EE4280 Lecture 9: Noise

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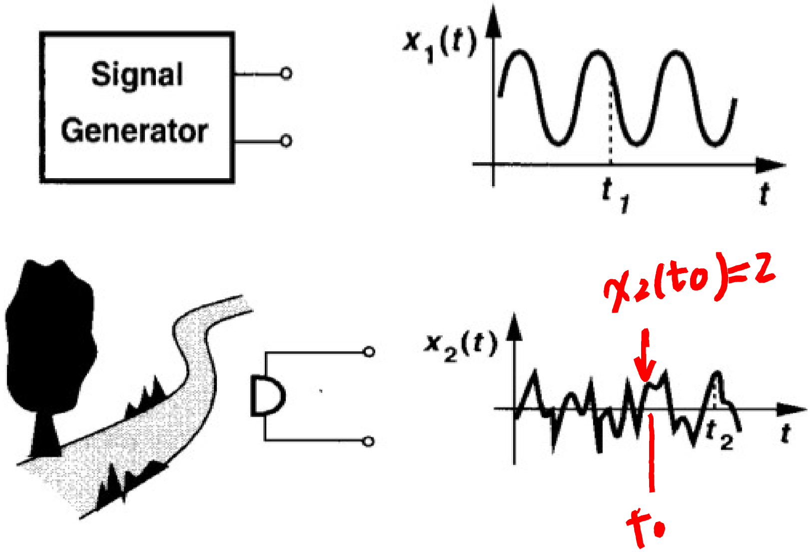
What is Noise?

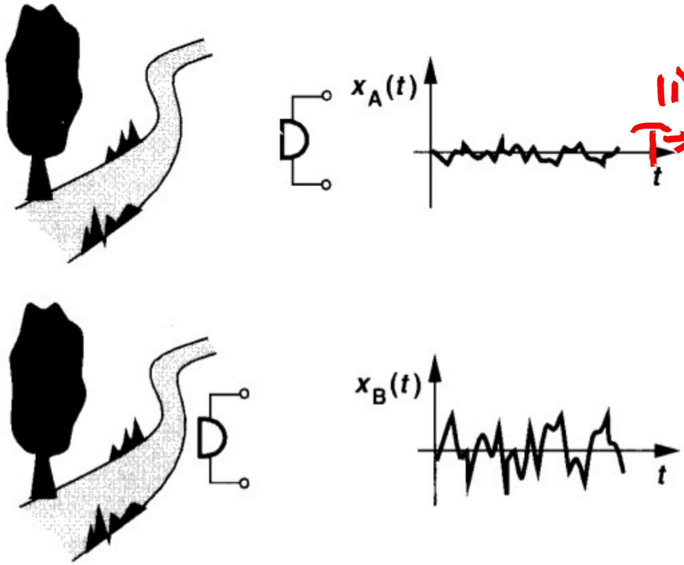
Noise is a random process. We consider a phenomenon random because we do not know everything about it, or simply because we do not <u>need</u> to know everything about it.



Since the instantaneous noise amplitude is not known, we resort to 'statistical' models, i.e., some properties that can be predicted.

Statistical Characterization

1. Mean and Average Power



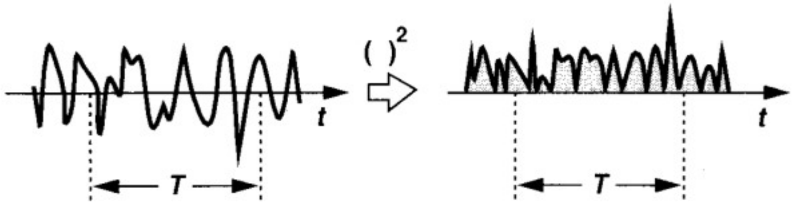
$$\lim_{T\to\infty} \frac{1}{T}\int_T x_A(t)\,dt = \overline{x_A} = 0$$

$$\lim_{T\to\infty} \frac{1}{T}\int_T x_A^2(t)\,dt = \overline{x_A^2} = 25\,\mu V^2$$

