



CHAPTER 4

Differential Amplifier

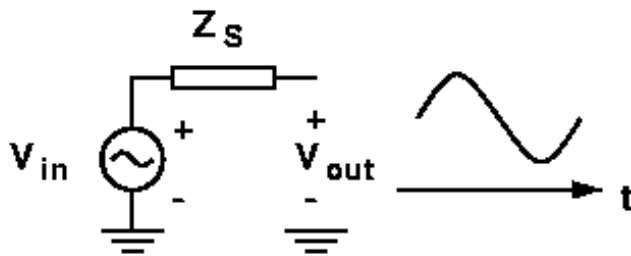
Outline

- 1. Single-Ended and Differential Operation**
2. Basic Differential Pair
3. Common-Mode Response
4. Differential Pair with MOS Loads
5. Gilbert Cell

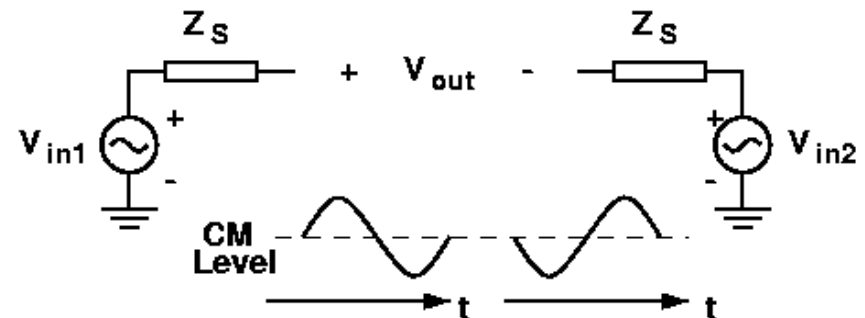
Single-Ended / Differential Amplifiers

amplitude相同, 相位相反

- A *single-ended signal* is defined as one that is measured with respect to a fixed potential, usually GND.
- A *differential signal* is defined as one that is measured between two nodes have *equal* and *opposite* signal.
- *Common mode level* : the *center* potential in difference signaling.



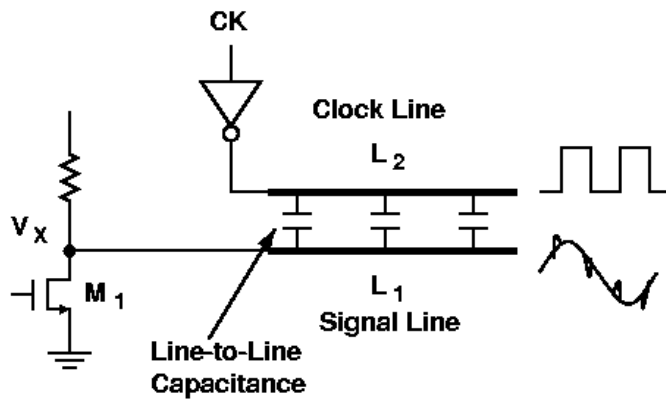
Single-ended signal



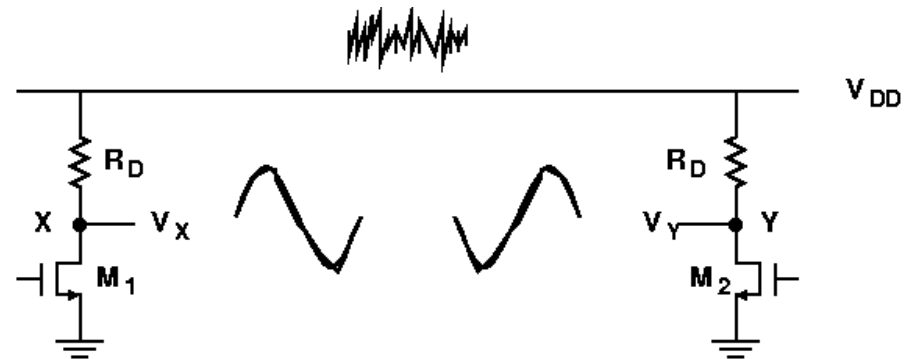
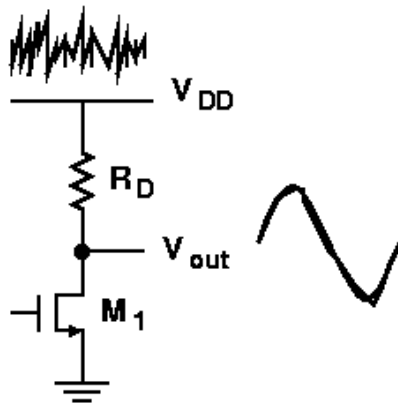
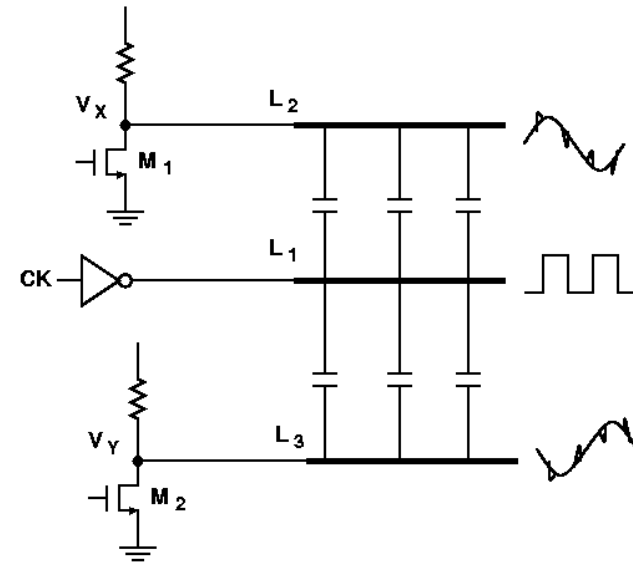
Differential signal

Single-Ended v.s. Differential Amp

- Differential signaling: higher immunity to environmental noise.
- Single-ended signal

