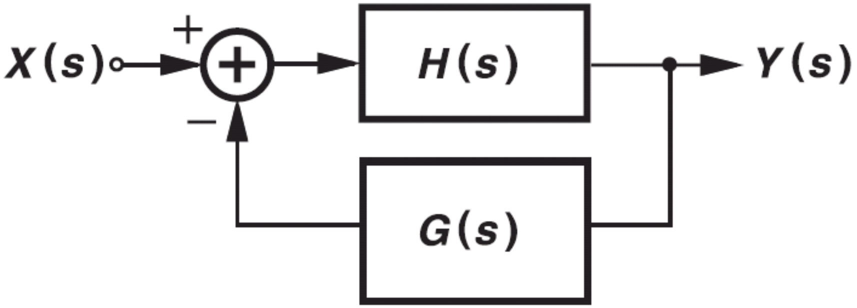
CHAPTER 8

Feedback

Outline

1. **General Consideration**
2. Feedback Topologies
3. Effect of Loading
4. Effect of Feedback on Noise

General Consideration

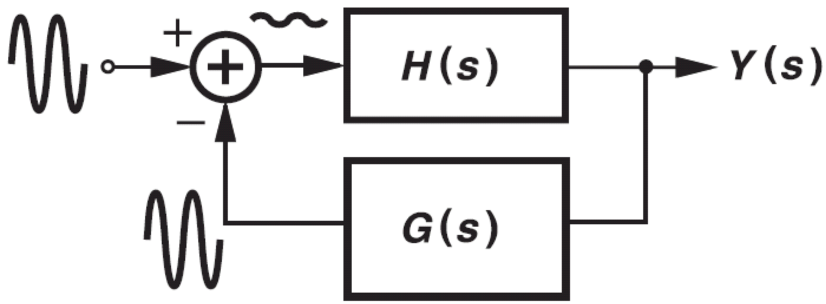


- H(s) : Feedforward network (Represents an amplifier)
- G(s) : Feedback network (β, feedback factor, freq. independent)

$$Y(s) = H(s)[X(s) - G(s)Y(s)], \qquad \frac{Y(s)}{X(s)} = \frac{H(s)}{1 + G(s)H(s)}$$

- X(s) – G(s)Y(s) : The input to H(s), also called feedback error
- H(s) : open loop transfer function
- Y(s)/X(s) : closed loop transfer function

General Consideration



- In a well designed negative feedback system, the error term is minimized, making the output G(s) an accurate copy of the input.
- Input of H(s) as “virtual ground”.
- Four elements in the feedback system
  - The feedforward amplifier.
  - A means of sensing the output.
  - The feedback network.
  - A means of generating the feedback error.

Properties of Feedback Circuits

- Gain Degeneration
- Terminal Impedance Modification
- Bandwidth Modification
- Nonlinearity Reduction

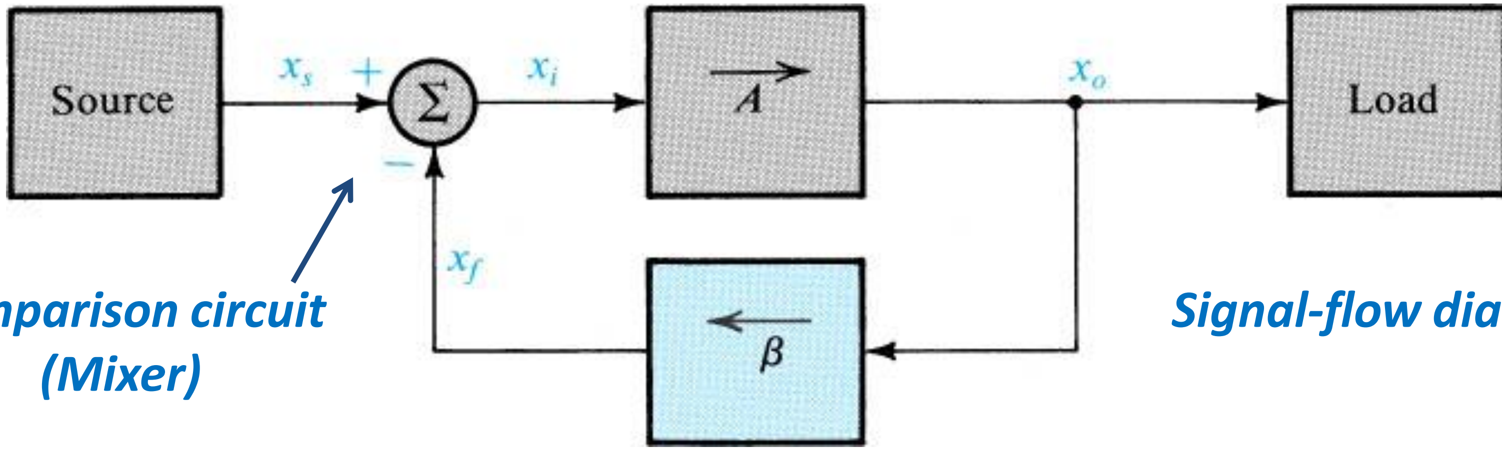
Properties of Feedback Circuits

Negative Feedback properties :

1. ***Desensitize the gain*** :
   - *make gain less sensitive to variations.*
2. ***Reduce nonlinear distortion*** :
   - *make gain independent of signal level.*
3. ***Reduce effect of noise*** :
   - *minimize unwanted signal contribution to output.*
4. ***Control the input and output impedance*** :
   - *use feedback to control impedance.*
5. ***Extend bandwidth** of the amplifier.*

*The basic idea of negative feedback is to **trade off gain** for other desirable properties, like increased input impedance, extended bandwidth… etc.*

Terminologies



Comparison circuit (Mixer)

Signal-flow diagram

1. *Open-loop Gain :* A
2. *Feedback Factor :* β
3. *Loop Gain :* $A\beta$
4. *Amount of Feedback :* $1+A\beta$
5. *Closed-Loop Gain :* $A_f = A/(1+A\beta)$

$$x_o = Ax_i \qquad x_f = \beta x_o \qquad x_i = x_s - x_f$$

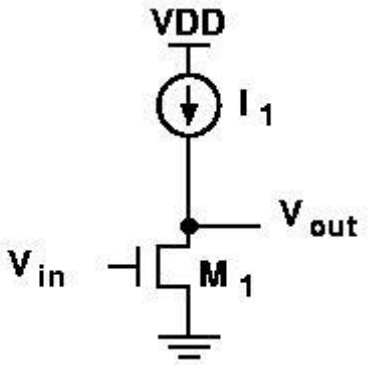
$$A_f \equiv \frac{x_o}{x_s} = \frac{A}{1+A\beta} \qquad A\beta >> 1,\ A_f \cong \frac{1}{\beta}$$

$$x_f = \frac{A\beta}{1+A\beta} x_s \cong x_s \qquad x_i = \frac{1}{1+A\beta} x_s \cong 0$$

*1. A_f is almost determined by β and **independent** of A, that is, the process variation.*
*2. β can be implemented by passive component and **accurate, predictable, stable**.*
*3. x_i : negative feedback **reduces** the input signal of the basic Amp by **(1+Aβ)***

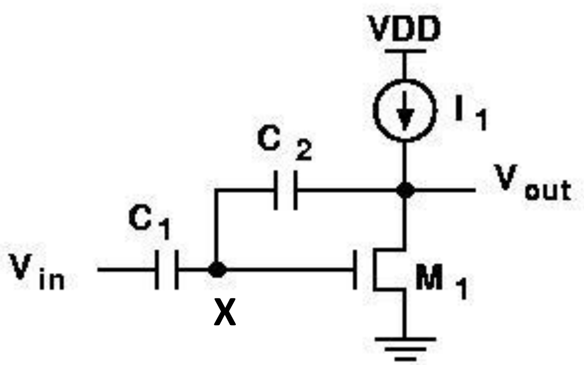
Gain Desensitization

- Gain desensitization



$$A(v) = -g_{m1} r_{o1}$$

- Poor definition of the gain : both g_{m1} and r_{o1} vary with process and temperature.
- For the CS amplifier with feedback (C_1 & C_2)
- The overall voltage gain of the circuit at low freq. such that C_2 does not load the output node



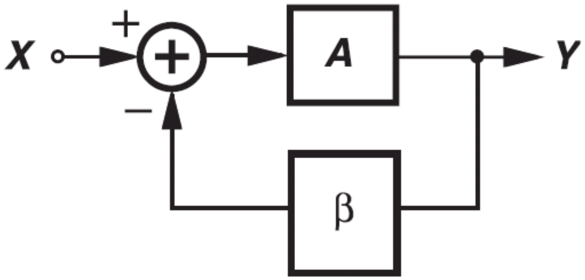
$$A(v) = \frac{V_{out}}{V_X} = -g_{m1} r_{o1} >> 1$$

$$(V_{out} - V_X) C_2 s = (V_X - V_{in}) C_1 s$$

$$\frac{V_{out}}{V_{in}} = -\frac{1}{\left(1 + \frac{1}{g_{m1} r_{O1}}\right) \frac{C_2}{C_1} + \frac{1}{g_{m1} r_{O1}}} \approx -\frac{C_1}{C_2} = -\frac{1/sC_2}{1/sC_1}$$

- Compared to $g_{m1} r_{o1}$, this gain can be controlled with much higher accuracy because it is given by the *ratio* of two capacitors – gain desensitization.

Gain Desensitization

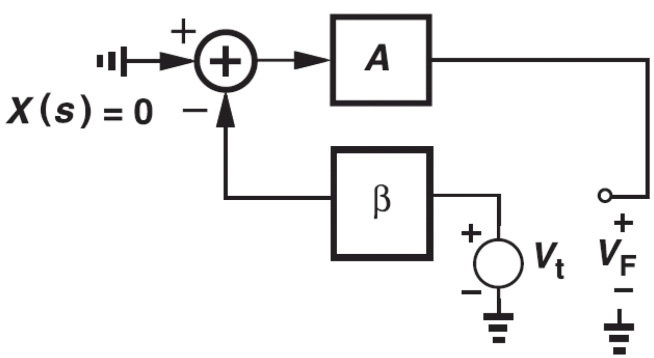


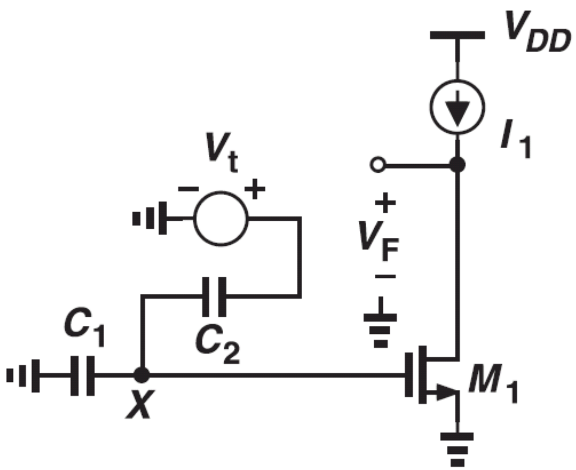
$$\frac{Y}{X} = \frac{A}{1+A\beta} \approx \frac{1}{\beta}\left(1-\frac{1}{A\beta}\right) \approx \frac{1}{\beta} \quad \text{if} \quad A\beta >> 1$$

- In a feedback system, the closed-loop gain is much less sensitive to device parameters than the open-loop gain is.
- The closed loop gain varies by a small percentage even if the open loop gain A varies a lot if the loop gain (βA) >> 1.
- The higher the loop gain (βA), the less sensitive Y/X will be to variations in *A*.
  - We begin with a high-gain amplifier and apply feedback to obtain a low, but less sensitive closed-loop gain.

Loop Gain

- Calculation of loop gain
  - Set the main input to zero.
  - Break the loop at some point.
  - Inject a test signal in the right direction.
  - Obtain the value that returns to the break point.

