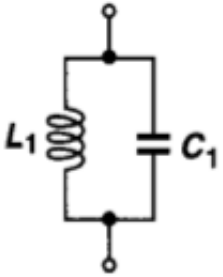
EE4280 Lecture 5:
LC Oscillator

Ping-Hsuan Hsieh (謝秉璇)
Delta Building R908
EXT 42590
phsieh@ee.nthu.edu.tw

Starting from LC Tank …

- **At resonant frequency of** $\omega_{res} = 1/\sqrt{L_1C_1}$
  - The inductor and the capacitor impedance are equal and opposite

- **Ideally without any loss, the impedance goes to infinity → infinite Q**
  - Inductive when $\omega < \omega_{res}$, voltage leads current by 90°
  - Capacitive when $\omega > \omega_{res}$, current leads voltage by 90°

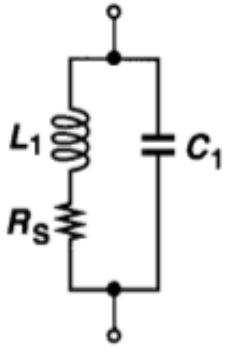


Capacitor and Inductor

Consider Loss in the Tank

◆ **In practice, devices suffer from resistive components**

- Series resistance of the metal wire can be modeled as:



$$Z_{eq}(s) = \frac{R_S + L_1 s}{1 + L_1 C_1 s^2 + R_S C_1 s}$$

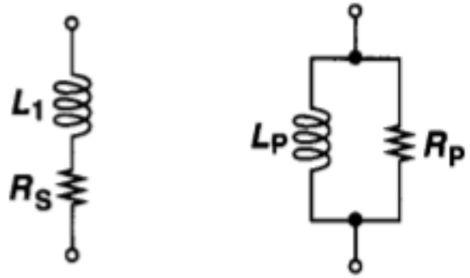
$$|Z_{eq}(s = j\omega)|^2 = \frac{R_S^2 + L_1^2\omega^2}{(1 - L_1 C_1 \omega^2)^2 + R_S^2 C_1^2 \omega^2}$$

Q of the inductor = $\omega L_1 / R_S$

- The impedance does not go to infinity → this circuit has a finite Q

→ The impedance peaked in the vicinity of $\omega = 1/\sqrt{L_1 C_1}$

- For easy analysis and to provide intuition:

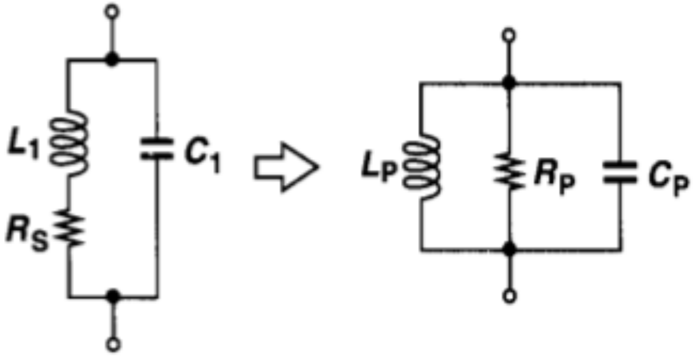


$$L_1 s + R_S = \frac{R_P L_P s}{R_P + L_P s}$$

$$\rightarrow \quad L_P = L_1\left(1 + \frac{R_S^2}{L_1^2\omega^2}\right)$$

RLC Tank

- As the operating frequency is high enough and $Q_L = \frac{\omega L_1}{R_S} >> 1$



$$L_P \approx L_1$$

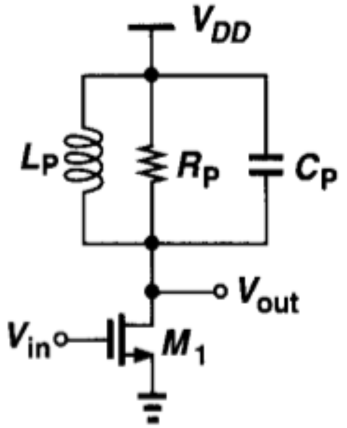
$$R_P \approx \frac{L_1^2\omega^2}{R_S} \approx Q^2 R_S$$