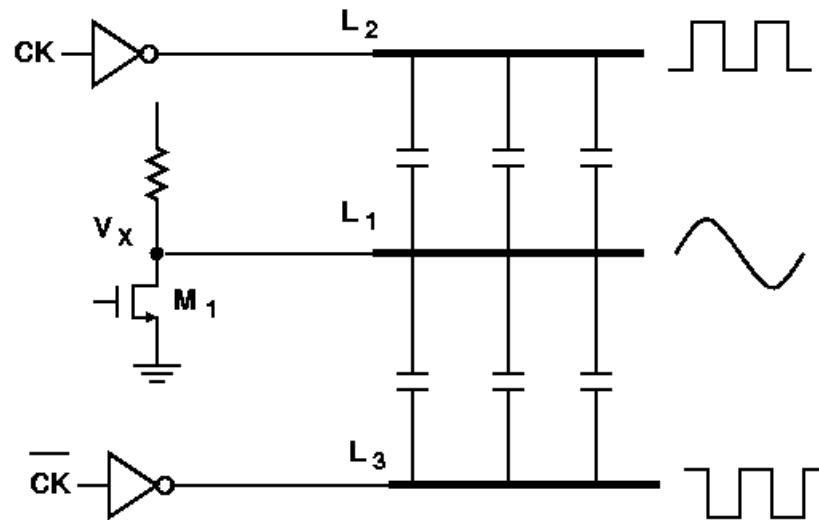


- Differential



Single-Ended v.s. Differential Amp

- Reduction of coupled noise by differential operation

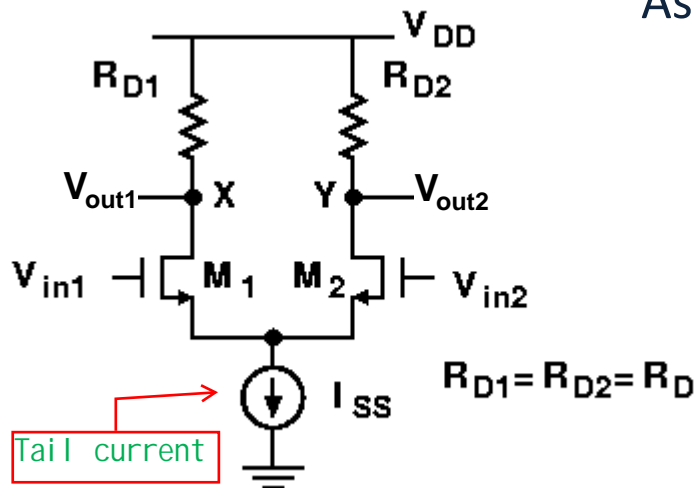
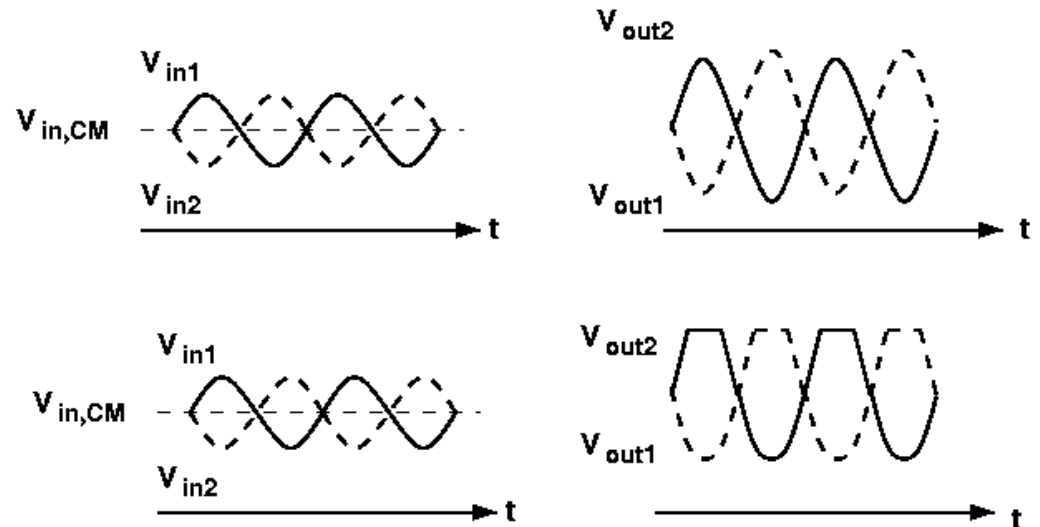
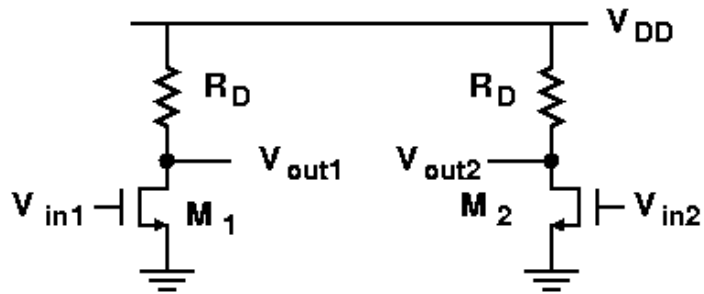


- Differential signaling : increase the maximum achievable voltage swings. The peak to peak swing is equal to **twice** that of a single-ended signal.
- Example :
 - Single-ended signal $V_o = 1 \sin \omega t$
 - Differential signal $V_{o+} = 1 \sin \omega t$ $V_{o-} = -1 \sin \omega t$ $V_{o+} - V_{o-} = 2 \sin \omega t$

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- 2. Basic Differential Pair**
3. Common-Mode Response
4. Differential Pair with MOS Loads
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Basic Differential Pair

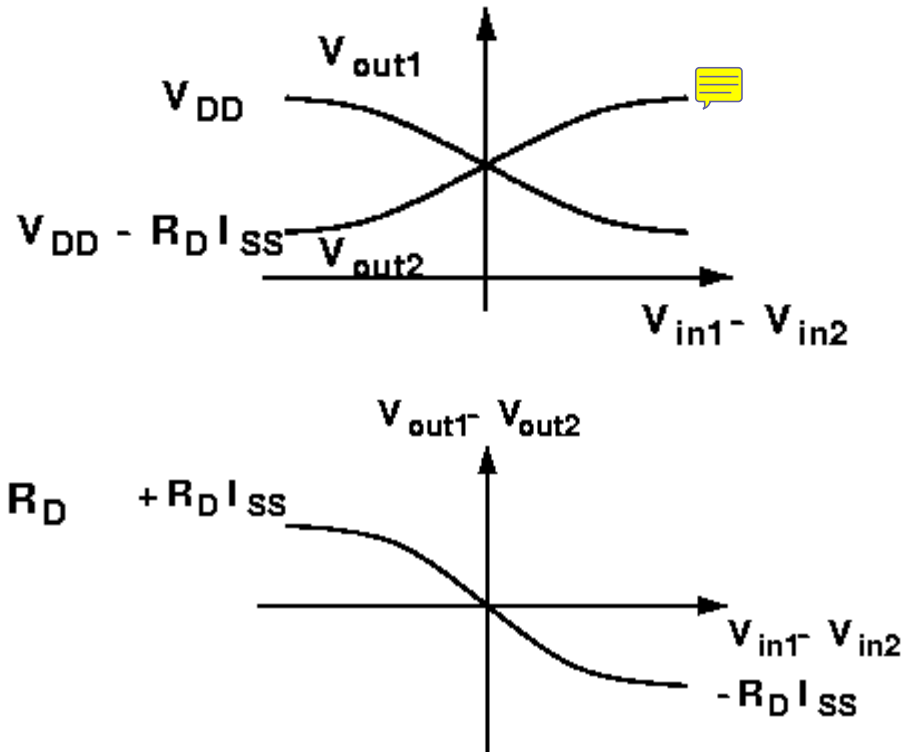
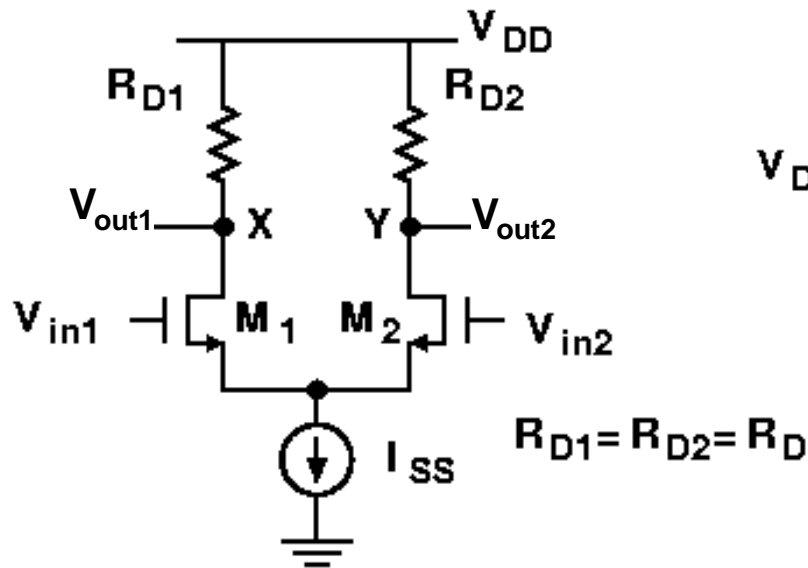


As the input CM level ($V_{in,cm}$) changes

- The variation of the g_m leads to a change in the small-signal gain
- Shift the output CM level from its ideal value lowers the maximum allowable output swings.
- To make the bias currents of the devices have minimal dependence on the input CM level
 → Differential pair with a current source bias

