CHAPTER 4

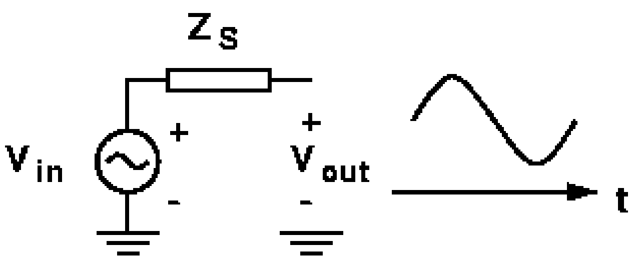
Differential Amplifier

Outline

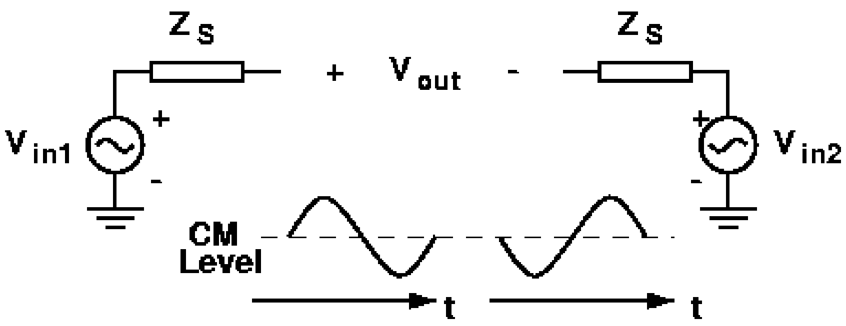
1. **Single-Ended and Differential Operation**
2. Basic Differential Pair
3. Common-Mode Response
4. Differential Pair with MOS Loads
5. Gilbert Cell

Single-Ended / Differential Amplifiers

- A *single-ended signal* is defined as one that is measured with respect to a fixed potential, usually GND.
- A *differential signal* is defined as one that is measured between two nodes have *equal* and *opposite* signal.
- *Common mode level* : the *center* potential in difference signaling.



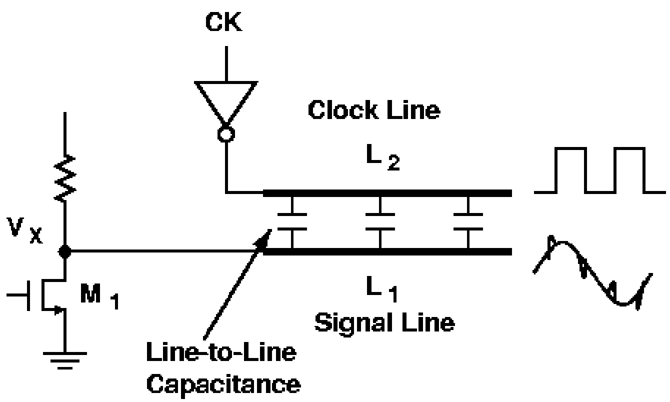
Single-ended signal

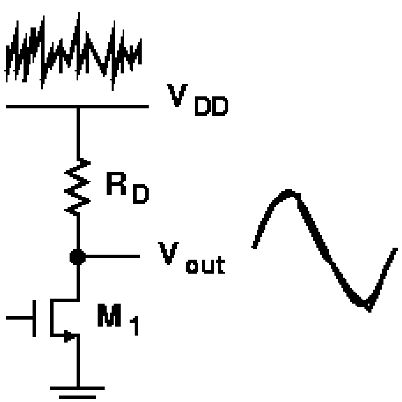


Differential signal

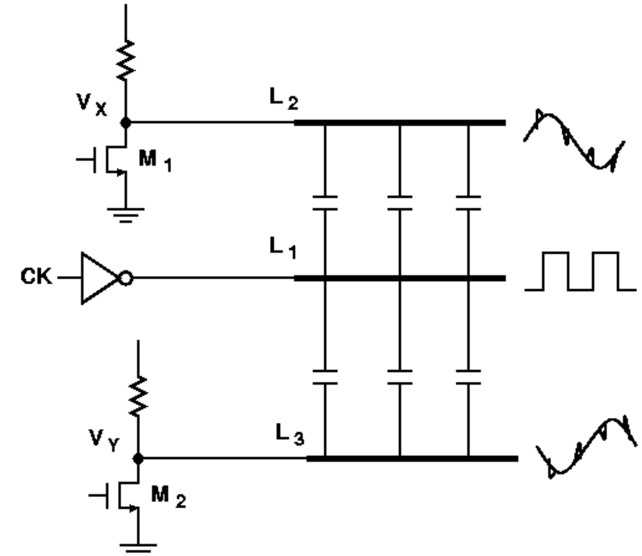
Single-Ended v.s. Differential Amp

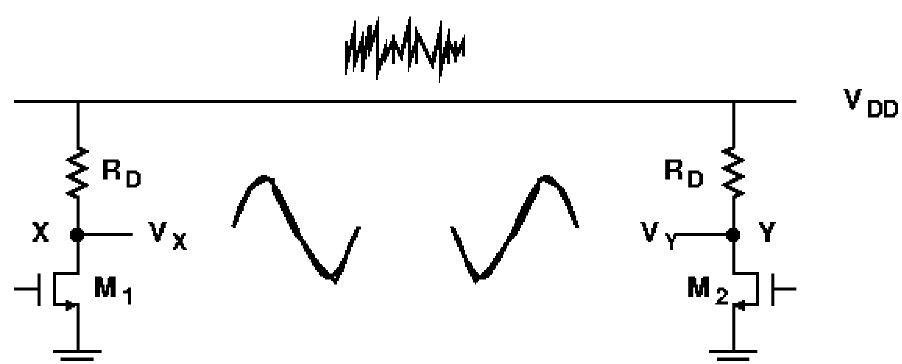
- Differential signaling: higher immunity to environmental noise.
- Single-ended signal





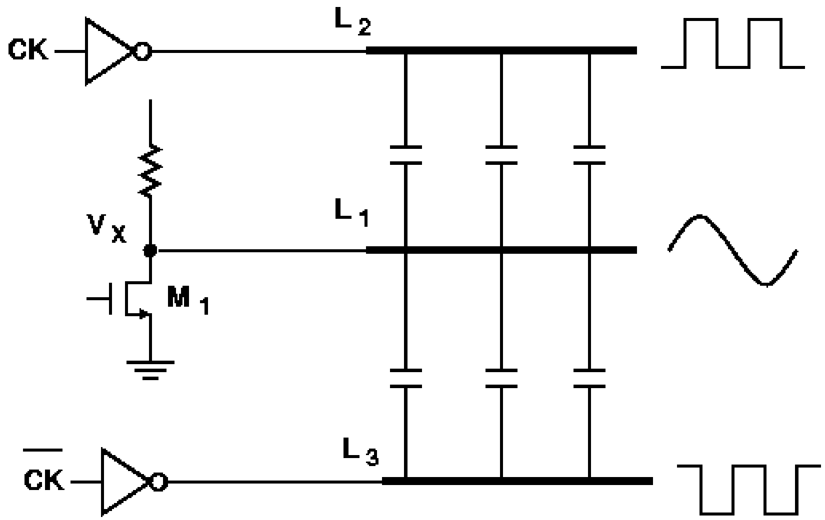
- Differential





Single-Ended v.s. Differential Amp

- Reduction of coupled noise by differential operation



- Differential signaling : increase the maximum achievable voltage swings. The peak to peak swing is equal to twice that of a single-ended signal.
- Example :
  - Single-ended signal $V_o = 1\sin\omega t$
  - Differential signal $V_{o+} = 1\sin\omega t$ $V_{o-} = -1\sin\omega t$ $V_{o+} - V_{o-} = 2\sin\omega t$