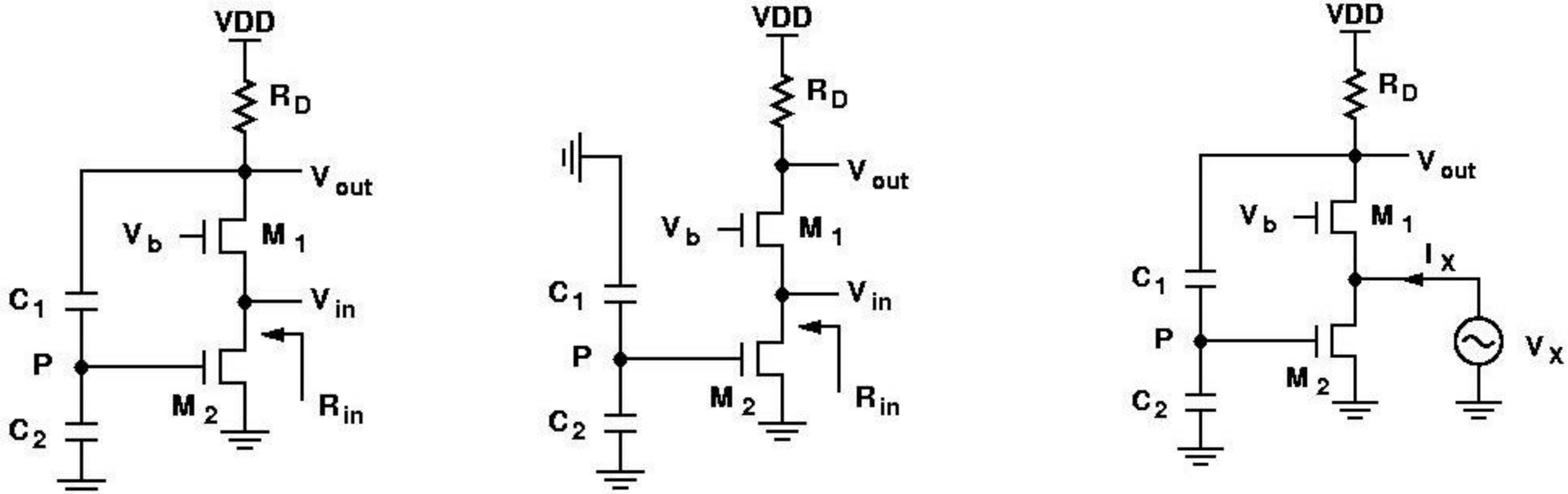




$$V_t \frac{C_2}{C_1 + C_2}(-g_{m1} r_{O1}) = V_F, \quad \frac{V_F}{V_t} = -\frac{C_2}{C_1 + C_2} g_{m1} r_{O1} = -A\beta$$

Input Impedance Modification

- Common gate circuit with feedback (capacitive voltage divider).



- ***The input resistance without feedback*** $R_{in,open} = (g_{m1} + g_{mb1})^{-1}$
- Consider the ***input resistance with feedback***, as

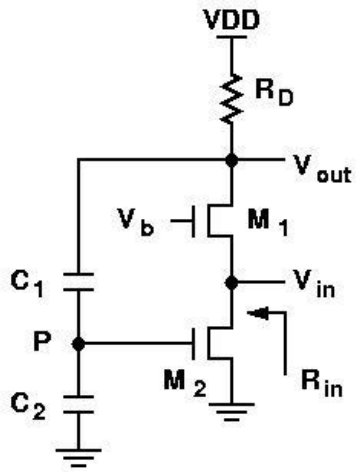
$$V_{out} = \left(g_{m1} + g_{mb1}\right) V_X R_D,\ V_P = V_{out} \frac{C_1}{C_1 + C_2} = (g_{m1} + g_{mb1}) V_X R_D \frac{C_1}{C_1 + C_2},\ I_{M2} = g_{m2}(g_{m1} + g_{mb1}) V_X R_D \frac{C_1}{C_1 + C_2}$$

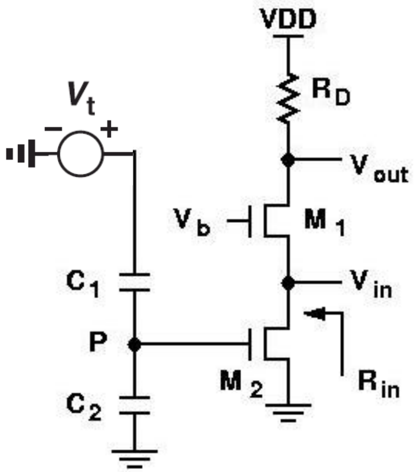
$$I_X = (g_{m1} + g_{mb1}) V_X + g_{m2}(g_{m1} + g_{mb1}) \frac{C_1}{C_1 + C_2} R_D V_X,\ R_{in,closed} = V_X / I_X = \frac{1}{g_{m1} + g_{mb1}} \frac{1}{1 + g_{m2} R_D \frac{C_1}{C_1 + C_2}}$$

- ***The Loop gain***

$$g_{m2} R_D \frac{C_1}{C_1 + C_2} = A\beta \qquad = R_{in,open} \frac{1}{1 + A\beta}$$

Loop Gain



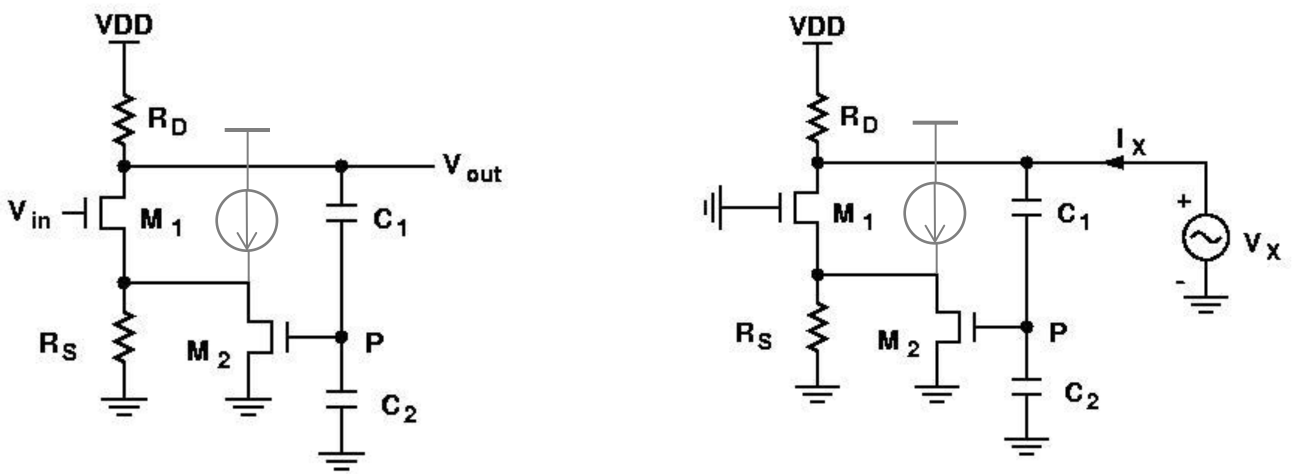


- The feedforward amplifier : M_1 and R_D ($A = R_D$)
- Output sensed by C_1 and C_2.
- The feedback network : C_1, C_2 and M_2 ($\beta = g_{m2}\frac{C_1}{C_1+C_2}$)
- The subtraction occurs in the current domain at the input terminal.
- Loop gain = $A\beta$

$$A\beta = R_D g_{m2}\frac{C_1}{C_1+C_2}, \quad \left(\frac{V_{out}}{V_t} = -A\beta\right)$$

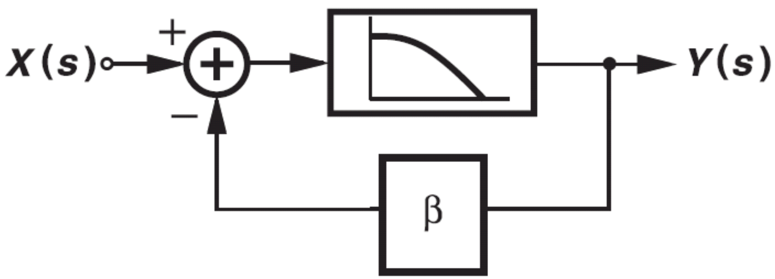
Output Impedance Modification

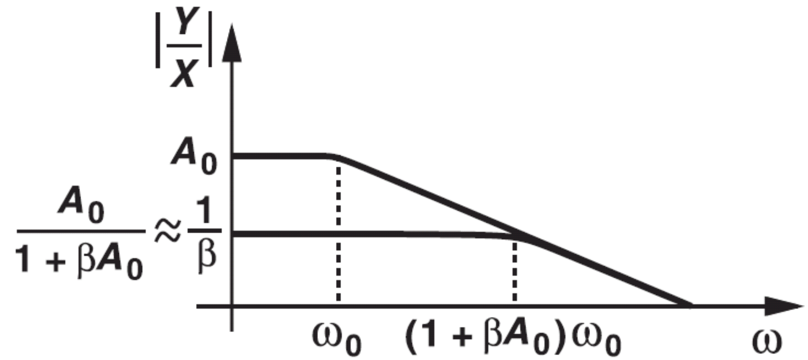
- Common source stage with feedback.
- Common source stage: M_1, R_S and $R_{D.}$
- Feedback network sense the V_{out}, returning a current equal to $\left[\frac{C_1}{C_1+C_2}\right]V_{out}g_{m2}$
- To find the output resistance at relatively low frequencies



$$I_{D1} = V_X \frac{C_1}{C_1+C_2} g_{m2} \frac{R_S}{R_S + \dfrac{1}{g_{m1}+g_{mb1}}}, \qquad I_X = \frac{V_X}{R_D} + I_{D1}, \qquad \frac{V_X}{I_X} = \frac{R_D}{1+\dfrac{g_{m2}R_S\left(g_{m1}+g_{mb1}\right)R_D}{\left(g_{m1}+g_{mb1}\right)R_S+1}\dfrac{C_1}{C_1+C_2}} = \frac{R_D}{1+A\beta}$$

Bandwidth Modification





- Suppose the feedforward amplifier has a one-pole transfer function $A(s)=\dfrac{A_0}{1+s/\omega_0}$
- The transfer function of the closed loop system is

$$\frac{Y}{X}(s)=\frac{\dfrac{A_0}{1+s/\omega_0}}{1+\beta\dfrac{A_0}{1+s/\omega_0}}=\frac{A_0}{1+\beta A_0+\dfrac{s}{\omega_0}}=\frac{\dfrac{A_0}{1+\beta A_0}}{1+\dfrac{s}{\omega_0\left(1+\beta A_0\right)}}$$

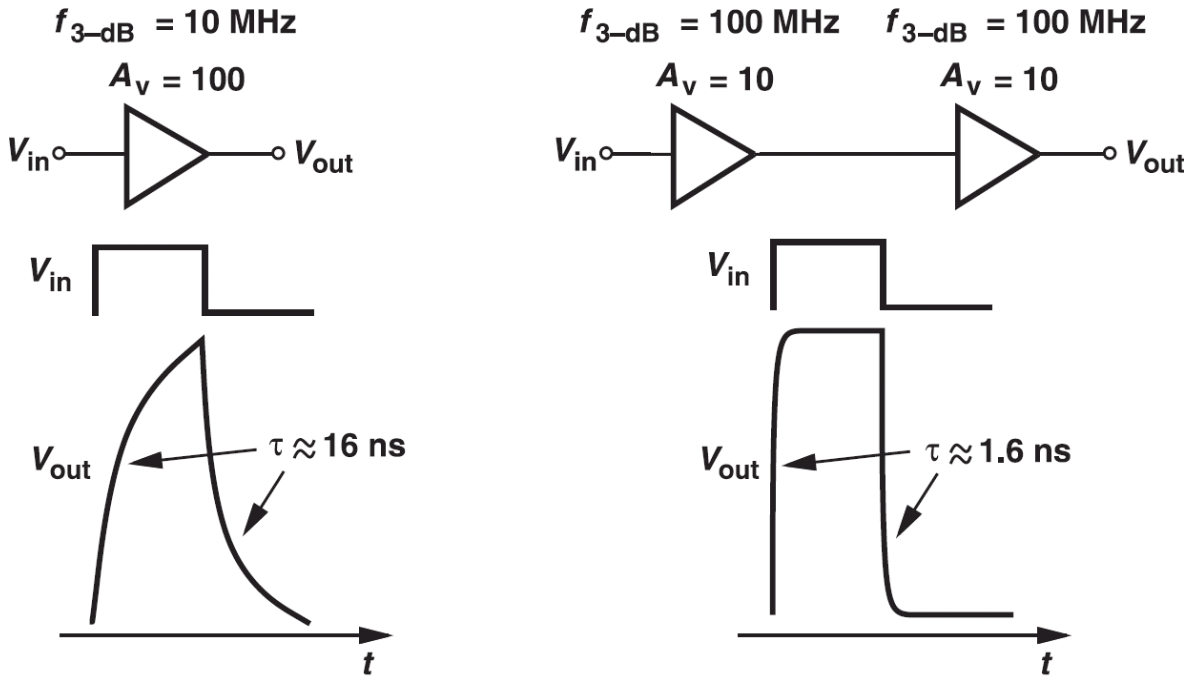
- The -3dB bandwidth has increased by a factor $1+\beta A_0$, albeit at the cost of a proportional reduction in the gain.
- If A is large, the closed loop gain remains approximately equal to $1/\beta$

Bandwidth Modification Example

- Suppose we need to amplify a 20-MHz square wave by a factor of 100 and maximum bandwidth but we have only a single-pole amplifier with an open loop gain of 100 and -3 dB bandwidth of 10 MHz.

