CHAPTER 3

Single Stage Amplifier

Outline

1. **Common-Source Amplifier**
2. Common-Source Amp with Source Degeneration
3. Common-Drain Amplifier
4. Common-Gate Amplifier
5. Cascode Amplifier

Vision

- An important part of a designer's job is to use proper approximations so as to create a simple mental picture of a complicated circuit.
- The intuition thus gained makes it possible to formulate the behavior of most circuits by inspection rather than by lengthy calculations

Basic Concepts

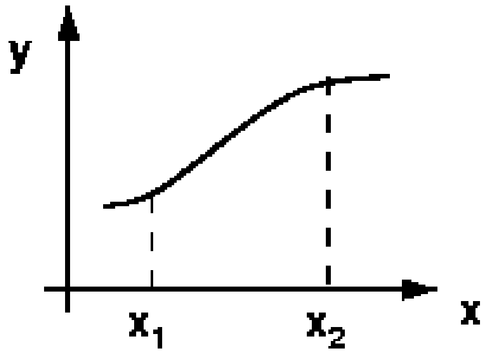
- The input-output characteristic of an amplifier is generally a nonlinear function

$$y(t) \approx \alpha_0 + \alpha_1 x(t) + \alpha_2 x^2(t) + \cdots + \alpha_n x^n(t) \qquad x_1 \le x \le x_2$$

- For a sufficiently narrow range of *x*

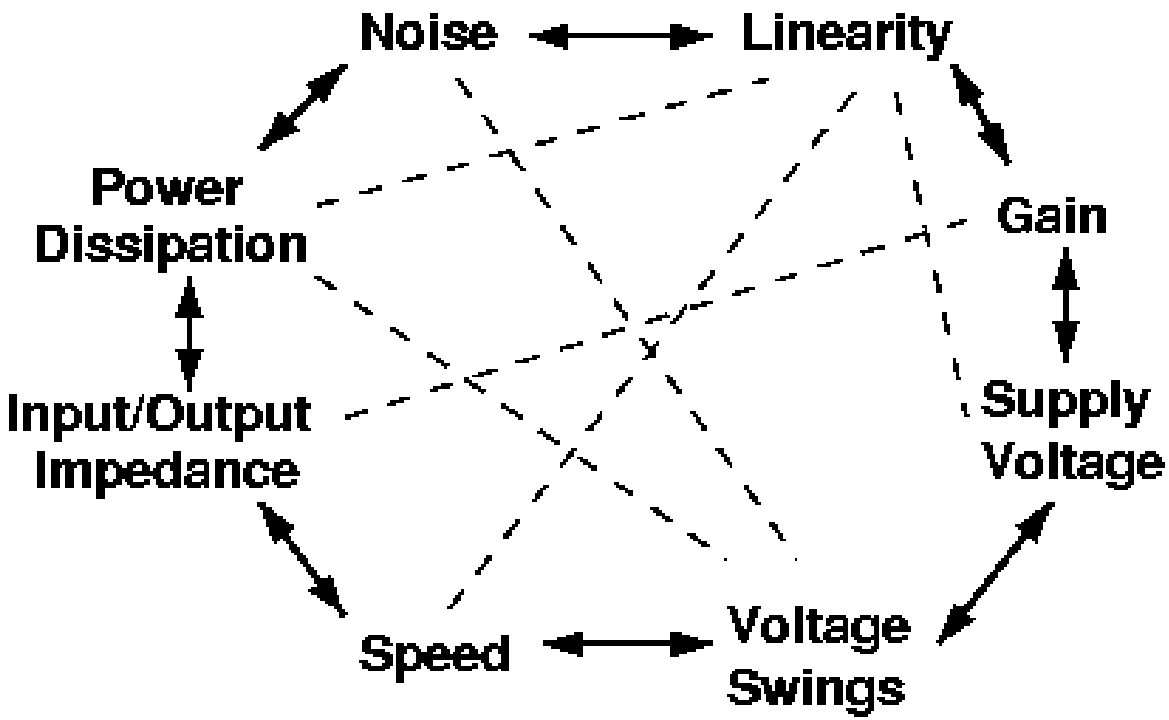
$$y(t) \approx \alpha_0 + \alpha_1 x(t), \quad \alpha_0\text{: operationg point, } \alpha_1\text{ : small signal gain}$$

- As *x(t)* increases in magnitude, higher order terms manifest themselves, leading to nonlinear distortion.
- Input-output characteristic of a nonlinear system

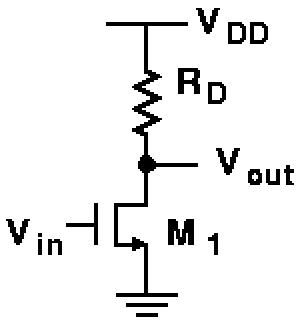


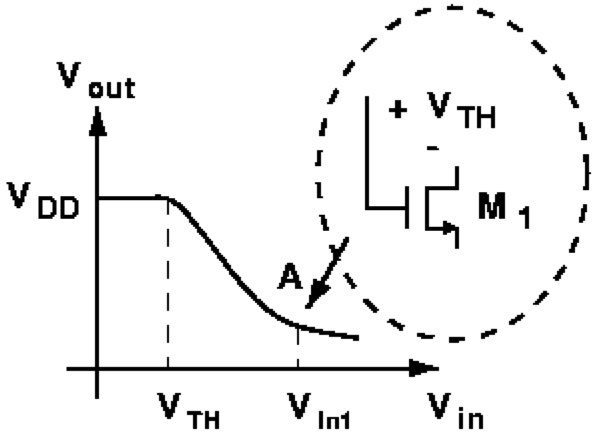
Analog Design Octagon

- Analog design octagon



Common Source Stage (I)





- M1 off

$$V_{in} \le V_{TH} \quad \Rightarrow \quad V_{out} = V_{DD}$$

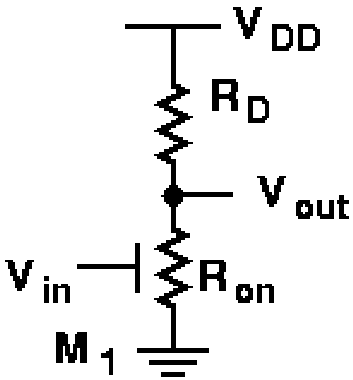
- M1 in the saturation region $\left(\text{Let} \quad V_{TH} \le V_{in} \le V_{in1} \quad \Rightarrow \quad V_{in} - V_{TH} \le V_{out} \right)$
  - To find V_{in1}

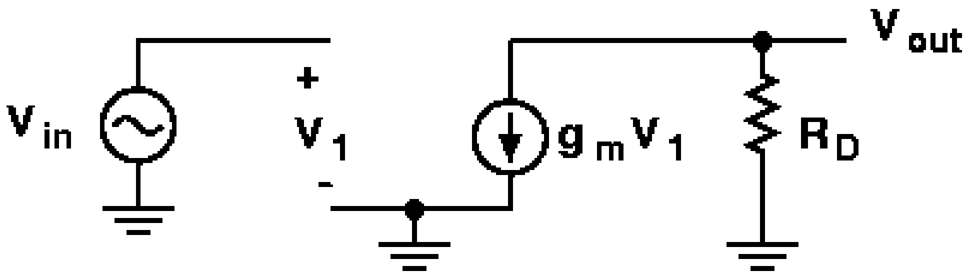
$$V_{in1} - V_{TH} = V_{DD} - R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left(V_{in1} - V_{TH} \right)^2$$

- M1 in the triode region $\left(V_{in} > V_{in1} \right)$

$$V_{out} = V_{DD} - R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[2\left(V_{in1} - V_{TH} \right) V_{out} - V_{out}^2 \right]$$

Common Source Amplifier (II)





- Since the transconductance drops in the triode region, (the r_o also becomes smaller), we usually ensure that

$$V_{out} > V_{in} - V_{TH}$$

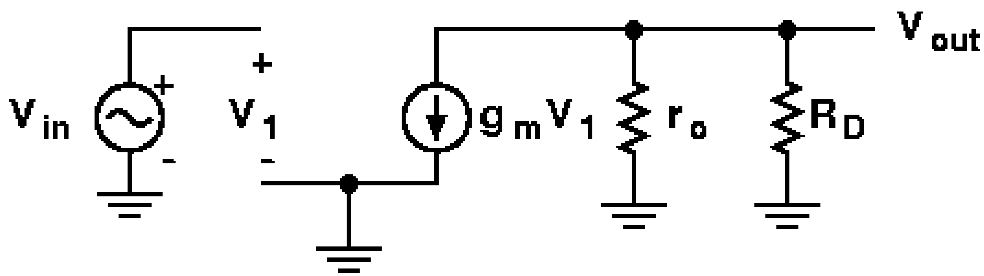
$$\text{As} \quad V_{out} = V_{DD} - R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{TH})^2$$

$$\Rightarrow \frac{\partial V_{out}}{\partial V_{in}} = -R_D \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{TH}) = -g_m R_D$$

$$\Rightarrow A_v = -g_m R_D$$

- Since g_m itself varies with the input signal, the gain of the circuit changes substantially if the signal swing is large.

