Chapter 1 – Electric Circuit Variables

Exercises

Exercise 1.2-1 Find the charge that has entered an element by time t when $i = 8t^2 - 4t$ A, $t \geq 0$. Assume $q(t) = 0$ for $t < 0$.

Answer: $q(t) = \frac{8}{3}t^3 - 2t^2$ C

Solution:

$$i(t) = 8t^2 - 4t \text{ A}$$

$$q(t) = \int_0^t i\,d\tau + q(0) = \int_0^t (8\tau^2 - 4\tau)\,d\tau + 0 = \frac{8}{3}\tau^3 - 2\tau^2\Big|_0^t = \underline{\frac{8}{3}t^3 - 2t^2 \quad \text{C}}$$

Exercise 1.2-2 The total charge that has entered a circuit element is $q(t) = 4 \sin 3t$ C when $t \geq 0$ and $q(t) = 0$ when $t < 0$. Determine the current in this circuit element for $t > 0$.

Answer: $i(t) = \frac{d}{dt} 4\sin 3t = 12\cos 3t$ A

Solution:

$$i(t) = \frac{dq}{dt} = \frac{d}{dt} 4\sin 3t = 12\cos 3t \quad \text{A}$$

Exercise 1.3-1 Which of the three currents, $i_1 = 45\ \mu$A, $i_2 = 0.03$ mA, and $i_3 = 25 \times 10^{-4}$ A, is largest?

Answer: i_3 is largest.

Solution:

$$i_1 = 45\ \mu\text{A} = 45 \times 10^{-6}\text{ A} < i_2 = 0.03\text{ mA} = .03 \times 10^{-3}\text{ A} = 3 \times 10^{-5}\text{ A} < i_3 = 25 \times 10^{-4}\text{ A}$$

Exercise 1.5-1 Figure E 1.5-1 shows four circuit elements identified by the letters *A*, *B*, *C*, and *D*.

(a) Which of the devices supply 12 W?
(b) Which of the devices absorb 12 W?
(c) What is the value of the power received by device *B*?
(d) What is the value of the power delivered by device *B*?
(e) What is the value of the power delivered by device *D*?

Answers: (a) *B* and *C*, (b) *A* and *D*, (c)–12 W, (c) 12 W, (e)–12 W

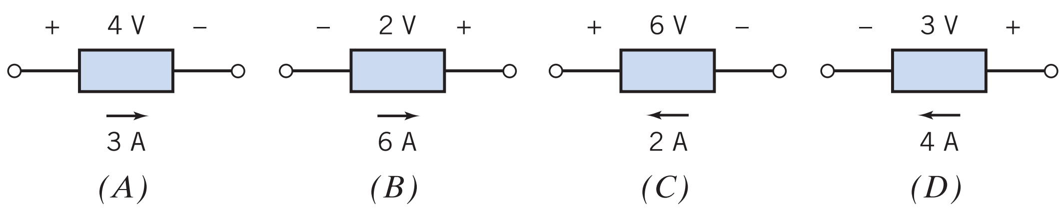


Figure E 1.5-1

Solution:
(a) *B* and *C.* The element voltage and current do not adhere to the passive convention in *B* and *C* so the product of the element voltage and current is the power supplied by these elements.

(b) *A* and *D*. The element voltage and current adhere to the passive convention in *A* and *D* so the product of the element voltage and current is the power delivered to, or absorbed by these elements.

(c) −12 W. The element voltage and current do not adhere to the passive convention in *B*, so the product of the element voltage and current is the power received by this element: (2 V)(6 A) = −12 W. The power supplied by the element is the negative of the power delivered to the element, 12 W.

(d) 12 W

(e) –12 W. The element voltage and current adhere to the passive convention in *D*, so the product of the element voltage and current is the power received by this element: (3 V)(4 A) = 12 W. The power supplied by the element is the negative of the power received to the element, −12 W.

Problems

Section 1-2 Electric Circuits and Current Flow

P1.2.1 The total charge that has entered a circuit element is $q(t) = 1.25(1–e^{-5t})$ when $t \geq 0$ and $q(t) = 0$ when $t < 0$. Determine the current in this circuit element for $t \geq 0$.

Answer: $i(t) = 6.25e^{-5t}$ A

Solution:

$$i(t) = \frac{d}{dt}1.25\left(1-e^{-5t}\right) = \underline{6.25\,e^{-5t}}\ \mathbf{A}$$

P 1.2-2 The current in a circuit element is $i(t) = 4(1–e^{-5t})$ A when $t \geq 0$ and $i(t) = 0$ when $t < 0$. Determine the total charge that has entered a circuit element for $t \geq 0$.

Hint: $q(0) = \int_{-\infty}^{t} i(\tau) = \int_{-\infty}^{0} 0 d\tau = 0$

Answer: $q(t) = 4t + 0.8e^{-5t} – 0.8$ C for $t \geq 0$

Solution:

$$q(t) = \int_0^t i(\tau)\,d\tau + q(0) = \int_0^t 4\left(1-e^{-5\tau}\right)d\tau + 0 = \int_0^t 4\,d\tau - \int_0^t 4e^{-5\tau}\,d\tau = \underline{4t + \frac{4}{5}e^{-5t} - \frac{4}{5}}\ \text{C}$$

P 1.2-3 The current in a circuit element is $i(t) = 4 \sin 3t$ A when $t \geq 0$ and $i(t) = 0$ when $t < 0$. Determine the total charge that has entered a circuit element for $t \geq 0$.

Hint: $q(0) = \int_{-\infty}^{t} i(\tau)d\tau = \int_{-\infty}^{0} 0\, d\tau = 0$

Solution:

$$q(t) = \int_0^t i(\tau)\,d\tau + q(0) = \int_0^t 4\sin 5\tau\,d\tau + 0 = -\frac{4}{5}\cos 3\tau\Big|_0^t = \underline{-\frac{4}{5}\cos 3t + \frac{4}{5}}\ \text{C}$$

P 1.2-4 The current in a circuit element is $i(t)=\begin{cases}0 & t<2\\ 2 & 2<t<4\\ -1 & 4<t<8\\ 0 & 8<t\end{cases}$ where the units of current are A and the units of time are s. Determine the total charge that has entered a circuit element for $t \geq 0$.

Answer:

$$q(t)=\begin{cases}0 & t<2\\ 2t-4 & 2<t<4\\ 8-t & 4<t<8\\ 0 & 8<t\end{cases}$$ where the units of charge are C.

Solution:

$q(t)=\int_{-\infty}^{t} i(\tau)\,d\tau=\int_{-\infty}^{t} 0\,d\tau=$ <u>0 C for $t \leq 2$</u> so $q(2) = 0$.

$q(t)=\int_{2}^{t} i(\tau)\,d\tau+q(2)=\int_{2}^{t} 2\,d\tau=2\tau\Big|_{2}^{t}=$ <u>$2t-4$ C for $2 \leq t \leq 4$</u>. In particular, $q(4) = 4$ C.

$q(t)=\int_{4}^{t} i(\tau)\,d\tau+q(4)=\int_{4}^{t} -1\,d\tau+4=-\tau\Big|_{4}^{t}+4=$ <u>$8-t$ C for $4 \leq t \leq 8$</u>. In particular, $q(8) = 0$ C.

$q(t)=\int_{8}^{t} i(\tau)\,d\tau+q(8)=\int_{8}^{t} 0\,d\tau+0=$ <u>0 C for $8 \leq t$</u>.

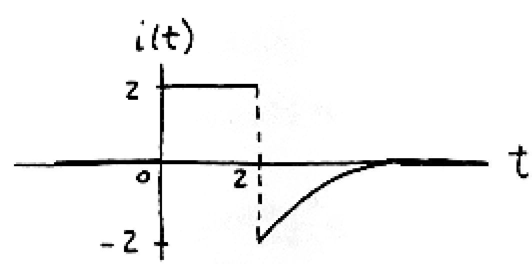
P 1.2-5 The total charge $q(t)$, in coulombs, that enters the terminal of an element is

$$q(t)=\begin{cases}0 & t<0\\ 2t & 0 \leq t \leq 2\\ 3+e^{-2t(t-2)} & t>2\end{cases}$$

Find the current $i(t)$ and sketch its waveform for $t \geq 0$.

Solution:

$$i(t)=\frac{dq(t)}{dt} \qquad i(t)=\begin{cases}0 & t<0\\ 2 & 0<t<2\\ \underline{-2e^{-2(t-2)}} & t>2\end{cases}$$



P 1.2-6 An electroplating bath, as shown in Figure P 1.2-6, is used to plate silver uniformly onto objects such as kitchen ware and plates. A current of 600 A flows for 20 minutes, and each coulomb transports 1.118 mg of silver. What is the weight of silver deposited in grams?

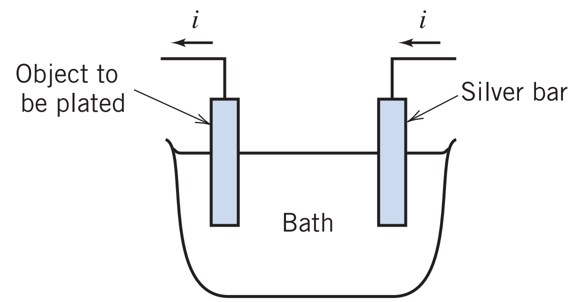


Figure P 1.2-6

Solution:

$$i = 450\ \text{A} = 450\ \frac{\text{C}}{\text{s}}$$

$$\text{Silver deposited} = 450\frac{\text{C}}{\text{s}}\times 20\,\text{min}\times 60\ \frac{\text{s}}{\text{min}}\times 1.118\ \frac{\text{mg}}{\text{C}} = 6.0372\times 10^{5}\ \text{mg} = \underline{603.72\ \text{g}}$$

P1.2-7 Find the charge $q(t)$ and sketch its waveform when the current entering a terminal of ab element is as shown in Figure P1.2-7. Assume that $q(t) = 0$ for $t < 0$.

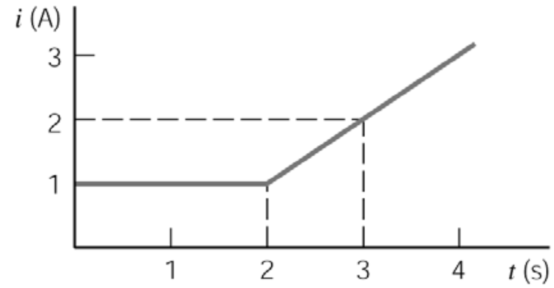


Figure P1.2-7

Solution:

$$i(t) = \begin{cases} 1 & \text{when} \quad 0 < t \le 2 \\ t-1 & \text{when} \quad 2 \le t \end{cases}$$

and
$$q(t) = \int_0^t i\,(\tau)d\tau + q(0) = \int_0^t i\,(\tau)d\tau$$

since $q(0) = 0$.

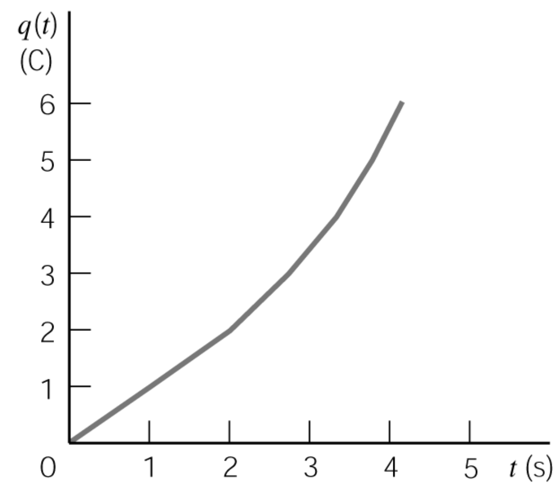
When $0 < t \le 2$, we have

$$q = \int_0^t 1\,d\tau = t \quad \text{C}$$

When $t \ge 2$, we have

$$q = \int_0^t i\,(\tau)d\tau = \int_0^2 1\,d\tau + \int_2^t (\tau - 1)\,d\tau$$
$$= \tau\Big|_0^2 + \frac{\tau^2}{2}\Big|_2^t - \tau\Big|_2^t = \frac{t^2}{2} - t + 2 \quad \text{C}$$

The sketch of $q(t)$ is shown to the right:.



Section 1-3 Systems of Units

P 1.3-1 A constant current of 3.2 μA flows through an element. What is the charge that has passed through the element in the first millisecond?
Answer: 3.2 nC

Solution:

$$\Delta q = i\,\Delta t = \left(3.2\times10^{-6}\ \text{A}\right)\left(1\times10^{-3}\ \text{s}\right) = 3.2\times10^{-9}\ \text{As} = 3.2\times10^{-9}\ \text{C} = \underline{3.2\times10^{-9}\ \text{nC}}$$

P 1.3-2 A charge of 45 nC passes through a circuit element during a particular interval of time that is 5 ms in duration. Determine the average current in this circuit element during that interval of time.
Answer: $i = 9\ \mu$A

Solution:

$$i = \frac{\Delta q}{\Delta t} = \frac{45\times10^{-9}}{5\times10^{-3}} = 9\times10^{-6} = \underline{9\ \mu\text{A}}$$

P 1.3-3 Ten billion electrons per second pass through a particular circuit element. What is the average current in that circuit element?
Answer: $i = 1.602$ nA

Solution

$$i = \left[10\,\text{billion}\ \frac{\text{electron}}{\text{s}}\right]\left[1.602\times10^{-19}\ \frac{\text{C}}{\text{electron}}\right] = \left[10\times10^{9}\ \frac{\text{electron}}{\text{s}}\right]\left[1.602\times10^{-19}\ \frac{\text{C}}{\text{electron}}\right]$$

$$= 10^{10}\times1.602\times10^{-19}\ \frac{\text{electron}}{\text{s}}\ \frac{\text{C}}{\text{electron}}$$

$$= 1.602\times10^{-9}\ \frac{\text{C}}{\text{s}} = \underline{1.602\quad \text{nA}}$$

P1.3-4 The charge flowing in a wire is plotted in Figure P1.3-4. Sketch the corresponding current.

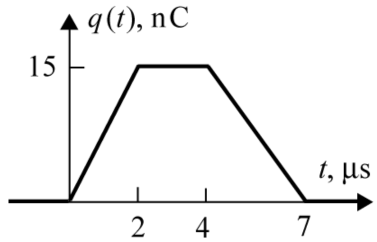
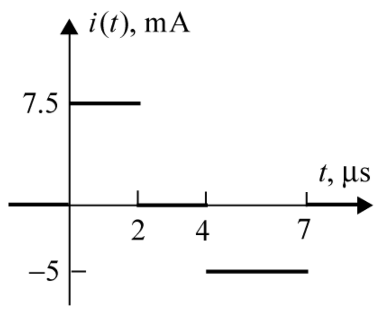


Figure P1.3-4

P1.3-4

$$i(t)=\frac{d}{dt}q(t)=\text{the slope of the } q \text{ versus } t \text{ plot}=\begin{cases}\dfrac{15\times10^{-9}}{2\times10^{-6}}=7.5\times10^{-3}=7.5\text{ mA when } 0<t<2\mu s\\ -\dfrac{15\times10^{-9}}{3\times10^{-6}}=-5\times10^{-3}=-5\text{ mA when } 4\mu s<t<7\mu s\\ 0 \quad \text{otherwise}\end{cases}$$



P1.3-5 The current in a circuit element is plotted in Figure P1.3-5. Sketch the corresponding charge flowing through the element for $t > 0$.

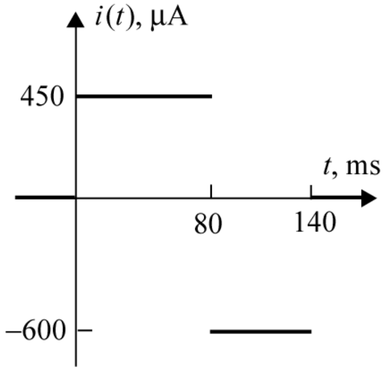


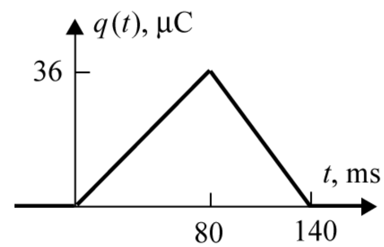
Figure P1.3-5

Solution:

$$q(t) = \int_0^t i(\tau)\, d\tau = \begin{cases} \int_0^t 450\mu\text{A}\, d\tau \quad \text{when } 0 < t < 80 \text{ ms} \\ \left(450\times10^{-6}\right)\left(80\times10^{-3}\right) + \int_{80 \text{ ms}}^t \left(-600\mu\text{A}\right) d\tau \quad \text{when } 80 \text{ ms} < t < 140 \text{ ms} \\ \left(450\times10^{-6}\right)\left(80\times10^{-3}\right) + \left(-600\times10^{-6}\right)\left(60\times10^{-3}\right) + \int_{140 \text{ ms}}^t 0\, d\tau \end{cases}$$

$$= \begin{cases} \left(450\times10^{-6}\right)t \quad \text{when } 0 < t < 80 \text{ ms} \\ \left(36\times10^{-6}\right) + \left(-600\times10^{-6}\right)t \quad \text{when } 80 \text{ ms} < t < 140 \text{ ms} \\ 0 \text{ C} \quad \text{when } 140 \text{ ms} < t \end{cases}$$

While $0 < t < 80$ ms $q(t)$ increases linearly from 0 to 36 μC and while $80 < t < 140$ ms $q(t)$ decreases linearly from 36 to 0 μC. Here's the sketch:



P1.3-6 The current in a circuit element is plotted in Figure P1.3-6. Determine the total charge that flows through the circuit element between 300 and 1200 μs.

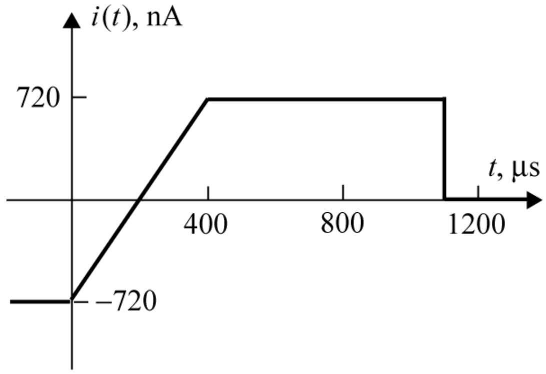
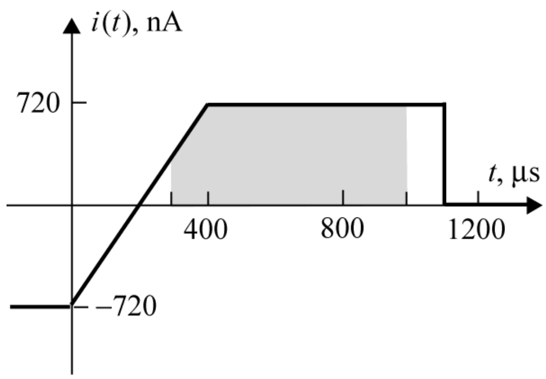


Figure P1.3-6

Solution:

$$q(t)=\int_{300\ \mu s}^{1000\ \mu s} i(\tau)\,d\tau = \text{"area under the curve between 300 } \mu\text{s and 1000 } \mu\text{s"}$$



$$q(t)=\left(\frac{360+720}{2}\times10^{-9}\right)\left(100\times10^{-6}\right)+\left(720\times10^{-9}\right)\left(600\times10^{-6}\right)=\left(54+432\right)\times10^{-12}=486\text{ pC}$$

Section 1-5 Power and Energy

P1.5-1 Figure P1.5-1 shows four circuit elements identified by the letters *A*, *B*, *C*, and *D*.

(*a*) Which of the devices supply 30 mW?
(*b*) Which of the devices absorb 0.03 W?
(*c*) What is the value of the power received by device *B*?
(*d*) What is the value of the power delivered by device *B*?
(*e*) What is the value of the power delivered by device *C*?

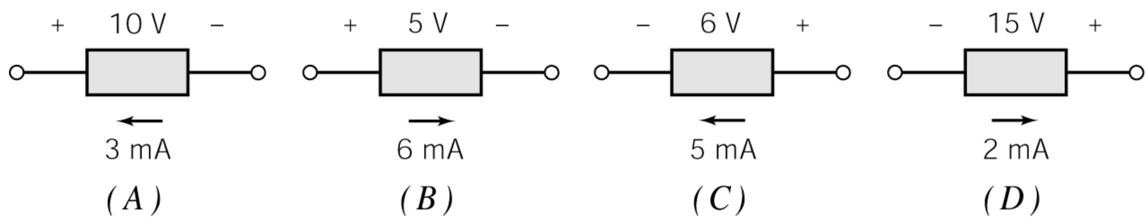


Figure P1.5-1

Solution:

(a) *A* and *D.* The element voltage and current do not adhere to the passive convention in Figures P1.5- *A* and *D* so the product of the element voltage and current is the power supplied by these elements.

(b) *B* and *C*. The element voltage and current adhere to the passive convention in Figures P1.5-1 *B* and *C* so the product of the element voltage and current is the power delivered to, or absorbed by these elements.

(c) 30 mW. The element voltage and current adhere to the passive convention in Figure P1.5-1*B*, so the product of the element voltage and current is the power received by this element: (5 V)(6 mA) = 30 mW. The power supplied by the element is the negative of the power received to the element, −30 W.

(d) −30 mW

(e) –30 mW. The element voltage and current adhere to the passive convention in Figure P1.5-1*C*, so the product of the element voltage and current is the power received by this element: (5 V)(6 mA) = 30 mW. The power supplied by the element is the negative of the power received to the element, −30 W.

P 1.5-2 An electric range has a constant current of 10 A entering the positive voltage terminal with a voltage of 110 V. The range is operated for two hours. (a) Find the charge in coulombs that passes through the range. (b) Find the power absorbed by the range. (c) If electric energy costs 12 cents per kilowatt-hour, determine the cost of operating the range for two hours.

Solution:

a.) $q = \int i\,dt = i\Delta t = (10\,\text{A})(2\,\text{hrs})(3600\,\text{s/hr}) \underline{= 7.2\times10^{4}\text{ C}}$

b.) $P = v\,i = (110\text{ V})(10\text{ A}) = \underline{1100\text{ W}}$

c.) $\text{Cost} = \dfrac{0.12\,\$}{\text{kWhr}}\times 1.1\,\text{kW}\times 2\,\text{hrs} = \underline{0.264\,\$}$

P 1.5-3 A walker's cassette tape player uses four AA batteries in series to provide 6 V to the player circuit. The four alkaline battery cells store a total of 200 watt-seconds of energy. If the cassette player is drawing a constant 10 mA from the battery pack, how long will the cassette operate at normal power?

Solution:

$$P = (6\text{V})(10\text{ mA}) = 0.06\text{ W}$$

$$\Delta t = \frac{\Delta w}{P} = \frac{200\text{ W}\cdot\text{s}}{0.06\text{ W}} = \underline{3.33\times10^{3}\text{ s}}$$

P 1.5-4 The current through and voltage across an element vary with time as shown in Figure P 1.5-4. Sketch the power delivered to the element for $t > 0$. What is the total energy delivered to the element between $t = 0$ and $t = 25$ s? The element voltage and current adhere to the passive convention.

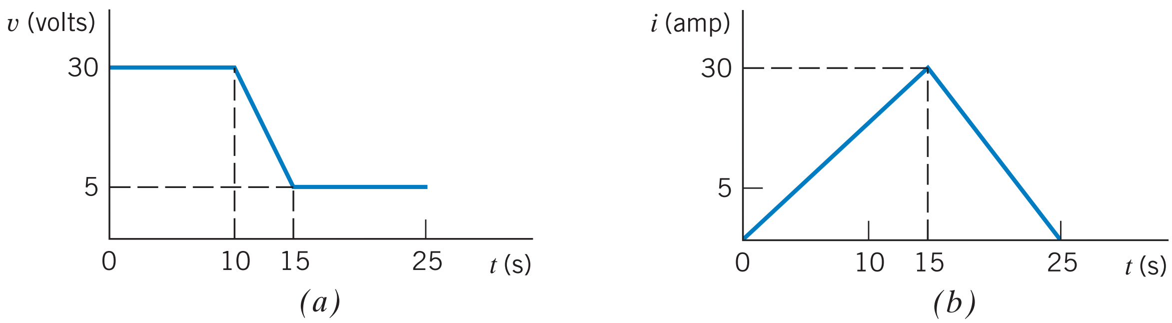


Figure P 1.5-4

Solution:

for $0 \le t \le 10$ s: $v = 30$ V and $i = \frac{30}{15} t = 2t$ A $\therefore$ $\underline{P = 30(2t) = 60t \text{ W}}$

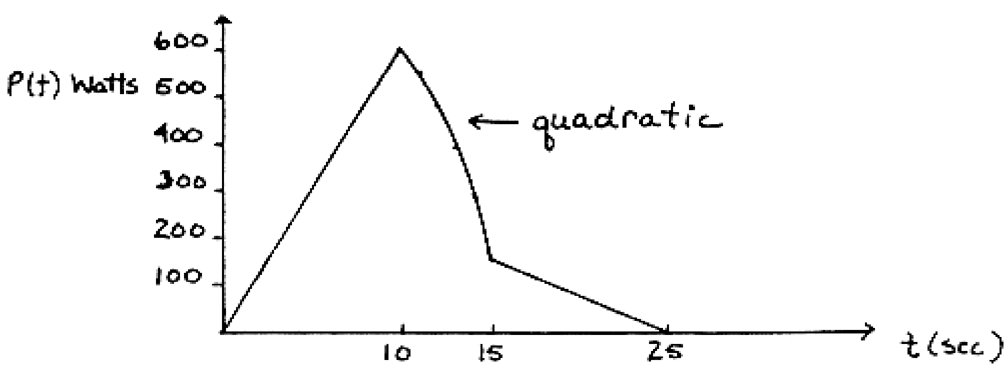
for $10 \le t \le 15$ s: $v(t) = -\frac{25}{5} t + b \Rightarrow v(10) = 30 \text{ V} \Rightarrow b = 80 \text{ V}$

$v(t) = -5t + 80$ and $i(t) = 2t$ A $\Rightarrow P = (2t)(-5t+80) = \underline{-10t^2 + 160t \text{ W}}$

for $15 \le t \le 25$ s: $v = 5$ V and $i(t) = -\frac{30}{10} t + b$ A

$i(25) = 0 \Rightarrow b = 75 \Rightarrow i(t) = -3t + 75$ A

$\therefore \underline{P = (5)(-3t+75) = -15t + 375 \text{ W}}$



$$\text{Energy} = \int P\,dt = \int_0^{10} 60t\,dt + \int_{10}^{15} (160t - 10t^2)\,dt + \int_{15}^{25} (375 - 15t)\,dt$$

$$= 30t^2 \Big|_0^{10} + 80t^2 - \frac{10}{3} t^3 \Big|_{10}^{15} + 375t - \frac{15}{2} t^2 \Big|_{15}^{25} = \underline{5833.\bar{3} \text{ J}}$$

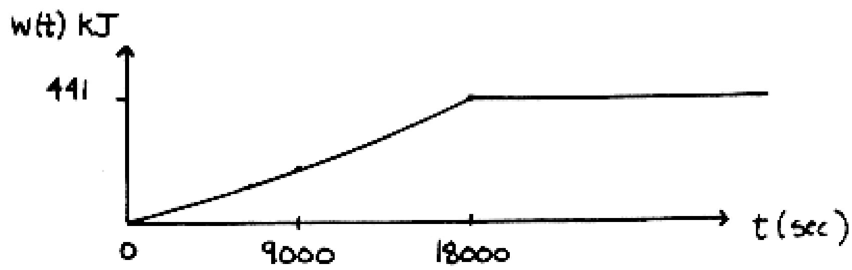
P 1.5-5 An automobile battery is charged with a constant current of 2 A for five hours. The terminal voltage of the battery is $v = 11 + 0.5t$ V for $t > 0$, where t is in hours. (a) Find the energy delivered to the battery during the five hours. (b) If electric energy costs 15 cents/kWh, find the cost of charging the battery for five hours.
Answer: (b) 1.84 cents

Solution:

a.) Assuming no more energy is delivered to the battery after 5 hours (battery is fully charged).

$$w = \int P dt = \int_0^t vi\, d\tau = \int_0^{5(3600)} 2\left(11 + \frac{0.5\,\tau}{3600}\right) d\tau = 22t + \frac{0.5}{3600}\tau^2 \Bigg|_0^{5(3600)}$$

$$= 441 \times 10^3 \text{ J} = \underline{441 \text{ kJ}}$$



b.) $\text{Cost} = 441\,\text{kJ} \times \dfrac{1 \text{ hr}}{3600\text{s}} \times \dfrac{15¢}{\text{kWhr}} = \underline{1.84¢}$

P 1.5-6 Find the power, $p(t)$, supplied by the element shown in Figure P 1.5-6 when $v(t) = 4 \sin 3t$ V and $i(t) = (1/12) \sin 3t$ A. Evaluate $p(t)$ at t=0.5 s and t = 1 s. Observe that the power supplied by this element has a positive value at some times and a negative value at other times.

Hint: $(\sin at)(\sin bt) = \frac{1}{2}(\cos(a-b)t - \cos(a+b)t)$

Answer: $p(t) = (1/6)\cos(6t)$ W, $p(0.5) = 0.0235$ W, $p(1) = -0.02466$ W

Solution:

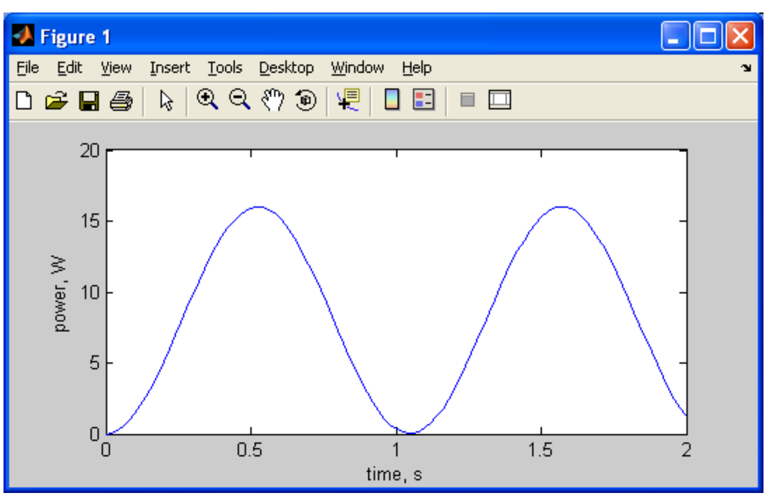
$$p(t) = v(t)\,i(t) = (4\cos 3t)\left(\frac{1}{12}\sin 3t\right) = \frac{1}{6}(\sin 0 + \sin 6t) = \underline{\frac{1}{6}\sin 6t \quad \text{W}}$$

$$p(0.5) = \frac{1}{6}\sin 3 = \underline{0.0235 \quad \text{W}}$$

$$p(1) = \frac{1}{6}\sin 6 = \underline{-0.0466 \quad \text{W}}$$

Here is a MATLAB program to plot $p(t)$:

```
clear
t0=0;                      % initial time
tf=2;                      % final time
dt=0.02;                   % time increment
t=t0:dt:tf;                % time

v=4*cos(3*t);              % device voltage
i=(1/12)*sin(3*t);         % device current

for k=1:length(t)
   p(k)=v(k)*i(k);         % power
end

plot(t,p)
xlabel('time, s');
ylabel('power, W')
```



P 1.5-7 Find the power, $p(t)$, supplied by the element shown in Figure P 1.5-6 when $v(t) = 8 \sin 3t$ V and $i(t) = 2 \sin 3t$ A.

Hint: $(\sin at)(\sin bt) = \frac{1}{2}(\cos(a-b)t - \cos(a+b)t)$

Answer: $p(t) = 8 - 8\cos 6t$ W

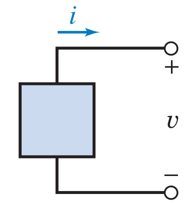


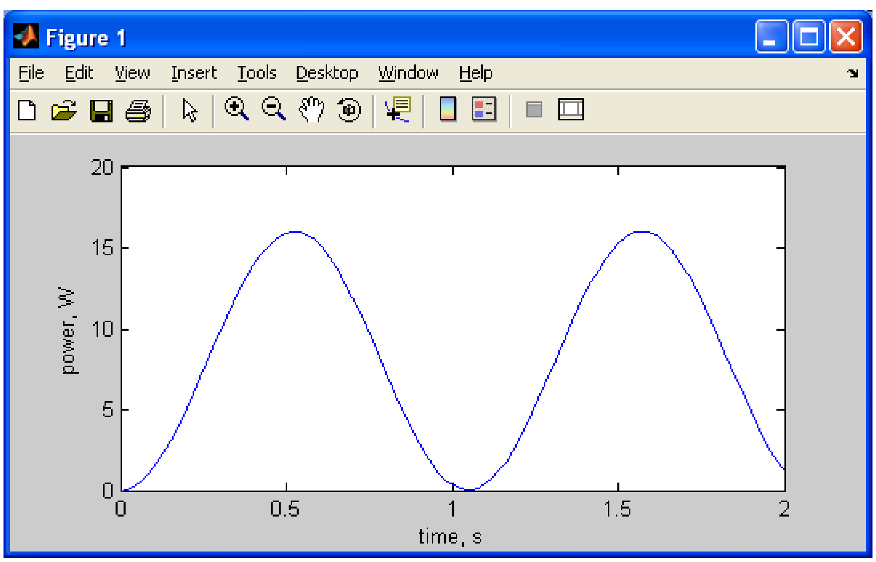
Figure P 1.5-7

Solution:

$$p(t) = v(t)\,i(t) = (8\sin 3t)(2\sin 3t) = 8(\cos 0 - \cos 6t) = \underline{8 - 8\cos 6t \quad \text{W}}$$

Here is a MATLAB program to plot $p(t)$:

```
clear

t0=0;                   % initial time
tf=2;                   % final time
dt=0.02;                % time increment
t=t0:dt:tf;             % time

v=8*sin(3*t);           % device voltage
i=2*sin(3*t);           % device current

for k=1:length(t)
   p(k)=v(k)*i(k);      % power
end

plot(t,p)
xlabel('time, s');
ylabel('power, W')
```



P 1.5-8 Find the power, $p(t)$, supplied by the element shown in Figure P 1.5-6. The element voltage is represented as $v(t) = 4(1 - e^{-2t})$V when $t \geq 0$ and $v(t) = 0$ when $t < 0$. The element current is represented as $i(t) = 2e^{-2t}$ A when $t \geq 0$ and $i(t) = 0$ when $t < 0$.

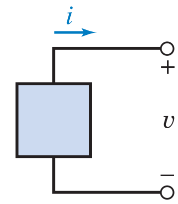


Figure P 1.5-7

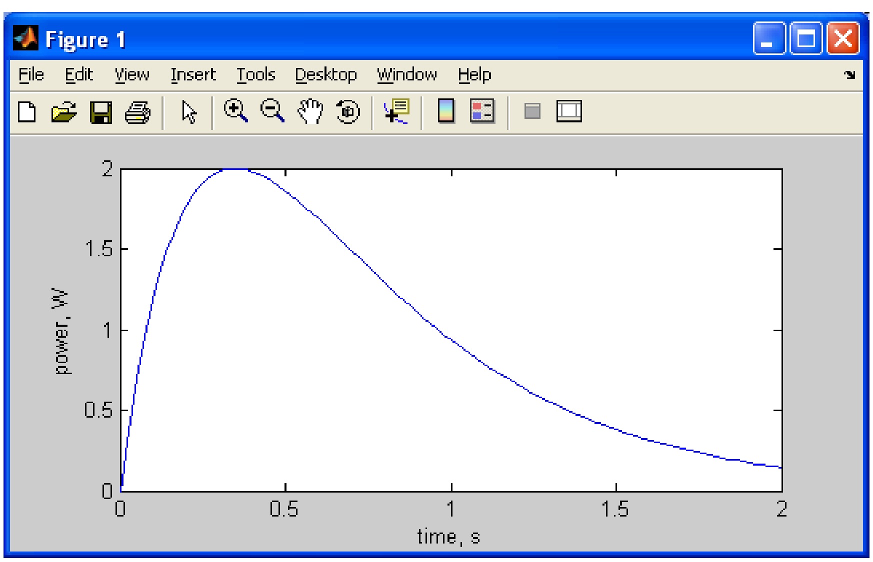
Answer: $p(t) = 8(1 - e^{-2t})e^{-2t}$ W

Solution:

$$p(t) = v(t)\,i(t) = 4\left(1 - e^{-2t}\right) \times 2\,e^{-2t} = \underline{8\left(1 - e^{-2t}\right)e^{-2t} \quad \text{W}}$$

Here is a MATLAB program to plot $p(t)$:

```
clear

t0=0;                      % initial time
tf=2;                      % final time
dt=0.02;                   % time increment
t=t0:dt:tf;                % time

v=4*(1-exp(-2*t));         % device voltage
i=2*exp(-2*t);             % device current

for k=1:length(t)
   p(k)=v(k)*i(k);         % power
end

plot(t,p)
xlabel('time, s');
ylabel('power, W')
```



P 1.5-9 The battery of a flashlight develops 3 V, and the current through the bulb is 200 mA. What power is absorbed by the bulb? Find the energy absorbed by the bulb in a five-minute period.

Solution:
The power is $P = V\,I = 3 \times 0.015 = \underline{0.045\text{ W}}$. Next, the energy is $w = P\Delta t = 0.045 \times 5 \times 60 = \underline{13.5\text{ J}}$.

P1.5-10 Medical researchers studying hypertension often use a technique called "2D gel electrophoresis" to analyze the protein content of a tissue sample. An image of a typical "gel" is shown in Figure 1.5-10*a*.

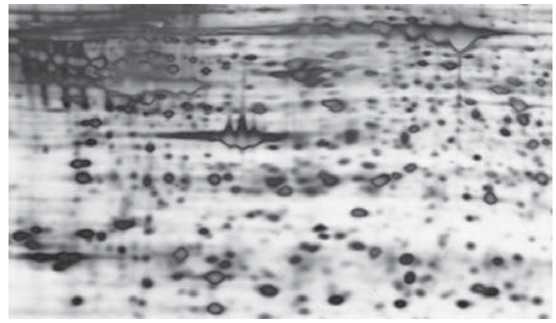
The procedure for preparing the gel uses the electric circuit illustrated in Figure 1.5-10*b*. The sample consists of a gel and a filter paper containing ionized proteins. A voltage source causes a large, constant voltage, 500 V, across the sample. The large, constant voltage moves the ionized proteins from the filter paper to the gel. The current in the sample is given by

$$i(t) = 2 + 30\,e^{-at} \text{ mA}$$

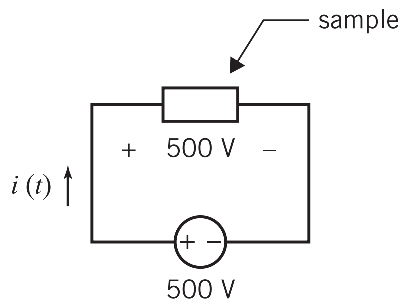
where t is the time elapsed since the beginning of the procedure and the value of the constant a is

$$a = 0.85 \ \frac{1}{\text{hr}}$$

Determine the energy supplied by the voltage source when the gel preparation procedure lasts 3 hours.



(a)

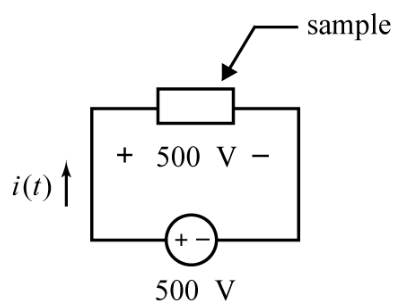


(b)

Figure 1.5-10 (*a*) An image of a gel and (*b*) the electric circuit used to preparation a gel.

Solution:

$$\begin{aligned} energy = w(t) = \int_0^T p(t)\,dt &= \int_0^T v(t)\,i(t)\,dt \\ &= \frac{500}{1000}\int_0^3 \left(2 + 30\,e^{-0.85t}\right) dt \\ &= \int_0^3 dt + 15\int_0^3 \left(e^{-0.85t}\right) dt \\ &= (3-0) + \frac{15}{-0.85}\left(e^{-2.55} - 1\right) \\ &= 3 + 16.3 = 19.3 \text{ J} \end{aligned}$$



Section 1.7 How Can We Check…?

P 1.7-1 Conservation of energy requires that the sum of the power absorbed by all of the elements in a circuit be zero. Figure P 1.7-1 shows a circuit. All of the element voltages and currents are specified. Are these voltage and currents correct? Justify your answer.

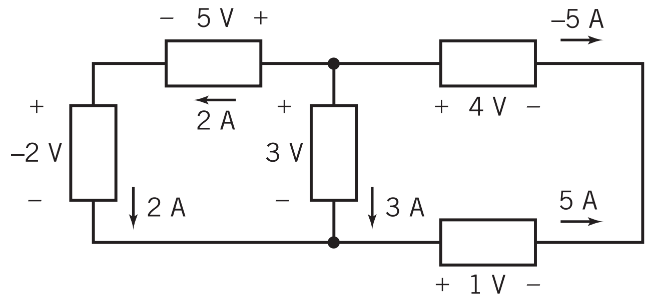


Figure P 1.7-1

Hint: Calculate the power absorbed by each element. Add up all of these powers. If the sum is zero, conservation of energy is satisfied and the voltages and currents are probably correct. If the sum is not zero, the element voltages and currents cannot be correct.

Solution:
Notice that the element voltage and current of each branch adhere to the passive convention. The sum of the powers absorbed by each branch are:

$$(-2\text{ V})(2\text{ A})+(5\text{ V})(2\text{ A})+(3\text{ V})(3\text{ A})+(4\text{ V})(-5\text{ A})+(1\text{ V})(5\text{ A}) = -4\text{ W} + 10\text{ W} + 9\text{ W} -20\text{ W} + 5\text{ W} = 0\text{ W}$$

The element voltages and currents satisfy conservation of energy and may be correct.

P 1.7-2 Conservation of energy requires that the sum of the power absorbed by all of the elements in a circuit be zero. Figure P 1.7-2 shows a circuit. All of the element voltages and currents are specified. Are these voltage and currents correct? Justify your answer.
Hint: Calculate the power absorbed by each element. Add up all of these powers. If the sum is zero, conservation of energy is satisfied and the voltages and currents are probably correct. If the sum is not zero, the element voltages and currents cannot be correct.

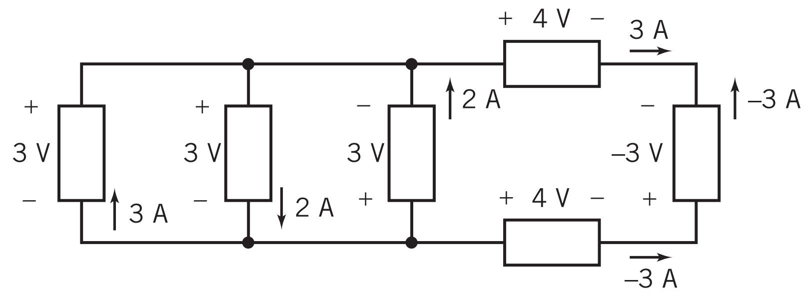


Figure P 1.7-2

Solution:
Notice that the element voltage and current of some branches do not adhere to the passive convention. The sum of the powers absorbed by each branch are:

-(3 V)(3 A)+(3 V)(2 A)+ (3 V)(2 A)+(4 V)(3 A)+(-3 V)(-3 A)+(4 V)(-3 A)
= -9 W + 6 W + 6 W + 12 W + 9 W -12 W
≠ 0 W

The element voltages and currents do not satisfy conservation of energy and cannot be correct.

P 1.7-3 The element currents and voltages shown in Figure P 1.7-3 are correct with one exception: the reference direction of exactly one of the element currents is reversed. Determine which reference direction has been reversed.

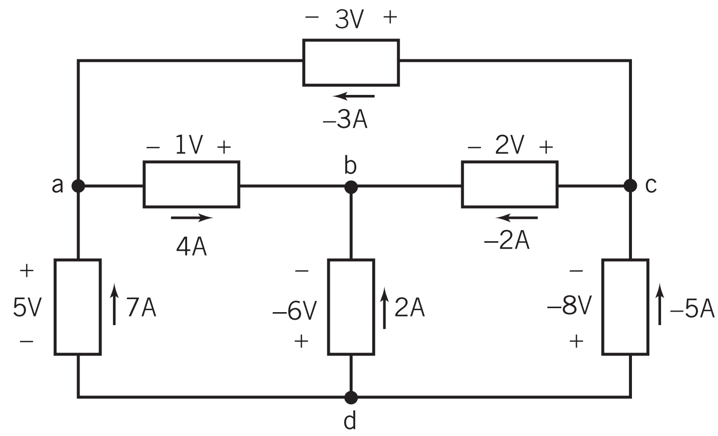


Figure P 1.7-3

Solution:
Let's tabulate the power received by each element. We'll identify each element by its nodes.

nodes		Power received, W
a	*c*	$(3)(-3)=-9$
a	*b*	$-(1)(4)=-4$
b	*c*	$(2)(-2)=-4$
a	*d*	$-(5)(7)=-35$
b	*d*	$(-6)(2)=-12$
c	*d*	$(-8)(-5)=40$

So

$$\textit{Total power received} = -(9+4+4+35+12)+40=-24\neq 0$$

Changing the current reference direction for a particular element will change the total power by twice the power of the particular element. Since the element connected between nodes b and d receives -12 W, changing the reference direction of its current will increase the total power received by 24 W, as required. After making that change

$$\textit{Total power received} = -(9+4+4+35)+(12+40)=0$$

We conclude that it is the reference direction of the element connected between nodes and b that has been reversed.

Design Problems

DP 1-1 A particular circuit element is available in three grades. Grade A guarantees that the element can safely absorb 1/2 W continuously. Similarly, Grade B guarantees that 1/4 W can be absorbed safely and Grade C guarantees that 1/8 W can be absorbed safely. As a rule, elements that can safely absorb more power are also more expensive and bulkier.

The voltage across an element is expected to be about 20 V and the current in the element is expected to be about 8 mA. Both estimates are accurate to within 25 percent. The voltage and current reference adhere to the passive convention.

Specify the grade of this element. Safety is the most important consideration, but don't specify an element that is more expensive than necessary.

Solution:

The voltage may be as large as 20(1.25) = 25 V and the current may be as large as (0.008)(1.25) = 0.01 A. The element needs to be able to absorb (25 V)(0.01 A) = 0.25 W continuously. A Grade B element is adequate, but without margin for error. Specify a Grade B device if you trust the estimates of the maximum voltage and current and a Grade A device otherwise.

DP 1-2 The voltage across a circuit element is $v(t) = 20\ (1 - e^{-8t})$V when $t \geq 0$ and $v(t) = 0$ when $t < 0$. The current in this element is $i(t) = 30e^{-8t}$ mA when $t \geq 0$ and $i(t) = 0$ when $t < 0$. The element current and voltage adhere to the passive convention. Specify the power that this device must be able to absorb safely.
Hint: Use MATLAB, or a similar program, to plot the power.

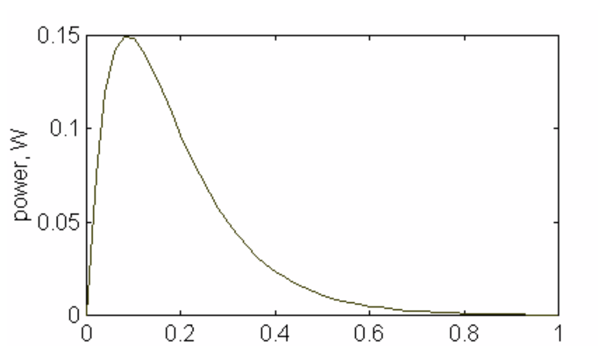
Solution:

$$p(t) = 20\left(1-e^{-8t}\right)\times 0.03\,e^{-8t} = \underline{0.6\left(1-e^{-8t}\right)e^{-8t} \quad \text{W}}$$

Here is a MATLAB program to plot $p(t)$:

```
clear

t0=0;                    % initial time
tf=1;                    % final time
dt=0.02;                 % time increment
t=t0:dt:tf;              % time

v=20*(1-exp(-8*t));      % device voltage
i=.030*exp(-8*t);        % device current

for k=1:length(t)
   p(k)=v(k)*i(k);       % power
end

plot(t,p)
xlabel('time, s');
ylabel('power, W')
```

Here is the plot:



The circuit element must be able to absorb 0.15 W.

Chapter 2 Circuit Elements

Exercises

Exercise 2.4-1 Find the power absorbed by a 100-ohm resistor when it is connected directly across a constant 10-V source.
Answer: 1-W

Solution:

$$P = \frac{v^2}{R} = \frac{(10)^2}{100} = \underline{1\ \text{W}}$$

Exercise 2.4-2 A voltage source v = 10 cos t V is connected across a resistor of 10 ohms. Find the power delivered to the resistor.
Answer: 10 $\cos^2 t$ W

Solution:

$$P = \frac{v^2}{R} = \frac{(10\ \cos\ t)^2}{10} = \underline{10\ \cos^2 t\ \text{W}}$$

Exercise 2.7-1 Find the power absorbed by the CCCS in Figure E 2.7-1.

Hint: The controlling element of this dependent source is a short circuit. The voltage across a short circuit is zero. Hence, the power absorbed by the controlling element is zero. How much power is absorbed by the controlled element?

Answer:–115.2 watts are absorbed by the CCCS. (The CCCS delivers +115.2 watts to the rest of the circuit.)

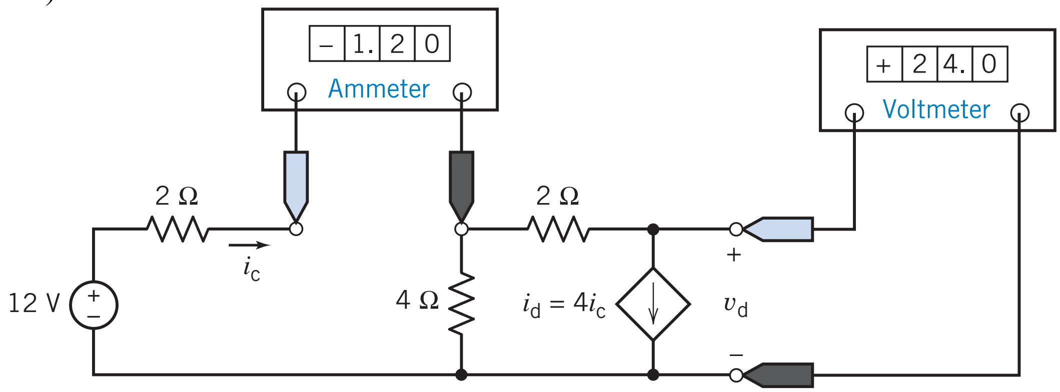


Figure E 2.7-1

Solution:

$$i_c = -1.2 \quad \text{A}, \; v_d = 24 \quad \text{V}$$
$$i_d = 4\,(-1.2) = -4.8 \quad \text{A}$$

i_d and v_d adhere to the passive convention so

$$P = v_d \, i_d = (24)\,(-4.8) = \underline{-115.2 \quad \text{W}}$$

is the power received by the dependent source

Exercise 2.8-1 For the potentiometer circuit of Figure 2.8-2, calculate the meter voltage, v_m, when $\theta = 45°$, $R_p = 20$ kΩ, and $I = 2$ mA.
Answer: $v_m = 5$ V

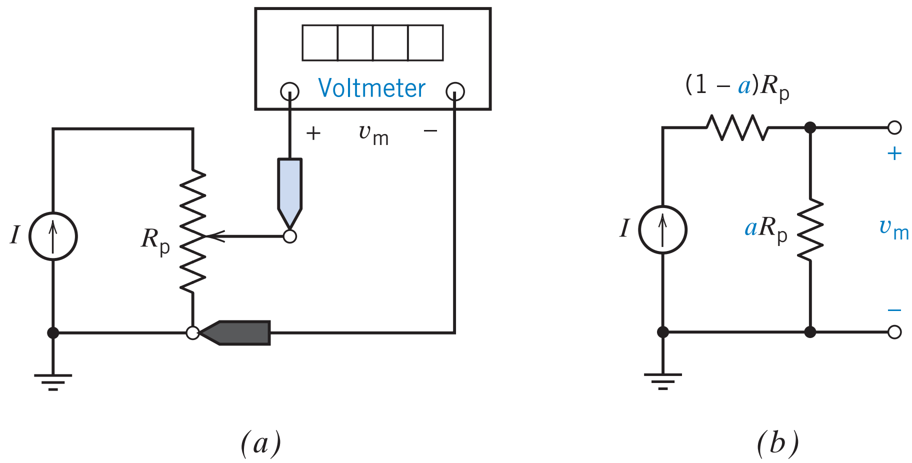


Figure 2.8-2

Solution:

$$\theta = 45^\circ,\ I = 2\ \text{mA},\ R_p = 20\ \text{k}\Omega$$

$$a = \frac{\theta}{360} \quad \Rightarrow \quad a\,R_p = \frac{45}{360}(20\ \text{k}\Omega) = 2.5\ \text{k}\Omega$$

$$v_m = (2\times10^{-3})(2.5\times10^{3}) = \underline{5\ \text{V}}$$

Exercise 2.8-2 The voltage and current of an AD590 temperature sensor of Figure 2.8-3 are 10 V and 280 μA, respectively. Determine the measured temperature.

Answer: $T = 280°$K, or approximately 6.85°C

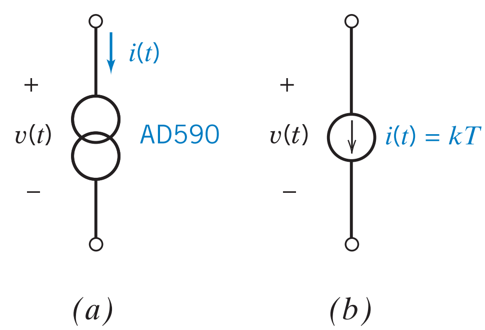


Figure 2.8-3

Solution:

$$v = 10\ \text{V},\ i = 280\ \mu\text{A},\ k = 1\frac{\mu\text{A}}{^\circ\text{K}}\ \text{for AD590}$$

$$i = kT \quad \Rightarrow \quad T = \frac{i}{k} = (280\,\mu\text{A})\left(1\ \frac{^\circ\text{K}}{\mu\text{A}}\right) = \underline{280^\circ\ \text{K}}$$

Exercise 2.9-1 What is the value of the current i in Figure E 2.9-1 at time $t = 4$ s?

Answer: $i = 0$ amperes at $t = 4$ s (both switches are open).

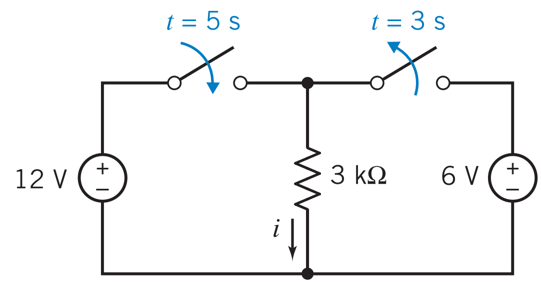


Figure E 2.9-1

Solution:

At $t = 4$ s both switches are open, so $i = 0$ A.

Exercise 2.9-2 What is the value of the voltage v in Figure E 2.9-2 at time $t = 4$ s? At $t = 6$ s?

Answer: $v = 6$ volts at $t = 4$ s, and $v = 0$ volts at $t = 6$ s.

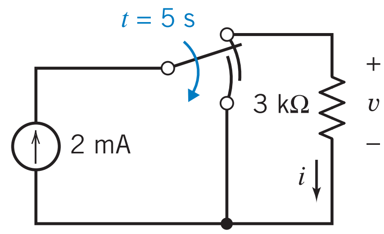


Figure E 2.9-2

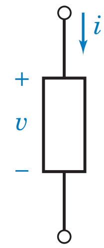
Solution:

At $t = 4$ s the switch is in the up position, so $v = i\,R = (2\text{ mA})(3\text{ k}\Omega) = \underline{6\text{V}}$.

At $t = 6$ s the switch is in the down position, so $v = 0$ V.

Section 2-2 Engineering and Linear Models

P 2.2-1 An element has voltage *v* and current *i* as shown in Figure P 2.2-1*a*. Values of the current *i* and corresponding voltage *v* have been tabulated as shown in Figure P 2.2-1*b*. Determine if the element is linear.



(*a*)

v, V	*i*, A
–3	–3
–4	–2
0	0
12	2
32	4
60	6

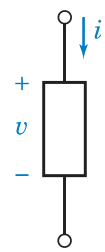
(*b*)

Figure P 2.2-1

Solution:

The element is not linear. For example, doubling the current from 2 A to 4 A does not double the voltage. Hence, the property of homogeneity is not satisfied.

P 2.2-2 A linear element has voltage *v* and current *i* as shown in Figure P 2.2-2*a*. Values of the current *i* and corresponding voltage *v* have been tabulated as shown in Figure P 2.2-2*b*. Represent the element by an equation that expresses *v* as a function of *i*. This equation is a model of the element. (a) Verify that the model is linear. (b) Use the model to predict the value of *v* corresponding to a current of $i = 40$ mA. (c) Use the model to predict the value of *i* corresponding to a voltage of $v = 4$ V.



(*a*)

v, V	*i*, A
–3.6	–30
2.4	20
6.0	50

(*b*)

Figure P 2.2-2*a*

Hint: Plot the data. We expect the data points to lie on a straight line. Obtain a linear model of the element by representing that straight line by an equation.

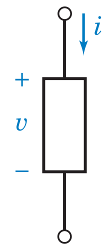
Solution:

(a) The data points do indeed lie on a straight line. The slope of the line is 0.12 V/A and the line passes through the origin so the equation of the line is $v = 0.12\,i$. The element is indeed linear.

(b) When $i = 40$ mA, v = (0.12 V/A)×(40 mA) = (0.12 V/A)×(0.04 A) = 4.8 mV

(c) When $v = 3 \text{ V},\ i = \dfrac{3}{0.12} = 25 \text{ A}$

P 2.2-3 A linear element has voltage *v* and current *i* as shown in Figure P 2.2-3*a*. Values of the current *i* and corresponding voltage *v* have been tabulated as shown in Figure P 2.2-3*b*. Represent the element by an equation that expresses *v* as a function of *i*. This equation is a model of the element. (a) Verify that the model is linear. (b) Use the model to predict the value of *v* corresponding to a current of *i* = 4 mA. (c) Use the model to predict the value of *i* corresponding to a voltage of *v* = 12 V.



(a)

v, V	*i*, mA
3.078	12
5.13	20
12.825	50

(b)

Figure P 2.2-3

Hint: Plot the data. We expect the data points to lie on a straight line. Obtain a linear model of the element by representing that straight line by an equation.

Solution:

(a) The data points do indeed lie on a straight line. The slope of the line is 256.5 V/A and the line passes through the origin so the equation of the line is $v = 256.5\,i$. The element is indeed linear.

(b) When *i* = 6 mA, *v* = (256.5 V/A)×(6 mA) = (256.5 V/A)×(0.006 A) = 1.054 V

(c) When $v = 12\ \text{V},\ i = \dfrac{12}{256.5} = 0.04678\ \text{A} = 46.78$ mA.

P 2.2-4 An element is represented by the relation between current and voltage as

$$v = 3i + 5$$

Determine whether the element is linear.

Solution:

Let $i = 1$ A, then $v = 3i + 5 = 8$ V. Next $2i = 2$A but $16 = 2v \neq 3(2i) + 5 = 11$.. Hence, the property of homogeneity is not satisfied. The element is not linear.

P 2.2-5 The circuit shown in Figure P 2.3-5 consists of a current source, a resistor, and element A.

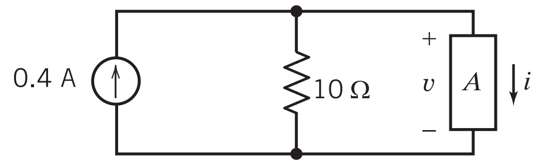


Figure P 2.3-5

Consider three cases.

(a) When element A is a 40-Ω resistor, described by $i = v / 40$, then the circuit is represented by

$$0.4=\frac{v}{10}+\frac{v}{40}$$

Determine the values of v and i. Notice that the above equation has a unique solution.

(b) When element A is a nonlinear resistor described by $i = v^2 / 2$, then the circuit is represented by

$$0.4=\frac{v}{10}+\frac{v^2}{2}$$

Determine the values of v and i. In this case there are two solutions of the above equation. Nonlinear circuits exhibit more complicated behavior than linear circuits.

(c) When element A is a nonlinear resistor described by $i = 0.8+\frac{v^2}{2}$, then the circuit is described by

$$0.4=\frac{v}{10}+0.8+\frac{v^2}{2}$$

Show that this equation has no solution. This result usually indicates a modeling problem. At least one of the three elements in the circuit has not been modeled accurately.

Solution:

(a)

$$0.4=\frac{v}{10}+\frac{v}{40}=\frac{v}{8} \quad \Rightarrow \quad v=3.2\ \text{V}$$

$$i=\frac{v}{40}=0.08\ \text{A}$$

(b)

$$0.4=\frac{v}{10}+\frac{v^2}{2} \quad \Rightarrow \quad v^2+\frac{v}{5}-0.8=0$$

Using the quadratic formula $v=\frac{-0.2\pm1.8}{2}=0.8,\ -1.0\ \text{V}$

When $v = 0.8$ V then $i=\frac{0.8^2}{2}=0.32\ \text{A}$. When $v = -1.0$ V then $i=\frac{(-1)^2}{2}=0.5\ \text{A}$.

(c)

$$0.4=\frac{v}{10}+0.8+\frac{v^2}{2} \quad \Rightarrow \quad v^2+\frac{v}{5}+0.8=0$$

Using the quadratic formula $v = \dfrac{-0.2 \pm \sqrt{0.04 - 3.2}}{2}$

So there is no real solution to the equation.

Section 2-4 Resistors

P 2.4-1 A current source and a resistor are connected in series in the circuit shown in Figure P 2.4-1. Elements connected in series have the same current, so $i = i_s$ in this circuit. Suppose that $i_s = 3$ A and $R = 7\ \Omega$. Calculate the voltage v across the resistor and the power absorbed by the resistor.

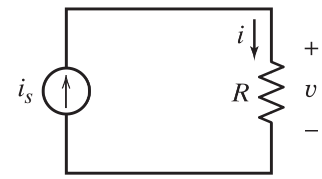
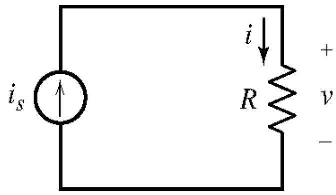


Figure P 2.4-1

Answer: $v = 21$ V and the resistor absorbs 63 W.

Solution:



$i = i_s = 3$ A and $v = Ri = 7 \times 3 = \underline{21\ \text{V}}$

v and i adhere to the passive convention

$\therefore P = v\,i = 21 \times 3 = \underline{63\ \text{W}}$

is the power absorbed by the resistor.

P 2.4-2 A current source and a resistor are connected in series in the circuit shown in Figure P 2.4-1. Elements connected in series have the same current, so $i = i_s$ in this circuit. Suppose that $i = 3$ mA and $v = 24$ V. Calculate the resistance R and the power absorbed by the resistor.

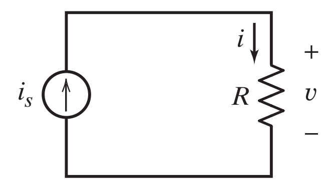
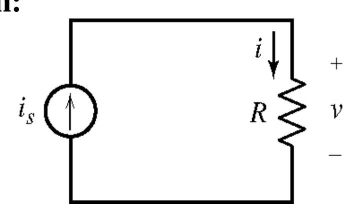


Figure P 2.4-1

Answer: $R = 8$ kΩ and the resistor absorbs 72 mW.

Solution:



$i = i_s = 3$ mA and $v = 48$ V

$$R = \frac{v}{i} = \frac{48}{0.003} = 16000 = \underline{16\ \text{K}\Omega}$$

$$P = (3\times10^{-3})\times48 = 144\times10^{-3} = \underline{144\ \text{mW}}$$

P 2.4-3 A voltage source and a resistor are connected in parallel in the circuit shown in Figure P 2.4-3. Elements connected in parallel have the same voltage, so $v = v_s$ in this circuit. Suppose that v_s = 10 V and R = 5 Ω. Calculate the current i in the resistor and the power absorbed by the resistor.

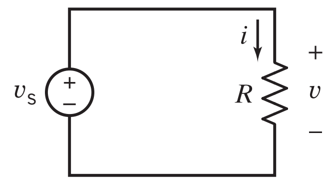
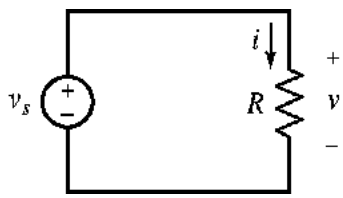


Figure P 2.4-3

Answer: i = 2 A and the resistor absorbs 20 W.

Solution:



$v = v_s = 10\text{ V}$ and $R = 5\ \Omega$

$$i = \frac{v}{R} = \frac{10}{5} = \underline{2\text{ A}}$$

v and i adhere to the passive convention

$\therefore\ p = v\,i = 2 \cdot 10 = \underline{20\text{ W}}$

is the power absorbed by the resistor

P 2.4-4 A voltage source and a resistor are connected in parallel in the circuit shown in Figure P 2.4-3. Elements connected in parallel have the same voltage, so $v = v_s$ in this circuit. Suppose that v_s = 24 V and i = 2 A. Calculate the resistance R and the power absorbed by the resistor.

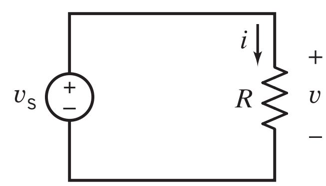
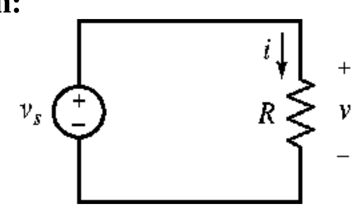


Figure P 2.4-3

Answer: R = 12 Ω and the resistor absorbs 48 W.

Solution:



$v = v_s = 24\text{ V}$ and $i = 3\text{ A}$

$$R = \frac{v}{i} = \frac{24\text{ V}}{3\text{ A}} = \underline{8\ \Omega}$$

$$p = v\,i = 24(3) = \underline{72\text{ W}}$$

P 2.4-5 A voltage source and two resistors are connected in parallel in the circuit shown in Figure P 2.4-5. Elements connected in parallel have the same voltage, so $v_1 = v_s$ and $v_2 = v_s$ in this circuit. Suppose that $v_s = 150$ V, $R_1 = 50\ \Omega$, and $R_2 = 25\ \Omega$. Calculate the current in each resistor and the power absorbed by each resistor.

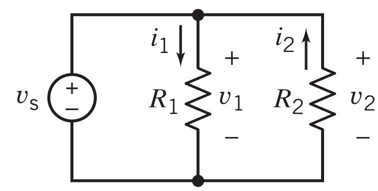
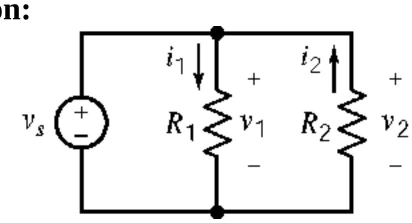


Figure P 2.4-5

Hint: Notice the reference directions of the resistor currents.
Answer: $i_1 = 3$ A and $i_2 = -6$ A. R_1 absorbs 450 W and R_2 absorbs 900 W.

Solution:



$v_1 = v_2 = v_s = 150$ V;

$R_1 = 50\ \Omega$; $R_2 = 25\ \Omega$

v_1 and i_1 adhere to the passive convention so

$$i_1 = \frac{v_1}{R_1} = \frac{150}{50} = \underline{3\text{ A}}$$

v_2 and i_2 do not adhere to the passive convention so $i_2 = -\dfrac{v_2}{R_2} = -\dfrac{150}{25} = \underline{-6\text{ A}}$

The power absorbed by R_1 is $P_1 = v_1 i_1 = 150 \cdot 3 = \underline{450\text{ W}}$

The power absorbed by R_2 is $P_2 = -v_2 i_2 = -150(-6) = \underline{900\text{ W}}$

P 2.4-6 A current source and two resistors are connected in series in the circuit shown in Figure P 2.4-6. Elements connected in series have the same current, so $i_1 = i_s$ and $i_2 = i_s$ in this circuit. Suppose that $i_s = 2$ A, $R_1 = 4\ \Omega$, and $R_2 = 8\ \Omega$. Calculate the voltage across each resistor and the power absorbed by each resistor.

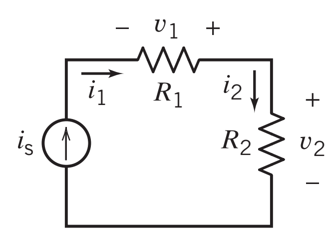
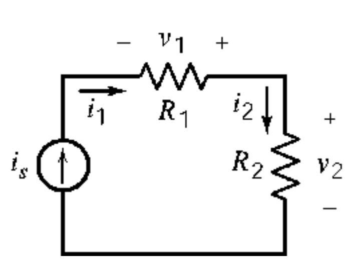


Figure P 2.4-6

Hint: Notice the reference directions of the resistor voltages.
Answer: $v_1 = -8$ V and $v_2 = 16$ V. R_1 absorbs 16 W and R_2 absorbs 32 W.

Solution:



$i_1 = i_2 = i_s = 25$ mA and $R_1 = 4\ \Omega$ and $R_2 = 8\ \Omega$

v_1 and i_1 do <u>not</u> adhere to the passive convention so

$v_1 = -R_1\ i_1 = -4(0.025) = \underline{-0.1\text{ V.}}$

The power absorbed by R_1 is

$P_1 = -v_1 i_1 = -(-0.1)(0.025) = \underline{2.5\text{ mW.}}$

v_2 and i_2 do adhere to the passive convention so $v_2 = R_2\ i_2 = 8(0.025) = \underline{0.2\text{ V.}}$

The power absorbed by R_2 is $P_2 = v_2 i_2 = (0.2)(0.025) = \underline{5\text{ mW.}}$

P 2.4-7 An electric heater is connected to a constant 250-V source and absorbs 1000 W. Subsequently, this heater is connected to a constant 210-V source. What power does it absorb from the 210-V source? What is the resistance of the heater?

Hint: Model the electric heater as a resistor.

Solution:

Model the heater as a resistor, then from $P = \dfrac{v^2}{R} \Rightarrow R = \dfrac{v^2}{P} = \dfrac{(250)^2}{1000} = 62.5\ \Omega$

with a 220 V source $\quad P = \dfrac{v^2}{R} = \dfrac{(220)^2}{62.5} = \underline{774.4\ \text{W}}$

P 2.4-8 The portable lighting equipment for a mine is located 100 meters from its dc supply source. The mine lights use a total of 5 kW and operate at 120 V dc. Determine the required cross-sectional area of the copper wires used to connect the source to the mine lights if we require that the power lost in the copper wires be less than or equal to 5 percent of the power required by the mine lights.

Hint: Model both the lighting equipment and the wire as resistors.

Solution:

The current required by the mine lights is: $i = \dfrac{P}{v} = \dfrac{5000}{120} = \dfrac{125}{3}$ A

Power loss in the wire is : $i^2\ R$

Thus the maximum resistance of the copper wire allowed is

$$R = \frac{0.05P}{i^2} = \frac{0.05\times 5000}{(125/3)^2} = 0.144\ \Omega$$

now since the length of the wire is $L = 2\times 100 = 200$ m$= 20{,}000$ cm

thus $R = \rho L / A$ with $\rho = 1.7\times 10^{-6}\Omega\cdot\text{cm}$ from Table 2.5–1

$$A = \frac{\rho L}{R} = \frac{1.7\times 10^{-6}\times 20{,}000}{0.144} = \underline{0.236\ \text{cm}^2}$$

***P 2.4-9** The resistance of a practical resistor depends on the nominal resistance and the resistance tolerance as follows:

$$R_{\text{nom}}\left(1-\frac{t}{100}\right) \le R \le R_{\text{nom}}\left(1+\frac{t}{100}\right)$$

where R_{nom} is the nominal resistance and t is the resistance tolerance expressed as a percentage. For example, a 100-Ω, 2 percent resistor will have a resistance given by

$$98\,\Omega \le R \le 102\,\Omega$$

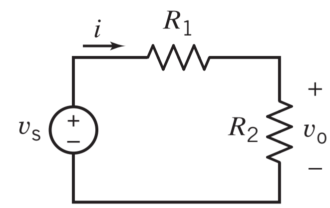


Figure P 2.4-9

The circuit shown in Figure P 2.4-9 has one input, v_s, and one output, v_o. The gain of this circuit is given by

$$\text{gain} = \frac{v_0}{v_s} = \frac{R_2}{R_1 + R_2}$$

Determine the range of possible values of the gain when R_1 is the resistance of a 100-Ω, 2 percent resistor and R_2 is the resistance of a 400-Ω, 5 percent resistor. Express the gain in terms of a nominal gain and a gain tolerance.

Solution:

$$0.7884 = \frac{380}{102+380} \le gain \le \frac{420}{98+420} = 0.8108$$

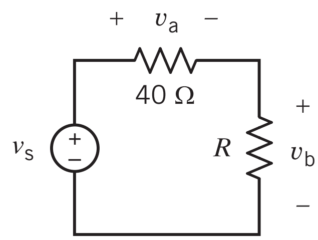
$$nominal\ gain = \frac{0.7884+0.8108}{2} = 0.7996$$

$$gain\ tolerance = \frac{0.7996-0.7884}{0.7996}\times 100 = \frac{0.8108-0.7996}{0.7996}\times 100 = 1.40\%$$

So

$$gain = 0.7996 \pm 1.40\%$$

P 2.4-10 The voltage source shown in Figure P 2.4-10 is an adjustable dc voltage source. In other words, the voltage v_s is a constant voltage, but the value of that constant can be adjusted. The tabulated data were collected as follows. The voltage, v_s, was set to some value, and the voltages across the resistor, v_a and v_b, were measured and recorded. Next, the value of v_s was changed, and the voltages across the resistors were measured again and recorded. This procedure was repeated several times. (The values of v_s were not recorded.) Determine the value of the resistance, R.



v_a, V	v_b, V
11.75	7.05
7.5	4.5
5.625	3.375
10	6
4.375	2.625

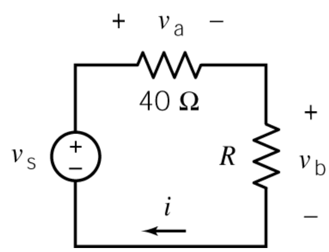
Figure P 2.4-10

Solution:

Label the current i as shown. That current is the element current in both resistors. First

$$i = \frac{v_a}{40}$$

Next $\quad v_b = R\,i = R\,\frac{v_a}{40} \quad \Rightarrow \quad R = 40\,\frac{v_b}{v_a}$



v_a, V	v_b, V
11.75	7.05
7.5	4.5
5.625	3.375
10	6
4.375	2.625

For example,

$$R = 40\,\frac{7.05}{11.75} = 24\ \Omega$$

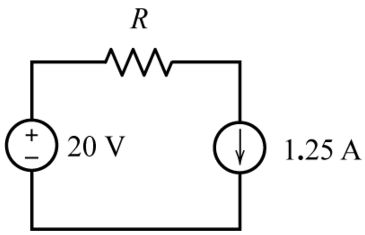


Figure P2.4-11

P2.4-11 Consider the circuit shown in Figure P2.4-11.

(a) Suppose the current source supplies 3.125 W of power. Determine the value of the resistance *R*.

(b) Suppose instead the resistance is *R* = 12 Ω. Determine the value of the power supplied by the current source.

Solution:

(a) Suppose the current source supplies 3.125 W of power.

$$1.25\,v_3 = 3.125 \quad \Rightarrow \quad v_3 = 2.5 \text{ V}$$

Then, using KVL

$$R = \frac{20 + 2.5}{1.25} = 18\ \Omega$$

(b) Suppose instead the resistance is *R* = 12 Ω. From KVL

$$1.25(12) - v_3 - 20 = 0 \quad \Rightarrow \quad v_3 = -5 \text{ V}$$

The value of the power supplied by the current source is

$$1.25\,v_3 = 1.25(-5) = -6.25 \text{ W}$$

P2.4-12. We will encounter "ac circuits" in Chapter 10. Frequently we analyze ac circuits using "phasors" and "impedances". Phasors are complex numbers that represents currents and voltages in an ac circuit. Impedances are complex numbers that describe ac circuit elements. (See Appendix B for a discussion of complex numbers.) Figure P2.4-11 shows a circuit element in an ac circuit. **I** and **V** are complex numbers representing the element current and voltage. **Z** is a complex number describing the element itself. "Ohm's law for ac circuits" indicates that

$$\mathbf{V} = \mathbf{Z}\,\mathbf{I}$$

(a) Suppose $\mathbf{V} = 12\angle 45°\ \text{V}$, $\mathbf{I} = B\angle\theta\ \text{A}$ and $\mathbf{Z} = 18 + j8\ \Omega$. Determine the values of B and θ.

(b) Suppose $\mathbf{V} = 48\angle 135°\ \text{V}$, $\mathbf{I} = 3\angle 15°\ \text{A}$ and $\mathbf{Z} = R + jX\ \Omega$. Determine the values of R and X.

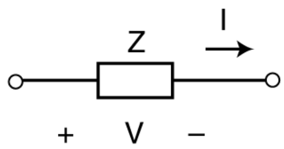


Figure P2.4-12

Solution:

(a)
$$12\angle 45° = (18 + j8)(B\angle\theta)$$

$$B\angle\theta = \frac{12\angle 45°}{18 + j8} = \frac{12\angle 45°}{19.7\angle 24°} = 0.609\angle 21°$$

so $B = 0.609$ A and $\theta = 21°$

(b)
$$48\angle 135° = (R + jX)(3\angle 15°)$$

$$R + jX = \frac{48\angle 75°}{3\angle 15°} = 18\angle 60° = 9 + j15.6\,\Omega$$

so $R = 9\ \Omega$ and $X = 15.6\ \Omega$

Section 2-5 Independent Sources

P 2.5-1 A current source and a voltage source are connected in parallel with a resistor as shown in Figure P 2.5-1. All of the elements connected in parallel have the same voltage, v_s in this circuit. Suppose that $v_s = 15$ V, $i_s = 3$ A, and $R = 5\ \Omega$. (a) Calculate the current i in the resistor and the power absorbed by the resistor. (b) Change the current source current to $i_s = 5$ A and recalculate the current, i, in the resistor and the power absorbed by the resistor.

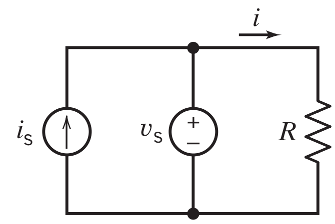


Figure P 2.5-1

Answer: $i = 3$ A and the resistor absorbs 45 W both when $i_s = 3$ A and when $i_s = 5$ A.

Solution:

(a) $i = \frac{v_s}{R} = \frac{15}{5} = \underline{3\ \text{A}}$ and $P = R\ i^2 = 5\,(3)^2 = \underline{45\ \text{W}}$

(b) i and P do not depend on i_s.

The values of i and P are 3 A and 45 W, both when $i_s = 3$ A and when $i_s = 5$ A.

P 2.5-2 A current source and a voltage source are connected in series with a resistor as shown in Figure P 2.5-2. All of the elements connected in series have the same current, i_s, in this circuit. Suppose that $v_s = 10$ V, $i_s = 2$ A, and $R = 5\ \Omega$. (a) Calculate the voltage v across the resistor and the power absorbed by the resistor. (b) Change the voltage source voltage to $v_s = 5$ V and recalculate the voltage, v, across the resistor and the power absorbed by the resistor.

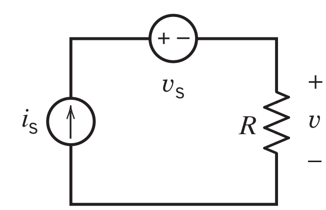


Figure P 2.5-2

Answer: $v = 10$ V and the resistor absorbs 20 W both when $v_s = 10$ V and when $v_s = 5$ V.

Solution:

(a) From Ohm's law $v = R\ i_s = 5(3) = 15$ V. (The resistor voltage does not depend on the voltage source voltage.) Next $P = \frac{v^2}{R} = \frac{15^2}{5} = \underline{45\ \text{W}}$.

(b) Since v and P do not depend on v_s the values of v and P are 15 V and 45 W both when v_s=10V and when v_s=5V.

P 2.5-3 The current source and voltage source in the circuit shown in Figure P 2.5-3 are connected in parallel so that they both have the same voltage, v_s. The current source and voltage source are also connected in series so that they both have the same current, i_s. Suppose that v_s = 12 V and i_s = 3 A. Calculate the power supplied by each source.
Answer: The voltage source supplies –36 W, and the current source supplies 36 W.

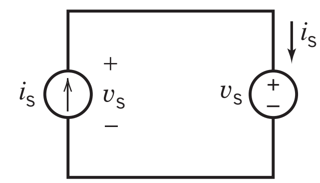


Figure P 2.5-3

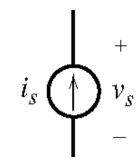
Solution:

Consider the current source:

i_s and v_s do not adhere to the passive convention,

so $P_{cs} = i_s v_s = 3 \cdot 12 = \underline{36 \text{ W}}$

is the power supplied by the current source.



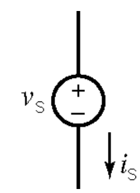
Consider the voltage source:

i_s and v_s do adhere to the passive convention,

so $P_{vs} = i_s\, v_s = 3 \cdot 12 = \underline{36 \text{ W}}$

is the power absorbed by the voltage source.

$\therefore$ The voltage source supplies −36 W.



P 2.5-4 The current source and voltage source in the circuit shown in Figure P 2.5-4 are connected in parallel so that they both have the same voltage, v_s. The current source and voltage source are also connected in series so that they both have the same current, i_s. Suppose that v_s = 12 V and i_s = 3 A. Calculate the power supplied by each source.

Answer: The voltage source supplies 36 W, and the current source supplies –36 W.

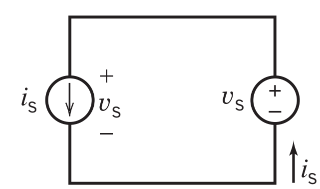
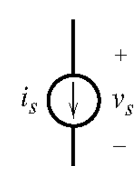


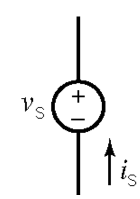
Figure P 2.5-4

Solution:

Consider the current source. i_S and v_S adhere to the passive convention so $P_{CS} = i_S\, v_S = 2(12) = 24$ W is the received by the current source. The current source supplies $\underline{-24 \text{ W}}$.



Consider the voltages source. i_S and v_S do not adhere to the passive convention so $P_{CS} = i_S\, v_S = 2(12) = \underline{24 \text{ W}}$ is the supplied by the voltage source.



P 2.5-5

(a) Find the power supplied by the voltage source shown in Figure P 2.5-5 when for $t \geq 0$ we have

$$v = 2\cos t \text{ V}$$

and

$$i = 10\cos t \text{ mA}$$

(b) Determine the energy supplied by this voltage source for the period $0 \leq t \leq 1$ s.

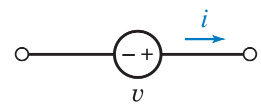


Figure P 2.5-5

Solution:

(a) $P = v\,i = (2\ \cos\ t)\ (10\ \cos\ t) = \underline{20\ \cos^2 t\ \ \text{mW}}$

(b) $w = \int_0^1 P\ dt\ =\ \int_0^1 20\ \cos^2 t\ dt = 20\left(\frac{1}{2}t + \frac{1}{4}\sin 2t\right)\Bigg|_0^1 = \underline{10 + 5\ \sin\ 2\ \text{mJ}}$

P 2.5-6 Figure P 2.5-6 shows a battery connected to a load. The load in Figure P 2.5-6 might represent automobile headlights, a digital camera, or a cell phone. The energy supplied by the battery to load is given by

$$w = \int_{t_1}^{t_2} vi\,dt$$

When the battery voltage is constant and the load resistance is fixed, then the battery current will be constant and

$$w = vi(t_2 - t_1)$$

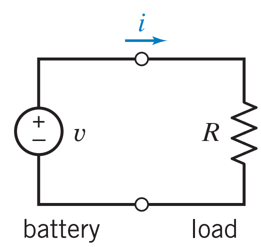


Figure P 2.5-6

The capacity of a battery is the product of the battery current and time required to discharge the battery. Consequently, the energy stored in a battery is equal to the product of the battery voltage and the battery capacity. The capacity is usually given with the units of Ampere-hours (Ah). A new 12-V battery having a capacity of 800 mAh is connected to a load that draws a current of 50 mA. (a) How long will it take for the load to discharge the battery? (b) How much energy will be supplied to the load during the time required to discharge the battery?

Solution:

(a) time to discharge $= \dfrac{\text{capacity}}{\text{current}} = \dfrac{800\text{ mAh}}{25\text{ mA}} = 32$ hours

(b) energy = (12 V) (0.025 A) (32 * 60*60 seconds) =34.56 kJ

Section 2-6 Voltmeters and Ammeters

P 2.6-1 For the circuit of Figure P 2.6-1:

(a) What is the value of the resistance *R*?

(b) How much power is delivered by the voltage source?

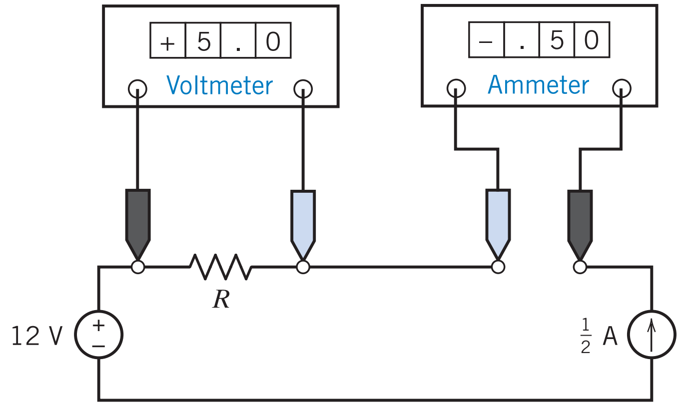


Figure P 2.6-1

Solution:

(a) $R = \dfrac{v}{i} = \dfrac{5}{0.5} = 10\ \Omega$

(b) The voltage, 12 V, and the current, 0.5 A, of the voltage source adhere to the passive convention so the power

$$P = 12\,(0.5) = 6\text{ W}$$

is the power received by the source. The voltage source delivers -6 W.

P 2.6-2 The current source in Figure P 2.6-2 supplies 40 W. What values do the meters in Figure P 2.6-2 read?

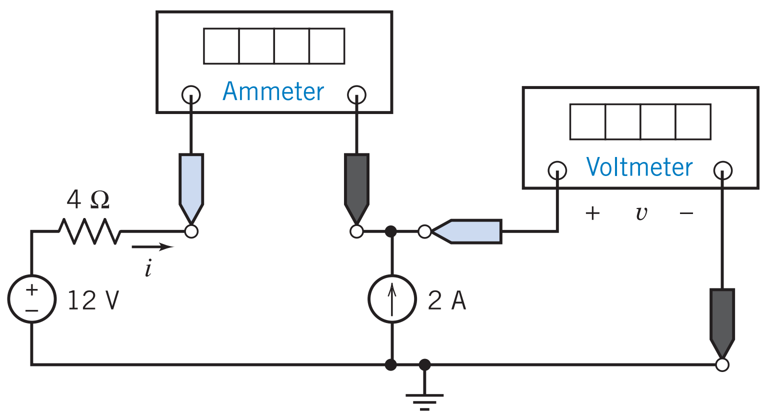


Figure P 2.6-2

Solution:

The voltmeter current is zero so the ammeter current is equal to the current source current except for the reference direction:

$$i = -2\text{ A}$$

The voltage v is the voltage of the current source. The power supplied by the current source is 40 W so

$$40 = 2\,v \quad \Rightarrow \quad v = 20\text{ V}$$

P 2.6-3 An ideal voltmeter is modeled as an open circuit. A more realistic model of a voltmeter is a large resistance. Figure P 2.6-3*a* shows a circuit with a voltmeter that measures the voltage v_m. In Figure P 2.6-3*b* the voltmeter is replaced by the model of an ideal voltmeter, an open circuit. Ideally, there is no current in the 100-Ω resistor and the voltmeter measures v_{mi} = 12 V, the ideal value of v_m. In Figure P 2.6-3*c* the voltmeter is modeled by the resistance R_m. Now the voltage measured by the voltmeter is

$$v_m = \left(\frac{R_m}{R_m + 100}\right) 12$$

As $R_m \to \infty$, the voltmeter becomes an ideal voltmeter and $v_m \to v_{mi} = 12$ V. When $R_m < \infty$, the voltmeter is not ideal and $v_m < v_{mi}$. The difference between v_m and v_{mi} is a measurement error caused by the fact that the voltmeter is not ideal.

(a) Express the measurement error that occurs when R_m = 900 Ω as a percent of v_{mi}.

(b) Determine the minimum value of R_m required to ensure that the measurement error is smaller than 2 percent of v_{mi}.

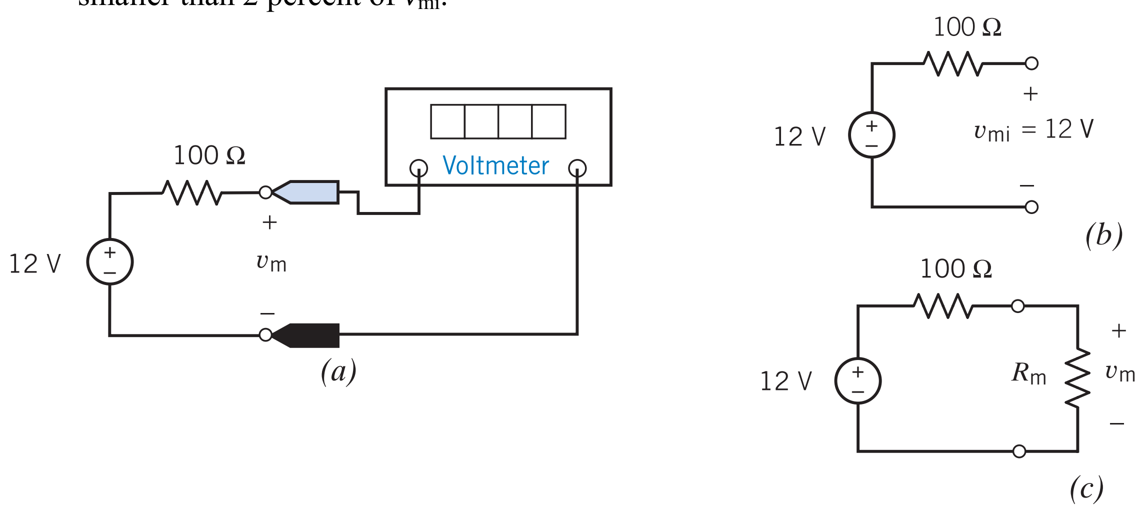


Figure P 2.6-3

Solution:

(a)

$$v_m = \left(\frac{900}{900+100}\right) 12 = 10.8 \text{ V}$$

$$\frac{12-10.8}{12} = 0.1 = 10\%$$

(b) We require

$$0.02 \ge \frac{12 - \left(\frac{R_m}{R_m+100}\right)12}{12} \quad \Rightarrow \quad \frac{R_m}{R_m+100} \ge 0.98 \quad \Rightarrow \quad R_m \ge 4900\ \Omega$$

(checked: LNAP 6/16/04)

2.6-4 An ideal ammeter is modeled as a short circuit. A more realistic model of an ammeter is a small resistance. Figure P 2.6-4*a* shows a circuit with an ammeter that measures the current i_m. In Figure P 2.6-4*b* the ammeter is replaced by the model of an ideal ammeter, a short circuit. Ideally, there is no voltage across the 1-kΩ resistor and the ammeter measures $i_{mi} = 2$ A, the ideal value of i_m. In Figure P 2.6-4*c* the ammeter is modeled by the resistance R_m. Now the current measured by the ammeter is

$$i_m = \left(\frac{1000}{1000 + R_m}\right)2$$

As $R_m \to 0$, the ammeter becomes an ideal ammeter and $i_m \to i_{mi} = 2$ A. When $R_m > 0$, the ammeter is not ideal and $i_m < i_{mi}$. The difference between i_m and i_{mi} is a measurement error caused by the fact that the ammeter is not ideal.

(a) Express the measurement error that occurs when $R_m = 10\ \Omega$ as a percent of i_{mi}.

(b) Determine the maximum value of R_m required to ensure that the measurement error is smaller than 5 percent.

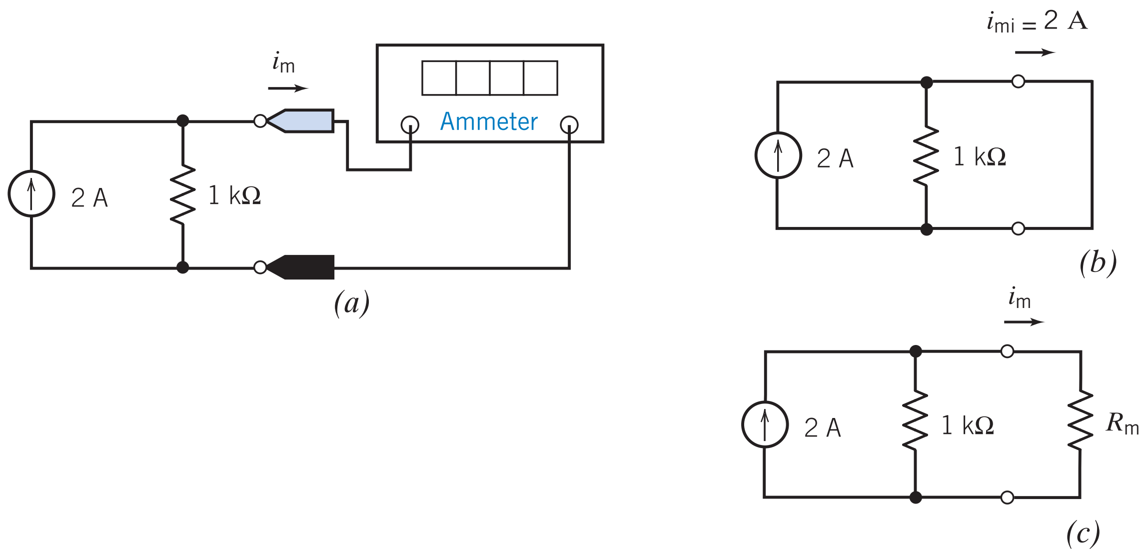


Figure P 2.6-4

Solution:

(a)

$$i_m = \left(\frac{1000}{1000+10}\right)2 = 1.98 \text{ A}$$

$$\%\text{ error} = \frac{2-1.98}{2} \times 100 = 0.99\%$$

(b)

$$0.05 \ge \frac{2-\left(\dfrac{1000}{1000+R_m}\right)2}{2} \quad \Rightarrow \quad \frac{1000}{1000+R_m} \ge 0.95 \quad \Rightarrow \quad R_m \le 52.63\ \Omega$$

(checked: LNAP 6/17/04)

P 2.6-5 The voltmeter in Figure P 2.6-5*a* measures the voltage across the current source. Figure P 2.6-5*b* shows the circuit after removing the voltmeter and labeling the voltage measured by the voltmeter as v_m. Also, the other element voltages and currents are labeled in Figure P 2.6-5*b*.

Given that

$$12 = v_R + v_m \text{ and } -i_R = i_s = 2\text{ A}$$

and

$$v_R = 25\, i_R$$

(a) Determine the value of the voltage measured by the meter.

(b) Determine the power supplied by each element.

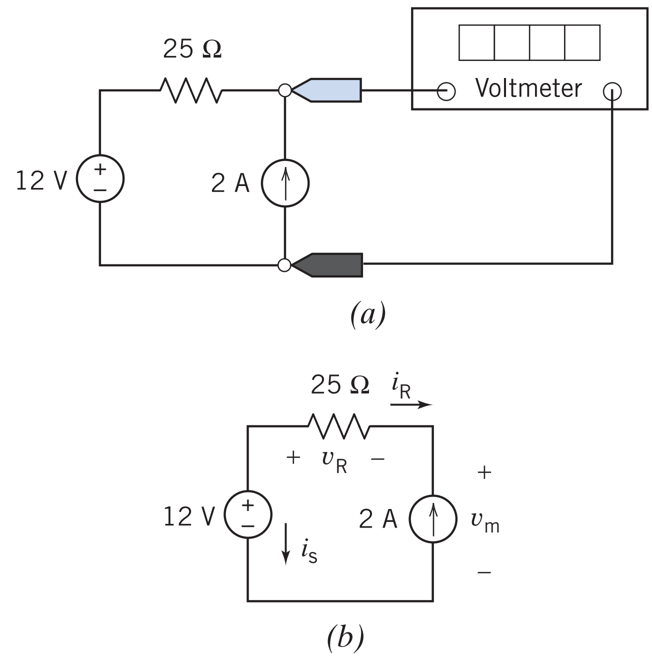


Figure P 2.6-5

Solution:

a.)

$$v_R = 25\, i_R = 25(-2) = -50 \text{ V}$$

$$v_m = 12 - v_R = 12 - (-50) = 62 \text{ V}$$

b.)

Element	**Power supplied**
voltage source	$-12(i_s) = -12(2) = -24$ W
current source	$62(2) = 124$ W
resistor	$-v_R \times i_R = -(-50)(-2) = -100$ W
total	0

P 2.6-6 The ammeter in Figure P 2.6-6*a* measures the current in the voltage source. Figure P 2.6-6*b* shows the circuit after removing the ammeter and labeling the current measured by the ammeter as i_m. Also, the other element voltages and currents are labeled in Figure P 2.6-6*b*.

Given that

$$2 + i_m = i_R \text{ and } v_R = v_s = 12\text{ V}$$

and $$v_R = 25 i_R$$

(a) Determine the value of the current measured by the meter.

(b) Determine the power supplied by each element.

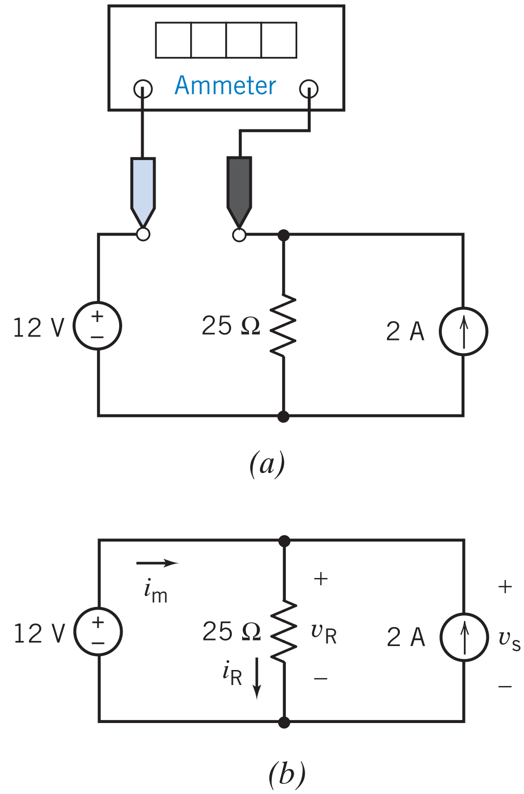


Figure P 2.6-6

Solution:

a.)

$$i_R = \frac{v_R}{25} = \frac{12}{25} = 0.48 \text{ A}$$

$$i_m = i_R - 2 = 0.48 - 2 = -1.52 \text{ A}$$

b.)

Element	**Power supplied**
voltage source	$12(i_m) = 12(-1.52) = -18.24$ W
current source	$v_s(2) = 12(2) = 24$ W
resistor	$-v_R \times i_R = -(12)(0.48) = -5.76$ W
total	0

Section 2-7 Dependent Sources

P 2.7-1 The ammeter in the circuit shown in Figure P 2.7-1 indicates that i_a = 2 A, and the voltmeter indicates that v_b = 8 V. Determine the value of *r*, the gain of the CCVS.
Answer: *r* = 4 V/A

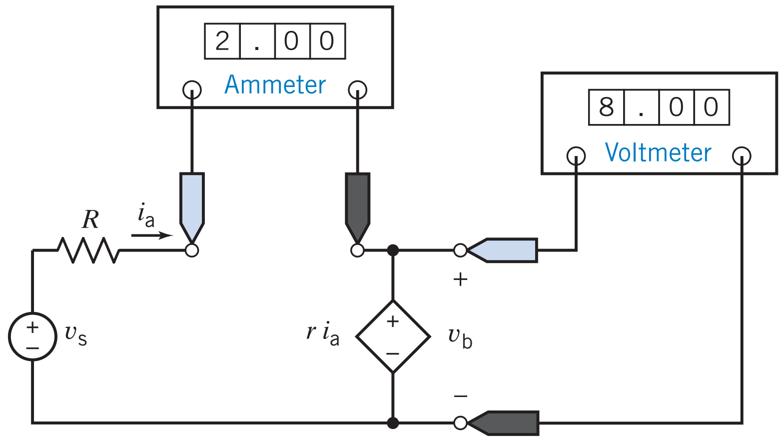
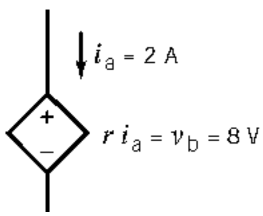


Figure P 2.7-1

Solution:



$$r = \frac{v_b}{i_a} = \frac{8}{2} = \underline{4\ \Omega}$$

P 2.7-2 The ammeter in the circuit shown in Figure P 2.7-2 indicates that i_a = 2 A, and the voltmeter indicates that v_b = 8 V. Determine the value of *g*, the gain of the VCCS.
Answer: *g* = 0.25 A/V

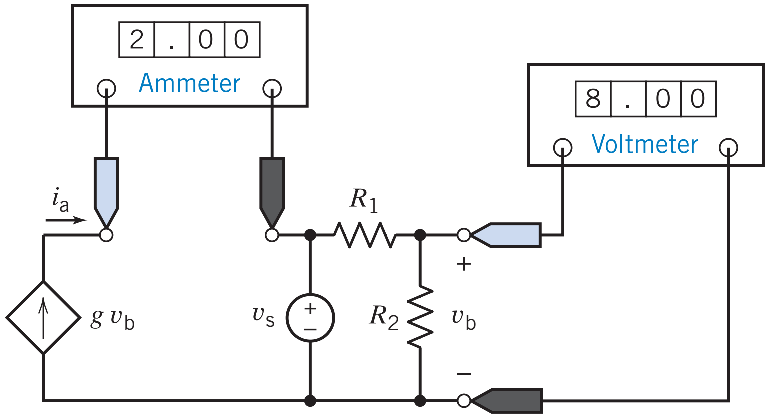


Figure P 2.7-2

Solution:

$$v_b = 8\text{ V}\;;\;\; g\, v_b = i_a = 2\text{ A}\;;\;\; g = \frac{i_a}{v_b} = \frac{2}{8} = \underline{0.25\ \frac{\text{A}}{\text{V}}}$$

P 2.7-3 The ammeters in the circuit shown in Figure P 2.7-3 indicate that i_a = 32 A and i_b = 8 A. Determine the value of *d*, the gain of the CCCS.

Answer: *d* = 4 A/A

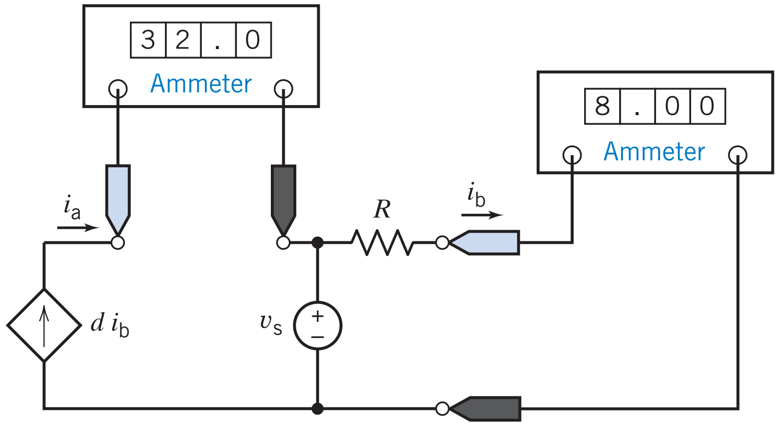


Figure P 2.7-3

Solution:

$$i_b = 8\text{ A}\;;\;\; d\, i_b = i_a = 32\text{A}\;;\;\; d = \frac{i_a}{i_b} = \frac{32}{8} = \underline{4\ \frac{\text{A}}{\text{A}}}$$

P 2.7-4 The voltmeters in the circuit shown in Figure P 2.7-4 indicate that v_a = 2 V and v_b = 8 V. Determine the value of *b*, the gain of the VCVS.

