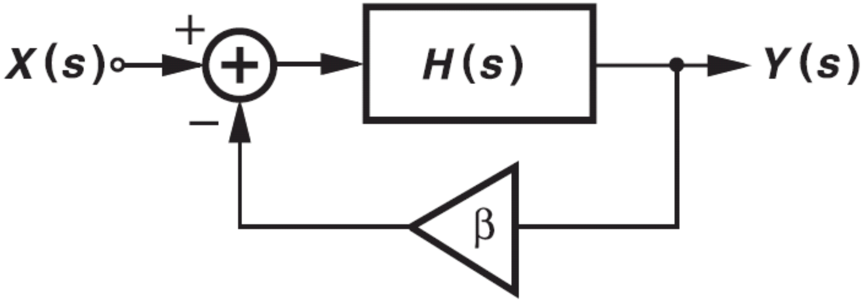
CHAPTER 10

Frequency Compensation

Outline

1. **General Consideration**
2. Phase Margin
3. Frequency Compensation
4. Compensation of Two-Stage Op Amps
5. Other Compensation Techniques

General Consideration



$$\frac{Y}{X}(s) = \frac{H(s)}{1+\beta H(s)}$$

- Consider the negative feedback system, where β is assumed constant
- If $\beta H(s = j\omega_1) = -1$, the gain goes to infinity, and the circuit can amplify its own noise until it eventually begins to oscillate.
- ***Barkhausen's Criteria*** : The circuit may oscillate at frequency ω_1 if

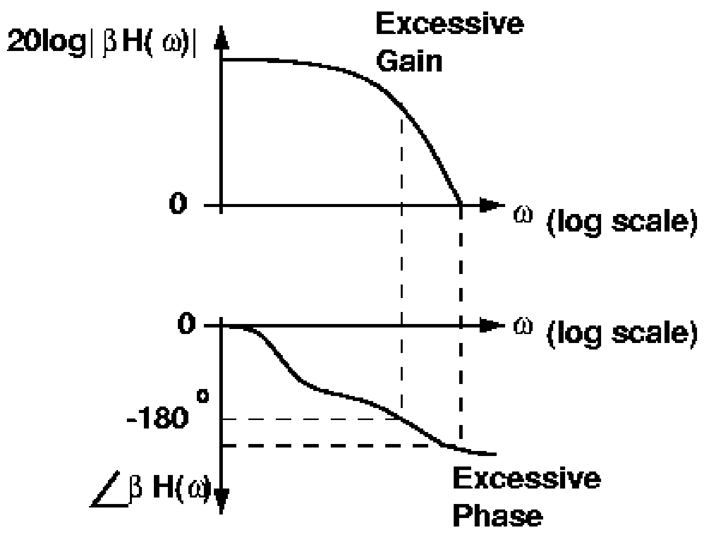
$$|\beta H(j\omega_1)| = 1 \qquad \angle \beta H(j\omega_1) = -180^o$$

- The total phase shift around the loop at ω_1 is 360°. (180° from negative feedback)
- The feedback signal add in phase to the original noise to allow oscillation buildup.

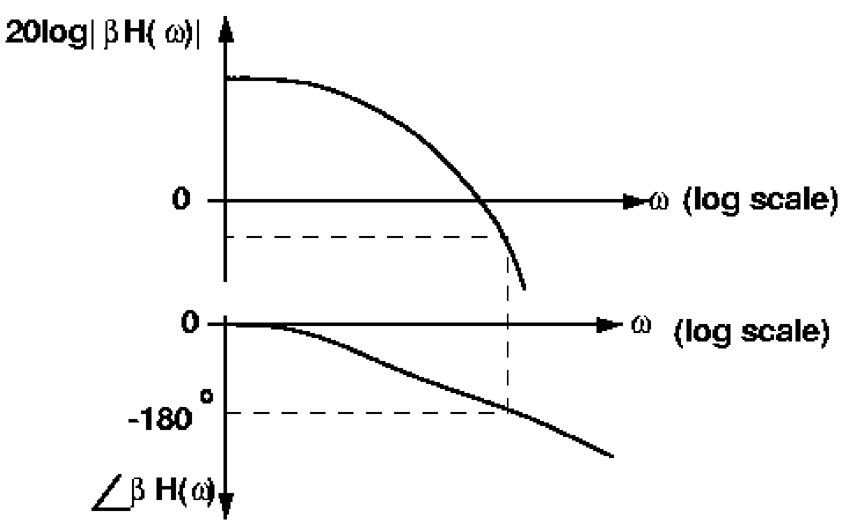
Bode Plot for Stability Analysis

- Assume that β is less than or equal to unity and does not depend on the frequency.
- The worst case stability corresponds to $\beta = 1$.
- We often analyze the magnitude and phase plots for $\beta H = H$.

Unstable



Stable

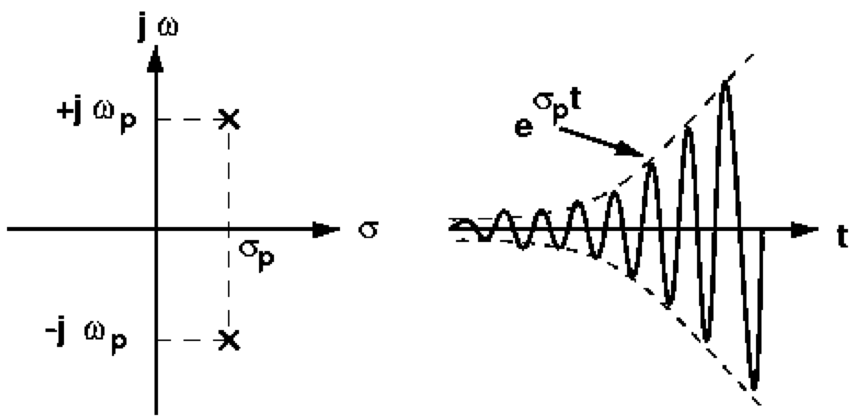


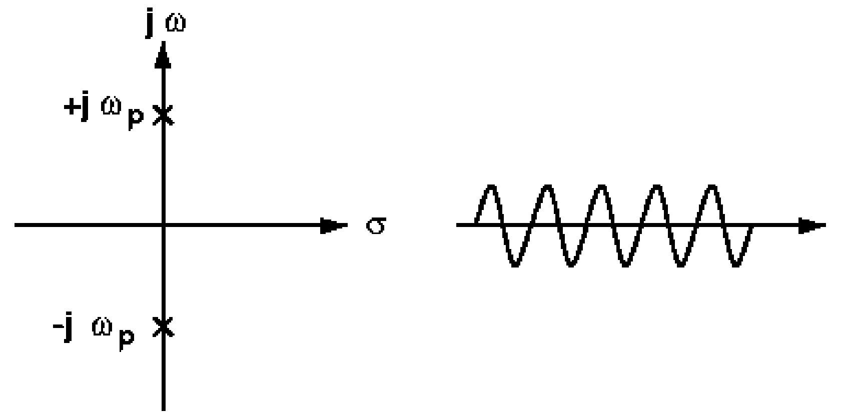
- In a stable system

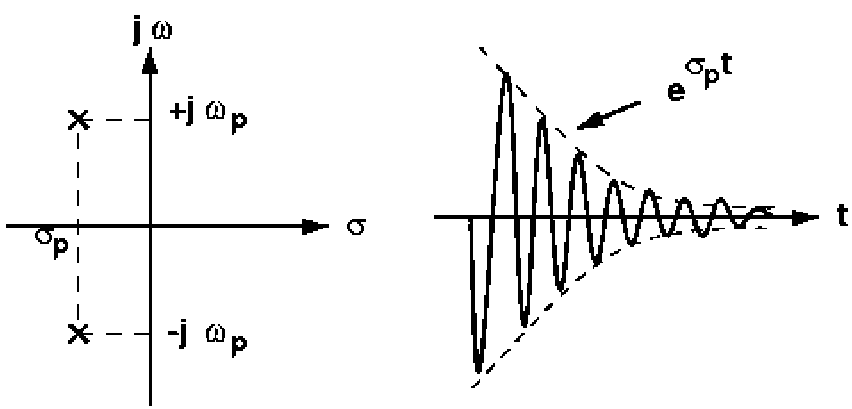
$$20\log\left|\beta H\left(j\omega_{180^o}\right)\right| < 0 \qquad \angle\beta H\left(j\omega_{unity}\right) > -180^o$$

$$\text{Gain Margin} = -20\log\left|\beta H\left(j\omega_{180^o}\right)\right| \qquad \text{Phase Margin} = 180^o + \angle\beta H\left(j\omega_{unity}\right)$$

Time Response vs. the Position of Poles







- Expressing each pole frequency as

$$s_p = j\omega_p + \sigma_p$$

- The impulse response of the system includes a term

$$\exp\left(j\omega_p + \sigma_p\right)t$$

Stability and Pole Location

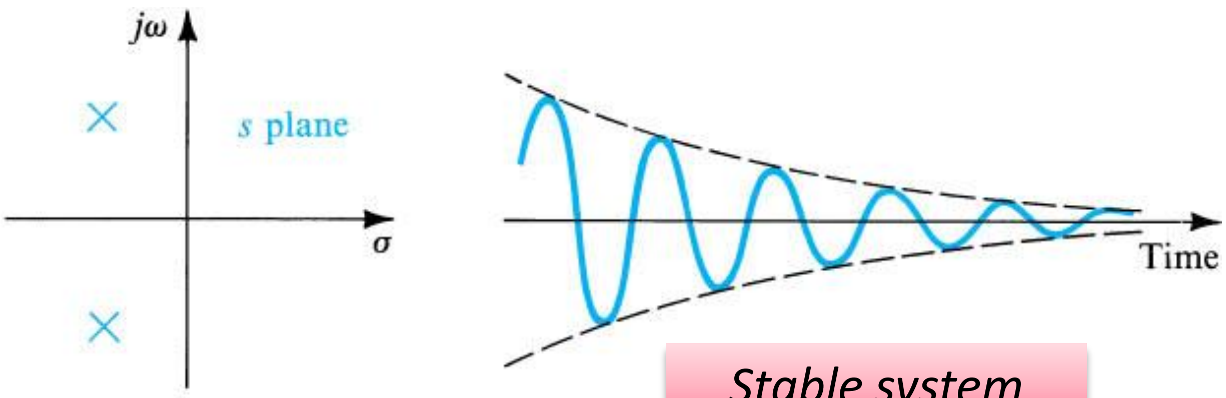
http://en.wikipedia.org/wiki/Laplace_transform

$$H(s) = \frac{1}{(s+\alpha)(s+\beta)}. \qquad h(t) = \mathcal{L}^{-1}\{H(s)\}.$$