- At $\omega = 1/\sqrt{L_1C_1}$, the tank reduces to a simple resistor

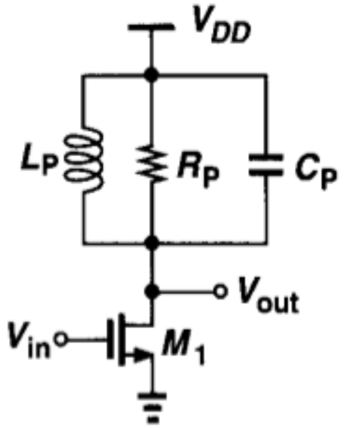
Common-Source Stage with LC Tank Load (I)

◆ **With a single stage**

- At low-frequencies
- At resonant frequency $\frac{\Delta V_{out}}{\Delta V_{in}} = -g_{m1} R_P$

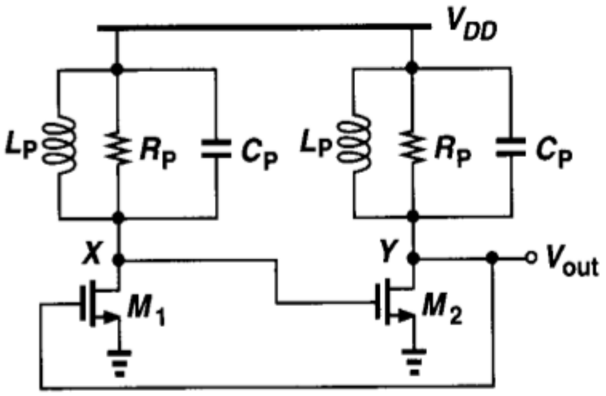


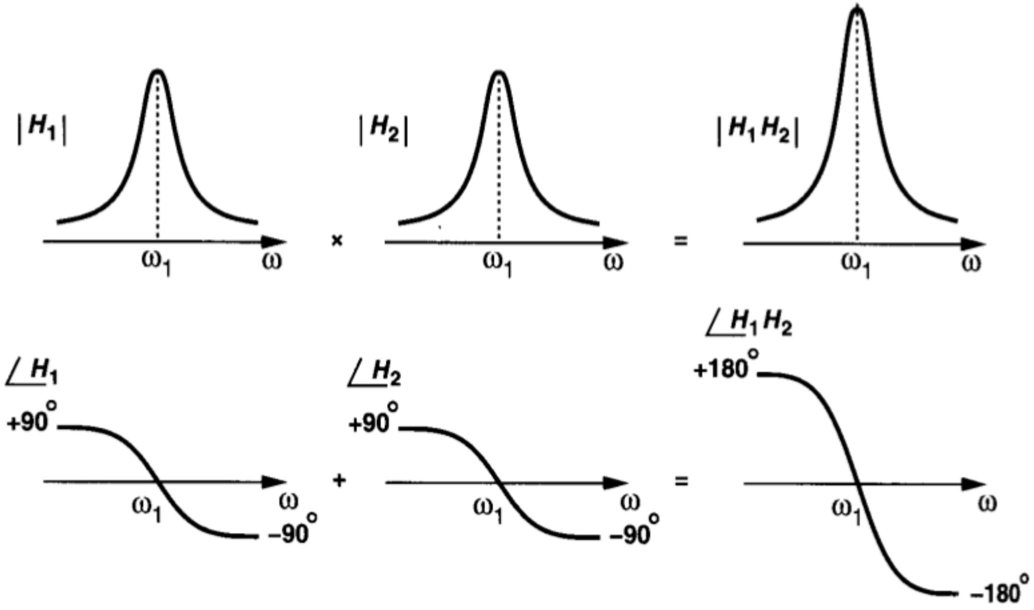
Common-Source Stage with LC Tank Load (II)



Two-Stage CS with LC Tank Load

- **Cross-coupled oscillator**





- **Two frequencies with 360° phase shift**
  - Does not latch up at low-frequencies
  - Provide zero *additional* phase shift at resonant frequency
- **Gain requirement**
  - $g_{m1}R_Pg_{m2}R_P \geq 1$
  - If the inductor dominates the quality factor