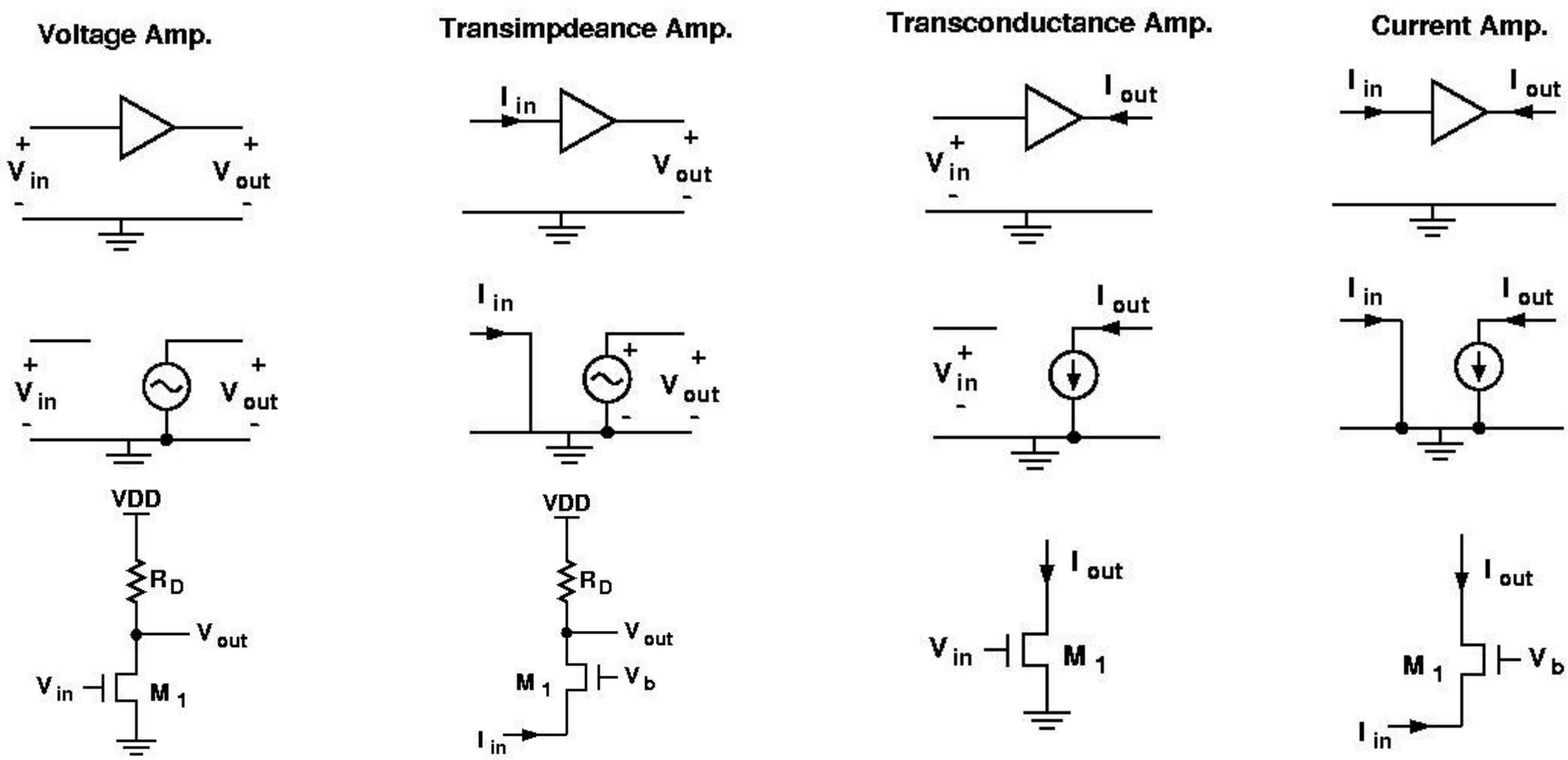
(a) With open-loop amplifier, the risetime and falltime is long: $\frac{1}{2\pi f_{3-dB}} \approx 16ns$

(b) Placing two of the amplifiers with feedback in cascade to achieve the same gain. The power dissipation is doubled.



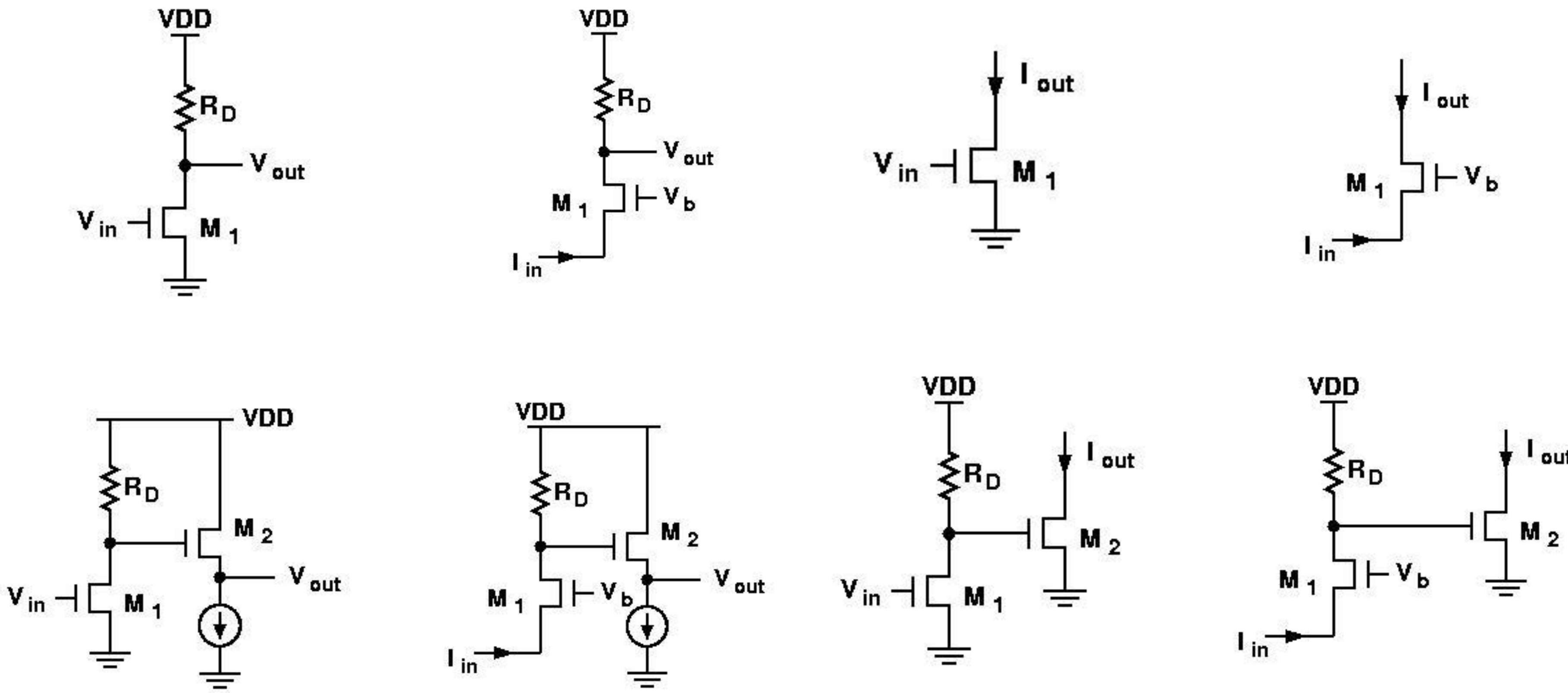
Types of Amplifiers

- Circuits sensing a voltage must exhibit a high Z_{in} (as a voltmeter), circuits sensing a current must provide a low Z_{in} (as a current meter).
- Circuits generating a voltage must exhibit a low Z_{out} (as a voltage source), circuits generating a current must provide a high Z_{out} (as a current source).



Amplifiers with Improved Performance

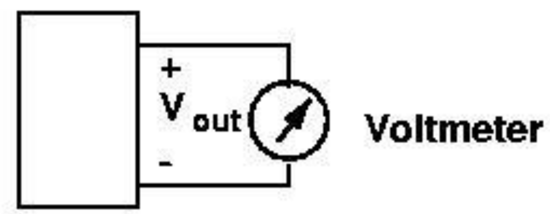
- The basic circuits may not provide adequate performance in many applications.
- Use modified circuits to alter the output impedance or increase the gain.



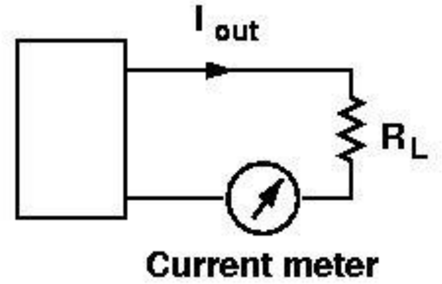
Sense and Return Mechanisms

- Four type of feedback : voltage-voltage (series-shunt), voltage-current (shunt-shunt), current-current (shunt-series), and current voltage (series-series).
- The ***first*** entry in each case denotes ***the quantity sensed at the output*** and the ***second*** the type of ***signal returned to the input***.

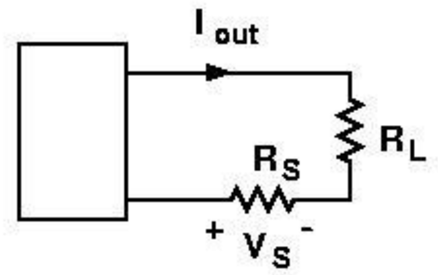
➢ Sensing a voltage by a voltmeter



➢ Sensing a current by a current meter



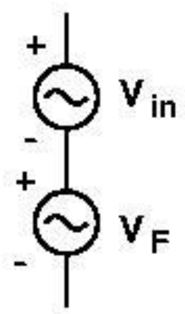
➢ Sensing a current by a small resistor



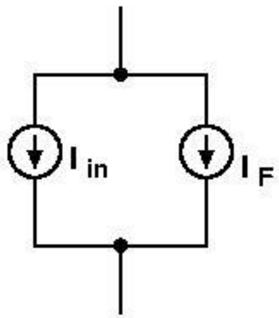
- To sense a current, a current meter is inserted in series with the signal.
- The addition of the feedback signal and the input signal can be performed in the voltage domain or current domain.

Return Mechanisms

- Voltage mode addition



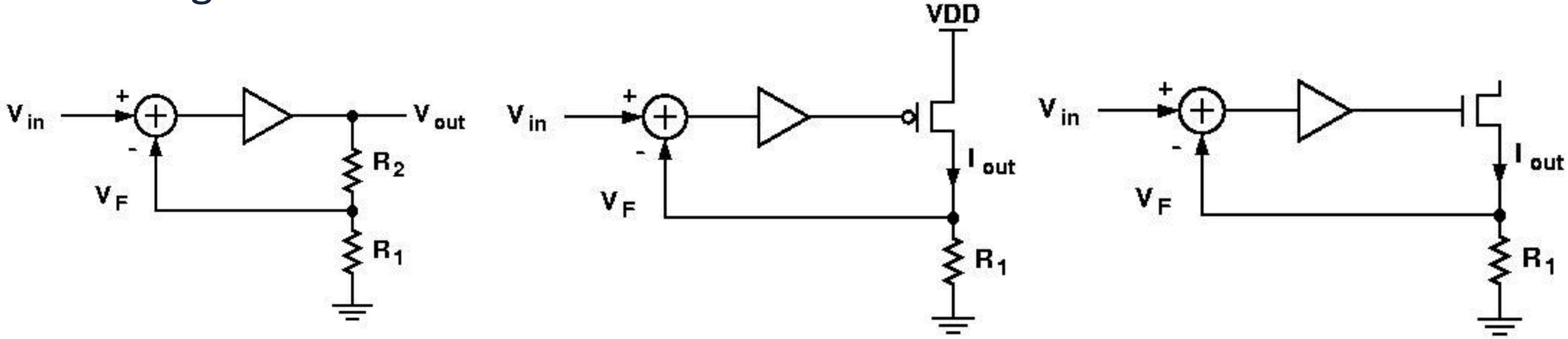
- Current mode addition



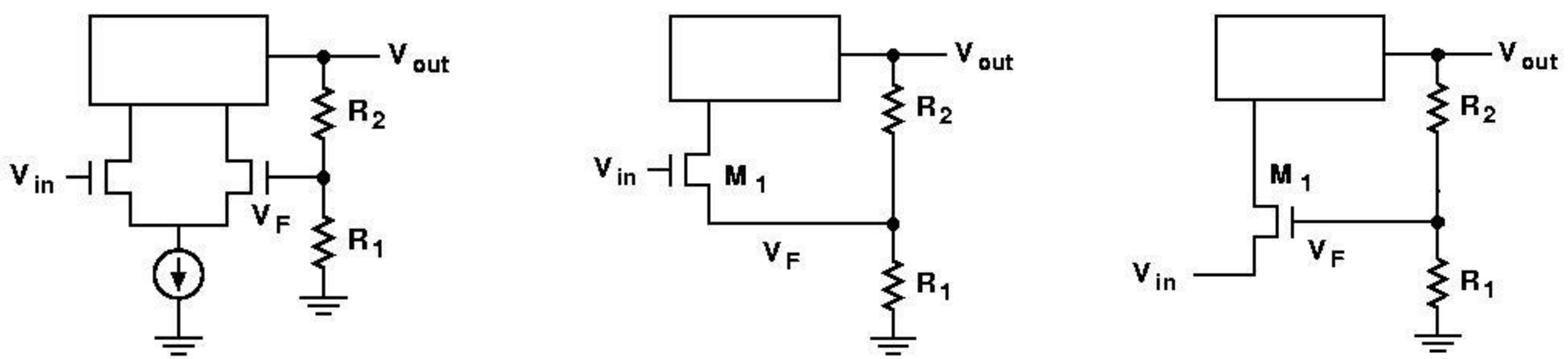
- To add two quantities, we place them in series if they are voltages and in parallel if they are current.
- The feedback network in reality introduces loading effects that must be taken into account.