Common Source Amplifier (III)



- To take channel length modulation effect into account :

$$V_{out} = V_{DD} - R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{TH})^2 (1 + \lambda V_{out})$$

- We have

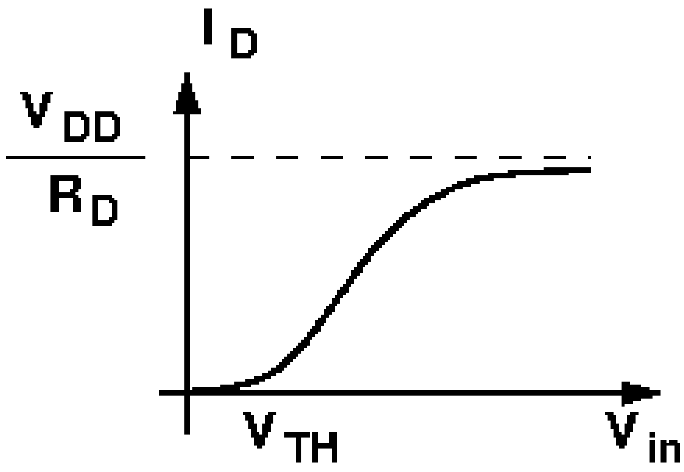
$$\frac{\partial V_{out}}{\partial V_{in}} = -R_D \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{TH})(1 + \lambda V_{out}) - R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{TH})^2 \lambda \frac{\partial V_{out}}{\partial V_{in}}$$

- As

$$I_D \approx \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{TH})^2 \Rightarrow A_v = -R_D g_m - R_D I_D \lambda A_v \Rightarrow A_v = -\frac{g_m R_D}{1 + R_D \lambda I_D}$$

$$\lambda I_D = \frac{1}{r_O} \Rightarrow A_v = -g_m \frac{r_O R_D}{r_O + R_D} = -g_m (r_O \parallel R_D)$$

Design Trade-off



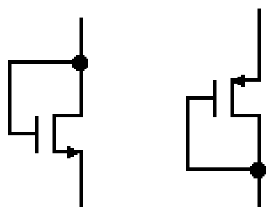
- To maximize gain

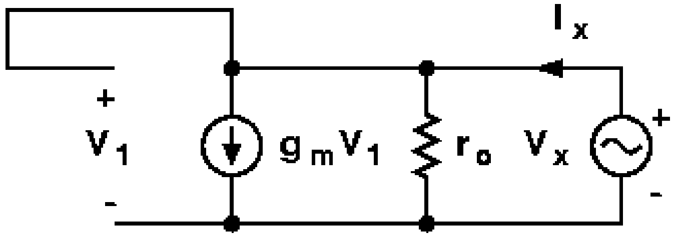
$$A_v = -\sqrt{2\mu_n C_{ox}\frac{W}{L}I_D}\,\frac{V_{RD}}{I_D} = -\sqrt{2\mu_n C_{ox}\frac{W}{L}}\,\frac{V_{RD}}{\sqrt{I_D}}$$

  - Increase *W/L* → greater device capacitance (*Gain* ↔ *BW*)
  - Higher V_{RD} → smaller voltage swing (*Gain* ↔ *Voltage swing*)
  - Reduce I_D while V_{RD} is constant → larger RC time constant at the output node (*Gain* ↔ *BW*)

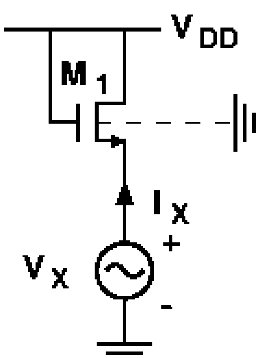
Diode Connected Load

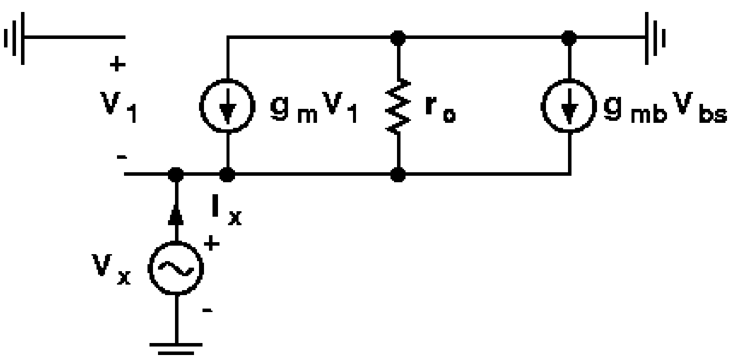
- In many CMOS technologies, it is difficult to fabricate resistors with tightly controlled values or a reasonable size. Replace R_D with a MOS transistor.





- Diode connected : gate and drain shorted → $V_{DS} = V_{GS} > V_{GS} - V_{TH}$ → the transistor always in saturation region.



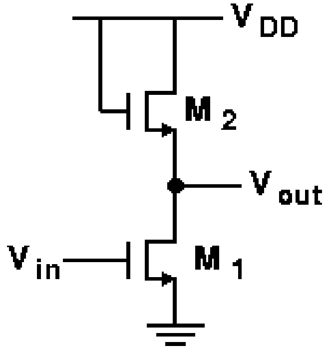


$$(g_m + g_{mb})V_X + \frac{V_X}{r_O} = I_X$$

$$\frac{V_X}{I_X} = \frac{1}{g_m + g_{mb} + r_O^{-1}}$$

$$R_L = \frac{1}{g_m + g_{mb}} \parallel r_O \approx \frac{1}{g_m + g_{mb}}$$

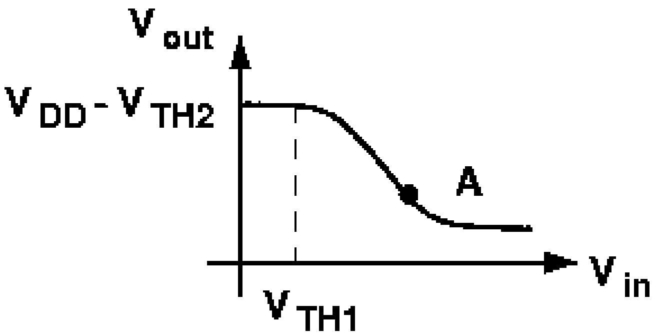
CS Stage + Diode Connected Load



$$A_v = -g_{m1}\frac{1}{g_{m2}+g_{mb2}} = -\frac{g_{m1}}{g_{m2}}\frac{1}{1+\eta} \qquad \eta = \frac{g_{mb2}}{g_{m2}}$$

$$A_v = -\frac{\sqrt{2\mu_n C_{ox}(W/L)_1 I_{D1}}}{\sqrt{2\mu_n C_{ox}(W/L)_2 I_{D2}}}\frac{1}{1+\eta} = -\frac{\sqrt{(W/L)_1}}{\sqrt{(W/L)_2}}\frac{1}{1+\eta}$$

- If the variation of η with the output voltage is neglected, the gain is independent of the bias current and voltages (so long as M_1 stays in saturation).
- Input-output characteristics of a CS stage with diode connected load.
- Operated at point A.

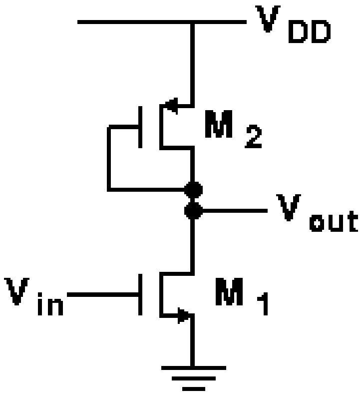


CS Stage + Diode-Connected PMOS

- The circuit is free from body effect.

$$A_v = -\sqrt{\frac{\mu_n (W/L)_1}{\mu_p (W/L)_2}} \qquad A_v \approx -\frac{|V_{GS2} - V_{TH2}|}{V_{GS1} - V_{TH1}}$$

$$\mu_n \left(\frac{W}{L}\right)_1 (V_{GS1} - V_{TH1})^2 \approx \mu_p \left(\frac{W}{L}\right)_2 (V_{GS2} - V_{TH2})^2$$



- Example :

If A_v = 10, V_{GS1}-V_{TH1} = 200 mV, → V_{GS2}-V_{TH2} = 2 V, V_{TH2} = 0.7 V → V_{GS2}= 2.7 V

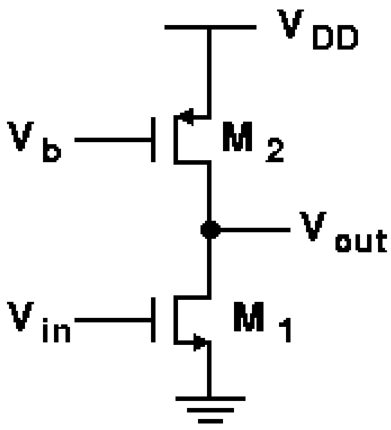
→ $V_{omax} = V_{DD} - V_{GS2}$ → *Trade-off between gain and output swing*

- To take the effect of channel length modulation effect into account

$$A_v \approx -g_{m1}\left(\frac{1}{g_{m2}} \parallel r_{o1} \parallel r_{o2}\right)$$

CS Stage + Current Source Load

- For resistor or diode connected load, increasing the load resistance limits the output voltage swing → CS stage with current source load.



