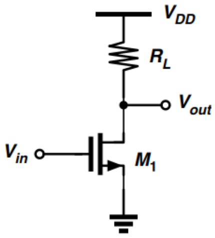
1. Consider a common-source amplifier like the following



a. The small-signal voltage gain with $V_{in} = V_{in0}$ is $\mu_n C_{ox}(W/L)(V_{in0} - V_{TH})(r_{o1}||R_L)$.

b. Input-referred thermal noise voltage?

$S_{v,out} = 4kTg_{m1}(R_L||r_o)^2 \Rightarrow S_{v,in} = 4kTg_{m1}^{-1}$

c. Input signal $V_m \cos\omega t$, what is the amplitudes of the fundamental and the second harmonic at the output?

$V_{DD} - V_{out} = 0.5\mu_n C_{ox}(W/L)(V_{in} - V_{TH})^2 R_L = f(V_{in})$

$f(V_{in}) = f(V_{in0}) + f'(V_{in0})(V_{in} - V_{in0}) + (1/2!)f''(V_{in0})(V_{in} - V_{in0})^2 \ldots$

$= f(V_{in0}) + [\mu_n C_{ox}(W/L)(V_{in0} - V_{TH})R_L][V_m \cos\omega t] + [0.5\mu_n C_{ox}(W/L)R_L][(V_m \cos\omega t)^2] + \cdots$

$\cos^2\omega t = 0.5(1 + \cos 2\omega t)$

⇨ The amplitude of the fundamental harmonic is $\mu_n C_{ox}(W/L)V_{ov}R_L V_m$

⇨ The amplitude of the second harmonic is $0.25\mu_n C_{ox}(W/L)R_L V_m^2$

d. Keep the total power consumption the same. The relationship between the harmonic distortion and the size of $M1$.

Keeping the power consumption same means that drain current remains the same. $I_D = 0.5\mu_n C_{ox}(W/L)V_{ov}$, $\frac{A_{HD2}}{A_F} = \frac{V_m}{4(V_{in0} - V_{TH})}$

$\Rightarrow$ If we increase the width of $M1$, the overdrive voltage will be decreased, then the ratio of harmonic distortion will be increased at the same time.

e. The relationship between the input-referred thermal noise voltage and the size of $M1$.

$S_{v,out} = 4kT(\gamma/g_{m1}^{-1} + 1/R_L)R_L^2, S_{v,out} = S_{v,in}A_v^2, A_v = g_{m1}R_L$

$S_{v,in} = 4kT\left(\frac{\gamma}{g_{m1}} + \frac{1}{g_{m1}^2 R_L}\right), g_{m1} = \mu_n C_{ox}(W/L)V_{ov}$ $\Rightarrow$ the input-referred thermal noise voltage would decrease with increasing the width of $M1$.

f. (Hspice) $V_{DD} = 1.8V, V_{in,DC} = 0.9V, V_{out,DC} = 0.9V$, and $I_{DC} = 1mA$

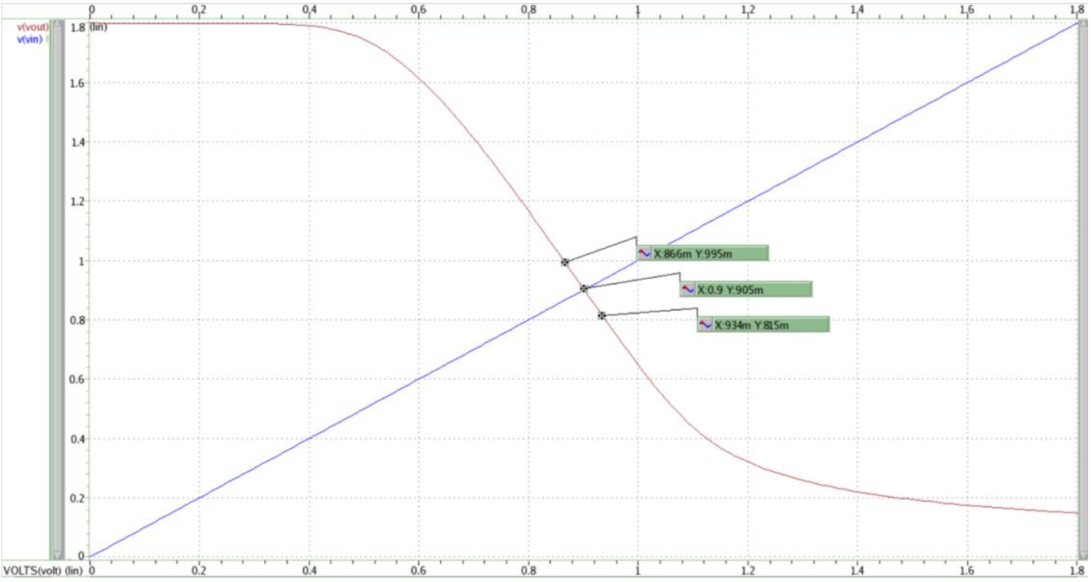
```
+0:vdd     =   1.8000  0:vin     = 900.0000m 0:vout    = 904.7484m
```

```
element  0:m1
model    0:n_18.1
region     Saturati
 id          1.0290m
```

