EE4280 Lecture 9: Noise

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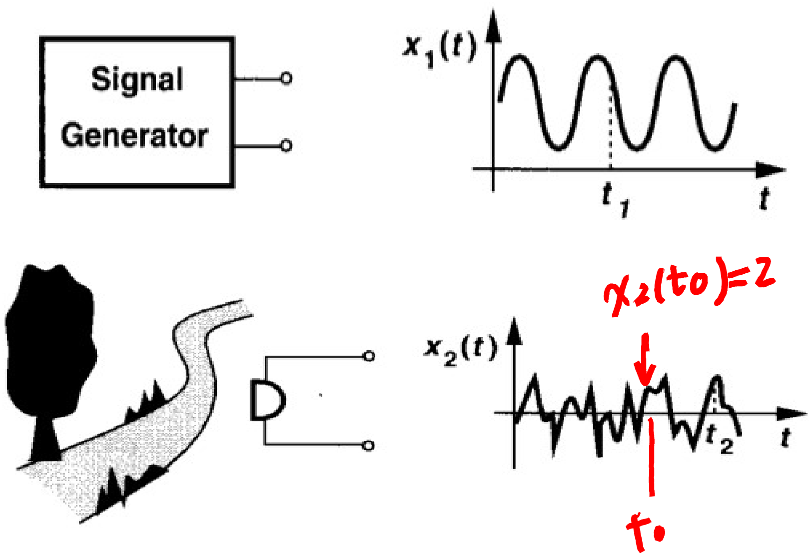
What is Noise?

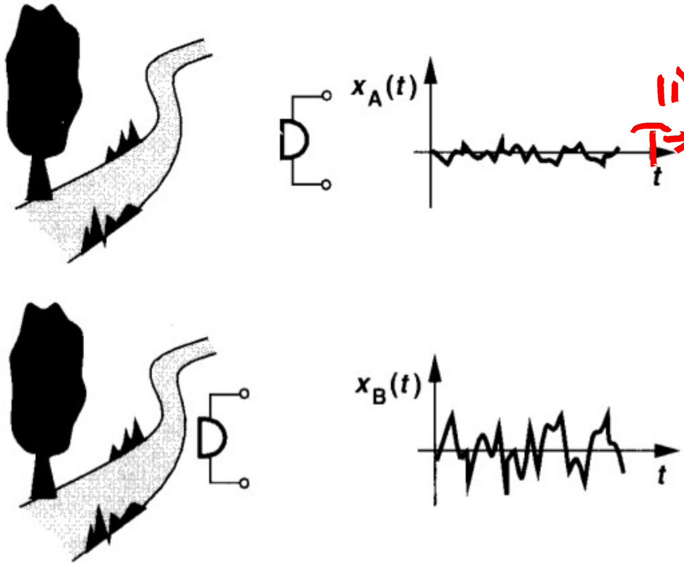
Noise is a random process. We consider a phenomenon random because we do not know everything about it, or simply because we do not need to know everything about it.



Since the instantaneous noise amplitude is not known, we resort to 'statistical' models, i.e., some properties that can be predicted.

Statistical Characterization

1. Mean and Average Power



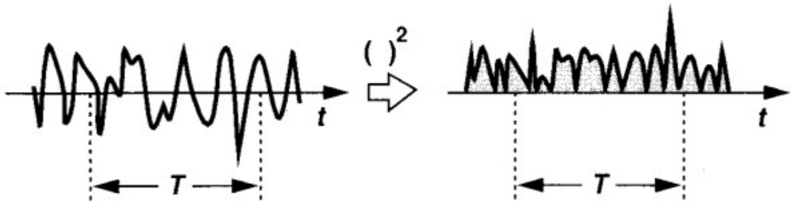
$$\lim_{T\to\infty}\frac{1}{T}\int_T x_A(t)\,dt = \overline{x_A} = 0$$

$$\lim_{T\to\infty}\frac{1}{T}\int_T x_A^2(t)\,dt = \overline{x_A^2} = 25\,\mu V^2$$

$$= (5\,mV_{rms})^2$$

$$\overline{x_B^2} = (15\,mV)^2$$

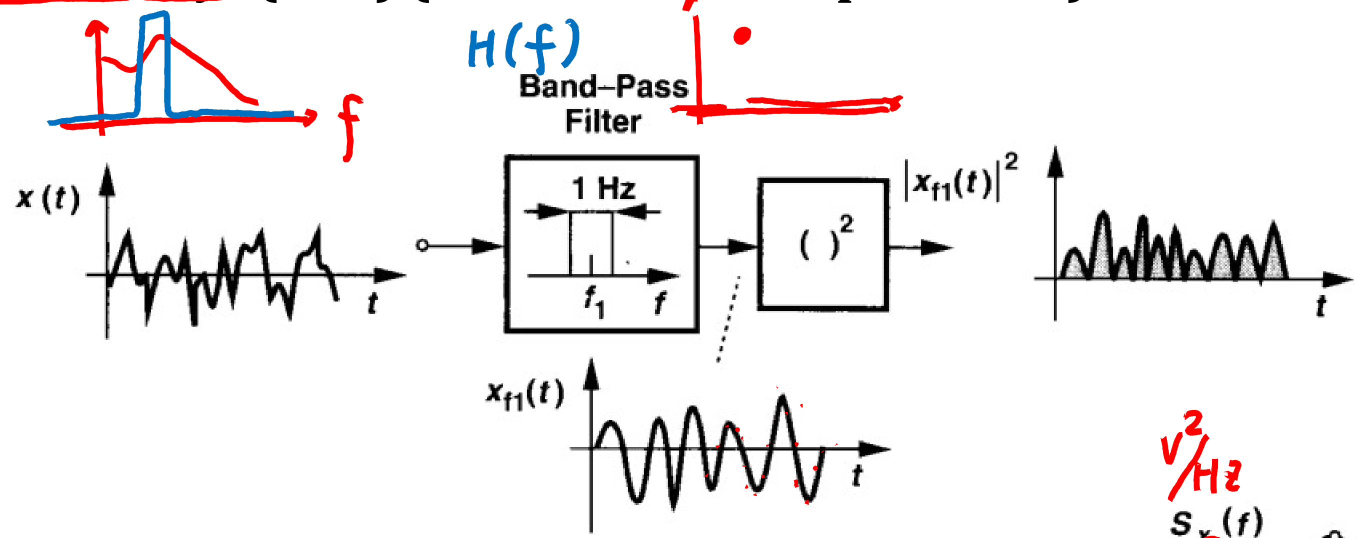
Larger fluctuations mean that the noise is 'stronger'



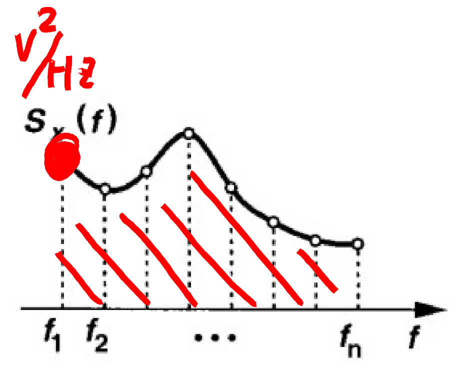
Statistical Characterization

2. Frequency Domain

For random signals, the concept of Fourier transform cannot be directly applied. But we still know that men carry less high-frequency components in their voice than women do. We define the "power spectral density" (PSD) (also called the "spectrum") as:



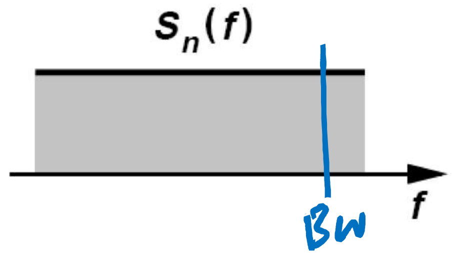
The PSD thus indicates how much power the signal carries in a small bandwidth around each frequency.



$$P_{av} = \lim_{T\to\infty} \frac{1}{T}\int_T x^2(t)\,dt = \int S_x(f)\,df$$

Statistical Characterization

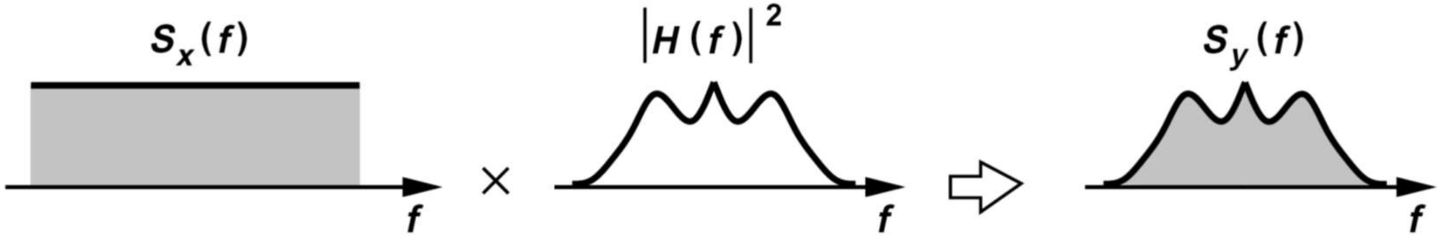
- What is the unit of $S_n(f)$?
- What is the total noise power?



A flat spectrum is called 'white'

- Is the total noise power infinite?