Answer: *b* = 4 V/V

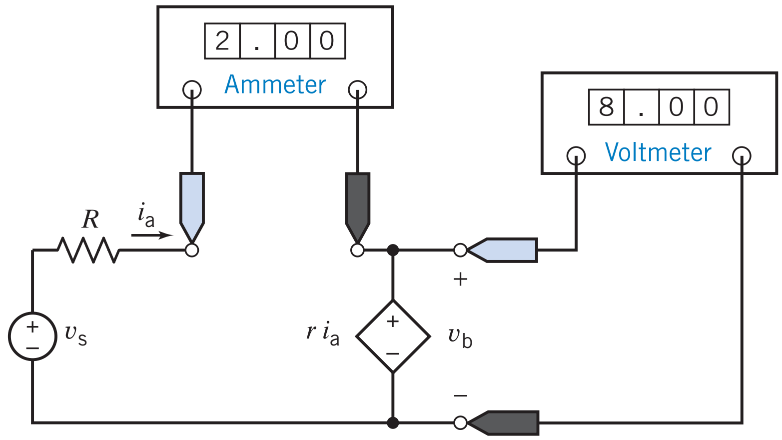


Figure P 2.7-4

Solution:

$$v_a = 2\text{ V}\ ;\quad b\, v_a = v_b = 8\text{ V}\ ;\quad b = \frac{v_b}{v_a} = \frac{8}{2} = 4\ \frac{\text{V}}{\text{V}}$$

P 2.7-5 The values of the current and voltage of each circuit element are shown in Figure P 2.7-5. Determine the values of the resistance, *R*, and of the gain of the dependent source, *A*.

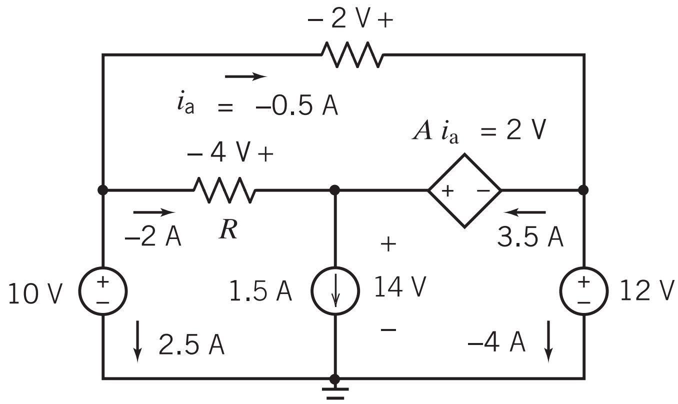


Figure P 2.7-5

Solution:

$$R = -\frac{4}{-2} = 2\ \Omega \quad \text{and} \quad A = \frac{2}{-0.5} = -4\ \frac{\text{V}}{\text{A}}$$

(checked: LNAP 6/6/04)

P 2.7-6 Find the power supplied by the VCCS in Figure P 2.7-6.
Answer: 17.6 watts are supplied by the VCCS. (–17.6 watts are absorbed by the VCCS.)

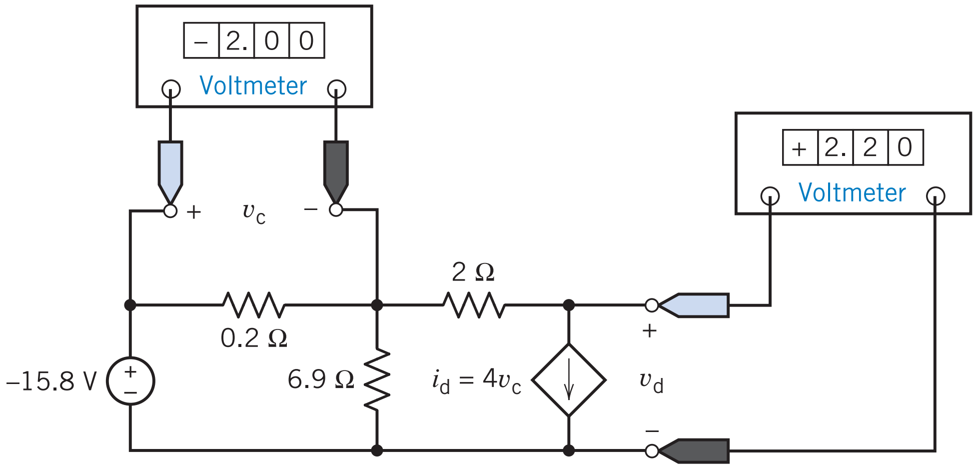


Figure P 2.7-6

Solution:

$$v_c = -2 \text{ V},\ i_d = 4\,v_c = -8 \text{ A and } v_d = 2.2 \text{ V}$$

i_d and v_d adhere to the passive convention so

$$P = v_d\ i_d = (2.2)\,(-8) = \underline{-17.6 \quad \text{W}}$$

is the power received by the dependent source. The power supplied by the dependent source is 17.6 W.

P2.7-7 The circuit shown in Figure P2.7-7 contains a dependent source. Determine the value of the gain k of that dependent source.

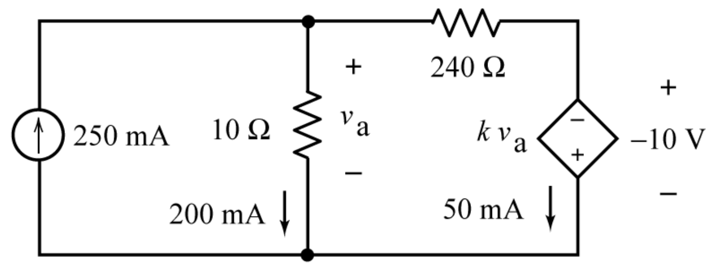


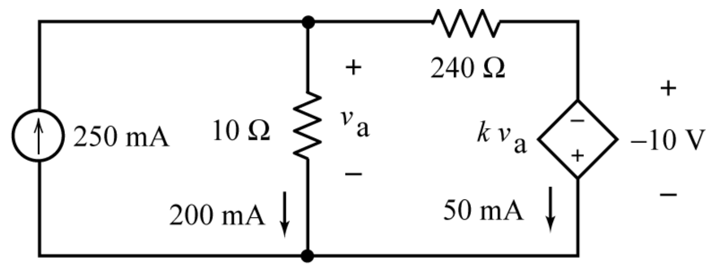
Figure P2.7-7

Solution:

$$v_a = 10(0.2) = 2 \text{ V}$$

$$k\,v_a = -(-10) = 10 \text{ V}$$

$$k = \frac{k\,v_a}{v_a} = \frac{10}{2} = 5 \ \frac{\text{V}}{\text{V}}$$



P2.7-8 The circuit shown in Figure P2.7-8 contains a dependent source. Determine the value of the gain k of that dependent source.

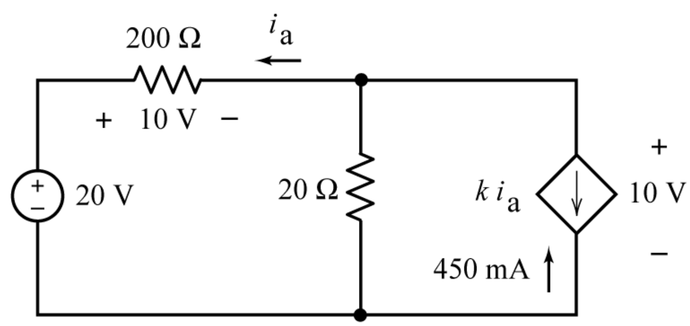


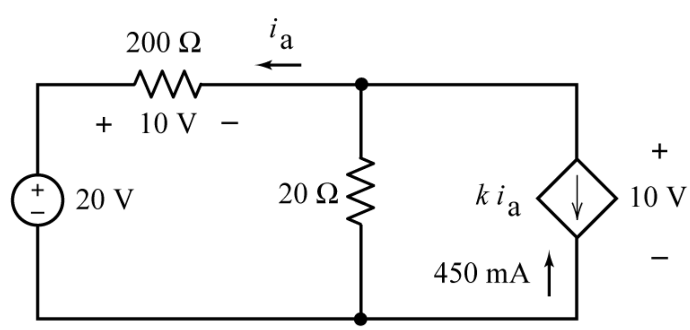
Figure P2.7-8

Solution:

$$i_a = -\frac{10}{200} = -0.05 \text{ A} = -50 \text{ mA}$$

$$k\,i_a = -450 \text{ mA}$$

$$k = \frac{k\,i_a}{i_a} = \frac{-450}{-50} = 9 \ \frac{\text{A}}{\text{A}}$$



P2.7-9 The circuit shown in Figure P2.7-9 contains a dependent source. The gain of that dependent source is

$$k = 25\ \frac{\text{V}}{\text{A}}$$

Determine the value of the voltage v_b.

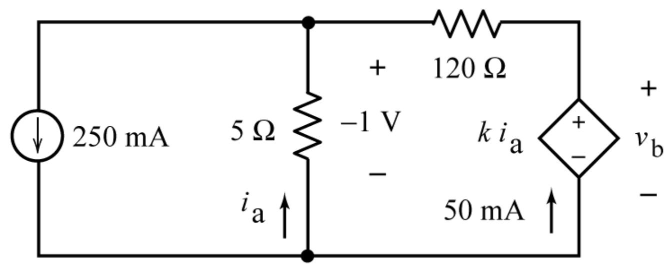


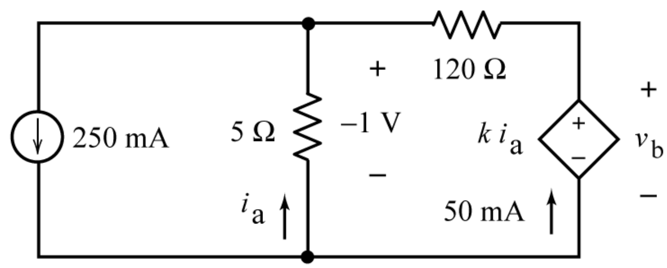
Figure P2.7-9

Solution:

$$i_a = -\frac{-1}{5} = 0.2\text{ A}$$

$$k = 25\ \frac{\text{V}}{\text{A}}$$

$$v_b = 25(0.2) = 5\text{ V}$$



P2.7-10 The circuit shown in Figure P2.7-10 contains a dependent source. The gain of that dependent source is

$$k = 90\ \frac{\text{mA}}{\text{V}} = 0.09\ \frac{\text{A}}{\text{V}}$$

Determine the value of the current i_b.

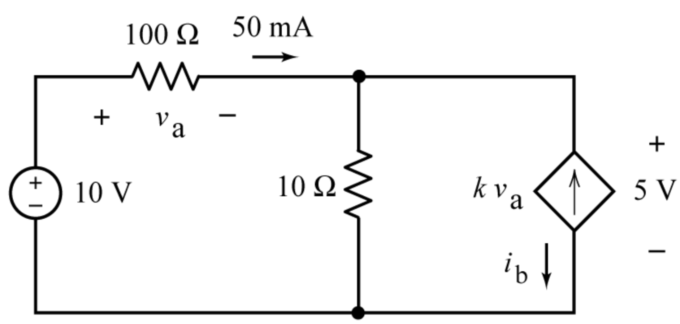


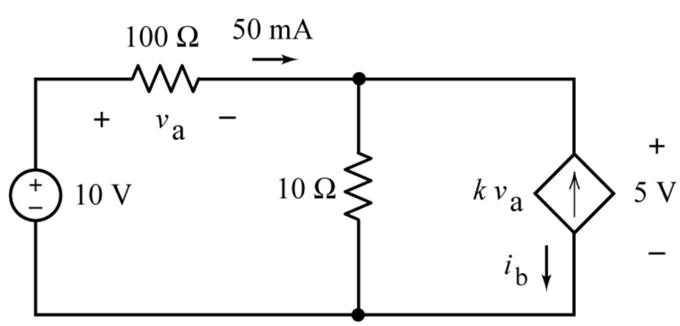
Figure P2.7-10

Solution:

$$v_a = 100(0.05) = 5\text{ V}$$

$$k = 90\ \frac{\text{mA}}{\text{V}} = 0.09\ \frac{\text{A}}{\text{V}}$$

$$i_b = -(0.09)(5) = -0.45\text{ A} = -450\text{ mA}$$



Section 2-8 Transducers

P 2.8-1 For the potentiometer circuit of Figure 2.8-2, the current source current and potentiometer resistance are 1.1 mA and 100 kΩ, respectively. Calculate the required angle, θ, so that the measured voltage is 23 V.

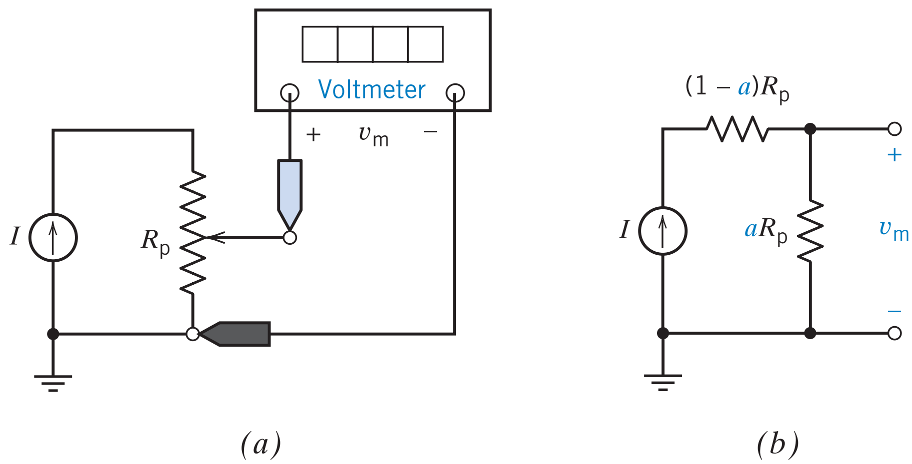


Figure 2.8-2

Solution: $$a = \frac{\theta}{360}\ , \quad \theta = \frac{360\ v_m}{R_p i} = \frac{(360)(23\text{ V})}{(100\text{ k}\Omega)(1.1\text{ mA})} = \underline{75.27^\circ}$$

P 2.8-2 An AD590 sensor has an associated constant $k = 1\frac{\mu A}{{}^\circ K}$. The sensor has a voltage v = 20 V; and the measured current, $i(t)$, as shown in Figure 2.8-3, is

$$4\ \mu\text{A} < i < 13\ \mu\text{A}$$

in a laboratory setting. Find the range of measured temperature.

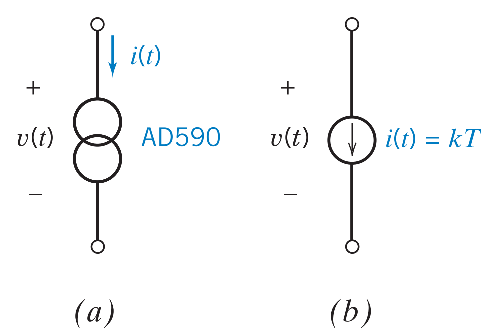


Figure 2.8-3

Solution:

$$\text{AD590}: k = 1\frac{\mu\text{A}}{{}^\circ\text{K}}\ ,$$

v=20 V (voltage condition satisfied)

$$\left.\begin{array}{l} 4\ \mu\text{A} < i < 13\ \mu\text{A} \\ T = \dfrac{i}{k} \end{array}\right\} \Rightarrow \underline{4^\circ\text{K} < T < 13^\circ\text{K}}$$

Section 2-9 Switches

P 2.9-1 Determine the current, i, at $t = 1$ s and at $t = 4$ s for the circuit of Figure P 2.9-1.

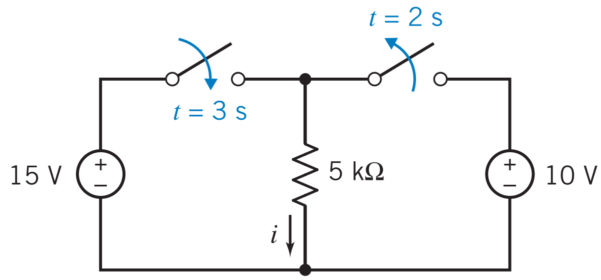
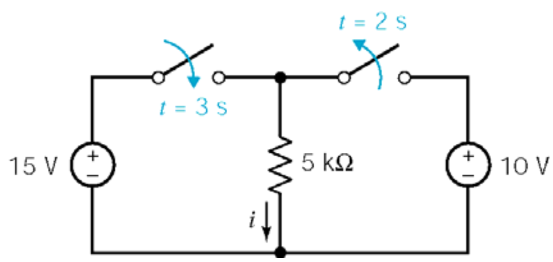


Figure P 2.9-1

Solution:



At $t = 1$ s the left switch is open and the right switch is closed so the voltage across the resistor is 10 V.

$$i = \frac{v}{R} = \frac{10}{5\times10^3} = \underline{2\ \text{mA}}$$

At $t = 4$ s the left switch is closed and the right switch is open so the voltage across the resistor is 15 V.

$$i = \frac{v}{R} = \frac{15}{5\times10^3} = \underline{3\ \text{mA}}$$

P 2.9-2 Determine the voltage, v, at $t = 1$ s and at $t = 4$ s for the circuit shown in Figure P 2.9-2.

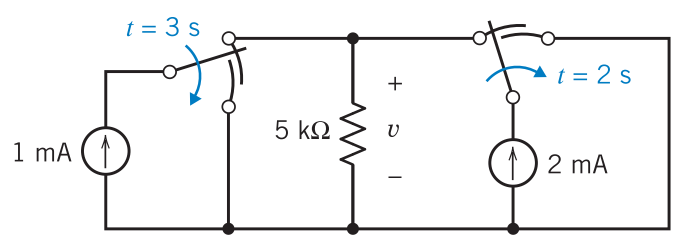
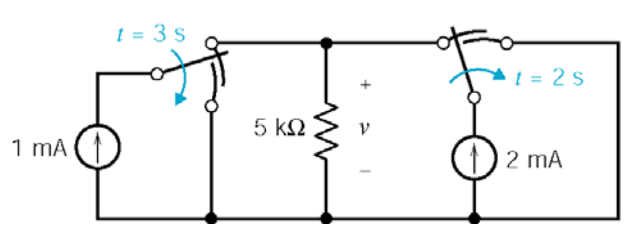


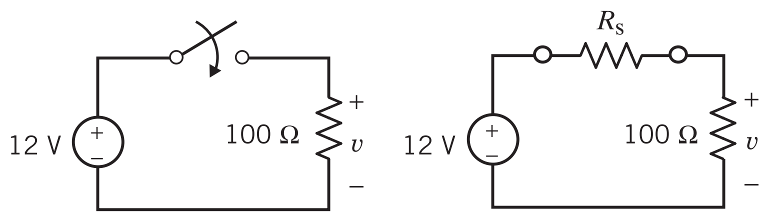
Figure P 2.9-2

Solution:

At $t = 1$ s the current in the resistor is 3 mA so $\underline{v = 15\text{ V}}$.

At $t = 4$ s the current in the resistor is 0 A so $\underline{v = 0\text{ V}}$.





(a) *(b)*

Figure P 2.9-3

P 2.9-3 Ideally an open switch is modeled as an open circuit and a closed switch is modeled as a closed circuit. More realistically, an open switch is modeled as a large resistance and a closed switch is modeled as a small resistance.

Figure P 2.9-3*a* shows a circuit with a switch. In figure P 2.9-3*b* the switch has been replaced with a resistance. In figure P 2.9-3*b* the voltage v is given by

$$v = \left(\frac{100}{R_s + 100}\right) 12$$

Determine the value of v for each of the following cases.

(a) The switch is closed and $R_s = 0$ (a short circuit).
(b) The switch is closed and $R_s = 5\ \Omega$.
(c) The switch is open and $R_s = \infty$ (an open circuit).
(d) The switch is open and $R_s = 10\ \text{k}\Omega$.

Solution:

(a) $v = 12$ V

(b) $v = \left(\frac{100}{105}\right) 12 = 11.43$ V

(c) $v = 0$ V

(d) $v = \left(\frac{100}{10100}\right) 12 = 0.1188 \simeq 0.12$ V

Section 2-10 How Can We Check…?

P 2.10-1 The circuit shown in Figure P 2.10-1 is used to test the CCVS. Your lab partner claims that this measurement shows that the gain of the CCVS is –20 V/A instead of +20 V/A. Do you agree? Justify your answer.

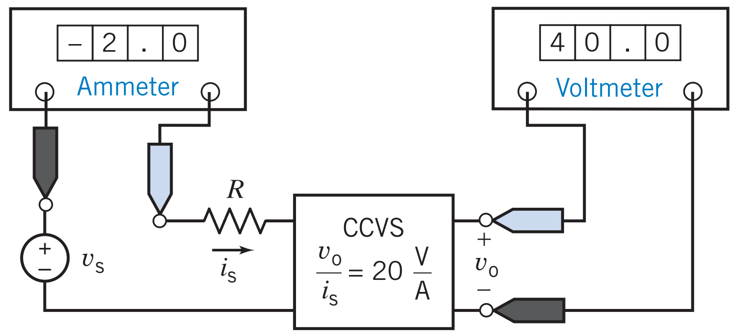


Figure P 2.10-1

Solution:

v_o=40 V and $i_s = -(-2) = 2$ A. (Notice that the ammeter measures $-i_s$ rather than i_s.)

So $$\frac{v_o}{i_s} = \frac{40}{2} = 20\ \frac{\text{V}}{\text{A}}$$

Your lab partner is wrong.

P 2.10-2 The circuit of Figure P 2.10-2 is used to measure the current in the resistor. Once this current is known, the resistance can be calculated as $R = \frac{v_s}{i}$. The circuit is constructed using a voltage source with v_s = 12 V and a 25-Ω, 1/2-W resistor. After a puff of smoke and an unpleasant smell, the ammeter indicates that i = 0 A. The resistor must be bad. You have more 25-Ω, 1/2-W resistors. Should you try another resistor? Justify your answer.

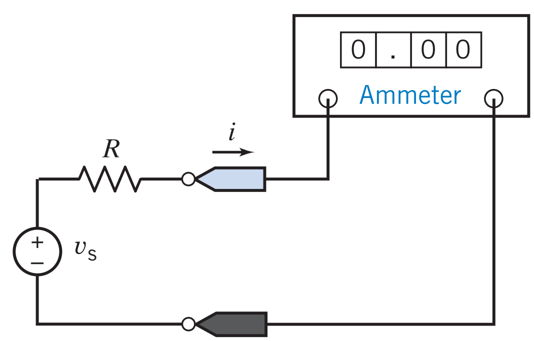


Figure P 2.10-2

Hint: 1/2-W resistors are able to safely dissipate one 1/2 W of power. These resistors may fail if required to dissipate more than 1/2 watt of power.

Solution:

We expect the resistor current to be $i = \frac{v_s}{R} = \frac{12}{25} = 0.48$ A. The power absorbed by this resistor will be $P = i\ v_s = (0.48)\ (12) = 5.76$ W.

A half watt resistor can't absorb this much power. You should not try another resistor.

Design Problems

DP 2-1 Specify the resistance *R* in Figure DP 2-1 so that both of the following conditions are satisfied:

1. $i > 40$ mA.

2. The power absorbed by the resistor is less than 0.5 W.

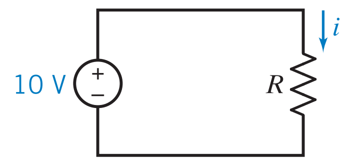


Figure DP 2-1

Solution:

$$1.)\ \frac{10}{R} > 0.04 \quad \Rightarrow \quad R < \frac{10}{0.04} = 250\,\Omega$$

$$2.)\ \frac{10^2}{R} < \frac{1}{2} \quad \Rightarrow \quad R > 200\,\Omega$$

Therefore 200 < R < 250 Ω. For example, R = 225 Ω.

DP 2-2 Specify the resistance *R* in Figure DP 2-2 so that both of the following conditions are satisfied:

1. $v > 40$ V.

2. The power absorbed by the resistor is less than 15 W.

Hint: There is no guarantee that specifications can always be satisfied.

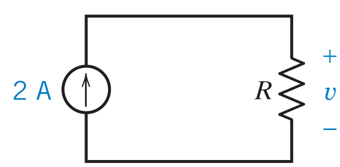


Figure DP 2-2

Solution:

$$1.)\ 2R > 40 \quad \Rightarrow \quad R > 20\,\Omega$$

$$2.)\ 2^2 R < 15 \quad \Rightarrow \quad R < \frac{15}{4} = 3.75\,\Omega$$

Therefore 20 < R < 3.75 Ω. These conditions cannot satisfied simultaneously.

DP 2-3 Resistors are given a power rating. For example, resistors are available with ratings of 1/8 W, 1/4 W, 1/2 W, and 1 W. A 1/2-W resistor is able to safely dissipate 1/2 W of power, indefinitely. Resistors with larger power ratings are more expensive and bulkier than resistors with lower power ratings. Good engineering practice requires that resistor power ratings be specified to be as large as, but not larger than, necessary.

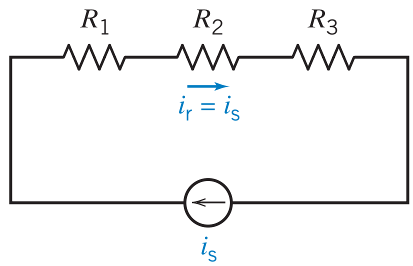


Figure DP 2-3

Consider the circuit shown in Figure DP 2-3. The values of the resistances are

$$R_1 = 1000\,\Omega,\ R_2 = 2000\,\Omega,\ \text{and}\ R_3 = 4000\,\Omega$$

The value of the current source current is

$$i_s = 30\,\text{mA}$$

Specify the power rating for each resistor.

Solution::

$$P_1 = (30\text{ mA})^2 \cdot (1000\ \Omega) = (.03)^2(1000) = 0.9\text{ W} < 1\text{ W}$$

$$P_2 = (30\text{ mA})^2 \cdot (2000\ \Omega) = (.03)^2(2000) = 1.8\text{ W} < 2\text{ W}$$

$$P_3 = (30\text{ mA})^2 \cdot (4000\ \Omega) = (.03)^2(4000) = 3.6\text{ W} < 4\text{ W}$$

Chapter 3 Resistive Circuits

Exercises

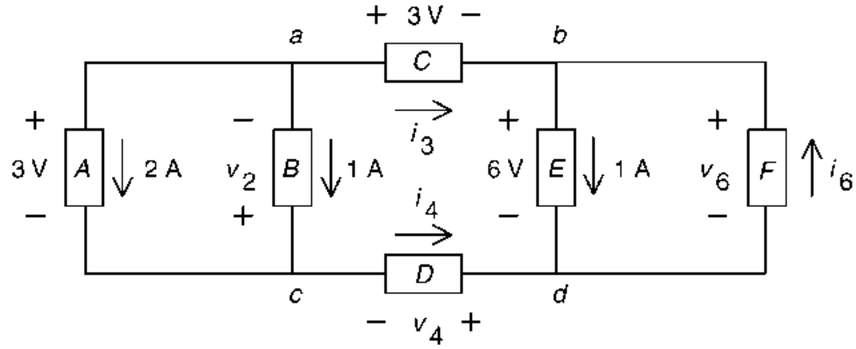


Figure E 3.2-1

Exercise 3.2-1 Determine the values of i_3, i_4, i_6, v_2, v_4, and v_6 in Figure E 3.2-1.

Answer: $i_3 = –3$ A, $i_4 = 3$ A, $i_6 = 4$ A, $v_2 = –3$ V, $v_4 = –6$ V, $v_6 = 6$ V

Solution:

Apply KCL at node *a* to get $2 + 1 + i_3 = 0 \Rightarrow i_3 = -3$ A

Apply KCL at node *c* to get $2 + 1 = i_4 \Rightarrow i_4 = 3$ A

Apply KCL at node *b* to get $i_3 + i_6 = 1 \Rightarrow -3 + i_6 = 1 \Rightarrow i_6 = 4$ A

Apply KVL to the loop consisting of elements *A* and *B* to get

$$-v_2 – 3 = 0 \Rightarrow v_2 = -3 \text{ V}$$

Apply KVL to the loop consisting of elements *C*, *E*, *D*, and *A* to get

$$3 + 6 + v_4 – 3 = 0 \Rightarrow v_4 = -6 \text{ V}$$

Apply KVL to the loop consisting of elements *E* and *F* to get

$$v_6 – 6 = 0 \Rightarrow v_6 = 6 \text{ V}$$

Check: The sum of the power supplied by all branches is

$$-(3)(2) + (-3)(1) – (3)(-3) + (-6)(3) – (6)(1) + (6)(4) = -6 - 3 + 9 - 18 - 6 + 24 = 0$$

Exercise 3.3-1 Determine the voltage measured by the voltmeter in the circuit shown in Figure E 3.3-1*a*.
Hint: Figure E3.3-1*b* shows the circuit after the ideal voltmeter has been replaced by the equivalent open circuit and a label has been added to indicate the voltage measured by the voltmeter, v_m.
Answer : $v_m = 2$ V

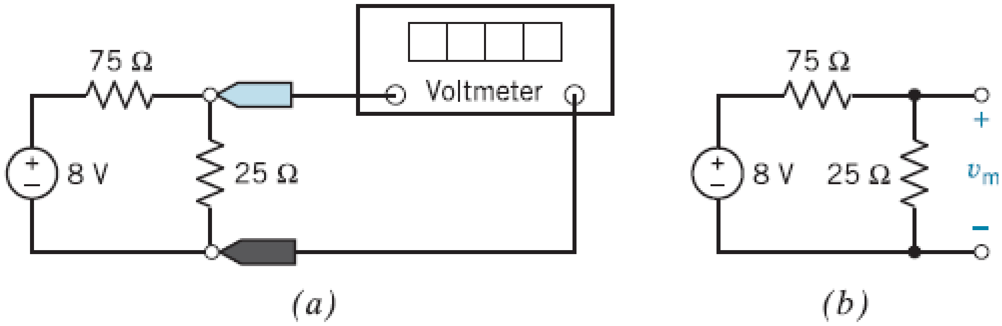


Figure E 3.3-1

Solution: From voltage division $\Rightarrow v_m = \dfrac{25}{25+75}(8) = 2$ V

Exercise 3.3-2 Determine the voltage measured by the voltmeter in the circuit shown in Figure E 3.3-2*a*.
Hint: Figure E 3.3-2*b* shows the circuit after the ideal voltmeter has been replaced by the equivalent open circuit and a label has been added to indicate the voltage measured by the voltmeter, v_m.
Answer**:** $v_m = -2$ V

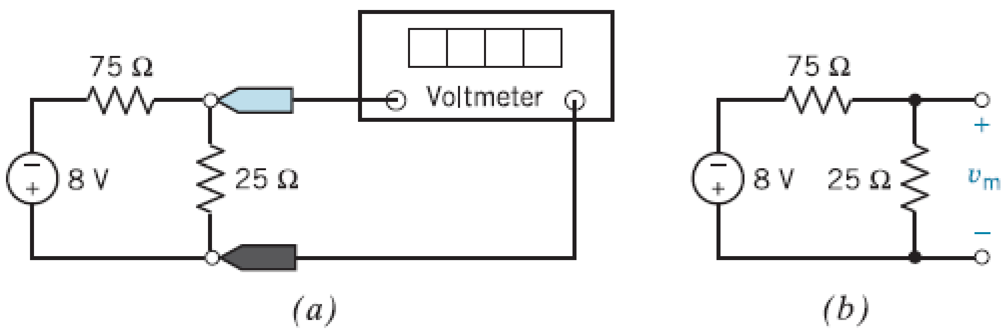


Figure E 3.3-2

Solution: From voltage division $\Rightarrow v_m = \dfrac{25}{25+75}(-8) = -2$ V

Exercise 3.4-1 A resistor network consisting of parallel resistors is shown in a package used for printed circuit board electronics in Figure E 3.4-1*a*. This package is only 2 cm × 0.7 cm, and each resistor is 1 kΩ. The circuit is connected to use four resistors as shown in Figure E 3.4-1*b*. Find the equivalent circuit for this network. Determine the current in each resistor when i_s = 1 mA.
Answer: $R_p = 250\ \Omega$

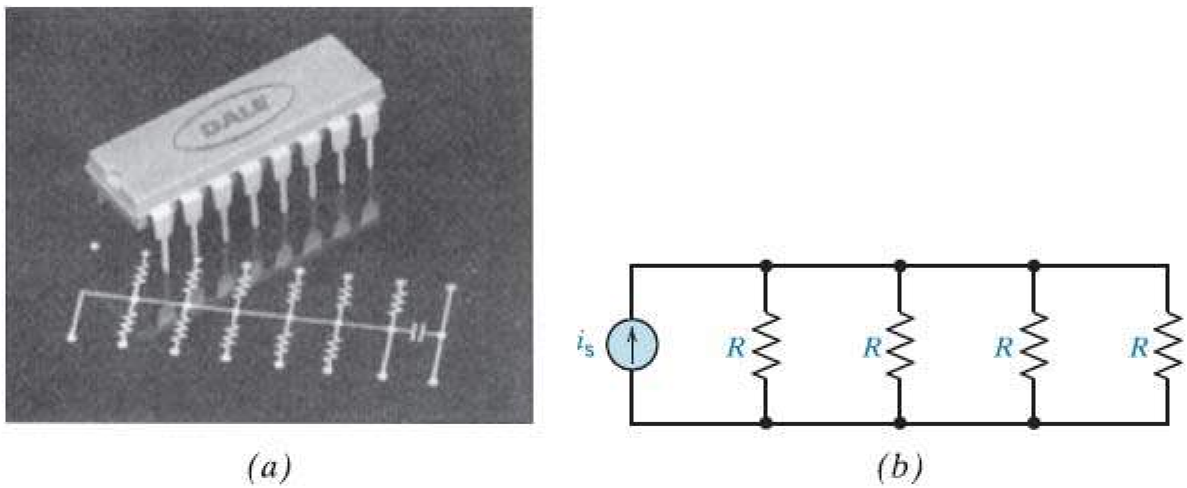
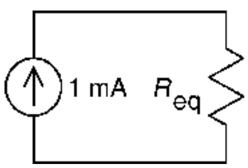


Figure E 3.4-1

Solution:



$$\frac{1}{R_{eq}} = \frac{1}{10^3}+\frac{1}{10^3}+\frac{1}{10^3}+\frac{1}{10^3}=\frac{4}{10^3} \Rightarrow R_{eq}=\frac{10^3}{4}=\underline{\frac{1}{4}\ \text{k}\Omega}$$

By current division, the current in each resistor $=\frac{1}{4}(10^{-3}) = \underline{\frac{1}{4}\ \text{mA}}$

Exercise 3.4-2 Determine the current measured by the ammeter in the circuit shown in Figure E 3.4-2*a*.

Hint: Figure E 3.4-2*b* shows the circuit after the ideal ammeter has been replaced by the equivalent short circuit and a label has been added to indicate the current measured by the ammeter, i_m.
Answer: $i_m = -1$ A

Solution:

From current division

$$i_m = \frac{10}{10+40}(-5) = -1 \quad \text{A}$$

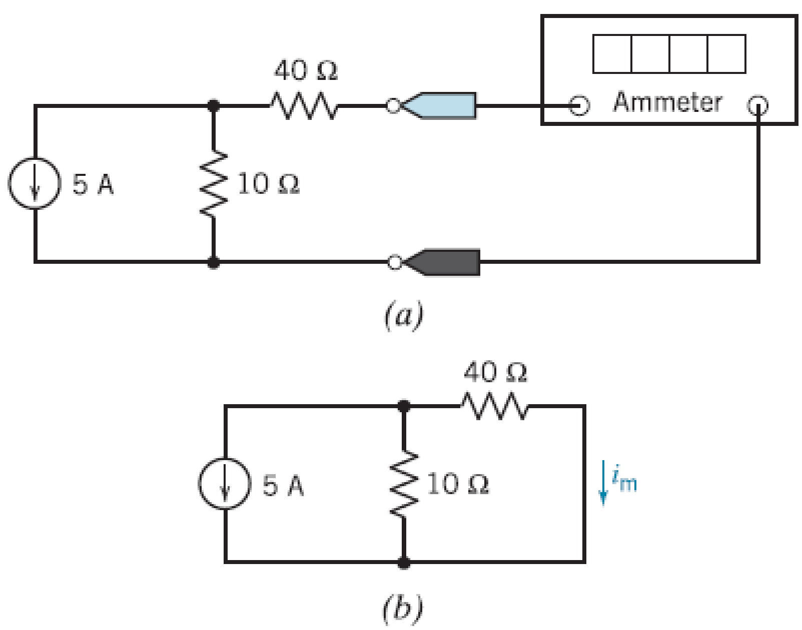


Figure E 3.4-2

Exercise 3.6-1 Determine the resistance measured by the ohmmeter in Figure E 3.6-1.

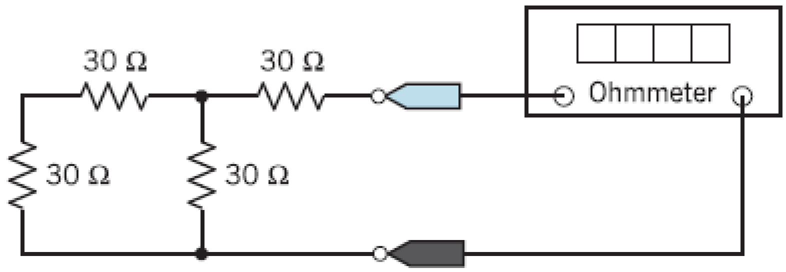


Figure E 3.6-1

Answer: $\frac{(30+30)\cdot 30}{(30+30)+30}+30=50\ \Omega$

Section 3-2 Kirchhoff's Laws

P 3.2-1 Consider the circuit shown in Figure P 3.2-1. Determine the values of the power supplied by branch *B* and the power supplied by branch *F*.

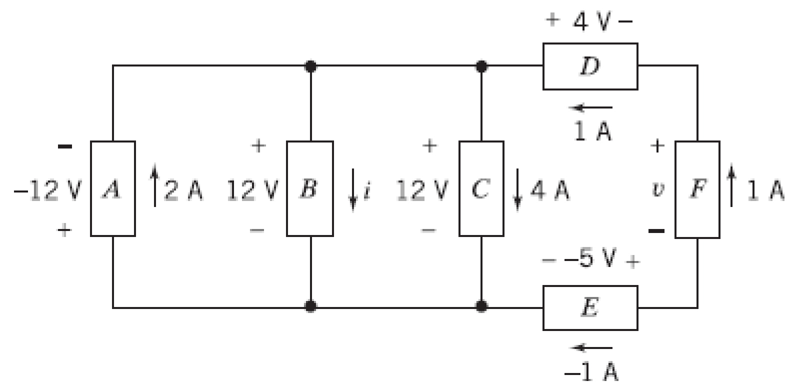
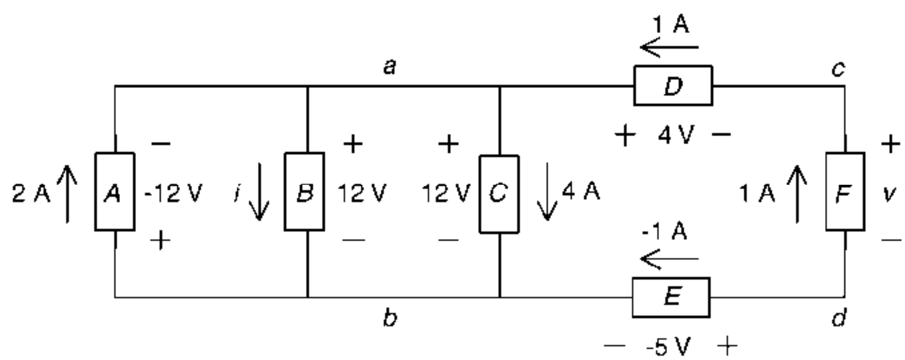


Figure P 3.2-1

Solution:



Apply KCL at node *a* to get $2 + 1 = i + 4 \Rightarrow i = -1$ A

The current and voltage of element *B* adhere to the passive convention so (12)(-1) = -12 W is power received by element *B*. The power supplied by element *B* is <u>12 W</u>.

Apply KVL to the loop consisting of elements *D*, *F*, *E*, and *C* to get

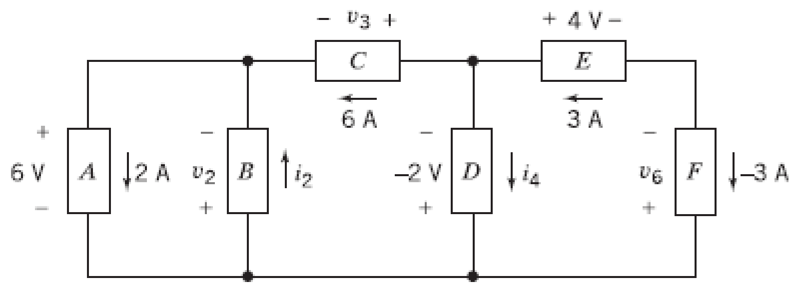
$$4 + v + (-5) - 12 = 0 \Rightarrow v = 13 \text{ V}$$

The current and voltage of element *F* do not adhere to the passive convention so (13)(1) = <u>13 W</u> is the power supplied by element *F*.

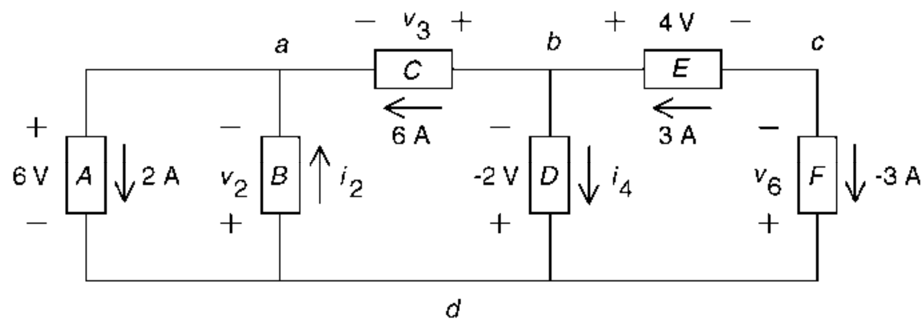
Check: The sum of the power supplied by all branches is

$$-(2)(-12) + \underline{12} - (4)(12) + (1)(4) + \underline{13} - (-1)(-5) = 24 + 12 - 48 + 4 + 13 - 5 = 0$$

P 3.2-2 Determine the values of i_2, i_4, v_2, v_3, and v_6 in Figure P 3.2-2.



Solution:



Apply KCL at node *a* to get $2 = i_2 + 6 = 0 \Rightarrow i_2 = -4$ A

Apply KCL at node *b* to get $3 = i_4 + 6 \Rightarrow i_4 = -3$ A

Apply KVL to the loop consisting of elements *A* and *B* to get

$$-v_2 - 6 = 0 \Rightarrow v_2 = -6 \text{ V}$$

Apply KVL to the loop consisting of elements *C*, *D*, and A to get

$$-v_3 - (-2) - 6 = 0 \Rightarrow v_4 = -4 \text{ V}$$

Apply KVL to the loop consisting of elements *E*, *F* and *D* to get

$$4 - v_6 + (-2) = 0 \Rightarrow v_6 = 2 \text{ V}$$

Check: The sum of the power supplied by all branches is

$$-(6)(2) - (-6)(-4) - (-4)(6) + (-2)(-3) + (4)(3) + (2)(-3) = -12 - 24 + 24 + 6 + 12 - 6 = 0$$

P 3.2-3 Consider the circuit shown in Figure P 3.2-3.

(a) Suppose that $R_1 = 6\ \Omega$ and $R_2 = 3\ \Omega$. Find the current i and the voltage v.

(b) Suppose, instead, that $i = 1.5$ A and $v = 2$ V. Determine the resistances R_1 and R_2.

(c) Suppose, instead, that the voltage source supplies 24 W of power and that the current source supplies 9 W of power. Determine the current i, the voltage v, and the resistances R_1 and R_2.

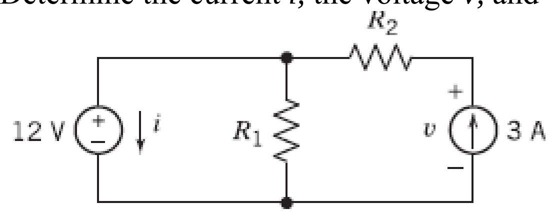
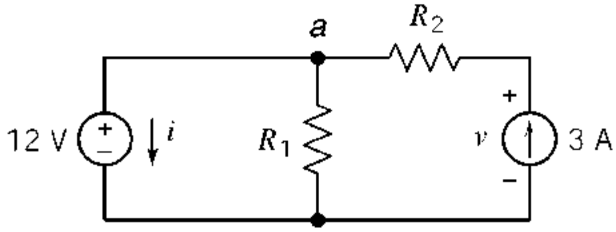


Figure P 3.2-3

Solution:



KVL : $-12 - R_2(3) + v = 0$ (outside loop)

$$v = 12 + 3R_2 \quad \text{or} \quad R_2 = \frac{v-12}{3}$$

KCL $\quad i + \frac{12}{R_1} - 3 = 0$ (top node)

$$i = 3 - \frac{12}{R_1} \quad \text{or} \quad R_1 = \frac{12}{3-i}$$

(a)

$$v = 12 + 3(4) = 24 \text{ V} \quad \text{and} \quad i = 3 - \frac{12}{8} = 1.5 \text{ A}$$

(b)

$$R_2 = \frac{42-12}{3} = \underline{10\,\Omega}\ ; \ R_1 = \frac{12}{3-2.25} = \underline{16\,\Omega}$$

(checked using LNAP 7/27/08)

(c) $24 = -12\, i$, because 12 and i adhere to the passive convention.

$$\therefore \underline{i = -2 \text{ A}} \quad \text{and} \quad R_1 = \frac{12}{3+2} = \underline{2.4\,\Omega}$$

$9 = 3v$, because 3 and v do not adhere to the passive convention

$$\therefore \underline{v = 3 \text{ V}} \quad \text{and} \quad R_2 = \frac{3-12}{3} = \underline{-3\,\Omega}$$

The situations described in (b) and (c) cannot occur if R_1 and R_2 are required to be nonnegative.

P 3.2-4 Determine the power absorbed by each of the resistors in the circuit shown in Figure P 3.2-4.

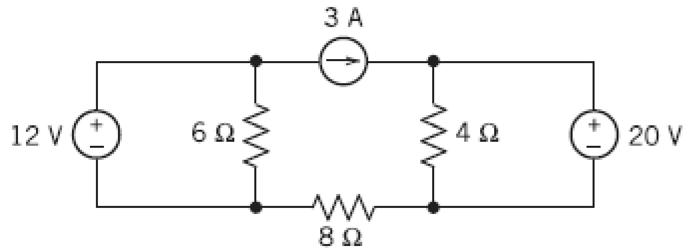
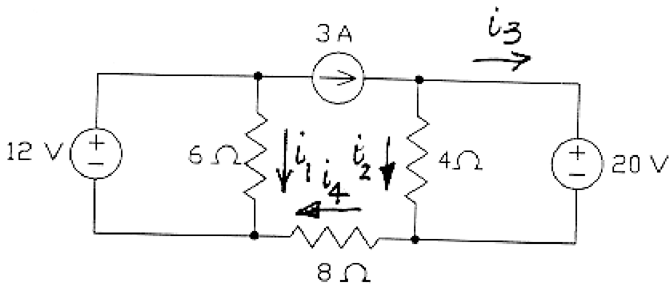


Figure P 3.2-4

Solution:



$$i_1 = \frac{12}{6} = 2 \text{ A}$$

$$i_2 = \frac{20}{4} = 5 \text{ A}$$

$$i_3 = 3 - i_2 = -2 \text{ A}$$

$$i_4 = i_2 + i_3 = 3 \text{ A}$$

Power absorbed by the 4 Ω resistor = $4 \cdot i_2^2$ = <u>100 W</u>

Power absorbed by the 6 Ω resistor = $6 \cdot i_1^2$ = <u>24 W</u>

Power absorbed by the 8 Ω resistor = $8 \cdot i_4^2$ = <u>72 W</u>

(checked using LNAP 8/16/02)

P 3.2-5 Determine the power absorbed by each of the resistors in the circuit shown in Figure P 3.2-5.
Answer: The 4-Ω resistor absorbs 16 W, the 6-Ω resistor absorbs 24 W, and the 8-Ω resistor absorbs 8 W.

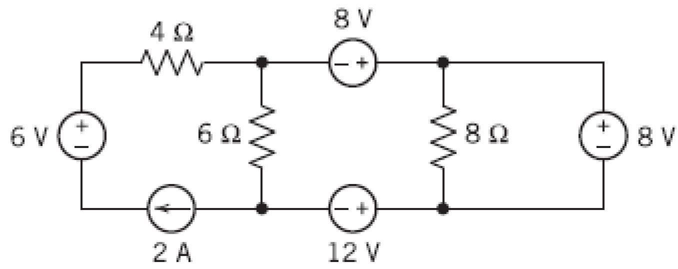
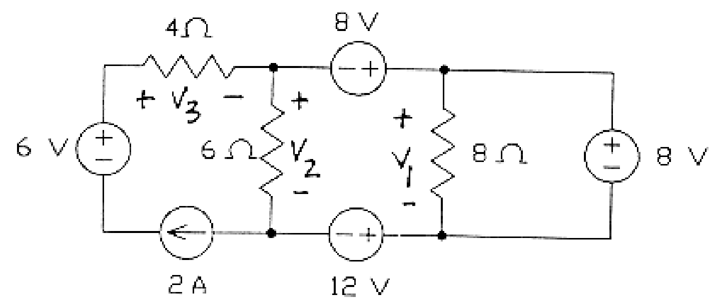


Figure P 3.2-5

Solution:



$$v_1 = 8\text{ V}$$
$$v_2 = -8+8+12 = 12\text{ V}$$
$$v_3 = 2\cdot 4 = 8\text{ V}$$
$$4\Omega:\quad P = \frac{v_3^2}{4} = \underline{16\text{ W}}$$
$$6\Omega:\quad P = \frac{v_2^2}{6} = \underline{24\text{ W}}$$
$$8\Omega:\quad P = \frac{v_1^2}{8} = \underline{8\text{ W}}$$

(checked using LNAP 8/16/02)

P 3.2-6 Determine the power supplied by each voltage source in the circuit of Figure P 3.2-6.

Answer: The 2-V voltage source supplies 2 mW and the 3-V voltage source supplies –6 mW.

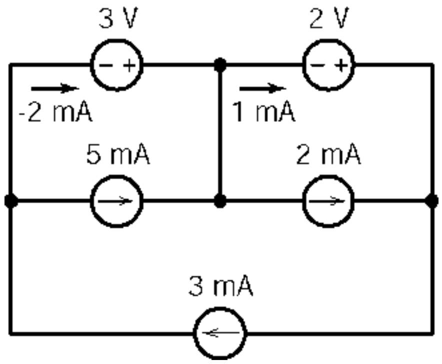
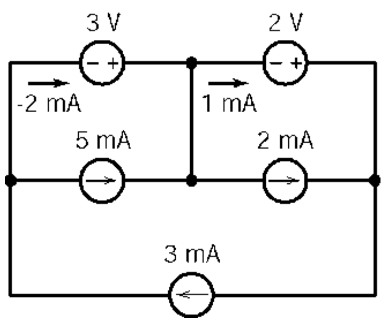


Figure P 3.2-6

Solution:



$$P_{2\text{ V}} = +\left[2\times\left(1\times10^{-3}\right)\right] = 2\times10^{-3} = 2 \text{ mW}$$

$$P_{3\text{ V}} = +\left[3\times\left(-2\times10^{-3}\right)\right] = -6\times10^{-3} = -6 \text{ mW}$$

(checked using LNAP 8/16/02)

P 3.2-7 What is the value of the resistance R in Figure P 3.2-7?
Hint: Assume an ideal ammeter. An ideal ammeter is equivalent to a short circuit.
Answer: $R = 4\ \Omega$

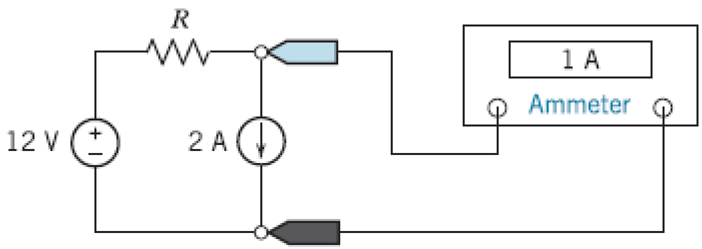
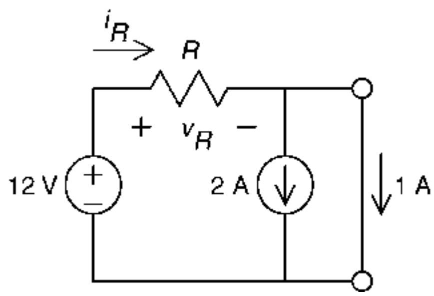


Figure P 3.2-7

Solution:



KCL: $i_R = 2 + 1 \quad \Rightarrow \quad i_R = 3\ \text{A}$

KVL: $v_R + 0 - 12 = 0 \quad \Rightarrow \quad v_R = 12\ \text{V}$

$$\therefore \quad R = \frac{v_R}{i_R} = \frac{12}{3} = 4\ \Omega$$

(checked using LNAP 8/16/02)

P 3.2-8 The voltmeter in Figure P 3.2-8 measures the value of the voltage across the current source to be 56 V. What is the value of the resistance *R*?

Hint: Assume an ideal voltmeter. An ideal voltmeter is equivalent to an open circuit.

Answer: $R = 10\ \Omega$

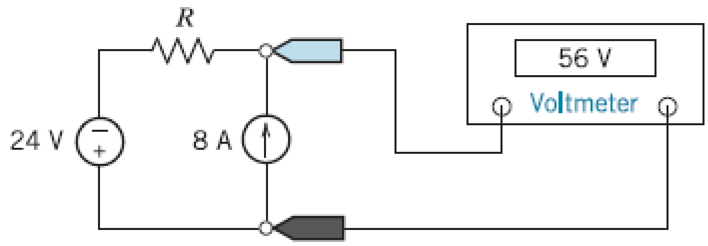
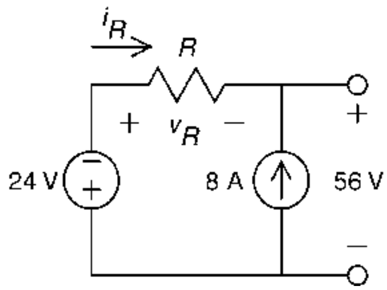


Figure P 3.2-8

Solution:



KVL: $v_R + 56 + 24 = 0 \Rightarrow v_R = -80\text{ V}$

KCL: $i_R + 8 = 0 \Rightarrow i_R = -8\text{ A}$

$\therefore \quad R = \dfrac{v_R}{i_R} = \dfrac{-80}{-8} = 10\ \Omega$

(checked using LNAP 8/16/02)

P 3.2-9 Determine the values of the resistances R_1 and R_2 in Figure P 3.2-9.

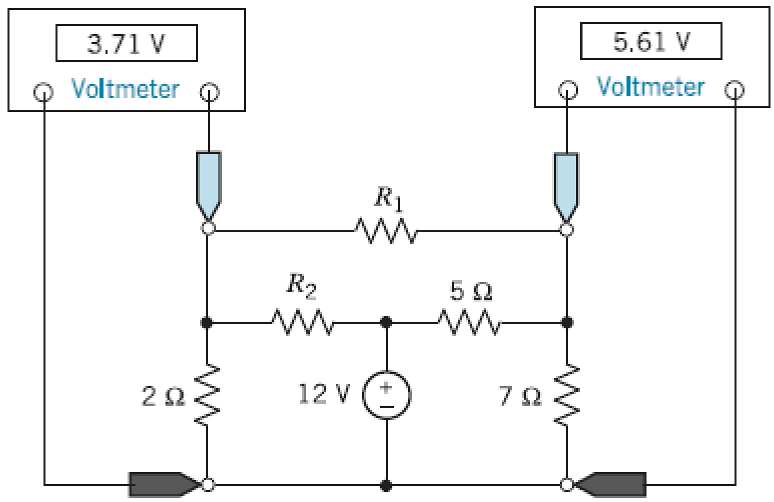
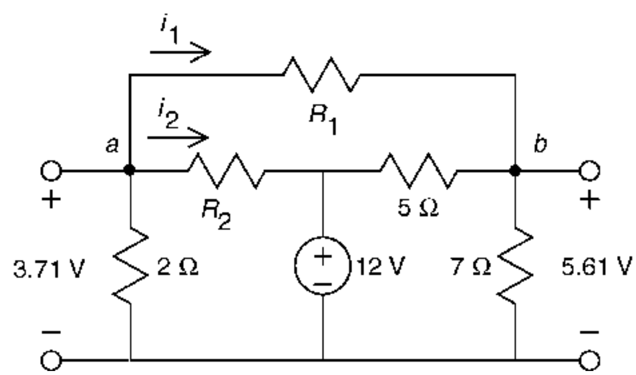


Figure P 3.2-9

Solution:



KCL at node *b*:

$$\frac{5.61}{7} = \frac{3.71 - 5.61}{R_1} + \frac{12 - 5.61}{5} \quad \Rightarrow \quad 0.801 = \frac{-1.9}{R_1} + 1.278$$

$$\Rightarrow \quad R_1 = \frac{1.9}{1.278 - 0.801} = 3.983 \approx 4\ \Omega$$

KCL at node *a*:

$$\frac{3.71}{2} + \frac{3.71 - 5.61}{4} + \frac{3.71 - 12}{R_2} = 0 \quad \Rightarrow \quad 1.855 + (-0.475) + \frac{-8.29}{R_2} = 0$$

$$\Rightarrow \quad R_2 = \frac{8.29}{1.855 - 0.475} = 6.007 \approx 6\ \Omega$$

(checked using LNAP 8/16/02)

P 3.2-10 The circuit shown in Figure P 3.2-10 consists of five voltage sources and four current sources. Express the power supplied by each source in terms of the voltage source voltages and the current source currents.

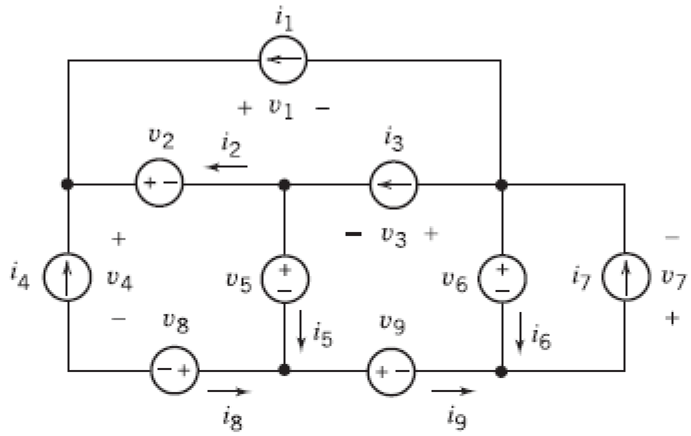


Figure P 3.2-10

Solution:

The subscripts suggest a numbering of the sources. Apply KVL to get

$$v_1 = v_2 + v_5 + v_9 - v_6$$

i_1 and v_1 do not adhere to the passive convention, so

$$p_1 = i_1 v_1 = i_1 \left(v_2 + v_5 + v_9 - v_6\right)$$

is the power supplied by source 1. Next, apply KCL to get

$$i_2 = -\left(i_1 + i_4\right)$$

i_2 and v_2 do not adhere to the passive convention, so

$$p_2 = i_2 v_2 = -\left(i_1 + i_4\right) v_2$$

is the power supplied by source 2. Next, apply KVL to get

$$v_3 = v_6 - \left(v_5 + v_9\right)$$

i_3 and v_3 adhere to the passive convention, so

$$p_3 = -i_3 v_3 = -i_3 \left(v_6 - \left(v_5 + v_9\right)\right)$$

is the power supplied by source 3. Next, apply KVL to get

$$v_4 = v_2 + v_5 + v_8$$

i_4 and v_4 do not adhere to the passive convention, so

$$p_4 = i_4 v_4 = i_4 \left(v_2 + v_5 + v_8\right)$$

is the power supplied by source 4. Next, apply KCL to get

$$i_5 = i_3 - i_2 = i_3 - \left(-\left(i_1 + i_4\right)\right) = i_1 + i_3 + i_4$$

i_5 and v_5 adhere to the passive convention, so

$$p_5 = -i_5 v_5 = -\left(i_1 + i_3 + i_4\right) v_5$$

is the power supplied by source 5. Next, apply KCL to get

$$i_6 = i_7 - \left(i_1 + i_3\right)$$

i_6 and v_6 adhere to the passive convention, so

$$p_6 = -i_6 v_6 = -\left(i_7 - \left(i_1 + i_3\right)\right) v_6$$

is the power supplied by source 6. Next, apply KVL to get

$$v_7 = -v_6$$

i_7 and v_7 adhere to the passive convention, so

$$p_7 = -i_7 v_7 = -i_7 \left(-v_6\right) = i_7 v_6$$

is the power supplied by source 7. Next, apply KCL to get

$$i_8 = -i_4$$

i_8 and v_8 do not adhere to the passive convention, so

$$p_8 = i_8 v_8 = \left(-i_4\right) v_8 = -i_4 v_8$$

is the power supplied by source 8. Finally, apply KCL to get

$$i_9 = i_1 + i_3$$

i_9 and v_9 adhere to the passive convention, so

$$p_9 = -i_9 v_9 = -\left(i_1 + i_3\right) v_9$$

is the power supplied by source 9.

(Check: $\sum_{n=1}^{9} p_n = 0$.)

P 3.2-11 Determine the power received by each of the resistors in the circuit shown in Figure P 3.2-11.

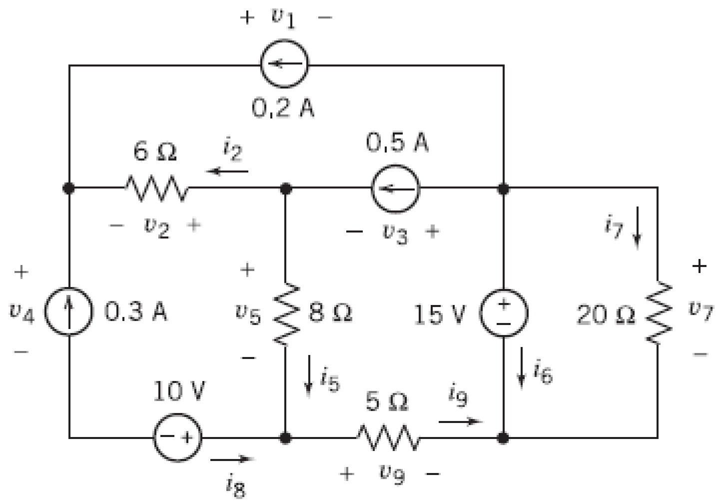


Figure P 3.2-11

Solution

The subscripts suggest a numbering of the circuit elements. Apply KCL to get

$$i_2 + 0.2 + 0.3 = 0 \quad \Rightarrow \quad i_2 = -0.5 \text{ A}$$

The power received by the 6 Ω resistor is

$$p_2 = 6 i_2^{\,2} = 6(-0.5)^2 = 1.5 \text{ W}$$

Next, apply KCL to get

$$i_5 = 0.2 + 0.3 + 0.5 = 1.0 \text{ A}$$

The power received by the 8 Ω resistor is

$$p_5 = 8 i_5^{\,2} = 8(1)^2 = 8 \text{ W}$$

Next, apply KVL to get

$$v_7 = 15 \text{ V}$$

The power received by the 20 Ω resistor is

$$p_7 = \frac{v_7^{\,2}}{20} = \frac{15^2}{20} = 11.25 \text{ W}$$

is the power supplied by source 7. Finally, apply KCL to get

$$i_9 = 0.2 + 0.5 = 0.7 \text{ A}$$

The power received by the 5 Ω resistor is

$$p_9 = 5 i_9^{\,2} = 5(0.7)^2 = 2.45 \text{ W}$$

P 3.2-12 Determine the voltage and current of each of the circuit elements in the circuit shown in Figure P 3.2-12.

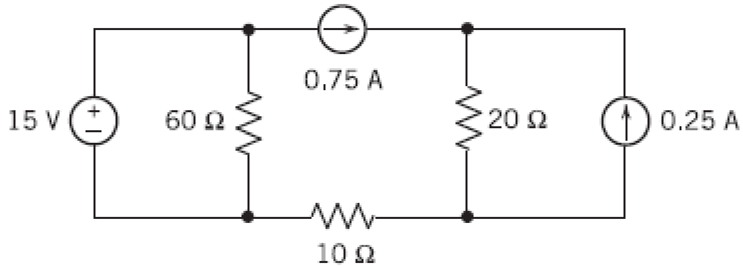
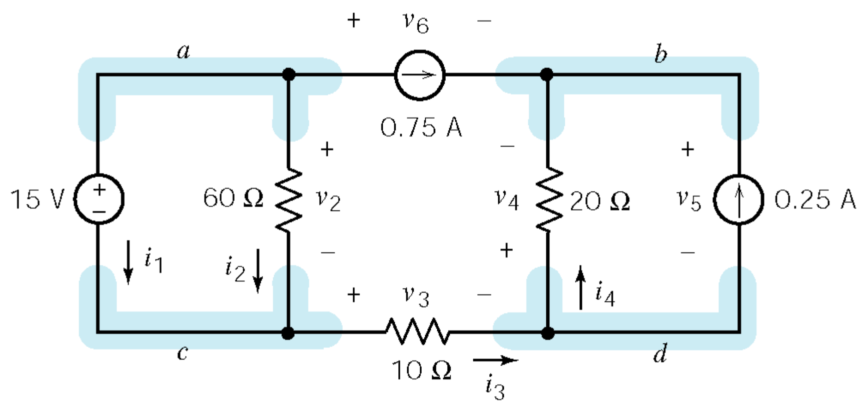


Figure P 3.2-12

Solution: We can label the circuit as follows:



The subscripts suggest a numbering of the circuit elements. Apply KCL at node *b* to get

$$i_4 + 0.25 + 0.75 = 0 \quad \Rightarrow \quad i_4 = -1.0 \text{ A}$$

Next, apply KCL at node *d* to get

$$i_3 = i_4 + 0.25 = -1.0 + 0.25 = -0.75 \text{ A}$$

Next, apply KVL to the loop consisting of the voltage source and the 60 Ω resistor to get

$$v_2 - 15 = 0 \quad \Rightarrow \quad v_2 = 15 \text{ V}$$

Apply Ohm's law to each of the resistors to get

$$i_2 = \frac{v_2}{60} = \frac{15}{60} = 0.25 \text{ A}, \qquad v_3 = 10\, i_3 = 10(-0.75) = -7.5 \text{ V}$$

and

$$v_4 = 20\, i_4 = 20(-1) = -20 \text{ V}$$

Next, apply KCL at node *c* to get

$$i_1 + i_2 = i_3 \quad \Rightarrow \quad i_1 = i_3 - i_2 = -0.75 - 0.25 = -1.0 \text{ A}$$

Next, apply KVL to the loop consisting of the 0.75 A current source and three resistors to get

$$v_6 - v_4 - v_3 - v_2 = 0 \quad \Rightarrow \quad v_6 = v_4 + v_3 + v_2 = -20 + (-7.5) + 15 = -12.5 \text{ V}$$

Finally, apply KVL to the loop consisting of the 0.25 A current source and the 20 Ω resistor to get

$$v_5 + v_4 = 0 \quad \Rightarrow \quad v_5 = -v_4 = -(-20) = 20 \text{ V}$$

(Checked: LNAPDC 8/28/04)

P 3.2-13 Determine the value of the current that is measured by the meter in Figure P 3.2-13.

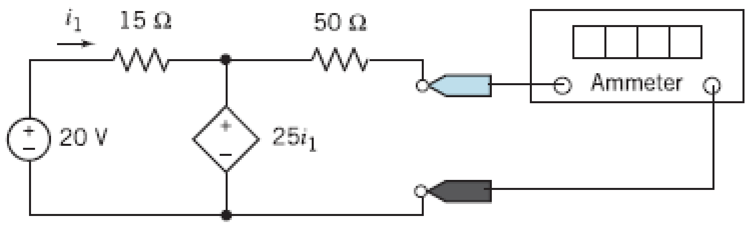
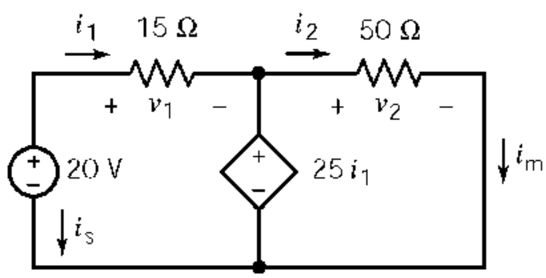


Figure P 3.2-13

Solution:
We can label the circuit as shown.



The subscripts suggest a numbering of the circuit elements. Apply KVL to node the left mesh to get

$$15 i_1 + 25 i_1 - 20 = 0 \quad \Rightarrow \quad i_1 = \frac{20}{40} = 0.5 \text{ A}$$

Apply KVL to node the left mesh to get

$$v_2 - 25 i_1 = 0 \quad \Rightarrow \quad v_2 = 25 i_1 = 25(0.5) = 12.5 \text{ V}$$

Apply KCL to get $i_m = i_2$. Finally, apply Ohm's law to the 50 Ω resistor to get

$$i_m = i_2 = \frac{v_2}{50} = \frac{12.5}{50} = 0.25 \text{ A}$$

(Checked: LNAPDC 9/1/04)

P 3.2-14 Determine the value of the voltage that is measured by the meter in Figure P 3.2-14.

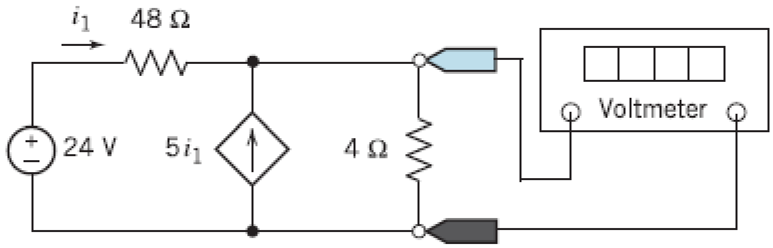
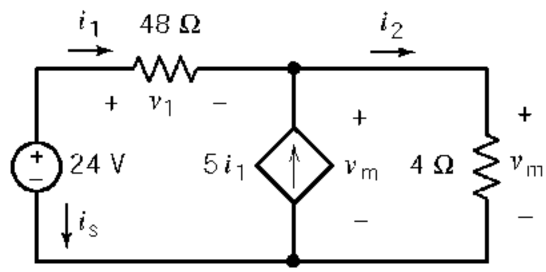


Figure P 3.2-14

Solution:

We can label the circuit as shown.



The subscripts suggest a numbering of the circuit elements. Ohm's law to the 48 Ω resistor to get

$$v_1 = 48 i_1$$

Apply KCL at the top node of the CCCS to get

$$i_1 + 5i_1 = i_2 \quad \Rightarrow \quad i_2 = 6i_1$$

Ohm's law to the 4 Ω resistor to get

$$v_m = 4i_2 = 4\left(6i_1\right) = 24 i_1$$

Apply KVL to the outside loop to get

$$v_1 + v_m - 24 = 0 \quad \Rightarrow \quad 48 i_1 + 24 i_1 = 24 \quad \Rightarrow \quad i_1 = \frac{24}{72} = \frac{1}{3} \text{ A}$$

Finally,

$$v_m = 24 i_1 = 24\left(\frac{1}{3}\right) = 8 \text{ V}$$

(Checked: LNAPDC 9/1/04)

P 3.2-15 Determine the value of the voltage that is measured by the meter in Figure P 3.2-15.

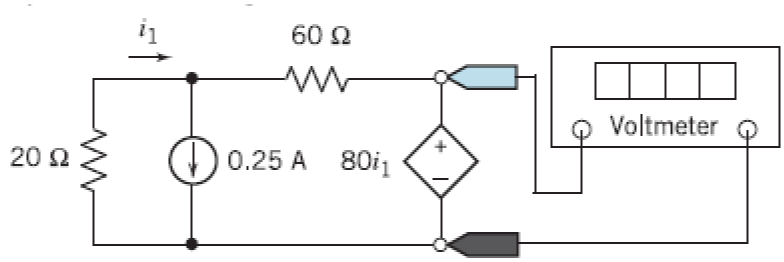
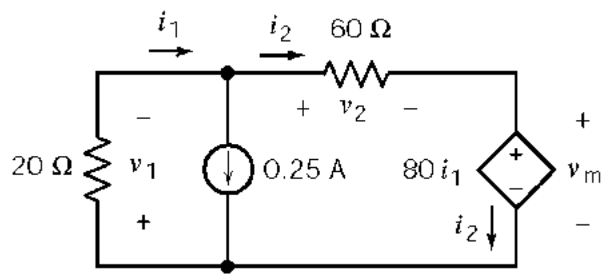


Figure P 3.2-15

Solution:

We can label the circuit as shown.



The subscripts suggest a numbering of the circuit elements. Apply KCL at the top node of the current source to get

$$i_1 = i_2 + 0.25$$

Apply Ohm's law to the resistors to get

$$v_1 = 20i_1 \quad \text{and} \quad v_2 = 60i_2 = 60(i_1 - 0.25) = 60i_1 - 15$$

Apply KVL to the outside to get

$$v_2 + 80i_1 + v_1 = 0 \quad \Rightarrow \quad (60i_1 - 15) + 80i_1 + 20i_1 = 0 \quad \Rightarrow \quad i_1 = \frac{15}{160} = 0.09375 \text{ A}$$

Finally, $$v_m = 80i_1 = 80(0.09375) = 7.5 \text{ V}$$

(Checked: LNAPDC 9/1/04)

P 3.2-16 The voltage source in Figure P 3.2-16 supplies 4.8 W of power. The current source supplies 3.6 W. Determine the values of the resistances, R_1 and R_2.

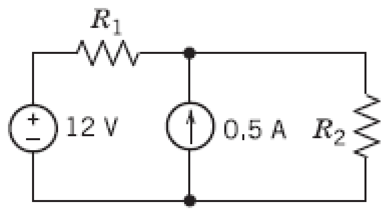
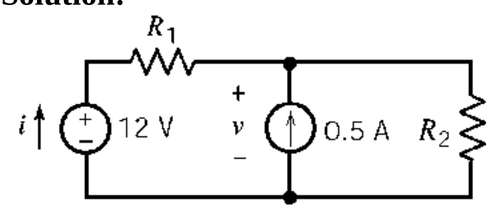


Figure P 3.2-16

Solution:



$$i=\frac{3.6}{12}=0.3\text{ A} \quad \text{and} \quad v=\frac{4.8}{0.5}=9.6\text{ V}$$

$$R_1=\frac{12-9.6}{0.3}=8\ \Omega \quad \text{and} \quad R_2=\frac{9.6}{0.3+0.5}=12\ \Omega$$

(Checked: LNAPDC 7/27/08)

P 3.2-17 Determine the current i in Figure P 3.3-17.
Answer: $i = 4$ A

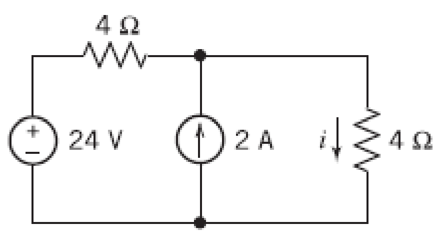
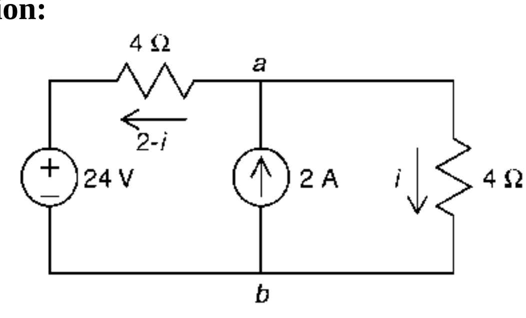


Figure P 3.3-17

Solution:



Apply KCL at node a to determine the current in the horizontal resistor as shown.

Apply KVL to the loop consisting of the voltages source and the two resistors to get

$-4(2-i) + 4(i) - 24 = 0 \Rightarrow i = 4$ A

P 3.2-18 Determine the value of the current i_m in Figure P 3.2-18*a*.

Hint: Apply KVL to the closed path a-b-d-c-a in Figure P 3.2-18*b* to determine v_a. Then apply KCL at node b to find i_m.

Answer: $i_m = 9$ A.

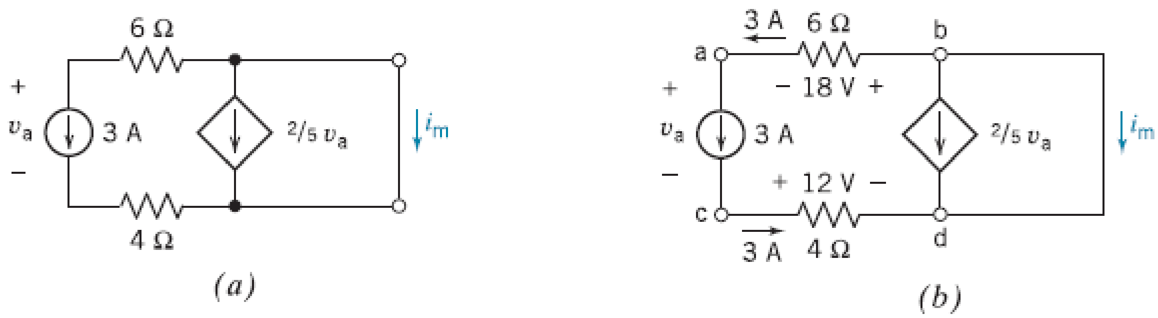


Figure P 3.2-18

Solution:

$$-18+0-12-v_a=0 \quad \Rightarrow \quad v_a=-30 \text{ V} \quad \text{and} \quad i_m=\frac{2}{5}v_a+3 \quad \Rightarrow \quad i_m=9 \text{ A}$$

P3.2-19 Determine the value of the voltage v_6 for the circuit shown in Figure P3.2-19.

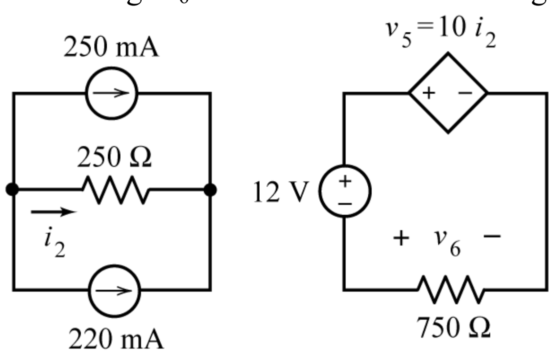


Figure P3.2-23

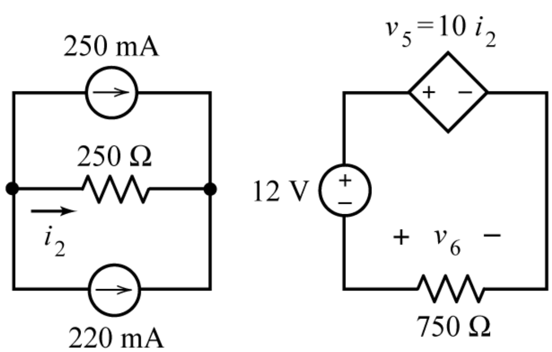
Solution:

Apply KCL at the left node:

$$0.25+i_2+0.22=0 \quad \Rightarrow \quad i_2=-0.47 \text{ A}$$

Use the element equation of the dependent source:

$$v_5=10\,i_2=10(-0.47)=-4.7 \text{ V}$$



Apply KVL to the right mesh

$$v_5-v_6-12=0 \quad \Rightarrow \quad v_6=v_5-12=-4.7-12=-16.7 \text{ V}$$

P3.2-20 Determine the value of the voltage v_6 for the circuit shown in Figure P3.2-20.

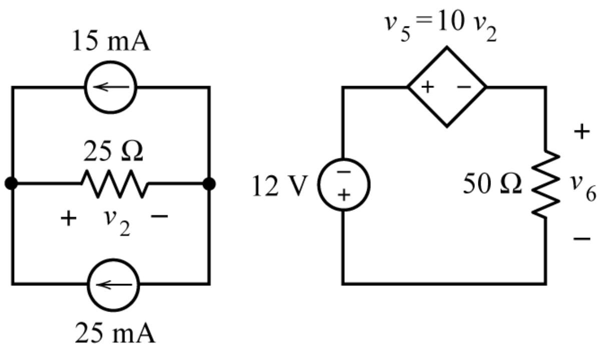
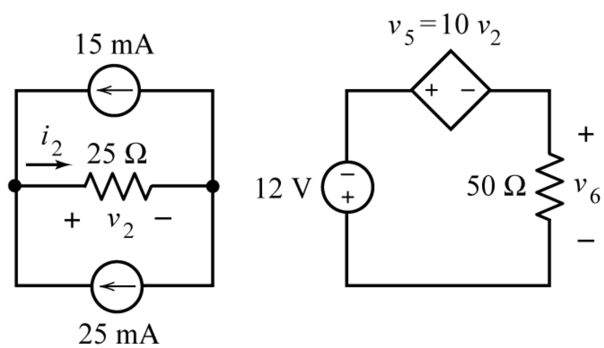


Figure P3.2-20

Solution:

Apply KCL at the left node:

$$0.015 + 0.025 = i_2 \quad \Rightarrow \quad i_2 = 0.040 \text{ A}$$



From Ohm's law

$$v_2 = 25 i_2 = 25(0.04) = 1 \text{ V}$$

Use the element equation of the dependent source:

$$v_5 = 10 v_2 = 10(1) = 10 \text{ V}$$

Apply KVL to the right mesh

$$v_5 + v_6 + 12 = 0 \quad \Rightarrow \quad v_6 = -v_5 - 12 = -10 - 12 = -22 \text{ V}$$

P3.2-21 Determine the value of the voltage v_5 for the circuit shown in Figure P3.2-21.

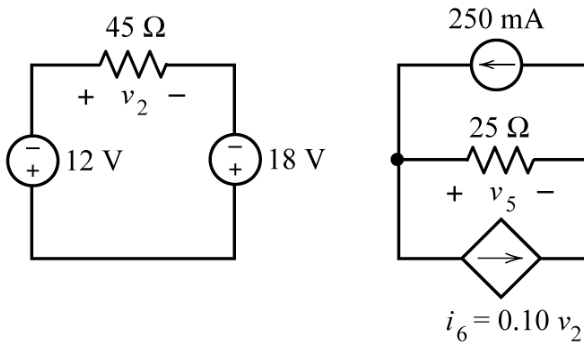
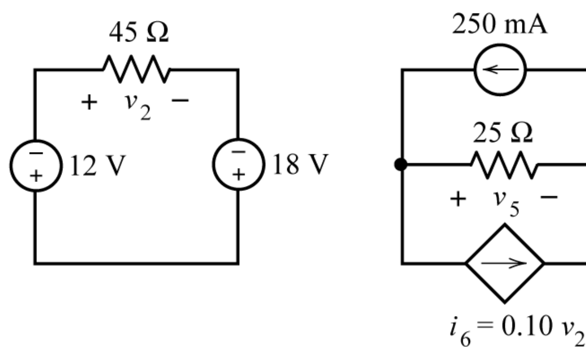


Figure P3.2-21

Solution:

Apply KVL to the left mesh:

$$v_2 - 18 + 12 = 0 \quad \Rightarrow \quad v_2 = 6 \text{ V}$$



Use the element equation of the dependent source:

$$i_6 = 0.10 v_2 = 0.10(6) = 0.6 \text{ A}$$

Apply KCL at the right node

$$\frac{v_5}{25} + i_6 = 0.25 \quad \Rightarrow \quad v_5 = 25(0.25 - i_6) = 25(0.25 - 0.6) = -8.75 \text{ V}$$

P3.2-22 Determine the value of the voltage v_5 for the circuit shown in Figure P3.2-22.

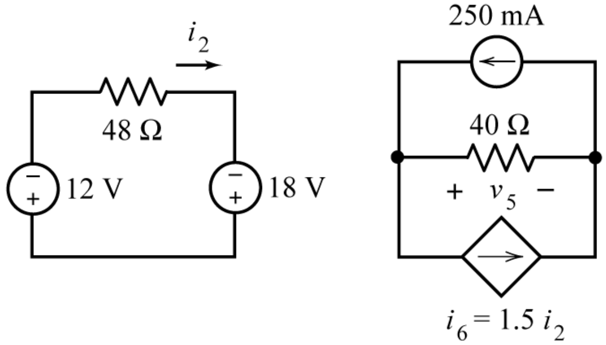


Figure P3.2-22

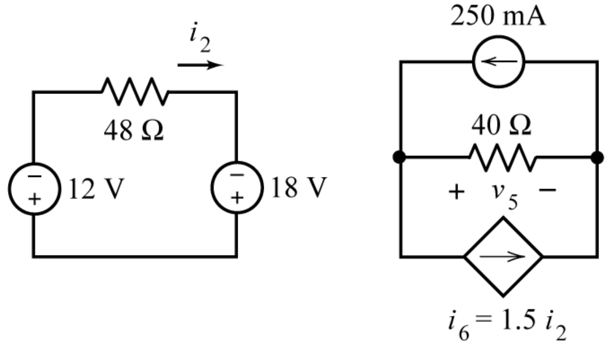
Solution:

Apply KVL to the left mesh:

$$48 i_2 - 18 + 12 = 0 \quad \Rightarrow \quad i_2 = 0.125 \text{ A}$$

Use the element equation of the dependent source:

$$i_6 = 1.5 i_2 = 1.5(0.125) = 0.1875 \text{ A}$$



Apply KCL at the right node

$$\frac{v_5}{40} + i_6 = 0.25 \quad \Rightarrow \quad v_5 = 40(0.25 - i_6) = 40(0.25 - 0.1875) = 2.5 \text{ V}$$

P3.2-23 Determine the value of the voltage v_6 for the circuit shown in Figure P3.2-23.

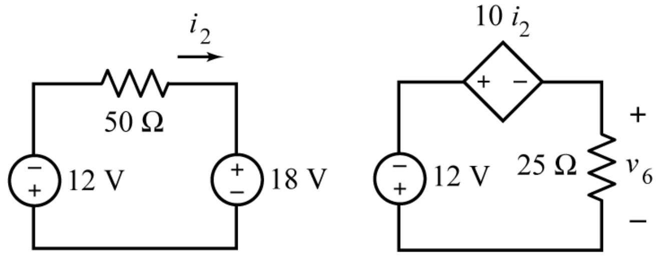
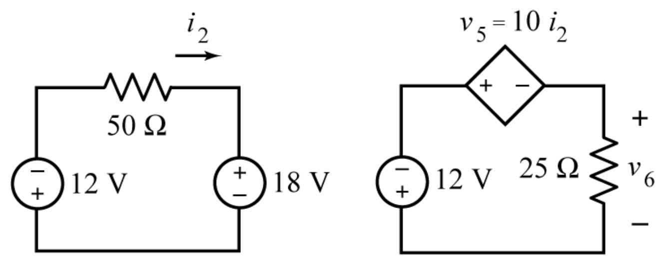


Figure P3.2-23

Solution:

Apply KVL to the left mesh:

$$50i_2+18+12=0 \;\;\Rightarrow\;\; i_2=\frac{-30}{50}=-0.6\text{ A}$$



Use the element equation of the dependent source:

$$v_5=10i_2=10(-0.6)=-6\text{ V}$$

Apply KVL to the right mesh

$$v_5+v_6+12=0 \;\;\Rightarrow\;\; v_6=-v_5-12=-(-6)-12=-6\text{ V}$$

P3.2-24 Determine the value of the voltage v_5 for the circuit shown in Figure P3.2-24.

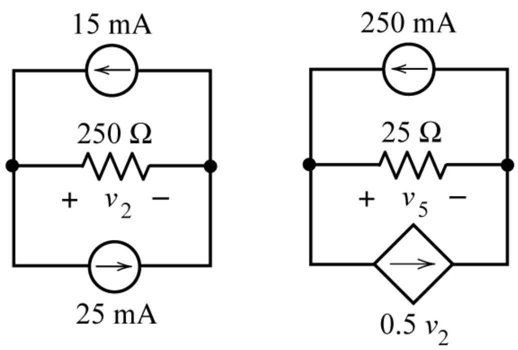
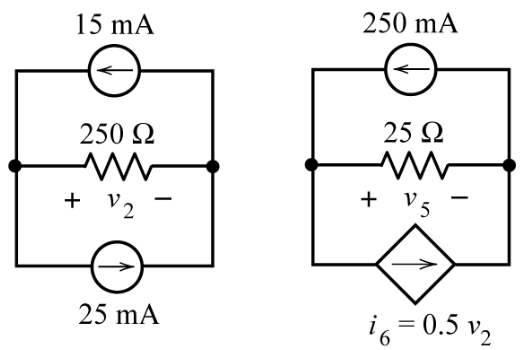


Figure P3.2-24

Solution:

Apply KCL at the left node:



$$0.015 = 0.025 + \frac{v_2}{250} \quad \Rightarrow \quad v_2 = 250(-0.01) = -2.5 \text{ V}$$

Use the element equation of the dependent source:

$$i_6 = 0.5 v_2 = 0.5(-2.5) = -1.25 \text{ A}$$

Apply KCL at the right node

$$\frac{v_5}{25} + i_6 = 0.25 \quad \Rightarrow \quad v_5 = 25\left(0.25 - i_6\right) = 25\left(0.25 - (-1.25)\right) = 37.5 \text{ V}$$

P3.2-25 The voltage source in the circuit shown in Figure P3.2-25 supplies 2 W of power. The value of the voltage across the 25 Ω is $v_5 = 4$ V. Determine the values of the resistance R_1 and of the gain, G, of the CCVS.

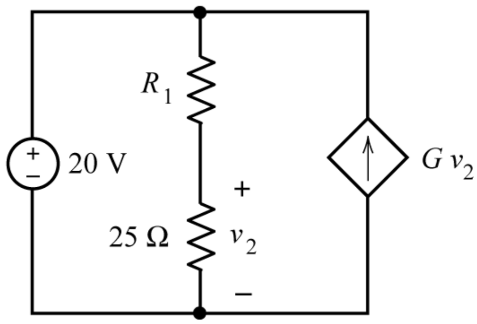
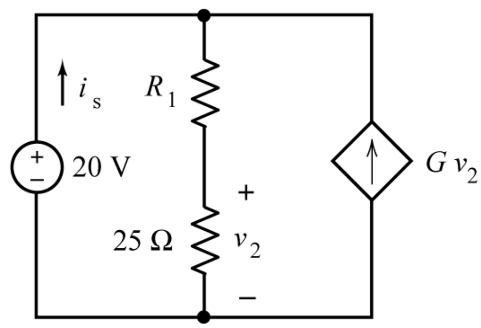


Figure P3.2-25

Solution:

The voltage source current is calculated from the values of the source voltage and power:

$$i_s = \frac{2}{20} = 0.1 \text{ A}$$



Apply KCL at the bottom node to get

$$i_s + G v_2 = \frac{v_2}{25} \quad \Rightarrow \quad G v_2 = \frac{4}{25} - 0.1 = 0.06 \text{ A}$$

Then

$$G = \frac{G v_2}{v_2} = \frac{0.06}{4} = 0.015 \text{ A/V} = 15 \text{ mA/V}$$

Next, use Ohm's law to determine the value of the resistance R_1:

$$R_1 = \frac{20 - v_2}{\frac{v_2}{25}} = \frac{20-4}{\frac{4}{25}} = 100 \ \Omega$$

P3.2-26 Consider the circuit shown in Figure P3.2-26. Determine the values of

(a) The current i_a in the 20-Ω resistor.

(b) The voltage v_b across the 10-Ω resistor.

(c) The current i_c in the independent voltage source.

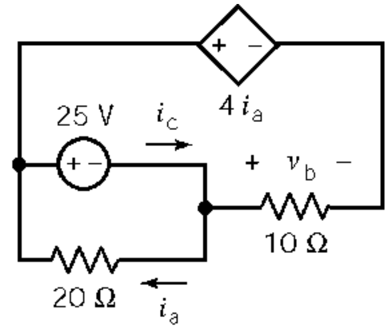
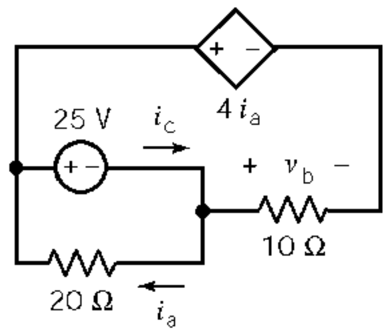


Figure P3.2-26

Solution:



(a) From Ohm's law $i_a = -\dfrac{25}{20} = -1.25\text{ A}$.

(b) From KVL $4i_a - v_b + 20i_a = 0 \;\Rightarrow\; v_b = 24i_a = 24(-1.25) = -30\text{ V}$

(c) From KCL $i_c = i_a + \dfrac{v_b}{10} = -1.25 + \dfrac{-30}{10} = -4.25\text{ A}$

P3.2-27 Consider the circuit shown in Figure 3.2-27.

a.) Determine the values of the resistances.

b.) Determine the values of the power supplied by each current source.

c.) Determine the values of the power received by each resistor.

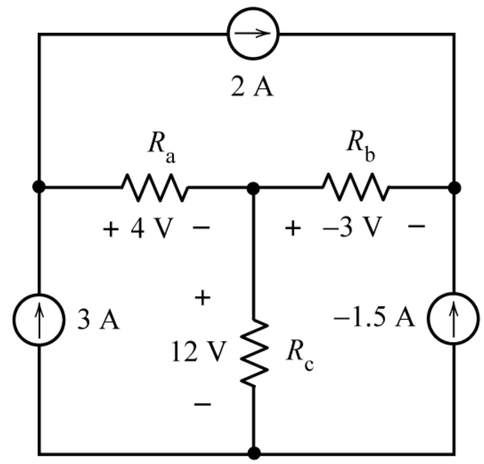
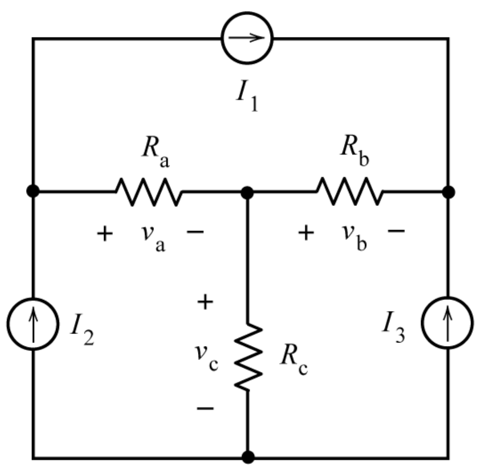


Figure 3.2-27

Solution:



Using KCL and Ohm's law:

$$v_a = R_a(I_2 - I_1),\quad v_b = -R_b(I_1 + I_3)$$

and

$$v_c = R_c(I_2 + I_3)$$

Using KVL, the power supplied the current sources are:

$I_2(v_a + v_c)$, $-I_1(v_a + v_b)$ and $I_3(-v_b + v_c)$

The power received the resistors are:

$$v_a(I_2 - I_1),\quad -v_b(I_1 + I_3)\ \text{ and }\ v_c(I_2 + I_3)$$

a.) $R_a = \dfrac{4}{3-2} = 4\ \Omega$, $R_b = -\dfrac{-3}{2+(-1.5)} = 6\ \Omega$

and $R_c = \dfrac{12}{3+(-1.5)} = 8\ \Omega$

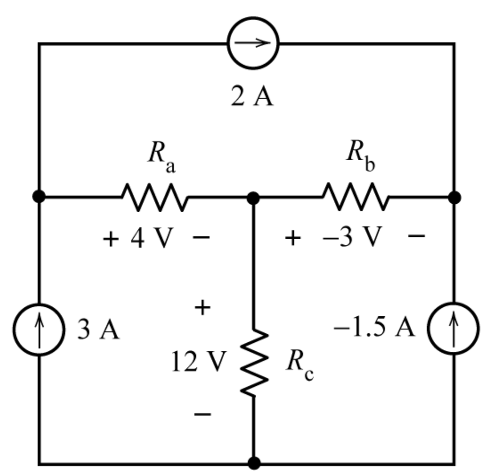
b.) $3(4+12) = 48$ W, $-2(4+(-3)) = -2$ W

and

$$-1.5(-(-3)+12) = -22.5 \text{ W}$$

c.) $4(3-2) = 4$ W, $-(-3)(2+(-1.5)) = 1.5$ W

and $12(3+(-1.5)) = 18$ W



P3.2-28 Consider the circuit shown in Figure 3.2-28.

a. Determine the value of the power supplied by each independent source.

b. Determine the value of the power received by each resistor.

c. Is power conserved?

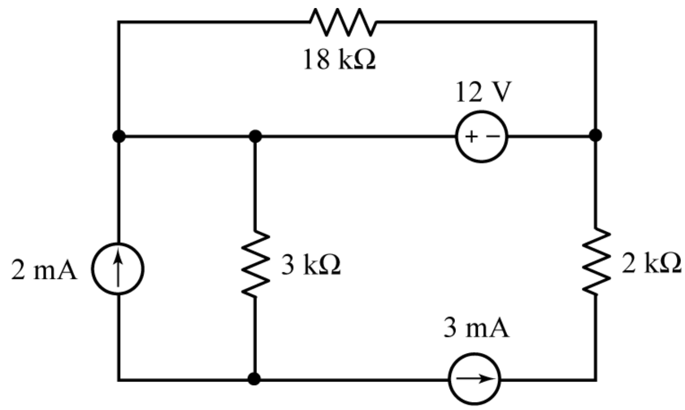
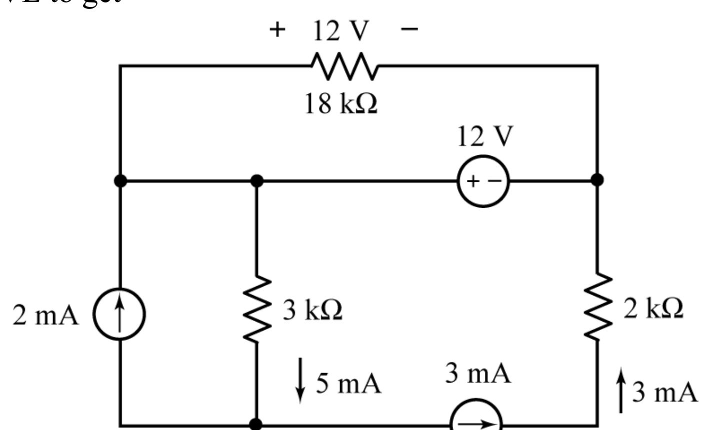
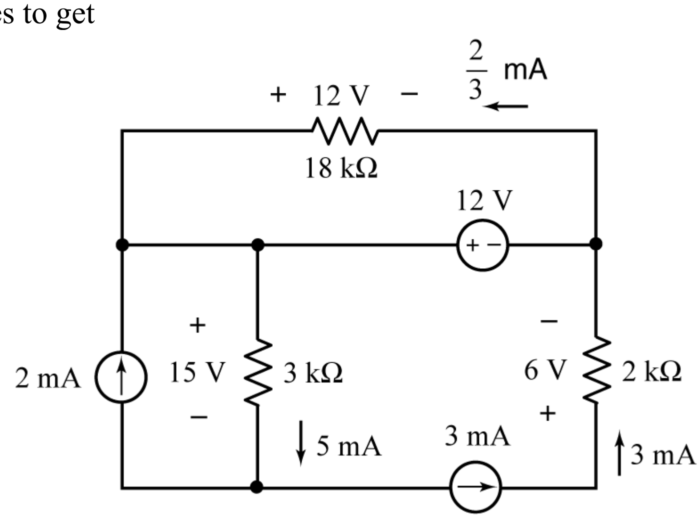


Figure 3.2-28

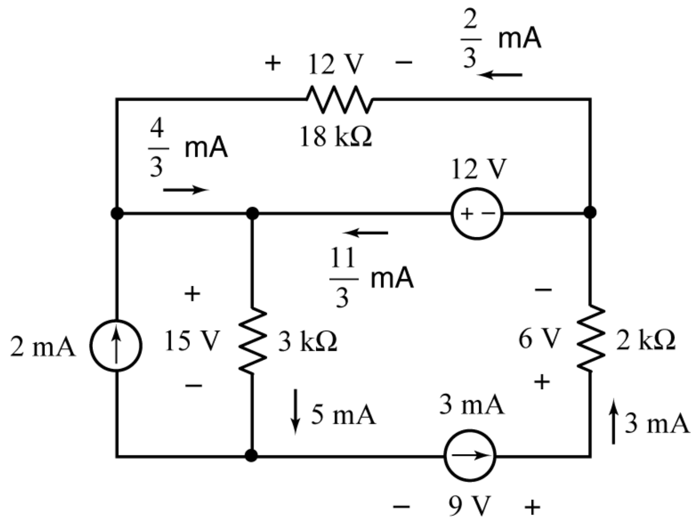
Solution:
Apply KCL twice and KVL to get



Apply Ohm's law 3 times to get



Apply KCL twice and KVL to get



a. Notice that the voltage and current reference directions for each independent source do not adhere to passive convention. Consequently the power supplied by the 2 mA current source is

$$15\,(2) = 30 \text{ mW}$$

The power supplied by the 3 mA current source is

$$9\,(3) = 27 \text{ mW}$$

The power supplied by the 12 V voltage source is

$$12\,(11/3) = 44 \text{ mW}$$

b. Notice that the voltage and current reference directions for each resistor do adhere to passive convention. Consequently the power received by the 18 kΩ resistor is

$$12\,(2/3) = 8 \text{ mW}$$

The power received by the 3 kΩ resistor is

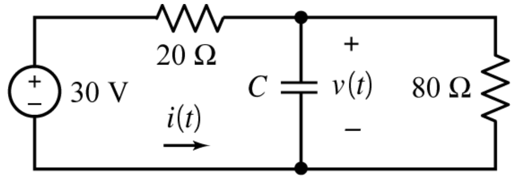
$$15\,(5) = 75 \text{ mW}$$

The power received by the 2 kΩ resistor is

$$6\,(3) = 18 \text{ mW}$$

c. Power is conserved; the sum of the powers supplied by the independent sources is equal to the sum of the powers received by the resistors.

$$30 + 27 + 44 = 101 \text{ mW} = 8 + 75 + 18$$

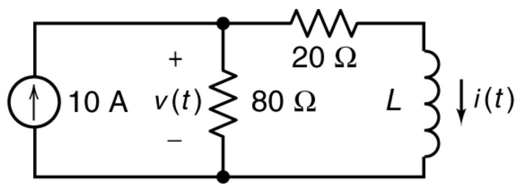


P3.2-29

P3.2-29 The voltage across the capacitor in Figure P3.2-29 is $v(t) = 24 - 10e^{-25t}$ V for $t \ge 0$. Determine the voltage source current $i(t)$ for $t > 0$.

Solution: Notice that $i(t)$ is the current in the 20 Ω resistor. Apply KVL to the left mesh to get

$$-20\,i(t) + \left[24 - 10e^{-25t}\right] - 30 = 0 \quad \Rightarrow \quad i(t) = \frac{\left[24 - 10e^{-25t}\right] - 30}{20} = -0.3 - 0.5e^{-25t} \quad \text{for } t > 0.$$



P3.2-30

P3.2-30 The current the inductor in Figure P3.2-30 is given by $i(t) = 8 - 6e^{-25t}$ A for $t \ge 0$. Determine the voltage $v(t)$ across the 80 Ω resistor for $t > 0$.

Solution: Notice that $i(t)$ is the current in the 20 Ω resistor. Apply KCL at the top node of the 80 Ω resistor to get

$$\frac{v(t)}{80} = 10 - \left(8 - 6e^{-25t}\right) \quad \Rightarrow \quad v(t) = 80\left[10 - \left(8 - 6e^{-25t}\right)\right] = 160 + 480e^{-25t} \quad \text{for } t \ge 0$$

Section 3-3 Series Resistors and Voltage Division

P 3.3-1 Use voltage division to determine the voltages v_1, v_2, v_3, and v_4 in the circuit shown in Figure P 3.3-1.

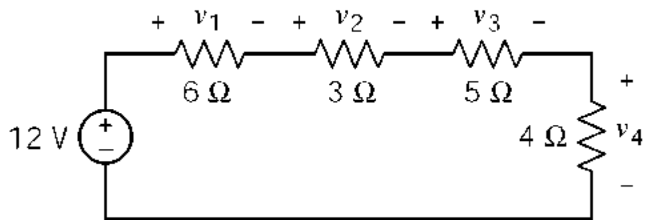
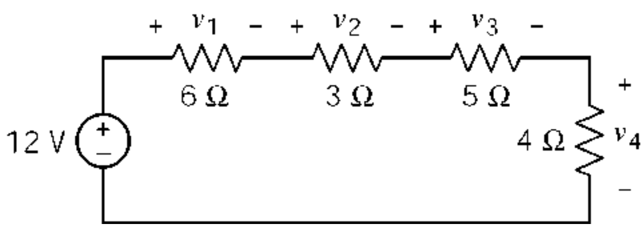


Figure P 3.3-1.

Solution:



$$v_1 = \frac{6}{6+3+5+4}\,12 = \frac{6}{18}\,12 = \underline{4\text{ V}}$$

$$v_2 = \frac{3}{18}\,12 = \underline{2\text{ V}}\ ;\ v_3 = \frac{5}{18}\,12 = \underline{\frac{10}{3}\text{ V}}$$

$$v_4 = \frac{4}{18}12 = \underline{\frac{8}{3}\text{ V}}$$

(checked using LNAP 8/16/02)

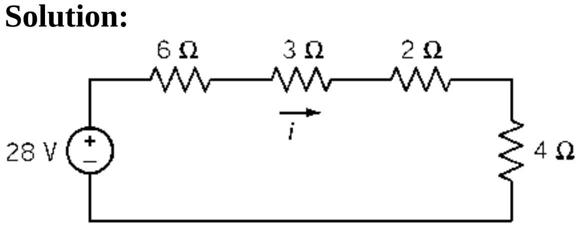
P 3.3-2 Consider the circuits shown in Figure P 3.3-2.