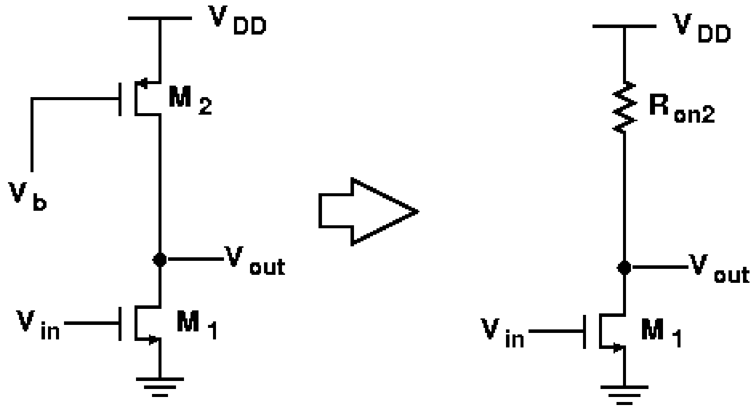
$$A_v = -g_{m1}(r_{O1} \| r_{O2})$$

$$|V_{DS2,\min}| = |V_{GS2} - V_{TH2}|$$

$$\lambda \propto 1/L \quad \Rightarrow \quad r_O \propto L/I_D$$

- The output bias voltage of the circuit needs a feedback loop to force V_{out} to a known value.
- If A_v ↑ → L ↑ → W ↑ (for constant I) → C_{load} ↑ → *Gain-Bandwidth Trade-off*
- Keep W constant, L ↑ → $V_{DS\min}$ ↑ → $V_{out,swing}$ ↓

CS Stage + Triode Load



- The gate of M2 is biased at a sufficiently low level, ensuring the load is in deep triode region for all output voltage swings.

$$V_{DD} - V_b - V_{TH} > V_{DD} - V_{out} \Rightarrow V_{out} - V_{TH} > V_b$$

$$R_{on2} = \frac{1}{\mu_p C_{ox}(W/L)_2(V_{DD} - V_b - |V_{THP}|)}$$

- Consume less voltage headroom than diode connected devices.
- *Drawback*
  - *R_{on2} depends on $\mu_p C_{ox}$, V_b, and V_{THP}, which vary with process and Temp.*
  - *Difficult to use.*

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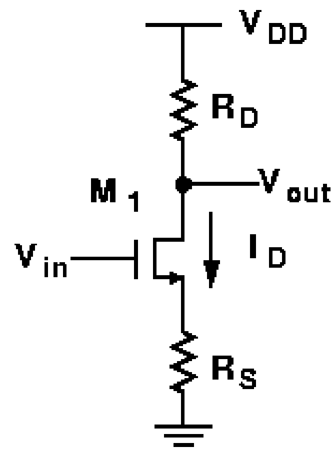
CS Stage + Source Degeneration (I)

- Common source Gain

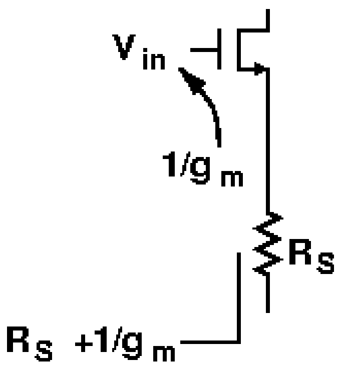
$$V_{out} = -I_D R_D \Rightarrow A_v = \frac{\partial V_{out}}{\partial V_{in}} = -\frac{\partial I_D}{\partial V_{in}} R_D = -G_m R_D$$



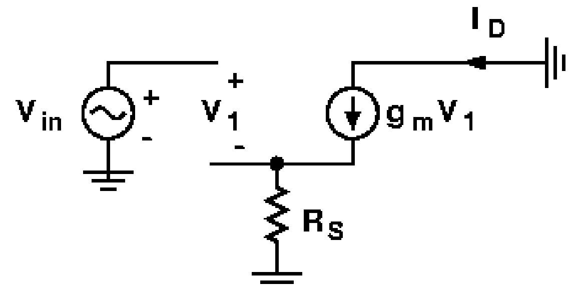
- Improve the linearity of the gain amplifier
  - Higher linearity, Lower gain

G_m : equivalent transconductance of circuit

g_m : transconductance of MOS



Let $I_D = f(V_{GS}) \Rightarrow G_m = \frac{\partial I_D}{\partial V_{in}} = \frac{\partial f}{\partial V_{GS}} \frac{\partial V_{GS}}{\partial V_{in}}$



For $V_{GS} = V_{in} - I_D R_S \Rightarrow \frac{\partial V_{GS}}{\partial V_{in}} = 1 - R_S \frac{\partial I_D}{\partial V_{in}}$

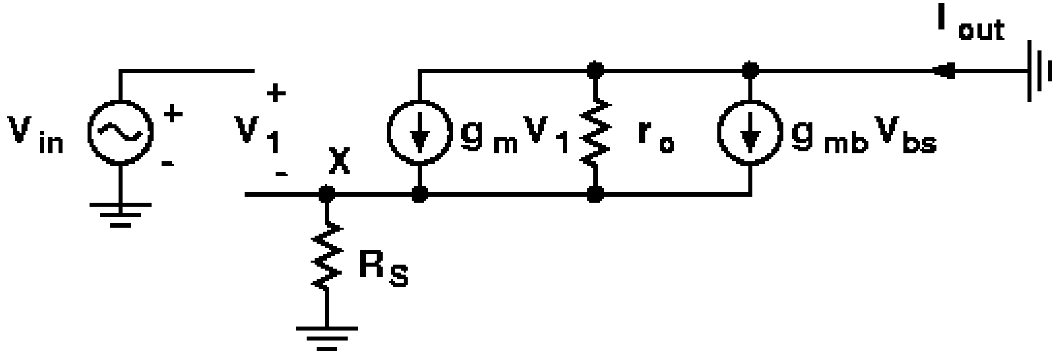
$$G_m = \left(1 - R_S \frac{\partial I_D}{\partial V_{in}}\right) \frac{\partial f}{\partial V_{GS}} = (1 - R_S G_m) g_m \Rightarrow G_m = \frac{g_m}{1 + g_m R_S}$$

$$A_v = -G_m R_D = \frac{-g_m R_D}{1 + g_m R_S} = -\frac{R_D}{1/g_m + R_S} \Rightarrow \text{For } R_S >> 1/g_m \quad G_m \approx 1/R_S$$

Linear!

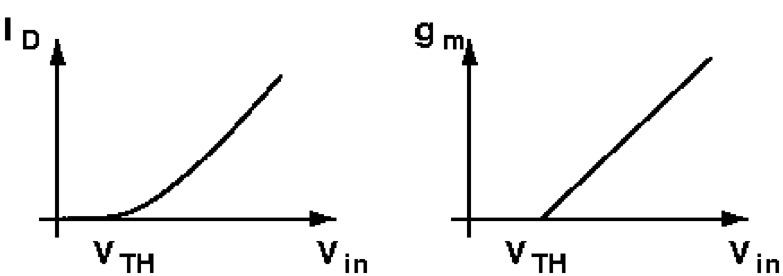
CS Stage + Source Degeneration (II)

- To take the body effect and channel length modulation effect into account

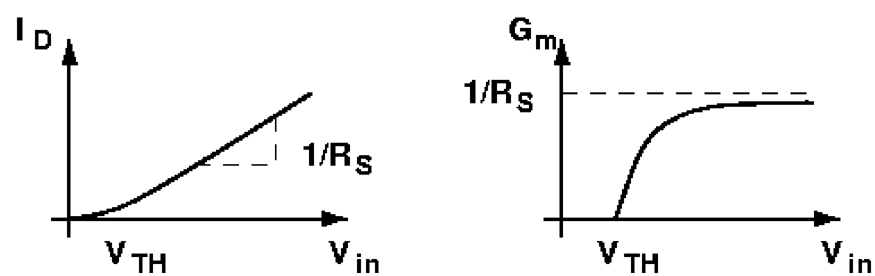


$$I_{out} = g_m V_1 - g_{mb} V_X - \frac{V_X}{r_O} = g_m (V_{in} - I_{out} R_S) + g_{mb}(-I_{out} R_S) - \frac{I_{out} R_S}{r_O}$$

$$G_m = \frac{I_{out}}{V_{in}} = \frac{g_m}{1 + (g_m + g_{mb}) R_S + R_S / r_O}$$