<u>**Important Theorem**</u>



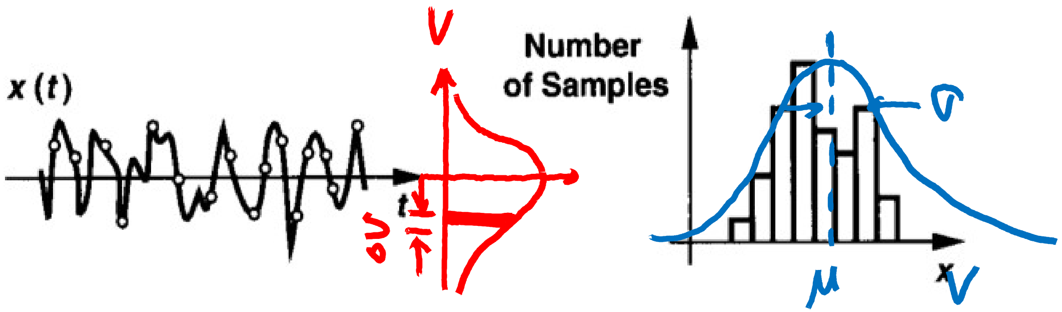
If a signal with spectrum $S_X(f)$ is applied to a linear time-invariant system with transfer function $H(s)$, the output spectrum is given by

$$S_Y(f) = S_X(f)|H(f)|^2$$

Statistical Characterization

3. Amplitude Distribution

By sampling the time-domain waveform for a long time, we can construct a "probability density function" (PDF). The PDF in essence indicates "how often" the amplitude is between certain limits.



For example, a Gaussian distribution is defined by a mean and a standard deviation. We say the noise amplitude rarely exceeds 4σ.

<u>Note:</u> Generally PDF and PSD bear no relationship.

Thermal Noise: Gaussian, white

Flicker Noise: Gaussian, not white

Correlated and Uncorrelated Sources

Can we use superposition for average noise power from a few noise components?

$$P_{av} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{+T/2} [x_1(t) + x_2(t)]^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int \left(x_1^2(t) + \underline{2x_1(t)x_2(t)} + x_2^2(t) \right) dt$$

$$x(t) = x_1(t) + x_2(t)$$

$$\lim_{T\to\infty} \frac{1}{T}\int_T x(t)\,dt = \lim_{T\to\infty}\frac{1}{T}\int_T \left(x_1(t) + x_2(t)\right) dt$$

$$= P_{av1} + P_{av2}$$

$$\overline{x(t)} = \overline{x_1(t)} + \overline{x_2(t)} = \emptyset$$

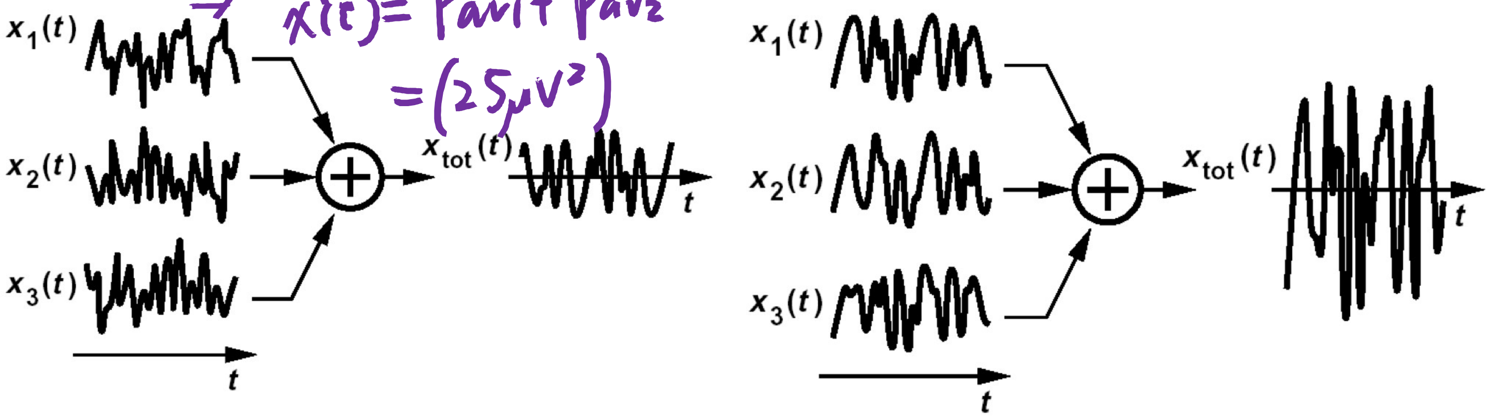
$$\overline{x_1^2(t)} = (3\,\text{mVrms})^2 \quad + \quad \overline{2x_1(t)x_2(t)} \rightarrow \emptyset$$

if $x_1(t)$ and $x_2(t)$ are independent to E.O.

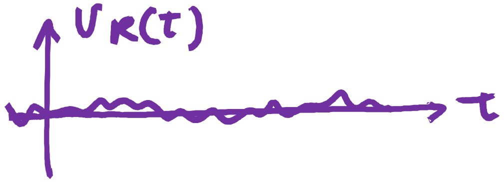
$$\overline{x_2^2(t)} = (4\,\text{mVrms})^2$$

We occasionally encounter correlated sources:

$$\Rightarrow \overline{x^2(t)} = P_{av1} + P_{av2} = (25\,\mu V^2)$$

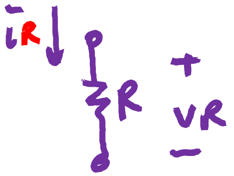


Types of Noise

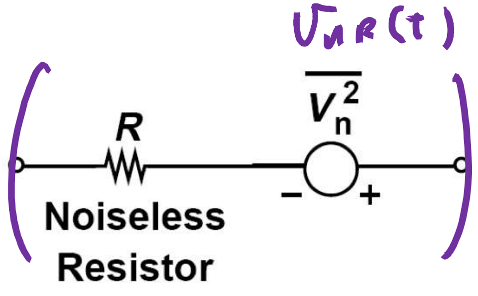


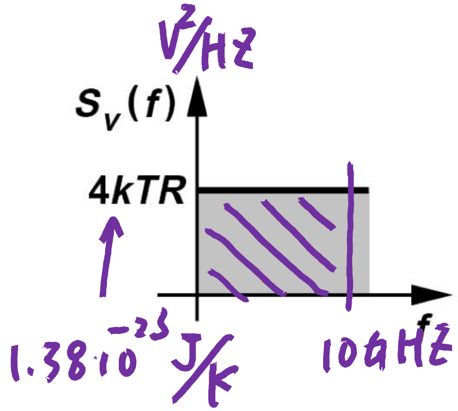
1. Thermal Noise in Resistors

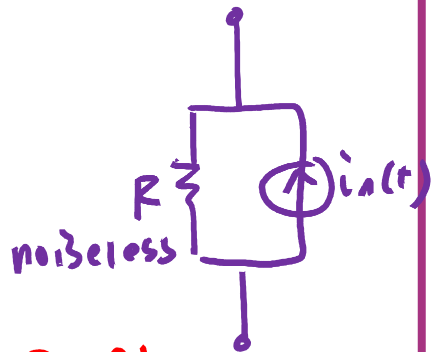
Random movement of charge carriers in a resistor causes fluctuations in the current. The PDF is Gaussian because there are so many carriers. The PSD is given by:



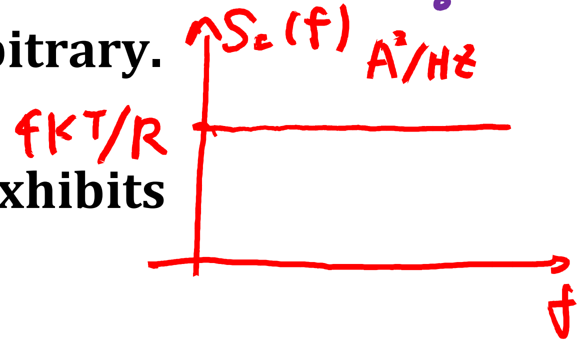
$V_R = i_R \cdot R$







Note that the polarity of the voltage source is arbitrary.



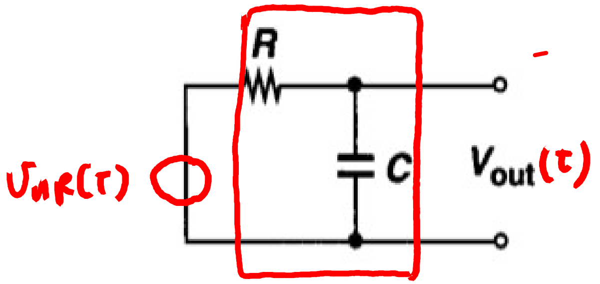
<u>Example:</u> A 50-Ω resistor at room temperature exhibits

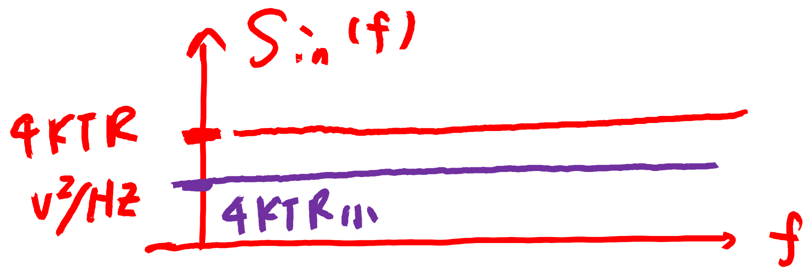
$$S_v(f) = 4\cdot K\cdot T\cdot 50 = (0.91nV)^2/Hz$$