Practical Examples

- Voltage can be sensed by a resistive / capacitive divider in parallel with the port.
- A current can be sensed by placing a resistor in series with the wire and sensing the voltage across it.

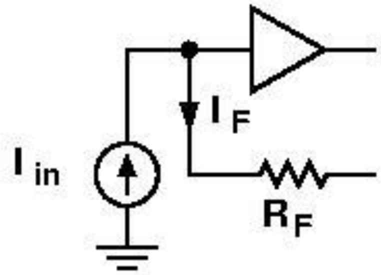


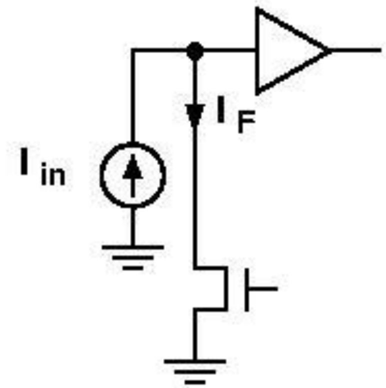
- To subtract two voltages, a differential pair or a single transistor can be used.



Sense and Return Mechanism

- Subtraction of currents can be accomplished as follows.





- In summary
  - For voltage subtraction, the input and feedback signals are applied to two distinct nodes.
  - For current subtraction, they are applied to a single node.
  - It help to identify the type of feedback

Outline

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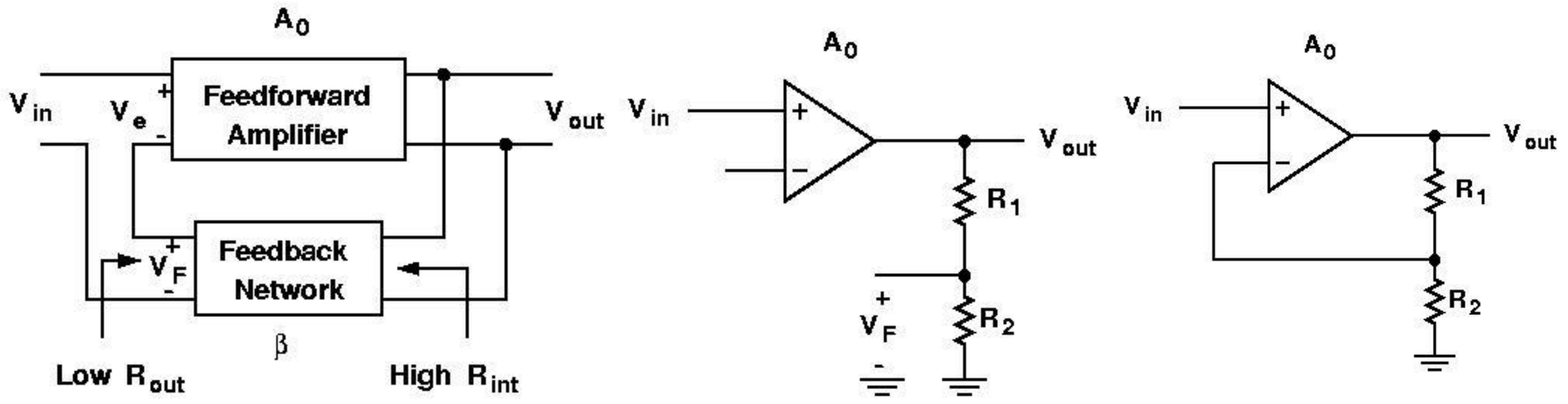
2. **Feedback Topologies**
3. Effect of Loading
4. Effect of Feedback on Noise

Voltage-Voltage (V-V) Feedback

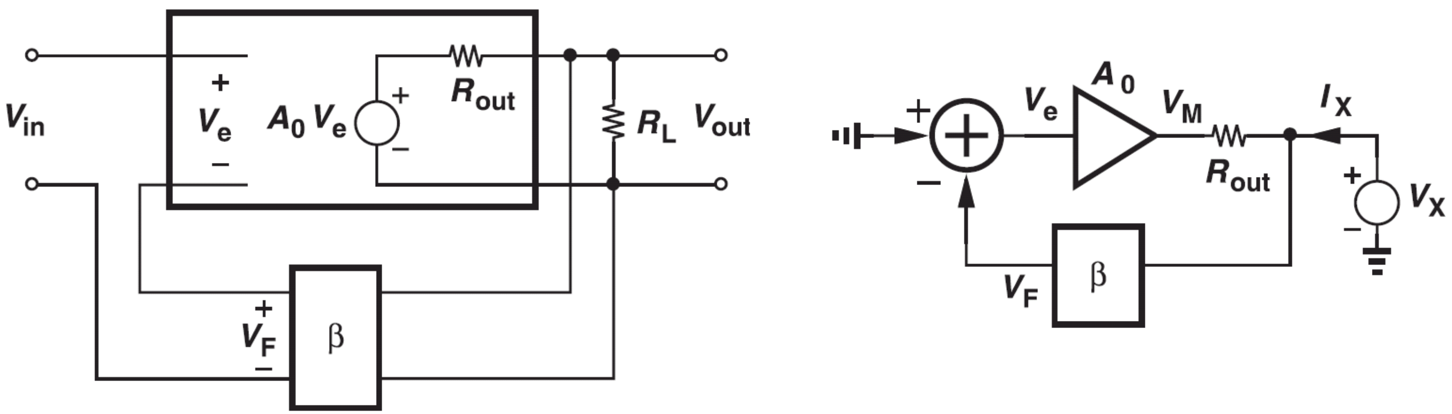
- Voltage-voltage feedback (series - shunt)
  - samples the output voltage and returns the feedback signal as a voltage.



- The feedback network is connected in parallel with the output and in series with the input port.
- An ideal feedback network in this case exhibits infinite input impedance and zero output impedance.

$$V_F = \beta V_{out} \qquad V_e = V_{in} - V_F \qquad V_{out} = A_0 (V_{in} - \beta V_{out}) \qquad \frac{V_{out}}{V_{in}} = \frac{A_0}{1 + \beta A_0} \qquad \beta = \frac{R_2}{R_1 + R_2}$$

Effect of V-V Feedback on R_{out}



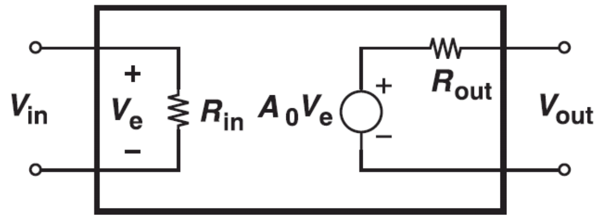
- If the amplifier is loaded by a resistor R_L
  - Consider a voltage amplifier without feedback (*open-loop* configuration), the output would drop in proportional to $R_L / (R_L + R_{out})$
  - Consider a feedback amplifier, if loop gain remains much greater than unity

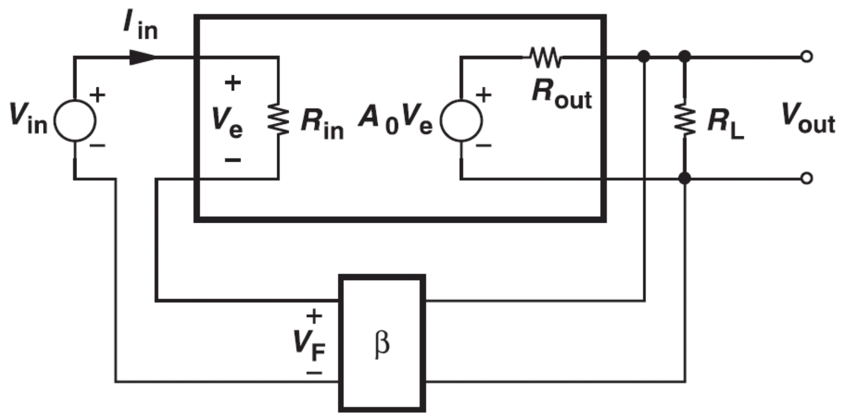
$$V_{out} / V_{in} \approx 1/\beta$$

  - The circuit stabilizes the output voltage amplitude despite load variations, it behaves as a voltage source, thus exhibiting a low output impedance.

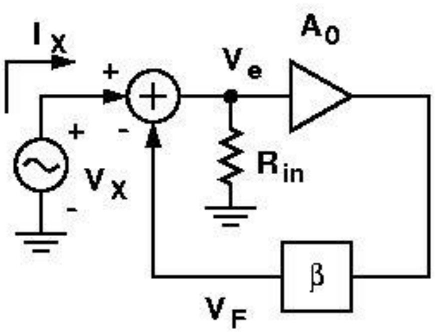
$$V_F = \beta V_X \qquad V_e = -\beta V_X \qquad V_M = -\beta A_0 V_X \qquad I_X = [V_X - (-\beta A_0 V_X)] / R_{out} \qquad \frac{V_X}{I_X} = \frac{R_{out}}{1+\beta A_0}$$

Effect of V-V Feedback on R_{in}





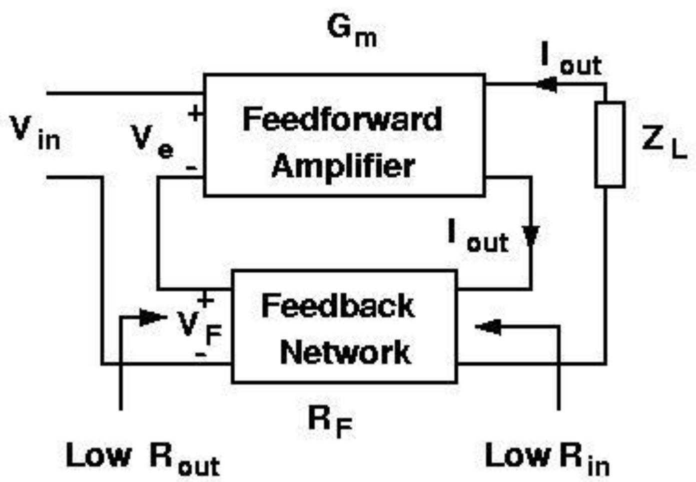
- For the open loop Amp, the R_{in} of the FF Amp sustains the entire V_{in}.
- For the closed loop Amp, the R_{in} of the FF Amp sustains only a fraction of V_{in}.
- The $I(R_{in})$ in the FB topology is less than that in the open-loop system.
- Returning a voltage quantity to the input increases the input impedance.