(a) Determine the value of the resistance R in Figure P 3.3-2b that makes the circuit in Figure P 3.3-2b equivalent to the circuit in Figure P 3.3-2a.

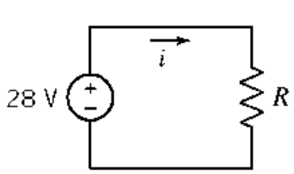
(b) Determine the current i in Figure P 3.3-2b. Because the circuits are equivalent, the current i in Figure P 3.3-2a is equal to the current i in Figure P 3.3-2b.

(c) Determine the power supplied by the voltage source.

Solution:



(*a*)



(*b*)

$$(a)\ \ R = 6+3+2+4 = \underline{15\ \Omega}$$

$$(b)\ \ i = \frac{28}{R} = \frac{28}{15} = \underline{1.867\text{ A}}$$

$$(c)\ \ p = 28 \cdot i = 28(1.867) = \underline{52.27\ \text{ W}}$$

(28 V and i do not adhere to the passive convention.)

(checked using LNAP 8/16/02)

P 3.3-3 The ideal voltmeter in the circuit shown in Figure P 3.3-3 measures the voltage v.

(a) Suppose $R_2 = 100\ \Omega$. Determine the value of R_1.

(b) Suppose, instead, $R_1 = 100\ \Omega$. Determine the value of R_2.

(c) Suppose, instead, that the voltage source supplies 1.2 W of power. Determine the values of both R_1 and R_2.

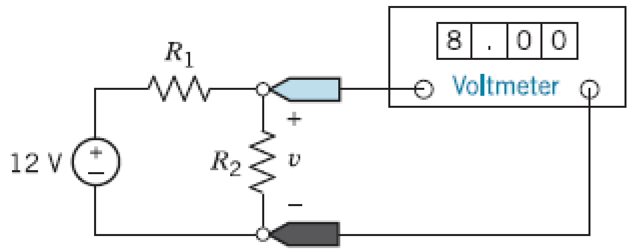
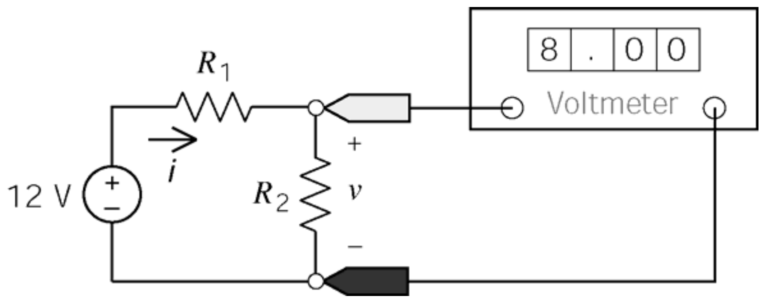


Figure P 3.3-3

Solution:



$$i\,R_2 = v = 8\text{ V}$$

$$12 = i\,R_1 + v = i\,R_1 + 8$$

$$\Rightarrow 4 = i\,R_1$$

$$(a)\ i = \frac{8}{R_2} = \frac{8}{50}\ ;\ R_1 = \frac{4}{i} = \frac{4\cdot 50}{8} = \underline{25\ \Omega}$$

$$(b)\ i = \frac{4}{R_1} = \frac{4}{50}\ ;\ R_2 = \frac{8}{i} = \frac{8\cdot 50}{4} = \underline{100\ \Omega}$$

$$(c)\ 1.2 = 12\ i \Rightarrow i = 0.1\text{ A}\ ;\ R_1 = \frac{4}{i} = \underline{40\ \Omega};\ R_2 = \frac{8}{i} = \underline{80\ \Omega}$$

(Checked using LNAP 8/16/02)

P 3.3-4 Determine the voltage v in the circuit shown in Figure P 3.3-4.

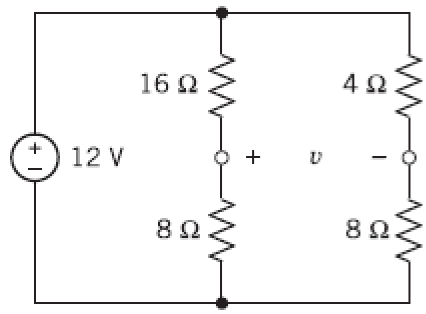
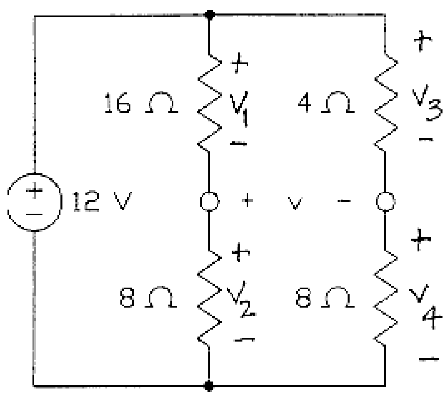


Figure P 3.3-4

Solution:



Voltage division

$$v_1 = \frac{16}{16+8} 12 \; = \; 8 \text{ V}$$

$$v_3 = \frac{4}{4+8} 12 \; = \; 4 \text{ V}$$

KVL: $v_3 - v - v_1 = 0$

$$\underline{v = -4 \text{ V}}$$

(checked using LNAP 8/16/02)

P 3.3-5 The model of a cable and load resistor connected to a source is shown in Figure P 3.3-5. Determine the appropriate cable resistance, R, so that the output voltage, v_o, remains between 9 V and 13 V when the source voltage, v_s, varies between 20 V and 28 V. The cable resistance can only assume integer values in the range $20 < R < 100\ \Omega$.

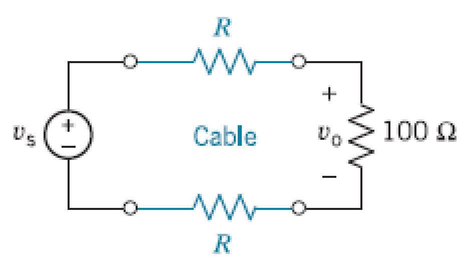


Figure P 3.3-5

Solution:

using voltage divider: $v_0 = \left(\frac{100}{100+2R}\right) v_s \quad \Rightarrow \quad R = 50\left(\frac{v_s}{v_o} - 1\right)$

$$\left.\begin{array}{l} \text{with } v_s = 20 \text{ V and } v_0 > 9 \text{ V}, \; R < 61.1\,\Omega \\ \text{with } v_s = 28 \text{ V and } v_0 < 13 \text{ V}, \; R > 57.7\,\Omega \end{array}\right\} \underline{R = 60\,\Omega}$$

P 3.3-6 The input to the circuit shown in Figure P 3.3-6 is the voltage of the voltage source, v_a. The output of this circuit is the voltage measured by the voltmeter, v_b. This circuit produces an output that is proportional to the input, that is

$$v_b = k\, v_a$$

where k is the constant of proportionality.

(a) Determine the value of the output, v_b, when $R = 240\ \Omega$ and $v_a = 18$ V.

(b) Determine the value of the power supplied by the voltage source when $R = 240\ \Omega$ and $v_a = 18$ V.

(c) Determine the value of the resistance, R, required to cause the output to be $v_b = 2$ V when the input is $v_a = 18$ V.

(d) Determine the value of the resistance, R, required to cause $v_b = 0.2 v_a$ (that is, the value of the constant of proportionality is $k = \frac{2}{10}$).

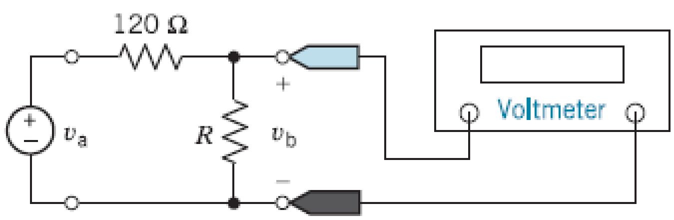
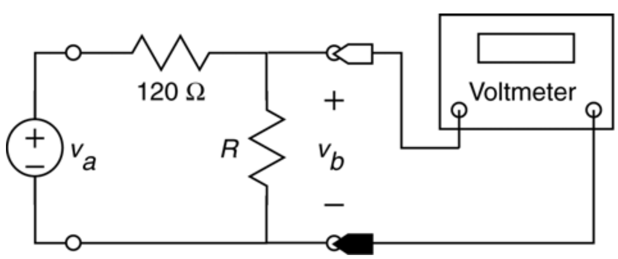


Figure P 3.3-6

Solution:



a.) $\left(\dfrac{180}{120+180}\right)18 = 10.8 \text{ V}$

b.) $18\left(\dfrac{18}{120+180}\right) = 1.08 \text{ W}$

c.) $\left(\dfrac{R}{R+120}\right)18 = 2 \quad\Rightarrow\quad 18\,R = 2\,R + 2(120) \quad\Rightarrow\quad R = 15\ \Omega$

d.) $0.2 = \dfrac{R}{R+120} \quad\Rightarrow\quad (0.2)(120) = 0.8\,R \quad\Rightarrow\quad R = 30\ \Omega$

(Checked using LNAP 8/16/02)

P 3.3-7 Determine the value of voltage *v* in the circuit shown in Figure P 3.3-7.

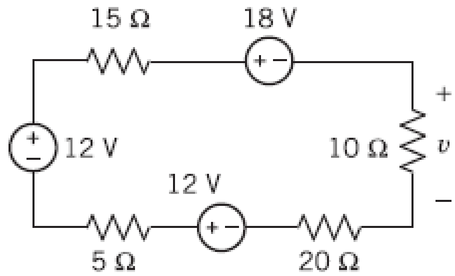


Figure P 3.3-7

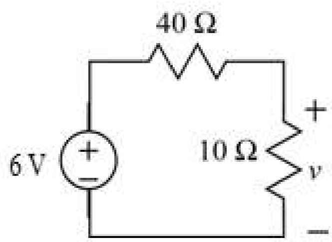
Solution:
All of the elements are connected in series.

Replace the series voltage sources with a single equivalent voltage having voltage

$$12 + 12 - 18 = 6 \text{ V}.$$

Replace the series 15 Ω, 5 Ω and 20 Ω resistors by a single equivalent resistance of

$$15 + 5 + 20 = 40 \ \Omega.$$



By voltage division

$$v = \left(\frac{10}{10+40}\right)6 = \frac{6}{5} = 1.2 \text{ V}$$

(Checked: LNAP 2/6/07)

P 3.3-8 Determine the power supplied by the dependent source in the circuit shown in Figure P 3.3-8.

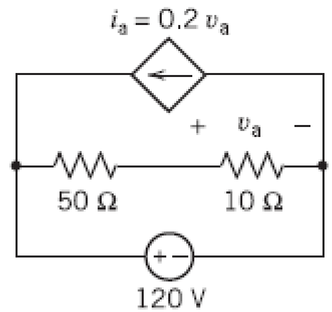


Figure P 3.3-8

Solution:

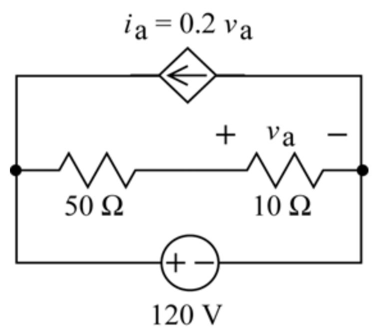
Use voltage division to get

$$v_a = \left(\frac{10}{10+50}\right)(120) = 20 \text{ V}$$

Then $$i_a = 0.2(20) = 4 \text{ A}$$

The power supplied by the dependent source is given by

$$p = (120)i_a = 480 \text{ W}$$



(Checked: LNAP 6/21/04)

P 3.3-9 A potentiometer can be used as a transducer to convert the rotational position of a dial to an electrical quantity. Figure P 3.3-9 illustrates this situation. Figure P 3.3-9*a* shows a potentiometer having resistance R_p connected to a voltage source. The potentiometer has three terminals, one at each end and one connected to a sliding contact called a wiper. A voltmeter measures the voltage between the wiper and one end of the potentiometer.

Figure P 3.3-9*b* shows the circuit after the potentiometer is replaced by a model of the potentiometer that consists of two resistors. The parameter a depends on the angle, θ, of the dial. Here $a = \frac{\theta}{360°}$, and θ is given in degrees. Also, in Figure P 3.3-9*b*, the voltmeter has been replaced by an open circuit and the voltage measured by the voltmeter, v_m, has been labeled. The input to the circuit is the angle θ, and the output is the voltage measured by the meter, v_m.

(a) Show that the output is proportional to the input.

(b) Let $R_p = 1$ kΩ and $v_s = 24$ V. Express the output as a function of the input. What is the value of the output when $\theta = 45°$? What is the angle when $v_m = 10$ V?

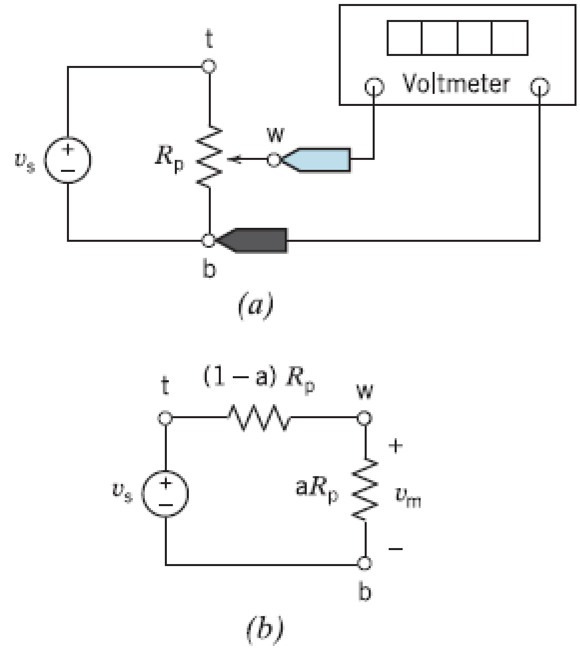
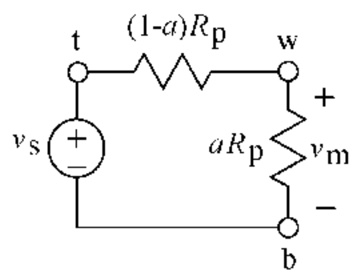


Figure P 3.3-9

Solution:

(a) Use voltage division to get



$$v_m = \frac{aR_p}{(1-a)R_p + R_p} v_s = a v_s$$

Therefore
$$v_m = \left(\frac{v_s}{360}\right)\theta$$

So the input is proportional to the input.

(b) When $v_s = 24$ V then $v_m = \left(\frac{1}{15}\right)\theta$. When $\theta = 45°$ then $v_m = 3$ V. When $v_m = 10$ V then $\theta = 150°$.

(Checked: LNAP 6/12/04)

P 3.3-10 Determine the value of the voltage measured by the meter in Figure P 3.3-10.

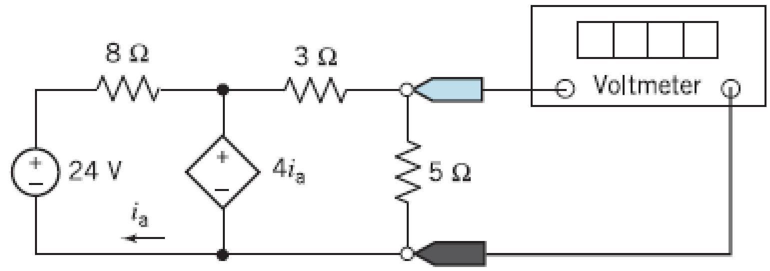
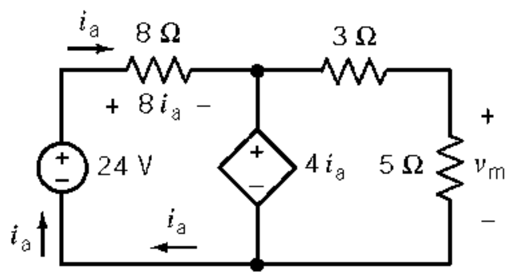


Figure P 3.3-10

Solution:

Replace the (ideal) voltmeter with the equivalent open circuit. Label the voltage measured by the meter. Label some other element voltages and currents.



Apply KVL the left mesh to get

$$8i_a + 4i_a - 24 = 0 \quad \Rightarrow \quad i_a = 2 \text{ A}$$

Use voltage division to get

$$v_m = \frac{5}{5+3} 4i_a = \frac{5}{5+3} 4(2) = 5 \text{ V}$$

(Checked using LNAP 9/11/04)

P 3.3-11 For the circuit of Figure P 3.3-11, find the voltage v_3 and the current i and show that the power delivered to the three resistors is equal to that supplied by the source.

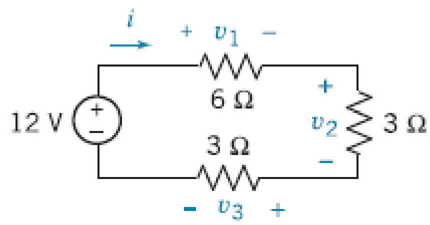
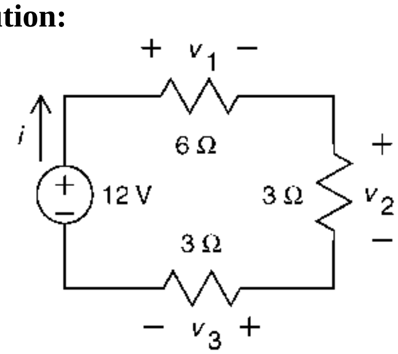


Figure P 3.3-11

Solution:



From voltage division $v_3 = 12\left(\frac{3}{3+9}\right) = \underline{3 \text{ V}}$

then $i = \frac{v_3}{3} = \underline{1 \text{ A}}$

The power absorbed by the resistors is: $(1^2)(6) + (1^2)(3) + (1^2)(3) = 12$ W

The power supplied by the source is (12)(1) = 12 W.

P 3.3-12 Consider the voltage divider shown in Figure P 3.3-12 when $R_1 = 6\ \Omega$. It is desired that the output power absorbed by $R_1 = 6\ \Omega$ be 6 W. Find the voltage v_o and the required source v_s.

Answer: $v_s = 14$ V, $v_o = 6$ V

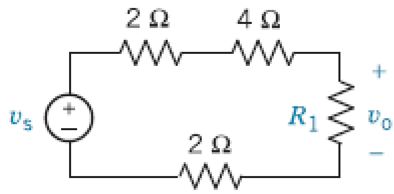
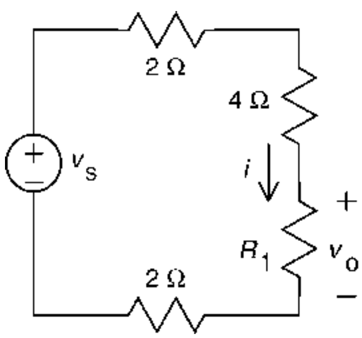


Figure P 3.3-12

Solution:



$P = 4.5$ W and $R_1 = 8\ \Omega$

$$i^2 = \frac{P}{R_1} = \frac{4.5}{8} = 0.5625 \text{ or } i = 0.75 \text{ A}$$

$v_0 = i\,R_1 = (0.75)\,(8) = \underline{6\text{ V}}$

from KVL: $-v_s + i\,(2+4+8+2) = 0$

$\Rightarrow\ v_s = 16\,i = 16(0.75) = 12$ V

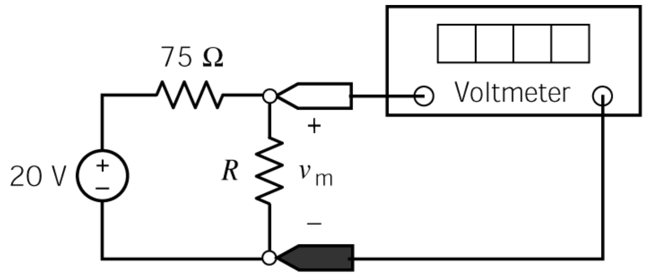


Figure P3.3-13

P3.3-13

Consider the voltage divider circuit shown in Figure P3.3-13. The resistor *R* represents a temperature sensor. The resistance *R*, in Ω, is related to the temperature *T*, in °C, by the equation

$$R = 50 + \frac{1}{2}T$$

a) Determine the meter voltage, v_m, corresponding to temperatures 0 °C, 75 °C and 100 °C.

b) Determine the temperature, *T*, corresponding to the meter voltages 8 V, 10 V and 15 V.

P3.3-13

Using voltage division $$v_m = \left(\frac{R}{75+R}\right)20$$

Solving for *R* yields $$R = \frac{75 v_m}{20 - v_m}$$

The temperature can be calculated from the resistance using

$$T = 2(R-50) = 2\left(\frac{75 v_m}{20-v_m} - 50\right) = \frac{150 v_m}{20-v_m} - 100$$

a) At 0 °C the resistance is *R* = 50 Ω so $v_m = \left(\frac{50}{75+50}\right)20 = 8\text{ V}$. At 75 °C the resistance is

R = 87.5 Ω so $v_m = \left(\frac{87.5}{75+87.5}\right)20 = 10.77\text{ V}$. At 100 °C the resistance is *R* = 100 Ω so

$$v_m = \left(\frac{100}{75+100}\right)20 = 11.43\text{ V}.$$

b) When $v_m = 8$ V, the temperature is $T = \frac{150(8)}{20-8} - 100 = 0\text{ °C}$. When $v_m = 10$ V, the temperature is $T = \frac{150(10)}{20-10} - 100 = 50\text{ °C}$. When $v_m = 15$ V, the temperature is $T = \frac{150(15)}{20-15} - 100 = 350\text{ °C}$.

P3.3-14 Consider the circuit shown in Figure P3.3-14.

(a) Determine the value of the resistance R required to cause $v_o = 17.07$ V.

(b) Determine the value of the voltage v_o when $R = 14\ \Omega$.

(c) Determine the power supplied by the voltage source when $v_o = 14.22$ V.

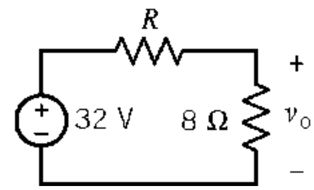


Figure P3.3-14

Solution:

Voltage division indicates that $v_o = \left(\frac{8}{8+R}\right)32$.

(a) When $v_o = 17.07$ V, then $17.07 = \left(\frac{8}{8+R}\right)32 \quad \Rightarrow \quad R = \frac{(8)32}{17.07} - 8 = 6.997 \simeq 7\ \Omega$.

(b) When $R = 14\ \Omega$ then $v_o = \left(\frac{8}{8+14}\right)32 = 11.6363$ V.

(c) The power supplied by the voltage source is given by $32\frac{v_o}{8} = 4v_o$ since $\frac{v_o}{8}$ is the current in each of the series elements. When $v_o = 14.22$ V the voltage source supplies 56.88 W.

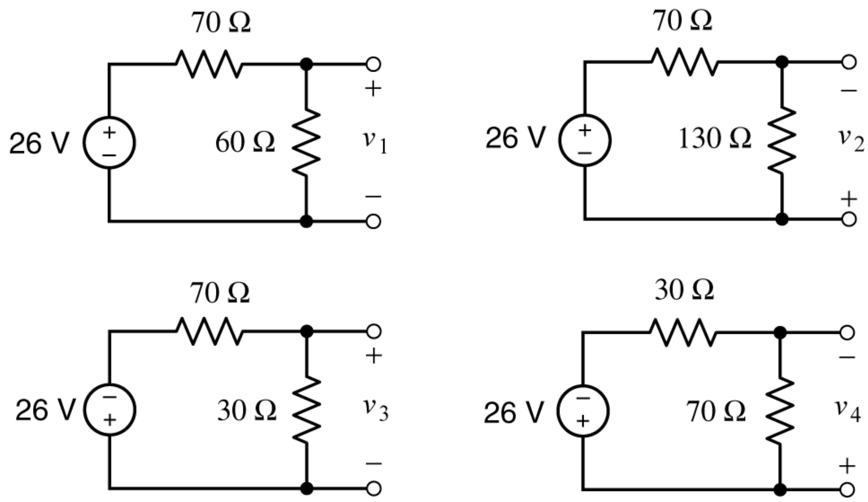


Figure P3.3-15

P3.3-15. Figure P3.3-15 shows four similar but slightly different circuits. Determine the values of the voltages v_1, v_2, v_3 and v_4.

Solution: Using voltage division:

$$v_1 = \left(\frac{60}{60+70}\right)26 = 12\text{ V}, \quad v_2 = -\left(\frac{130}{70+130}\right)26 = -16.9\text{ V},$$

$$v_3 = -\left(\frac{30}{70+30}\right)26 = -7.8\text{ V} \quad \text{and} \quad v_4 = -\left(\frac{70}{30+70}\right)26 = 18.2\text{ V}$$

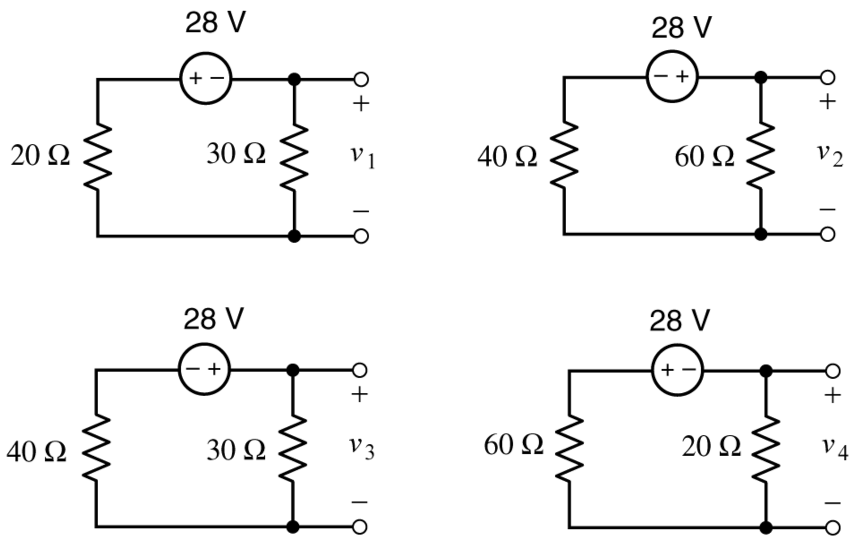


Figure P3.3-16

P3.3-16. Figure P3.3-16 shows four similar but slightly different circuits. Determine the values of the voltages v_1, v_2, v_3 and v_4.

Solution: Using voltage division:

$$v_1 = -\left(\frac{30}{20+30}\right)28 = -16.8 \text{ V}, \quad v_2 = -\left(\frac{60}{40+60}\right)28 = 16.8 \text{ V},$$

$$v_3 = \left(\frac{30}{40+30}\right)28 = 12 \text{ V} \quad \text{and} \quad v_4 = -\left(\frac{20}{60+20}\right)28 = -7 \text{ V}$$

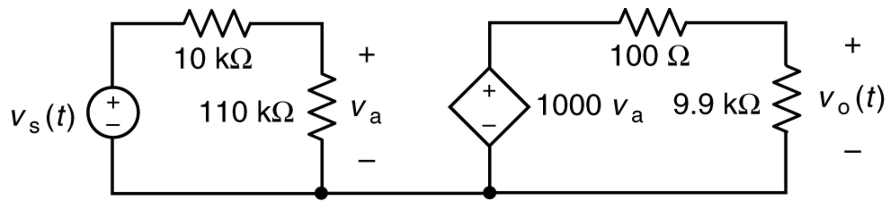


Figure P3.3-17

P3.3-17. The input to the circuit shown in Figure P3.3-19 is the voltage source voltage

$$v_s(t) = 12\cos(377t) \text{ mV}$$

The output is the voltage $v_o(t)$. Determine $v_o(t)$.

P3.3-17

Using voltage division: $v_a(t) = \frac{110}{10+110} 12\cos(377t) = 11\cos(377t) \text{ mV}$

Using voltage division again: $v_o(t) = \frac{9900}{100+9900} 1000\, v_a(t)$

Therefore:

$$v_o(t) = \frac{9900}{100+9900}(1000)11\cos(377t) = 10890\cos(377t) \text{ mV}$$

$$= 10.89\cos(377t) \text{ V}$$

Section 3-4 Parallel Resistors and Current Division

P 3.4-1 Use current division to determine the currents i_1, i_2, i_3, and i_4 in the circuit shown in Figure P 3.4-1.

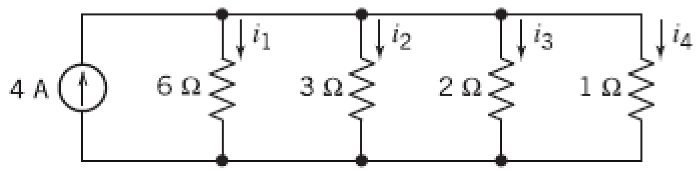
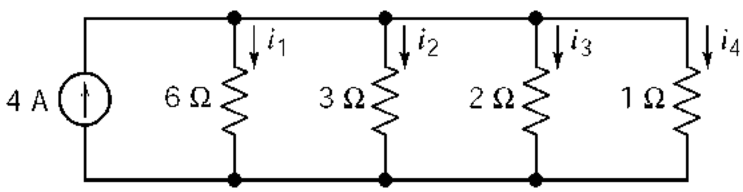


Figure P 3.4-1.

Solution:



$$i_1 = \frac{\frac{1}{6}}{\frac{1}{6}+\frac{1}{3}+\frac{1}{2}+\frac{1}{1}} 4 = \frac{1}{1+2+3+6} 4 = \underline{\frac{1}{3} \text{ A}}$$

$$i_2 = \frac{\frac{1}{3}}{\frac{1}{6}+\frac{1}{3}+\frac{1}{2}+\frac{1}{1}} 4 = \underline{\frac{2}{3} \text{ A}};$$

$$i_3 = \frac{\frac{1}{2}}{\frac{1}{6}+\frac{1}{3}+\frac{1}{2}+\frac{1}{1}} 4 = \underline{1 \text{ A}}$$

$$i_4 = \frac{1}{\frac{1}{6}+\frac{1}{3}+\frac{1}{2}+1} 4 = \underline{2 \text{ A}}$$

P 3.4-2 Consider the circuits shown in Figure P 3.4-2.

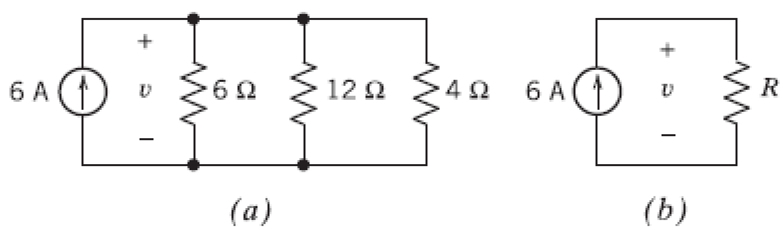


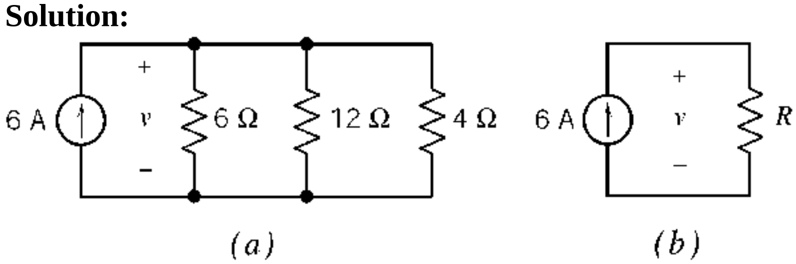
Figure P 3.4-2

(a) Determine the value of the resistance R in Figure P 3.4-2b that makes the circuit in Figure P 3.4-2b equivalent to the circuit in Figure P 3.4-2a.

(b) Determine the voltage v in Figure P 3.4-2b. Because the circuits are equivalent, the voltage v in Figure P 3.4-2a is equal to the voltage v in Figure P 3.4-2b.

(c) Determine the power supplied by the current source.

Solution:



$$(a) \quad \frac{1}{R} = \frac{1}{6} + \frac{1}{12} + \frac{1}{4} = \frac{1}{2} \Rightarrow \underline{R = 2\,\Omega}$$

$$(b) \quad v = 6 \cdot 2 = \underline{12 \text{ V}}$$

$$(c) \quad p = 6 \cdot 12 = \underline{72 \text{ W}}$$

P 3.4-3 The ideal voltmeter in the circuit shown in Figure P 3.4-3 measures the voltage *v*.

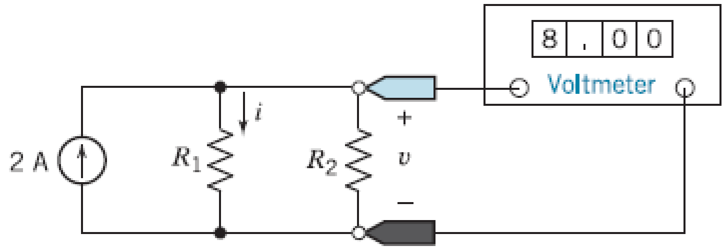
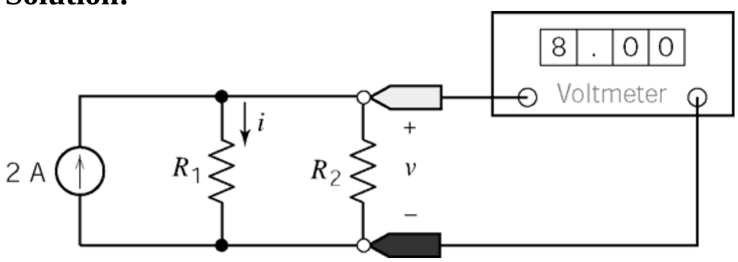


Figure P 3.4-3

(a) Suppose $R_2 = 12\ \Omega$. Determine the value of R_1 and of the current i.
(b) Suppose, instead, $R_1 = 12\ \Omega$. Determine the value of R_2 and of the current i.
(c) Instead, choose R_1 and R_2 to minimize the power absorbed by any one resistor.

Solution:



$$i = \frac{8}{R_1} \text{ or } R_1 = \frac{8}{i}$$

$$8 = R_2(2-i) \Rightarrow i = 2 - \frac{8}{R_2} \text{ or } R_2 = \frac{8}{2-i}$$

$$(a)\ i = 2 - \frac{8}{6} = \underline{\frac{2}{3}\text{ A}}\ ;\ R_1 = \frac{8}{\frac{2}{3}} = \underline{12\ \Omega}$$

$$(b)\ i = \frac{8}{6} = \underline{\frac{4}{3}\text{ A}}\ ;\ R_2 = \frac{8}{2-\frac{4}{3}} = \underline{12\ \Omega}$$

(c) $R_1 = R_2$ will cause $i = \frac{1}{2} 2 =$ 1 A. The current in both R_1 and R_2 will be 1 A.

$$2 \cdot \frac{R_1 R_2}{R_1 + R_2} = 8\ ;\quad R_1 = R_2 \quad \Rightarrow \quad 2 \cdot \frac{1}{2} R_1 = 8 \Rightarrow R_1 = 8 \therefore \underline{R_1 = R_2 = 8\,\Omega}$$

P 3.4-4 Determine the current i in the circuit shown in Figure P 3.4-4.

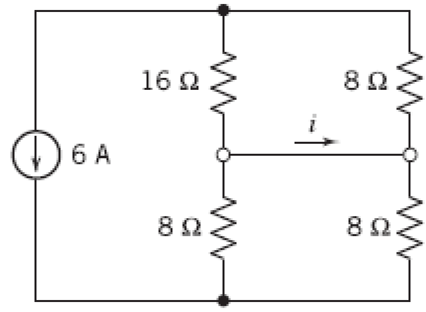
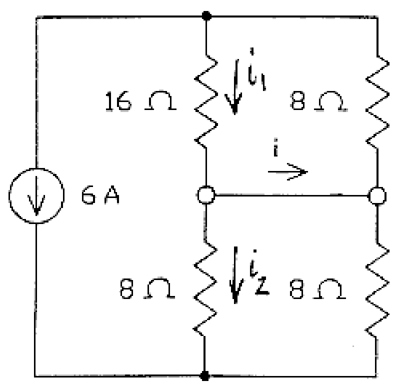


Figure P 3.4-4

Solution:



Current Division:

$$i_1 = \frac{8}{16+8}(-6) = -2 \text{ A}$$

$$i_2 = \frac{8}{8+8}(-6) = -3 \text{ A}$$

KCL:

$$i = i_1 - i_2 = -2 - (-3) = 1 \text{ A}$$

P 3.4-5 Consider the circuit shown in Figure P 3.4-5 when 4 Ω $\le R_1 \le$ 6 Ω and R_2 = 10 Ω. Select the source i_s so that v_o remains between 9 V and 13 V

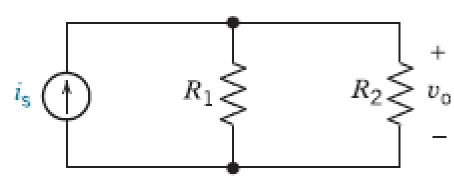
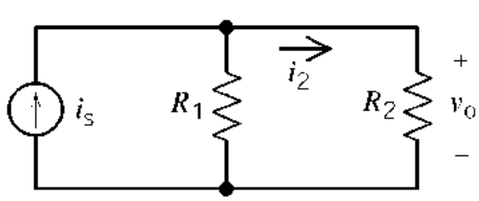


Figure P 3.4-5

Solution:



current division: $i_2 = \left(\dfrac{R_1}{R_1 + R_2}\right) i_s$ and

Ohm's Law: $v_o = i_2 R_2$ yields

$$i_s = \left(\frac{v_o}{R_2}\right)\left(\frac{R_1 + R_2}{R_1}\right)$$

plugging in $R_1 = 4\Omega$, $v_o > 9$ V gives $i_s > 3.15$ A

and $R_1 = 6\Omega$, $v_o < 13$ V gives $i_s < 3.47$ A

So any $\underline{3.15 \text{ A} < i_s < 3.47 \text{ A}}$ keeps $9 \text{ V} < v_o < 13 \text{ V}$.

P 3.4-6 Figure P 3.4-6 shows a transistor amplifier. The values of R_1 and R_2 are to be selected. Resistances R_1 and R_2 are used to bias the transistor, that is, to create useful operating conditions. In this problem, we want to select R_1 and R_2 so that $v_b = 5$ V. We expect the value of i_b to be approximately 10 μA. When $i_1 \le 10 i_b$, it is customary to treat i_b as negligible, that is, to assume $i_b = 0$. In that case R_1 and R_2 comprise a voltage divider.

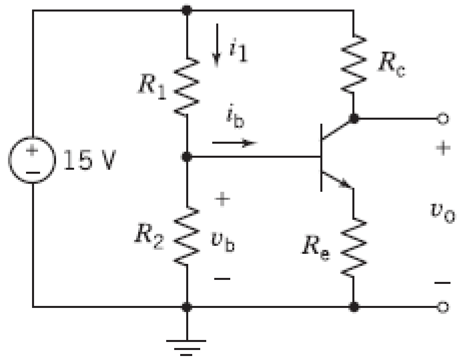


Figure P 3.4-6

(a) Select values for R_1 and R_2 so that $v_b = 5$ V and the total power absorbed by R_1 and R_2 is no more than 5 mW.

(b) An inferior transistor could cause i_b to be larger than expected. Using the values of R_1 and R_2 from part (a), determine the value of v_b that would result from $i_b = 15\ \mu$A.

Solution:

(a) To insure that i_b is negligible we require

$$i_1 = \frac{15}{R_1 + R_2} \ge 10\left(10 \times 10^{-6}\right) = 10^{-3}$$

so

$$R_1 + R_2 \le 150\ \text{k}\Omega$$

To insure that the total power absorbed by R_1 and R_2 is no more than 5 mW we require

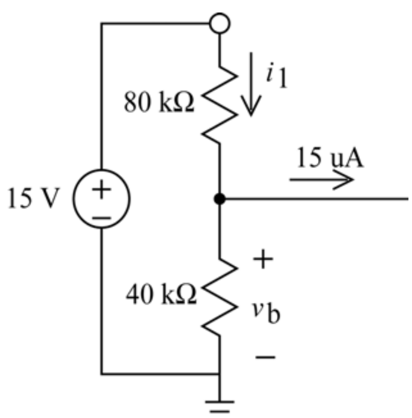
$$\frac{15^2}{R_1 + R_2} \le 5 \times 10^{-3} \quad \Rightarrow \quad R_1 + R_2 \ge 45\ \text{k}\Omega$$

Next to cause $v_b = 5$ V we require

$$5 = v_b = \frac{R_2}{R_1 + R_2} 15 \quad \Rightarrow \quad R_1 = 2R_2$$

For example, $R_1 = 40\ \text{k}\Omega,\ R_2 = 80\ \text{k}\Omega,$ satisfy all three requirements.

(b)



KVL gives $\left(80 \times 10^3\right) i_1 + v_b - 15 = 0$

KCL gives $i_1 = \dfrac{v_b}{40 \times 10^3} + 15 \times 10^{-6}$

Therefore $\left(80 \times 10^3\right)\left(\dfrac{v_b}{40 \times 10^3} + 15 \times 10^{-6}\right) + v_b = 15$

Finally $3v_b + 1.2 = 15 \quad \Rightarrow \quad v_b = \dfrac{13.8}{3} = 4.6$ V

P 3.4-7 Determine the value of the current *i* in the circuit shown in Figure P 3.4-7.

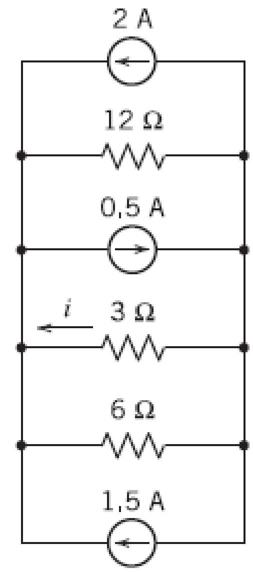
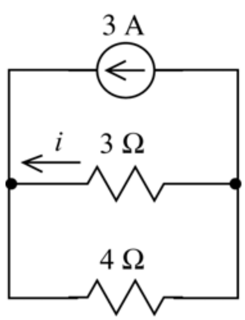


Figure P 3.4-7

Solution:

All of the elements of this circuit are connected in parallel. Replace the parallel current sources by a single equivalent 2 – 0.5 + 1.5 = 3 A current source. Replace the parallel 12 Ω and 6 Ω resistors by a single $\frac{12\times 6}{12+6}=4\ \Omega$ resistor.



By current division

$$i=-\left(\frac{4}{3+4}\right)3=-\frac{12}{7}=-1.714\text{ A}$$

(checked: LNAP 62/6/07)

P 3.4-8 Determine the value of the voltage v in Figure P 3.4-8.

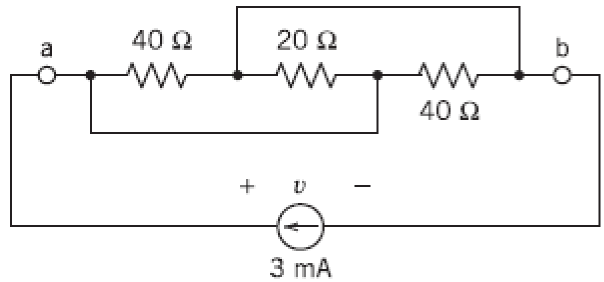
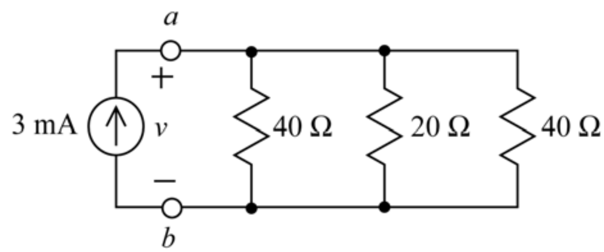


Figure P 3.4-8

Solution:
Each of the resistors is connected between nodes a and b. The resistors are connected in parallel and the circuit can be redrawn like this:



Then $$40 \| 20 \| 40 = 10\ \Omega$$

so $$v = 10(0.003) = 0.03 = 30\text{ mV}$$

(checked: LNAP 6/21/04)

P 3.4-9 Determine the power supplied by the dependent source in Figure P 3.4-9.

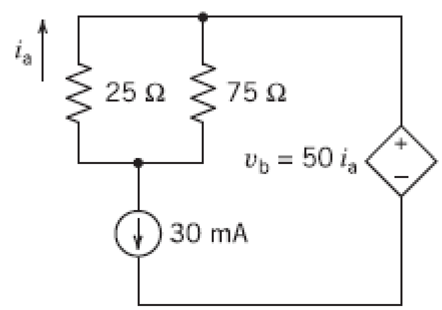
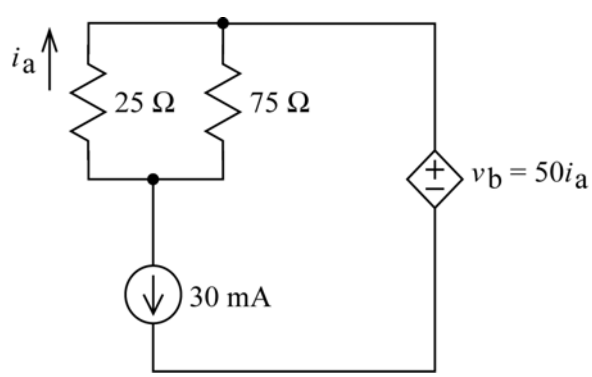


Figure P 3.4-9

Solution:
Use current division to get

$$i_a = -\frac{75}{25+75}\left(30\times10^{-3}\right) = -22.5\text{ mA}$$

so $$v_b = 50\left(-22.5\times10^{-3}\right) = -1.125\text{ V}$$



The power supplied by the dependent source is given by

$$p = -\left(30\times10^{-3}\right)(-1.125) = 33.75\text{ mW}$$

(checked: LNAP 6/12/04)

.

P 3.4-10 Determine the values of the resistances R_1 and R_2 for the circuit shown in Figure P 3.4-10.

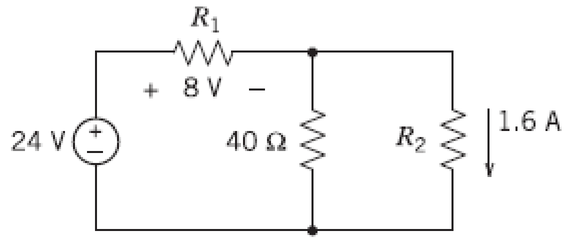


Figure P 3.4-10

Solution:
Using voltage division

$$8 = \frac{R_1}{R_1 + \frac{40 R_2}{R_2 + 40}} \times 24 \quad \Rightarrow \quad \frac{1}{3} = \frac{R_1 (R_2 + 40)}{R_1 R_2 + 40(R_1 + R_2)}$$

$$\Rightarrow \quad R_1 R_2 + 40(R_1 + R_2) = 3 R_1 R_2 + 120 R_1 \quad \Rightarrow \quad R_1 = \frac{40 R_2}{2 R_2 + 80}$$

Using KVL

$$24 = 8 + R_2 (1.6) \quad \Rightarrow \quad R_2 = 10\ \Omega$$

Then

$$R_1 = \frac{40(10)}{2(10) + 80} = 4\ \Omega$$

P 3.4-11 Determine the values of the resistances R_1 and R_2 for the circuit shown in Figure P 3.4-11.

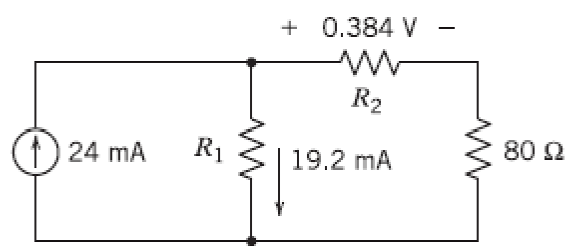


Figure P 3.4-11

Solution:
Using KCL

$$.024 = 0.0192 + \frac{0.384}{R_2} \quad \Rightarrow \quad R_2 = \frac{0.384}{0.0048} = 80\ \Omega$$

Using current division

$$\frac{0.384}{R_2} = \frac{R_1}{R_1 + (R_2 + 80)} \times 0.024 \quad \Rightarrow \quad 16 = \frac{R_1 R_2}{R_1 + R_2 + 80} = \frac{80 R_1}{R_1 + 160} \quad \Rightarrow \quad R_1 = 40\ \Omega$$

P 3.4-12 Determine the value of the current measured by the meter in Figure P 3.4-12.

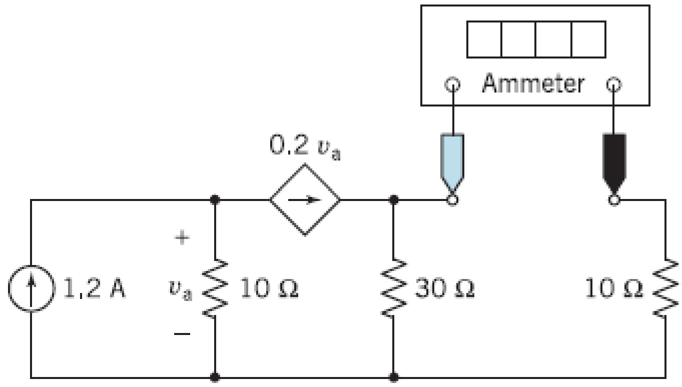
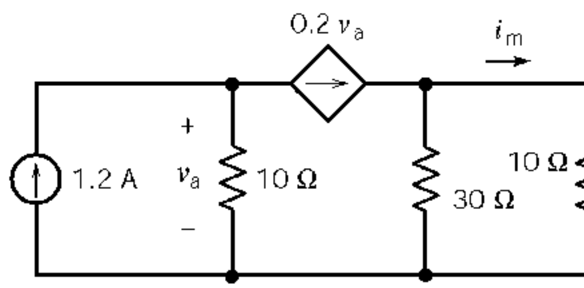


Figure P 3.4-12

Solution:

Replace the (ideal) ammeter with the equivalent short circuit. Label the current measured by the meter.



Apply KCL at the left node of the VCCS to get

$$1.2 = \frac{v_a}{10} + 0.2\,v_a = 0.3\,v_a \quad \Rightarrow \quad v_a = \frac{1.2}{0.3} = 4\text{ V}$$

Use current division to get

$$i_m = \frac{30}{30+10}\,0.2\,v_a = \frac{30}{30+10}\,0.2(4) = 0.6\text{ A}$$

(checked using LNAP 9/11/04)

P3.4-13 Consider the combination of resistors shown in Figure P3.4-13. Let R_p denote the equivalent resistance.

(a) Suppose $20\ \Omega \le R \le 320\ \Omega$. Determine the corresponding range of values of R_p.

(b) Suppose instead $R = 0$ (a short circuit). Determine the value of R_p.

(c) Suppose instead $R = \infty$ (an open circuit). Determine the value of R_p.

(d) Suppose instead the equivalent resistance is $R_p = 40\ \Omega$. Determine the value of R.

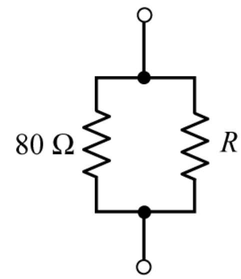


Figure P3.4-13

Solution:

(a) First, when $R = 20\ \Omega$ then $R_p = 80 \| 20 = \dfrac{80(20)}{80+20} = 16\ \Omega$. Next, when $R = 320\ \Omega$ then

$R_p = 80 \| 320 = \dfrac{80(320)}{80+320} = 64\ \Omega$. Consequently

$$16\ \Omega \le R_p \le 64\ \Omega.$$

(b) When $R = 0$ then $R_p = 80 \| 0 = \dfrac{80(0)}{80+0} = 0\ \Omega$.

(c) When $R = \infty$ then $R_p = \dfrac{80(\infty)}{80+\infty} = \dfrac{80}{\dfrac{80}{\infty}+1} = 80\ \Omega$.

(d) When $R_p = 40\ \Omega$ then $40 = 80 \| R = \dfrac{80R}{80+R} \Rightarrow 80 + R = 2R \Rightarrow R = 80\ \Omega$

P3.4-14 Consider the combination of resistors shown in Figure P3.4-14. Let R_p denote the equivalent resistance.

(a) Suppose $40\ \Omega \le R \le 400\ \Omega$. Determine the corresponding range of values of R_p.

(b) Suppose instead $R = 0$ (a short circuit). Determine the value of R_p.

(c) Suppose instead $R = \infty$ (an open circuit). Determine the value of R_p.

(d) Suppose instead the equivalent resistance is $R_p = 80\ \Omega$. Determine the value of R.

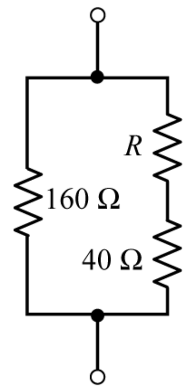


Figure P3.4-14

Solution:

(a) First, when $R = 40\ \Omega$ then $R_p = 160 \,||\, (40+40) = 160 \,||\, 80 = \dfrac{160(80)}{160+80} = \dfrac{160}{3} = 53.33\ \Omega$. Next, when $R = 400\ \Omega$ then $R_p = 160 \,||\, (40+400) = 160 \,||\, 440 = \dfrac{160(440)}{160+440} = \dfrac{16(44)}{6} = 117.33\ \Omega$.

Consequently $$53.33\ \Omega \le R_p \le 117.33\ \Omega.$$

(b) When $R = 0$ then $R_p = 160 \,||\, (0+40) = 160 \,||\, 40 = \dfrac{160(40)}{160+40} = \dfrac{16(4)}{2} = 32\ \Omega$.

(c) When $R = \infty$ then $R_p = 160 \,||\, (\infty+40) = 160 \,||\, \infty = \dfrac{1}{\dfrac{1}{160}+\dfrac{1}{\infty}} = 160\ \Omega$.

(d) When $R_p = 80\ \Omega$ then

$$80 = 160 \,||\, (R+40) = \frac{160(R+40)}{160+R+40} \quad \Rightarrow \quad R+200 = \frac{160}{80}(R+40) = 2R+80 \quad \Rightarrow \quad R = 120\ \Omega.$$

P3.4-15 Consider the combination of resistors shown in Figure P3.4-15. Let R_p denote the equivalent resistance.

(a) Suppose $50\ \Omega \le R \le 800\ \Omega$. Determine the corresponding range of values of R_p.

(b) Suppose instead $R = 0$ (a short circuit). Determine the value of R_p.

(c) Suppose instead $R = \infty$ (an open circuit). Determine the value of R_p.

(d) Suppose instead the equivalent resistance is $R_p = 150\ \Omega$. Determine the value of R.

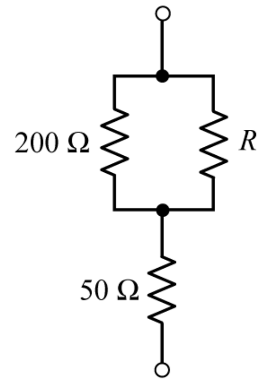


Figure P3.4-15

Solution:

(a) First, when $R = 50\ \Omega$ then $R_p = 50 + (200 \,||\, 50) = 50 + \dfrac{200(50)}{200+50} = 50 + \dfrac{200}{5} = 90\ \Omega$. Next, when $R = 800\ \Omega$ then $R_p = 50 + (200 \,||\, 800) = 50 + \dfrac{200(800)}{200+800} = 50 + \dfrac{800}{5} = 210\ \Omega$.

Consequently $$90\ \Omega \le R_p \le 210\ \Omega.$$

(b) When $R = 0$ then $R_p = 50 + (200 \,||\, 0) = 50 + \dfrac{200(0)}{200+0} = 50\ \Omega$.

(c) When $R = \infty$ then $R_p = 50 + (200 \,||\, \infty) = 50 + \dfrac{200(\infty)}{200+\infty} = 50 + \dfrac{200}{\dfrac{200}{\infty}+1} = 250\ \Omega$.

(d) When $R_p = 150\ \Omega$ then

$$150 = 50 + (200 \,||\, R) = 50 + \frac{200R}{200+R} \quad \Rightarrow \quad 100(200+R) = 200R \quad \Rightarrow \quad R = 200\ \Omega.$$

P3.4-16 The input to the circuit shown in Figure P3.4-16 is the source current, i_s. The output is the current measured by the meter, i_o. A current divider connects the source to the meter. Given the following observations:

A. The input $i_s = 5$ A causes the output to be $i_o = 2$ A.
B. When $i_s = 2$ A the source supplies 48 W.

Determine the values of the resistances R_1 and R_2.

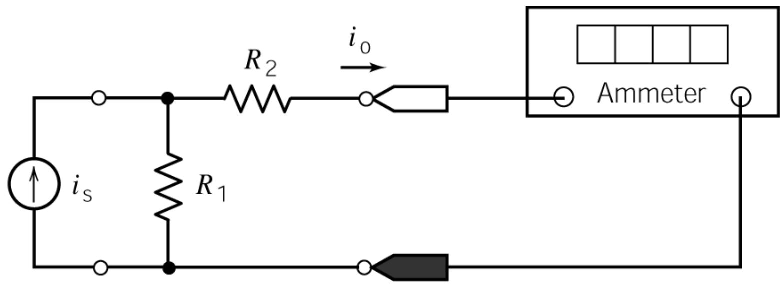


Figure P3.4-16

Solution:

From current division, $i_o = \left(\frac{R_1}{R_1+R_2}\right) i_s$. When $i_s = 5$ A and $i_o = 2$ A then $\frac{2}{5} = \frac{R_1}{R_1+R_2}$ so $2(R_1+R_2) = 5R_1$ or $2R_2 = 3R_1$.

The power supplied by the source is given by $i_s\left[\left(\frac{R_1 R_2}{R_1+R_2}\right) i_s\right]$. When $i_s = 2$ A the source supplies 48 W, so $48 = 2\left[\left(\frac{R_1 R_2}{R_1+R_2}\right) 2\right] \Rightarrow 12 = \frac{R_1 R_2}{R_1+R_2}$.

Combining these results gives $12 = \frac{R_1\left(\frac{3}{2}R_1\right)}{R_1+\left(\frac{3}{2}R_1\right)} = \frac{\frac{3}{2}R_1}{\frac{5}{2}} = \frac{3}{5}R_1 \Rightarrow R_1 = \frac{5}{3}(12) = 20\ \Omega$ and $\frac{3R_1}{2} = 30\ \Omega$.

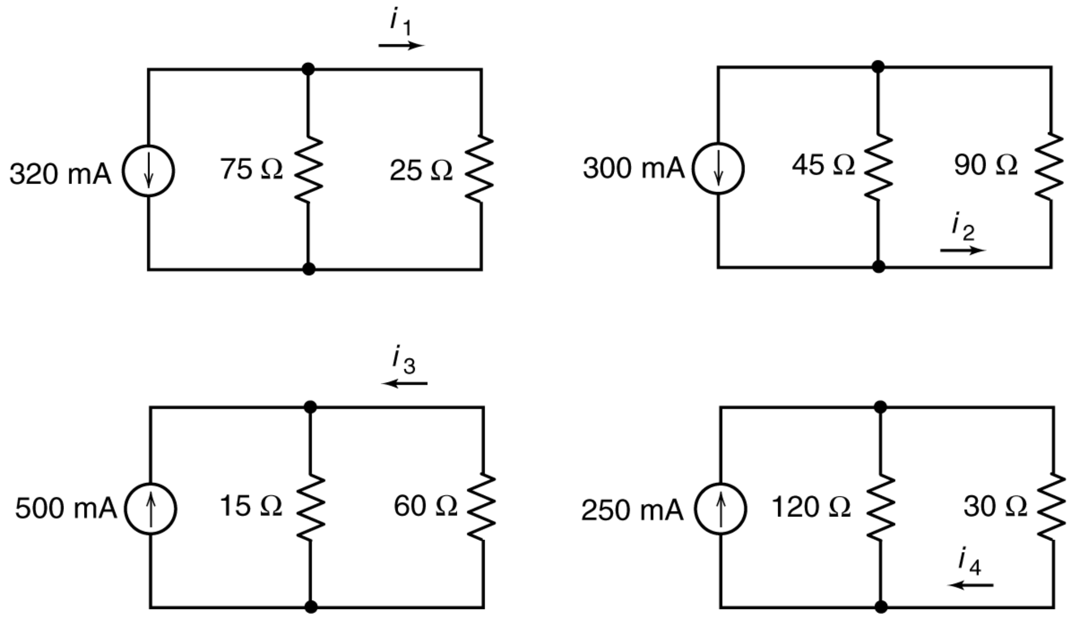


Figure P3.4-17

P3.4-17. Figure P3.4-17 shows four similar but slightly different circuits. Determine the values of the currents i_1, i_2, i_3 and i_4.

Solution:
Using current division:

$$i_1 = -\left(\frac{75}{75+25}\right)320 = -240 \text{ mA}, \quad i_2 = \left(\frac{45}{45+90}\right)300 = 100 \text{ mA},$$

$$i_3 = -\left(\frac{15}{15+60}\right)500 = -100 \text{ mA} \quad \text{and} \quad i_4 = \left(\frac{120}{120+30}\right)250 = 200 \text{ mA}$$

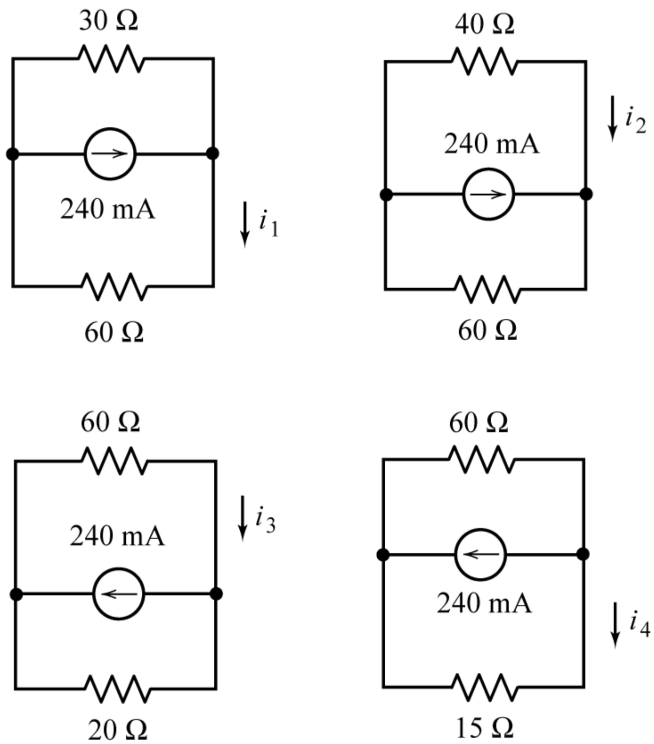


Figure P3.4-18

P3.4-18. Figure P3.4-18 shows four similar but slightly different circuits. Determine the values of the currents i_1, i_2, i_3 and i_4.

Solution:
Using current division:

$$i_1 = \left(\frac{30}{30+60}\right) 240 = 80 \text{ mA}, \quad i_2 = -\left(\frac{60}{60+40}\right) 240 = -144 \text{ mA},$$

$$i_3 = \left(\frac{20}{60+20}\right) 240 = 60 \text{ mA} \quad \text{and} \quad i_4 = -\left(\frac{60}{60+15}\right) 240 = -192 \text{ mA}$$

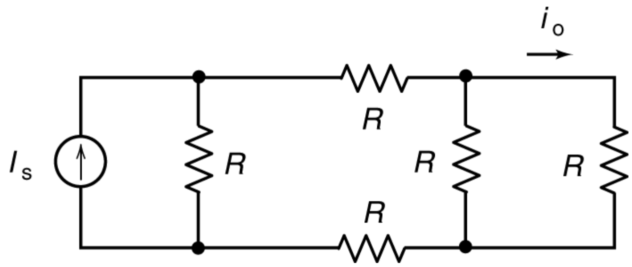


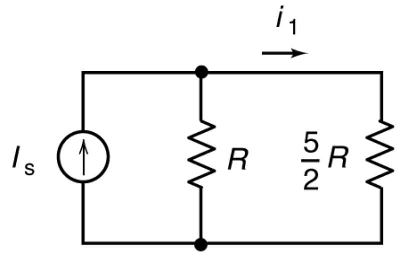
Figure P3.4-19

P3.4-19. The input to the circuit shown in Figure P3.4-19 is the current source current I_s. The output is the current i_o. The output of this circuit is proportion to the input, that is

$$i_o = k\, I_s$$

Determine the value of the constant of proportionality, *k*.

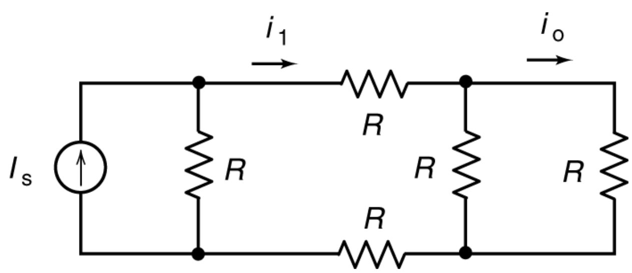
Solution:
Replace six resistors at the right of the circuit by an equivalent resistance to get



Using current division:

$$i_1 = \left(\frac{\frac{5}{2}R}{R + \frac{5}{2}R} \right) I_s = \frac{2}{7} I_s$$

Return to the original circuit



Using current division:

$$i_o = \frac{R}{R+R} i_1 = \frac{1}{2} i_1 = \frac{1}{2}\left(\frac{2}{7} I_s\right) = \frac{1}{7} I_s$$

Therefore
$$k = \frac{1}{7}$$

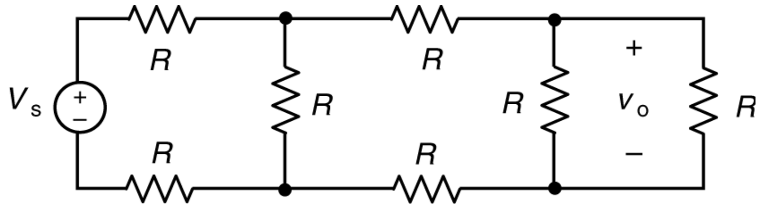


Figure P3.4-20

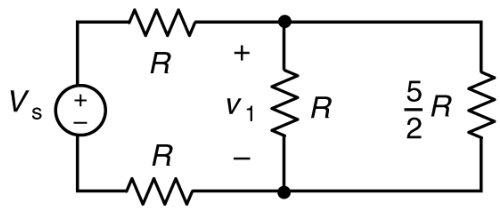
P3.4-20. The input to the circuit shown in Figure P3.4-20 is the voltage source voltage V_s. The output is the voltage v_o. The output of this circuit is proportion to the input, that is

$$v_o = k V_s$$

Determine the value of the constant of proportionality, *k*.

Solution:

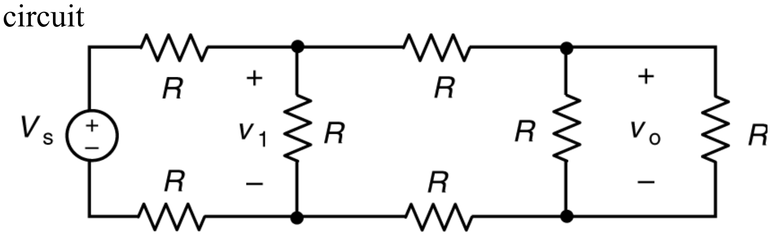
Replace six resistors at the right of the circuit by an equivalent resistance to get



Using voltage division:

$$v_1 = \left(\frac{R \,||\, \frac{5}{2} R}{2R + R \,||\, \frac{5}{2} R} \right) V_s \left(\frac{\frac{5}{7} R}{2R + \frac{5}{7} R} \right) V_s = \frac{5}{19} V_s$$

Return to the original circuit



Using current division:

$$v_o = \frac{\frac{R}{2}}{2R + \frac{R}{2}} v_1 = \frac{1}{5} v_1 = \frac{1}{5}\left(\frac{5}{19} I_s \right) = \frac{1}{19} V_s$$

Therefore

$$k = \frac{1}{19}$$

Section 3-5 Series Voltage Sources and Parallel Current Sources

P 3.5-1 Determine the power supplied by each source in the circuit shown in Figure P 3.5-1.

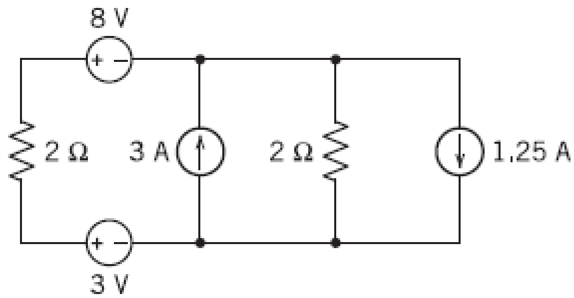
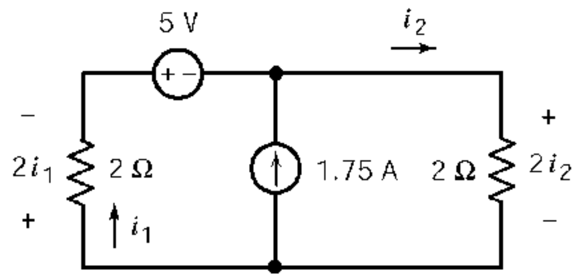


Figure P 3.5-1

Solution:

The voltage sources are connected in series and can be replaced by a single equivalent voltage source. Similarly, the parallel current sources can be replaced by an equivalent current source.

After doing so, and labeling the resistor currents, we have the circuit shown.



Apply KCL at the top node of the current source to get

$$i_1 + 1.75 = i_2$$

Apply KVL to the outside loop to get

$$5 + 2i_2 + 2i_1 = 0$$

so $$5 + 2(i_1 + 1.75) + 2i_1 = 0 \quad \Rightarrow \quad i_1 = -\frac{8.5}{4} = -2.125 \text{ A}$$

and $$i_2 = -2.125 + 1.75 = -0.375 \text{ A}$$

The power supplied by each sources is:

Source	Power delivered
8-V voltage source	$-8i_1 = 17$ W
3-V voltage source	$3i_1 = -6.375$ W
3-A current source	$3 \times 2i_2 = -2.25$ W
1.25-A current source	$-1.25 \times 2i_2 = 0.9375$ W

(Checked using LNAP, 9/14/04)

P 3.5-2 Determine the power supplied by each source in the circuit shown in Figure P 3.5-2.

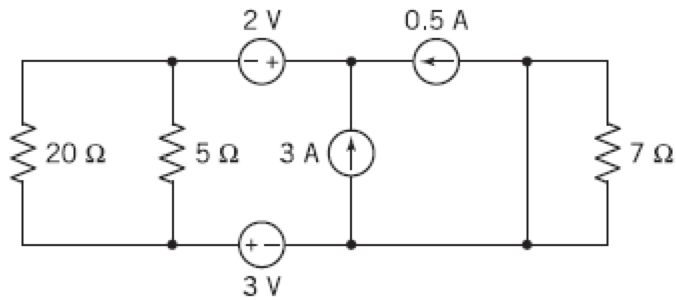


Figure P 3.5-2

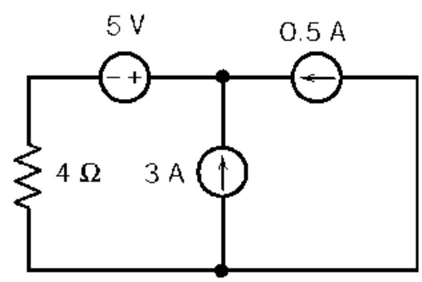
Solution:

The 20-Ω and 5-Ω resistors are connected in parallel. The equivalent resistance is $\frac{20 \times 5}{20+5} = 4\ \Omega$.

The 7-Ω resistor is connected in parallel with a short circuit, a 0-Ω resistor. The equivalent resistance is $\frac{0 \times 7}{0+7} = 0\ \Omega$, a short circuit.

The voltage sources are connected in series and can be replaced by a single equivalent voltage source.

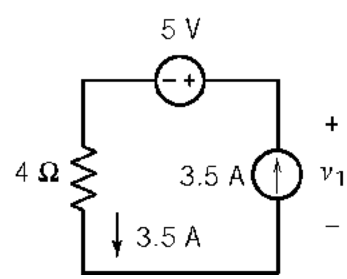
After doing so, and labeling the resistor currents, we have the circuit shown.



The parallel current sources can be replaced by an equivalent current source.

Apply KVL to get

$$-5 + v_1 - 4(3.5) = 0 \quad \Rightarrow \quad v_1 = 19\ \text{V}$$



The power supplied by each sources is:

Source	Power delivered
8-V voltage source	$-2(3.5) = -7$ W
3-V voltage source	$-3(3.5) = -10.5$ W
3-A current source	$3 \times 19 = 57$ W
0.5-A current source	$0.5 \times 19 = 9.5$ W

(Checked using LNAP, 9/15/04)

P 3.5-3 Determine the power received by each resistor in the circuit shown in Figure P 3.5-3.

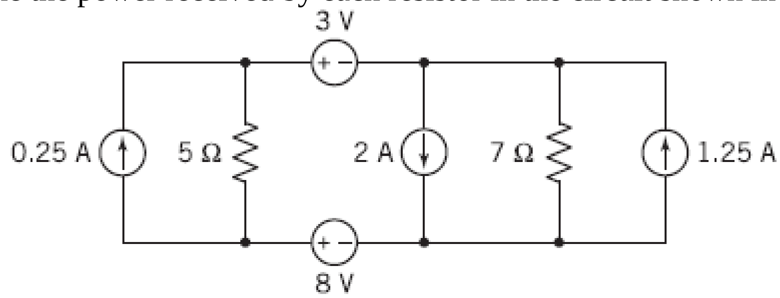
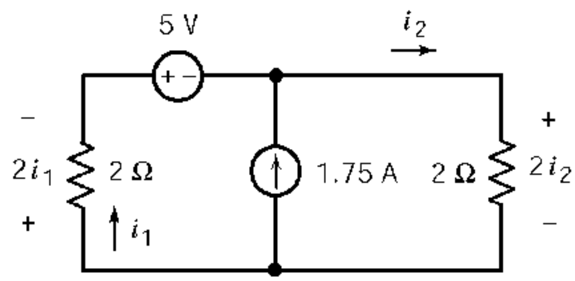


Figure P 3.5-3

Solution:

The voltage sources are connected in series and can be replaced by a single equivalent voltage source. Similarly, the parallel current sources can be replaced by an equivalent current source.

After doing so, and labeling the resistor currents, we have the circuit shown.



Apply KCL at the top node of the current source to get

$$i_1 + 1.75 = i_2$$

Apply KVL to the outside loop to get

$$5 + 2i_2 + i_1 = 0$$

so

$$5 + 2(i_1 + 1.75) + 2i_1 = 0 \quad \Rightarrow \quad i_1 = -\frac{8.5}{4} = -2.125 \text{ A}$$

and

$$i_2 = -2.125 + 1.75 = -0.375 \text{ A}$$

The power supplied by each sources is:

Source	Power delivered
8-V voltage source	$-8i_1 = 17$ W
3-V voltage source	$3i_1 = -6.375$ W
3-A current source	$3 \times 2i_2 = -2.25$ W
1.25-A current source	$-1.25 \times 2i_2 = 0.9375$ W

(Checked using LNAP, 9/14/04)

Section 3-6 Circuit Analysis

P 3.6-1 The circuit shown in Figure P 3.6-1*a* has been divided into two parts. In Figure P 3.6-1*b*, the right-hand part has been replaced with an equivalent circuit. The left-hand part of the circuit has not been changed.

(a) Determine the value of the resistance R in Figure P 3.6-1*b* that makes the circuit in Figure P 3.6-1*b* equivalent to the circuit in Figure P 3.6-1*a*.

(b) Find the current i and the voltage v shown in Figure P 3.6-1*b*. Because of the equivalence, the current i and the voltage v shown in Figure P 3.6-1*a* are equal to the current i and the voltage v shown in Figure P 3.6-1*b*.

(c) Find the current i_2 shown in Figure P 3.6-1*a* using current division.

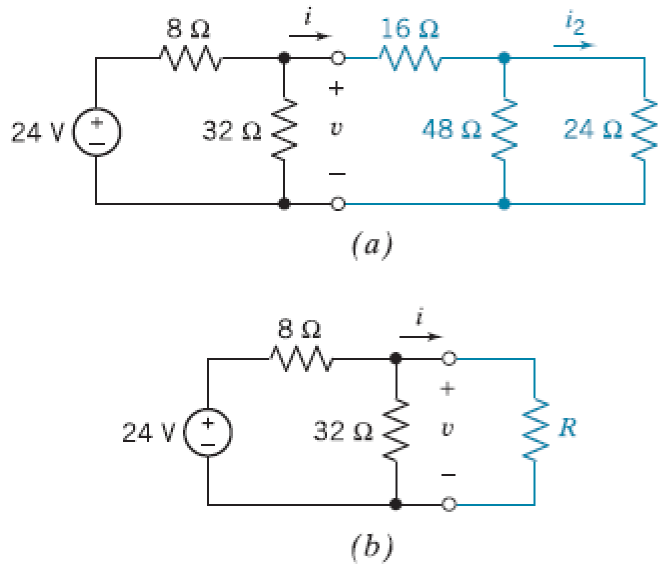
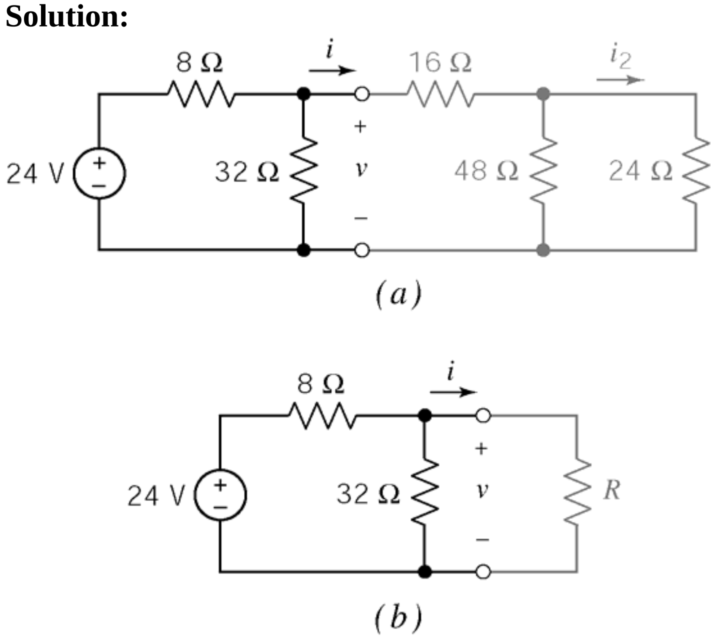


Figure P 3.6-1

Solution:



$$(a)\quad R = 16+\frac{48\cdot 24}{48+24} = \underline{32\ \Omega}$$

$$(b)\quad v = \frac{\dfrac{32\cdot 32}{32+32}}{8+\dfrac{32\cdot 32}{32+32}}24 = \underline{16\text{ V}}\ ;$$

$$i=\frac{16}{32}=\underline{\frac{1}{2}\text{ A}}$$

$$(c)\quad i_2 = \frac{48}{48+24}\cdot\frac{1}{2} = \underline{\frac{1}{3}\text{ A}}$$

P 3.6-2 The circuit shown in Figure P 3.6-2*a* has been divided into three parts. In Figure P 3.6-2*b,* the rightmost part has been replaced with an equivalent circuit. The rest of the circuit has not been changed. The circuit is simplified further in Figure 3.6-2*c*. Now the middle and rightmost parts have been replaced by a single equivalent resistance. The leftmost part of the circuit is still unchanged.

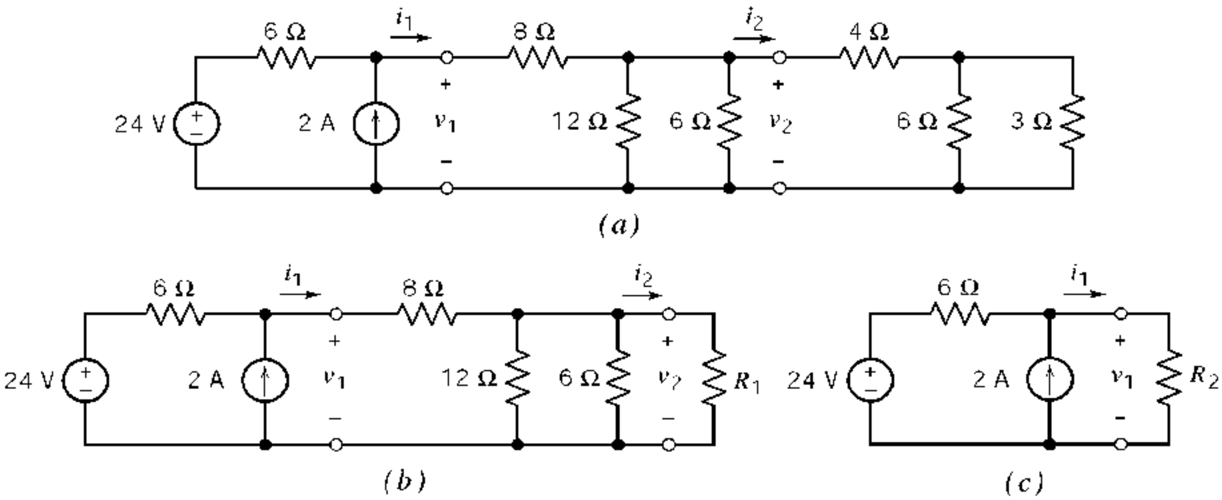


Figure P 3.6-2

(a) Determine the value of the resistance R_1 in Figure P 3.6-2*b* that makes the circuit in Figure P 3.6-2*b* equivalent to the circuit in Figure P 3.6-2*a*.

(b) Determine the value of the resistance R_2 in Figure P 3.6-2*c* that makes the circuit in Figure P 3.6-2*c* equivalent to the circuit in Figure P 3.6-2*b*.

(c) Find the current i_1 and the voltage v_1 shown in Figure P 3.6-2*c*. Because of the equivalence, the current i_1 and the voltage v_1 shown in Figure P 3.6-2*b* are equal to the current i_1 and the voltage v_1 shown in Figure P 3.6-2*c*.
Hint: $24 = 6(i_1 - 2) + i_1 R_2$

(d) Find the current i_2 and the voltage v_2 shown in Figure P 3.6-2*b*. Because of the equivalence, the current i_2 and the voltage v_2 shown in Figure P 3.6-2*a* are equal to the current i_2 and the voltage v_2 shown in Figure P 3.6-2*b*.
Hint: Use current division to calculate i_2 from i_1.

(e) Determine the power absorbed by the 3-Ω resistance shown at the right of Figure P 3.6-2*a*.

Solution:

$$(a)\ R_1 = 4 + \frac{3 \cdot 6}{3+6} = \underline{6\,\Omega}$$

$$(b)\ \frac{1}{R_p} = \frac{1}{12} + \frac{1}{6} + \frac{1}{6} \Rightarrow R_p = 2.4\,\Omega \quad \text{then} \quad R_2 = 8 + R_p = \underline{10.4\,\Omega}$$

$$(c)\ \text{KCL: } i_2 + 2 = i_1 \quad \text{and} \quad -24 + 6\,i_2 + R_2 i_1 = 0$$

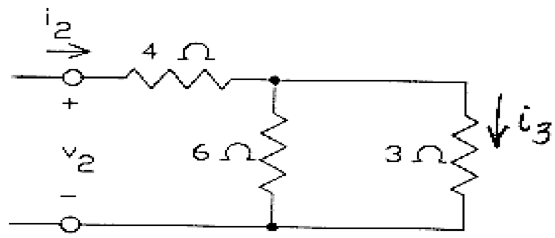
$$\Rightarrow \ -24 + 6\,(i_1 - 2) + 10.4 i_1 = 0$$

$$\Rightarrow \ i_1 = \frac{36}{16.4} = \underline{2.195\text{ A}} \ \Rightarrow \ v_1 = i_1\ R_2 = 2.2\,(10.4) = \underline{22.83\text{ V}}$$

$$(d)\ \ i_2 = \frac{\frac{1}{6}}{\frac{1}{6}+\frac{1}{6}+\frac{1}{12}}(2.195) = \underline{0.878\text{ A}},$$

$$v_2 = (0.878)(6) = \underline{5.3\text{ V}}$$

$$(e)\ \ i_3 = \frac{6}{3+6} i_2 = 0.585\text{ A} \quad \Rightarrow \quad P = 3\, i_3^{\,2} = \underline{1.03\text{ W}}$$



P 3.6-3 Find i using appropriate circuit reductions and the current divider principle for the circuit of Figure P 3.6-3.

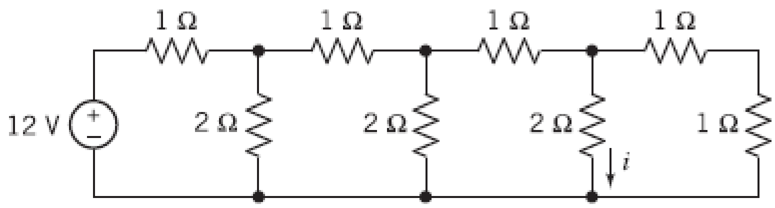
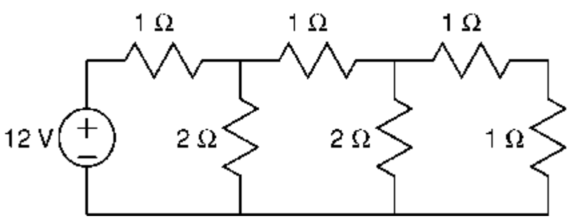
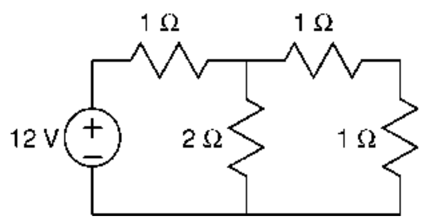


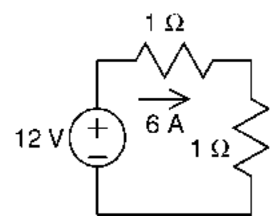
Figure P 3.6-3

Solution:

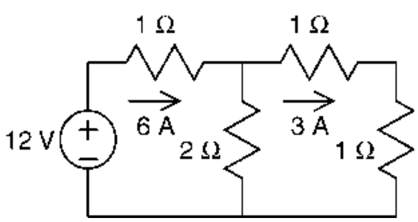
Reduce the circuit from the right side by repeatedly replacing series 1 Ω resistors in parallel with a 2 Ω resistor by the equivalent 1 Ω resistor

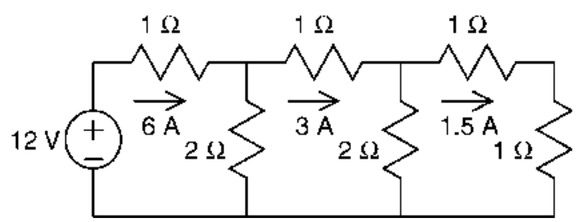


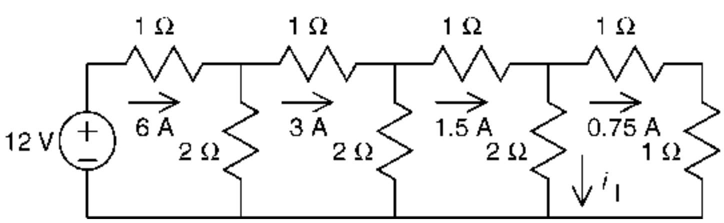




This circuit has become small enough to be easily analyzed. The vertical 1 Ω resistor is equivalent to a 2 Ω resistor connected in parallel with series 1 Ω resistors:







$$i_1 = \frac{1+1}{2+(1+1)}(1.5) = 0.75 \text{ A}$$

P 3.6-4

(a) Determine values of R_1 and R_2 in Figure P 3.6-4*b* that make the circuit in Figure P 3.6-4*b* equivalent to the circuit in Figure P 3.6-4*a*.

(b) Analyze the circuit in Figure P 3.6-4*b* to determine the values of the currents i_a and i_b

(c) Because the circuits are equivalent, the currents i_a and i_b shown in Figure P 3.6-4*b* are equal to the currents i_a and i_b shown in Figure P 3.6-4*a*. Use this fact to determine values of the voltage v_1 and current i_2 shown in Figure P 3.6-4*a*.

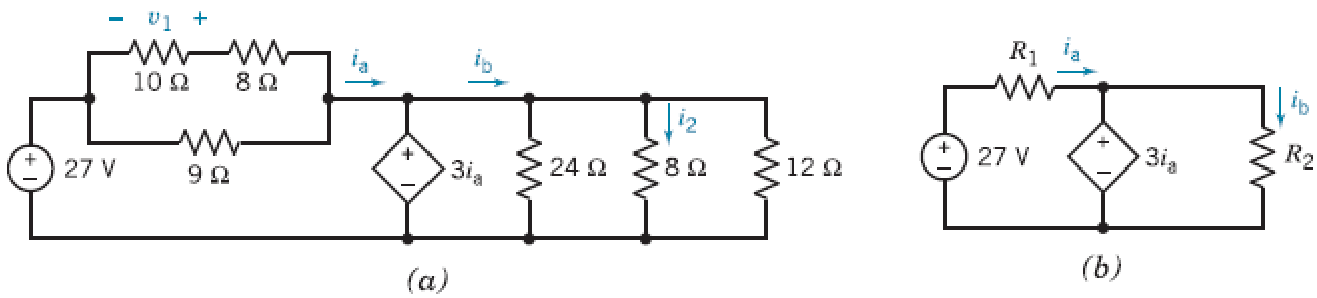
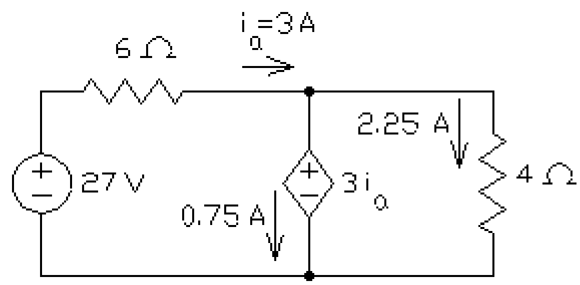


Figure P 3.6-4

Solution:

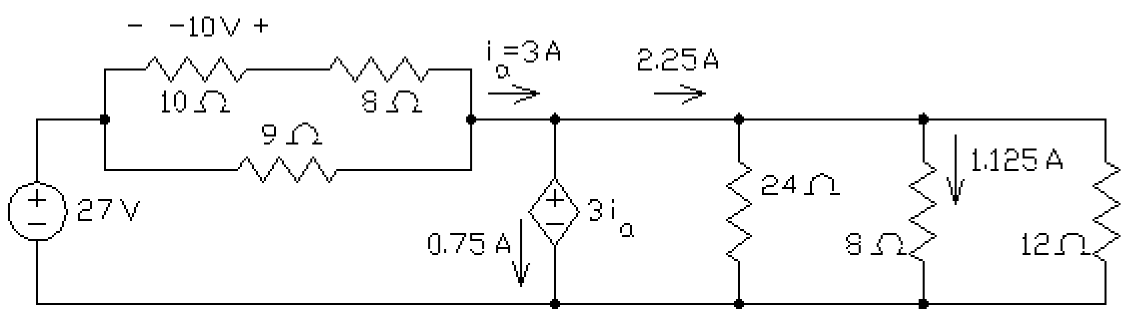
(a) $$\frac{1}{R_2} = \frac{1}{24} + \frac{1}{12} + \frac{1}{8} \quad \Rightarrow \quad R_2 = 4\,\Omega \quad \text{and} \quad R_1 = \frac{(10+8)\cdot 9}{(10+8)+9} = 6\,\Omega$$

(b)



First, apply KVL to the left mesh to get $-27 + 6i_a + 3i_a = 0 \quad \Rightarrow \quad i_a = 3\text{ A}$. Next, apply KVL to the left mesh to get $4i_b - 3i_a = 0 \quad \Rightarrow \quad i_b = 2.25\text{ A}$.

(c)



$$i_2 = \frac{\frac{1}{8}}{\frac{1}{24} + \frac{1}{8} + \frac{1}{12}} 2.25 = 1.125\text{ A} \quad \text{and} \quad v_1 = -(10)\left[\frac{9}{(10+8)+9}3\right] = -10\text{ V}$$

P 3.6-5 The voltmeter in the circuit shown in Figure P 3.6-5 shows that the voltage across the 30-Ω resistor is 6 volts. Determine the value of the resistance R_1.

Hint: Use the voltage division twice.

Answer: $R_1 = 40\ \Omega$

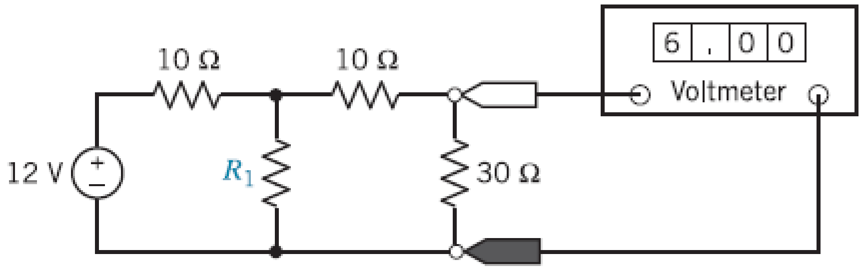
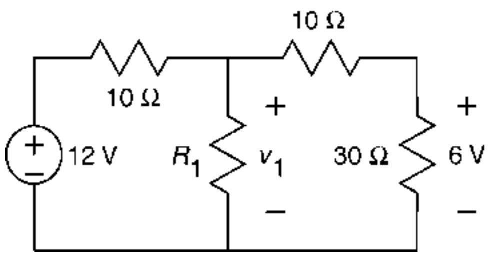
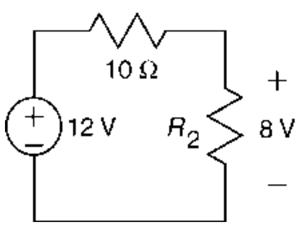


Figure P 3.6-5

P3.6-5



$$\frac{30}{10+30} v_1 = 6 \quad \Rightarrow \quad v_1 = 8\text{ V}$$



$$\frac{R_2}{R_2+10} 12 = 8 \quad \Rightarrow \quad R_2 = 20\,\Omega$$

$$20 = \frac{R_1(10+30)}{R_1+(10+30)} \quad \Rightarrow \quad R_1 = 40\,\Omega$$

Alternate values that can be used to change the numbers in this problem:

meter reading, V	Right-most resistor, Ω	R_1, Ω
6	30	40
4	30	10
4	20	15
4.8	20	30

P 3.6-6 Determine the voltages v_a and v_c and the currents i_b and i_d for the circuit shown in Figure P 3.6-6.

Answer: $v_a = -2$ V, $v_c = 6$ V, $i_b = -16$ mA, and $i_d = 2$ mA

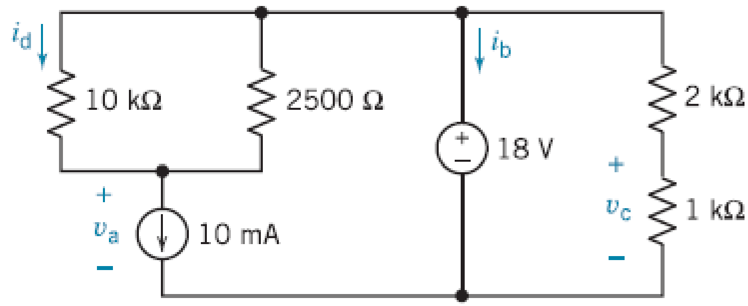
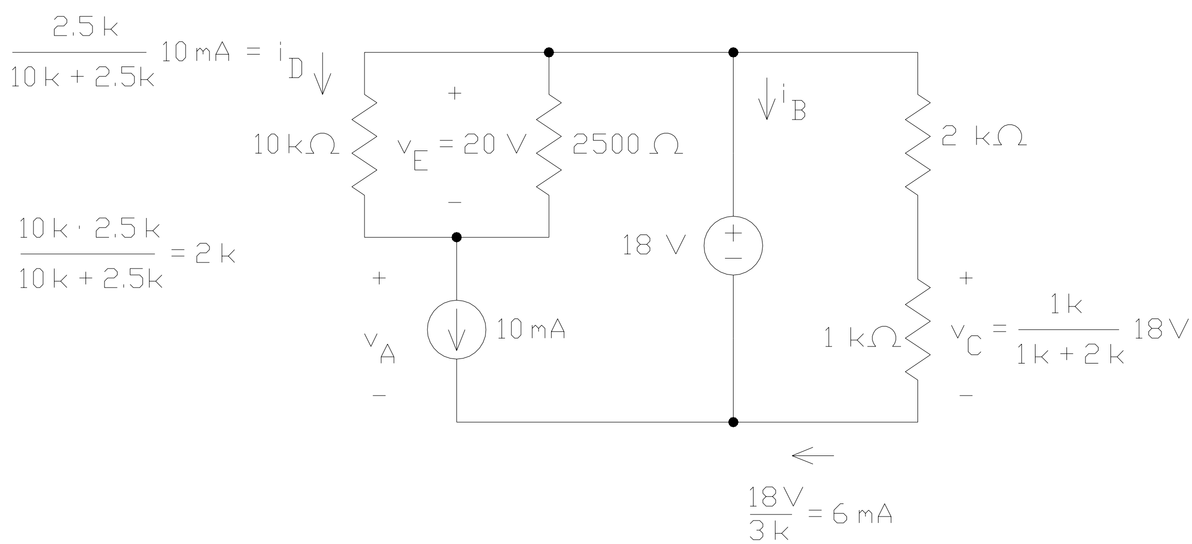


Figure P 3.6-6

Solution:



P 3.6-7 Determine the value of the resistance R in Figure P 3.6-7.
Answer: $R = 28\ \text{k}\Omega$

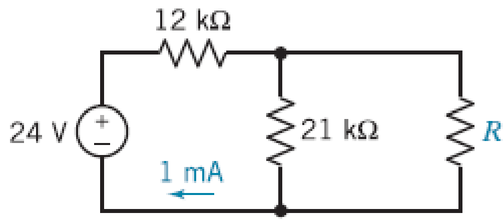
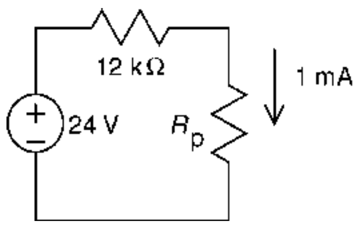


Figure P 3.6-7

Solution:



$$1\times10^{-3} = \frac{24}{12\times10^{3} + R_p} \Rightarrow R_p = 12\times10^{3} = 12\ \text{k}\Omega$$

$$12\times10^{3} = R_p = \frac{\left(21\times10^{3}\right)R}{\left(21\times10^{3}\right)+R} \quad\Rightarrow\quad R = 28\ \text{k}\Omega$$

P 3.6-8 Most of us are familiar with the effects of a mild electric shock. The effects of a severe shock can be devastating and often fatal. Shock results when current is passed through the body. A person can be modeled as a network of resistances. Consider the model circuit shown in Figure P 3.6-8. Determine the voltage developed across the heart and the current flowing through the heart of the person when he or she firmly grasps one end of a voltage source whose other end is connected to the floor.

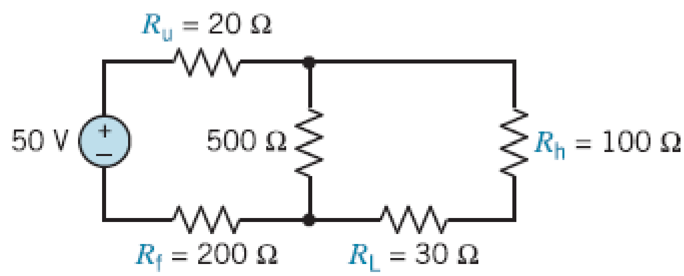
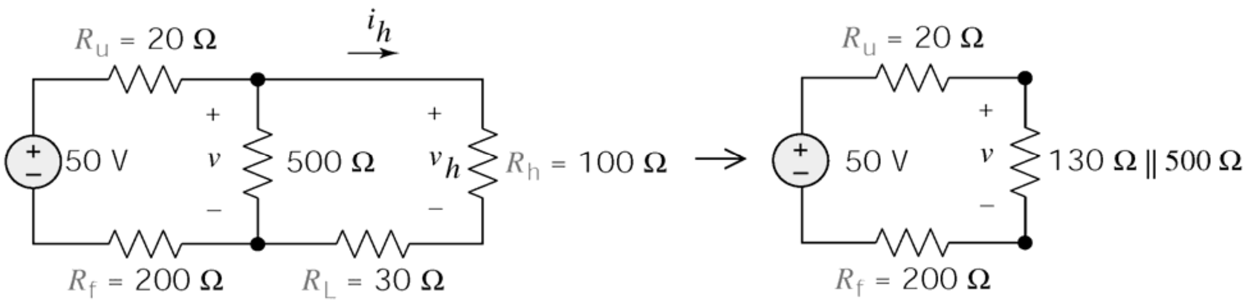


Figure P 3.6-8

The heart is represented by R_h. The floor has resistance to current flow equal to R_f, and the person is standing barefoot on the floor. This type of accident might occur at a swimming pool or boat dock. The upper-body resistance R_u and lower-body resistance R_L vary from person to person.

Solution:



$$\text{Voltage division } \Rightarrow v = 50\left(\frac{130\|500}{130\|500+200+20}\right) = 15.963\ V$$

$$\therefore v_h = v\left(\frac{100}{100+30}\right) = (15.963)\left(\frac{10}{13}\right) = \underline{12.279\ \text{V}}$$

$$\therefore\ i_h = \frac{v_h}{100} = \underline{.12279\ \text{A}}$$

P 3.6-9 Determine the value of the current i in Figure 3.6-9.

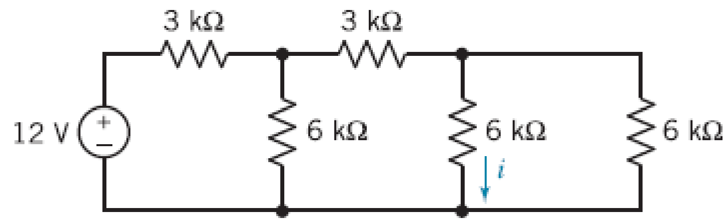
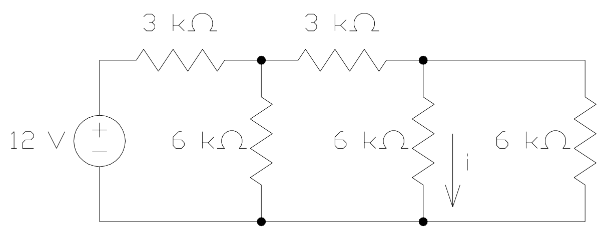
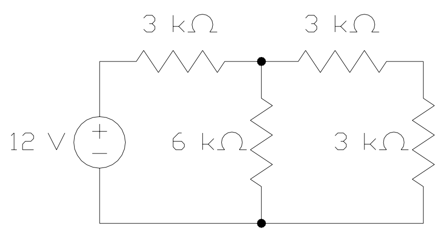


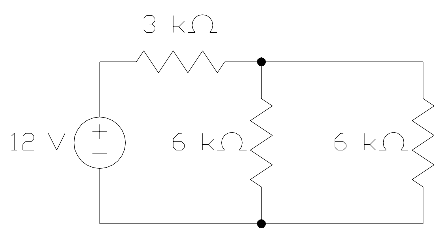
Figure 3.6-9

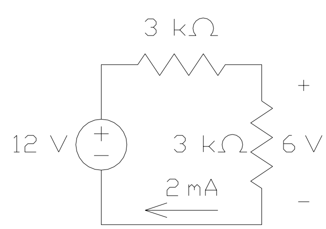
Answer: $i = 0.5$ mA

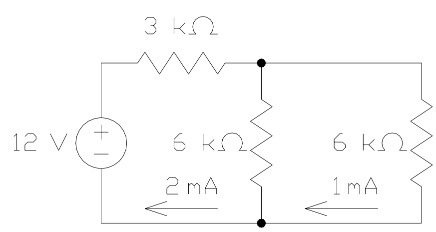
Solution:

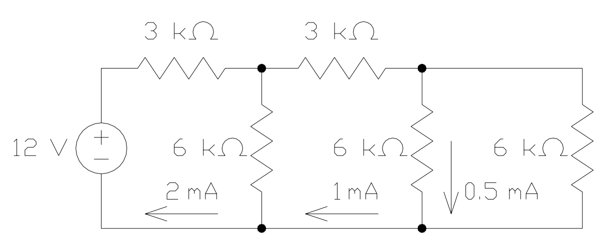












P 3.6-10 Determine the values of i_a, i_b, and v_c in Figure P 3.6-10.

