

A close-up, high-angle photograph of a green printed circuit board (PCB) with intricate white and silver traces and components. The lighting is dramatic, highlighting the texture and depth of the board's surface.

CHAPTER 6

Frequency Response of Amplifiers

Outline

- 1. Miller Effect / Poles**
2. Common-Source (CS) Stage
3. Source Follower (SF)
4. Common-Gate (CG) Stage
5. Cascode Stage
6. Differential Pair (DP)

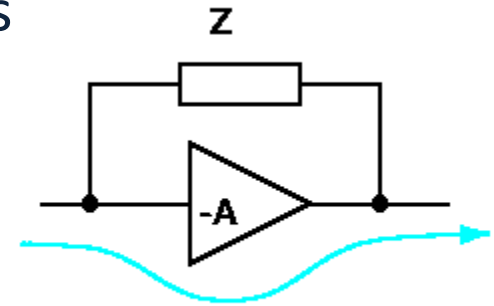
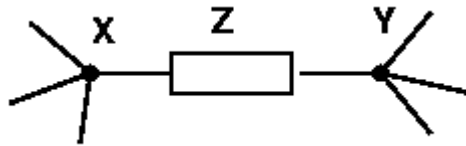
Why Frequency Response

- In most analog circuits, the speed trades with many other parameters such as gain, power dissipation, and noise.
- Transient signal varied with time need to consider the capacitive and inductive effect inherent in circuit.

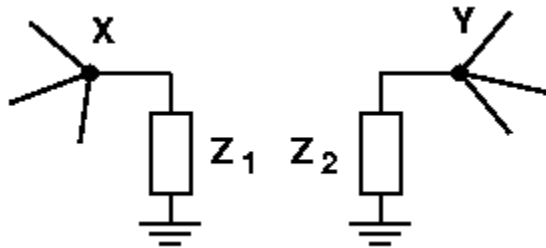
Miller Effect

- For the current flowing through Z from X to Y is

$$\frac{V_X - V_Y}{Z} = \frac{V_X}{Z_1}$$



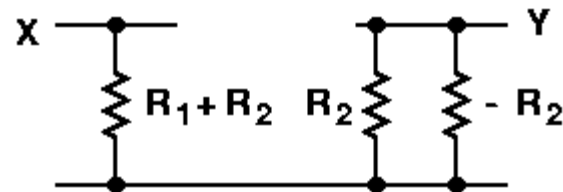
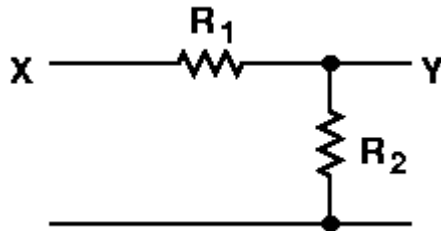
- That is



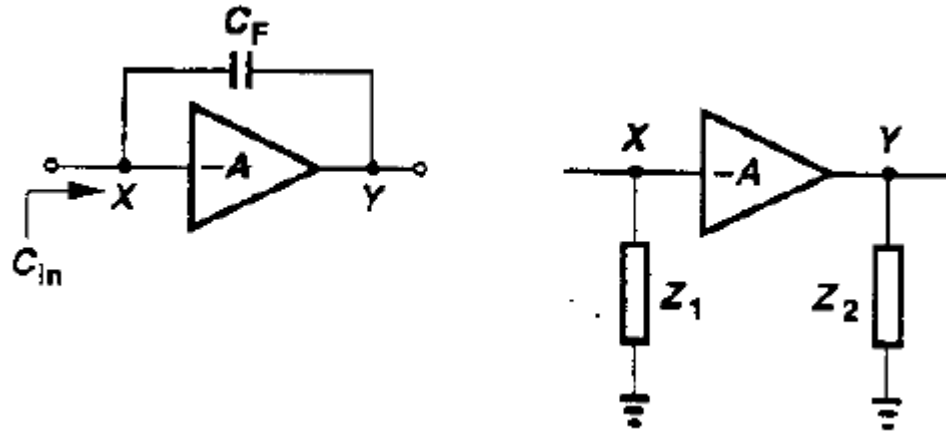
$$Z_1 = \frac{Z}{1 - \frac{V_Y}{V_X}}$$

$$Z_2 = \frac{Z}{1 - \frac{V_X}{V_Y}}$$

- If the impedance Z forms the only signal path between X and Y , then the conversion is often invalid.



Miller Effect of Feedback Capacitor

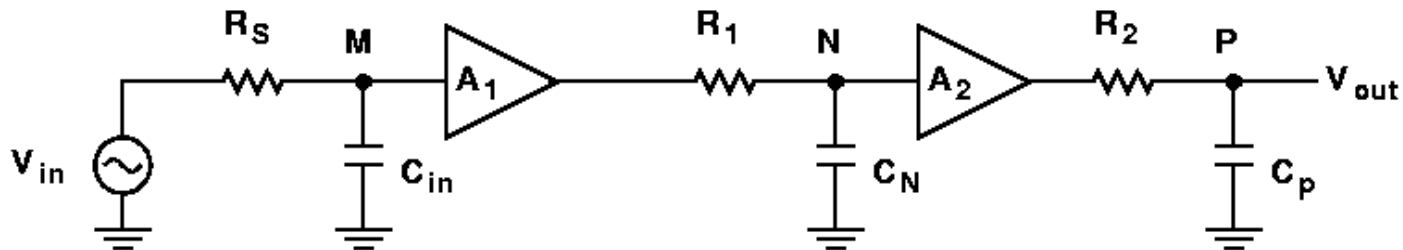


$$Z_1 = \frac{Z}{1 - A_v} = \frac{Z}{1 + A} = \frac{1}{sC_F(1 + A)}, \quad C_1 = C_F(1 + A)$$

$$Z_2 = \frac{Z}{1 - (1/A_v)} = \frac{Z}{1 + (1/A)} = \frac{1}{sC_F(1 + A^{-1})}, \quad C_2 = C_F(1 + A^{-1})$$

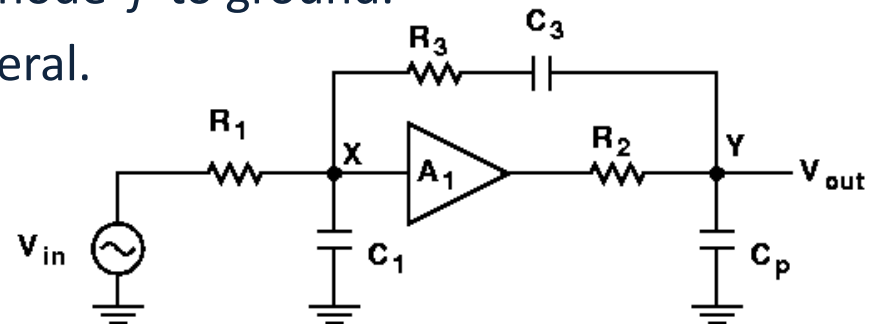
Association of Poles with Nodes

- Assume A_1 and A_2 are ideal amplifiers, R_1 and R_2 model the output resistance of each stage, C_{in} and C_N represent the input capacitance of each stage, and C_p denotes the load capacitance.



$$\frac{V_{out}}{V_{in}}(s) = \frac{A_1}{1 + R_S C_{in} s} \cdot \frac{A_2}{1 + R_1 C_N s} \cdot \frac{1}{1 + R_2 C_p s}$$

- Each pole with one node of the circuit, i.e., $\omega_j = \tau_j^{-1}$, where τ_j is the product of the capacitance and resistance seen at node j to ground.
- The above statement is not valid in general.
 - The location of the pole is difficult to calculate.