Differential Amp – Qualitative Analysis

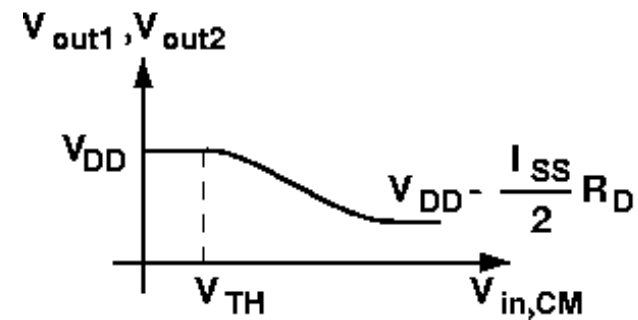
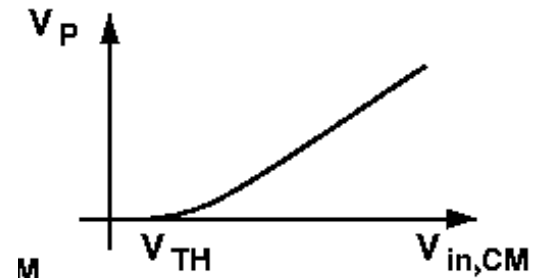
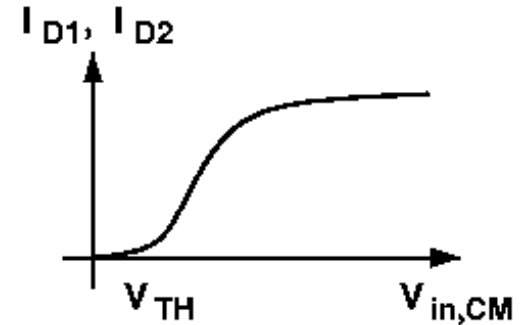
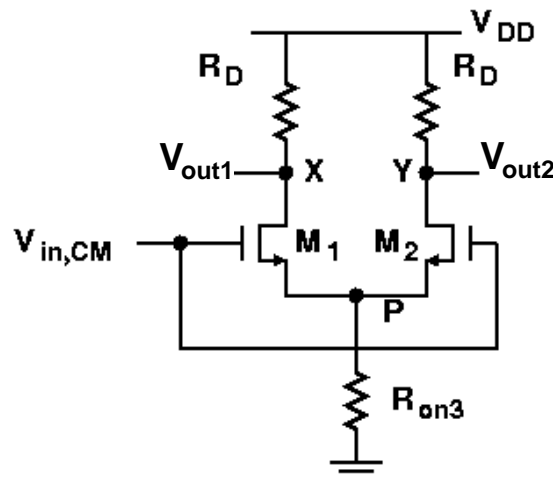
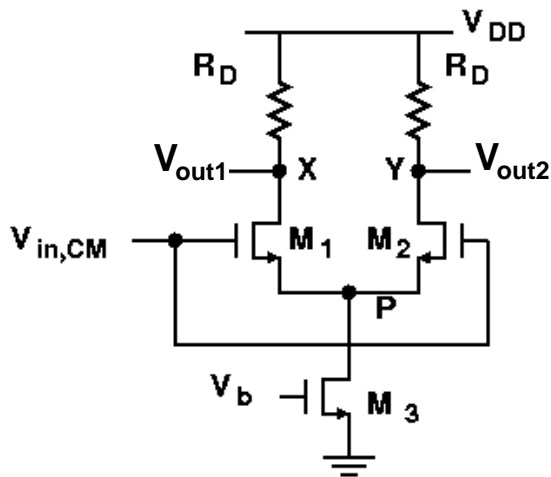
- The maximum and minimum levels at the output are well defined.
- The small-signal gain is maximum for $V_{in1} = V_{in2}$.
- The circuit becomes more nonlinear as the input swings increase.



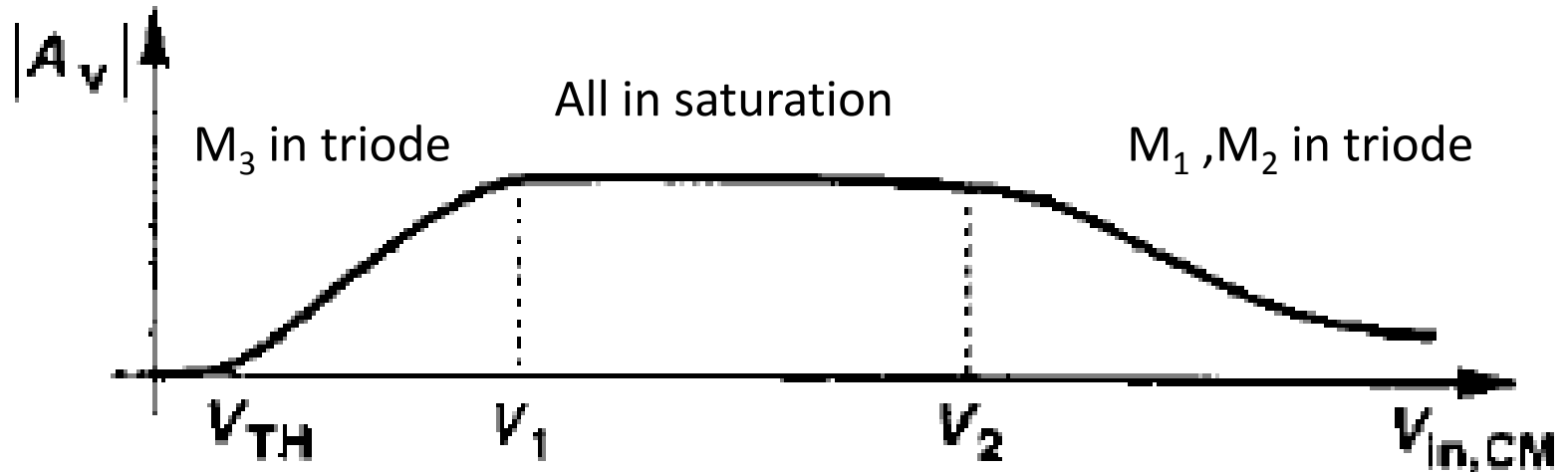
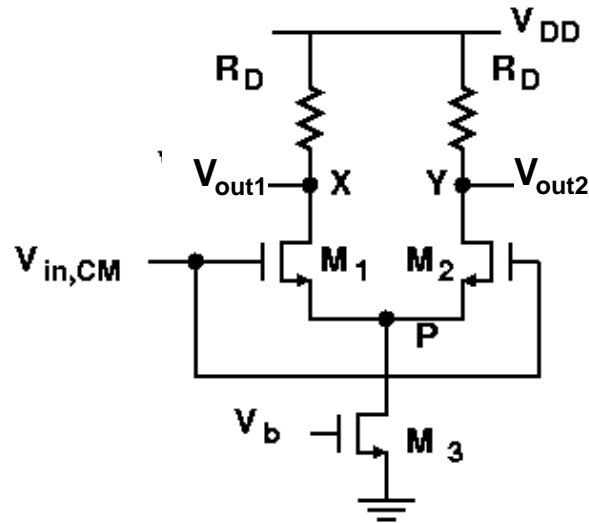
Input Common-Mode Range

- The tail current source is to suppress the effect of input CM level variations.
- For proper operation, M1, M2 and M3 should be operated in saturation region.
- If $V_{in,CM}$ rises further, then M1 and M2 will enter the triode region.

$$V_{GS1} + (V_{GS3} - V_{TH3}) \leq V_{in,CM} \leq \min \left[V_{DD} - R_D \frac{I_{SS}}{2} + V_{TH}, V_{DD} \right]$$



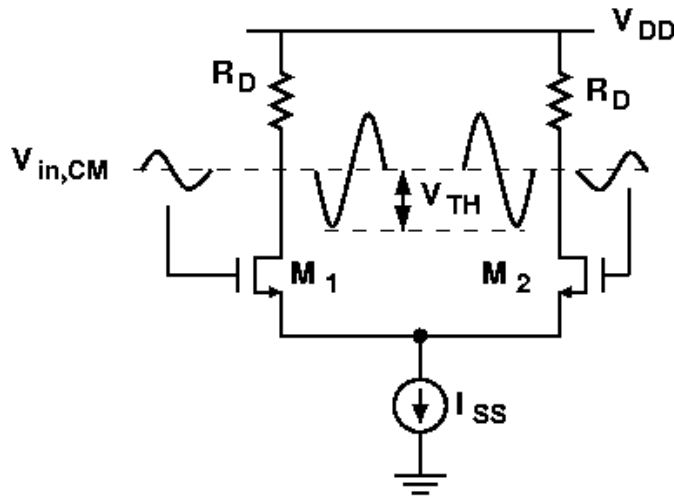
A_v vs. $V_{in,CM}$



Input CM vs Output Swing



- The higher the input CM level, the smaller allowable output swing



$$V_{out1} = V_{DD} - R_{D1} I_{D1}$$

$$V_{out2} = V_{DD} - R_{D2} I_{D2}$$

$$\text{if } R_{D1} = R_{D2} = R_D$$

$$V_{out1} - V_{out2} = R_D (I_{D2} - I_{D1})$$

$$V_{in1} - V_{in2} = V_{GS1} - V_{GS2} = \Delta V$$

$$I_{SS} = I_{D1} + I_{D2}$$

$$(V_{GS} - V_{TH})^2 = \frac{I_D}{\frac{1}{2} \mu_n C_{ox} \frac{W}{L}}, \quad V_{GS} = \sqrt{\frac{I_D}{\frac{1}{2} \mu_n C_{ox} \frac{W}{L}}} + V_{TH}, \quad V_{in1} - V_{in2} = \sqrt{\frac{2I_{D1}}{\mu_n C_{ox} \frac{W}{L}}} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox} \frac{W}{L}}}$$

