EE4280 Lecture 3:
Oscillator

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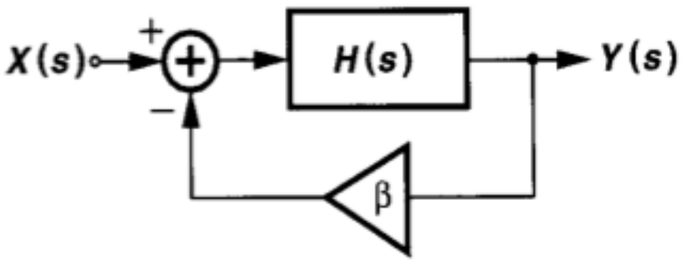
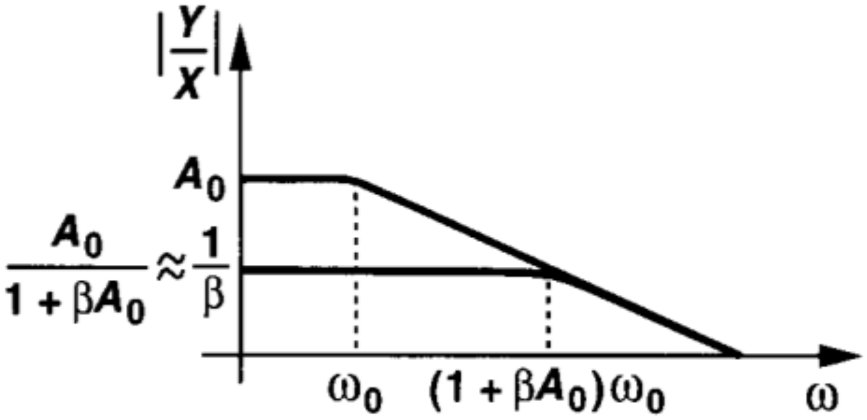
Negative Feedback

Closed-loop transfer function vs. open-loop transfer function

- Gain desensitization
- Bandwidth extension



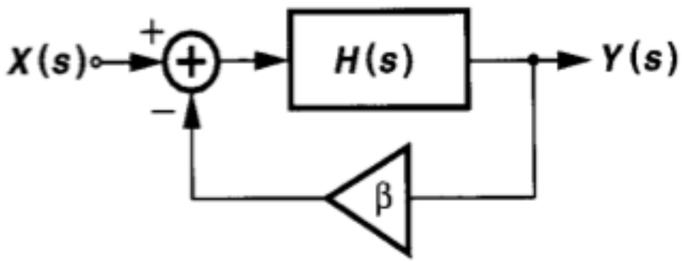
$$\frac{Y}{X}(s) = \frac{H(s)}{1+\beta H(s)}$$