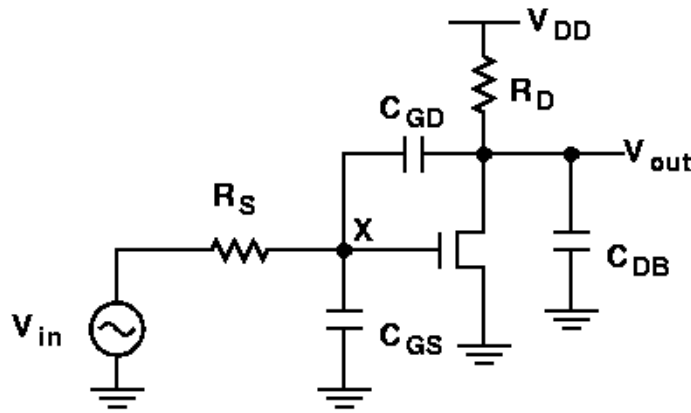


Outline

1. Miller Effect / Poles
- 2. Common-Source (CS) Stage**
3. Source Follower (SF)
4. Common-Gate (CG) Stage
5. Cascode Stage
6. Differential Pair (DP)

Common Source Stage

- Provides voltage gain and high input impedance.



$$\omega_{in} = \frac{1}{R_S [C_{GS} + (1 + g_m R_D) C_{GD}]}$$

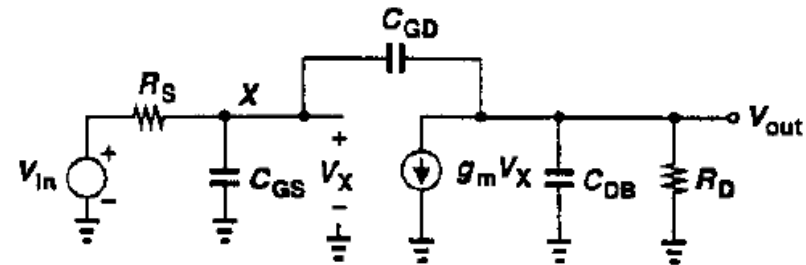
$$\omega_{out} = \frac{1}{R_D (C_{DB} + C_{GD})}$$

- To obtain the exact transfer function

$$\frac{V_X - V_{in}}{R_S} + V_X C_{GS} s + (V_X - V_{out}) C_{GD} s = 0,$$

$$(V_{out} - V_X) C_{GD} s + g_m V_X + V_{out} \left(\frac{1}{R_D} + C_{DB} s \right) = 0,$$

$$\frac{V_{out}}{V_{in}}(s) = \frac{(C_{GD} s - g_m) R_D}{R_S R_D \xi s^2 + [R_S (1 + g_m R_D) C_{GD} + R_S C_{GS} + R_D (C_{GD} + C_{DB})] s + 1}$$



$$\xi = C_{GS} C_{GD} + C_{GS} C_{DB} + C_{GD} C_{DB}$$

Dominant Pole Approximation

- Two independent capacitors (initial conditions) in the circuit yield a second order differential equation for the time response.

$$\begin{aligned} \frac{V_{out}}{V_{in}}(s) &= \frac{(C_{GD}s - g_m)R_D}{R_S R_D \xi s^2 + [R_S(1 + g_m R_D)C_{GD} + R_S C_{GS} + R_D(C_{GD} + C_{DB})]s + 1} \\ &= -g_m R_D \frac{(1 - C_{GD}s / g_m)}{R_S R_D \xi s^2 + [R_S(1 + g_m R_D)C_{GD} + R_S C_{GS} + R_D(C_{GD} + C_{DB})]s + 1} \\ &= -g_m R_D \frac{(1 - s / z_1)}{\left(\frac{s}{\omega_{p1}} + 1\right)\left(\frac{s}{\omega_{p2}} + 1\right)} = -g_m R_D \frac{(1 - s / z_1)}{\frac{s^2}{\omega_{p1}\omega_{p2}} + \left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right)s + 1} \end{aligned}$$

- If $|\omega_{p1}| \ll |\omega_{p2}|$

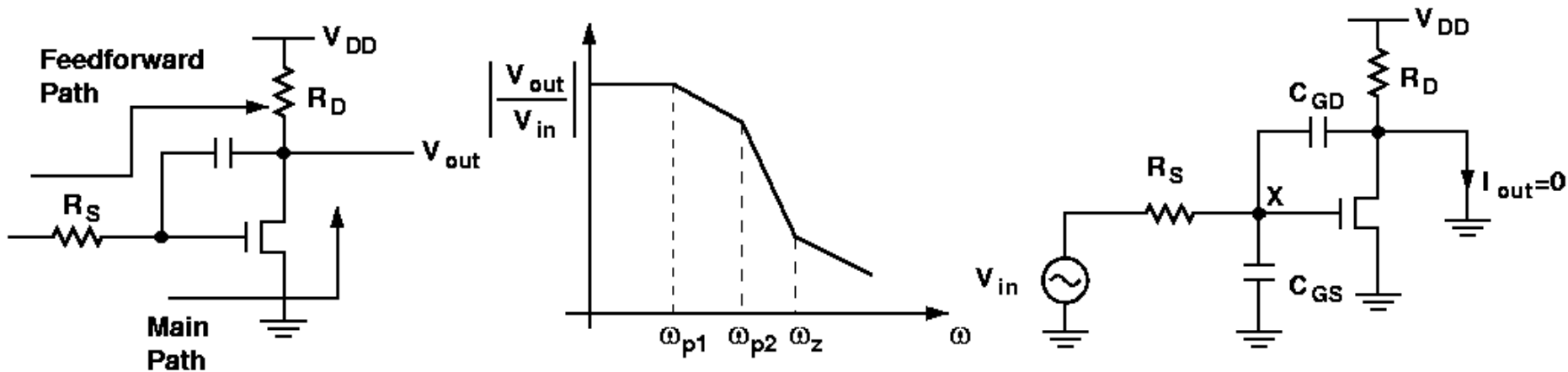
$$z_1 = \frac{g_m}{C_{GD}}, \quad \omega_{p1} = \frac{1}{R_S(1 + g_m R_D)C_{GD} + R_S C_{GS} + R_D(C_{GD} + C_{DB})}$$

$$\omega_{p2} = \frac{1}{\omega_{p1}} \cdot \frac{1}{R_S R_D (C_{GS} C_{GD} + C_{GS} C_{DB} + C_{GD} C_{DB})} \cong \frac{R_S C_{GS}}{R_S R_D (C_{GS} C_{GD} + C_{GS} C_{DB})} = \frac{1}{R_D (C_{GD} + C_{DB})}$$

$$\text{If } C_{GS} \gg (1 + g_m R_D)C_{GD} + R_D(C_{GD} + C_{DB}) / R_S$$

Right-Half Plane Zero

- C_{GD} provides a feed forward path that conducts the input signal to the output at very high frequencies.
- A zero in the right half plane causes stability issue in feedback Amp.



