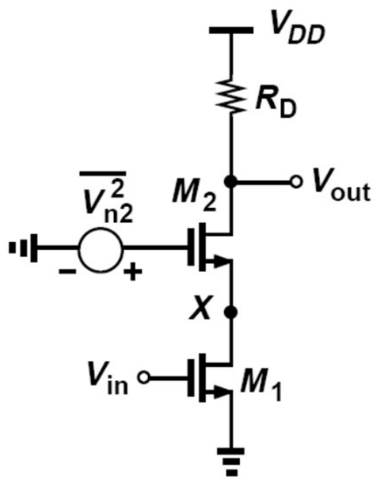


The effect of noise of M2



Cascode Stage

With capacitance at node X

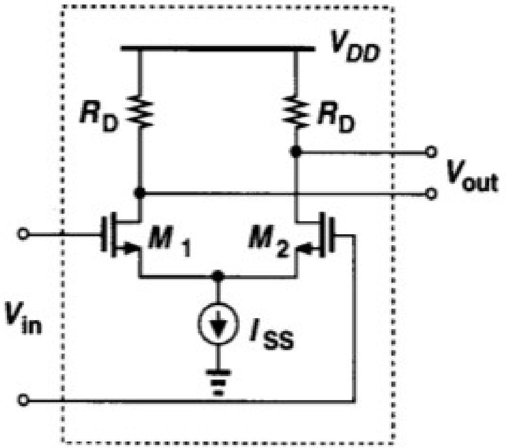


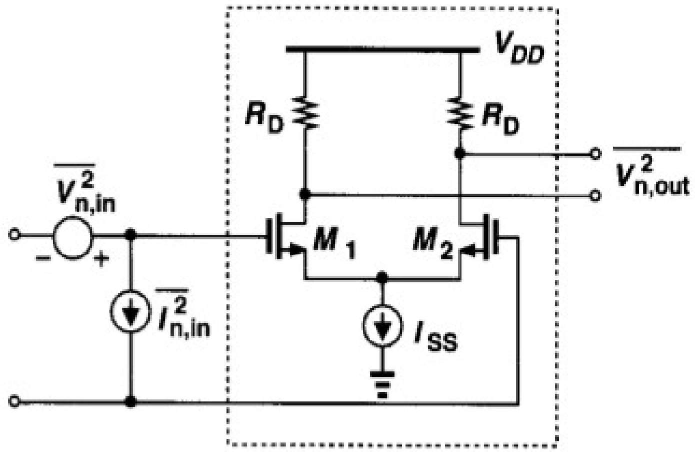
At high frequencies:

- **Output noise is increased**
- **The gain from V_{in} to V_{out} is decreased**

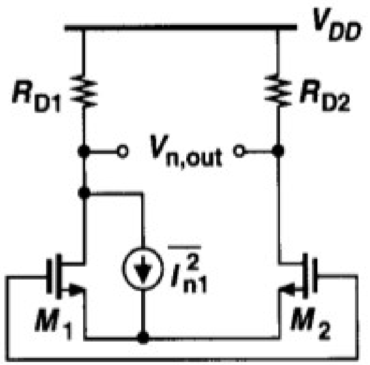
Differential Pair

Consider differential signals

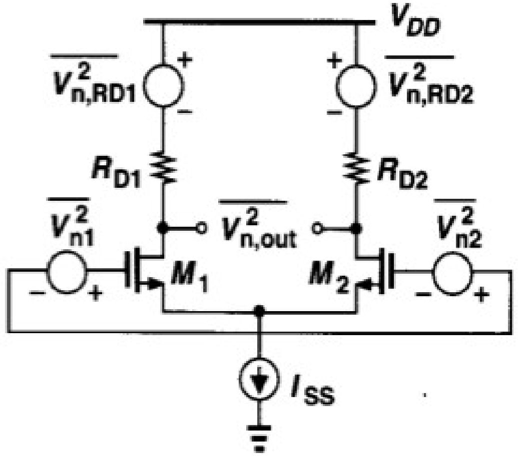




Since the four noise generators are uncorrelated, we can use superposition for the powers. Noise from M_1:



Differential Pair



$$\overline{V^2_{n,out}} =$$

$$\overline{V^2_{n,in}} =$$

Thus, the input-referred noise voltage of a diff pair is 40% larger than that of a common-source stage – probably the only disadvantage of differential operation.

Noise from I_{SS}

- **Common-mode disturbance**
- **Modulation of g_m and therefore voltage gain**