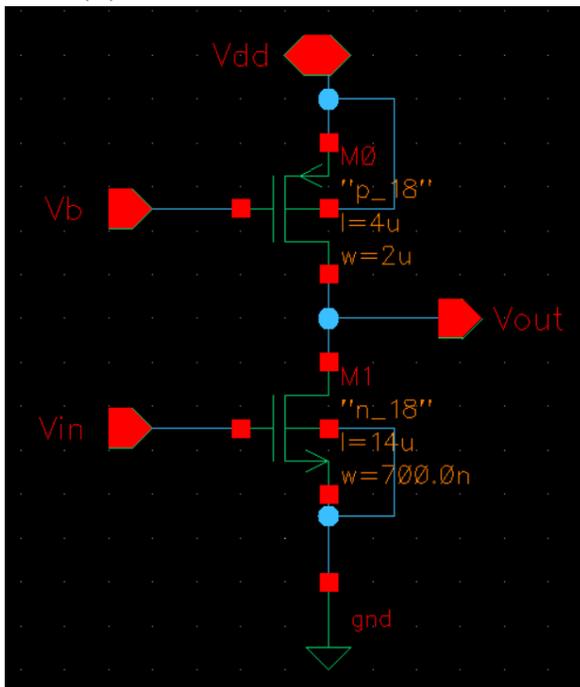


HW4

1.

(a)



```

hw4_1a
.proTECT
.lib 'cic018.' TT
.unprotect
.temp 25
.option post
M0 Vout Vb Vdd Vdd P_18 W=2u L=4u m=1
M1 Vout Vin gnd gnd N_18 W=0.7u L=14u m=1
Vdd Vdd gnd DC=1.8
Vb Vb gnd DC=1.239743
Vin Vin GND DC=0.5 AC=1
.op
*.dc Vb 0 1.8 0.01
.noise V(Vout) Vin 10000
.tf V(Vout) Vin
.ac dec 100 10 10G
.end
    
```

```

subckt
element 0:m0 0:m1
model 0:p_18.1 0:n_18.1
region Saturati Saturati
id -190.4922n 190.4918n
ibs 2.603e-23 -6.915e-23
ibd 105.0938a -122.1942a
vgs -560.2570m 500.0000m
vds -799.7521m 1.0002
vbs 0. 0.
vth -472.0084m 332.5823m
vdsat -111.7565m 152.7538m
vod -88.2486m 167.4177m
beta 35.1905u 14.6485u
gam_eff 557.0847m 507.4459m
gm 2.7155u 1.9752u
gds 2.3660n 5.5667n
gmb 835.9004n 385.7082n
cdtot 2.3778f 1.0028f
cgtot 48.6526f 63.8668f
cstot 54.3516f 65.9235f
cbtot 20.8013f 18.6462f
cgs 42.9754f 58.0608f
cgd 721.9569a 197.5977a
    
```

```

**** small-signal transfer characteristics
v(vout)/vin = -248.8710
input resistance at vin = 1.000e+20
output resistance at v(vout) = 125.9989x

+0:v_b = 1.2397 0:vdd = 1.8000 0:vin = 500.0000m
+0:vout = 1.0002
    
```

Result :

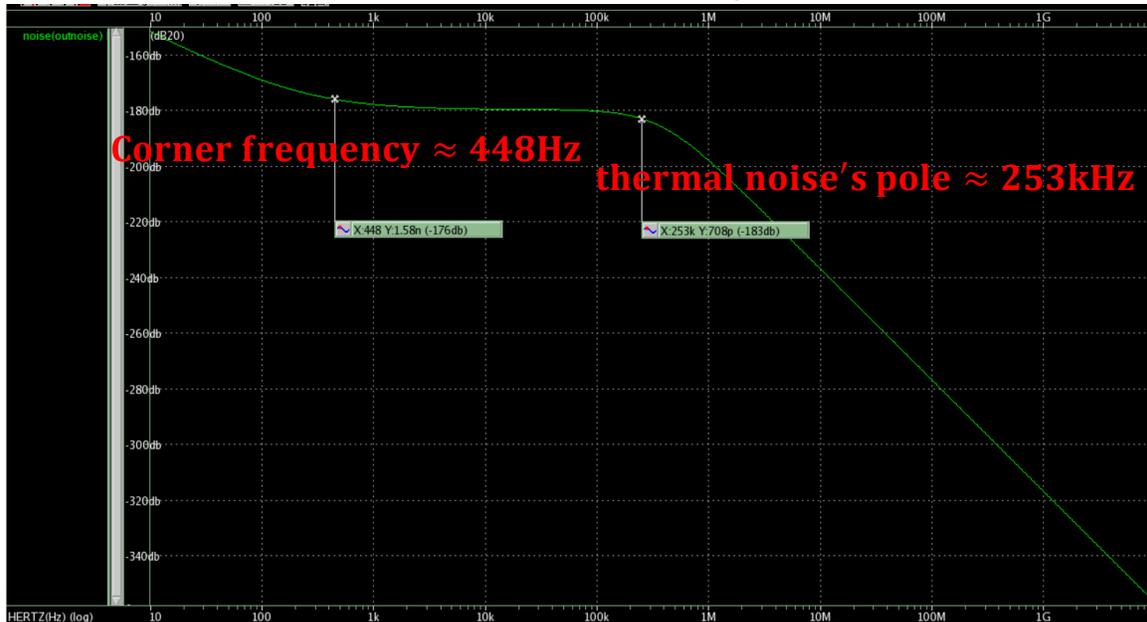
$V_{out} = 1.0002V (\approx 1V)$; $|gain| = 248.8710 (> 120)$