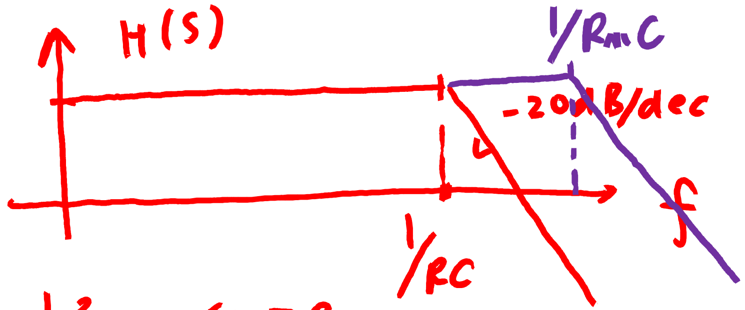
If the resistor is used in a system with 10-GHz bandwidth, then it contributes a total rms voltage of

$$(0.91nV)^2/Hz \cdot 10GHz = (91\mu V_{rms})^2$$

Example: Noise Spectrum and Total Noise Power

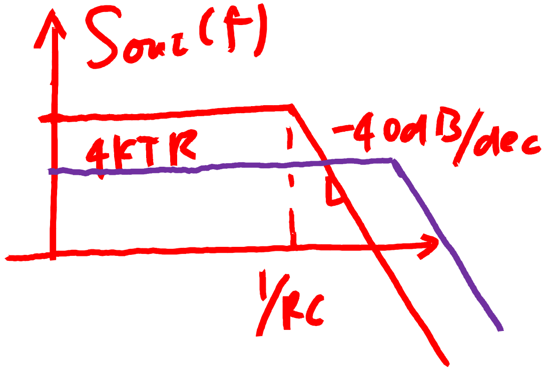






$$\frac{V_{out}}{V_R}(s) = \frac{1}{1+sRC}$$

$$S_{out}(f) = S_{in}(f)\left|\frac{1}{1+\underset{\uparrow j\omega}{s}RC}\right|^2 = \frac{4KTR}{1+(\omega RC)^2}$$



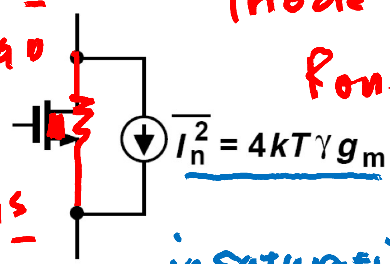
$$P_{n,out} = \int S_{out}(f)\,df = \frac{KT}{C}$$

For R=50 Ω and C=1 pF: $= (6.25\,mV_{rms})^2$

Trade-offs between noise, area, speed, and power

Types of Noise

2. Thermal Noise in Transistors



Triode region

$R_{on} = 1/\mu C_{ox}\frac{W}{L}(V_{GS}-V_{th})$

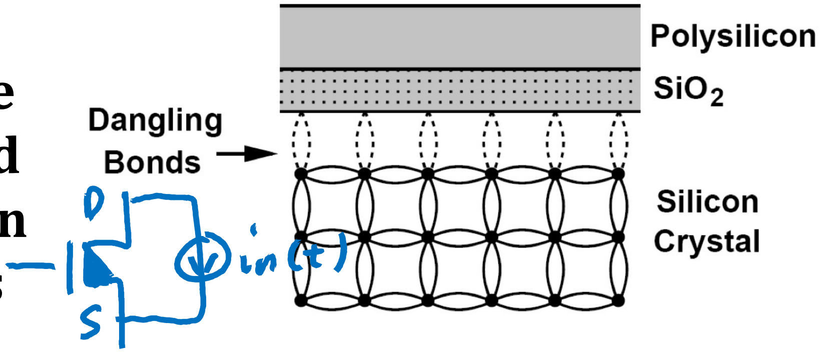
$S_{in}(f) = 4kT/R_{on}$ A^2/Hz

$= 4kT\mu C_{ox}\frac{W}{L}V_{ov}$

in saturation

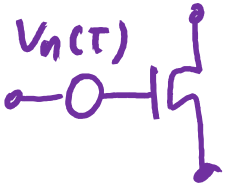
3. Flicker Noise in Transistors

In MOSFETs, the extra energy states at the interface between silicon and oxide trap and release carriers randomly and at different rates. The noise in the drain current is Gaussian, but its spectrum is given by:

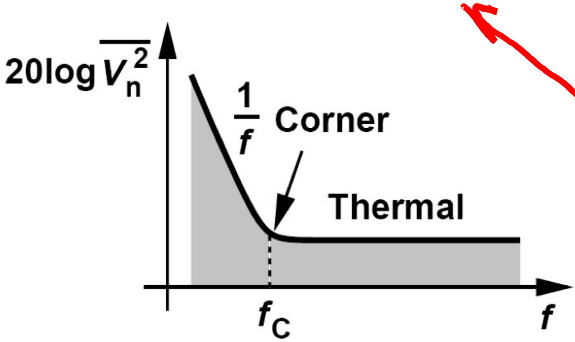


$S_{in}(f) \propto 4kT\,V_{ov}$ A^2/Hz

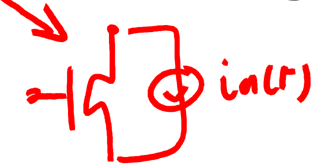
$= 4kT\gamma\mu C_{ox}\frac{W}{L}V_{ov}$ g_m



$S_{vnf}(f) = \frac{K}{C_{ox}\cdot W\cdot L\cdot f}$



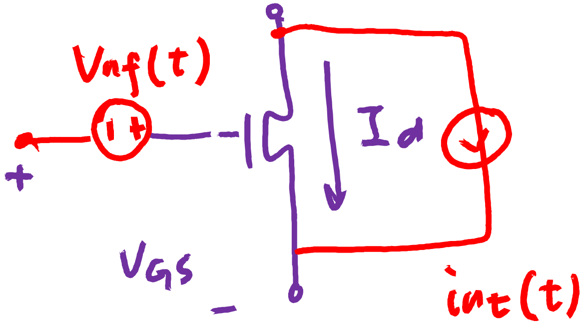
Where k is a constant and its value [hea]vily depends on how 'clean' the [...]. We often characterize the [...] noise by considering [...]quency.