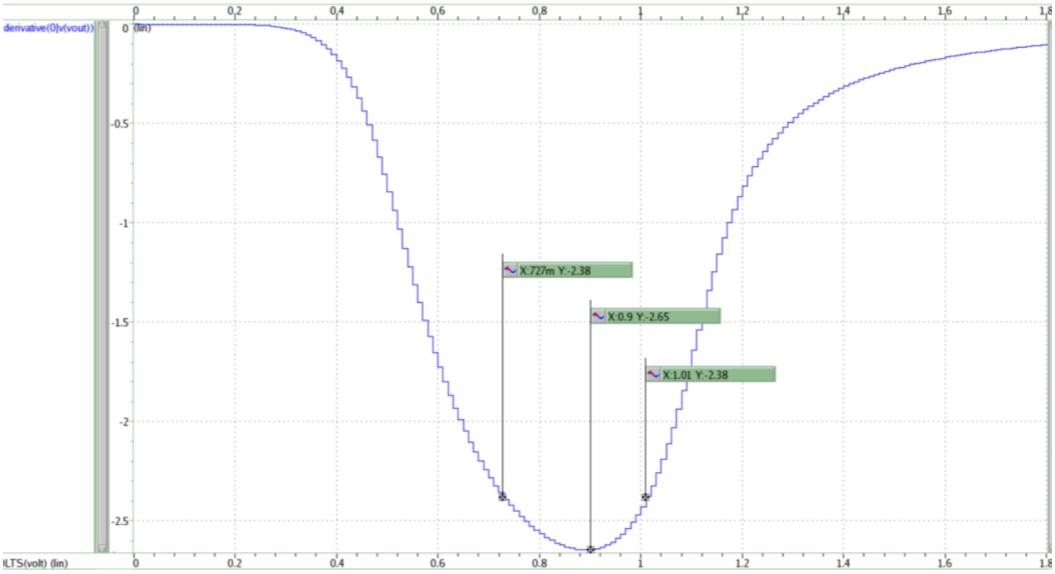
$V_{DD} = 1.8V, V_{in,DC} = 0.9V, V_{out,DC} = 0.904V$, and $I_{DC} = 1.03mA$

g. With minimum channel length ($L = 0.18\mu m$), the overdrive voltage of M1 is 410mV, and the small-signal gain is -2.65.

h. With dc analysis, the plot of the transfer function from V_{in} to V_{out} by changing V_{in} from 0V to 1.8V is