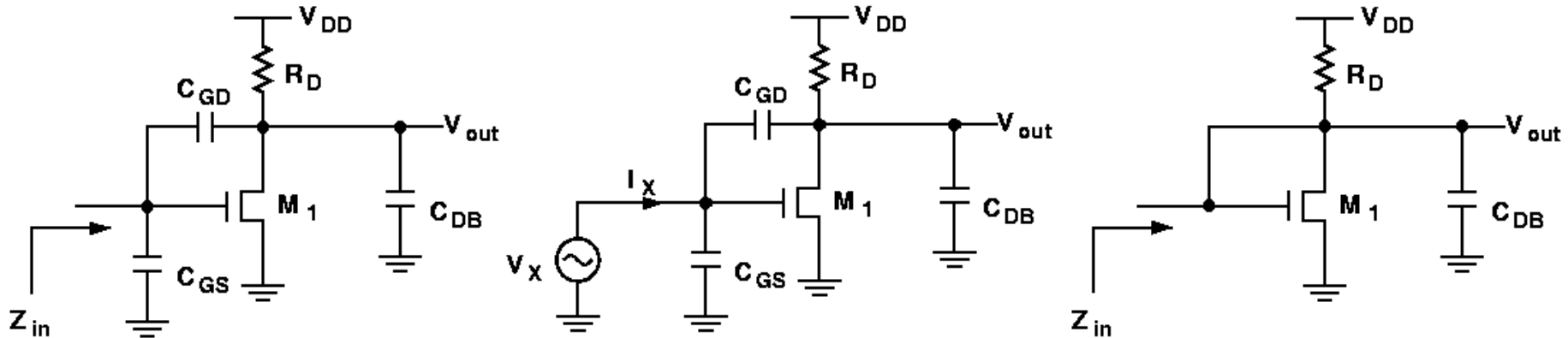
- The zero, s_z , can be computed by noting that the $V_{out}(s)/V_{in}(s)$ must drop to zero for $s = s_z$.
- The currents through C_{GD} and M_1 are equal and opposite.

$$V_x C_{GD} s_z = g_m V_x$$

$$s_z = \frac{g_m}{C_{GD}}$$

Input Impedance of a CS Stage

- As a first-order approximation $Z_{in} = \{ [C_{GS} + (1 + g_m R_D)] C_{GD} \}^{-1}$



- At high frequencies, take the effect of the output node into account

$$\text{Ignore } C_{GS}, (I_X - g_m V_X) \frac{R_D}{1 + R_D C_{DB} s} + \frac{I_X}{C_{GD} s} = V_X \Rightarrow \frac{V_X}{I_X} = \frac{1 + R_D (C_{GD} + C_{DB}) s}{C_{GD} s (1 + g_m R_D + R_D C_{DB} s)}$$

- At low frequencies

$$Z_{in} \approx \frac{1}{s C_{GS}} \parallel \frac{1}{s C_{GD} (1 + g_m R_D)}$$

- At high frequencies

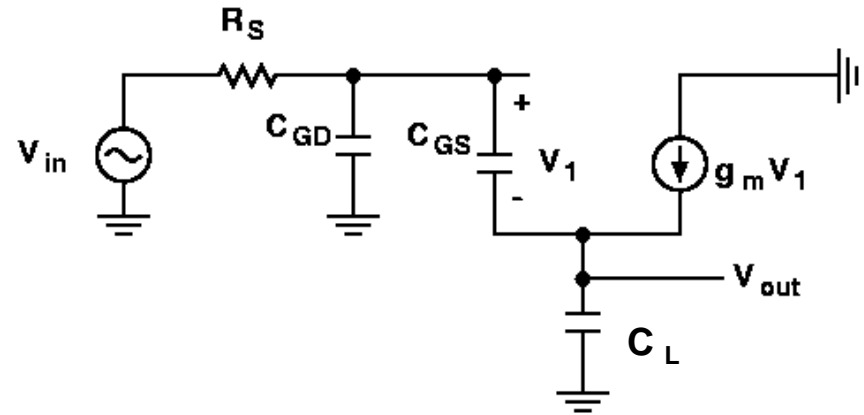
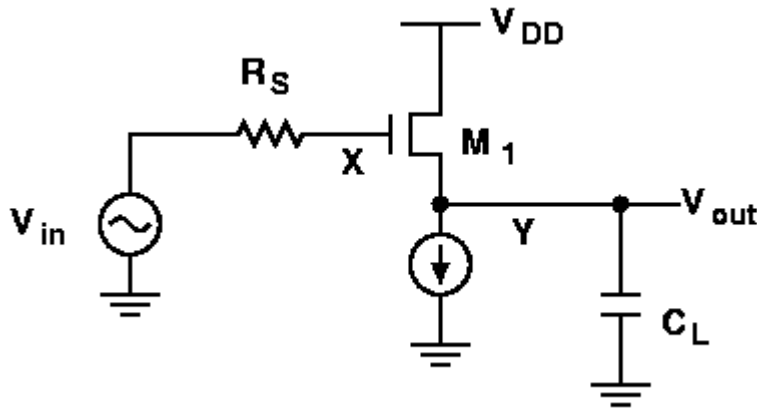
$$Z(C_{GD}) = 0, \quad Z_{in} \approx \frac{1}{s C_{GS}} \parallel \frac{1}{g_m} \parallel R_D$$

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Source Follower

- Source followers are occasionally used as level shifters or buffers.



$$V_1 C_{GS} s + g_m V_1 = V_{out} s C_L \quad V_1 = \frac{s C_L}{g_m + s C_{GS}} V_{out} \quad V_{in} = R_S [V_1 C_{GS} s + (V_1 + V_{out}) C_{GD} s] + V_1 + V_{out}$$

$$\begin{aligned} \frac{V_{out}}{V_{in}}(s) &= \frac{g_m + C_{GS} s}{R_S (C_{GS} C_L + C_{GS} C_{GD} + C_{GD} C_L) s^2 + (g_m R_S C_{GD} + C_L + C_{GS}) s + g_m} \\ &= \frac{(1 + s C_{GS} / g_m)}{s^2 R_S (C_{GS} C_L + C_{GS} C_{GD} + C_{GD} C_L) / g_m + s (g_m R_S C_{GD} + C_L + C_{GS}) / g_m + 1} \end{aligned}$$

- The signal conducted by C_{GS} at high frequencies adds with the same polarity to the signal produced by the intrinsic transistor. (*left half plane zero*)

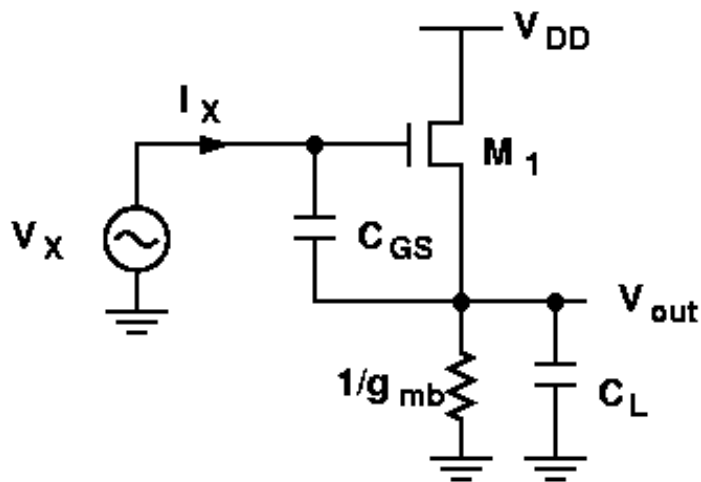
Input Impedance of Source Follower

$$\frac{V_{out}}{V_{in}}(s) = \frac{(1 + sC_{GS} / g_m)}{s^2 R_S (C_{GS} C_L + C_{GS} C_{GD} + C_{GD} C_L) / g_m + s(g_m R_S C_{GD} + C_L + C_{GS}) / g_m + 1}$$

- If the two poles are far apart

$$\omega_{p1} \approx \frac{g_m}{g_m R_S C_{GD} + C_L + C_{GS}} = \frac{1}{R_S C_{GD} + \frac{C_L + C_{GS}}{g_m}} \quad \text{If } R_S = 0, \text{ then } \omega_{p1} = \frac{g_m}{C_L + C_{GS}}$$

- Input impedance (ignore C_{GD})



$$V_X = \frac{I_X}{C_{GS} s} + \left(I_X + \frac{g_m I_X}{C_{GS} s} \right) \left(\frac{1}{g_{mb}} \parallel \frac{1}{s C_L} \right)$$

$$Z_{in} = \frac{1}{C_{GS} s} + \left(1 + \frac{g_m}{C_{GS} s} \right) \frac{1}{g_{mb} + C_L s}$$