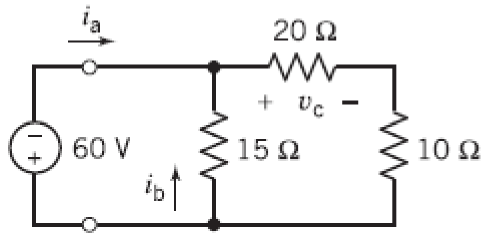
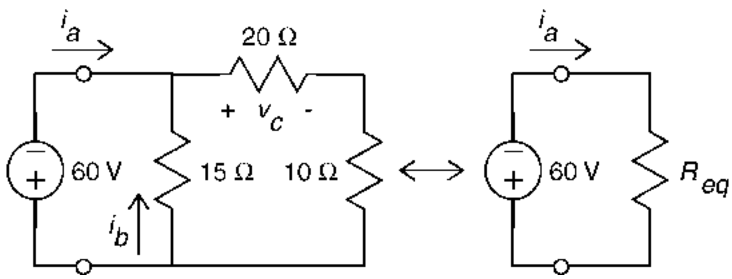


Figure P 3.6-10

Solution:



$$R_{eq} = \frac{15\left(20+10\right)}{15+\left(20+10\right)} = 10\ \Omega$$

$$i_a = -\frac{60}{R_{eq}} = -6\text{ A},\quad i_b = \left(\frac{30}{30+15}\right)\left(\frac{60}{R_{eq}}\right) = 4\text{ A},\quad v_c = \left(\frac{20}{20+10}\right)\left(-60\right) = -40\text{ V}$$

P 3.6-11 Find i and $R_{eq\ a\text{–}b}$ if $v_{ab} = 40$ V in the circuit of Figure P 3.6-11.
Answer: $R_{eq\ a\text{–}b} = 8\ \Omega$, $i = 5/6$ A

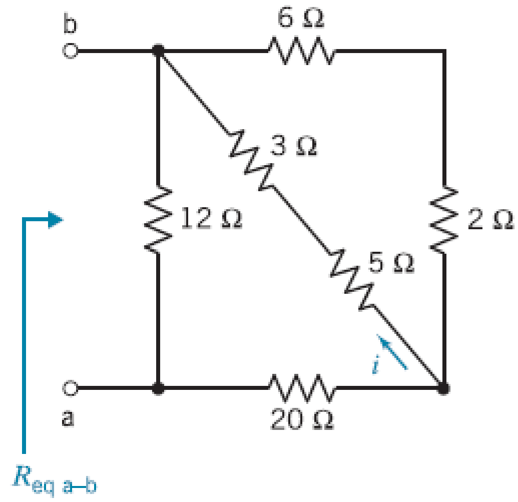
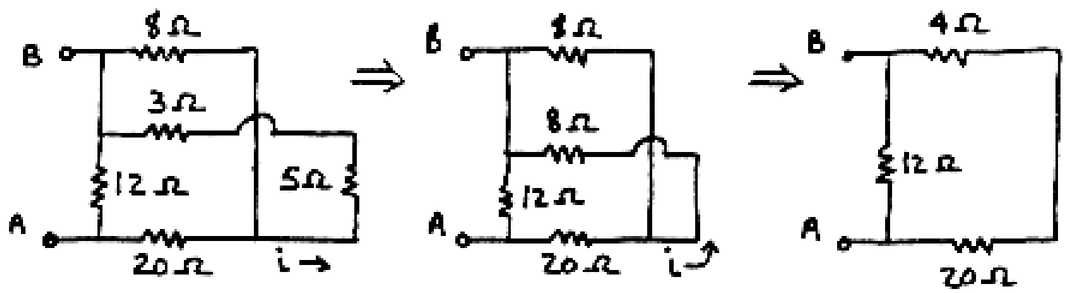


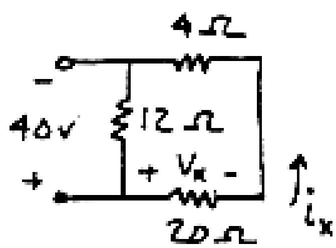
Figure P 3.6-11

Solution:
a)



$$R_{eq} = 24 \| 12 = \frac{(24)(12)}{24 + 12} = \underline{8\ \Omega}$$

b)



from voltage division:

$$v_x = 40\left(\frac{20}{20+4}\right) = \frac{100}{3}\text{ V} \therefore i_x = \frac{\frac{100}{3}}{20} = \frac{5}{3}\text{ A}$$

from current division: $i = i_x \left(\frac{8}{8+8}\right) = \underline{\frac{5}{6}\text{ A}}$

P 3.6-12 The ohmmeter in Figure P 3.6-12 measures the equivalent resistance, R_{eq}, of the resistor circuit. The value of the equivalent resistance, R_{eq}, depends on the value of the resistance R.

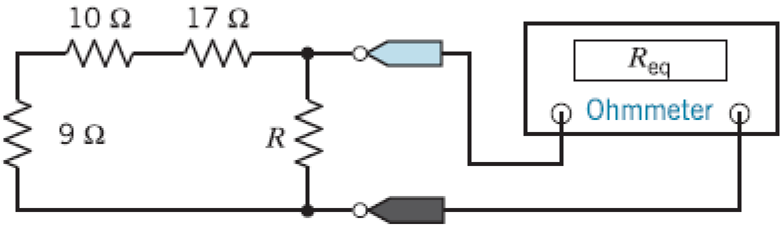
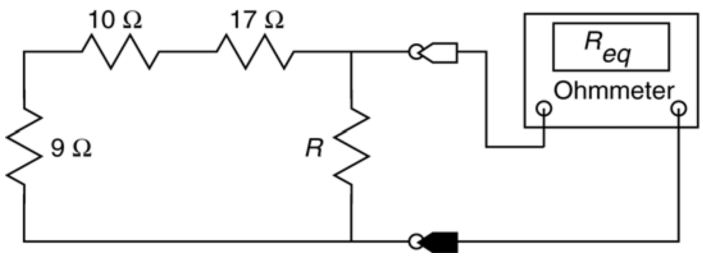


Figure P 3.6-12

(a) Determine the value of the equivalent resistance, R_{eq}, when $R = 18\ \Omega$.
(b) Determine the value of the resistance R required to cause the equivalent resistance to be $R_{eq} = 18\ \Omega$.

Solution:



$$9+10+17=36\ \Omega$$

a.) $$\frac{36(9)}{36+9}=7.2\ \Omega$$

b.) $$\frac{36\,R}{36+R}=12 \quad\Rightarrow\quad 24\,R=(12)(36) \quad\Rightarrow\quad R=18\ \Omega$$

P 3.6-13 Find the R_{eq} at terminals a–b in Figure P 3.6-13. Also determine i, i_1, and i_2.
Answer: $R_{eq} = 8\ \Omega$, $i = 5$ A, $i_1 = 5/3$ A, $i_2 = 5/2$ A

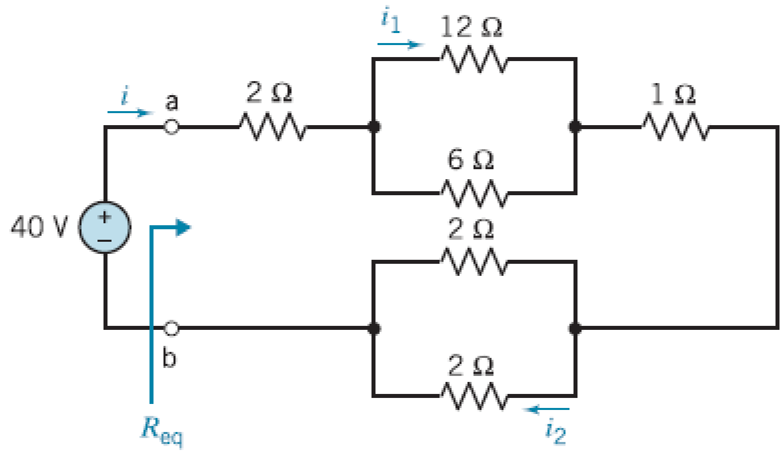


Figure P 3.6-13

Solution:

$$R_{eq} = 2+1+\left(6\|12\right)+\left(2\|2\right)=3+4+1=\underline{8\ \Omega} \quad \text{so} \quad i=\frac{40}{R_{eq}}=\frac{40}{8}=\underline{5\ \text{A}}$$

Using current division

$$i_1 = i\left(\frac{6}{6+12}\right) = (5)\left(\tfrac{1}{3}\right) = \underline{\tfrac{5}{3}\ \text{A}} \quad \text{and} \quad i_2 = i\left(\frac{2}{2+2}\right) = (5)\left(\tfrac{1}{2}\right) = \underline{\tfrac{5}{2}\ \text{A}}$$

P 3.6-14 All of the resistances in the circuit shown in Figure P 3.6-14 are multiples of R. Determine the value of R.

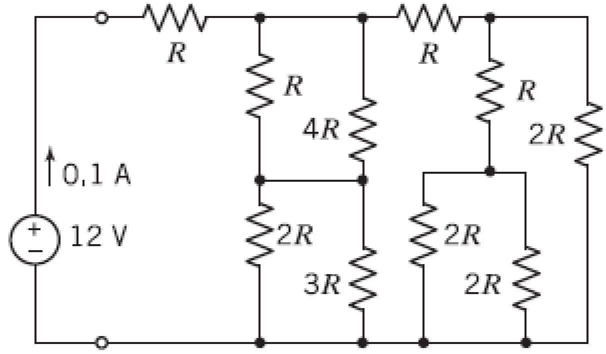


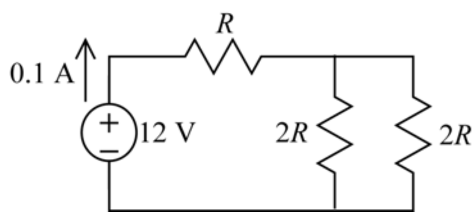
Figure P 3.6-14

Solution:

$$\left(R \,||\, 4R\right)+\left(2R \,||\, 3R\right)=\frac{4}{5}R+\frac{6}{5}R=2R$$

$$R+\left(2R \,||\, \left(R+\left(2R \,||\, 2R\right)\right)\right)=R+\left(2R \,||\, 2R\right)=2R$$

So the circuit is equivalent to



Then

$$12=0.1\left(R+\left(2R \,||\, 2R\right)\right)=0.1\left(2R\right) \quad \Rightarrow \quad R=60\ \Omega$$

(checked: ELAB 5/31/04)

P 3.6-15 The circuit shown in Figure P 3.6-15 contains seven resistors, each having resistance *R*. The input to this circuit is the voltage source voltage, v_s. The circuit has two outputs, v_a and v_b. Express each output as a function of the input.

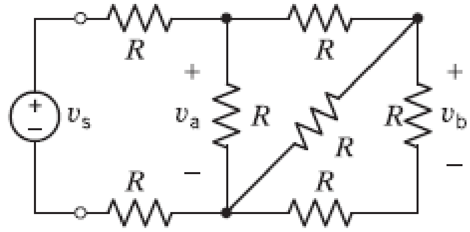
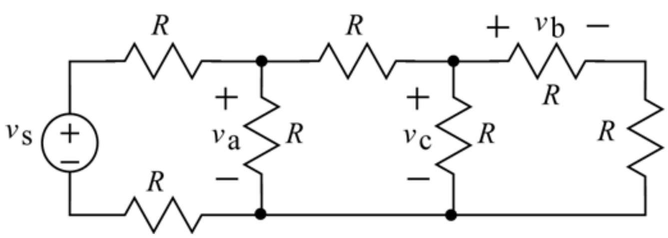


Figure P 3.6-15

Solution:

The circuit can be redrawn as



$$v_a = \frac{R \| \left(R + (R \| 2R)\right)}{2R + R \| \left(R + (R \| 2R)\right)} v_s = \frac{5}{21} v_s$$

$$v_c = \frac{R \| 2R}{R + (R \| 2R)} v_s = \frac{2}{5} v_a = \frac{2}{21} v_s$$

$$v_b = \frac{R}{R + R} v_c = \frac{1}{2} v_c = \frac{1}{21} v_s$$

(Checked using LNAP 5/23/04)

P 3.6-16 The circuit shown in Figure P 3.6-16 contains three 10-Ω, 1/4-W resistors. (Quarter-watt resistors can dissipate 1/4 W safely.) Determine the range of voltage source voltages, v_s, such that none of the resistors absorbs more than 1/4 W of power.

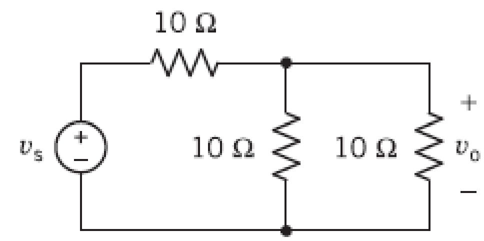


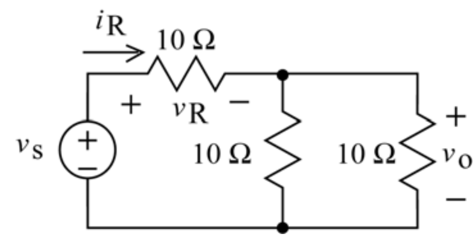
Figure P 3.6-16

Solution:

$$v_o = \frac{(10 \| 10)}{10 + (10 \| 10)} v_s = \frac{5}{15} v_s = \frac{v_s}{3}$$

$$v_R + v_o - v_s = 0 \quad \Rightarrow \quad v_R = \frac{2}{3} v_s$$

$$i_R = \frac{v_R}{10} = \frac{2}{30} v_s$$



$$P = \left(\frac{2}{30} v_s\right)^2 (10) = \frac{4}{90} v_s^{\,2} \le \frac{1}{4} \quad \Rightarrow \quad |v_s| \le \sqrt{\frac{90}{16}} = \frac{3\sqrt{10}}{4} = 2.37 \text{ V}$$

(checked: LNAP 5/31/04)

P 3.6-17 The four resistors shown in Figure P 3.6-17 represent strain gauges. Strain gauges are transducers that measure the strain that results when a resistor is stretched or compressed. Strain gauges are used to measure force, displacement, or pressure. The four strain gauges in Figure P 3.6-17 each have a nominal (unstrained) resistance of 120 **Ω** and can each absorb 0.2 mW safely. Determine the range of voltage source voltages, v_s, such that no strain gauge absorbs more than 0.2 mW of power.

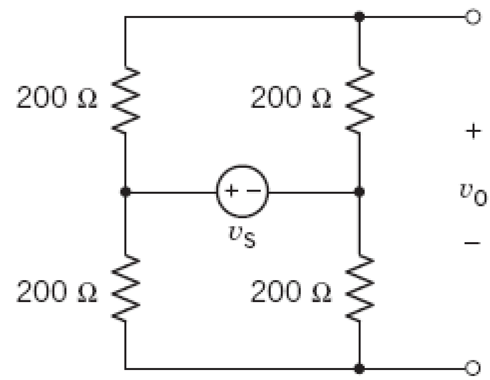


Figure P 3.6-17

Solution:

The voltage across each strain gauge is $\frac{v_s}{2}$ so the current in each strain gauge is $\frac{v_s}{400}$. The power dissipated by each resistor is given by $\frac{v_s}{2}\left(\frac{v_s}{400}\right)=\frac{{v_s}^2}{800}$ so we require $0.5\times 10^{-3}\le\frac{{v_s}^2}{800}$ or $\left|v_s\right|\le\sqrt{0.4}=0.6325\text{ V}$.

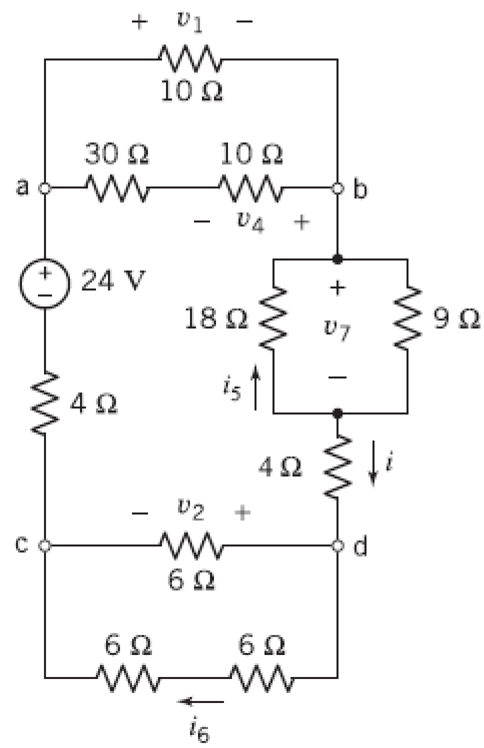
(checked: LNAP 6/9/04)

P 3.6-18 The circuit shown in Figure P 3.6-18*b* has been obtained from the circuit shown in Figure P 3.6-18*a* by replacing series and parallel combinations of resistances by equivalent resistances.

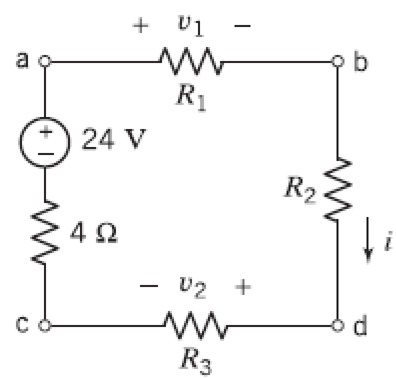
(a) Determine the values of the resistances R_1, R_2, and R_3 in Figure P 3.6-18*b* so that the circuit shown in Figure P 3.6-18*b* is equivalent to the circuit shown in Figure P 3.6-18*a*.

(b) Determine the values of v_1, v_2, and i in Figure P 3.6-18*b*.

(c) Because the circuits are equivalent, the values of v_1, v_2, and i in Figure P 3.6-18*a* are equal to the values of v_1, v_2, and i in Figure P 3.6-18*b*. Determine the values of v_4, i_5, i_6, and v_7 in Figure P 3.6-18*a*.



(a)



(b)

Figure P 3.6-18

Solution:

(a)

$$R_1 = 10 \,||\, (30+10) = 8\ \Omega, \quad R_2 = 4 + (18 \,||\, 9) = 10\ \Omega \quad \text{and}$$

$$R_3 = 6 \,||\, (6+6) = 4\ \Omega$$

(b)

$$i = 1\ \text{A}, \quad v_1 = 8\ \text{V} \quad \text{and} \quad v_2 = 4\ \text{V}$$

(c)

$$v_4 = -\frac{10}{10+30} 8 = -2\ \text{V}, \quad i_5 = -\frac{9}{9+18} 1 = -\frac{1}{3}\ \text{A},$$

$$v_7 = -18\left(-\frac{1}{3}\right) = +6\ \text{V} \quad \text{and} \quad i_6 = \frac{4}{12} = \frac{1}{3}\ \text{A}$$

(checked: LNAP 6/6/04)

P 3.6-19 Determine the values of v_1, v_2, i_3, v_4, v_5, and i_6 in Figure P 3.6-19.

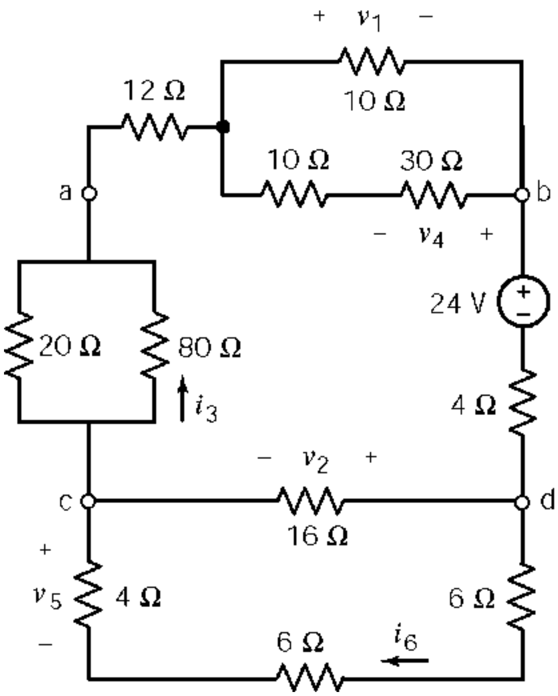
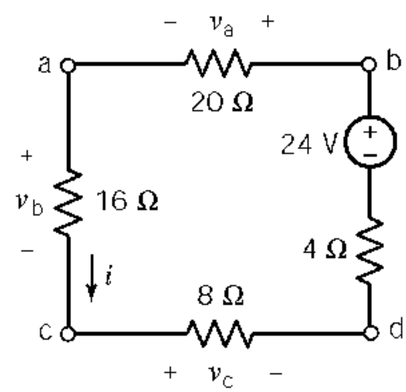


Figure P 3.6-19

Solution:

Replace series and parallel combinations of resistances by equivalent resistances. Then KVL gives



$$(20+4+8+16)i = 48 \quad \Rightarrow \quad i = 0.5 \text{ A}$$

$$v_a = 20i = 10 \text{ V}, \; v_b = 16i = 8 \text{ V and } v_c = 8i = 4 \text{ V}$$

Compare the original circuit to the equivalent circuit to get

$$v_1 = -\left(\frac{10 \,||\, (10+30)}{12+10 \,||\, (10+30)}\right) v_a = -\left(\frac{8}{12+8}\right) 10 = -4 \text{ V}$$

$$v_2 = -v_c = -4 \text{ V}$$

$$i_3 = -\left(\frac{20}{20+80}\right) i = -\left(\frac{1}{5}\right)(0.5) = -0.1 \text{ A}$$

$$v_4 = -\left(\frac{30}{10+30}\right) v_1 = -\left(\frac{1}{4}\right)(-4) = 1 \text{ V}$$

$$v_5 = \left(\frac{4}{5+6+6}\right) v_c = \left(\frac{1}{4}\right)(4) = 1 \text{ V}$$

$$i_6 = -\left(\frac{16}{16+(4+6+6)}\right) i = -\left(\frac{1}{2}\right)(0.5) = -0.25 \text{ A}$$

(checked: LNAP 6/10/04)

P 3.6-20 Determine the values of i, v, and R_{eq} by the circuit model shown in Figure P 3.6-20, given that v_{ab} = 18 V.

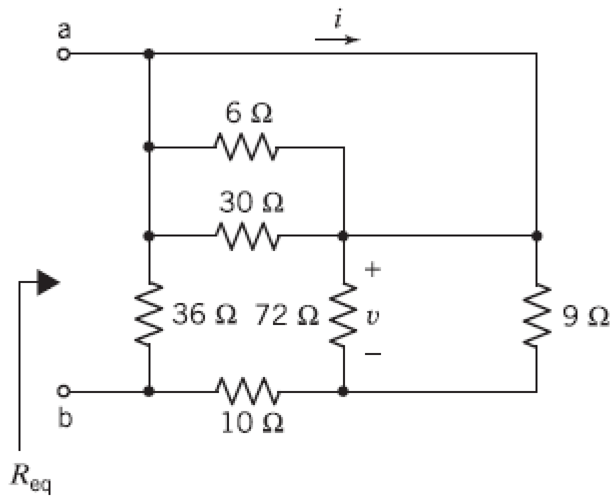
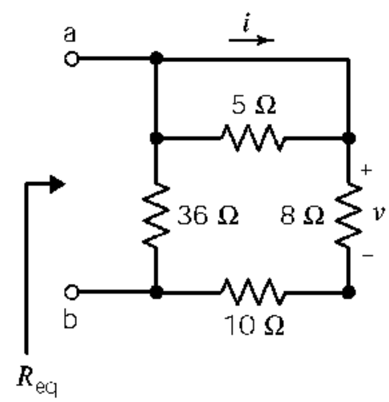


Figure P 3.6-20

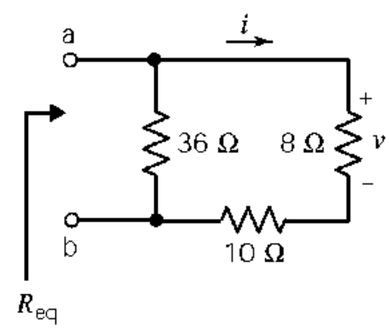
Solution:

Replace parallel resistors by equivalent resistors:

$$6 \parallel 30 = 5\ \Omega \quad \text{and} \quad 72 \parallel 9 = 8\ \Omega$$



A short circuit in parallel with a resistor is equivalent to a short circuit.



$$R_{eq} = 36 \parallel (8+10) = 12\ \Omega$$

Using voltage division when v_{ab} = 18 V:

$$v = \frac{8}{8+10} v_{ab} = \frac{4}{9}(18) = 8\ \text{V}$$

$$i = \frac{v}{8} = 1\ \text{A}$$

(checked: LNAP 6/21/04)

P 3.6-22 Determine the value of the resistance R in the circuit shown in Figure P 3.6-22, given that $R_{eq} = 9\ \Omega$.

Answer: $R = 15\ \Omega$

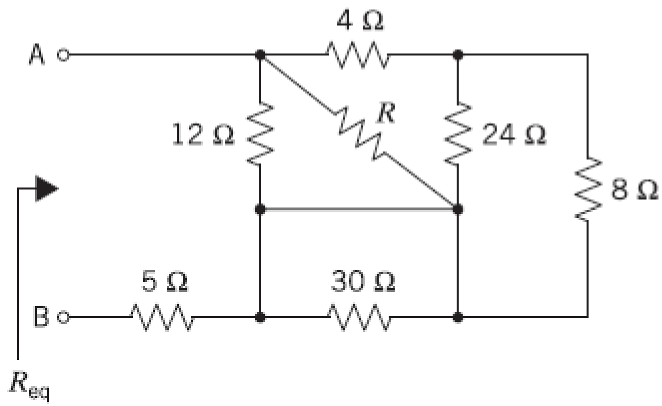
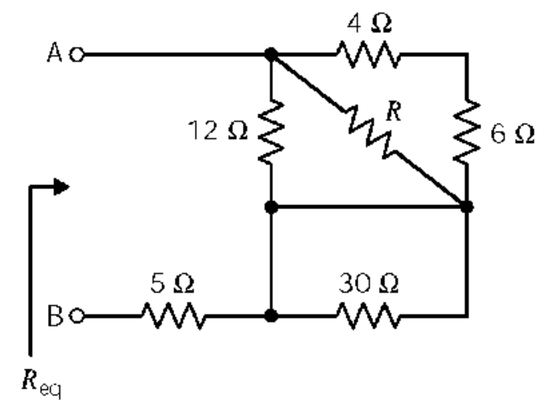


Figure P 3.6-22

Solution:

Replace parallel resistors by an equivalent resistor:

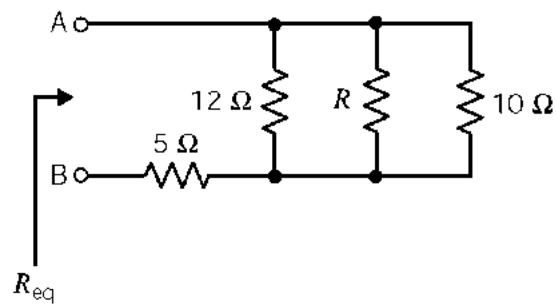
$$8 \parallel 24 = 6\ \Omega$$



A short circuit in parallel with a resistor is equivalent to a short circuit.

Replace series resistors by an equivalent resistor:

$$4+6 = 10\ \Omega$$



Now

$$9 = R_{eq} = 5 + (12 \parallel R \parallel 10)$$

so

$$4 = \frac{R \times \frac{60}{11}}{R + \frac{60}{11}} \quad \Rightarrow \quad R = 15\ \Omega$$

(checked: LNAP 6/21/04)

P 3.6-22 Determine the value of the resistance R in the circuit shown in Figure P 3.6-22, given that $R_{eq} = 50\ \Omega$.

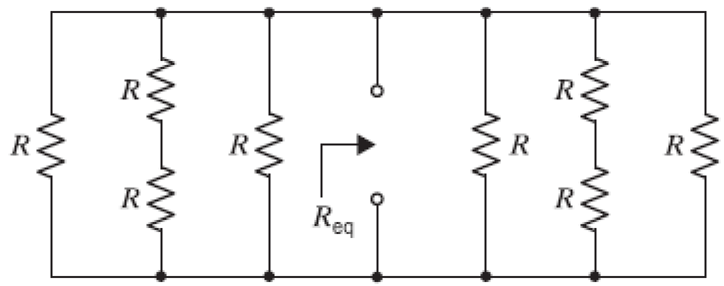


Figure P 3.6-22

Solution:

$$R_{eq} = \left(R \,||\, (R+R) \,||\, R\right) || \left(R \,||\, (R+R) \,||\, R\right)$$

$$R \,||\, (R+R) \,||\, R = 2R \,||\, \frac{R}{2} = \frac{2}{5}R$$

$$R_{eq} = \frac{2}{5}R \,||\, \frac{2}{5}R = \frac{R}{5} \quad \Rightarrow \quad R = 5\,R_{eq} = 200\ \Omega$$

(checked: LNAP 6/21/04)

P 3.6-23 Determine the values of r, the gain of the CCVS, and g, the gain of the VCCS, for the circuit shown in Figure P 3.6-23.

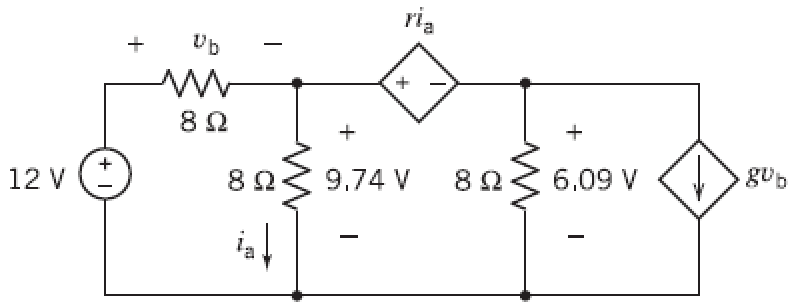


Figure P 3.6-23

Solution:

$$i_a = \frac{9.74}{8} = 1.2175 \text{ A}$$

$$9.74 - 6.09 = r\,i_a = r\left(\frac{9.74}{8}\right) \quad \Rightarrow \quad r = \left(\frac{9.74\text{-}6.09}{9.74}\right)8 = 3\ \frac{\text{V}}{\text{A}}$$

$$v_b = 12 - 9.74 = 2.26 \text{ V}$$

$$gv_b + \frac{6.09}{8} + \frac{9.74}{8} - \frac{2.26}{8} = 0 \quad \Rightarrow \quad gv_b = -1.696 \text{ A}$$

$$g = \frac{gv_b}{v_b} = \frac{-6.696}{2.26} = -0.75$$

(checked: LNAP 6/21/04)

P 3.6-24 The input to the circuit in Figure P 3.6-24 is the voltage of the voltage source, v_s. The output is the voltage measured by the meter, v_o. Show that the output of this circuit is proportional to the input. Determine the value of the constant of proportionality.

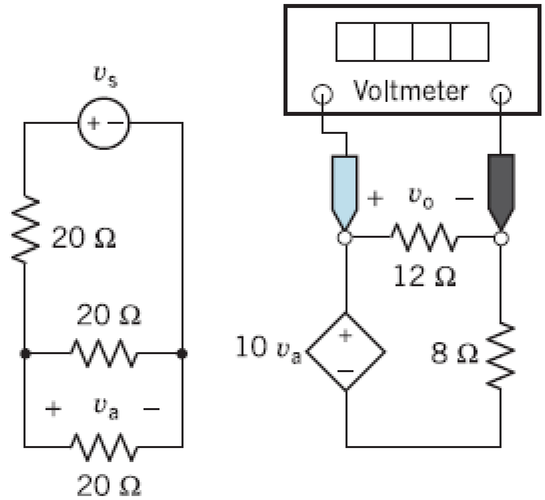


Figure P 3.6-24

Solution:

$$v_a = \frac{20 \,||\, 20}{20 + (20 \,||\, 20)} v_s = \frac{1}{3} v_s$$

$$v_o = \left(\frac{12}{12+8}\right)(10 v_a) = \frac{3}{5} \times 10 \times \frac{1}{3} v_s = 2 v_s$$

So v_o is proportional to v_s and the constant of proportionality is $2 \ \frac{\text{V}}{\text{V}}$.

P 3.6-25 The input to the circuit in Figure P 3.6-25 is the voltage of the voltage source, v_s. The output is the current measured by the meter, i_o. Show that the output of this circuit is proportional to the input. Determine the value of the constant of proportionality.

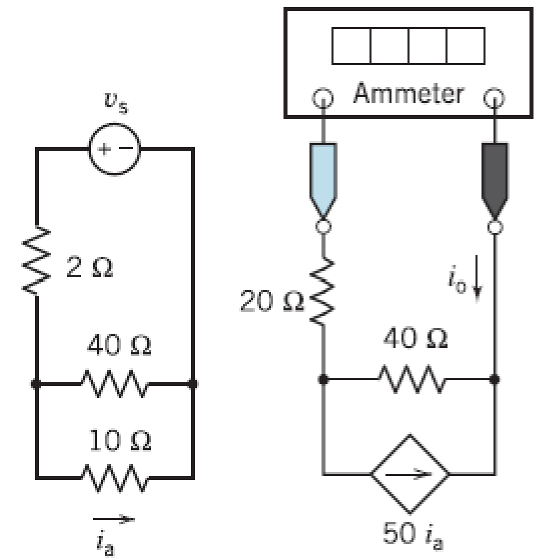


Figure P 3.6-25

Solution:

$$i_a = \left(\frac{40}{40+10}\right)\frac{v_s}{2+(40\,||\,10)} = \left(\frac{4}{5}\right)\left(\frac{v_s}{10}\right) = \frac{4}{50}v_s$$

$$i_o = -\left(\frac{40}{20+40}\right)(50 i_a) = -\frac{100}{3}\left(\frac{4}{50}\right)v_s = -\frac{8}{3}v_s$$

The output is proportional to the input and the constant of proportionality is $-\frac{8}{3}\ \frac{\text{A}}{\text{V}}$.

P 3.6-26 Determine the voltage measured by the voltmeter in the circuit shown in Figure P 3.6-26.

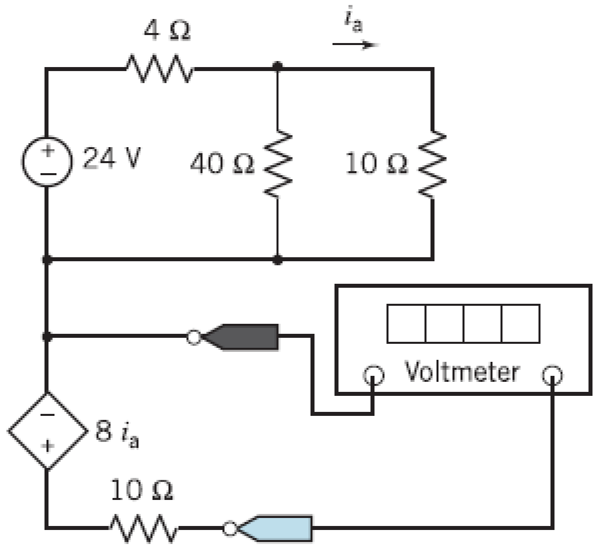
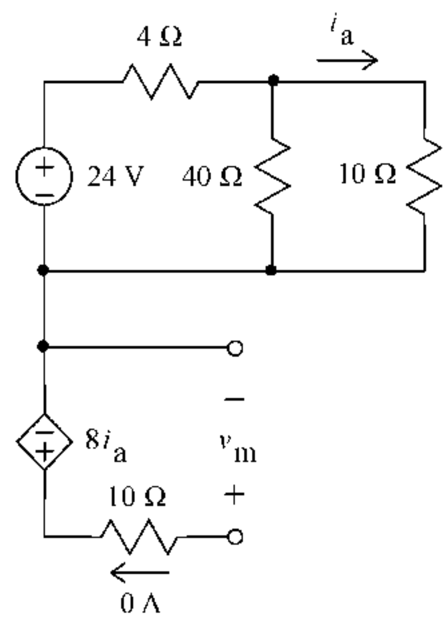


Figure P 3.6-26

Solution:

Replace the voltmeter by the equivalent open circuit and label the voltage measured by the meter as v_m.



The 10-Ω resistor at the right of the circuit is in series with the open circuit that replaced the voltmeter so it's current is zero as shown. Ohm's law indicates that the voltage across that 10-Ω resistor is also zero. Applying KVL to the mesh consisting of the dependent voltage source, 10-Ω resistor and open circuit shows that

$$v_m = 8\, i_a$$

The 10-Ω resistor and 40-Ω resistor are connected in parallel. The parallel combination of these resistors is equivalent to a single resistor with a resistance equal to

$$\frac{40 \times 10}{40 + 10} = 8\ \Omega$$

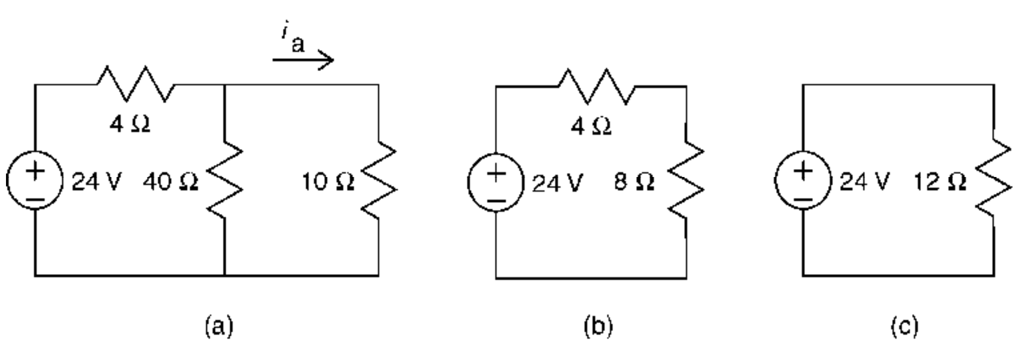
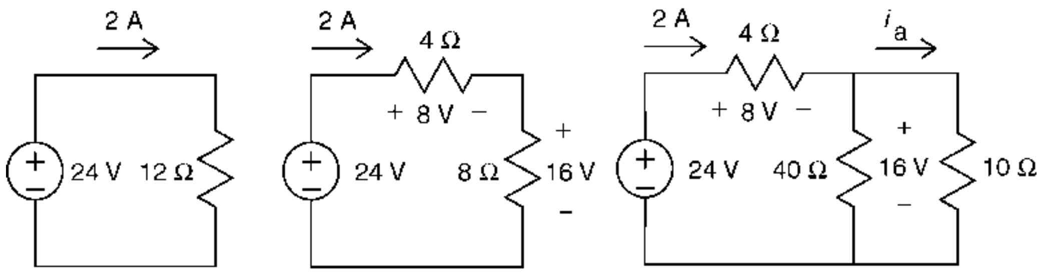


Figure a shows part of the circuit. In Figure b, an equivalent resistor has replaced the parallel resistors. Now the 4-Ω resistor and 8-Ω resistor are connected in series. The series combination of these resistors is equivalent to a single resistor with a resistance equal to $4+8=12\ \Omega$. In Figure c, an equivalent resistor has replaced the series resistors.

Here the same three circuits with the order reversed. The earlier sequence of figures illustrates the process of simplifying the circuit by repeatedly replacing series or parallel resistors by an equivalent resistor. This sequence of figures illustrates an analysis that starts with the simplified circuit and works toward the original circuit.



(a) (b) (c)

Consider Figure a. Using Ohm's law, we see that the current in the 12-Ω resistor is 2 A. The current in the voltage source is also 2 A. Replacing series resistors by an equivalent resistor does not change the current or voltage of any other element of the circuit, so the current in the voltage source must also be 2 A in Figure b. The currents in resistors in Figure b are equal to the current in the voltage source. Next, Ohm's law is used to calculate the resistor voltages as shown in Figure b.

Replacing parallel resistors by an equivalent resistor does not change the current or voltage of any other element of the circuit, so the current in the 4-Ω resistor in Figure c must be equal to the current in the 4-Ω resistor in Figure b. Using current division in Figure c are yields

$$i_a = \left(\frac{40}{40+10}\right) 2 = 1.6 \text{ A}$$

Finally,

$$v_m = 8\, i_a = 8 \times 1.6 = 12.8 \text{ V}$$

P 3.6-27 Determine the current measured by the ammeter in the circuit shown in Figure P 3.6-27.

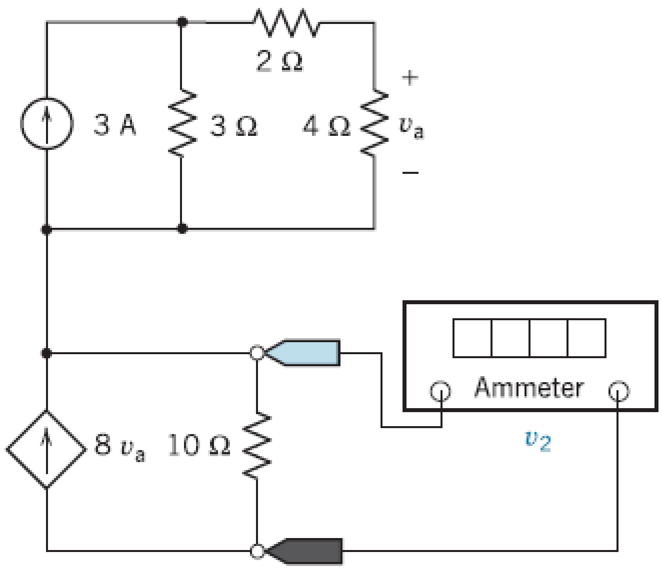


Figure P 3.6-27

Solution:

Replace the ammeter by the equivalent short circuit and label the current measured by the meter as i_m.

The 10-Ω resistor at the right of the circuit is in parallel with the short circuit that replaced the ammeter so it's voltage is zero as shown. Ohm's law indicates that the current in that 10-Ω resistor is also zero. Applying KCL at the top node of that 10-Ω resistor shows that

$$i_m = 0.8\, v_a$$

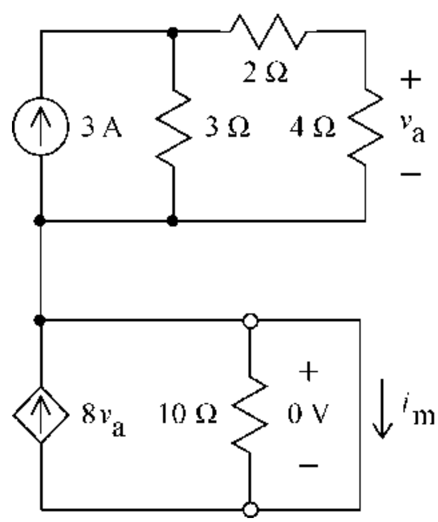
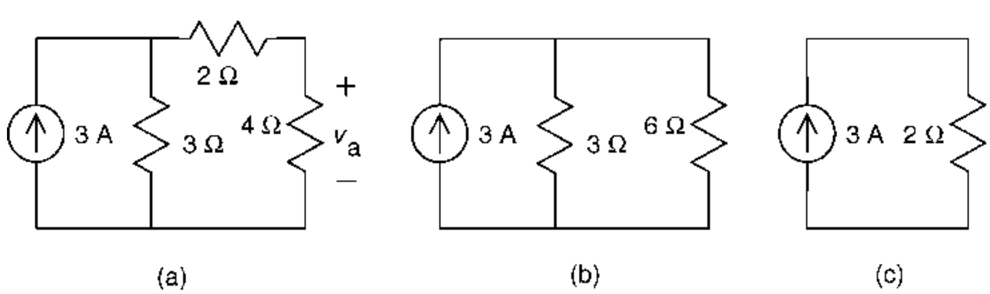


Figure a shows part of the circuit. The 2-Ω resistor and 4-Ω resistor are connected in series. The series combination of these resistors is equivalent to a single 6-Ω resistor.

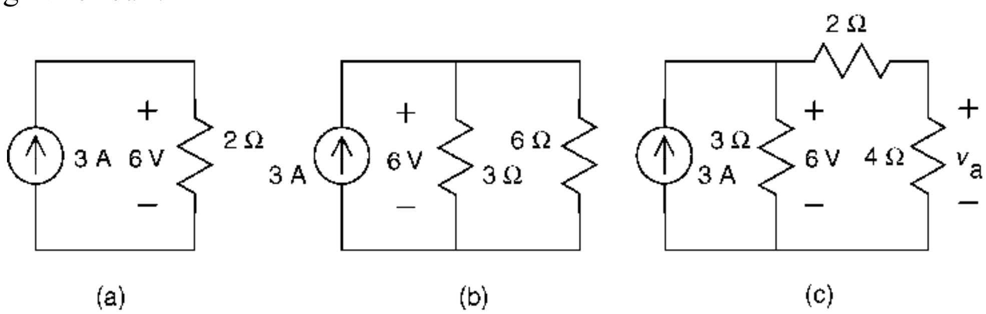


In Figure b, an equivalent resistor has replaced the series resistors. Now the 3-Ω resistor and 6-Ω resistor are connected in parallel. The parallel combination of these resistors is equivalent to a single resistor with a resistance equal to

$$\frac{3\times 6}{3+6}=2\ \Omega$$

In Figure c, an equivalent resistor has replaced the parallel resistors.

Here the same three circuits with the order reversed. The earlier sequence of figures illustrates the process of simplifying the circuit by repeatedly replacing series or parallel resistors by an equivalent resistor. This sequence of figures illustrates an analysis that starts with the simplified circuit and works toward the original circuit.



Consider Figure a. Using Ohm's law, we see that the voltage across the 2-Ω resistor is 6 V. The voltage across the current source is also 6 V. Replacing parallel resistors by an equivalent resistor does not change the current or voltage of any other element of the circuit, so the voltage across the current source must also be 6 V in Figure b. The voltage across each resistor in Figure b is equal to the voltage across the current source.

Replacing series resistors by an equivalent resistor does not change the current or voltage of any other element of the circuit, so the voltage across the 3-Ω resistor in Figure c must be equal to the voltage across the 3-Ω resistor in Figure b. Using voltage division in Figure c yields

$$v_a=\left(\frac{4}{2+4}\right)6=4\ \text{V}$$

Finally,

$$i_m=0.8\,v_a=0.8\times 4=3.2\ \text{V}$$

P 3.6-28 Determine the value of the resistance R that causes the voltage measured by the voltmeter in the circuit shown in Figure P 3.6-28 to be 6 V.

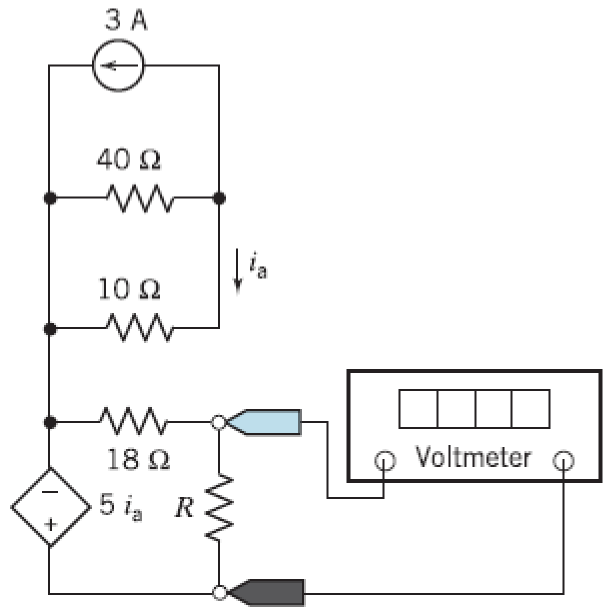


Figure P 3.6-28

Solution:
Use current division in the top part of the circuit to get

$$i_a = \left(\frac{40}{40+10}\right)(-3) = -2.4 \text{ A}$$

Next, denote the voltage measured by the voltmeter as v_m and use voltage division in the bottom part of the circuit to get

$$v_m = \left(\frac{R}{18+R}\right)(-5\,i_a) = \left(\frac{-5\,R}{18+R}\right)i_a$$

Combining these equations gives:

$$v_m = \left(\frac{-5\,R}{18+R}\right)(-2.4) = \frac{12\,R}{18+R}$$

When $v_m = 6$ V,

$$6 = \frac{12\,R}{18+R} \quad \Rightarrow \quad R = \frac{6\times 18}{12-6} = 18 \;\Omega$$

P 3.6-29 The input to the circuit shown in Figure P 3.6-29 is the voltage of the voltage source, v_s. The output is the current measured by the meter, i_m.

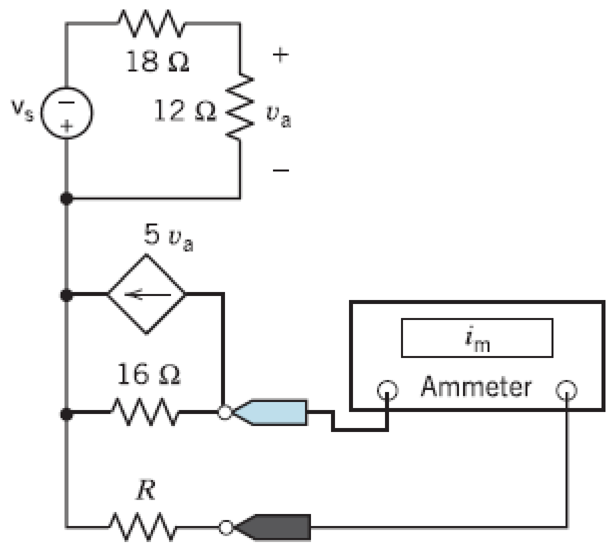


Figure P 3.6-29

(a) Suppose $v_s = 15$ V. Determine the value of the resistance R that causes the value of the current measured by the meter to be $i_m = 5$ A.

(b) Suppose $v_s = 15$ V and $R = 24\ \Omega$. Determine the current measured by the ammeter.

(c) Suppose $R = 24\ \Omega$. Determine the value of the input voltage, v_s, that causes the value of the current measured by the meter to be $i_m = 3$ A.

Soluton:

Use voltage division in the top part of the circuit to get

$$v_a = \left(\frac{12}{12+18}\right)(-v_s) = -\frac{2}{5}v_s$$

Next, use current division in the bottom part of the circuit to get

$$i_m = -\left(\frac{16}{16+R}\right)(5\,v_a) = \left(-\frac{80}{16+R}\right)v_a$$

Combining these equations gives:

$$i_m = \left(-\frac{80}{16+R}\right)\left(-\frac{2}{5}v_s\right) = \left(\frac{32}{16+R}\right)v_s$$

a. When $v_s = 15$ V and $i_m = 12$ A

$$12 = \left(\frac{32}{16+R}\right)15 \quad\Rightarrow\quad 192 + 12\,R = 480 \quad\Rightarrow\quad R = \frac{288}{12} = 24\ \Omega$$

b. When $v_s = 15$ V and $R = 80\ \Omega$

$$i_m = \left(\frac{32}{16+80}\right)15 = 5\ \text{A}$$

c. When $i_m = 3$ A and $R = 24\ \Omega$

$$3 = \left(\frac{32}{16+24}\right)v_s = \frac{4}{5}v_s \quad\Rightarrow\quad v_s = \frac{15}{4} = 3.75\ \text{V}$$

P 3.6-30 The ohmmeter in Figure P 3.6-30 measures the equivalent resistance of the resistor circuit connected to the meter probes.

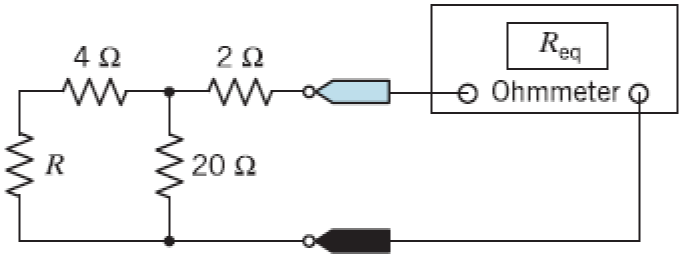


Figure P 3.6-31

(a) Determine the value of the resistance R required to cause the equivalent resistance to be R_{eq} = 12 **Ω**.
(b) Determine the value of the equivalent resistance when R = 14 Ω.

Solution:

$$R_{eq} = \left(\left(R+4\right) \| 20\right)+2 = \frac{\left(R+4\right)\times 20}{\left(R+4\right)+20}+2 = \frac{20\,R+80}{R+24}+2$$

(a) $$12 = \frac{20\,R+80}{R+24}+2 \quad \Rightarrow \quad 10 = \frac{20\,R+80}{R+24} \quad \Rightarrow \quad R+24 = 2\,R+8 \quad \Rightarrow \quad R = 16\ \Omega$$

(b) $$R_{eq} = \frac{20\left(14\right)+80}{14+24}+2 = 11.5\ \Omega$$

(Checked: LNAPDC 9/28/04)

P 3.6-31 The voltmeter in Figure P 3.6-31 measures the voltage across the current source.

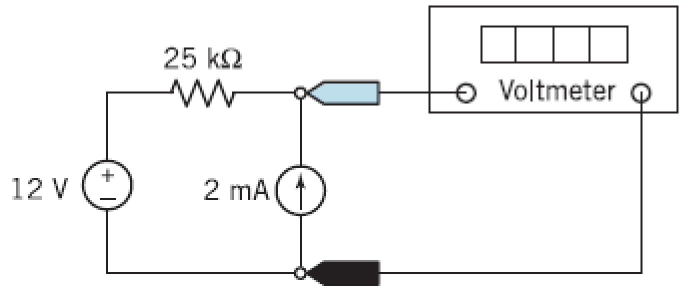
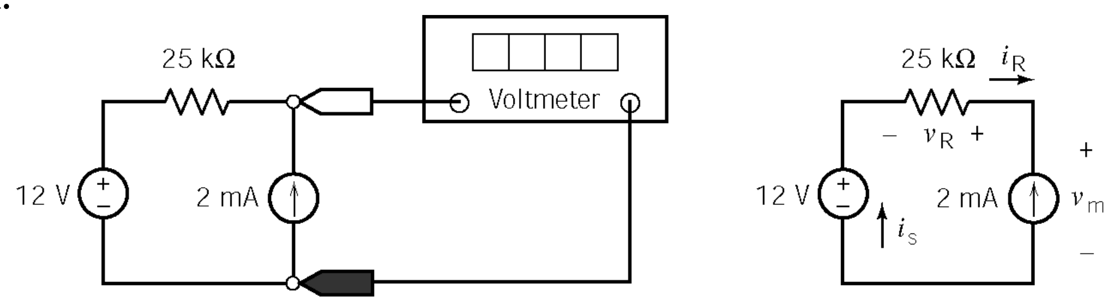


Figure P 3.6-31

(a) Determine the value of the voltage measured by the meter.
(b) Determine the power supplied by each circuit element.

Solution:



(*a*) (*b*)

Replace the ideal voltmeter with the equivalent open circuit and label the voltage measured by the meter. Label the element voltages and currents as shown in (***b***).

<table>
<tr>
<td>

Using units of V, A, Ω and W:

a.) Determine the value of the voltage measured by the meter.

Kirchhoff's laws give

$$12 + v_R = v_m \text{ and } -i_R = -i_s = 2 \times 10^{-3} \text{ A}$$

Ohm's law gives

$$v_R = -\left(25 \times 10^3\right) i_R$$

Then

$$v_R = -\left(25 \times 10^3\right) i_R = -\left(25 \times 10^3\right)\left(-2 \times 10^{-3}\right) = 50 \text{ V}$$

$$v_m = 12 + v_R = 12 + 50 = 62 \text{ V}$$

</td>
<td>

Using units of V, mA, kΩ and mW:

a.) Determine the value of the voltage measured by the meter.

Kirchhoff's laws give

$$12 + v_R = v_m \text{ and } -i_R = -i_s = 2 \text{ mA}$$

Ohm's law gives

$$v_R = -25\, i_R$$

Then

$$v_R = -25\, i_R = -25(-2) = 50 \text{ V}$$

$$v_m = 12 + v_R = 12 + 50 = 62 \text{ V}$$

</td>
</tr>
</table>

b.) Determine the power supplied by each element.

voltage source	$12(i_s) = -12(-2\times10^{-3})$ $= -24\times10^{-3}$ W
current source	$62(2\times10^{-3}) = 124\times10^{-3}$ W
resistor	$v_R\, i_R = 50(-2\times10^{-3})$ $= -100\times10^{-3}$ W
total	0

b.) Determine the power supplied by each element.

voltage source	$12(i_s) = -12(-2)$ $= -24$ mW
current source	$62(2) = 124$ mW
resistor	$v_R\, i_R = 50(-2)$ $= -100$ mW
total	0

P 3.6-32 Determine the resistance measured by the ohmmeter in Figure P 3.6-32.

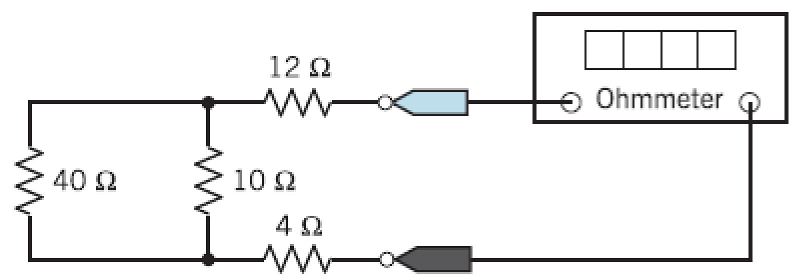


Figure P 3.6-32

Solution:

$$12 + \frac{40\times10}{40+10} + 4 = 12\ \Omega$$

P 3.6-33 Determine the resistance measured by the ohmmeter in Figure P 3.6-33.

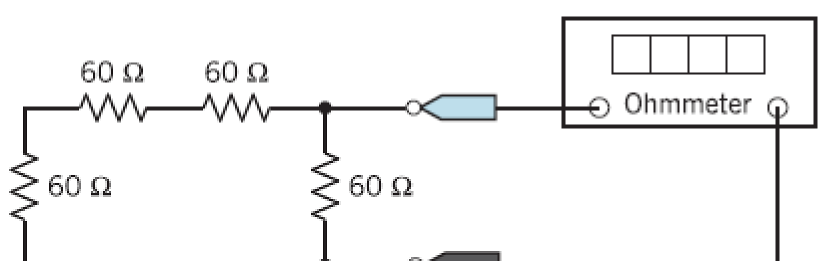


Figure P 3.6-33

Solution:

$$\frac{(60+60+60)\times60}{(60+60+60)+60} = 45\ \Omega$$

P3.6-34

Consider the circuit shown in Figure P3.6-34. Given the values of the following currents and voltages:

$$i_1 = 0.625 \text{ A}, \ v_2 = -25 \text{ V}, \ i_3 = -1.25 \text{ A} \ \text{ and } \ v_4 = -18.75 \text{ V}$$

Determine the values of R_1, R_2, R_3 and R_4.

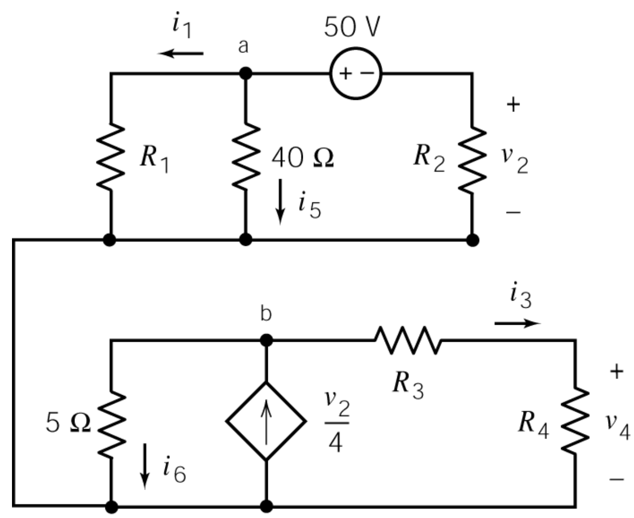
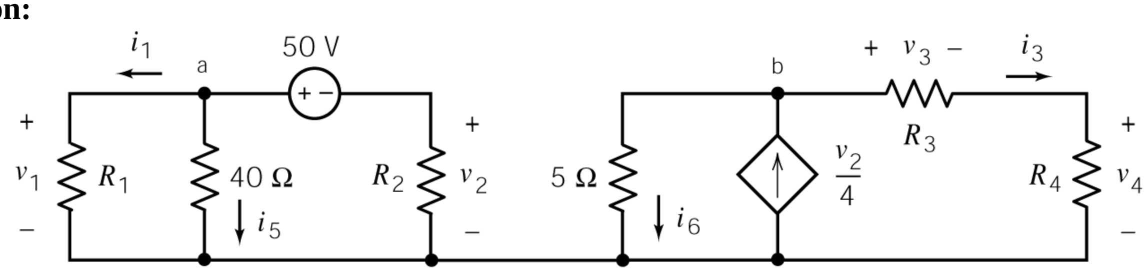


Figure P3.6-34

Solution:



From KVL $\quad 50 + v_2 - v_1 = 0 \ \Rightarrow \ v_1 = 50 + (-25) = 25 \text{ V}$

From Ohm's law $\quad R_1 = \dfrac{v_1}{i_1} = \dfrac{25}{0.625} = 40 \ \Omega$

From KCL

$$i_1 + i_5 + i_2 = 0 \ \Rightarrow \ i_2 = -(i_1 + i_5) = -\left(0.625 + \frac{v_1}{40}\right) = -\left(0.625 + \frac{25}{40}\right) = -1.25 \text{ A}$$

From Ohm's law $\quad R_2 = \dfrac{v_2}{i_2} = \dfrac{-25}{-1.25} = 20 \ \Omega$

From KCL $\quad \dfrac{v_2}{4} = i_6 + i_3 \ \Rightarrow \ i_6 = -i_3 + \dfrac{v_2}{4} = -(-1.25) + \dfrac{-25}{4} = -5 \text{ A}$

From KVL $\quad v_3 + v_4 - 5i_6 = 0 \ \Rightarrow \ v_3 = -v_4 + 5i_6 = -(-18.75) + 5(-5) = -6.25 \text{ V}$

From Ohm's law $\quad R_3 = \dfrac{v_3}{i_3} = \dfrac{-6.25}{-1.25} = 5 \ \Omega \ \text{ and } \ R_4 = \dfrac{v_4}{i_3} = \dfrac{-18.75}{-1.25} = 15 \ \Omega$

P3.6-35

Consider the circuits shown in Figure P3.6-35. The equivalent circuit on the right is obtained from the original circuit on the left by replacing series and parallel combinations of resistors by equivalent resistors. The value of the current in the equivalent circuit is $i_s = 0.8$ A. Determine the values of R_1, R_2, R_5, v_2 and i_3.

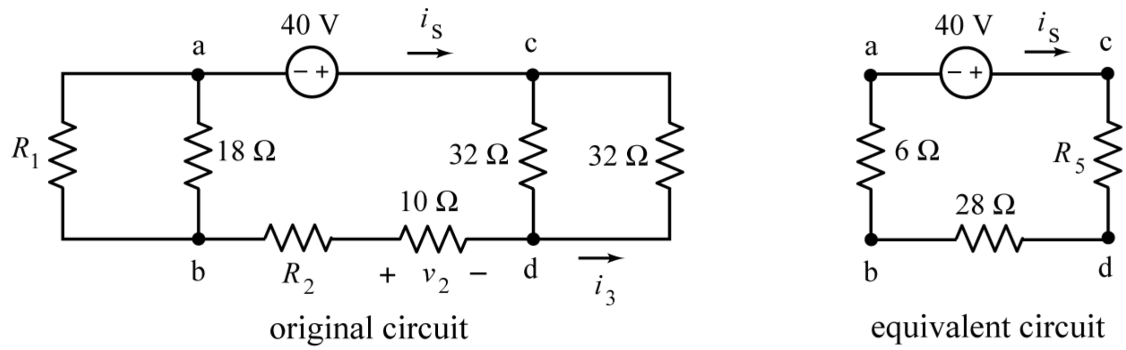


Figure P3.6-35

Solution:

$$R_1 \,||\, 18 = 6 \quad \Rightarrow \quad \frac{18\,R_1}{18+R_1} = 6 \quad \Rightarrow \quad 3\,R_1 = 18 + R_1 \quad \Rightarrow \quad R_1 = 9\ \Omega$$

$$R_2 + 10 = 28 \quad \Rightarrow \quad R_2 = 18\ \Omega$$

$$40 = \left(6 + 28 + R_5\right) i_s \quad \Rightarrow \quad \frac{40}{0.8} = 34 + R_5 \quad \Rightarrow \quad R_5 = 50 - 34 = 16\ \Omega$$

$$v_2 = -10\, i_s = -10\left(0.8\right) = -8\text{ V} \;\text{ and }\; i_3 = -\frac{32}{32+32} i_s = -\frac{0.8}{2} = -0.4\text{ A}$$

P3.6-36 Consider the circuit shown in Figure P3.6-36. Given

$$v_2 = \frac{2}{3} v_s,\ i_3 = \frac{1}{5} i_1 \text{ and } v_4 = \frac{3}{8} v_2 .$$

Determine the values of R_1, R_2 and R_4.

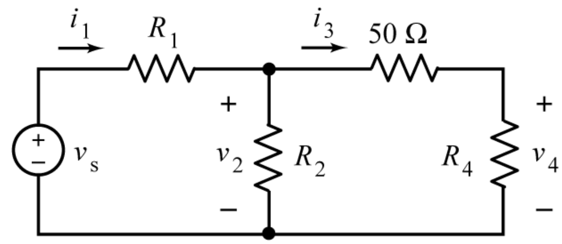


Figure P3.6-36

Hint: Interpret $v_2 = \frac{2}{3} v_s$, $i_3 = \frac{1}{5} i_1$ and $v_4 = \frac{3}{8} v_2$ as current and voltage division.

Solution:

From voltage division $v_4 = \frac{R_4}{50 + R_4} v_2$

so $$\frac{R_4}{50+R_4} = \frac{3}{8} \Rightarrow 8R_4 = 3(50+R_4) \Rightarrow R_4 = \frac{150}{8-3} = 30\ \Omega .$$

From current division $i_3 = \frac{R_2}{R_2 + (50+R_4)} i_1 = \frac{R_2}{R_2+80} i_1$

so $$\frac{R_2}{R_2+80} = \frac{1}{5} \Rightarrow 5R_2 = R_2 + 80 \Rightarrow R_2 = 20\ \Omega .$$

Notice that $R_2 \,||\, (50+R_4) = 20 \,||\, (50+30) = 20 \,||\, 80 = 16\ \Omega$. From voltage division

$$v_1 = \frac{R_2 \,||\, (50+R_4)}{R_1 + (R_2 \,||\, (50+R_4))} v_s = \frac{16}{R_1+16} v_s$$

so $$\frac{16}{R_1+16} = \frac{2}{3} \Rightarrow 48 = 2(R_1+16) \Rightarrow R_1 = \frac{48-32}{2} = 8\ \Omega .$$

P3.6-37 Consider the circuit shown in Figure P3.6-37. Given

$$i_2 = \frac{2}{5} i_s,\ \ v_3 = \frac{2}{3} v_1 \text{ and } i_4 = \frac{4}{5} i_2 .$$

Determine the values of R_1, R_2 and R_4.

Hint: Interpret $i_2 = \frac{2}{5} i_s$, $v_3 = \frac{2}{3} v_1$ and $i_4 = \frac{4}{5} i_2$ as current and voltage division.

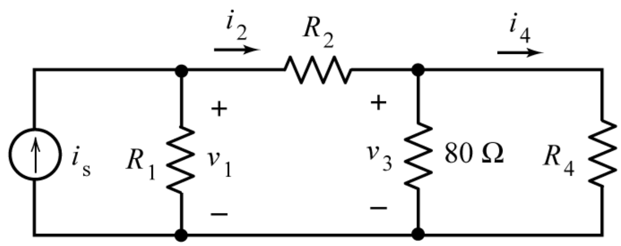


Figure P3.6-37

Solution:

From current division $i_4 = \dfrac{80}{80+R_4} i_2$

so $$\frac{80}{80+R_4} = \frac{4}{5} \Rightarrow 400 = 4\left(80+R_4\right) \Rightarrow R_4 = \frac{400-320}{4} = 20\ \Omega .$$

From voltage division $v_3 = \dfrac{80 \,||\, R_4}{R_2+\left(80 \,||\, R_4\right)} v_1 = \dfrac{80 \,||\, 20}{R_2+\left(80 \,||\, 20\right)} v_1 = \dfrac{16}{R_2+16} v_1$

so $$\frac{16}{R_2+16} = \frac{2}{3} \Rightarrow 48 = 2\left(R_2+16\right) \Rightarrow R_2 = \frac{48-32}{2} = 8\ \Omega .$$

Notice that $R_2+\left(80 \,||\, R_4\right) = 8+\left(80 \,||\, 20\right) = 8+16 = 24\ \Omega$. From current division

$$i_1 = \frac{R_1}{R_1+\left(R_2+\left(80 \,||\, R_4\right)\right)} i_s = \frac{R_1}{R_1+24} i_s$$

so $$\frac{R_1}{R_1+24} = \frac{2}{5} \Rightarrow 5R_1 = 2\left(R_1+24\right) \Rightarrow R_1 = \frac{48}{3} = 16\ \Omega$$

P3.6-38 Consider the circuit shown in Figure P3.6-38.

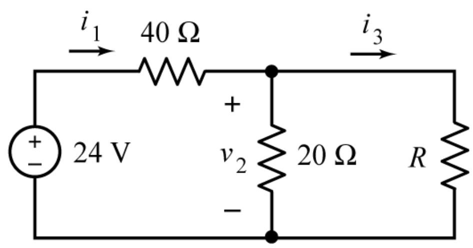


Figure P3.6-38

(a) Suppose $i_3 = \frac{1}{3} i_1$. What is the value of the resistance *R*?

(b) Suppose instead $v_2 = 4.8$ V. What is the value of the equivalent resistance of the parallel resistors?

(c) Suppose instead *R* = 20 Ω. What is the value of the current in the 40 Ω resistor?

Hint: Interpret $i_3 = \frac{1}{3} i_1$ as current division.

Solution:

(a) From current division $i_3 = \dfrac{20}{20+R} i_1$ so $\dfrac{20}{20+R} = \dfrac{1}{3} \Rightarrow 60 = 20 + R \Rightarrow R = 40\ \Omega$.

(b) From voltage division $v_2 = \dfrac{R_p}{40+R_p} 24$

so $$4.8 = \frac{R_p}{40+R_p} 24 \Rightarrow \frac{4.8}{24}\left[40 + R_p\right] = R_p \Rightarrow R_p = \frac{(0.2)40}{1-0.2} = 10\ \Omega.$$

(c) $$i_1 = \frac{24}{40 + (20 \| 20)} = \frac{24}{40+10} = \frac{24}{50} = 0.48\ \text{A}$$

P3.6-39 Consider the circuit shown in Figure P3.6-39.

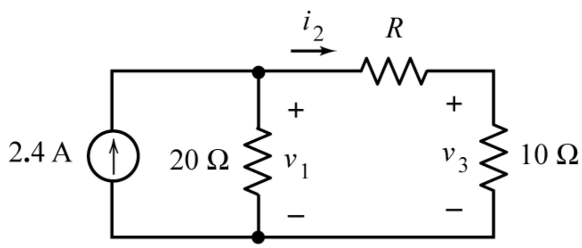


Figure P3.6-39

(a) Suppose $v_3 = \frac{1}{4} v_1$. What is the value of the resistance *R*?

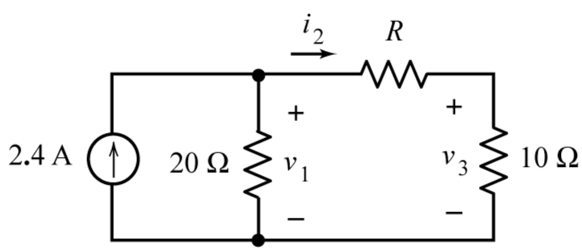
(b) Suppose $i_2 = 1.2$ A. What is the value of the resistance *R*?

(c) Suppose $R = 70\ \Omega$. What is the voltage across the 20 Ω resistor?

(d) Suppose $R = 30\ \Omega$. What is the value of the current in this 30 Ω resistor?

Hint: Interpret $v_3 = \frac{1}{4} v_1$ as voltage division.

Solution:



(a) From voltage division $v_3 = \frac{10}{10+R} v_1$ so $\frac{10}{10+R} = \frac{1}{4} \Rightarrow 40 = 10 + R \Rightarrow R = 30\ \Omega$.

(b) $$1.2 = \frac{20}{20+(R+10)} 2.4 = \frac{20}{R+30} 2.4 \Rightarrow R+30 = \frac{20(2.4)}{1.2} = 40 \Rightarrow R = 10\ \Omega$$

(c) $$20 \,||\, (70+10) = \frac{20(80)}{20+80} = 16\ \Omega \quad \text{so} \quad v_1 = (16)2.4 = 38.4\ \text{V}$$

(d) $$i_2 = \frac{20}{20+(R+10)} 2.4 = \frac{20}{20+(30+10)} 2.4 = \frac{20}{60} 2.4 = 0.8\ \text{A}$$

P3.6-40 Consider the circuit shown in Figure P3.6-40. Given that the voltage of the dependent voltage source is $v_a = 8$ V, determine the values of R_1 and v_o.

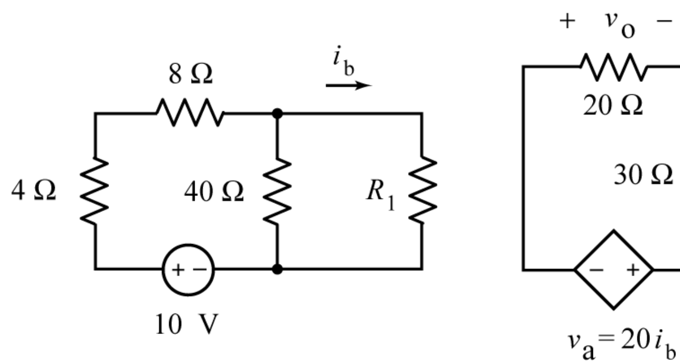


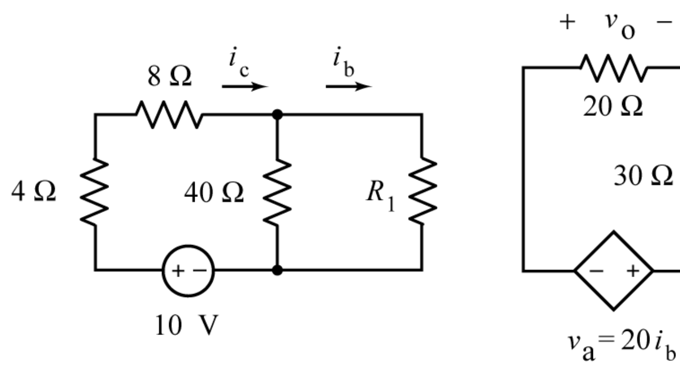
Figure P3.6-40

Solution:

First,

$$v_o = -\frac{20}{20+30} 8 = -3.2 \text{ V}$$

Next,



$$\frac{8}{20} = i_b = \frac{40}{40+R_1} i_c = \frac{40}{40+R_1}\left(\frac{10}{12+40 \| R_1}\right) = \frac{40}{40+R_1}\left(\frac{10}{12+\dfrac{40 R_1}{40+R_1}}\right) = \frac{400}{12(40+R_1)+40 R_1} = \frac{400}{480+52 R_1}$$

then

$$\frac{8}{20} = \frac{400}{480+52 R_1} \quad \Rightarrow \quad 480+52 R_1 = \frac{400(20)}{8} = 1000 \quad \Rightarrow \quad \frac{1000-480}{52} = 10 \ \Omega$$

P3.6-41 Consider the circuit shown in Figure P3.6-41. Given that the current of the dependent current source is $i_a = 2$ A, determine the values of R_1 and i_o.

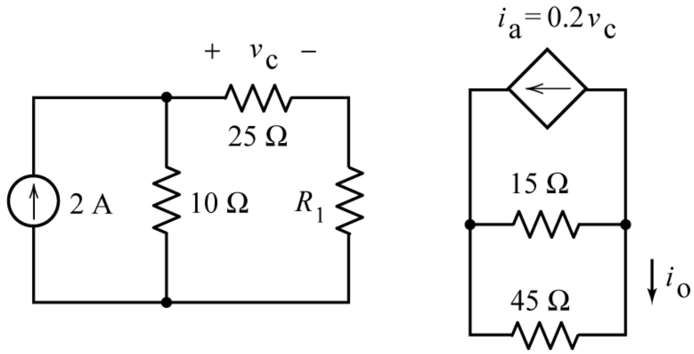
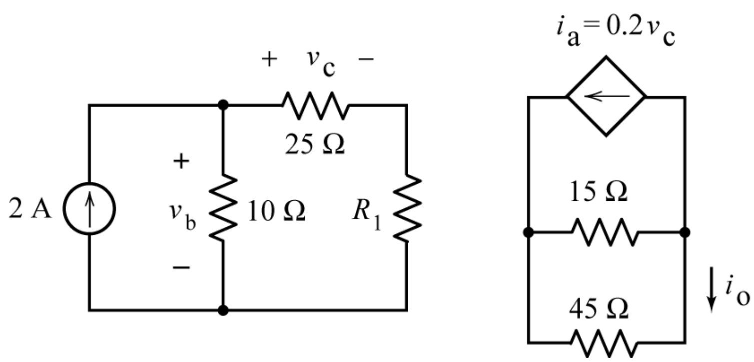


Figure P3.6-41

Solution:
First,

$$i_o = -\frac{15}{15+45} 2 = -0.5 \text{ A}$$

Next,



$$\frac{2}{0.2} = v_c = \frac{25}{25+R_1} v_b = \frac{25}{25+R_1}\left(2\left(10 \,\|\, 25+R_1\right)\right) = \frac{50}{25+R_1}\left(\frac{10\left(25+R_1\right)}{10+\left(25+R_1\right)}\right) = \frac{500}{35+R_1}$$

then

$$\frac{2}{0.2} = \frac{500}{35+R_1} \quad \Rightarrow \quad 35+R_1 = 50 \quad \Rightarrow \quad R_1 = 15 \ \Omega$$

P3.6-42 Determine the values of i_a, i_b, i_2, and v_1 in the circuit shown in Figure P3.6-42.

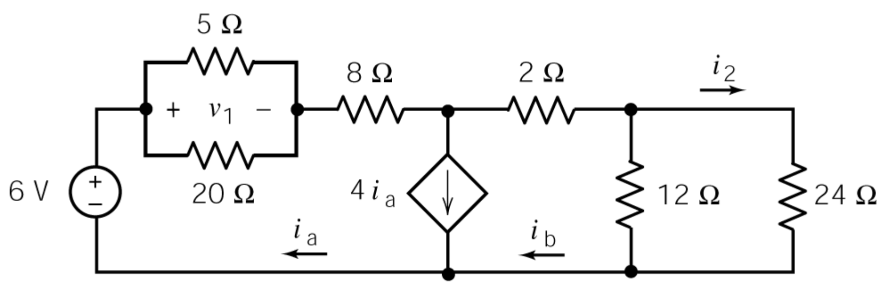
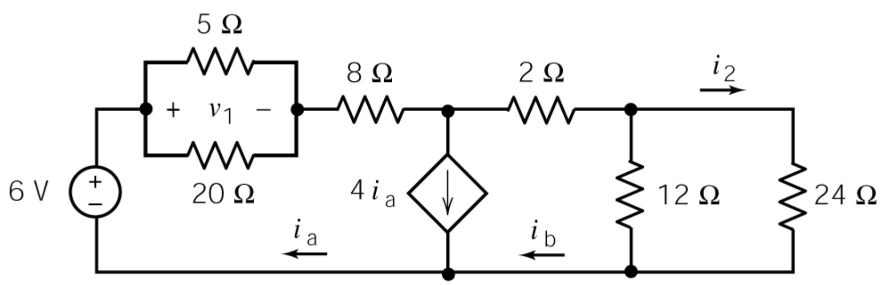
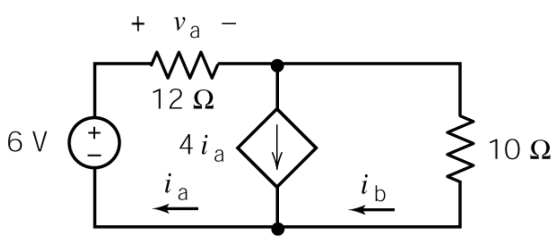


Figure P3.6-42

Solution:



Use equivalent resistances to reduce the circuit to



From KCL $i_b = 4i_a + i_a \quad \Rightarrow \quad i_b = -3i_a$.

From KVL

$$12i_a + 10i_b - 6 = 0 \quad \Rightarrow \quad 12i_a + 10(-3i_a) = 6$$

So $i_a = -\frac{1}{3}$ A, $v_a = -4$ A and $i_b = 1$ A .

Returning our attention to the original circuit, notice that i_a and i_b were not changed when the circuit was reduced. Now $v_1 = (5 \| 20)i_a = (4)(-0.333) = -1.333$ V and $i_2 = \frac{12}{12+24} i_b = 0.333$ A .

P3.6-43 The Ohmmeter in Figure P3.6-43 measures R_{eq}, the equivalent resistance of the part of the circuit to the left of the terminals.

(a) Suppose R_{eq} = 12 Ω. Determine the value of the resistance *R*.
(b) Suppose instead that *R* = 14 Ω. Determine the value of the equivalent resistance R_{eq}.

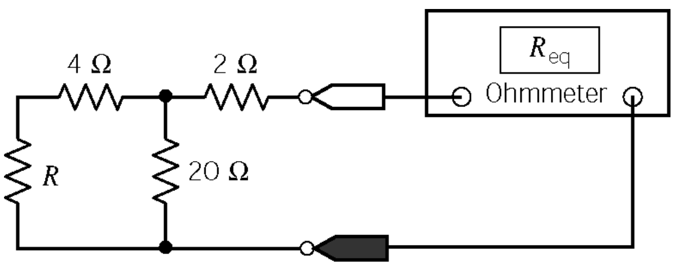


Figure P3.6-43

Solution:

$$R_{eq} = 2 + \left(20 \,||\, (4+R)\right).$$

(a) When $R_{eq} = 12\ \Omega$ then

$$12 = 2 + \left(20 \,||\, (4+R)\right) \;\Rightarrow\; 10 = \frac{20(4+R)}{20+(4+R)} \;\Rightarrow\; 24 + R = 2(4+R) \;\Rightarrow\; R = 16\ \Omega$$

(b) When $R = 14\ \Omega$ then

$$R_{eq} = 2 + \left(20 \,||\, (4+14)\right) = 2 + \left(20 \,||\, 18\right) = 2 + \frac{20(18)}{20+18} = 11.4736\ \Omega$$

P3.6-43. Determine the values of the resistance R and current i_a in the circuit shown in Figure P3.6-43.

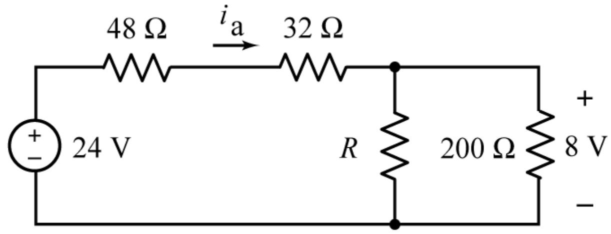
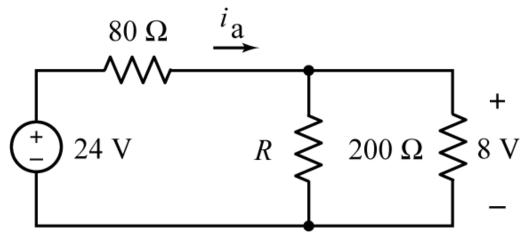


Figure P3.6-43

Solution. Replace the series resistors by an equivalent resistor. Then use KVL to write



$$80i_a + 8 - 24 = 0 \quad \Rightarrow \quad i_a = \frac{24-8}{80} = 0.2 \text{ A}$$

Use KCL to write

$$\frac{24-8}{80} = \frac{8}{R} + \frac{8}{200} \quad \Rightarrow \quad \frac{8}{R} = \frac{16}{80} - \frac{8}{200} = 0.16 \quad \Rightarrow \quad R = \frac{8}{0.16} = 50 \ \Omega$$

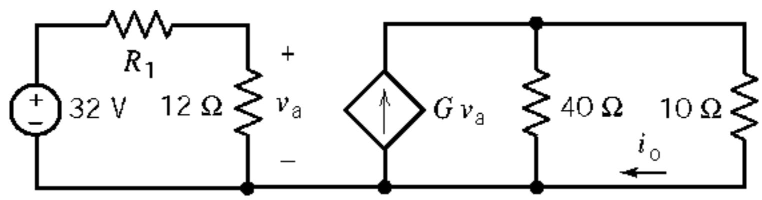


Figure P3.6-44

P3.6-44 The input to the circuit shown in Figure P3.6-44 is the voltage of the voltage source, 32 V. The output is the current in the 10 Ω resistor, i_o. Determine the values of the resistance, R_1, and of the gain of the dependent source, G, that cause both the value of voltage across the 12 Ω to be v_a = 10.38 V and the value of the output current to be i_o = 0.4151 A.

Solution:

Using voltage division

$$10.38 = v_a = \frac{12}{R_1 + 12}(32) \quad \Rightarrow \quad R_1 + 12 = \frac{12(32)}{10.38} = 36.9942 \approx 37 \ \Omega \quad \Rightarrow \quad R_1 = 25 \ \Omega$$

Using current division

$$0.4151 = i_o = \frac{40}{40+10} G v_a = (0.8)G(10.38) \quad \Rightarrow \quad G = \frac{0.4151}{(0.8)10.38} = 0.05 \ \frac{\text{A}}{\text{V}}$$

P3.6-45 The equivalent circuit in Figure 3.6-45 is obtained from the original circuit by replacing series and parallel combinations of resistors by equivalent resistors. The values of the currents in the equivalent circuit are $i_a = 3.5$ A and $i_b = -1.5$ A. Determine the values of the voltages v_1 and v_2 in the original circuit.

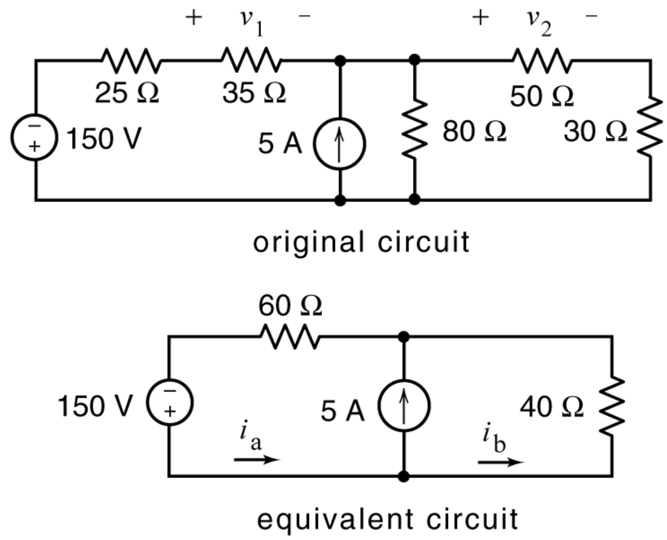
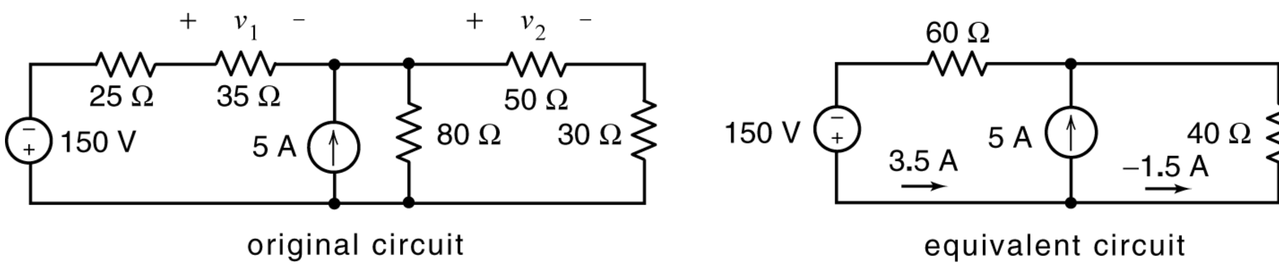
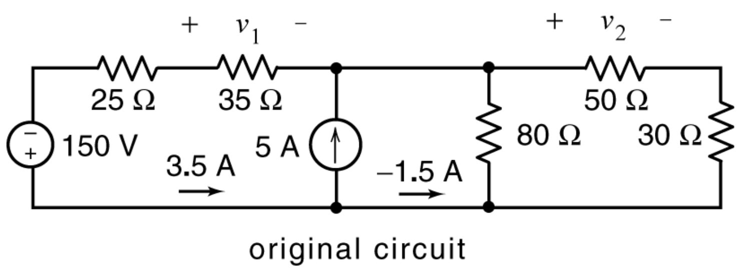


Figure P3.6-45

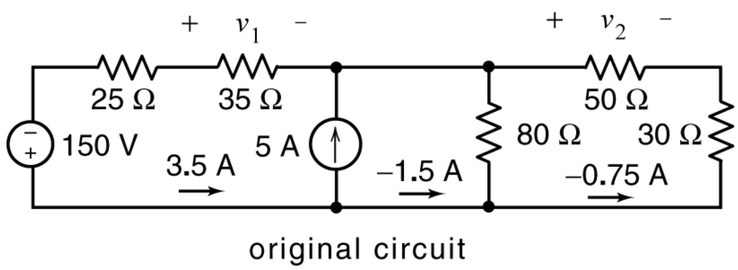
Solution:



Label the currents in the equivalent circuit that correspond to the give currents in the equivalent circuit:



Use current division:



Using Ohm's law:

$$v_1 = -35 i_a = -35(3.5) = -122.5 \text{ V} \quad \text{and} \quad v_2 = -50(-0.75) = 37.5 \text{ V}$$

P3.6-46 Figure 3.6-46 shows three separate, similar circuits. In each a 12 V source is connected to a subcircuit consisting of three resistors. Determine the values of the voltage source currents i_1, i_2 and i_3. Conclude that while the voltage source voltage is 12 V in each circuit, the voltage source current depends on the subcircuit connected to the voltage source.

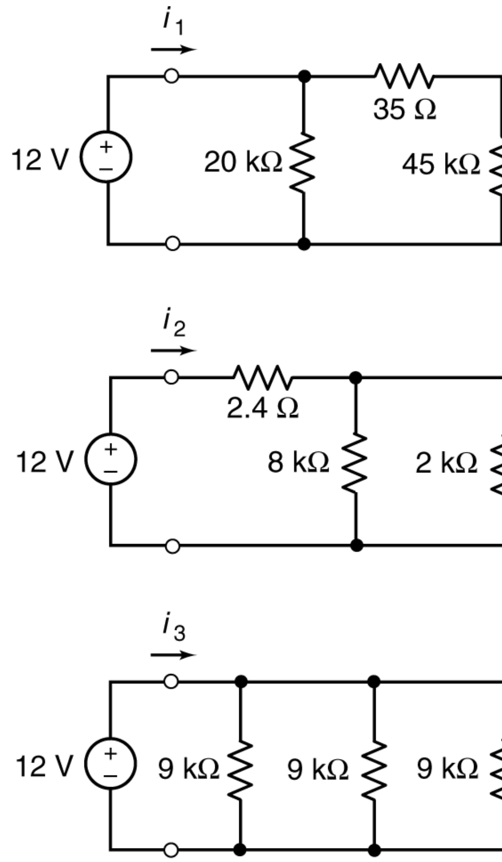
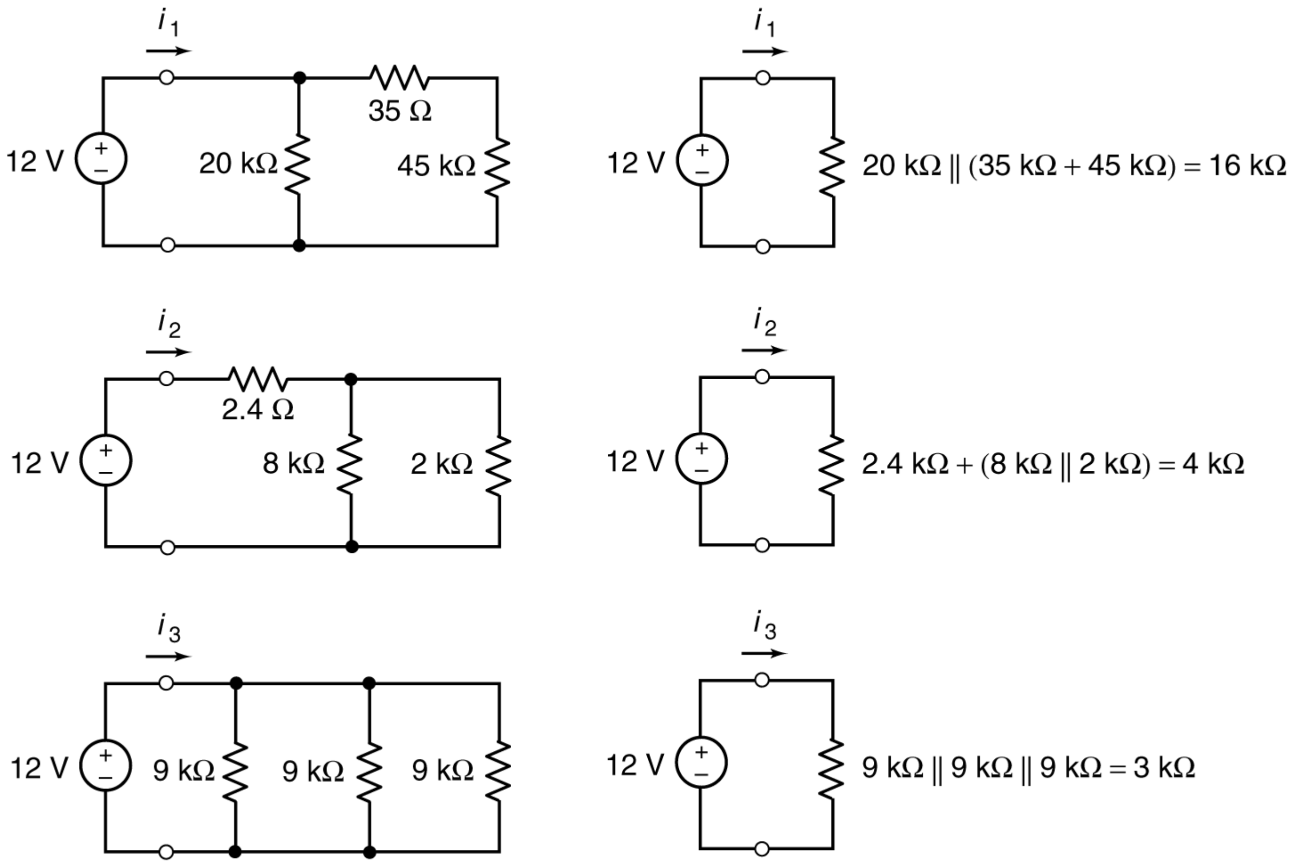


Figure P3.6-46

Solution:

Replace the resistor subcircuit by an equivalent resistor in each circuit:



Using Ohm's law:

$$i_1 = \frac{12}{16\text{ k}\Omega} = 0.75\text{ mA}\ ,\ i_2 = \frac{12}{4\text{ k}\Omega} = 3\text{ mA} \quad \text{and} \quad i_3 = \frac{12}{3\text{ k}\Omega} = 4\text{ mA}$$

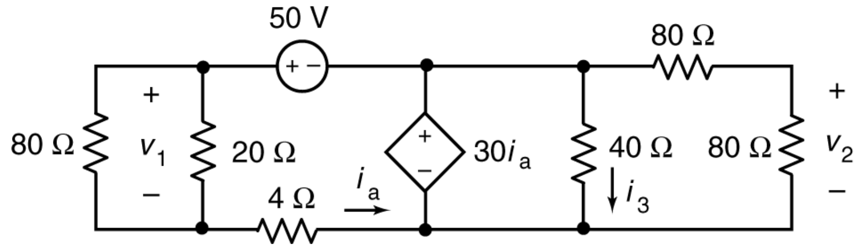
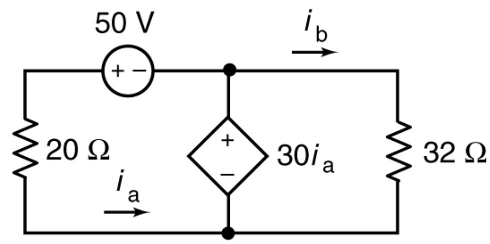


Figure P3.6-47

P3.6-47 Determine the values of the voltages, v_1 and v_2, and of the current, i_3, in the circuit shown in Figure P3.6-47.

Solution:
Replace series and parallel combinations of resistors by equivalent resistors to get

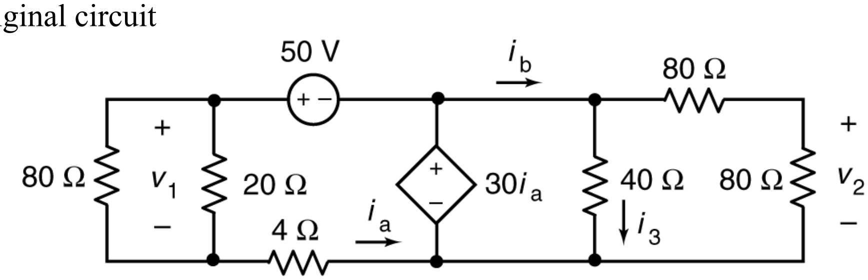


($4+(80\,||\,20)=4+16=20\ \Omega$ and $40||(80+80)=40\,||\,160=32\ \Omega$.) Next, apply KVL to the left mesh to get

$$50+30i_a-20i_a=0 \quad\Rightarrow\quad i_a=\frac{50}{20-10}=-5\text{ A} \quad\text{and}\quad 30i_a=-150\text{ V}$$

Ohm's law gives
$$i_b=\frac{30i_a}{32}=\frac{-150}{32}=-4.6875\ \text{ A}$$

Label i_b on the original circuit



Finally
$$v_1=(80\,||\,20)i_a=16(-5)=-80\text{ V},\ v_2=\frac{1}{2}(30i_a)=-75\text{ V}$$

and
$$i_3=\frac{80+80}{40+(80+80)}i_b=\frac{4}{5}(-4.6875)=-3.75\text{ A}$$

Section 3-7 Analyzing Resistive Circuits using MATLAB

P3.7-1 Determine the power supplied by each of the sources, independent and dependent, in the circuit shown in Figure P3.7-1.
Hint: Use the guidelines given in Section 3.7 to label the circuit diagram. Use MATLAB to solve the equations representing the circuit.

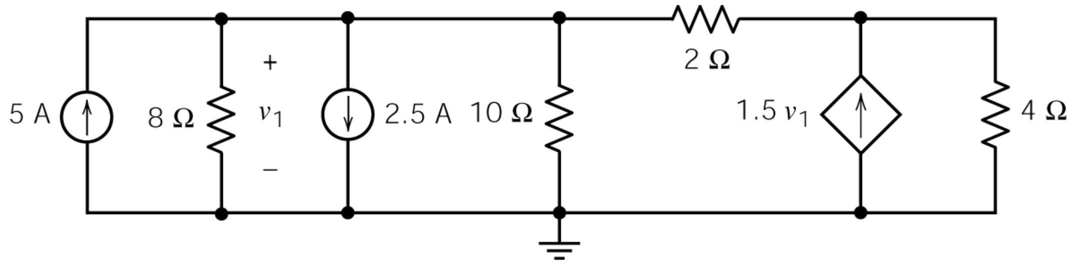
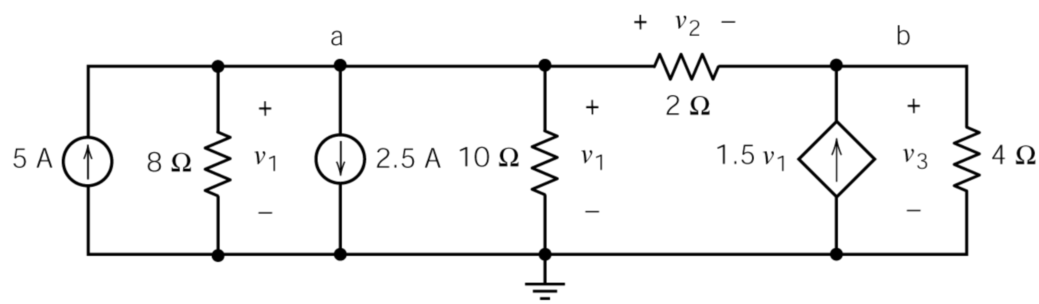


Figure 3.7-1

Solution:
We'll begin by choosing the bottom node to be the reference node. Next we'll label the other nodes and some element voltages:



Notice that the 8 Ω resistor, the 10 Ω resistor and the two independent current sources are all connected in parallel. Consequently, the element voltages of theses elements can be labeled so that they are equal. Similarly, the 4 Ω resistor and the dependent current source are connected in parallel so their voltages can be labeled so as to be equal.

Using Ohm's Law we see that the current directed downward in the 8 Ω resistor is $\frac{v_1}{8}$, current directed downward in the 10 Ω resistor is $\frac{v_1}{10}$, and the current directed from left to right in the 2 Ω resistor is $\frac{v_2}{2}$. Applying Kirchhoff's Current Law (KCL) at node a gives

$$5 = \frac{v_1}{8} + 2.5 + \frac{v_1}{10} + \frac{v_2}{2} \quad \Rightarrow \quad 0.225\,v_1 + 0.5\,v_2 = 2.5 \qquad (1)$$

Using Ohm's Law we see that the current directed downward in the 4 Ω resistor is $\frac{v_3}{4}$. Applying Kirchhoff's Current Law (KCL) at node a gives

$$\frac{v_2}{2} + 1.5v_1 = \frac{v_3}{4} \quad \Rightarrow \quad 1.5v_1 + 0.5v_2 - 0.25v_3 = 0 \qquad (2)$$

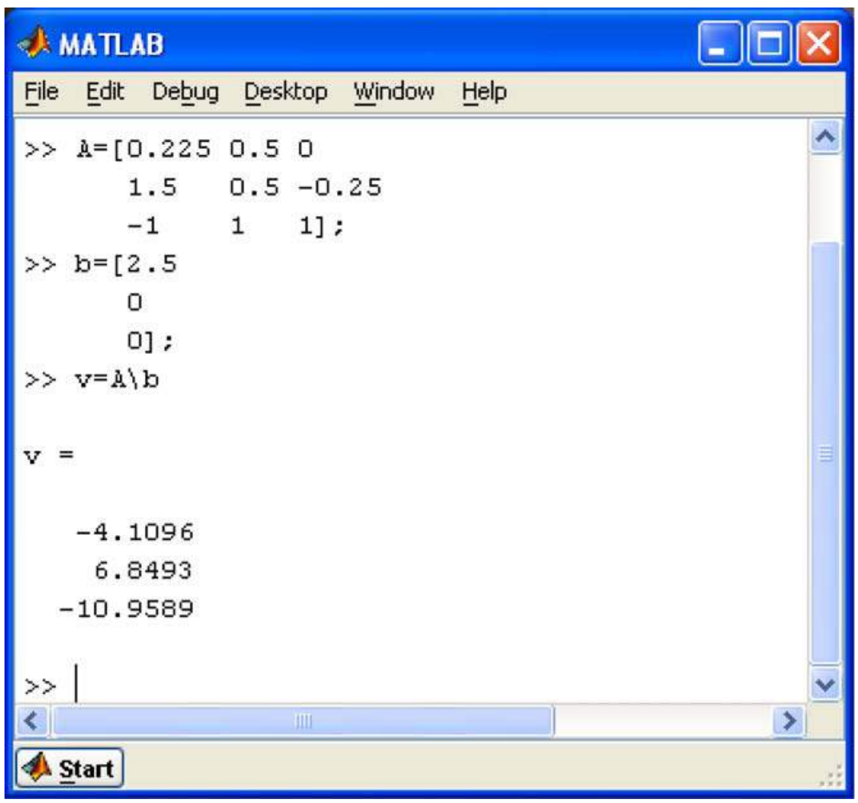
Applying Kirchhoff's Voltage Law (KVL) to the mesh consisting of the 10 Ω resistor, the 2 Ω resistor and the dependent source to get

$$v_2 + v_3 - v_1 = 0 \qquad (3)$$

Equations 1, 2 and 3 comprise a set of three simultaneous equations in the three unknown voltages v_1, v_2 and v_3. We can write these equations in matrix form as

$$\begin{bmatrix} 0.225 & 0.5 & 0 \\ 1.5 & 0.5 & -0.25 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 0 \\ 0 \end{bmatrix}$$

We can solve this matrix equation using MATLAB:



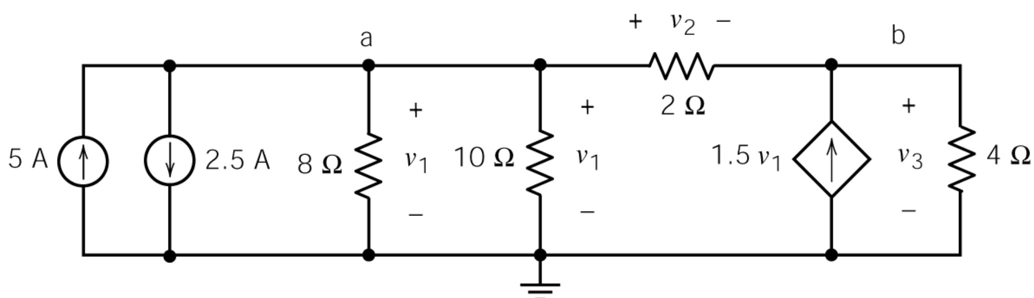
Hence

$$v_1 = -4.1096 \text{ V}, \quad v_2 = 6.8493 \text{ V} \quad \text{and} \quad v_3 = -10.9589 \text{ V}$$

The power supplied by the 5 A current source is $5v_1 = 5(-4.1096) = -20.548$ W. The power supplied by the 2.5 A current source is $-2.5v_1 = -2.5(-4.1096) = 10.247$ W. The power supplied by the dependent current source is $(1.5v_1)v_3 = 1.5(-4.1096)(-10.9589) = 67.555$ W.