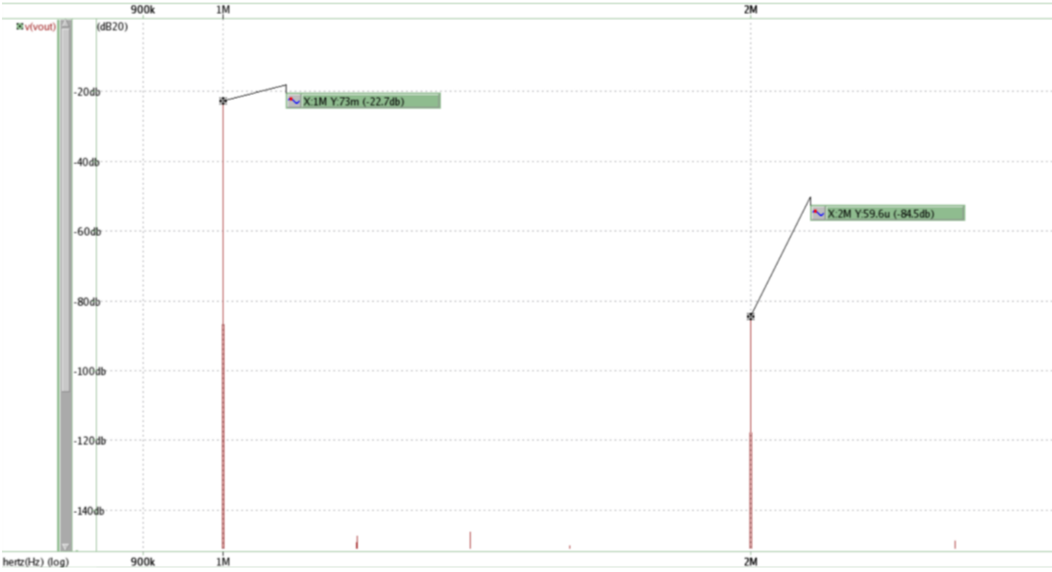
i. The input voltage range that the small-signal voltage gain is within +/–10% of that at the operating point is 274mV.



j. The small-signal gain decreases at the lower and upper bounds of this range because the gain at lower bound follows the equation that $gain = \mu_n C_{ox}(W/L)V_{ov}R_{out}$, which means that the gain will be increase with increasing the value of input voltage. However, when the input voltage larger than the upper bound of this range, the drain current (I_D) increasing leads to the voltage cross R_{out} increase, and this would lower the voltage between drain and source (V_{DS}). By the equation that $gain = \mu_n C_{ox}(W/L)V_{ov}(1 + \lambda V_{DS})R_{out}$, the gain will be decreased when V_{DS} decrease. That is the reason why the small-signal gain decreases at the lower and upper bounds of this range.

k. $V_{ov} = 410\text{mV}$ (from 1g), $A_F = g_m R_L V_m, V_{HD2} = 0.25 g_m R_L V_m^2$ (from 1c), $V_m = 25\text{mV}$. The expected power ratio of the second harmonic to the fundamental signal is $\left(\frac{V_m}{4(V_{ino} - V_{TH})}\right)^2 = 2.32 \times 10^{-4}$

l. (Hspice) Input signal is $0.025\sin(2\pi \times 1M \times t)$



- The coefficient of harmonic distortion at 1Meg Hz is -22.7dB.
- The coefficient of harmonic distortion at 2Meg Hz is -84.5dB.

m. What is the simulated ratio between the power of the second harmonic to that of the fundamental?

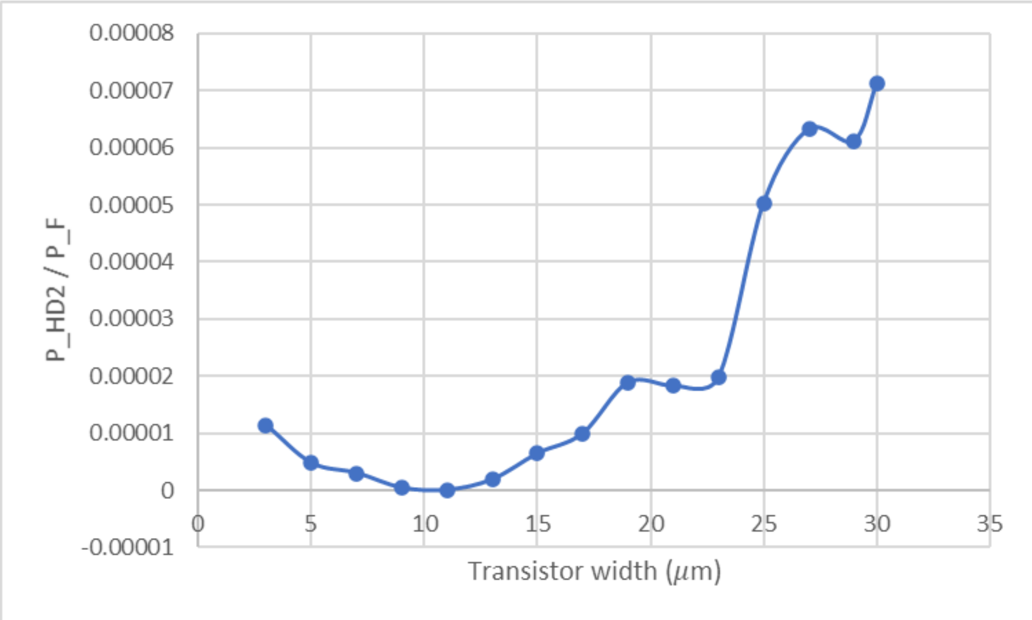
frequency index	frequency (hz)	fft_mag (dB)	fft_mag	fft_phase (deg)
500	1.0000x	-22.7301	73.0288m	89.9970
1000	2.0000x	-84.4914	59.6255u	179.9834
1500	3.0000x	-94.1120	19.6969u	89.9671
2000	4.0000x	-126.9219	450.7173n	-160.6407m