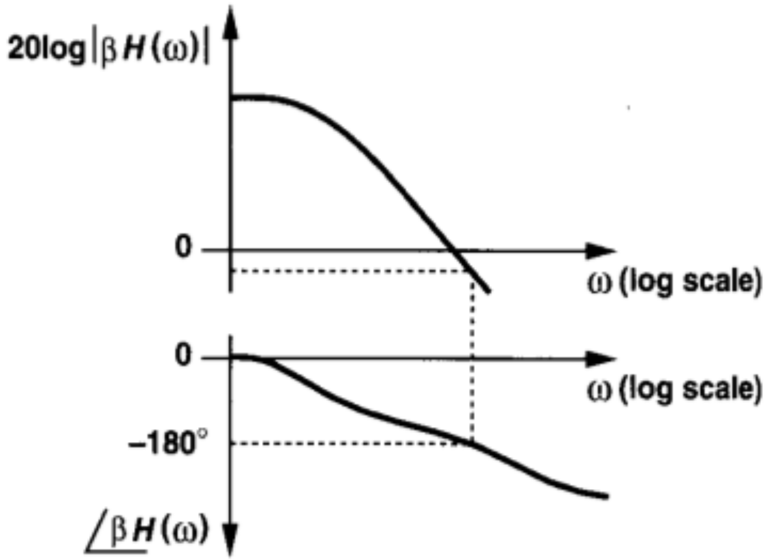
Stability in Negative Feedback

As the operating frequency increases

- The open-loop gain decreases
- The open-loop phase shift increases



$$\frac{Y}{X}(s) = \frac{H(s)}{1 + \beta H(s)}$$

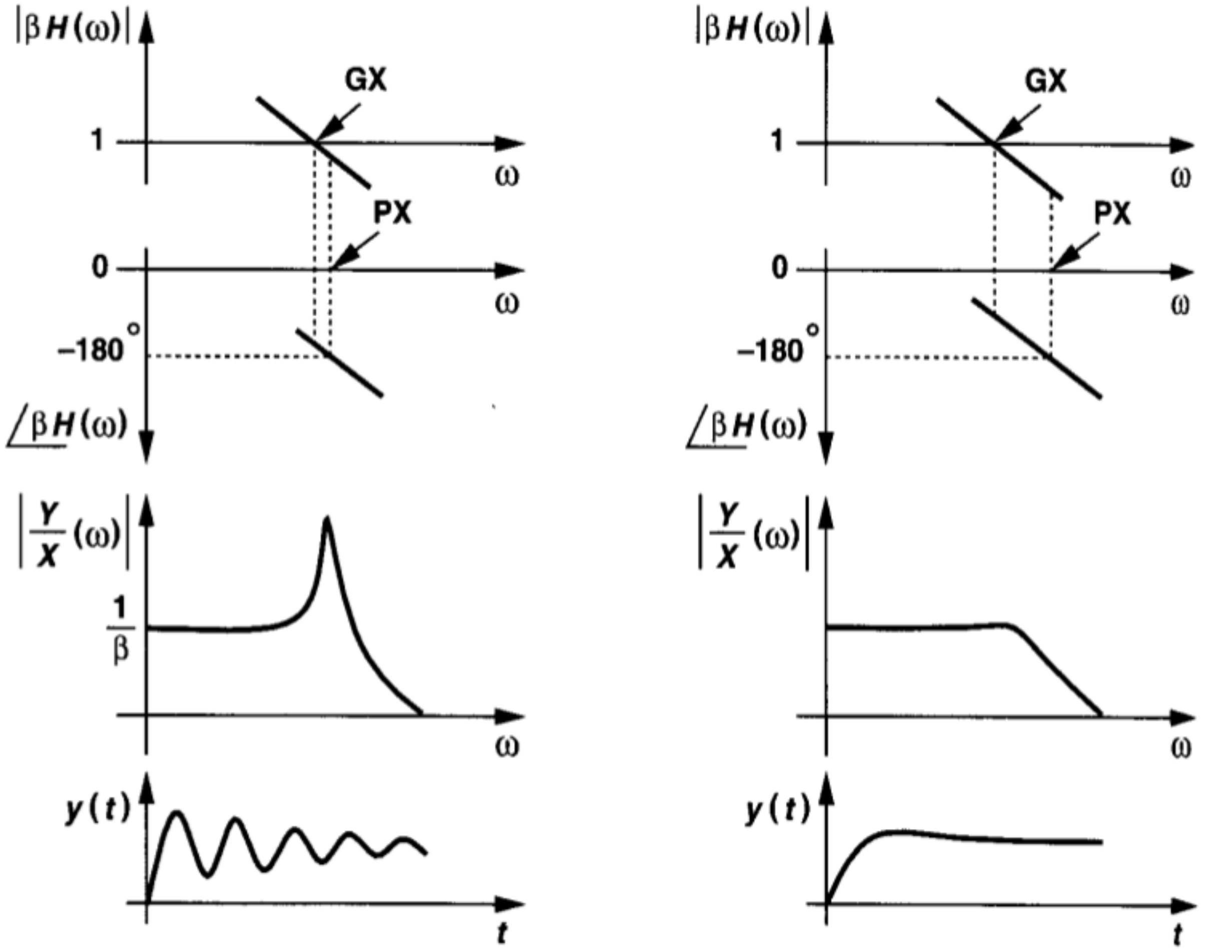


In fact, we care about the frequencies at which

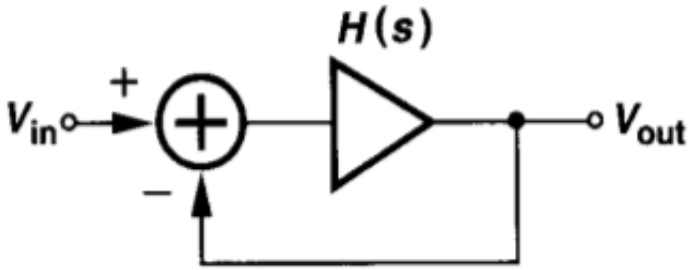
- The open-loop gain drops to 0 dB
- The open-loop phase delay is 180°

