CHAPTER 6

Frequency Response of Amplifiers

Outline

1. **Miller Effect / Poles**
2. Common-Source (CS) Stage
3. Source Follower (SF)
4. Common-Gate (CG) Stage
5. Cascode Stage
6. Differential Pair (DP)

Why Frequency Response

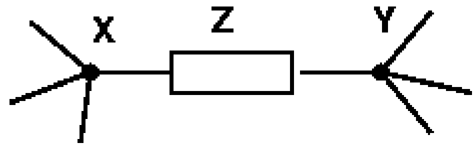
- In most analog circuits, the speed trades with many other parameters such as gain, power dissipation, and noise.

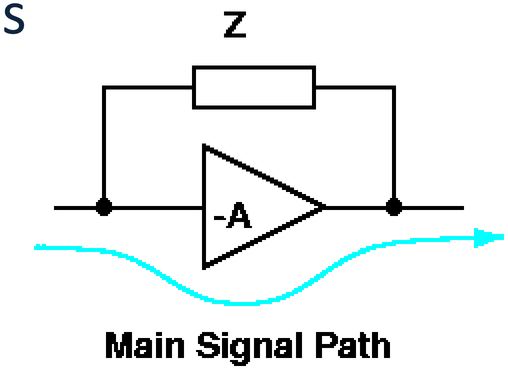
- Transient signal varied with time need to consider the capacitive and inductive effect inherent in circuit.

Miller Effect

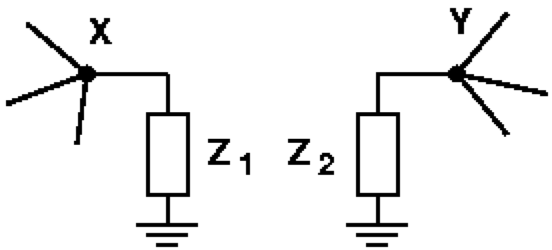
- For the current flowing through *Z* from *X* to *Y* is

$$\frac{V_X - V_Y}{Z} = \frac{V_X}{Z_1}$$



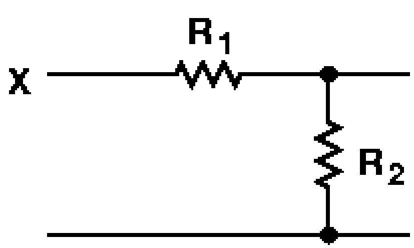


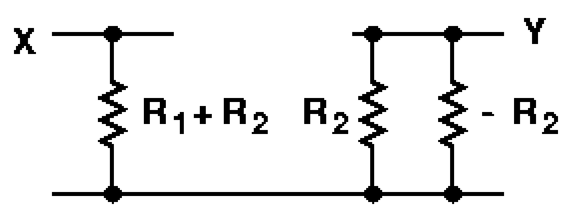
- That is



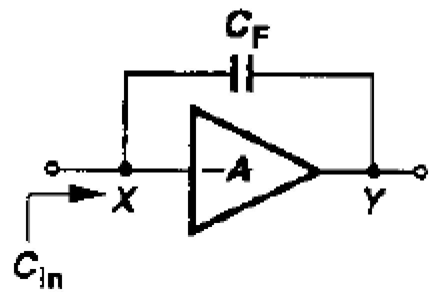
$$Z_1 = \frac{Z}{1 - \frac{V_Y}{V_X}} \qquad Z_2 = \frac{Z}{1 - \frac{V_X}{V_Y}}$$

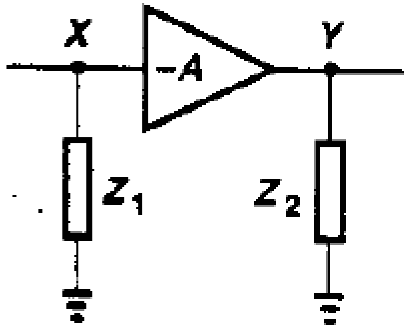
- If the impedance *Z* forms the only signal path between *X* and *Y* , then the conversion is often invalid.





Miller Effect of Feedback Capacitor



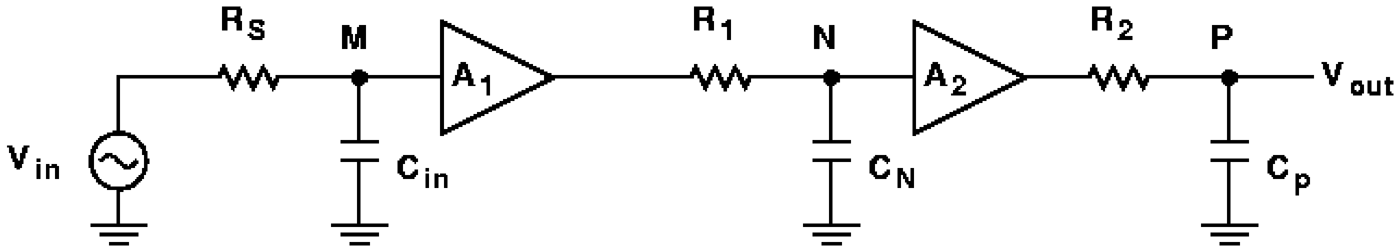


$$Z_1 = \frac{Z}{1-A_v} = \frac{Z}{1+A} = \frac{1}{sC_F(1+A)}, \quad C_1 = C_F(1+A)$$

$$Z_2 = \frac{Z}{1-(1/A_v)} = \frac{Z}{1+(1/A)} = \frac{1}{sC_F(1+A^{-1})}, \quad C_2 = C_F(1+A^{-1})$$

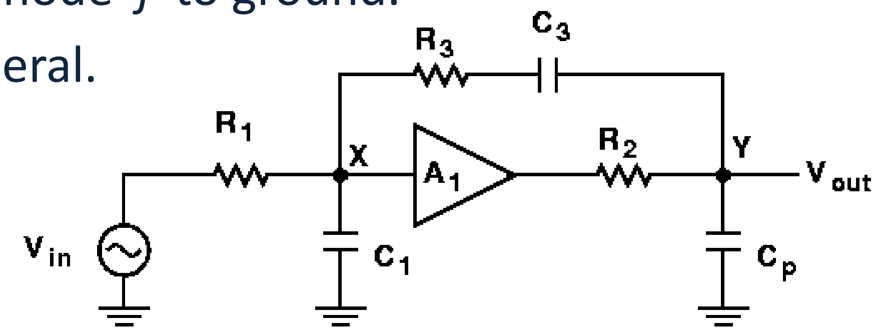
Association of Poles with Nodes

- Assume A_1 and A_2 are ideal amplifiers, R_1 and R_2 model the output resistance of each stage, C_{in} and C_N represent the input capacitance of each stage, and C_p denotes the load capacitance.



$$\frac{V_{out}}{V_{in}}(s) = \frac{A_1}{1+R_S C_{in} s} \cdot \frac{A_2}{1+R_1 C_N s} \cdot \frac{1}{1+R_2 C_p s}$$

- Each pole with one node of the circuit, i.e., $\omega_j = \tau_j^{-1}$, where τ_j is the product of the capacitance and resistance seen at node j to ground.
- The above statement is not valid in general.
  - The location of the pole is difficult to calculate.