- Power ratio $= \left(\frac{A_{HD2}}{A_F}\right)^2 = \frac{(59.6255u)^2}{(73.2088m)^2} = 6.67 \times 10^{-7}$

n. How is this number compared to the prediction in question1)-k? What are the possible reasons for this discrepancy?

- Hand-calculation: 2.32×10^{-4}, Simulation: 6.67×10^{-7}
- Suppose that $f(V_{in}) = f(V_{ino}) + \alpha_1 \cos\omega t + \alpha_2 \cos 2\omega t, \omega = 1\text{MHz}$
  By hand-calculation, $\alpha_1 = 109.34m, \ \alpha_2 = 5.885n$
- The possible reason for this discrepancy is that we have the different coefficient by hand-calculation and simulation. In addition, we only consider the first two terms of the Taylor expansion, so the results must be very different.

o. Change the size of $M1$ from the width in question 1)-g to 30µm with step size of 2µm, adjust $V_{in,CM}$ accordingly so that the power consumption stays constant. Repeat the previous simulations with a sinusoidal input amplitude of 25mV at frequency of 1MHz. Plot the ratio between the power of the second harmonic to that of the fundamental vs. transistor width.



p. How is the result compared to that in question 1)-d? Explain the possible reasons for this discrepancy and elaborate your arguments.
In question 1)-d, we need to keep the power consumption same, which means that drain current remains the same. Therefore, we concluded that the ratio of harmonic distortion would be increased with increasing the transistor width in the condition that the power consumption stays constant.
In this plot, we can find the same result compare to the conclusion made in question 1)-d.

q. What is the simulated output noise voltage and input-referred noise (both in terms of V^2/Hz) voltage at low frequencies? What is the total output noise voltage and total input-referred noise voltage (both in terms of V_{rms}) if a noise bandwidth of 1GHz is assumed?

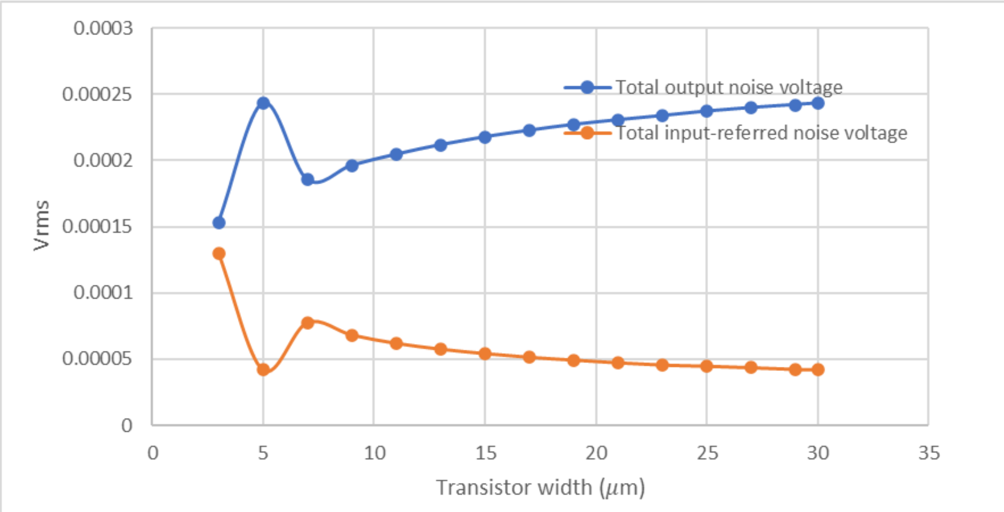
```
**** total output noise voltage          =  36.6238a      sq v/hz
                                         =   6.0518n      v/rt hz

     transfer function value:
       v(vout)/vin                       =   2.6452
     equivalent input noise at vin
                                         =   2.2878n      /rt hz
```