Consider an amplifier with a complex-conjugate pole pair : $s = \sigma_0 \pm j\omega_n$

$$v(t) = e^{\sigma_0 t}[e^{j\omega_n t} + e^{-j\omega_n t}] = 2e^{\sigma_0 t}\cos(\omega_n t)$$

A sinusoidal signal with an envelope $e^{\sigma_0 t}$



Stable system

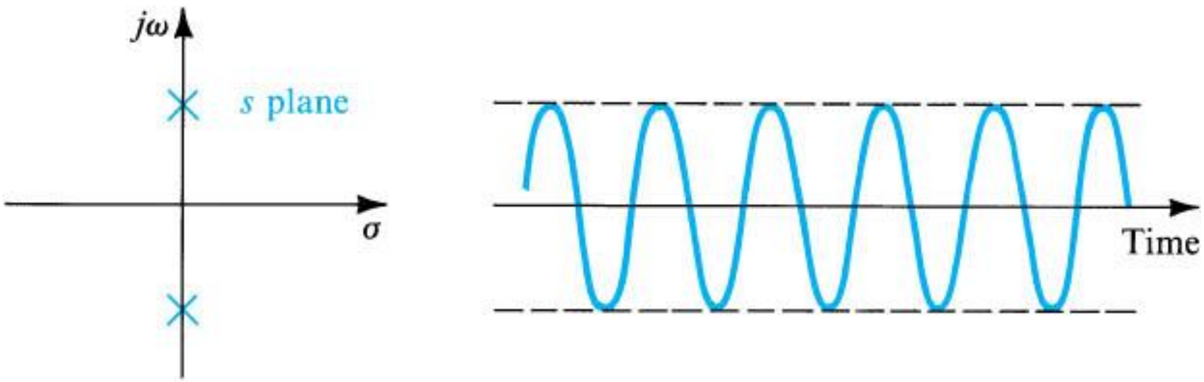
Left-plane pole $\sigma_0 < 0$ *: Oscillation **decays** exponentially to zero.*

Stability and Pole Location

Right-plane pole $\sigma_0 > 0$ *: Oscillation **grows** exponentially to nonlinear.*

Unstable system

jw axis pole $\sigma_0 = 0$ *: Oscillation **sustained.***



Oscillation system

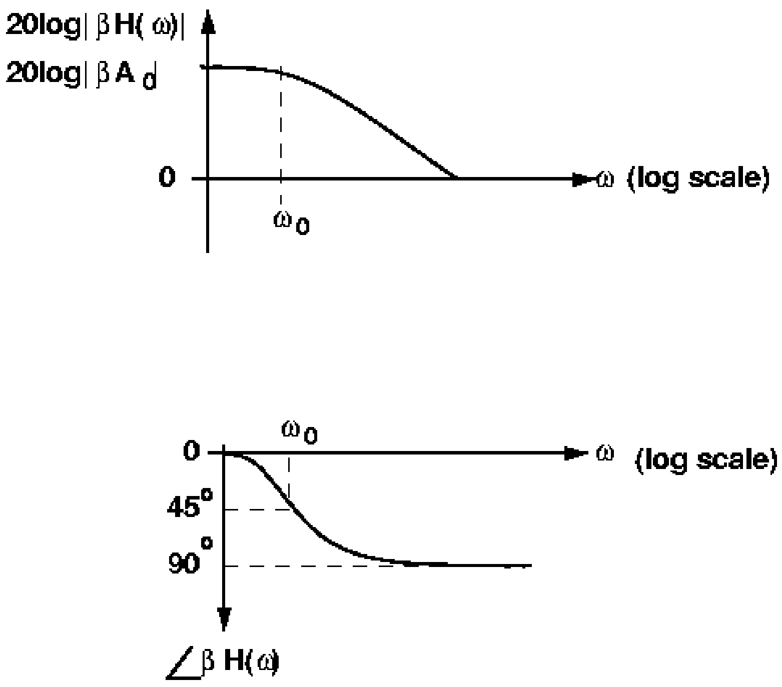
*The existence of any **righ-half-plane** poles results in **instability***

Bode Plots of Loop Gain

- Consider an one pole amplifier

$$H(s)=\frac{A_0}{\left(1+\frac{s}{\omega_0}\right)}$$

$$\frac{Y}{X}(s)=\frac{\frac{A_0}{1+\beta A_0}}{1+\frac{s}{\omega_0\left(1+\beta A_0\right)}}$$



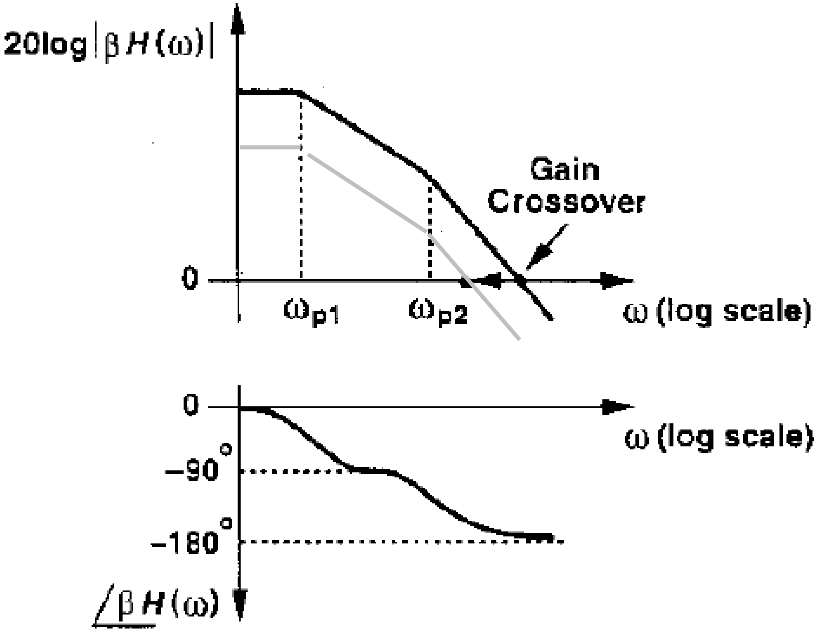
- A single pole cannot contribute a phase shift greater than 90^o, and the system is unconditionally stable for all non-negative values of β.

Two-pole Systems

- Consider a two-pole system

$$H(s) = \frac{A_0}{\left(1 + s/\omega_{P1}\right)\left(1 + s/\omega_{P2}\right)}$$

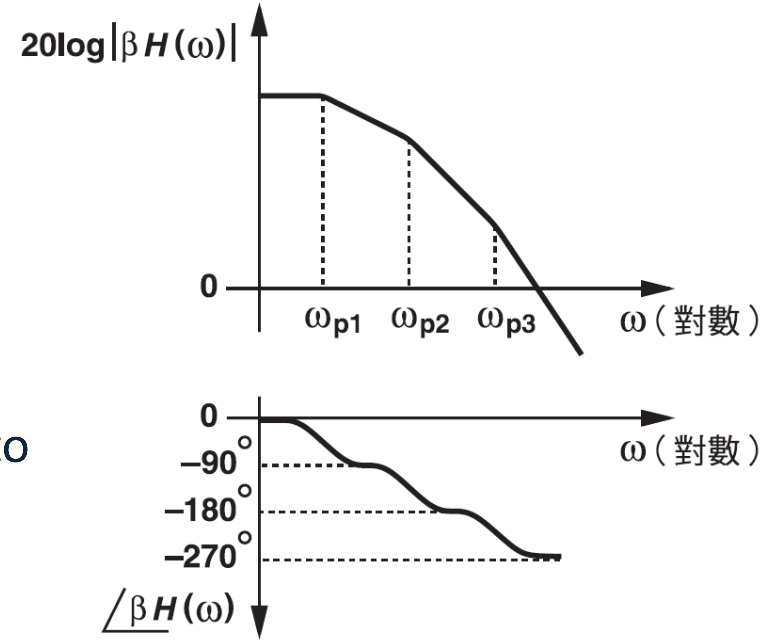
$$\frac{Y}{X}(s) = \frac{A_0}{\left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{s}{\omega_{p2}}\right) + \beta A_0}$$



- Maximum phase shift = 180°
- As the feedback becomes weaker, the gain crossover point moves toward the origin.
- The stability is obtained at the cost of weaker feedback.

Multipole Systems

- Consider a 3-pole system
- Maximum phase shift = 270°
- The phase begins to change at 0.1 of pole frequency whereas the magnitude begins to drop only near the pole frequency.
- Additional poles(and zeros) impact the phase to a much greater extent than magnitude.
- The stability is obtained at the cost of weaker feedback.



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2. **Phase Margin**
3. Frequency Compensation
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Phase Margin

- If $\angle \beta H(j\omega_u) = -175^o, \quad \beta H(j\omega_1) = 1 \times \exp(-j175^o)$

$$\frac{Y}{X}(j\omega_1) = \frac{H(j\omega_1)}{1+\beta H(j\omega_1)} = \frac{\frac{1}{\beta}\exp(-j175^o)}{1+\exp(-j175^o)} = \frac{1}{\beta} \cdot \frac{-0.9962 - j0.0872}{0.0038 - j0.0872}$$

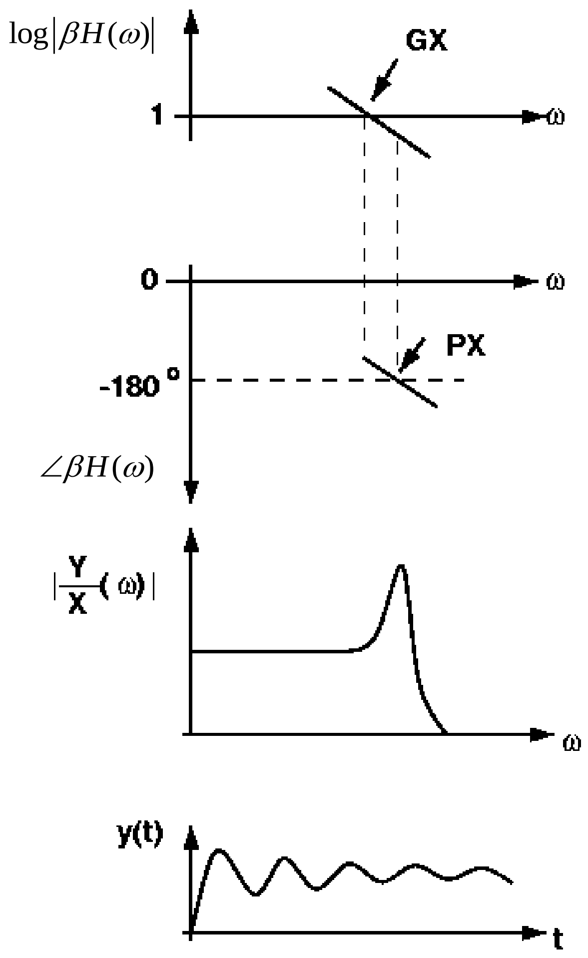
$$\left|\frac{Y}{X}(j\omega_1)\right| = \frac{1}{\beta} \cdot \frac{1}{0.0872} \approx \frac{11.5}{\beta}$$