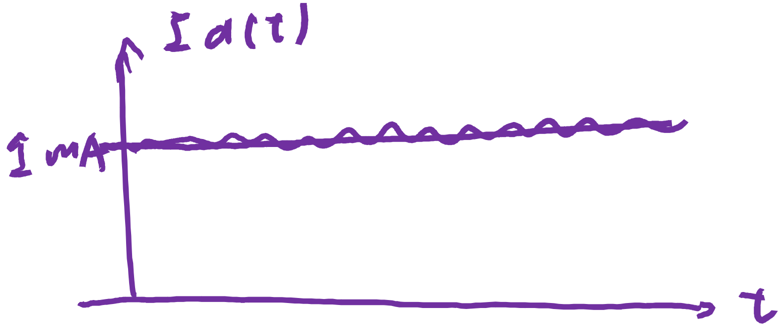


$S_{inf}(f) = \frac{K g_m^2}{C_{ox}WLf}$ A^2/Hz

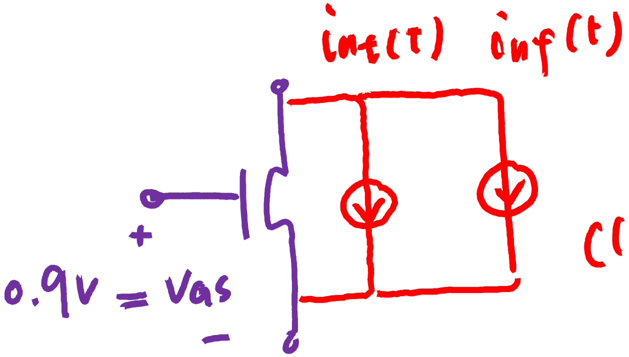
Example: A Current Source

Total noise in the drain current for a band from 1 kHz to 1 MHz

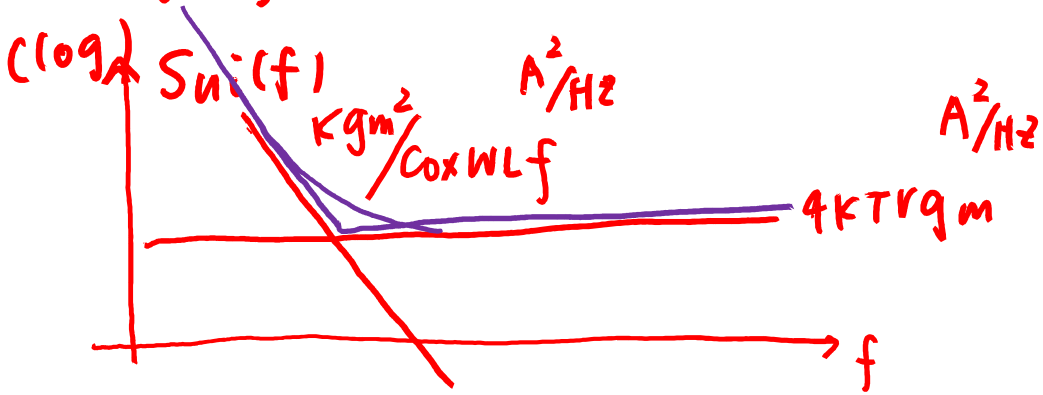


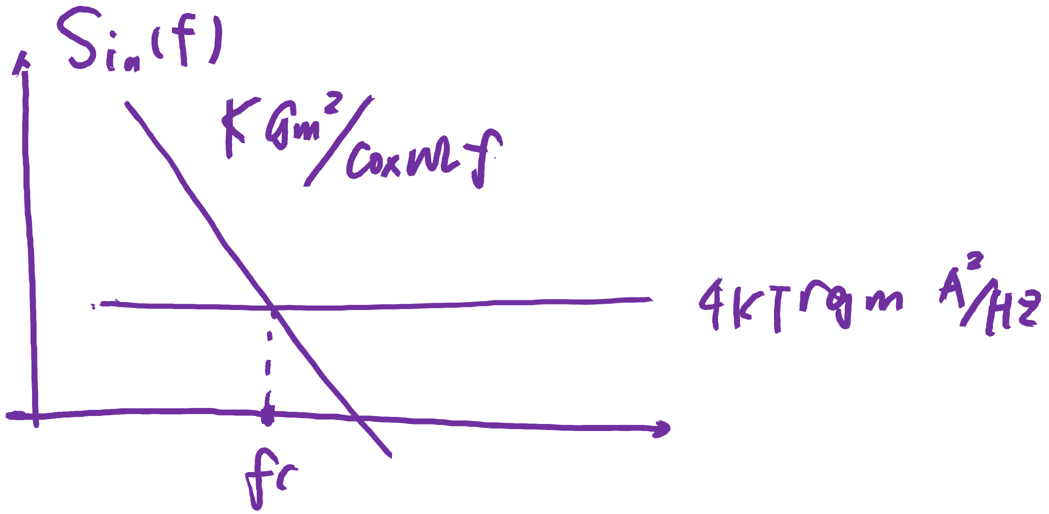


$$i_d(t) = 1\,mA + \underline{i_{n1}(t) + i_{n2}(t)}$$



$$i_n(t) = i_{n1}(t) + i_{n2}(t)$$





find out f_c such that $\frac{Kg_m^2}{C_{ox}WLf_c} = 4kT\gamma g_m$

$$f_c = \frac{K}{C_{ox}WL} g_m \frac{1}{4kT\gamma}$$

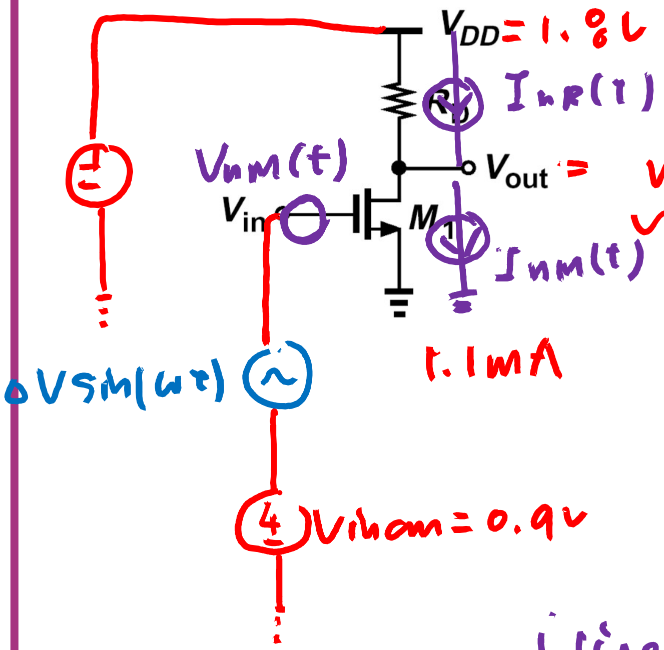
$10kHz \sim 10MHz$

from 1kHz to 1MHz

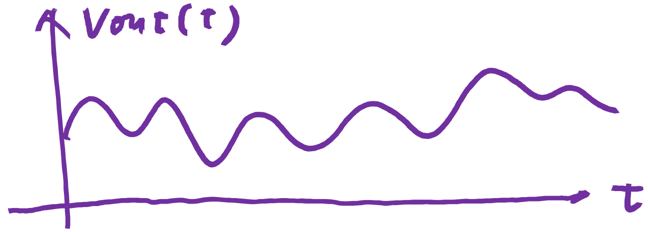
$$\overline{I_{n,tot}^2} = \int_{1k}^{1M} S_{in}(f)df = 4kT\gamma g_m(10^6 - 10^3) + \frac{Kg_m^2}{C_{ox}WL}(\ln 10^6 - \ln 10^3)$$

Common-Source Stage

$$A_v = \frac{\partial V_{out}}{\partial V_{in}}$$



$$V_{out} = \underbrace{V_{DD} - I_D \cdot R_D}_{1.2V} \underbrace{- g_m R \cdot \partial v \sin(\omega t)}_{\partial V_{out}(t)} + V_{outn}(t)$$



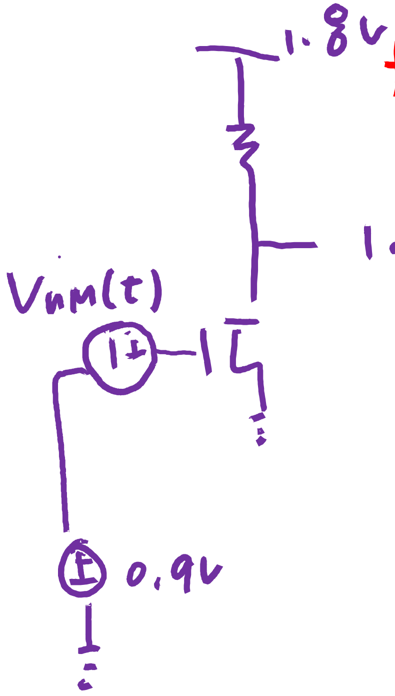
(linear approximation)

We are applying small-signal analysis with superposition for all the sources, including ∂V_{in}, I_{nm}, I_{nR}, V_{nm}, ...

$$V_{outn}(t) = V_{nm}(t) \cdot g_m R - I_{nR}(t) \cdot R + I_{nm}(t) \cdot R$$

$$S_{voutn}(f) = S_{vnm}(f) \cdot (g_m R)^2 + S_{InR}(f) R^2 + S_{Inm}(f) \cdot R^2$$

flicker noise

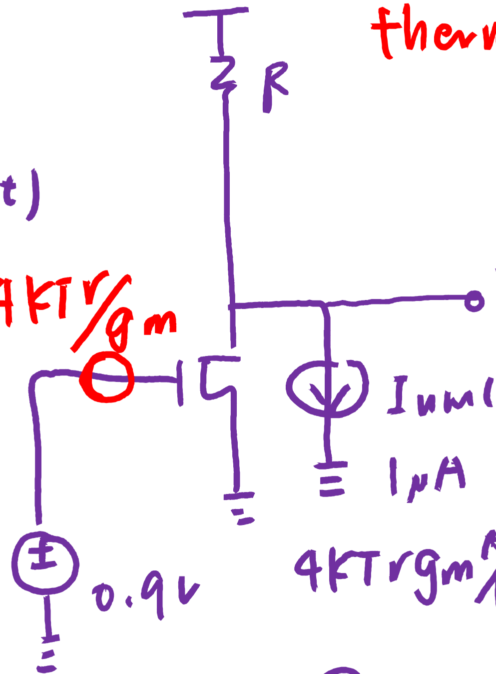


$1.2 - \underline{g_m R\, V_{nm}(t)}$

$\Downarrow$

$\frac{\Delta V_{out}}{V_{nm}}$

thermal noise

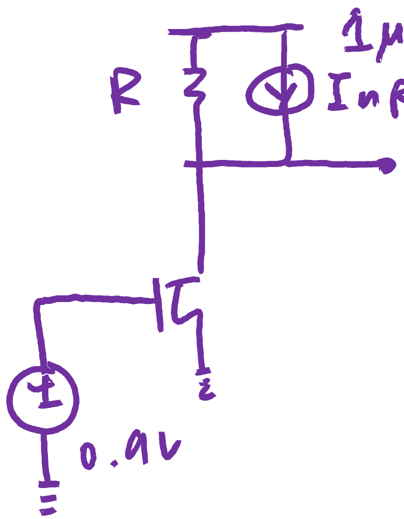


$V_{out}(t) = 1.2V - I_{nm}(t)\cdot \underline{\underline{R}}$

$4KT\gamma g_m\ A^2/Hz$ gain of $\frac{\Delta V_{out}}{I_{nm}}$

$S_{vnout}(f)$

$= 4KT\gamma g_m \cdot \underline{\underline{R}}^2$



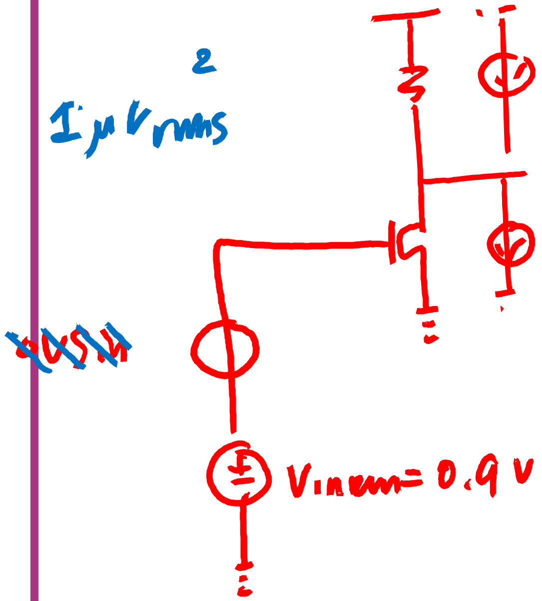
$V_{out} = 1.2 + I_{nR}(t)\cdot \underline{\underline{R}}$

