[redacted]

1. CS stage

a) It's a typical CS stage amplifier, and we know it very well.

i) $S_{out}(f) = 4kTR_L = 4x(1.38xE\text{-}23)x300x600 = 9.9E\text{-}18$ (V^2/Hz)

ii) $\overline{V_{output,n}}^2 = S_{out}(f) \times 1GHz = 9.9\,nV^2 = (99u\,V_{rms})^2$

iii) $\overline{V_{output,n,RL}}^2 = \frac{kT}{C} = 41.4\,nV^2 = (0.2m\,V_{rms})^2$

b)

i) DC analysis

Size : W/L = 8um/0.18um.

Power : 2.3mW

```
****** transient analysis tnom=  25.000 temp=  27.000 ******
total_avg_pwr_uw=   2.2333k  from=   0.           to= 100.0000n
```

Output DC voltage : 1.05V

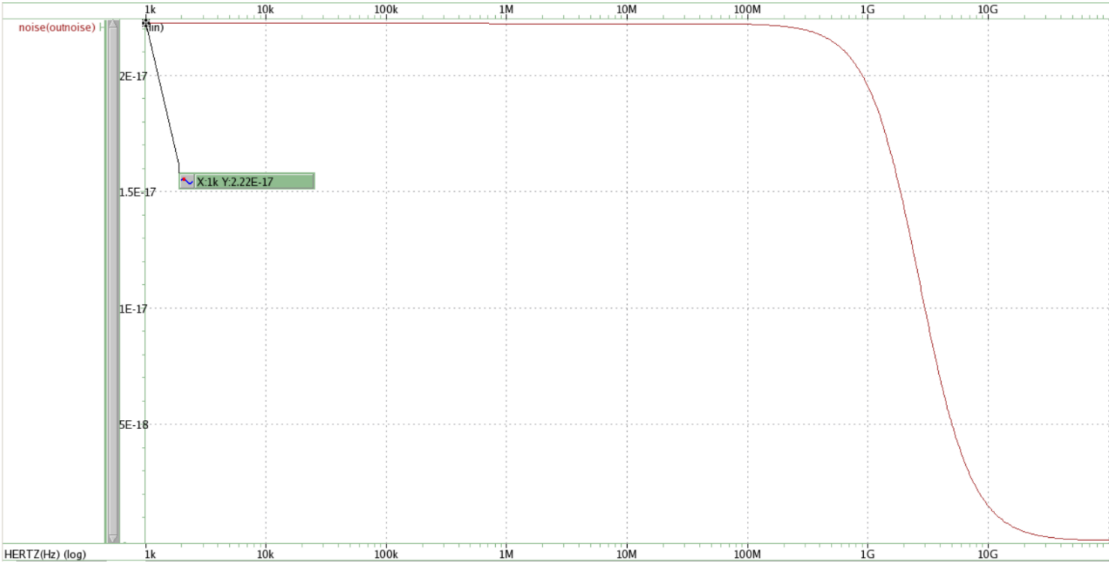
```
 node      =voltage       node     =voltage       node     =voltage

+0:vdd     =    1.8000  0:vin     = 950.0000m 0:vout     =    1.0556
+0:vss     =    0.
```

ii) AC analysis

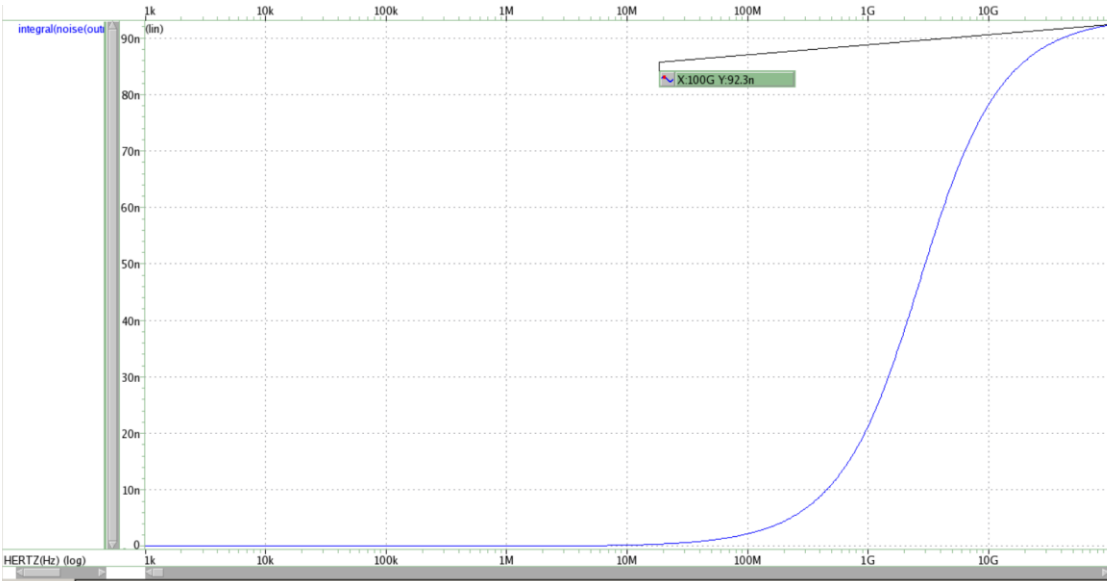
At f=1kHz, gain=2(or 6dB), 3-dB bandwidth = 2.69GHz