$$I_{D1} - I_{D2} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{in2}) \sqrt{\frac{4I_{SS}}{\mu_n C_{ox} \frac{W}{L}} - (V_{in1} - V_{in2})^2}, \quad \frac{\partial \Delta I_D}{\partial \Delta V_{in}} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \frac{\frac{4I_{SS}}{\mu_n C_{ox} \frac{W}{L}} - 2\Delta V_{in}^2}{\sqrt{\frac{4I_{SS}}{\mu_n C_{ox} \frac{W}{L}} - \Delta V_{in}^2}}$$

Maximum Input Range

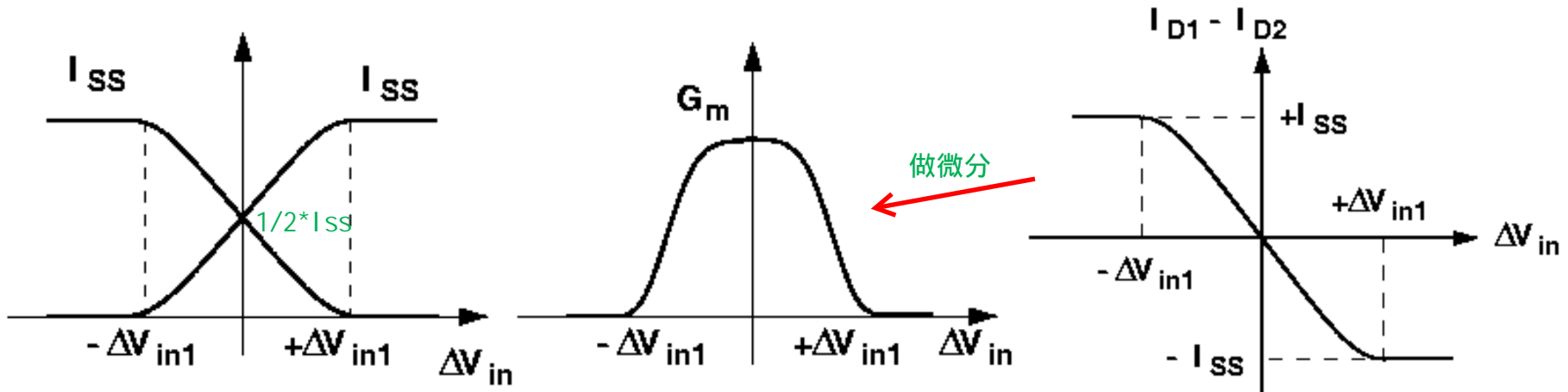
- For $\Delta V_{in} = 0$

$$G_m = \sqrt{\mu_n C_{ox} (W/L) I_{SS}}, \quad V_{out1} - V_{out2} = R_D \Delta I = R_D G_m \Delta V_{in}$$

$$|A_v| = \sqrt{\mu_n C_{ox} (W/L) I_{SS}} R_D = G_m R_D$$

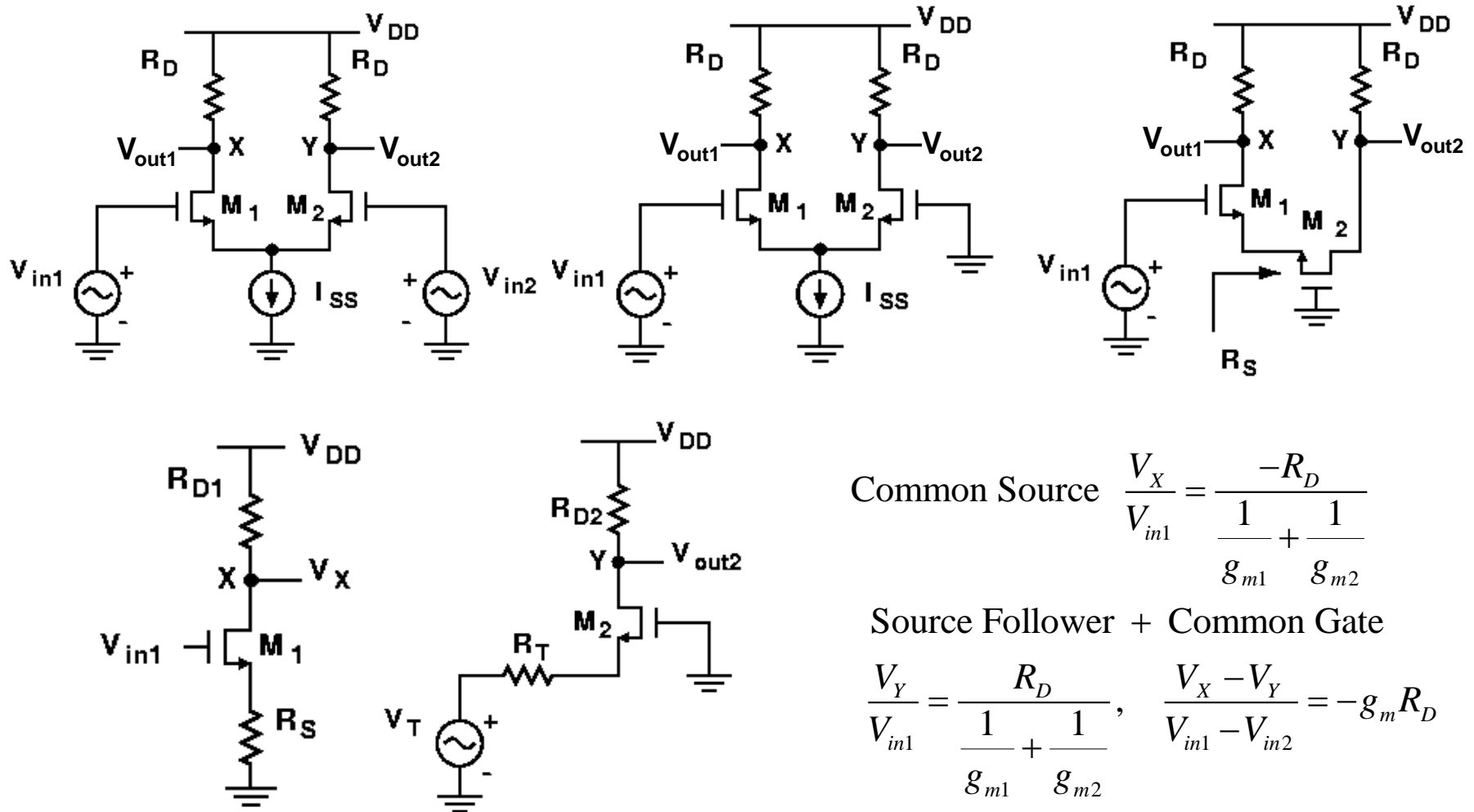
- G_m falls to zero for $\Delta V_{in} = \sqrt{2I_{SS}/(\mu_n C_{ox} W/L)} = \Delta V_{in1}$
- ΔV_{in1} represents the maximum differential input that the circuit can handle.
- For a zero differential input,

$$I_{D1} = I_{D2} = I_{SS}/2 \quad (V_{GS1} - V_{TH})_{1,2} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{W}{L}}} = \frac{\Delta V_{in1}}{\sqrt{2}}$$