Observation: Changing the order of the 8 Ω resistor, the 10 Ω resistor and the two independent current sources only changes the order of the terms in the KCL equation at node a. We know that addition is commutative, so change the order of the terms will not affect the values of the voltages v_1, v_2 and v_3.

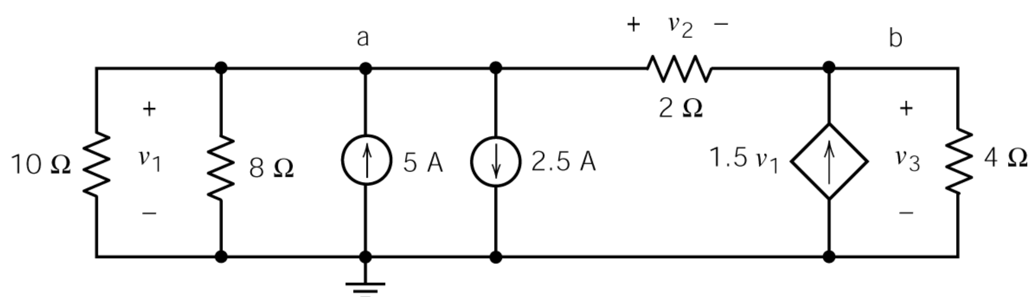
For example, if the positions of the 2.5 A current source and 8 Ω resistor are swithched:



The KCL equation at node a is

$$5 = 2.5 + \frac{v_1}{8} + \frac{v_1}{10} + \frac{v_2}{2} \quad \Rightarrow \quad 0.225v_1 + 0.5v_2 = 2.5$$

Similarly, when the circuit is drawn as



The KCL equation at node a is

$$5 = \frac{v_1}{10} + \frac{v_1}{8} + 2.5 + \frac{v_2}{2} \quad \Rightarrow \quad 0.225v_1 + 0.5v_2 = 2.5$$

The changes do not affect the values of the voltages v_1, v_2 and v_3.

P3.7-2 Determine the power supplied by each of the sources, independent and dependent, in the circuit shown in Figure P3.7-2.
Hint: Use the guidelines given in Section 3.7 to label the circuit diagram. Use MATLAB to solve the equations representing the circuit.

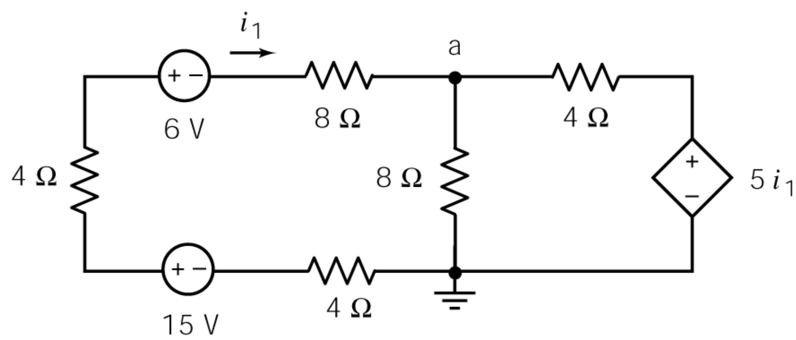
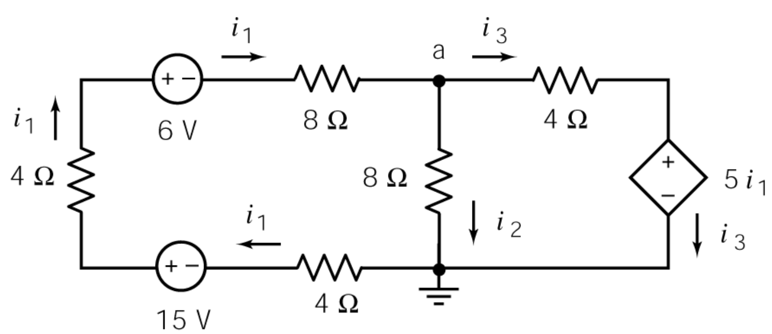


Figure 3.7-2

Solution:
We'll begin by choosing the bottom node to be the reference node. Next we'll label the other nodes and some element currents:



Notice that two 4 Ω resistors, an 8 Ω resistor and the two independent voltage sources are all connected in series. Consequently, the element currents of theses elements can be labeled so that they are equal. Similarly, a 4 Ω resistor and the dependent voltage source are connected in series so their currents can be labeled so as to be equal.

The current in each resistor has been labeled so we can use Ohm's Law to calculate resistor voltages from the resistor currents and the resistances. Apply Kirchhoff's Voltage Law (KVL) to the left mesh to get

$$6+8i_1+8i_2+4i_1-15+4i_1=0 \quad \Rightarrow \quad 16i_1+8i_2=9 \tag{1}$$

Apply Kirchhoff's Voltage Law (KVL) to the right mesh to get

$$4i_3+5i_1-8i_2=0 \tag{2}$$

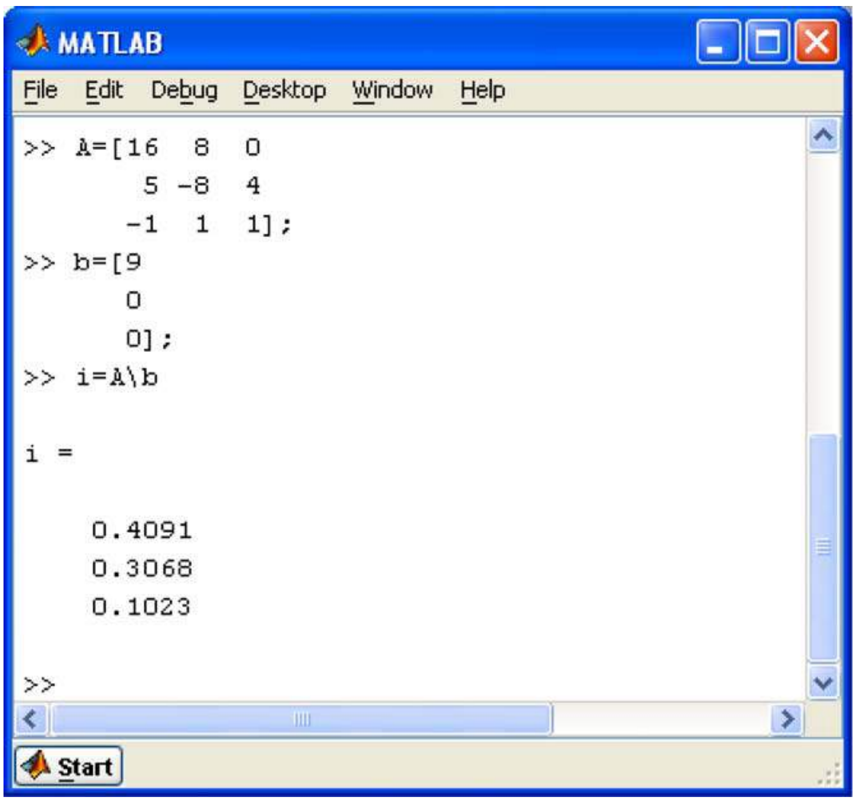
Applying Kirchhoff's Current Law (KCL) at node a to get

$$i_1=i_2+i_3 \quad \Rightarrow \quad -i_1+i_2+i_3=0 \tag{3}$$

Equations 1, 2 and 3 comprise a set of three simultaneous equations in the three unknown voltages v_1, v_2 and v_3. We can write these equations in matrix form as

$$\begin{bmatrix} 16 & 8 & 0 \\ 5 & -8 & 4 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \\ 0 \end{bmatrix}$$

We can solve this matrix equation using MATLAB:

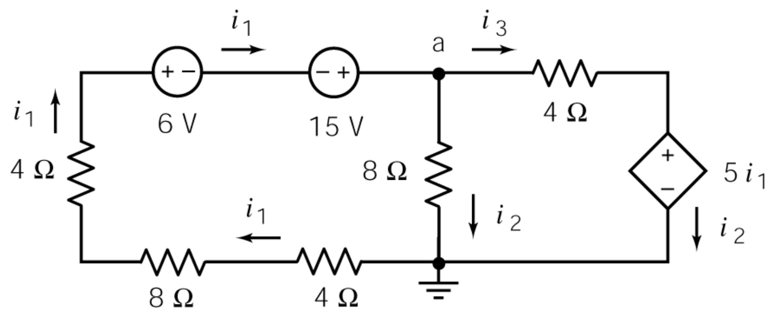


Hence

$$i_1 = 0.4091 \text{ A}, \quad i_2 = 0.3068 \text{ A} \quad \text{and} \quad i_3 = 0.1023 \text{ A}$$

The power supplied by the 15 V voltage source is $15 i_1 = 15(0.4091) = 6.1365$ W . The power supplied by the 6 V voltage source is $-6 i_1 = -6(0.4091) = -2.4546$ W . The power supplied by the dependent voltage source is $-(5 i_2) i_3 = -5(0.3068)(0.1023) = 0.1569$ W .

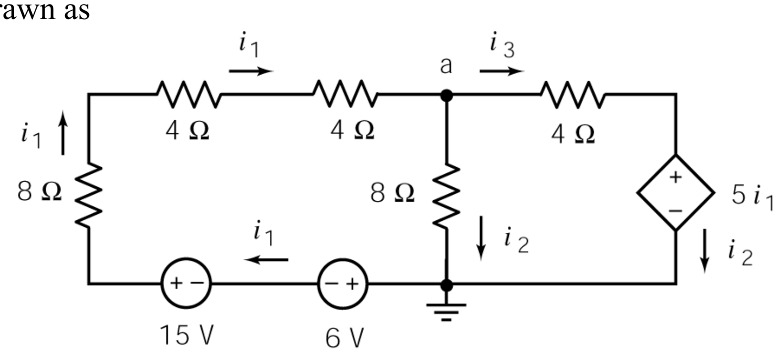
Observation: Changing the order of the two 4 Ω resistors, an 8 Ω resistor and the two independent voltage sources in the left mesh changes the order of the terms in the KVL equation for that mesh. We know that addition is commutative, so change the order of the terms will not affect the values of the currents i_1, i_2 and i_3. For example, when the circuit is drawn as



The KVL equation for the left mesh is

$$6-15+8i_2+4i_1+8i_1+4i_1=0 \quad \Rightarrow \quad 16i_1+8i_2=9$$

When the circuit is drawn as



The KVL equation for the left mesh is

$$4i_1+4i_1+8i_2+6-15+8i_1=0 \quad \Rightarrow \quad 16i_1+8i_2=9$$

These changes do not affect the values of the currents i_1, i_2 and i_3.

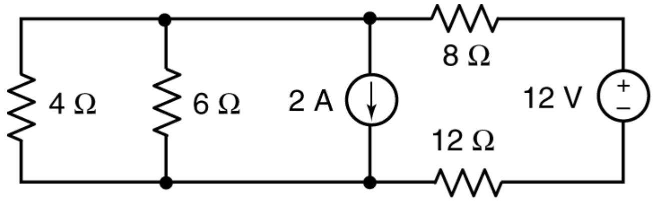
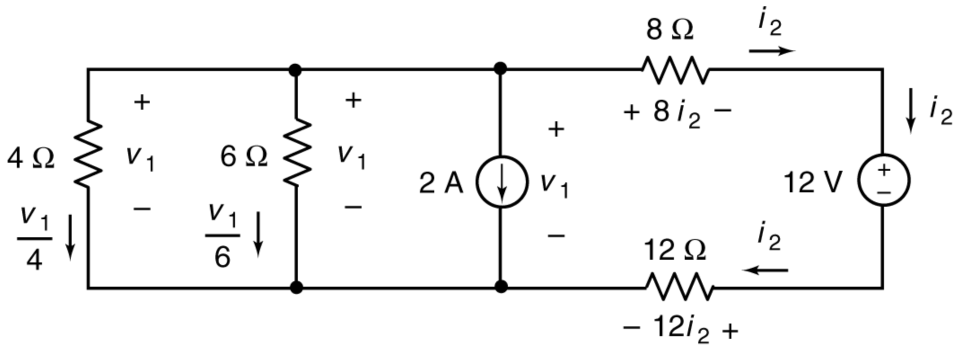


Figure P3.7-3

P3.7-3. Determine the power supplied by each of the independent sources in the circuit shown in Figure P3.7-3.

P3.7-3

Label the element currents and voltages as suggested in Table 3.7-1 Guidelines for Labeling Circuit Variables:



Apply KCL at the top left node: $\dfrac{v_1}{4}+\dfrac{v_1}{4}+2+i_2=0$

Apply KVL to the left mesh: $8i_2-12+12i_2-v_1=0$

In matrix form:
$$\begin{bmatrix} \frac{1}{4}+\frac{1}{6} & 1 \\ -1 & 8+12 \end{bmatrix}\begin{bmatrix} v_1 \\ i_2 \end{bmatrix}=\begin{bmatrix} -2 \\ 12 \end{bmatrix}$$

Solving using MATLAB:
$$\begin{bmatrix} v_1 \\ i_2 \end{bmatrix}=\begin{bmatrix} -5.5714 \\ 0.3214 \end{bmatrix}$$

The current source **supplies** $-2v_1=-2(-5.5714)=11.1428\text{ W}$

The voltage source supplies $12i_2=12(0.3214)=3.8568\text{ W}$

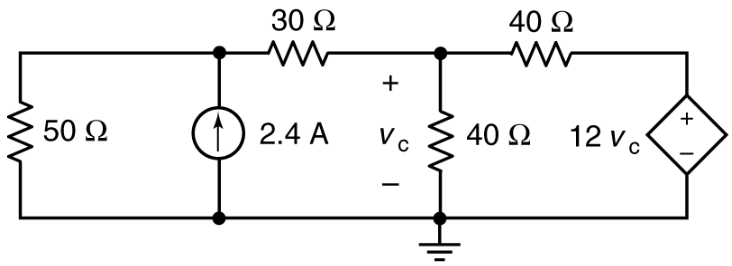
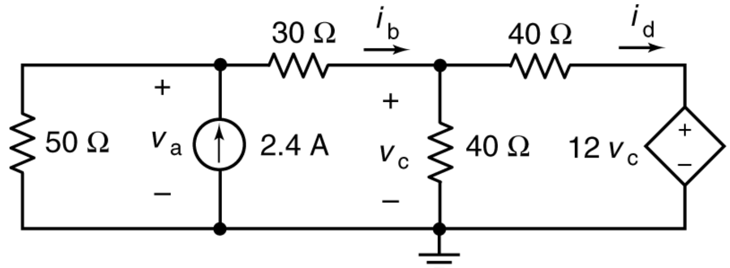


Figure P3.7-4

P3.7-4. Determine the power supplied by each of the sources in the circuit shown in Figure P3.7-4.

P3.7-4

Label the element currents and voltages as suggested in Table 3.7-1 Guidelines for Labeling Circuit Variables:



Apply KVL to the loop consisting of 30 Ω, 40 Ω and 50 Ω resistors: $30\, i_b + v_c - v_a = 0$

Apply KVL to the right mesh: $v_c = 40\, i_d + 12\, v_c$

Apply KCL at the top node of the current source: $\frac{v_a}{50} + i_b = 2.4$

Apply KCL at the top node of a 40 Ω resistor: $i_b = \frac{v_c}{40} + i_d$

In matrix form:

$$\begin{bmatrix} -1 & 30 & 1 & 0 \\ 0 & 0 & 11 & 40 \\ 0.02 & 1 & 0 & 0 \\ 0 & -1 & 0.025 & 1 \end{bmatrix} \begin{bmatrix} v_a \\ i_b \\ v_c \\ i_d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2.4 \\ 0 \end{bmatrix}$$

Solving using MATLAB:

$$\begin{bmatrix} v_a \\ i_b \\ v_c \\ i_d \end{bmatrix} = \begin{bmatrix} 41.0526 \\ 1.5789 \\ -6.3158 \\ 1.7368 \end{bmatrix}$$

The current source **supplies** $2.4\, v_a = 2.4(41.0526) = 98.5262 \text{ W}$

The VCVS supplies $-(12\, v_c)\, i_d = -12(-6.3158)(1.7368) = 131.6314 \text{ W}$

Section 3-8 How Can We Check …

P 3.8-1 A computer analysis program, used for the circuit of Figure P 3.8-1, provides the following branch currents and voltages:

$$i_1 = -0.833 \text{ A}, \; i_2 = -0.333 \text{ A}, \; i_3 = -1.167 \text{ A},$$

and

$$v = -2.0 \text{ V}.$$

Are these answers correct?

Hint: Verify that KCL is satisfied at the center node and that KVL is satisfied around the outside loop consisting of the two 6-Ω resistors and the voltage source.

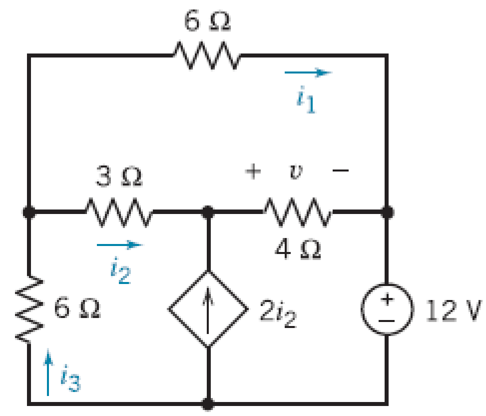


Figure P 3.8-1

Solution:

KCL at node a: $i_3 = i_1 + i_2$

$$-1.167 = -0.833 + (-0.333)$$

$$-1.167 = -1.166 \;\; \text{OK}$$

KVL loop consisting of the vertical 6 Ω resistor, the 3 Ω and4Ω resistors, and the voltage source:

$$6i_3 + 3i_2 + v + 12 = 0$$

yields $\underline{v = -4.0 \text{ V}}$ $\underline{\text{not}}$ $v = -2.0$ V

The answers are not correct.

P 3.8-2 The circuit of Figure P 3.8-2 was assigned as a homework problem. The answer in the back of the textbook says the current, *i,* is 1.25 A. Verify this answer using current division.

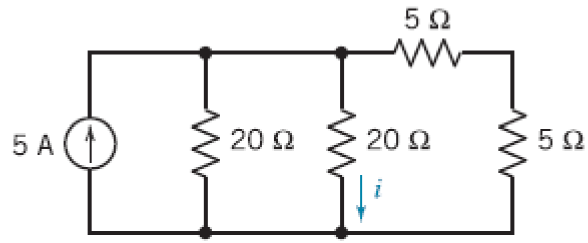


Figure P 3.8-2

Solution:

Apply current division to get: $$i=\left(\frac{\frac{1}{20}}{\frac{1}{20}+\frac{1}{20}+\frac{1}{5+5}}\right)5=\left(\frac{1}{4}\right)5=1.25\text{ A}\quad\surd$$

The answer is correct.

P 3.8-3 The circuit of Figure P 3.8-3 was built in the lab and v_o was measured to be 6.25 V. Verify this measurement using the voltage divider principle.

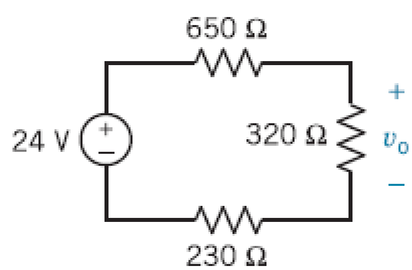


Figure P 3.8-3

Solution:

Apply voltage division to get: $$v=\left(\frac{320}{650+320+230}\right)24=\left(\frac{320}{1200}\right)24=6.4\text{ V}\neq 6.25\text{ V}$$

The measurement is not correct.

P 3.8-4 The circuit of Figure P 3.8-4 represents an auto's electrical system. A report states that

$$i_H = 9 \text{ A}, \quad i_B = -9 \text{ A}, \quad \text{and} \quad i_A = 19.1 \text{ A}.$$

Verify that this result is correct.

Hint: Verify that KCL is satisfied at each node and that KVL is satisfied around each loop.

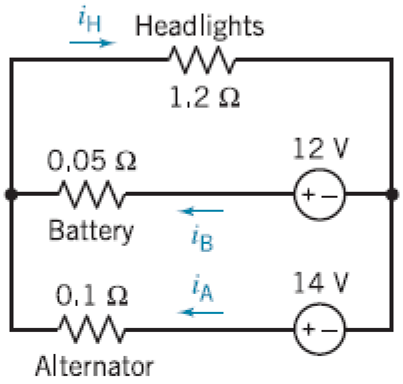


Figure P 3.8-4

Solution:

KVL bottom loop: $-14+0.1i_A+1.2i_H=0$

KVL right loop: $-12+0.05i_B+1.2i_H = 0$

KCL at left node: $i_A+i_B=i_H$

Solving the three above equations yields:

$\underline{i_A=16.8 \text{ A}}$ $\quad \underline{i_H=10.3 \text{ A}}$ $\quad$ and $\quad \underline{i_B=-6.49 \text{ A}}$

These are not the given values. Consequently***, the report is incorrect***.

P 3.8-5 Computer analysis of the circuit in Figure P 3.8-5 shows that

$$i_a = -0.5 \text{ mA and } i_b = -2 \text{ mA}.$$

Was the computer analysis done correctly?

Hint: Verify that the KVL equations for all three meshes are satisfied when $i_a = -0.5$ mA and $i_b = -2$ mA.

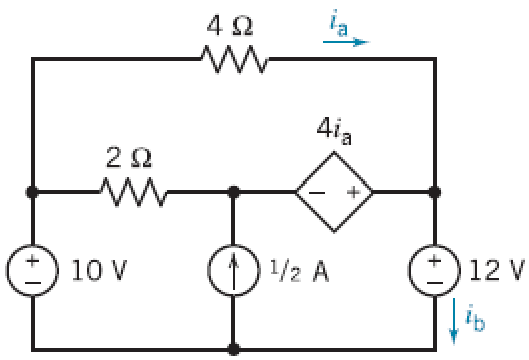


Figure P 3.8-5

Solution:

Top mesh: $0=4\,i_a+4\,i_a+2\left(i_a+\frac{1}{2}-i_b\right)=10(-0.5)+1-2(-2)$

Lower left mesh: $v_s=10+2\left(i_a+0.5-i_b\right)=10+2(2)=14 \text{ V}$

Lower right mesh: $v_s+4\,i_a=12 \quad \Rightarrow \quad v_s=12-4(-0.5)=14 \text{ V}$

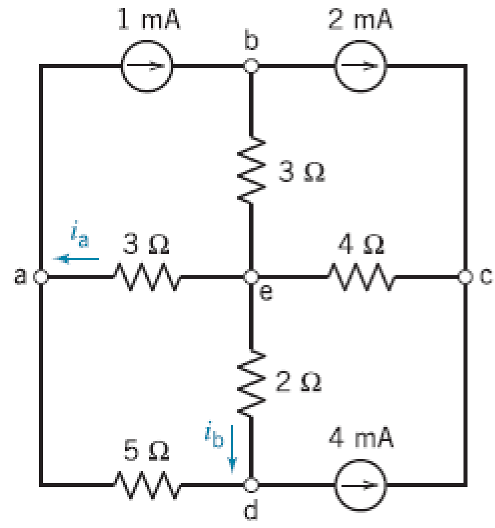
The KVL equations are satisfied so ***the analysis is correct***.

P 3.8-6 Computer analysis of the circuit in Figure P 3.8-6. shows that

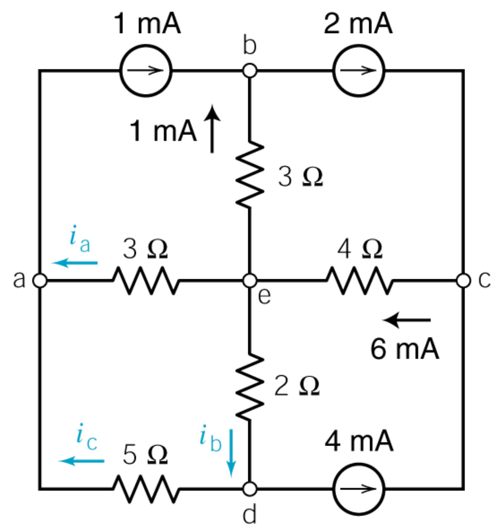
$$i_a = 0.5 \text{ mA and } i_b = 4.5 \text{ mA}.$$

Was the computer analysis done correctly?

Hint: First, verify that the KCL equations for all five nodes are satisfied when $i_a = 0.5$ mA and $i_b = 4.5$ mA. Next, verify that the KVL equation for the lower left mesh (a-e-d-a) is satisfied. (The KVL equations for the other meshes aren't useful because each involves an unknown voltage.)



Solution: Apply KCL at nodes b and c to label the circuit as



Apply KCL to get

$$i_c + 4 = 4.5 \quad \Rightarrow \quad i_c = 0.5 \text{ A} \qquad \text{(node d)}$$

$$i_a + i_c = 1 \quad \Rightarrow \quad 0.5 + 0.5 = 1 \checkmark \qquad \text{(node a)}$$

$$6 = 1 + i_a + i_b = 1 + 0.5 + 4.5 \checkmark \qquad \text{(node e)}$$

Apply KVL to get to mesh (a-e-d-a)

$$-3(0.5) + 2(4.5) - 5(0.5) = -1.5 + 9 - 2.5 \neq 0$$

The given currents ***do not satisfy*** these five Kirchhoff's laws equations and therefore ***are not correct***.

P 3.8-7 Verify that the element currents and voltages shown in Figure P 3.8-7 satisfy Kirchhoff's laws:

(a) Verify that the given currents satisfy the KCL equations corresponding to nodes a, b, and c.

(b) Verify that the given voltages satisfy the KVL equations corresponding to loops a-b-d-c-a and a-b-c-d-a.

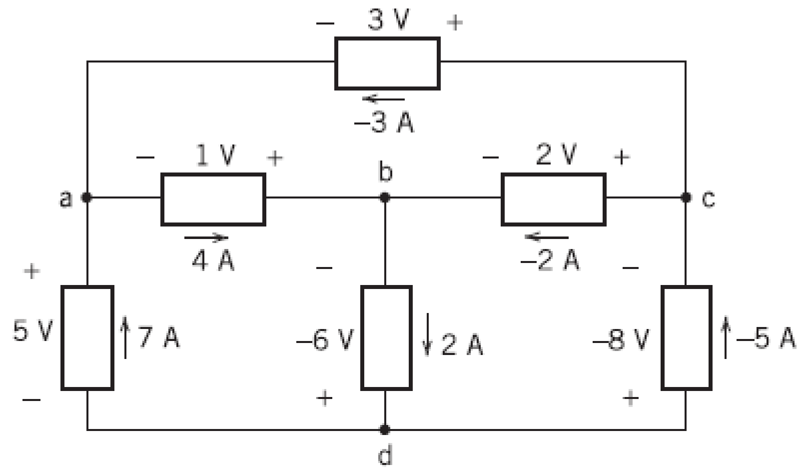


Figure P 3.8-7

Solution:

(a)

$$7+(-3)=4 \qquad \text{(node } a\text{)}$$
$$4+(-2)=2 \qquad \text{(node } b\text{)}$$
$$-5=-2+(-3) \qquad \text{(node } c\text{)}$$

(b)

$$-1-(-6)+(-8)+3=0 \qquad \text{(loop } a\text{-}b\text{-}d\text{-}c\text{-}a\text{)}$$
$$-1-2-(-8)-5=0 \qquad \text{(loop } a\text{-}b\text{-}c\text{-}d\text{-}a\text{)}$$

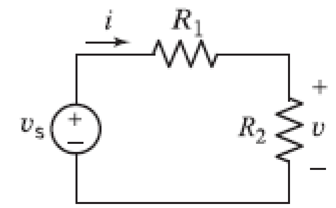
The given currents and voltages satisfy these five Kirchhoff's laws equations

P 3.8-8 Figure P 3.8-8 shows a circuit and some corresponding data. The tabulated data provides values of the current, i, and voltage, v, corresponding to several values of the resistance R_2.

(a) Use the data in rows 1 and 2 of the table to find the values of v_s and R_1.

(b) Use the results of part (a) to verify that the tabulated data are consistent.

(c) Fill in the missing entries in the table.



(a)

R_2, Ω	i, A	v, V
0	2.4	0
10	1.2	12
20	0.8	16
30	?	18
40	0.48	?

(b)

Figure P 3.8-8

Solution:

(a)

$$i = \frac{v_s}{R_1 + R_2}$$

from row 1

$$2.4 = \frac{v_s}{R_1}$$

from row 2

$$1.2 = \frac{v_s}{R_1 + 10}$$

so

$$2.4 R_1 = v_s = 1.2\left(R_1 + 10\right) \quad \Rightarrow \quad R_1 = 10\ \Omega$$

then

$$v_s = 2.4\left(10\right) = 24\ \text{V}$$

(b)

$$i = \frac{24}{10 + R_2} \quad \text{and} \quad v = \frac{24 R_2}{10 + R_2}$$

When $R_2 = 20\ \Omega$ then $i = \frac{24}{30} = 0.8\ \text{A}$ and $v = \frac{480}{30} = 16\ \text{V}$.

When $R_2 = 30\ \Omega$ then $v = \frac{720}{40} = 18\ \text{V}$.

When $R_2 = 40\ \Omega$ the $i = \frac{24}{50} = 0.48\ \text{A}$.

(c) When $R_2 = 30\ \Omega$ then $i = \frac{24}{40} = 0.6\ \text{A}$.

When $R_2 = 40$ W then $v = \frac{960}{50} = 19.2\ \text{V}$.

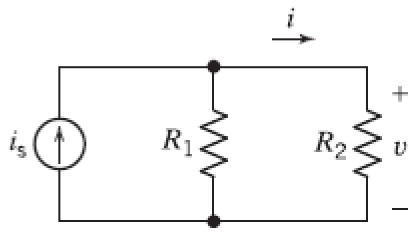
(checked: LNAP 6/21/04)

P 3.8-9 Figure P 3.8-9 shows a circuit and some corresponding data. The tabulated data provide values of the current, i, and voltage, v, corresponding to several values of the resistance R_2.

(a) Use the data in rows 1 and 2 of the table to find the values of i_s and R_1.

(b) Use the results of part (a) to verify that the tabulated data are consistent.

(c) Fill in the missing entries in the table.



(a)

R_2, Ω	i, A	v, V
10	4/3	40/3
20	6/7	120/7
40	1/2	20
80	?	?

(b)

Figure P 3.8-9

Solution:

(a)
$$i=\frac{R_1}{R_1+R_2}i_s$$

From row 1 $\quad \frac{4}{3}=\frac{R_1}{R_1+10}i_s \quad \Rightarrow \quad 4R_1+40=3R_1i_s$

From row 2 $\quad \frac{6}{7}=\frac{R_1}{R_1+20}i_s \quad \Rightarrow \quad 6R_1+120=7R_1i_s$

So $\quad \frac{4R_1+40}{3R_1}=i_s=\frac{6R_1+120}{7R_1} \quad \Rightarrow \quad 28R_1+280=18R_1+360 \quad \Rightarrow \quad R_1=8\ \Omega$

Then $\quad \frac{4}{3}=\frac{8}{8+10}i_s \quad \Rightarrow \quad i_s=3\text{ A}$

(b)
$$i=\frac{8}{8+R_2}(3)=\frac{24}{8+R_2} \quad \text{and} \quad v=R_2\,i=\frac{24R_2}{8+R_2}$$

When R_2 = 40 Ω then $i=\frac{24}{48}=0.5\text{ A}$ and $v=\frac{960}{48}=20\text{ V}$. These are the values in the table so tabulated data is consistent.

(c) When R_2 = 80 Ω then $i=\frac{24}{88}=\frac{3}{11}\text{ A}$ and $v=\frac{24(80)}{88}=\frac{240}{11}\text{ V}$.

(checked: LNAP 6/21/04)

Design Problems

DP 3-1 The circuit shown in Figure DP 3.1 uses a potentiometer to produce a variable voltage. The voltage v_m varies as a knob connected to the wiper of the potentiometer is turned. Specify the resistances R_1 and R_2 so that the following three requirements are satisfied:

1. The voltage v_m varies from 8 V to 12 V as the wiper moves from one end of the potentiometer to the other end of the potentiometer.
2. The voltage source supplies less than 0.5 W of power.
3. Each of R_1, R_2, and R_P dissipates less than 0.25 W.

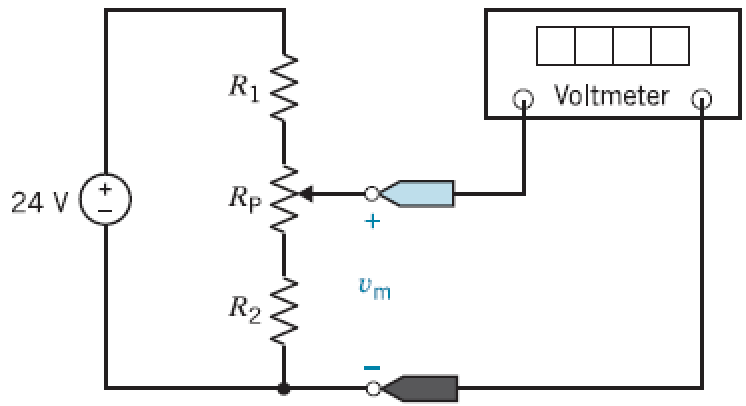
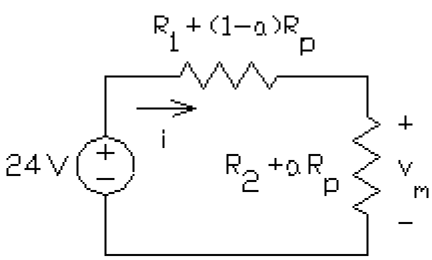


Figure DP 3.1

Solution:



Using voltage division:

$$v_m = \frac{R_2 + aR_p}{R_1 + (1-a)R_p + R_2 + aR_p} 24 = \frac{R_2 + aR_p}{R_1 + R_2 + R_p} 24$$

$$v_m = 8 \text{ V when a} = 0 \Rightarrow \frac{R_2}{R_1 + R_2 + R_p} = \frac{1}{3}$$

$$v_m = 12 \text{ V when a} = 1 \Rightarrow \frac{R_2 + R_p}{R_1 + R_2 + R_p} = \frac{1}{2}$$

The specification on the power of the voltage source indicates

$$\frac{24^2}{R_1 + R_2 + R_p} \le \frac{1}{2} \quad \Rightarrow \quad R_1 + R_2 + R_p \ge 1152\,\Omega$$

Try $R_p = 2000\ \Omega$. Substituting into the equations obtained above using voltage division gives $3R_2 = R_1 + R_2 + 2000$ and $2\left(R_2 + 2000\right) = R_1 + R_2 + 2000$. Solving these equations gives $R_1 = 6000\,\Omega$ and $R_2 = 4000\,\Omega$.

With these resistance values, the voltage source supplies 48 mW while R_1, R_2 and R_p dissipate 24 mW, 16 mW and 8 mW respectively. Therefore the design is complete.

DP 3-2 The resistance R_L in Figure DP 3.2 is the equivalent resistance of a pressure transducer. This resistance is specified to be 200 Ω ± 5 percent. That is, 190 Ω ≤ R_L ≤ 210 Ω. The voltage source is a 12 V ± 1 percent source capable of supplying 5 W. Design this circuit, using 5 percent, 1/8-watt resistors for R_1 and R_2, so that the voltage across R_L is

$$v_o = 4\text{V} \pm 10\%$$

(A 5 percent, 1/8-watt 100-Ω resistor has a resistance between 95 and 105 Ω and can safely dissipate 1/8-W continuously.)

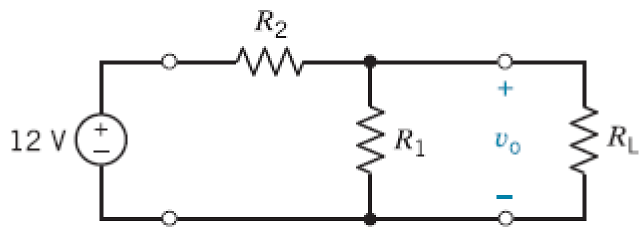


Figure DP 3.2

Solution:

Try $R_1 = \infty$. That is, R_1 is an open circuit. From KVL, 8 V will appear across R_2. Using voltage division, $\frac{200}{R_2+200}12 = 4 \quad \Rightarrow \quad R_2 = 400\ \Omega$. The power required to be dissipated by R_2 is $\frac{8^2}{400} = 0.16\text{ W} < \frac{1}{8}\text{ W}$. To reduce the voltage across any one resistor, let's implement R_2 as the series combination of two 200 Ω resistors. The power required to be dissipated by each of these resistors is $\frac{4^2}{200} = 0.08\text{ W} < \frac{1}{8}\text{ W}$.

Now let's check the voltage:

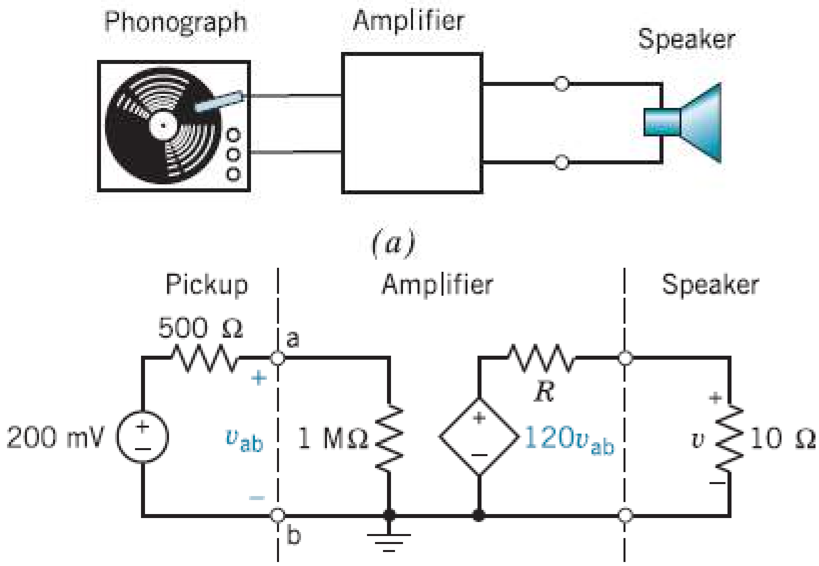
$$11.88\frac{190}{190+420} < v_0 < 12.12\frac{210}{210+380}$$

$$3.700 < v_0 < 4.314$$

$$4 - 7.5\% < v_0 < 4 + 7.85\%$$

Hence, v_o = 4 V ± 8% and the design is complete.

DP 3-3 A phonograph pickup, stereo amplifier, and speaker are shown in Figure DP 3.3*a* and redrawn as a circuit model as shown in Figure DP 3.3*b*. Determine the resistance *R* so that the voltage *v* across the speaker is 16 V. Determine the power delivered to the speaker.



(b)

Figure DP 3.3

Solution:

$$V_{ab} \cong 200 \text{ mV}$$

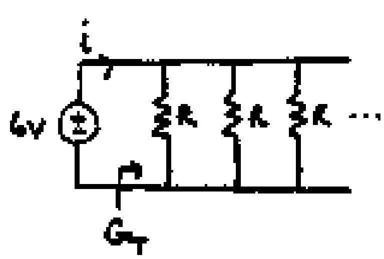
$$v = \frac{10}{10+R}\; 120\, V_{ab} \;= \frac{10}{10+R}\,(120)\,(0.2)$$

$$\text{let } v = 16 \;=\; \frac{240}{10+R} \;\Rightarrow\; \underline{R = 5\,\Omega}$$

$$\therefore\; \text{P} = \frac{16^2}{10} = \underline{25.6\text{W}}$$

DP 3-4 A Christmas tree light set is required that will operate from a 6-V battery on a tree in a city park. The heavy-duty battery can provide 9A for the four-hour period of operation each night. Design a parallel set of lights (select the maximum number of lights) when the resistance of each bulb is 12 Ω.

Solution:



$$i = G_T v = \frac{N}{R} v \quad \text{where } G_T = \sum_{n=1}^{N} \frac{1}{R_n} = N\left(\frac{1}{R}\right)$$

$$\therefore\; N = \frac{iR}{v} = \frac{(9)(12)}{6} = \underline{18 \text{ bulbs}}$$

DP 3-5 The input to the circuit shown in Figure DP 3.5 is the voltage source voltage, v_s. The output is the voltage v_o. The output is related to the input by

$$v_o = \frac{R_2}{R_1 + R_2} v_s = g v_s$$

The output of the voltage divider is proportional to the input. The constant of proportionality, g, is called the gain of the voltage divider and is given by

$$g = \frac{R_2}{R_1 + R_2}$$

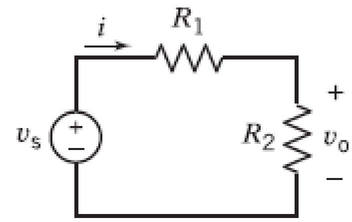


Figure DP 3.5

The power supplied by the voltage source is

$$p = v_s i_s = v_s \left(\frac{v_s}{R_1 + R_2} \right) = \frac{v_s^2}{R_1 + R_2} = \frac{v_s^2}{R_{in}}$$

where

$$R_{in} = R_1 + R_2$$

is called the input resistance of the voltage divider.

(a) Design a voltage divider to have a gain, $g = 0.65$.
(b) Design a voltage divider to have a gain, $g = 0.65$, and an input resistance, $R_{in} = 2500\ \Omega$.

Solution:

Notice that
$$g = \frac{R_2}{R_1 + R_2} \quad \Rightarrow \quad g\, R_1 = (1-g) R_2$$

Thus either resistance can be determined from the other resistance and the gain of the voltage divider. Also

$$g = \frac{R_2}{R_1 + R_2} = \frac{R_2}{R_{in}} \quad \Rightarrow \quad R_2 = g\, R_{in}$$

Consequently
$$g\, R_1 = (1-g) R_2 = (1-g)\, g\, R_{in} \quad \Rightarrow \quad R_1 = (1-g) R_{in}$$

(a) The solution of this problem is not unique. Given any value of R_1, we can determine a value of R_2 that will cause $g = 0.65$. Let's pick a convenient value for R_1, say

$$R_1 = 100\ \Omega$$

Then
$$g\, R_1 = (1-g) R_2 \quad \Rightarrow \quad R_2 = \frac{g\, R_1}{1-g} = \frac{0.65 \times 100}{1-0.65} = 186\ \Omega$$

(b)
$$R_2 = g\, R_{in} = 0.65 \times 2500 = 1625\ \Omega$$

and
$$R_1 = (1-g) R_{in} = (1-0.65) 2500 = 875\ \Omega$$

DP 3-6 The input to the circuit shown in Figure DP 3.6 is the current source current, i_s. The output is the current i_o. The output is related to the input by

$$i_o = \frac{R_1}{R_1 + R_2} i_s = g i_s$$

The output of the current divider is proportional to the input. The constant of proportionality, g, is called the gain of the current divider and is given by

$$g = \frac{R_1}{R_1 + R_2}$$

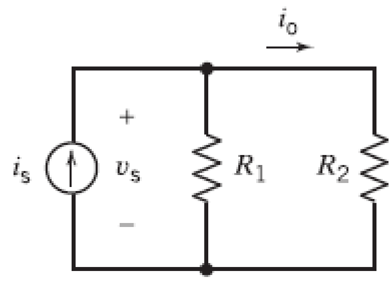


Figure DP 3.6

The power supplied by the current source is

$$p = v_s i_s = \left[i_s \left(\frac{R_1 R_2}{R_1 + R_2} \right) \right] i_s = \frac{R_1 R_2}{R_1 + R_2} i_s^2 = R_{in} i_s^2$$

where
$$R_{in} = \frac{R_1 R_2}{R_1 + R_2}$$

is called the input resistance of the current divider.

(a) Design a current divider to have a gain, $g = 0.65$.
(b) Design a current divider to have a gain, $g = 0.65$, and an input resistance, $R_{in} = 10000\ \Omega$.

Solution:

Notice that
$$g = \frac{R_1}{R_1 + R_2} \quad \Rightarrow \quad g\,R_2 = (1-g)\,R_1$$

Thus either resistance can be determined from the other resistance and the gain of the current divider. Also

$$g = \frac{R_1}{R_1 + R_2} = \frac{R_{in}}{R_2} \quad \Rightarrow \quad R_2 = \frac{R_{in}}{g}$$

Consequently
$$(1-g)\,R_1 = g\,R_2 = R_{in} \quad \Rightarrow \quad R_1 = \frac{R_{in}}{(1-g)}$$

Thus specified values of g and R_{in} uniquely determine the required values of R_1 and R_2.

(a) The solution of this problem is not unique. Given any value of R_1, we can determine a value of R_2 that will cause $g = 0.65$. Let's pick a convenient value for R_1, say

$$R_1 = 100\ \Omega$$

Then
$$g\,R_2 = (1-g)\,R_1 \quad \Rightarrow \quad R_2 = \frac{(1-g)\,R_1}{g} = \frac{(1-0.65)\times 100}{0.65} = 54\ \Omega$$

(b)
$$R_2 = \frac{R_{in}}{g} = \frac{10000}{0.65} = 15385\ \Omega$$

and
$$R_1 = \frac{R_{in}}{(1-g)} = \frac{10000}{(1-0.65)} = 28571\ \Omega$$

DP 3-7 Design the circuit shown in Figure DP 3-7 to have an output $v_o = 8.5$ V when the input is $v_s = 12$ V. The circuit should require no more than 1 mW from the voltage source.

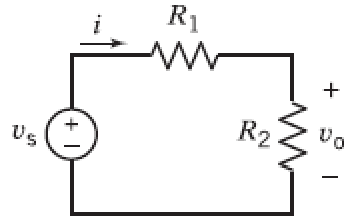


Figure DP 3.7

Solution:

The required gain is $$g = \frac{v_o}{v_s} = \frac{8.5}{12} = 0.7083$$

Consequently, $$0.7083\,R_1 = (1-0.7083)\,R_2 \quad \Rightarrow \quad R_2 = 2.428\,R_1$$

It is also required that

$$0.001 \ge \frac{12^2}{R_1 + R_2} \quad \Rightarrow \quad R_1 + R_2 \ge 144000 \quad \Rightarrow \quad 3.428\,R_1 \ge 144000 \quad \Rightarrow \quad R_1 \ge 42007\ \Omega$$

For example, $$R_1 = 45\ \text{k}\Omega \quad \text{and} \quad R_2 = 109.26\ \text{k}\Omega$$

DP 3-8 Design the circuit shown in Figure DP 3.8 to have an output $i_o = 1.8$ mA when the input is $i_s = 5$ mA. The circuit should require no more than 1 mW from the current source.

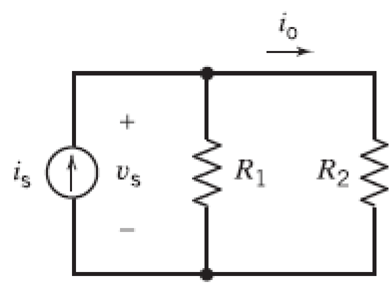


Figure DP 3.8

Solution:

The required gain is $$g = \frac{i_o}{i_s} = \frac{1.8}{5} = 0.36$$

Consequently, $$0.36\,R_2 = (1-0.36)\,R_1 \quad \Rightarrow \quad R_2 = 1.778\,R_1$$

It is also required that

$$0.001 \ge 0.005^2 \frac{R_1 R_2}{R_1 + R_2} \quad \Rightarrow \quad \frac{R_1 + R_2}{R_1 R_2} = \frac{2.778\,R_1}{1.778\,R_1^2} \ge \frac{0.005^2}{0.001} = 0.025$$

$$\Rightarrow \quad R_1 \le \frac{2.778}{1.778(0.025)} = 62.5\ \Omega$$

For example, $$R_1 = 60\ \Omega \quad \text{and} \quad R_2 = 106.7\ \Omega$$

DP 3.9 A thermistor is a temperature dependent resistor. The thermistor resistance, R_T, is related to the temperature by the equation

$$R_{\text{T}} = R_{\text{T}}\, e^{\beta(1/T - 1/T_{\text{o}})}$$

where T has units of °K and R is in Ohms. R_0 is resistance at temperature T_0 and the parameter β is in °K. For example, suppose that a particular thermistor has a resistance R_0 = 620 Ω at the temperature T_0 = 20 °C = 293 °K and β = 3330 °K. At T = 70 °C = 343 °K the resistance of this thermistor will be

$$R_{\text{T}} = 620\, e^{3330(1/342 - 1/293)} = 121.68\ \ \Omega$$

In Figure DP 3-9 this particular thermistor in used in a voltage divider circuit. Specify the value of the resistor R that will cause the voltage v_T across the thermistor to be 4 V when the temperature is 100 °C.

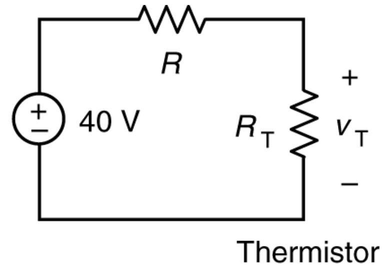


Figure DP 3-9

Solution

At $T = 373$°K $$R_{\text{T}} = 620\, e^{3330(1/372 - 1/293)} = 54.17\ \ \Omega$$

Using voltage division

$$4 = v_{\text{T}} = \frac{54.17}{R + 54.17}(40) \;\Rightarrow\; R + 54.17 = \frac{54.17}{4}(40)$$

$$R = 541.7 - 54.17 = 487.54\ \Omega$$

DP3-10 The circuit shown in Figure DP 3-10 contains a thermistor that has a resistance $R_0 = 620\ \Omega$ at the temperature $T_0 = 20\ °C = 293\ °K$ and $\beta = 3330\ °K$. (See problem DP 3-9.) Design this circuit (that is, specify the values of R and V_s) so that the thermistor voltage is $v_T = 4$ V when $T = 100\ °C$ and $v_T = 20$ V when $T = 0\ °C$.

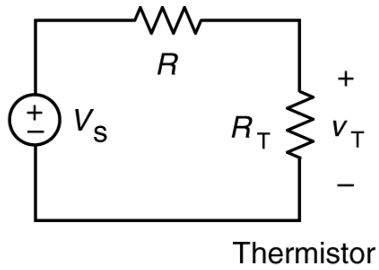


Figure DP 3-10

Solution

At $T = 0°C = 273°K$ $\qquad R_T = 620\,e^{3330(1/273-1/293)} = 1425.6\ \Omega$

At $T = 100°C = 373°K$ $\qquad R_T = 620\,e^{3330(1/373-1/293)} = 54.17\ \Omega$

Using voltage division

$$4 = v_T = \frac{54.17}{R+54.17}(V_s) \quad\Rightarrow\quad R+54.17 = \frac{54.17}{4}(V_s)$$

$$20 = v_T = \frac{1425.6}{R+1425.6}(V_s) \quad\Rightarrow\quad R+1425.6 = \frac{1425.6}{20}(V_s)$$

In matrix form:

$$\begin{bmatrix} \frac{54.17}{4} & -1 \\ \frac{1425.6}{20} & -1 \end{bmatrix} \begin{bmatrix} V_s \\ R \end{bmatrix} = \begin{bmatrix} 54.17 \\ 1425.6 \end{bmatrix}$$

Solving gives $\qquad V_s = 23.7528\ \text{V}$ and $R = 267.5029\ \Omega$

DP3-11 The circuit shown in Figure DP 3-11 is designed help orange grower protect their crops against frost by sounding an alarm when the temperature falls below freezing. It contains a thermistor that has a resistance R_0 = 620 Ω at the temperature T_0 = 20 °C = 293 °K and β = 3330 °K. (See problem DP 3-9.)

The alarm will sound when the voltage at the – input of the comparator is less than the voltage at the + input. Using voltage division twice, we see that the alarm sounds whenever

$$\frac{R_2}{R_T + R_2} < \frac{R_4}{R_3 + R_4}$$

Determine values of R_2, R_3 and R_4 that cause the alarm to sound whenever the temperature is below freezing.

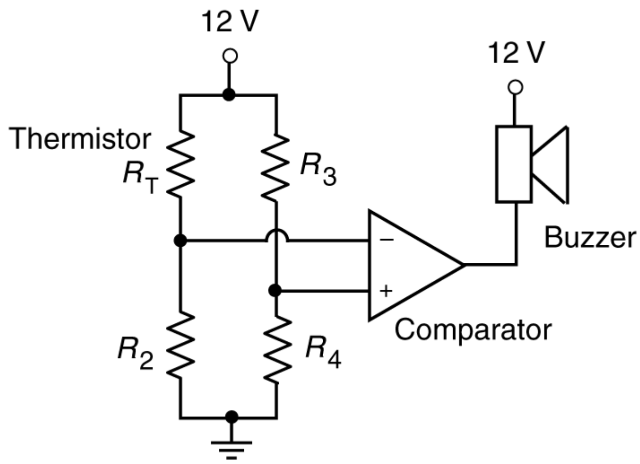


Figure DP 3-11

Solution:
The solution is not unique. For example, pick R_3 = 30 kΩ and R_4 = 10 kΩ. The alarm turns on and off when

$$\frac{R_2}{R_T + R_2} = \frac{10}{30+10} = 0.25$$

At freezing the temperature is T = 0°C = 273°K and the thermistor resistance is

$$R_T = 620\,e^{3330(1/273 - 1/293)} = 1425.6\ \ \Omega$$

At freezing, we have

$$\frac{R_2}{1425.6 + R_2} = 0.25 \quad \Rightarrow \quad 0.75\,R_2 = 356.4$$

$$R_2 = 475.2\ \Omega$$

Let's check. At –2 °C

$$R_T = 620\,e^{3330(1/271 - 1/293)} = 1560\ \ \Omega$$

$$\frac{475.2}{1560 + 475.2} = 0.2335 < 0.25$$

so the alarm is on. At 2 °C

$$R_T = 620\,e^{3330(1/275 - 1/293)} = 1305\ \ \Omega$$

$$\frac{475.2}{1305 + 475.2} = 0.2669 > 0.25$$

so the alarm is off.

Chapter 4 Exercises

Exercise 4.2-1 Determine the node voltages, v_a and v_b, for the circuit of Figure E 4.2-1.

Answer: $v_a = 3$ V and $v_b = 11$ V

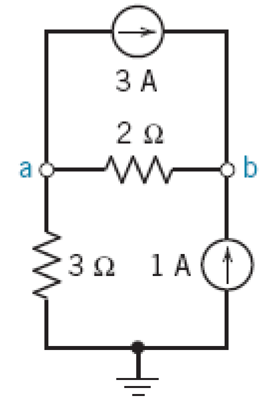


Figure E 4.2-1

Solution:

KCL at a: $$\frac{v_a}{3} + \frac{v_a - v_b}{2} + 3 = 0 \;\Rightarrow\; 5\,v_a - 3\,v_b = -18$$

KCL at b: $$\frac{v_b - v_a}{2} - 3 - 1 = 0 \;\Rightarrow\; v_b - v_a = 8$$

Solving these equations gives: $v_a = 3$ V and $v_b = 11$ V

Exercise 4.2-2 Determine the node voltages, v_a and v_b, for the circuit of Figure E 4.2-2.

Answer: $v_a = -4/3$ V and $v_b = 4$ V

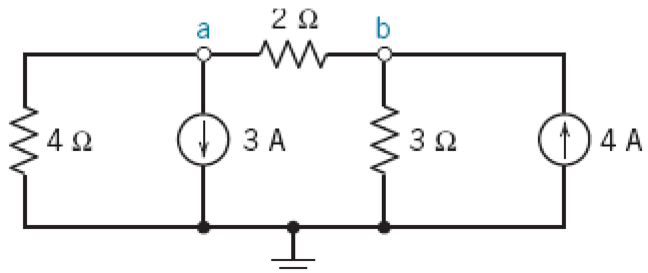


Figure E 4.2-2

Solution:

KCL at a: $$\frac{v_a}{4} + \frac{v_a - v_b}{2} + 3 = 0 \;\Rightarrow\; 3\,v_a - 2\,v_b = -12$$

KCL at b: $$\frac{v_b}{3} - \frac{v_a - v_b}{2} - 4 = 0 \;\Rightarrow\; -3\,v_a + 5\,v_b = 24$$

Solving: $v_a = -4/3$ V and $v_b = 4$ V

Exercise 4.3-1 Find the node voltages for the circuit of Figure E 4.3-1.

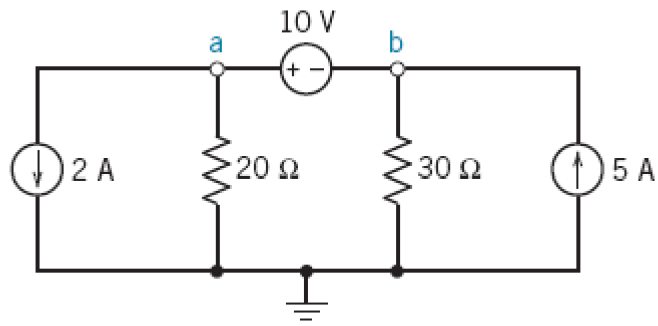


Figure E 4.3-1

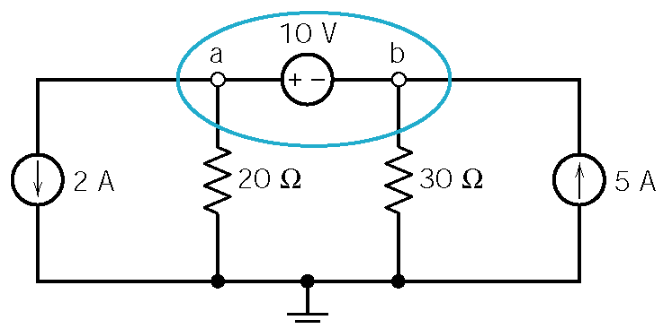
Hint: Write a KCL equation for the supernode corresponding to the 10-V voltage source.

Answer:

$$2+\frac{v_b+10}{20}+\frac{v_b}{30}=5 \quad\Rightarrow\quad v_b=30\text{ V} \quad\text{and}\quad v_a=40\text{ V}$$

Solution:

Apply KCL to the supernode to get



$$2+\frac{v_b+10}{20}+\frac{v_b}{30}=5$$

Solving:

$$v_b=30\text{ V and } v_a=v_b+10=40\text{ V}$$

Exercise 4.3-2 Find the voltages v_a and v_b for the circuit of Figure E 4.3-2.

Answer: $\frac{(v_b+8)-(-12)}{10}+\frac{v_b}{40}=3 \quad\Rightarrow\quad v_b=8\text{ V} \;\text{ and }\; v_a=16\text{ V}$

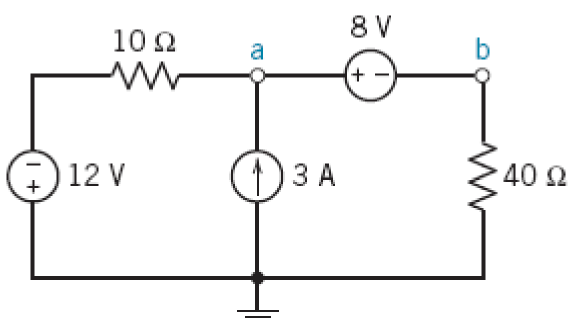


Figure E 4.3-2

Solution:

$$\frac{\left(v_b+8\right)-\left(-12\right)}{10}+\frac{v_b}{40}=3 \quad\Rightarrow\quad v_b=8\text{ V and } v_a=16\text{ V}$$

Exercise 4.4-1 Find the node voltage v_b for the circuit shown in Figure E 4.4-2.

Hint: Apply KCL at node a to express i_a as a function of the node voltages. Substitute the result into $v_b = 4i_a$ and solve for v_b.

Answer: $-\frac{6}{8}+\frac{v_b}{4}-\frac{v_b}{12}=0 \;\Rightarrow\; v_b = 4.5 \text{ V}$

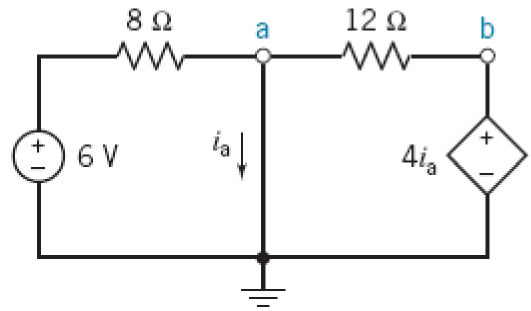


Figure E 4.4-2

Solution:

Apply KCL at node a to express i_a as a function of the node voltages. Substitute the result into $v_b = 4\,i_a$ and solve for v_b .

$$\frac{6}{8}+\frac{v_b}{12}=i_a \;\Rightarrow\; v_b = 4i_a = 4\left(\frac{9+v_b}{12}\right) \;\Rightarrow\; v_b = 4.5 \text{ V}$$

Exercise 4.4-2 Find the node voltages for the circuit shown in Figure E 4.4-2.

Hint: The controlling voltage of the dependent source is a node voltage, so it is already expressed as a function of the node voltages. Apply KCL at node a.

Answer: $\frac{v_a-6}{20}+\frac{v_a-4v_a}{15}=0 \;\Rightarrow\; v_a = -2 \text{ V}$

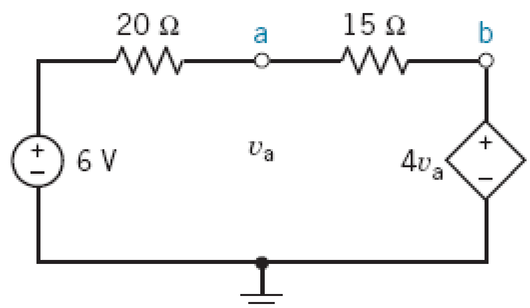


Figure E 4.4-2

Solution:

The controlling voltage of the dependent source is a node voltage so it is already expressed as a function of the node voltages. Apply KCL at node a.

$$\frac{v_a-6}{20}+\frac{v_a-4\,v_a}{15}=0 \;\Rightarrow\; v_a = -2 \text{ V}$$

Exercise 4.5-1 Determine the value of the voltage measured by the voltmeter in Figure E 4.5-1.

Answer: –1 V

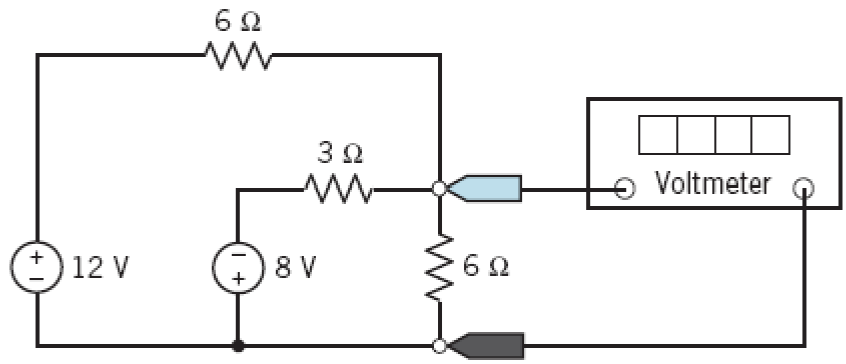
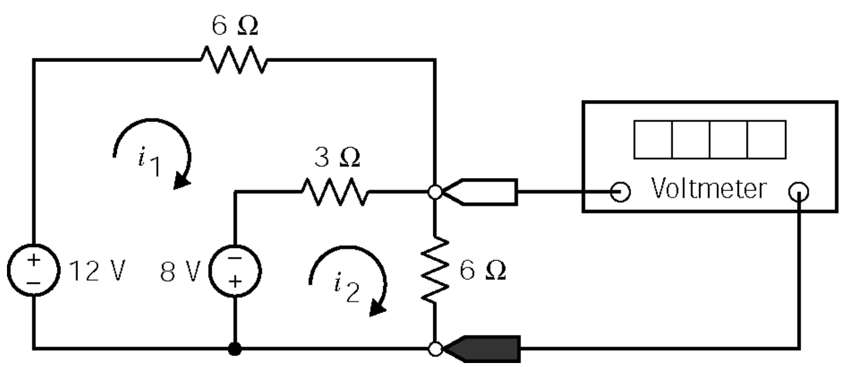


Figure E 4.5-1

Solution:



Mesh equations:

$$-12+6\,i_1+3\left(i_1-i_2\right)-8=0 \;\Rightarrow\; 9\,i_1-3\,i_2=20$$

$$8-3\left(i_1-i_2\right)+6\,i_2=0 \;\Rightarrow\; -3\,i_1+9\,i_2=-8$$

Solving these equations gives:

$$i_1=\frac{13}{6}\text{ A and } i_2=-\frac{1}{6}\text{ A}$$

The voltage measured by the meter is $6\,i_2 = -1$ V.

Exercise 4.6-1 Determine the value of the voltage measured by the voltmeter in Figure E 4.6-1.

Hint: Write and solve a single mesh equation to determine the current in the 3-Ω resistor.

Answer: –4 V

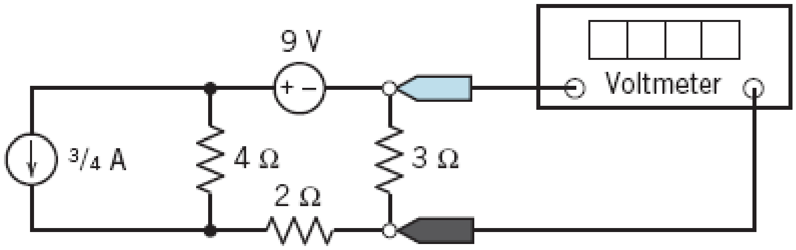
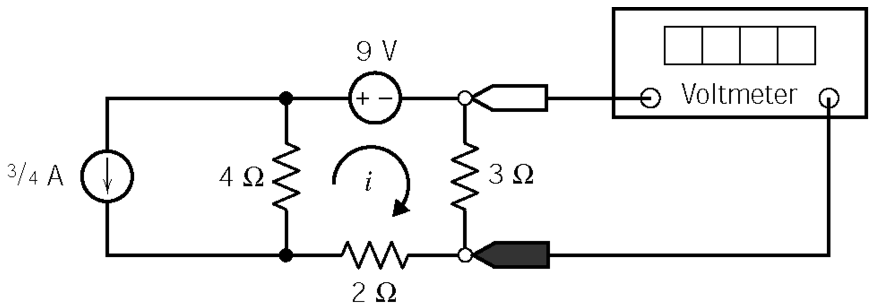


Figure E 4.6-1

Solution:



Mesh equation: $9+3\,i+2\,i+4\left(i+\frac{3}{4}\right)=0 \;\Rightarrow\; (3+2+4)\,i=-9-3 \;\Rightarrow\; i=\frac{-12}{9}$ A

The voltmeter measures $3\,i=-4$ V

Exercise 4.6-2 Determine the value of the current measured by the ammeter in Figure E 4.6-2.

Hint: Write and solve a single mesh equation.

Answer: –3.67 A

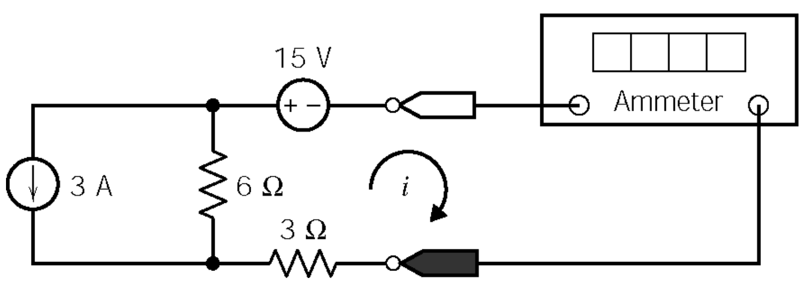


Figure E 4.6-2

Solution:

Mesh equation: $15+3\,i+6\,(i+3)=0 \;\Rightarrow\; (3+6)\,i=-15-6\,(3) \;\Rightarrow\; i=\frac{-33}{9}=-3\frac{2}{3}$ A

Section 4-2 Node Voltage Analysis of Circuits with Current Sources

P 4.2-1 The node voltages in the circuit of Figure P 4.2-1 are

$v_1 = -4$ V and $v_2 = 2$ V.

Determine *i*, the current of the current source.
Answer: $i = 1.5$ A

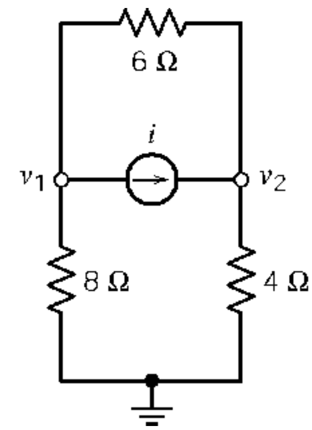


Figure P 4.2-1

Solution:

KCL at node 1: $$0 = \frac{v_1}{8} + \frac{v_1 - v_2}{6} + i = \frac{-4}{8} + \frac{-4-2}{6} + i = -1.5 + i \quad \Rightarrow \quad i = 1.5 \text{ A}$$

(checked using LNAP 8/13/02)

P 4.2-2 Determine the node voltages for the circuit of Figure P 4.2-2.

Answer: $v_1 = 2$ V, $v_2 = 30$ V, and $v_3 = 24$ V

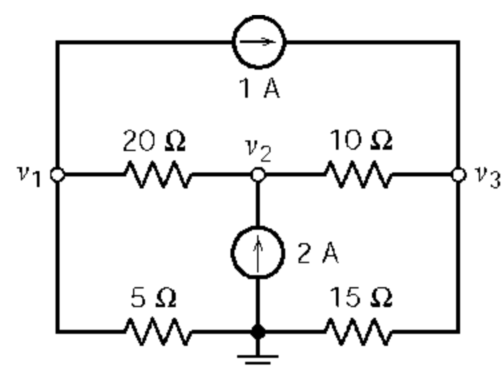


Figure P 4.2-2

Solution:

KCL at node 1: $$\frac{v_1 - v_2}{20} + \frac{v_1}{5} + 1 = 0 \quad \Rightarrow \quad 5v_1 - v_2 = -20$$

KCL at node 2: $$\frac{v_1 - v_2}{20} + 2 = \frac{v_2 - v_3}{10} \quad \Rightarrow \quad -v_1 + 3v_2 - 2v_3 = 40$$

KCL at node 3: $$\frac{v_2 - v_3}{10} + 1 = \frac{v_3}{15} \quad \Rightarrow \quad -3v_2 + 5v_3 = 30$$

Solving gives $v_1 = 2$ V, $v_2 = 30$ V and $v_3 = 24$ V.

(checked using LNAP 8/13/02)

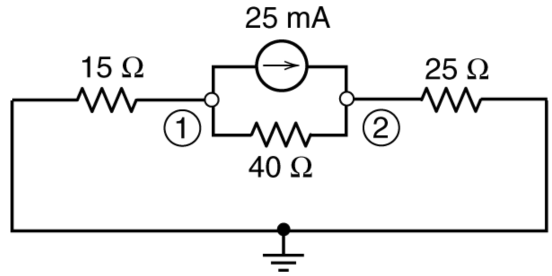
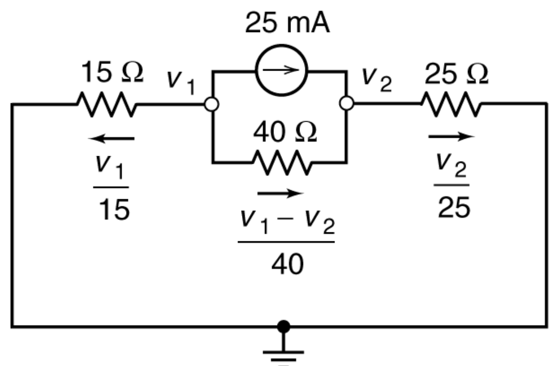


Figure P4.2-3

P4.2-3 The encircled numbers in the circuit shown Figure P4.2-3 are node numbers. Determine the values of the corresponding node voltages, v_1 and v_2.

Solution:
First, express the resistor currents in terms of the node voltages:



Apply KCL at node 1 to get $$\frac{v_1}{15}+0.025+\frac{v_1-v_2}{40}=0$$

Multiply both sides by 40 and simplify to get

$$\frac{11}{3}v_1-v_2=-1$$

Apply KCL at node 2 to get $$0.025+\frac{v_1-v_2}{40}=\frac{v_2}{25}$$

Multiply both sides by 40 and simplify to get

$$-v_1+\frac{13}{5}v_2=1$$

Solving, we get $v_1=-0.1875\text{ V}$ and $v_2=0.3125\text{ V}$

P 4.2-4 Consider the circuit shown in Figure P 4.2-4. Find values of the resistances R_1 and R_2 that cause the voltages v_1 and v_2 to be $v_1 = 1$ V and $v_2 = 2$ V.

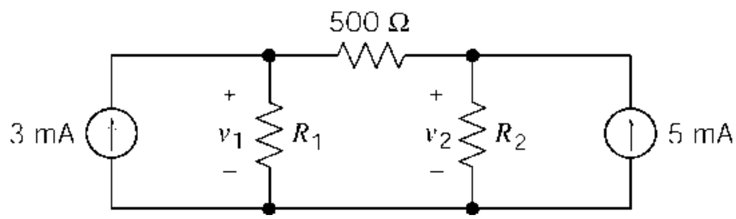


Figure P 4.2-4

Solution:

Write the node equations:

$$-.003 + \frac{v_1}{R_1} + \frac{v_1 - v_2}{500} = 0$$

$$-\frac{v_1 - v_2}{500} + \frac{v_2}{R_2} - .005 = 0$$

When $v_1 = 1$ V, $v_2 = 2$ V

$$-.003 + \frac{1}{R_1} + \frac{-1}{500} = 0 \quad \Rightarrow \quad R_1 = \frac{1}{.003 + \frac{1}{500}} = \underline{200\ \Omega}$$

$$-\frac{-1}{500} + \frac{2}{R_2} - .005 = 0 \quad \Rightarrow \quad R_2 = \frac{2}{.005 - \frac{1}{500}} = \underline{667\ \Omega}$$

(checked using LNAP 8/13/02)

P 4.2-5 Find the voltage v for the circuit shown in Figure P 4.2-5.
Answer: $v = 21.7$ mV

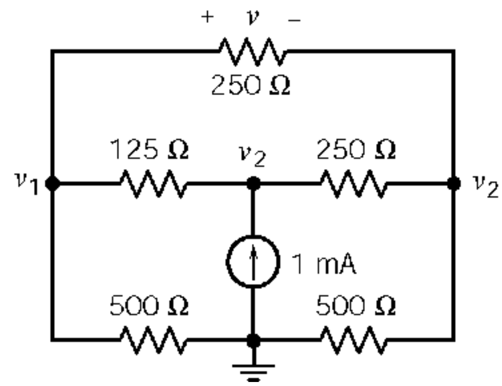


Figure P 4.2-5

Solution:

Write node equations:

$$\frac{v_1}{500} + \frac{v_1 - v_2}{125} + \frac{v_1 - v_3}{250} = 0$$

$$-\frac{v_1 - v_2}{125} - .001 + \frac{v_2 - v_3}{250} = 0$$

$$-\frac{v_2 - v_3}{250} - \frac{v_1 - v_3}{250} + \frac{v_3}{500} = 0$$

Solving gives: $v_1 = 0.261$ V, $v_2 = 0.337$ V, $v_3 = 0.239$ V

Finally: $v = v_1 - v_3 = \underline{0.022 \text{ V}}$

(checked using LNAP 8/13/02)

P 4.2-6 Simplify the circuit shown in Figure P 4.2-6 by replacing series and parallel resistors with equivalent resistors; then analyze the simplified circuit by writing and solving node equations.

(a) Determine the power supplied by each current source.

(b) Determine the power received by the 12-Ω resistor.

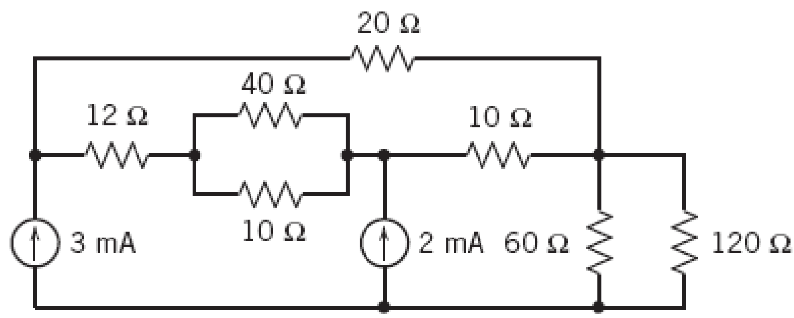
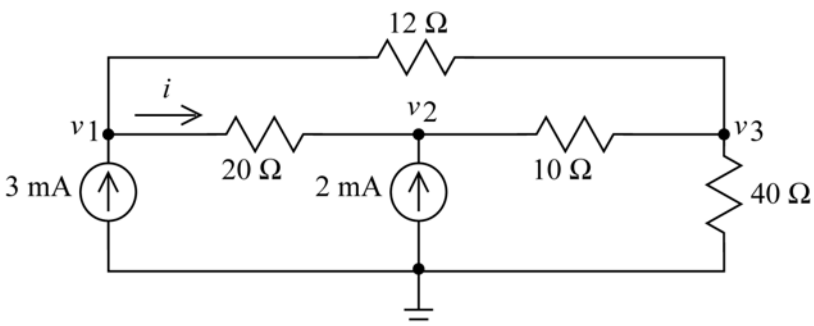


Figure P 4.2-6

Solution: Replacing series and parallel resistors with equivalent resistors we get



$$12\ \Omega + (40\ \Omega \,||\, 10\ \Omega) = 20\ \Omega$$

$$60\ \Omega \,||\, 120\ \Omega = 40\ \Omega$$

The node equations are

$$3\times10^{-3} = \frac{v_1 - v_2}{20} + \frac{v_1 - v_3}{20} \quad \Rightarrow \quad 0.06 = 2v_1 - (v_2 - v_3)$$

$$2\times10^{-3} + \frac{v_1 - v_2}{20} = \frac{v_2 - v_3}{10} \quad \Rightarrow \quad 0.04 = -v_1 + 3v_2 - 2v_3$$

$$\frac{v_2 - v_3}{10} + \frac{v_1 - v_3}{20} = \frac{v_3}{40} \quad \Rightarrow \quad 0 = -(2v_1 + 4v_2) + 7v_3$$

Solving, e.g. using MATLAB, gives

$$\begin{bmatrix} 2 & -1 & -1 \\ -1 & 3 & -2 \\ -2 & -4 & +7 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} .06 \\ .04 \\ 0 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0.244 \\ 0.228 \\ 0.200 \end{bmatrix}$$

(a) The power supplied by the 3 mA current source is $(3\times10^{-3})(0.244) = 0.732$ mW. The power supplied by the 2 mA source is $(2\times10^{-3})(0.228) = 0.456$ mW.

(b) The current in the 12 Ω resistor is equal to the current $i = \frac{v_1 - v_2}{20} = \frac{0.244 - 0.228}{20} = 0.8\text{ mA}$ so the power received by the 12 Ω resistor is $(0.8\times10^{-3})^2(12) = 7.68\times10^{-b} = 7.68\ \mu\text{W}$.

(checked: LNAP and MATLAB 5/31/04)

P 4.2-7 The node voltages in the circuit shown in Figure P 4.2-7 are $v_a = 7$ V and $v_b = 10$ V. Determine values of the current source current, i_s, and the resistance, R.

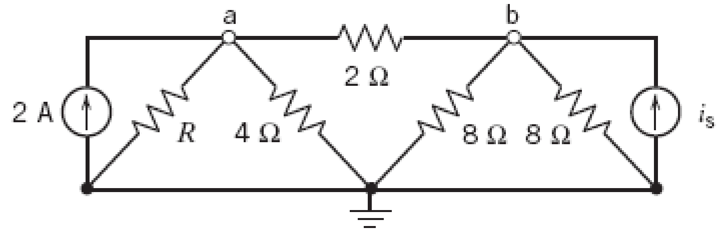


Figure P 4.2-7

Solution

Apply KCL at node a to get

$$2 = \frac{v_a}{R} + \frac{v_a}{4} + \frac{v_a - v_b}{2} = \frac{7}{R} + \frac{7}{4} + \frac{7-10}{2} = \frac{7}{R} + \frac{1}{4} \quad \Rightarrow \quad R = 4\ \Omega$$

Apply KCL at node b to get

$$i_s + \frac{v_a - v_b}{2} = \frac{v_b}{8} + \frac{v_b}{8} = i_s + \frac{7-10}{2} = \frac{10}{8} + \frac{10}{8} \quad \Rightarrow \quad i_s = 4\ \text{A}$$

(checked: LNAP 6/21/04)

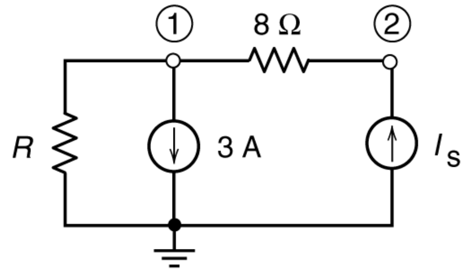


Figure P4.2-8

P4.2-8 The encircled numbers in the circuit shown Figure P4.2-8 are node numbers. The corresponding node voltages are v_1 and v_2.The node equation representing this circuit is

$$\begin{bmatrix} 0.225 & -0.125 \\ -0.125 & 0.125 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$$

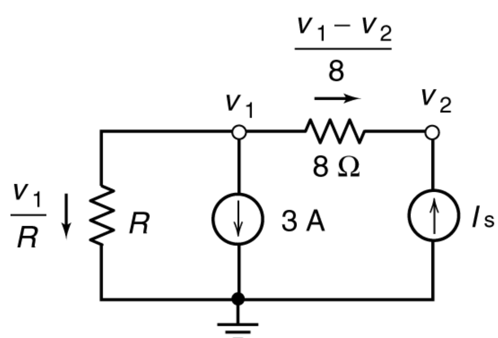
***a*)** Determine the values of R and I_s in Figure P4.2-8

***b*)** Determine the value of the power supplied by the 3 A current source.

Solution:

***a*)** The node equations representing the circuit are

$$\frac{v_1}{R}+\frac{v_1-v_2}{8}+3=0 \quad \text{and} \quad \frac{v_1-v_2}{8}+I_s=0$$



In matrix form, we have

$$\begin{bmatrix} 0.125+\frac{1}{R} & -0.125 \\ 0.125 & -0.125 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -3 \\ -I_s \end{bmatrix}$$

After multiplying the 2[nd] row by -1, we have

$$\begin{bmatrix} 0.125+\frac{1}{R} & -0.125 \\ -0.125 & 0.125 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -3 \\ I_s \end{bmatrix}$$

Comparing to the given node equation, we see that

$$\frac{1}{R}=0.1 \quad \Rightarrow \quad R=10\ \Omega \quad \text{and} \quad I_s=2\ \text{A}$$

***b*)** Solving the given node equations, we get

$$v_1=-10\ \text{V} \quad \text{and} \quad v_2=6\ \text{V}$$

The power supplied by the 3 A current source is given by

$$-v_1(3)=-(-10)(3)=30\ \text{W}$$

Section 4-3 Node Voltage Analysis of Circuits with Current and Voltage Sources

P 4.3-1 The voltmeter in Figure P 4.3-1 measures v_c, the node voltage at node c. Determine the value of v_c.

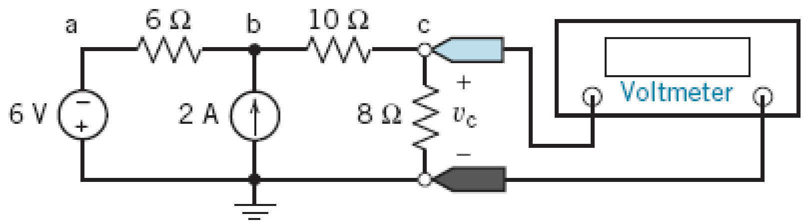
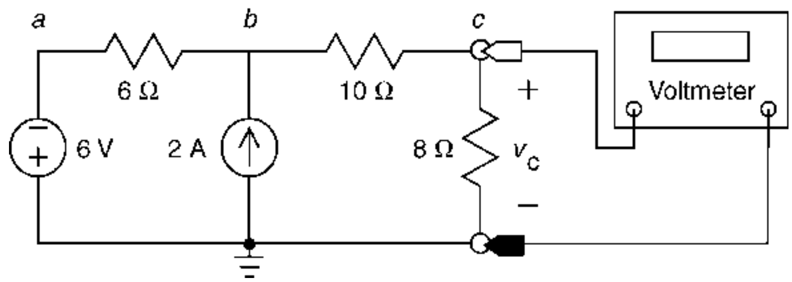


Figure P 4.3-1

Solution:



Express the voltage of the voltage source in terms of its node voltages:

$$0 - v_a = 6 \quad \Rightarrow \quad v_a = -6 \text{ V}$$

KCL at node b:

$$\frac{v_a - v_b}{6} + 2 = \frac{v_b - v_c}{10} \quad \Rightarrow \quad \frac{-6 - v_b}{6} + 2 = \frac{v_b - v_c}{10} \quad \Rightarrow \quad -1 - \frac{v_b}{6} + 2 = \frac{v_b - v_c}{10} \quad \Rightarrow \quad 30 = 8\,v_b - 3\,v_c$$

KCL at node c:

$$\frac{v_b - v_c}{10} = \frac{v_c}{8} \quad \Rightarrow \quad 4\,v_b - 4\,v_c = 5\,v_c \quad \Rightarrow \quad v_b = \frac{9}{4}\,v_c$$

Finally:

$$30 = 8\left(\frac{9}{4}\,v_c\right) - 3\,v_c \quad \Rightarrow \quad v_c = 2 \text{ V}$$

(checked using LNAP 8/13/02)

P 4.3-2 The voltages v_a, v_b, v_c, and v_d in Figure P 4.3-2 are the node voltages corresponding to nodes a, b, c, and d. The current i is the current in a short circuit connected between nodes b and c. Determine the values of v_a, v_b, v_c, and v_d and of i.

Answer: $v_a = -12$ V, $v_b = v_c = 4$ V, $v_d = -4$ V, $i = 2$ mA

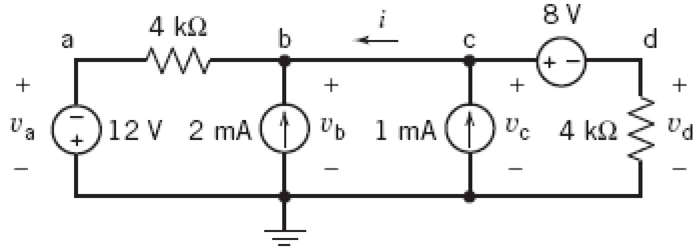
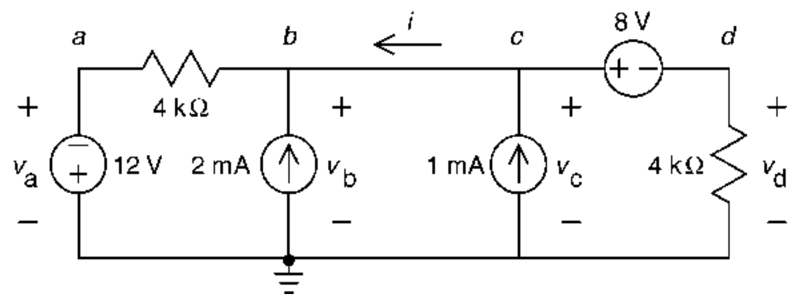


Figure P 4.3-2

Solution:



Express the branch voltage of each voltage source in terms of its node voltages to get:

$$v_a = -12 \text{ V}, \;\; v_b = v_c = v_d + 8$$

KCL at node b:

$$\frac{v_b - v_a}{4000} = 0.002 + i \;\; \Rightarrow \;\; \frac{v_b - (-12)}{4000} = 0.002 + i \;\; \Rightarrow \;\; v_b + 12 = 8 + 4000\, i$$

KCL at the supernode corresponding to the 8 V source:

$$0.001 = \frac{v_d}{4000} + i \;\; \Rightarrow \;\; 4 = v_d + 4000\, i$$

so $$v_b + 4 = 4 - v_d \;\; \Rightarrow \;\; (v_d + 8) + 4 = 4 - v_d \;\; \Rightarrow \;\; v_d = -4 \text{ V}$$

Consequently $v_b = v_c = v_d + 8 = 4$ V and $i = \dfrac{4 - v_d}{4000} = 2$ mA

(checked using LNAP 8/13/02)

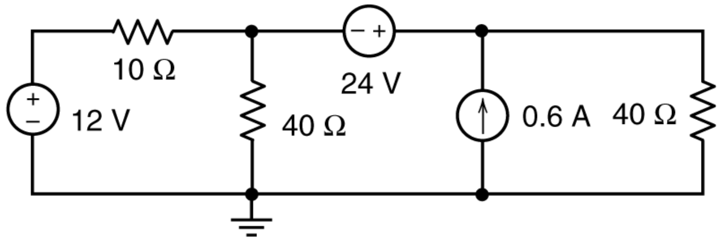
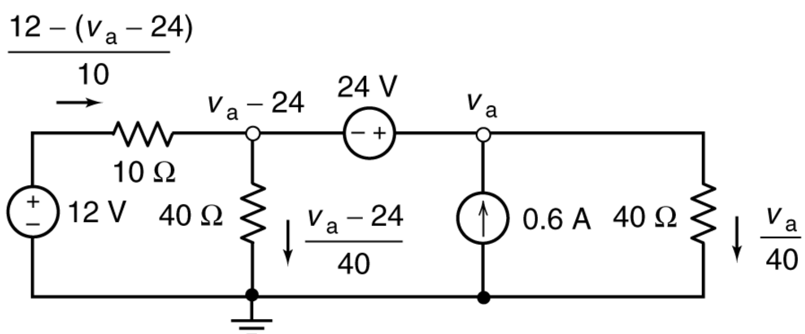


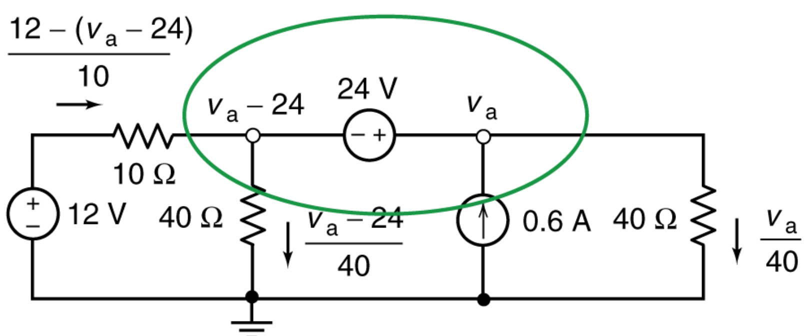
Figure P4.3-3

P4.3-3. Determine the values of the power supplied by each of the sources in the circuit shown in Figure P4.3-3.

Solution: First, label the node voltages. Next, express the resistor currents in terms of the node voltages.



Identify the supernode corresponding to the 24 V source



Apply KCL to the supernode to get

$$\frac{12-(v_a-24)}{10}+0.6=\frac{v_a-24}{40}+\frac{v_a}{40} \Rightarrow 196=6v_a \Rightarrow v_a=32\text{ V}$$

The 12 V source supplies $12\left(\frac{12-(v_a-24)}{10}\right)=12\left(\frac{12-(32-24)}{10}\right)=4.8\text{ W}$

The 24 V source supplies $24\left(-0.6+\frac{v_a}{40}\right)=24\left(-0.6+\frac{32}{40}\right)=4.8\text{ W}$

The current source supplies $0.6v_a=0.6(32)=19.2\text{ W}$

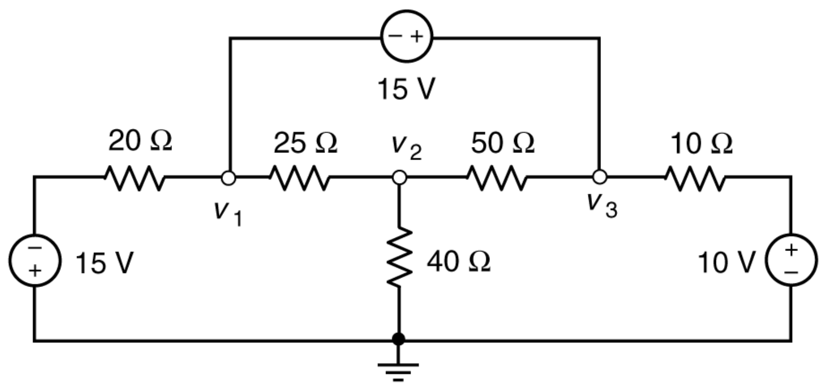
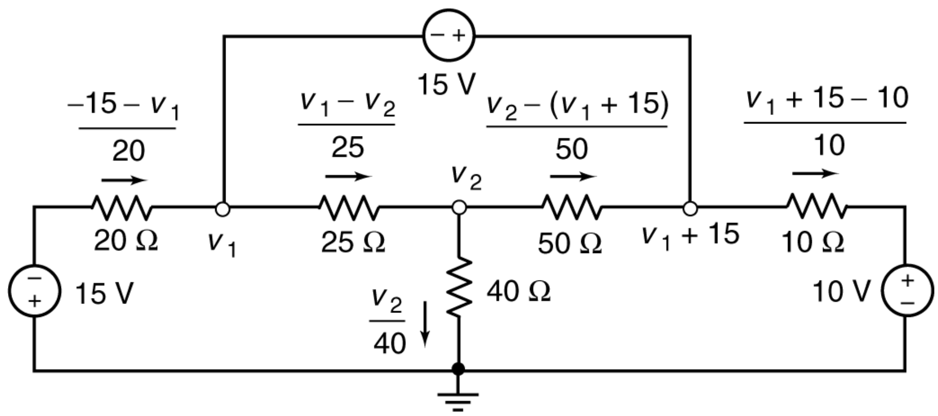


Figure P4.3-4

P4.3-4. Determine the values of the node voltages, v_1, v_2 and v_3 in the circuit shown in Figure P4.3-13.

Solution:

First, express the resistor currents in terms of the node voltages:



Apply KCL to the supernode to get

$$\frac{-15-v_1}{20}+\frac{v_2-(v_1+15)}{50}=\frac{v_1-v_2}{25}+\frac{v_1+5}{10} \quad \Rightarrow \quad 0.21v_1-0.06v_2=-1.55$$

Apply KCL at node 2 to get $\frac{v_1-v_2}{25}=\frac{v_2}{40}+\frac{v_2-(v_1+15)}{50} \quad \Rightarrow \quad -0.06v_1+0.085v_2=0.3$

In matrix form:

$$\begin{bmatrix} 0.21 & -0.06 \\ -0.06 & 0.085 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}=\begin{bmatrix} -1.55 \\ 0.3 \end{bmatrix}$$

Solving using MATLAB: $v_1=-7.9825$ V and $v_2=-2.1053$ V

P 4.3-5 The voltages v_a, v_b, and v_c in Figure P 4.3-5 are the node voltages corresponding to nodes a, b, and c. The values of these voltages are:

$$v_a = 12 \text{ V},\ v_b = 9.882 \text{ V}, \quad \text{and} \quad v_c = 5.294 \text{ V}$$

Determine the power supplied by the voltage source.

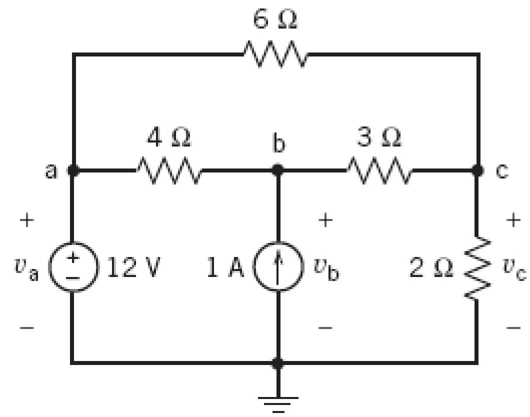
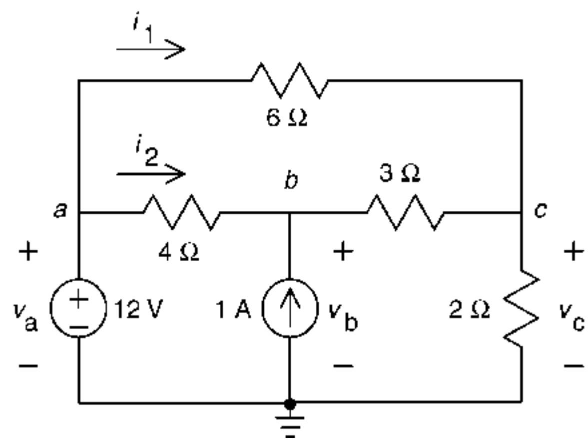


Figure P 4.3-5

Solution:



The power supplied by the voltage source is

$$v_a\left(i_1+i_2\right)=v_a\left(\frac{v_a-v_b}{4}+\frac{v_a-v_c}{6}\right)=12\left(\frac{12-9.882}{4}+\frac{12-5.294}{6}\right)$$

$$=12(0.5295+1.118)=12(1.648)=19.76 \text{ W}$$

(checked using LNAP 8/13/02)

P 4.3-6 The voltmeter in the circuit of Figure P 4.3-6 measures a node voltage. The value of that node voltage depends on the value of the resistance *R*.

(a) Determine the value of the resistance *R* that will cause the voltage measured by the voltmeter to be 4 V.

(b) Determine the voltage measured by the voltmeter when $R = 1.2\ \text{k}\Omega = 1200\ \Omega$.

Answer: (a) 6 kΩ (b) 2 V

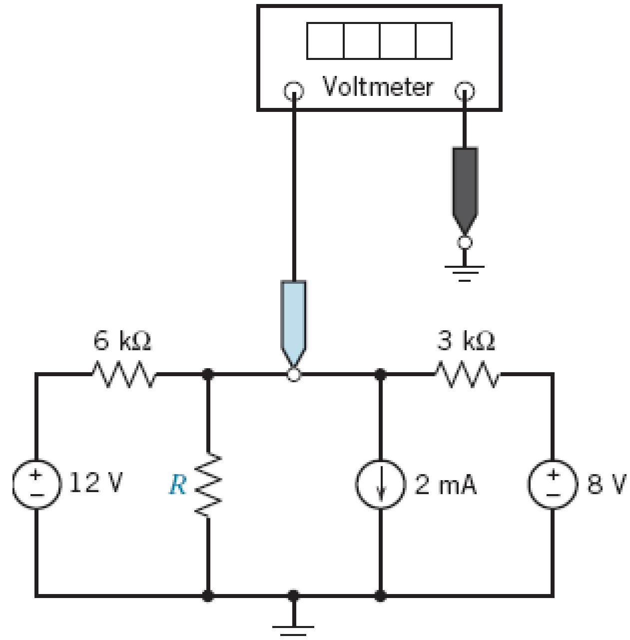
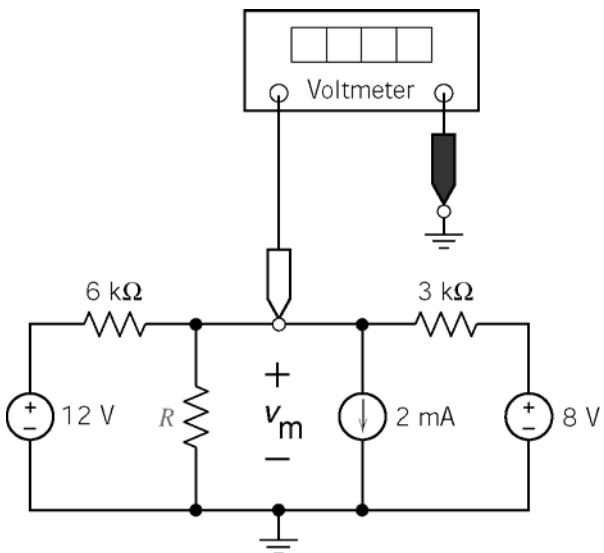


Figure P 4.3-6

Solution:
Label the voltage measured by the meter. Notice that this is a node voltage.



Write a node equation at the node at which the node voltage is measured.

$$-\left(\frac{12 - v_m}{6000}\right) + \frac{v_m}{R} + 0.002 + \frac{v_m - 8}{3000} = 0$$

That is

$$\left(3 + \frac{6000}{R}\right) v_m = 16 \;\Rightarrow\; R = \frac{6000}{\frac{16}{v_m} - 3}$$

(a) The voltage measured by the meter will be 4 volts when $R = 6\ \text{k}\Omega$.

(b) The voltage measured by the meter will be 2 volts when $R = 1.2\ \text{k}\Omega$.

P 4.3-7 Determine the values of the node voltages, v_1 and v_2, in Figure P 4.3-7. Determine the values of the currents i_a and i_b.

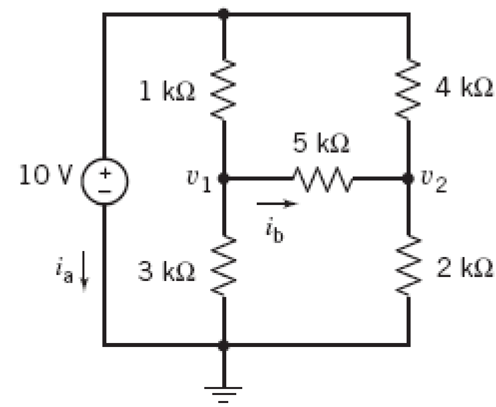
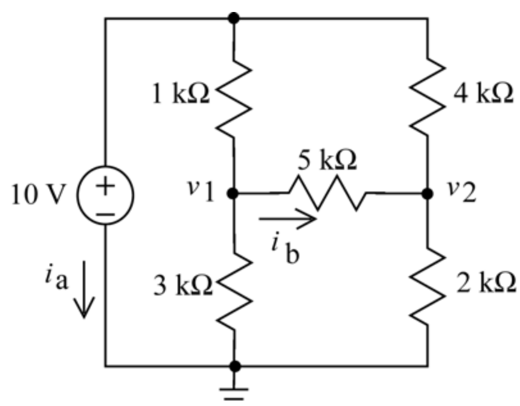


Figure P 4.3-7

Solution:



Apply KCL at nodes 1 and 2 to get

$$\frac{10-v_1}{1000} = \frac{v_1}{3000} + \frac{v_1 - v_2}{5000} \quad \Rightarrow \quad 23v_1 - 3v_2 = 150$$

$$\frac{10-v_2}{4000} + \frac{v_1 - v_3}{5000} = \frac{v_3}{2000} \quad \Rightarrow \quad -4v_1 + 19v_3 = 50$$

Solving, e.g. using MATLAB, gives

$$\begin{bmatrix} 23 & -3 \\ -4 & 19 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 150 \\ 50 \end{bmatrix} \quad \Rightarrow \quad v_1 = 7.06 \text{ V and } v_1 = 4.12 \text{ V}$$

Then $$i_b = \frac{v_1 - v_2}{5000} = \frac{7.06 - 4.12}{5000} = 0.588 \text{ mA}$$

Apply KCL at the top node to get

$$i_a = \frac{v_1 - 10}{1000} + \frac{v_2 - 10}{4000} = \frac{7.06-10}{1000} + \frac{4.12-10}{4000} = -4.41 \text{ mA}$$

(checked: LNAP 5/31/04)

P 4.3-8 The circuit shown in Figure P 4.3-8 has two inputs, v_1 and v_2, and one output, v_o. The output is related to the input by the equation

$$v_o = av_1 + bv_2$$

where a and b are constants that depend on R_1, R_2 and R_3.

(a) Determine the values of the coefficients a and b when $R_1 = 10\ \Omega$, $R_2 = 40\ \Omega$ and $R_3 = 8\ \Omega$.
(b) Determine the values of the coefficients a and b when $R_1 = R_2$ and $R_3 = R_1 \parallel R_2$.

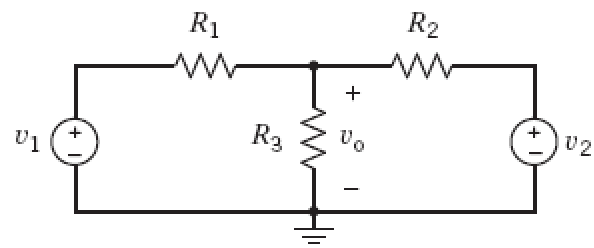


Figure P 4.3-8

Solution:

$$\frac{v_o}{R_3} + \frac{v_o - v_1}{R_1} + \frac{v_o - v_2}{R_2} = 0 \quad \Rightarrow \quad v_o = \frac{v_1}{1 + \frac{R_1}{R_2} + \frac{R_1}{R_3}} + \frac{v_2}{1 + \frac{R_2}{R_1} + \frac{R_2}{R_3}}$$

(a) When $R_1 = 10\ \Omega$, $R_2 = 40\ \Omega$ and $R_3 = 8\ \Omega$

$$v_o = \frac{v_1}{1 + \frac{1}{4} + \frac{5}{4}} + \frac{v_2}{1 + 4 + 5} = 0.4v_1 + 0.1v_2$$

So $a = 0.4$ and $b = 0.1$.

(b) When $R_1 = R_2$ and $R_3 = R_1 \parallel R_2 = R_1 / 2$

$$v_o = \frac{v_1}{1+1+2} + \frac{v_2}{1+1+2} = 0.25v_1 + 0.25v_2$$

So $a = 0.25$ and $b = 0.25$.

(checked: LNAP 5/31/04)

P 4.3-9 Determine the values of the node voltages of the circuit shown in Figure P 4.3-9.