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2. **Basic Differential Pair**
3. Common-Mode Response
4. Differential Pair with MOS Loads
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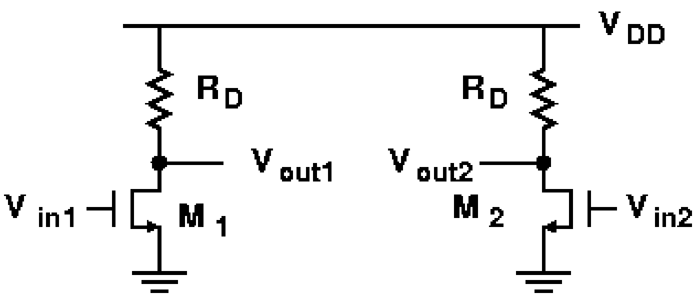
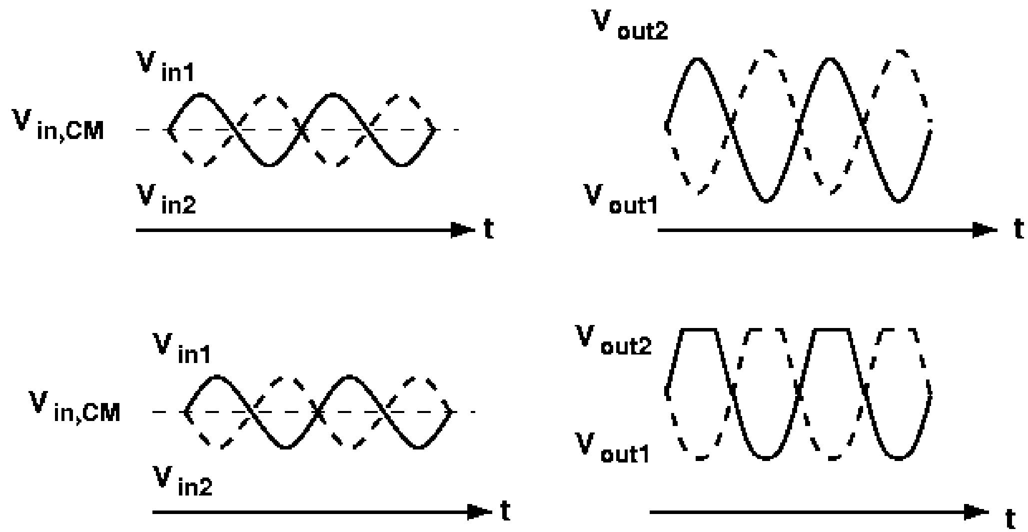
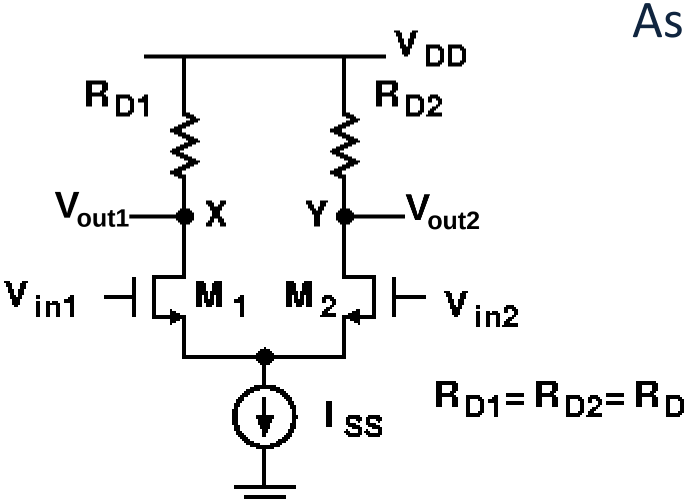
Basic Differential Pair







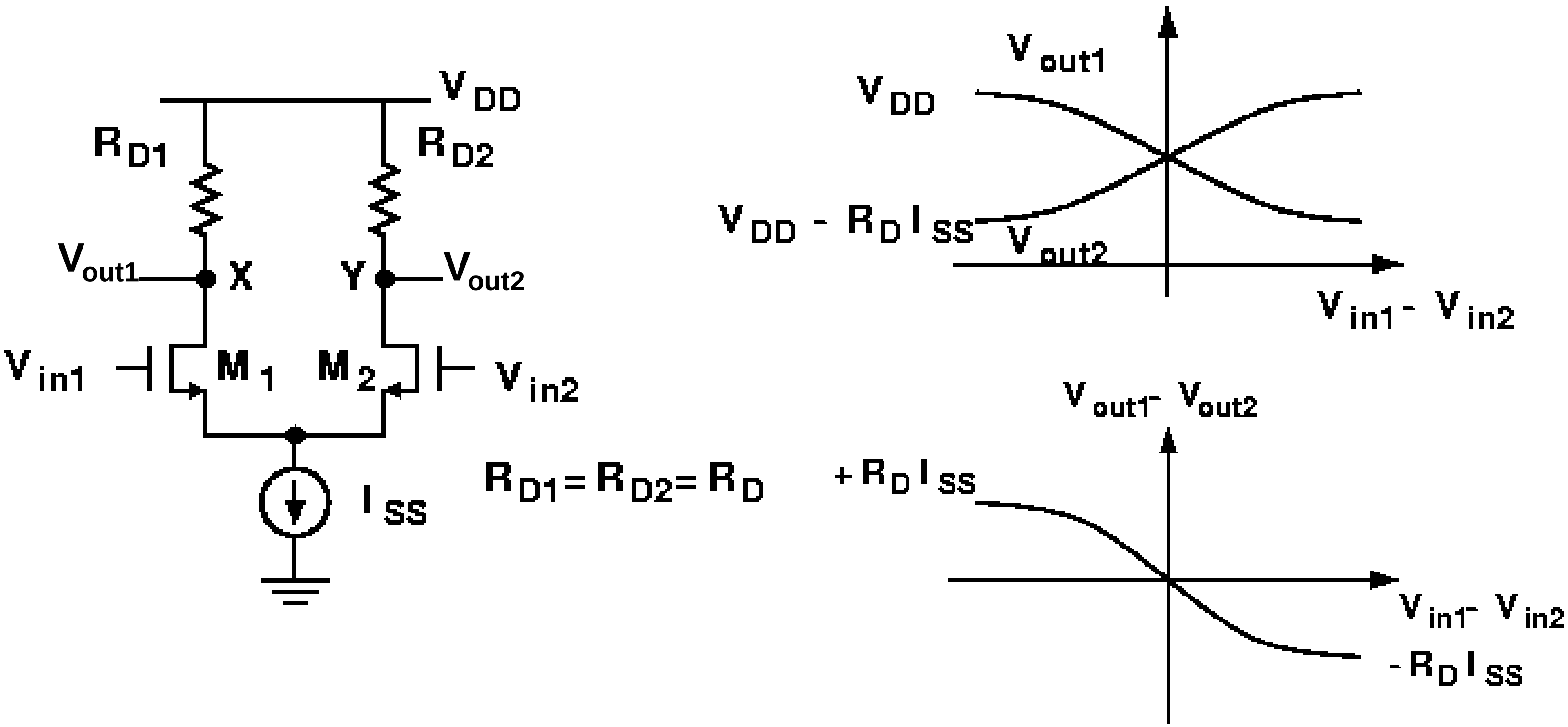
As the input CM level ($V_{in,cm}$) changes

– The variation of the g_m leads to a change in the small-signal gain
– Shift the output CM level from its ideal value lowers the maximum allowable output swings.
– To make the bias currents of the devices have minimal dependence on the input CM level

→ Differential pair with a current source bias

Differential Amp – Qualitative Analysis

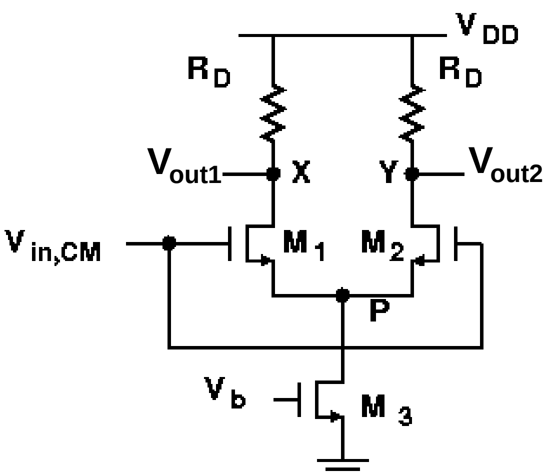
- The maximum and minimum levels at the output are well defined.
- The small-signal gain is maximum for $V_{in1} = V_{in2}$.
- The circuit becomes more nonlinear as the input swings increases.