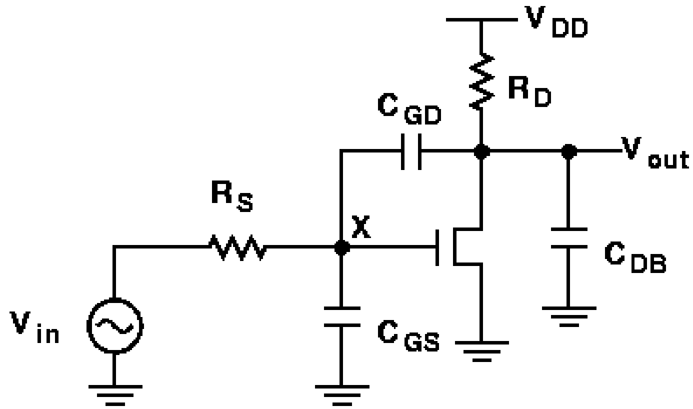


Outline

1. Miller Effect / Poles
2. **Common-Source (CS) Stage**
3. Source Follower (SF)
4. Common-Gate (CG) Stage
5. Cascode Stage
6. Differential Pair (DP)

Common Source Stage

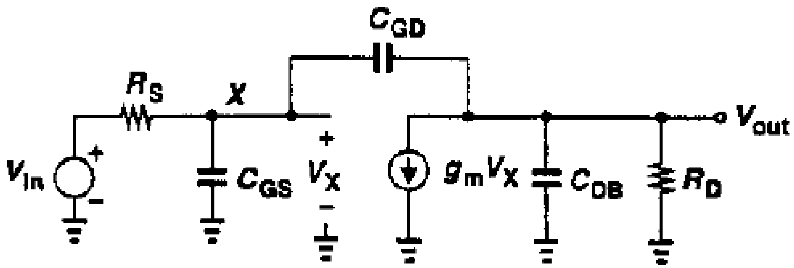
- Provides voltage gain and high input impedance.



$$\omega_{in} = \frac{1}{R_S[C_{GS} + (1 + g_m R_D)C_{GD}]}$$

$$\omega_{out} = \frac{1}{R_D(C_{DB} + C_{GD})}$$

- To obtain the exact transfer function



$$\frac{V_X - V_{in}}{R_S} + V_X C_{GS} s + (V_X - V_{out}) C_{GD} s = 0,$$

$$(V_{out} - V_X) C_{GD} s + g_m V_X + V_{out}(\frac{1}{R_D} + C_{DB} s) = 0,$$

$$\xi = C_{GS}C_{GD} + C_{GS}C_{DB} + C_{GD}C_{DB}$$

$$\frac{V_{out}}{V_{in}}(s) = \frac{(C_{GD}s - g_m)R_D}{R_S R_D \xi s^2 + [R_S(1 + g_m R_D)C_{GD} + R_S C_{GS} + R_D(C_{GD} + C_{DB})]s + 1}$$

Dominant Pole Approximation

- Two independent capacitors (initial conditions) in the circuit yield a second order differential equation for the time response.

$$\frac{V_{out}}{V_{in}}(s)=\frac{(C_{GD}s-g_m)R_D}{R_SR_D\xi s^2+[R_S(1+g_mR_D)C_{GD}+R_SC_{GS}+R_D(C_{GD}+C_{DB})]s+1}$$

$$=-g_mR_D\frac{(1-C_{GD}s/g_m)}{R_SR_D\xi s^2+[R_S(1+g_mR_D)C_{GD}+R_SC_{GS}+R_D(C_{GD}+C_{DB})]s+1}$$

$$=-g_mR_D\frac{(1-s/z_1)}{\left(\frac{s}{\omega_{p1}}+1\right)\left(\frac{s}{\omega_{p2}}+1\right)}=-g_mR_D\frac{(1-s/z_1)}{\frac{s^2}{\omega_{p1}\omega_{p2}}+\left(\frac{1}{\omega_{p1}}+\frac{1}{\omega_{p2}}\right)s+1}$$

- If $|\omega_{p1}| << |\omega_{p2}|$

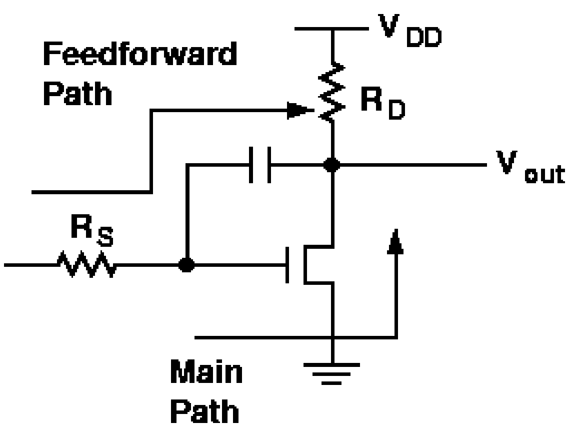
$$z_1=\frac{g_m}{C_{GD}},\quad \omega_{p1}=\frac{1}{R_S(1+g_mR_D)C_{GD}+R_SC_{GS}+R_D(C_{GD}+C_{DB})}$$

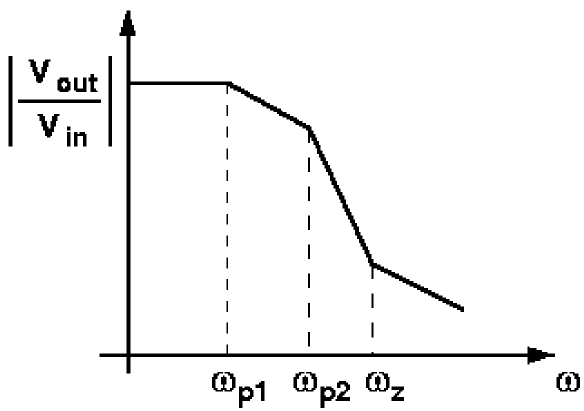
$$\omega_{p2}=\frac{1}{\omega_{p1}}\cdot\frac{1}{R_SR_D(C_{GS}C_{GD}+C_{GS}C_{DB}+C_{GD}C_{DB})}\cong\frac{R_SC_{GS}}{R_SR_D(C_{GS}C_{GD}+C_{GS}C_{DB})}=\frac{1}{R_D(C_{GD}+C_{DB})}$$

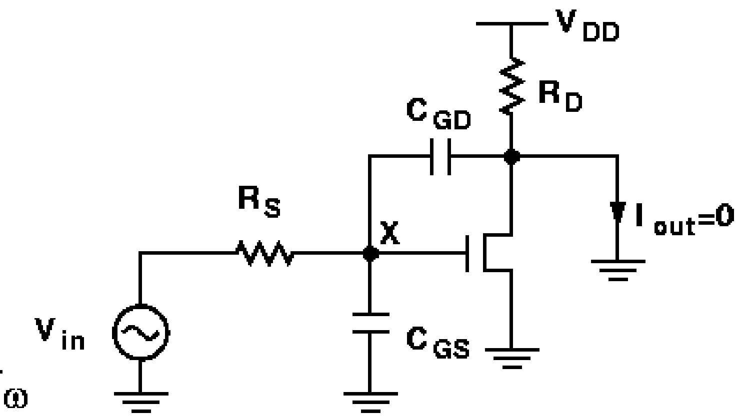
$$If\ \ C_{GS}>>(1+g_mR_D)C_{GD}+R_D(C_{GD}+C_{DB})/R_S$$

Right-Half Plane Zero

- C_{GD} provides a feed forward path that conducts the input signal to the output at very high frequencies.
- A zero in the right half plane causes stability issue in feedback Amp.