- Since at low frequencies, $\left| Y/X \right| \approx 1/\beta$, the closed-loop frequency response exhibits a sharp peak in the vicinity of $\omega = \omega_1$

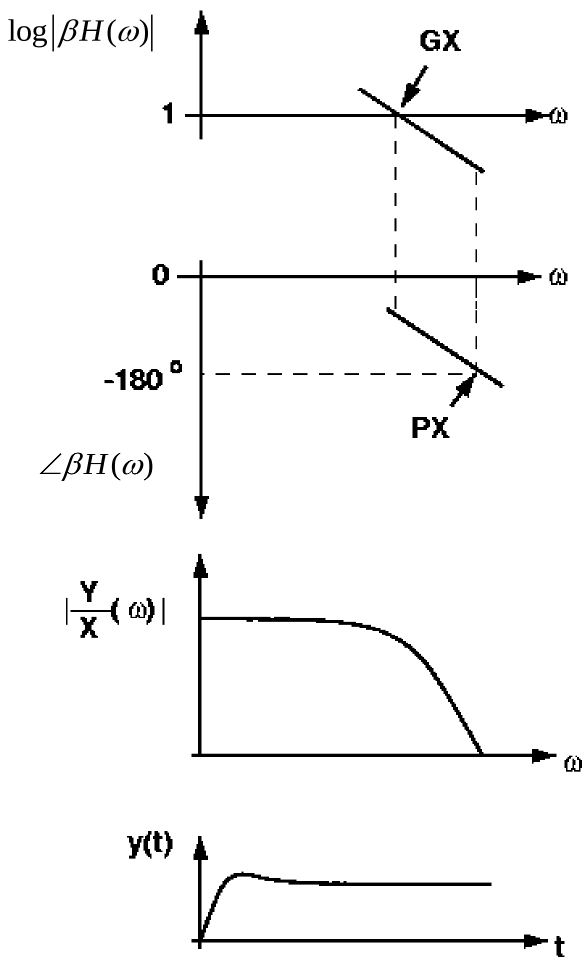
- The closed loop system is near oscillation and its step response exhibits a very underdamped behavior.

Closed-Loop Freq. and Time Response

- For a small phase margin

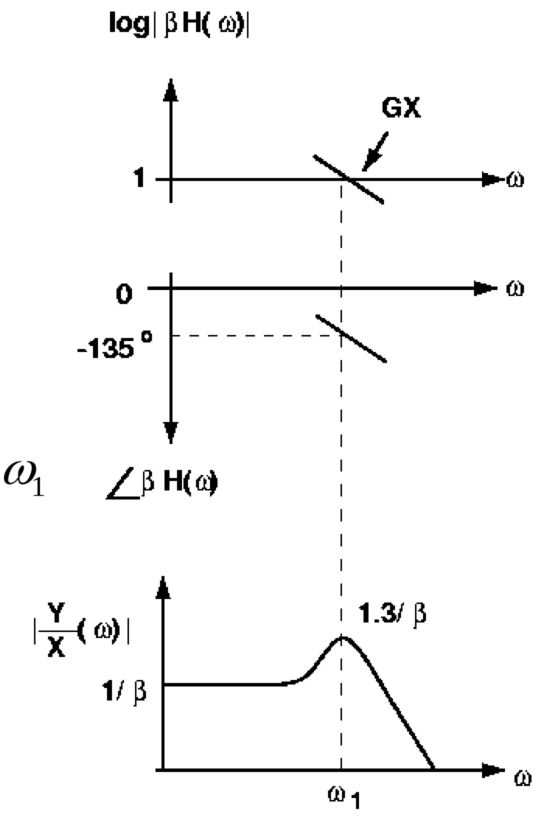


- For a large phase margin



- The greater the phase margin, the more stable the feedback system.

CL Freq. Response with 45° PM

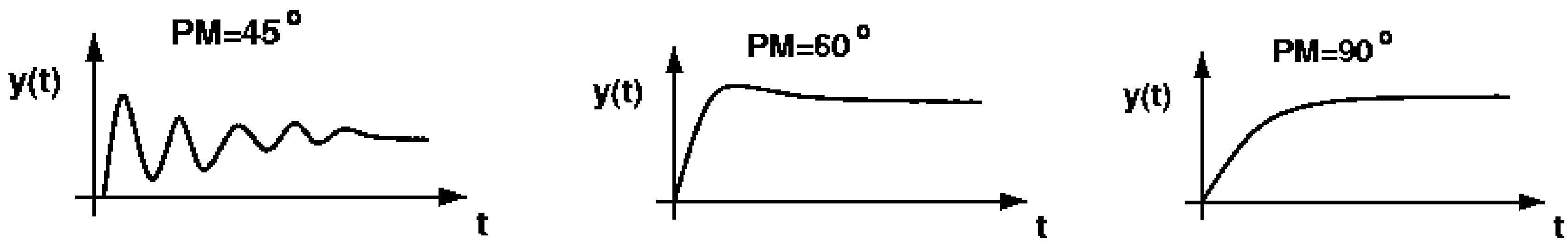


- For PM = 45°, $\angle\beta H(\omega_1) = -135^o$ $\quad |\beta H(\omega_1)| = 1$

$$\frac{Y}{X} = \frac{H(j\omega_1)}{1+1\times\exp(-j135^o)} = \frac{H(j\omega_1)}{0.29-0.71j}$$

$$\left|\frac{Y}{X}\right| = \frac{1}{\beta}\cdot\frac{1}{|0.29-0.71j|} \approx \frac{1.3}{\beta}$$

  – The feedback system suffers from a 30% peak at $\omega = \omega_1$
- For PM = 60°, the peaking is negligible $\left|\frac{Y}{X}\right| = \frac{1}{\beta}$
- The concept of phase margin is well-suited to the design of circuits that process small signals, not large signals.



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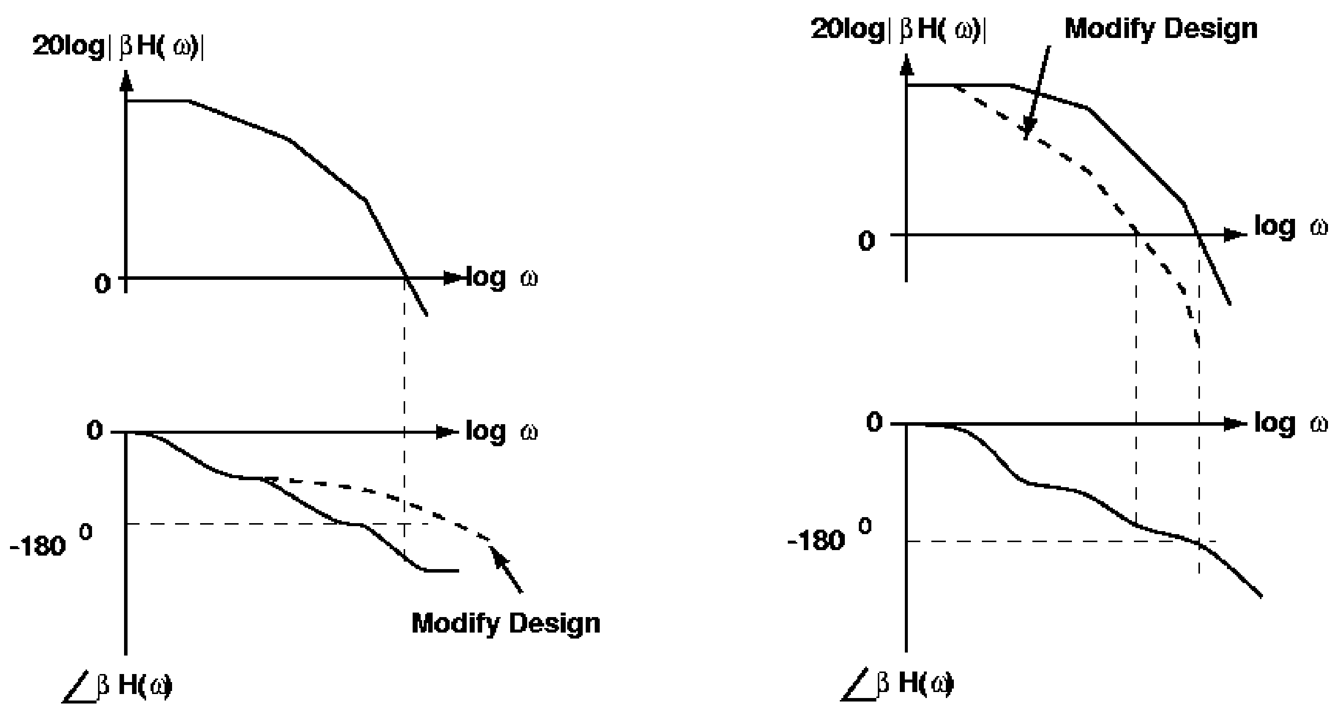
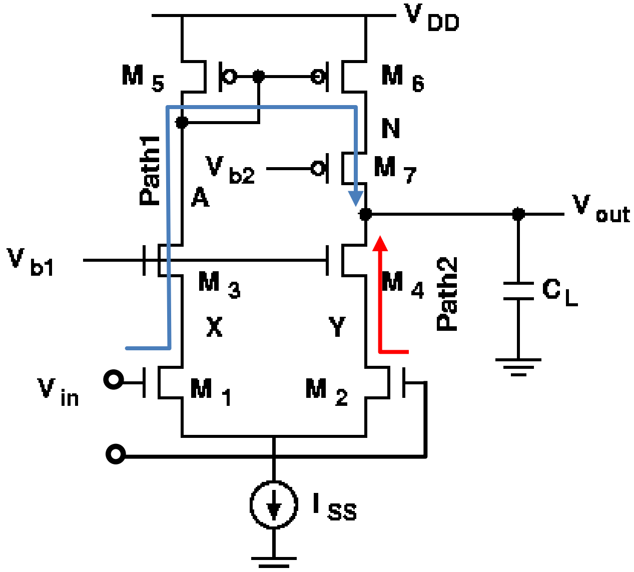
Frequency Compensation

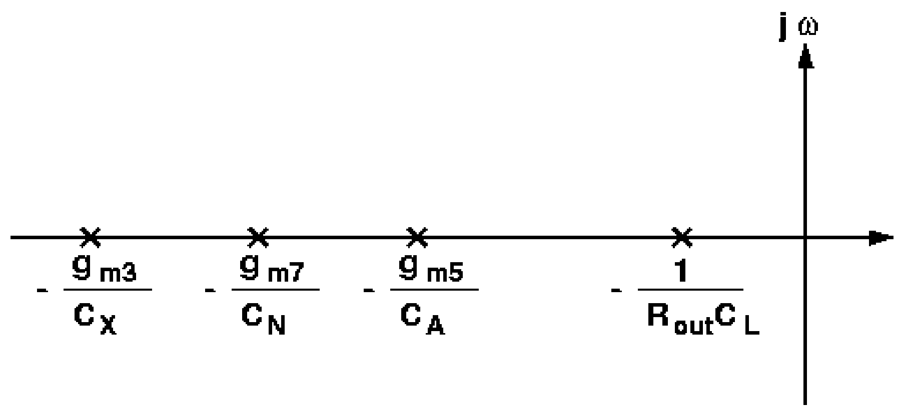
- Frequency compensation : their open loop transfer function must be modified such that the closed-loop circuit is stable and the time response is well behaved.
- Frequency compensation can be achieved by
  - Minimizing the overall phase shift, thus pushing the *phase crossover out*.
  - Dropping the gain, thereby pushing the gain *crossover in*.