PMOS、NMOS 的 c_{dtot} , c_{gtot} , c_{stot} , c_{btot} 皆 $< 80fF$

PMOS、NMOS 都在 saturation region

Comment :

做法：一開始我先透過調整 NMOS 和 PMOS 的 W、L 並跑 Vb 從 0~1.8V 找出最大的 gain > 120 和 NMOS、PMOS 的電容 cgtot、cstot、cdtot、cbtot 沒有超過 80fF 者，再細調 Vb 使得 Vout 的 DC voltage 落在 1V 附近。

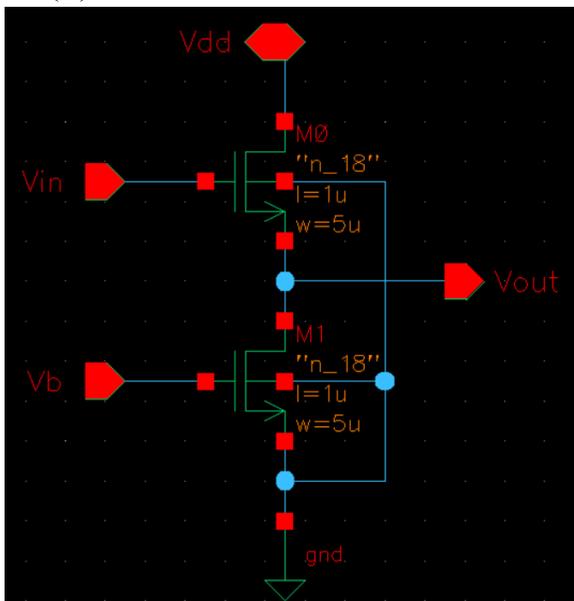


Comment :

output noise 在低頻主要是被 flicker noise 影響，在高頻則主要是被 thermal noise 影響。

corner frequency 是 flicker noise 漸進線和 thermal noise 漸近線的交點，而 thermal noise pole 則是在高頻時曲線改變的點（-3dB）。

(b)



```

hw3_1b
.protect
.lib 'cic018.' TT
.unprotect
.temp 25
.option post
M0 Vdd Vin Vout gnd N_18 W=5u L=1u m=1
M1 Vout Vb gnd gnd N_18 W=5u L=1u m=1
Vdd Vdd gnd DC=1.8
Vb Vb gnd DC=0.42
Vin Vin gnd DC=1 AC=1
.op
*.dc Vin 0 1.8 0.001
.noise V(Vout) Vin 10000
.ac dec 100 10 10G
.end
    
```

```

subckt
element 0:m0 0:m1
model 0:n_18.1 0:n_18.1
region Saturati Saturati
id 3.2488u 3.2488u
ibs -212.7605a -5.749e-22
ibd -765.7194a -212.7594a
vgs 499.8590m 420.0000m
vds 1.2999 500.1410m
vbs -500.1410m 0.
vth 472.7113m 388.0421m
vdsat 78.5413m 77.3394m
vod 27.1478m 31.9579m
beta 1.5669m 1.5516m
gam_eff 519.9037m 507.4460m
gm 63.6509u 62.3685u
gds 792.2613n 699.5570n
gmb 9.8318u 12.8926u
cdtot 5.8360f 6.9729f
cgtot 27.7923f 29.1474f
cstot 28.8383f 31.5855f
cbtot 15.4524f 19.0719f
cgs 22.4608f 23.4572f
cgd 1.7730f 1.8124f
    
```

```

*** total output noise voltage = 293.3893a sq v/hz
                                = 17.1286n v/rt hz
transfer function value:
v(vout)/vin = 848.9671m
equivalent input noise at vin = 20.1758n /rt hz
    
```

```

***** operating point information tnom= 25.000 temp= 25.000 *****
***** operating point status is all simulation time is 0.
node =voltage node =voltage node =voltage
    
```

```

+0:vb = 420.0000m 0:vdd = 1.8000 0:vin = 1.0000
+0:vout = 500.1410m
    
```