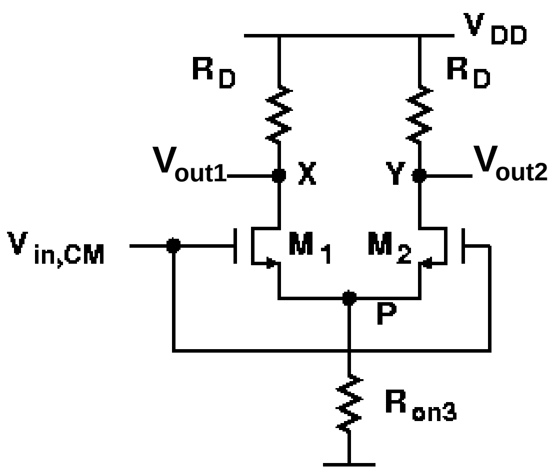


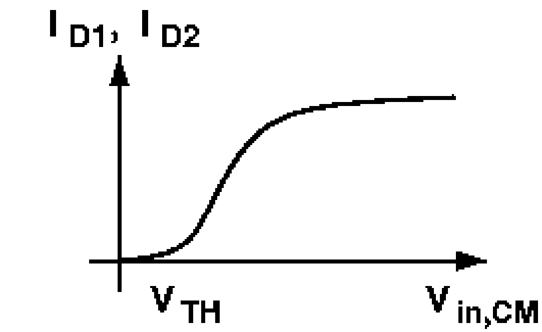
Input Common-Mode Range

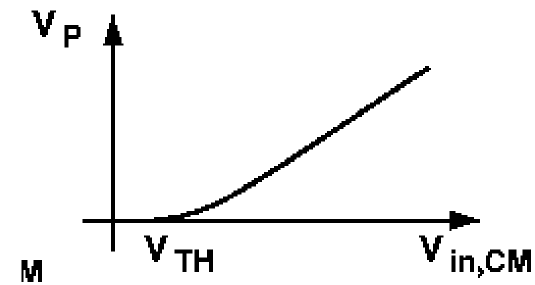
- The tail current source is to suppress the effect of input CM level variations.
- For proper operation, M1, M2 and M3 should be operated in saturation region.
- If $V_{in,CM}$ rises further, then M1 and M2 will enter the triode region.

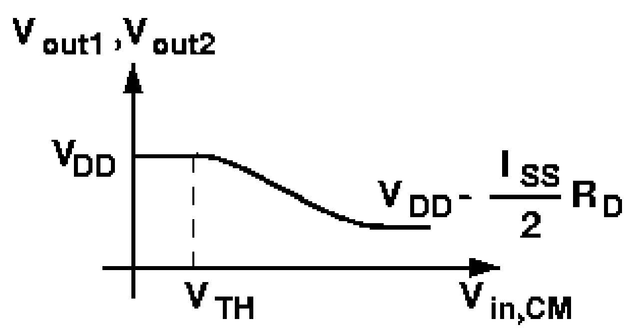
$$V_{GS1} + (V_{GS3} - V_{TH3}) \le V_{in,CM} \le \min\left[V_{DD} - R_D \frac{I_{SS}}{2} + V_{TH}, V_{DD}\right]$$



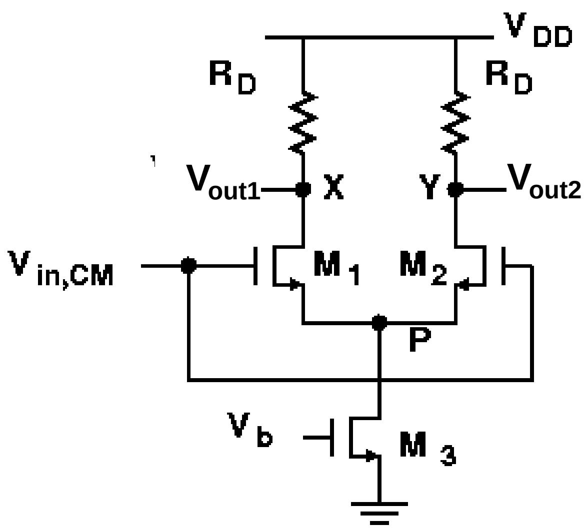


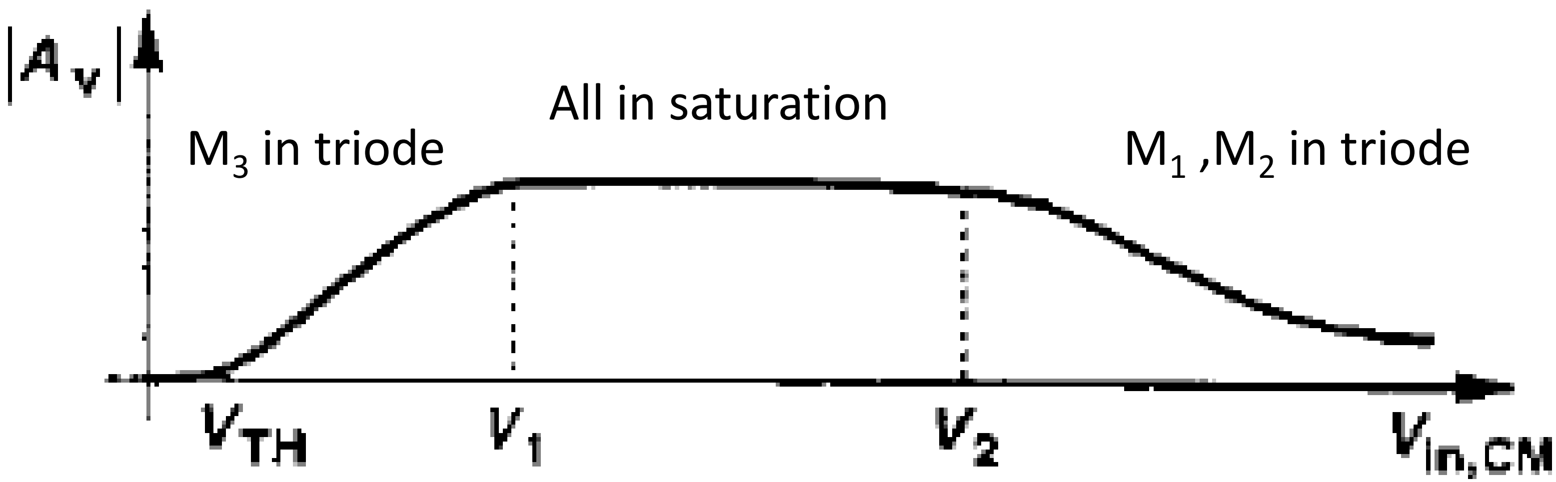






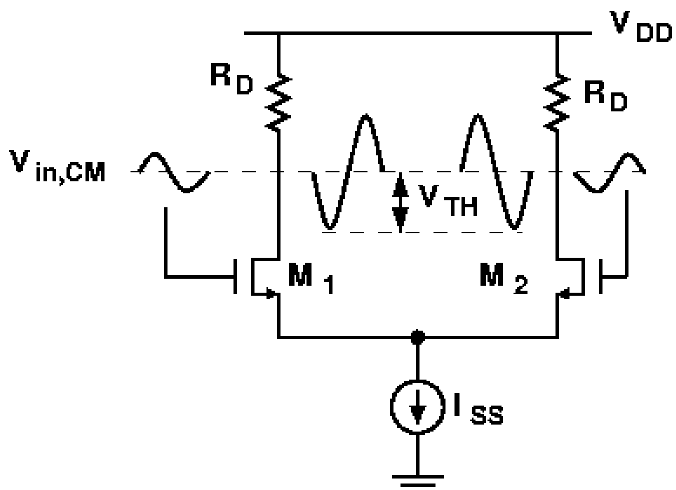
A_v vs. $V_{in.CM}$


V DD
R D
R D
Vout1
X
Y
Vout2
V in,CM
M 1
M 2
P
V b
M 3


|A v|
All in saturation
M3 in triode
M1 ,M2 in triode
VTH
V1
V2
Vin,CM

Input CM vs Output Swing

- The higher the input CM level, the smaller allowable output swing



$$V_{out1} = V_{DD} - R_{D1} I_{D1}$$
$$V_{out2} = V_{DD} - R_{D2} I_{D2}$$
$$\text{if } R_{D1} = R_{D2} = R_D$$
$$V_{out1} - V_{out2} = R_D (I_{D2} - I_{D1})$$
$$V_{in1} - V_{in2} = V_{GS1} - V_{GS2} = \Delta V$$
$$I_{SS} = I_{D1} + I_{D2}$$

$$(V_{GS} - V_{TH})^2 = \frac{I_D}{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}}, \quad V_{GS} = \sqrt{\frac{I_D}{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}}} + V_{TH}, \quad V_{in1} - V_{in2} = \sqrt{\frac{2I_{D1}}{\mu_n C_{ox}\frac{W}{L}}} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox}\frac{W}{L}}}$$

$$I_{D1} - I_{D2} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in1} - V_{in2})\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (V_{in1} - V_{in2})^2}, \qquad \frac{\partial \Delta I_D}{\partial \Delta V_{in}} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\frac{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - 2\Delta V_{in}^2}{\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - \Delta V_{in}^2}}$$