Single-Ended Telescopic OP Amp.

- Frequency compensation : their open loop transfer function must be modified such that the closed-loop circuit is stable and the time response is well behaved.

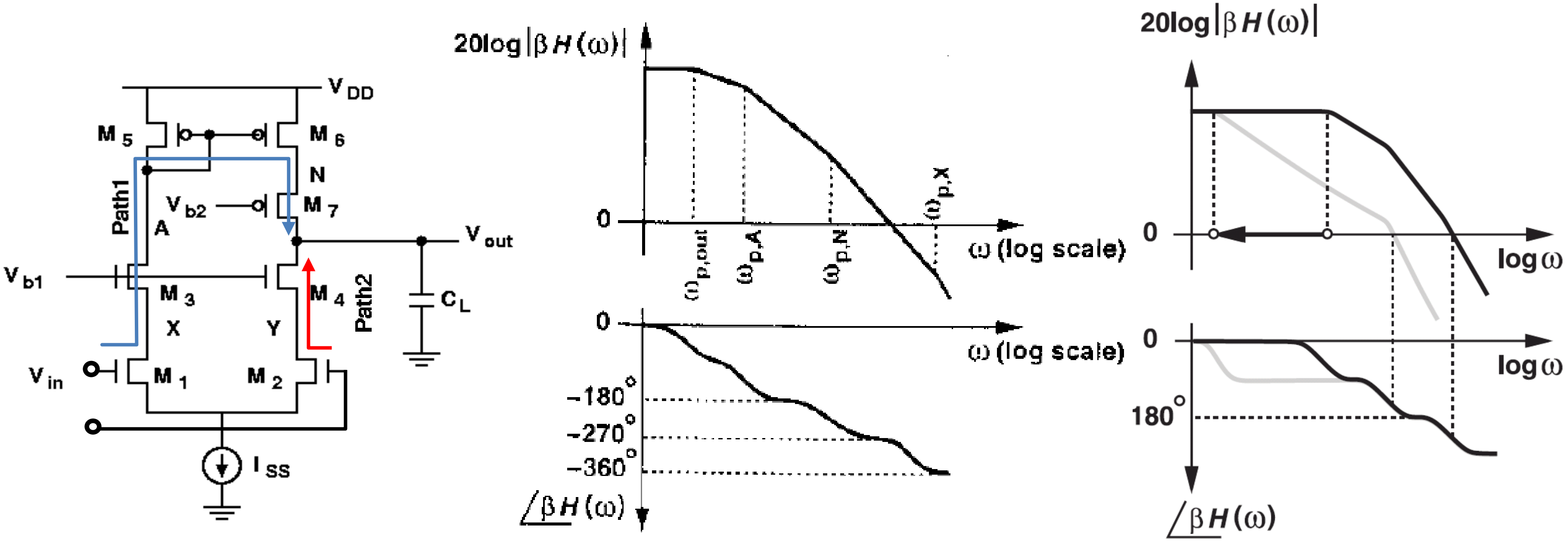




- Path 1 : a high-frequency pole at the source of M_3, a mirror pole at node A, another high-frequency pole at the source of M_7, and a pole at the output.
- Path2 : a high-frequency pole at the source of M_4, and a pole at the output.
- Dominant pole (d.p.) : $\omega_{p,out}$ usually sets the open loop 3-dB bandwidth.
- Non-dominant pole : the closet pole to the origin after the d.p. is at node A.

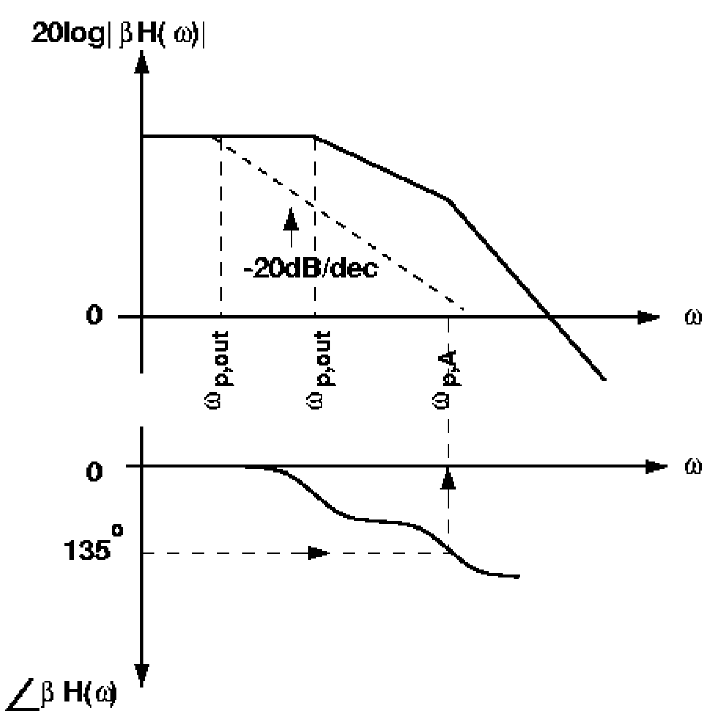
$$C_A = C_{GS5} + C_{GS6} + C_{DB5} + 2C_{GD6} + C_{DB3} + C_{GD3}$$

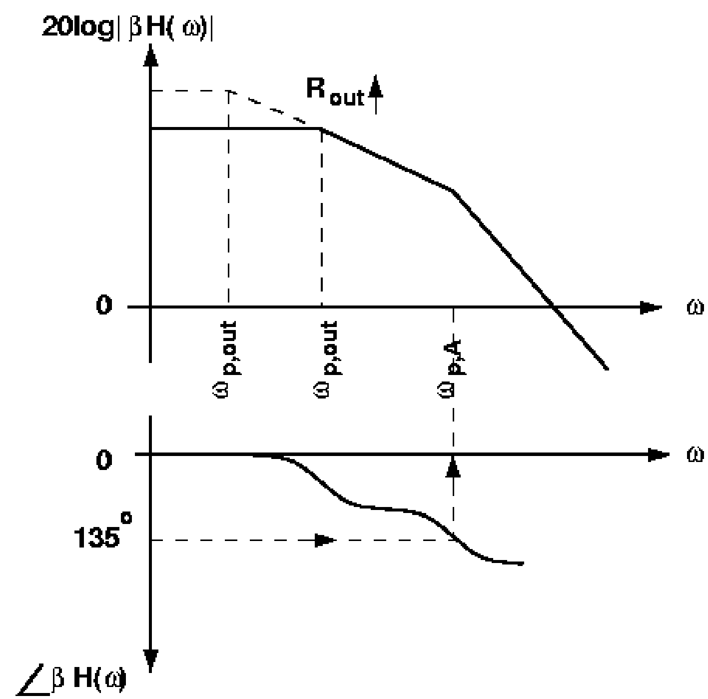
Bode Plots of Telescopic OP Amp



- The mirror pole typically limits the phase margin.
- The circuit contains a zero at $2\omega_{p,A}$. (ignore it now for simplicity)
- Force the loop gain to drop → the gain crossover point moves toward the origin.
- The dominant pole can be pushed to a lower freq. by increasing the load cap.
- The unity-gain bandwidth of the compensated op amp is equal to the frequency of the 1st non-dominant pole.

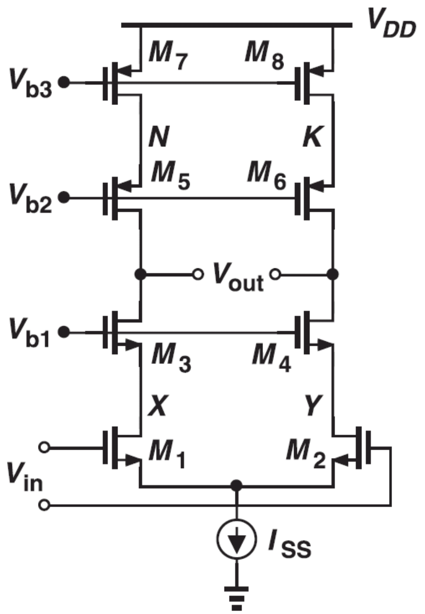
Bode Plots of Loop Gain for Higher R_o



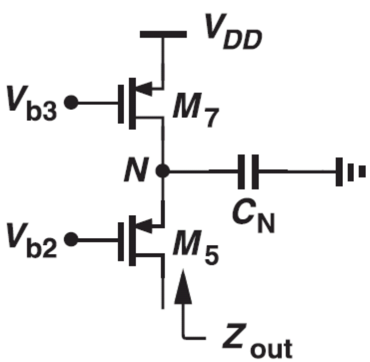


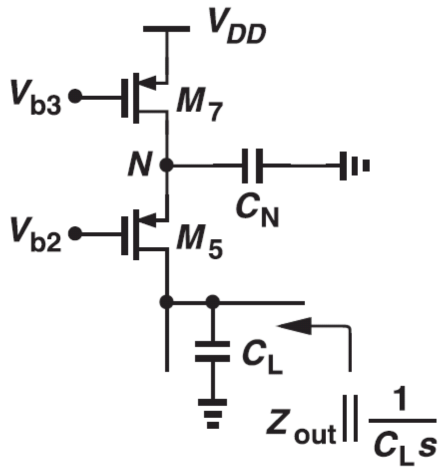
- Translating the dominant pole toward the origin for 45° phase margin, the unity gain frequency is equal to the frequency of the first non-dominant pole.
- Increasing R_{out} does not compensate the op amp.
- To achieve a wideband in a feedback system, the 1st nondominant pole must be as far as possible → The mirror pole is undesirable.