Gain Margin and Phase Margin

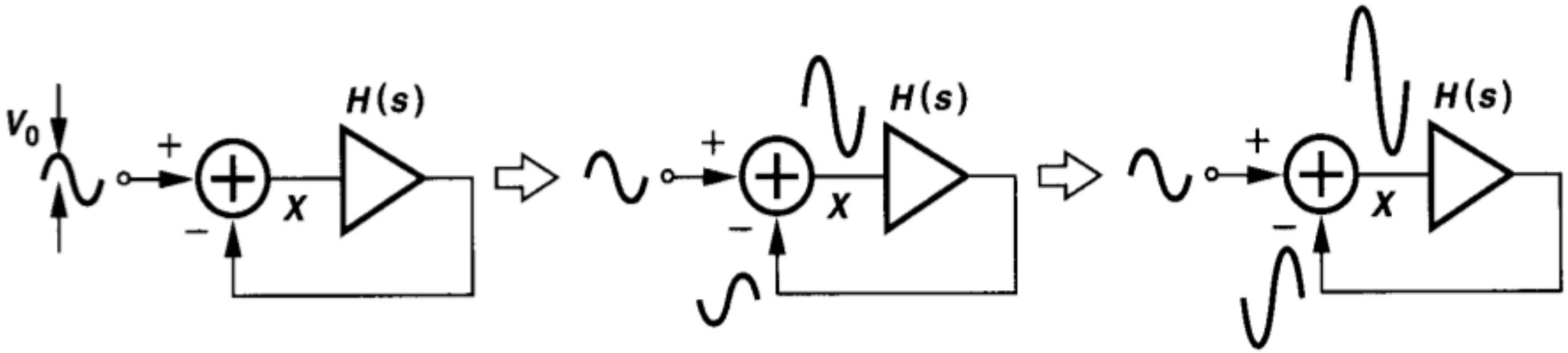
- **Insufficient phase margin results in peaking in closed-loop transfer function and ringing in time-domain step response**



What happens with zero phase margin?



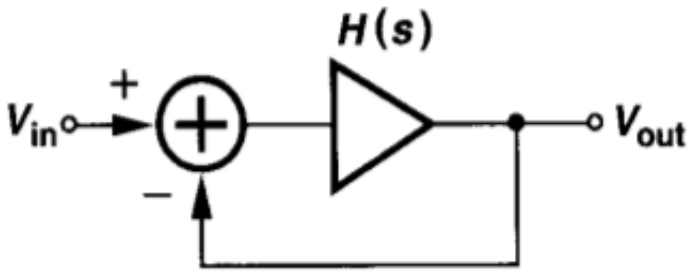
- **In other words, if when** $s = j\omega_0$, $H(j\omega_0) = -1$



- The closed-loop gain approaches infinity at ω_0
- The circuit generates an output signal without input – autonomous

Oscillator – Barkhausen Criteria

- **For an open-loop transfer function that satisfies two conditions:**



$$|H(j\omega_0)| \geq 1$$

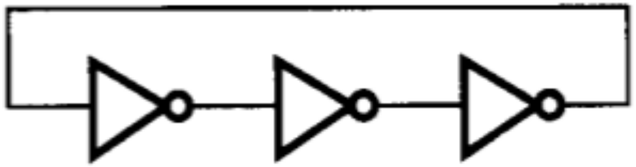
$$\angle H(j\omega_0) = 180^\circ$$

- **The circuit may oscillate at ω_0**
  - These conditions are necessary but not sufficient
  - In order to ensure oscillation in the presence of PVT variations, we typically choose the loop gain to be at least twice or three times the required value

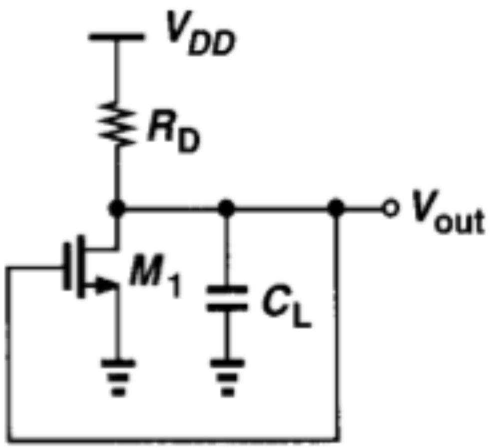
  - Negative feedback at low frequency
  - Total phase shift of 360° at ω_0 → positive feedback at ω_0
    → additional *frequency-dependent* phase delay that is 180° at ω_0

Ring Oscillator (I)

- **A number of gain stages in a loop – a ring**

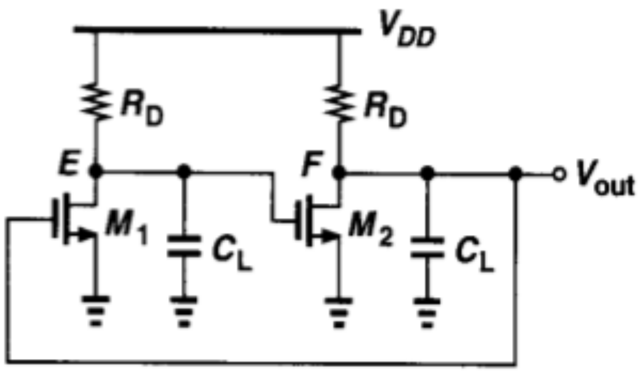


- **Starting from a single-stage of common-source amplifier**



Ring Oscillator (II)