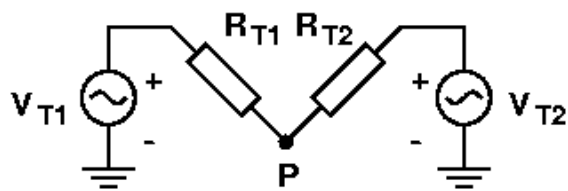
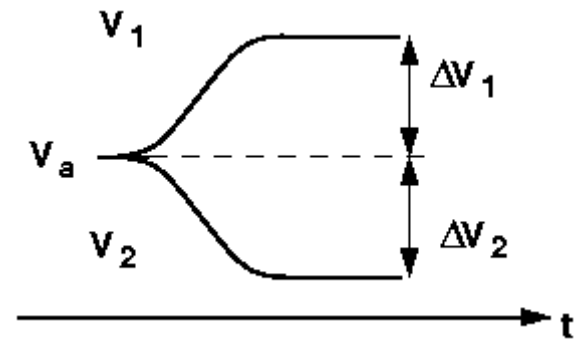
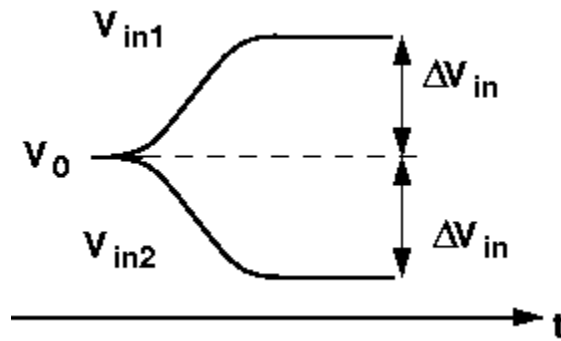
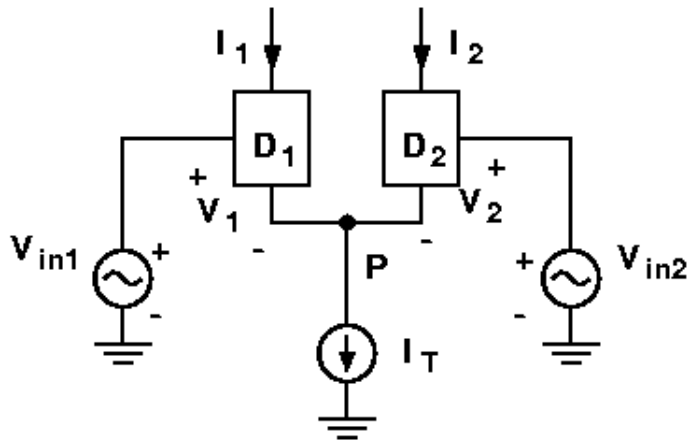
Voltage Gain of Differential Amplifier

- Method 1 : *By linear superposition*

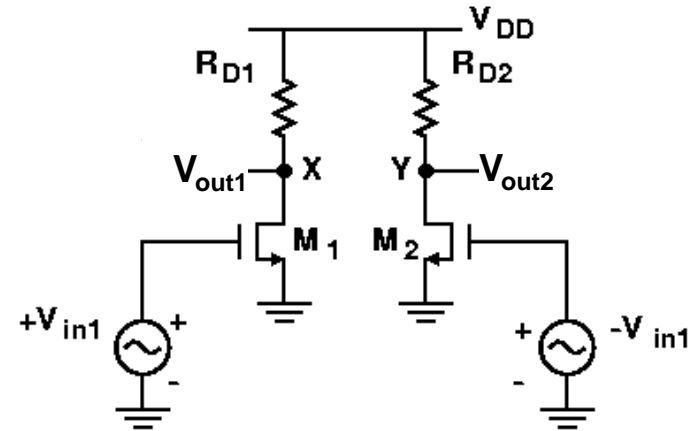
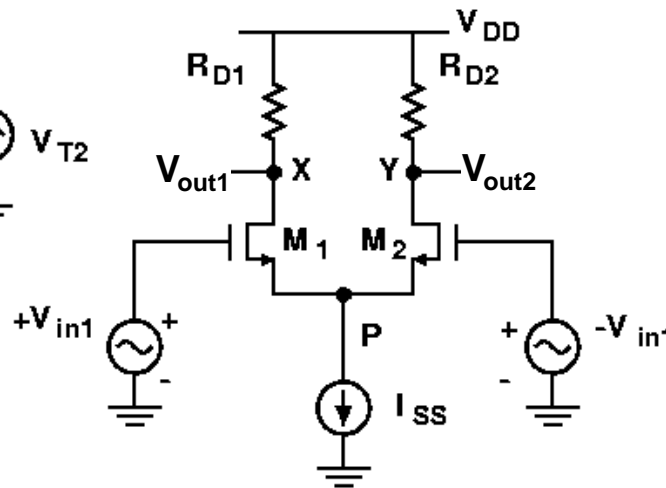


Equivalent Half Circuit

- Node P is a virtual ground 大家比較喜歡virtual ground的概念



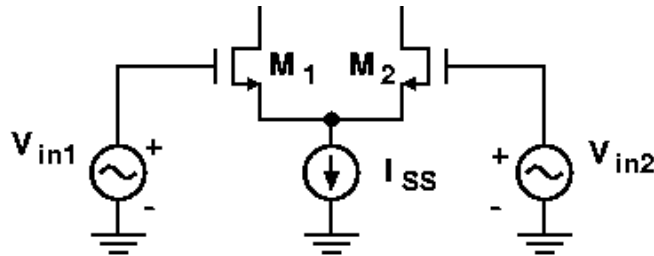
點virtual ground



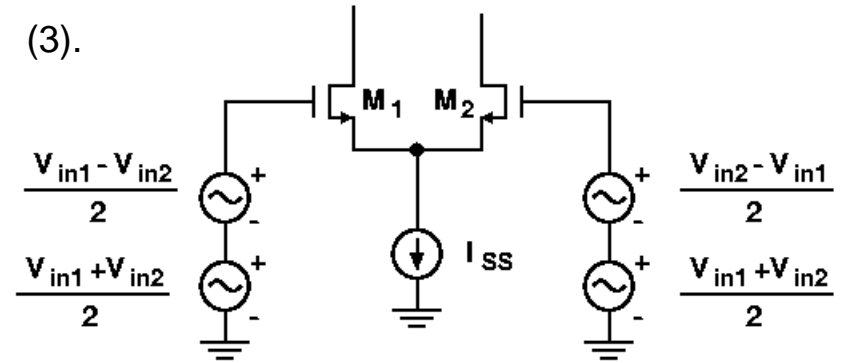
Equivalent Circuit

- Conversion of arbitrary inputs to differential and common-mode components

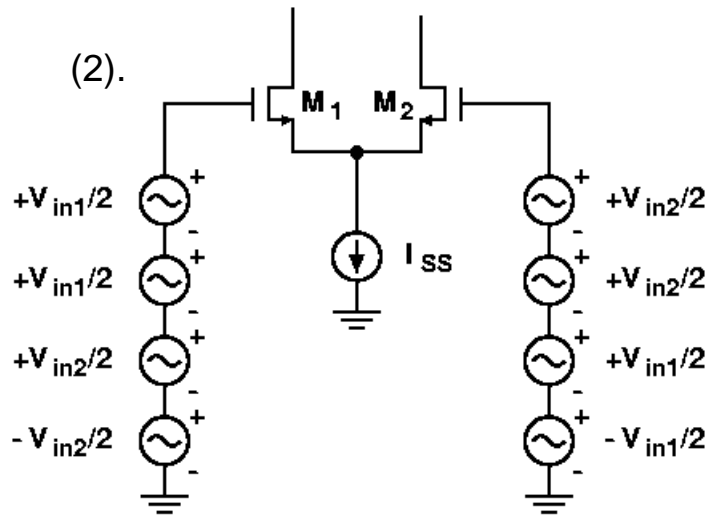
(1).