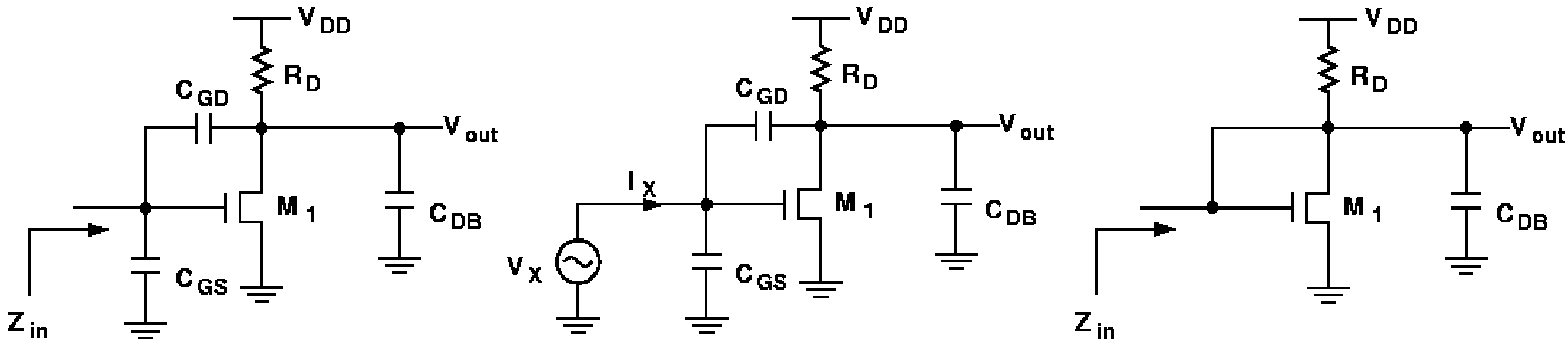
- The zero, s_z , can be computed by noting that the $V_{out}(s)/V_{in}(s)$ must drop to zero for $s = s_z$.
- The currents through C_{GD} and M_1 are equal and opposite.

$$V_x C_{GD} s_z = g_m V_x$$

$$s_z = \frac{g_m}{C_{GD}}$$

Input Impedance of a CS Stage

- As a first-order approximation $Z_{in} = \left\{ \left[C_{GS} + (1 + g_m R_D) \right] C_{GD} \right\}^{-1}$



- At high frequencies, take the effect of the output node into account

$$Ignore\ \ C_{GS},\ \left(I_X - g_m V_X\right)\frac{R_D}{1+R_D C_{DB} s} + \frac{I_X}{C_{GD} s} = V_X \Rightarrow \frac{V_X}{I_X} = \frac{1+R_D(C_{GD}+C_{DB})s}{C_{GD}s(1+g_m R_D + R_D C_{DB} s)}$$

- At low frequencies $Z_{in} \approx \frac{1}{sC_{GS}} \parallel \frac{1}{sC_{GD}(1+g_m R_D)}$

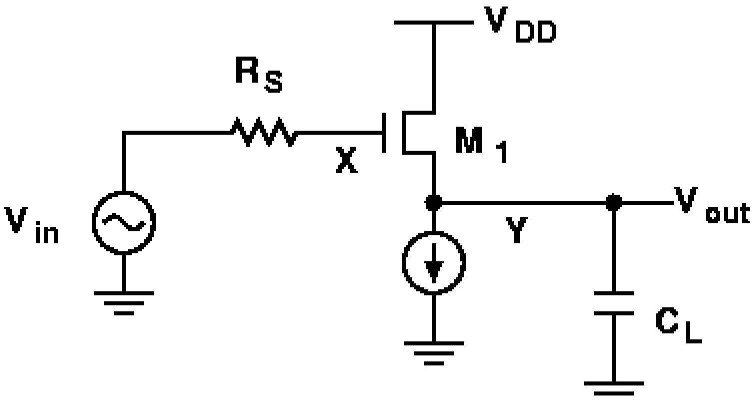
- At high frequencies $Z(C_{GD}) = 0, \quad Z_{in} \approx \frac{1}{sC_{GS}} \parallel \frac{1}{g_m} \parallel R_D$

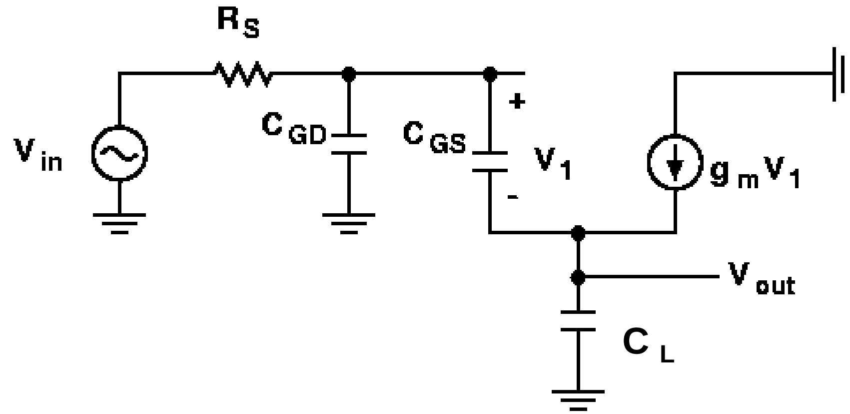
Outline

1. Miller Effect / Poles
2. Common-Source (CS) Stage
3. **Source Follower (SF)**
4. Common-Gate (CG) Stage
5. Cascode Stage
6. Differential Pair (DP)

Source Follower

- Source followers are occasionally used as level shifters or buffers.





$$V_1 C_{GS} s + g_m V_1 = V_{out} s C_L \qquad V_1 = \frac{sC_L}{g_m + sC_{GS}} V_{out} \qquad V_{in} = R_S [V_1 C_{GS} s + (V_1 + V_{out}) C_{GD} s] + V_1 + V_{out}$$

$$\frac{V_{out}}{V_{in}}(s) = \frac{g_m + C_{GS} s}{R_S (C_{GS} C_L + C_{GS} C_{GD} + C_{GD} C_L) s^2 + (g_m R_S C_{GD} + C_L + C_{GS}) s + g_m}$$

$$= \frac{(1 + sC_{GS} / g_m)}{s^2 R_S (C_{GS} C_L + C_{GS} C_{GD} + C_{GD} C_L) / g_m + s (g_m R_S C_{GD} + C_L + C_{GS}) / g_m + 1}$$

➢ The signal conducted by C_{GS} at high frequencies adds with the same polarity to the signal produced by the intrinsic transistor. (*left half plane* zero)

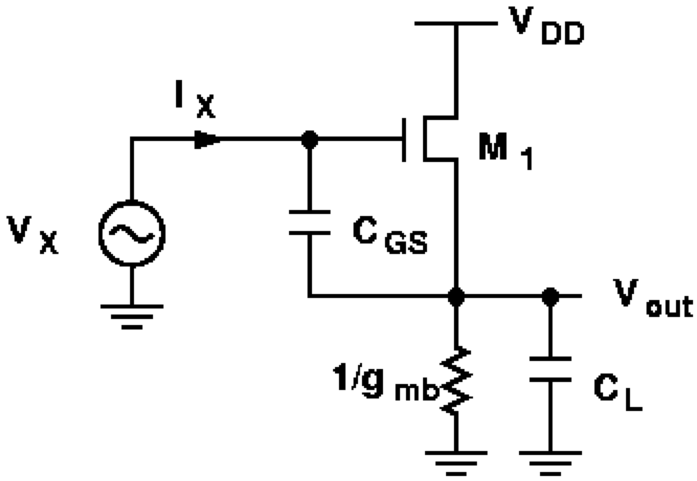
Input Impedance of Source Follower

$$\frac{V_{out}}{V_{in}}(s)=\frac{(1+sC_{GS}/g_m)}{s^2R_S\left(C_{GS}C_L+C_{GS}C_{GD}+C_{GD}C_L\right)/g_m+s\left(g_mR_SC_{GD}+C_L+C_{GS}\right)/g_m+1}$$

- If the two poles are far apart

$$\omega_{p1}\approx\frac{g_m}{g_mR_SC_{GD}+C_L+C_{GS}}=\frac{1}{R_SC_{GD}+\dfrac{C_L+C_{GS}}{g_m}}\qquad If\ R_S=0,\ \text{then}\ \omega_{p1}=\frac{g_m}{C_L+C_{GS}}$$

- Input impedance (ignore C_{GD})



$$V_X=\frac{I_X}{C_{GS}s}+\left(I_X+\frac{g_mI_X}{C_{GS}s}\right)\left(\frac{1}{g_{mb}}\,\|\,\frac{1}{sC_L}\right)$$

$$Z_{in}=\frac{1}{C_{GS}s}+\left(1+\frac{g_m}{C_{GS}s}\right)\frac{1}{g_{mb}+C_Ls}$$