- Output noise voltage $= 36.6 \times 10^{-18}\ (V^2/Hz)$
- Input-referred noise voltage $= 5.23 \times 10^{-18}\ (V^2/Hz)$

- Total output noise voltage $= \sqrt{V_{n,out}^2 \times 1G} = 0.1913m\ V_{rms}$
- Total input-referred noise voltage $= \sqrt{V_{n,in}^2 \times 1G} = 72.32\mu\ V_{rms}$

r. Change the size of $M1$ from the width in question 1)-g to 30μm with step size of 2μm, adjust $V_{in,CM}$ accordingly so that the power consumption stays constant. Repeat the noise simulations and plot the total output noise voltage and total input-referred noise voltage (both in terms of V_{rms}) vs. transistor width.



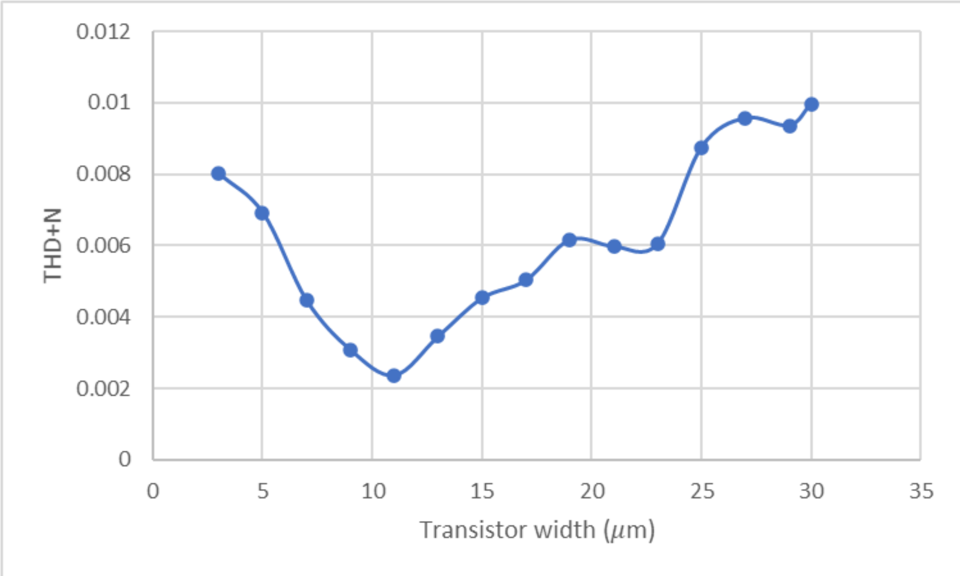
s. How is the result compared to that in question 1)-e? Explain the possible reasons for this discrepancy and elaborate your arguments.

$$S_{v,in} = 4kT\left(\frac{\gamma}{g_{m1}} + \frac{1}{g_{m1}^2 R_L}\right), g_{m1} = \mu_n C_{ox}(W/L)V_{ov} = 2I_D/V_{ov}, S_{v,out} = S_{v,in}A_v^2$$

- By the equation of $S_{v,in}$, we can find that $S_{v,in}$ decrease with increasing the transistor width (since power consumption is constant also means that drain current is constant). And the result of simulation also meets this analysis.
- By the equation of $S_{v,out}$, we can find that $S_{v,out}$ increase with increasing the transistor width. And the result of simulation also meets this analysis.