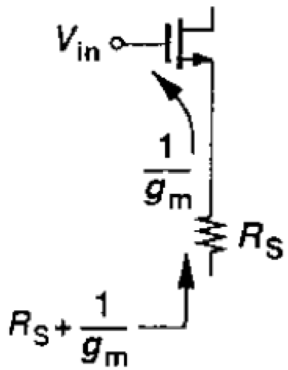
Common source amp



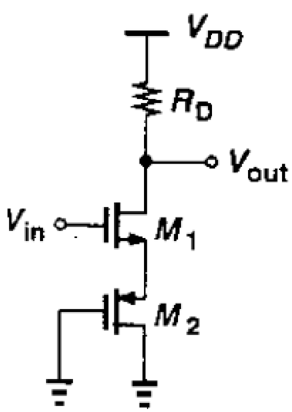
Common source amp + source degeneration

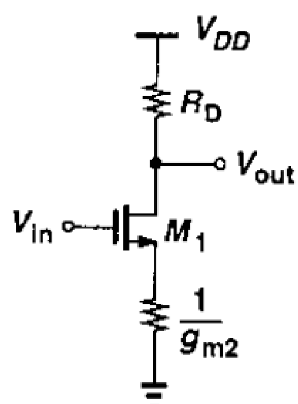
Formulate Gain by Inspection

- Magnitude of gain as the resistance seen at the drain node divided by the total resistance in the source path



$$A_v = -\frac{R_D}{1/g_m + R_S}$$

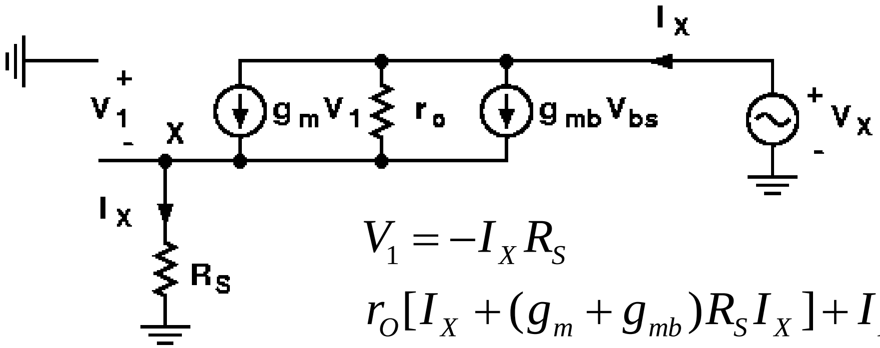




$$A_v = -\frac{R_D}{1/g_{m1} + 1/g_{m2}}$$

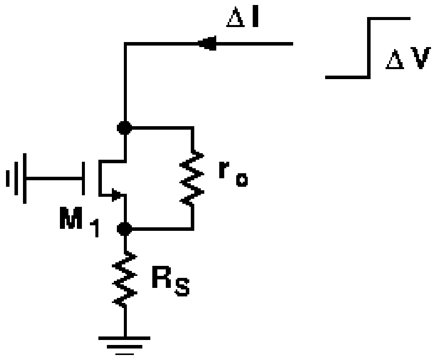
CS Stage + Source Degeneration (III)

- R_{out} of CS + Source degeneration



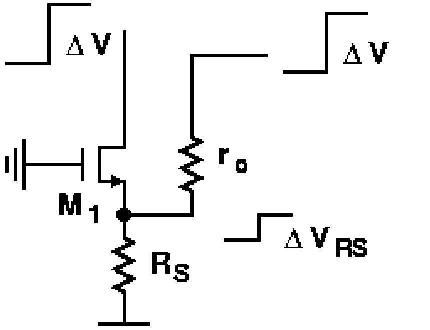
$$V_1 = -I_X R_S$$

$$r_O[I_X + (g_m + g_{mb})R_S I_X] + I_X R_S = V_X$$



$$R_{out} = \left[1 + (g_m + g_{mb})R_S\right]r_O + R_S = \left[1 + (g_m + g_{mb})r_O\right]R_S + r_O$$

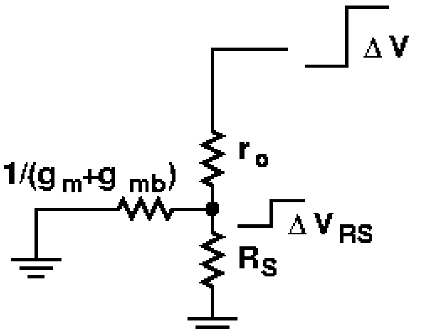
$$R_{out} \approx \left[1 + (g_m + g_{mb})R_S\right]r_O$$



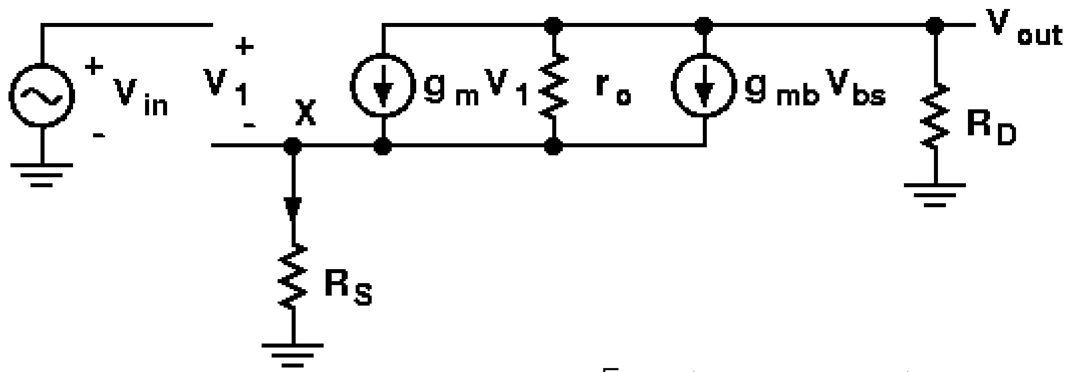
- By Inspection

$$\Delta V_{RS} = \Delta V \frac{\frac{1}{g_m + g_{mb}} \| R_S}{\frac{1}{g_m + g_{mb}} \| R_S + r_O}, \ \Delta I = \frac{\Delta V_{RS}}{R_S} = \Delta V \frac{1}{[1 + \left(g_m + g_{mb}\right) R_S] r_O + R_S}$$

$$\frac{\Delta V}{\Delta I} = \left[1 + (g_m + g_{mb})R_S\right]r_O + R_S$$



CS Stage + Source Degeneration (IV)



$$I_{r_O} = -\frac{V_{out}}{R_D} - (g_m V_1 + g_{mb} V_{bs}) = -\frac{V_{out}}{R_D} - \left[g_m \left(V_{in} + V_{out}\frac{R_S}{R_D} \right) + g_{mb} V_{out}\frac{R_S}{R_D} \right]$$

$$V_{out} = I_{r_O} r_O - \frac{V_{out}}{R_D} R_S = -\frac{V_{out}}{R_D} r_O - \left[g_m \left(V_{in} + V_{out}\frac{R_S}{R_D} \right) + g_{mb} V_{out}\frac{R_S}{R_D} \right] r_O - V_{out}\frac{R_S}{R_D}$$

- Voltage gain with r_o & g_{mb}

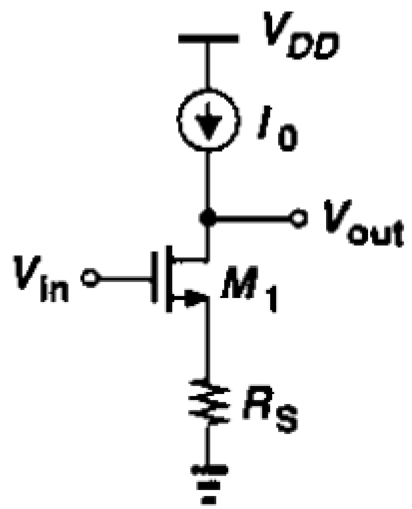
$$\frac{V_{out}}{V_{in}} = -\frac{g_m r_O R_D}{R_D + R_S + r_O + (g_m + g_{mb}) R_S r_O}$$

$$= -\frac{g_m r_O}{R_S + r_O + (g_m + g_{mb}) R_S r_O} \cdot \frac{R_D \left[R_S + r_O + (g_m + g_{mb}) R_S r_O \right]}{R_D + R_S + r_O + (g_m + g_{mb}) R_S r_O}$$

$$= -G_{meff} R_O = -G_{meff} \{ R_D \,||\, [R_S + r_O + (g_m + g_{mb}) R_S r_O] \}$$