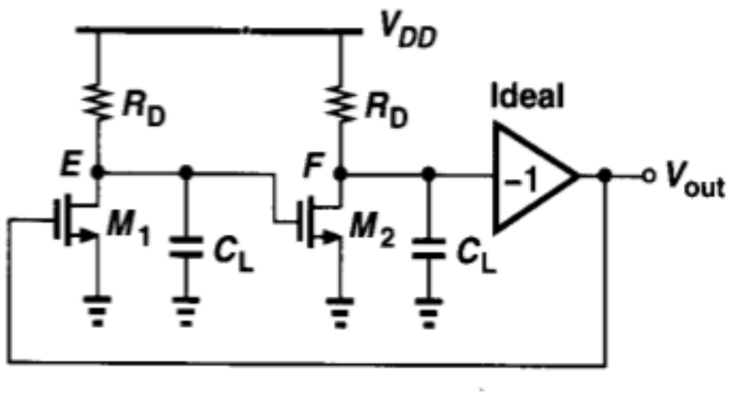
- **Two stages of common-source amplifiers**



- Positive feedback near zero frequency
  → The circuit latches up rather than oscillates
  → We would like to have negative feedback at low frequencies

Ring Oscillator (III)

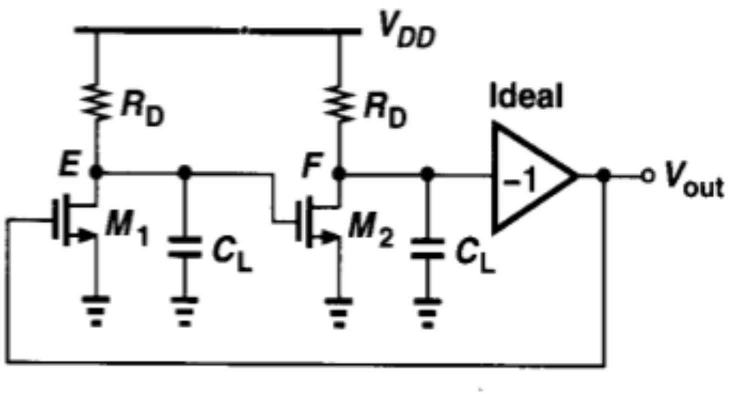
- **Inserting an ideal inverter (with zero phase shift at all frequencies)**

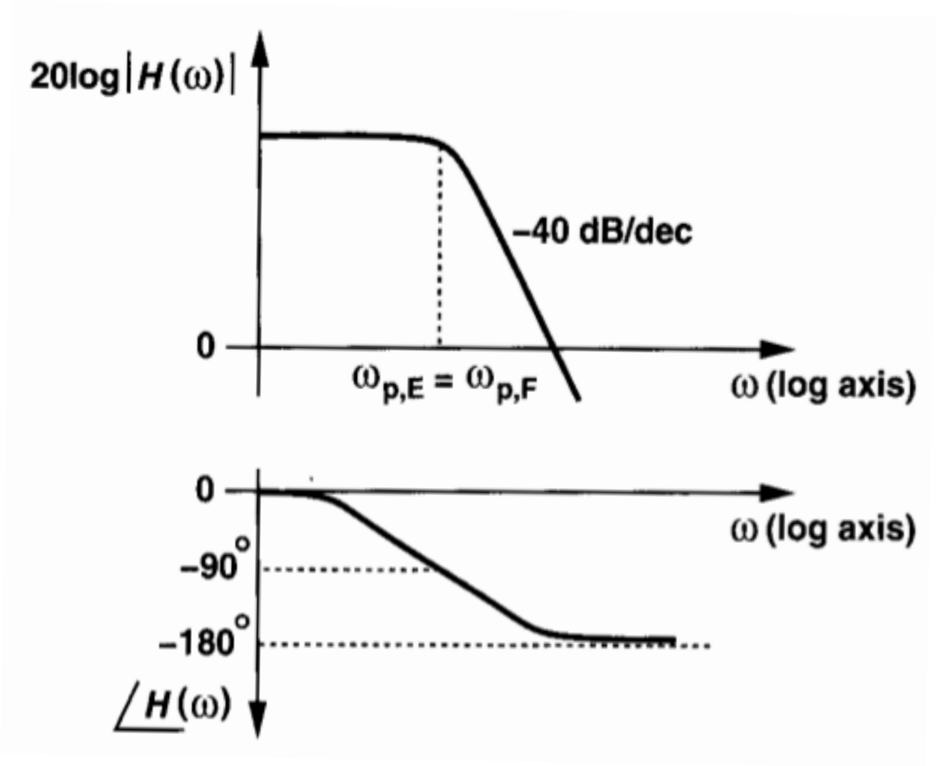


→ Greater phase shift around the loop is required

Ring Oscillator (III)