At low frequencies, $g_{mb} \gg |C_L s|$

$$Z_{in} = \frac{1}{C_{GS} s} \left(1 + \frac{g_m}{g_{mb}} \right) + \frac{1}{g_{mb}}$$

$$C_{in} = C_{GS} \frac{g_{mb}}{g_{mb} + g_m}$$

Input Impedance of Source Follower

- At relatively **low frequencies**, the equivalent input $C_{in} = \frac{C_{GS}g_{mb}}{g_m + g_{mb}}$
 - By Miller effect

$$A_v = \frac{g_m}{g_m + g_{mb}} \quad C_{eq} = C_{GS} \left[1 - \frac{g_m}{g_m + g_{mb}} \right] = \frac{C_{GS}g_{mb}}{g_m + g_{mb}}$$

- The overall input capacitance is equal to C_{GD} plus a fraction of C_{GS} .
- At **high frequencies**,

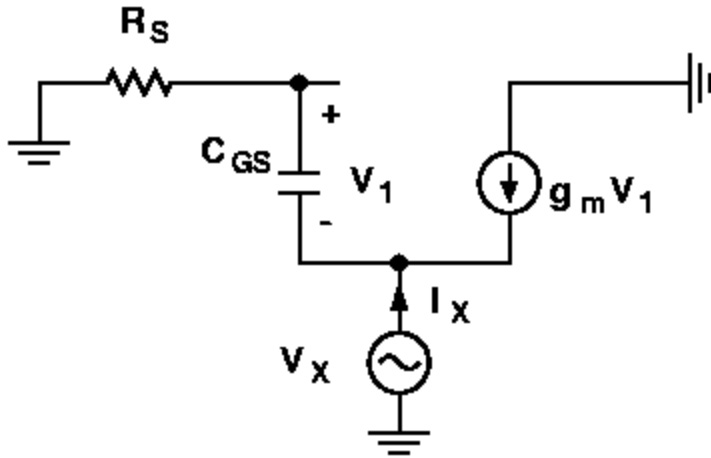
$$Z_{in} = \frac{1}{C_{GS}s} + \left(1 + \frac{g_m}{C_{GS}s} \right) \frac{1}{g_{mb} + C_Ls}$$

$$g_{mb} \ll |sC_L| \quad \text{and} \quad Z_{in} \approx \frac{1}{sC_{GS}} + \frac{1}{sC_L} + \frac{g_m}{s^2C_{GS}C_L} = \frac{1}{j\omega C_{GS}} + \frac{1}{j\omega C_L} - \frac{g_m}{\omega^2 C_{GS}C_L}$$

- The input impedance consists of the series combination of capacitors C_{GS} and C_L and a negative resistance.
- The negative resistance property can be utilized in oscillators.

Output Impedance of Source Follower

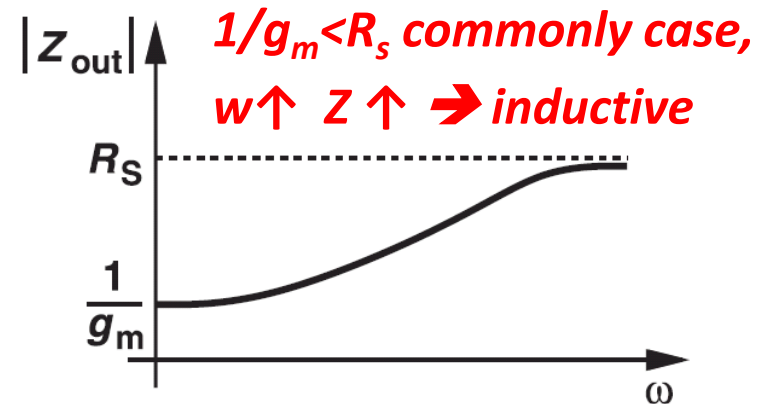
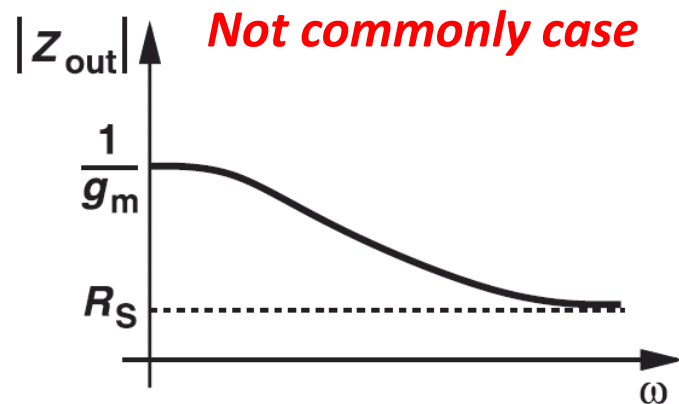
- Neglecting g_{mb} , C_{SB} & C_{GD}



$$V_1 C_{GS} s + g_m V_1 = -I_X, \quad V_1 C_{GS} s R_S + V_1 = -V_X$$

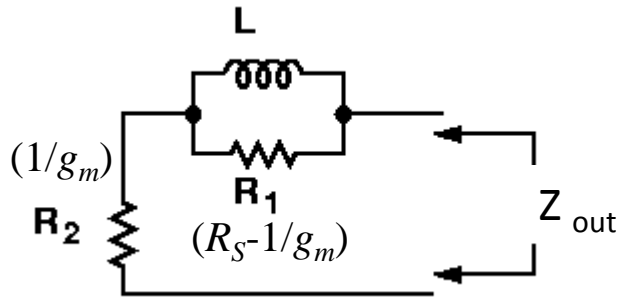
$$Z_{out} = \frac{V_X}{I_X} = \frac{s R_S C_{GS} + 1}{g_m + s C_{GS}}$$

- At low frequencies, $Z_{out} \approx 1/g_m$
- At high frequencies, $Z_{out} \approx R_S$ with $Z(C_{GS}) = 0$
- If $1/g_m < R_S$, the output impedance contains an inductive component.



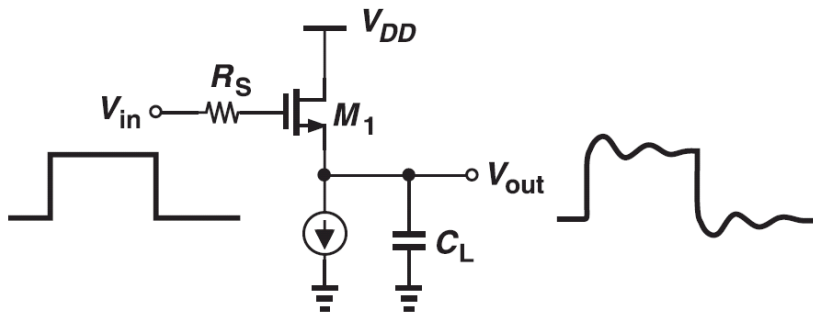
Output Impedance of SF

- Equivalent network of output impedance



- At low frequencies, $Z_{out} = 1/g_m$ when $\omega = 0$
- At high frequencies, $Z_{out} = R_S$ when $\omega = \infty$
- Therefore, $R_2 = 1/g_m$ $R_1 = R_S - 1/g_m$

$$Z_{out} - \frac{1}{g_m} = \frac{sC_{GS}(R_S - 1/g_m)}{g_m + sC_{GS}}, \quad \frac{1}{Z_{out} - \frac{1}{g_m}} = \frac{1}{R_S - 1/g_m} + \frac{1}{\frac{sC_{GS}}{g_m}(R_S - 1/g_m)}, \quad L = \frac{C_{GS}}{g_m}(R_S - 1/g_m)$$



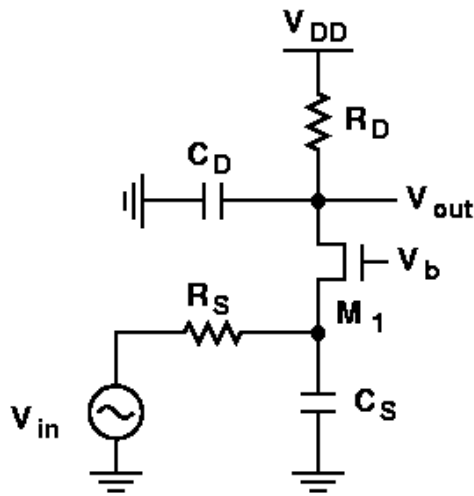
- If a source follower is driven by a large resistance, then it exhibits substantial inductive behavior.
- This effect manifests itself as ringing in the step response. If it drive a large C_L .