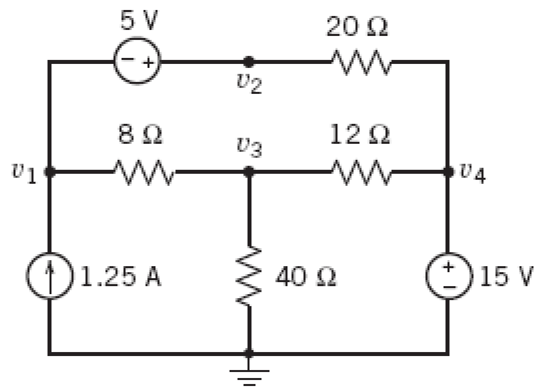


Figure P 4.3-9

Solution:
Express the voltage source voltages as functions of the node voltages to get

$$v_2 - v_1 = 5 \text{ and } v_4 = 15$$

Apply KCL to the supernode corresponding to the 5 V source to get

$$1.25 = \frac{v_1 - v_3}{8} + \frac{v_2 - 15}{20} = 0 \quad \Rightarrow \quad 80 = 5v_1 + 2v_2 - 5v_3$$

Apply KCL at node 3 to get

$$\frac{v_1 - v_3}{8} = \frac{v_3}{40} + \frac{v_3 - 15}{12} \quad \Rightarrow \quad -15v_1 + 28v_3 = 150$$

Solving, e.g. using MATLAB, gives

$$\begin{bmatrix} -1 & 1 & 0 \\ 5 & 2 & -5 \\ -15 & 0 & 28 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 80 \\ 150 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 22.4 \\ 27.4 \\ 17.4 \end{bmatrix}$$

So the node voltages are:

$$v_1 = 22.4 \text{ V}, v_2 = 27.4 \text{ V}, v_3 = 17.4 \text{ V, and } v_4 = 15$$

(checked: LNAP 6/9/04)

P 4.3-10 Figure P 4.3-10 shows a measurement made in the laboratory. Your lab partner forgot to record the values of R_1, R_2, and R_3. He thinks that the two resistors were 10-kΩ resistors and the other was a 5-kΩ resistor. Is this possible? Which resistor is the 5-kΩ resistor?

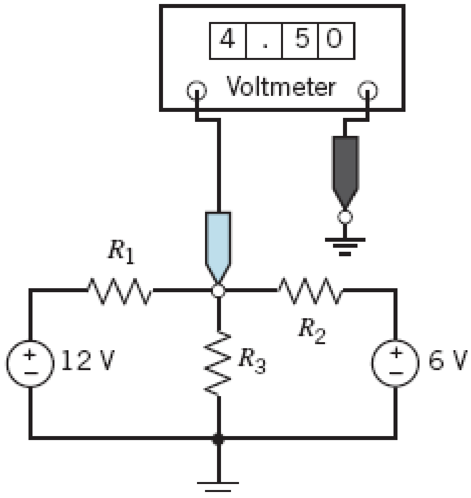


Figure P 4.3-10

Solution:
Write a node equation to get

$$-\left(\frac{12-4.5}{R_1}\right)+\frac{4.5}{R_3}+\frac{4.5-6}{R_2}=0 \quad \Rightarrow \quad -\frac{7.5}{R_1}+\frac{4.5}{R_3}-\frac{1.5}{R_2}=0$$

Notice that $\frac{7.5}{R_1}$ is either 0.75 mA or 1.5 mA depending on whether R_1 is 10 kΩ or 5 kΩ. Similarly, $\frac{4.5}{R_3}$ is either 0.45 mA or 0.9 mA and $\frac{1.5}{R_2}$ is either 0.15 mA or 0.3 mA. Suppose R_1 and R_2 are 10 kΩ resistors and R_3 is a 5 kΩ resistor. Then

$$-\frac{7.5}{R_1}+\frac{4.5}{R_3}-\frac{1.5}{R_2}=-0.75+0.9-0.15=0$$

It is possible that two of the resistors are 10 kΩ and the third is 5 kΩ. R_3 is the 5 kΩ resistor.

(checked: LNAP 6/9/04)

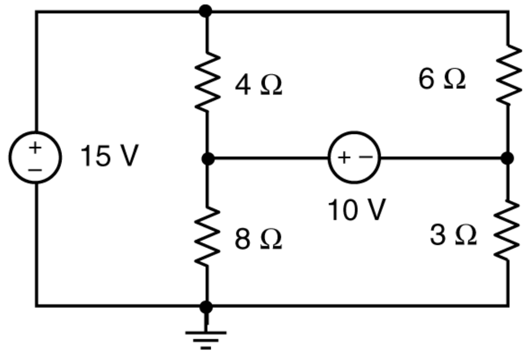
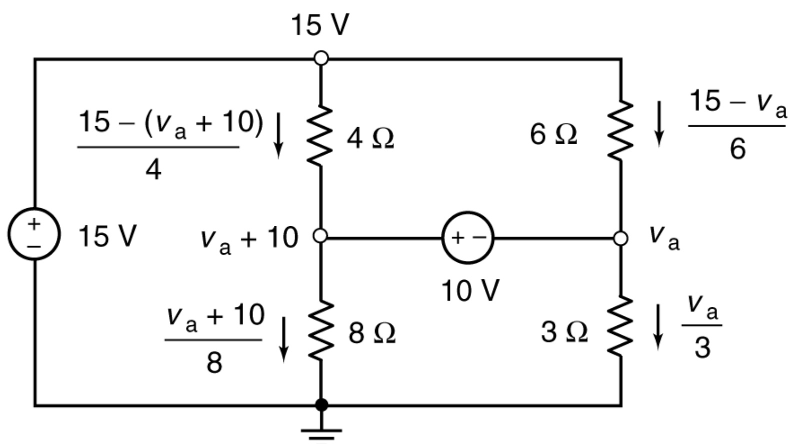


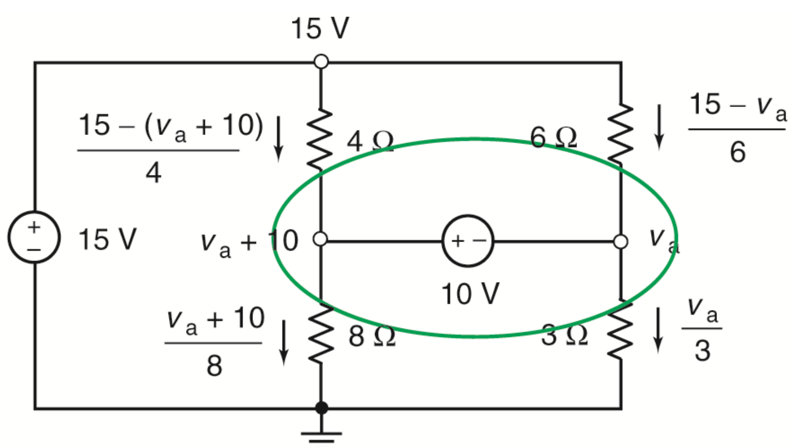
Figure P4.3-11

P4.3-11. Determine the values of the power supplied by each of the sources in the circuit shown in Figure P4.3-11.

Solution: First, label the node voltages. Next, express the resistor currents in terms of the node voltages.



Identify the supernode corresponding to the 10 V source



Apply KCL to the supernode to get

$$\frac{15-\left(v_a+10\right)}{4}+\frac{15-v_a}{6}=\frac{v_a+10}{8}+\frac{v_a}{3} \quad\Rightarrow\quad 60=21v_a \quad\Rightarrow\quad v_a=2.857\text{ V}$$

The 15 V source supplies

$$15\left(\frac{15-(v_a+10)}{4}+\frac{15-v_a}{6}\right)=15\left(\frac{15-12.857}{4}+\frac{15-2.857}{6}\right)=15(2.56)=38.4 \text{ W}$$

The 10 V source supplies $10\left(\frac{15-v_a}{6}+\frac{v_a}{3}\right)=10\left(\frac{15-2.857}{6}+\frac{2.857}{3}\right)=10(1.071)=10.71 \text{ W}$

P 4.3-12 Determine the values of the node voltages of the circuit shown in Figure P 4.3-12.

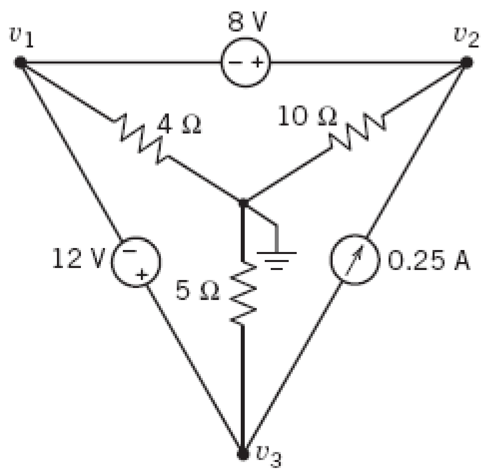


Figure P 4.3-12

Solution:

Express the voltage source voltages in terms of the node voltages:

$$v_2-v_1=8 \text{ and } v_3-v_1=12$$

Apply KVL to the supernode to get

$$\frac{v_2}{10}+\frac{v_1}{4}+\frac{v_3}{5}=0 \quad\Rightarrow\quad 2v_2+5v_1+4v_3=0$$

so $$2(8+v_1)+5v_1+4(12+v_1)=0 \quad\Rightarrow\quad v_1=-\frac{64}{11} \text{ V}$$

The node voltages are

$$v_1=-5.818 \text{ V}$$
$$v_2=2.182 \text{ V}$$
$$v_3=6.182 \text{ V}$$

(checked: LNAP 6/21/04)

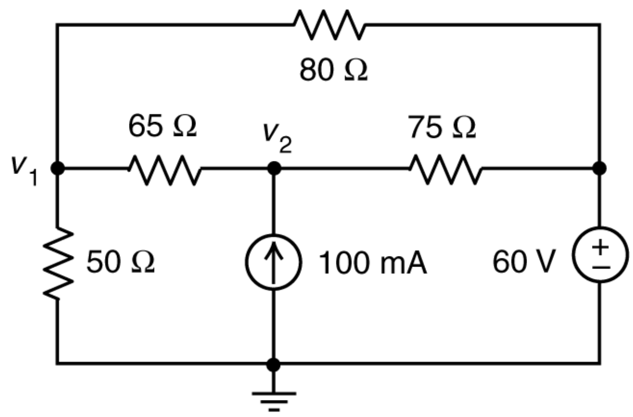


Figure P4.3-13

P4.3-13. Determine the values node voltages, v_1 and v_2, in the circuit shown in Figure P4.3-13.

Solution: Apply KCL at node 1 to get

$$\frac{v_1}{50}+\frac{v_1-v_2}{65}+\frac{v_1-60}{80}=0 \quad\Rightarrow\quad \left(\frac{1}{50}+\frac{1}{65}+\frac{1}{80}\right)v_1-\left(\frac{1}{65}\right)v_2=\frac{60}{80}$$

Apply KCL at node 2 to get

$$0.1=\frac{v_2-v_1}{65}+\frac{v_2-60}{75}= \quad\Rightarrow\quad -\left(\frac{1}{65}\right)v_1+\left(\frac{1}{65}+\frac{1}{75}\right)v_2=0.1$$

In matrix form

$$\begin{bmatrix}\frac{1}{50}+\frac{1}{65}+\frac{1}{80} & -\frac{1}{65}\\ -\frac{1}{65} & \frac{1}{65}+\frac{1}{75}\end{bmatrix}\begin{bmatrix}v_1\\ v_2\end{bmatrix}=\begin{bmatrix}\frac{60}{80}\\ 0.1\end{bmatrix}$$

Solving, we get v_1 = 13.2356 V and v_2 = 22.3456 V

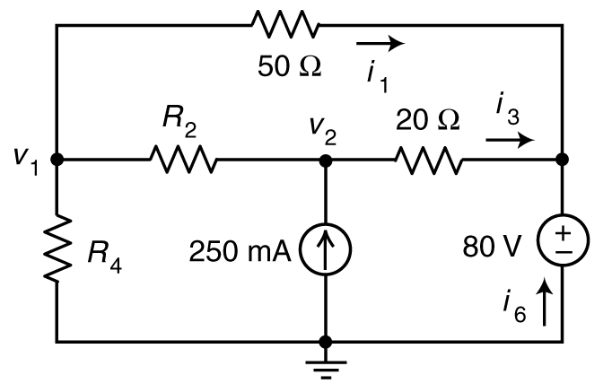


Figure P4.3-14

P4.3-14. The voltage source in the circuit shown in Figure P4.3-14 supplies 83.802 W. The current source supplies 17.572 W. Determine the values of the node voltages v_1 and v_2.

Solution: From the power supplied by the current source we calculate

$$17.572 = v_2\left(0.25\right) \quad \Rightarrow \quad v_2 = \frac{17.572}{0.25} = 70.288 \text{ V}$$

Using Ohm's law
$$i_3 = \frac{70.288 - 80}{20} = -0.4856 \text{ A}$$

From the power supplied by the voltage source we calculate

$$83.802 = 80\, i_6 \quad \Rightarrow \quad i_6 = \frac{83.802}{80} = 1.0475 \text{ V}$$

$$i_1 = -\left(i_3 + i_6\right) = -\left(-0.4856 + 1.0475\right) = -0.5619 \text{ A}$$

Using Ohm's law
$$-0.5619 = \frac{v_1 - 80}{50} \quad \Rightarrow \quad v_1 = 80 + 50\left(-0.5619\right) = 51.905 \text{ V}$$

In summary
$v_1 = 51.905$ V and $v_2 = 70.288$ V

Section 4-4 Node Voltage Analysis with Dependent Sources

P 4.4-1 The voltages v_a, v_b, and v_c in Figure P 4.4-1 are the node voltages corresponding to nodes a, b, and c. The values of these voltages are:

$$v_a = 8.667 \text{ V},\ v_b = 2 \text{ V}, \quad \text{and} \quad v_c = 10 \text{ V}$$

Determine the value of A, the gain of the dependent source.

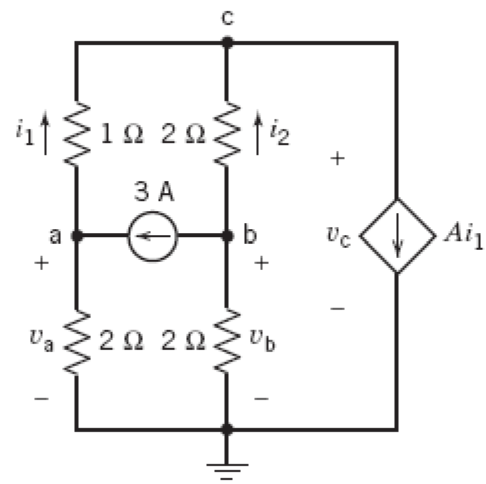
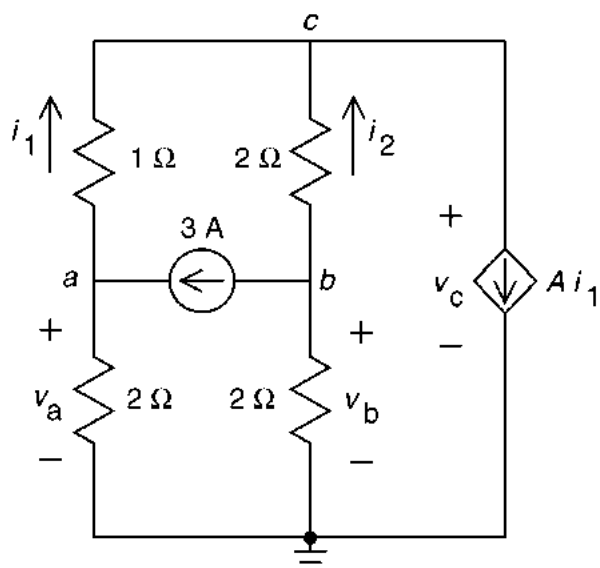


Figure P 4.4-1

Solution:



Express the resistor currents in terms of the node voltages:

$$i_1 = \frac{v_a - v_c}{1} = 8.667 - 10 = -1.333 \text{ A} \quad \text{and}$$

$$i_2 = \frac{v_b - v_c}{2} = \frac{2-10}{2} = -4 \text{ A}$$

Apply KCL at node c:

$$i_1 + i_2 = A\, i_1 \quad \Rightarrow \quad -1.333 + (-4) = A\,(-1.333)$$

$$\Rightarrow \quad A = \frac{-5.333}{-1.333} = 4$$

(checked using LNAP 8/13/02)

P 4.4-2 Find i_b for the circuit shown in Figure P 4.4-2.

Answer: $i_b = -12$ mA

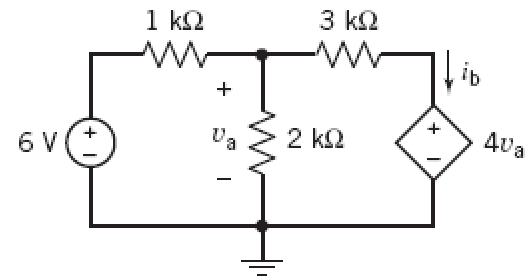
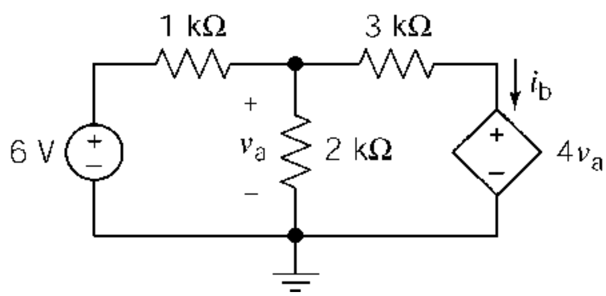


Figure P 4.4-2

Solution:



Write and solve a node equation:

$$\frac{v_a - 6}{1000} + \frac{v_a}{2000} + \frac{v_a - 4v_a}{3000} = 0 \quad \Rightarrow \quad v_a = 12 \text{ V}$$

$$i_b = \frac{v_a - 4v_a}{3000} = \underline{-12 \text{ mA}}$$

(checked using LNAP 8/13/02)

P 4.4-3 Determine the node voltage v_b for the circuit of Figure P 4.4-3.
Answer: $v_b = 1.5$ V

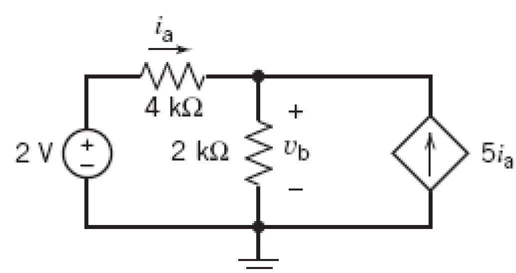
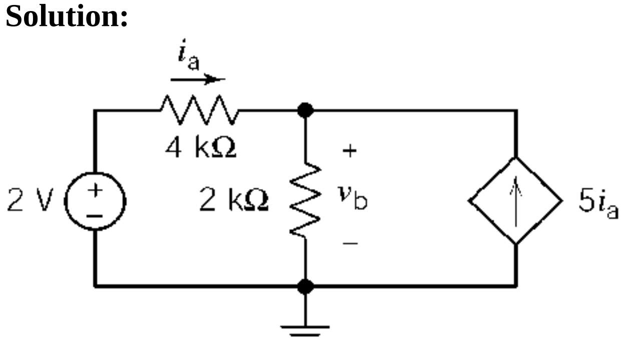


Figure P 4.4-3

Solution:



First express the controlling current in terms of the node voltages:

$$i_a = \frac{2 - v_b}{4000}$$

Write and solve a node equation:

$$-\frac{2 - v_b}{4000} + \frac{v_b}{2000} - 5\left(\frac{2 - v_b}{4000}\right) = 0 \quad \Rightarrow \quad \underline{v_b = 1.5 \text{ V}}$$

(checked using LNAP 8/14/02)

P 4.4-4 The circled numbers in Figure P 4.4-4 are node numbers. The node voltages of this circuit are v_1 = 10 V, v_2 = 14 V, and v_3 = 12 V.

(a) Determine the value of the current i_b.

(b) Determine the value of r, the gain of the CCVS.

Answers: (a) –2 A (b) 4 V/A

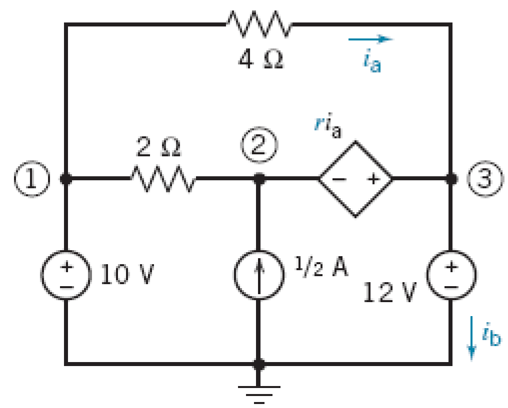
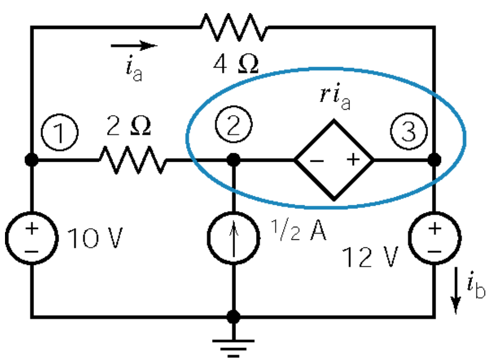


Figure P 4.4-4

Solution:



Apply KCL to the supernode of the CCVS to get

$$\frac{12-10}{4}+\frac{14-10}{2}-\frac{1}{2}+i_b=0 \quad \Rightarrow \quad i_b=-2 \text{ A}$$

Next

$$\left.\begin{array}{l} i_a=\dfrac{10-12}{4}=-\dfrac{1}{2} \\ r\,i_a=12-14 \end{array}\right\} \quad \Rightarrow \quad r=\frac{-2}{-\frac{1}{2}}=4\ \frac{\text{V}}{\text{A}}$$

(checked using LNAP 8/14/02)

P 4.4-5 Determine the value of the current i_x in the circuit of Figure P 4.4-5.

Answer: i_x = 2.4 A

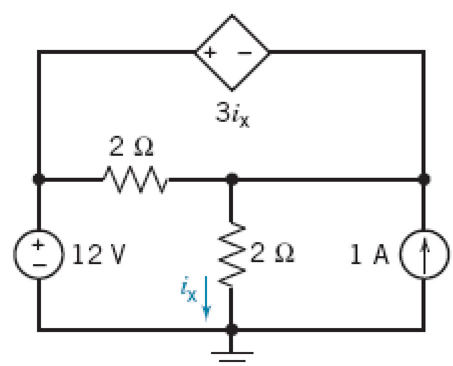
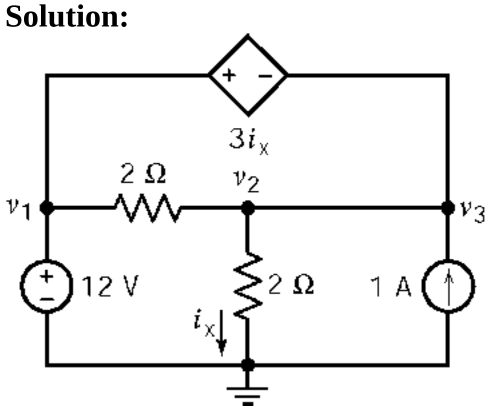


Figure P 4.4-5

Solution:



First, express the controlling current of the CCVS in terms of the node voltages: $i_x=\dfrac{v_2}{2}$

Next, express the controlled voltage in terms of the node voltages:

$$12-v_2=3i_x=3\frac{v_2}{2} \quad \Rightarrow v_2=\frac{24}{5} \text{ V}$$

so i_x = 12/5 A = 2.4 A.

(checked using ELab 9/5/02)

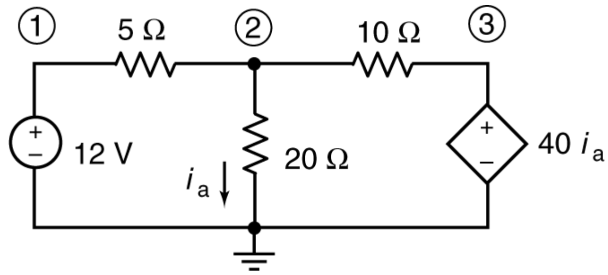
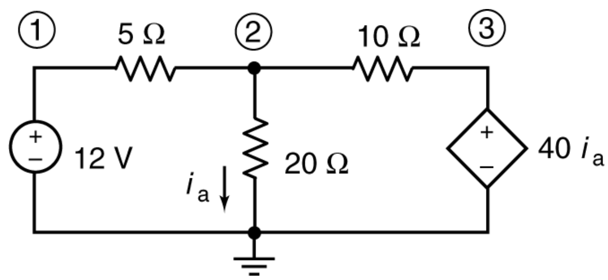


Figure P4.4-6

P4.4-6 The encircled numbers in the circuit shown Figure P4.4-6 are node numbers. Determine the value of the power supplied by the CCVS .

P4.4-6

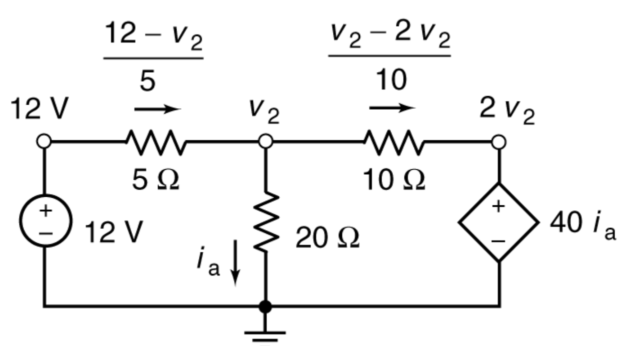


First, express the contolling current of the CCVS in terms of the node voltages:

$$i_a = \frac{v_2}{20}$$

Notice that $\quad v_1 = 12 \text{ V} \quad \text{and} \quad v_3 = 40\, i_a = 40\left(\frac{v_2}{20}\right) = 2\, v_2$

Next, express the resistor currents in terms of the node voltages:



Apply KCL at node 2 to get

$$\frac{12 - v_2}{5} + \frac{v_2}{20} + \frac{v_2 - 2\,v_2}{10} = 0 \quad \Rightarrow \quad v_2 = 16 \text{ V}$$

Then $\quad i_a = \frac{v_2}{20} = \frac{16}{20} = 0.8 \text{ A} \quad \text{and} \quad v_3 = 40\, i_a = 40(0.8) = 32 \text{ V}$

The CCVS supplies $\quad v_3\left(\frac{v_3 - v_2}{10}\right) = 32\left(\frac{32 - 16}{10}\right) = 51.2 \text{ W}$

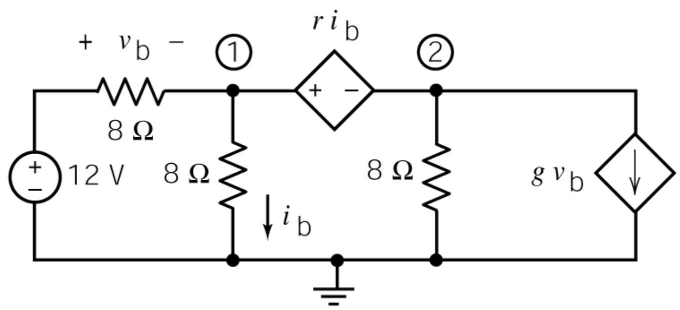


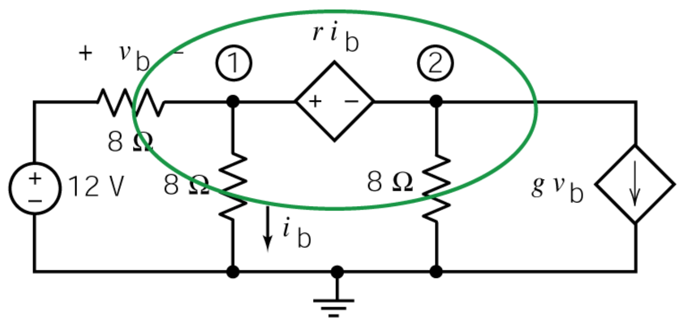
Figure P4.4-7

P4.4-7 The encircled numbers in the circuit shown Figure 4.4-27 are node numbers. The corresponding node voltages are:

$$v_1 = 9.74 \text{ V} \quad \text{and} \quad v_2 = 6.09 \text{ V}$$

Determine the values of the gains of the dependent sources, *r* and *g*.

Solution:



Using Ohm's law, $i_b = \frac{v_1}{8} = \frac{9.74}{8} = 1.2175 \text{ A}$. Using KVL, the voltage across the CCVS is

$$r\,i_b = v_1 - v_2 = 9.74 - 6.09 = 3.65 \text{ V}$$

Then
$$r = \frac{r\,i_b}{i_b} = \frac{3.65}{1.2175} = 2.9979 \text{ V/A}$$

Using KVL, $v_b = 12 - v_1 = 12 - 9.74 = 2.26 \text{ V}$. Apply KCL to the supernode corresponding to the CCVS to get

$$\frac{12 - v_1}{8} = \frac{v_1}{8} + \frac{v_2}{8} + g\,v_b \quad \Rightarrow \quad \frac{12 - 9.74}{8} = \frac{9.74}{8} + \frac{6.09}{8} + g\,v_b \quad \Rightarrow \quad g\,v_b = -1.6963 \text{ A}$$

Then
$$g = \frac{g\,v_b}{v_b} = \frac{-1.6963}{2.26} = -0.7506 \text{ A/V}$$

P 4.4-8 Determine the value of the power supplied by the dependent source in Figure P 4.4-8.

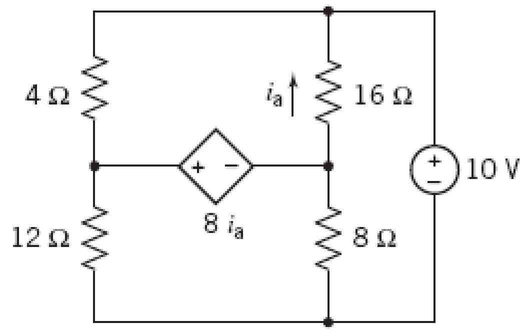
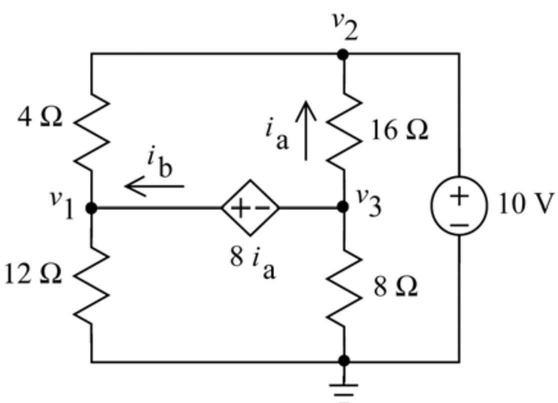


Figure P 4.4-8

Solution:

Label the node voltages.



First, v_2 = 10 V due to the independent voltage source. Next, express the controlling current of the dependent source in terms of the node voltages:

$$i_a = \frac{v_3 - v_2}{16} = \frac{v_3 - 10}{16}$$

Now the controlled voltage of the dependent source can be expressed as

$$v_1 - v_3 = 8\ i_a = 8\left(\frac{v_3 - 10}{16}\right) \quad \Rightarrow \quad v_1 = \frac{3}{2} v_3 - 5$$

Apply KCL to the supernode corresponding to the dependent source to get

$$\frac{v_1 - v_2}{4} + \frac{v_1}{12} + \frac{v_3 - v_2}{16} + \frac{v_3}{8} = 0$$

Multiplying by 48 and using v_2 = 10 V gives

$$16v_1 + 9v_3 = 150$$

Substituting the earlier expression for v_1

$$16\left(\frac{3}{2}v_3 - 5\right) + 9v_3 = 150 \quad \Rightarrow \quad v_3 = 6.970 \text{ V}$$

Then v_1 = 5.455 V and i_a = -0.1894 A. Applying KCL at node 2 gives

$$\frac{v_1}{12} = i_b + \frac{10 - v_1}{4} \quad \Rightarrow \quad 12\ i_b = -3 + 4\ v_1 = -30 + 4(5.455)$$

So $$i_b = -0.6817 \text{ A.}$$

Finally, the power supplied by the dependent source is

$$p = \left(8\ i_a\right) i_b = 8\left(-0.1894\right)\left(-0.6817\right) = 1.033 \text{ W}$$

(checked: LNAP 5/24/04)

P 4.4-9 The node voltages in the circuit shown in Figure P 4.4-9 are

$$v_1 = 4 \text{ V}, v_2 = 0 \text{ V}, \quad \text{and} \quad v_3 = -6 \text{ V}$$

Determine the values of the resistance, R, and of the gain, b, of the CCCS.

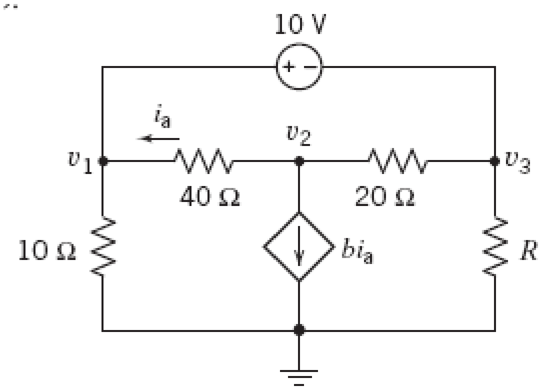
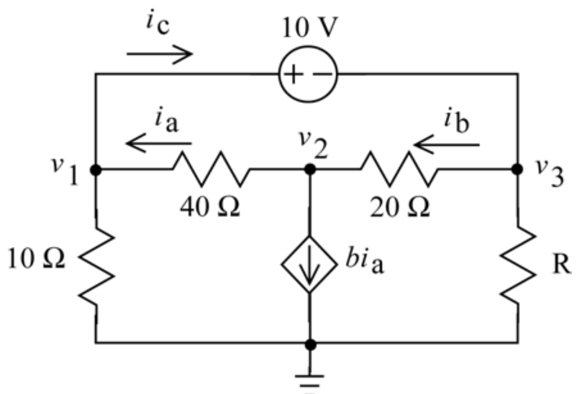


Figure P 4.4-9

Solution:



Apply KCL at node 2:

$$i_a + bi_a = i_b = \frac{v_3 - v_2}{20} = \frac{-6-(0)}{20} = -0.3 \text{ A}$$

but

$$i_a = \frac{v_2 - v_1}{40} = \frac{0-4}{40} = -0.1$$

so

$$(1+b)(-0.1) = (-0.3) \quad \Rightarrow \quad b=2 \ \frac{\text{A}}{\text{A}}$$

Next apply KCL to the supernode corresponding to the voltage source.

$$\frac{v_1}{10} + 2\, i_a + \frac{v_3}{R} = 0 \quad \Rightarrow \quad \frac{4}{10} + 2(-0.1) + \frac{-6}{R} = 0 \quad \Rightarrow \quad R = \frac{6}{.2} = 30 \ \Omega$$

(checked: LNAP 6/9/04)

P 4.4-10 The value of the node voltage at node *b* in the circuit shown in Figure P 4.4-10 is v_b = 18 V.

(a) Determine the value of *A*, the gain of the dependent source.

(b) Determine the power supplied by the dependent source.

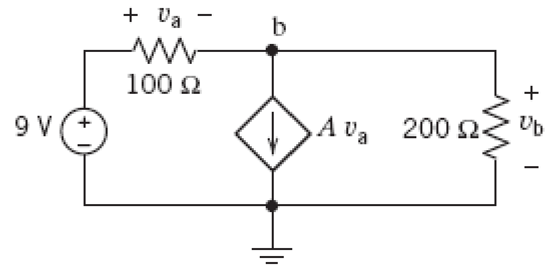


Figure P 4.4-10

Solution:

(a) Express the controlling voltage of the dependent source in terms of the node voltages:

$$v_a = 9 - v_b$$

Apply KCL at node b to get

$$\frac{9-v_b}{100} = A\left(9-v_b\right)+\frac{v_b}{200} \quad \Rightarrow \quad A=\frac{18-3v_b}{200\left(9-v_b\right)}=0.02$$

(b) The power supplied by the dependent source is

$$-\left(Av_a\right)v_b = -\left(0.02\left(9-18\right)\right)\left(18\right)=3.24 \text{ W}$$

(checked: LNAP 6/06/04)

P 4.4-11 Determine the power supplied by the dependent source in the circuit shown in Figure P 4.4-11.

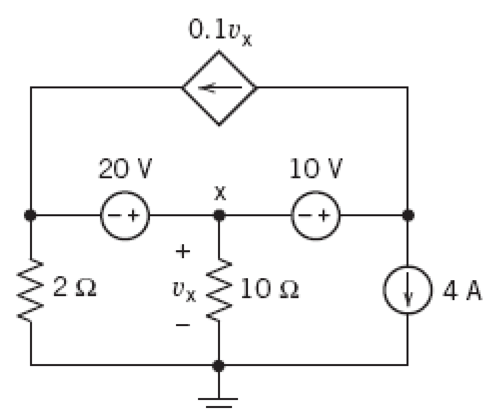


Figure P 4.4-11

Solution:

This circuit contains two ungrounded voltage sources, both incident to node x. In such a circuit it is necessary to merge the supernodes corresponding to the two ungrounded voltage sources into a single supernode. That single supernode separates the two voltage sources and their nodes from the rest of the circuit. It consists of the two resistors and the current source. Apply KCL to this supernode to get

$$\frac{v_x-20}{2}+\frac{v_x}{10}+4=0 \quad \Rightarrow \quad v_x = 10 \text{ V}.$$

The power supplied by the dependent source is

$$\left(0.1\,v_x\right)\left(-30\right) = -30 \text{ W}.$$

(checked: LNAP 6/6/04)

P 4.4-12 Determine values of the node voltages, v_1, v_2, v_3, v_4, and v_5, in the circuit shown in Figure P 4.4-12.

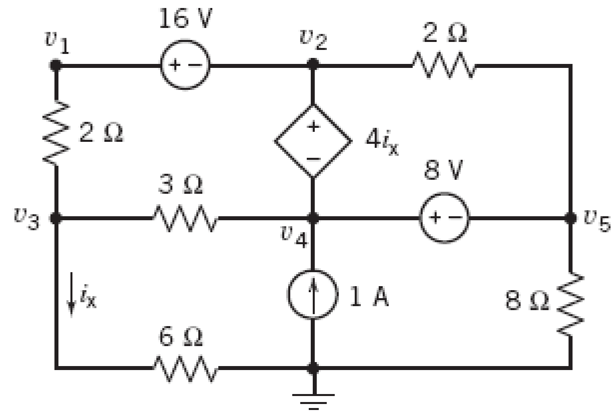
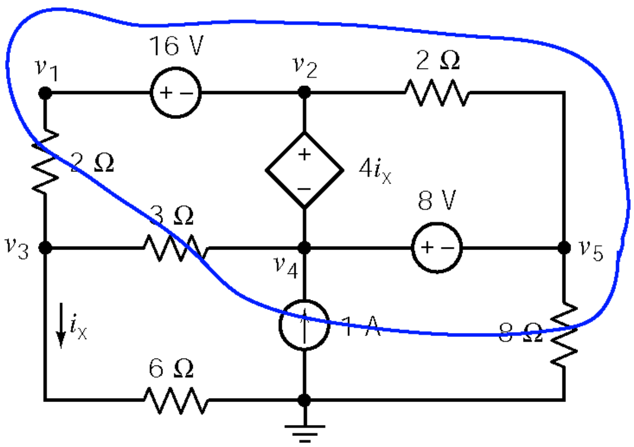


Figure P 4.4-12.

Solution:



Express the voltages of the independent voltage sources in terms of the node voltages

$$v_1 - v_2 = 16 \text{ and } v_4 - v_5 = 8$$

Express the controlling current of the dependent source in terms of the node voltages

$$i_x = \frac{v_3}{6}$$

Express the controlled voltage of the dependent source in terms of the node voltages

$$v_2 - v_4 = 4i_x = 4\left(\frac{v_3}{6}\right) \quad \Rightarrow \quad -6v_2 + 4v_3 + 6v_4 = 0$$

Apply KCL to the supernode to get

$$\frac{v_1 - v_3}{2} + \frac{v_4 - v_3}{3} + \frac{v_5}{8} = 1 \quad \Rightarrow \quad 12v_1 - 20v_3 + 8v_4 + 3v_5 = 24$$

Apply KCL at node 3 to get

$$\frac{v_3 - v_1}{2} + \frac{v_3}{6} + \frac{v_3 - v_4}{3} = 0 \quad \Rightarrow \quad -3v_1 + 6v_2 - 2v_4 = 0$$

Solving, e.g. using MATLAB, gives

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & -6 & 4 & 6 & 0 \\ 12 & 0 & -20 & 8 & 3 \\ -3 & 0 & 6 & -2 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix} = \begin{bmatrix} 16 \\ 8 \\ 0 \\ 24 \\ 0 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix} = \begin{bmatrix} 24 \\ 8 \\ 12 \\ 0 \\ -8 \end{bmatrix}$$

(checked: LNAP 6/13/04)

P 4.4-13 Determine values of the node voltages, v_1, v_2, v_3, v_4, and v_5, in the circuit shown in Figure P 4.4-13.

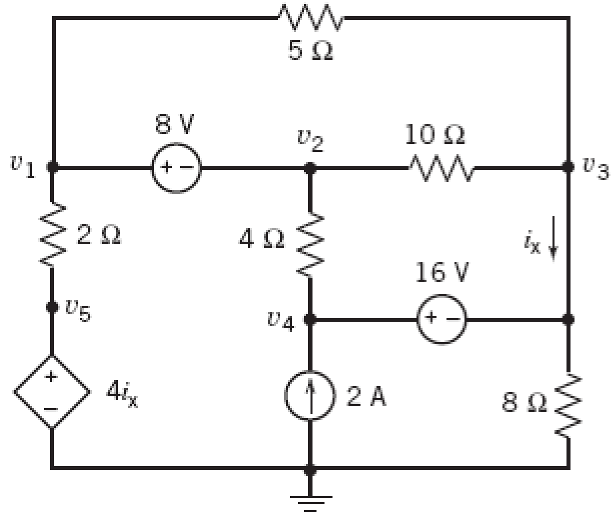


Figure P 4.4-13

Solution:
Express the voltage source voltages in terms of the node voltages:

$$v_1 - v_2 = 8 \text{ and } v_4 - v_3 = 16$$

Express the controlling current of the dependent source in terms of the node voltages:

$$i_x = \frac{v_2 - v_3}{10} + \frac{v_1 - v_3}{5} = 0.2v_1 + 0.1v_2 - 0.3v_3$$

Express the controlled voltage of the dependent source in terms of the node voltages:

$$v_5 = 4i_x = 0.8v_1 = 0.4v_2 - 1.2v_3 \quad \Rightarrow \quad 0.8v_1 + 0.4v_2 - 1.2v_3 - v_5 = 0$$

Apply KVL to the supernodes

$$\frac{v_1 - v_5}{2} + \frac{v_2 - v_4}{4} + \frac{v_2 - v_3}{10} + \frac{v_1 - v_3}{5} = 0 \quad \Rightarrow \quad 14v_1 + 7v_2 - 6v_3 - 5v_4 - 10v_5 = 0$$

$$\frac{v_4 - v_2}{4} + \frac{v_3}{8} + \frac{v_3 - v_2}{10} + \frac{v_3 - v_1}{5} = 2 \quad \Rightarrow \quad -8v_1 - 14v_2 + 17v_3 + 10v_4 = 80$$

Solving, e.g. using MATLAB, gives

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0.8 & 0.4 & -1.2 & 0 & -1 \\ 14 & 7 & -6 & -5 & -10 \\ -8 & -14 & 17 & 10 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix} = \begin{bmatrix} 8 \\ 16 \\ 0 \\ 0 \\ 80 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix} = \begin{bmatrix} 11.32 \\ 3.32 \\ 2.11 \\ 18.11 \\ 7.85 \end{bmatrix}$$

(checked: LNAP 6/13/04)

P 4.4-14 The voltages v_1, v_2, v_3, and v_4 are the node voltages corresponding to nodes 1, 2, 3, and 4 in Figure P 4.4-14. Determine the values of these node voltages.

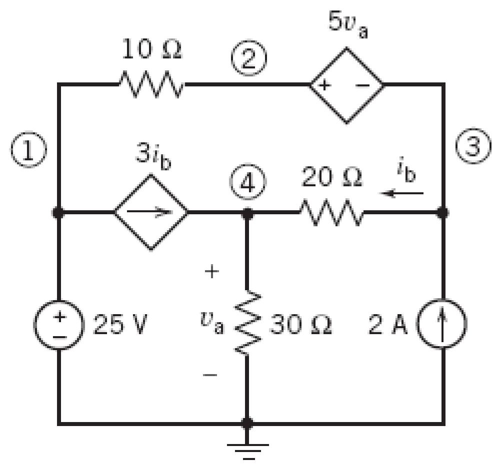
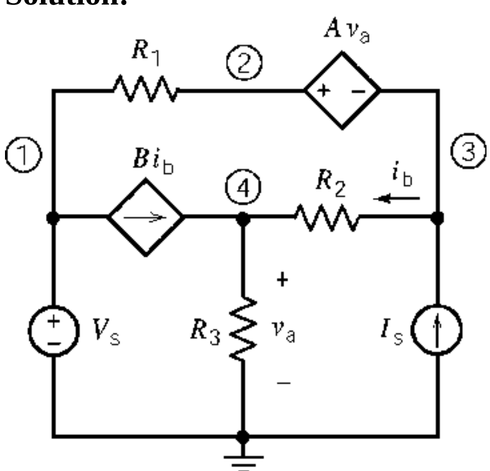


Figure P 4.4-14

Solution:



Express the controlling voltage and current of the dependent sources in terms of the node voltages:

$$v_a = v_4 \quad \text{and} \quad i_b = \frac{v_3 - v_4}{R_2}$$

Express the voltage source voltages in terms of the node voltages:

$$v_1 = V_s \quad \text{and} \quad v_2 - v_3 = A\,v_a = A\,v_4$$

Apply KCL to the supernode corresponding to the dependent voltage source

$$\frac{v_2 - v_1}{R_1} + \frac{v_3 - v_4}{R_2} = I_s \quad \Rightarrow \quad -R_2\,v_1 + R_2\,v_2 + R_1\,v_3 - R_1\,v_4 = R_1\,R_2\,I_s$$

Apply KCL at node 4:

$$B\frac{v_3 - v_4}{R_2} + \frac{v_3 - v_4}{R_2} = \frac{v_4}{R_3} \quad \Rightarrow \quad (B+1)v_3 - \left(B+1+\frac{R_2}{R_3}\right)v_4 = 0$$

Organizing these equations into matrix form:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -A \\ -R_2 & R_2 & R_1 & -R_1 \\ 0 & 0 & B+1 & -\left(B+1+\dfrac{R_2}{R_3}\right) \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} V_s \\ 0 \\ R_1\,R_2\,I_s \\ 0 \end{bmatrix}$$

With the given values:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -5 \\ -20 & 20 & 10 & -10 \\ 0 & 0 & 4 & -4.667 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 25 \\ 0 \\ 400 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 25 \\ 44.4 \\ 8.4 \\ 7.2 \end{bmatrix}$$

(Checked using LNAP 9/29/04)

P 4.4-15 The voltages v_1, v_2, v_3, and v_4 in Figure P 4.4-15 are the node voltages corresponding to nodes 1, 2, 3, and 4. The values of these voltages are

$v_1 = 10$ V, $v_2 = 75$ V, $v_3 = -15$ V, and $v_4 = 22.5$ V

Determine the values of the gains of the dependent sources, A and B, and of the resistance R_1.

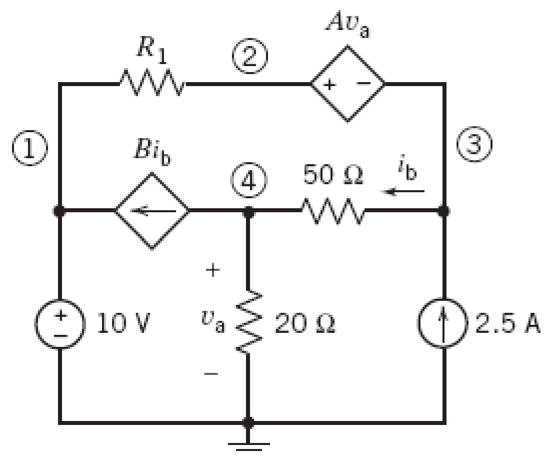


Figure P 4.4-15

Solution:

Express the controlling voltage and current of the dependent sources in terms of the node voltages:

$$v_a = v_4 = 22.5 \text{ V}$$

and
$$i_b = \frac{v_3 - v_4}{R_2} = \frac{-15 - 22.5}{50} = -0.75$$

Express the dependent voltage source voltage in terms of the node voltages:

$$v_2 - v_3 = A v_a = A v_4$$

so
$$A = \frac{v_2 - v_3}{v_4} = \frac{75 - (-15)}{22.5} = 4 \text{ V/V}$$

Apply KCL to the supernode corresponding to the dependent voltage source

$$\frac{v_2 - v_1}{R_1} + \frac{v_3 - v_4}{R_2} = I_s \quad \Rightarrow \quad \frac{75 - 10}{R_1} + \frac{-15 - 22.5}{50} = 2.5 \quad \Rightarrow \quad R_1 = 20\ \Omega$$

Apply KCL at node 4:

$$\frac{v_3 - v_4}{R_2} = \frac{v_4}{R_3} + B\frac{v_3 - v_4}{R_2} \quad \Rightarrow \quad \frac{-15 - 22.5}{50} = \frac{22.5}{20} + B\frac{-15 - 22.5}{50} \quad \Rightarrow \quad B = 2.5 \text{ A/A}$$

(Checked using LNAP 9/29/04)

P 4.4-16 The voltages v_1, v_2, and v_3 in Figure P 4.4-16 are the node voltages corresponding to nodes 1, 2, and 3. The values of these voltages are

$$v_1 = 12 \text{ V}, v_2 = 21 \text{ V}, \quad \text{and} \quad v_3 = -3 \text{ V}$$

(a) Determine the values of the resistances R_1 and R_2.

(b) Determine the power supplied by each source.

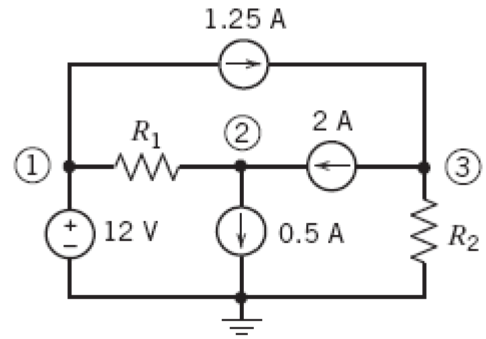


Figure P 4.4-16

Solution:

(a) $R_1 = \dfrac{v_2 - v_1}{2-0.5} = \dfrac{21-12}{1.5} = 6\ \Omega$ and $R_2 = \dfrac{v_2}{1.25-2} = \dfrac{-3}{-0.75} = 4\ \Omega$

(b) The power supplied by the voltage source is $12(0.5+1.25-2) = -3$ W. The power supplied by the 1.25-A current source is $1.25(-3-12) = -18.75$ W. The power supplied by the 0.5-A current source is $-0.5(21) = -10.5$ W. The power supplied by the 2-A current source is $2(21-(-3)) = 48$ W.

P 4.4-17 The voltages v_1, v_2, and v_3 in Figure P 4.4-17 are the node voltages corresponding to nodes 1, 2, and 3. The values of these voltages are

$$v_1 = 12 \text{ V}, v_2 = 9.6 \text{ V}, \quad \text{and} \quad v_3 = -1.33 \text{ V}$$

(a) Determine the values of the resistances R_1 and R_2.

(b) Determine the power supplied by each source.

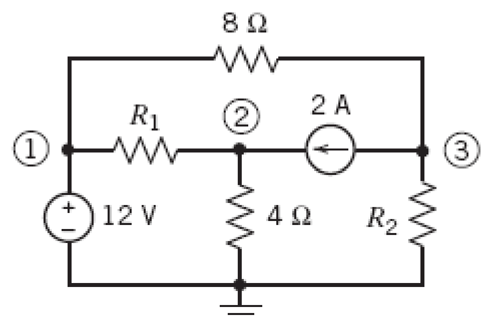
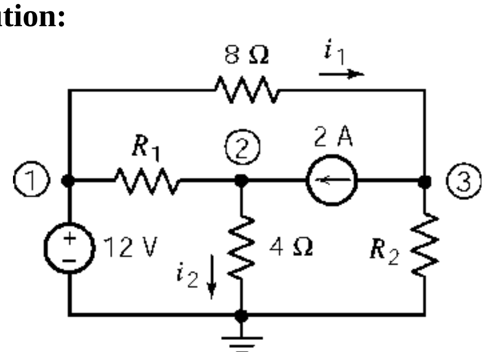


Figure P 4.4-18

Solution:



$$i_1 = \frac{12-(-1.33)}{8} = 1.666 \text{ A}$$

and

$$i_2 = \frac{9.6}{4} = 2.4 \text{ A}$$

(a) $R_1 = \dfrac{v_2 - v_1}{2-i_1} = \dfrac{9.6-12}{2-2.4} = 6\ \Omega$ and

$$R_2 = \frac{v_3}{i_1 - 2} = \frac{-1.33}{1.666-2} = 3.98 \simeq 4\ \Omega$$

(b) The power supplied by the voltage source is $12(2.4+1.66-2) = 24.7$ W. The power supplied by the current source is $2(9.6-(-1.33)) = 21.9$ W.

(Checked using LNAP 10/2/04)

P4.4-18

The voltages v_2, v_3 and v_4 for the circuit shown in Figure P4.4-18 are:

$$v_2 = 16 \text{ V}, \; v_3 = 8 \text{ V} \quad \text{and} \quad v_4 = 6 \text{ V}$$

Determine the values of the following:

(a) The gain, *A*, of the VCVS
(b) The resistance R_5
(c) The currents i_b and i_c
(d) The power received by resistor R_4

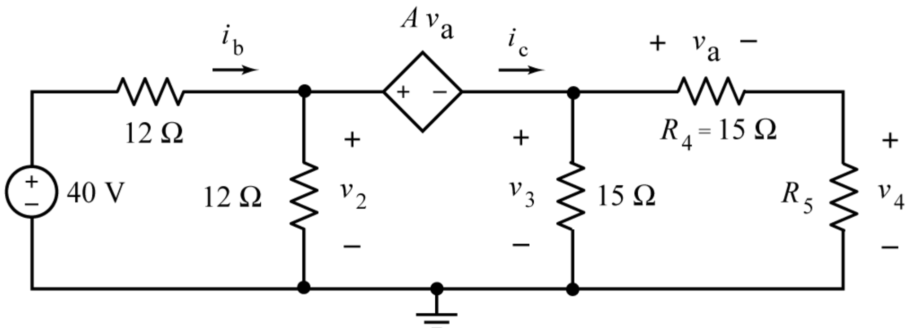


Figure P4.4-18

Solution:

Given the node voltages $v_2 = 16 \text{ V}, \; v_3 = 8 \text{ V} \quad \text{and} \quad v_4 = 6 \text{ V}$

$$A = \frac{A v_a}{v_a} = \frac{16-8}{8-6} = 4 \; \frac{\text{V}}{\text{V}}$$

$$R_5 \left(\frac{v_3 - v_4}{15} \right) = v_4 \quad \Rightarrow \quad R_5 = \frac{15(6)}{8-6} = 45 \; \Omega,$$

$$i_b = \frac{40-24}{12} = 2 \text{ A} \;\; \text{and} \;\; i_c = \frac{40-16}{12} - \frac{16}{12} = 0.6667 \text{ A}$$

$$p_4 = \frac{{v_a}^2}{15} = \frac{2^2}{15} = 0.2667 \text{ W}$$

P4.4-19

Determine the values of the node voltages v_1 and v_2 for the circuit shown in Figure P4.4-19.

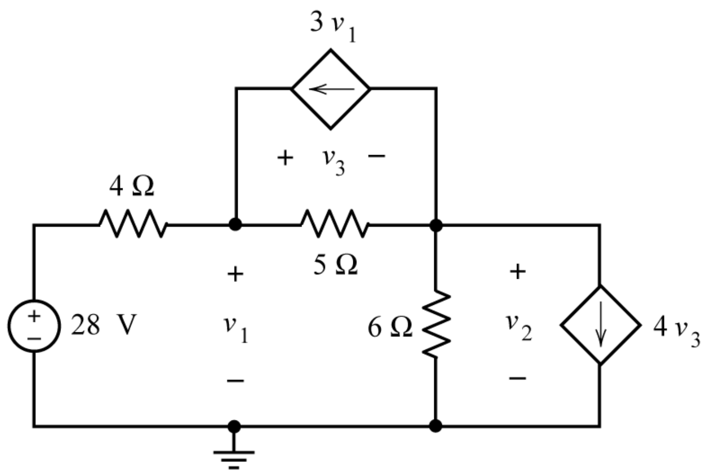


Figure P4.4-19

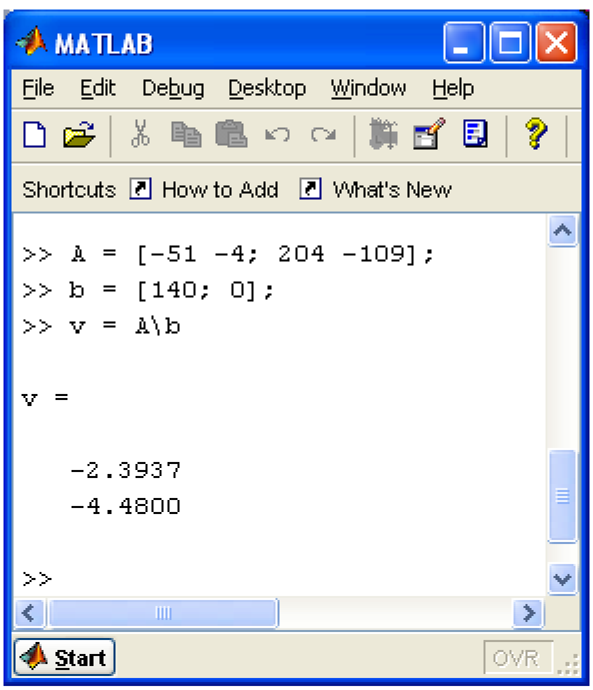
P4.4-19

The node equations are

$$\frac{28-v_1}{4}+3v_1=\frac{v_1-v_2}{5} \quad\Rightarrow\quad 5(28-v_1)+20(3v_1)=4(v_1-v_2) \quad\Rightarrow\quad 140=-51v_1-4v_2$$

and
$$\frac{v_1-v_2}{5}=3v_1+\frac{v_2}{6}+4v_3=3v_1+\frac{v_2}{6}+4(v_1-v_2) \quad\Rightarrow\quad 0=204v_1-109v_2$$

Using MATLAB to solve these equations:



Consequently $v_1=-2.3937$ V and $v_2=-4.4800$ V

P4.4-20

The encircled numbers in Figure P4.4-20 are node numbers. Determine the values of v_1, v_2 and v_3, the node voltages corresponding to nodes 1, 2 and 3.

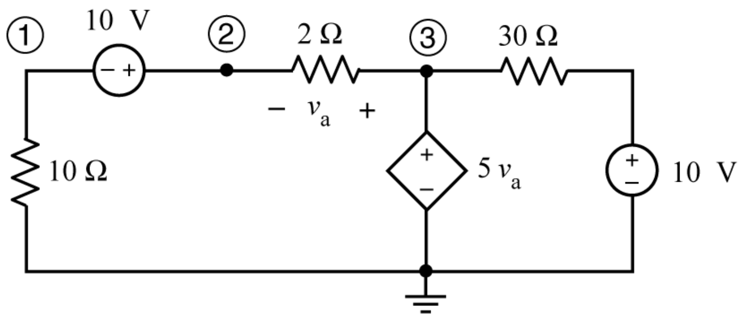


Figure P4.4-20

Solution:

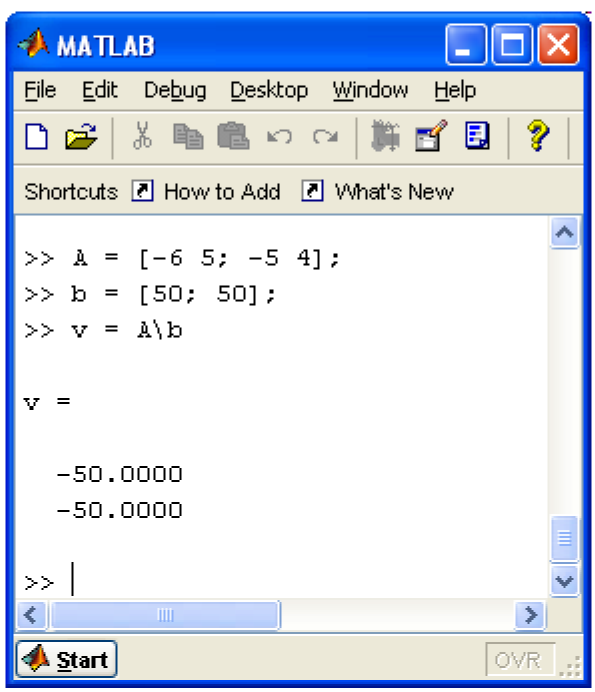
Apply KCL to the supernode corresponding to the horizontal voltage source to get

$$\frac{v_1}{10} = \frac{v_a}{2} = \frac{v_3 - v_2}{2} = \frac{v_3 - (v_1 + 10)}{2} \quad \Rightarrow \quad v_1 = 5\left(v_3 - (v_1 + 10)\right) \quad \Rightarrow \quad 50 = -6\,v_1 + 5\,v_3$$

Looking at the dependent source we notice that

$$v_3 = 5\,v_a = 5\left(v_3 - v_2\right) = 5\left(v_3 - (v_1 + 10)\right) \quad \Rightarrow \quad 50 = -5\,v_1 + 4\,v_3$$

Using MATLAB to solve these equations:



Consequently $\quad v_1 = -50 \text{ V}$ and $v_3 = -50 \text{ V}$

Then $\quad v_2 = v_1 + 10 = -40 \text{ V}$

P4.4-21

Determine the values of the node voltages v_1, v_2 and v_3 for the circuit shown in Figure P4.4-21.

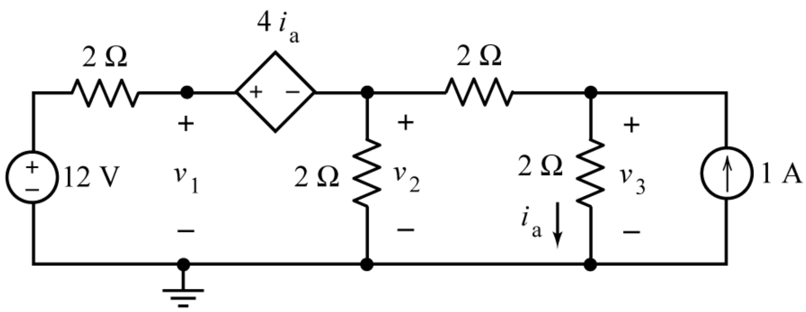


Figure P4.4-21

Solution:

Represent the controlling current of the dependent source in terms of the node voltages: $i_a = \dfrac{v_3}{2}$

Represent the controlled voltage of the dependent source in terms of the node voltages:

$$4i_a = v_1 - v_2 \quad \Rightarrow \quad = 4\left(\frac{v_3}{2}\right) = v_1 - v_2 \quad \Rightarrow \quad 0 = v_1 - v_2 - 2v_3$$

Apply KCL to the supernode corresponding to the dependent voltage source:

$$\frac{12 - v_1}{2} = \frac{v_2}{2} + \frac{v_2 - v_3}{2} \quad \Rightarrow \quad 12 - v_1 = v_2 + v_2 - v_3 \quad \Rightarrow \quad 12 = v_1 + 2v_2 - v_3$$

Apply KCL to top node of the current source:

$$\frac{v_2 - v_3}{2} + 1 = \frac{v_3}{2} \quad \Rightarrow \quad v_2 - v_3 + 2 = v_3 \quad \Rightarrow \quad 2 = -v_2 + 2v_3$$

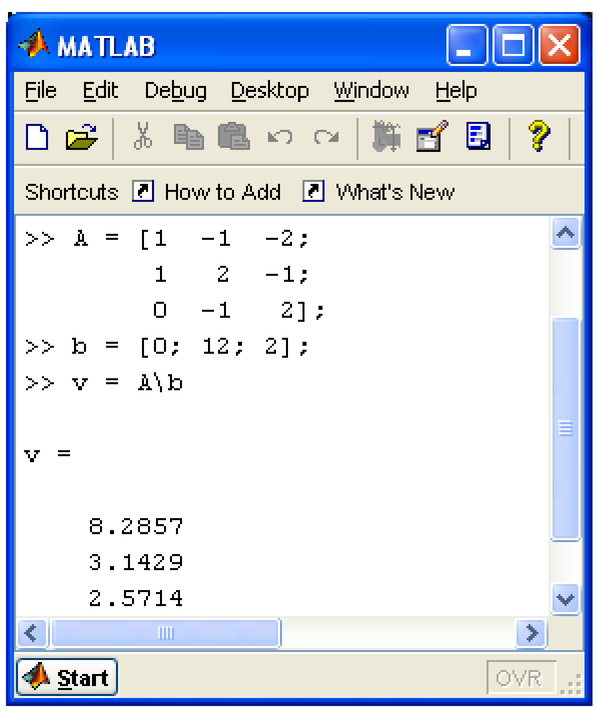
Solving these equations using MATLAB gives

$v_1 = 8.2857$ V ,

$v_2 = 3.1459$ V

and

$v_3 = 2.5714$ V



P4.4-22

Determine the values of the node voltages v_1, v_2 and v_3 for the circuit shown in Figure P4.4-22.

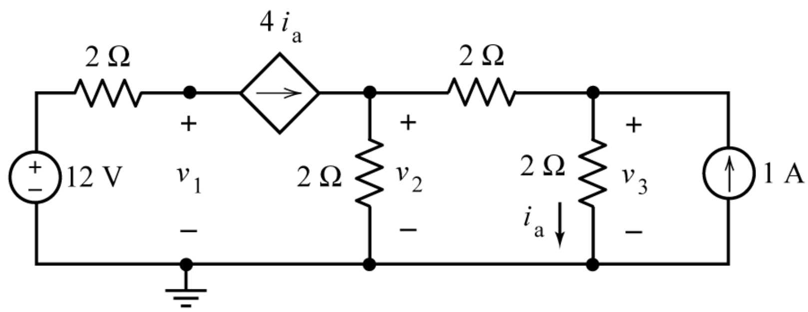


Figure P4.4-22

Solution:

The node equations are:

$$\frac{12-v_1}{2}=4i_a=4\left(\frac{v_3}{2}\right) \Rightarrow 12-v_1=4v_3 \Rightarrow 12=v_1+4v_3$$

$$4i_a=\frac{v_2}{2}+\frac{v_2-v_3}{2} \Rightarrow 4\left(\frac{v_3}{2}\right)=\frac{v_2}{2}+\frac{v_2-v_3}{2} \Rightarrow 0=2v_2-5v_3$$

$$\frac{v_2-v_3}{2}+1=\frac{v_3}{2} \Rightarrow v_2-v_3+2=v_3 \Rightarrow 2=-v_2+2v_3$$

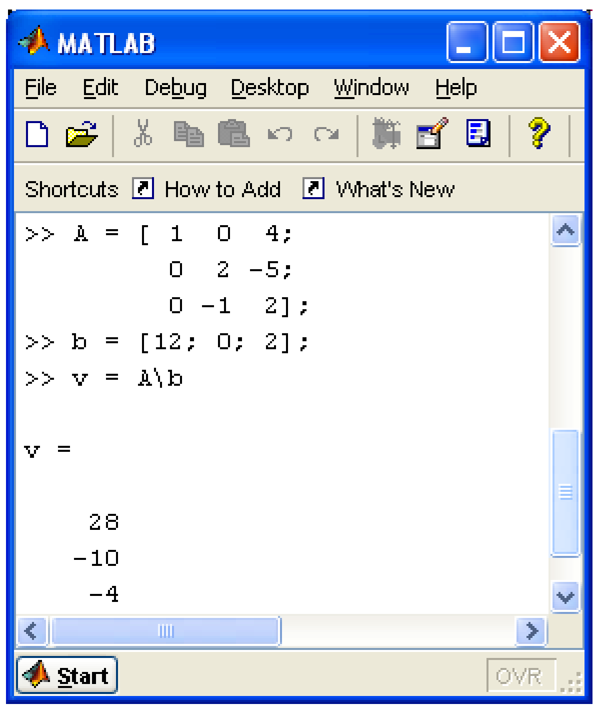
Solving these equations using MATLAB gives

$$v_1=28 \text{ V},$$

$$v_2=-10 \text{ V}$$

and

$$v_3=-4 \text{ V}$$



Section 4-5 Mesh Current Analysis with Independent Voltage Sources

P 4.5-1 Determine the mesh currents, i_1, i_2, and i_3, for the circuit shown in Figure P 4.5-1.

Answers: $i_1 = 3$ A, $i_2 = 2$ A, and $i_3 = 4$ A

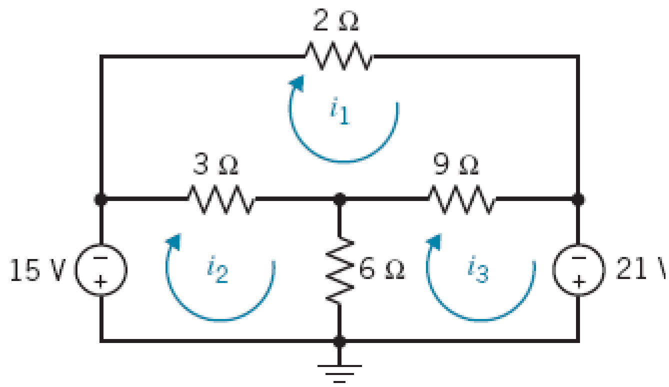


Figure P 4.5-1

Solution:

The mesh equations are

$$2\,i_1 + 9\,(i_1 - i_3) + 3(i_1 - i_2) = 0$$
$$15 - 3\,(i_1 - i_2) + 6\,(i_2 - i_3) = 0$$
$$-6\,(i_2 - i_3) - 9\,(i_1 - i_3) - 21 = 0$$

or

$$14\,i_1 - 3\,i_2 - 9\,i_3 = 0$$
$$-3\,i_1 + 9\,i_2 - 6\,i_3 = -15$$
$$-9\,i_1 - 6\,i_2 + 15\,i_3 = 21$$

so

$$i_1 = 3 \text{ A}, \; i_2 = 2 \text{ A and } i_3 = 4 \text{ A}.$$

(checked using LNAP 8/14/02)

P 4.5-2 The values of the mesh currents in the circuit shown in Figure P 4.5-2 are

$i_1 = 2$ A, $i_2 = 3$ A, and $i_3 = 4$ A.

Determine the values of the resistance R and of the voltages v_1 and v_2 of the voltage sources.

Answers: $R = 12\ \Omega$, $v_1 = -4$ V, and $v_2 = -28$ V

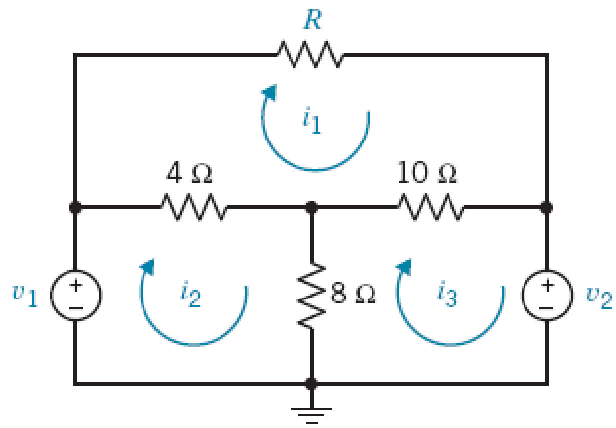


Figure P 4.5-2

Solution:

The mesh equations are:

Top mesh: $$4\,(2-3)+R(2)+10\,(2-4)=0$$

so $$R = 12\ \Omega.$$

Bottom, right mesh: $$8\,(4-3)+10\,(4-2)+v_2=0$$

so $$v_2 = -28\text{ V}.$$

Bottom left mesh $$-v_1+4\,(3-2)+8\,(3-4)=0$$

so $$v_1 = -4\text{ V}.$$

(checked using LNAP 8/14/02)

P 4.5-3 The currents i_1 and i_2 in Figure P 4.5-3 are the mesh currents. Determine the value of the resistance R required to cause $v_a = -6$ V.

Answer: $R = 4\ \Omega$

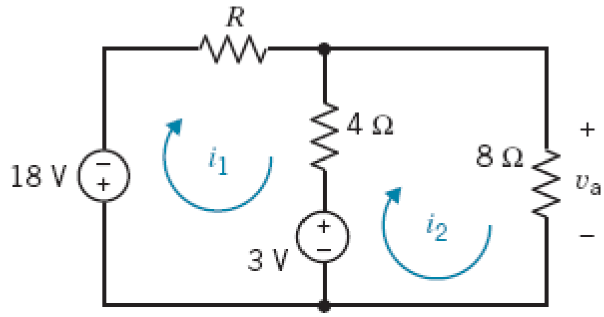
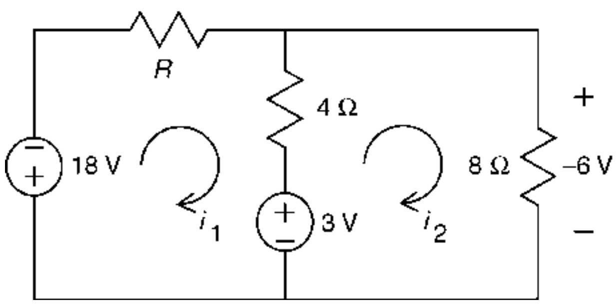


Figure P 4.5-3

Solution:



Ohm's Law: $i_2 = \dfrac{-6}{8} = -0.75$ A

KVL for loop 1:

$$R\, i_1 + 4\left(i_1 - i_2\right) + 3 + 18 = 0$$

KVL for loop 2

$$+(-6) - 3 - 4\left(i_1 - i_2\right) = 0$$

$$\Rightarrow \quad -9 - 4\left(i_1 - (-0.75)\right) = 0$$

$$\Rightarrow \quad i_1 = -3 \text{ A}$$

$$R(-3) + 4\left(-3 - (-0.75)\right) + 21 = 0 \quad \Rightarrow \quad R = 4\ \Omega$$

(checked using LNAP 8/14/02)

P 4.5-4 Determine the mesh currents, i_a and i_b, in the circuit shown in Figure P 4.5-4.

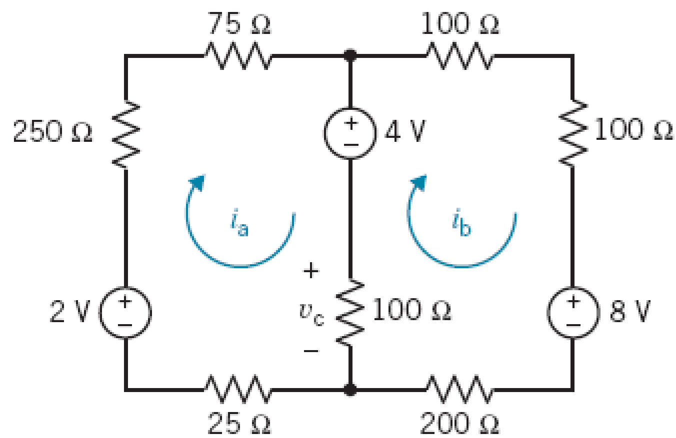


Figure P 4.5-4

Solution:

KVL loop 1:

$$25\, i_a - 2 + 250\, i_a + 75\, i_a + 4 + 100\, (i_a - i_b) = 0$$

$$450\, i_a - 100\, i_b = -2$$

KVL loop 2:

$$-100(i_a - i_b) - 4 + 100\, i_b + 100\, i_b + 8 + 200\, i_b = 0$$

$$-100\, i_a + 500\, i_b = -4$$

Solving these equations: $\underline{i_a = -6.5 \text{ mA}}$, $\underline{i_b = -9.3 \text{ mA}}$

(checked using LNAP 8/14/02)

P 4.5-5 Find the current i for the circuit of Figure P 4.5-5.

Hint: A short circuit can be treated as a 0-V voltage source.

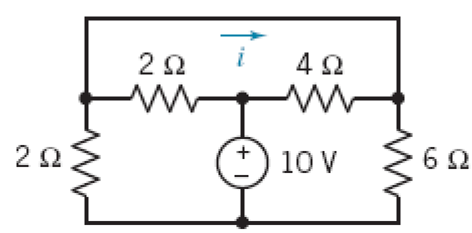
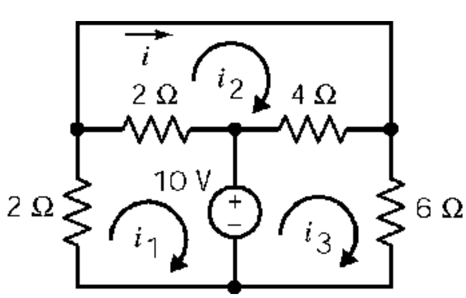


Figure P 4.5-5

Solution:



Mesh Equations:

$$\text{mesh 1}: \; 2i_1 + 2\,(i_1 - i_2) + 10 = 0$$

$$\text{mesh 2}: \; 2(i_2 - i_1) + 4\,(i_2 - i_3) = 0$$

$$\text{mesh 3}: \; -10 + 4\,(i_3 - i_2) + 6\, i_3 = 0$$

Solving:

$$i = i_2 \;\Rightarrow\; i = -\frac{5}{17} = -0.294 \text{ A}$$

(checked using LNAP 8/14/02)

P 4.5-6 Simplify the circuit shown in Figure P 4.5-6 by replacing series and parallel resistors by equivalent resistors. Next, analyze the simplified circuit by writing and solving mesh equations.

(a) Determine the power supplied by each source.

(b) Determine the power absorbed by the 30-Ω resistor.

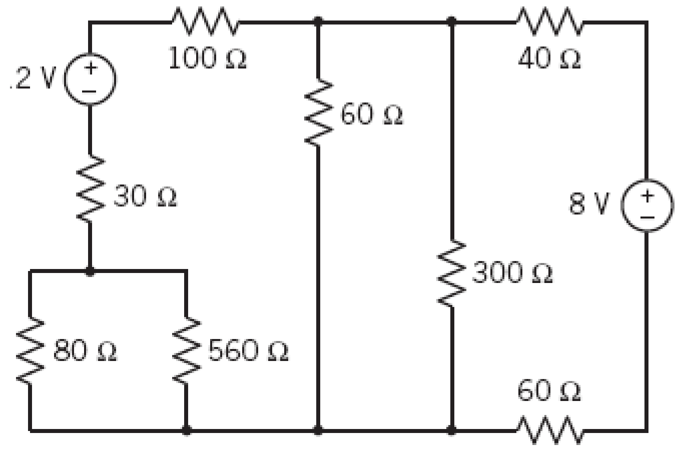
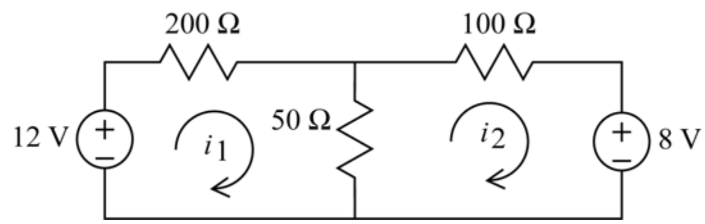


Figure P 4.5-6

Solution: Replace series and parallel resistors with equivalent resistors:

$$60\ \Omega \| 300\ \Omega = 50\ \Omega,\quad 40\ \Omega + 60\ \Omega = 100\ \Omega \quad \text{and} \quad 100\ \Omega + 30\ \Omega + (80\ \Omega \| 560\ \Omega) = 200\ \Omega$$

so the simplified circuit is



The mesh equations are

$$200 i_1 + 50(i_1 - i_2) - 12 = 0$$
$$100 i_2 + 8 - 50(i_1 - i_2) = 0$$

or

$$\begin{bmatrix} 250 & -50 \\ -50 & 150 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 12 \\ -8 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0.04 \\ -0.04 \end{bmatrix}$$

The power supplied by the 12 V source is $12 i_1 = 12(0.04) = 0.48$ W. The power supplied by the 8 V source is $-8 i_2 = -8(-0.04) = 0.32$ W. The power absorbed by the 30 Ω resistor is

$$i_1^2(30) = (0.04)^2(30) = 0.048 \text{ W}.$$

(checked: LNAP 5/31/04)

Section 4-6 Mesh Current Analysis with Voltage and Current Sources

P 4.6-1 Find i_b for the circuit shown in Figure P 4.6-1.

Answer: $i_b = 0.6$ A

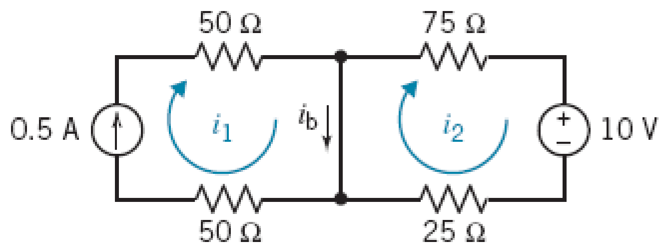


Figure P 4.6-1

Solution:

$$\text{mesh 1:} \quad i_1 = \frac{1}{2} \text{ A}$$

$$\text{mesh 2:} \quad 75\, i_2 + 10 + 25\, i_2 = 0 \quad \Rightarrow i_2 = -0.1 \text{ A}$$

$$i_b = i_1 - i_2 = \underline{0.6 \text{ A}}$$

(checked using LNAP 8/14/02)

P 4.6-2 Find v_c for the circuit shown in Figure P 4.6-2.

Answer: $v_c = 15$ V

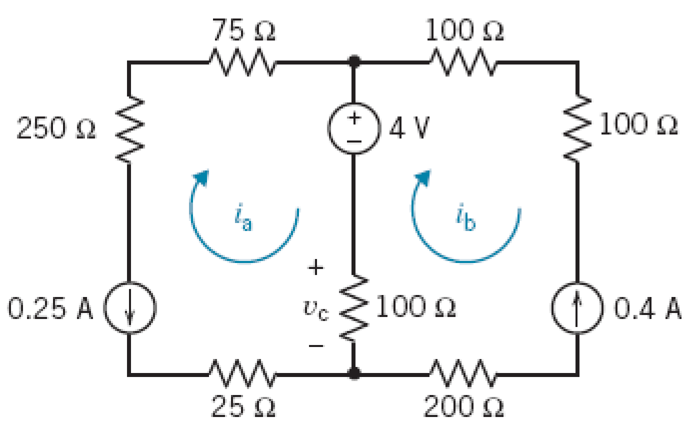


Figure P 4.6-2

Solution:

Mesh currents:

$$\text{mesh a:} \quad i_a = -0.25 \text{ A}$$

$$\text{mesh b:} \quad i_b = -0.4 \text{ A}$$

Ohm's Law:

$$v_c = 100(i_a - i_b) = 100(0.15) = \underline{15 \text{ V}}$$

(checked using LNAP 8/14/02)

P 4.6-3 Find v_2 for the circuit shown in Figure P 4.6-3.

Answer: $v_2 = 2$ V

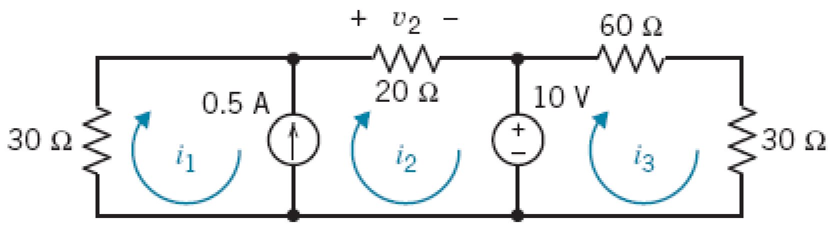
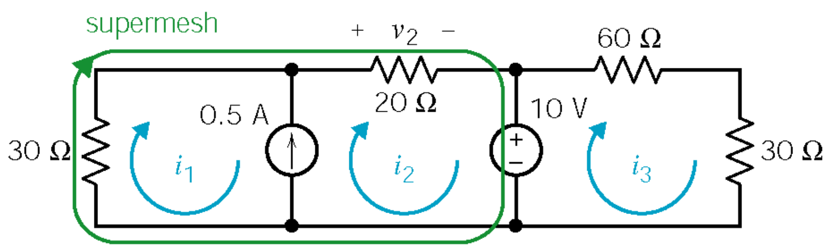


Figure P 4.6-3

Solution:



Express the current source current as a function of the mesh currents:

$$i_1 - i_2 = -0.5 \Rightarrow i_1 = i_2 - 0.5$$

Apply KVL to the supermesh:

$$30\, i_1 + 20\, i_2 + 10 = 0 \quad \Rightarrow \quad 30\,(i_2 - 0.5) + 20 i_2 = -10$$

$$50\, i_2 - 15 = -10 \quad \Rightarrow \quad i_2 = \frac{5}{50} = .1 \text{ A}$$

$$i_1 = -.4 \text{ A} \quad \text{and} \quad v_2 = 20\, i_2 = \underline{2\text{ V}}$$

(checked using LNAP 8/14/02)

P 4.6-4 Find v_c for the circuit shown in Figure P 4.6-4.

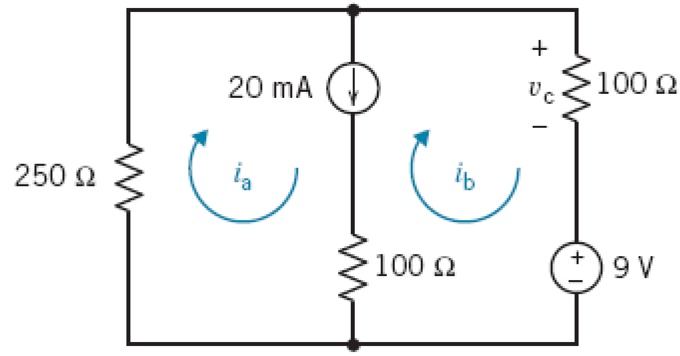


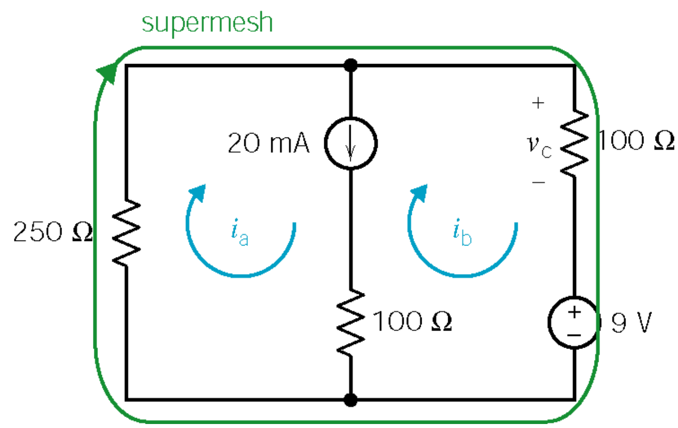
Figure P 4.6-4

Solution:

Express the current source current in terms of the mesh currents:

$$i_b = i_a - 0.02$$

Apply KVL to the supermesh:



$$250\; i_a + 100\;(i_a - 0.02) + 9 = 0$$
$$\therefore i_a = -.02 \text{ A} = -20 \text{ mA}$$
$$v_c = 100(i_a - 0.02) = \underline{-4 \text{ V}}$$

(checked using LNAP 8/14/02)

P 4.6-5 Determine the value of the voltage measured by the voltmeter in Figure P 4.6-5.

Answer: 8 V

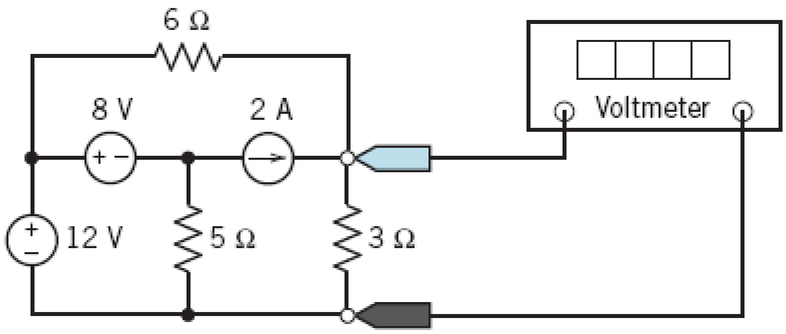
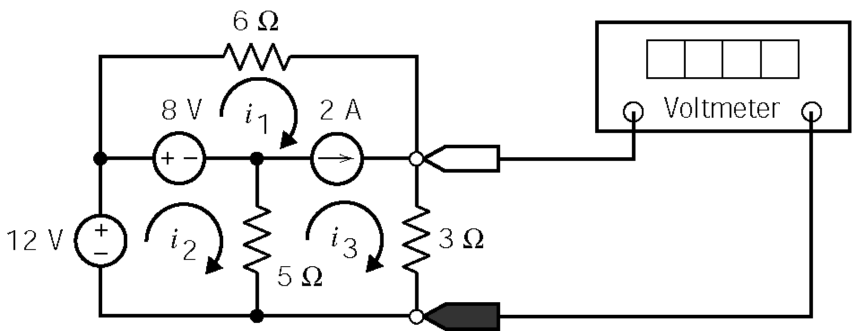


Figure P 4.6-5

Solution: Label the mesh currents:



Express the current source current in terms of the mesh currents:

$$i_3 - i_1 = 2 \quad \Rightarrow \quad i_1 = i_3 - 2$$

Supermesh: $$6\,i_1 + 3\,i_3 - 5\left(i_2 - i_3\right) - 8 = 0 \quad \Rightarrow \quad 6\,i_1 - 5\,i_2 + 8\,i_3 = 8$$

Lower, left mesh: $$-12 + 8 + 5\left(i_2 - i_3\right) = 0 \quad \Rightarrow \quad 5\,i_2 = 4 + 5\,i_3$$

Eliminating i_1 and i_2 from the supermesh equation:

$$6\left(i_3 - 2\right) - \left(4 + 5\,i_3\right) + 8\,i_3 = 8 \quad \Rightarrow \quad 9\,i_3 = 24$$

The voltage measured by the meter is: $3\,i_3 = 3\left(\frac{24}{9}\right) = 8\text{ V}$

(checked using LNAP 8/14/02)

P 4.6-6 Determine the value of the current measured by the ammeter in Figure P 4.6-6.

Hint: Write and solve a single mesh equation.

Answer: –5/6 A

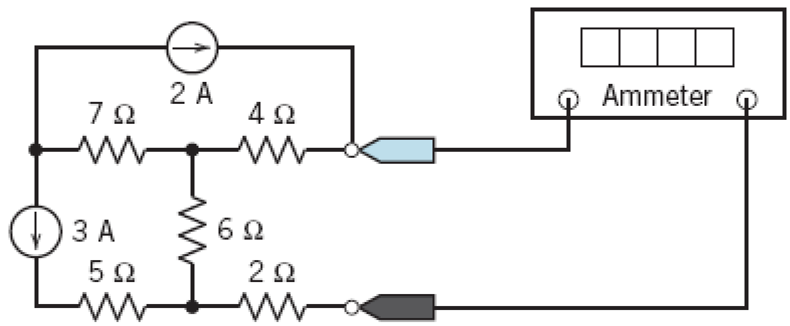
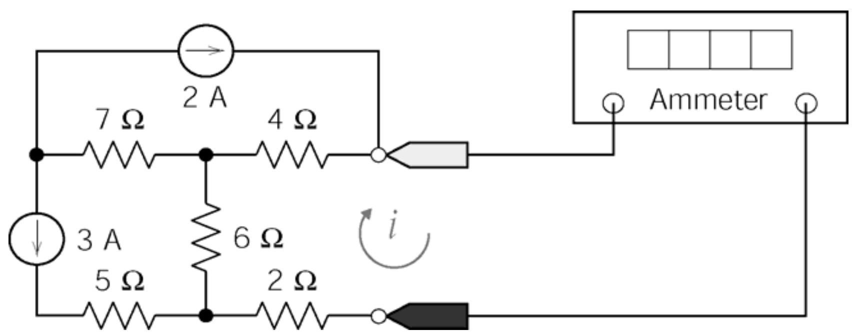


Figure P 4.6-6

Solution:



Mesh equation for right mesh:

$$4(i-2)+2\,i+6(i+3)=0 \quad \Rightarrow \quad 12\,i-8+18=0 \quad \Rightarrow \quad i=-\frac{10}{12}\ \text{A}=-\frac{5}{6}\ \text{A}$$

(checked using LNAP 8/14/02)

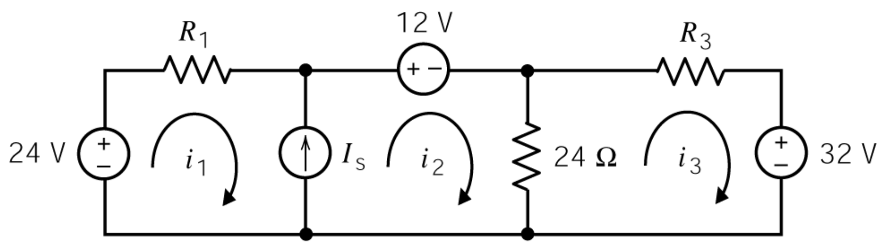


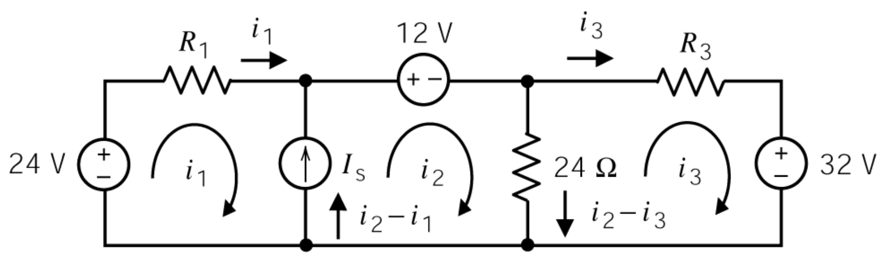
Figure P4.6-7

P4.6-7 The mesh currents are labeled in the circuit shown Figure 4.6-7. The value of these mesh currents are:

$$i_1 = -1.1014 \text{ A}, \; i_2 = 0.8986 \text{ A and } \; i_3 = -0.2899 \text{ A}$$

a.) Determine the values of the resistances R_1 and R_3.
b.) Detemine the value of the current sourc current.
c.) Determine the value of the power supplied by the 12 V voltage source.

Solution: Label the resistor currents and the current source currrents in terms of the mesh currents:



a.) Apply KVL to the supermesh corresponding to the current source to get

$$R_1 i_1 + 12 + 24(i_2 - i_3) - 24 = 0 \quad \Rightarrow \quad R_1 = \frac{12 - 24(i_2 - i_3)}{i_1} = \frac{12 - 24(0.8986 - (-0.2899))}{-1.1014} = 15 \ \Omega$$

Apply KVL to the rightmost mesh to get

$$R_3 i_3 + 32 - 24(i_2 - i_3) = 0 \quad \Rightarrow \quad R_3 = \frac{-32 + 24(i_2 - i_3)}{i_3} = \frac{-32 + 24(0.8986 - (-0.2899))}{-0.2899} = 12 \ \Omega$$

b.)

$$I_s = i_2 - i_1 = 0.8986 - (-1.1014) = 2 \text{ A}$$

c.) Noticing that 12 V and i_2 adhere to the passive convention, the power supplied by the 12 V voltage source is

$$-12 i_2 = -12(0.8986) = -10.783 \text{ W}.$$

P 4.6-8 Determine values of the mesh currents, i_1, i_2, and i_3, in the circuit shown in Figure P 4.6-8.

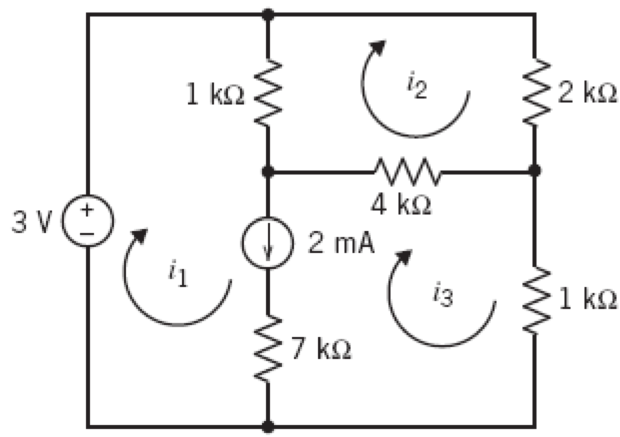


Figure P 4.6-8

Solution: Use units of V, mA and kΩ. Express the currents to the supermesh to get

$$i_1 - i_3 = 2$$

Apply KVL to the supermesh to get

$$4(i_3 - i_3) + (1)i_3 - 3 + (1)(i_1 - i_2) = 0 \quad \Rightarrow \quad i_1 - 5i_2 + 5i_3 = 3$$

Apply KVL to mesh 2 to get

$$2i_2 + 4(i_2 - i_3) + (1)(i_2 - i_1) = 0 \quad \Rightarrow \quad (-1)i_1 + 7i_2 - 4i_3 = 0$$

Solving, e.g. using MATLAB, gives

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & -5 & 5 \\ -1 & 7 & -4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$$

(checked: LNAP 6/21/04)

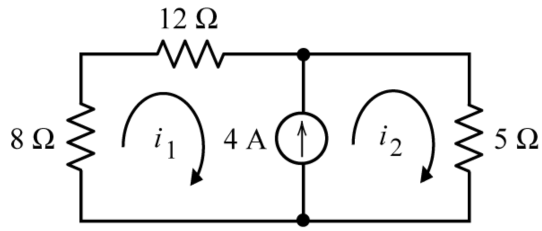


Figure P4.6-9

P4.6-9 The mesh currents are labeled in the circuit shown Figure 4.6-9. Determine the value of the mesh currents i_1 and i_2.

Solution: Determine the value of the mesh currents i_1 and i_2.

$$i_2 = i_1 + 4 \text{ A}, \quad 8i_1 + 12i_1 + 5i_2 = 0 \quad \Rightarrow \quad 8i_1 + 12i_1 + 5(4 + i_1) = 0 \quad \Rightarrow \quad i_1 = -0.8 \text{ A}$$

and

$$i_2 = i_1 + 4 = -0.8 + 4 = 3.2 \text{ A}$$

P 4.6-10 The mesh currents in the circuit shown in Figure P 4.6-10 are

$i_1 = -2.2213$ A, $i_2 = 0.7787$ A, and $i_3 = 0.0770$ A

(a) Determine the values of the resistances R_1 and R_3.

(b) Determine the value of the power supplied by the current source.

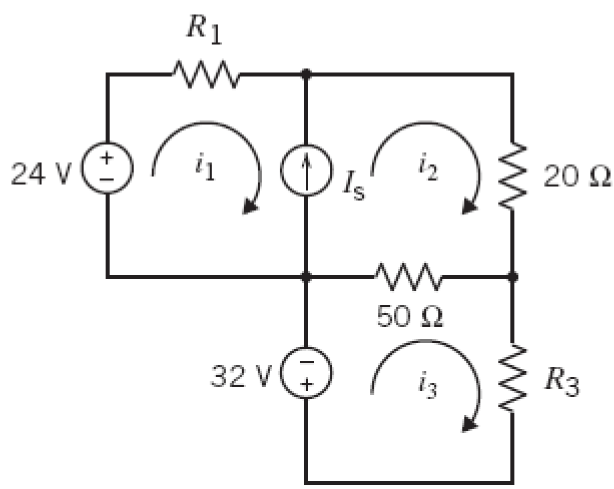


Figure P 4.6-10

Solution:

(a)

$$50(i_3 - i_2) + R_3 i_3 + 32 = 0 \quad \Rightarrow \quad 50(0.0770 - 0.7787) + R_3(0.0770) + 32 = 0$$

$$\Rightarrow \quad R_3 = 40 \ \Omega$$

$$i_1 R_1 + 20 i_2 + 50(i_2 - i_3) - 24 = 0 \quad \Rightarrow \quad R_1(-2.2213) + 20(0.7787) + 50(0.7787 - 0.0770) = 24$$

$$\Rightarrow \quad R_1 = 12 \ \Omega$$

(b)

$$I_s = i_2 - i_1 = 0.7787 - (-2.2213) = 3 \text{ A}$$

The power supplied by the current source is

$$p = I_s(24 - R_1 i_1) = 3(24 - 12(-2.2213)) = 152 \text{ W}$$

(checked: LNAP 6/19/04)

P 4.6-11 Determine the value of the voltage measured by the voltmeter in Figure P 4.6-11.

Hint: Apply KVL to a supermesh to determine the current in the 2-Ω resistor.

Answer: 4/3 V

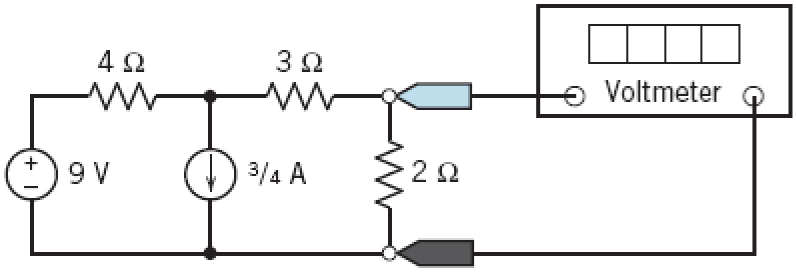
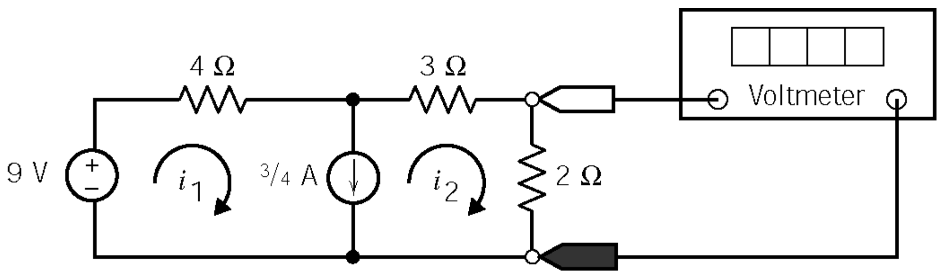


Figure P 4.6-11

Solution:



Express the current source current in terms of the mesh currents: $\frac{3}{4}=i_1-i_2 \quad \Rightarrow \quad i_1=\frac{3}{4}+i_2$.

Apply KVL to the supermesh: $-9+4i_1+3\,i_2+2\,i_2=0 \quad \Rightarrow \quad 4\left(\frac{3}{4}+i_2\right)+5\,i_2=9 \quad \Rightarrow \quad 9\,i_2=6$

so $i_2=\frac{2}{3}$ A and the voltmeter reading is $2\,i_2=\frac{4}{3}$ V

P 4.6-12 Determine the value of the current measured by the ammeter in Figure P 4.6-12.

Hint: Apply KVL to a supermesh.

Answer: –0.333 A

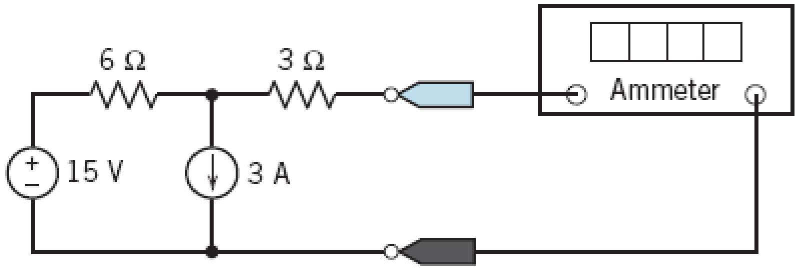
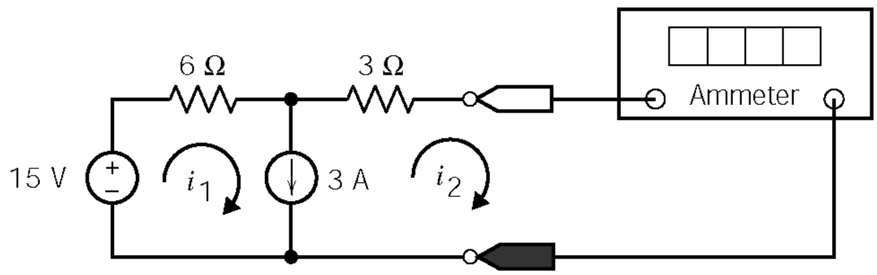


Figure P 4.6-12

Solution:



Express the current source current in terms of the mesh currents: $3 = i_1 - i_2 \quad \Rightarrow \quad i_1 = 3 + i_2$.

Apply KVL to the supermesh: $-15 + 6\, i_1 + 3\, i_2 = 0 \quad \Rightarrow \quad 6\left(3 + i_2\right) + 3\, i_2 = 15 \quad \Rightarrow \quad 9\, i_2 = -3$

Finally, $i_2 = -\dfrac{1}{3}$ A is the current measured by the ammeter.

