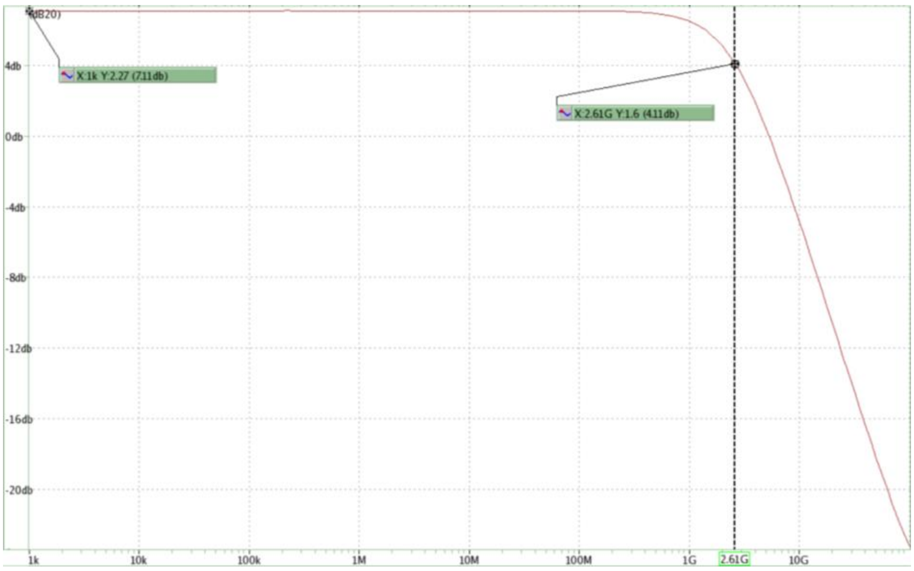
1. Consider a common-source amplifier with resistor load (R_L) of 600Ω, $V_{DD} = 1.8V, V_{in,DC} = 0.9V, temp = 27°, k = 1.38 \times 10^{-23}$ J/K
   a. Consider only the thermal noise of R_L
      i. Calculate the output noise power (in terms of V^2/Hz).
      $S_{v,R_L} = 4kTR_L = (3.15nV)^2/Hz = 9.94 \times 10^{-18}\ V^2/Hz$
      ii. Calculate the total rms output noise voltage over the frequency range from DC to 1 GHz.
      $9.94 \times 10^{-18}\ (V^2/Hz) \times 10^9\ (Hz) = 9.94 \times 10^{-9}\ V_{rms}^2$
      iii. With load capacitor (C_L) of 100 fF, calculate the total rms output noise voltage over the entire frequency range.
      $$P_{n,out} = \frac{kT}{C} = 4.14 \times 10^{-8}\ V_{rms}^2$$
   b. Hspice
      i. With dc analysis, report the following of your final design.
      $Gain = -2.27,\ 3 - dB\ bandwidth = 2.6G\ Hz$
      $(W/L)_1 = 10u/0.2u,\ Power\ consumption = 2.15mW$
      $V_{out,DC} = 1.0851V$
      ii. With ac analysis from 1 kHz to 100 GHz, plot the frequency response from V_{in}, to V_{out} over 1 kHz to 100 GHz.



      iii. With noise analysis from 1 kHz to 100 GHz, report the following.

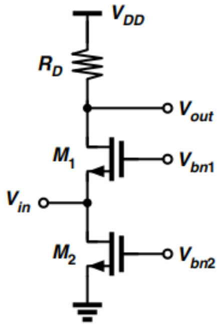
File	Equation	Specification		Result		Pass/Fail
		Min	Max	Value	Mean	
D0:1_a.ac0	integ(abs('0|noise(outnoise)'), 1k, 100g)			98n		

- The total rms output noise voltage over the frequency range from 1kHz to 1GHz is 98n V_{rms}^2
- The reason why there are difference between hand calculations and the result of simulation is that we ignored the effect of flicker

noise, which plays an important role on the low frequency, and this will, and the noise come from transistor. Moreover, the frequency range in hand calculation is from DC to 1 GHz, and the frequency range in simulation is from 1kHz to 100GHz.
I also Integrate the above waveform (output noise PSD) over 1 kHz to 1 GHz, and the result is 23.1n V_{rms}^2, which is closer to the result of hand calculation.