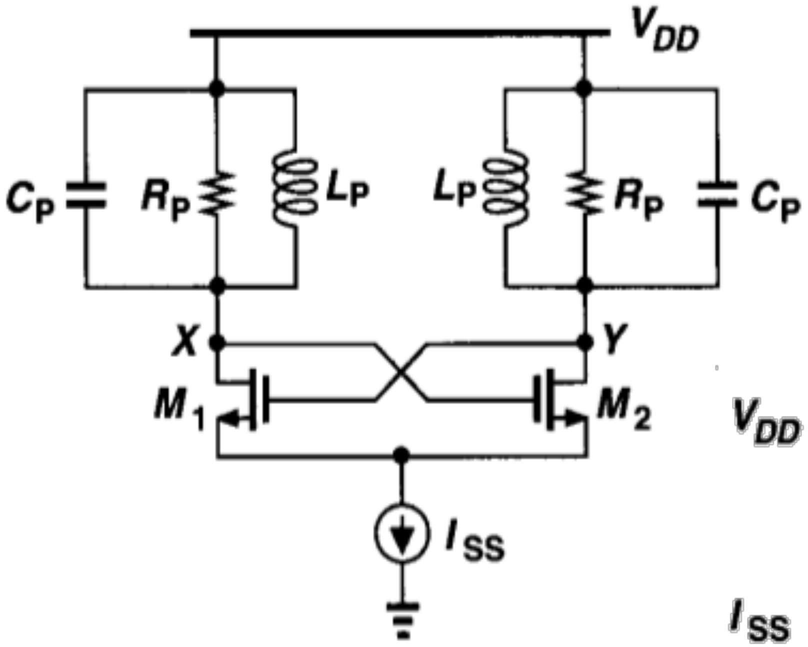
$$R_P \approx \frac{L_1^2\omega^2}{R_S} = \frac{L_1}{C_1R_S} \text{ at } \omega = \frac{1}{\sqrt{L_1C_1}}$$

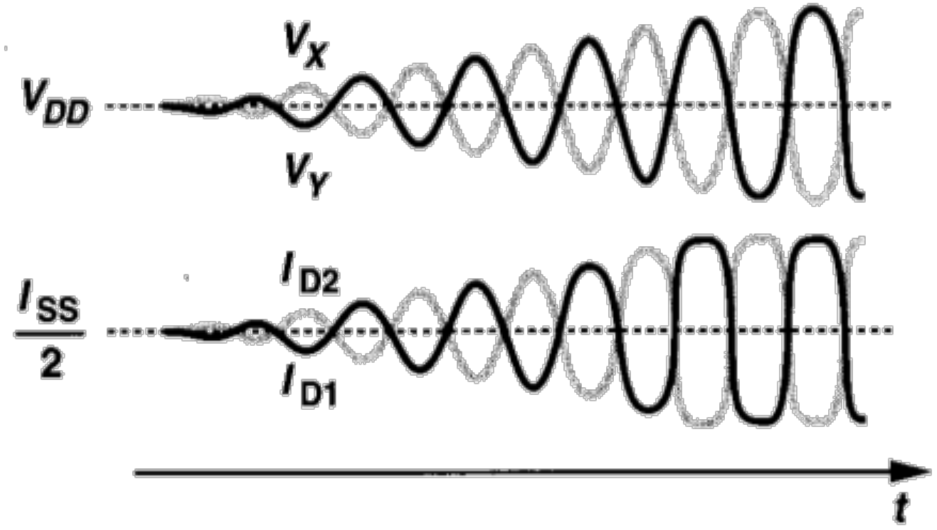
→ larger inductor is preferred to save power

→ The drain current (bias condition) and output swing depend on V_{DD}

Cross-Coupled Oscillator (I)

