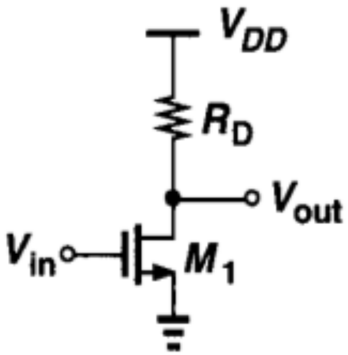
EE4280 Lecture 1:
Nonlinearity

Ping-Hsuan Hsieh (謝秉璇)
Delta Building R908
EXT 42590
phsieh@ee.nthu.edu.tw

Nonlinearity

Nonlinear characteristic deviates from a straight line as the input swing increases



Nonlinearity

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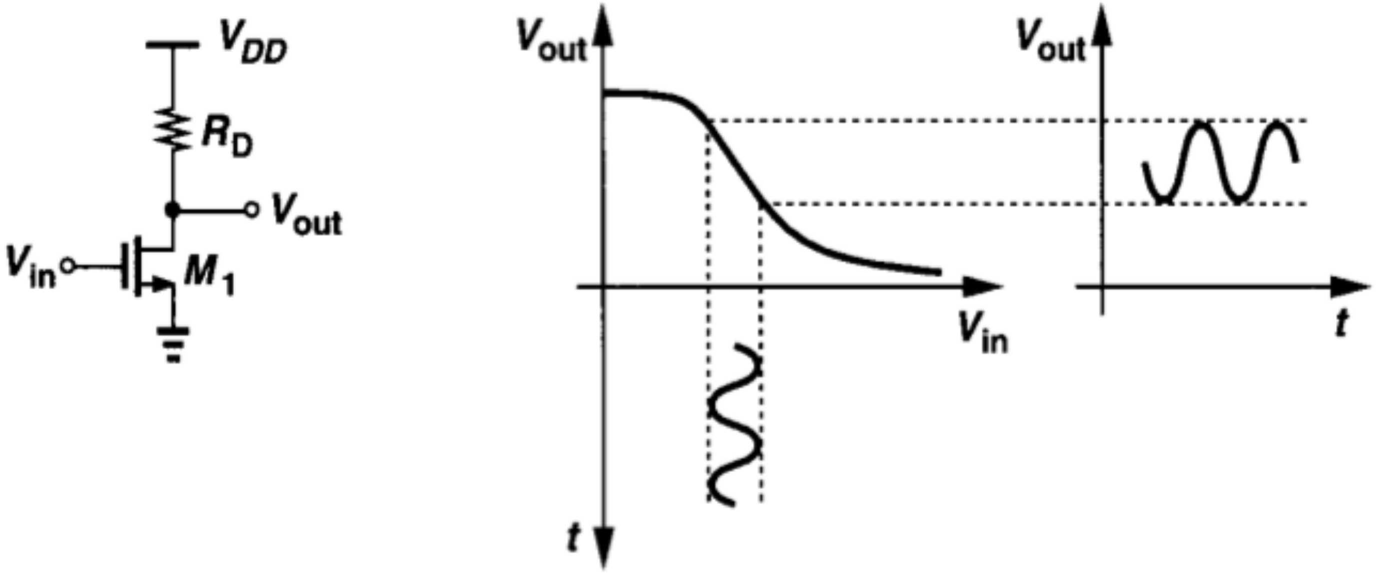
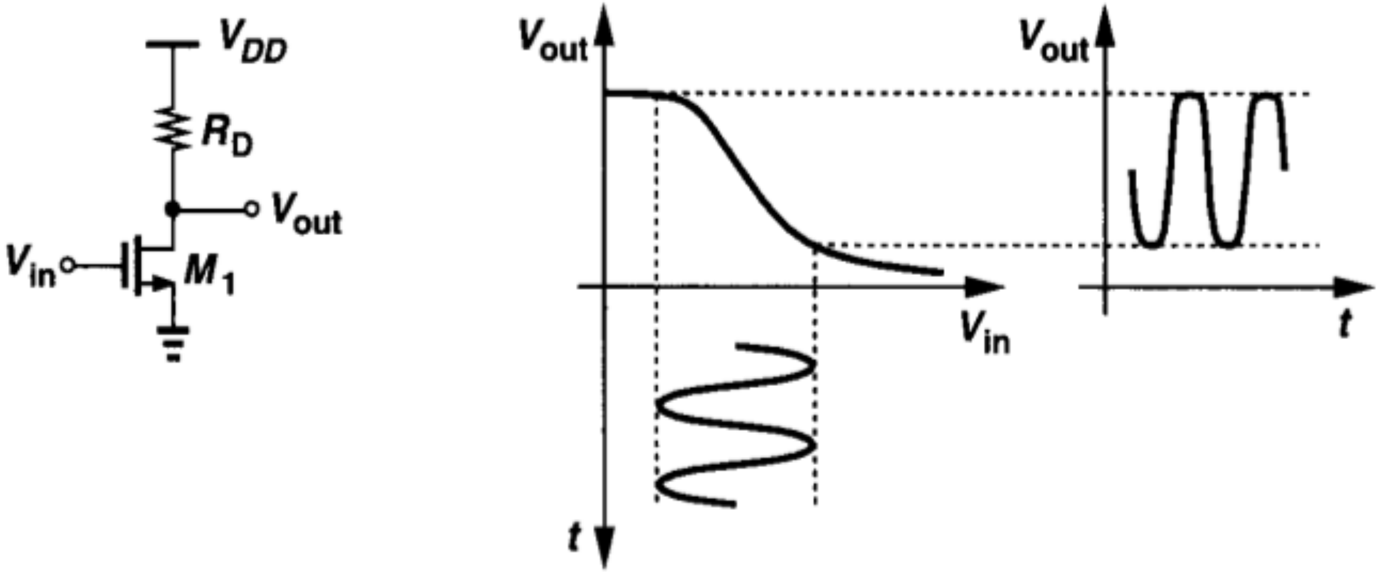