$$= (5\,mV_{rms})^2$$

$$\overline{x_B^2} = (15\,mV)^2$$

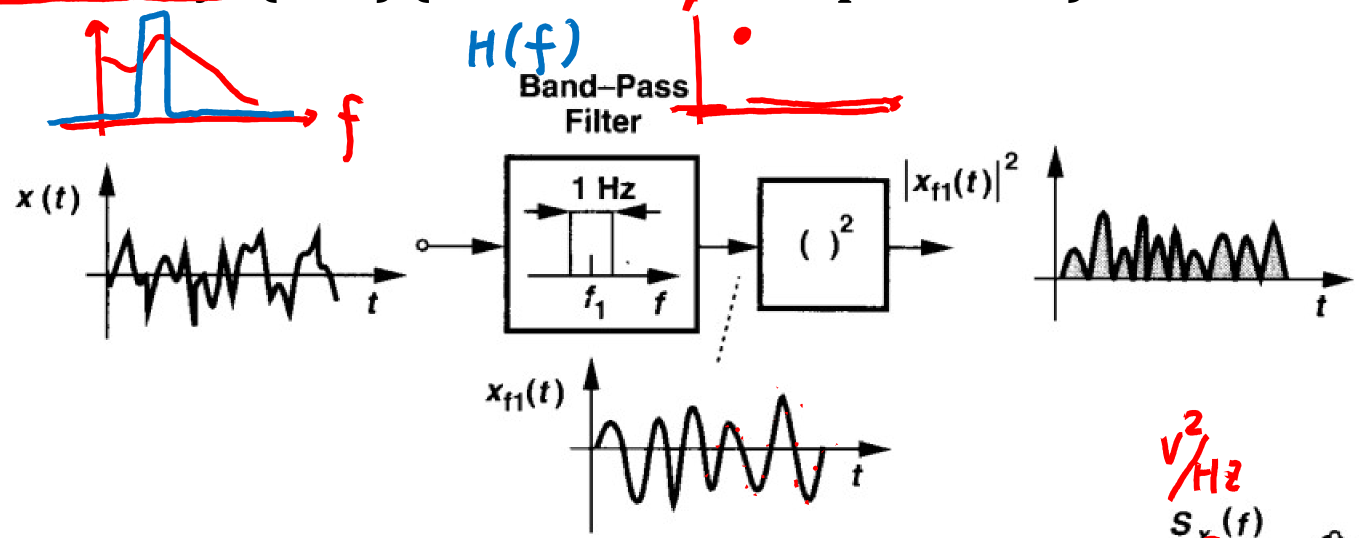
Larger fluctuations mean that the noise is 'stronger'



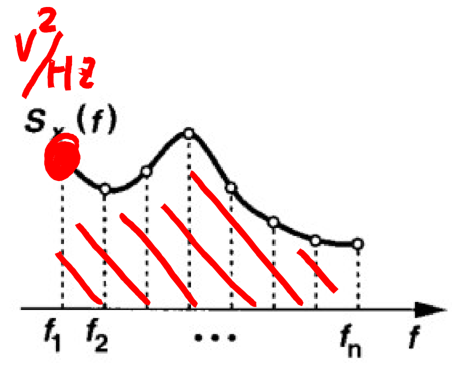
Statistical Characterization

2. Frequency Domain

For random signals, the concept of Fourier transform cannot be directly applied. But we still know that men carry less high-frequency components in their voice than women do. We define the "power spectral density" (PSD) (also called the "spectrum") as:



The PSD thus indicates how much power the signal carries in a small bandwidth around each frequency.



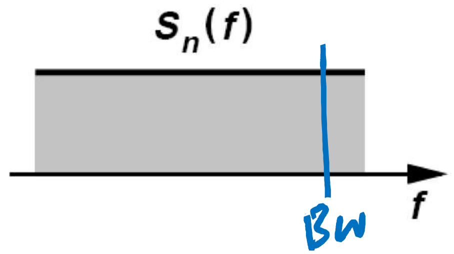
$$P_{av} = \lim_{T \to \infty} \frac{1}{T} \int_T x^2(t)\, dt = \int S_x(f)\, df$$