- With tail current I_{SS}

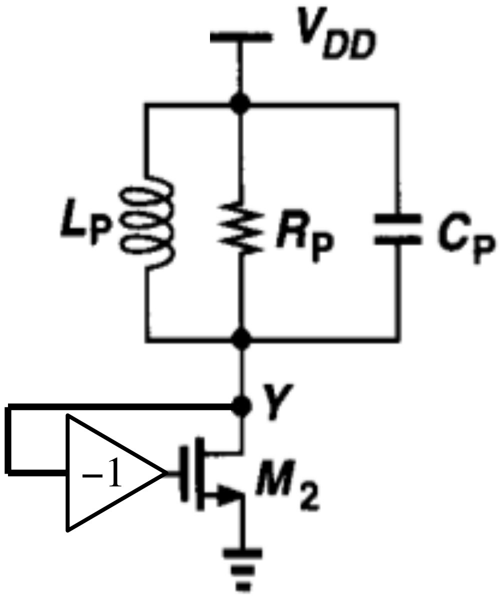




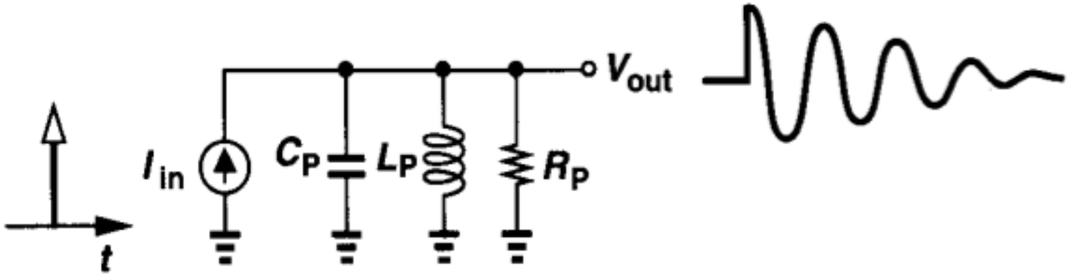
- Common mode at ~V_{DD}
- I_{SS} and R_P determine oscillation amplitude

Negative Resistance (I)

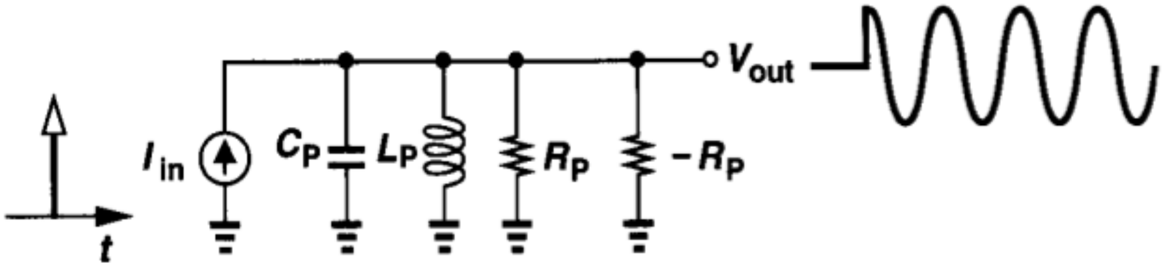
- **Half-circuit of the cross-coupled oscillator**



- Without bottom part of the circuit – only the RLC tank (lossy tank)



- Oscillation can be sustained with negative resistance



Negative Resistance (II)

- **The negative resistance has to be strong enough to sustain oscillation**

- **A positive feedback structure may create negative resistance**

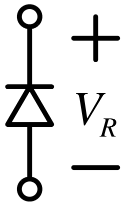
LC Voltage-Controlled Oscillator

◆ **The resonant frequency**

- Little dependence on bias current and transistor transconductance

→ Voltage-controlled capacitor → varactor

- For example: a reversed-biased *pn* junction



$$C_{var} = \frac{C_0}{(1 + \frac{V_R}{\phi_B})^m}$$

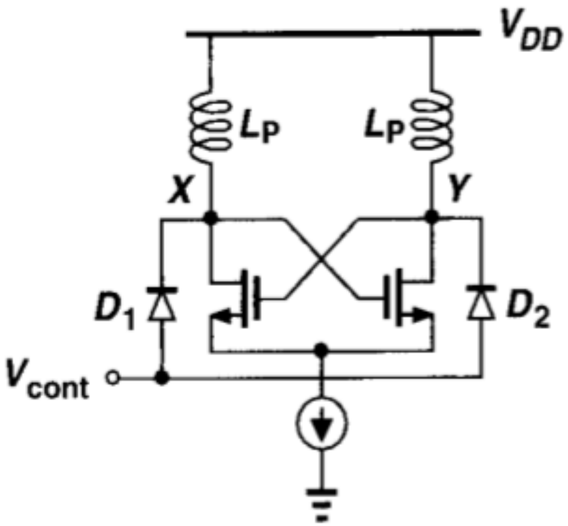
- Limited range of V_R results in limited capacitance range.
- Furthermore, to increase the operating frequency, C_0 is minimized.