- **Inserting an ideal inverter (with zero phase shift at all frequencies)**

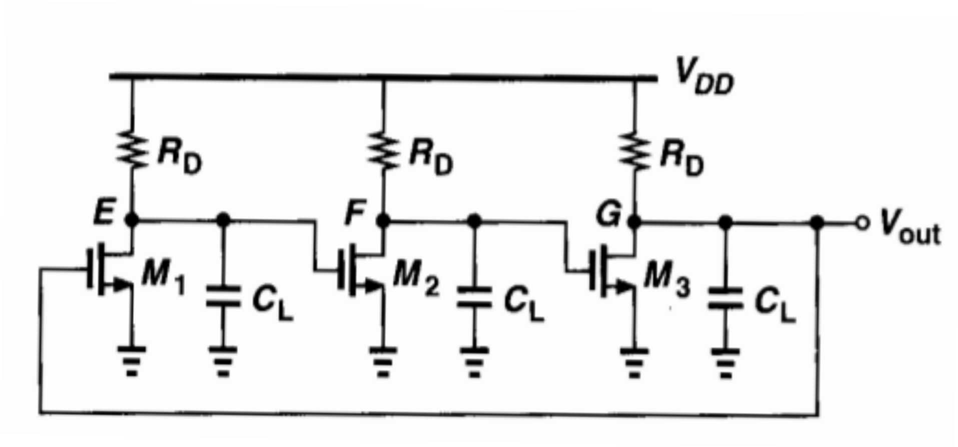




→ Greater phase shift around the loop is required

3-Stage Ring Oscillator (I)

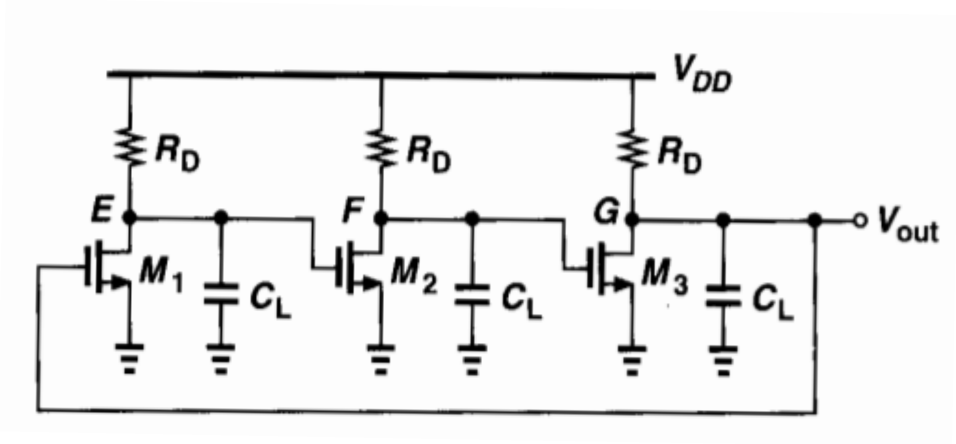
- **Negative feedback at low frequencies**
- **At** $\omega = \omega_{p,E} \left(= \omega_{p,F} = \omega_{p,G} \right)$
- **At** $\omega = \infty$



$H(s) =$

3-Stage Ring Oscillator (II)

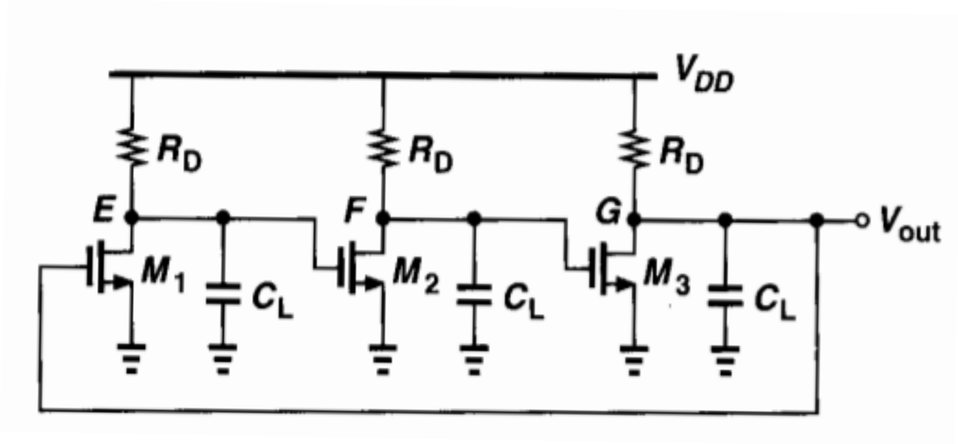
- **Gain requirement to meet Barkhausen Criteria**