Fully Differential Telescopic Op Amp



- This topology avoids the mirror pole, thereby exhibiting stable behavior for a greater bandwidth.
- One dominant pole at the output node and only one nondominant pole arising from node X (or Y).
- Capacitance at node N, $C_N = C_{GS5} + C_{SB5} + C_{GD7} + C_{DB7}$, shunts the output resistance of M_7 at high frequencies, thereby dropping the output impedance of the cascode.
- The pole in the PMOS cascode is merged with the output pole, thus creating no additional pole.





$$Z_{out} = (1 + g_{m5} r_{O5}) Z_N + r_{O5} \ ,$$

$$Z_N = r_{O7} \,\|\, (C_N s)^{-1},$$

$$Z_{out} \approx (1 + g_{m5} r_{O5}) \frac{r_{O7}}{1 + s r_{O7} C_N}$$

$$Z_{out} \,\|\, \frac{1}{sC_L} = \frac{(1 + g_{m5} r_{O5}) r_{O7}}{[(1 + g_{m5} r_{O5}) r_{O7} C_L + r_{O7} C_N] s + 1}$$

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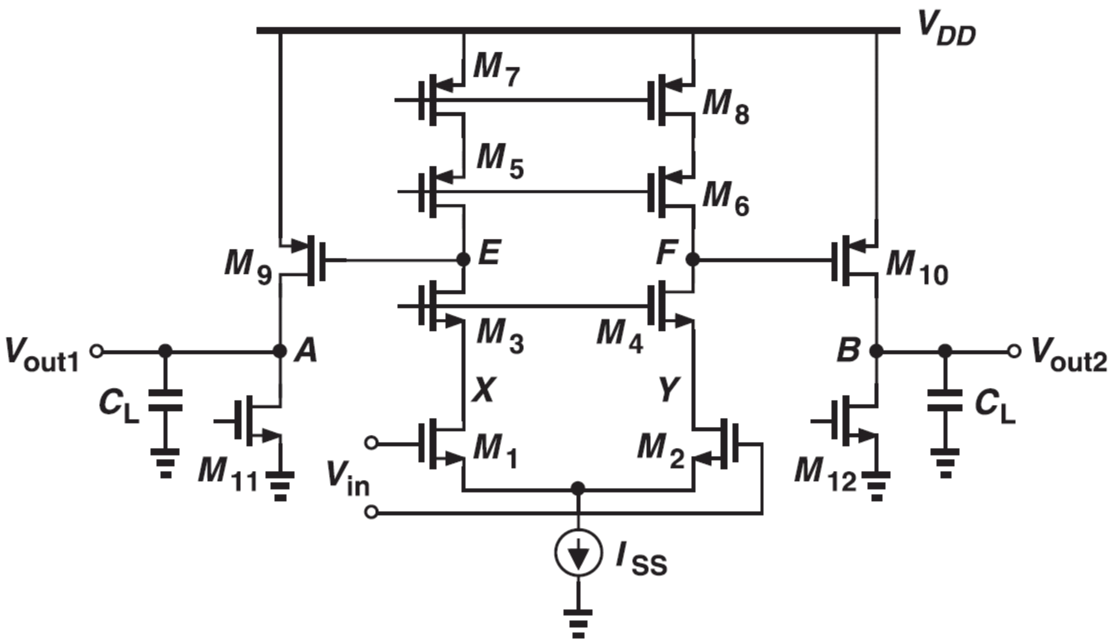
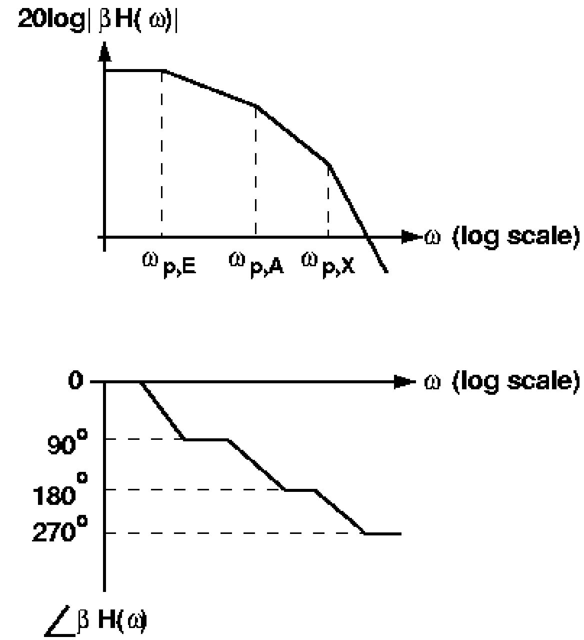
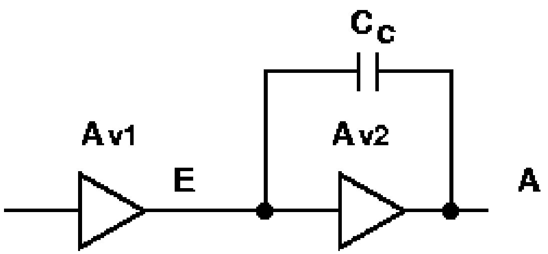
Compensation of Two Stage Op Amps

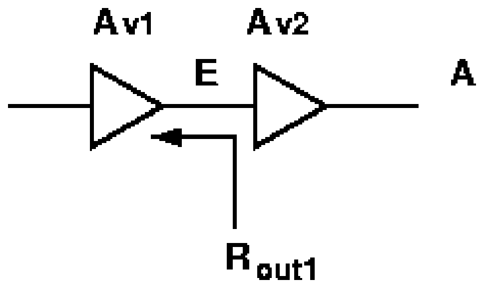


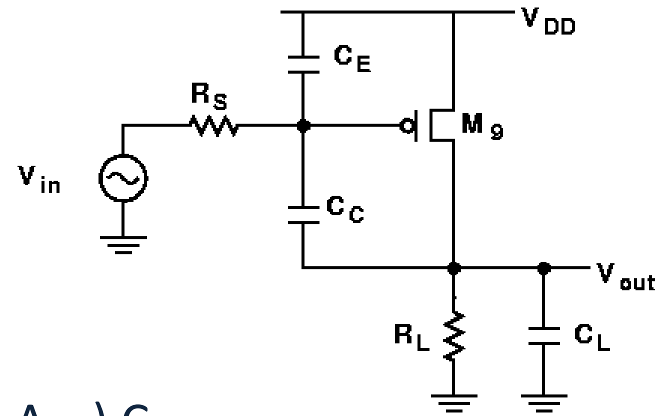


- We identify three poles : a pole at *X* (or *Y*), another at *E* (or *F*), and a third at *A* (or *B*).
- Since the poles at *E* and *A* are relatively close to the origin, the phase approaches -180° well below the third pole.
- If the magnitude of $\omega_{P,E}$ is to be reduced, the available bandwidth is limited to approximately $\omega_{P,A}$, a low value. A very large compensation capacitor is required.

Miller Comp. of a Two-Stage Op Amp







- Create a large capacitance at node E, equal to $(1 + A_{v2})\, C_C$.
- The corresponding pole is moved to $1/R_{out1}\left[C_E + (1 + A_{v2})C_C\right]$
- Miller compensation
  - Lowering the required capacitor value.
  - Moves the output pole away from the origin.
- If R_S denotes the output resistance of the first stage, $R_L = r_{O9} \,||\, r_{O11}$.

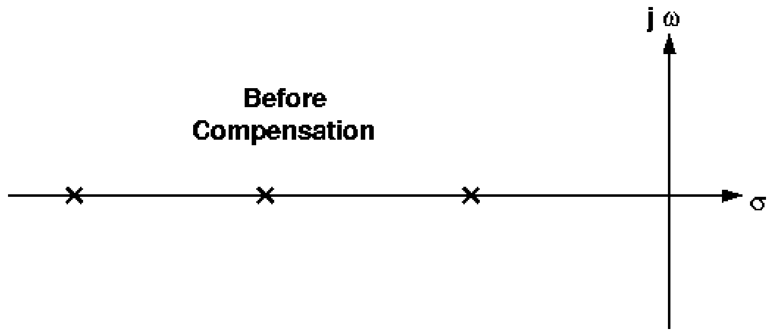
$$|\omega_{p1}| << |\omega_{p2}|$$

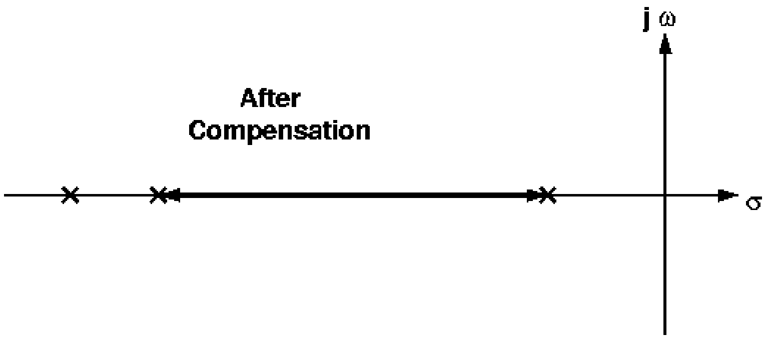
Ref: p.174

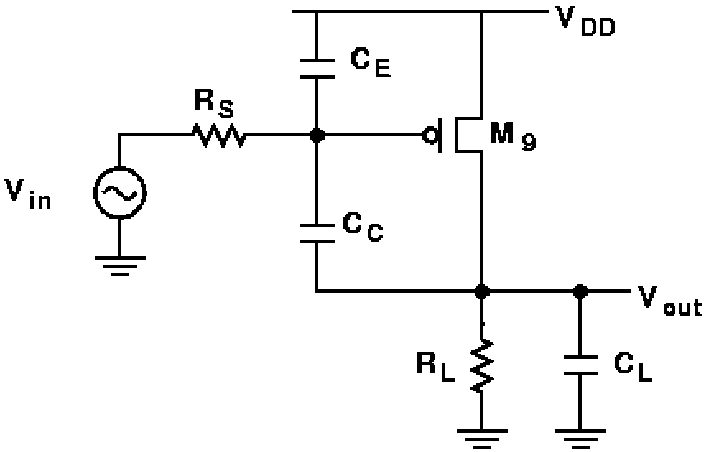
$$\omega_{p1} \approx \frac{1}{R_S\left[(1+g_{m9}R_L)(C_C + C_{GD9}) + C_E\right] + R_L(C_C + C_{GD9} + C_L)}$$

$$\omega_{p2} \approx \frac{R_S\left[(1+g_{m9}R_L)(C_C + C_{GD9}) + C_E\right] + R_L(C_C + C_{GD9} + C_L)}{R_S R_L\left[(C_C + C_{GD9})C_E + (C_C + C_{GD9})C_L + C_E C_L\right]}$$

Pole Splitting







- Before compensation, ω_{p1} and ω_{p2} are of the same order of magnitude.