At low frequencies, $g_{mb}>>\ |C_Ls|$

$$Z_{in}=\frac{1}{C_{GS}s}\left(1+\frac{g_m}{g_{mb}}\right)+\frac{1}{g_{mb}}$$

$$C_{in}=C_{GS}\frac{g_{mb}}{g_{mb}+g_m}$$

Input Impedance of Source Follower

- At relatively low frequencies, the equivalent input C_{in} = $\dfrac{C_{GS}g_{mb}}{g_m+g_{mb}}$
  - By Miller effect

$$A_v=\frac{g_m}{g_m+g_{mb}} \qquad C_{eq}=C_{GS}\left[1-\frac{g_m}{g_m+g_{mb}}\right]=\frac{C_{GS}g_{mb}}{g_m+g_{mb}}$$

  - The overall input capacitance is equal to C_{GD} plus a fraction of C_{GS}.
- At high frequencies,

$$Z_{in}=\frac{1}{C_{GS}s}+\left(1+\frac{g_m}{C_{GS}s}\right)\frac{1}{g_{mb}+C_Ls}$$

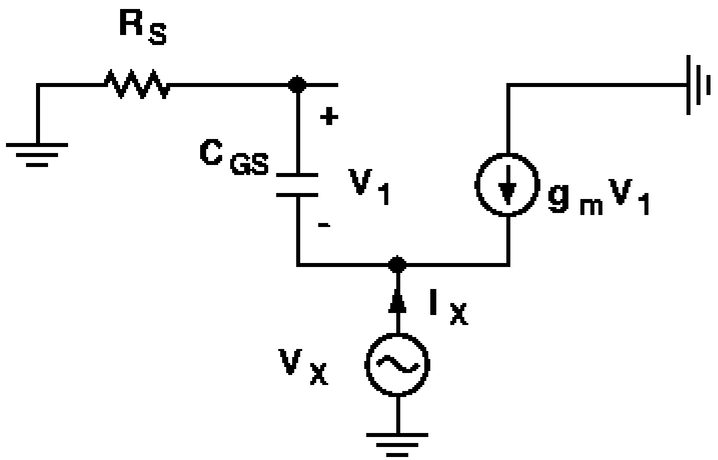
$$g_{mb}<<\left|sC_L\right| \text{ and } Z_{in}\approx\frac{1}{sC_{GS}}+\frac{1}{sC_L}+\frac{g_m}{s^2C_{GS}C_L}=\frac{1}{j\omega C_{GS}}+\frac{1}{j\omega C_L}-\frac{g_m}{\omega^2C_{GS}C_L}$$

  - The input impedance consists of the series combination of capacitors C_{GS} and C_L and a negative resistance.
  - The negative resistance property can be utilized in oscillators.

Output Impedance of Source Follower

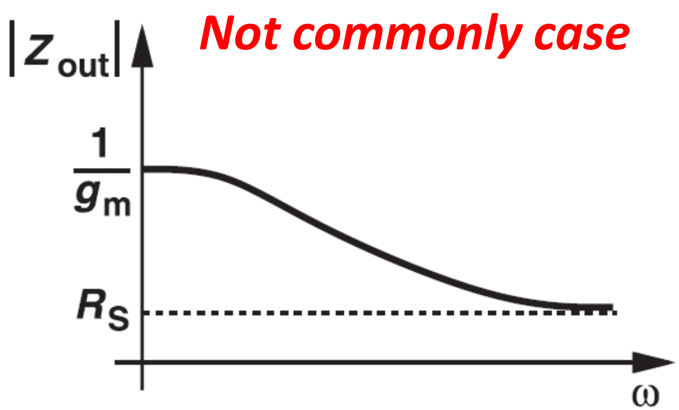
- Neglecting g_{mb} , C_{SB} & C_{GD}

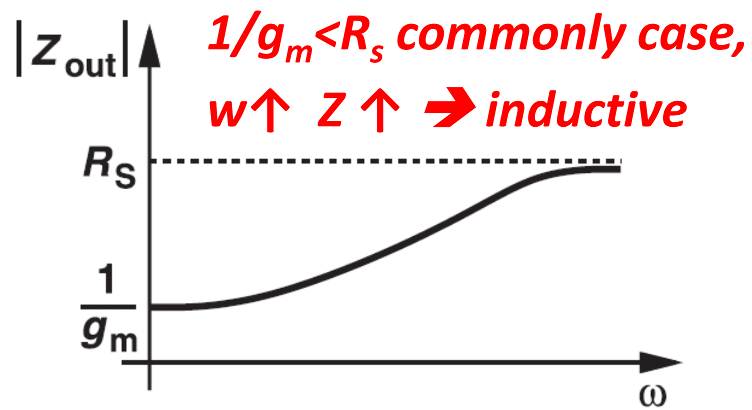
$$V_1 C_{GS} s + g_m V_1 = -I_X , \quad V_1 C_{GS} s R_S + V_1 = -V_X$$



$$Z_{out} = \frac{V_X}{I_X} = \frac{sR_S C_{GS} + 1}{g_m + sC_{GS}}$$

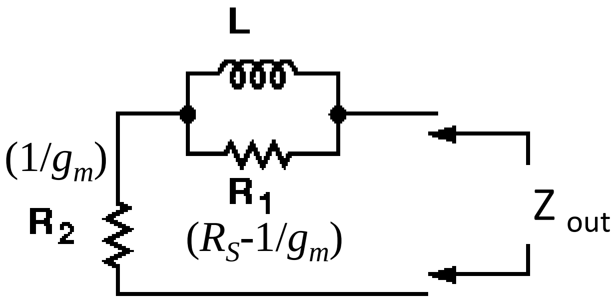
- At low frequencies, $Z_{out} \approx 1/g_m$
- At high frequencies, $Z_{out} \approx R_S$ with $Z(C_{GS}) = 0$
- If $1/g_m < R_S$, the output impedance contains an inductive component.





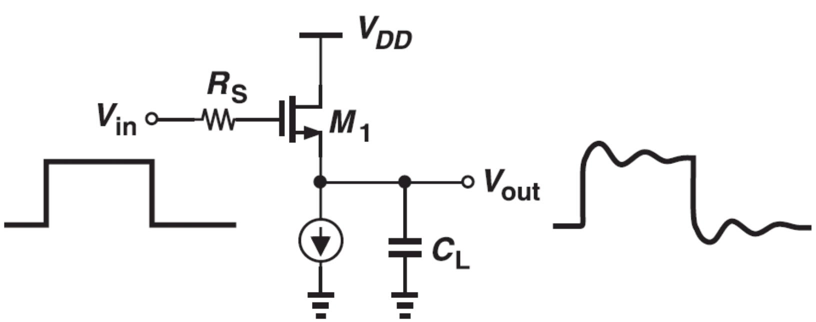
Output Impedance of SF

- Equivalent network of output impedance



- At low frequencies, $Z_{out} = 1/g_m$ when $\omega = 0$
- At high frequencies, $Z_{out} = R_S$ when $\omega = \infty$
- Therefore, $R_2 = 1/g_m \quad R_1 = R_S - 1/g_m$

$$Z_{out} - \frac{1}{g_m} = \frac{sC_{GS}(R_S - 1/g_m)}{g_m + sC_{GS}}, \quad \frac{1}{Z_{out} - \frac{1}{g_m}} = \frac{1}{R_S - 1/g_m} + \frac{1}{\frac{sC_{GS}}{g_m}(R_S - 1/g_m)}, \quad L = \frac{C_{GS}}{g_m}(R_S - 1/g_m)$$



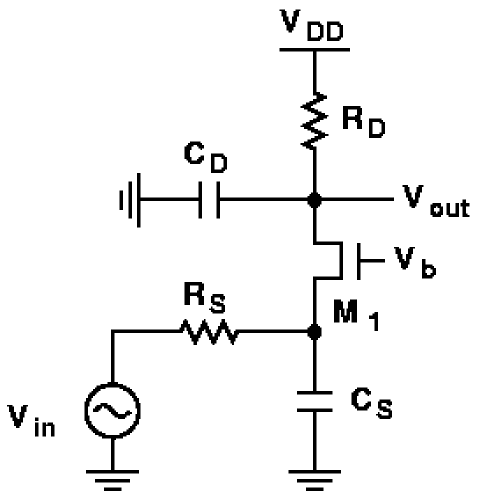
- If a source follower is driven by a large resistance, then it exhibits substantial inductive behavior.
- This effect manifests itself as ringing in the step response. If it drive a large C_L.