CS Stage + Source Degeneration (V)

- I_0 = constant, $I(R_S)$ = constant, small-signal voltage drop across R_S = 0



$$A_v = -\frac{g_m r_O}{R_S + [1 + (g_m + g_{mb}) R_S] r_O} \{R_S + [1 + (g_m + g_{mb}) R_S] r_O\}$$

$$= -g_m r_O = \text{ intrinsic gain, independent of } R_S$$

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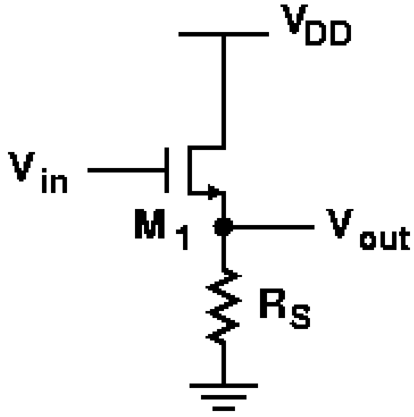
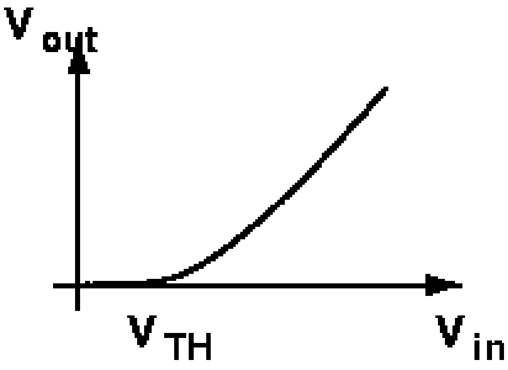
CD Stage: Source Follower (I)

- The source follower can operate as a voltage buffer – High input impedance, low output impedance.
- Gain ≈ 1, but not equal to 1 even with R_S = infinity.





$$\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{in}-V_{TH}-V_{out}\right)^2 R_S = V_{out}$$

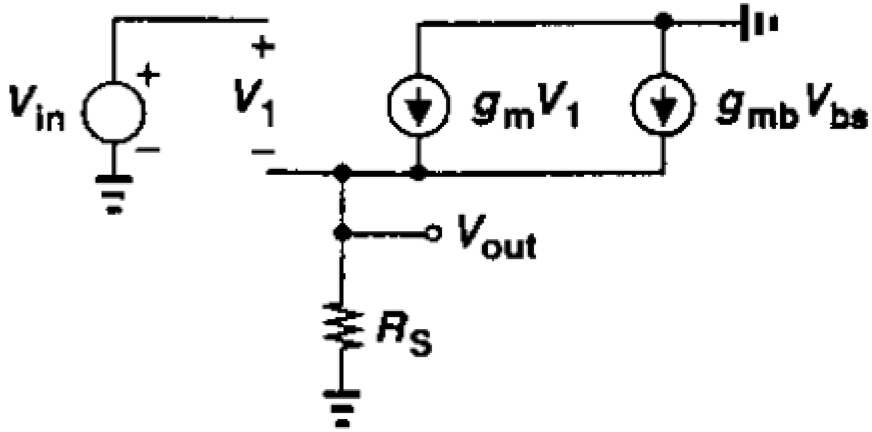
$$\frac{\partial V_{out}}{\partial V_{in}} = \frac{\mu_n C_{ox}\frac{W}{L}\left(V_{in}-V_{TH}-V_{out}\right)R_S}{1+\mu_n C_{ox}\frac{W}{L}\left(V_{in}-V_{TH}-V_{out}\right)R_S(1+\eta)}$$

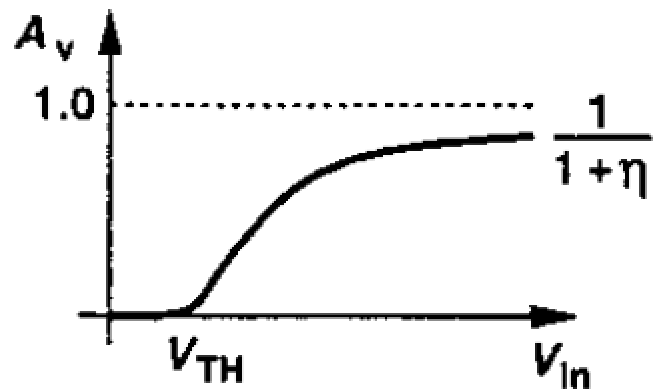
$$g_m = \mu_n C_{ox}\frac{W}{L}\left(V_{in}-V_{TH}-V_{out}\right)$$

$$A_v = \frac{g_m R_S}{1+(g_m+g_{mb})R_S}$$

CD: Small-signal equivalent circuit

- Calculate the voltage gain by small-signal equivalent circuit of source follower with body effect





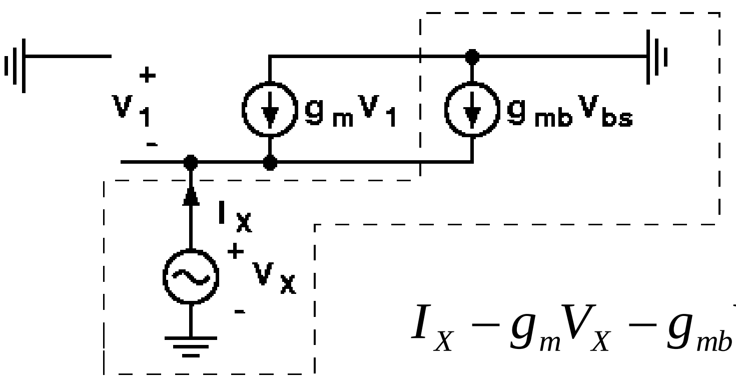
$$V_{in} - V_1 = V_{out}, \quad V_{bs} = -V_{out}, \quad g_m V_1 - g_{mb} V_{out} = V_{out} / R_S$$

$$A_v = \frac{V_{out}}{V_{in}} = \frac{g_m R_S}{1 + (g_m + g_{mb}) R_S}$$

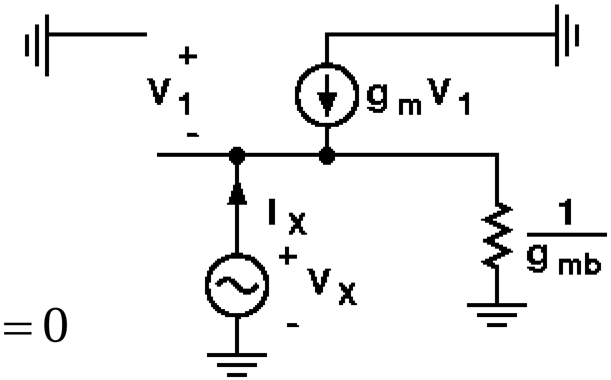
$$V_{in} \uparrow I_D \uparrow g_m \uparrow ==> A_v \approx \frac{g_m}{g_m + g_{mb}} = \frac{1}{1+\eta}$$