Maximum Input Range

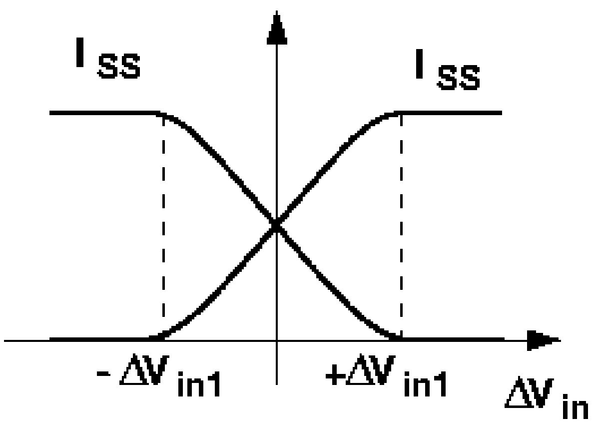
- For $\Delta V_{in} = 0$

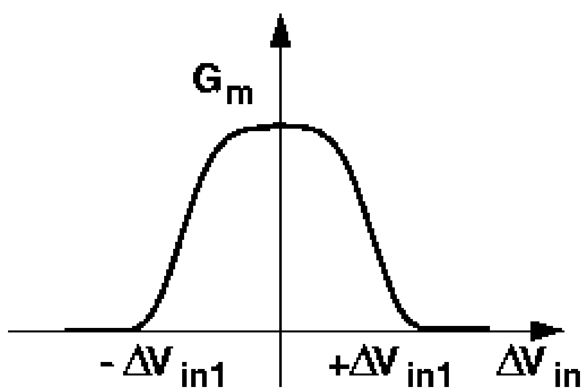
$$G_m = \sqrt{\mu_n C_{ox}(W/L)I_{SS}}, \qquad V_{out1} - V_{out2} = R_D \Delta I = R_D G_m \Delta V_{in}$$

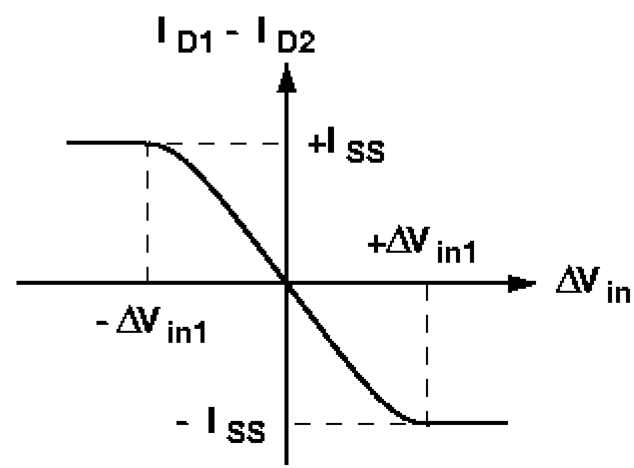
$$|A_v| = \sqrt{\mu_n C_{ox}(W/L)I_{SS}}R_D = G_m R_D$$

- G_m falls to zero for $\Delta V_{in} = \sqrt{2I_{SS}/(\mu_n C_{ox} W/L)} = \Delta V_{in1}$
- ΔV_{in1} represents the maximum differential input that the circuit can handle.
- For a zero differential input,

$$I_{D1} = I_{D2} = I_{SS}/2 \qquad (V_{GS1} - V_{TH})_{1,2} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} = \frac{\Delta V_{in1}}{\sqrt{2}}$$