Outline

1. Miller Effect / Poles
2. Common-Source (CS) Stage
3. Source Follower (SF)
4. **Common-Gate (CG) Stage**
5. Cascode Stage
6. Differential Pair (DP)

Common-Gate Stage

- Neglect channel length modulation effect



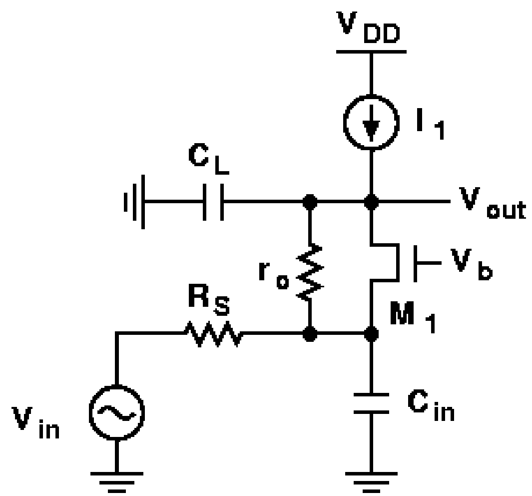
$$\frac{V_{out}}{V_{in}}(s) = \frac{(g_m + g_{mb})R_D}{1+(g_m + g_{mb})R_S} \frac{1}{\left(1+\frac{C_S}{g_m + g_{mb} + R_S^{-1}} s\right)(1+R_D C_D s)}$$

- ➢ It exhibits no Miller multiplication of capacitances, potentially achieving wide band.
- ➢ Low Input impedance may load the preceding stage.
- ➢ The DC level of the input signal must be quite low.

$$if\ \lambda \neq 0, \quad Z_{in} \approx \frac{Z_L}{(g_m + g_{mb})r_O} + \frac{1}{g_m + g_{mb}}, \quad Z_L = R_D \| [1/sC_D]$$

- Since Z_{in} now depends on Z_L, it is difficult to associate a pole with the input node.

Input Impedance of Common Gate

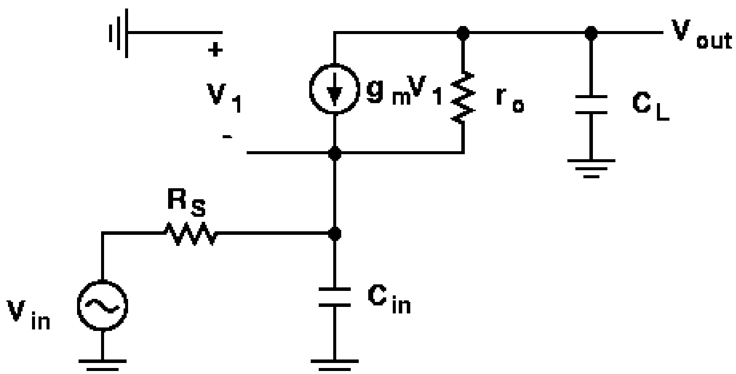


$$(-V_{out}C_L s + V_1 C_{in} s)R_S + V_{in} = -V_1$$

$$V_1 = -\frac{-V_{out}C_L s R_S + V_{in}}{1 + C_{in}R_S s}, \quad r_O(-V_{out}C_L s - g_m V_1) - V_1 = V_{out}$$

$$\frac{V_{out}}{V_{in}}(s) = \frac{1 + g_m r_O}{r_O C_L C_{in} R_S s^2 + [r_O C_L + C_{in}R_S + (1 + g_m r_O)C_L R_S]s + 1}$$

$$\frac{V_{out}}{V_{in}} = 1 + g_m r_O \, (@\,\text{Low Freq}: s = jw = 0)$$



- The body effect can be included by simply replacing g_m with $g_m + g_{mb}$.

$$Z_{in} \approx \frac{Z_L}{(g_m + g_{mb})r_O} + \frac{1}{g_m + g_{mb}}$$

$$Z_L = \frac{1}{sC_L}, \quad Z_{in} = \frac{1}{g_m + g_{mb}} + \frac{1}{sC_L} \cdot \frac{1}{(g_m + g_{mb})r_O}$$

$$\text{As } C_L \text{ or } s \text{ increases, } Z_{in} = \frac{1}{g_m + g_{mb}}$$

$$\text{input pole } \omega_{p,in} = \frac{1}{\left(R_S \parallel \frac{1}{g_m + g_{mb}}\right)C_{in}}, \text{independent of } C_L$$

Outline

1. Miller Effect / Poles
2. Common-Source (CS) Stage
3. Source Follower (SF)
4. Common-Gate (CG) Stage
5. **Cascode Stage**
6. Differential Pair (DP)

Cascode Stage

- Common source + common gate gain stage.
- Increase the voltage gain of amplifiers and the output impedance of current source.

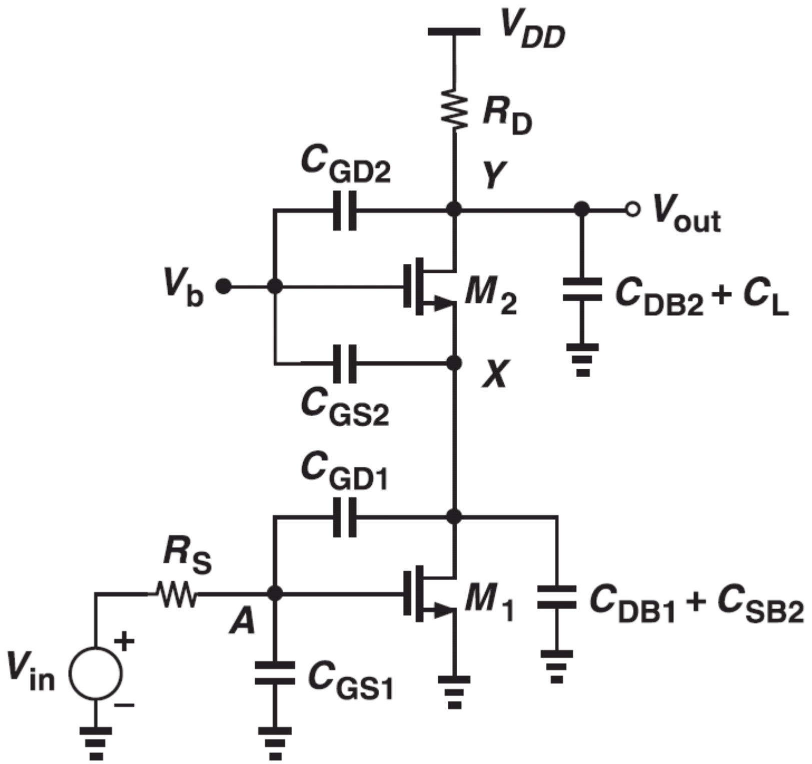
$$Z_{out} = (1 + g_{m2} r_{O2}) Z_X + r_{O2}$$

- Providing shielding – suppressing the Miller effect.
- The low frequency gain from M_1 gate to drain

$$A_v = -\frac{g_{m1}}{g_{m2} + g_{mb2}} \approx -1$$

- The input pole is at

$$\omega_{p,A} = \frac{1}{R_S\left[C_{GS1} + \left(1 + \frac{g_{m1}}{g_{m2} + g_{mb2}}\right) C_{GD1}\right]}$$