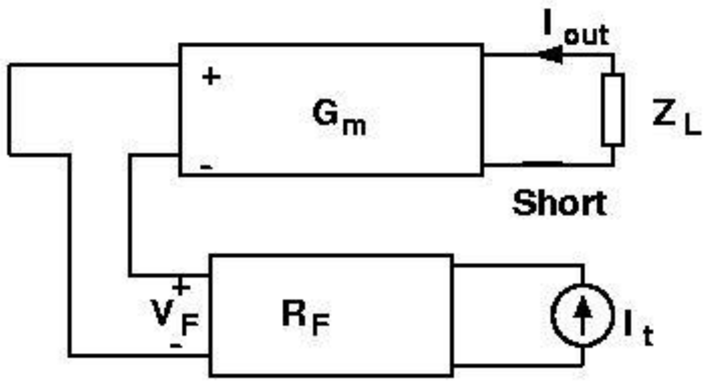


$$V_e = I_X R_{in} \qquad V_F = \beta A_0 I_X R_{in} \qquad V_e = V_X - V_F = V_X - \beta A_0 I_X R_{in}$$

$$I_X R_{in} = V_X - \beta A_0 I_X R_{in} \qquad \frac{V_X}{I_X} = R_{in}(1+\beta A_0)$$

Current-Voltage (I-V) Feedback





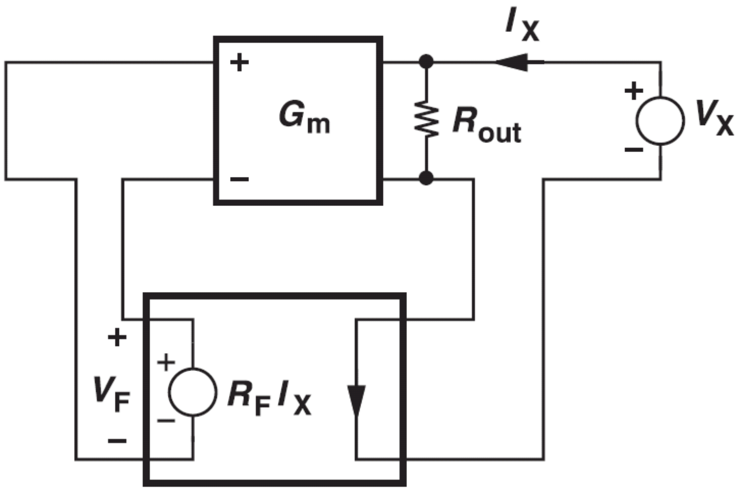
- Sense the output current to perform feedback. (series – series)
- The current is usually sensed by placing a small resistor in series with the output and using the voltage across the resistor as the feedback information.
- The feedback factor β (R_F) .

$$V_F = R_F I_{out}, \quad V_e = V_{in} - R_F I_{out}, \quad I_{out} = G_m \left(V_{in} - R_F I_{out}\right) \quad \frac{I_{out}}{V_{in}} = \frac{G_m}{1 + G_m R_F}$$

- An ideal FB network in this case exhibits zero input and output impedance .
- The loop gain = $G_m R_F$. $V_F = R_F I_t, \quad I_{out} = -G_m R_F I_t, \quad -\frac{I_{out}}{I_t} = G_m R_F$

R_{in}/R_{out} of I-V Feedback Amplifier

- Output resistance of a current-voltage feedback amplifier

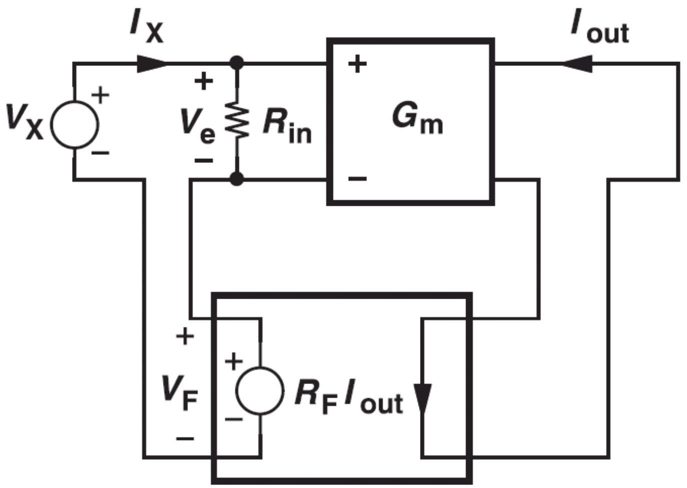


$$V_F = R_F I_X$$

$$-R_F I_X G_m = I_X - V_X / R_{out}$$

$$\frac{V_X}{I_X} = R_{out}(1 + G_m R_F)$$

- Input resistance of a current-voltage feedback amplifier

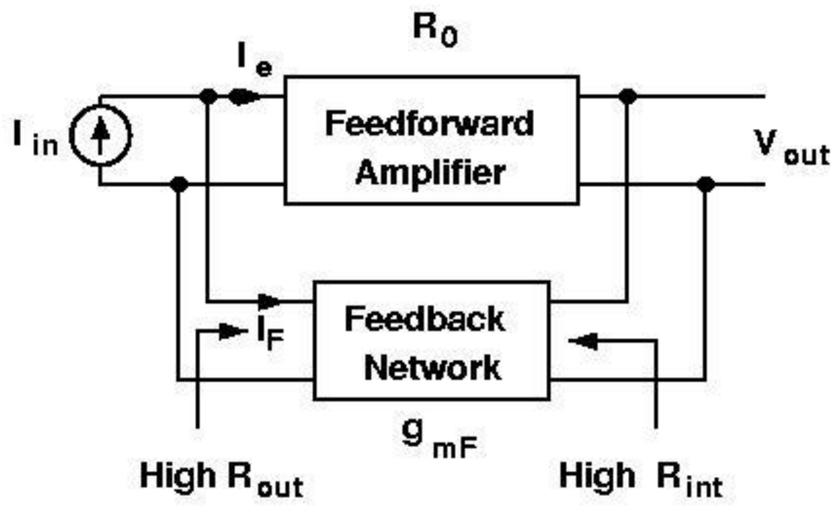


$$R_{in} I_X G_m = I_{out}$$

$$V_e = V_X - G_m R_F I_X R_{in}$$

$$\frac{V_X}{I_X} = R_{in}(1 + G_m R_F)$$

Voltage-Current (V-I) Feedback



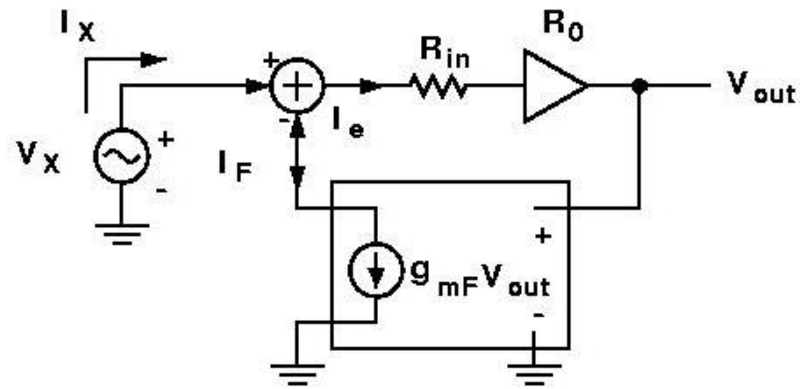
- The output voltage is sensed and a proportional current is returned to the summing point at the input. (shunt – shunt)
- The feedforward path incorporates a transimpedance amplifier with gain R_0.
- The feedback factor has a dimension of conductance.
- The feedback network ideally exhibiting infinite input and output impedance.

$$I_F = g_{mF} V_{out} \qquad I_e = I_{in} - I_F \qquad V_{out} = R_0 I_e = R_0 \left(I_{in} - g_{mF} V_{out} \right) \qquad \frac{V_{out}}{I_{in}} = \frac{R_0}{1 + g_{mF} R_0}$$

- The Loop gain : $g_{mF} R_0$

R_{in}/R_{out} of V-I Feedback Amplifier

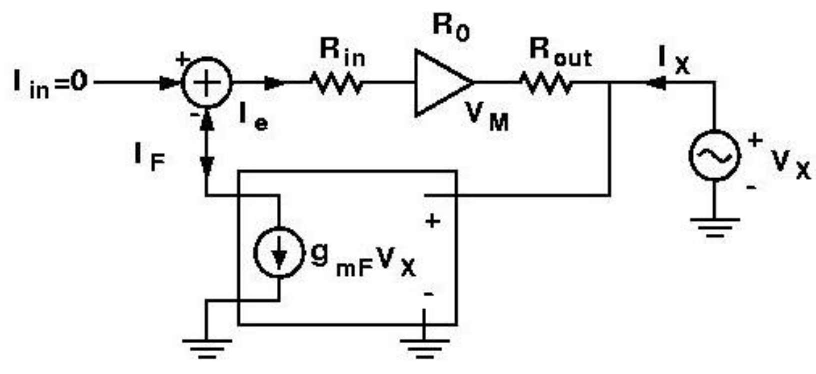
- R_{in} of a voltage-current feedback amplifier.
- The R_{in} of R_0 is placed in series because an ideal transimpedance amplifier exhibits a zero input impedance.



$$I_F = I_X - V_X / R_{in} \qquad (V_X / R_{in}) R_0 g_{mF} = I_F$$

$$\frac{V_X}{I_X} = \frac{R_{in}}{1 + R_0 g_{mF}}$$

- Output impedance of a voltage-current feedback amplifier.

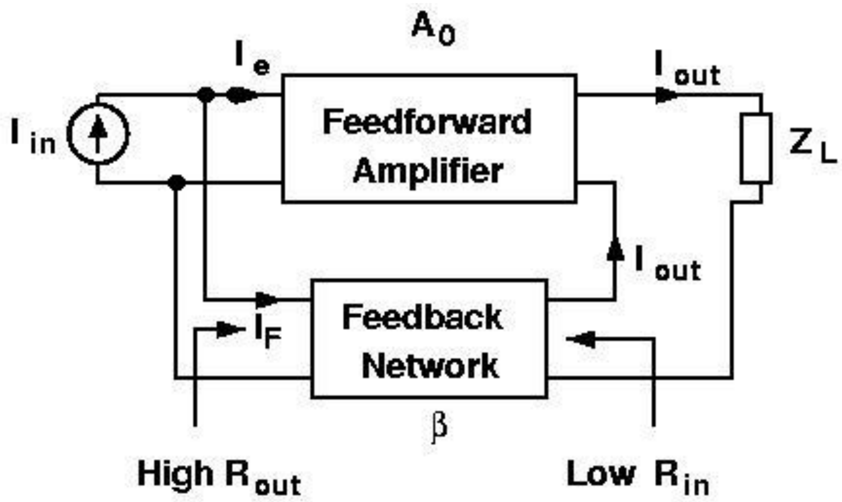


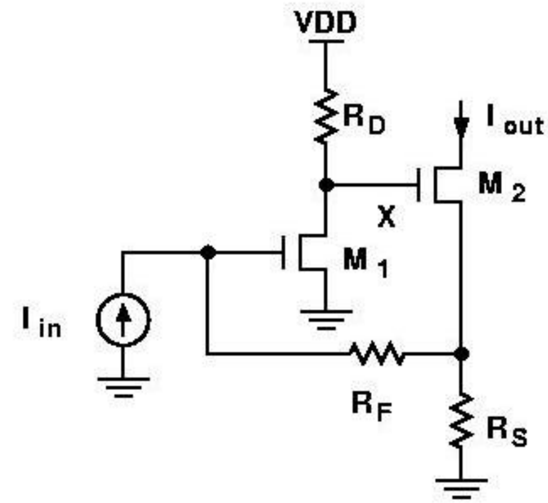
$$I_F = V_X g_{mF} \qquad I_e = -I_F \qquad V_M = -R_0 g_{mF} V_X$$

$$I_X = (V_X - V_M) / R_{out} = (V_X + g_{mF} R_0 V_X) / R_{out}$$

$$\frac{V_X}{I_X} = \frac{R_{out}}{1 + g_{mF} R_0}$$

Current-Current (I-I) Feedback





- The feedforward amplifier is characterized by a current gain A_I. (shunt – series)
- The feedback network by a current ratio β.
- The closed loop current gain is $$A_{if} = \frac{A_i}{1 + A_i\beta}$$
- The input resistance is $$R_{if} = \frac{R_i}{1 + A_i\beta}$$
- The output resistance is $$R_{of} = (1 + A_i\beta)R_{out}$$

Outline

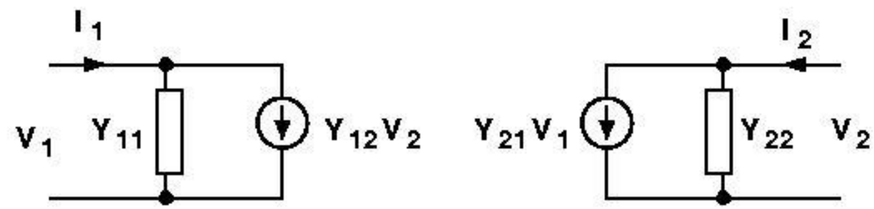
1. General Consideration
2. Feedback Topologies
3. **Effect of Loading**
4. Effect of Feedback on Noise