iii) Noise analisis

Output noise PSD:



Integral of PSD



$$\overline{V_{output,n,total}}^2 = 93.2\ nV^2$$

From a-iii, $\overline{V_{output,n,RL}}^2 = 41.4\ nV^2$, the difference comes from the nosie of M1.

We can get $\overline{V_{output,n,M1}}^2 = (93.2 - 41.4)nV^2 = 51.8\ nV^2 = (0.23m\ V_{rms})^2$

Lastly, $\overline{V_{input,n,total}}^2 = \frac{93.2nV^2}{gain^2} = \frac{93.2nV^2}{2^2} = \ 23.3nV^2 = (0.15m\ V_{rms})^2$

2. CG stage

a) gain = $g_{m1}R_D$

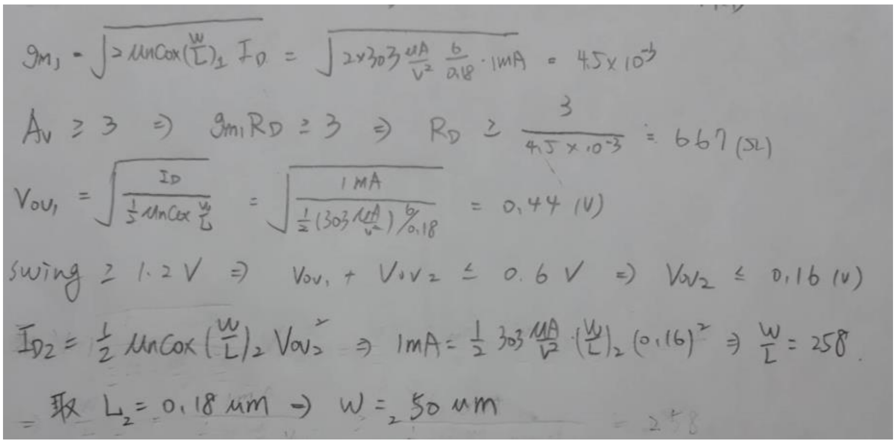
b) output swing = VDD – V_{ov1} – V_{ov2}

c) By the textbook:

input-referred noise voltage = $\left(\sqrt{4kT\gamma\frac{1}{g_{m1}} + 4kT\frac{1}{{g_{m1}}^2 R_D}}\ \ V/\sqrt{Hz}\right)^2$

input-referred noise current = $\left(\sqrt{4kT\gamma g_{m2} + 4kT\frac{1}{R_D}}\ \ A/\sqrt{Hz}\right)^2$

d)



$g_{m1} = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 I_D} = \sqrt{2\times 303\frac{\mu A}{V^2}\cdot\frac{6}{0.18}\cdot 1mA} = 4.5\times10^{-3}$

$A_v \geq 3 \Rightarrow g_{m1}R_D \geq 3 \Rightarrow R_D \geq \frac{3}{4.5\times10^{-3}} = 667\ (\Omega)$

$V_{ov1} = \sqrt{\frac{I_D}{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}}} = \sqrt{\frac{1mA}{\frac{1}{2}(303\frac{\mu A}{V^2})\frac{6}{0.18}}} = 0.44\ (V)$

swing $\geq 1.2V \Rightarrow V_{ov1} + V_{ov2} \leq 0.6V \Rightarrow V_{ov2} \leq 0.16\ (V)$