- **For small input swing, the output is a reasonable replica of the input**
  - Small-signal gain is related to the slope at a given bias point

Nonlinearity

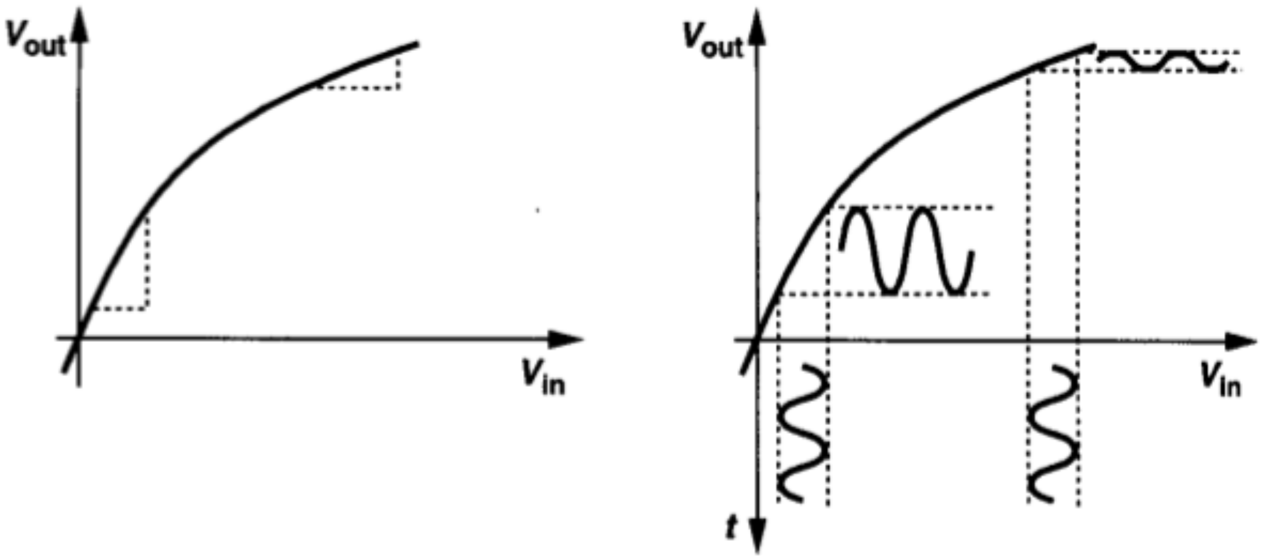
Nonlinear characteristic deviates from a straight line as the input swing increases