R_{out} of Source Follower

- Body effect decrease R_{out} of source follower

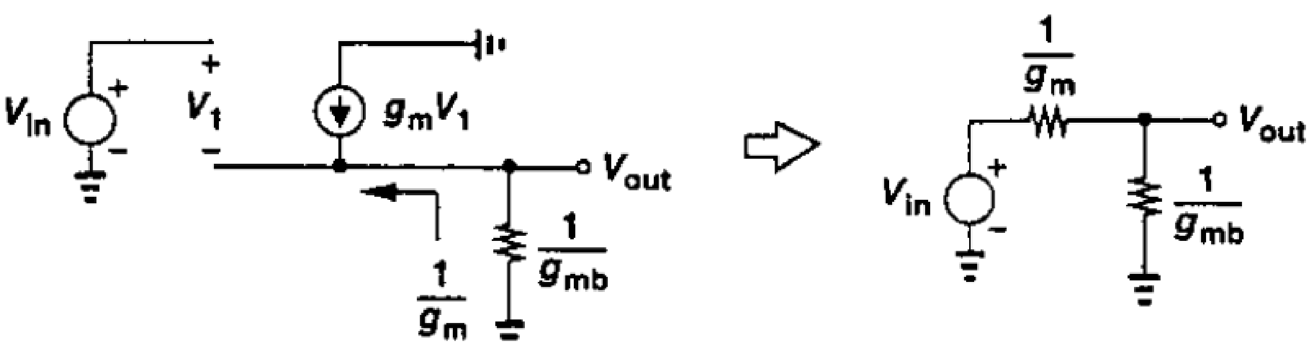


$$I_X - g_m V_X - g_{mb} V_X = 0$$



$$R_{out} = \frac{1}{g_m + g_{mb}}$$

- Less-than-unity voltage gain of source follower with body effect



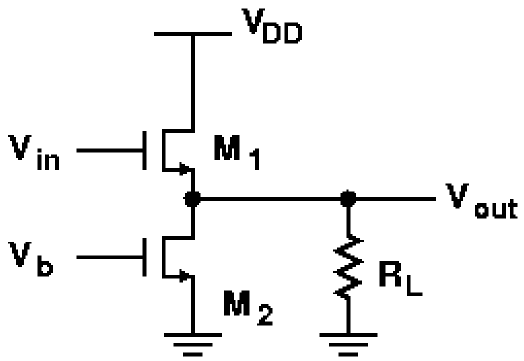
$$R_S = \infty$$

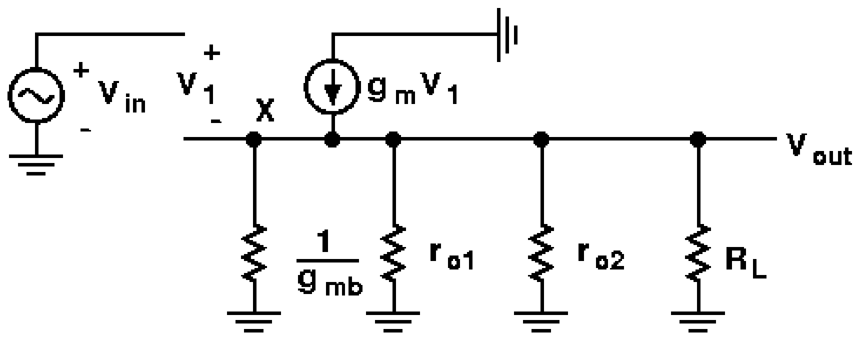
$$A_v = \frac{1/g_{mb}}{1/g_m + 1/g_{mb}}$$

$$= \frac{g_m}{g_m + g_{mb}}$$

Source Follower with r_o

- Source follower with finite channel-length modulation

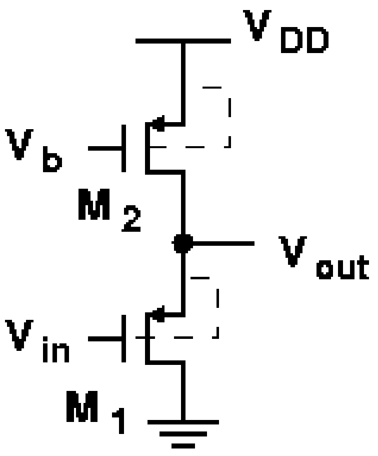


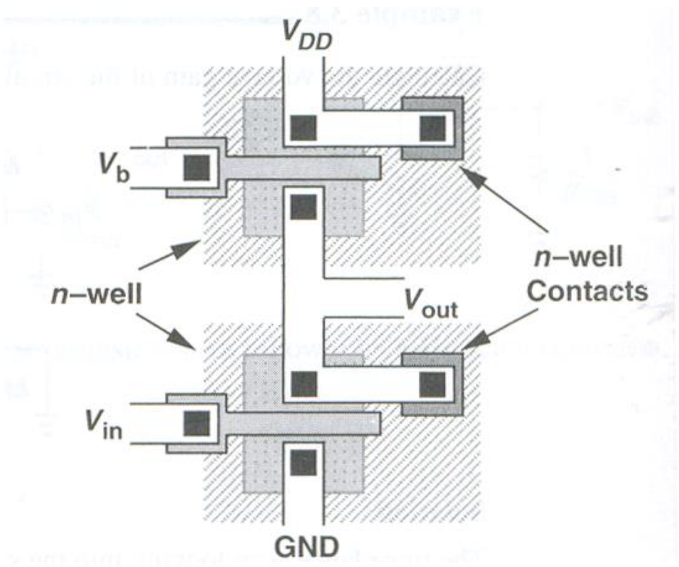


$$A_v = \frac{\frac{1}{g_{mb}} \| r_{O1} \| r_{O2} \| R_L}{\frac{1}{g_{mb}} \| r_{O1} \| r_{O2} \| R_L + \frac{1}{g_m}}$$

Source Follower Drawback

- Voltage headroom consumption due to level shift.
- Nonlinearity
  - Nonlinear dependence of V_{TH} upon the source potential.
  - r_O of the transistor also changes substantially with V_{DS}.
- PMOS source follower with no body effect





- Higher output impedance using PMOS source follower.

Outline

1. Common-Source Amplifier
2. Common-Source Amp with Source Degeneration
3. Common-Drain Amplifier
4. **Common-Gate Amplifier**
5. Cascode Amplifier

CG: Common-Gate Stage

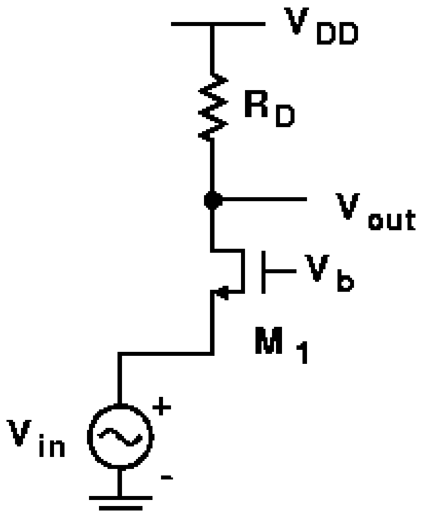
- If M_1 is saturated, the V_{out} can be expressed as

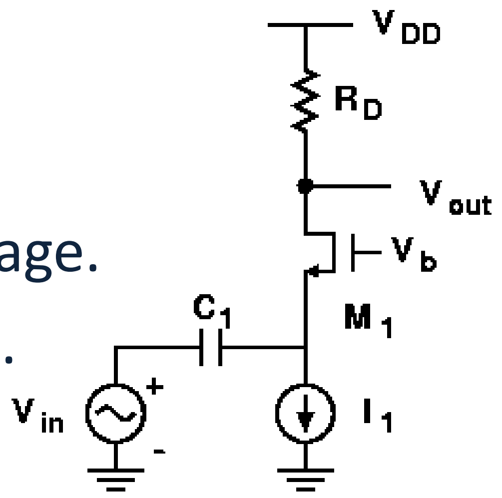
$$V_{out} = V_{DD} - \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_b - V_{in} - V_{TH})^2 R_D$$

$$\frac{\partial V_{out}}{\partial V_{in}} = -\mu_n C_{ox}\frac{W}{L}(V_b - V_{in} - V_{TH})\left(-1 - \frac{\partial V_{TH}}{\partial V_{in}}\right)R_D$$

$$\text{For} \quad \frac{\partial V_{TH}}{\partial V_{in}} = \frac{\partial V_{TH}}{\partial V_{SB}} = \eta$$

$$\frac{\partial V_{out}}{\partial V_{in}} = \mu_n C_{ox}\frac{W}{L}(V_b - V_{in} - V_{TH})(1+\eta)R_D = g_m(1+\eta)R_D$$



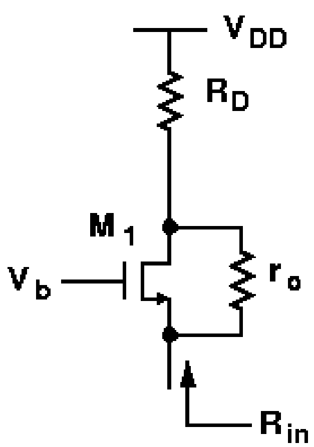


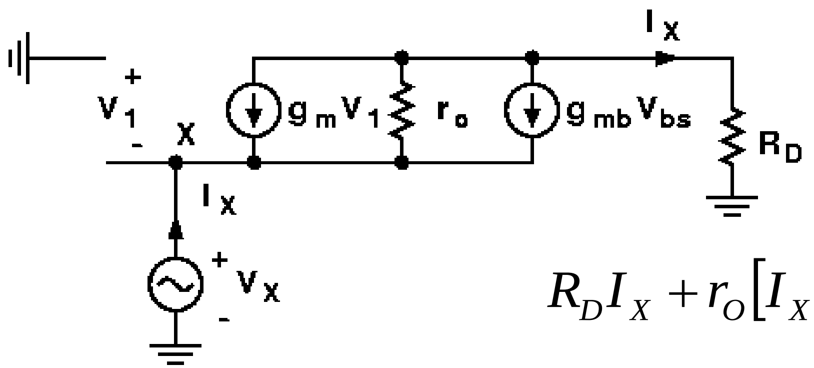
- Body effect increases the equivalent g_m of the stage.
- Body effect deceases the input impedance of CG.