- The total rms output noise voltage contributed by the transistor is $98n - 9.94n = 88.1n\ V_{rms}^2$
- $\overline{V_{n,out}^2} = A_v^2 \overline{V_{n,in}^2} \Rightarrow \overline{V_{n,in}^2} = \sqrt{98n/(-2.27)^2} = 1.38 \times 10^4\ V_{rms}$

2. Assume $\lambda = \gamma = 0$. Consider the following circuit (Ignore channel-length modulation & body effect)



a. Calculate the dc voltage gain.
$\lambda = 0 \longrightarrow r_o = 0 \Rightarrow V_{out} = -V_{in} g_{m1} R_D \Rightarrow A_v = -g_{m1} R_D$

b. Calculate the output swing.
$V_{out} \geq V_{ov1} + V_{ov2}, V_{out} \leq V_{DD}$
$\Rightarrow V_{out,swing} = V_{DD} - (V_{bn1} - V_{in} - V_{TH1}) - (V_{bn2} - V_{TH2})$

c. Calculate the input-referred thermal noise voltage (in terms of $V/\sqrt{Hz}$) and input-referred thermal noise current (in terms of $A/\sqrt{Hz}$).

i. Input-referred thermal noise voltage

$S_{v,Vout} = S_{v,R_D} + S_{v,M1} + S_{v,M2} = (4kT/R_D) \times R_D^2 + (4kT\gamma/g_{m2}^{-1}) \times R_D^2$

$$S_{v,Vin} = \frac{S_{v,Vout}}{A_v^2} = 4kT\left(\frac{1}{g_{m1}^2 R_D} + \frac{g_{m2}\gamma}{g_{m1}^2}\right) \Rightarrow \overline{V_{n,in}} = \sqrt{4kT\left(\frac{1}{g_{m1}^2 R_D} + \frac{\gamma}{g_{m1}}\right)}$$

ii. Input-referred thermal noise current

When calculate $\overline{I_{n,in}^2}$, we need to open the input.

$$S_{v,Iin} = S_{v,Vout}/R_{out}^2 = 4kT\left(R_D^{-1} + \gamma g_{m2}\right) \Rightarrow \overline{I_{n,in}} = \sqrt{4kT(R_D^{-1} + \gamma g_{m2})}$$

d. Use $\mu_n C_{ox} = 303\ \mu A/V^2$ and $\mu_p C_{ox} = 91\ \mu A/V^2$ for the calculation. Set $V_{DD} = 1.8V. I_{D,M1} = I_{RD} = 1\ mA$ and $(W/L)_1 = 6\ \mu m/0.18\ \mu m$. Design

R_D and $(W/L)_2$ so that the dc voltage gain is at least 3 V/V, output swing is at least 1.2 V, and the input referred thermal noise voltage and current are minimized. Describe how the circuit is designed.

i. DC voltage gain is at least 3 V/V

$A_v = -g_{m1}R_D, g_{m1} = \mu_n C_{ox}(W/L)_1 V_{ov1}$

$I_{D,M1} = 0.5\mu_n C_{ox}(W/L)_1 V_{ov1}^2 = 1mA \Rightarrow V_{ov1} = 0.445 \Rightarrow g_{m1} = 4.5m\ \Omega^{-1}$

$\Longrightarrow |Gain| = g_{m1}R_D \geq 3 \Longrightarrow R_D \geq 666.67\Omega$

ii. Output swing is at least 1.2 V

$V_{out,swing} = V_{DD} - V_{ov1} - V_{ov2} = 1.8 - 0.445 - V_{ov2} \geq 1.2\ V$

$\Longrightarrow V_{ov2} \leq 0.155\ V$

$I_{D,M2} = 0.5\mu_n C_{ox}(W/L)_2 V_{ov2}^2 = 1mA$

iii. Input referred thermal noise voltage and current are minimized

$S_{n,Vin,R_D} = 4kTR_D/(g_{m1}R_D)^2$

$\Longrightarrow$ My design is that $V_{ov2}(= V_{bn2} - V_{TH2}) = 0.122V$, $(W/L)_2 = (70\mu/0.18\mu)$, and $R_D = 1500\Omega$.