Two-Port Network Models

- For current-voltage feedback (Z)

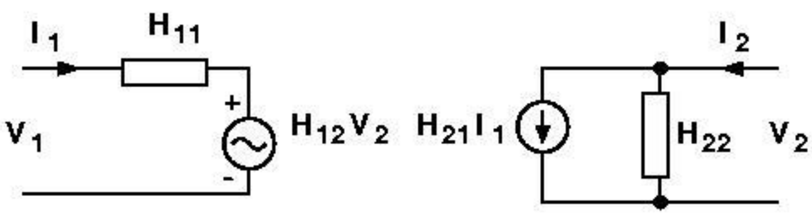
$$V_1 = Z_{11}I_1 + Z_{12}I_2$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

- For voltage-current feedback (Y)



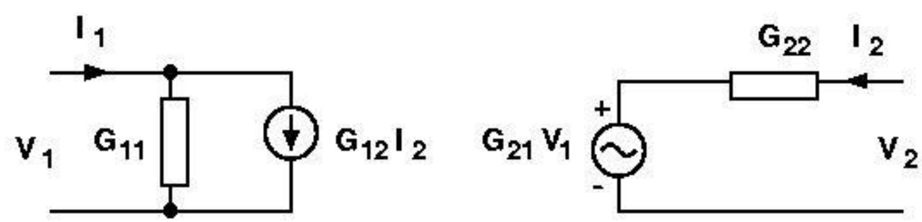
$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

- For current-current feedback (H)



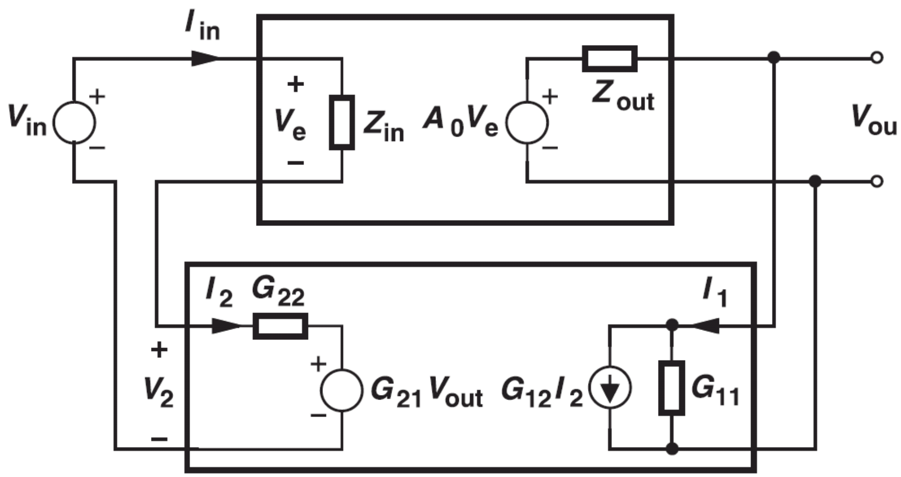
$$V_1 = H_{11}I_1 + H_{12}V_2$$
$$I_2 = H_{21}I_1 + H_{22}V_2$$

- For voltage-voltage feedback (G)

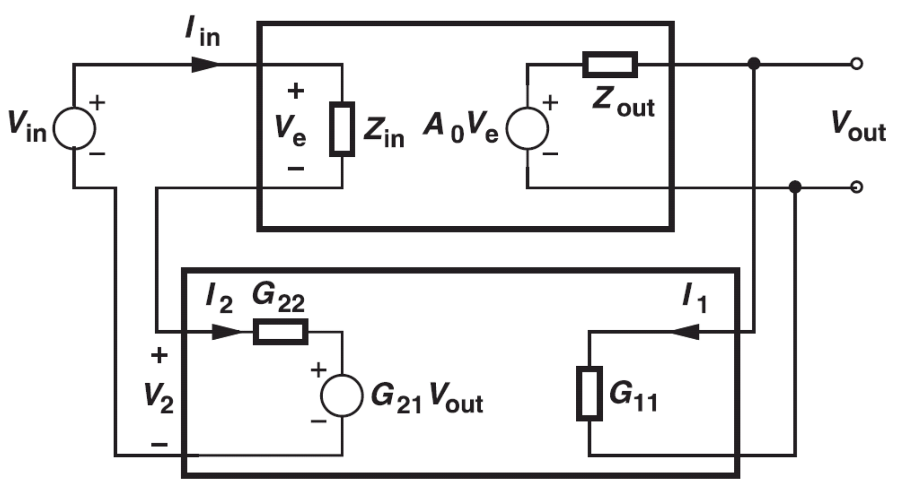


$$I_1 = G_{11}V_1 + G_{12}I_2$$
$$V_2 = G_{21}V_1 + G_{22}I_2$$

Loading in V-V Feedback



(a)



(b)

- If A_0 is large, the signal amplified by A_0 is much greater than the contribution of $G_{12}I_2$.

$$V_e = (V_{in} - G_{21}V_{out})\frac{Z_{in}}{Z_{in}+G_{22}}$$

$$(V_{in} - G_{21}V_{out})\frac{Z_{in}}{Z_{in}+G_{22}}A_0\frac{G_{11}^{-1}}{G_{11}^{-1}+Z_{out}} = V_{out}$$

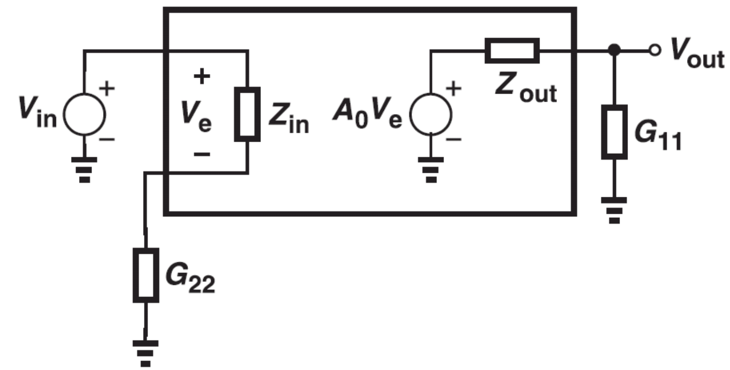
$$\frac{V_{out}}{V_{in}} = \frac{A_0\frac{Z_{in}}{Z_{in}+G_{22}}\frac{G_{11}^{-1}}{G_{11}^{-1}+Z_{out}}}{1+\frac{Z_{in}}{Z_{in}+G_{22}}\frac{G_{11}^{-1}}{G_{11}^{-1}+Z_{out}}G_{21}A_0}$$

If $G_{11}^{-1} = \infty, \quad G_{22} = 0,$

$V_{out}/V_{in} = A_0/(1+G_{21}A_0)$

Loading in a V-V Feedback Circuit

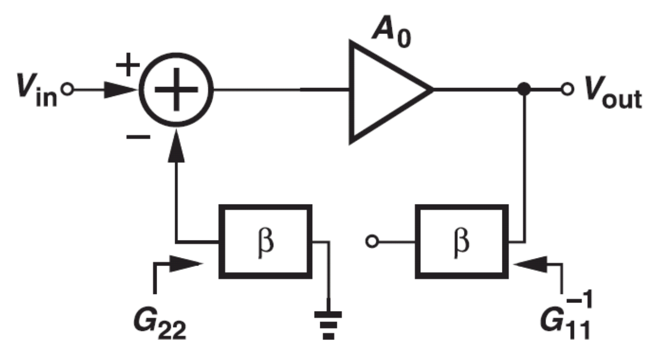
- If we define the open-loop gain in the presence of loading as



$$A_{v,open} = \frac{Z_{in}}{Z_{in} + G_{22}} \frac{G_{11}^{-1}}{G_{11}^{-1} + Z_{out}} A_0$$

- The finite input and output impedances of the feedback network reduces the output voltage and the voltages seen by the input of the main amplifier.
- G_{11} is obtained by leaving the output of the feedback network open.

$$G_{11} = \left. \frac{I_1}{V_1} \right|_{I_2=0}$$

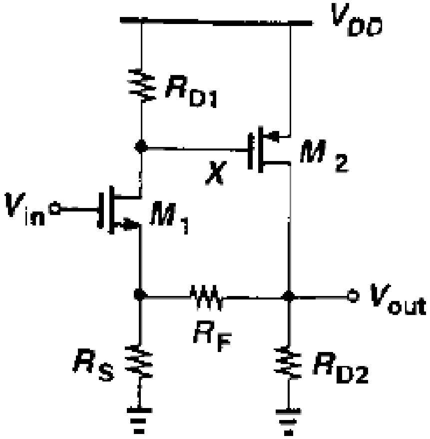


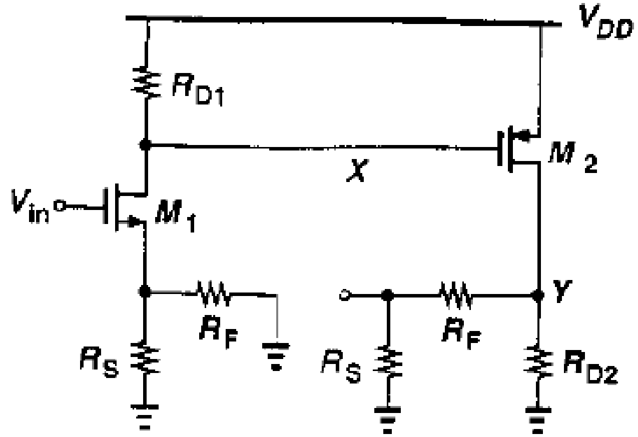
- G_{22} is calculated by shorting the input of the feedback network.

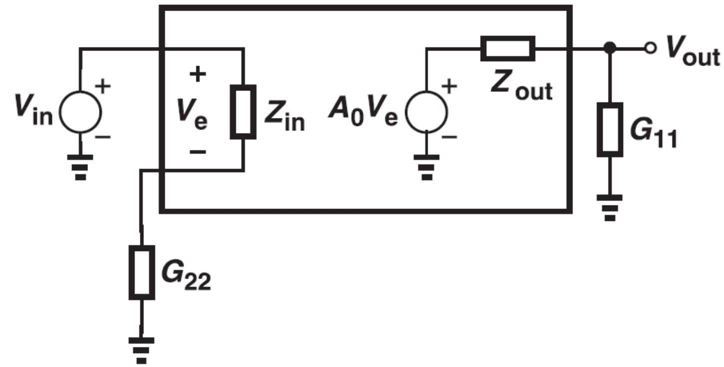
$$G_{22} = \left. \frac{V_2}{I_2} \right|_{V_1=0}$$

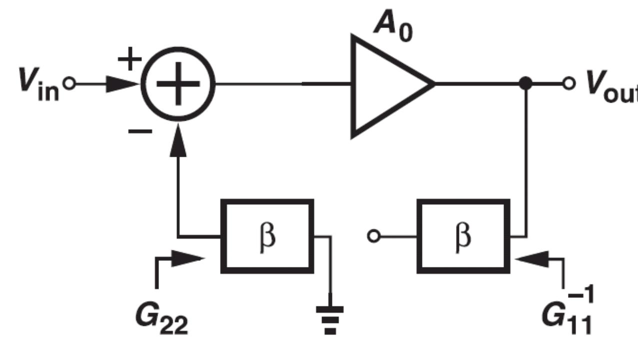
- Consider the loading effect $\frac{V_{out}}{V_{in}} = \frac{A_{v,open}}{1 + A_{v,open} G_{21}}$

Example of V-V Feedback









$$G_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0} = R_S \parallel R_F \qquad G_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0} = \left(R_S + R_F\right)^{-1}$$

$$G_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0} = \frac{R_S}{R_S + R_F} = \beta$$

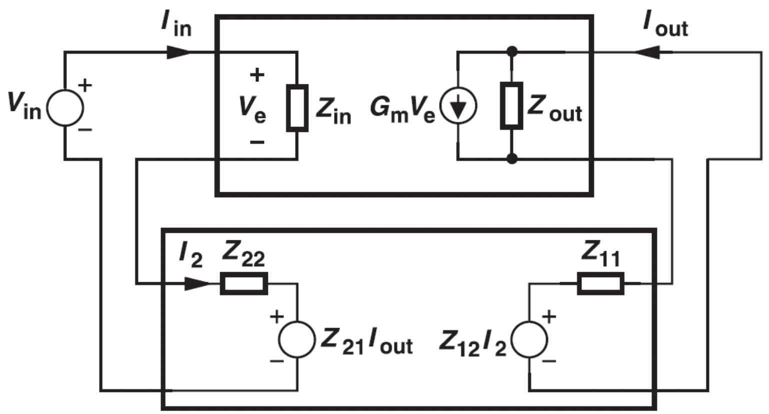
$$A_{v,open} = \frac{-R_{D1}}{R_F \parallel R_S + 1/g_{m1}} \{g_{m2}[R_{D2} \parallel (R_F + R_S)]\}$$

$$A_{v,closed} = \frac{A_{v,open}}{1 + G_{21} A_{v,open}}$$

$$G_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0} \qquad G_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0}$$