$$Z_{in} = \frac{1}{g_m + g_{mb}} = \frac{1}{g_m(1+\eta)}$$

CG Stage- Input Impedance

- By taking into account both the output impedance of the transistor r_o , find the input impedance Z_{in}:





$$R_D I_X + r_O\left[I_X - (g_m + g_{mb})V_X\right] = V_X$$

$$Z_{in} = \frac{V_X}{I_X} = \frac{R_D + r_O}{1 + (g_m + g_{mb})r_O} \approx \frac{R_D}{(g_m + g_{mb})r_O} + \frac{1}{g_m + g_{mb}}$$

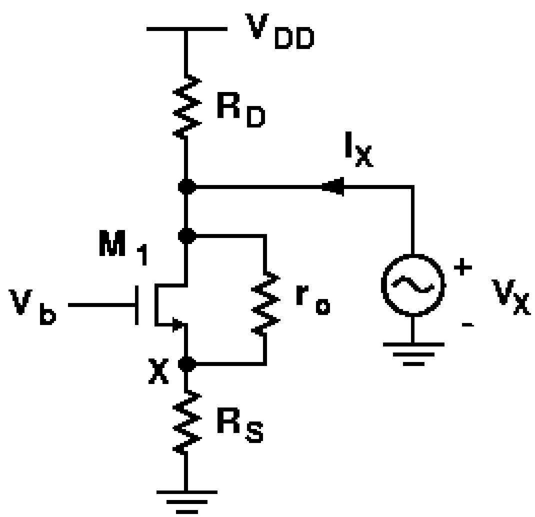
- For $R_D = 0$, same as source follower

$$Z_{in} = \frac{V_X}{I_X} = \frac{r_O}{1 + (g_m + g_{mb})r_O} = \frac{1}{g_m + g_{mb} + 1/r_O} = r_O \parallel \frac{1}{g_m} \parallel \frac{1}{g_{mb}}$$

- For $R_D = \infty$, $Z_{in} = \infty$

CG Stage- Output Impedance

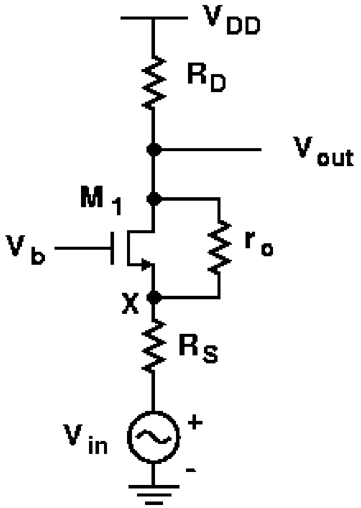
- The output impedance is similar to that of a common source gain stage with source degeneration. R_S is the impedance of signal source.

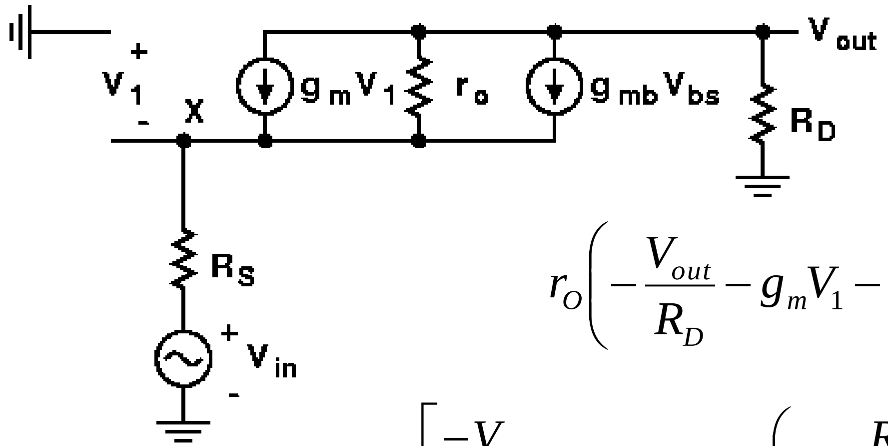


$$R_{out} = \{[1+(g_m + g_{mb})r_O]R_S + r_O\} \parallel R_D$$

CG Stage- Voltage gain

- Voltage gain is similar to CS + Source degeneration, it's slightly higher due to body effect





$$V_1 - \frac{V_{out}}{R_D} R_S + V_{in} = 0$$

$$r_O\left(-\frac{V_{out}}{R_D} - g_m V_1 - g_{mb} V_1\right) - \frac{V_{out}}{R_D} R_S + V_{in} = V_{out}$$

$$r_O\left[\frac{-V_{out}}{R_D} - (g_m + g_{mb})\left(V_{out}\frac{R_S}{R_D} - V_{in}\right)\right] - \frac{V_{out} R_S}{R_D} + V_{in} = V_{out}$$

$$\frac{V_{out}}{V_{in}} = \frac{1 + (g_m + g_{mb}) r_O}{r_O + (g_m + g_{mb}) r_O R_S + R_S + R_D} R_D = \frac{1 + (g_m + g_{mb}) r_O}{r_O + (g_m + g_{mb}) r_O R_S + R_S} \frac{[r_O + (g_m + g_{mb}) r_O R_S + R_S] R_D}{r_O + (g_m + g_{mb}) r_O R_S + R_S + R_D}$$

$$= \frac{1 + (g_m + g_{mb}) r_O}{r_O + (g_m + g_{mb}) r_O R_S + R_S} R_{out}$$

$$\left.\frac{V_{out}}{V_{in}}\right|_{CS+SD} = \frac{g_m r_O}{r_O + (g_m + g_{mb}) r_O R_S + R_S} \frac{[r_O + (g_m + g_{mb}) r_O R_S + R_S] R_D}{r_O + (g_m + g_{mb}) r_O R_S + R_S + R_D}$$

Outline

1. Common-Source Amplifier
2. Common-Source Amp with Source Degeneration
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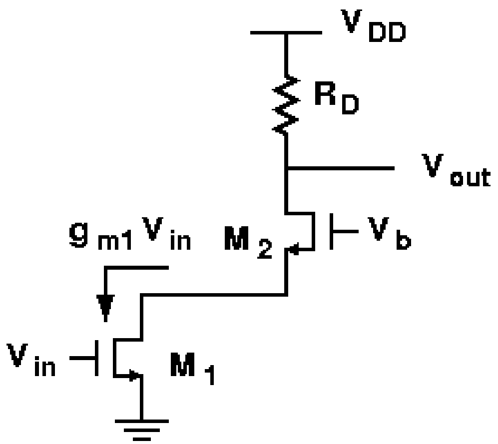
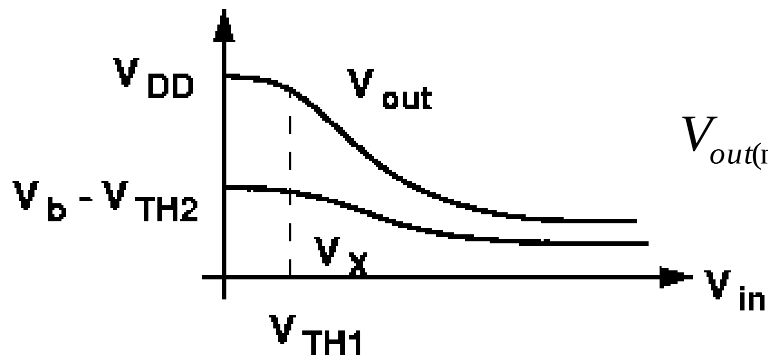
4. Common-Gate Amplifier
5. **Cascode Amplifier**

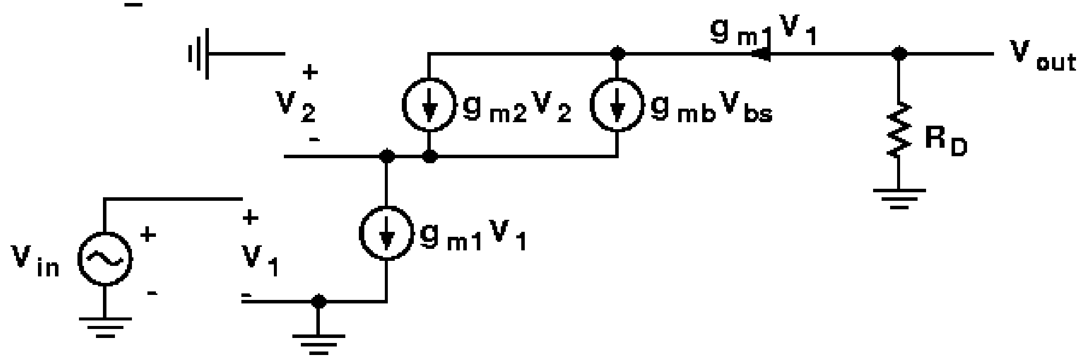
CAS: Cascode Stage (I)

- Cascade of a CS stage and a CG stage → a high output impedance.





$$V_{out(\min)} = V_{DS1(sat)} + V_{DS2(sat)}$$



$$A_v = -g_{m1} R_D \quad (r_o = \infty)$$

- Without consideration of r_o, The voltage gain is independent of the transconductance and body effect of M2.