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Common-Gate Stage

- Neglect channel length modulation effect



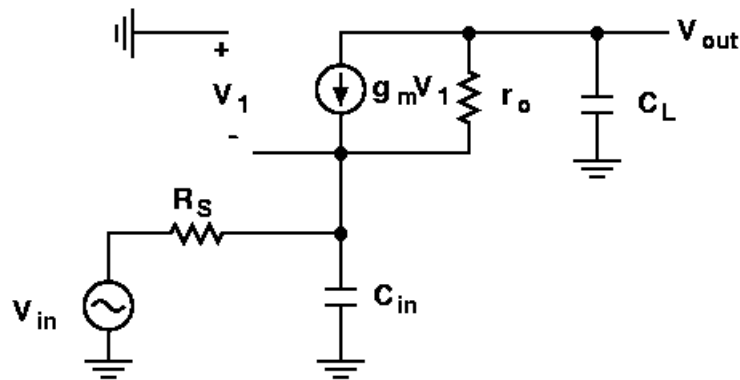
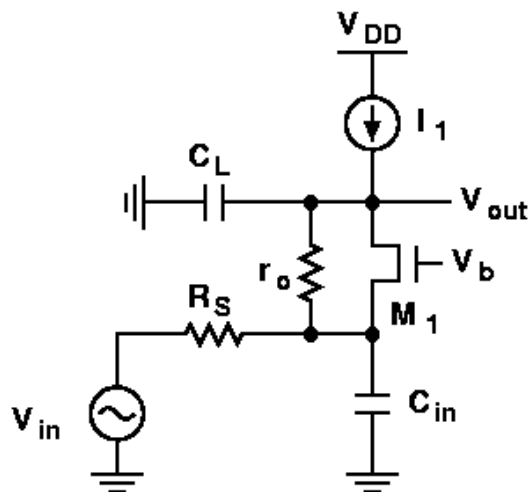
$$\frac{V_{out}(s)}{V_{in}} = \frac{(g_m + g_{mb})R_D}{1 + (g_m + g_{mb})R_S} \frac{1}{\left(1 + \frac{C_S}{g_m + g_{mb} + R_S^{-1}}s\right)(1 + R_D C_D s)}$$

- It exhibits no Miller multiplication of capacitances, potentially achieving **wide band**.
- Low Input impedance may load the preceding stage.
- The DC level of the input signal must be quite low.

$$\text{if } \lambda \neq 0, \quad Z_{in} \approx \frac{Z_L}{(g_m + g_{mb})r_O} + \frac{1}{g_m + g_{mb}}, \quad Z_L = R_D \parallel [1/sC_D]$$

- Since Z_{in} now depends on Z_L , it is difficult to associate a pole with the input node.

Input Impedance of Common Gate



$$(-V_{out} C_L s + V_1 C_{in} s) R_S + V_{in} = -V_1$$

$$V_1 = -\frac{-V_{out} C_L s R_S + V_{in}}{1 + C_{in} R_S s}, \quad r_o(-V_{out} C_L s - g_m V_1) - V_1 = V_{out}$$

$$\frac{V_{out}}{V_{in}}(s) = \frac{1 + g_m r_o}{r_o C_L C_{in} R_S s^2 + [r_o C_L + C_{in} R_S + (1 + g_m r_o) C_L R_S] s + 1}$$

$$\frac{V_{out}}{V_{in}} = 1 + g_m r_o \quad (@ \text{ Low Freq : } s = j\omega = 0)$$

- The body effect can be included by simply replacing g_m with $g_m + g_{mb}$.

$$Z_{in} \approx \frac{Z_L}{(g_m + g_{mb}) r_o} + \frac{1}{g_m + g_{mb}}$$

$$Z_L = \frac{1}{s C_L}, \quad Z_{in} = \frac{1}{g_m + g_{mb}} + \frac{1}{s C_L} \cdot \frac{1}{(g_m + g_{mb}) r_o}$$

$$\text{As } C_L \text{ or } s \text{ increases, } Z_{in} = \frac{1}{g_m + g_{mb}}$$

$$\text{input pole } \omega_{p,in} = \frac{1}{\left(R_S \parallel \frac{1}{g_m + g_{mb}} \right) C_{in}}, \text{ independent of } C_L$$

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Cascode Stage

- Common source + common gate gain stage.
- Increase the voltage gain of amplifiers and the output impedance of current source.

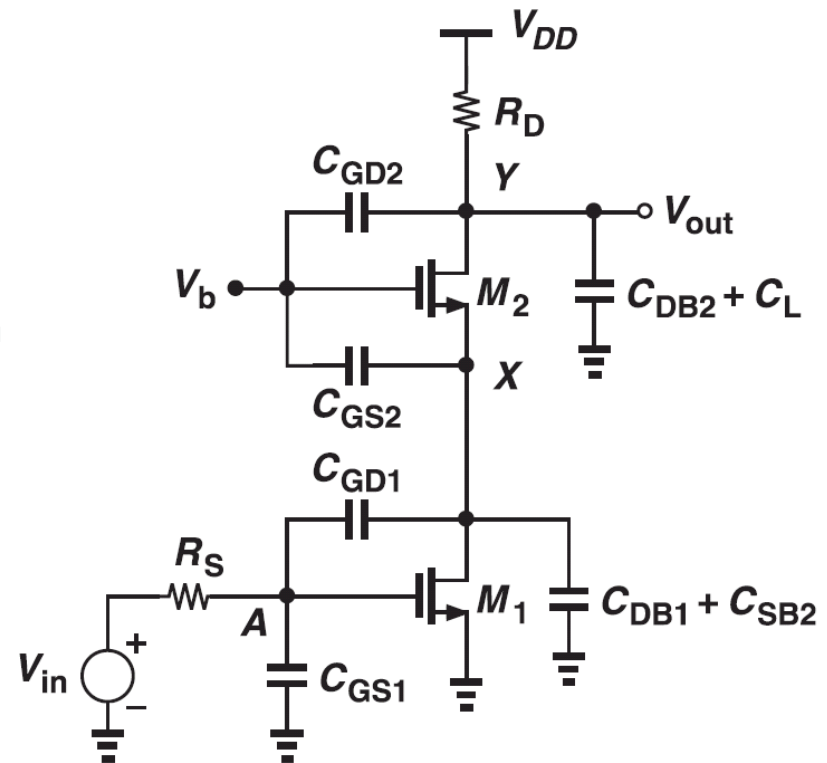
$$Z_{out} = (1 + g_{m2}r_{O2})Z_X + r_{O2}$$

- Providing shielding – suppressing the Miller effect.
- The low frequency gain from M_1 gate to drain

$$A_v = -\frac{g_{m1}}{g_{m2} + g_{mb2}} \approx -1$$

- The input pole is at

$$\omega_{p,A} = \frac{1}{R_S \left[C_{GS1} + \left(1 + \frac{g_{m1}}{g_{m2} + g_{mb2}} \right) C_{GD1} \right]}$$