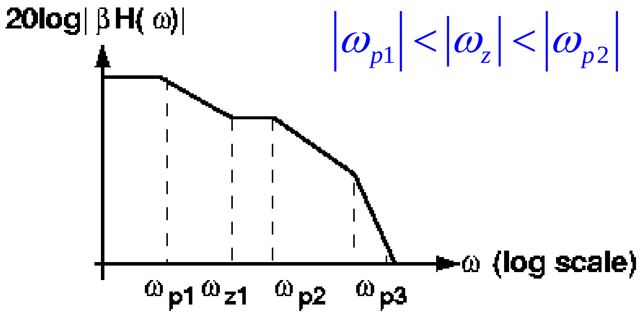
$$C_C = 0, \qquad \omega_{p2} \approx 1/R_L C_L$$

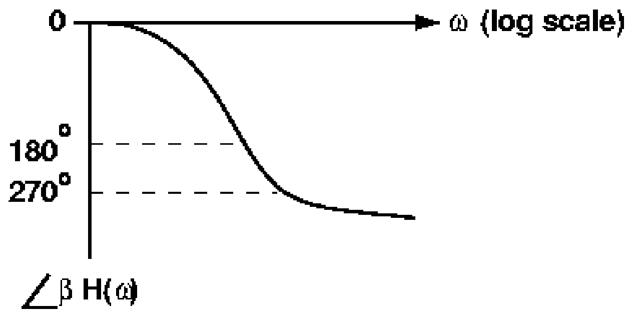
- After compensation ,

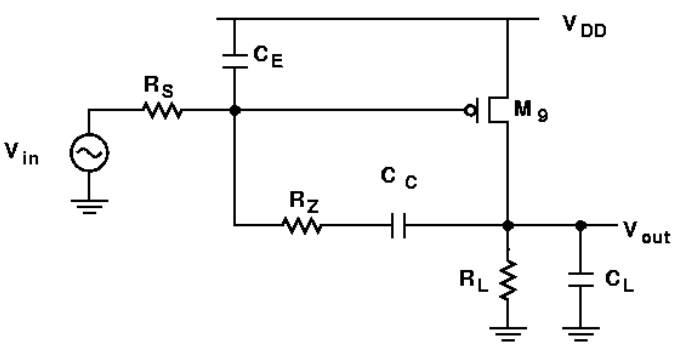
$$\text{If } C_C + C_{GD9} >> C_E, \quad \omega_{p2} \approx \frac{g_{m9}}{C_E + C_L} \approx \frac{g_{m9}}{C_L}$$

- Miller compensation increases the magnitude of the output pole by roughly a factor of $g_{m9} R_L$.
- At high frequencies, C_C provides a low impedance between the gate and drain of M_9. The resistance seen by C_L becomes $R_S \parallel g_{m9}^{-1} \parallel R_L \approx g_{m9}^{-1}$.

Effect of Right Half Plane Zero







- A right-half plane zero at $\omega_z = g_{m9} / (C_C + C_{GD9})$
- Numerator is $(1 - s / \omega_z)$
- The phase shift is $-\tan^{-1}(\omega / \omega_z)$ ➔ negative
- Zero slow down the drop of magnitude.
- The stability degrades considerably.
- The right half–plane zero in a two-stage CMOS op amps is a serious issue because gm is relatively small and C_C is usually large.
- A series resistor to eliminate the right-half plane zero.

$$\omega_z \approx \frac{1}{C_C\left(g_{m9}^{-1} - R_Z\right)}$$

- The output stage now exhibits three poles.
- To cancel the first non-dominant pole

$$\frac{1}{C_C\left(g_{m9}^{-1} - R_Z\right)} = \frac{-g_{m9}}{C_L + C_E}, \quad R_Z = \frac{C_L + C_E + C_C}{g_{m9} C_C} \approx \frac{C_L + C_C}{g_{m9} C_C}$$

Resistor for Miller Compensation

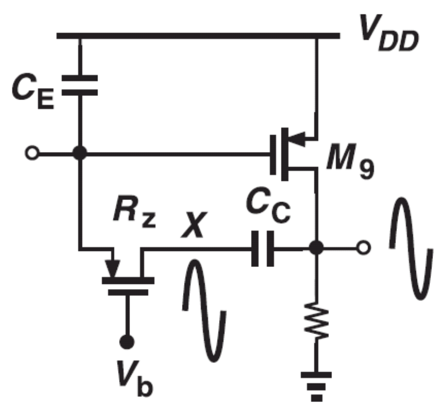
- Typically the feedback R_Z is realized by a MOS transistor in the triode region.
- R_Z changes substantially as output voltage excursions are coupled through C_C to node X, thereby degrading the large-signal settling response.
- Let

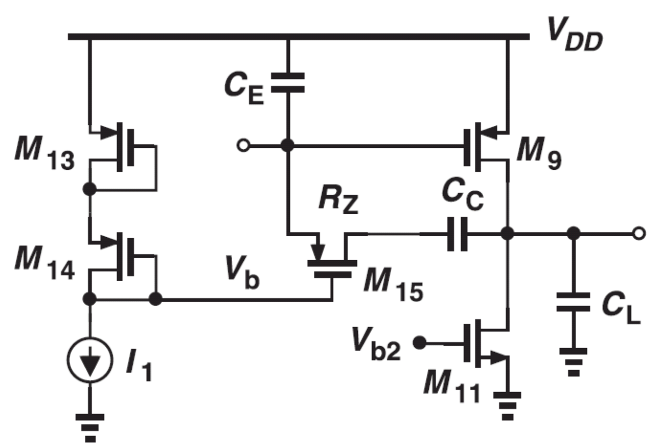
$$V_{GS13} = V_{GS9} \Longrightarrow V_{GS15} = V_{GS14}, \quad g_{m14} = \mu_p C_{ox} (W/L)_{14} (V_{GS14} - V_{TH14}),$$

$$R_{on15} = \left[\mu_p C_{ox} (W/L)_{15} (V_{GS15} - V_{TH15}) \right]^{-1}, R_{on} = g_{m14}^{-1} \frac{(W/L)_{14}}{(W/L)_{15}}$$

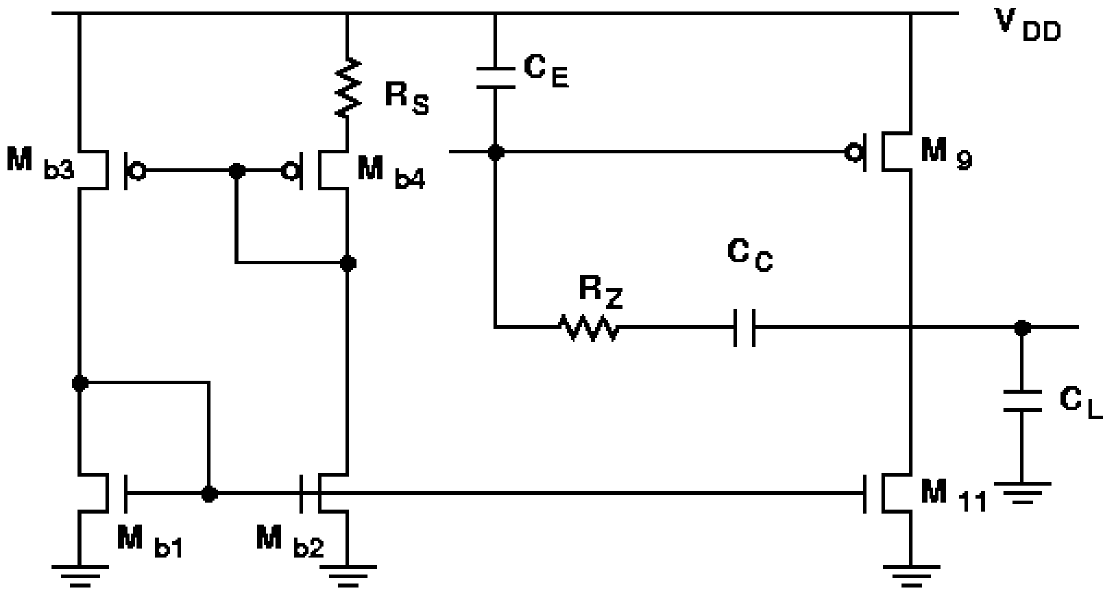
- For pole zero cancellation to occur

$$g_{m14}^{-1} \frac{(W/L)_{14}}{(W/L)_{15}} = g_{m9}^{-1} \left(1 + \frac{C_L}{C_C} \right), \qquad (W/L)_{15} = \sqrt{(W/L)_{14} (W/L)_9} \sqrt{\frac{I_{D9}}{I_{D14}}} \frac{C_C}{C_C + C_L}$$





Defining g_{m9} with respect to R_S

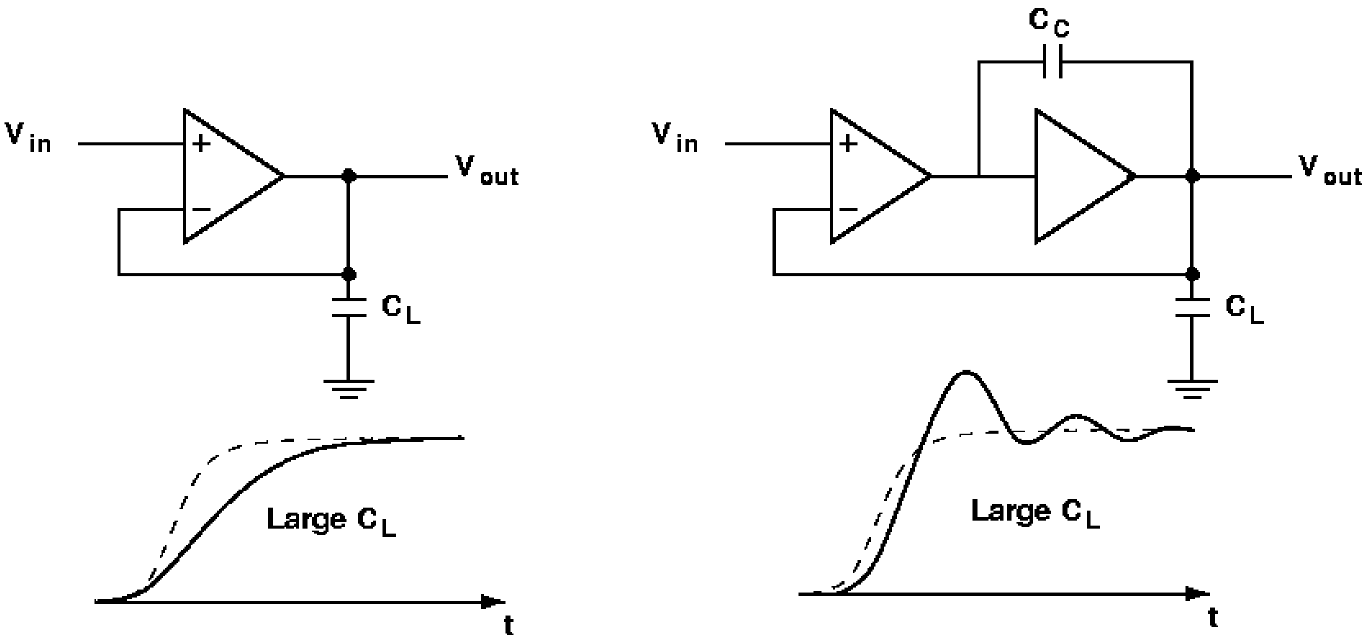


- Use a simple resistor for R_Z and define g_{m9} as a resistor that closely match R_Z.
- Incorporating M_{b1}-M_{b4} along with R_S to generate (p.379)

$$I_b \propto R_S^{-2} \qquad\qquad g_{m9} \propto \sqrt{I_{D9}} \propto \sqrt{I_{D11}} \propto R_S^{-1}$$

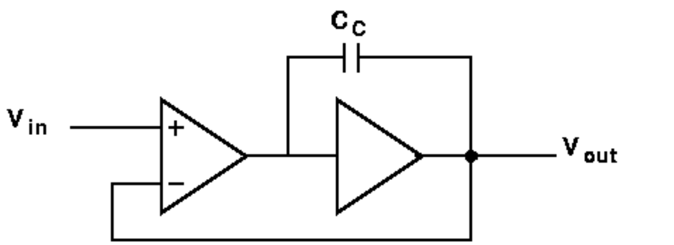
- Proper ratio-ing of R_Z and R_S with temperature and process variations.
- Short channel effects may deviate from the square low regime and create errors.