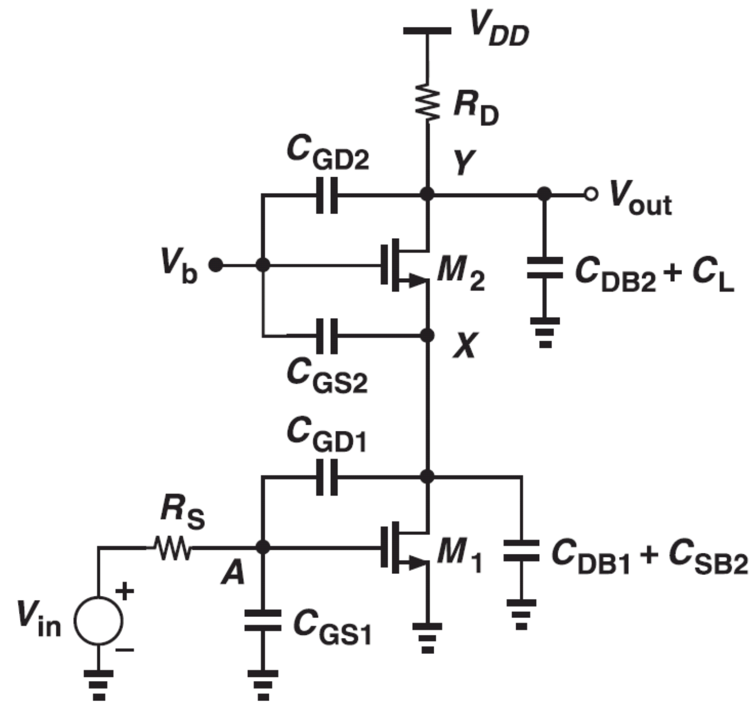


Cascode Stage

- The total capacitance at node X is (typically chosen farther from the origin than the other two)

$$C_X = 2C_{GD1} + C_{DB1} + C_{SB2} + C_{GS2}$$

$$\omega_{p,X} = \frac{1}{R_X C_X} = \frac{g_{m2} + g_{mb2}}{2C_{GD1} + C_{DB1} + C_{SB2} + C_{GS2}}$$



- The output pole at node Y is

$$\omega_{p,Y} = \frac{1}{R_D\left(C_{DB2} + C_L + C_{GD2}\right)}$$

- Output impedance of a cascode current source (neglect C_Y, C_{GD1})

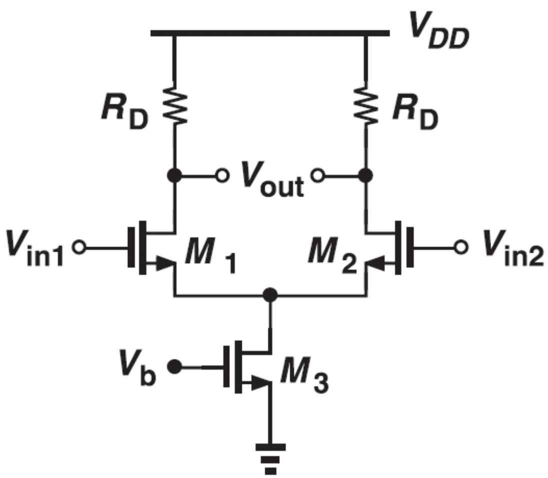
$$Z_{out} = (1 + g_{m2} r_{O2}) Z_X + r_{O2}, Z_X = r_{O1} \,||\, (C_X s)^{-1}$$

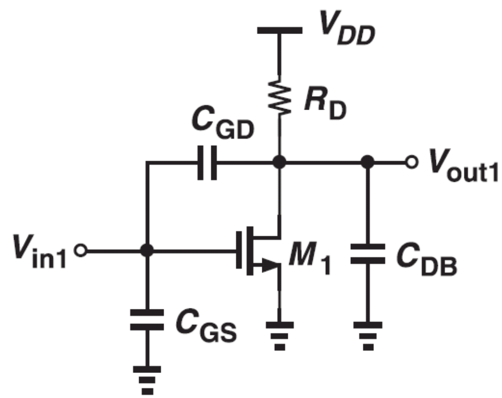
$$pole = (r_{O1} C_X)^{-1}$$

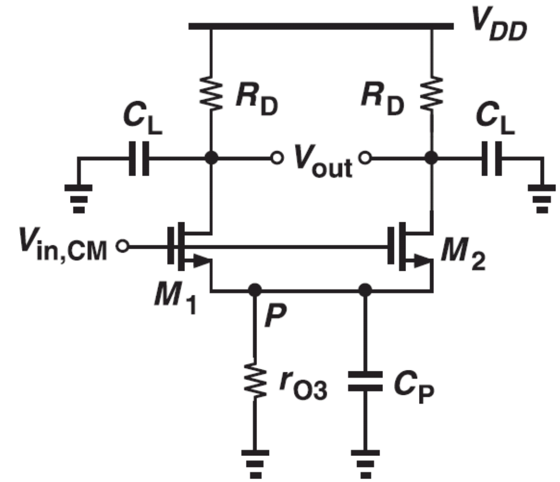
Outline

1. Miller Effect / Poles
2. Common-Source (CS) Stage
3. Source Follower (SF)
4. Common-Gate (CG) Stage
5. Cascode Stage
6. **Differential Pair (DP)**

Differential Pair





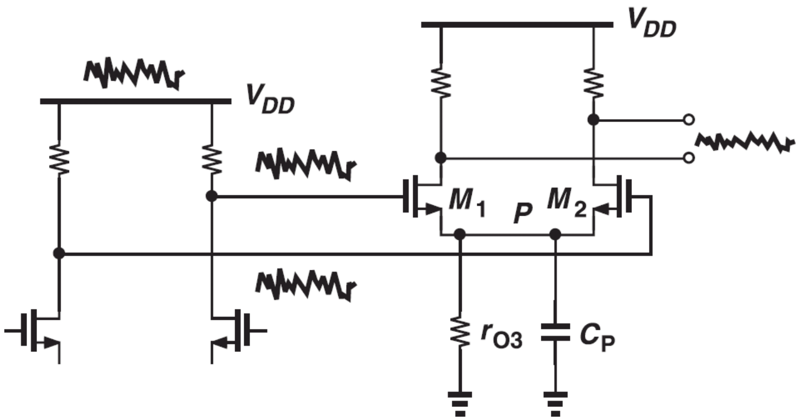


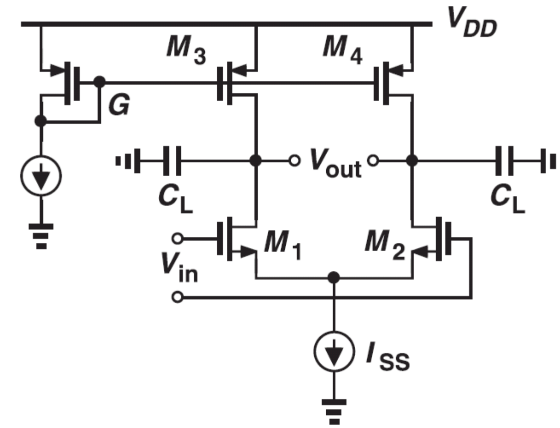
- For differential signals, the response is identical to that of a CS stage.
- For CM gain, the total capacitance at node P determines the HF gain.

$$A_{v,CM} = -\Delta g_m \left[R_D \,\|\, \left(\frac{1}{C_L s} \right) \right] \Big/ \left\{ \left(g_{m1} + g_{m2} \right) \left[r_{O3} \,\|\, \left(\frac{1}{C_P s} \right) \right] + 1 \right\}$$

- If the output pole >> the pole at the tail node of current source, the CM rejection of the circuit degrades considerably at high frequencies (HF).
- If the V_{DD} contains HF noise and the circuit exhibits mismatches, the resulting CM disturbance at node P leads to a differential noise component at the output.