$$\frac{A_0^3}{\left[\sqrt{1+(\frac{\omega_{osc}}{\omega_0})^2}\right]^3} = 1$$

3-Stage Ring Oscillator (II)

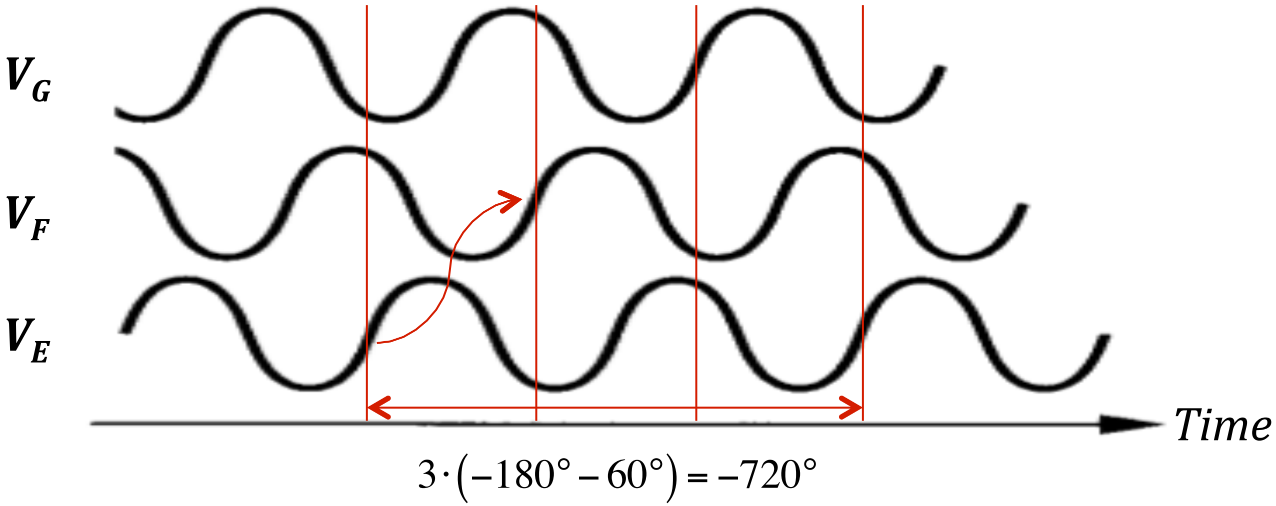
- **Gain requirement to meet Barkhausen Criteria**



$$\frac{A_0^3}{\left[\sqrt{1+(\frac{\omega_{osc}}{\omega_0})^2}\right]^3} = 1$$

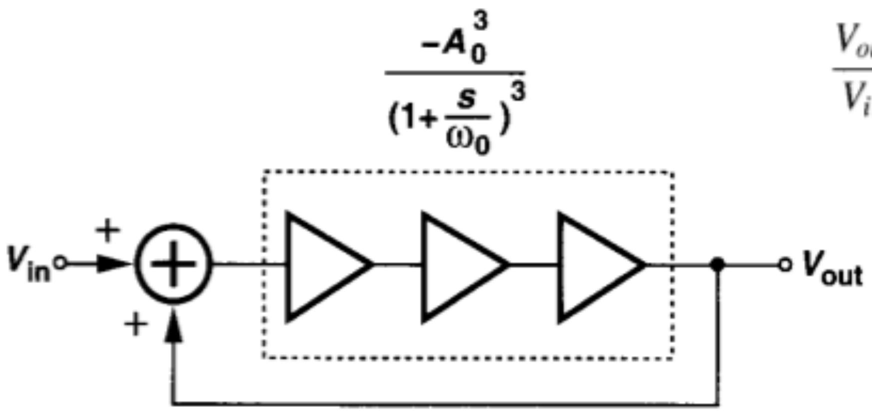
$$A_0 = 2$$

- **Waveforms at node E, F, and G**



3-Stage Ring Oscillator (III)