e. Hspice

i. Keep the device sizes unchanged. Adjust the bias voltages ($V_{in,DC}$, V_{bn1}, and V_{bn2}) so that no DC current flows through V_{in}, and the bias current is less than 1 mA while maintaining all transistors in saturation.

```
subckt
element  0:m1        0:m2
model    0:n_18.1    0:n_18.1
region     Saturati   Saturati
 id        704.4097u  700.5893u
 ibs      -1.685e-19 -1.451e-19
 ibd      -374.6739a   -1.3934f
 vgs       900.0000m  577.0000m
 vds       543.3855m  200.0000m
 vbs         0.         0.
 vth       503.7659m  512.7566m
 vdsat     273.6230m  119.6630m
 vod       396.2341m   64.2434m
 beta       11.6160m  137.9506m
 gam eff   507.4528m  507.4465m
 gm          2.4968m    9.4838m
 gds       243.2238u  899.7118u
 gmb       366.1920u    1.4339m
 cdtot       8.3362f  103.6703f
 cgtot      11.2185f  124.2395f
 cstot      16.0386f  176.3318f
 cbtot      14.8579f  178.5529f
 cgs         8.1220f   86.5888f
 cgd         2.1908f   25.8128f
```

```
****     small-signal transfer characteristics

     v(vout)/vin                            =    3.0088
     input resistance at            vin     =  344.2079
     output resistance at v(vout)           =    1.0994k
```

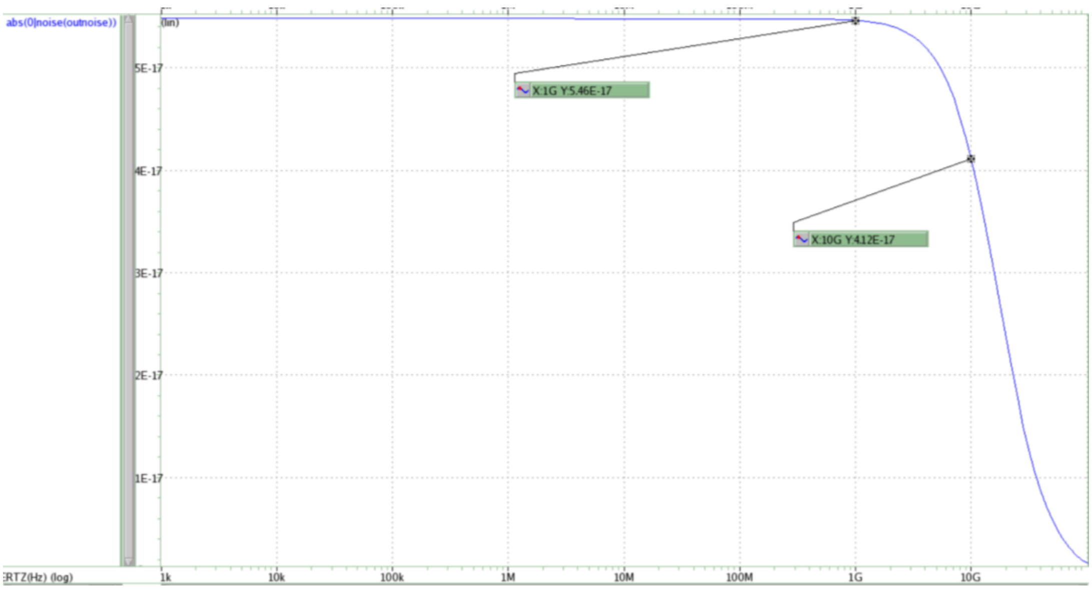
```
subckt
element  0:vin       0:vbn1     0:vbn2      0:vdd       0:vss
 volts    200.0000m    1.1000   577.0000m    1.8000      0.
 current    3.8204u   0.         0.       -704.4097u  704.4097u
 power   -764.0811n   0.         0.          1.2679m    0.

    total voltage source power dissipation=    1.2672m       watts
```

ii. With dc analysis, report the following.