Cascode Stage

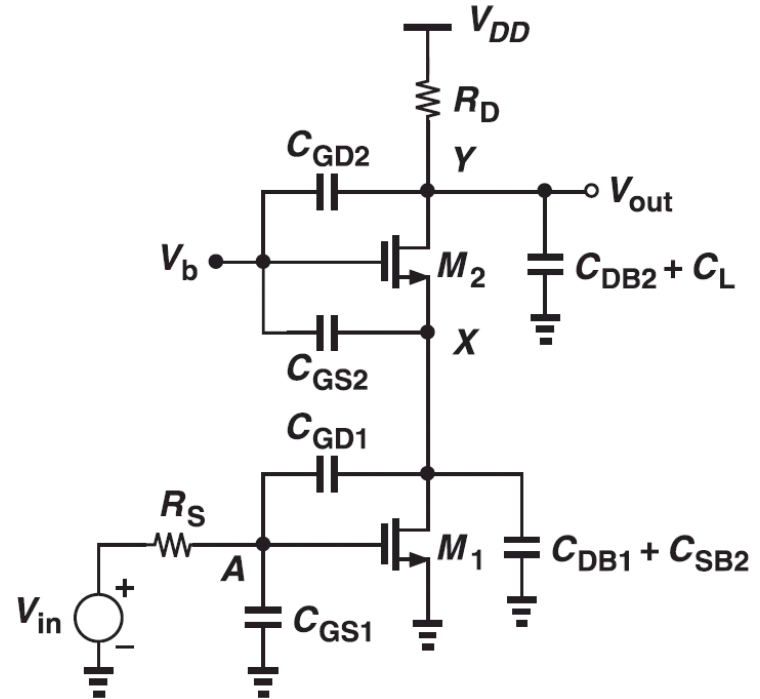
- The total capacitance at node X is (typically chosen farther from the origin than the other two)

$$C_X = 2C_{GD1} + C_{DB1} + C_{SB2} + C_{GS2}$$

$$\omega_{p,X} = \frac{1}{R_X C_X} = \frac{g_{m2} + g_{mb2}}{2C_{GD1} + C_{DB1} + C_{SB2} + C_{GS2}}$$

- The output pole at node Y is

$$\omega_{p,Y} = \frac{1}{R_D (C_{DB2} + C_L + C_{GD2})}$$



- Output impedance of a cascode current source (neglect C_Y , C_{GD1})

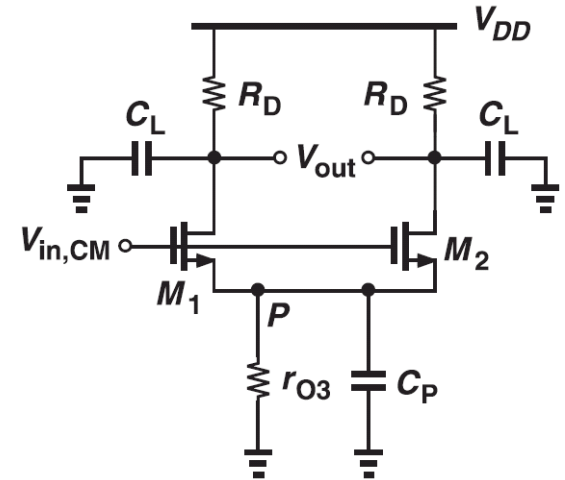
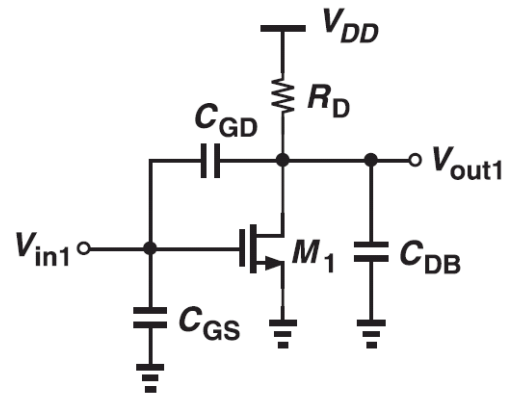
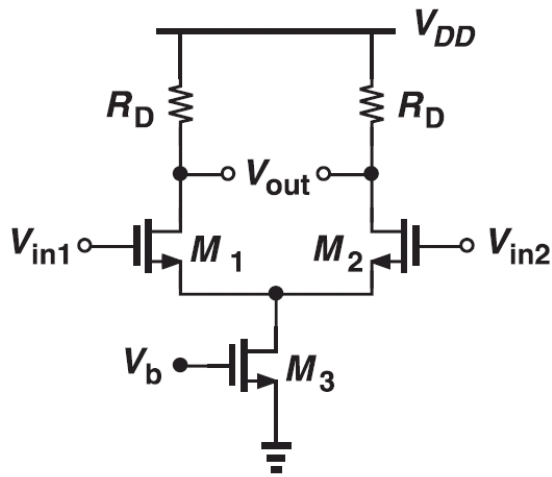
$$Z_{out} = (1 + g_{m2} r_{O2}) Z_X + r_{O2}, Z_X = r_{O1} \parallel (C_X s)^{-1}$$

$$pole = (r_{O1} C_X)^{-1}$$

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Differential Pair

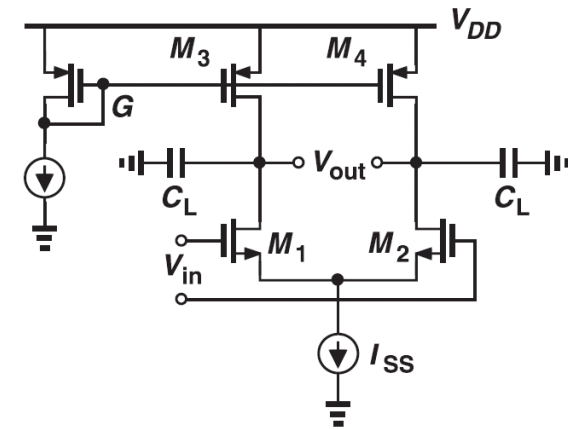
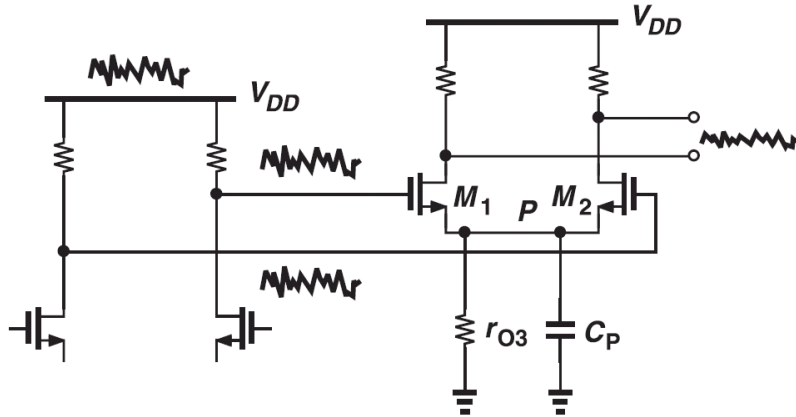


- For differential signals, the response is identical to that of a CS stage.
- For CM gain, the total capacitance at node P determines the HF gain.

$$A_{v,CM} = -\Delta g_m \left[R_D \parallel \left(\frac{1}{C_{LS}} \right) \right] / \left\{ (g_{m1} + g_{m2}) \left[r_{O3} \parallel \left(\frac{1}{C_{PS}} \right) \right] + 1 \right\}$$

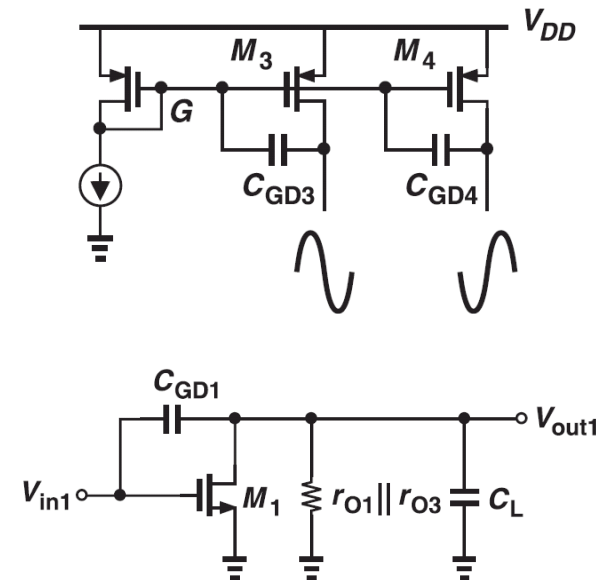
- If the output pole \gg the pole at the tail node of current source, the CM rejection of the circuit degrades considerably at high frequencies (HF).
- If the V_{DD} contains HF noise and the circuit exhibits mismatches, the resulting CM disturbance at node P leads to a differential noise component at the output.