$I_{D2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 V_{ov2}^2 \Rightarrow 1mA = \frac{1}{2}\, 303\frac{\mu A}{V^2}\left(\frac{W}{L}\right)_2 (0.16)^2 \Rightarrow \frac{W}{L} = 258$

取 $L_2 = 0.18\ um \rightarrow W \approx 50\ um$

e)

i)

Gain = 3

```
****      small-signal transfer characteristics

     v(vout)/vin                                   =     3.0332
     input resistance at              vin          =   308.5876
     output resistance at v(vout)                  =   989.3366
```

DC current through Vin = -5 uA, which is small.

```
subckt
element   0:vin       0:vbn1      0:vbn2      0:vdd       0:vss
 volts     250.0000m    1.1700    600.0000m     1.8000      0.
 current    -5.5330u    0.          0.        -703.8386u  709.3716u
 power       1.3832u    0.          0.           1.2669m    0.
```

Bias current < 1mA, ouput swing = 1.8 – (0.387+0.088) = 1.33 V

```
subckt
element  0:m1        0:m2
model    0:n_18.1    0:n_18.1
region      Saturati    Saturati
 id         703.8386u  709.3716u
 ibs       -172.6252a -1.477e-19
 ibd       -658.8887a   -1.2513f
 vgs        920.0000m  600.0000m
 vds        705.3936m  250.0000m
 vbs       -250.0000m    0.
 vth        532.9514m  511.1554m
 vdsat      273.0098m  133.2226m
 vod        387.0486m   88.8446m
 beta        11.6024m   98.4985m
 gam eff    514.0252m  507.4467m
 gm           2.5766m    8.5836m
```

ii) Bias point: V_{bn1}=1.17 V, V_{bn2}=0.6V, $V_{in,DC}$=0.25V

Power: 1.3mW

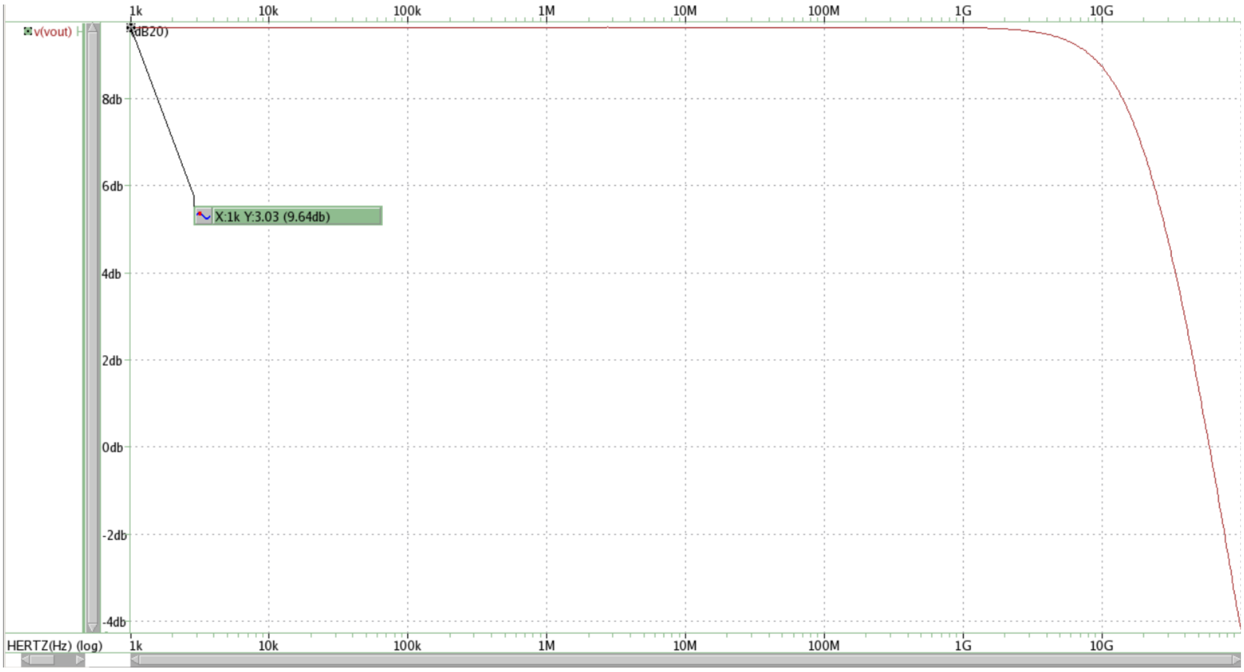
```
****** transient analysis tnom=  25.000 temp=  27.000 ******
total_avg_pwr_uw=    1.2669k  from=    0.           to= 100.0000n
```