- **For small input swing, the output is a reasonable replica of the input**
- **For large input swings, most amplifiers experience gain compression (instead of expansion)**
  - The output exhibits "saturated" levels due to supply voltage or bias current

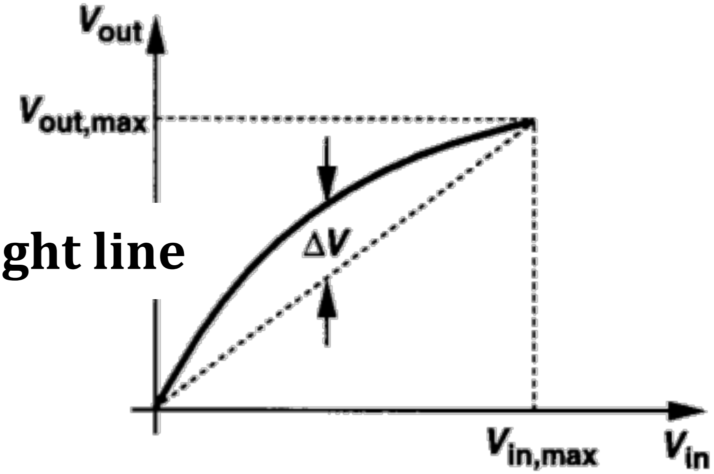
Nonlinearity

Can be viewed as variation of the slope (small-signal gain) with the input level (common-mode)



To Quantify Nonlinearity (I)

- **Taylor Expansion**



- **Maximum deviation from an ideal straight line**

  $\Delta V / V_{out,\max}$ **for certain** $V_{in,\max}$

To Quantify Nonlinearity (II)

- **A single tone test**

$$\Delta x = A\cos\omega_1 t$$

$$\Delta y = \alpha_1 A\cos\omega_1 t + \alpha_2 A^2\cos^2\omega_1 t + \alpha_3 A^3\cos^3\omega_1 t + \ldots$$

- **Total harmonic distortion**

$$THD =$$

To Quantify Nonlinearity (III)

- **1-dB compression point**

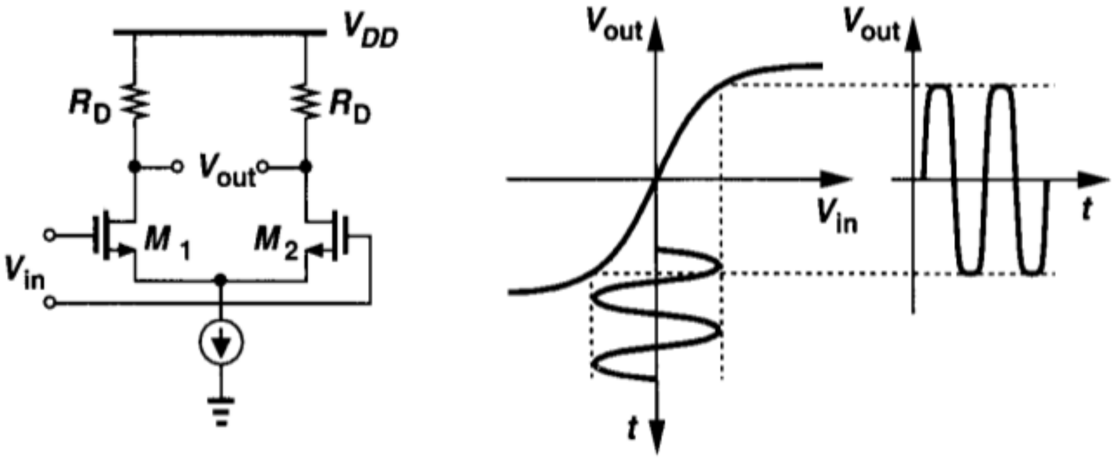
The signal at ω_1

$$\Delta y = \left(\alpha_1 A + \frac{3}{4} \alpha_3 A^3 \right) \cos \omega_1 t$$

The input level where the gain has dropped by 1dB

Nonlinearity of Differential Circuits (I)