gain of $\frac{\Delta V_{out}}{I_{nR}}$

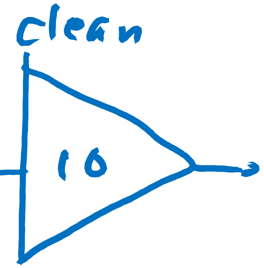
$V_{out\,n}(t) = -V_{nm}(t)\cdot g_m R$

$S_{v_n out}(f) =$

$4KT\gamma/g_m\,(g_m R)^2$



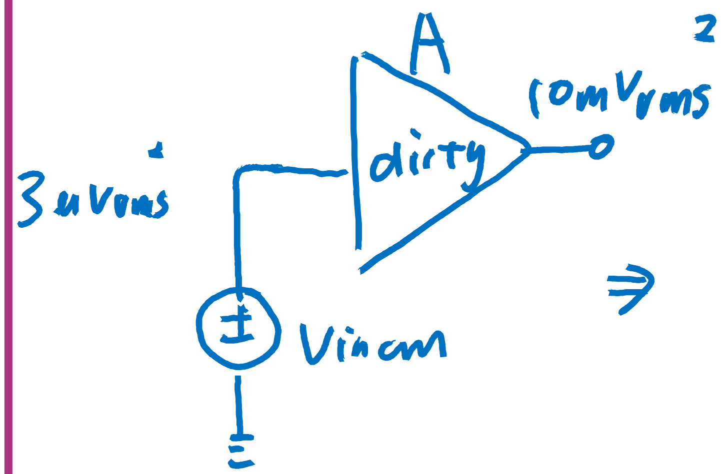
$$V_{out}(t) = 1.2 - \cancel{g_m R \cdot V_s \sin(t)} + V_{nout}(t)$$



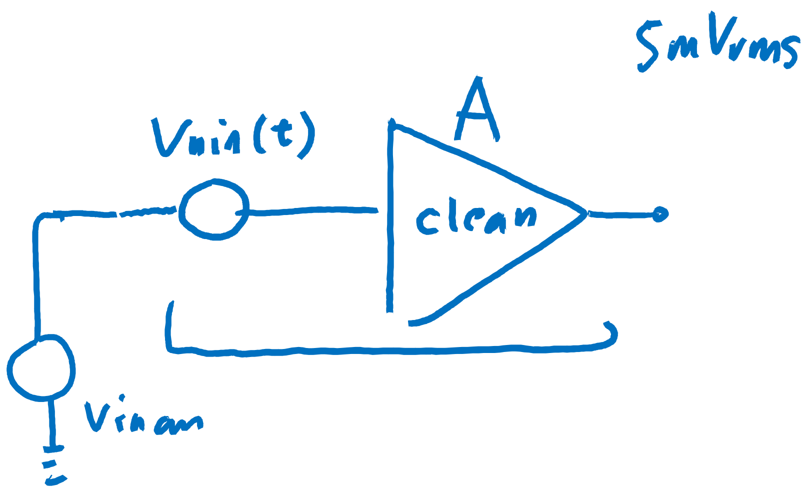
for noise

$$S_{vnout}(f) = S_{Inm}(f) \cdot R^2 + S_{nR}(f) \cdot R^2$$

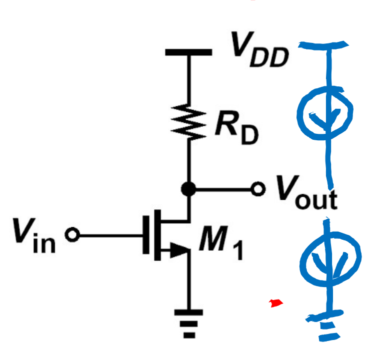
$$= 4kT\gamma g_m R^2 + 4kT/R \cdot R^2 \quad V^2/Hz$$



$\Rightarrow$

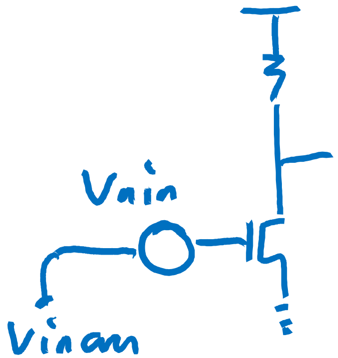


Common-Source Stage



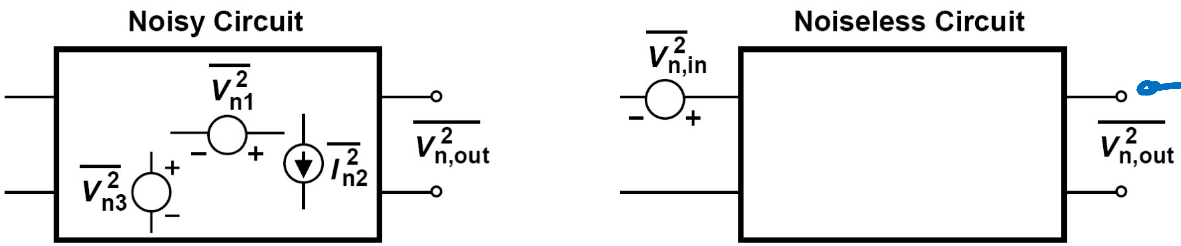
$S_{nR}(f) = 4kT/R$

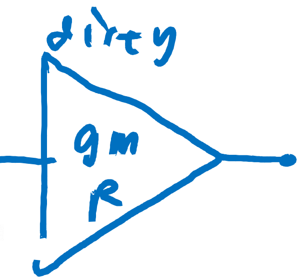
$S_{nm}(f) = 4kT\gamma g_m$



$$S_{\overline{V^2_{n,out}}}(f) = 4kT \cdot R + 4kT\gamma g_m R^2$$

<u>Input-Referred Noise</u> is the noise voltage or current that, when applied to the input of the noiseless circuit, generates the same output noise as the actual circuit does.



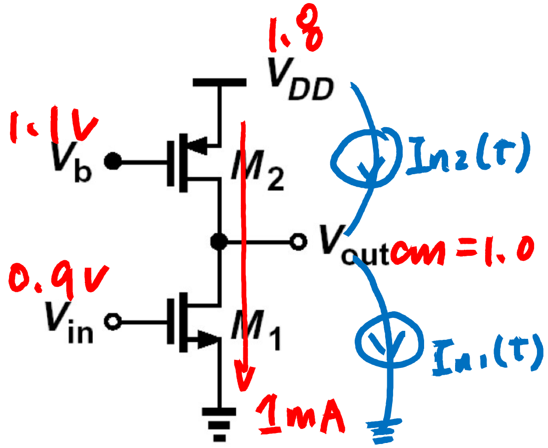


$$S_{\overline{V^2_{n,in}}}(f) = \frac{4kTR}{(g_m R)^2} + \frac{4kT\gamma}{g_m}$$

Common-Source Stage

$$\overline{V_{n,in}^2} = 4kT\left(\frac{2}{3g_m} + \frac{1}{g_m^2 R_D}\right) + \frac{K}{C_{ox}WL}\frac{1}{f}$$

Why does the noise decrease as R_D increases?



With a current-source load:

$$S_{\overline{V_{n,out}^2}}(f) = (4kT\gamma g_{m1} + 4kT\gamma g_{m2})\left|H^2(f)\right|$$

$$\overline{V_{n,in}^2} = \frac{4kT\gamma}{g_{m1}} + \frac{4kT\gamma g_{m2}}{g_{m1}^2} \qquad \left|\frac{R_{out}}{1+sC_L R_{out}}\right|^2$$

$$S_{In1}(f) = 4kT\gamma g_{m1} \quad A^2/Hz$$

Consider BW limitation from C_L and a low-frequency signal V_m at input:

$$\overline{V_{nout}^2} \quad P_{n,out} = \gamma(g_{m1}+g_{m2})(r_{o1}//r_{o2})\frac{kT}{C_L} \qquad \int \frac{(r_{o1}//r_{o2})^2}{1+\omega^2 C_L^2 (r_{o1}//r_{o2})^2}\,df$$