Effect of High-Frequency Supply Noise

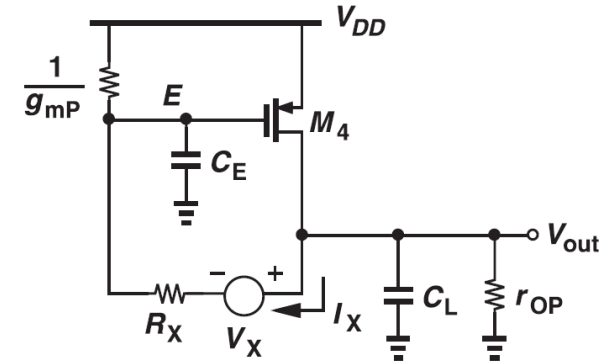
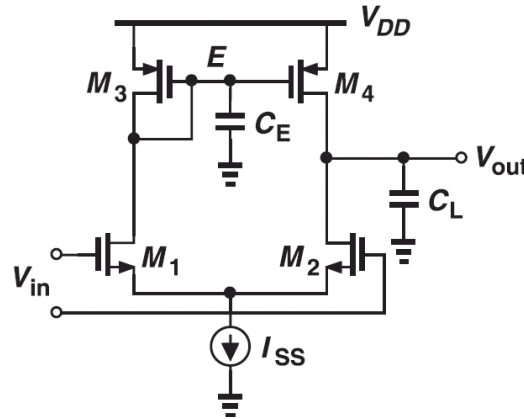
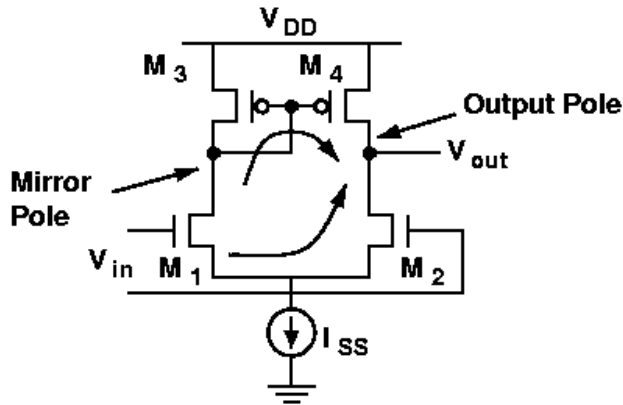


- Trade-off between voltage headroom and CMRR.
- To minimize voltage headroom, $M_3 \uparrow$, $C_p \uparrow$, high frequency CMRR \downarrow .
- For differential output signals, C_{GD3} and C_{GD4} conduct equal and opposite currents to G, making this node an AC ground.
- The dominant pole is at

$$\omega_p = \frac{1}{[r_{O1} \parallel r_{O3}]C_L}$$



DP with Active Current Mirror



- This topology contains two signal paths with different transfer functions.
- Pole at node E (*Mirror pole*)

$$\omega_{pE} = \frac{g_{m3}}{C_E}, \quad C_E \approx C_{GS3} + C_{GS4} + C_{DB3} + C_{DB1} + 2C_{GD1} + g_{m4}(r_{O4} \parallel r_{O2})C_{GD4}$$

$$V_X = g_{mN}r_{ON}V_{in}, \quad R_X = 2r_{ON}, \quad V_E = (V_{out} - V_X) \frac{(C_E s + g_{mP})^{-1}}{(C_E s + g_{mP})^{-1} + R_X}, \quad -g_{m4}V_E - I_X = V_{out}(sC_L + r_{OP}^{-1})$$

$$\frac{V_{out}}{V_{in}} = \frac{g_{mN}r_{ON}r_{OP}(2g_{mP} + sC_E)}{2r_{OP}r_{ON}C_EC_Ls^2 + [(2r_{ON} + r_{OP})C_E + r_{OP}(1 + 2g_{mP}r_{ON})C_L]s + 2g_{mP}(r_{ON} + r_{OP})}$$

$$\omega_{p1} \approx \frac{2g_{mP}(r_{ON} + r_{OP})}{(2r_{ON} + r_{OP})C_E + r_{OP}(1 + 2g_{mP}r_{ON})C_L} \approx \frac{1}{(r_{ON} \parallel r_{OP})C_L} \quad \text{if } 2g_{mP}r_{ON} \gg 1 \quad \omega_{p2} \approx \frac{g_{mP}}{C_E}$$

DP with Active Current Mirror

- To find zero

- Consider that the circuit consists of a “slow path” (M_1 , M_3 , and M_4) in parallel with a “fast path” (M_1 , and M_2)

- Slow path

$$A_{v1} = \frac{A_0}{\left(1 + \frac{s}{p_1}\right)\left(1 + \frac{s}{p_2}\right)}$$