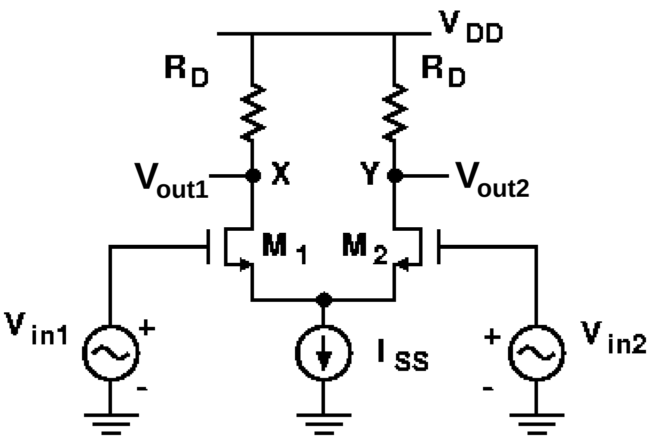


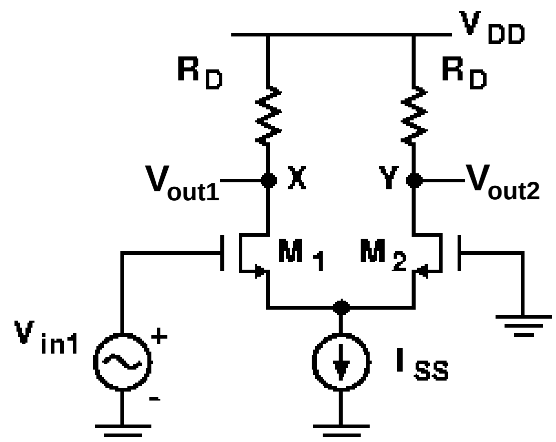


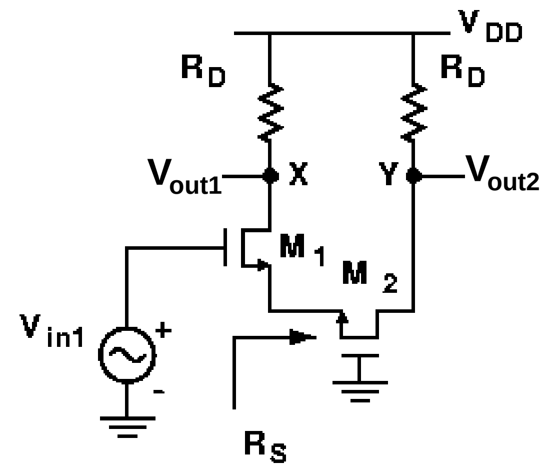


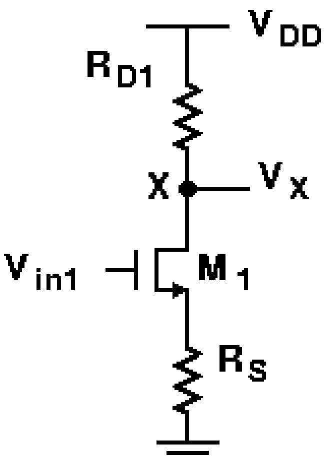
Voltage Gain of Differential Amplifier

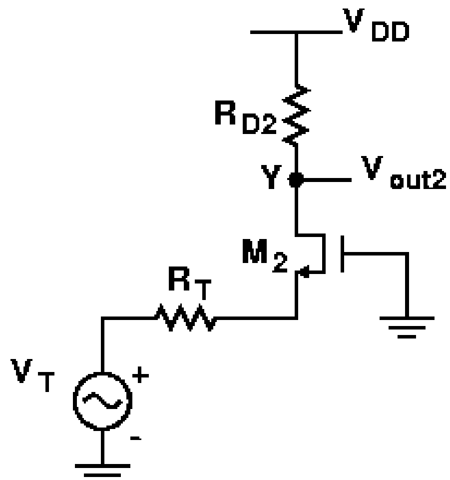
- Method 1 : *By linear superposition*











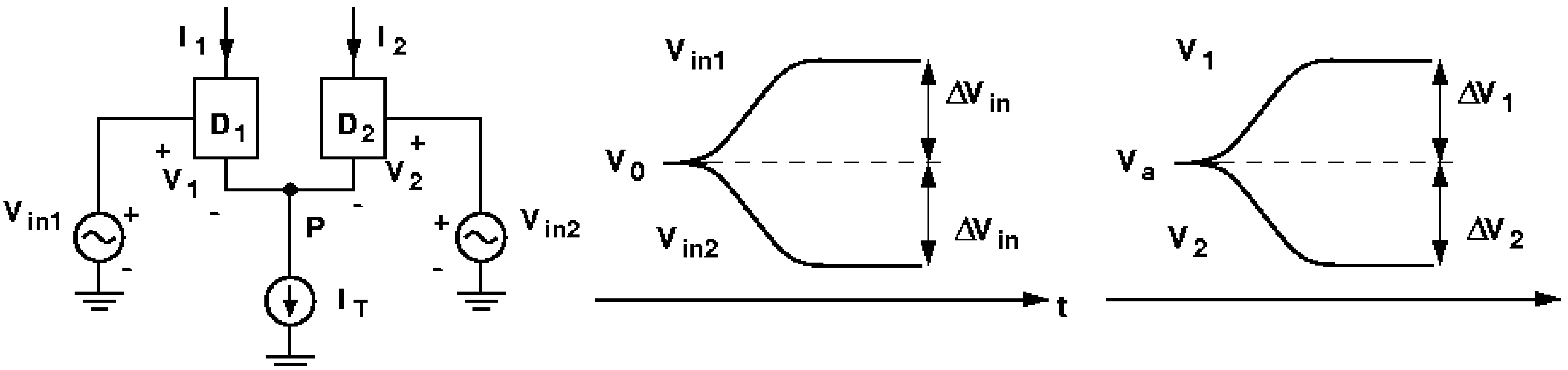
Common Source $\dfrac{V_X}{V_{in1}} = \dfrac{-R_D}{\dfrac{1}{g_{m1}} + \dfrac{1}{g_{m2}}}$

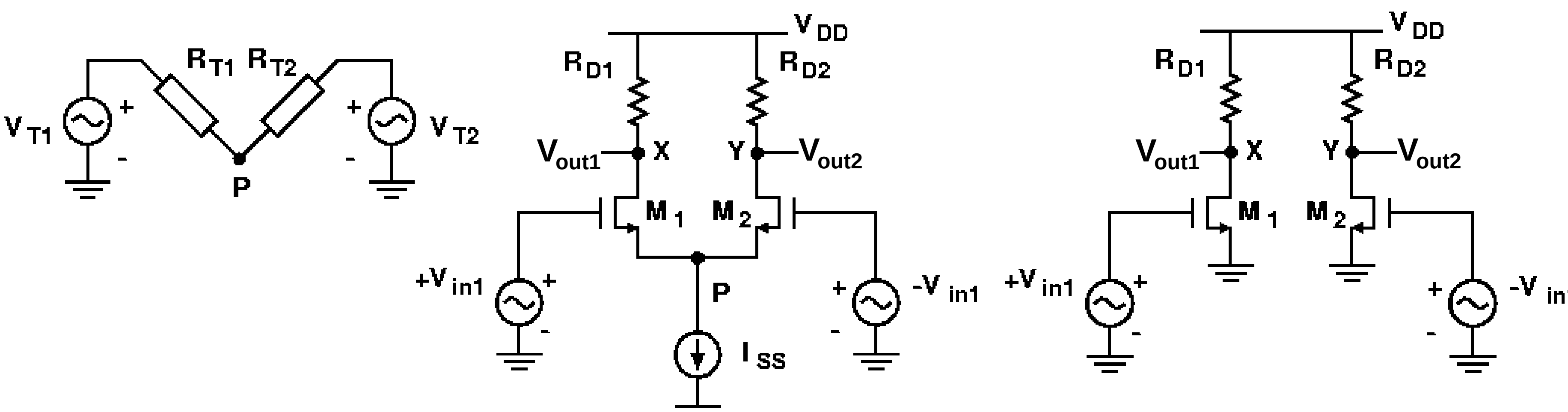
Source Follower + Common Gate

$$\frac{V_Y}{V_{in1}} = \frac{R_D}{\dfrac{1}{g_{m1}} + \dfrac{1}{g_{m2}}}, \qquad \frac{V_X - V_Y}{V_{in1} - V_{in2}} = -g_m R_D$$

Equivalent Half Circuit

- Node P is a virtual ground

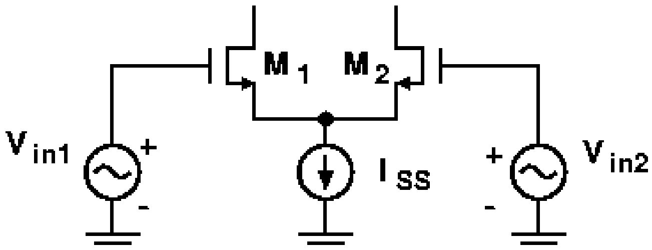




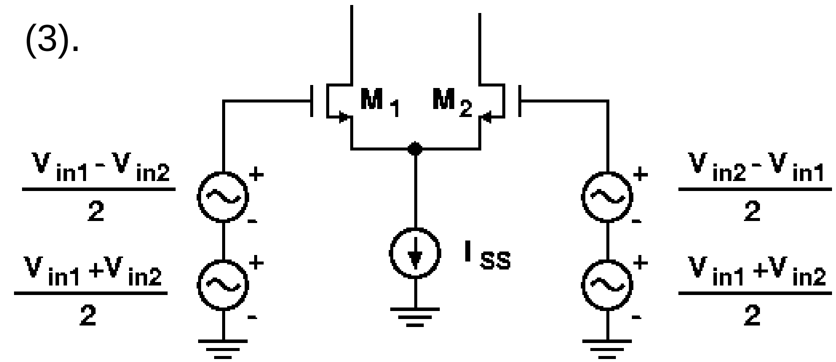
Equivalent Circuit

- Conversion of arbitrary inputs to differential and common-mode components

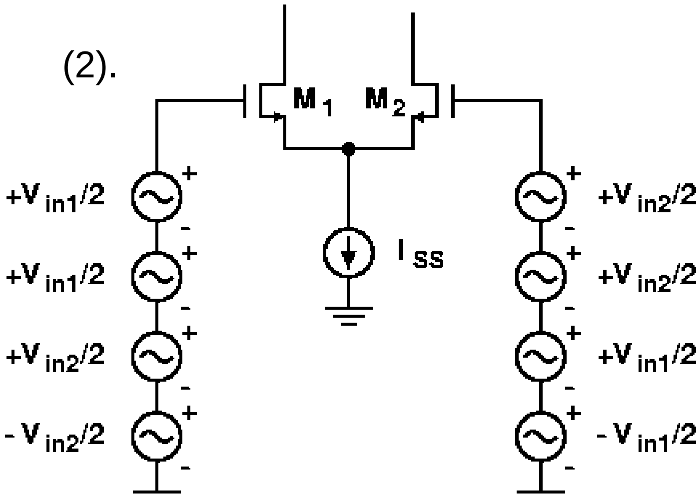
(1).