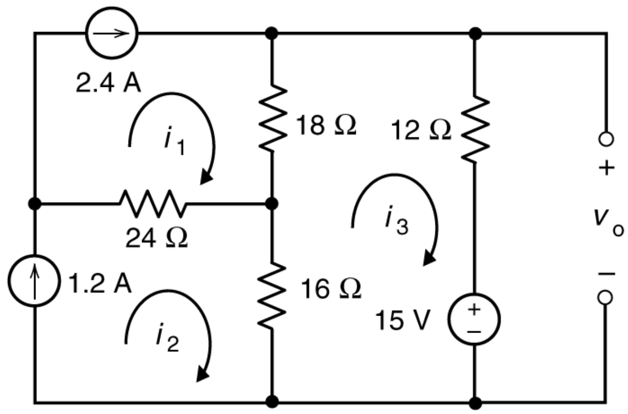
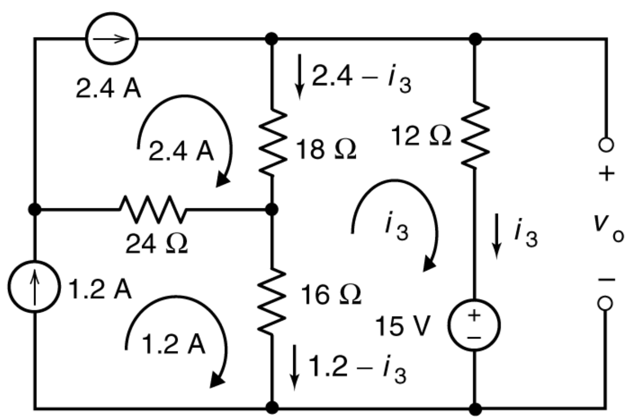


Figure P4.6-13

P4.6-13 Determine the values of the mesh currents i_1, i_2 and i_3 and the output voltage v_o in the circuit shown Figure 4.6-13.

Solution: Notice that the current source are each in a single mesh. Consequently, $i_1 = 2.4$ A and $i_2 = 1.2$ A. Label the resistor currents in terms of the mesh currents:



Apply KVL to mesh 3 to get

$$12 i_3 + 15 - 16(1.2 - i_3) - 18(2.4 - i_3) = 0 \quad \Rightarrow \quad 46 i_3 = 47.4 \quad \Rightarrow \quad i_3 = 1.0304 \text{ A}$$

Apply KVL to the rightmost mesh to get

$$v_o - 15 - 12 i_3 = 0 \quad \Rightarrow \quad v_o = 15 + 12(1.0304) = 27.3648 \text{ V}$$

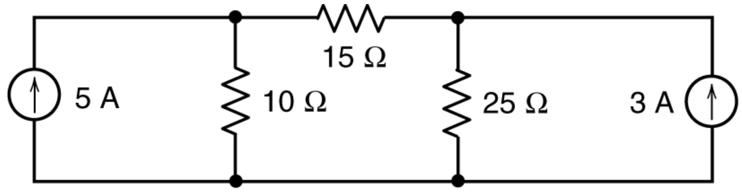
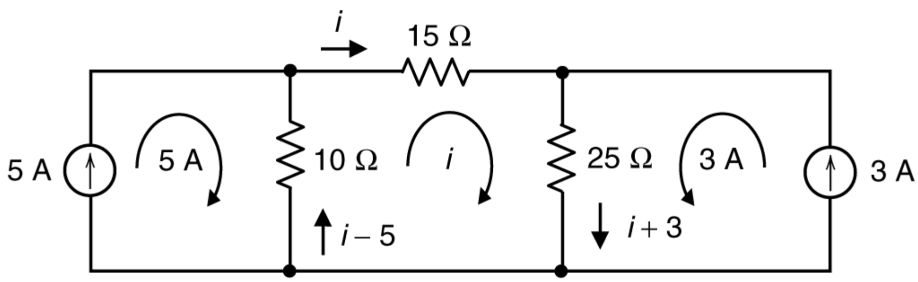


Figure P4.6-14

P4.6-14 Determine the values of the power supplied by the sources in the circuit shown Figure P4.6-14.

Solution: First, label the mesh currents, taking advantage of the current sources. Next, express the resistor currents in terms of the mesh currents:



Apply KVL to the middle mesh: $15i + 25(i+3) + 10(i-5) = 0 \Rightarrow i = -\frac{1}{2}$ A

The 5 A current source supplies $5(10)(i-5) = 5(10)\left(-\frac{1}{2} - 5\right) = 275$ W

The 3 A current source supplies $3(25)(i+3) = 3(25)\left(-\frac{1}{2} + 3\right) = 187.5$ W

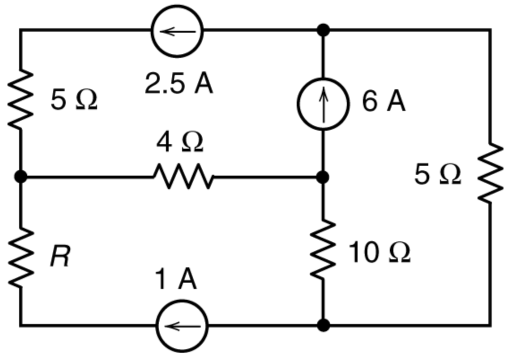
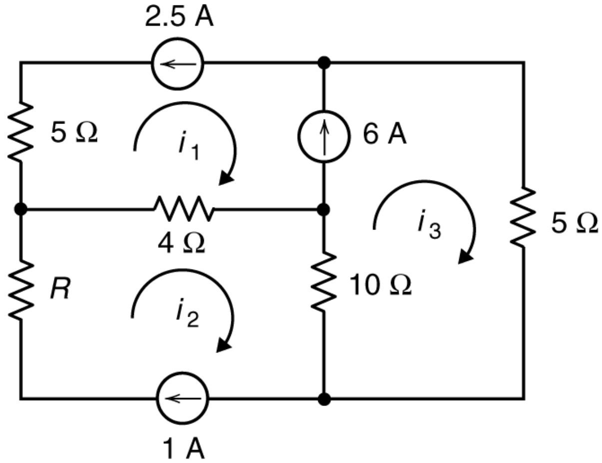


Figure P4.6-15

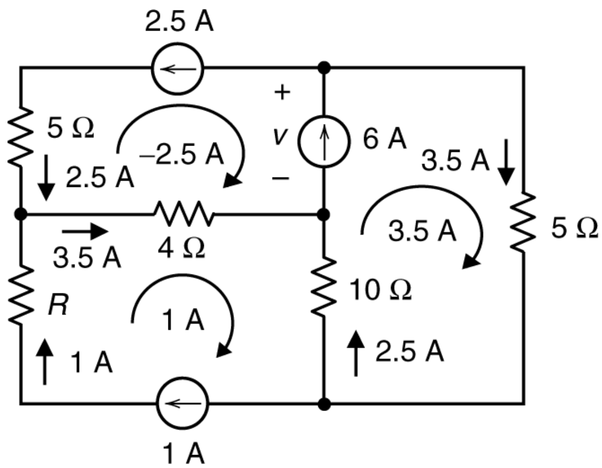
P4.6-15 Determine the values of the resistance *R* and of the power supplied by the 6 A current source in the circuit shown Figure P4.6-15.

Solution: First, label the mesh currents:



Notice that $i_1 = -2.5\text{ A},\ i_2 = 1\text{ A}$ and $6 = i_3 + 2.5 \Rightarrow i_3 = 3.5\text{ A}$

Next, express the resistor currents in terms of the mesh currents:



Apply KVL to the bottom, left mesh: $4(3.5) - 10(2.5) + R(1) = 0 \Rightarrow R = 11\ \Omega$

Apply KVL to the right mesh $3.5(5) + 2.5(10) - v = 0 \Rightarrow v = 42.5\text{ V}$

The 6 A current source supplies $6v = 6(42.5) = 255\text{ W}$

Section 4-7 Mesh Current Analysis with Dependent Sources

P 4.7-1 Find v_2 for the circuit shown in Figure P 4.7-1.

Answer: $v_2 = 10$ V

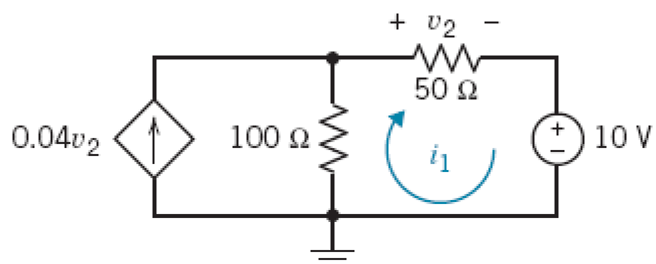


Figure P 4.7-1

Solution:

Express the controlling voltage of the dependent source as a function of the mesh current

$$v_2 = 50\, i_1$$

Apply KVL to the right mesh:

$$-100\ (0.04(50 i_1) - i_1) + 50 i_1 + 10 = 0 \Rightarrow i_1 = 0.2 \text{ A}$$
$$v_2 = 50\ i_1 = 10 \text{ V}$$

(checked using LNAP 8/14/02)

P4.7-2 Determine the values of the power supplied by the voltage source and by the CCCS in the circuit shown Figure P4.7-2

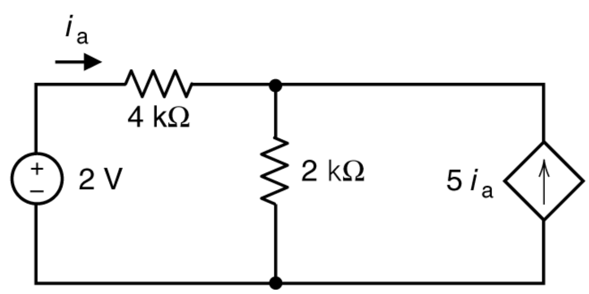
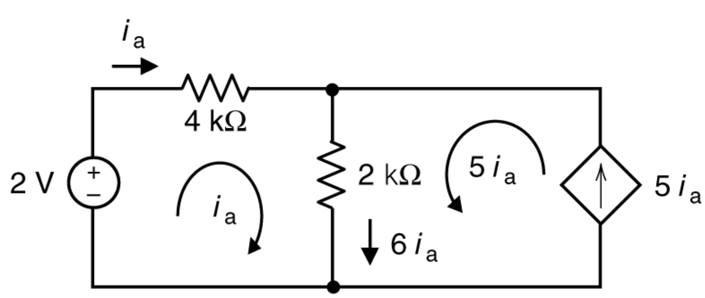


Figure P4.7-2

Solution: First, label the mesh currents, taking advantage of the current sources. Next, express the resistor currents in terms of the mesh currents:



Apply KVL to the left mesh: $4000\, i_a + 2000\left(6\, i_a\right) - 2 = 0 \quad \Rightarrow \quad i_a = \dfrac{1}{8} = 0.125 \text{ mA}$

The 2 A voltage source supplies $2\, i_a = 2\left(0.125 \times 10^{-3}\right) = 0.25$ mW

The CCCS supplies $\left(5\, i_a\right)\left[\left(2000\right)\left(6\, i_a\right)\right] = \left(60 \times 10^{3}\right)\left(0.125 \times 10^{-3}\right)^{2} = 0.9375 \times 10^{-3} = 0.9375$ mW

P 4.7-3 Find v_o for the circuit shown in Figure P 4.7-3.

Answer: $v_o = 2.5$ V

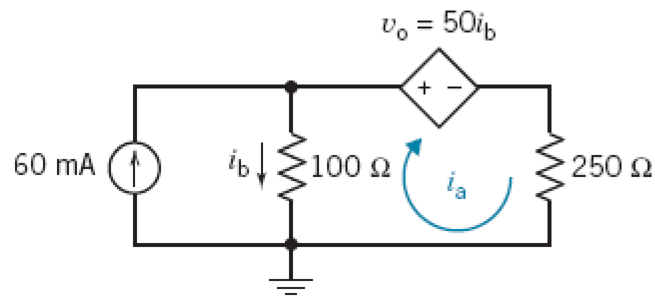


Figure P 4.7-3

Solution: Express the controlling current of the dependent source as a function of the mesh current:

$$i_b = .06 - i_a$$

Apply KVL to the right mesh:

$$-100\ (0.06 - i_a) + 50\ (0.06 - i_a) + 250\ i_a = 0 \quad \Rightarrow \quad i_a = 10 \text{ mA}$$

Finally: $$v_o = 50\, i_b = 50\,(0.06 - 0.01) \ = 2.5 \text{ V}$$

(checked using LNAP 8/14/02)

P 4.7-4 Determine the mesh current i_a for the circuit shown in Figure P 4.7-4.

Answer: $i_a = -24$ mA

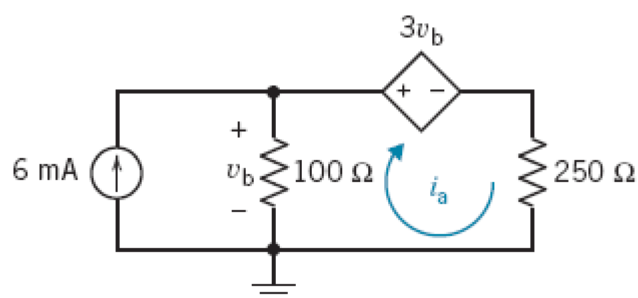


Figure P 4.7-4

Solution: Express the controlling voltage of the dependent source as a function of the mesh current:

$$v_b = 100\ (.006 - i_a)$$

Apply KVL to the right mesh:

$$-100\ (.006 - i_a) + 3\left[100(.006 - i_a)\right]\ + 250\, i_a \ = \ 0 \quad \Rightarrow \quad \underline{i_a = -24 \text{ mA}}$$

(checked using LNAP 8/14/02)

P 4.7-5 Although scientists continue to debate exactly why and how it works, the process of utilizing electricity to aid in the repair and growth of bones—which has been used mainly with fractures—may soon be extended to an array of other problems, ranging from osteoporosis and osteoarthritis to spinal fusions and skin ulcers.

An electric current is applied to bone fractures that have not healed in the normal period of time. The process seeks to imitate natural electrical forces within the body. It takes only a small amount of electric stimulation to accelerate bone recovery. The direct current method uses an electrode that is implanted at the bone. This method has a success rate approaching 80 percent.

The implant is shown in Figure P 4.7-5*a* and the circuit model is shown in Figure P 4.7-5*b*. Find the energy delivered to the cathode during a 24-hour period. The cathode is represented by the dependent voltage source and the 100-kΩ resistor.

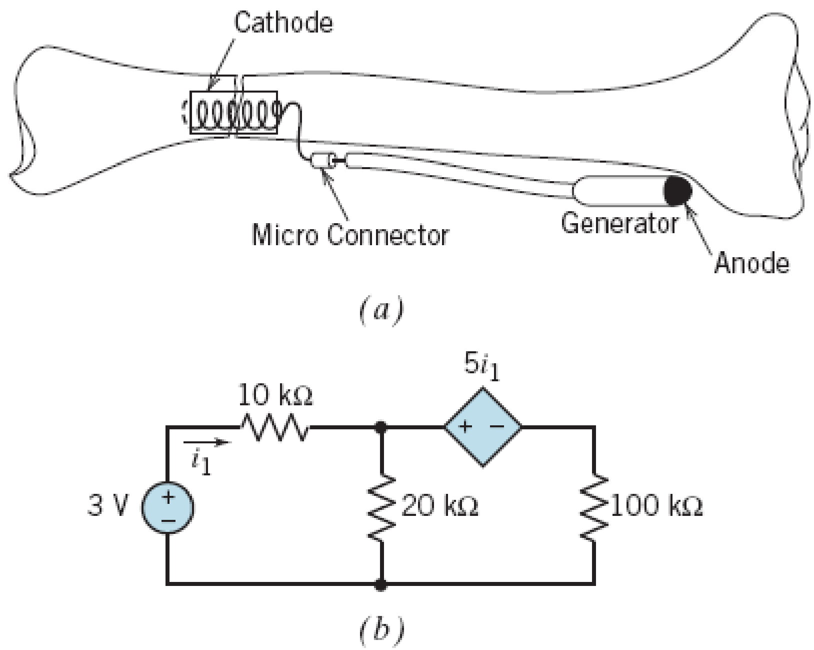
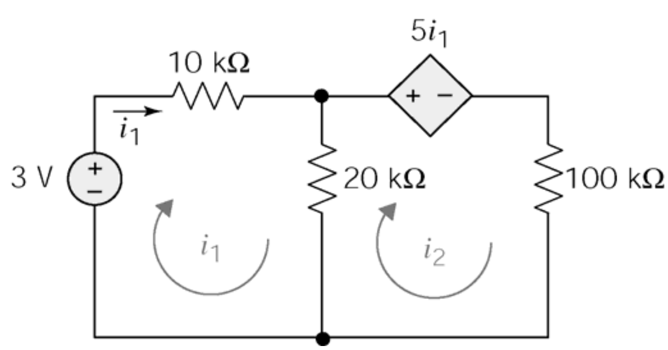


Figure P 4.7-5

Solution:



Apply KVL to left mesh : $-3+10\times10^3\, i_1+20\times10^3\left(i_1-i_2\right)=\ 0 \Rightarrow \underline{30\times10^3\, i_1-20\times10^3\, i_2=3}$ (1)

Apply KVL to right mesh : $5\times10^3\, i_1+100\times10^3\, i_2+20\times10^3\left(i_2-i_1\right)=0 \Rightarrow \underline{\ i_1\ =\ 8i_2\ }$ (2)

Solving (1) & (2) simultaneously $\Rightarrow\ i_1=\dfrac{6}{55}\text{ mA},\ i_2=\dfrac{3}{220}\text{ mA}$

$$\text{Power delivered to cathode} = (5i_1)\ (i_2) + 100(i_2)^2$$

$$= 5\left(\tfrac{6}{55}\right)\left(\tfrac{3}{220}\right) + 100\left(\tfrac{3}{220}\right)^2 = 0.026 \text{ mW}$$

$$\therefore \text{ Energy in 24 hr.} = \left(2.6\times10^{-5}\text{ W}\right)(24\text{ hr})\left(3600\ \tfrac{\text{s}}{\text{hr}}\right) = 2.25\text{ J}$$

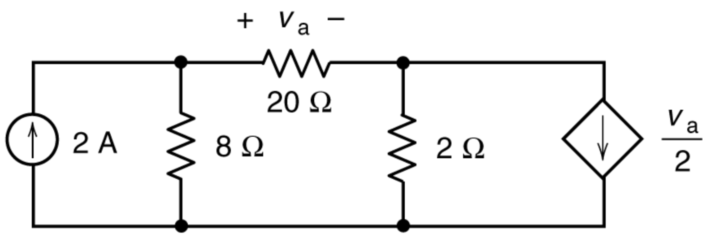
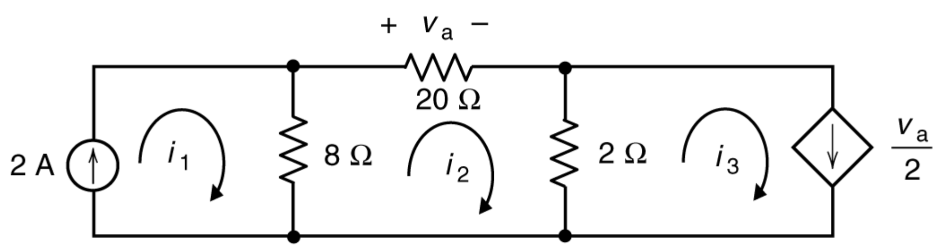


Figure P4.7-6

P4.7-6 Determine the value of the power supplied by the VCCS in the circuit shown Figure P4.7-6.

Solution: First, label the mesh currents.

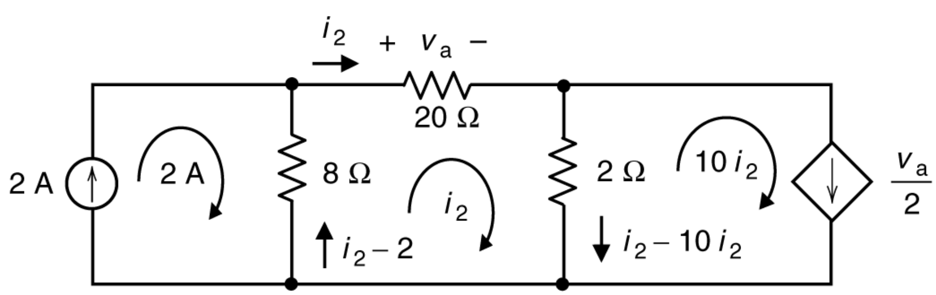


Next, express the controlling voltage of the VCCS in terms of the mesh currents:

$$v_a = 20\,i_2$$

Notice that $i_1 = 2\text{ A}$ and $i_3 = \dfrac{v_a}{2} = 10\,i_2$

Next, express the resistor currents in terms of the mesh currents:



Apply KVL to the middle mesh: $20\,i_2 + 2\left(i_2 - 10\,i_2\right) + 8\left(i_2 - 2\right) = 0 \;\Rightarrow\; i_2 = 1.6\text{ A}$

Consequently $v_a = 20\,i_2 = 20(1.6) = 32\text{ V}$ and $i_3 = \dfrac{v_a}{2} = \dfrac{32}{2} = 16\text{ A}$

The VCCS supplies $\dfrac{v_a}{2}\left[2\left(i_3 - i_2\right)\right] = \dfrac{32}{2}(2)(16 - 1.6) = 460.8\text{ W}$

P 4.7-7 The currents i_1, i_2 and i_3 are the mesh currents of the circuit shown in Figure P 4.7-7. Determine the values of i_1, i_2, and i_3.

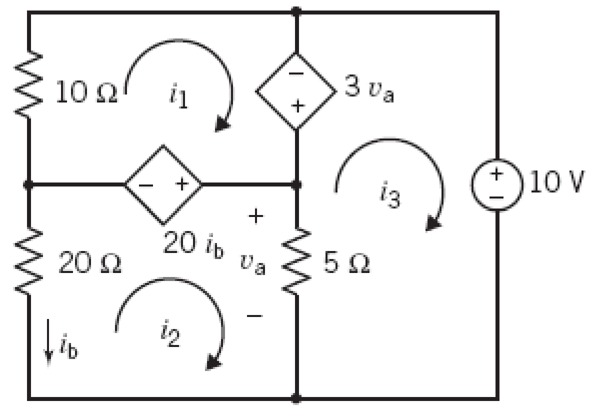


Figure P 4.7-7

Solution:

Express v_a and i_b, the controlling voltage and current of the dependent sources, in terms of the mesh currents

$$v_a = 5(i_2 - i_3) \text{ and } i_b = -i_2$$

Next express 20 i_b and 3 v_a, the controlled voltages of the dependent sources, in terms of the mesh currents

$$20\ i_b = -20\ i_2 \text{ and } 3\ v_a = 15(i_2 - i_3)$$

Apply KVL to the meshes

$$-15(i_2 - i_3) + (-20\ i_2) + 10\ i_1 = 0$$
$$-(-20\ i_2) + 5(i_2 - i_3) + 20\ i_2 = 0$$
$$10 - 5(i_2 - i_3) + 15\ (i_2 - i_3) = 0$$

These equations can be written in matrix form

$$\begin{bmatrix} 10 & -35 & 15 \\ 0 & 45 & -5 \\ 0 & 10 & -10 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -10 \end{bmatrix}$$

Solving, e.g. using MATLAB, gives

$$i_1 = -1.25 \text{ A, } i_2 = +0.125 \text{ A, and } i_3 = +1.125 \text{ A}$$

(checked: MATLAB & LNAP 5/19/04)

P 4.7-8 Determine the value of the power supplied by the dependent source in Figure P 4.7-8.

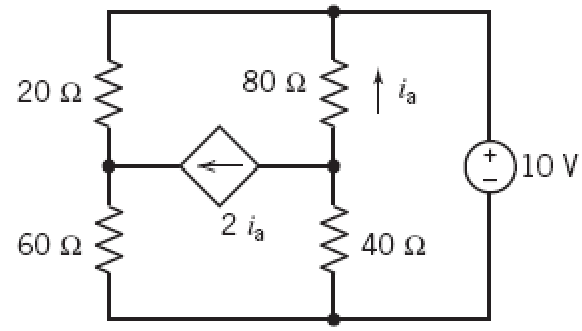
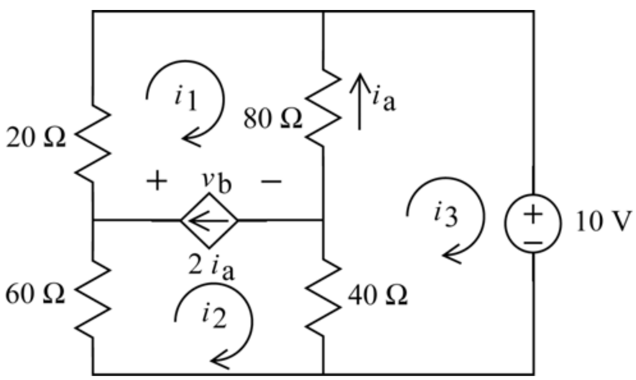


Figure P 4.7-8

Solution: Label the mesh currents:



Express i_a, the controlling current of the CCCS, in terms of the mesh currents

$$i_a = i_3 - i_1$$

Express 2 i_a, the controlled current of the CCCS, in terms of the mesh currents:

$$i_1 - i_2 = 2\ i_a = 2(i_3 - i_1) \quad \Rightarrow \quad 3\ i_1 - i_2 - 2\ i_3 = 0$$

Apply KVL to the supermesh corresponding to the CCCS:

$$80(i_1 - i_3) + 40(i_2 - i_3) + 60\ i_2 + 20\ i_1 = 0 \quad \Rightarrow \quad 100 i_1 + 100 i_2 - 120 i_3 = 0$$

Apply KVL to mesh 3

$$10 + 40(i_3 - i_2) + 80(i_3 - i_1) = 0 \quad \Rightarrow \quad -80\ i_1 - 40\ i_2 + 120\ i_3 = -10$$

These three equations can be written in matrix form

$$\begin{bmatrix} 3 & -1 & -2 \\ 100 & 100 & -120 \\ -80 & -40 & 120 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -10 \end{bmatrix}$$

Solving, e.g. using MATLAB, gives

$$i_1 = -0.2 \text{ A}, \ i_2 = -0.1 \text{ A and } i_3 = -0.25 \text{ A}$$

Apply KVL to mesh 2 to get

$$v_b + 40(i_2 - i_3) + 60 i_2 = 0 \quad \Rightarrow \quad v_b = -40(-0.1 - (-0.25)) - 60(-0.1) = 0 \text{ V}$$

So the power supplied by the dependent source is $p = v_b(2 i_a) = 0$ W.

(checked: LNAP 6/7/04)

P 4.7-9 Determine the value of the resistance R in the circuit shown in Figure P 4.7-9.

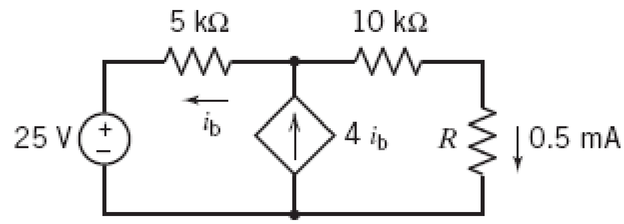


Figure P 4.7-9

Solution:

Notice that i_b and 0.5 mA are the mesh currents. Apply KCL at the top node of the dependent source to get

$$i_b + 0.5\times10^{-3} = 4\,i_b \quad\Rightarrow\quad i_b = \frac{1}{6}\text{ mA}$$

Apply KVL to the supermesh corresponding to the dependent source to get

$$-5000\,i_b + (10000+R)\left(0.5\times10^{-3}\right) - 25 = 0$$

$$-5000\left(\frac{1}{6}\times10^{-3}\right) + (10000+R)\left(0.5\times10^{-3}\right) = 25$$

$$R = \frac{\frac{125}{6}}{0.5\times10^{-3}} = 41.67\text{ k}\Omega$$

(checked: LNAP 6/21/04)

P 4.7-10 The circuit shown in Figure P 4.7-10 is the small signal model of an amplifier. The input to the amplifier is the voltage source voltage, v_s. The output of the amplifier is the voltage v_o.

(a) The ratio of the output to the input, v_o/v_s, is called the gain of the amplifier. Determine the gain of the amplifier.

(b) The ratio of the current of the input source to the input voltage, i_b/v_s, is called the input resistance of the amplifier. Determine the input resistance.

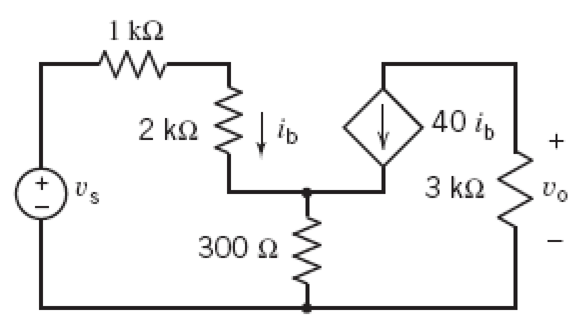


Figure P 4.7-10

Solution: The controlling and controlled currents of the CCCS, i_b and 40 i_b, are the mesh currents. Apply KVL to the left mesh to get

$$1000\,i_b + 2000\,i_b + 300\left(i_b + 40 i_b\right) - v_s = 0 \quad\Rightarrow\quad 15300 i_b = v_s$$

The output is given by $$v_o = -3000\left(40\,i_b\right) = -120000\,i_b$$

(a) The gain is $$\frac{v_o}{v_s} = -\frac{120000}{15300} = -7.84\text{ V/V}$$

(b) The input resistance is $$\frac{v_s}{i_b} = 15300\ \Omega$$

(checked: LNAP 5/24/04)

P 4.7-11 Determine the values of the mesh currents of the circuit shown in Figure P 4.7-11.

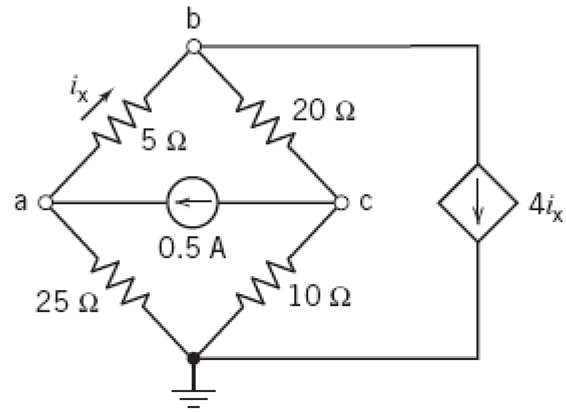
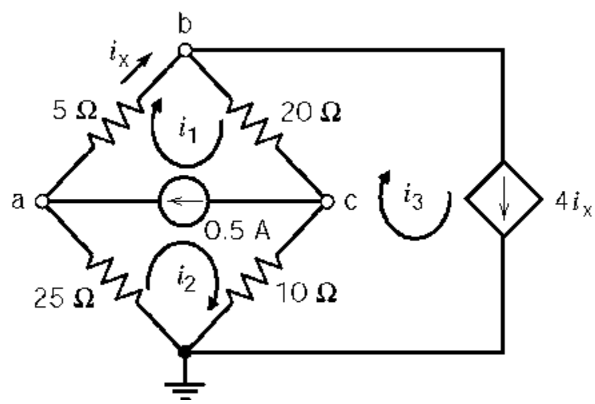


Figure P 4.7-11

Solution:
Label the mesh currents.



Express i_x in terms of the mesh currents:

$$i_x = i_1$$

Express $4i_x$ in terms of the mesh currents:

$$4i_x = i_3$$

Express the current source current in terms of the mesh currents to get:

$$0.5 = i_1 - i_2 \quad \Rightarrow \quad i_2 = i_x - 0.5$$

Apply KVL to supermesh corresponding to the current source to get

$$5i_1 + 20\left(i_1 - i_3\right) + 10\left(i_2 - i_3\right) + 25i_2 = 0$$

Substituting gives

$$5i_x + 20\left(-3i_x\right) + 10\left(i_x - 0.5 - 4i_x\right) + 25\left(i_x - 0.5\right) = 0 \quad \Rightarrow \quad i_x = -\frac{35}{120} = -0.29167$$

So the mesh currents are

$$i_1 = i_x = -0.29167 \quad \text{A}$$
$$i_2 = i_x - 0.5 = -0.79167 \quad \text{A}$$
$$i_3 = 4i_x = -1.1667 \quad \text{A}$$

(checked: LNAP 6/21/04)

P 4.7-12 The currents i_1, i_2, and i_3 are the mesh currents corresponding to meshes 1, 2, and 3 in Figure P 4.7-12. Determine the values of these mesh currents.

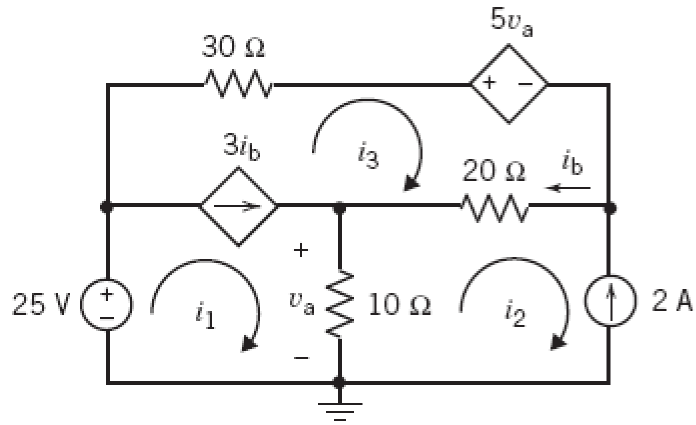
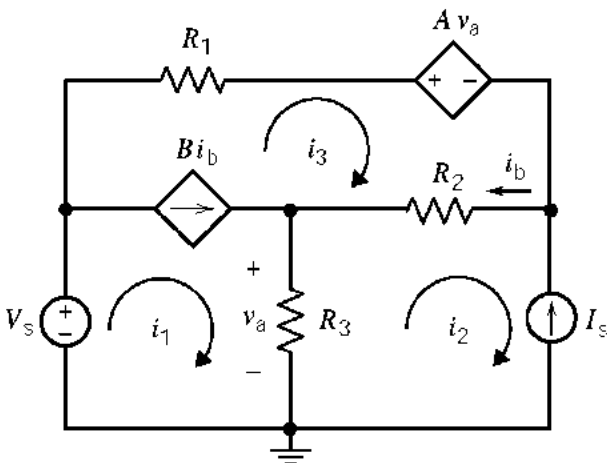


Figure P 4.7-12

Solution:



Express the controlling voltage and current of the dependent sources in terms of the mesh currents:

$$v_a = R_3(i_1 - i_2) \quad \text{and} \quad i_b = i_3 - i_2$$

Express the current source currents in terms of the mesh currents:

$$i_2 = -I_s \quad \text{and} \quad i_1 - i_3 = B\,i_b = B(i_3 - i_2)$$

Consequently

$$i_1 - (B+1)i_3 = B\,I_s$$

Apply KVL to the supermesh corresponding to the dependent current source

$$R_1\,i_3 + A R_3(i_1 - i_2) + R_2(i_3 - i_2) + R_3(i_1 - i_2) - V_s = 0$$

or

$$(A+1)R_3\,i_1 - (R_2 + (A+1)R_3)i_2 + (R_1 + R_2)i_3 = V_s$$

Organizing these equations into matrix form:

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & -(B+1) \\ (A+1)R_3 & -(R_2+(A+1)R_3) & R_1+R_2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} -I_s \\ B\,I_s \\ V_s \end{bmatrix}$$

With the given values:

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & -4 \\ 60 & -80 & 50 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 6 \\ 25 \end{bmatrix} \Rightarrow \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} -0.8276 \\ -2 \\ -1.7069 \end{bmatrix} \text{ A}$$

(Checked using LNAP 9/29/04)

P 4.7-13 The currents i_1, i_2, and i_3 are the mesh currents corresponding to meshes 1, 2, and 3 in Figure P 4.7-13. The values of these currents are

$i_1 = -1.375$ A, $i_2 = -2.5$ A, and $i_3 = -3.25$ A

Determine the values of the gains of the dependent source, A and B.

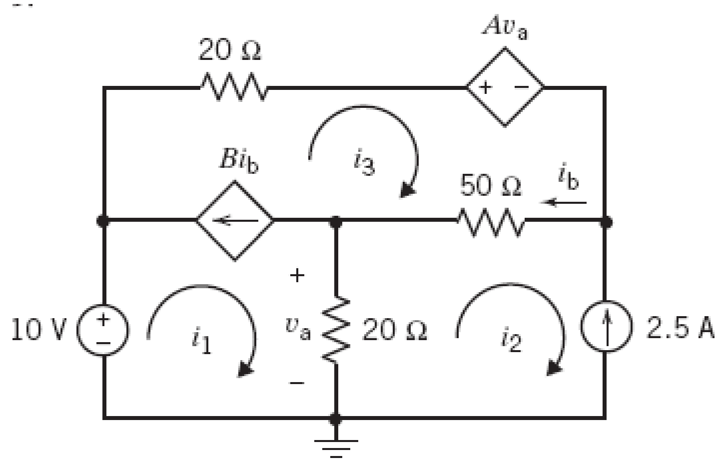


Figure P 4.7-13

Solution: Express the controlling voltage and current of the dependent sources in terms of the mesh currents:

$$v_a = 20(i_1 - i_2) = 20(-1.375-(-2.5)) = 22.5$$

and

$$i_b = i_3 - i_2 = -3.25-(-2.5) = -0.75 \text{ A}$$

Express the current source currents in terms of the mesh currents:

$$i_2 = -2.5 \text{ A}$$

and

$$i_3 - i_1 = B\, i_b \quad \Rightarrow \quad -1.375-(-2.5) = B(-0.75) \quad \Rightarrow \quad B = 2.5 \text{ A/A}$$

Apply KVL to the supermesh corresponding to the dependent current source

$$0 = 20\, i_3 + A v_a + 50\, i_b + v_a - 10 = 20\,(-3.25) + A(22.5) + 50(-0.75) + 22.5 - 10 \quad \Rightarrow \quad A = 4 \text{ V/V}$$

(Checked using LNAP 9/29/04)

P 4.7-14 Determine the current *i* in the circuit shown in Figure P 4.7-14.

Answer: $i = 3$ A

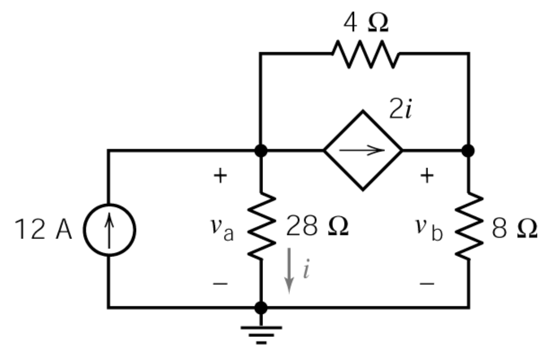


Figure P 4.7-14

Solution:

Label the node voltages as shown. The controlling currents of the CCCS is expressed as $i = \frac{v_a}{28}$.

The node equations are $$12 = \frac{v_a}{28} + \frac{v_a - v_b}{4} + \frac{v_a}{14}$$

and $$\frac{v_a - v_b}{4} + \frac{v_a}{14} = \frac{v_b}{8}$$

Solving the node equations gives $v_a = 84$ V and $v_b = 72$ V . Then $i = \frac{v_a}{28} = \frac{84}{28} = 3$ A .

(checked using LNAP 6/16/05)

P4.7-15 Determine the values of the mesh currents i_1 and i_2 for the circuit shown in Figure P4.7-15.

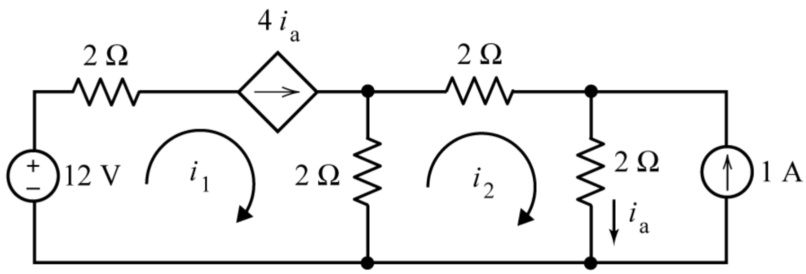


Figure P4.7-15

Solution: Expressing the dependent source currents in terms of the mesh currents we get:

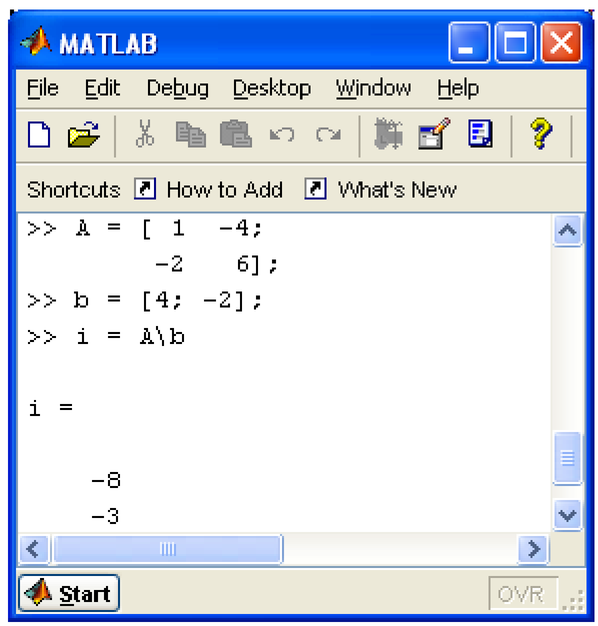
$$i_1 = 4i_a = 4(i_2 + 1) \quad \Rightarrow \quad 4 = i_1 - 4i_2$$

Apply KVL to mesh 2 to get

$$2i_2 + 2(i_2 + 1) - 2(i_1 - i_2) = 0 \quad \Rightarrow \quad -2 = -2i_1 + 6i_2$$

Solving these equations using MATLAB we get

$i_1 = -8$ A and $i_2 = -3$ A



P4.7-16 Determine the values of the mesh currents i_1 and i_2 for the circuit shown in Figure P4.7-16 **.**

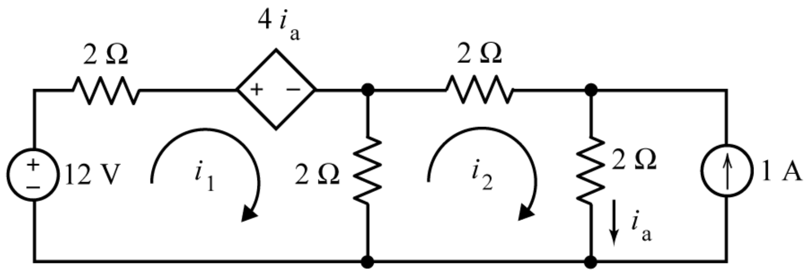


Figure P4.7-16

Apply KVL to mesh 1 to get

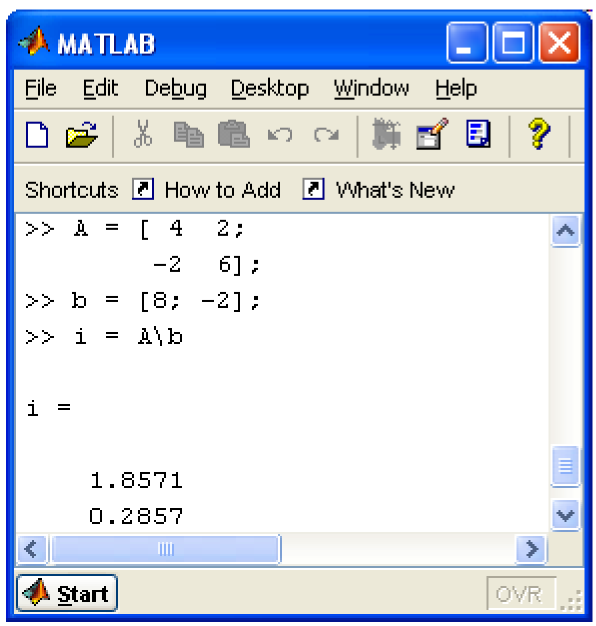
$$2i_1 + 4i_a + 2(i_1 - i_2) - 12 = 0 \quad \Rightarrow \quad 2i_1 + 4(i_2 + 1) + 2(i_1 - i_2) - 12 = 0 \quad \Rightarrow \quad 8 = 4i_1 + 2i_2$$

Apply KVL to mesh 2 to get

$$2i_2 + 2(i_2 + 1) - 2(i_1 - i_2) = 0 \quad \Rightarrow \quad -2 = -2i_1 + 6i_2$$

Solving these equations using MATLAB we get

$i_1 = 1.8571$ A and $i_2 = 0.2857$ A



Section 4.8 The Node Voltage Method and Mesh Current Method Compared

P 4.8-2 The circuit shown in Figure P 4.8-2 has two inputs, v_s and i_s, and one output v_o. The output is related to the inputs by the equation

$$v_o = a i_s + b v_s$$

where a and b are constants to be determined. Determine the values a and b by

(a) writing and solving mesh equations and

(b) writing and solving node equations.

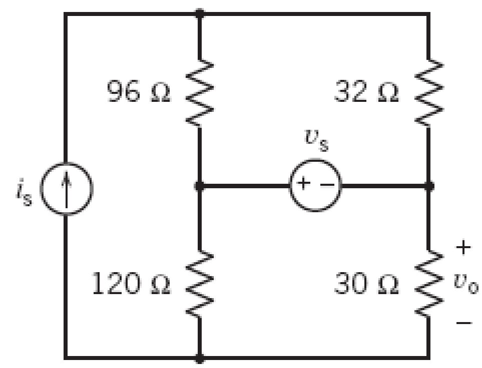
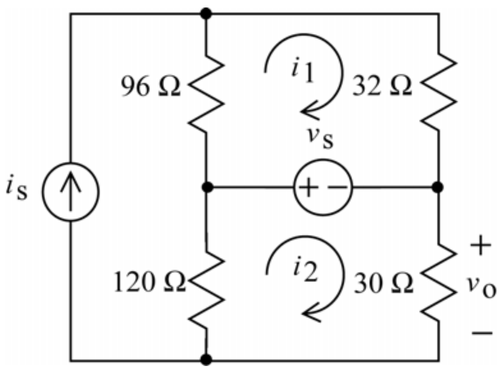


Figure P 4.8-2

Solution:

(a)



Apply KVL to meshes 1 and 2:

$$32 i_1 - v_s + 96\left(i_1 - i_s\right) = 0$$

$$v_s + 30 i_2 + 120\left(i_2 - i_s\right) = 0$$

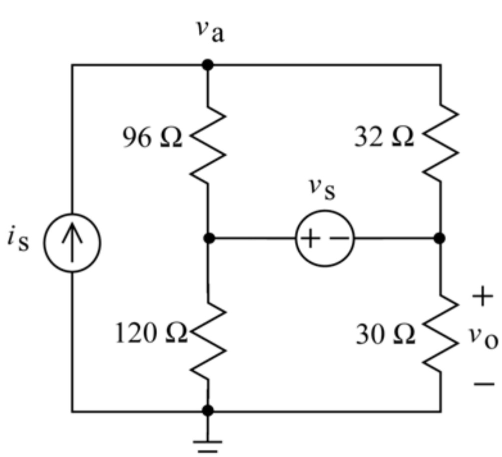
$$150 i_2 = +120 i_s - v_s$$

$$i_2 = \frac{4}{5} i_s - \frac{v_s}{150}$$

$$v_o = 30 i_2 = 24 i_s - \frac{1}{5} v_s$$

So a = 24 and b = -.02.

(b)



Apply KCL to the supernode corresponding to the voltage source to get

$$\frac{v_a - \left(v_s + v_o\right)}{96} + \frac{v_a - v_o}{32} = \frac{v_s + v_o}{120} + \frac{v_o}{30}$$

So

$$i_s = \frac{v_s + v_o}{120} + \frac{v_o}{30} = \frac{v_s}{120} + \frac{v_o}{24}$$

Then

$$v_o = 24 i_s - \frac{1}{5} v_s$$

So a = 24 and b = -0.2.

(checked: LNAP 5/24/04)

P 4.8-3 Determine the power supplied by the dependent source in the circuit shown in Figure P 4.8-3 by writing and solving

(a) node equations and

(b) mesh equations.

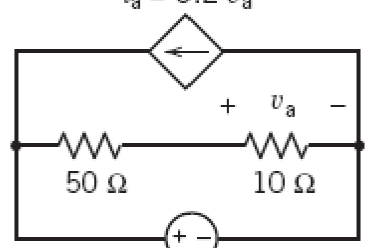
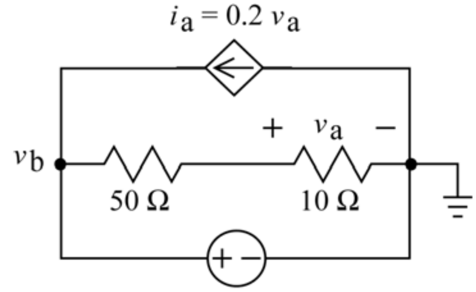


Figure P 4.8-3

Solution:

(a) Label the reference node and node voltages.



$$v_b = 120 \text{ V}$$

due to the voltage source.

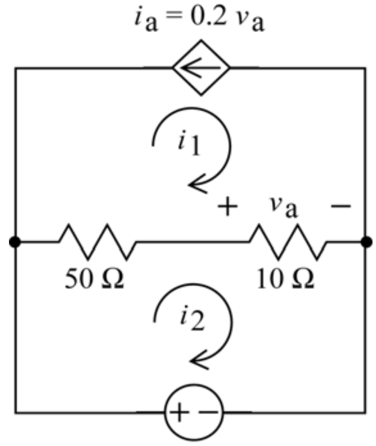
Apply KCL at the node between the resistors to get

$$\frac{v_b - v_a}{50} = \frac{v_a}{10} \quad \Rightarrow \quad v_a = 20 \text{ V}$$

and the power supplied by the dependent source is

$$p = v_b i_a = (120)\left[0.2(20)\right] = 480 \text{ W}$$

(b) Label the mesh currents. Express the controlling voltage of the dependent source in terms of the mesh current to get



$$v_a = 10\left(i_2 - i_1\right)$$

Express the controlled current of the dependent source in terms of the mesh currents to get

$$-i_1 = i_a = 0.2\left[10\left(i_2 - i_1\right)\right] = 2i_2 - 2i_1 \quad \Rightarrow \quad i_1 = 2i_2$$

Apply KVL to the bottom mesh to get

$$50\left(i_2 - i_1\right) + 10\left(i_2 - i_1\right) - 120 = 0 \quad \Rightarrow \quad i_2 - i_1 = 2$$

So $\quad i_2 - 2i_2 = 2 \quad \Rightarrow \quad i_2 = -2 \text{ A} \quad \Rightarrow \quad i_1 = -4 \text{ A}$

Then $\quad v_a = 10\left(-2-(-4)\right) = 20 \text{ V}$ and $i_a = 0.2(20) = 4 \text{ A}$

The power supplied by the dependent source is

$$p = 120\left(i_a\right) = 120(4) 480 \text{ W}$$

(checked: LNAP 6/21/04)

Section 4.8 The Node Voltage Method and Mesh Current Method Compared

P 4.8-2 The circuit shown in Figure P 4.8-2 has two inputs, v_s and i_s, and one output v_o. The output is related to the inputs by the equation

$$v_o = a i_s + b v_s$$

where a and b are constants to be determined. Determine the values a and b by

(a) writing and solving mesh equations and

(b) writing and solving node equations.

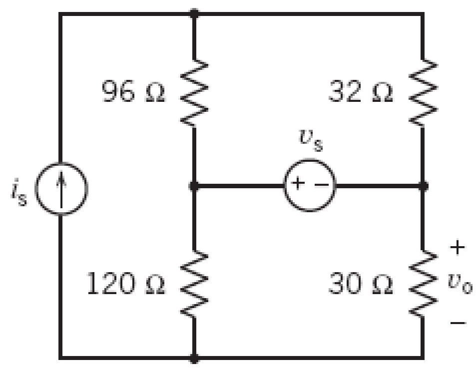
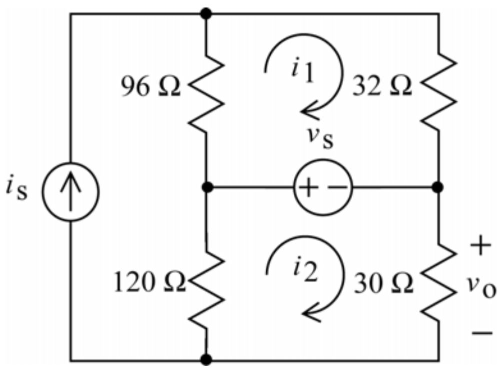


Figure P 4.8-2

Solution:

(a)

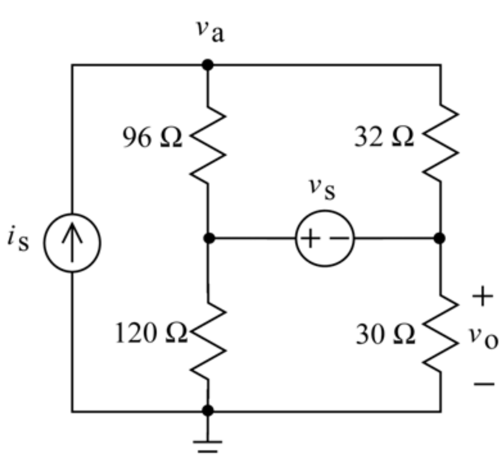


Apply KVL to meshes 1 and 2:

$$32i_1 - v_s + 96\left(i_1 - i_s\right) = 0$$
$$v_s + 30i_2 + 120\left(i_2 - i_s\right) = 0$$
$$150i_2 = +120i_s - v_s$$
$$i_2 = \frac{4}{5}i_s - \frac{v_s}{150}$$
$$v_o = 30i_2 = 24i_s - \frac{1}{5}v_s$$

So a = 24 and b = -.02.

(b)



Apply KCL to the supernode corresponding to the voltage source to get

$$\frac{v_a - \left(v_s + v_o\right)}{96} + \frac{v_a - v_o}{32} = \frac{v_s + v_o}{120} + \frac{v_o}{30}$$

So

$$i_s = \frac{v_s + v_o}{120} + \frac{v_o}{30} = \frac{v_s}{120} + \frac{v_o}{24}$$

Then

$$v_o = 24i_s - \frac{1}{5}v_s$$

So a = 24 and b = -0.2.

(checked: LNAP 5/24/04)

P 4.8-3 Determine the power supplied by the dependent source in the circuit shown in Figure P 4.8-3 by writing and solving

(a) node equations and

(b) mesh equations.

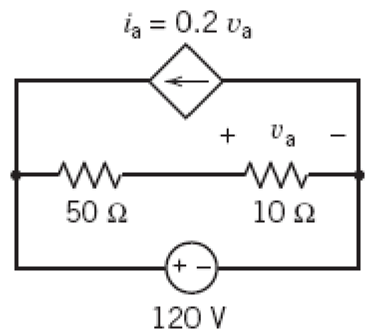


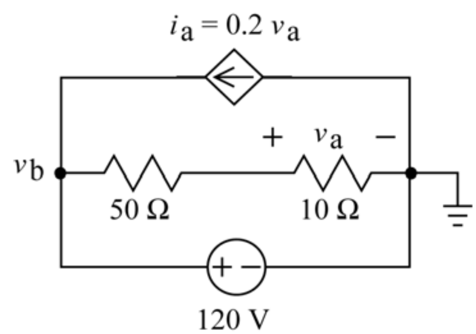
Figure P 4.8-3

Solution:

(a) Label the reference node and node voltages.

$$v_b = 120 \text{ V}$$

due to the voltage source.



Apply KCL at the node between the resistors to get

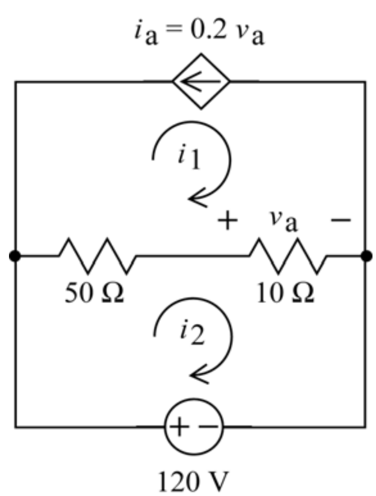
$$\frac{v_b - v_a}{50} = \frac{v_a}{10} \quad \Rightarrow \quad v_a = 20 \text{ V}$$

and the power supplied by the dependent source is

$$p = v_b i_a = (120)\left[0.2(20)\right] = 480 \text{ W}$$

(b) Label the mesh currents. Express the controlling voltage of the dependent source in terms of the mesh current to get

$$v_a = 10\left(i_2 - i_1\right)$$



Express the controlled current of the dependent source in terms of the mesh currents to get

$$-i_1 = i_a = 0.2\left[10\left(i_2 - i_1\right)\right] = 2i_2 - 2i_1 \quad \Rightarrow \quad i_1 = 2i_2$$

Apply KVL to the bottom mesh to get

$$50\left(i_2 - i_1\right) + 10\left(i_2 - i_1\right) - 120 = 0 \quad \Rightarrow \quad i_2 - i_1 = 2$$

So $\quad i_2 - 2i_2 = 2 \quad \Rightarrow \quad i_2 = -2 \text{ A} \quad \Rightarrow \quad i_1 = -4 \text{ A}$

Then $\quad v_a = 10\left(-2 - (-4)\right) = 20 \text{ V}$ and $i_a = 0.2(20) = 4 \text{ A}$

The power supplied by the dependent source is

$$p = 120\left(i_a\right) = 120(4)480 \text{ W}$$

(checked: LNAP 6/21/04)

Section 4.8 The Node Voltage Method and Mesh Current Method Compared

P 4.8-2 The circuit shown in Figure P 4.8-2 has two inputs, v_s and i_s, and one output v_o. The output is related to the inputs by the equation

$$v_o = a i_s + b v_s$$

where a and b are constants to be determined. Determine the values a and b by

(a) writing and solving mesh equations and

(b) writing and solving node equations.

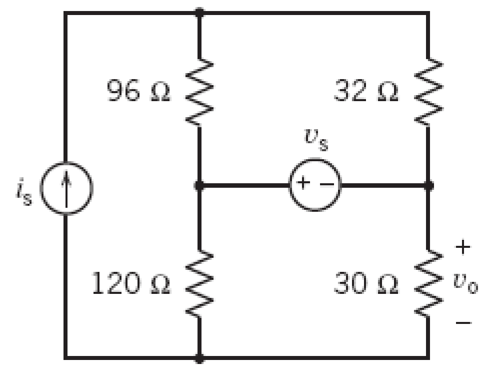
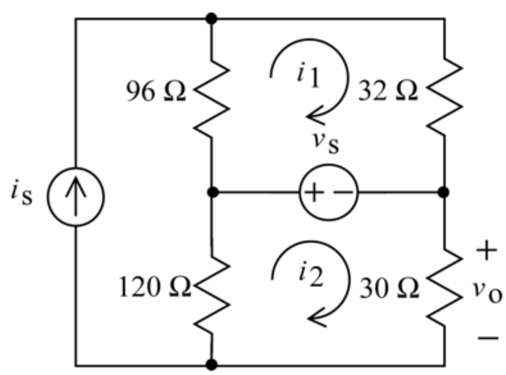


Figure P 4.8-2

Solution:

(a)



Apply KVL to meshes 1 and 2:

$$32 i_1 - v_s + 96\left(i_1 - i_s\right) = 0$$

$$v_s + 30 i_2 + 120\left(i_2 - i_s\right) = 0$$

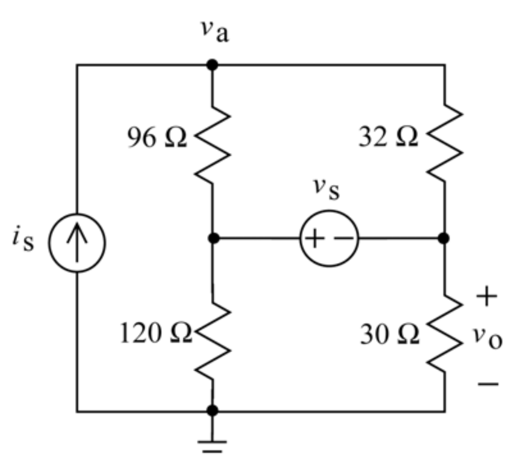
$$150 i_2 = +120 i_s - v_s$$

$$i_2 = \frac{4}{5} i_s - \frac{v_s}{150}$$

$$v_o = 30 i_2 = 24 i_s - \frac{1}{5} v_s$$

So a = 24 and b = -.02.

(b)



Apply KCL to the supernode corresponding to the voltage source to get

$$\frac{v_a - \left(v_s + v_o\right)}{96} + \frac{v_a - v_o}{32} = \frac{v_s + v_o}{120} + \frac{v_o}{30}$$

So

$$i_s = \frac{v_s + v_o}{120} + \frac{v_o}{30} = \frac{v_s}{120} + \frac{v_o}{24}$$

Then

$$v_o = 24 i_s - \frac{1}{5} v_s$$

So a = 24 and b = -0.2.

(checked: LNAP 5/24/04)

P 4.8-3 Determine the power supplied by the dependent source in the circuit shown in Figure P 4.8-3 by writing and solving

(a) node equations and

(b) mesh equations.

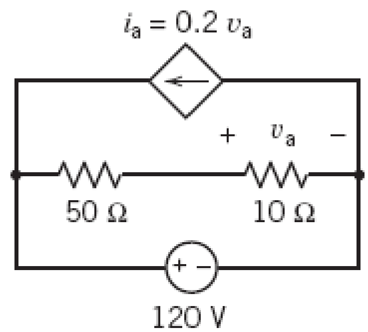
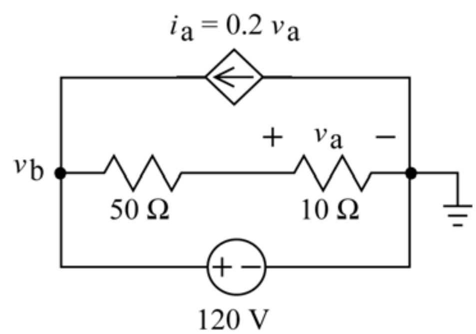


Figure P 4.8-3

Solution:

(a) Label the reference node and node voltages.



$$v_b = 120\text{ V}$$

due to the voltage source.

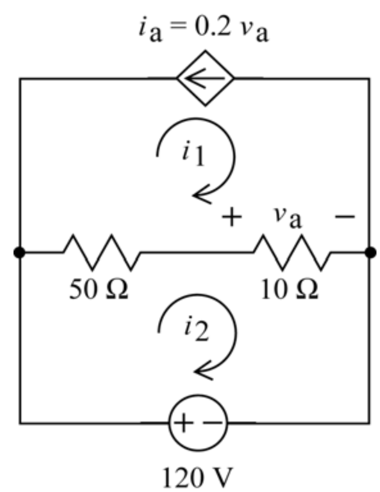
Apply KCL at the node between the resistors to get

$$\frac{v_b - v_a}{50} = \frac{v_a}{10} \quad \Rightarrow \quad v_a = 20\text{ V}$$

and the power supplied by the dependent source is

$$p = v_b i_a = (120)[0.2(20)] = 480\text{ W}$$

(b) Label the mesh currents. Express the controlling voltage of the dependent source in terms of the mesh current to get



$$v_a = 10(i_2 - i_1)$$

Express the controlled current of the dependent source in terms of the mesh currents to get

$$-i_1 = i_a = 0.2[10(i_2 - i_1)] = 2i_2 - 2i_1 \quad \Rightarrow \quad i_1 = 2i_2$$

Apply KVL to the bottom mesh to get

$$50(i_2 - i_1) + 10(i_2 - i_1) - 120 = 0 \quad \Rightarrow \quad i_2 - i_1 = 2$$

So $\quad i_2 - 2i_2 = 2 \quad \Rightarrow \quad i_2 = -2\text{ A} \quad \Rightarrow \quad i_1 = -4\text{ A}$

Then $\quad v_a = 10(-2-(-4)) = 20\text{ V}$ and $i_a = 0.2(20) = 4\text{ A}$

The power supplied by the dependent source is

$$p = 120(i_a) = 120(4) 480\text{ W}$$

(checked: LNAP 6/21/04)

Section 4.8 The Node Voltage Method and Mesh Current Method Compared

P 4.8-2 The circuit shown in Figure P 4.8-2 has two inputs, v_s and i_s, and one output v_o. The output is related to the inputs by the equation

$$v_o = ai_s + bv_s$$

where a and b are constants to be determined. Determine the values a and b by

(a) writing and solving mesh equations and

(b) writing and solving node equations.

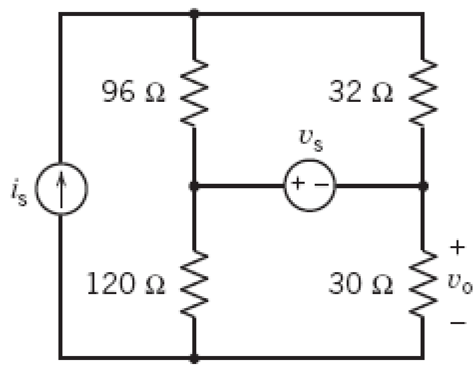
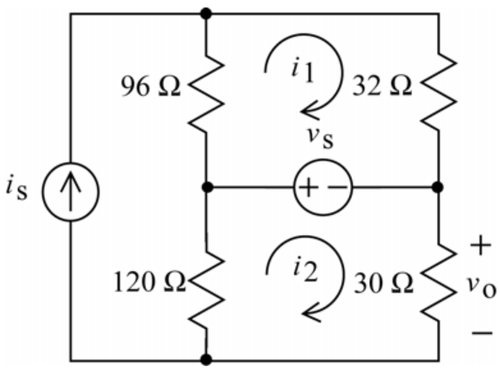


Figure P 4.8-2

Solution:

(a)



Apply KVL to meshes 1 and 2:

$$32i_1 - v_s + 96(i_1 - i_s) = 0$$

$$v_s + 30i_2 + 120(i_2 - i_s) = 0$$

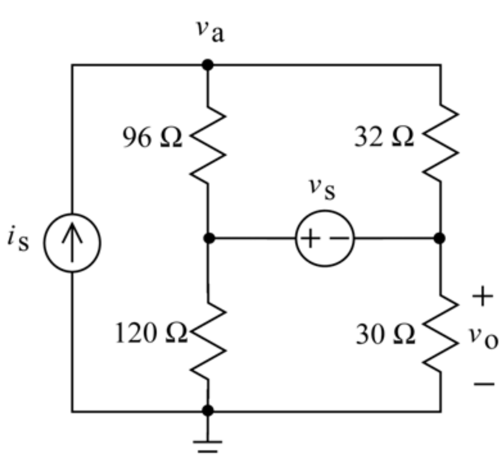
$$150i_2 = +120i_s - v_s$$

$$i_2 = \frac{4}{5}i_s - \frac{v_s}{150}$$

$$v_o = 30i_2 = 24i_s - \frac{1}{5}v_s$$

So a = 24 and b = -.02.

(b)



Apply KCL to the supernode corresponding to the voltage source to get

$$\frac{v_a - (v_s + v_o)}{96} + \frac{v_a - v_o}{32} = \frac{v_s + v_o}{120} + \frac{v_o}{30}$$

So

$$i_s = \frac{v_s + v_o}{120} + \frac{v_o}{30} = \frac{v_s}{120} + \frac{v_o}{24}$$

Then

$$v_o = 24i_s - \frac{1}{5}v_s$$

So a = 24 and b = -0.2.

(checked: LNAP 5/24/04)

P 4.8-3 Determine the power supplied by the dependent source in the circuit shown in Figure P 4.8-3 by writing and solving

(a) node equations and

(b) mesh equations.

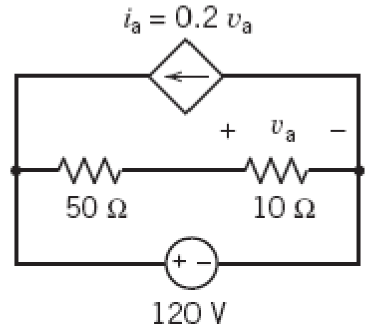
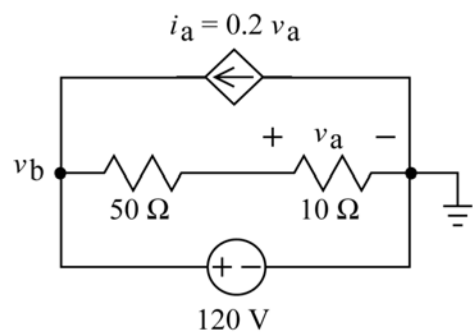


Figure P 4.8-3

Solution:

(a) Label the reference node and node voltages.



$$v_b = 120\text{ V}$$

due to the voltage source.

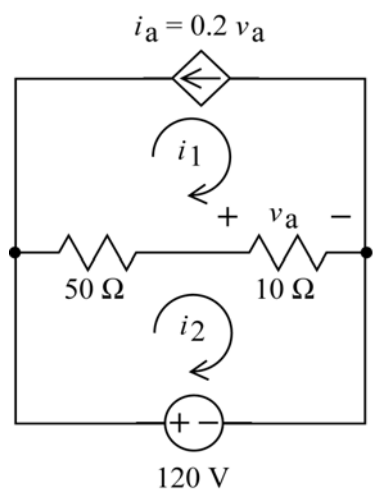
Apply KCL at the node between the resistors to get

$$\frac{v_b - v_a}{50} = \frac{v_a}{10} \quad \Rightarrow \quad v_a = 20\text{ V}$$

and the power supplied by the dependent source is

$$p = v_b i_a = (120)\left[0.2(20)\right] = 480\text{ W}$$

(b) Label the mesh currents. Express the controlling voltage of the dependent source in terms of the mesh current to get



$$v_a = 10(i_2 - i_1)$$

Express the controlled current of the dependent source in terms of the mesh currents to get

$$-i_1 = i_a = 0.2\left[10(i_2 - i_1)\right] = 2i_2 - 2i_1 \quad \Rightarrow \quad i_1 = 2i_2$$

Apply KVL to the bottom mesh to get

$$50(i_2 - i_1) + 10(i_2 - i_1) - 120 = 0 \quad \Rightarrow \quad i_2 - i_1 = 2$$

So $\quad i_2 - 2i_2 = 2 \quad \Rightarrow \quad i_2 = -2\text{ A} \quad \Rightarrow \quad i_1 = -4\text{ A}$

Then $\quad v_a = 10(-2-(-4)) = 20\text{ V}$ and $i_a = 0.2(20) = 4\text{ A}$

The power supplied by the dependent source is

$$p = 120(i_a) = 120(4)480\text{ W}$$

(checked: LNAP 6/21/04)

Section 4.9 Circuit Analysis Using MATLAB

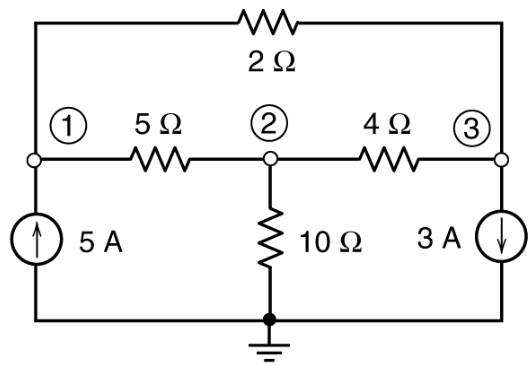
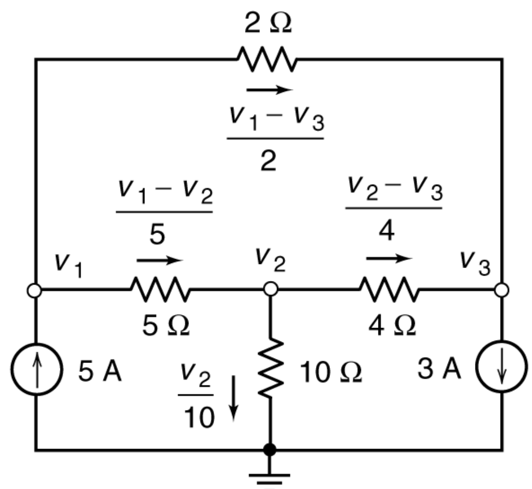


Figure P4.9-1

P4.9-1. The encircled numbers in the circuit shown Figure P4.9-1 are node numbers. Determine the values of the corresponding node voltages, v_1, v_2 and v_3.

Solution: First, express the resistor currents in terms of the node voltages:



Apply KCL at node 1 to get $5=\frac{v_1-v_2}{5}+\frac{v_1-v_3}{2} \quad\Rightarrow\quad 0.7\,v_1-0.2\,v_2-0.5\,v_3=5$

Apply KCL at node 2 to get $\frac{v_1-v_2}{5}=\frac{v_2}{10}+\frac{v_2-v_3}{4} \quad\Rightarrow\quad -0.2\,v_1+0.55\,v_2-0.25\,v_3=0$

Apply KCL at node 3 to get $\frac{v_2-v_3}{4}+\frac{v_1-v_3}{2}=3 \quad\Rightarrow\quad -0.5\,v_1-0.25\,v_2+0.75\,v_3=-3$

In matrix form:

$$\begin{bmatrix} 0.7 & -0.2 & -0.5 \\ -0.2 & 0.55 & -0.25 \\ -0.5 & -0.25 & 0.75 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}=\begin{bmatrix} 5 \\ 0 \\ -3 \end{bmatrix}$$

Solving using MATLAB: $v_1=28.1818\ \text{V},\ v_2=20\ \text{V}$ and $v_3=21.4545$

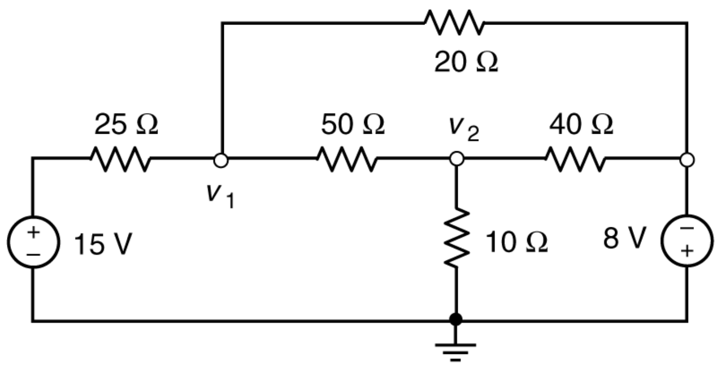
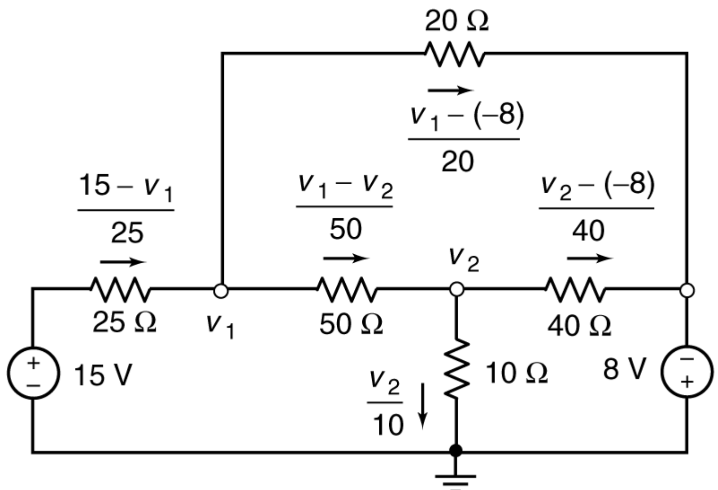


Figure P4.9-2

P4.9-2. Determine the values of the node voltages, v_1 and v_2, in the circuit shown Figure P4.9-2.

Solution: First, express the resistor currents in terms of the node voltages:



Apply KCL at node 1 to get $\dfrac{15-v_1}{25}=\dfrac{v_1-v_2}{50}+\dfrac{v_1+8}{20} \quad\Rightarrow\quad 0.11v_1-0.02v_2=0.2$

Apply KCL at node 2 to get $\dfrac{v_1-v_2}{50}=\dfrac{v_2}{10}+\dfrac{v_2+8}{40} \quad\Rightarrow\quad -0.02v_1+0.145v_2=-0.2$

In matrix form:

$$\begin{bmatrix} 0.11 & -0.02 \\ -0.02 & 0.145 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}=\begin{bmatrix} 0.2 \\ -0.2 \end{bmatrix}$$

Solving using MATLAB: $v_1=1.6077\ \text{V}$ and $v_2=-1.1576\ \text{V}$

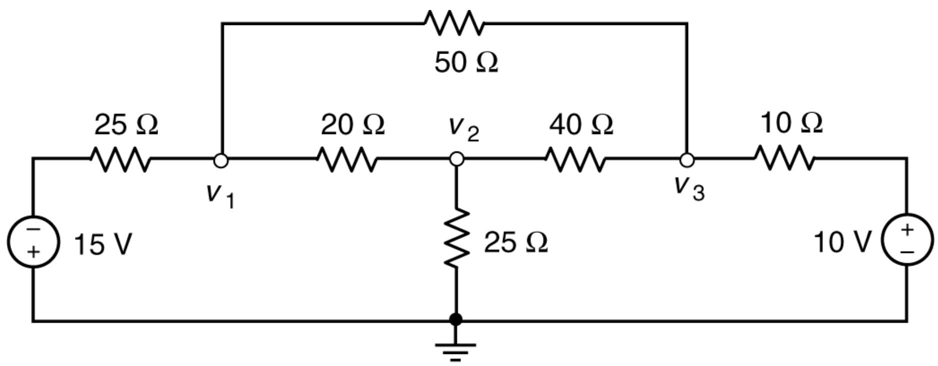
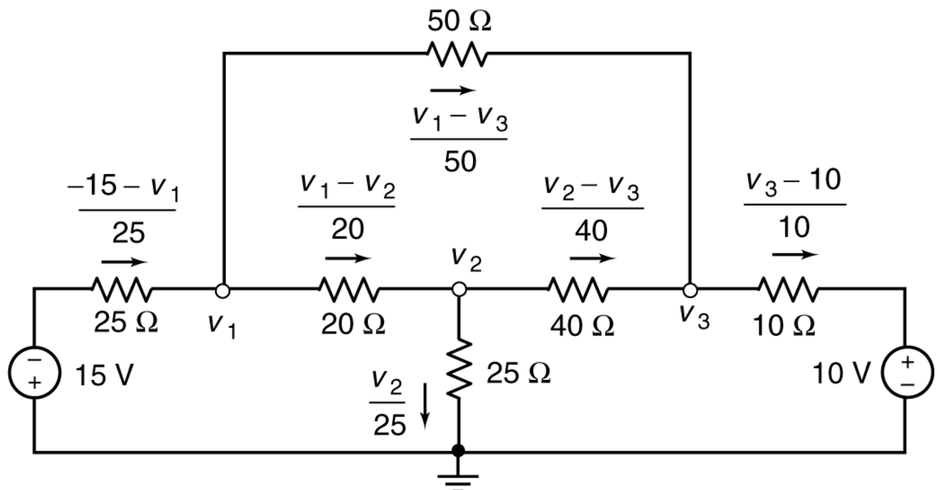


Figure P4.9-3

P4.9-3. Determine the values of the node voltages, v_1, v_2 and v_3 in the circuit shown in Figure P4.9-3.

Solution: First, express the resistor currents in terms of the node voltages:



Apply KCL at node 1 to get $\frac{-15-v_1}{25} = \frac{v_1 - v_2}{20} + \frac{v_1 - v_3}{50} \Rightarrow 0.11v_1 - 0.05v_2 - 0.02v_3 = -0.6$

Apply KCL at node 2 to get $\frac{v_1 - v_2}{20} = \frac{v_2}{25} + \frac{v_2 - v_3}{40} \Rightarrow -0.05v_1 + 0.115v_2 - 0.025v_3 = 0$

Apply KCL at node 3 to get $\frac{v_1 - v_2}{50} + \frac{v_2 - v_3}{40} = \frac{v_3 - 10}{10} \Rightarrow -0.02v_1 - 0.025v_2 + 0.145v_3 = 1$

In matrix form:
$$\begin{bmatrix} 0.11 & -0.05 & -0.02 \\ -0.05 & 0.115 & -0.025 \\ -0.02 & -0.025 & 0.145 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -0.6 \\ 0 \\ 1 \end{bmatrix}$$

Solving using MATLAB: $v_1 = 1.6077 \text{ V}$ and $v_2 = -1.1576 \text{ V}$

P4.9-4 Determine the node voltages, v_1 and v_2, for the circuit shown in Figure P4.9-4.

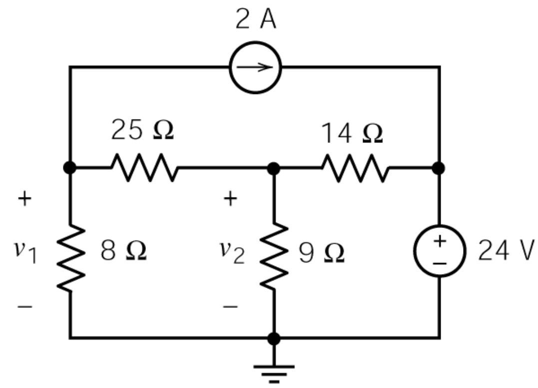
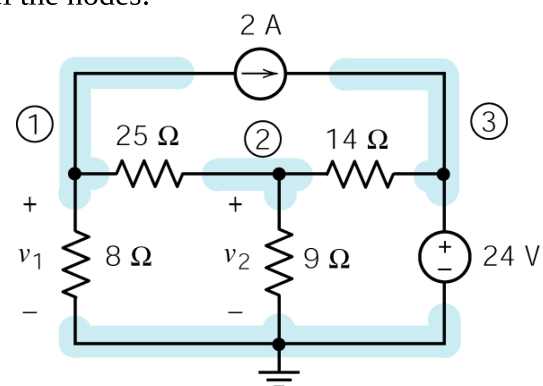


Figure P.4.9-4

Solution: Emphasize and label the nodes:



Notice the 24 V source connected between node 3 and the reference node. Consequently

$$v_3 = 24 \text{ V}$$

Apply KCL at node 1 to get

$$\frac{v_1}{8} + \frac{v_1 - v_2}{25} + 2 = 0$$

In this equation $\frac{v_1}{8}$ is the current directed downward in the 8 Ω resistor and $\frac{v_1 - v_2}{25}$ is the current directed from left to right in the 25 Ω resistor. We will simplify this equation by doing two things:

1. Multiplying each side by $8 \times 25 = 200$ to eliminate fractions.

2. Move the terms that don't involve node voltages to the right side of the equation.

The result is

$$33v_1 - 8v_2 = -400$$

Next, apply KCL at node 2 to get

$$\frac{v_2}{9} + \frac{v_2 - 24}{14} = \frac{v_1 - v_2}{25}$$

In this equation $\frac{v_2}{9}$ is the current directed downward in the 9 Ω resistor, $\frac{v_2 - 24}{14}$ is the current directed from left to right in the 14 Ω resistor and $\frac{v_1 - v_2}{25}$ is the current directed from left to right in the 25 Ω resistor. We will simplify this equation by doing two things:

1. Multiplying each side by $8 \times 25 \times 14 = 2800$ to eliminate fractions.

2. Move the terms that involve node voltages to the left side of the equation and move the terms that don't involve node voltages to the right side of the equation.

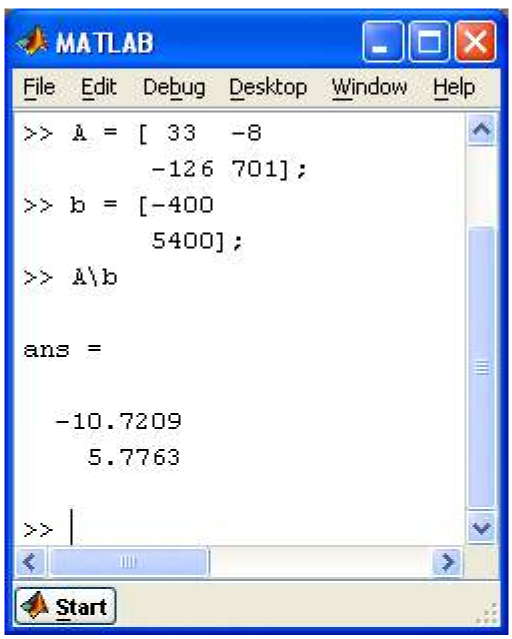
The result is

$$-(9\times 14)v_1 + (9\times 14 + 25\times 14 + 25\times 9)v_2 = 24\times 25\times 9 \quad \Rightarrow \quad -126 v_1 + 701 v_2 = 5400$$

The simultaneous equations can be written in matrix form

$$\begin{aligned} 33 v_1 - 8 v_2 &= -400 \\ -126 v_1 + 701 v_2 &= 5400 \end{aligned} \quad \Rightarrow \quad \begin{bmatrix} 33 & -8 \\ -126 & 701 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -400 \\ 5400 \end{bmatrix}$$

We can use MATLAB to solve the matrix equation:



Then $$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -10.7209 \\ 5.7763 \end{bmatrix}$$

That is, the node voltages are $v_1 = -10.7209$ V and $v_2 = 5.7763$ V.

P4.9-5 Determine the mesh currents, i_1 and i_2, for the circuit shown in Figure P4.9-5.

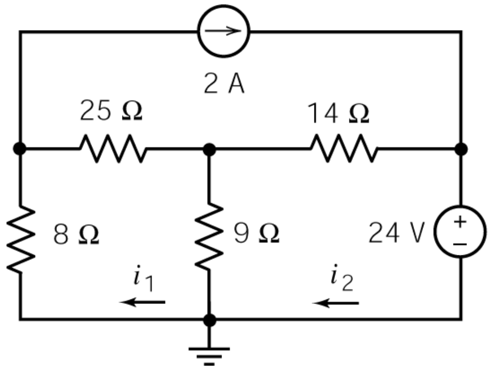
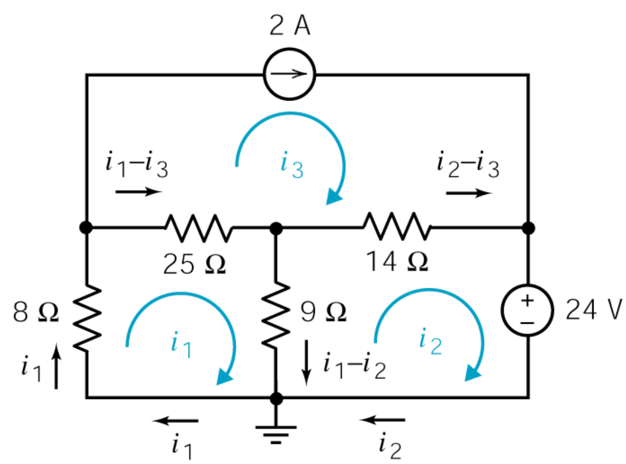


Figure P4.9.5

Solution: Label the label the mesh currents. Then, label the element currents in terms of the mesh currents:



Notice that the 2 A source on the outside of the circuit is in mesh 3 and that the currents 2 A and i_3 have the same direction. Consequently

$$i_3 = 2 \text{ A}$$

Apply KVL to mesh 1 to get

$$25(i_1 - i_3) + 9(i_1 - i_2) + 8i_1 = 0$$

In this equation $25(i_1 - i_3)$ is the voltage across the 25 Ω resistor (+ on the left), $9(i_1 - i_2)$ is the voltage across the 9 Ω resistor (+ on top) and $8i_1$ is the voltage across the 8 Ω resistor (+ on bottom). Substituting $i_3 = 2$ A and doing a little algebra gives

$$42i_1 - 9i_2 = 50$$

Next, apply KVL to mesh 2 to get

$$14(i_2 - i_3) + 24 - 9(i_1 - i_2) = 0$$

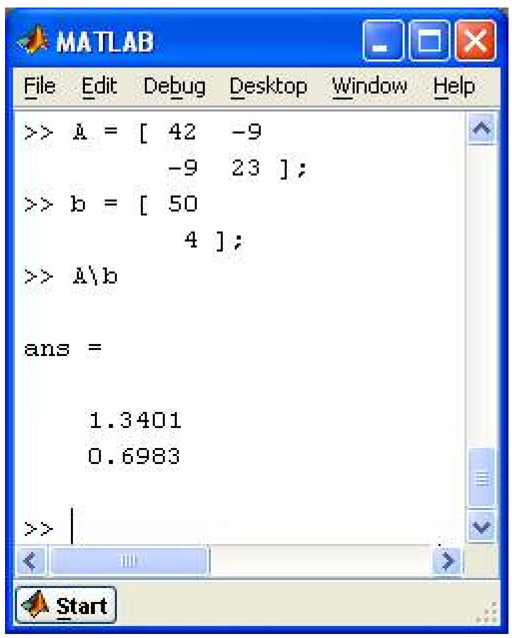
In this equation $14\left(i_2 - i_3\right)$ is the voltage across the 14 Ω resistor (+ on the left), 24 is the voltage source voltage and $9\left(i_1 - i_2\right)$ is the voltage across the 9 Ω resistor (+ on top). Substituting $i_3 = 2$ A and doing a little algebra gives

$$-9 i_1 + 23 i_2 = -24 + 14(2) = 4$$

The simultaneous equations can be written in matrix form

$$\begin{array}{l} 42 i_1 - 9 i_2 = 50 \\ -9 i_1 + 23 i_2 = 4 \end{array} \quad \Rightarrow \quad \begin{bmatrix} 42 & -9 \\ -9 & 23 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 50 \\ 4 \end{bmatrix}$$

We can use MATLAB to solve the matrix equation:



Then $$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1.3401 \\ 0.6983 \end{bmatrix}$$

That is, the mesh currents are $i_1 = 1.3401$ A and $i_2 = 0.6983$ A .

P4.9-6 Represent the circuit shown in Figure P4.9-6 by the matrix equation

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -40 \\ -228 \end{bmatrix}$$

Determine the values of the coefficients a_{11}, a_{12}, a_{21} and a_{22}.

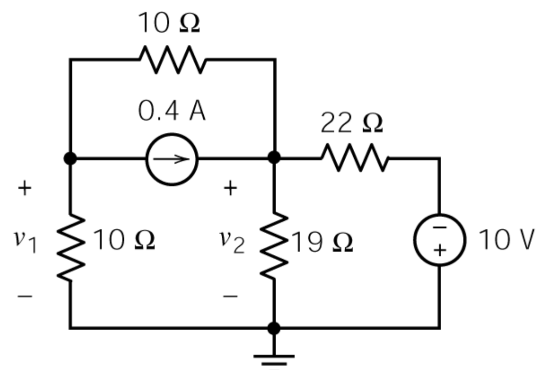
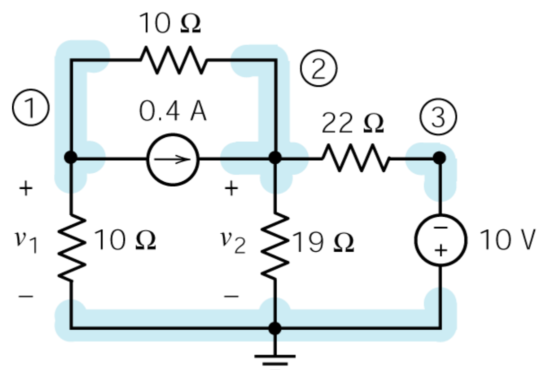


Figure P4.9-6

Solution: Emphasize and label the nodes:



Noticing the 10 V source connected between node 3 and the reference node, we determine that node voltage at node 3 is

$$v_3 = -10 \text{ V}$$

Apply KCL at node 1 to get $\quad \dfrac{v_1}{10} + 0.4 + \dfrac{v_1 - v_2}{10} = 0$

In this equation $\dfrac{v_1}{10}$ is the current directed downward in the vertical 10 Ω resistor and $\dfrac{v_1 - v_2}{10}$ is the current directed from left to right in the horizontal 10 Ω resistor. We will simplify this equation by doing two things:

1. Multiplying each side by 10 to eliminate fractions.

2. Move the terms that don't involve node voltages to the right side of the equation.

The result is $$2v_1 - v_2 = -4$$

Next, apply KCL at node 2 to get

$$\frac{v_2}{19} + \frac{v_2 - (-10)}{22} = \frac{v_1 - v_2}{10} + 0.4$$

In this equation $\frac{v_2}{19}$ is the current directed downward in the 19 Ω resistor, $\frac{v_2 - (-10)}{22}$ is the current directed from left to right in the 22 Ω resistor and $\frac{v_1 - v_2}{10}$ is the current directed from left to right in the horizontal 10 Ω resistor. We will simplify this equation by doing two things:

1. Multiplying each side by $19 \times 22 \times 10 = 4180$ to eliminate fractions.

2. Move the terms that involve node voltages to the left side of the equation and move the terms that don't involve node voltages to the right side of the equation.

The result is

$$-(19 \times 22)v_1 + (19 \times 10 + 22 \times 10 + 19 \times 22)v_2 = -10 \times 10 \times 19 + 0.4 \times 19 \times 22 \times 10$$

$$\Rightarrow \quad -418v_1 + 828v_2 = -228$$

Comparing our equations to the given equations, we see that we need to multiply both sides of our first equation by 10. Then

$$\begin{aligned} 20v_1 - 10v_2 &= -40 \\ -418v_1 + 828v_2 &= -228 \end{aligned} \quad \Rightarrow \quad \begin{bmatrix} 20 & -10 \\ -418 & 828 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -40 \\ -228 \end{bmatrix}$$

Comparing coefficients gives

$$a_{11} = 20, \; a_{12} = -10, \; a_{21} = -418 \text{ and } a_{22} = 828.$$

P4.9-7 Represent the circuit shown in Figure P4.9-7 by the matrix equation

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 10 \end{bmatrix}$$

Determine the values of the coefficients a_{11}, a_{12}, a_{21} and a_{22}.

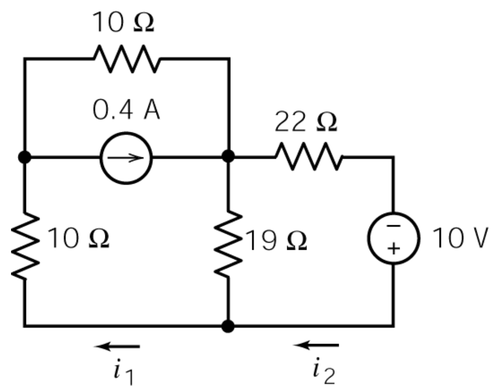
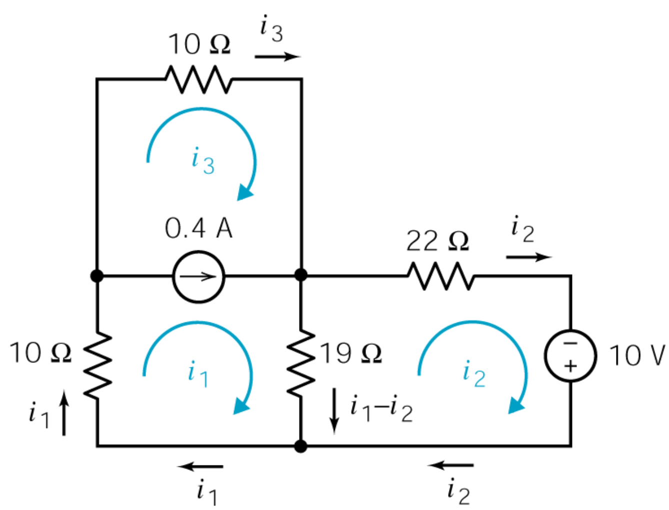


Figure P4.9-7

Solution: Label the label the mesh currents. Then, label the element currents in terms of the mesh currents:



Notice that the 0.4 A source on the inside of the circuit is in both mesh 1 and mesh 3. Mesh current i_1 is directed in the same way as current source current but the mesh current i_3 is directed opposite to the current source current. Consequently

$$i_1 - i_3 = 0.4 \text{ A}$$

The current source is in both mesh 1 and mesh 3 so we apply KVL to the supermesh corresponding to the current source (i.e. the perimeter of meshes 1 and 3). The result is

$$10\,i_3 + 19\left(i_1 - i_2\right) + 10\,i_1 = 0$$

In this equation $10 i_3$ is the voltage across the horizontal 10 Ω resistor (+ on the left), $19\left(i_1 - i_2\right)$ is the voltage across the 19 Ω resistor (+ on top) and $10 i_1$ is the voltage across the vertical 10 Ω resistor (+ on bottom). Substituting $i_3 = i_1 - 0.4$ and doing a little algebra gives

$$39 i_1 - 19 i_2 = 4$$

Next, apply KVL to mesh 2 to get

$$22 i_2 - 10 - 19\left(i_1 - i_2\right) = 0$$

In this equation $22 i_2$ is the voltage across the 22 Ω resistor (+ on the left), 10 is the voltage source voltage and $19\left(i_1 - i_2\right)$ is the voltage across the 19 Ω resistor (+ on top). Doing a little algebra gives

$$-19 i_1 + 41 i_2 = 10$$

To summarize, the circuit is represented by the simultaneous equations:

$$\begin{aligned} 39 i_1 - 19 i_2 &= 4 \\ -19 i_1 + 41 i_2 &= 10 \end{aligned} \quad \Rightarrow \quad \begin{bmatrix} 39 & -19 \\ -19 & 41 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 10 \end{bmatrix}$$

Comparing these equations to the given equations shows

$$a_{11} = 39, \; a_{12} = -19, \; a_{21} = -19 \text{ and } a_{22} = 41.$$

P4.9-8 Determine the values of the power supplied by each of the sources for the circuit shown in Figure P4.9-8.

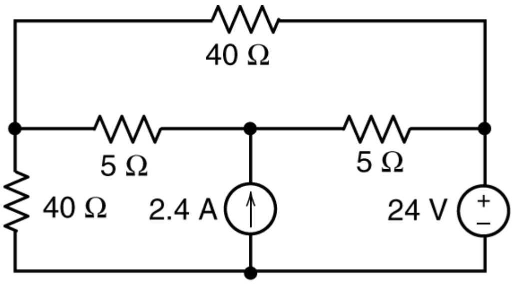
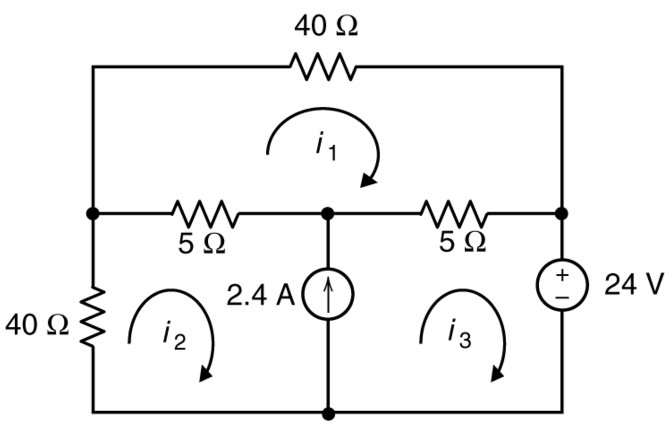
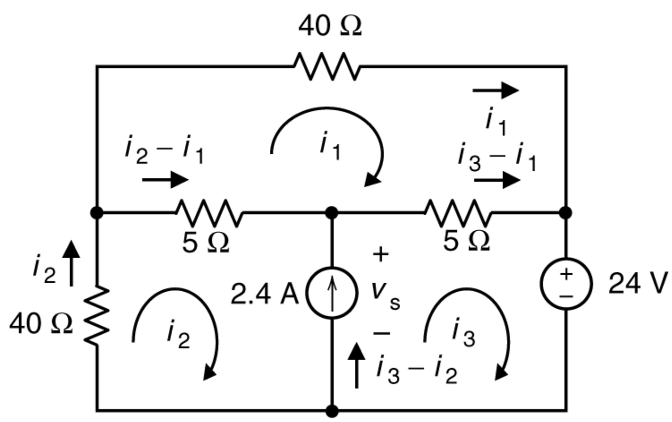


Figure P.4.9-8

Solution: First, label the mesh currents and then label the element currents:





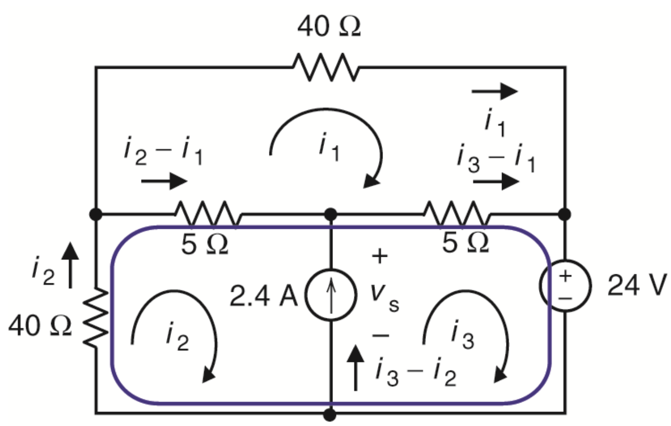
Notice the 2.4 A source in both mesh 2 and mesh 3. We have

$$i_3 - i_2 = 2.4 \text{ A}$$

Apply KVL to mesh 1 to get

$$40i_1 - 5(i_3 - i_1) - 5(i_2 - i_1) = 0 \quad \Rightarrow \quad 50i_1 - 5i_2 - 5i_3 = 0$$

Identify the supermesh corresponding to the 2.4 A current source:



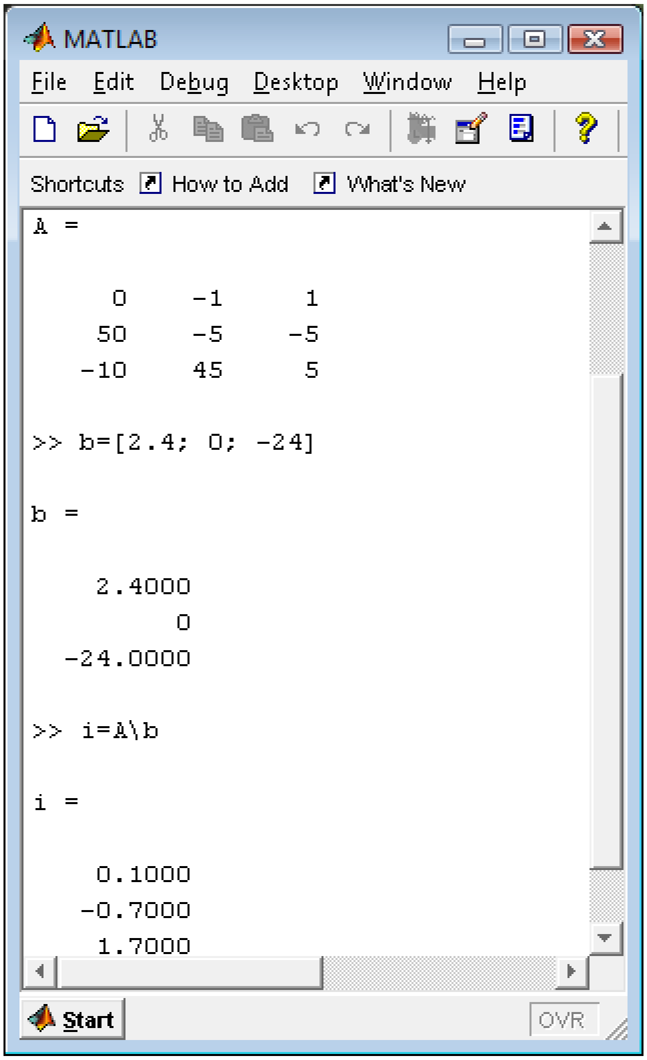
Apply KVL to the supermesh to get

$$5(i_2 - i_1) + 5(i_3 - i_1) + 24 + 40i_2 = 0 \quad \Rightarrow \quad -10i_1 + 45i_2 + 5i_3 = -24$$

Writing the mesh equations in matrix form gives

$$\begin{bmatrix} 0 & -1 & 1 \\ 50 & -5 & -5 \\ -10 & 45 & 5 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 2.4 \\ 0 \\ -24 \end{bmatrix}$$

Solving using MATLAB:



That is, the mesh currents are $i_1 = 0.1$ A, $i_2 = -0.7$ A and $i_3 = 1.7$ A.