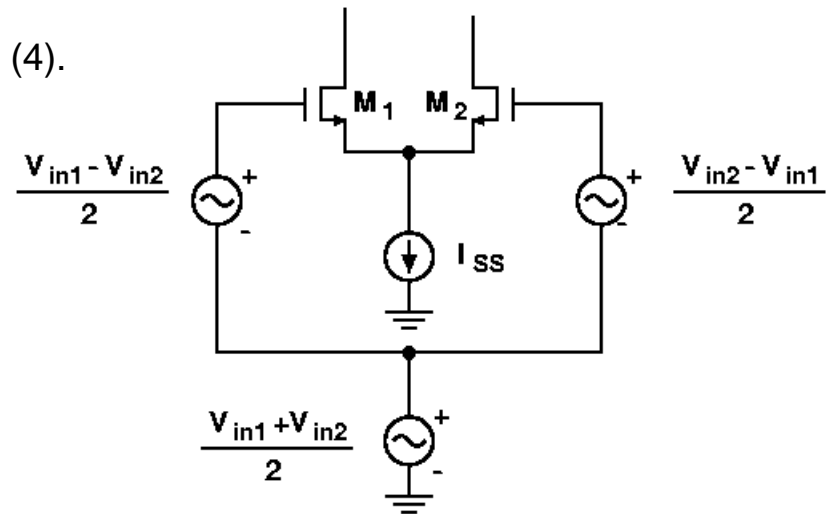
(3).



(2).



(4).

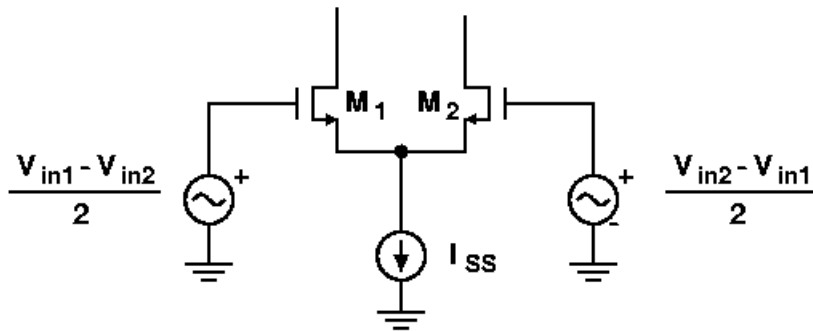


$$V_{in1} = \frac{V_{in1} - V_{in2}}{2} + \frac{V_{in1} + V_{in2}}{2}$$

$$V_{in2} = \frac{V_{in2} - V_{in1}}{2} + \frac{V_{in1} + V_{in2}}{2}$$

Superposition of CM and DM Signal

- For differential mode operation

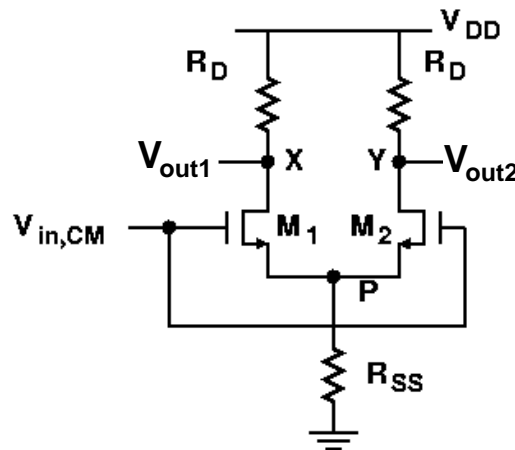
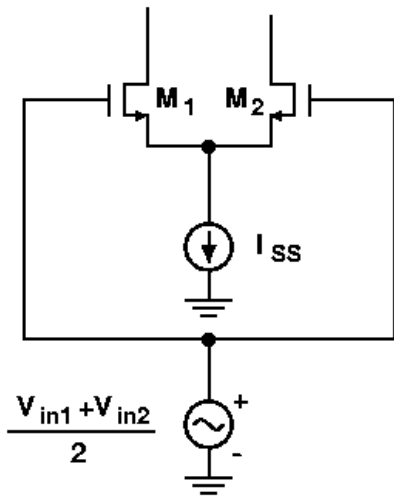


$$V_X = -g_m (R_D \parallel r_{O1}) \frac{V_{in1} - V_{in2}}{2}$$

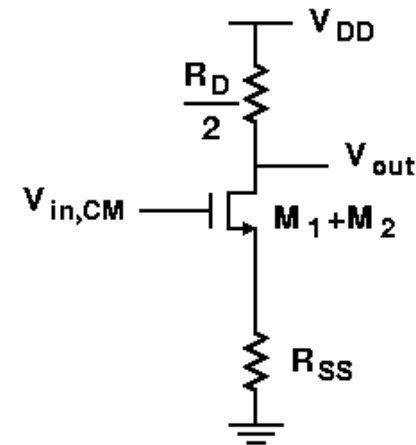
$$V_Y = -g_m (R_D \parallel r_{O2}) \frac{V_{in2} - V_{in1}}{2}$$

$$V_X - V_Y = -g_m (R_D \parallel r_{O}) (V_{in1} - V_{in2})$$

- For common mode operation



$$A_{v,CM} = \frac{V_{out}}{V_{in,CM}} = -\frac{R_D / 2}{1 / (2g_m) + R_{SS}}$$



Property of Differential Structure

- Advantage

- ⊕ Reject common mode noise, including power supply noise.
- ⊕ Reduce the **even-order harmonic** distortion
- ⊕ Increase output voltage swing

- Disadvantage

- ⊕ Double the power consumption and circuitry.
- ⊕ Need common mode voltage stabilization.

$$y_+ = \alpha_1(x_1) + \alpha_2(x_1)^2 + \alpha_3(x_1)^3 + \dots$$

$$y_- = \alpha_1(-x_1) + \alpha_2(-x_1)^2 + \alpha_3(-x_1)^3 + \dots$$

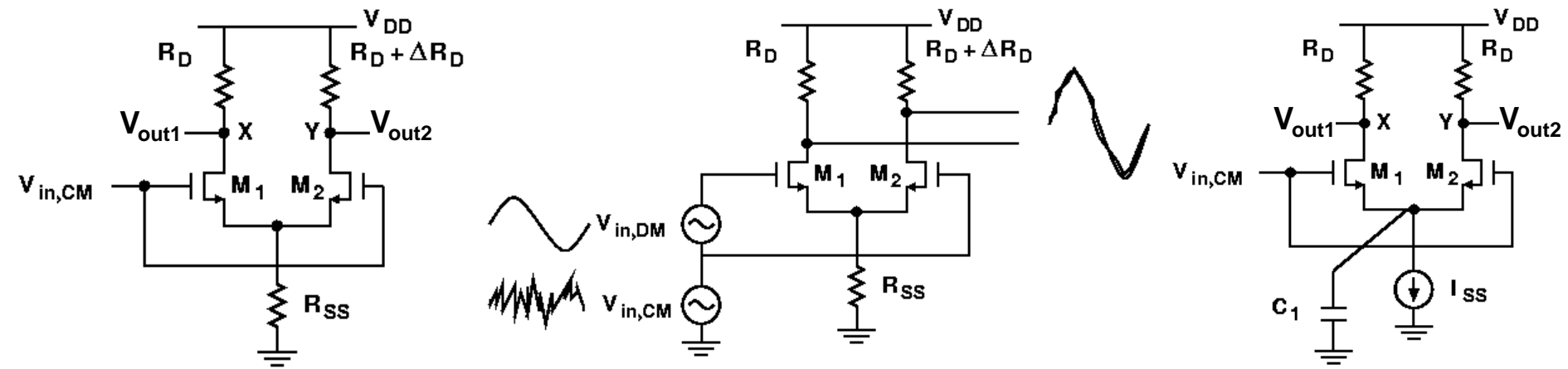
$$Y = y_+ - y_- = 2\alpha_1(x_1) + 2\alpha_3(x_1)^3 + \dots$$