Result :

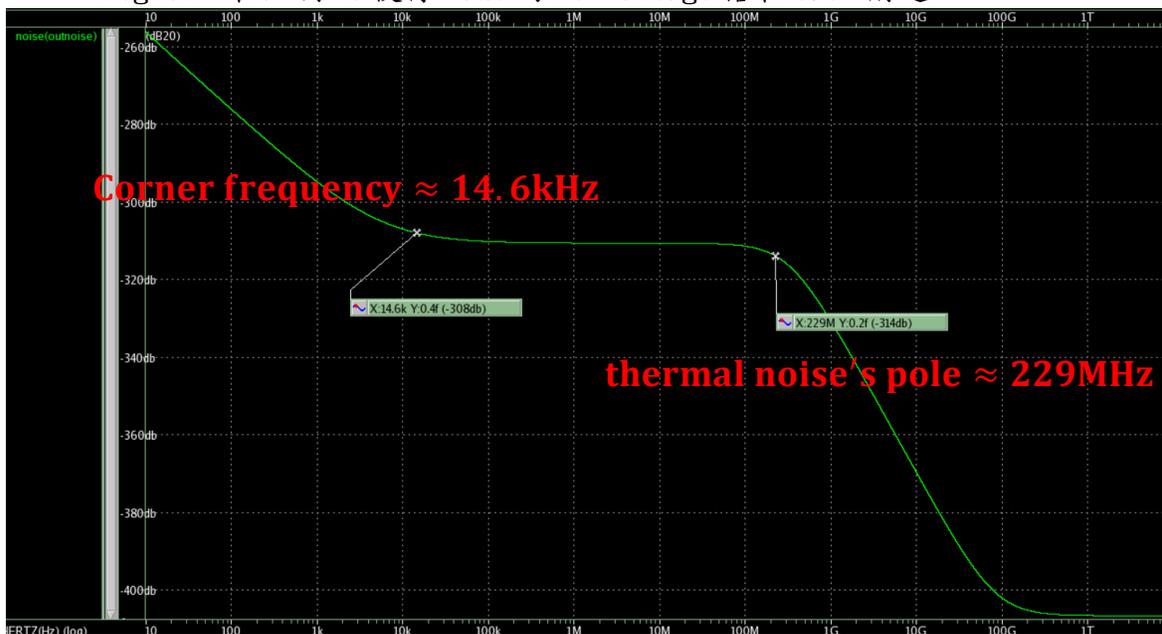
$V_{out} = 500.1410\text{mV} (\approx 0.5\text{V}) ; |gain| = 848.9761\text{m} (> 0.8)$

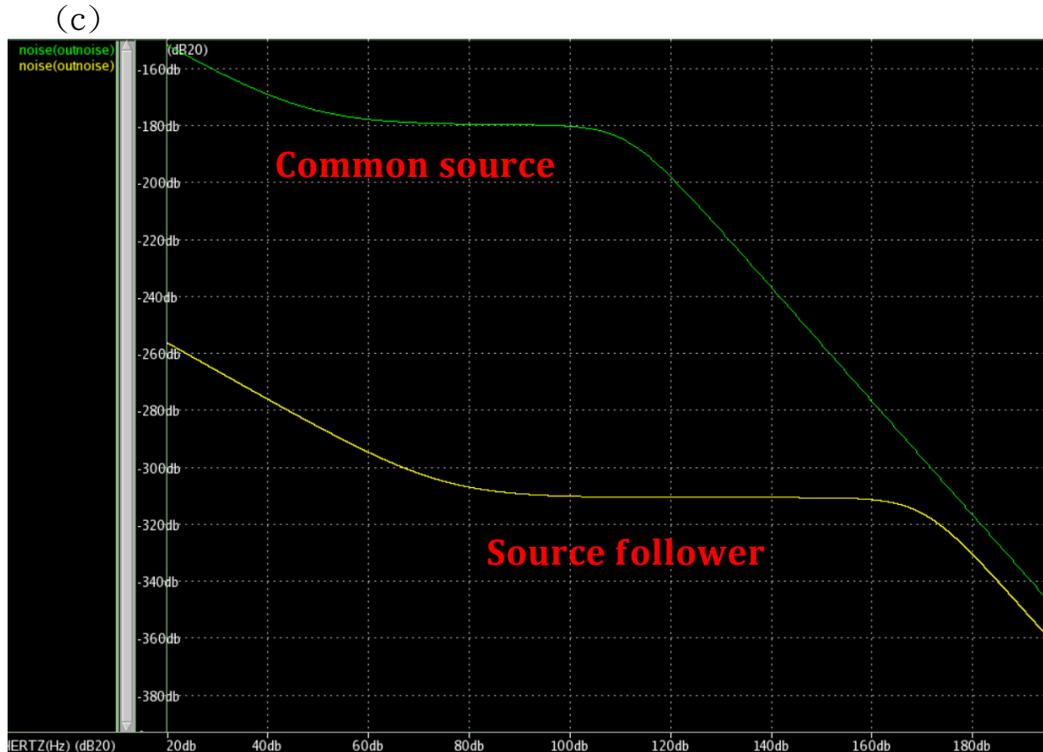
兩顆 NMOS 的 c_{dtot} , c_{gtot} , c_{stot} , c_{btot} 皆 $< 80\text{fF}$