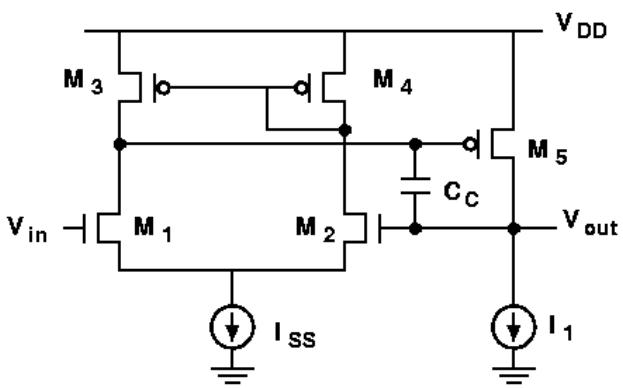
Increased Load Cap. on Step Response

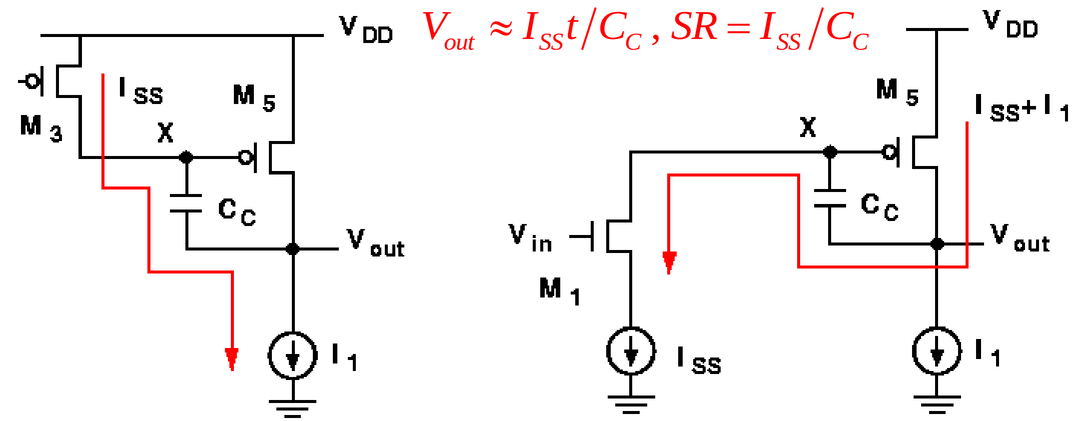


- In a two-stage op-amp, a higher load capacitance presented to the second stage moves the second pole toward the origin, degrading the phase margin.
  - The response approaches an oscillatory behavior if the load capacitance seen by the two-stage op amp increases.
- In one-stage op amps, a higher load capacitance brings the dominant pole closer to the origin, improving the phase margin.
  - Making the feedback system more overdamped.

Slewing in Two-Stage Op Amps







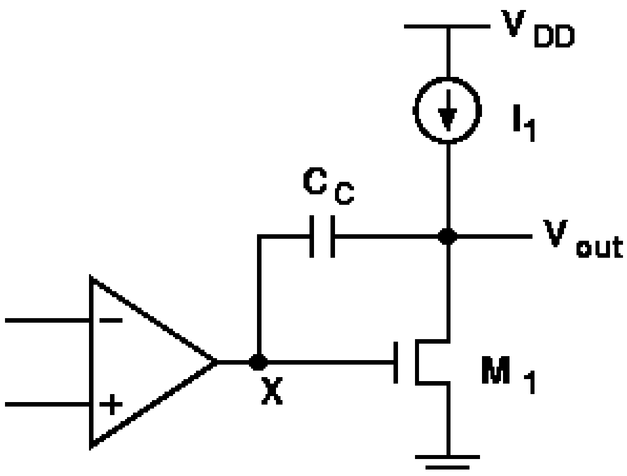
- C_C is charged by a constant I_{SS} if parasitic cap. at node *X* are negligible.
- The gain of the output stage makes node *X* a virtual ground.

- For the positive slew rate, M5 must provide two currents : $I_{SS} + I_1$
  - If M5 not wide enough to sustain $I_{SS} + I_1$ in saturation, then V_X drops significantly, possibly driving M_1 into triode region.
- For the negative slew rate, I_1 must support both I_{SS} and I_{D5}.
  - If $I_1 = I_{SS}$, V_x rises so as to turn off *M5*.
  - If $I_1 < I_{SS}$, M_3 enters the triode region and the slew rate is given by I_{D3}/C_C.

Outline

1. General Consideration
2. Phase Margin
3. Frequency Compensation
4. Compensation of Two-Stage Op Amps
5. **Other Compensation Techniques**

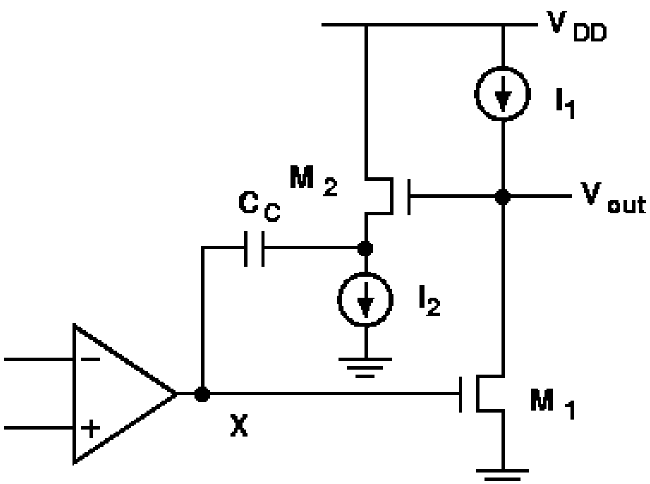
Other Compensation Techniques

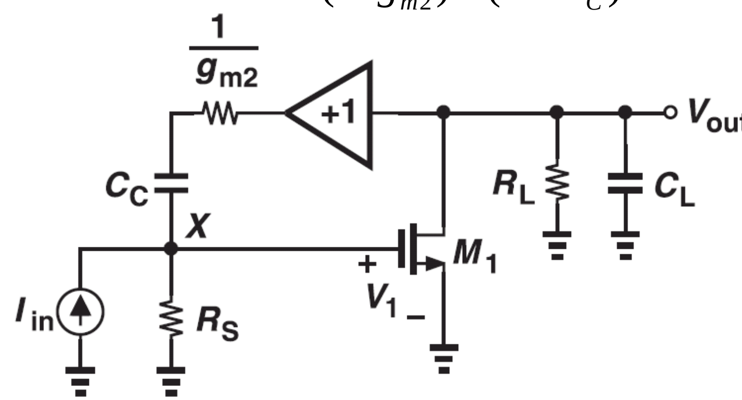


- The feedforward path formed by the comp. capacitor causes a right-half plane zero.
- If C_C could conduct current from the output node to node X but not vice versa, then the zero would move to a very high frequency.
  - Inserting a source follower in series with the capacitor.

$$-g_{m1}V_1 = V_{out}\left(R_L^{-1} + C_L s\right), \qquad V_1 = \frac{-V_{out}}{g_{m1}R_L}\left(1 + sR_L C_L\right)$$

$$\frac{V_{out} - V_1}{(1/g_{m2}) + (1/sC_C)} + I_{in} = \frac{V_1}{R_S}$$





Compensation Using Source Follower

- We have

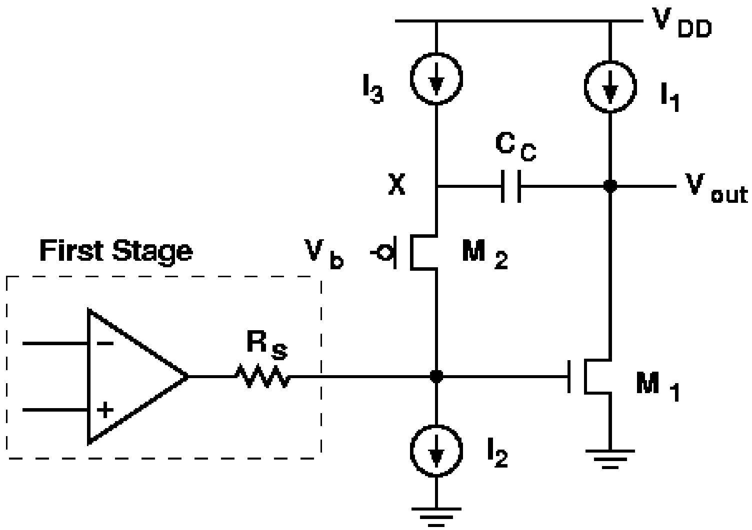
$$\frac{V_{out}}{I_{in}} = \frac{-g_{m1}R_L R_S\left(g_{m2}+sC_C\right)}{R_L C_C C_L\left(1+g_{m2}R_S\right)s^2+\left[\left(1+g_{m1}g_{m2}R_L R_S\right)C_C+g_{m2}R_L C_L\right]s+g_{m2}}$$

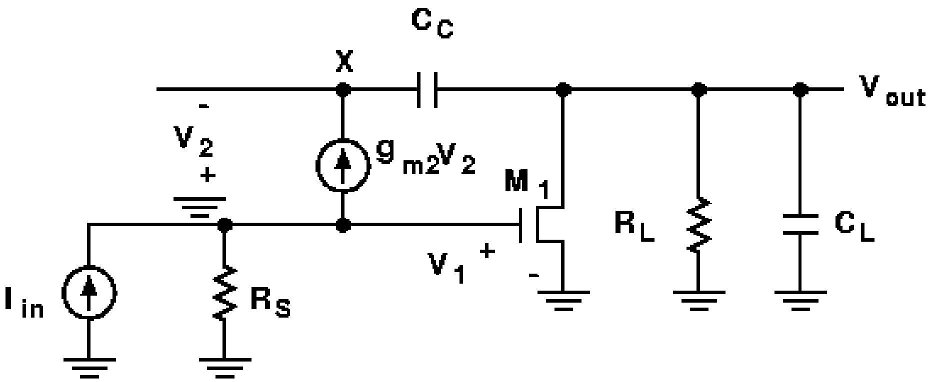
$$If \qquad 1+g_{m2}R_S >> 1 \qquad \left(1+g_{m1}g_{m2}R_L R_S\right)C_C >> g_{m2}R_L C_L$$

$$\omega_{p1} \approx \frac{g_{m2}}{g_{m1}g_{m2}R_L R_S C_C} \approx \frac{1}{g_{m1}R_L R_S C_C} \qquad \omega_{p2} \approx \frac{g_{m1}g_{m2}R_L R_S C_C}{R_L C_L C_C g_{m2}R_S} \approx \frac{g_{m1}}{C_L}$$

- Thus, the circuit contains a zero in the left half plane, which can be chosen to cancel one of the poles.
- The new values of ω_{p1}, ω_{p2} are similar to those obtained by simple Miller approximation.
- The source follower limits the lower end of the output voltage to $V_{GS2}+V_{I2}$.
- It is desirable to utilize the compensation capacitor to isolate the DC levels in the active feedback stage from that at the output.