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Common Mode Response

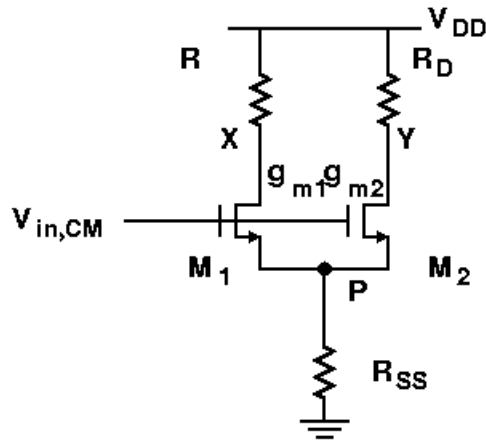
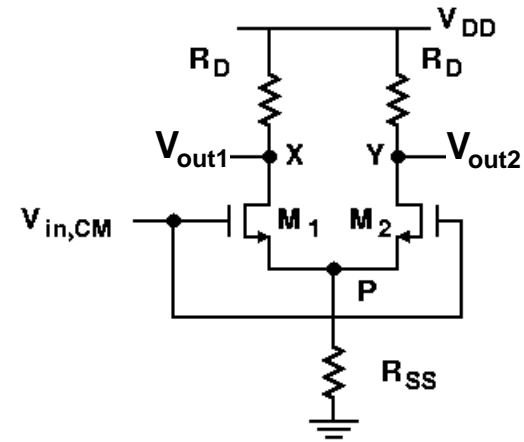
- In a symmetric circuit, input CM variations disturb the bias points, altering the small signal gain and possibly limit the output voltage swings.
- If the circuit is not symmetric, input CM variations will *lead to the variation of the differential output* → *Common mode to differential conversion*.



$$\Delta V_X = -\Delta V_{in,CM} \frac{g_m}{1 + 2g_m R_{SS}} R_D, \quad \Delta V_Y = -\Delta V_{in,CM} \frac{g_m}{1 + 2g_m R_{SS}} (R_D + \Delta R_D)$$

- The common-mode to differential conversion becomes significant at high frequencies since \$R_{SS}\$ is shunted by \$C_1\$.

Differential Pair Sensing CM Input



$$I_{D1} = g_{m1}(V_{in,CM} - V_P)$$

$$I_{D2} = g_{m2}(V_{in,CM} - V_P)$$

$$(g_{m1} + g_{m2})(V_{in,CM} - V_P)R_{SS} = V_P$$

$$V_P = \frac{(g_{m1} + g_{m2})R_{SS}}{(g_{m1} + g_{m2})R_{SS} + 1} V_{in,CM}$$

$$V_X = -g_{m1}(V_{in,CM} - V_P)R_D = \frac{-g_{m1}}{(g_{m1} + g_{m2})R_{SS} + 1} R_D V_{in,CM}$$

$$V_Y = -g_{m2}(V_{in,CM} - V_P)R_D = \frac{-g_{m2}}{(g_{m1} + g_{m2})R_{SS} + 1} R_D V_{in,CM}$$

$$V_X - V_Y = -\frac{g_{m1} - g_{m2}}{(g_{m1} + g_{m2})R_{SS} + 1} R_D V_{in,CM}$$

$$A_{CM-DM} = -\frac{g_{m1} - g_{m2}}{(g_{m1} + g_{m2})R_{SS} + 1} R_D = \frac{-\Delta g_m R_D}{(g_{m1} + g_{m2})R_{SS} + 1}$$

$$\Delta g_m \uparrow R_{SS} \downarrow A_{CM-DM} \uparrow$$

Common-Mode Rejection Ratio

- The undesirable differential component produced by CM variations must be normalized to the wanted differential output resulting from amplification.
- The common mode rejection ratio – CMRR

$$CMRR = \left| \frac{A_{DM}}{A_{CM-DM}} \right|$$

- If only g_m mismatch is considered ($V_{in1} = -V_{in2}$) (use p.13 method)

$$|A_{DM}| = \frac{R_D}{2} \frac{g_{m1} + g_{m2} + 4g_{m1}g_{m2}R_{SS}}{(g_{m1} + g_{m2})R_{SS} + 1}$$

- The CMRR can be derived as

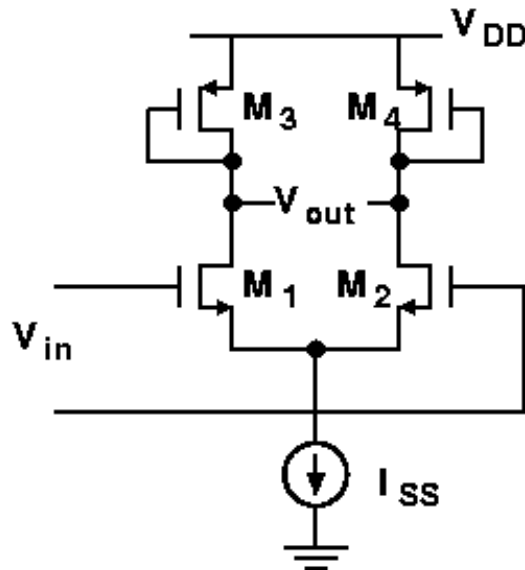
$$CMRR = \frac{g_{m1} + g_{m2} + 4g_{m1}g_{m2}R_{SS}}{2\Delta g_m} \approx \frac{g_m}{\Delta g_m} (1 + 2g_m R_{SS}), \quad g_m = \frac{g_{m1} + g_{m2}}{2}$$

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Differential Pair with MOS Loads

- Diode load

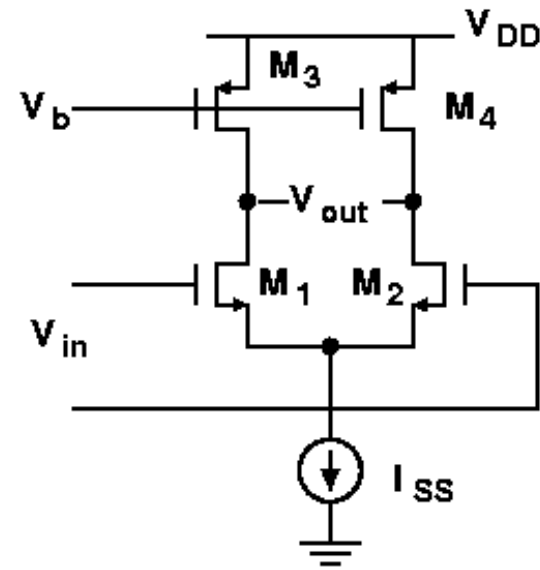


$$A_v = -g_{mN} (g_{mP}^{-1} \parallel r_{ON} \parallel r_{OP})$$

$$\approx -\frac{g_{mN}}{g_{mP}} \approx -\sqrt{\frac{\mu_n (W/L)_N}{\mu_p (W/L)_P}}$$

Output CM level = $V_{DD} - V_{GSP}$

- Current source load

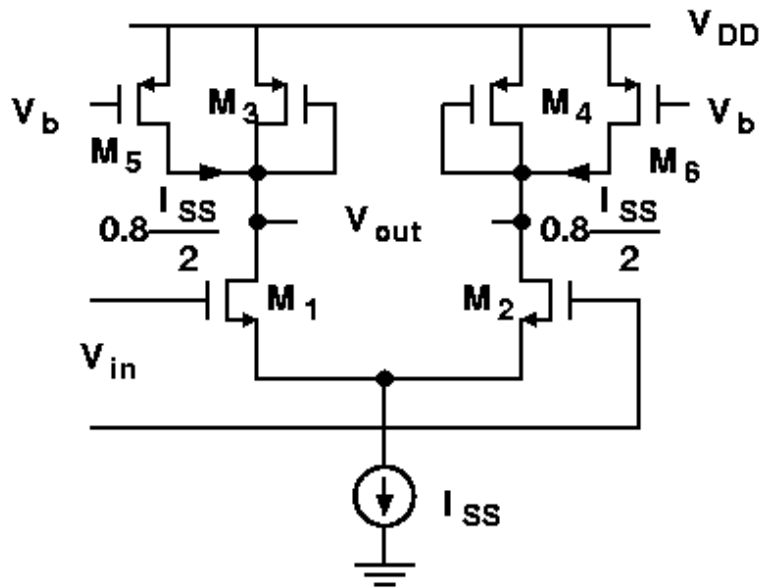


$$A_v = -g_{mN} (r_{ON} \parallel r_{OP})$$

Output CM not well defined
Need CM Feedback circuit

Differential Pair with MOS Loads

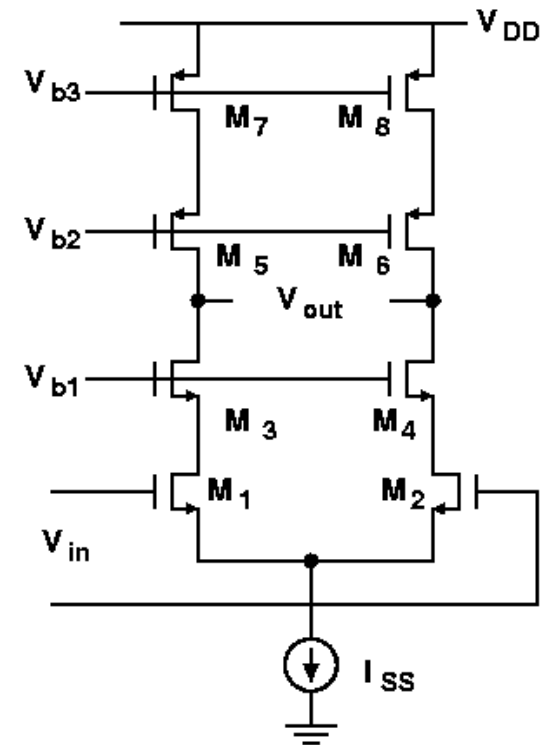
- Addition of current sources to increase the voltage gain