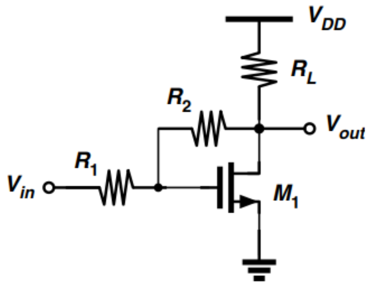
t. Based on the results of the previous two question sets, what is the optimal size for $M1$ that gives the minimum THD+N ratio? Notice that the noise power should be calculated at output node in this case.

$$THD + N = \frac{\sum_{n=2}^{\infty} \text{harmonic powers} + \text{noise power}}{\text{fundamental power}} \approx \frac{\text{second harmonic powers} + \text{noise power}}{\text{fundamental power}}$$

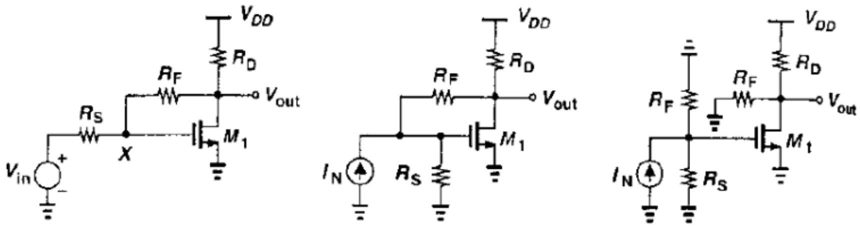


The optimal size of M1 is $(W/L) = (12\mu m/0.18\mu m)$.

2. Consider a common-source amplifier placed in a negative feedback loop like the following.



a. Loop gain (βA)? Closed-loop gain ($A/(1+A\beta)$)?



⇨ $A = -(R_1||R_2)g_{m1}(R_2||R_L), \beta = -1/R_2, g_{m1} = \mu_n C_{ox}(W/L)V_{ov}$

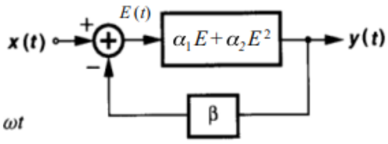
⇨ Loop gain $(\beta A) = \frac{R_1}{R_1+R_2}g_{m1}(R_2||R_L)$

⇨ Closed-loop gain $(R) = \frac{V_{out}}{I_N} = \frac{A}{1+A\beta} = \frac{-(R_1||R_2)g_{m1}(R_2||R_L)}{1+\frac{R_1}{R_1+R_2}g_{m1}(R_2||R_L)}$

b. Input signal $V_m \cos\omega t$, what is the amplitudes of the fundamental and the second harmonic at the output?

- $\frac{V_{out}}{V_{in}} = \frac{V_{out}}{I_N}\frac{1}{R_1} = \frac{1}{R_1}\frac{-(R_1||R_2)g_{m1}(R_2||R_L)}{1+\frac{R_1}{R_1+R_2}g_{m1}(R_2||R_L)}$

- For input-output characteristic, $A = \alpha_1 E + \alpha_2 E^2$, and $y(t) = a\cos\omega t + b\cos 2\omega t$(fundamental and second harmonic)



$\Rightarrow E = x(t) - \beta y(t) = (V_m - \beta a)\cos\omega t - \beta b\cos 2\omega t$

$\Rightarrow @\omega_1: a = (\alpha_1(V_m - a) - \alpha_2(V_m - a)b) \approx \alpha_1(V_m - a) \approx \frac{\alpha_1}{1+\alpha_1\beta}V_m$

$@\omega_2: b = -\alpha_1 b + \frac{\alpha_2(V_m - a)^2}{2} \approx \frac{\alpha_2 V_m^2}{2[1(1+\alpha_1\beta)^3]}$

- $\Delta V_{out} = -(R_1||R_2)(\mu_n C_{ox}\left(\frac{W}{L}\right)V_{ov})(R_2||R_L)(x(t) - \beta y(t)) +$

$(\frac{1}{2!})(-(R_1||R_2)(\mu_n C_{ox}\left(\frac{W}{L}\right))(R_2||R_L)\left(x(t) - \beta y(t)\right)^2)$

$\Rightarrow \alpha_1 = -(R_1||R_2)(\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{in} - V_{TH}))(R_2||R_L)$

$\alpha_2 = -\frac{1}{2}(R_1||R_2)(\mu_n C_{ox}\left(\frac{W}{L}\right))(R_2||R_L), \beta = -\frac{1}{R_2}$

$\Rightarrow \text{For } (V_{out} - I_N), \frac{A_{HD2}}{A_F}|_{V_{out}-I_N} = \frac{b}{a} = \frac{\alpha_2 V_m}{2\alpha_1(1+\alpha_1\beta)^2}$