V_{rms}^2

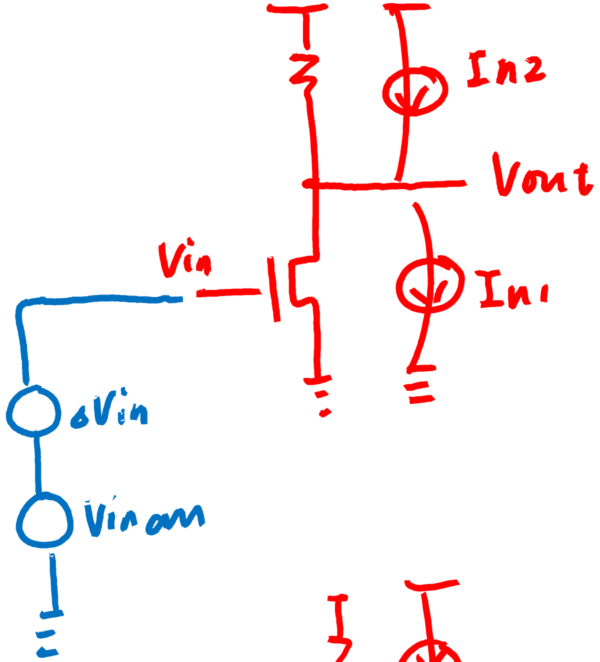
$$SNR_{out} =$$

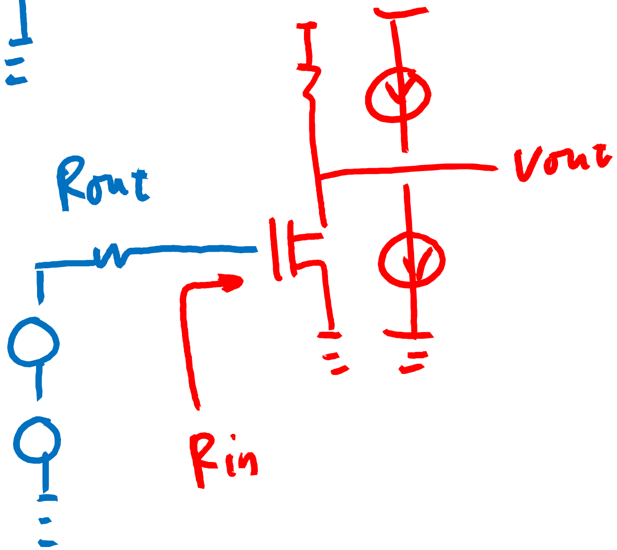
$\Delta V_{in} = V_m \sin(\omega t) \rightarrow \Delta V_{out} = V_m \cdot g_m \cdot (r_{o1}//r_{o2}) \cdot \sin(\omega t)$

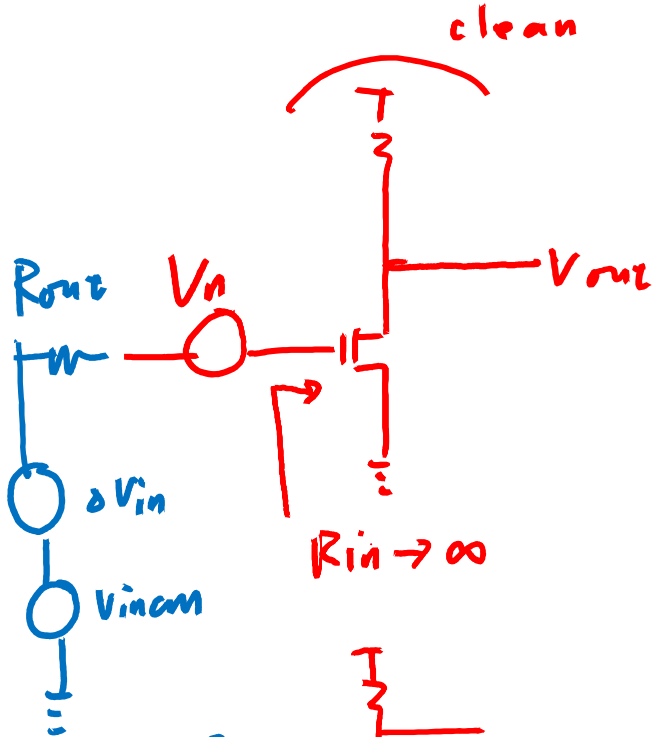
How to reduce the noise?

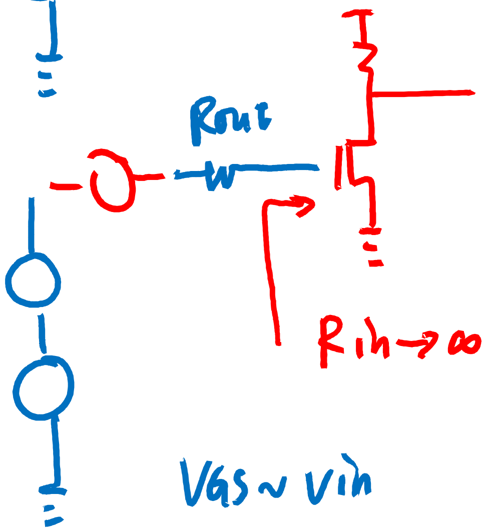
$$\frac{V_m^2\, g_{m1}^2 (r_{o1}//r_{o2})}{2} \Big/ \gamma(g_{m1}+g_{m2})\frac{kT}{C_L}$$

Trade-offs between speed, power, and voltage headroom





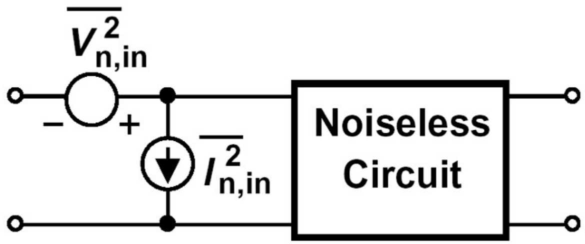




$V_{GS} \sim v_{in}$

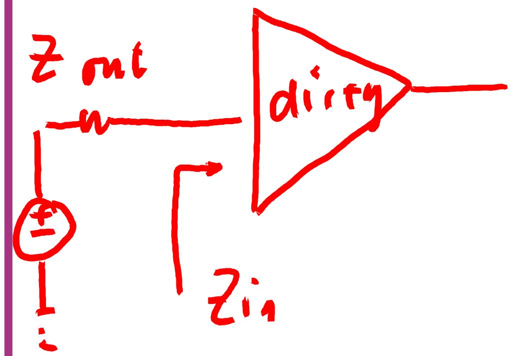
only when $R_{in} \gg R_{out}$

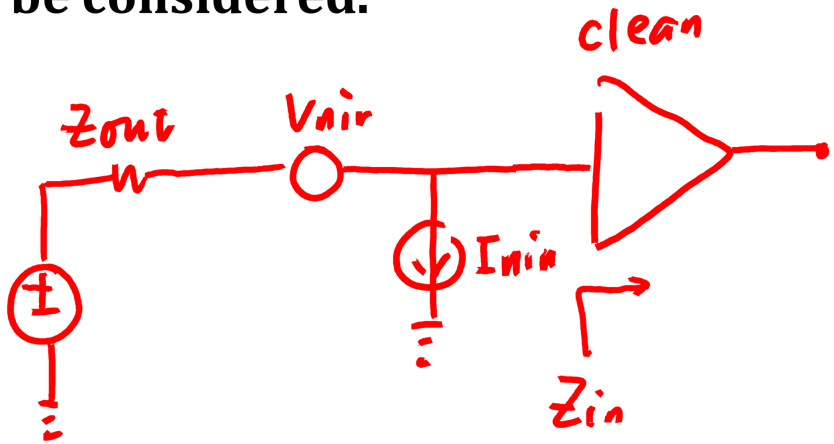
Input-Referred Noise



In general, we need both a voltage source and a current source at the input to model the circuit noise.

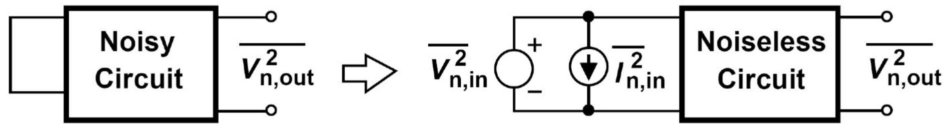
If the source impedance is high with respect to the input impedance of the circuit, then both must be considered.



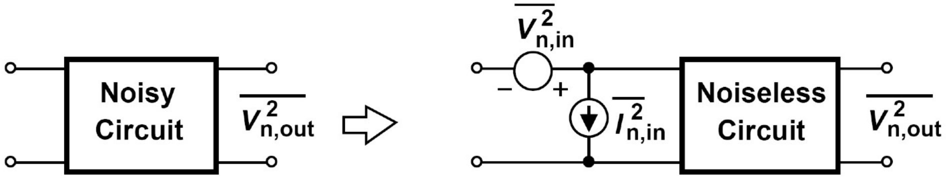


How To Calculate Input-Referred Noise?

With Z_S=0:



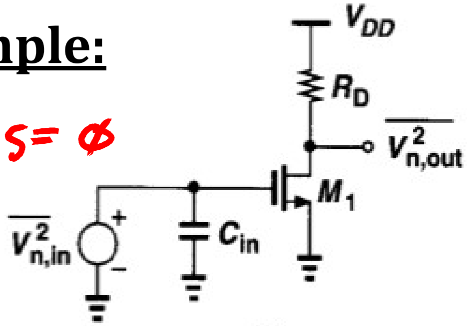
With Z_S=∞:



$S_{nout}(f) = 4kT\gamma g_m R_D^2 + 4kTR_D$

<u>Example:</u>

$Z_S = 0$

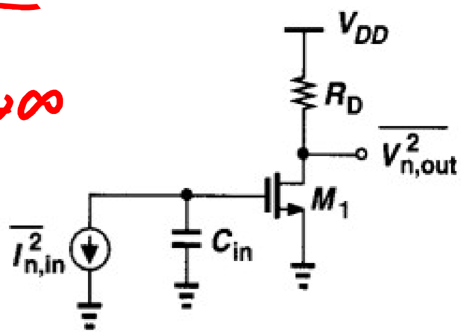


$S_{\overline{V^2_{n,in}}}(f)\cdot(g_{m1}R_D)^2 = S_{nout}(f)$

$\cancel{\overline{V^2_{n,out}} = \overline{V^2_{n,in}}}$

$\cancel{\overline{V^2_{n,out}} =}$

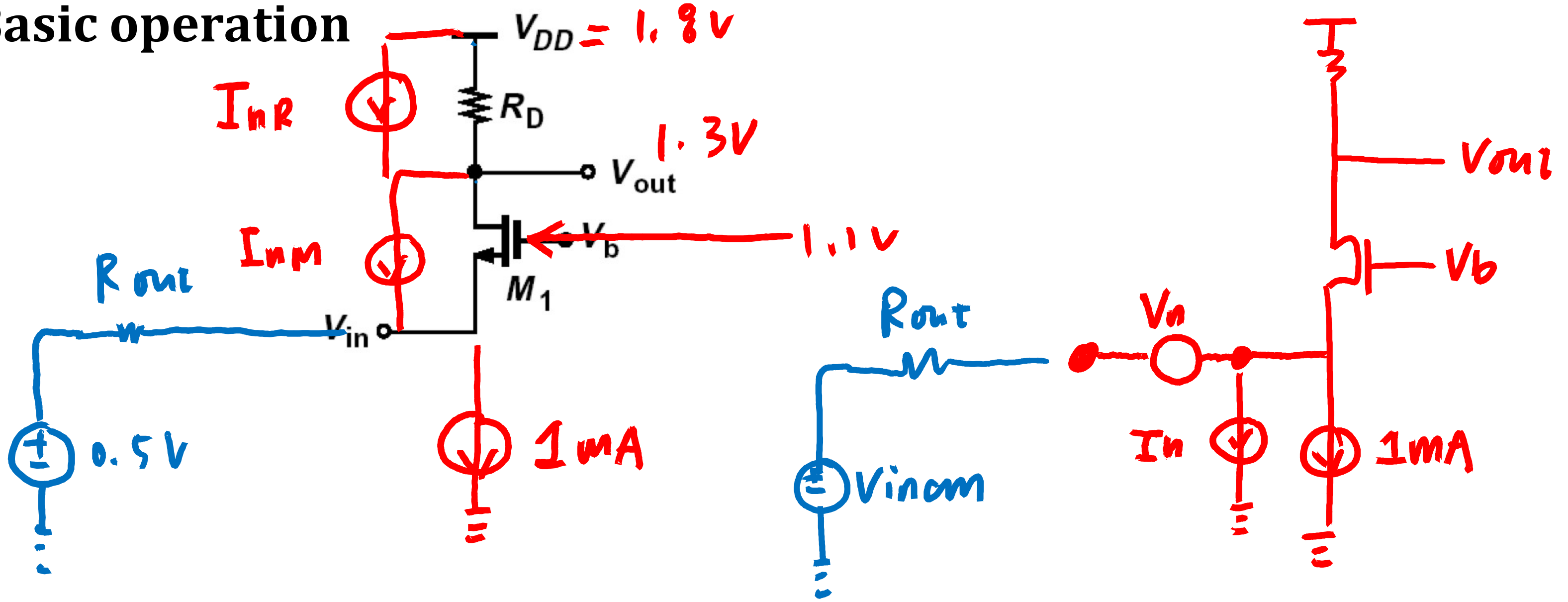
$Z_S \to \infty$



$S_{\overline{I^2_{n,in}}}(f)\cdot\left(\frac{1}{\omega C_{in}}\right)^2(g_{m1}R_D)^2 = S_{nout}(f)$

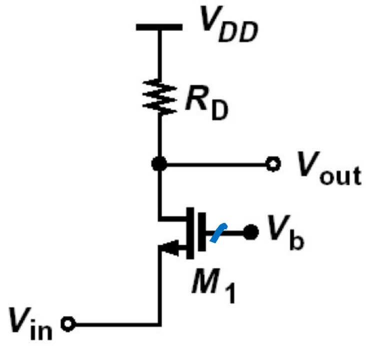
Common-Gate Stage

Basic operation



Common-Gate Stage

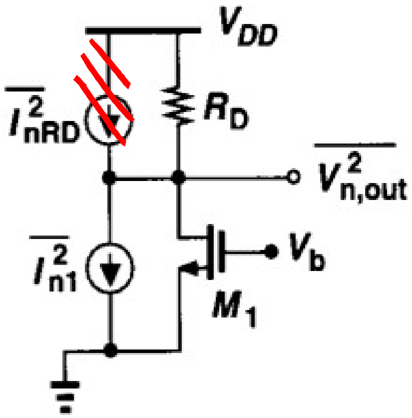
Due to the low input impedance, the input-referred noise current is not negligible even at low frequencies

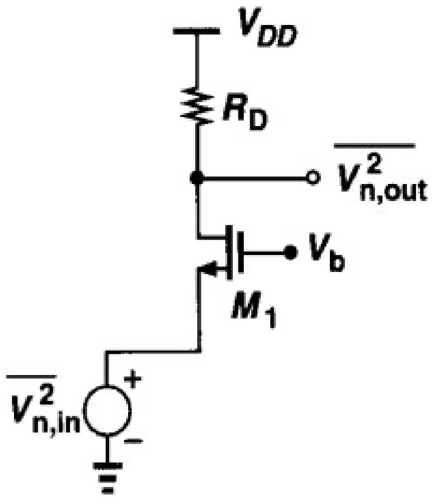


Common-Gate Stage

For Z_S=0

Case #1

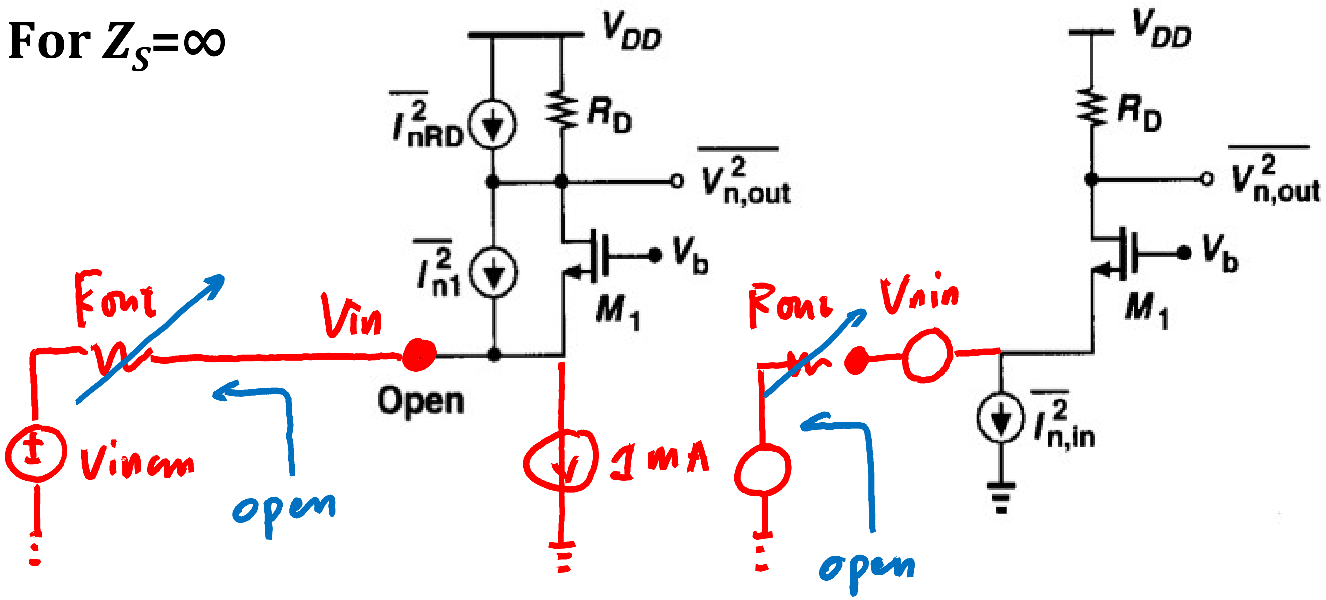




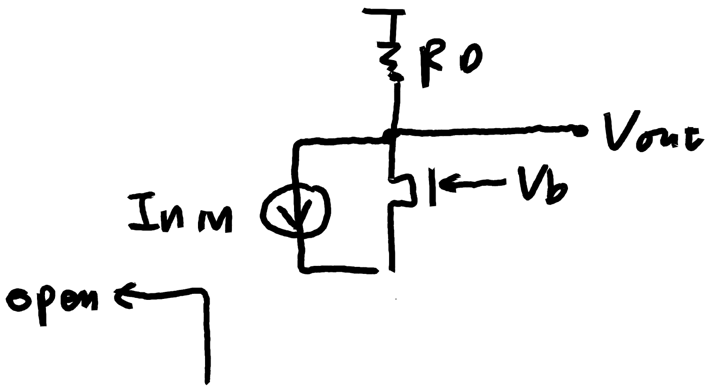
$$S_{vnout}(f) = S_{InR}(f)\,R_D^2 + S_{InM}(f)\cdot R_D^2 = S_{vnin}(f)\cdot(g_m+g_{mb})^2 R_D^2$$

Common-Gate Stage

For $Z_S=\infty$



$$S_{vnout}(f) = S_{InR}(f)\cdot R_D^2 \quad = \quad S_{Inin}(f)\cdot R_D^2$$

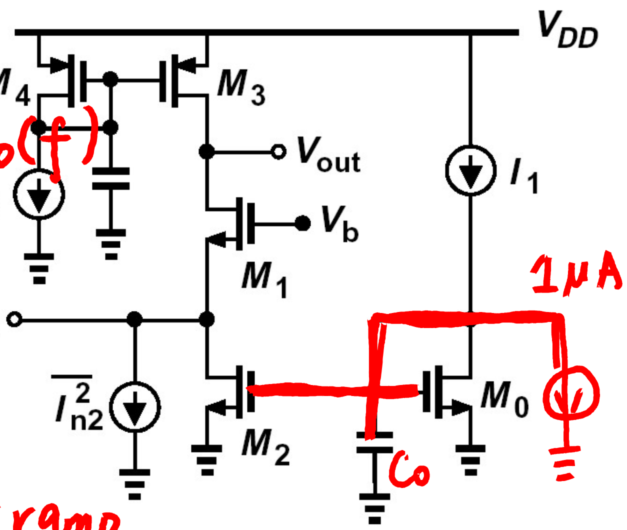


Current Source

The bias current source often contributes significant noise.