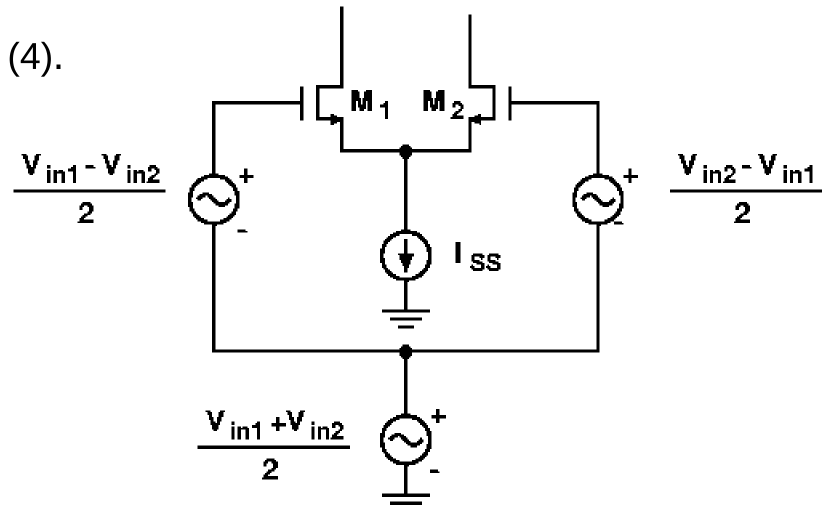
(3).



(2).



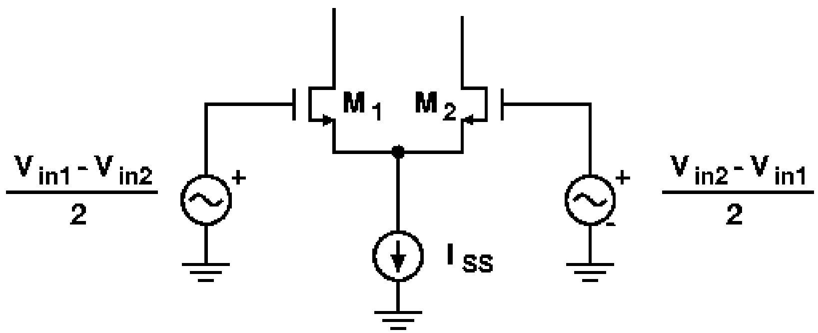
(4).



$$V_{in1} = \frac{V_{in1} - V_{in2}}{2} + \frac{V_{in1} + V_{in2}}{2} \qquad V_{in2} = \frac{V_{in2} - V_{in1}}{2} + \frac{V_{in1} + V_{in2}}{2}$$

Superposition of CM and DM Signal

- For differential mode operation



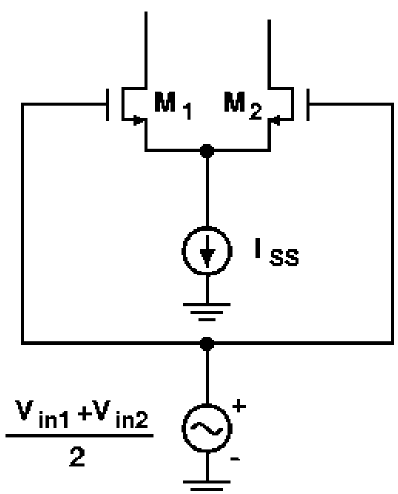
$$V_X = -g_m(R_D \parallel r_{O1})\frac{V_{in1}-V_{in2}}{2}$$

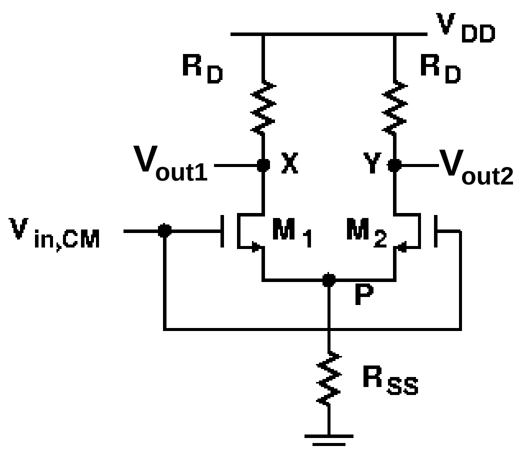
$$V_Y = -g_m(R_D \parallel r_{O2})\frac{V_{in2}-V_{in1}}{2}$$

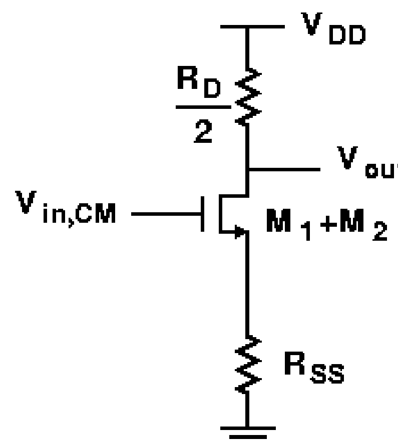
$$V_X - V_Y = -g_m(R_D \parallel r_O)(V_{in1}-V_{in2})$$

- For common mode operation

$$A_{v,CM} = \frac{V_{out}}{V_{in,CM}} = -\frac{R_D/2}{1/(2g_m)+R_{SS}}$$







Property of Differential Structure

- Advantage
  - Reject common mode noise, including power supply noise.
  - Reduce the even-order harmonic distortion
  - Increase output voltage swing

- Disadvantage
  - Double the power consumption and circuitry.
  - Need common mode voltage stabilization.

$$y_+ = \alpha_1(x_1) + \alpha_2(x_1)^2 + \alpha_3(x_1)^3 + ...$$

$$y_- = \alpha_1(-x_1) + \alpha_2(-x_1)^2 + \alpha_3(-x_1)^3 + ...$$

$$Y = y_+ - y_- = 2\alpha_1(x_1) + 2\alpha_3(x_1)^3 + ...$$

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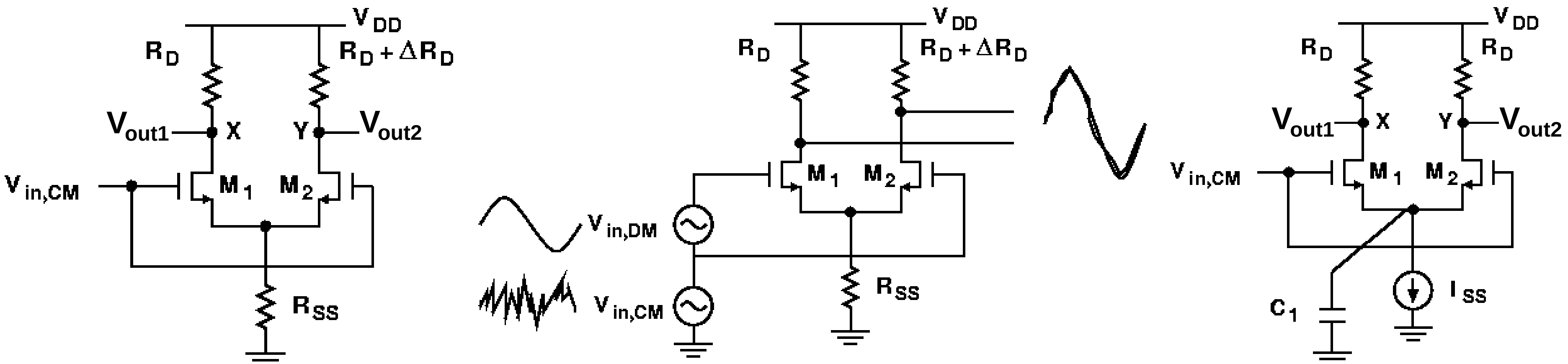
Common Mode Response

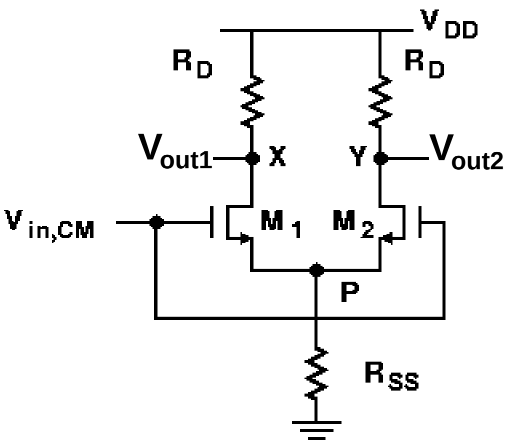
- In a symmetric circuit, input CM variations disturb the bias points, altering the small signal gain and possibly limit the output voltage swings.
- If the circuit is not symmetric, input CM variations will *lead to the variation of the differential output → Common mode to differential conversion*.