兩顆 NMOS 都在 saturation region

Comment :

做法：一開始我先透過調整 M0 和 M1 的 W、L，使得 c_{gtot} 、 c_{stot} 、 c_{dtot} 、 c_{btot} 沒有超過 80fF ，再來調整 V_b 促使 M0 和 M1 都進入 saturation region，在細調 V_b 使得 V_{out} 的 DC voltage 落在 0.5V 附近。





Result :

此圖我們可以發現，CS(Common source)有比較大的 output flicker noise，SF(Source follower)有比較大的 thermal noise's pole。

Comment :

1. CS(Common source)有比較大的 output flicker noise :

可以先從 flicker noise 的公式來看 $\overline{v_n^2} = \frac{K}{C_{ox}WL} \frac{1}{f} \times Av^2$ ，當WL乘積越

大，flicker noise 越小，我們假設兩者的 C_{ox} 和頻率都相同，因此計算 CS 的 WL 乘積 $(M0 + M1) = 1/(2u \times 4u) + 1/(0.7u \times 14u) = 2.27 \times 10^{11}$

SF 的 WL 乘積 $(M0 + M1) = 1/(5u \times 1u) + 1/(5u \times 1u) = 4 \times 10^{11}$

→ (input referred flicker noise)_{SF} > (input referred flicker noise)_{CS}，然而 $(gain)_{SF} \ll (gain)_{CS}$ ，因此 $(output\ flicker\ noise)_{SF} > (output\ flicker\ noise)_{CS}$ 。

2. SF(Source follower)有比較大的 thermal noise's pole :

可以從 output pole 看出 thermal noise's pole 的大小

CS 的 output pole 表示為 $\omega_{out_CS} = 1/R_{out}(C_{d0tot} + C_{d1tot})$

→ $C_{d0tot} + C_{d1tot} = 2.3778f + 1.0028f = 3.3806f$; $R_{out_CS} = r_{on} || r_{op}$

SF 的 output pole 表示為 $\omega_{out_CS} = 1/R_{out}(C_{s0tot} + C_{d1tot})$

→ $C_{s0tot} + C_{d1tot} = 28.8383f + 6.9729 = 35.8112f$; $R_{out_CS} = \frac{1}{gm_0} || r_{on}$

$\frac{1}{gm} \ll r_o$ → (thermal noise's pole)_{SF} > (thermal noise's pole)_{CS}

(d) total output noise power of CS and SF

File	Equation	Specification		Result		Pass/Fail
		Min	Max	Value	Mean	
D0:hw4_1a.ac0	<code>integ(0 noise(outnoise)',0,25.3M)</code>			603u		Common source Source follower
D1:hw4_1b.ac0	<code>integ(1 noise(outnoise)',0,22900M)</code>			152n		

Result :

Common source 的 total output noise power = 603uW

Source follower 的 total output noise power = 152nW

→ Common source 的 total output noise power 較大

Comment :

做法 :

對 output noise spectrum 從 frequency=0 積分到 frequency 大約等於 100 倍的 thermal noise's pole frequency 得到 total output noise power , 因為此時的 noise 趨於穩定且值很小。

分析 :Output noise power 的公式: $P_{n,out} = \int_{f_1}^{f_2} v_{n,out}^2 df$ CS 的 output noise 可以寫成 $\overline{v_{n,out}^2} = 4kT\gamma(g_{m0} + g_{m1})(r_{o0} || r_{o1})^2$ SF 的 output noise 可以寫成 $\overline{v_{n,out}^2} = 4kT\gamma(g_{m0} + g_{m1})(r_{o1} || 1/g_{m0})^2$ 由於 $r_o \gg \frac{1}{g_m}$, 且 $r_{o0CS} 、 r_{o1CS} > r_{o1SF}$ → $\overline{v_{n,out,CS}^2} \approx 4kT\gamma(g_{m0} + g_{m1})(r_o/2)^2$; $\overline{v_{n,out,SF}^2} \approx 4kT\gamma(g_{m0} + g_{m1})\left(\frac{1}{g_m}\right)^2$ → $\overline{v_{n,out,CS}^2} > \overline{v_{n,out,SF}^2}$