$\alpha_1|_{V_{out}-V_{in}} = \alpha_1|_{V_{out}-I_N} \times \frac{1}{R_1}, \alpha_2|_{V_{out}-V_{in}} = \alpha_2|_{V_{out}-I_N} \times \left(\frac{1}{R_1}\right)^2$

$\Rightarrow \text{For } (V_{out} - V_{in}), \frac{A_{HD2}}{A_F}|_{V_{out}-V_{in}} = \frac{\alpha_2 V_m}{2\alpha_1(1+\alpha_1\beta)^2} \times \frac{1}{R_1}$

c. Compare to the results in 1-c, how much improvement in linearity do we get from the negative feedback structure? What is the cost for this linearity improvement?

- Not only fundamental harmonic distortion but also second harmonic distortion are improved by the negative feedback structure.
- For fundamental harmonic distortion, this term is pretty similar to gain, such as $A_{F,CS} = \mu_n C_{ox}(W/L)V_{ov}R_L V_m = gain_{CS}V_m$ and $A_{F,FB} = \frac{\alpha_1}{1+\alpha_1\beta}V_m = gain_{FB}V_m$. We can find that $A_{F,CS}$ is larger than $A_{F,FB}$ due to the gain difference.
- For the amplitudes of the fundamental and the second harmonic at the output, the feedback one is smaller than the CS one since the dominator of the one in feedback in much larger than numerator compared to that in CS.

- However, the cost for this improvement is that the power dissipation would be higher, and the closed-loop gain is lower than the origin.

d. With loop gain >> 1 and $R_2 = 10R_1$, what is the small-signal closed-loop gain of the amplifier?

⇨ Closed-loop gain $\frac{V_{out}}{V_{in}} = \frac{V_{out}}{I_N}\frac{1}{R_1} = \frac{1}{R_1}\frac{-(R_1||R_2)g_{m1}(R_2||R_L)}{1+\frac{R_1}{R_1+R_2}g_{m1}(R_2||R_L)} \approx 10$

e. (Hspice) $V_{DD} = 1.8V, V_{in,DC} = 0.9V, V_{out,DC} = 0.9V,\ I_{DC} = 1mA,$ $R_2 = 10R_1, \text{and } R_1 + R_2 > 10R_L$

```
   node      =voltage       node      =voltage       node     =voltage

+0:vdd      =   1.8000  0:vgl      = 900.3275m 0:vin      = 900.0000m
+0:vout     = 903.6023m 0:vss      =   0.
subckt
element  0:rl        0:r2        0:rl
 r value  870.0000     8.7000k  870.0000
 v drop   896.3977m    3.2749m  327.4856u
 current    1.0303m  376.4202n  376.4202n
 power    923.5963u    1.2327n  123.2722p

subckt
element   0:m1
model     0:n_18.1
region      Saturati
 id          1.0300m
```

$V_{DD} = 1.8V, V_{in,DC} = 0.9V, V_{out,DC} = 0.903V, \text{and } I_{DC} = 1.03mA$

f. With minimum channel length, what are the size and the simulated overdrive voltage of $M1$? What are the values of $R1$, $R2$?

With minimum channel length ($L = 0.18\mu m$), the overdrive voltage of M1 is 409mV, the small-signal gain is -1.76, $R_1 = 870\Omega$, and $R_2 = 8.7k\Omega$.

g. What is the simulated small-signal voltage gain? How is this number compared to that predicted in question 2-d? Explain the possible reasons for this discrepancy and elaborate your arguments.

- For hand-calculation, closed-loop gain $= \frac{1}{R_1}\frac{-(R_1||R_2)g_{m1}(R_2||R_L)}{1+\frac{R_1}{R_1+R_2}g_{m1}(R_2||R_L)} =$ 2.03 with gm = 3.56m (Ω^{-1}), and the value of gm is given by the simulation.
- The results of gain obtained by hand-calculation and simulation are very similar, but the result is far from the result in 2-d because we assume loop gain is much larger than 1 in 2-d.
- However, the loop gain in this circuit is 0.26. Therefore, this is the reason for this discrepancy.

h. Based on the simulated small-signal voltage gain of the CS amplifier with and without feedback (in question set 1), how much improvement in linearity (in terms of the ratio of the power of the second harmonic to that of the fundamental) would you expect if the same input signal is applied to the two amplifiers?