$$I_D(M_3, M_4) = 0.2 \frac{I_{SS}}{2},$$

$g_m(M_3, M_4)$ decreased to be 1/5, Gain increased x5

- Cascode differential pair



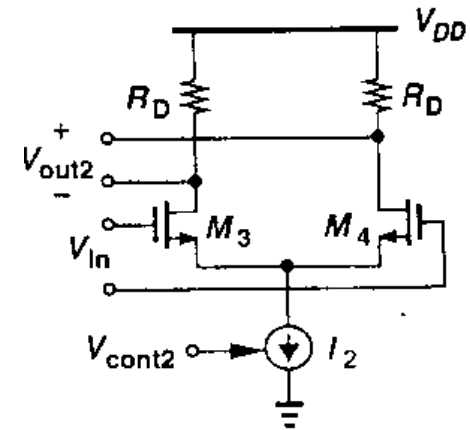
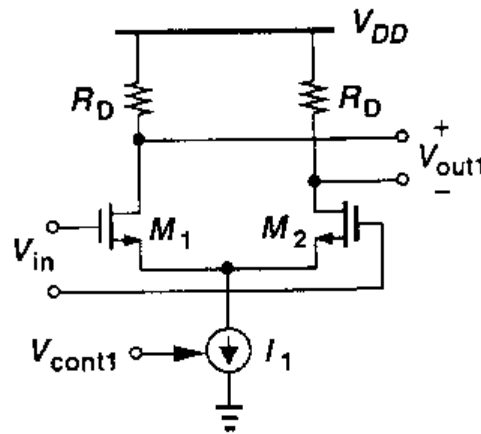
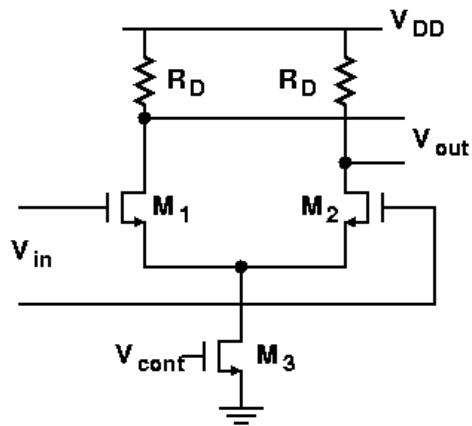
$$A_v \approx -g_{m1} [(g_{m3} r_{O3} r_{O1}) \parallel (g_{m5} r_{O5} r_{O7})]$$

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Gilbert Cell as a Variable Gain Amp

- The small-signal gain is a function of tail current
- The two transistors in a differential pair provide means of steering the tail current to one of two destinations



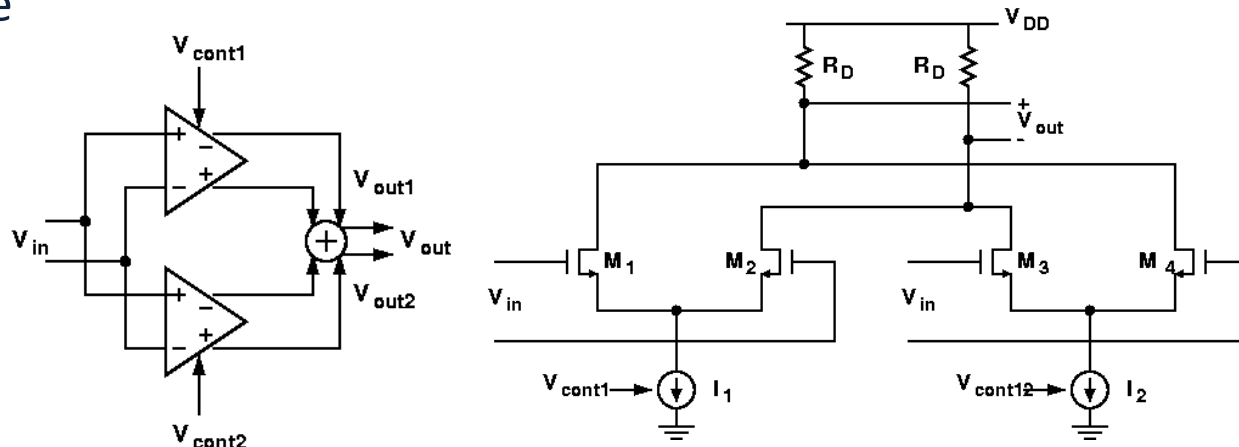
$$\frac{V_{out1}}{V_{in}} = -g_m R_D, \quad \frac{V_{out2}}{V_{in}} = g_m R_D$$

$$V_{out1} = R_D I_{D1} - R_D I_{D2}, \quad V_{out2} = R_D I_{D4} - R_D I_{D3}$$

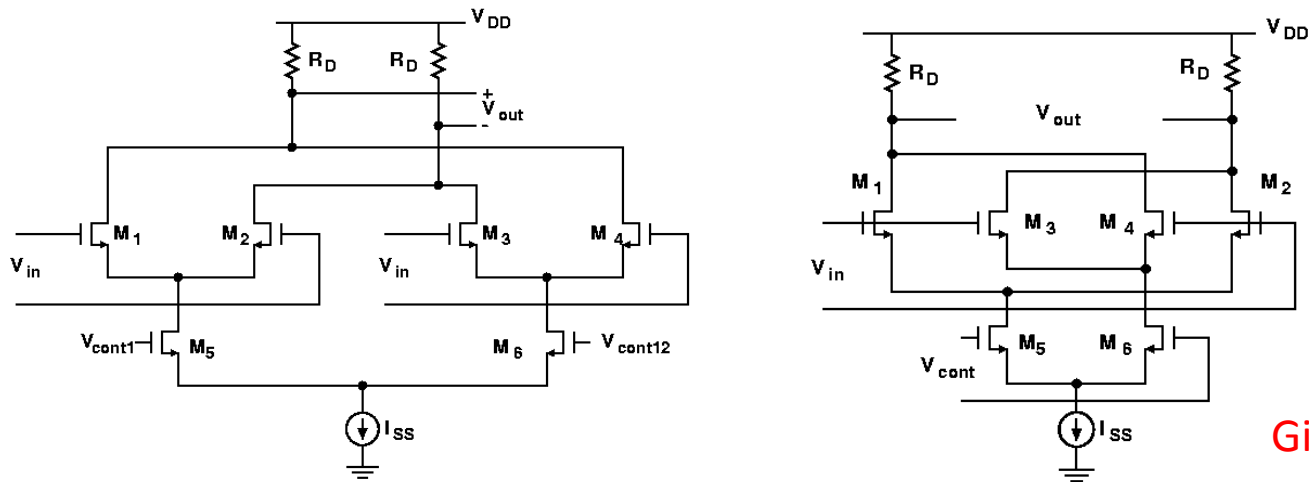
$$V_{out} = V_{out1} + V_{out2} = A_1 V_{in} + A_2 V_{in} = R_D (I_{D1} + I_{D4}) - R_D (I_{D2} + I_{D3})$$

Gilbert Cell as a Variable Gain Amp

- An amplifier whose gain can continuously varied from a negative value to a positive value



$$V_{out} = V_{out1} + V_{out2} = A_1 V_{in} + A_2 V_{in} = R_D (I_{D1} + I_{D4}) - R_D (I_{D2} + I_{D3})$$



Gilbert Cell - 1968