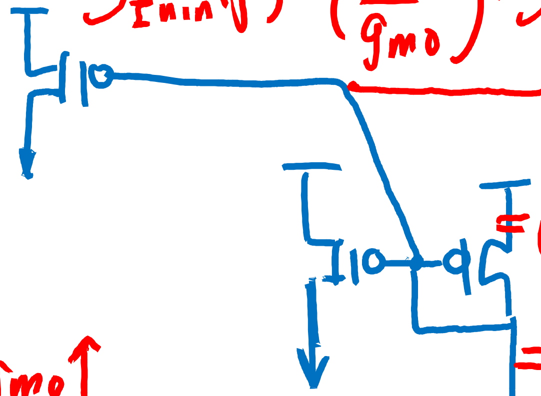
Noise from M_0 has no impact on V_{out} when $Z_S = \emptyset$

Example: only when $Z_S = \infty$



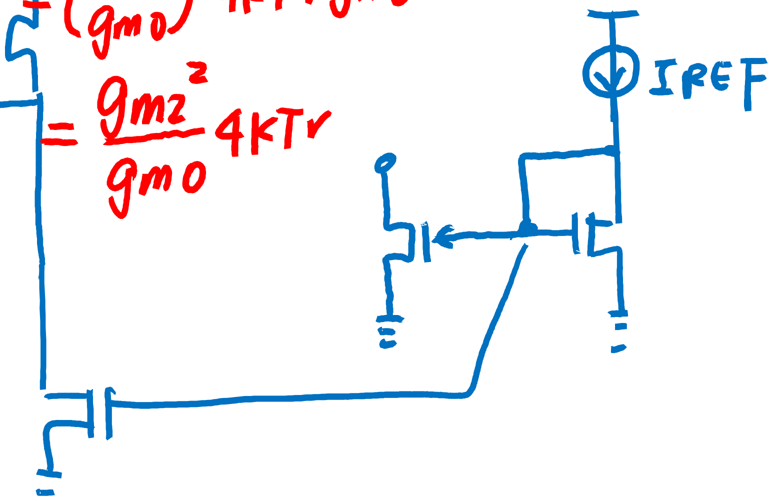
- Capacitor C_0 shunts the noise generated by M_0 to ground. $S_{Vnout}(f) = \left(\frac{g_{m2}}{g_{m0}} \cdot |Z_{out}|\right)^2 S_{InM0}(f)$
- Noise from M2:

$$S_{Inin}(f) = \left(\frac{g_{m2}}{g_{m0}}\right)^2 \cdot S_{InM0}(f)$$



$$= \left(\frac{g_{m2}}{g_{m0}}\right)^2 \cdot 4kT\gamma g_{m0}$$

$$= \frac{g_{m2}^2}{g_{m0}} 4kT\gamma$$

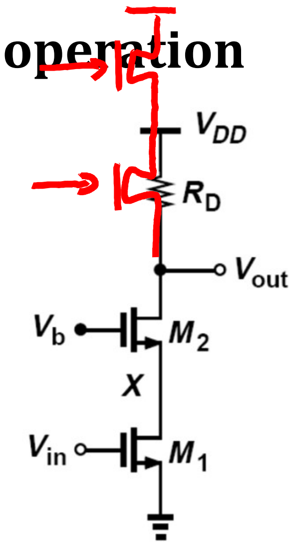


⇒ we'd prefer $g_{m0}\uparrow$

$g_{m2}\downarrow$

Cascode Stage

Basic operation



$R_D // r_{o1} \cdot (g_{m2} r_{o2})$

$$A = \frac{\Delta V_{out}}{\Delta V_{in}} = g_{m1} \cdot (R_{out})$$

$$S_{V_{n,out}}(f) = S_{I_{n,R}} \cdot R_{out}^2$$

$$+ \frac{4kT\gamma}{g_{m1}} \left(g_{m1}^2 R_{out}^2 \right)$$

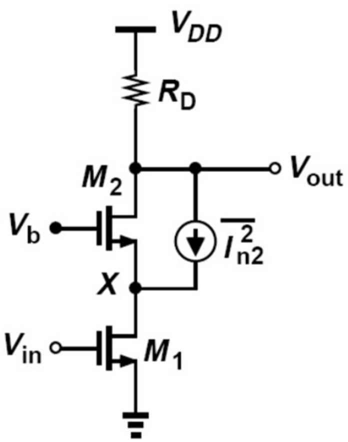
$$+ \emptyset$$

noise from cascode devices

does not show up at the output

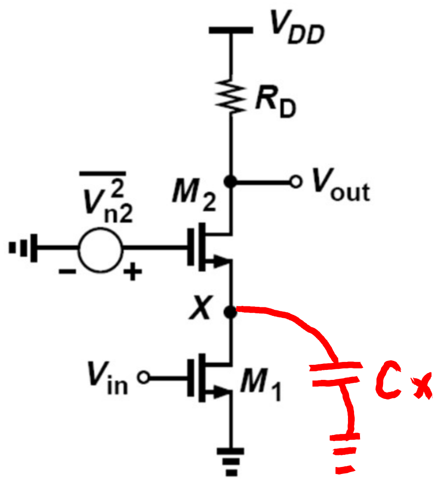
(by ignoring r_{o1})

The effect of noise of M2



Cascode Stage

With capacitance at node X



$$\frac{V_{out}}{V_{n2}} = \frac{\frac{1}{g_{m2}}}{\frac{1}{g_{m2}} + \frac{1}{sC_x}} \cdot g_{m2} \cdot R_{out}$$

At high frequencies:

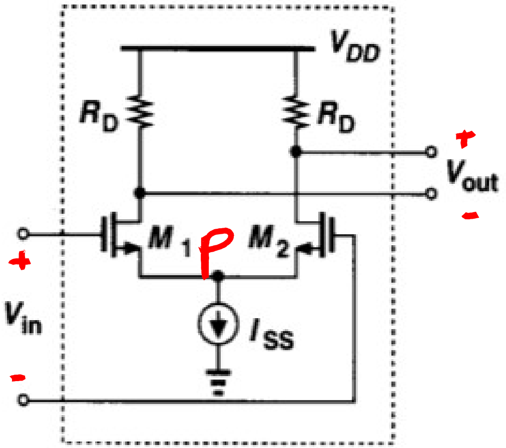
- **Output noise is increased**
- **The gain from V_{in} to V_{out} is decreased**

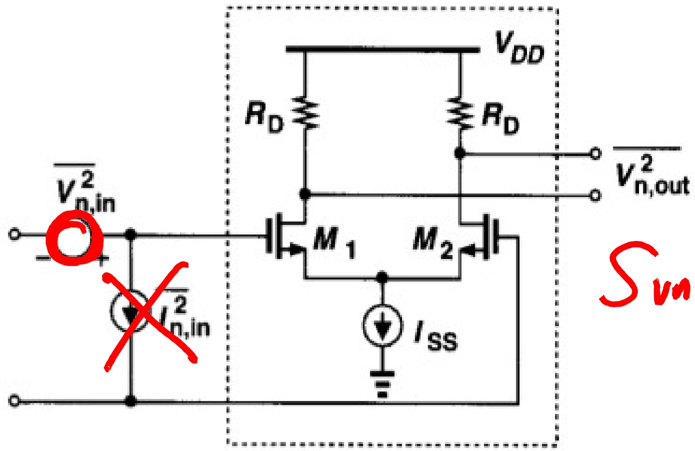
$\rightarrow$ SNR $\downarrow$

Differential Pair

$$A_v = \frac{\Delta V_{out}}{\Delta V_{in}} = (g_{m12}) \cdot R_D$$

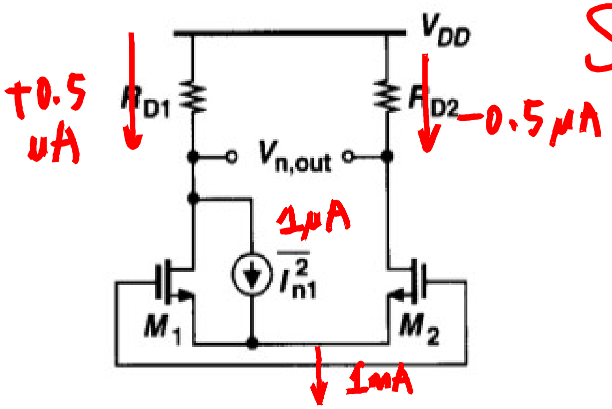
Consider differential signals





$$S_{vnout} = S_{vin} (g_{m12} R_D)^2$$

Since the four noise generators are uncorrelated, we can use superposition for the powers. Noise from M_1:



8KTR

$$S_{vnout}(f) = 4KTR_2 + 4KTR_1$$

$$+ 4KT\gamma g_{m1} \cdot R_D^2 + 4KT\gamma g_{m2} \cdot R_D^2$$