- $V_{in,DC} = 0.2V, V_{bn1} = 1.1V, \text{and } V_{bn2} = 0.577V$
- $\text{Power consumption} = 1.2672mW$
- $V_{out,DC} = 743.38mV$

iii. With ac analysis from 1 kHz to 100 GHz, plot the frequency response from V_{in}, to V_{out} over 1 kHz to 100 GHz.

iv. With noise analysis from 1 kHz to 100 GHz, report the following.



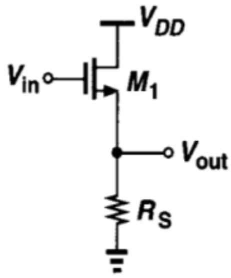
- $\overline{V^2_{n,out}} = A_v^2\overline{V^2_{n,in}}, \overline{V^2_{n,in}}, = \sqrt{4kTR_D/(g_{m1}R_D)^2}$
- At $f = 1GHz$

  By simulation, $\overline{V^2_{n,out}} = 5.47 \times 10^{-17} \Rightarrow \overline{V^2_{n,in}} = 2.46nV/\sqrt{Hz}$

  By hand calculation, $\overline{V^2_{n,in}} = 1.73\ nV/\sqrt{Hz}$ with $\gamma = 2/3$

- At $f = 10\text{GHz}$
  By simulation, $\overline{V_{n,out}^2} = 4.12 \times 10^{-17} \Rightarrow \overline{V_{n,in}^2} = 2.13\text{nV}/\sqrt{\text{Hz}}$
  By hand calculation, $\overline{V_{n,in}^2} = 1.73\ \text{nV}/\sqrt{\text{Hz}}$
- There is error between simulation and hand calculation since we ignore flicker noise. If I replace the value of g_{m1} with the real value, $g_{m1} = 2.5\text{m}\Omega^{-1}$, then we can get $\overline{V_{n,in}^2} = 2.48\ \text{nV}/\sqrt{\text{Hz}}$, which is similar to the result of simulation.
- $\overline{V_{n,in,1GHz}^2}$ is larger than $\overline{V_{n,in,10GHz}^2}$ since flicker noise plays an important role on the low frequency.

3. Calculate the input-referred thermal noise voltage (in terms of $V/\sqrt{\text{Hz}}$) of the following circuit. Assume $\lambda = \gamma = 0$.

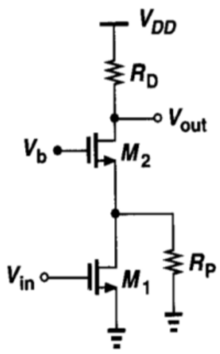


$$A_v = g_{m1} \times (g_{m1}^{-1}||R_S) = \frac{g_{m1}R_S}{1 + g_{m1}R_S} \approx 1$$

$$S_{v,vout} = 4kT\frac{(R_S||g_{m1}^{-1})^2}{R_S} + 4kT\gamma g_{m1}(R_S||g_{m1}^{-1})^2$$

$$\overline{V_{n,in}} = \sqrt{S_{v,vout}/A_v^2} \approx \frac{1+g_{m1}R_S}{g_{m1}R_S}\sqrt{4kT(\frac{(R_S||g_{m1}^{-1})^2}{R_S} + \gamma g_{m1}(R_S||g_{m1}^{-1})^2)}$$

4. Assume $\lambda = \gamma = 0$. Calculate the input-referred thermal noise voltage (in terms of $V/\sqrt{\text{Hz}}$) of the following circuit with and without R_P.



a. Without R_p

$A_v = -g_{m1}(g_{m2}r_{o2}r_{o1}||R_D) = -g_{m1}R_D \ \ (\lambda = 0)$

$S_{v,vout} = 4kTR_D + 4kT\gamma g_{m1}R_D^2$ (Due to $V_{M2,S}$ is a floating point)