Output DC voltage: 955mV

```
+0:vbn1     =    1.1700  0:vbn2     = 600.0000m 0:vdd      =    1.8000
+0:vin      = 250.0000m 0:vout     = 955.3936m 0:vss      =    0.
```

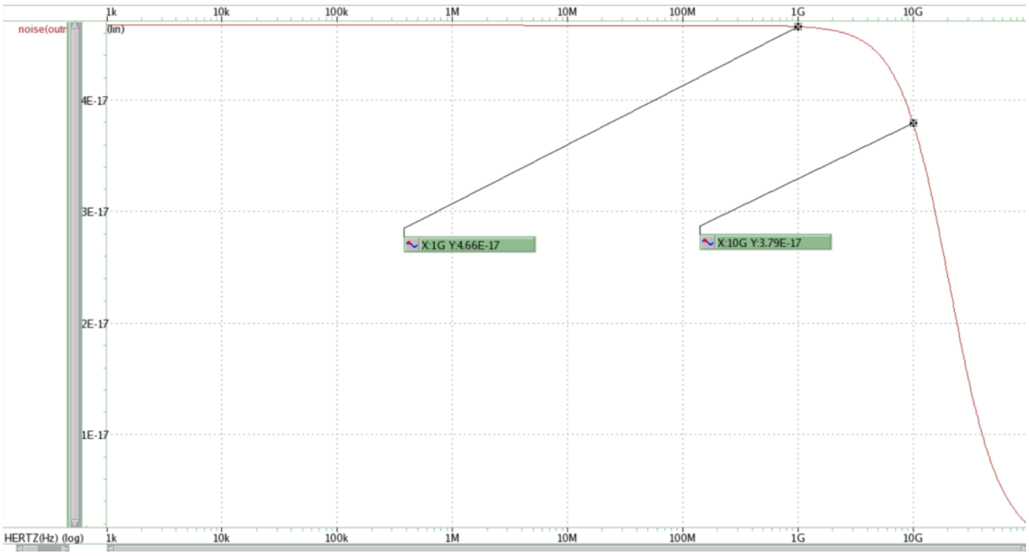
iii) AC kHz,analysis

(x,y) = (1kHz , 9.64dB)



iv) NOISE analysis

output noise PSD



At 1GHz, (x,y)=(1GHz , 4.66E-17); at 10GHz, (x,y)=(10GHz, 3.8E-17)

Input noise PSD

At 1GHz, (x,y)=(1GHz , 5.1E-18); at 10GHz, (x,y)=(10GHz, 5.1E-18)

From 2-d, with gm1=2.57m, RD=1200 ohm

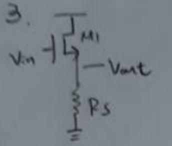
$$\overline{V_{input,n,total}}^2 = 4\mathrm{kT}\gamma\frac{1}{g_{m1}} + 4\mathrm{kT}\frac{1}{{g_{m1}}^2 R_D} = 6.38\times10^{-18}V^2$$

The noise derived from 2-d's equation is larger. Discrepency may come from:

1) There is still DC current flowing through Vin, so the formula in 2-d cannot entirely represent the noise behavior.
2) Second-order effect, such as body effect, causes the error.

計算題:

3.



$$A_v = \frac{g_m(R_S // r_o)}{1+g_m(R_S//r_o)} = \frac{g_m R_S}{1+g_m R_S} \quad \text{as } r_o \to \infty$$

$$\overline{V^2_{out,RS,n}} = \overline{I^2_{RS,n}} \cdot (R_S // 1/g_{m1})^2 = 4KT\,1/R_S \cdot (R_S // 1/g_{m1})^2$$