Statistical Characterization

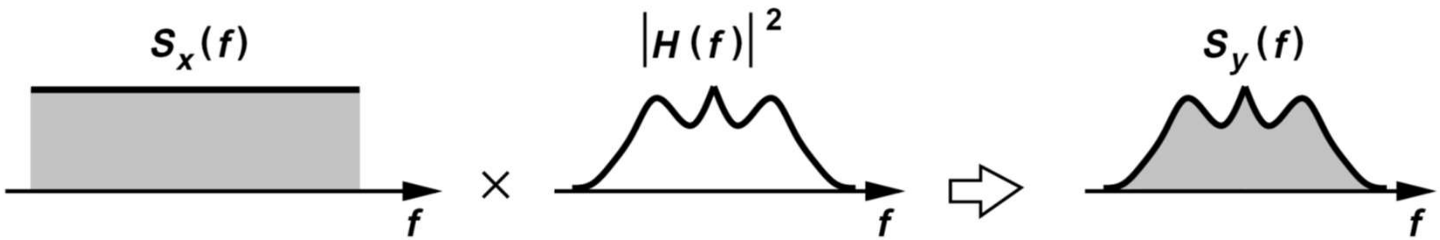
- **What is the unit of $S_n(f)$?**
- **What is the total noise power?**

A flat spectrum is called 'white'

- **Is the total noise power infinite?**



<u>Important Theorem</u>



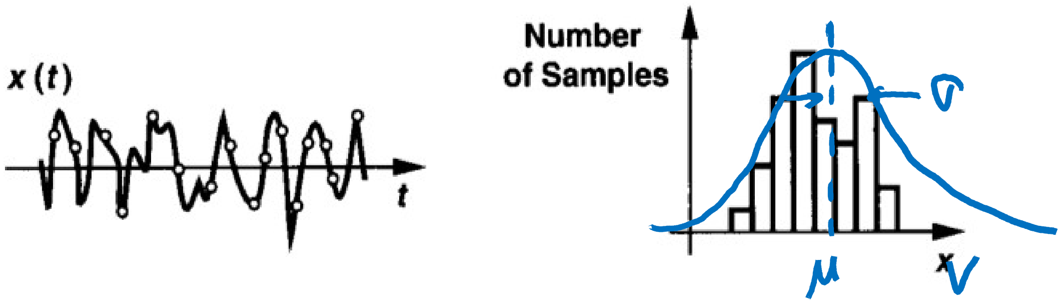
If a signal with spectrum $S_X(f)$ is applied to a linear time-invariant system with transfer function $H(s)$, the output spectrum is given by

$$S_Y(f) = S_X(f)|H(f)|^2$$

Statistical Characterization

3. Amplitude Distribution

By sampling the time-domain waveform for a long time, we can construct a "probability density function" (PDF). The PDF in essence indicates "how often" the amplitude is between certain limits.



For example, a Gaussian distribution is defined by a mean and a standard deviation. We say the noise amplitude rarely exceeds 4σ.

<u>Note:</u> Generally PDF and PSD bear no relationship.

Thermal Noise: Gaussian, white

Flicker Noise: Gaussian, not white

Correlated and Uncorrelated Sources

Can we use superposition for average noise power from a few noise components?

$$P_{av} = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{+T/2}[x_1(t)+x_2(t)]^2dt$$

$x(t) = x_1(t) + x_2(t)$

$= \lim_{T\to\infty}\frac{1}{T}\int\left(x_1^2(t) + \underline{2x_1(t)x_2(t)} + x_2^2(t)\right)dt$

$\lim_{T\to\infty}\frac{1}{T}\int_T x(t)dt = \lim_{T\to\infty}\frac{1}{T}\int_T\left(x_1(t)+x_2(t)\right)dt$

$= P_{av1} + P_{av2}$

$\overline{x(t)} = \overline{x_1(t)} + \overline{x_2(t)} = \emptyset$

$\overline{x_1^2(t)} = (3\text{mV}_{rms})^2 + \overline{2x_1(t)x_2(t)} \to \emptyset$