CAS: Cascode Stage (II)

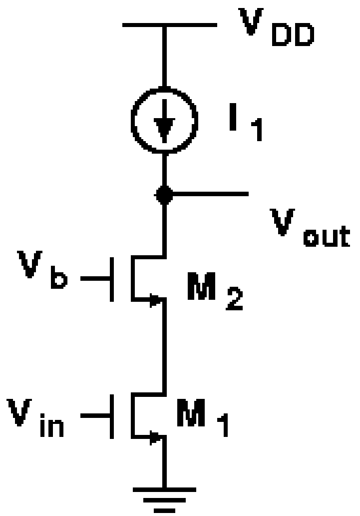
- If both M_1 and M_2 operate in saturation.

$$G_m \approx g_{m1}$$

$$R_{out} = [1 + (g_{m2} + g_{mb2}) r_{O2}] r_{O1} + r_{O2}$$

$$R_{out} \approx (g_{m2} + g_{mb2}) r_{O2} r_{O1}$$

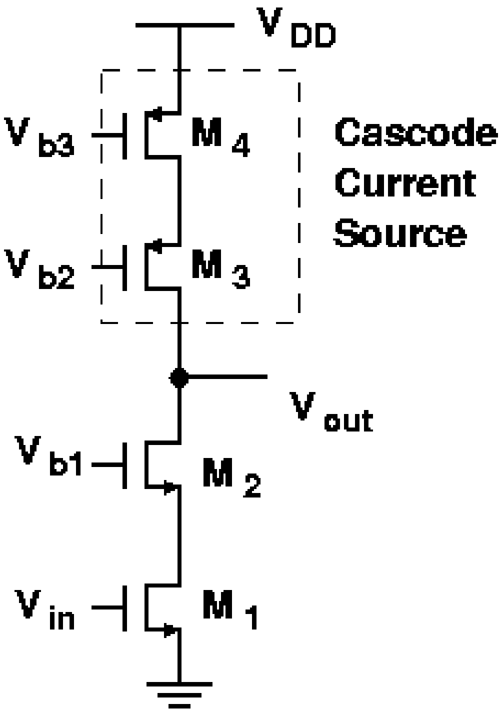
$$A_v = -(g_{m2} + g_{mb2}) r_{O2} g_{m1} r_{O1}$$



- The maximum voltage gain is roughly equal to the square of the intrinsic gain of the transistors

NMOS CAS Amp + PMOS CAS Load

- Cascode as a constant current source with high output impedance
- The maximum output swing is equal to



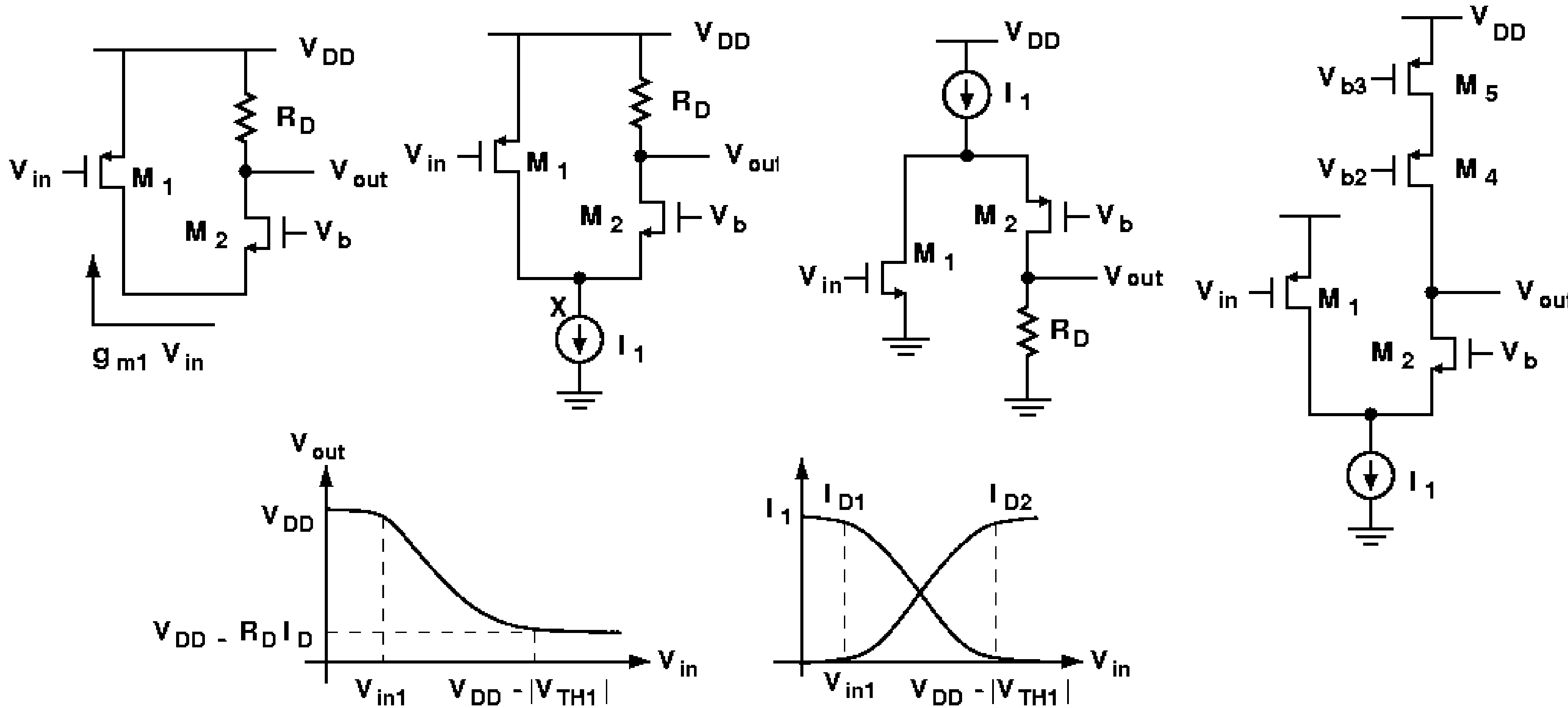
$$V_{out,swing} = V_{DD} - V_{DS1} - V_{DS2} - V_{SD3} - V_{SD4}$$

$$R_{out} = \{[1+(g_{m2}+g_{mb2})r_{O2}]r_{O1}+r_{O2}\}$$
$$\| \{[1+(g_{m3}+g_{mb3})r_{O3}]r_{O4}+r_{O3}\}$$

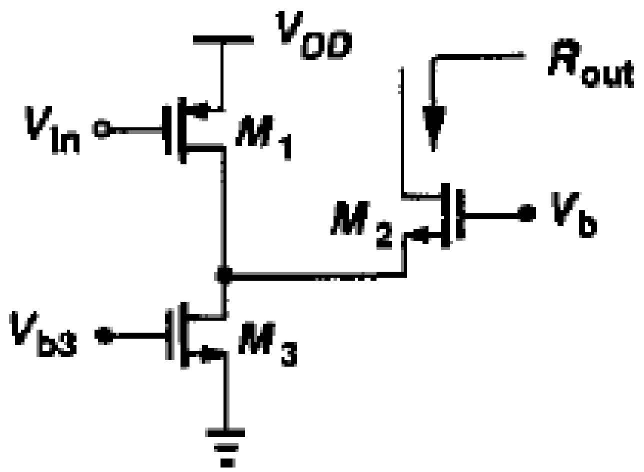
$$A_v \approx -g_{m1}[(g_{m2}r_{O2}r_{O1}) \| (g_{m3}r_{O3}r_{O4})]$$

Folded Cascode

- A PMOS-NMOS combination.
- The total bias current in this case must be higher to achieve comparable performance.



R_{out} of Folded-Cascode



$$R_{out} = \left[1 + \left(g_{m2} + g_{mb2}\right) r_{O2}\right](r_{O1} \parallel r_{O3}) + r_{O2}$$

Designer's Intuition

- Simulation is essential because the behavior of short-channel MOSFET can't be predicted accurately by hand calculations.
- Don't avoids a simple and intuitive analysis of the circuit and skip the task of gaining inside, you can't interpret the simulate results intelligently.
- Don't let the computer think for you!