- **Differential circuits exhibit an “odd-symmetric” input/output characteristics, i.e.,** $f(-x) = -f(x)$

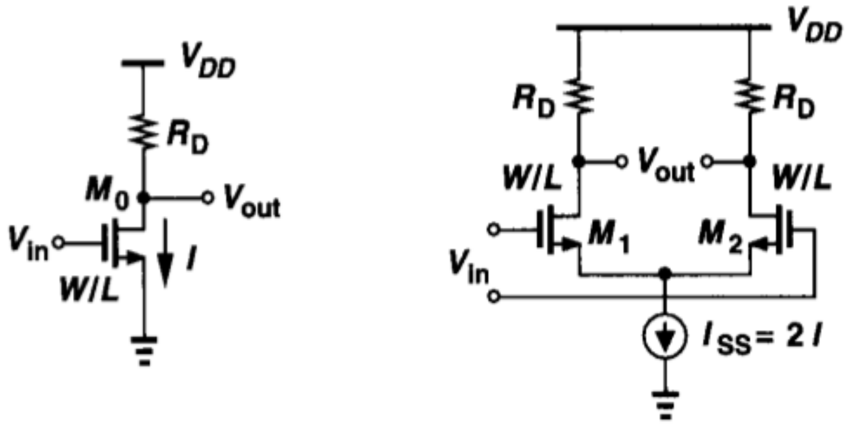


- **For the Taylor expansion to be an odd function, all the even-order terms musts be zero.**

$$\Delta y = \alpha_1 \Delta x + \alpha_3 (\Delta x)^3 + \alpha_5 (\Delta x)^5 + \ldots$$

- **A differential circuit produces no even-order harmonics**

Nonlinearity in Differential Circuits (II)



- **Single-ended and differential amplifiers with the same voltage gain**

$$|A_v| \approx g_m R_D$$
$$= \mu_n C_{ox} \frac{W}{L}(V_{GS} - V_{TH}) R_D$$

- **For the single-ended case:**

$$V_{DD} - V_{out} = I_D \cdot R_D$$

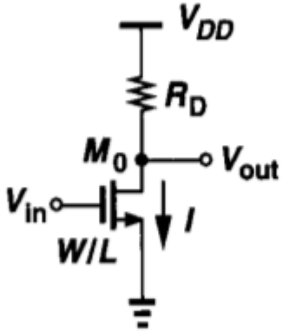
With $V_{in} = V_{GS} + V_m \cos\omega t$

We have $V_{DD} - V_{out} = \frac{1}{2}\mu_n C_{ox} \frac{W}{L}\left(V_{GS} + V_m \cos\omega t - V_{th}\right)^2 R_D$

Nonlinearity for Differential Circuits (III)

- **For the single-ended case:**

$$V_{DD} - V_{out} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{GS} + V_m \cos\omega t - V_{th}\right)^2 R_D$$

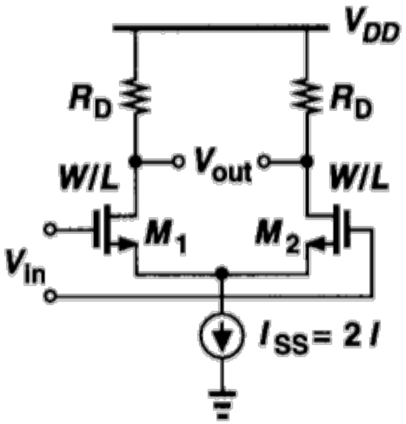


- **The second harmonic distortion:**

$$\frac{A_{HD2}}{A_F} =$$

Nonlinearity for Differential Circuits (IV)

- **For the differential amplifier:**



$$\Delta V_{out} = \left(I_{D1} - I_{D2}\right) R_D$$

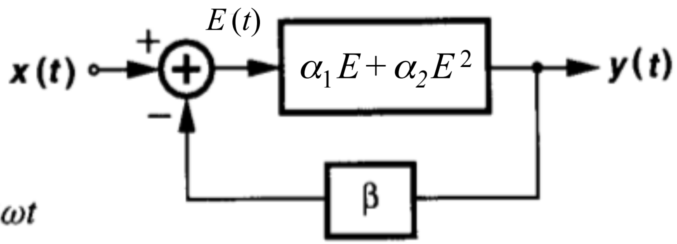
From Chapter 4: $$= \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\Delta V_{in}\sqrt{4\left(V_{GS} - V_{th}\right)^2 - \Delta V_{in}^2}$$