- For CS, $\alpha_1 = \mu_n C_{ox}\left(\frac{W}{L}\right)V_{ov}R_L, \alpha_2 = 0.5\mu_n C_{ox}\left(\frac{W}{L}\right)R_L$

$\Longrightarrow \frac{A_{HD2}}{A_F}|_{CS} = \frac{\alpha_2}{2\alpha_1}V_m = \frac{V_m}{4(V_{ino}-V_{TH})}.$

- $\frac{A_{HD2}}{A_F}|_{FB} = \frac{\alpha_2}{2\alpha_1(1+\alpha_1\beta)^2R_1}V_m, \alpha_1 = 2V_{ov}\alpha_2$
- The expected improvement in linearity of power is $\left(\frac{1}{(1+\alpha_1\beta)^2R_1}\right)^2$.
  - $\alpha_1\beta = [(R_1||R_2)(\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{in} - V_{TH}))(R_2||R_L)] \times \frac{1}{R_2} = 0.36$
  - The expected linear improvement is $\left(\frac{1}{(1+\alpha_1\beta)^2R_1}\right)^2 = 3.84 \times 10^{-7}$

i. Feed the amplifier with a sinusoidal input signal. Set the amplitude to 25mV and frequency to 1MHz. With transient simulations for at least 500 periods, perform dft on the output waveform over a time period when the wave-form becomes steady (after at least 100 periods) and plot the result with y-axis in dB20 scale. Place markers at 1MHz and 2MHz. Use ".option accurate" in your simulation. Set time step to less than 1ns. Zoom in to [0 3] MHz for x-axis and [−150 0] dB for y-axis.

- The coefficient of harmonic distortion at 1Meg Hz is -22.7dB.
- The coefficient of harmonic distortion at 2Meg Hz is -84.5dB.

j. What is the simulated ratio between the power of the second harmonic to that of the fundamental?

frequency index	frequency (hz)	fft_mag (dB)	fft_mag	fft_phase (deg)
500	1.0000x	-26.2277	48.8221m	89.9927
1000	2.0000x	-114.5711	1.8683u	179.9011
1500	3.0000x	-105.4943	5.3123u	89.1865
2000	4.0000x	-142.6537	73.6738n	-32.3263m

- Power Ratio $= \left(\frac{A_{HD2}}{A_F}\right)^2 = \frac{(1.8683u)^2}{(48.8221m)^2} = 1.464 \times 10^{-9}$

k. How is this result compared to that in question 2)-b? How is this improvement over a CS amplifier without feedback compared to that predicted in question 2)-h? What are the possible reasons for this discrepancy?

- By hand-calculation (in power),
  - $\left(\frac{A_{HD2}}{A_F}|_{FB}\right)^2 = \left(\frac{\alpha_2}{2\alpha_1(1+\alpha_1\beta)^2R_1}V_m\right)^2 = 9.02\times10^{-11}$

    $\alpha_1 = -(R_1||R_2)(\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{in}-V_{TH}))(R_2||R_L) = -3145.7$

    $\alpha_2 = -\frac{1}{2}(R_1||R_2)(\mu_n C_{ox}\left(\frac{W}{L}\right))(R_2||R_L) = 3845.6$

    $\beta = -\frac{1}{R_2} = -\frac{1}{8700}, V_m = 0.025$
  - $\left(\frac{A_{HD2}}{A_F}|_{CS}\right)^2 = \left(\frac{V_m}{4(V_{ino}-V_{TH})}\right)^2 = 2.32\times10^{-4}$
  - Linear improvement of power by hand-calculation is 3.89×10^{-7}
- By simulation (in power),
  - With feedback: $\left(\frac{A_{HD2}}{A_F}|_{FB}\right)^2 = 1.464\times10^{-9}$
  - Without feedback: $\left(\frac{A_{HD2}}{A_F}|_{CS}\right)^2 = 6.67\times10^{-7}$
  - Linear improvement of power by simulation is 2.2×10^{-3}
- We can find that the results of linear improvement by hand-calculation and simulation are totally difference. The result from simulation is much better than hand-calculation.
- The reason I think is that we use some approximations to get the hand-calculation result. Furthermore, in the simulation, I didn't keep drain current in 1mV precisely, and this could lead to the error of the equation.