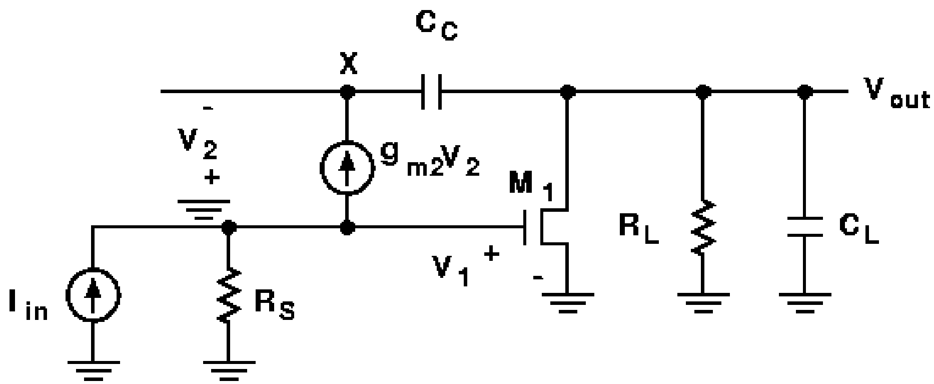
Compensation Using a CG Stage





- C_C and the common gate stage M_2 converts the output voltage swing to a current, returning the result to the gate of M_1.
- If V_1 changes by ΔV, V_{out} changes by $A_v \Delta V$
- The current flows through the capacitor is nearly equal to $A_v \Delta V C_C s$ because $1/g_{m2}$ can be relatively small.

Compensation Using a CG Stage



$$V_{out} + \frac{g_{m2}V_2}{sC_C} = -V_2,\ V_2 = -V_{out}\frac{sC_C}{sC_C + g_{m2}}$$

$$g_{m1}V_1 + V_{out}\left(\frac{1}{R_L} + sC_L\right) = g_{m2}V_2,\ I_{in} = \frac{V_1}{R_S} + g_{m2}V_2$$

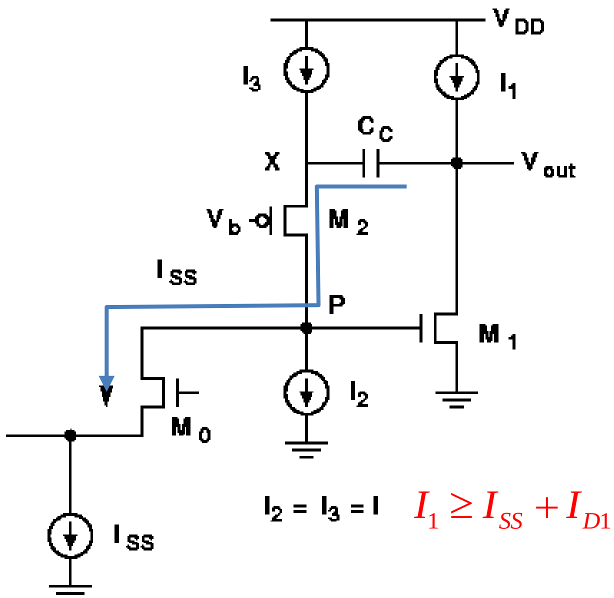
$$\frac{V_{out}}{I_{in}} = \frac{-g_{m1}R_SR_L\left(g_{m2} + sC_C\right)}{R_LC_CC_Ls^2 + \left[\left(1 + g_{m1}R_S\right)g_{m2}R_LC_C + C_C + g_{m2}R_LC_L\right]s + g_{m2}}$$

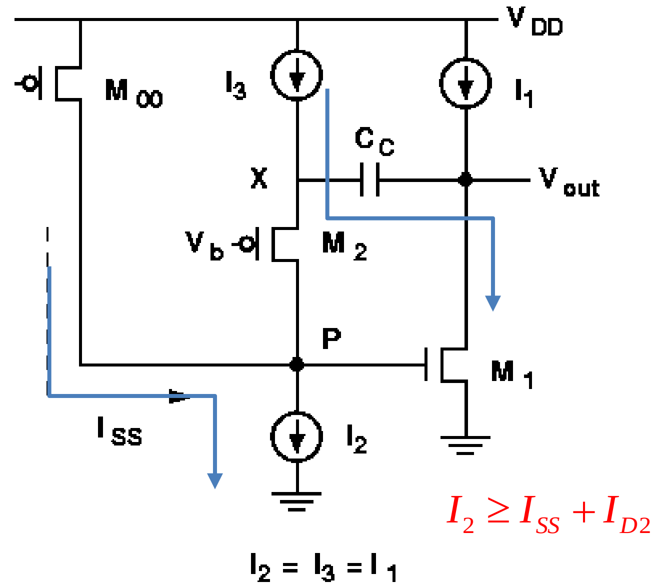
$$\omega_{p1} \approx \frac{1}{g_{m1}R_LR_SC_C}$$

$$\omega_{p2} \approx \frac{g_{m2}R_Sg_{m1}}{C_L}$$

- Contains a left-plane zero
- The second pole has considerably risen in magnitude – by a factor of $g_{m2}R_S$.
  - At very high frequencies, the feedback loop consisting of M_2 and R_S lowers the output resistance by the same factor.
  - If the capacitance at the gate of M1 is taken into account, pole splitting is less pronounced.
  - This technique can potentially provide a high bandwidth in 2-stage op-amp.

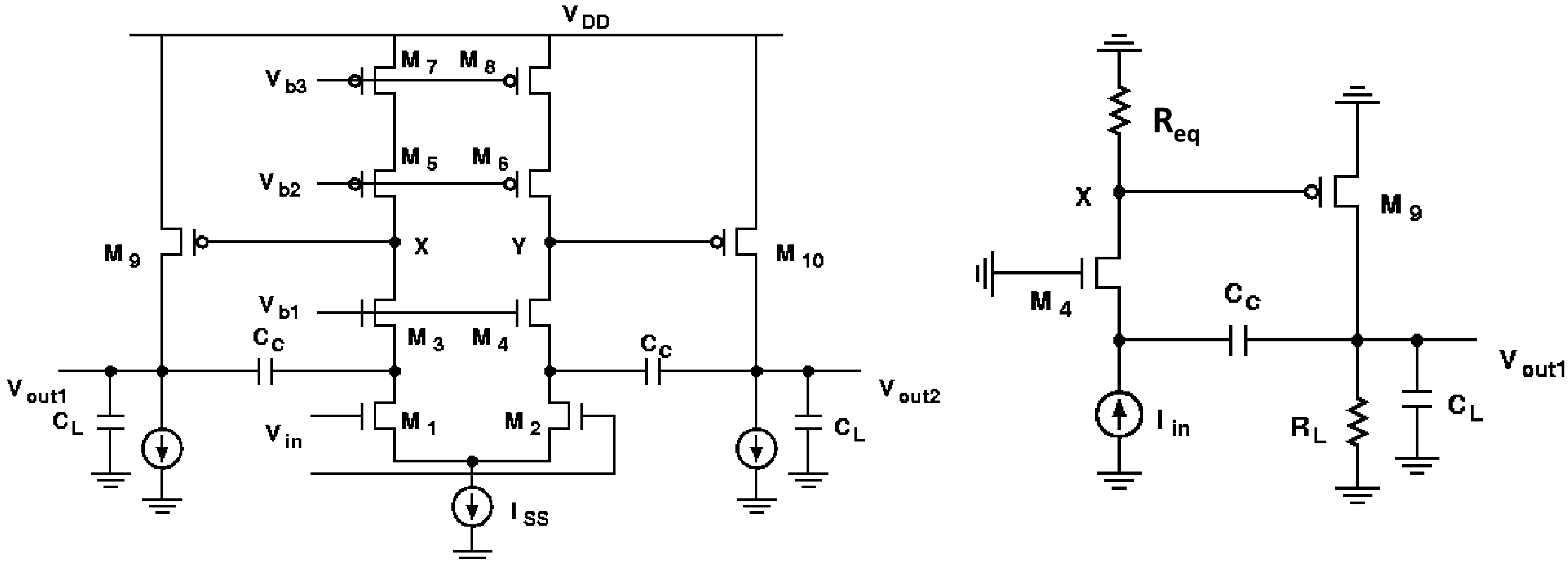
Slew Rate Analysis





- For positive slewing, M2 and hence I_1 must support I_{SS}, requiring that
  - If $I_1 < I_{SS} + I_{D1}$, then V_p drops, turning M_1 off.
  - If $I_1 < I_{SS}$, M_0 and its tail current source must enter the triode region, yielding a slew rate equal to I_1/C_C .
- For negative slewing, I_2 must support both I_{SS} and I_{D2}.
  - As I_{SS} flows into P → V_{GS1}↑ → I_{D1}↑ → V_X↓ → P is a virtual ground node.
  - I_3 and I_2 must be as large as I_{SS}, *raising the power dissipation.*

Alternative Comp. of 2-Stage Op-Amp



- The zero appears at $(g_{m4}R_{eq})(g_{m9}/C_C)$
- The dominant pole is located at approximately $\dfrac{1}{R_{eq}g_{m9}R_L C_C}$
- The first non-dominant pole is given by $\dfrac{g_{m4}g_{m9}R_{eq}}{C_L}$