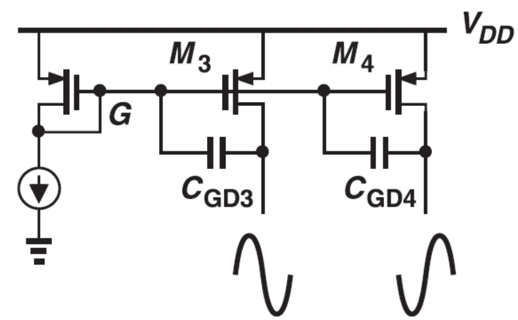
Effect of High-Frequency Supply Noise

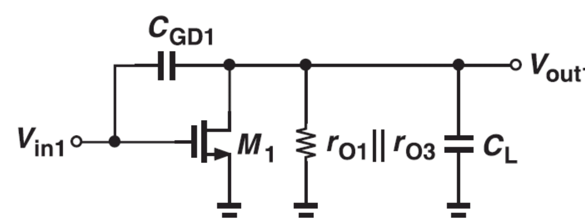




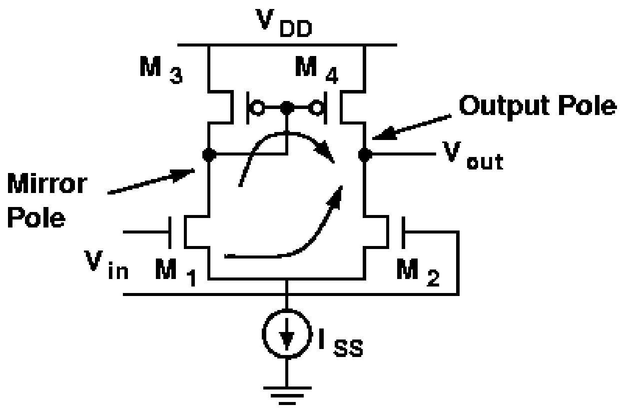
- Trade-off between voltage headroom and CMRR.
- To minimize voltage headroom, M_{tail} ↑, C_p ↑,high frequency CMRR ↓.
- For differential output signals, C_{GD3} and C_{GD4} conduct equal and opposite currents to G, making this node an AC ground.
- The dominant pole is at

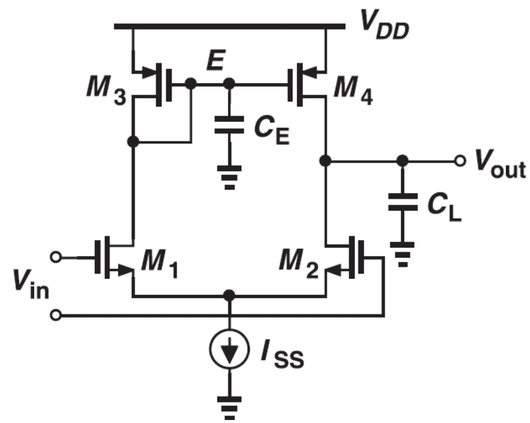
$$\omega_p = \frac{1}{[r_{O1} \parallel r_{O3}]C_L}$$

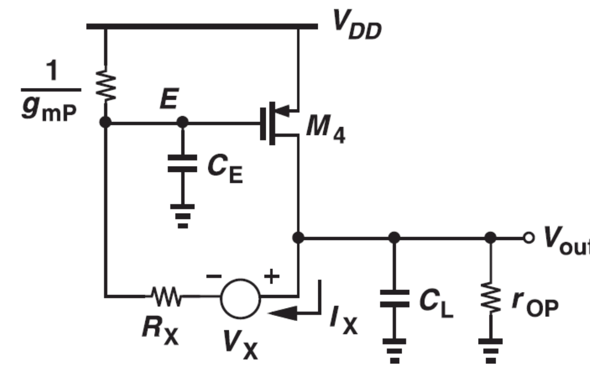




DP with Active Current Mirror







- This topology contains two signal paths with different transfer functions.
- Pole at node *E (Mirror pole)*

$$\omega_{pE} = \frac{g_{m3}}{C_E}, \quad C_E \approx C_{GS3} + C_{GS4} + C_{DB3} + C_{DB1} + 2C_{GD1} + g_{m4}\left(r_{O4} \parallel r_{O2}\right)C_{GD4}$$

$$V_X = g_{mN} r_{ON} V_{in}, \;\; R_X = 2r_{ON}, \; V_E = \left(V_{out} - V_X\right)\frac{\left(C_E s + g_{mP}\right)^{-1}}{\left(C_E s + g_{mP}\right)^{-1} + R_X}, \quad -g_{m4}V_E - I_X = V_{out}\left(sC_L + r_{OP}^{-1}\right)$$

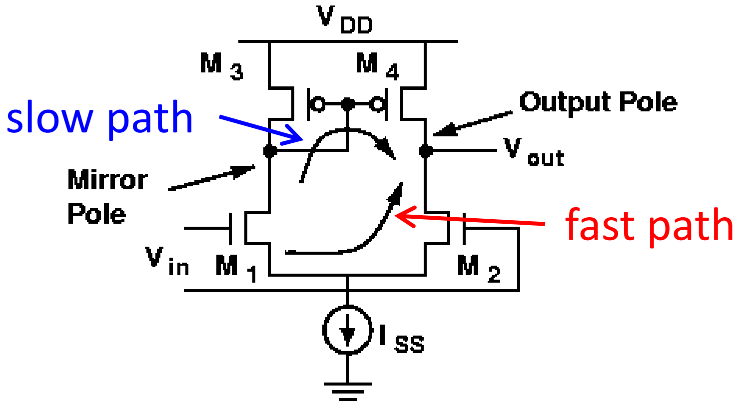
$$\frac{V_{out}}{V_{in}} = \frac{g_{mN} r_{ON} r_{OP}\left(2g_{mP} + sC_E\right)}{2r_{OP} r_{ON} C_E C_L s^2 + \left[\left(2r_{ON} + r_{OP}\right)C_E + r_{OP}\left(1 + 2g_{mP} r_{ON}\right)C_L\right]s + 2g_{mp}\left(r_{ON} + r_{OP}\right)}$$

$$\omega_{P1} \approx \frac{2g_{mP}\left(r_{ON} + r_{OP}\right)}{\left(2r_{ON} + r_{OP}\right)C_E + r_{OP}(1 + 2g_{mp} r_{ON})C_L} \approx \frac{1}{\left(r_{ON} \parallel r_{OP}\right)C_L} \quad \text{if} \quad 2g_{mP} r_{ON} >> 1 \qquad \omega_{p2} \approx \frac{g_{mP}}{C_E}$$

DP with Active Current Mirror

- To find zero
  - Consider that the circuit consists of a "slow path" (M_1, M_3, and M_4) in parallel with a "fast path" (M_1, and M_2)
  - Slow path

$$A_{v1} = \frac{A_0}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)}$$



  - Fast path

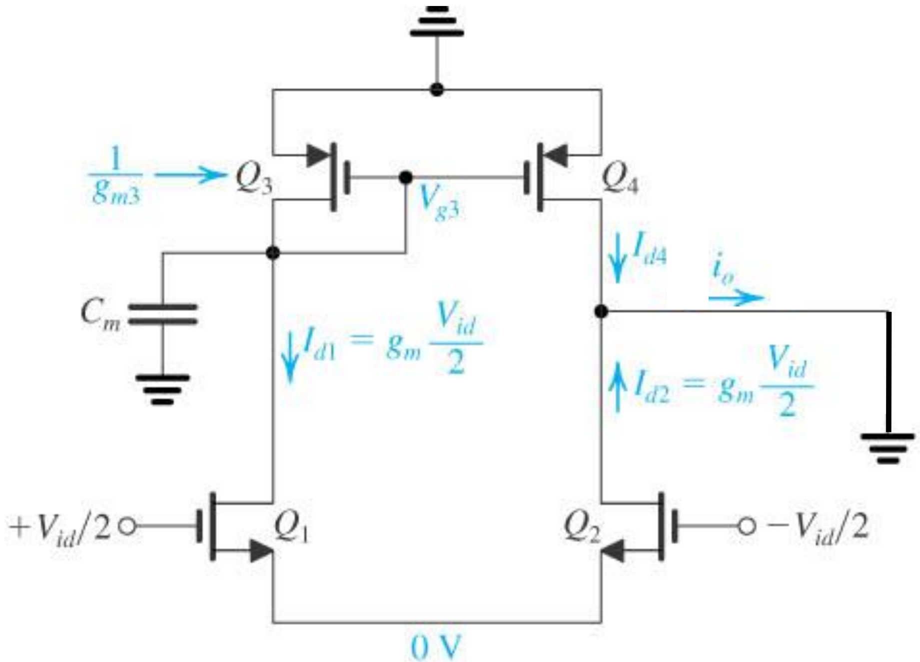
$$A_{v2} = \frac{A_0}{\left(1+\frac{s}{p_1}\right)}$$