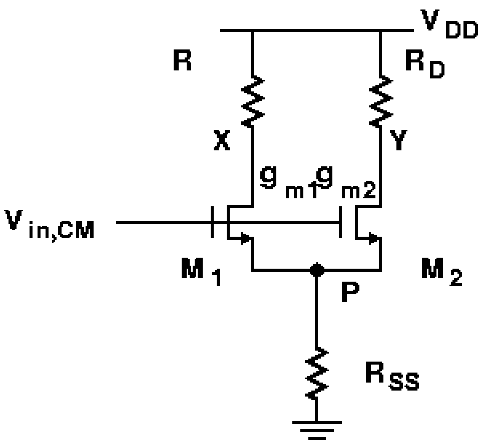


$$\Delta V_X = -\Delta V_{in,CM} \frac{g_m}{1+2g_m R_{SS}} R_D, \quad \Delta V_Y = -\Delta V_{in,CM} \frac{g_m}{1+2g_m R_{SS}} (R_D + \Delta R_D)$$

- The common-mode to differential conversion becomes significant at high frequencies since R_{SS} is shunted by C_1.

Differential Pair Sensing CM Input





$$I_{D1} = g_{m1}(V_{in,CM} - V_P)$$

$$I_{D2} = g_{m2}(V_{in,CM} - V_P)$$

$$(g_{m1} + g_{m2})(V_{in,CM} - V_P)R_{SS} = V_P$$

$$V_P = \frac{(g_{m1} + g_{m2})R_{SS}}{(g_{m1} + g_{m2})R_{SS} + 1} V_{in,CM}$$

$$V_X = -g_{m1}(V_{in,CM} - V_P)R_D = \frac{-g_{m1}}{(g_{m1} + g_{m2})R_{SS} + 1} R_D V_{in,CM}$$

$$V_Y = -g_{m2}(V_{in,CM} - V_P)R_D = \frac{-g_{m2}}{(g_{m1} + g_{m2})R_{SS} + 1} R_D V_{in,CM}$$

$$V_X - V_Y = -\frac{g_{m1} - g_{m2}}{(g_{m1} + g_{m2})R_{SS} + 1} R_D V_{in,CM}$$

$$A_{CM-DM} = -\frac{g_{m1} - g_{m2}}{(g_{m1} + g_{m2})R_{SS} + 1} R_D = \frac{-\Delta g_m R_D}{(g_{m1} + g_{m2})R_{SS} + 1}$$

$\Delta g_m \uparrow R_{SS} \downarrow A_{CM-DM} \uparrow$

Common-Mode Rejection Ratio

- The undesirable differential component produced by CM variations must be normalized to the wanted differential output resulting from amplification.
- The common mode rejection ratio – CMRR

$$CMRR = \left| \frac{A_{DM}}{A_{CM-DM}} \right|$$

- If only g_m mismatch is considered ($V_{in1} = -V_{in2}$) (use p.13 method)

$$\left|A_{DM}\right| = \frac{R_D}{2} \frac{g_{m1} + g_{m2} + 4g_{m1}g_{m2}R_{SS}}{(g_{m1} + g_{m2})R_{SS} + 1}$$

- The CMRR can be derived as

$$CMRR = \frac{g_{m1} + g_{m2} + 4g_{m1}g_{m2}R_{SS}}{2\Delta g_m} \approx \frac{g_m}{\Delta g_m}(1 + 2g_m R_{SS}), \quad g_m = \frac{g_{m1} + g_{m2}}{2}$$

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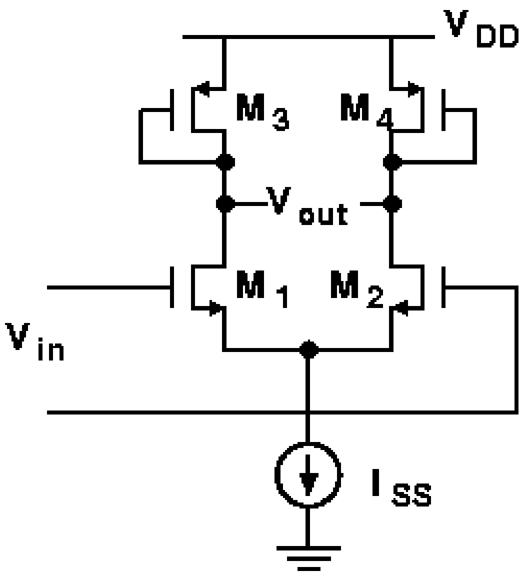
Differential Pair with MOS Loads

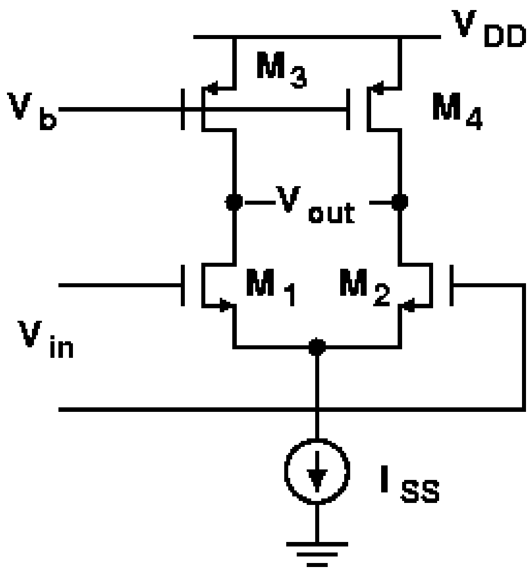
- Diode load



$$A_v = -g_{mN}\left(g_{mP}^{-1} \parallel r_{ON} \parallel r_{OP}\right)$$
$$\approx -\frac{g_{mN}}{g_{mP}} \approx -\sqrt{\frac{\mu_n (W/L)_N}{\mu_p (W/L)_P}}$$

Output CM level = V_{DD} - V_{GSP}

- Current source load

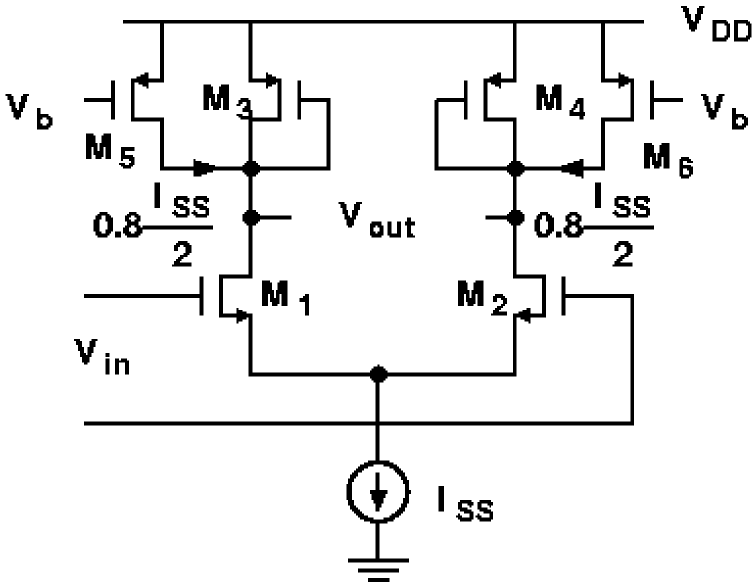


$$A_v = -g_{mN}\left(r_{ON} \parallel r_{OP}\right)$$

Output CM not well defined
Need CM Feedback circuit

Differential Pair with MOS Loads

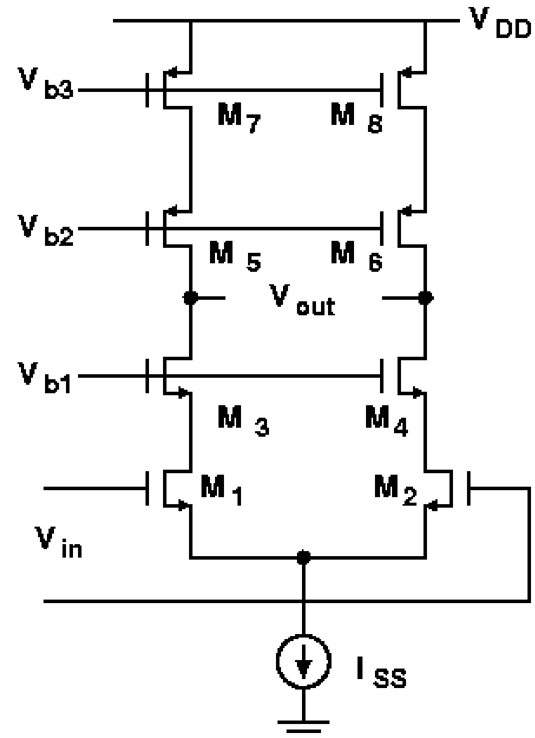
- Addition of current sources to increase the voltage gain