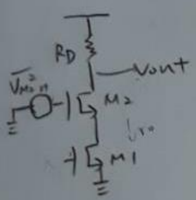
$$\overline{V^2_{in,RS,n}} = \frac{1}{A_v^2}\overline{V^2_{out,RS,n}} = \left(\frac{1+g_m R_S}{g_m R_S}\right)^2 4KT\,1/R_S \left(\frac{R_S}{1+g_m R_S}\right)^2 = 4KT\frac{1}{R_S}\frac{1}{g_{m1}^2}$$

$$\therefore \overline{V^2_{in,total,n}} = \overline{V^2_{in,RS,n}} + \overline{V^2_{in,M_1,n}}$$

$$= 4KT\frac{1}{R_S}\left(\frac{1}{g_{m1}}\right)^2 + 4KT\gamma\,1/g_m \Rightarrow 4KT\frac{1}{g_{m1}}\left(\gamma + \frac{1}{g_{m1}R_S}\right)\ (V/\sqrt{Hz})$$ #

4. (1) w/o. Rp

Consider M_2



1° M_2's input 到 Vout 的 voltage gain $gain_{M_2}$ is

$$|gain\,M_2| = \frac{g_{m2}r_{o2}}{R_2}(R_D // R_2)$$

where $R_2 = [1+g_{m2}r_{o2}]r_{o1} + r_{o2} \simeq g_{m2}r_{o1}r_{o2}$

$$\Rightarrow |gain_{M_2}| \simeq \frac{g_{m2}r_{o2}}{g_{m2}r_{o1}r_{o2}}(R_D // g_{m2}r_{o1}r_{o2}) = \frac{1}{r_{o1}}(R_D // g_{m2}r_{o1}r_{o2})$$

$\therefore$ M_2 在 Vout 造成的 noise $\overline{V^2_{out\,M_2,n}} = \overline{V^2_{M_2,n}} \cdot (gain_{M_2})^2$

此 noise refer 回 M_1's input 的 input-refered noise

為 $\overline{V^2_{in,M_2,n}} = \overline{V^2_{out\,M_2,n}} \cdot \left(\frac{1}{gain_{M_1}}\right)^2 = \overline{V^2_{M_2,n}}\left(\frac{gain_{M_2}}{gain_{M_1}}\right)^2$

$$(gain\,M_1)^2 = \left(g_{m1}\cdot(R_D // g_{m2}r_{o1}r_{o2})\right)^2$$

$$\therefore \overline{V^2_{in\,M_2,n}} = \overline{V^2_{M_2,n}}\,\frac{(R_D//g_{m2}r_{o1}r_{o2})^2/r_{o1}^2}{g_{m1}^2\cdot(R_D//g_{m2}r_{o1}r_{o2})^2} = \left(\frac{1}{g_{m1}r_{o1}}\right)^2$$

when $r_o \to \infty$, $\overline{V^2_{in\,M_2,n}} \to 0$

2° R_D 造成的 noise $= \overline{I^2_{RD,n}} \cdot R_D^2 \Rightarrow \overline{V^2_{out,RD,n}} = 4KTR_D$.

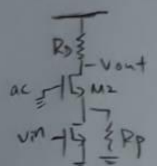
3° M_1 可直接 infer 至 input $\Rightarrow \overline{V^2_{in\,M_1}} = 4KT\gamma\,1/g_{m1}$

$$\therefore \overline{V^2_{in,total,n}} = \overline{V^2_{in\,M_1,n}} + \overline{V^2_{in\,M_2,n}} + \overline{V^2_{in\,RD,n}}$$

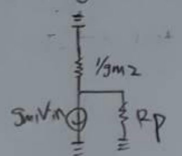
$$= 4KT\gamma\,1/g_{m1} + 0 + \frac{4KTR_D}{(g_{m1}R_D)^2} \Rightarrow 4KT\left(\frac{\gamma}{g_{m1}} + \frac{1}{g_{m1}^2 R_D}\right)\ (V/\sqrt{Hz})$$ #

(2) with R_P.

1° we need to calculate $gain_{M_1}$, $gain_{M_2}$ first.



For $gain_{M_1}$，小訊號 V_{in} 造成 M_2 current 的變動為



$$(g_{m1}V_{in})\frac{R_P}{1/g_{m2}+R_P} = \frac{V_{in}\cdot g_{m1}g_{m2}R_P}{1+g_{m2}R_P}$$

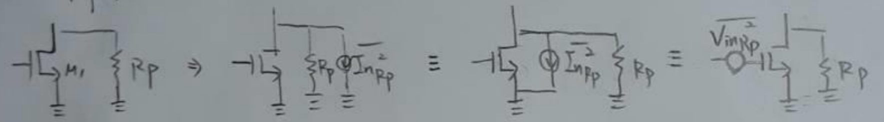
而 $V_{out} = R_D\cdot\left(\frac{V_{in}\,g_{m1}g_{m2}R_P}{1+g_{m2}R_P}\right)$ $\therefore gain_{M_1} = \frac{R_D\,g_{m1}g_{m2}R_P}{1+g_{m2}R_P}$.