Loading in I-V Feedback

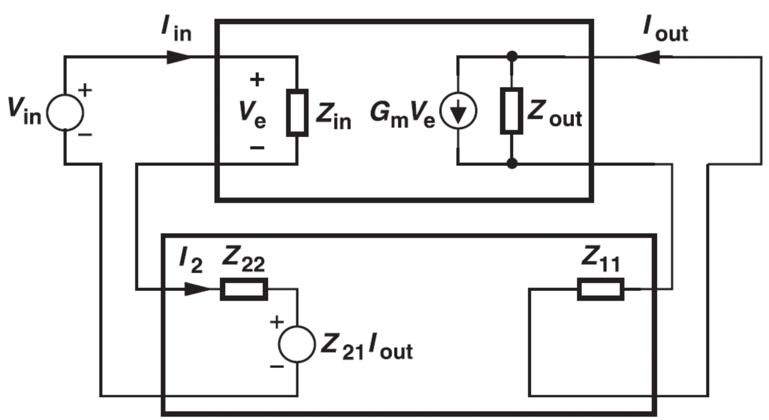
- Replacing the feedback network by a Z model, and neglect the source $Z_{12}I_2$

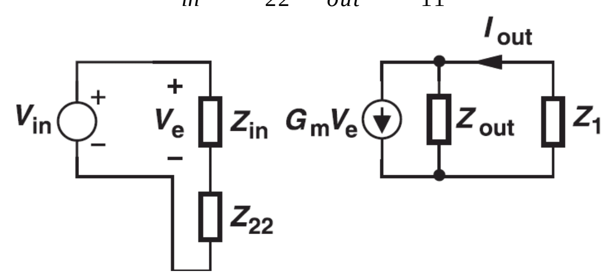


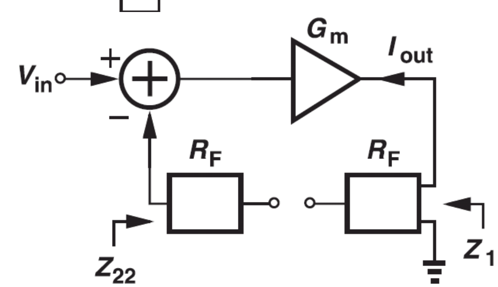
(a)

$$(V_{in} - Z_{21}I_{out})\frac{Z_{in}}{Z_{in}+Z_{22}}G_m\frac{Z_{out}}{Z_{out}+Z_{11}} = I_{out}$$

$$\frac{I_{out}}{V_{in}} = \frac{\frac{Z_{in}}{Z_{in}+Z_{22}}\frac{Z_{out}}{Z_{out}+Z_{11}}G_m}{1+\frac{Z_{in}}{Z_{in}+Z_{22}}\frac{Z_{out}}{Z_{out}+Z_{11}}G_mZ_{21}}$$



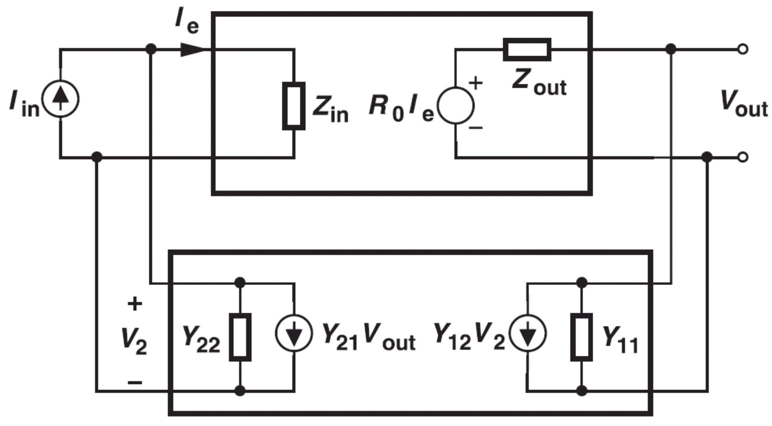




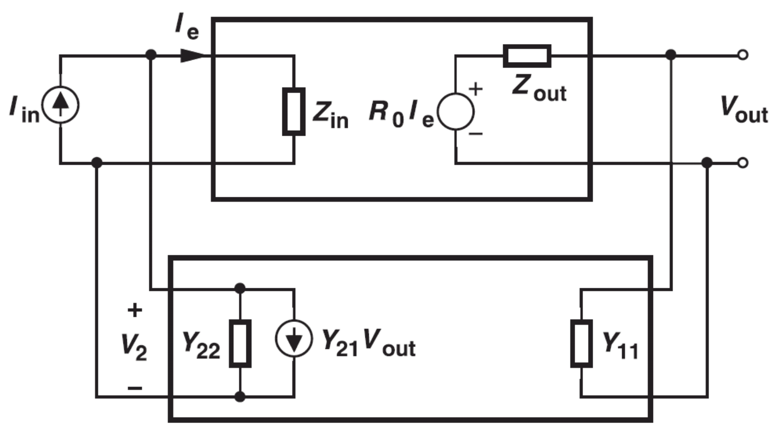
- The loaded open loop gain is equal to $$G_{m,open} = \frac{Z_{in}}{Z_{in}+Z_{22}}\frac{Z_{out}}{Z_{out}+Z_{11}}G_m$$

Loading in V-I Feedback

- Replacing the feedback network by a Y model, and neglect the source $Y_{12}V_2$

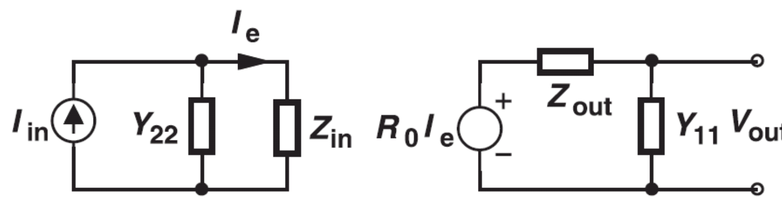


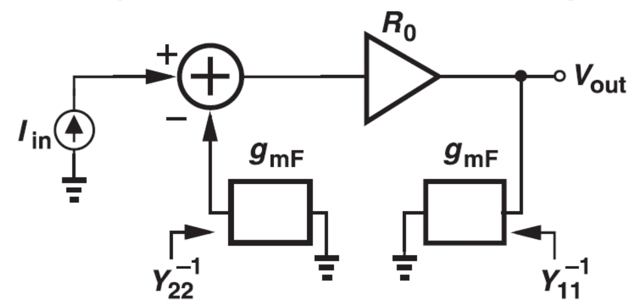
(a)



$$\frac{V_{out}}{I_{in}} = \frac{\frac{Y_{22}^{-1}}{Y_{22}^{-1}+Z_{in}} R_0 \frac{Y_{11}^{-1}}{Y_{11}^{-1}+Z_{out}}}{1+\frac{Y_{22}^{-1}}{Y_{22}^{-1}+Z_{in}} R_0 \frac{Y_{11}^{-1}}{Y_{11}^{-1}+Z_{out}} Y_{21}}$$

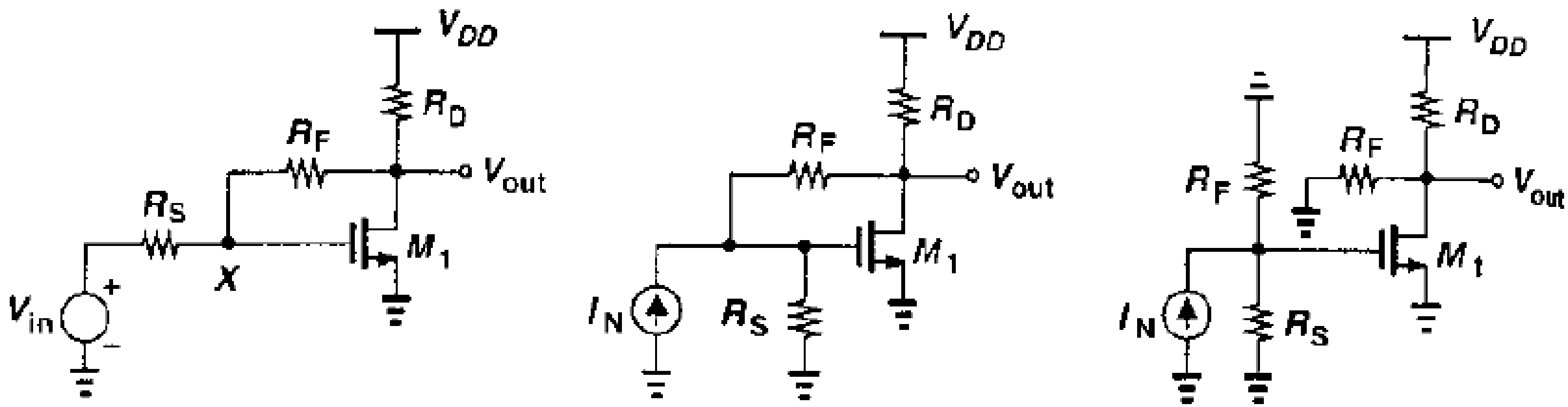
$$(I_{in} - Y_{21}V_{out})\frac{Y_{22}^{-1}}{Y_{22}^{-1}+Z_{in}} R_0 \frac{Y_{11}^{-1}}{Y_{11}^{-1}+Z_{out}} = V_{out}$$





- The loaded open loop gain is equal to $R_{0,open} = \frac{Y_{22}^{-1}}{Y_{22}^{-1}+Z_{in}} \frac{Y_{11}^{-1}}{Y_{11}^{-1}+Z_{out}} R_0$

Example of V-I Feedabck

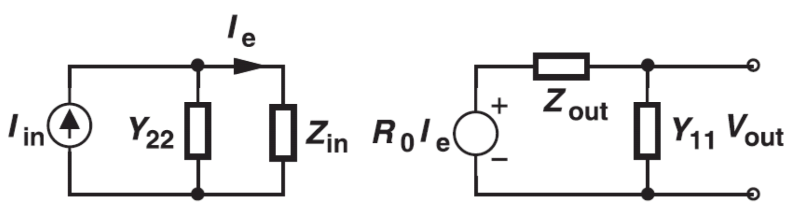


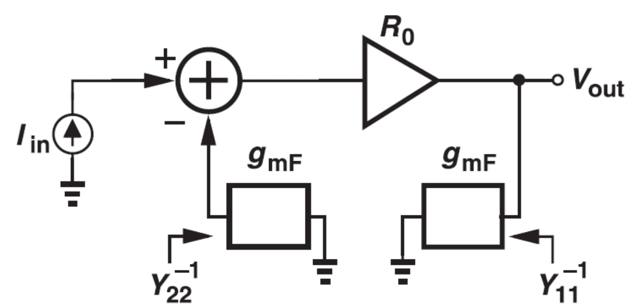
$$Y_{22} = \frac{I_2}{V_2}\bigg|_{V_1=0} = 1/R_F,\ Y_{11} = \frac{I_1}{V_1}\bigg|_{V_2=0} = 1/R_F,\ Y_{21} = \frac{I_2}{V_1}\bigg|_{V_2=0} = -1/R_F,$$

$$R_{0,open} = \frac{V_{out}}{I_N}\bigg|_{open} = -(R_S \parallel R_F) g_m (R_F \parallel R_D)$$

$$R_{0,closed} = \frac{R_{0,open}}{1+Y_{21}R_{0,open}} = \frac{-(R_S \parallel R_F) g_m (R_F \parallel R_D)}{1+g_m (R_F \parallel R_D) R_S/(R_S+R_F)}$$