- **Closed-loop transfer function (based on linear model)**



$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{\frac{-A_0^3}{(1+s/\omega_0)^3}}{1+\frac{A_0^3}{(1+s/\omega_0)^3}} = \frac{-A_0^3}{(1+s/\omega_0)^3 + A_0^3}$$

$$= \frac{-A_0^3}{(1+\frac{s}{\omega_0}+A_0)\left[(1+\frac{s}{\omega_0})^2 - (1+\frac{s}{\omega_0})A_0 + A_0^2\right]}$$

$$s_1 = (-A_0 - 1)\omega_0$$

$$s_{2,3} = [\frac{A_0(1 \pm j\sqrt{3})}{2} - 1]\omega_0.$$

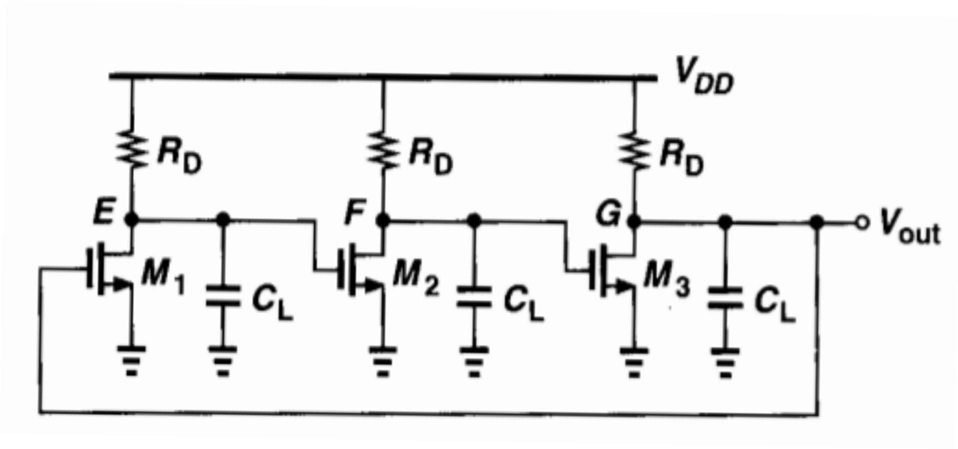
$$V_{out}(t) = a\exp(\frac{A_0-2}{2}\omega_0 t)\cos(\frac{A_0\sqrt{3}}{2}\omega_0 t) + b\exp\left((-A_0-1)\omega_0 t\right)$$

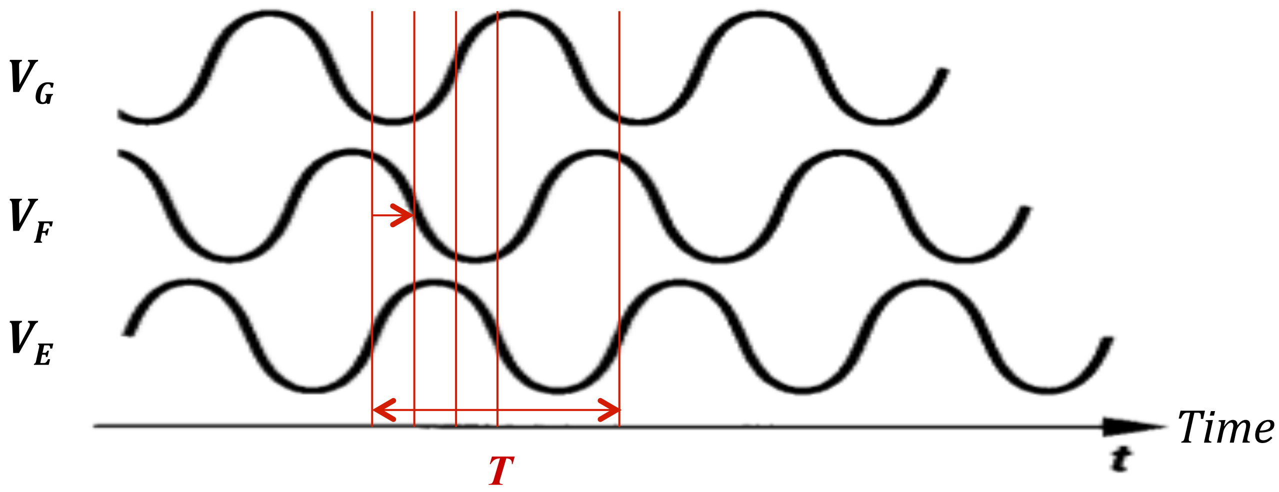
- A three-order system → right-half plane poles with $A_0 > 2$

→ The exponential envelope grows to infinity

3-Stage Ring Oscillator (IV)