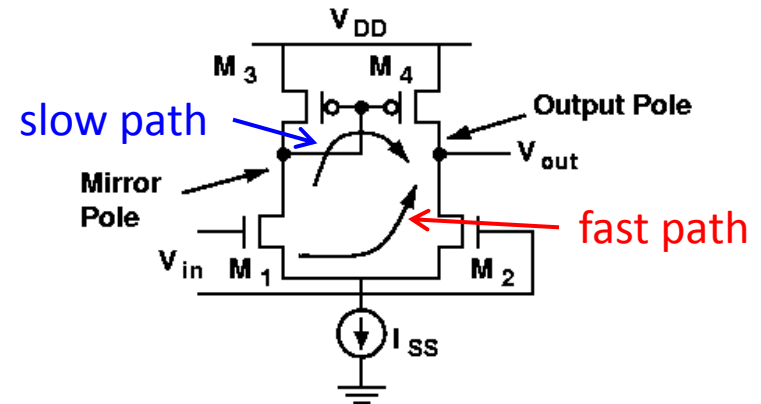
- Fast path

$$A_{v2} = \frac{A_0}{\left(1 + \frac{s}{p_1}\right)}$$

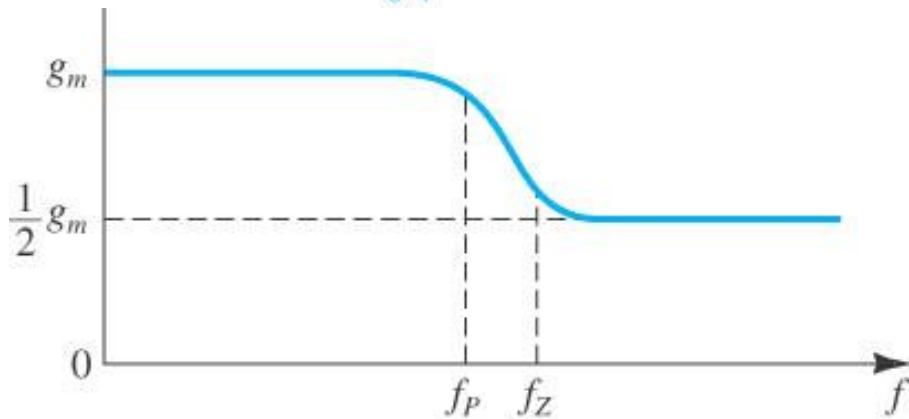
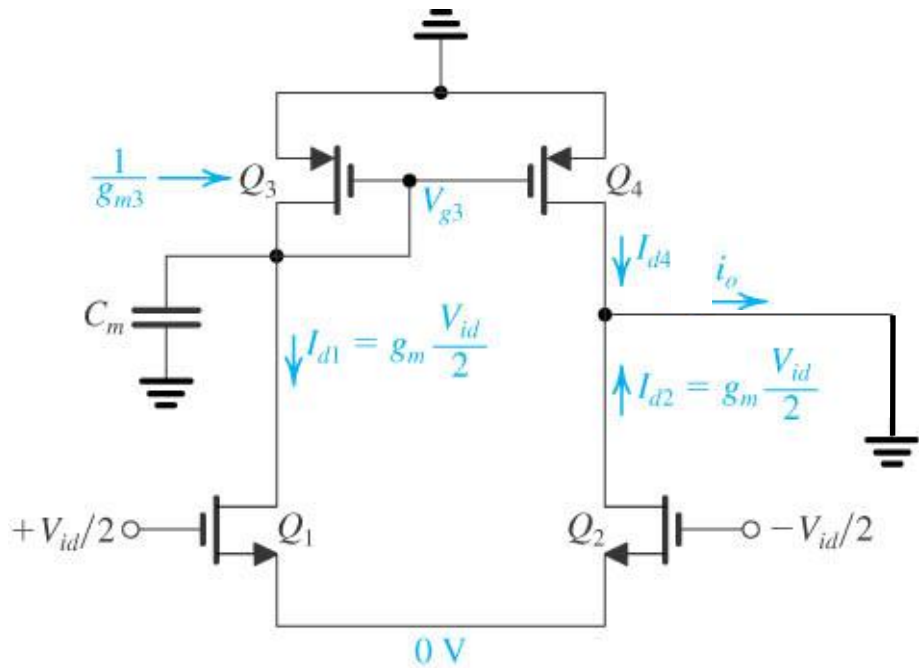
- We have

$$\frac{V_{out}}{V_{in}} = A_{v1} + A_{v2} = \frac{A_0}{1 + \frac{s}{p_1}} \left(1 + \frac{1}{1 + \frac{s}{p_2}}\right) = \frac{A_0 \left(2 + \frac{s}{p_2}\right)}{\left(1 + \frac{s}{p_1}\right)\left(1 + \frac{s}{p_2}\right)} \Rightarrow z = 2p_2$$

- Fully differential pair has no mirror pole, which is better than differential-to-single ended amp.



G_m Frequency Response



At low frequency: C_m is OPEN

$$i_{d4} = i_{d1} = g_m \frac{V_{id}}{2}, i_{d2} = g_m \frac{V_{id}}{2}$$

$$i_o = i_{d4} + i_{d2} = g_m V_{id}$$

$$G_m = i_o / V_{id} = g_m$$

At high frequency: C_m is SHORT

$$V_{g3} = 0, i_{d4} = 0, i_o = i_2 = g_m \frac{V_{id}}{2}$$

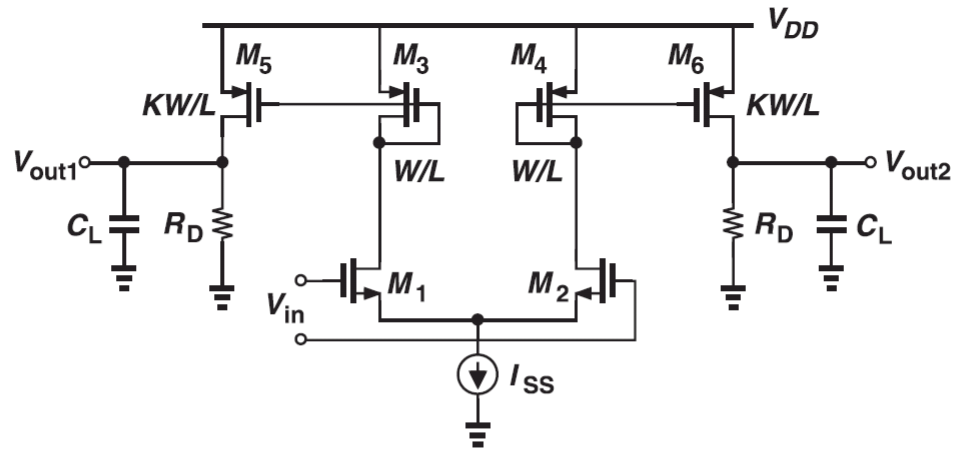
$$G_m = i_o / V_{id} = g_m / 2$$

G_m transfer function

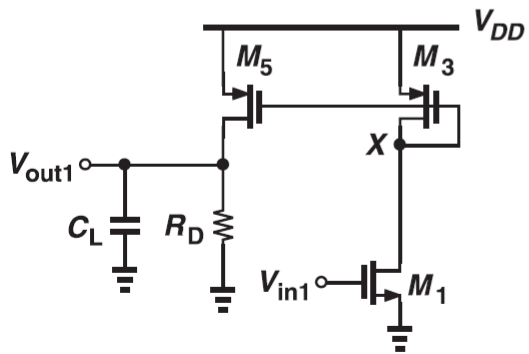
$$G_m \equiv \frac{i_o}{V_{id}} = g_m \left(\frac{1 + sC_m / 2g_{m3}}{1 + sC_m / g_{m3}} \right)$$

$$f_P = \frac{g_{m3}}{2\pi C_m}, f_Z = \frac{2g_{m3}}{2\pi C_m}$$

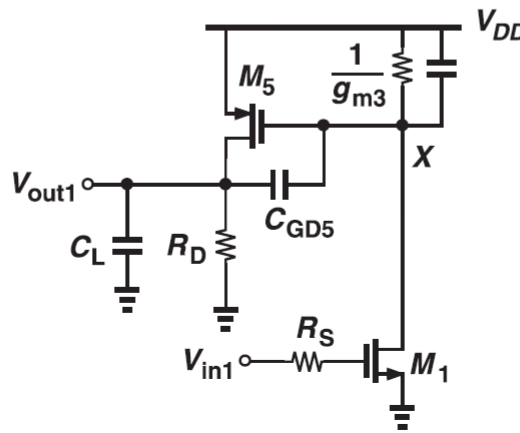
Differential Amplifier



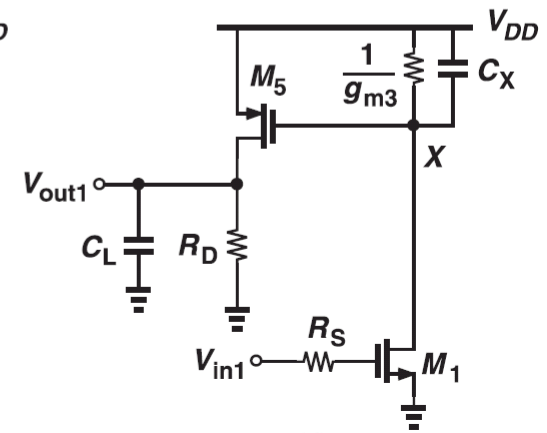
(a)



(b)



(c)



(d)

$$\frac{V_{out1}}{V_X}(s) = -g_{m5}R_D \frac{1}{1 + sR_D C_L}$$