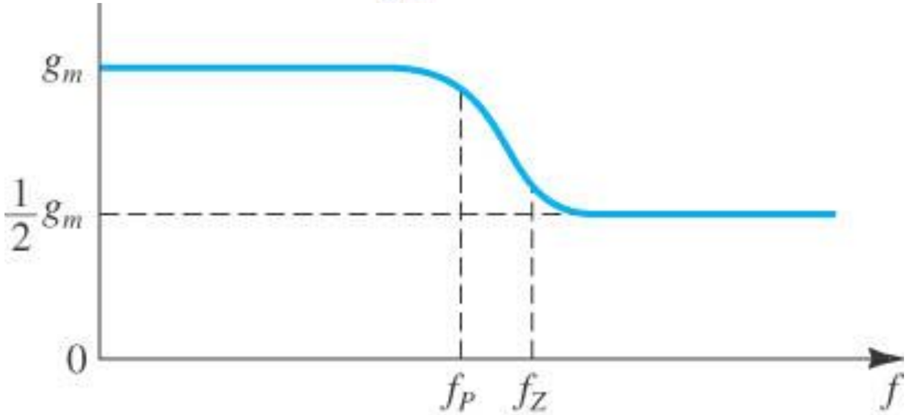
  - We have

$$\frac{V_{out}}{V_{in}} = A_{v1} + A_{v2} = \frac{A_0}{1+\frac{s}{p_1}}\left(1+\frac{1}{1+\frac{s}{p_2}}\right) = \frac{A_0\left(2+\frac{s}{p_2}\right)}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)} \quad \Rightarrow \quad z = 2p_2$$

  - Fully differential pair has no mirror pole, which is better than differential-to-single ended amp.

G_m Frequency Response





At low frequency: C_m is OPEN

$$i_{d4} = i_{d1} = g_m \frac{V_{id}}{2}, i_{d2} = g_m \frac{V_{id}}{2}$$

$$i_o = i_{d4} + i_{d2} = g_m V_{id}$$

$$G_m = i_o / V_{id} = g_m$$

At high frequency: C_m is SHORT

$$V_{g3} = 0, i_{d4} = 0, i_o = i_2 = g_m \frac{V_{id}}{2}$$

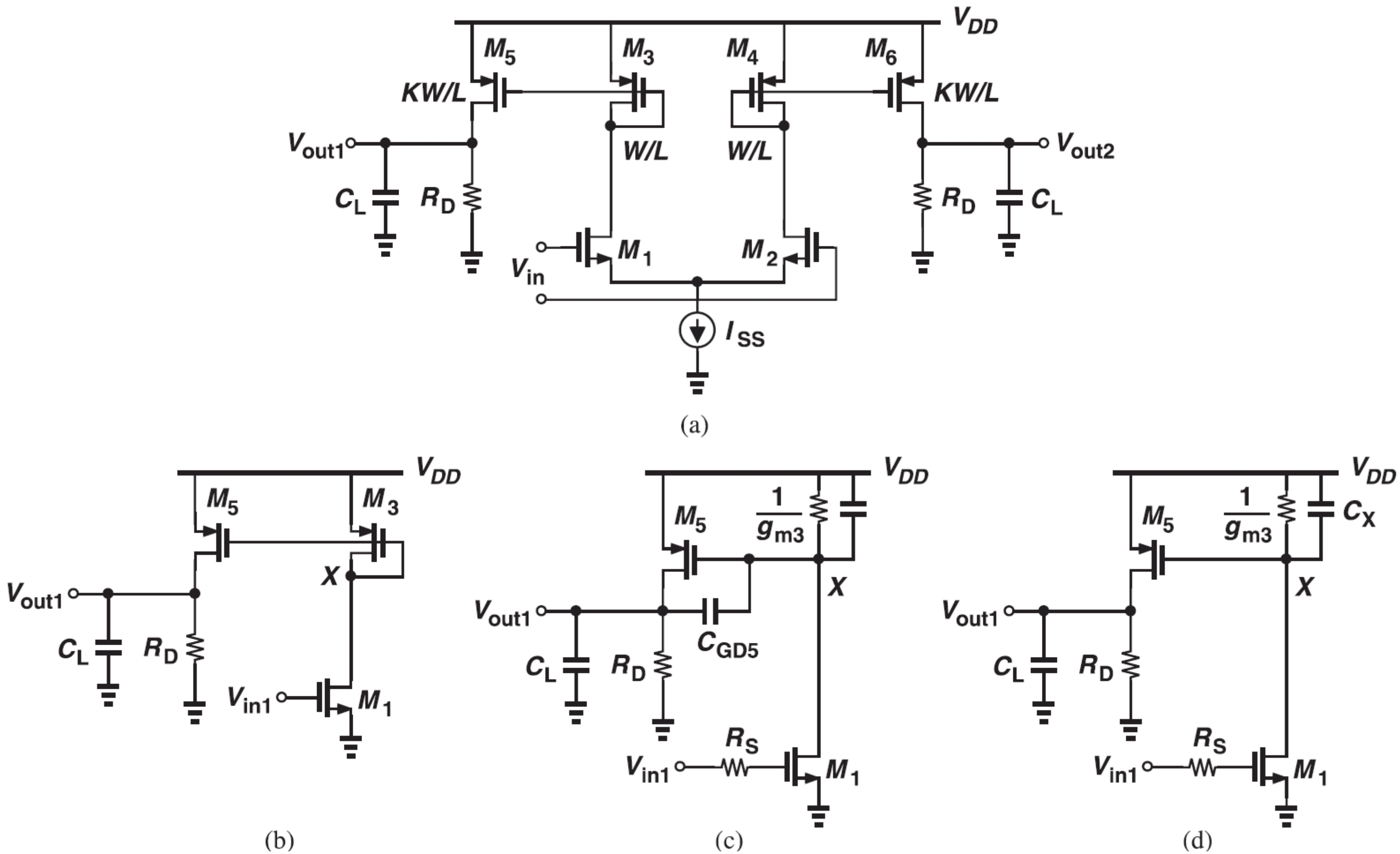
$$G_m = i_o / V_{id} = g_m / 2$$

G_m transfer function

$$G_m \equiv \frac{i_o}{V_{id}} = g_m \left(\frac{1 + sC_m / 2g_{m3}}{1 + sC_m / g_{m3}}\right)$$

$$f_P = \frac{g_{m3}}{2\pi C_m}, f_Z = \frac{2g_{m3}}{2\pi C_m}$$

Fully Differential Amplifier



$$\frac{V_{out1}}{V_X}(s) = -g_{m5} R_D \frac{1}{1 + sR_D C_L}$$

$$C_X \approx C_{GS3} + C_{GS5} + C_{DB3} + C_{GD5}(1 + g_{m5} R_D) + C_{DB1}$$