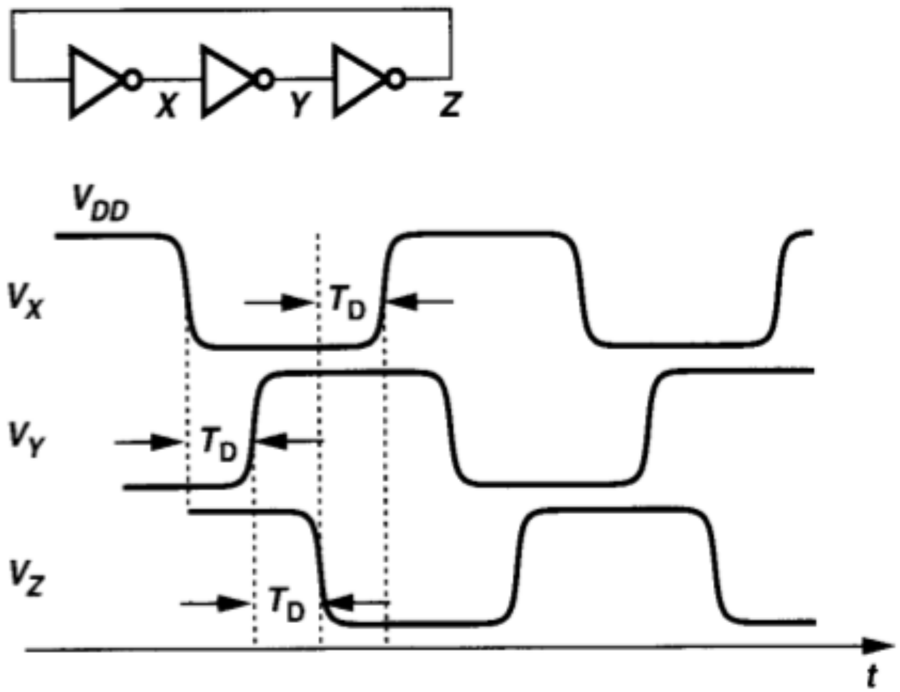
- **3 identical delay stages → oscillation period of** $\dfrac{4\pi}{\sqrt{3}\cdot A_0\omega_0}$
  - From small-signal linear analysis near the bias point

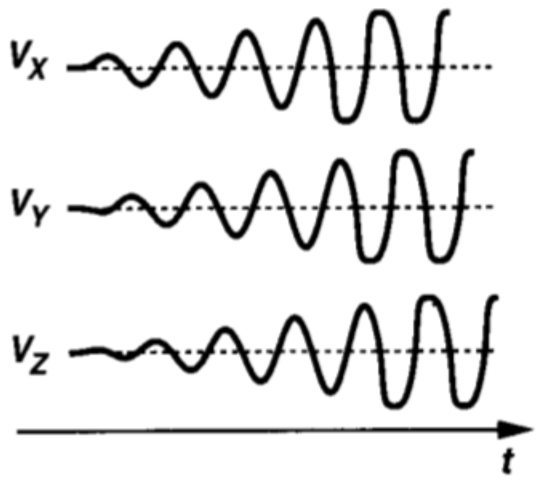




3-Stage Ring Oscillator (V)

- **3 identical delay stages → oscillation period of** $6 \cdot T_D$
- From the large-signal, nonlinear current driving load capacitances

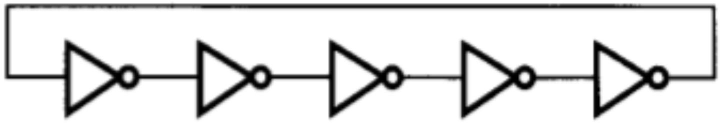




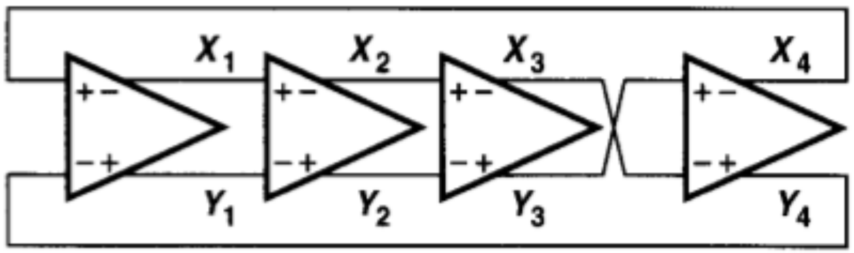
- When the circuit is released with all the inverters at the trip point, the oscillation begins with a frequency of , but as the amplitude grows and the circuit becomes nonlinear, the frequency shifts to

Ring Oscillator

- **Odd-number of stages**



- **Even-number of stages**



- Speed, power, noise immunity, etc.