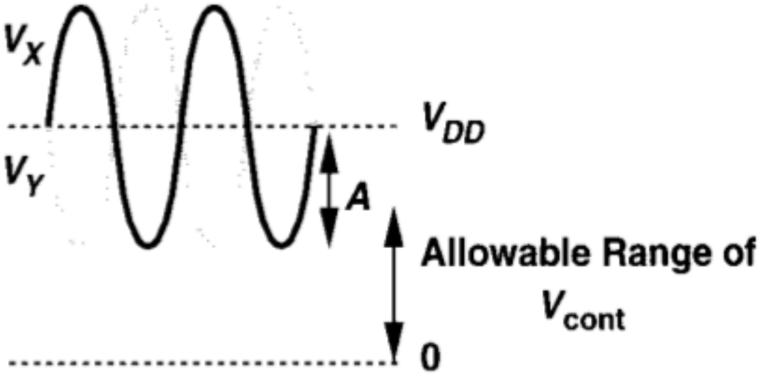
→ Trade-off between operating frequency and tuning range

Adding Varactors to Cross-Coupled Oscillator

- **To avoid forward biasing the two diodes**

→ Trade-off between signal swing and tuning range





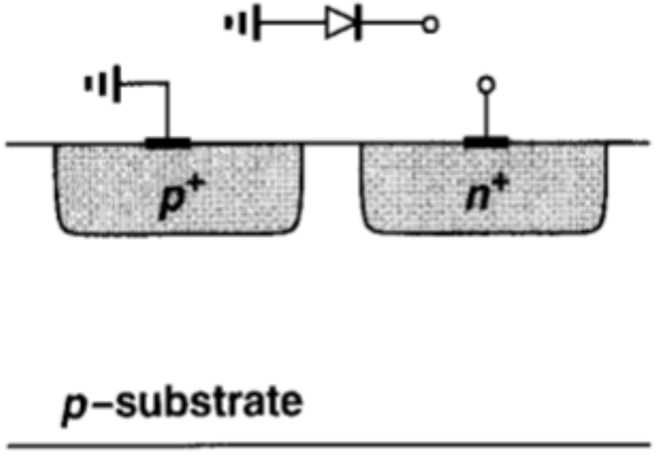
- **Capacitance depends on signal level and varies over time**

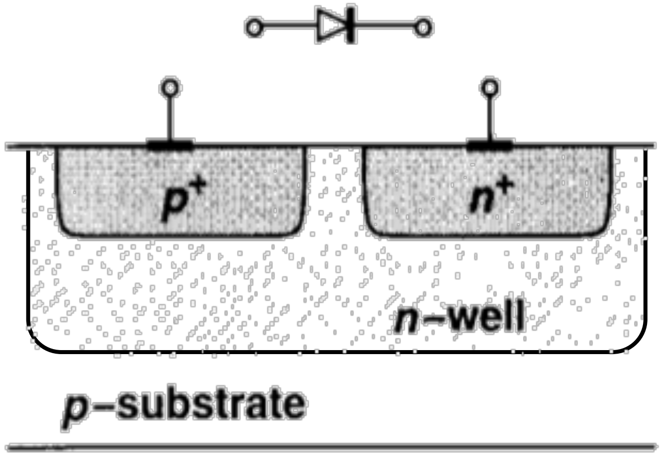
→ Average value (depending on V_{cont}) determines operating frequency

→ Oscillation waveform is distorted slightly

Varactor Diode in CMOS Technology

◆ **PN junction**





- Anode has to be grounded

→ Not tunable

- High resistivity in n-well
- High capacitance between n-well and ground

→ Fixed capacitance on signal nodes

→ Degrading tuning range