→ CS 的 output noise spectrum 會一直在 SF 的 output noise spectrum 上方

→ CS 的 total noise power 較大。

(e) total input referred noise power of CS and SF

Result :

input referred noise power 的公式: $P_{n,in} = \frac{P_{n,out}}{A_v^2}$ CS input referred noise power = $603u / (-248.8710)^2 = 9,7357nW$ SF input referred noise power = $152n / (848.9671m)^2 = 210.8928nW$

→ source follower 有較大的 input referred noise power

Comment :

分析 :我們可以先從由 $P_{n,out}$ 除以 $(gain)^2$ 求得 $P_{n,in}$ 探討:由於 $(gain)_{CS} \gg (gain)_{SF}$, 因此 $P_{n,out,SF}$ 除以 $(gain)_{SF}^2$ 得到的 $P_{n,in,SF}$ 會比 $P_{n,in,CS}$ 來的大。

再來我們可以從 input referred noise 探討:

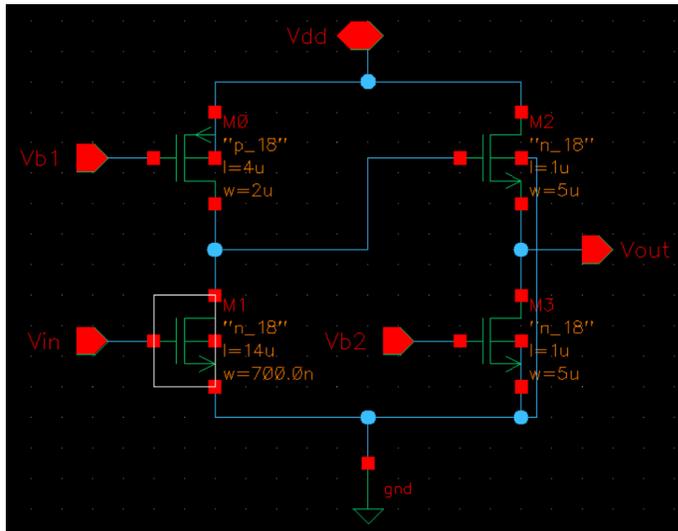
CS 的 input referred noise 可以寫成 $\overline{v_{n,in}^2} = 4kT\gamma(g_{m0} + g_{m1}) / (g_{m0})^2$ SF 的 input referred noise 可以寫成 $\overline{v_{n,in}^2} = 4kT\gamma(g_{m0} + g_{m1}) / (g_{m0})^2$ 由於 $g_{m0} \approx g_{m1} \rightarrow \overline{v_{n,in,SF}^2} \approx 4kT\gamma(2/g_{m0})$

$$\overline{v_{n,in_CS}^2} = 4kT\gamma \times 877007 : \overline{v_{n,in_SF}^2} = 4kT\gamma \times 31421$$

$$P_{n,in} = \int_{f_1}^{f_2} v_{n,in}^2 df$$

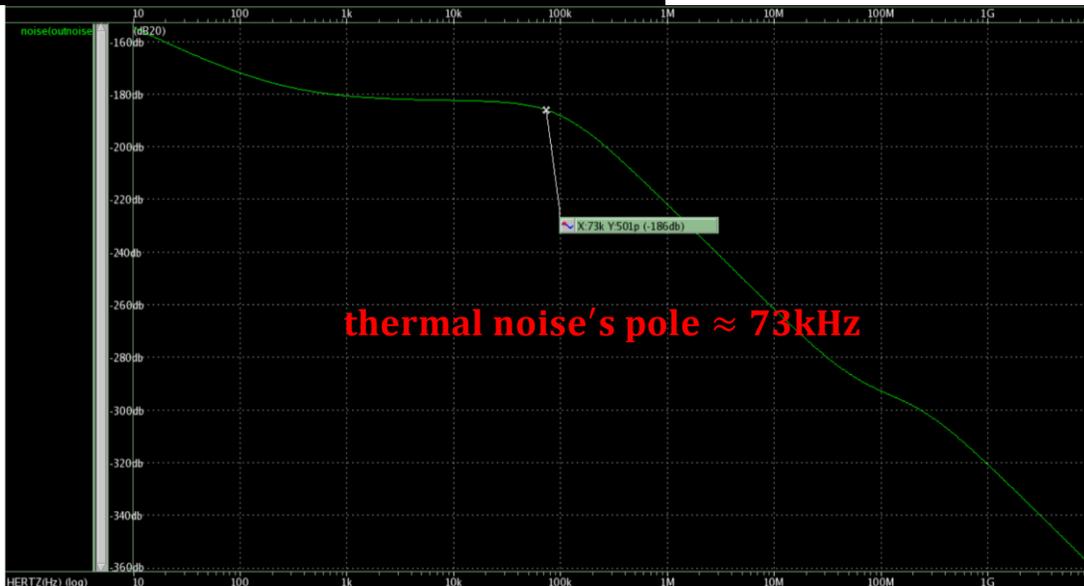
→ source follower 有較大的 input referred noise power