And $gain_{M_2} = \frac{R_D}{1/g_{m2}+R_P}$ (the gain of the CS stage with source degeneration)

$$\Rightarrow \frac{gain_{M_2}}{gain_{M_1}} = \frac{R_D/(1/g_{m2}+R_P)}{R_D g_{m1}g_{m2}R_P/(1+g_{m2}R_P)} = \frac{1}{g_{m1}R_P}$$

$$\therefore \overline{V_{in\,M_2.n}^2} = \overline{V_{M_2.n}^2}\left(\frac{gain_{M_2}}{gain_{M_1}}\right)^2 = 4KT\gamma\,1/g_{m2}\left(\frac{1}{g_{m1}R_P}\right)^2$$

2° R_P.



$$\overline{V_{inR_P}^2} = \frac{\overline{I_{nR_P}^2}}{g_{m1}^2}$$

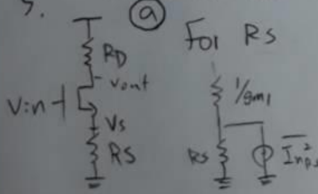
3° R_D

$$\overline{V_{in\,R_D.n}^2} = \overline{V_{out\,R_D.n}^2}\cdot\left(\frac{1}{gain_{M_1}}\right)^2 = 4KTR_D\cdot\left(\frac{1+g_{m2}R_P}{R_D\,g_{m1}g_{m2}R_P}\right)^2 = 4KT\frac{1}{R_D}\left(\frac{1+g_{m2}R_P}{g_{m1}g_{m2}R_P}\right)^2$$

$$\therefore \overline{V_{in,total.n}^2} = \overline{V_{in\,M_1.n}^2} + \overline{V_{in\,M_2\,n}^2} + \overline{V_{in\,R_P\,n}^2} + \overline{V_{in\,R_D.n}^2}$$

$$\Rightarrow \sqrt{4KT\gamma\,1/g_{m1} + 4KT\gamma\,1/g_{m2}\left(\frac{1}{g_{m1}R_P}\right)^2 + 4KT\frac{1}{R_P}\frac{1}{g_{m1}^2} + 4KT\frac{1}{R_D}\left(\frac{1+g_{m2}R_P}{g_{m1}g_{m2}R_P}\right)^2}\ \left(V/\sqrt{Hz}\right)$$ #

5. (a)



For R_S

$$\overline{V_{out.R_S.n}^2} = \overline{I_{nR_S}^2}\left(\frac{R_S}{R_S+1/g_{m1}}\right)^2\cdot R_D^2 = \overline{I_{nR_S}^2}\left(\frac{g_{m1}R_S}{1+g_{m1}R_S}\right)^2 R_D^2$$

$$\therefore \overline{V_{in\,R_S.n}^2} = \left(\frac{1+g_{m1}R_S}{g_{m1}R_D}\right)^2 \overline{I_{nR_S}^2}\left(\frac{g_{m1}R_SR_D}{1+g_{m1}R_S}\right)^2 = \frac{4KT}{R_S}\cdot R_S^2 = 4KTR_S$$

$$\therefore \overline{V_{in.n\,total}^2} \Rightarrow \sqrt{4KT\gamma\,1/g_{m1} + 4KTR_S + \frac{4KTR_D}{(g_{m1}R_D)^2}(1+g_{m1}R_S)^2}\ \left(V/\sqrt{HZ}\right)$$ #

(b) If $4KTR_S = 4KT\gamma\,1/g_{m1} \Rightarrow R_S g_{m1} = \gamma$

$$\frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_{ov}^2 = \frac{V_S}{R_S} \Rightarrow \frac{1}{2}g_{m1}V_{ov} = \frac{V_S}{R_S} \quad \left(\because g_{m1} = \mu_n C_{ox}\frac{W}{L}V_{ov}\right)$$

$$\therefore \frac{V_S}{V_{ov}} = \frac{g_{m1}R_S}{2} = \frac{1}{2}\gamma$$