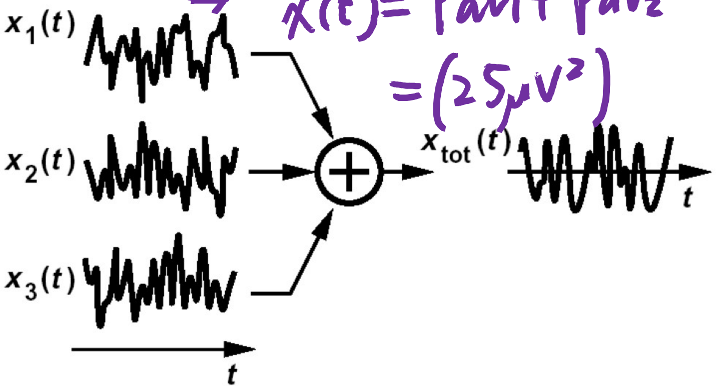
$\overline{x_2^2(t)} = (4\text{mV}_{rms})^2$

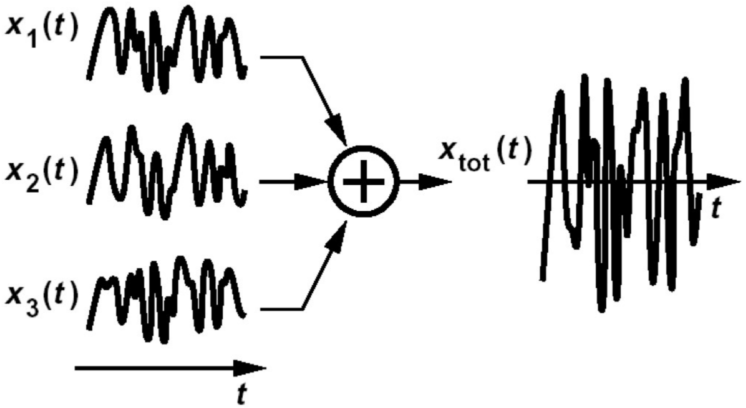
if $x_1(t)$ and $x_2(t)$ are independent to E.O.

We occasionally encounter correlated sources:

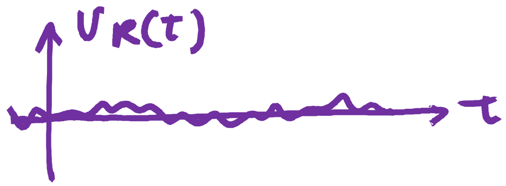
$\Rightarrow \overline{x^2(t)} = P_{av1} + P_{av2}$

$= (25\mu V^2)$



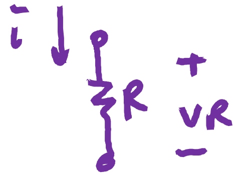


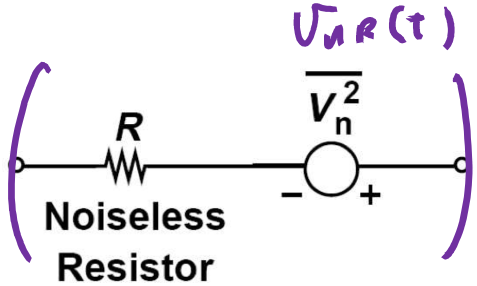
Types of Noise

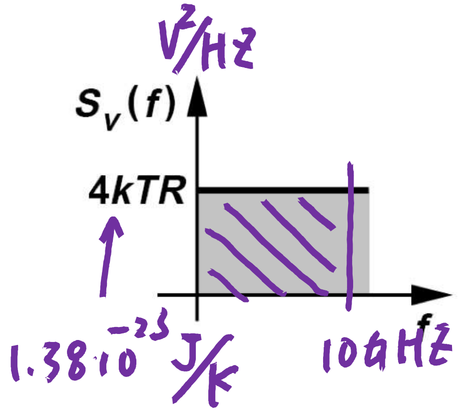


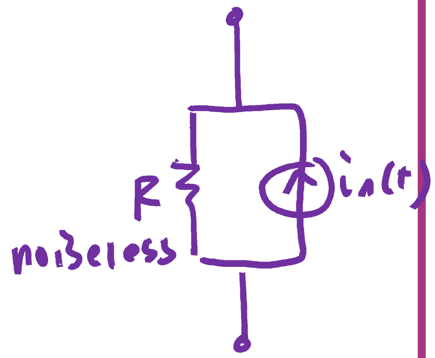
1. Thermal Noise in Resistors

Random movement of charge carriers in a resistor causes fluctuations in the current. The PDF is Gaussian because there are so many carriers. The PSD is given by:









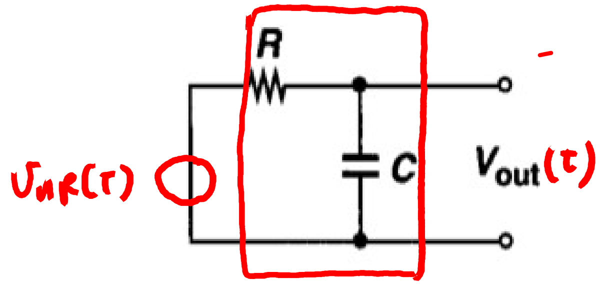
Note that the polarity of the voltage source is arbitrary.

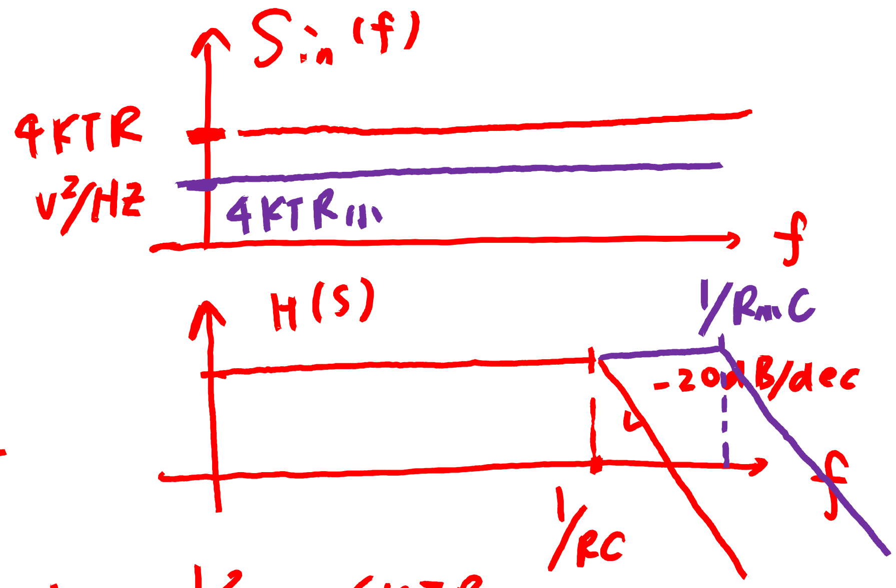
<u>Example:</u> A 50-Ω resistor at room temperature exhibits

$$S_v(f) = 4 \cdot K \cdot T \cdot 50 = (.91\,nV)^2/Hz$$

If the resistor is used in a system with 10-GHz bandwidth, then it contributes a total rms voltage of $(.91\,nV)^2/Hz \cdot 10\,GHz = (91\,\mu V_{rms})^2$

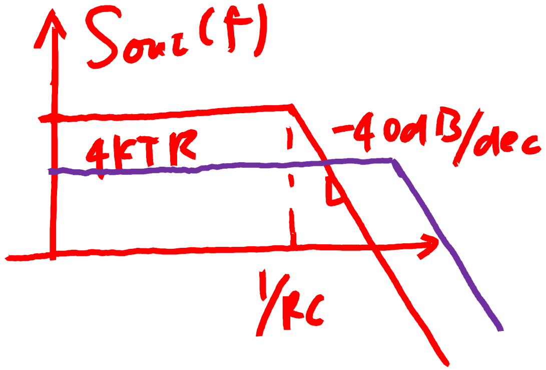
Example: Noise Spectrum and Total Noise Power





$$\frac{V_{out}}{V_R}(s) = \frac{1}{1+sRC}$$

$$S_{out}(f) = S_{in}(f)\left|\frac{1}{1+\underset{\uparrow\, j\omega}{s}RC}\right|^2 = \frac{4KTR}{1+(\omega RC)^2}$$



$$P_{n,out} = \int S_{out}(f)\,df = \frac{KT}{C}$$

For R=50 Ω and C=1 pF: $= (6.25\,mV_{rms})^2$

Trade-offs between noise, area, speed, and power