$\overline{V_{n,in}} = \sqrt{S_{v,vout}/A_v^2} \approx \sqrt{4kT((g_{m1}^2R_D)^{-1} + \gamma g_{m1}^{-1})}$

b. With R_p

$A_v = -g_{m1}(g_{m2}r_{o2}(r_{o1}||R_P)||R_D) = -g_{m1}R_D \ \ (\lambda = 0)$

Use Superposition to get output noise ($S_{v,vout}$)

i. By R_D: $S_{v,vout1} = (4kT/R_D) \times R_D^2$

ii. By M2: $S_{v,vout2} = (4kT\gamma/g_{m2}^{-1})\left(\frac{g_{m2}^{-1}}{R_P+g_{m2}^{-1}}\right)^2 (R_D)^2$

iii. By M1: $S_{v,vout3} = (4kT\gamma/g_{m1}^{-1})\left(\frac{R_P}{R_P+g_{m2}^{-1}}\right)^2 (R_D)^2$

iv. By R_P: $S_{v,vout4} = (4kT/R_P)\left(\frac{R_P}{R_P+g_{m2}^{-1}}\right)^2 (R_D)^2$

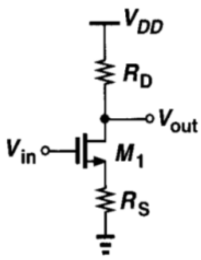
$\Rightarrow S_{v,vout} = S_{v,vout1} + S_{v,vout2} + S_{v,vout3} + S_{v,vout4}$

$$= 4kT(R_D + (\gamma g_{m2}R_D^2)\left(\frac{g_{m2}^{-1}}{R_P+g_{m2}^{-1}}\right)^2 + (\gamma g_{m1}R_D^2)\left(\frac{R_P}{R_P+g_{m2}^{-1}}\right)^2 + \left(\frac{R_D^2}{R_P}\right)\left(\frac{g_{m2}^{-1}}{R_P+g_{m2}^{-1}}\right)^2)$$

$\Rightarrow \overline{V_{n,in}} = \sqrt{\frac{S_{v,vout}}{A_v^2}}$

$$= \frac{1}{g_{m1}R_D}\sqrt{4kT(R_D + (\gamma g_{m2}R_D^2)\left(\frac{g_{m2}^{-1}}{R_P+g_{m2}^{-1}}\right)^2 + (\gamma g_{m1}R_D^2)\left(\frac{R_P}{R_P+g_{m2}^{-1}}\right)^2 + \left(\frac{R_D^2}{R_P}\right)\left(\frac{g_{m2}^{-1}}{R_P+g_{m2}^{-1}}\right)^2)}$$

5. Assume $\lambda = \gamma = 0$.



a. Calculate the input-referred thermal noise voltage (in terms of $V/\sqrt{Hz}$).

$A_v = -\frac{g_{m1}R_D}{1+g_{m1}R_S}$

$$S_{v,vout} = (4kT/R_D + 4kT\gamma g_{m1}\left(\frac{g_{m1}^{-1}}{g_{m1}^{-1}+R_S}\right)^2 + \frac{4kT}{R_S}\left(\frac{R_S}{g_{m1}^{-1}+R_S}\right)^2)(R_D)^2$$

$$\overline{V_{n,in}} = \sqrt{S_{v,vout}/A_v^2} = \frac{1+g_{m1}R_S}{g_{m1}R_D}\sqrt{4kT(R_D + \gamma g_{m1}\left(\frac{g_{m1}^{-1}}{g_{m1}^{-1}+R_S}\right)^2 + \frac{R_D^2}{R_S}\left(\frac{R_S}{g_{m1}^{-1}+R_S}\right)^2)}$$

b. If the thermal noise contributed by R_S is the same as that contributed from $M1$, how is the dc voltage drop across R_S compared to the overdrive voltage of $M1$?

$$4kT\gamma g_{m1}\left(\frac{g_{m1}^{-1}}{g_{m1}^{-1}+R_S}\right)^2 (R_D)^2 = \frac{4kT}{R_S}\left(\frac{R_S}{g_{m1}^{-1}+R_S}\right)^2 (R_D)^2, g_{m1} = \frac{2I_D}{V_{ov1}}$$

$$g_{m1} = \frac{2I_D}{V_{ov1}} = \frac{\gamma}{R_S} \Rightarrow I_D R_S = \frac{\gamma}{2}V_{ov1}$$