(f) CS + SF



```
**** small-signal transfer characteristics
v(vout)/vin = -211.2860
input resistance at vin = 1.000e+20
output resistance at v(vout) = 13.3379k

node =voltage node =voltage node =voltage
+0:net14 = 1.0002 0:vb1 = 1.2397 0:vb2 = 420.0000m
+0:vdd = 1.8000 0:vin = 500.0000m 0:vout = 500.3514m
```



File	Equation	Specification		Result		Pass/Fail
		Min	Max	Value	Mean	
D0:hw4_1f.ac0	integ(0 noise(outnoise)',0,7300k)/(-211.2860^2)			2.75n		

Result :

$$\text{gain} = -211.2860 \approx -211.2832(-228.8710 \times 848.9671\text{m})$$

$$V_{out} = 500.3514\text{mV} \approx 0.5\text{V}$$

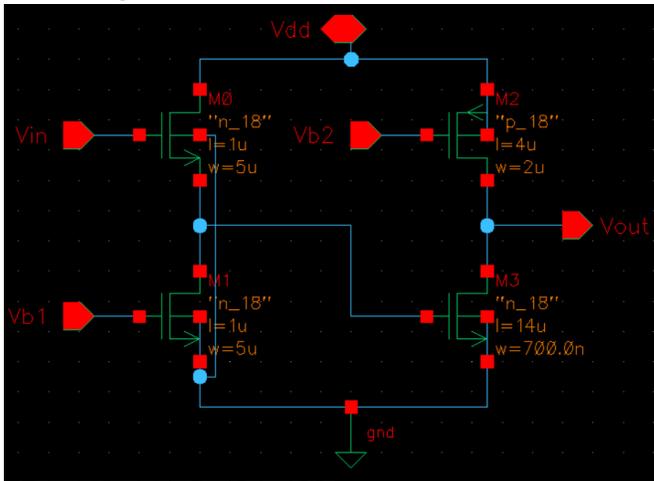
$$P_{n,in} = 2.75\text{uW}$$

Comment :

做法 :

我們先從 output noise 的圖找到 thermal noise's pole frequency，並對 output noise spectrum 從 frequency=0 積分到 frequency 大約等於 100 倍的 thermal noise's pole frequency 得到 total output noise power，再從求得的 $P_{n,out}$ 除以 $(gain)^2$ 得到 $P_{n,in}$ 。

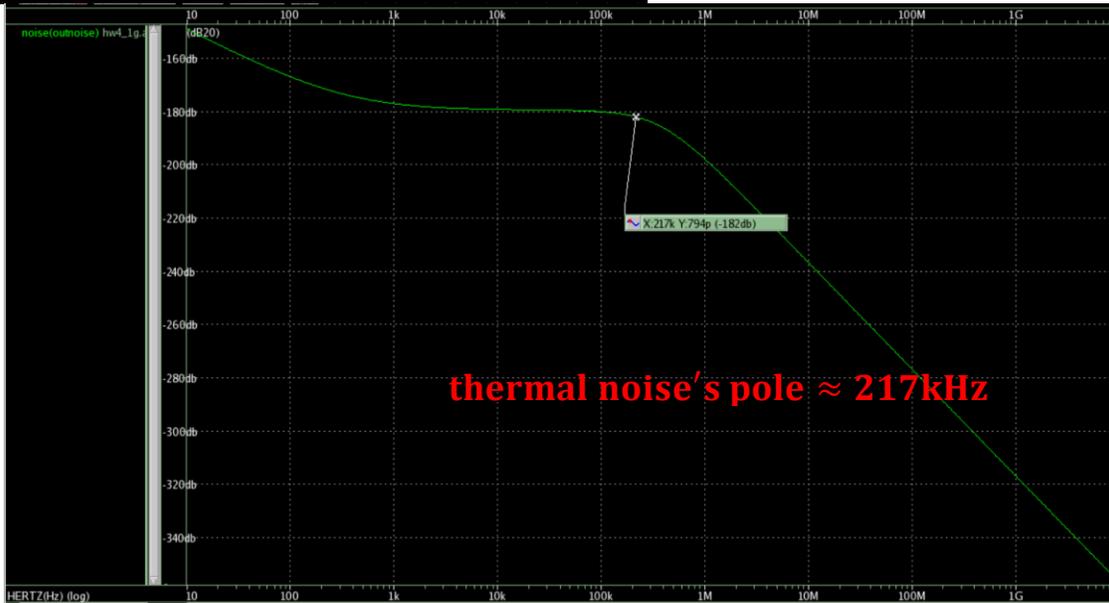
(g) CS + SF



**** small-signal transfer characteristics

v(vout)/vin = -212.6072
 input resistance at vin = 1.000e+20
 output resistance at v(vout) = 126.7744x