$$I_D(\mathrm{M}_3, \mathrm{M}_4) = 0.2\frac{I_{SS}}{2}\ ,$$

$g_m(\mathrm{M}_3, \mathrm{M}_4)$ decresed to be 1/5, Gain increased x5

- Cascode differential pair

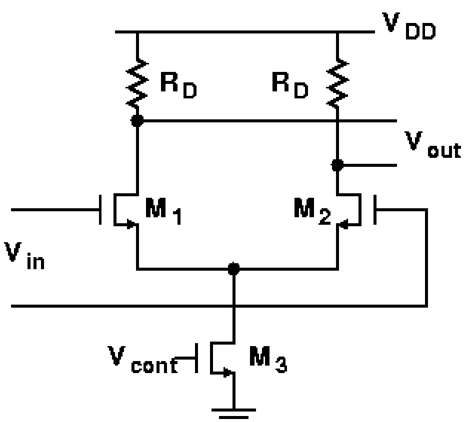


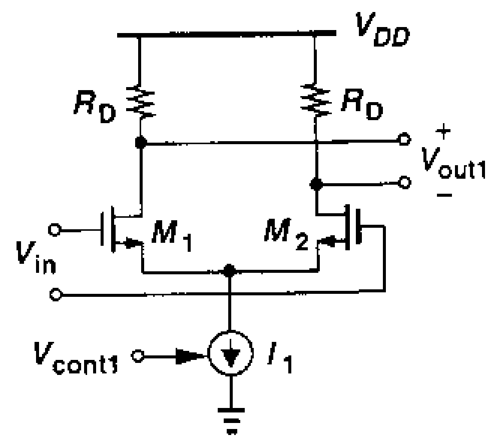
$$A_v \approx -g_{m1}\left[(g_{m3}r_{O3}r_{O1}) \parallel (g_{m5}r_{O5}r_{O7})\right]$$

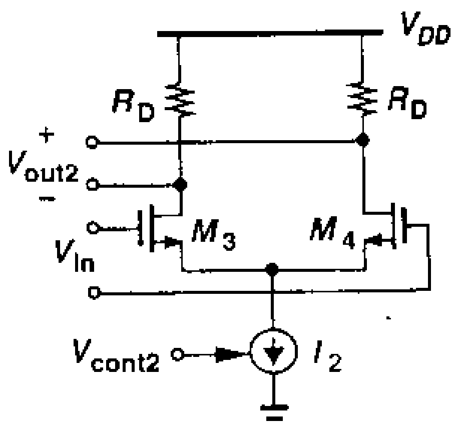
Outline

Gilbert Cell as a Variable Gain Amp

- The small-signal gain is a function of tail current
- The two transistors in a differential pair provide means of steering the tail current to one of two destinations







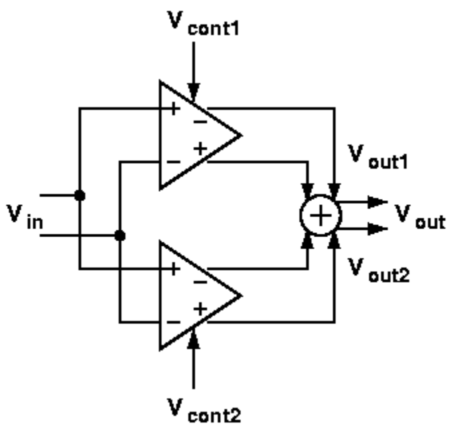
$$\frac{V_{out1}}{V_{in}} = -g_m R_D, \quad \frac{V_{out2}}{V_{in}} = g_m R_D$$

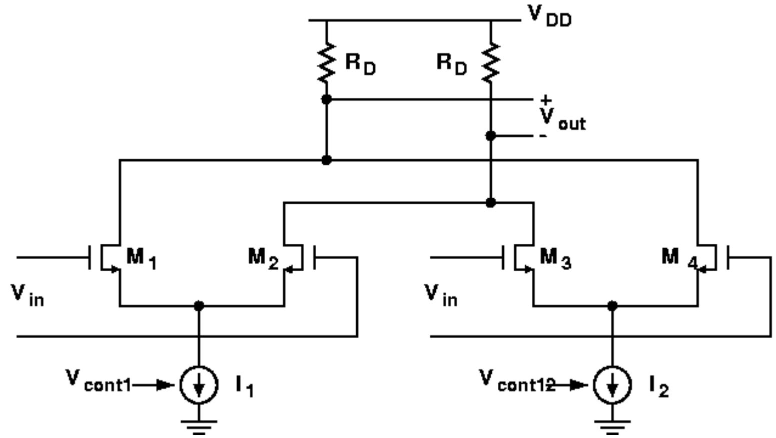
$$V_{out1} = R_D I_{D1} - R_D I_{D2}, \quad V_{out2} = R_D I_{D4} - R_D I_{D3}$$

$$V_{out} = V_{out1} + V_{out2} = A_1 V_{in} + A_2 V_{in} = R_D (I_{D1} + I_{D4}) - R_D (I_{D2} + I_{D3})$$

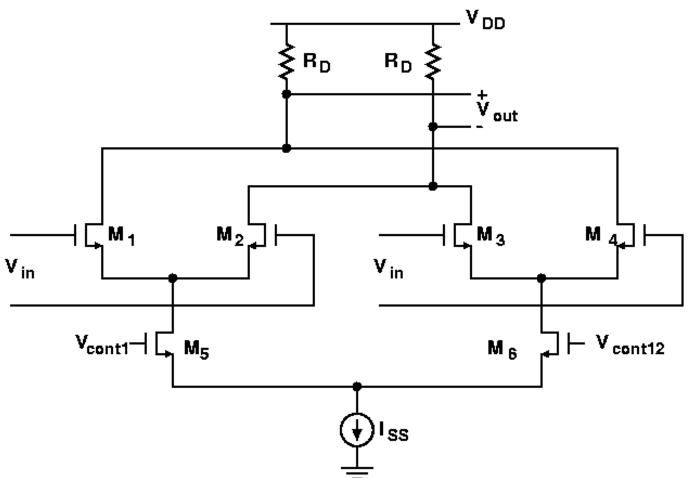
Gilbert Cell as a Variable Gain Amp

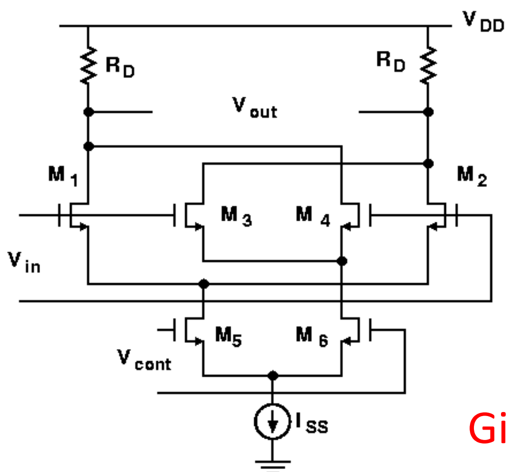
- An amplifier whose gain can continuously varied from a negative value to a positive value





$$V_{out} = V_{out1} + V_{out2} = A_1 V_{in} + A_2 V_{in} = R_D(I_{D1} + I_{D4}) - R_D(I_{D2} + I_{D3})$$





Gilbert Cell - 1968