- **The third harmonic distortion:**

$$\frac{A_{HD3}}{A_F} \approx$$

Effect of Negative Feedback on Nonlinearity (I)

- **Negative feedback makes the closed-loop gain relatively independent of the op amp's open-loop gain → Gain Desensitization**
- **Nonlinearity can be viewed as small-signal gain variation with input level → suppressed by negative feedback as well**
- **Consider an open-loop gain of**



With $x(t) = V_m \cos\omega t$

and if the output can be approximated as $y \approx a\cos\omega t + b\cos 2\omega t$

$E(t) =$

$y(t) =$

Effect of Negative Feedback on Nonlinearity (II)

With $$\begin{cases} a = (\alpha_1 - \alpha_2 \beta b)(V_m - \beta a) \\ b = -\alpha_1 \beta b + \dfrac{\alpha_2 (V_m - \beta a)^2}{2} \end{cases}$$

- **The second harmonic distortion:** $\frac{A_{HD2}}{A_F} = \frac{b}{a} =$
- **Compared to the open-loop case:**

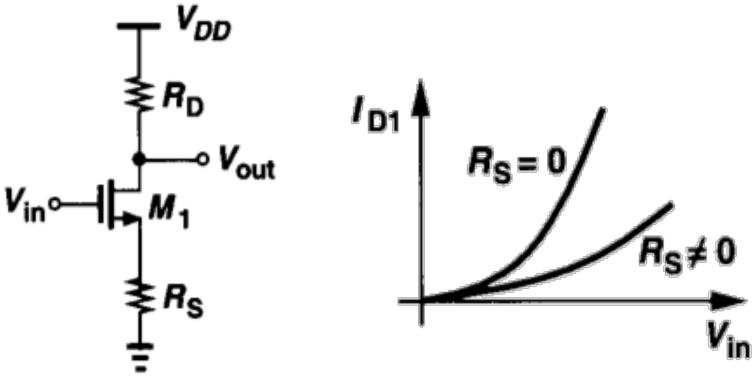
With the same input swing:

With the same output swing:

Linearization Technique (I)

- **To reduce the dependence of gain on input level**
  → To reduce the dependence of gain on <u>transistor bias current</u>

- **Source degeneration effectively reduce the signal swing on V_{GS}**



$$\frac{\Delta I_D}{\Delta V_{in}} =$$

- **Trade-off between linearity, noise, power dissipation, and gain**

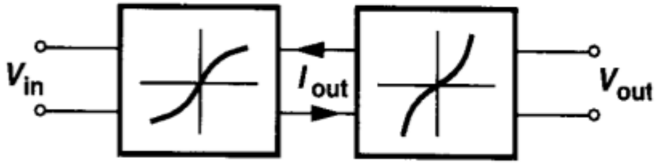
Linearization Technique (II)

- **For an output swing that corresponds to** I_D variation from $0.75I_1$ to $1.25I_1$
  - For common-source without degeneration:
  - For source degeneration of $g_mR_S = 2$ at $I_D = I_1$

Linearization Technique (III)

- **Post correction**
  - A common-source amplifier is in fact a voltage-to-current converter followed by a current-to-voltage converter

$$\Delta V_{in} \rightarrow \Delta I_D \rightarrow \Delta V_{out}$$

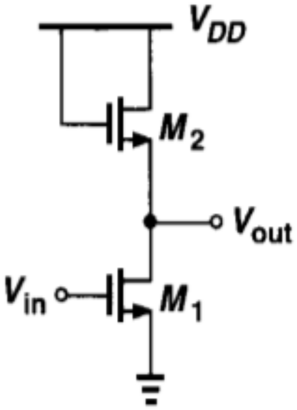


For $\Delta I_D = f\left(\Delta V_{in}\right)$

If we have $\Delta V_{in} = A \cdot f^{-1}\left(\Delta I_D\right)$

Linearization Technique (IV)

- **Common-source with diode-connected load**



- **Some of the design considerations include**
  - Body-effect that degrades the linearity
  - Limited voltage headroom