node	=voltage	node	=voltage	node	=voltage
+0:net03	= 500.1410m	0:vb1	= 420.0000m	0:vb2	= 1.2397
+0:vdd	= 1.8000	0:vin	= 1.0000	0:vout	= 965.0567m



thermal noise's pole \approx 217kHz

File	Equation	Specification		Result		Pass/Fail
		Min	Max	Value	Mean	
D1:hw4_1g.ac0	inteq(1 noise(outnoise)',0,7300k)/(-211.6072^2)			13.5n		

Result :

$$gain = -212.6072 \approx 211.2832(-228.8710 \times 848.9671m)$$

$$V_{out} = 965.0567mV \approx 1V$$

$$P_{n,in} = 13.5nW$$

Comment :

做法 :

我們先從 output noise 的圖找到 thermal noise's pole frequency，並對 output noise spectrum 從 frequency=0 積分到 frequency 大約等於 100 倍的 thermal noise's pole frequency 得到 total output noise power，再從求得的 $P_{n,out}$ 除以 $(gain)^2$ 得到 $P_{n,in}$ 。

(h) difference between (f)(SF+CS) and (g)(CS+SF)

Result :

SF + CS 有比較大的 input referred noise power

Comment :

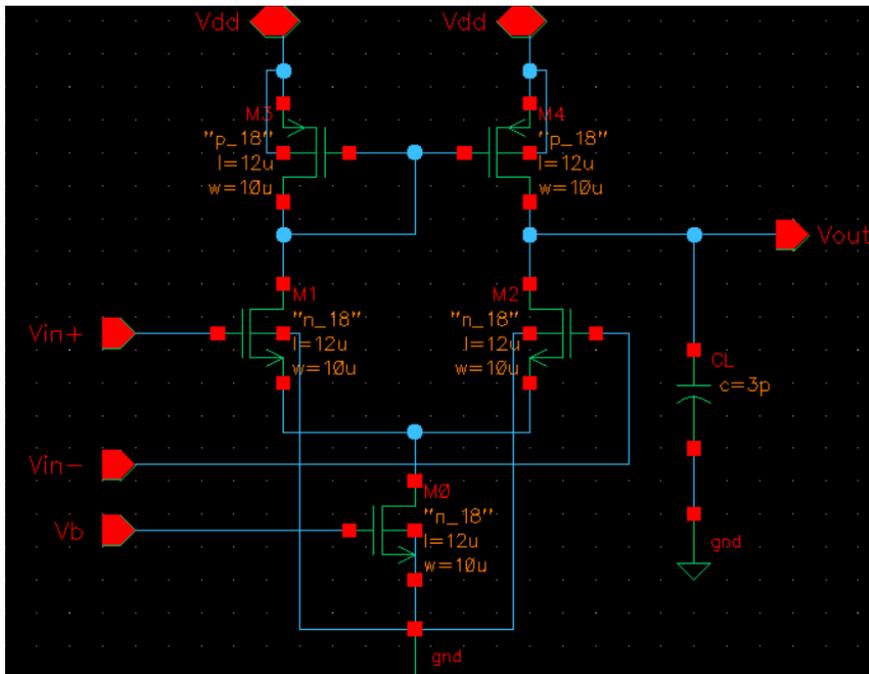
可以從 input referred noise 的公式來看

$$\overline{v_{in}^2} = \frac{\overline{v_{n,out0}^2}}{A_0^2} + \frac{\overline{v_{n,out1}^2}}{(A_0 A_1)^2} \quad A_0: \text{第一級的 gain} : A_1: \text{第二級的 gain}$$

由於 $(gain)_{SF} < (gain)_{CS}$ ，因此 $\overline{v_{in,SF+CS}^2} > \overline{v_{in,CS+SF}^2}$

2.