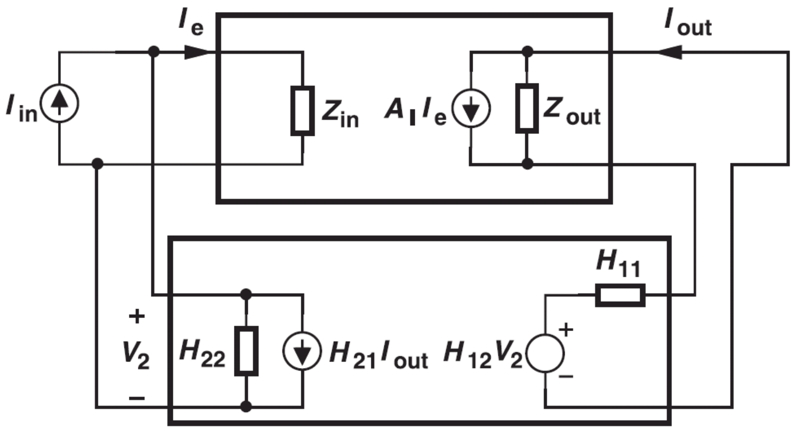
$$\frac{V_{out}}{V_{in}} = \frac{V_{out}}{I_N}\frac{1}{R_S} = \frac{R_{0,closed}}{R_S} = \frac{1}{R_S}\frac{-(R_S \parallel R_F) g_m (R_F \parallel R_D)}{1+g_m (R_F \parallel R_D) R_S/(R_S+R_F)}$$





Loading in I-I Feedback

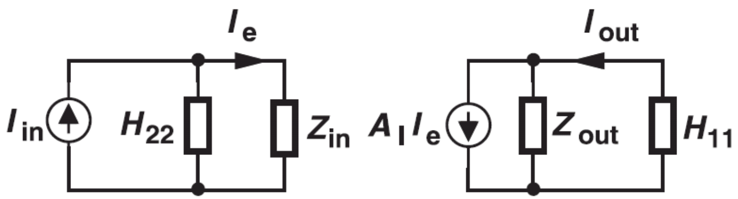
- Replacing feedback network by an H model. Neglecting the effect of $H_{12}V_2$

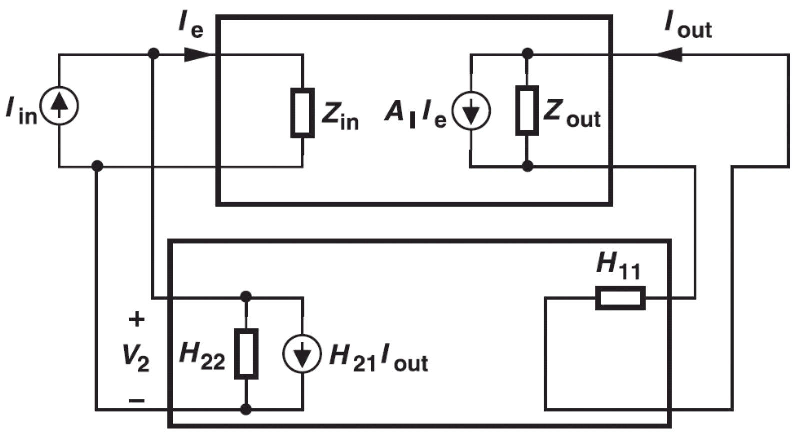


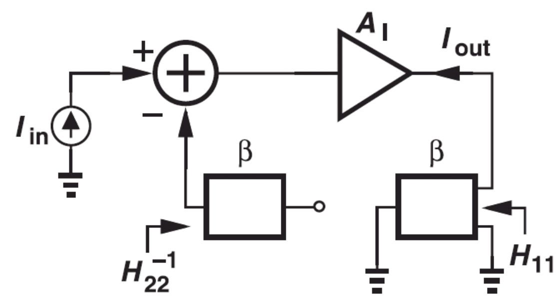
(a)

$$\left(I_{in} - H_{21}I_{out}\right)\frac{H_{22}^{-1}}{H_{22}^{-1}+Z_{in}}A_I\frac{Z_{out}}{H_{11}+Z_{out}} = I_{out}$$

$$\frac{I_{out}}{I_{in}} = \frac{\frac{H_{22}^{-1}}{H_{22}^{-1}+Z_{in}}A_I\frac{Z_{out}}{H_{11}+Z_{out}}}{1+\frac{H_{22}^{-1}}{H_{22}^{-1}+Z_{in}}A_I\frac{Z_{out}}{H_{11}+Z_{out}}H_{21}}$$



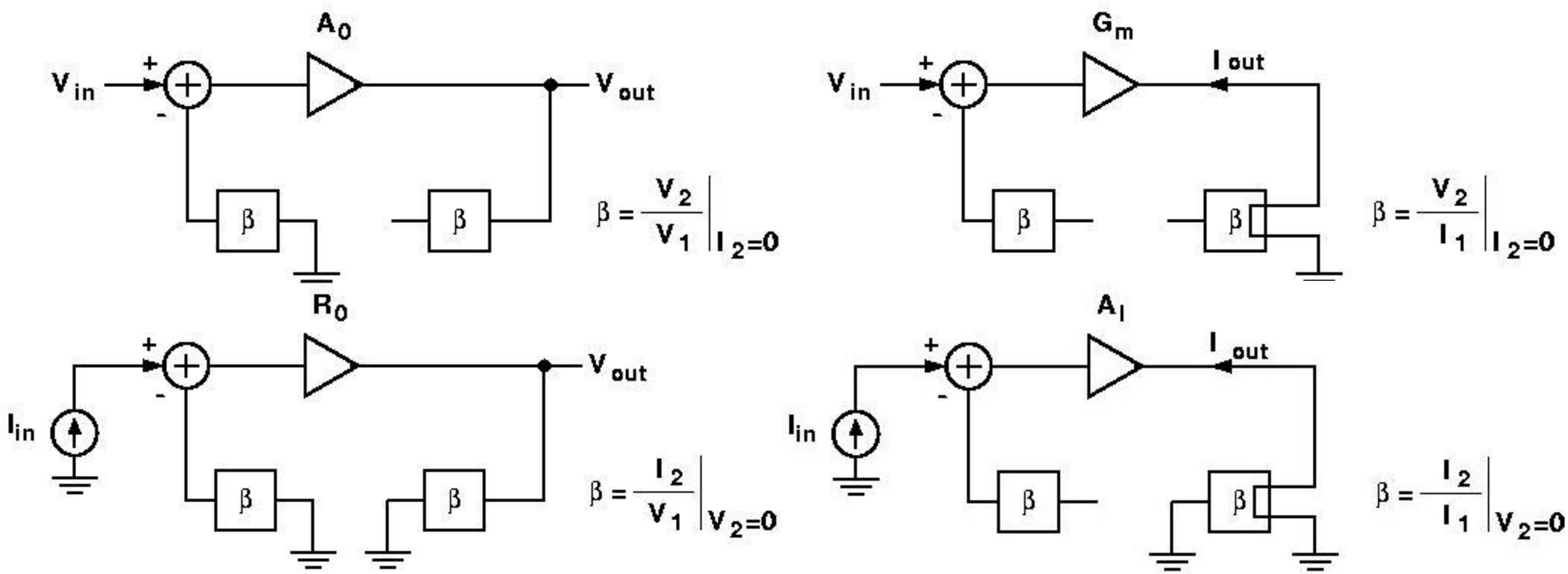




- The loaded open loop gain is equal to $$A_{I,open} = \frac{H_{22}^{-1}}{H_{22}^{-1}+Z_{in}}A_I\frac{Z_{out}}{H_{11}+Z_{out}}$$

Summary of Loading Effects

- The analysis is carried out in three steps
  - Open the loop with proper loading and calculate the open-loop gain A_{OL}, and the open-loop input and output impedances.
  - Determine the feedback ratio β and hence the loop gain $\beta A_{OL.}$
  - Calculate the closed-loop gain and input and output impedances by scaling the open loop values by a factor of $1+\beta A_{OL}$

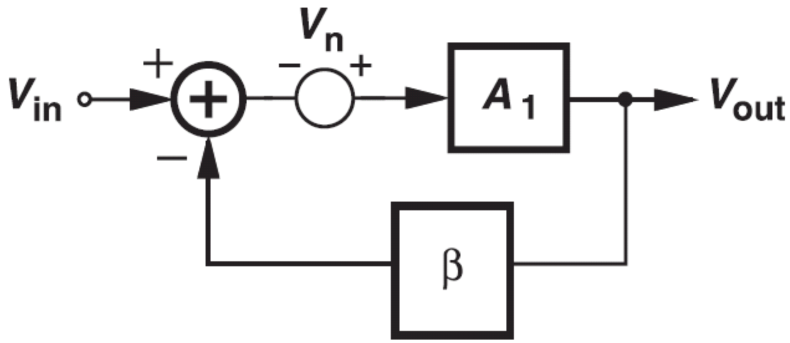


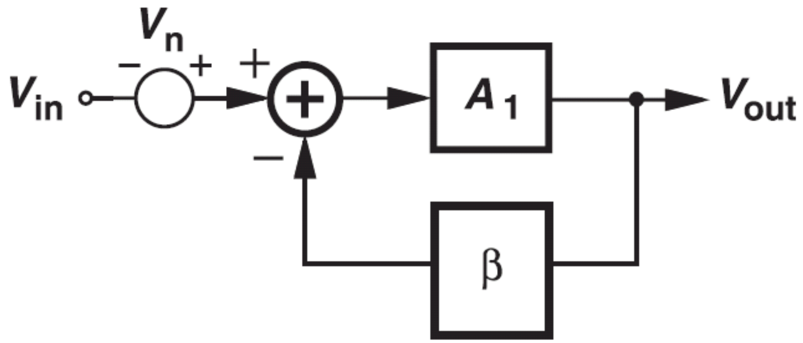
Outline

1. General Consideration
2. Feedback Topologies
3. Effect of Loading
4. **Effect of Feedback on Noise**

Effect of Feedback on Noise

- Feedback does not improve the noise performance of circuits.
- Assume the open-loop voltage amplifier A1 is characterized by only an input-referred noise voltage and the feedback network is noiseless.

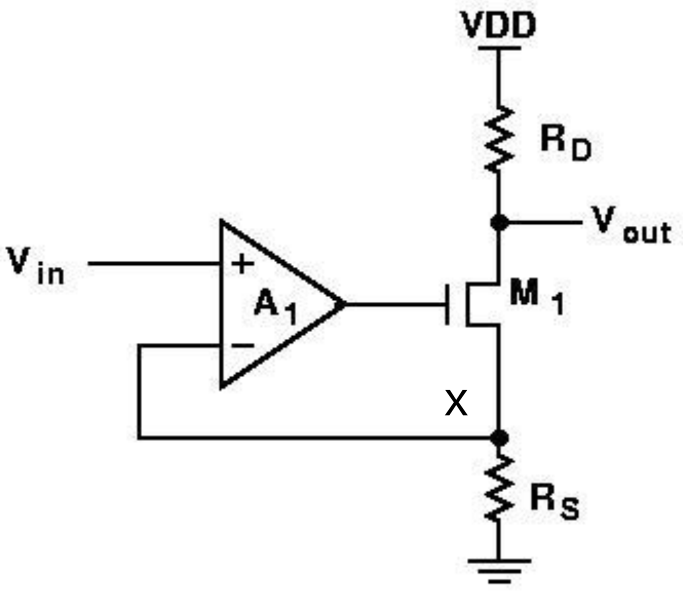




$$(V_{in} - \beta V_{out} + V_n)A_1 = V_{out} \qquad V_{out} = (V_{in} + V_n)\frac{A_1}{1+\beta A_1}$$

- In practice, the feedback network itself may contain resistors or transistors, degrading the overall noise performance.

Effect of Feedback on Noise



$$\left[\left(V_{in}-V_x\right)A_1-V_x\right]\cdot g_m=\frac{V_x}{R_s}$$

$$V_x=\frac{g_m A_1 V_{in}}{\frac{1}{R_s}+g_m\left(1+A_1\right)},\qquad V_o=-\frac{R_D}{R_S}V_X$$

$$\left.\frac{V_o}{V_{in}}\right|_{closed}=-\frac{g_m R_D A_1}{1+g_m R_S\left(1+A_1\right)},\qquad \left|V_{n,in,closed}\right|=\frac{\left|V_{n,R_D}\right|}{A_1 R_D}\left[\frac{1}{g_m}+\left(1+A_1\right)R_S\right]$$

$$\left.\frac{V_o}{V_{in}}\right|_{open}=-\frac{g_m R_D A_1}{1+g_m R_S},\qquad \left|V_{n,in,open}\right|=\frac{\left|V_{n,R_D}\right|}{A_1 R_D}\left[\frac{1}{g_m}+R_S\right]$$

- As

$$A_1\to\infty,\quad \left|V_{n,in,closed}\right|\to\left|V_{n,RD}\right|\frac{R_S}{R_D}\ \text{whereas}\ \left|V_{n,in,open}\right|\to 0$$