(a)



small-signal transfer characteristics

$v(vout)/vin+$	=	222.1521
input resistance at $vin+$	=	1.000e+20
output resistance at $v(vout)$	=	8.9305x

Result :

5 顆 MOS 的尺寸都是 $W = 10\mu m, L = 12\mu m$; $V_b = 0.5V$
 V_{in+} : DC = 0.9V, AC = 0.5V V_{in-} : DC = 0.9V, AC = -0.5V

Comment :

先把全部的 MOS 的尺寸 default 成 $W=10\mu\text{m}$, $L=1\mu\text{m}$ ，為了調高 gain，我們增加 MOS 的長度達到 $L=12\mu\text{m}$ ，gain 達到 222.1521。

$$(20 \log|\text{gain}| = 46.93 > 40\text{dB})$$

(b)

```

subckt
element 0:m0      0:m1      0:m2      0:m3      0:m4
model 0:n_18.1  0:n_18.1  0:n_18.1  0:p_18.1  0:p_18.1
region Saturati Saturati Saturati Saturati Saturati
id      3.7930u    1.8965u    1.8965u    -1.8965u   -1.8965u
ibs     -6.138e-22  -304.8433a -304.8433a  1.899e-22  1.899e-22
ibd     -304.8424a -829.2956a -829.2956a  353.4442a  353.4442a
vgs     500.0000m  508.1710m  508.1710m  -734.0669m -734.0669m
vds     391.8290m  674.1040m  674.1040m  -734.0669m -734.0669m
vbs     0.        -391.8290m -391.8290m  0.         0.
vth     316.2203m  391.2768m  391.2768m  -461.1448m -461.1448m
vdsat   164.6968m  124.4757m  124.4757m  -240.8060m -240.8060m
vod     183.7797m  116.8941m  116.8941m  -272.9221m -272.9221m
beta    249.1578u  250.9764u  250.9764u  57.7247u   57.7247u
gam_eff 507.4459m  517.4270m  517.4270m  557.0847m  557.0847m
gm      36.5785u   24.9879u   24.9879u   12.5077u   12.5077u
gds     173.0278n  101.6442n  101.6442n  10.7816n   10.7816n
gmb     7.1970u   4.1550u    4.1550u    3.9420u    3.9420u
cdtot   18.7717f    12.9431f   12.9431f   12.4626f   12.4626f
cgtot   785.9233f    764.7052f  764.7052f  765.1800f  765.1800f
cstot   808.6181f    773.8813f  773.8813f  861.4278f  861.4278f
cbtot   229.4802f    196.1299f  196.1299f  274.5269f  274.5269f
cgs     715.4331f    697.9644f  697.9644f  703.1946f  703.1946f
cgd     4.7792f     2.3749f    2.3749f    4.0704f    4.0704f

```

此 DP 包含兩條路徑：slow path:經過 M3, M4；fast path:經過 M2

$$\text{gain} = A_{\text{slow path}} + A_{\text{fast path}} = \frac{A_0}{1 + \frac{s}{p_1}} \left(\frac{1}{1 + \frac{s}{p_2}} + 1 \right) = \frac{A_0(2 + \frac{s}{p_2})}{(1 + \frac{s}{p_1})(1 + \frac{s}{p_2})}$$

p_1 是指 output pole； p_2 是指 mirrored pole(M3、M4 交界點 E)；

p_3 是指 pseudo ground pole (pg pole)

由上述 gain 我們可以知到 first zero(z_1) = $2p_2$ ， $\omega_E = \omega_{p_2} = 1/R_E C_E$

$$R_E = \frac{1}{g_{m3}} || r_{o1} = \frac{1}{12.5077\text{u}} || \frac{1}{101.6442\text{n}} = 79306.26559 \Omega$$

$$C_E \approx C_{g3 \text{ tot}} + C_{g4 \text{ tot}} + C_{d1 \text{ tot}} + C_{d2} = 1.5562462\text{pF}$$

$$\omega_{p1} \approx 1/(r_{ON} || r_{OP})C_L = 37475.5 \text{ rad/s} = 5964.4\text{Hz}$$

$$\omega_{p2} = 1/R_E C_E = 8.1024 \times 10^6 \text{ rad/s} = 1.2895\text{MHz}$$

$$\omega_{p3} = 1/R_{pg} C_{pg} = 37.146901 \times 10^6 \text{ rad/s} = 5.912112\text{MHz}$$

$$\omega_{z1} = 16.2048 \times 10^6 \text{ rad/s} = 2.579\text{MHz}$$

poles (rad/sec)		poles (hertz)	
real	imag	real	imag
-37.0372k	0.	-5.89465k	0.
-8.11930x	0.	-1.29223x	0.
-37.4646x	0.	-5.96268x	0.

zeros (rad/sec)		zeros (hertz)	
real	imag	real	imag
-16.4867x	0.	-2.62395x	0.
-35.7263x	0.	-5.68601x	0.
2.75627g	0.	438.674x	0.

105060012 張育崧

(紅框 $\rightarrow \omega_{p1}$; 藍框 $\rightarrow \omega_{p2}$; 綠框 $\rightarrow \omega_{p3}$; 黑框 $\rightarrow \omega_{z1}$)

Comment :

算出來的結果與模擬的結果差不多