The 24 V source supplies $\quad -24\,i_3 = (-24)(1.7) = -40.8 \text{ W}$

The power supplied by the current source depends on v_s, the voltage across the current source. Apply KVL to mesh 3 to get

$$5(i_3 - i_1) + 24 - v_s = 0 \quad \Rightarrow \quad v_s = 5(1.7 - 0.1) + 24 = 32 \text{ V}$$

The current source supplies $\quad 2.4\,v_s = 2.4(32) = 76.8 \text{ W}$

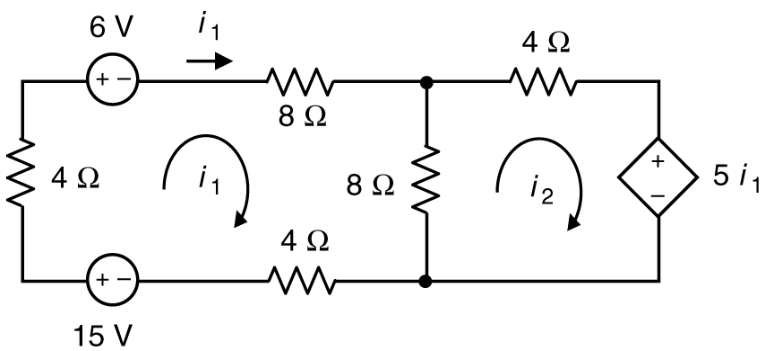
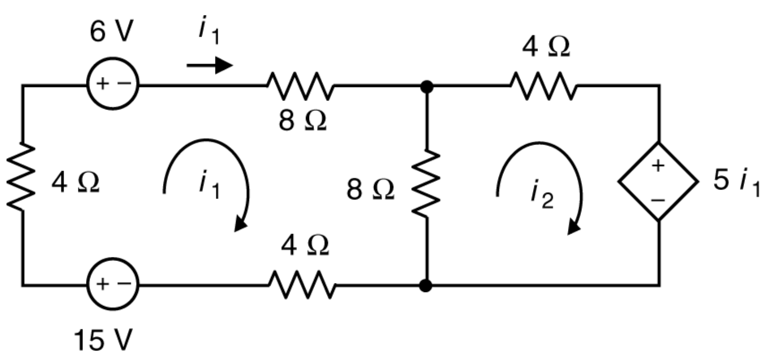


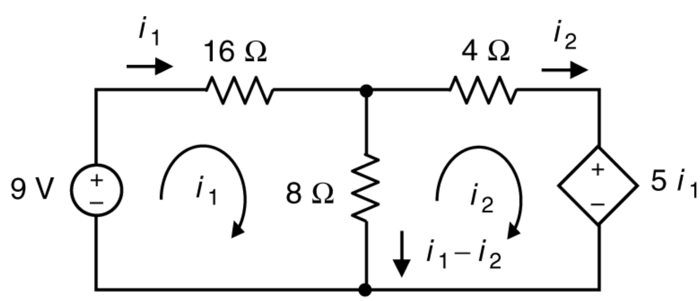
Figure P4.9-9

P4.9-9 The mesh currents are labeled in the circuit shown Figure 4.9-9. Determine the value of the mesh currents i_1 and i_2.

Solution: Determine the value of the mesh currents i_1 and i_2.



Replace series resistors with an equivalent resistor and series voltage sources with and equivalent voltage source to get



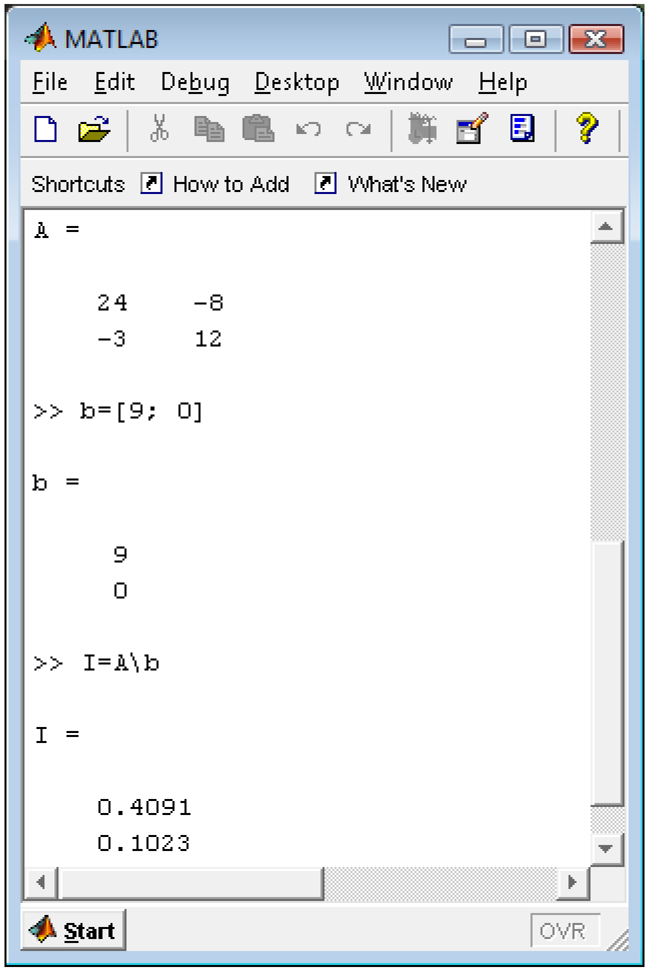
Apply KVL to mesh 1 $\quad 16i_1 + 8(i_1 - i_2) - 9 = 0 \quad \Rightarrow \quad 24i_1 - 8i_2 = 9$

Apply KVL to mesh 2 $\quad 4i_2 + 5i_1 - 8(i_1 - i_2) = 0 \quad \Rightarrow \quad -3i_1 + 12i_2 = 0$

In matrix form

$$\begin{bmatrix} 24 & -8 \\ -3 & 12 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \end{bmatrix}$$

Solving using MATLAB



So the mesh currents are $i_1 = 0.4091$ A and $i_2 = 0.1023$ A

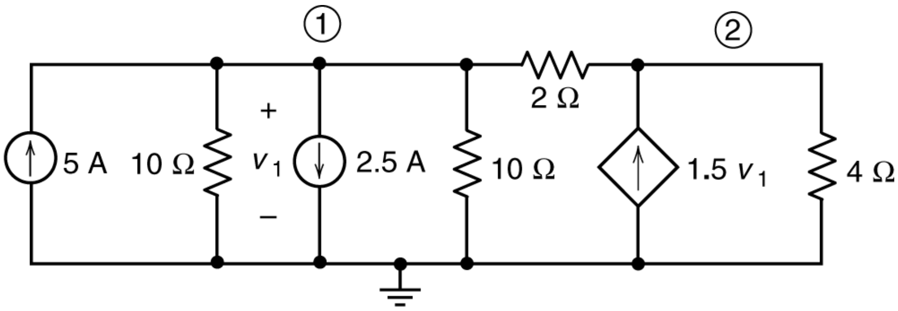
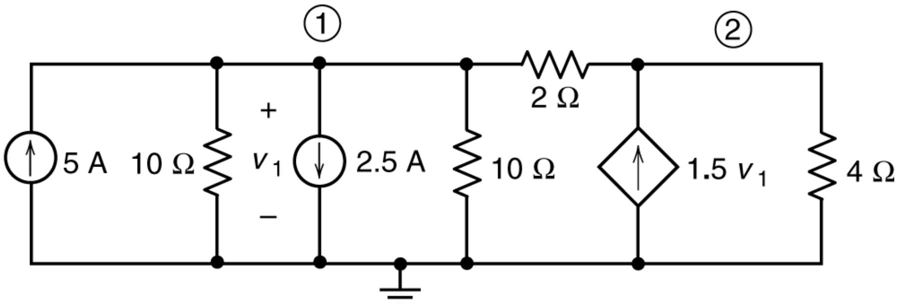


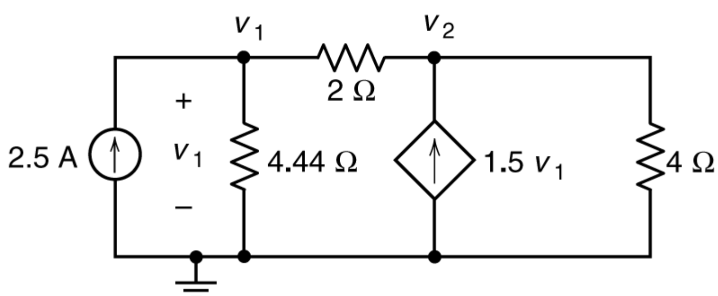
Figure P4.9-10

P4.9-10 The encircled numbers in the circuit shown Figure P4.9-10 are node numbers. Determine the values of the corresponding node voltages, v_1 and v_2.

Solution: Determine the value of the node voltages, v_1 and v_2.



Replace parallel resistors with an equivalent resistor and parallel sources with and equivalent current source to get

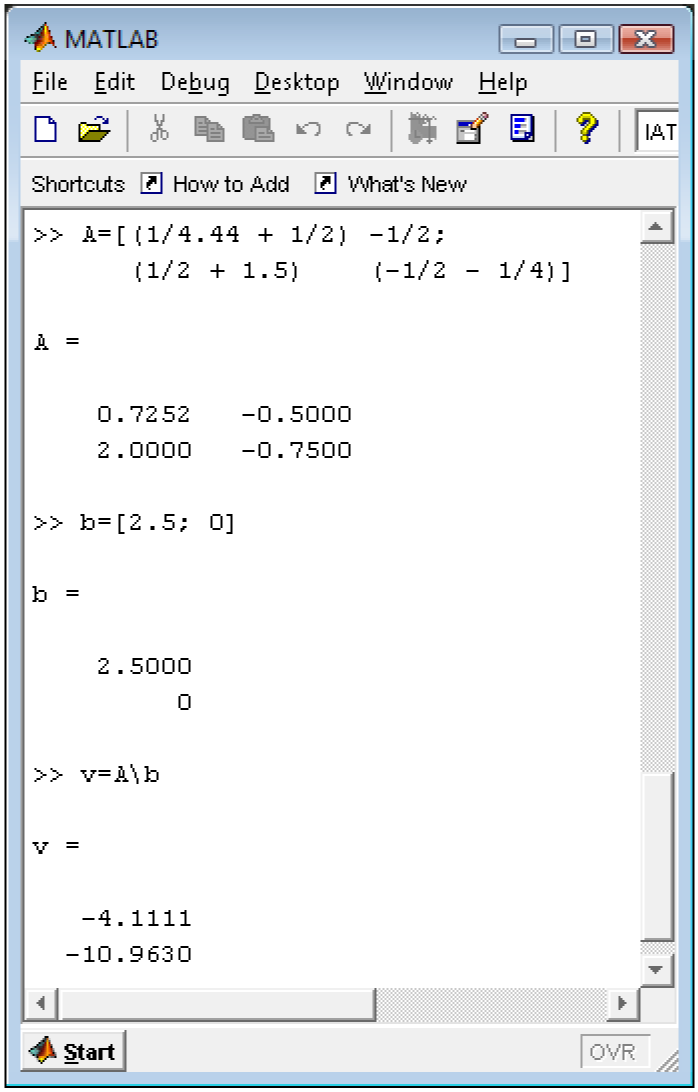


Apply KCL at node 1 $$2.5 = \frac{v_1}{4.44} + \frac{v_1 - v_2}{2} = 0$$

Apply KCL at node 2 $$\frac{v_1 - v_2}{2} + 1.5 v_1 = \frac{v_2}{4}$$

In matrix form $$\begin{bmatrix} \frac{1}{4.44} + \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} + 1.5 & -\frac{1}{2} - \frac{1}{4} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 0 \end{bmatrix}$$

Solving using MATLAB



So the node voltages are $v_1 = -4.1111$ V and $v_2 = -10.9630$ V

Section 4.10 How Can We Check … ?

P 4.10-1 Computer analysis of the circuit shown in Figure P 4.10-1 indicates that the node voltages are

$v_a = 5.2$ V, $v_b = -4.8$ V, and $v_c = 3.0$ V.

Is this analysis correct?

Hint: Use the node voltages to calculate all the element currents. Check to see that KCL is satisfied at each node.

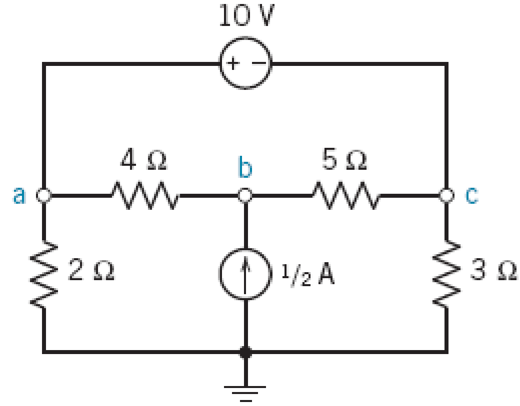


Figure P 4.10-1

Solution:

Apply KCL at node *b*:

$$\frac{v_b - v_a}{4} - \frac{1}{2} + \frac{v_b - v_c}{5} = 0$$

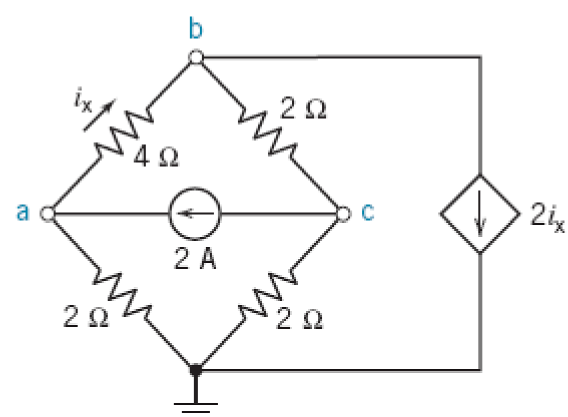
$$\frac{-4.8 - 5.2}{4} - \frac{1}{2} + \frac{-4.8 - 3.0}{5} \neq 0$$

The given voltages do not satisfy the KCL equation at node *b*. They are **not correct.**

P 4.10-2 An old lab report asserts that the node voltages of the circuit of Figure P 4.10-2 are

$v_a = 4$ V, $v_b = 20$ V, and $v_c = 12$ V.

Are these correct?



Solution:

Apply KCL at node *a*:

$$-\left(\frac{v_b - v_a}{4}\right) - 2 + \frac{v_a}{2} = 0$$

$$-\left(\frac{20-4}{4}\right) - 2 + \frac{4}{2} = -4 \neq 0$$

The given voltages do not satisfy the KCL equation at node *a*. They are **not correct.**

P 4.10-3 Your lab partner forgot to record the values of R_1, R_2, and R_3. He thinks that two of the resistors in Figure P 4.10-3 had values of 10 kΩ and that the other had a value of 5 kΩ. Is this possible? Which resistor is the 5-kΩ resistor?

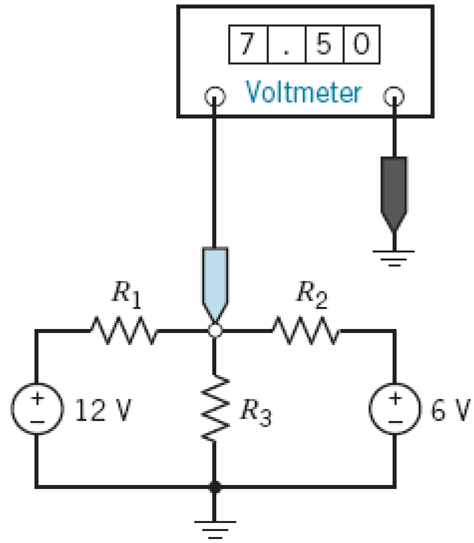


Figure P 4.10-3

Solution:

Writing a node equation:

$$-\left(\frac{12-7.5}{R_1}\right)+\frac{7.5}{R_3}+\frac{7.5-6}{R_2}=0$$

So

$$-\frac{4.5}{R_1}+\frac{7.5}{R_3}+\frac{1.5}{R_2}=0$$

There are only three cases to consider. Suppose $R_1 = 5\ \text{k}\Omega$ and $R_2 = R_3 = 10\ \text{k}\Omega$. Then

$$-\frac{4.5}{R_1}+\frac{7.5}{R_3}+\frac{1.5}{R_2}=\frac{-0.9+0.75+0.15}{1000}=\ 0$$

This choice of resistance values corresponds to branch currents that satisfy KCL. Therefore, it is indeed possible that two of the resistances are 10 kΩ and the other resistance is 5 kΩ. The 5 kΩ is R_1.

P 4.10-4 Computer analysis of the circuit shown in Figure P 4.10-4 indicates that the mesh currents are

$$i_1 = 2\text{ A},\ i_2 = 4\text{ A, and } i_3 = 3\text{ A}.$$

Verify that this analysis is correct.

Hint: Use the mesh currents to calculate the element voltages. Verify that KVL is satisfied for each mesh.

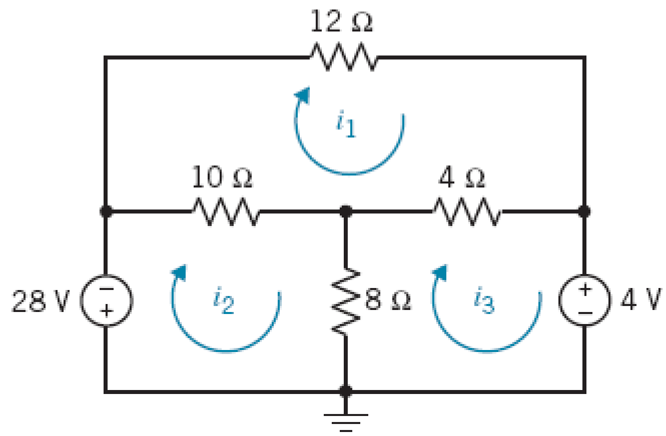


Figure P 4.10-4

Solution: Applying KVL to each mesh:

Top mesh: $$10\,(2-4)+12(2)+4\,(2-3)=0$$

Bottom right mesh $$8\,(3-4)+4\,(3-2)+4=0$$

Bottom, left mesh: $$28+10\,(4-2)+8\,(4-3)\neq 0$$
(Perhaps the polarity of the 28 V source was entered incorrectly.)

KVL is not satified for the bottom, left mesh so the computer analysis is **not correct**.

PSpice Problems

SP 4-1 Use PSpice to determine the node voltages of the circuit shown in Figure SP 4-1

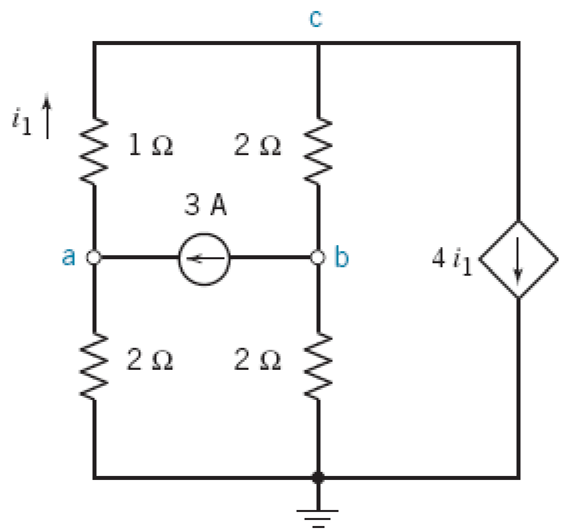
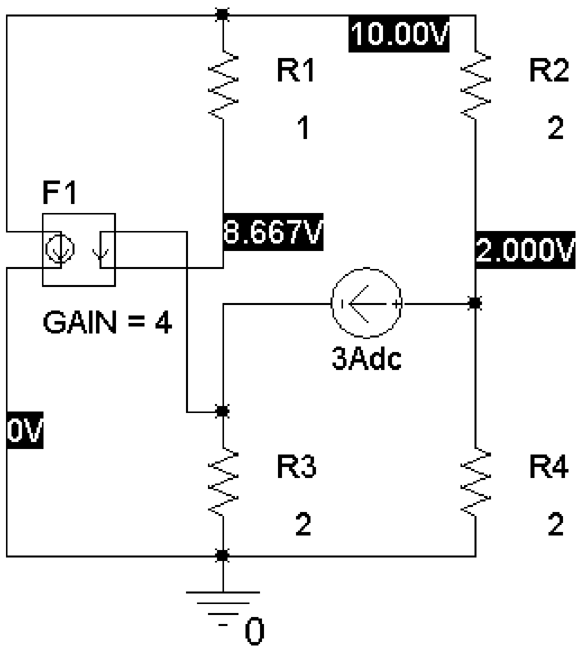


Figure SP 4-1

Solution: The PSpice schematic after running a “Bias Point” simulation:



SP 4-2 Use PSpice to determine the mesh currents of the circuit shown in Figure SP 4-2.

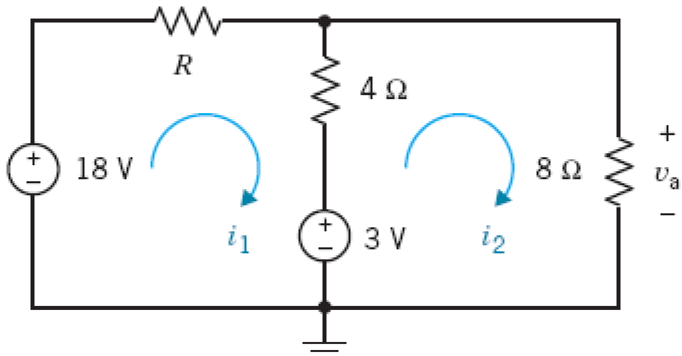
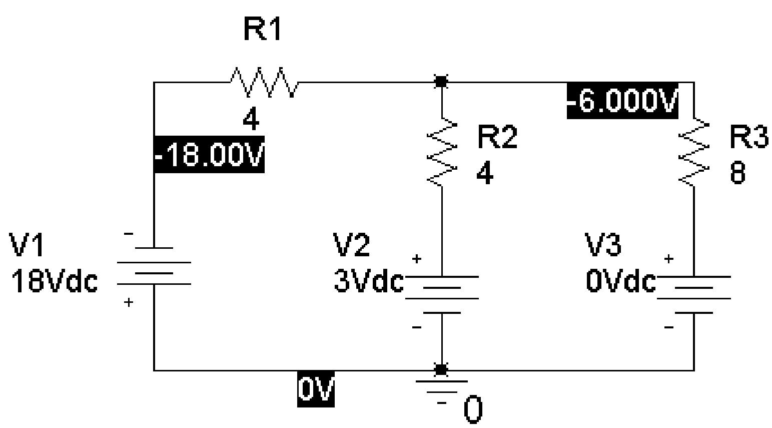


Figure SP 4-2

Solution: The PSpice schematic after running a "Bias Point" simulation:



From the PSpice output file:

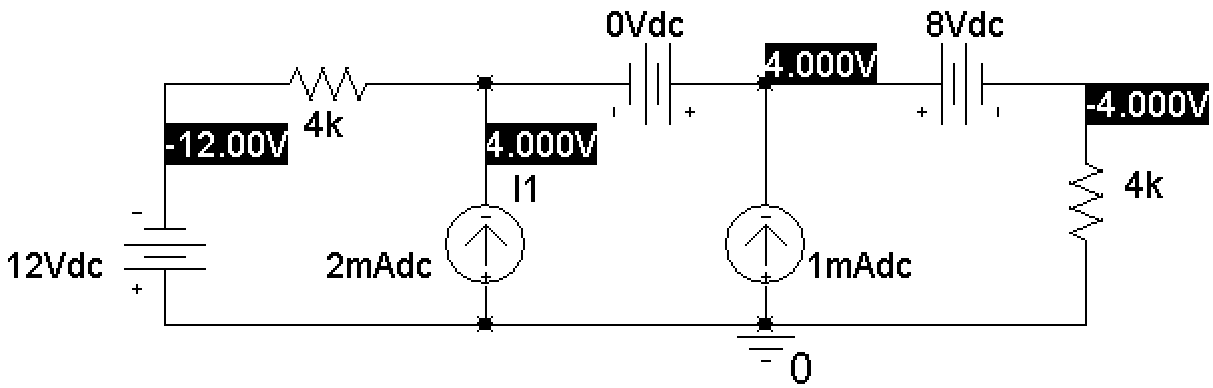
VOLTAGE SOURCE CURRENTS
NAME CURRENT

NAME	CURRENT
V_V1	-3.000E+00
V_V2	-2.250E+00
V_V3	**-7.500E-01**

The voltage source labeled V3 is a short circuit used to measure the mesh current. The mesh currents are $i_1 = -3$ A (the current in the voltage source labeled V1) and $i_2 = -0.75$ A (the current in the voltage source labeled V3).

SP 4-3 The voltages v_a, v_b, v_c, and v_d in Figure SP 4-3 are the node voltages corresponding to nodes a, b, c and d. The current *i* is the current in a short circuit connected between nodes b and c. Use PSpice to determine the values of v_a, v_b, v_c, and v_d and of *i*.

Solution: The PSpice schematic after running a "Bias Point" simulation:



The PSpice output file:

```
**** INCLUDING sp4_2-SCHEMATIC1.net ****
* source SP4_2
V_V4        0 N01588 12Vdc
R_R4        N01588 N01565  4k
V_V5        N01542 N01565 0Vdc
R_R5        0 N01516  4k
V_V6        N01542 N01516 8Vdc
I_I1        0 N01565 DC 2mAdc
I_I2        0 N01542 DC 1mAdc

VOLTAGE SOURCE CURRENTS
   NAME        CURRENT

   V_V4       -4.000E-03
   V_V5        2.000E-03
   V_V6       -1.000E-03
```

From the PSpice schematic: v_a = –12 V, v_b = v_c = 4 V, v_d = –4 V. From the output file: *i* = 2 mA.

SP 4-4 Determine the current, *i*, shown in Figure SP 4-4.

Answer: i = 0.56 A

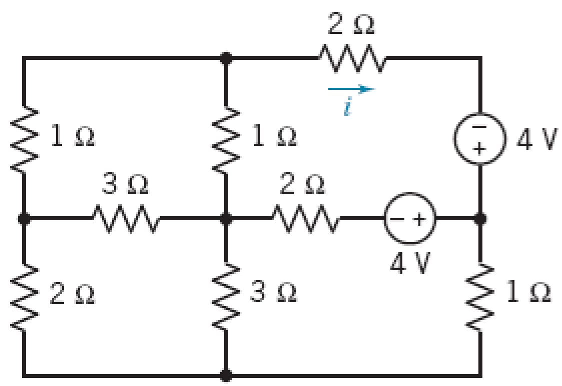
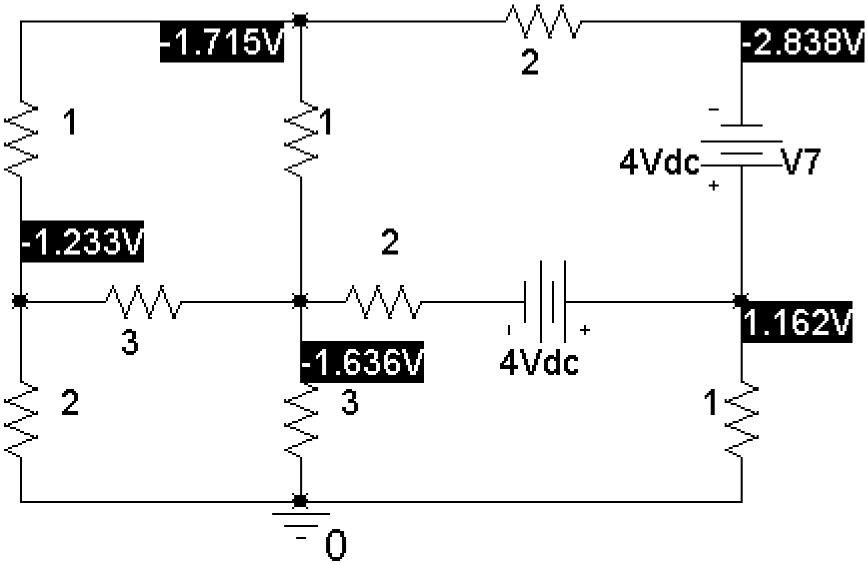


Figure SP 4-4

Solution: The PSpice schematic after running a "Bias Point" simulation:



The PSpice output file:

VOLTAGE SOURCE CURRENTS
NAME CURRENT

V_V7 -5.613E-01
V_V8 -6.008E-01

The current of the voltage source labeled V7 is also the current of the 2 Ω resistor at the top of the circuit. However this current is directed from right to left in the 2 Ω resistor while the current i is directed from left to right. Consequently, i = +5.613 A.

Design Problems

DP 4-1 An electronic instrument incorporates a 15-V power supply. A digital display is added that requires a 5-V power supply. Unfortunately, the project is over budget and you are instructed to use the existing power supply. Using a voltage divider, as shown in Figure DP 4-1, you are able to obtain 5 V. The specification sheet for the digital display shows that the display will operate properly over a supply voltage range of 4.8 V to 5.4 V. Furthermore, the display will draw 300 mA (I) when the display is active and 100 mA when quiescent (no activity).

(a) Select values of R_1 and R_2 so that the display will be supplied with 4.8 V to 5.4 V under all conditions of current I.

(b) Calculate the maximum power dissipated by each resistor, R_1 and R_2, and the maximum current drawn from the 15-V supply.

(c) Is the use of the voltage divider a good engineering solution? If not, why? What problems might arise?

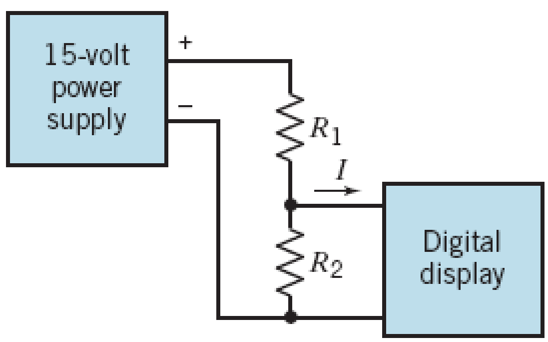
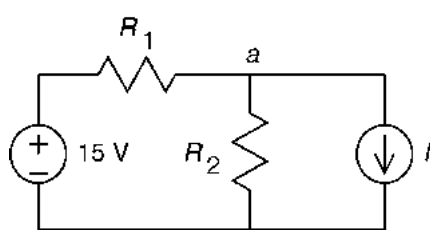


Figure DP 4-1

Solution:

Model the circuit as:



(a) We need to keep v_2 across R_2 in the range $4.8 \le v_2 \le 5.4$

$$\text{For } I = \begin{cases} 0.3\text{ A} & \text{display is active} \\ 0.1\text{ A} & \text{display is not active} \end{cases}$$

KCL at a: $\dfrac{v_2-15}{R_1}+\dfrac{v_2}{R_2}+I=0$

Assumed that maximum I results in minimum v_2 and visa-versa.

Then

$$v_2 = \begin{cases} 4.8 \text{ V} & \text{when } I=0.3 \text{ A} \\ 5.4 \text{ V} & \text{when } I=0.1 \text{ A} \end{cases}$$

Substitute these corresponding values of v_2 and I into the KCL equation and solve for the resistances

$$\frac{4.8-15}{R_1}+\frac{4.8}{R_2}+0.3=0$$

$$\frac{5.4-15}{R_1}+\frac{5.4}{R_2}+0.1=0$$

$$\Rightarrow \underline{R_1 = 7.89\ \Omega}, \ \underline{R_2 = 4.83\ \Omega}$$

(b) $$I_{R_{1\max}} = \frac{15-4.8}{7.89} = 1.292 \text{ A} \Rightarrow P_{R_{1\max}} = (1.292)^2(7.89) = 13.17 \text{ W}$$

$$I_{R_{2\max}} = \frac{5.4}{4.83} = 1.118 \text{ A} \Rightarrow P_{R_{2\max}} = \frac{(5.4)^2}{4.83} = 6.03 \text{ W}$$

maximum supply current $= I_{R_{1\max}} = 1.292$ A

(c) No; if the supply voltage (15V) were to rise or drop, the voltage at the display would drop below 4.8V or rise above 5.4V.

The power dissipated in the resistors is excessive. Most of the power from the supply is dissipated in the resistors, not the display.

DP 4-2 For the circuit shown in Figure DP 4-2, it is desired to set the voltage at node a equal to 0 V in order to control an electric motor. Select voltages v_1 and v_2 in order to achieve $v_a = 0$ V when v_1 and v_2 are less than 20 V and greater than zero and $R = 2\ \Omega$.

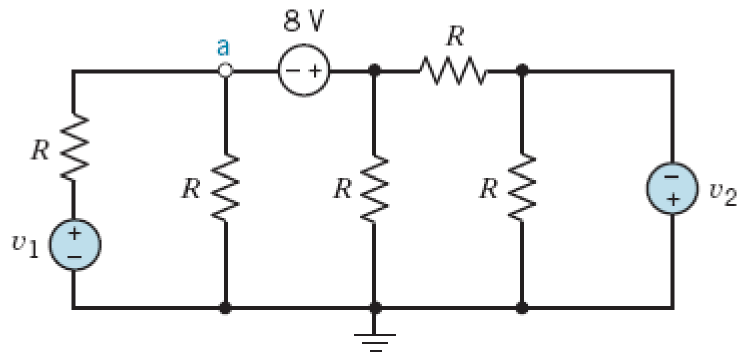


Figure DP 4-2

Solution:

Express the voltage of the 8 V source in terms of its node voltages to get $v_b - v_a = 8$. Apply KCL to the supernode corresponding to the 8 V source:

$$\frac{v_a - v_1}{R} + \frac{v_a}{R} + \frac{v_b}{R} + \frac{v_b - (-v_2)}{R} = 0 \quad \Rightarrow \quad 2\,v_a - v_1 + 2\,v_b + v_2 = 0$$

$$\Rightarrow \quad 2\,v_a - v_1 + 2\,(v_a + 8) + v_2 = 0$$

$$\Rightarrow \quad 4\,v_a - v_1 + v_2 + 16 = 0$$

$$\Rightarrow \quad v_a = \frac{v_1 - v_2}{4} - 4$$

Next set $v_a = 0$ to get

$$0 = \frac{v_1 - v_2}{4} - 4 \quad \Rightarrow \quad v_1 - v_2 = 16 \text{ V}$$

For example, $v_1 = 18$ V and $v_2 = 2$ V.

DP 4-3 A wiring circuit for a special lamp in a home is shown in Figure DP 4-3. The lamp has a resistance of 2 Ω, and the designer selects R = 100 Ω. The lamp will light when $I \geq$ 50 mA but will burn out when $I >$ 75 mA.

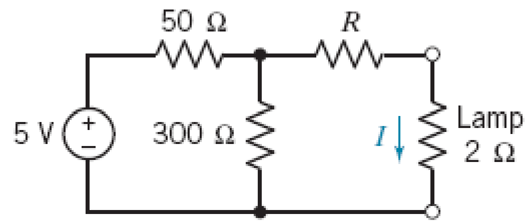


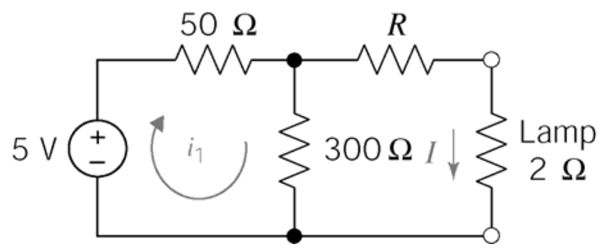
Figure DP 4-3

(a) Determine the current in the lamp and determine if it will light for R = 100 Ω.

(b) Select R so that the lamp will light but will not burn out if R changes by ± 10 percent because of temperature changes in the home.

Solution:

(a)



Apply KCL to left mesh: $-5+50i_1+300\ (i_1-I)=0$

Apply KCL to right mesh: $(R+2)\ I+300\ (I-i_1)=\ 0$

Solving for I: $I=\dfrac{150}{1570\ +\ 35\ R}$

We desire 50 mA ≤ I ≤ 75 mA so if R = 100 Ω, then I = 29.59 mA ⇒ l amp so the lamp will not light.

(b) From the equation for I, we see that decreasing R increases I:

$$\text{try } R = 50\ \Omega\ \Rightarrow\ I = 45 \text{ mA (won't light)}$$

$$\text{try } R = 25\Omega\ \Rightarrow\ I = 61 \text{ mA}\ \Rightarrow\ \underline{\text{will light}}$$

Now check R±10% to see if the lamp will light and not burn out:

$$\left.\begin{array}{l}-10\%\ \rightarrow\ 22.5\Omega\ \rightarrow\ I = 63.63 \text{ mA}\\ +10\%\ \rightarrow\ 27.5\Omega\ \rightarrow\ I = 59.23 \text{ mA}\end{array}\right\}\begin{array}{l}\text{lamp will}\\ \text{stay on}\end{array}$$

DP 4-4 In order to control a device using the circuit shown in Figure DP 4-4, it is necessary that v_{ab} = 10 V. Select the resistors when it is required that all resistors be greater than 1 Ω and $R_3 + R_4 = 20$ Ω.

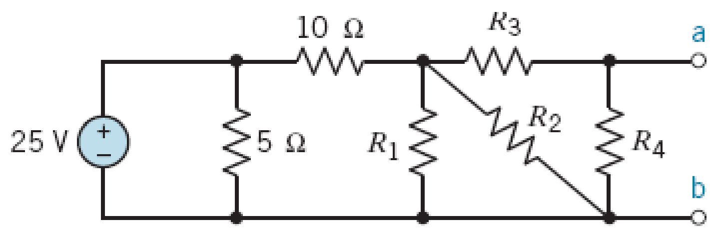
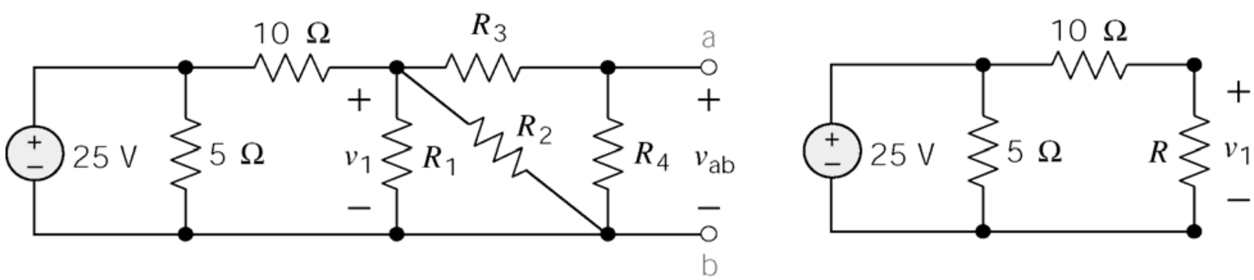


Figure DP 4-4

Solution:



Equivalent resistance: $R = R_1 \parallel R_2 \parallel \left(R_3 + R_4\right)$

Voltage division in the equivalent circuit: $v_1 = \dfrac{R}{10+R}(25)$

We require v_{ab} = 10 V. Apply the voltage division principle in the left circuit to get:

$$10 = \frac{R_4}{R_3 + R_4}\ v_1 = \frac{R_4}{R_3 + R_4} \times \frac{\left(R_1 \| R_2 \| \ (R_3 + R_4)\right)}{10 \ + \ \left(R_1 \| R_2 \| \ (R_3 + R_4)\right)} \times 25$$

This equation does not have a unique solution. Here's one solution:

$$\text{choose } R_1 = R_2 \ = \ 25\,\Omega \text{ and } R_3 + R_4 = 20\,\Omega$$

$$\text{then } 10 = \frac{R_4}{20} \times \frac{\left(12.5 \| 20\ \right)}{10 \ + \ \left(12.5 \| 20\ \right)} \times 25 \ \Rightarrow \ \underline{R_4 = 18.4\Omega}$$

$$\text{and } R_3 + R_4 = 20 \Rightarrow \underline{R_3 = 1.6\,\Omega}$$

DP 4-5 The current i shown in the circuit of Figure DP 4-5 is used to measure the stress between two sides of an earth fault line. Voltage v_1 is obtained from one side of the fault, and v_2 is obtained from the other side of the fault. Select the resistances R_1, R_2, and R_3 so that the magnitude of the current i will remain in the range between 0.5 mA and 2 mA when v_1 and v_2 may each vary independently between +1 V and +2 V ($1\text{ V} \le v_n \le 2\text{ V}$).

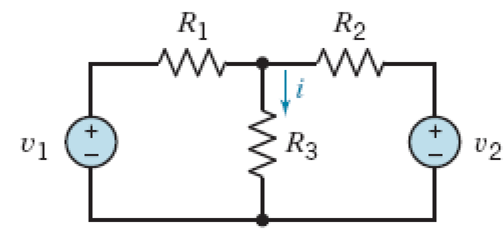
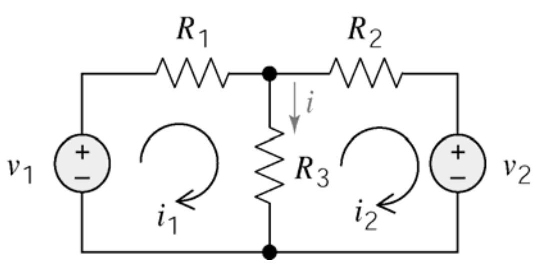


Figure DP 4-5 A circuit for earth fault-line stress measurement.

Solution:



Apply KCL to the left mesh:

$$\left(R_1+R_3\right) i_1 - R_3 i_2 - v_1 = 0$$

Apply KCL to the left mesh:

$$-R_3 i_1 + \left(R_2+R_3\right) i_2 + v_2 = 0$$

Solving for the mesh currents using Cramer's rule:

$$i_1 = \frac{\begin{bmatrix} v_1 & -R_3 \\ -v_2 & \left(R_2+R_3\right) \end{bmatrix}}{\Delta} \text{ and } i_2 = \frac{\begin{bmatrix} (R_1+R_3) & v_1 \\ -R_3 & -v_2 \end{bmatrix}}{\Delta}$$

$$\text{where } \Delta = \left(R_1+R_3\right)\left(R_2+R_3\right) - R_3^{\,2}$$

Try $R_1 = R_2 = R_3 = 1\text{ k}\Omega = 1000\ \Omega$. Then $\Delta = 3\text{ M}\Omega$. The mesh currents will be given by

$$i_1 = \frac{\left[2v_1 - v_2\right] 1000}{3\times 10^6} \text{ and } i_2 = \frac{\left[-2v_2 + v_1\right] 1000}{3\times 10^6} \Rightarrow i = i_1 - i_2 = \frac{v_1+v_2}{3000}$$

Now check the extreme values of the source voltages:

$$\text{if } v_1 = v_2 = 1\text{ V} \Rightarrow i = 2/3 \text{ mA} \quad \textbf{okay}$$

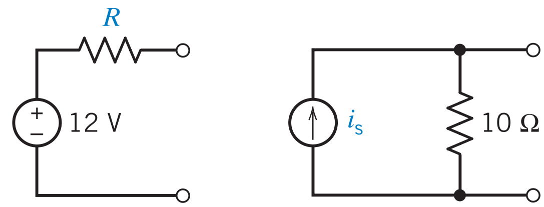
$$\text{if } v_1 = v_2 = 2\text{ V} \Rightarrow i = 4/3 \text{ mA} \quad \textbf{okay}$$

Chapter 5 Circuit Theorems

Exercises

Exercise 5.2-1 Determine values of R and i_s so that the circuits shown in Figures E 5.2-1*a*,*b* are equivalent to each other due to a source transformation.

Answer: $R = 10\ \Omega$ and $i_s = 1.2$ A



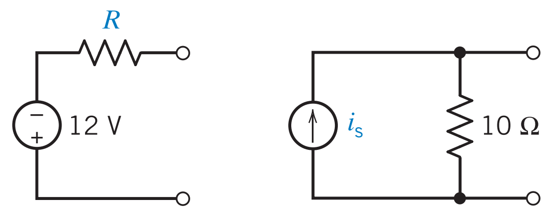
(*a*) (*b*)

Figures E 5.2-1

Exercise 5.2-2 Determine values of R and i_s so that the circuits shown in Figures E 5.2-2*a*,*b* are equivalent to each other due to a source transformation.

Hint: Notice that the polarity of the voltage source in Figure E 5.2-2*a* is not the same as in Figure E 5.2-1*a*.

Answer: $R = 10\ \Omega$ and $i_s = -1.2$ A

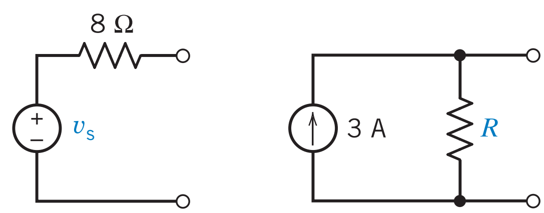


(*a*) (*b*)

Figures E 5.2-2

Exercise 5.2-3 Determine values of R and v_s so that the circuits shown in Figures E 5.2-3*a*,*b* are equivalent to each other due to a source transformation.

Answer: $R = 8\ \Omega$ and $v_s = 24$ V



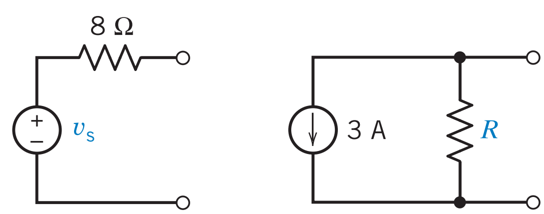
(*a*) (*b*)

Figure E 5.2-3

Exercise 5.2-4 Determine values of R and v_s so that the circuits shown in Figures E 5.2-4*a*,*b* are equivalent to each other due to a source transformation.

Hint: Notice that the reference direction of the current source in Figure E 5.2-4*b* is not the same as in Figure E 5.2-3*b*.

Answer: $R = 8\ \Omega$ and $v_s = -24$ V



(*a*) (*b*)

Figure E 5.2-4

Exercise 5.4-1 Determine values of R_t and v_{oc} that cause the circuit shown in Figure E 5.4-1*b* to be the Thévenin equivalent circuit of the circuit in Figure E 5.4-1*a*.

Answer: $R_t = 8\ \Omega$ and $v_{oc} = 2$ V

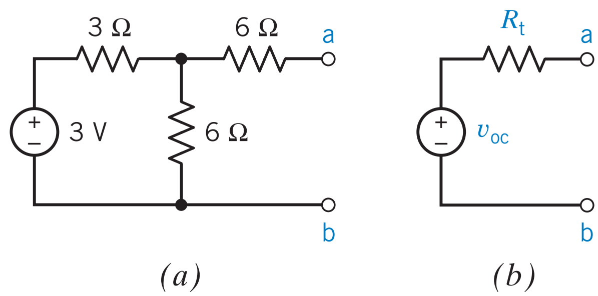
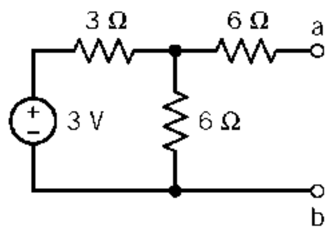
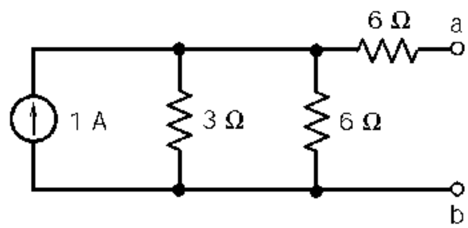
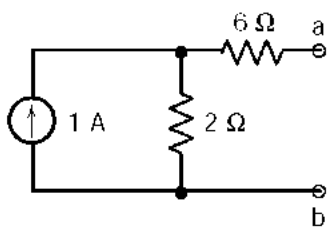


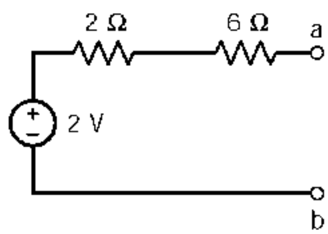
Figure E 5.2-1

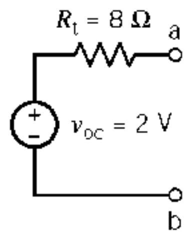
Solution:











Exercise 5.4-2 Determine values of R_t and v_{oc} that cause the circuit shown in Figure E 5.4-2*b* to be the Thévenin equivalent circuit of the circuit in Figure E 5.4-2*a*.

Answer: $R_t = 3\ \Omega$ and $v_{oc} = -6$ V

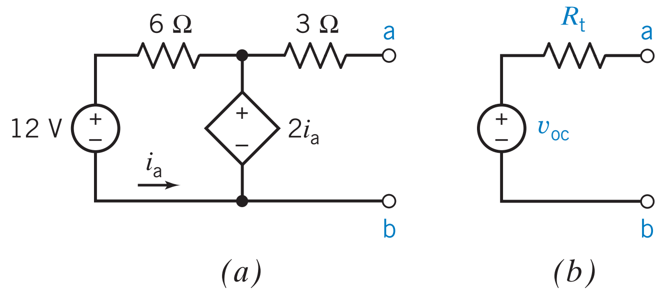
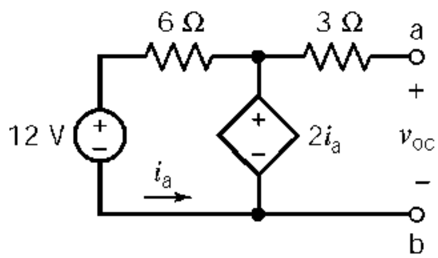


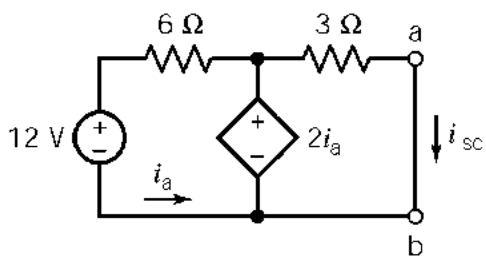
Figure E 5.2-2

Solution:



$$i_a = \frac{2\,i_a - 12}{6} \quad \Rightarrow \quad i_a = -3 \text{ A}$$

$$v_{oc} = 2\,i_a = -6 \text{ V}$$



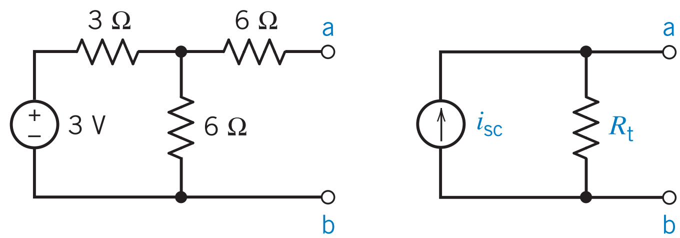
$$12 + 6\,i_a = 2\,i_a \quad \Rightarrow \quad i_a = -3 \text{ A}$$

$$3\,i_{sc} = 2\,i_a \quad \Rightarrow \quad i_{sc} = \frac{2}{3}(-3) = -2 \text{ A}$$

$$R_t = \frac{-6}{-2} = 3\,\Omega$$

Exercise 5.5-1 Determine values of R_t and i_{sc} that cause the circuit shown in Figure E 5.5-1*b* to be the Norton equivalent circuit of the circuit in Figure E 5.5-1*a*.

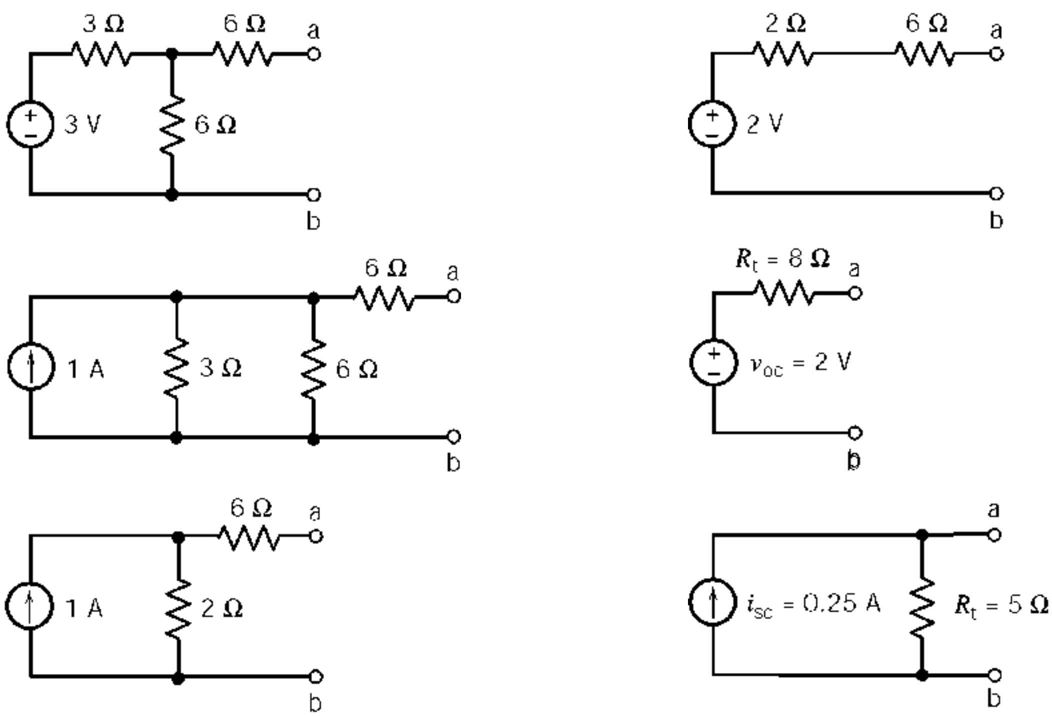
Answer: $R_t = 8\ \Omega$ and $i_{sc} = 0.25$ A



(a) *(b)*

Figure E 5.5-1

Solution:



Exercise 5.6-1 Find the maximum power that can be delivered to R_L for the circuit of Figure E 5.6-1 using a Thévenin equivalent circuit.

Answer: 9 W when $R_L = 4\ \Omega$

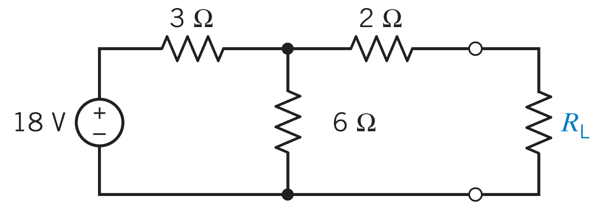
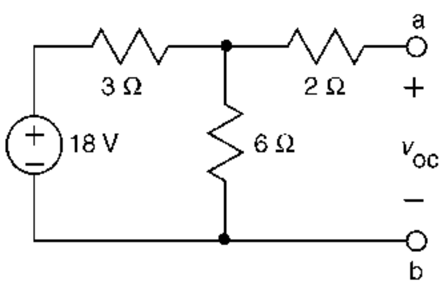
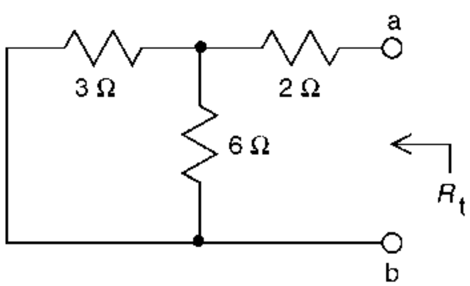


Figure E 5.6-1

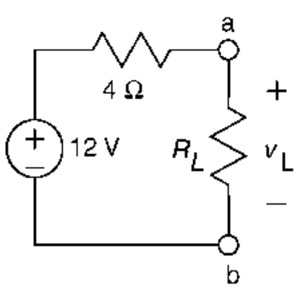
Solution:



$$v_{oc} = \frac{6}{6+3}(18) = 12\ \text{V}$$



$$R_t = 2 + \frac{(3)(6)}{3+6} = 4\ \Omega$$



For maximum power, we require

$$R_L = R_t = 4\ \Omega$$

Then

$$p_{\max} = \frac{{v_{oc}}^2}{4\,R_t} = \frac{12^2}{4(4)} = 9\ \text{W}$$

Section 5-2: Source Transformations

P 5.2-1 The circuit shown in Figure P 5.2-1*a* has been divided into two parts. The circuit shown in Figure P 5.2-1*b* was obtained by simplifying the part to the right of the terminals using source transformations. The part of the circuit to the left of the terminals was not changed.

(a) Determine the values of R_t and v_t in Figure P 5.2-1*b*.

(b) Determine the values of the current i and the voltage v in Figure P 5.2-1*b*. The circuit in Figure P 5.2-1*b* is equivalent to the circuit in Figure P 5.2-1*a*. Consequently, the current i and the voltage v in Figure P 5.2-1*a* have the same values as do the current i and the voltage v in Figure P 5.2-1*b*.

(c) Determine the value of the current i_a in Figure P 5.2-1*a*.

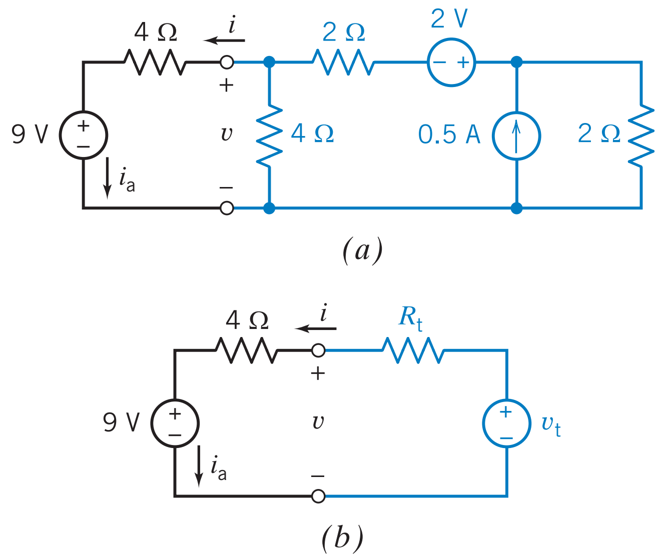
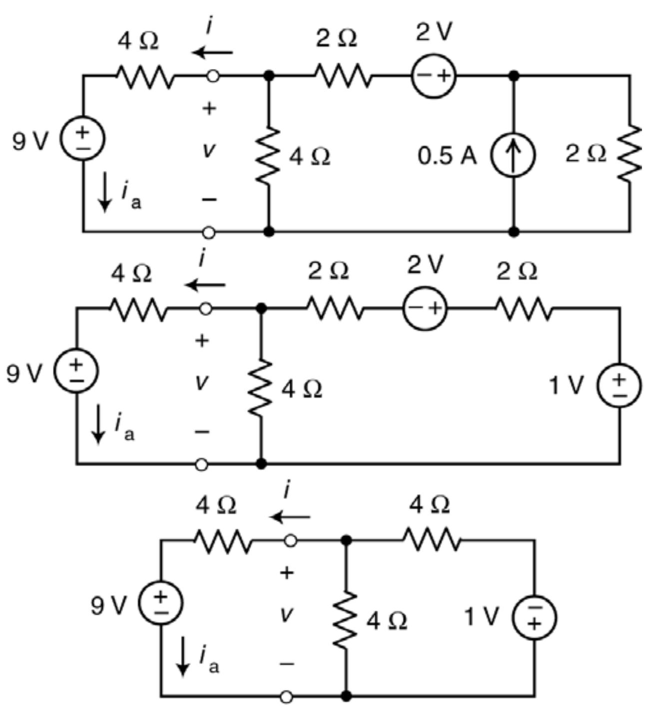
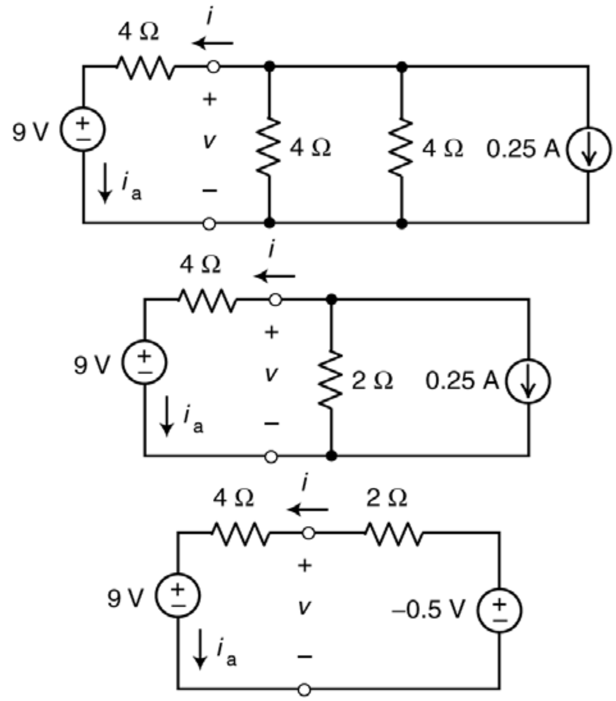


Figure P 5.2-1

Solution:

(a)





$$\therefore\ R_t = 2\ \Omega$$
$$v_t = -0.5\ \text{V}$$

(b) $-9 - 4i - 2i + (-0.5) = 0$

$$i = \frac{-9 + (-0.5)}{4+2} = -1.58\ \text{A}$$

$$v = 9 + 4\,i = 9 + 4(-1.58) = 2.67\ \text{V}$$

(c) $i_a = i = -1.58$ A

(checked using LNAP 8/15/02)

P 5.2-2 Consider the circuit of Figure P 5.2-2. Find i_a by simplifying the circuit (using source transformations) to a single-loop circuit so that you need to write only one KVL equation to find i_a.

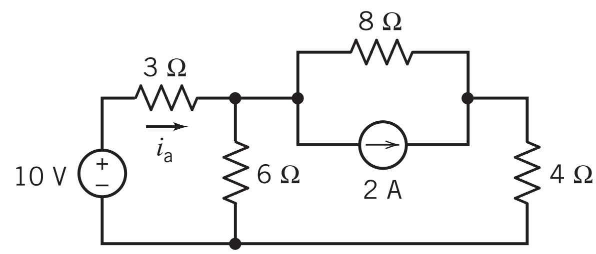
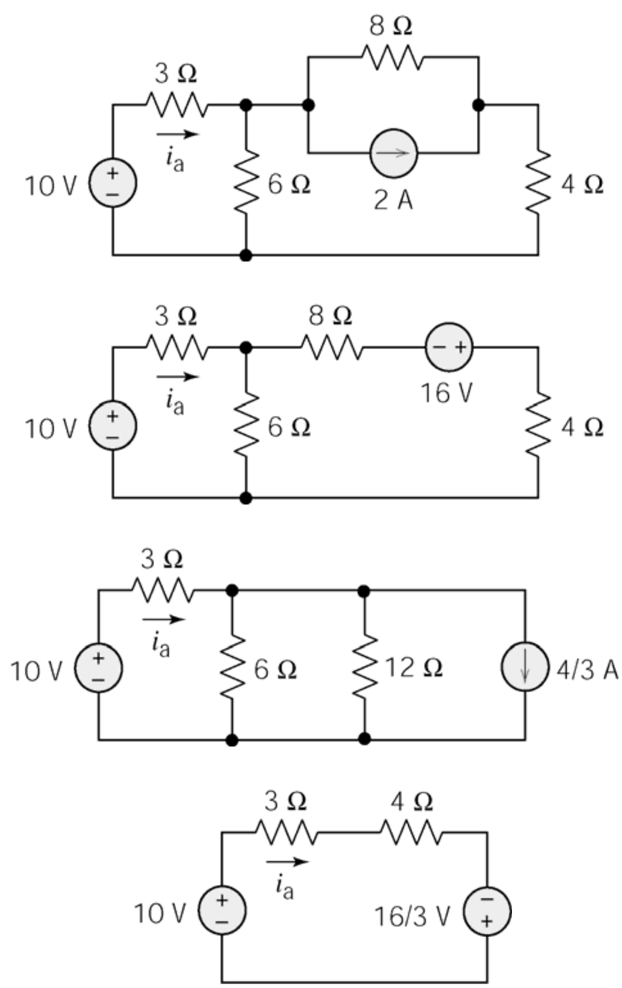


Figure P 5.2-2

Solution:



Finally, apply KVL: $-10+3\,i_a+4\,i_a-\dfrac{16}{3}=0 \quad \therefore \; \underline{i_a = \;2.19 \text{ A}}$

(checked using LNAP 8/15/02)

P 5.2-3 Find v_o using source transformations if $i = 5/2$ A in the circuit shown in Figure P 5.2-3.

Hint: Reduce the circuit to a single mesh that contains the voltage source labeled v_o.

Answer: $v_o = 28$ V

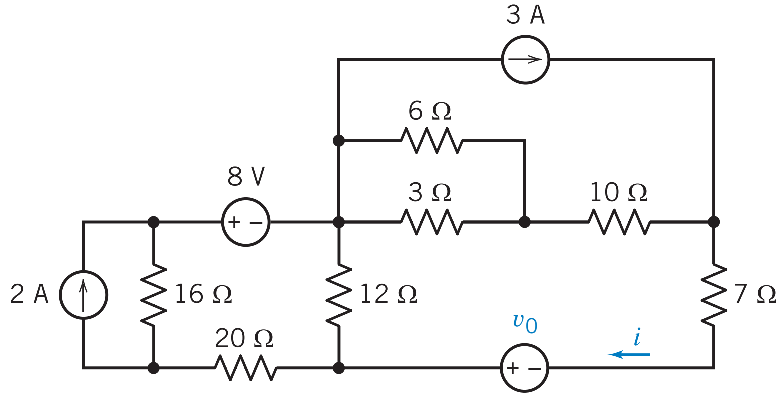
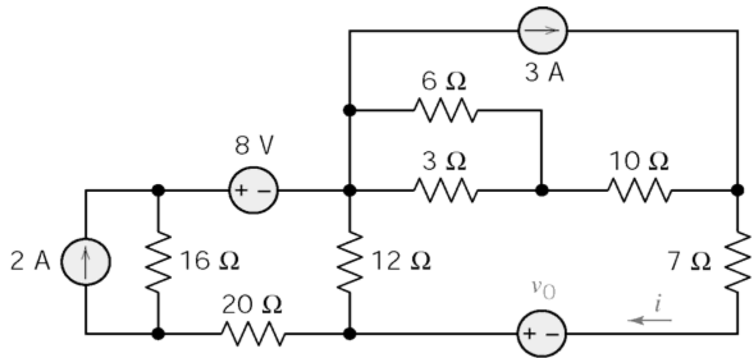
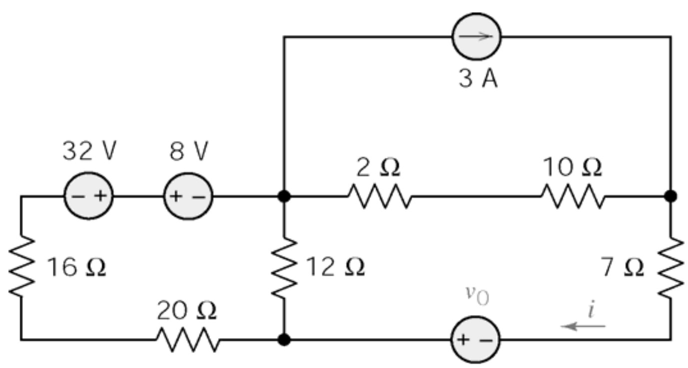


Figure P 5.2-3

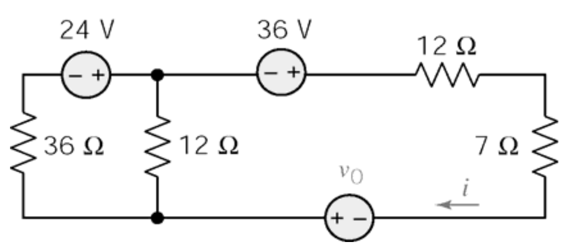
Solution:



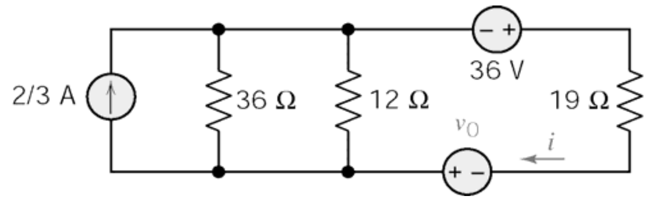
Source transformation at left; equivalent resistor for parallel 6 and 3 Ω resistors:



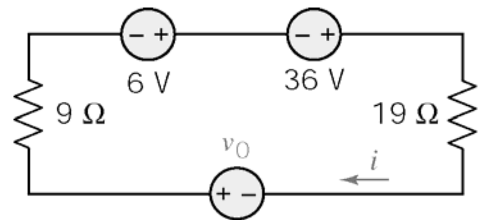
Equivalents for series resistors, series voltage source at left; series resistors, then source transformation at top:



Source transformation at left; series resistors at right:



Parallel resistors, then source transformation at left:



Finally, apply KVL to loop

$$-6 + i\ (9+19) - 36 - v_o = 0$$

$$i = 5/2\ \Rightarrow v_o = -42 + 28\ (5/2) = 28\text{ V}$$

(checked using LNAP 8/15/02)

P 5.2-4 Determine the value of the current i_a in the circuit shown in Figure P 5.2-4.

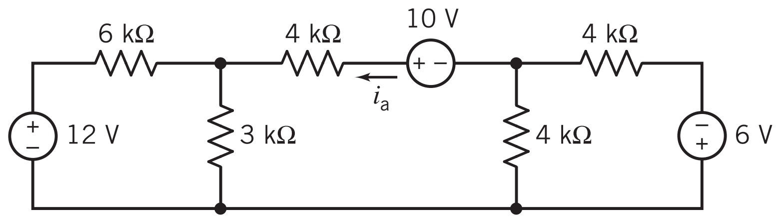
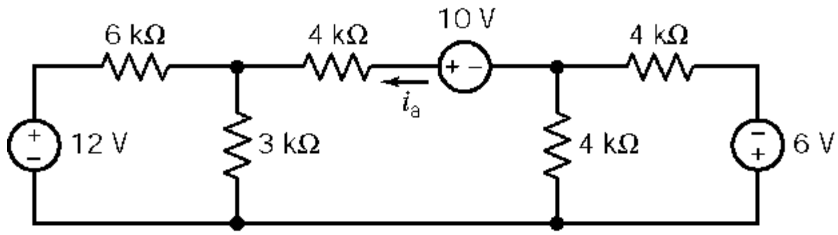
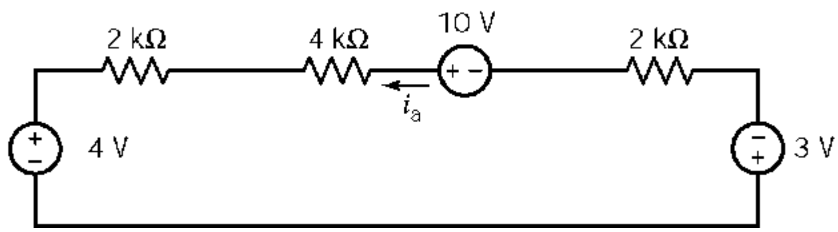


Figure P 5.2-4

Solution:





$$-4 - 2000\, i_a - 4000\, i_a + 10 - 2000\, i_a - 3 = 0$$

$$\therefore\ i_a = 375\ \mu\text{A}$$

(checked using LNAP 8/15/02)

P 5.2-5 Use source transformations to find the current i_a in the circuit shown in Figure P 5.2-5.

Answer: $i_a = 1$ A

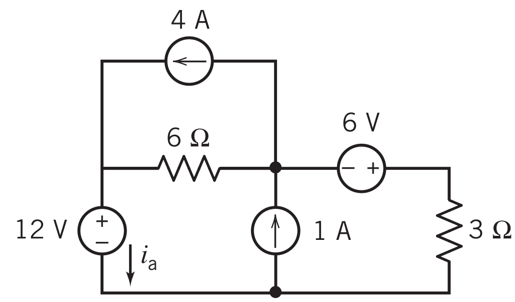
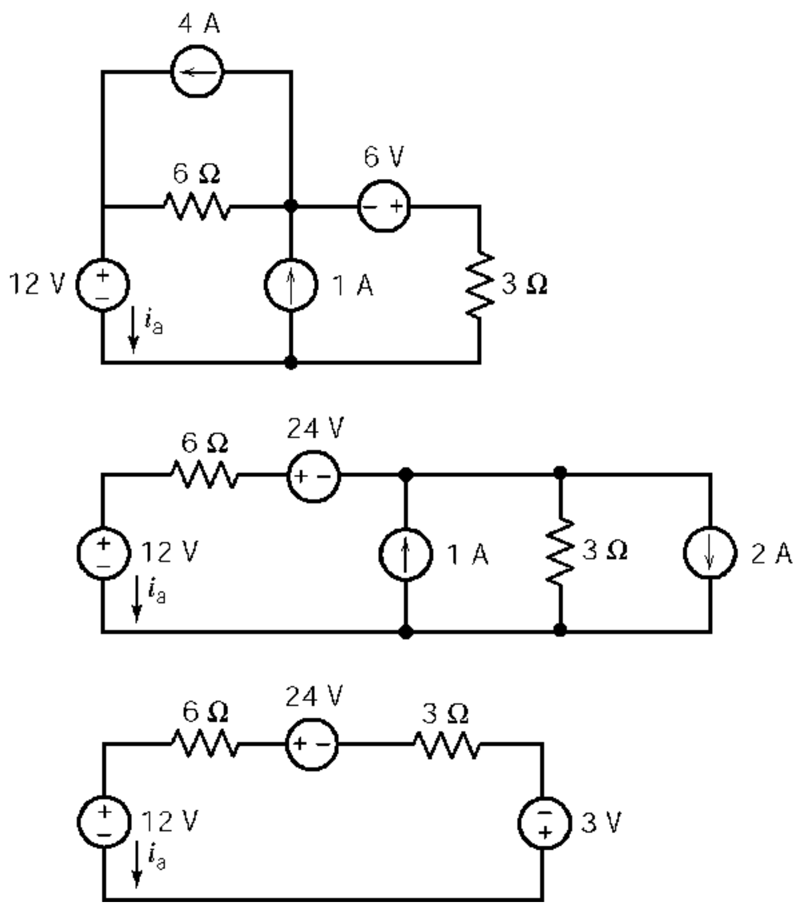


Figure P 5.2-5.

Solution:



$$-12-6\,i_a + 24 - 3\,i_a - 3 = 0 \Rightarrow i_a = 1 \text{ A}$$

(checked using LNAP 8/15/02)

P 5.2-6 Use source transformations to find the value of the voltage v_a in Figure P 5.2-6.

Answer: $v_a = 7$ V

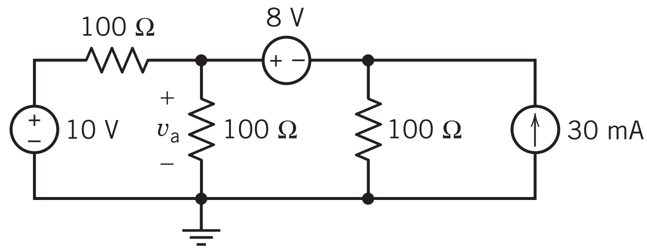
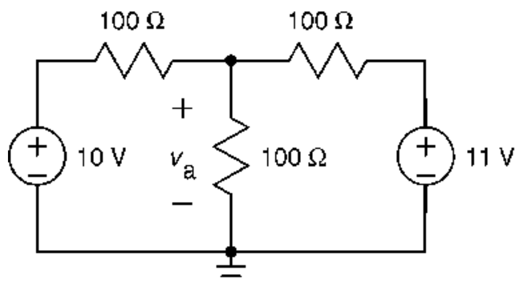


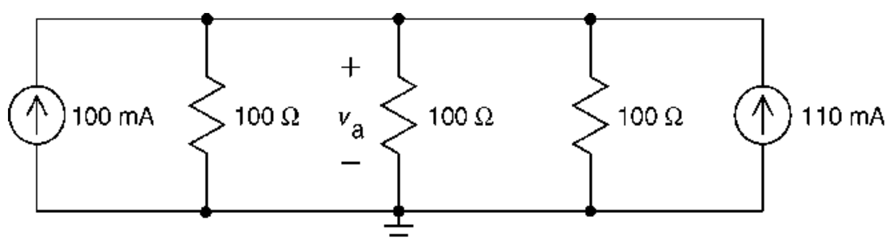
Figure P 5.2-6

Solution:

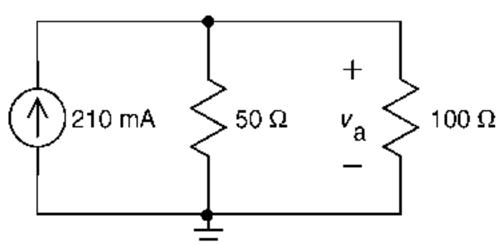
A source transformation on the right side of the circuit, followed by replacing series resistors with an equivalent resistor:



Source transformations on both the right side and the left side of the circuit:



Replacing parallel resistors with an equivalent resistor and also replacing parallel current sources with an equivalent current source:



Finally,

$$v_a = \frac{50(100)}{50+100}(0.21) = \frac{100}{3}(0.21) = 7 \text{ V}$$

(checked using LNAP 8/15/02)

P5.2-7

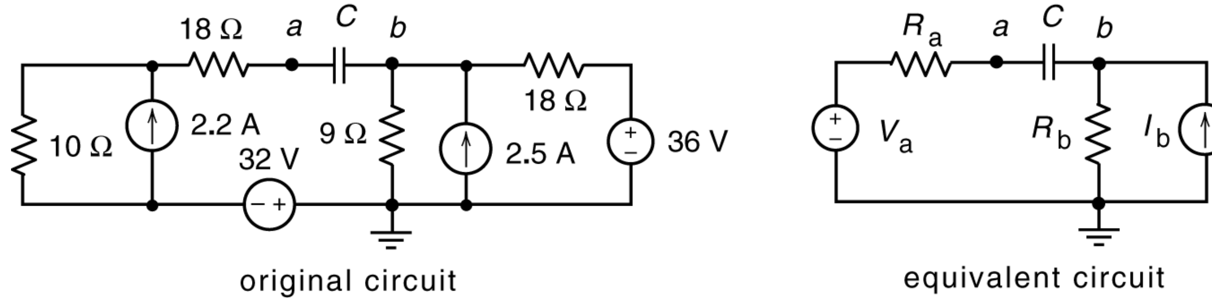
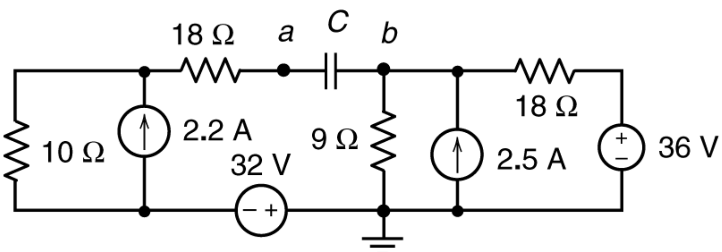


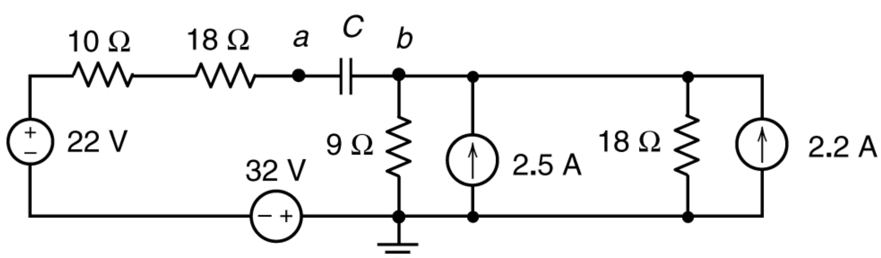
Figure P5.2-7

The equivalent circuit in Figure P5.2-7 is obtained from the original circuit using source transformations and equivalent resistances. (The lower case letters *a* and *b* identify the nodes of the capacitor in both the original and equivalent circuits.) Determine the values of R_a, V_a, R_b and I_b in the equivalent circuit.

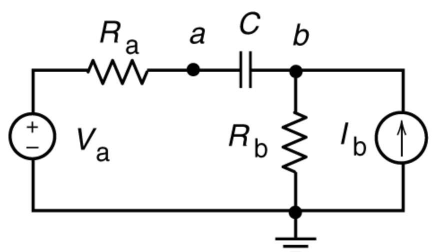
Solution



Performing a source transformation at each end of the circuit yields



Thenx



where

$$V_a = 2.2(10) - 32 = -10 \text{ V}, \ R_a = 18 + 10 = 28\ \Omega, \ R_b = 18 \,||\, 9 = 6\ \Omega \ \text{and} \ \ I_b = 2.5 + \frac{36}{18} = 4.5 \text{ A}$$

P 5.2-8 The circuit shown in Figure P 5.2-8 contains an unspecified resistance *R*.
(a) Determine the value of the current *i* when $R = 4\ \Omega$.
(b) Determine the value of the voltage *v* when $R = 8\ \Omega$.
(c) Determine the value of *R* that will cause $i = 1$ A.
(d) Determine the value of *R* that will cause $v = 16$ V.

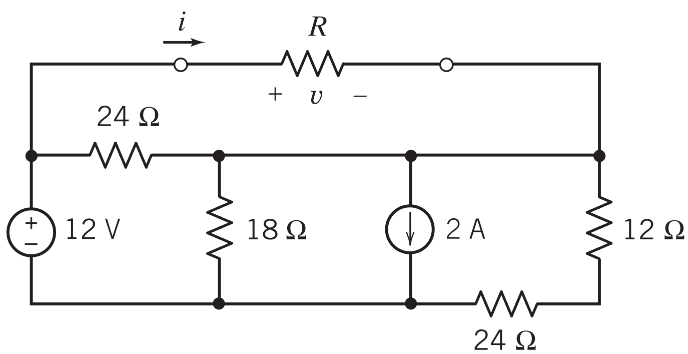
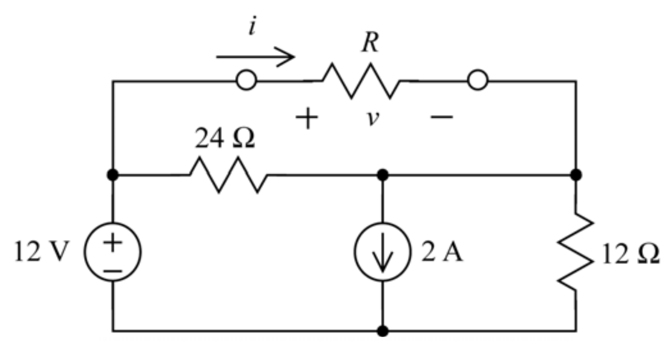


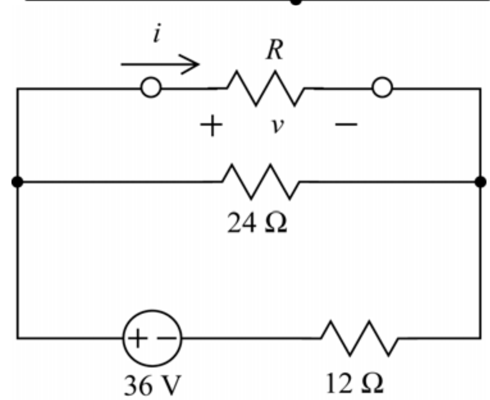
Figure P 5.2-8

Solution:



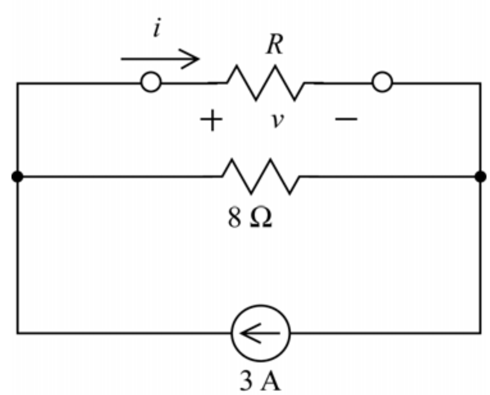
Replace series and parallel resistors by an equivalent resistor.

$$18 \,||\, (12+24) = 12\ \Omega$$



Do a source transformation, then replace series voltage sources by an equivalent voltage source.

Do two more source transformations



Now current division gives

$$i = \left(\frac{8}{8+R}\right)3 = \frac{24}{8+R}$$

Then Ohm's Law gives

$$v = Ri = \frac{24R}{8+R}$$

(a) $i = \dfrac{24}{8+4} = 2\ \text{A}$

(b) $v = \dfrac{24(8)}{8+8} = 12\ \text{V}$

(c) $1 = \dfrac{24}{8+R} \quad \Rightarrow \quad R = 16\ \Omega$

(d) $16 = \dfrac{24R}{8+R} \quad \Rightarrow \quad R = 16\ \Omega$

(checked: LNAP 6/9/04)

P 5.2-9 Determine the value of the power supplied by the current source in the circuit shown in Figure P 5.2-9.

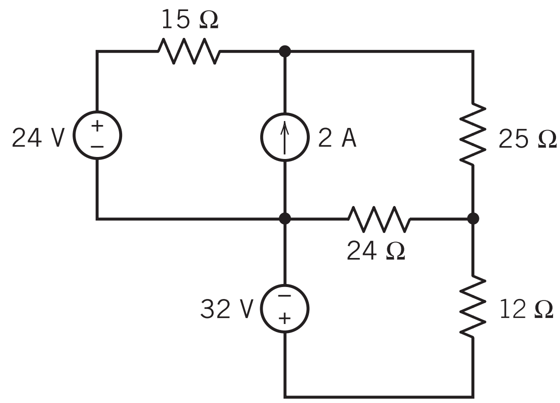
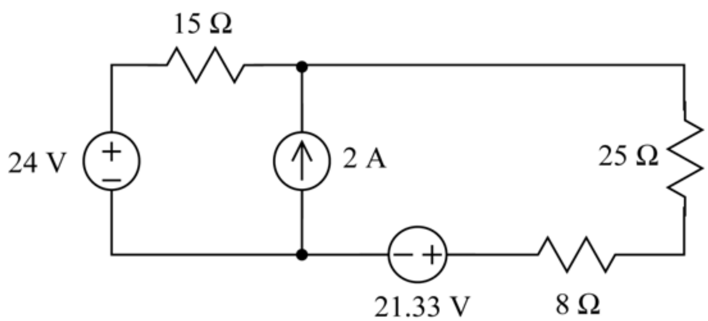
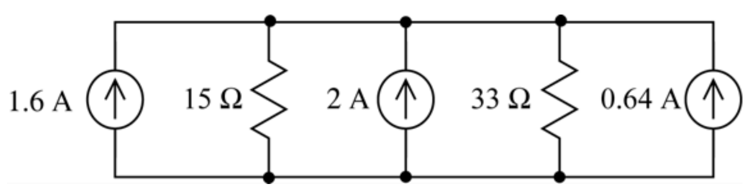
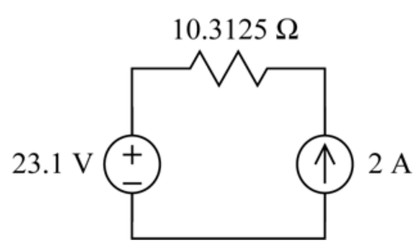


Figure P 5.2-9

Solution:
Use source transformations and equivalent resistances to reduce the circuit as follows







The power supplied by the current source is given by

$$p = \left[23.1 + 2(10.3125)\right]2 = 87.45 \text{ W}$$

Section 5-3 Superposition

P5.3-1

The inputs to the circuit shown in Figure P5.3-1 are the voltage source voltages v_1 and v_2.. The output of the circuit is the voltage v_o. The output is related to the inputs by

$$v_o = a\, v_1 + b\, v_2$$

where a and b are constants. Determine the values of a and b.

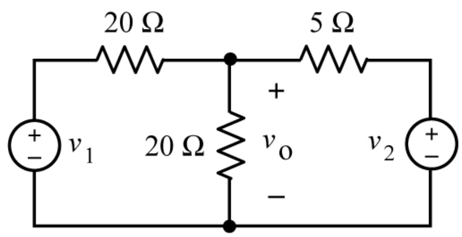
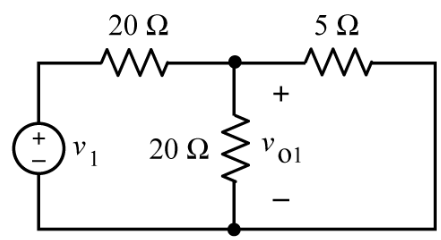


Figure P5.3-1

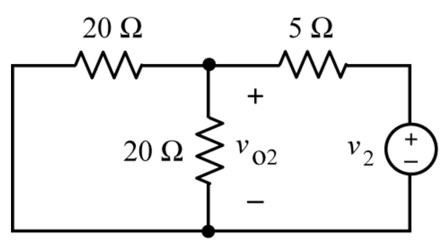
Solution:

Let $v_{o1} = a\, v_1$ be the output when $v_2 = 0$. In this case, the right voltage source acts like a short circuit so we have the circuit show to the right. Then



$$v_{o1} = \frac{20\,||\,5}{20+(20\,||\,5)} v_1 = \frac{4}{20+4} v_1 = \frac{1}{6} v_1 \quad \Rightarrow \quad a = \frac{1}{6}$$

Let $v_{o2} = b\, v_2$ be the output when $v_1 = 0$. In this case, the left voltage source acts like a short circuit so we have the circuit show to the right. Then



$$v_{o2} = \frac{20\,||\,20}{5+(20\,||\,20)} v_2 = \frac{10}{5+10} v_2 = \frac{2}{3} v_2 \quad \Rightarrow \quad b = \frac{2}{3}$$

P5.3-2

A particular linear circuit has two inputs, v_1 and v_2, and one output, v_o. Three measurements are made. The first measurement shows that the output is $v_o = 4$ V when the inputs are $v_1 = 2$ V and $v_2 = 0$. The second measurement shows that the output is $v_o = 10$ V when the inputs are $v_1 = 0$ and $v_2 = -2.5$ V. In the third measurement the inputs are $v_1 = 3$ V and $v_2 = 3$ V. What is the value of the output in the third measurement?

Solution:

The output of a linear circuit is a linear combination of the inputs:

$$v_o = a_1 v_1 + a_2 v_2$$

From the first two measurements we have:

$$\begin{pmatrix} 4 \\ 10 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & -2.5 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \quad \Rightarrow \quad \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

Now the output of the third measurement can be determine to be

$$v_o = a_1(3) + a_2(3) = (2)(3) + (-4)(3) = -6 \text{ V}$$

P5.3-3

The circuit shown in Figure P5.3-3 has two inputs, v_s and i_s, and one output i_o. The output is related to the inputs by the equation

$$i_o = a\,i_s + b\,v_s$$

Given the following two facts:

The output is $i_o = 0.45$ A when the inputs are $i_s = 0.25$ A and $v_s = 15$ V.

and

The output is $i_o = 0.30$ A when the inputs are $i_s = 0.50$ A and $v_s = 0$ V.

Determine the values of the constants a and b and the values of the resistances are R_1 and R_2.

Answers: $a = 0.6$ A/A, $b = 0.02$ A/V, $R_1 = 30\ \Omega$ and $R_2 = 20\ \Omega$.

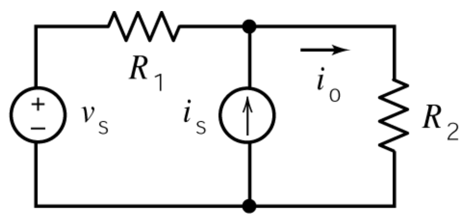
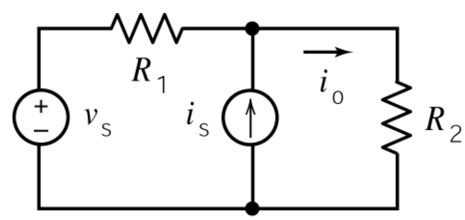


Figure P5.3-3

Solution:



From the 1st fact:

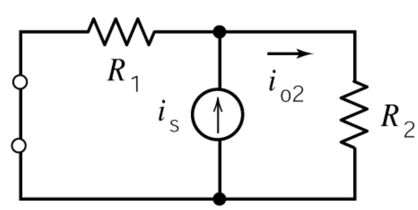
$$0.45 = a(0.25) + b(15)$$

From the 2nd fact:

$$0.30 = a(0.50) + b(0) \quad \Rightarrow \quad a = \frac{0.30}{0.50} = 0.60$$

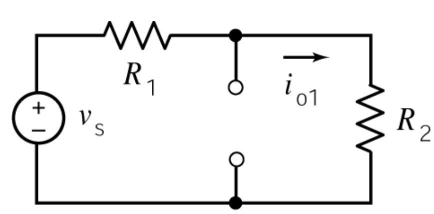
Substituting gives $\quad 0.45 = (0.60)(0.25) + b(15) \quad \Rightarrow \quad b = \dfrac{0.45 - (0.60)(0.25)}{15} = 0.02$

Next, consider the circuit:



$$a\,i_s = i_{o1} = i_o\Big|_{v_s=0} = \left(\frac{R_1}{R_1 + R_2}\right) i_s$$

so $\quad 0.60 = \dfrac{R_1}{R_1 + R_2} \quad \Rightarrow \quad 2\,R_1 = 3\,R_2$



and $\quad b\,v_s = i_{o2} = i_o\Big|_{i_s=0} = \dfrac{v_s}{R_1 + R_2}$

so $\quad 0.02 = \dfrac{1}{R_1 + R_2} \quad \Rightarrow \quad R_1 + R_2 = \dfrac{1}{0.02} = 50\ \Omega$

Solving these equations gives $R_1 = 30\ \Omega$ and $R_2 = 20\ \Omega$.

P 5.3-4 Use superposition to find the value of the voltage v in Figure P 5.3-4.

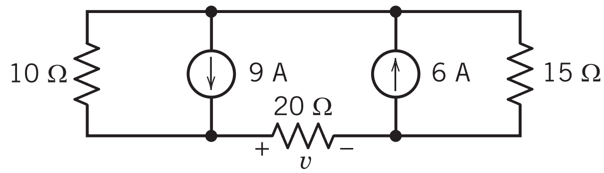
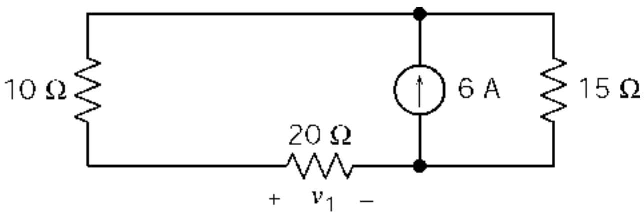


Figure P 5.3-4

Solution:

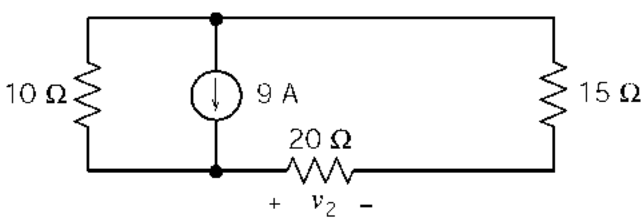
Consider 6 A source only (open 9 A source)



Use current division:

$$\frac{v_1}{20} = 6\left[\frac{15}{15 + 30}\right] \Rightarrow \underline{v_1 = 40 \text{ V}}$$

Consider 9 A source only (open 6 A source)



Use current division:

$$\frac{v_2}{20} = 9\left[\frac{10}{10 + 35}\right] \Rightarrow \underline{v_2 = 40 \text{ V}}$$

$$\underline{\therefore\ v = v_1 + v_2 = 40 + 40 = 80 \text{ V}}$$

(checked using LNAP 8/15/02)

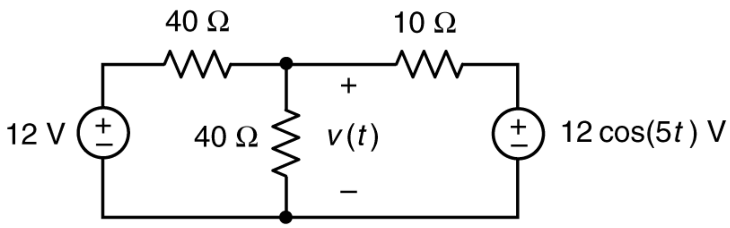
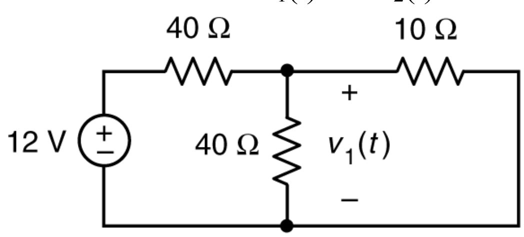
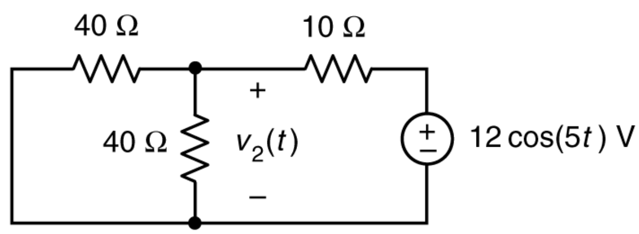


Figure P5.3-5

P5.3-5 Determine $v(t)$, the voltage across the vertical resistor in the circuit in Figure P5.3-5.

Solution;
We'll use superposition. Let $v_1(t)$ the be the part of $v(t)$ due to the voltage source acting alone. Similarly, let $v_2(t)$ the be the part of $v(t)$ due to the voltage source acting alone. We can use these circuits to calculate $v_1(t)$ and $v_2(t)$.





Notice that $v_1(t)$ is the voltage across parallel resistors. Using equivalent resistance, we calculate $40||10 = 8\ \Omega$. Next, using voltage division we calculate

$$v_1(t) = \frac{8}{8+40}(12) = 2\text{ V}$$

Similarly $v_2(t)$ is the voltage across parallel resistors Using equivalent resistance we first determine $40||40 = 20\ \Omega$ and then calculate

$$v_2(t) = \frac{20}{10+20}(12\cos(5t)) = 8\cos(5t)\text{ V}$$

Using superposition $$v(t) = v_1(t) + v_2(t) = 2 + 8\cos(5t)\text{ V}$$

P 5.3-6 Use superposition to find the value of the current i in Figure P 5.3-6.

Answer: i = 3.5 mA

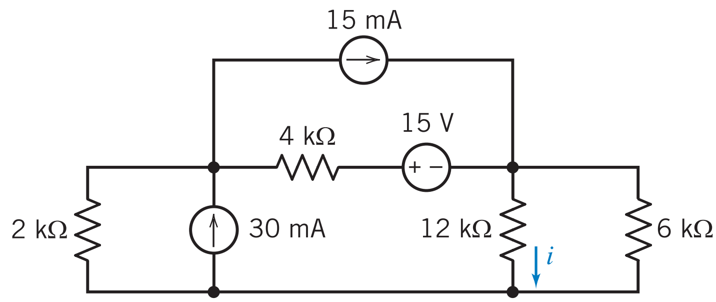
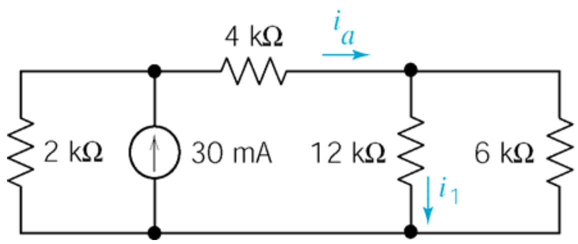
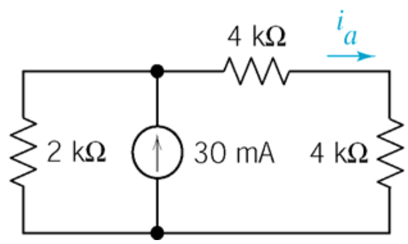


Figure P5.3-6

Solution:

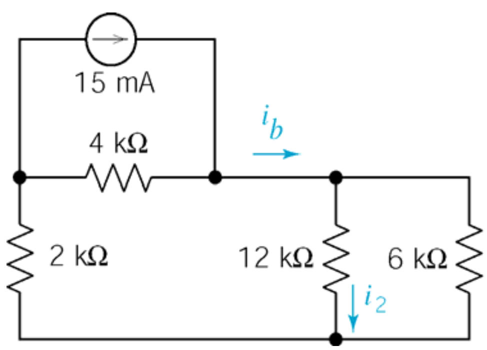
Consider 30 mA source only (open 15 mA and short 15 V sources). Let i_1 be the part of i due to the 30 mA current source.

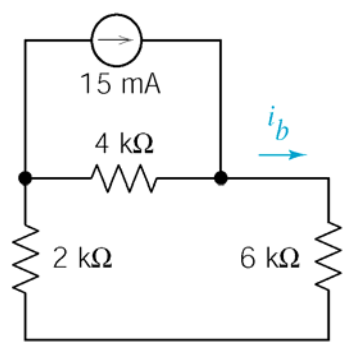




$$i_a = 30\left(\frac{2}{2+8}\right) = 6 \text{ mA} \Rightarrow i_1 = i_a\left(\frac{6}{6+12}\right) = \underline{2 \text{ mA}}$$

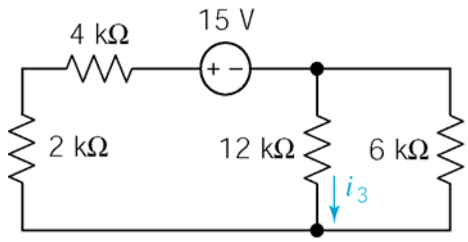
Consider 15 mA source only (open 30 mA source and short 15 V source) Let i_2 be the part of i due to the 15 mA current source.

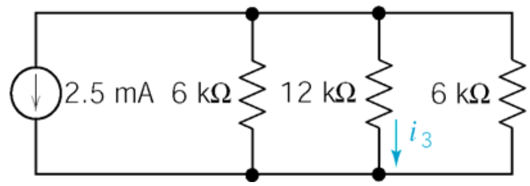




$$i_b = 15\left(\frac{4}{4+6}\right) = 6 \text{ mA} \Rightarrow i_2 = i_b\left(\frac{6}{6+12}\right) = \underline{2 \text{ mA}}$$

Consider 15 V source only (open both current sources). Let i_3 be the part of i due to the 15 V voltage source.





$$i_3 = -2.5\left(\frac{6\|6}{(6\|6)+12}\right) = -10\left(\frac{3}{3+12}\right) = \underline{-0.5 \text{ mA}}$$

Finally, $$\underline{i = i_1 + i_2 + i_3 = 2 + 2 - 0.5 = 3.5 \text{ mA}}$$

(checked using LNAP 8/15/02)

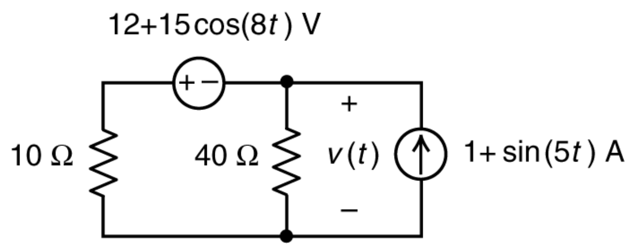
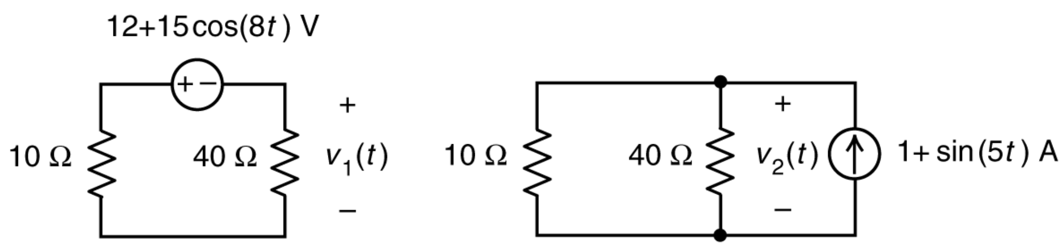


Figure P5.3-7

P5.3-7 Determine $v(t)$, the voltage across the 40 Ω resistor in the circuit in Figure P5.3-7.

Solution:
We'll use superposition. Let $v_1(t)$ the be the part of $v(t)$ due to the voltage source acting alone. Similarly, let $v_2(t)$ the be the part of $v(t)$ due to the voltage source acting alone. We can use these circuits to calculate $v_1(t)$ and $v_2(t)$.



Using voltage division we calculate

$$v_1(t) = -\frac{40}{10+40}\left(12+15\cos(8t)\right) = -9.6 - 12\cos(8t)$$

Using equivalent resistance we first determine 10||40 = 8 Ω and then calculate

$$v_2(t) = 8\left(1+\sin(5t)\right) = 8 + 8\sin(5t)$$

Using superposition $$v(t) = v_1(t) + v_2(t) = -1.6 + 8\sin(5t) - 12\cos(8t) \text{ V}$$

P 5.3-8 Use superposition to find the value of the current i_x in Figure P 5.3-8.

Answer: $i = 3.5$ mA

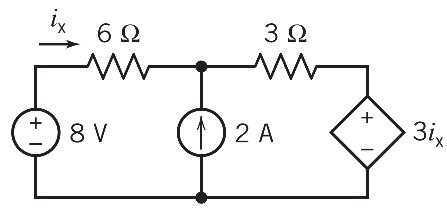
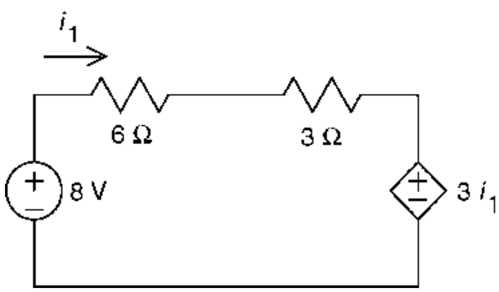


Figure P5.3-8

Solution:

Consider 8 V source only (open the 2 A source)



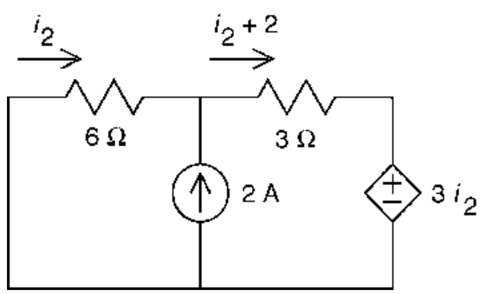
Let i_1 be the part of i_x due to the 8 V voltage source.

Apply KVL to the supermesh:

$$6\left(i_1\right)+3\left(i_1\right)+3\left(i_1\right)-8=0$$

$$i_1=\frac{8}{12}=\frac{2}{3}\text{ A}$$

Consider 2 A source only (short the 8 V source)



Let i_2 be the part of i_x due to the 2 A current source.

Apply KVL to the supermesh:

$$6\left(i_2\right)+3\left(i_2+2\right)+3\,i_2=0$$

$$i_2=\frac{-6}{12}=-\frac{1}{2}\text{ A}$$

Finally, $$i_x=i_1+i_2=\frac{2}{3}-\frac{1}{2}=\frac{1}{6}\text{ A}$$

P 5.3-9 The input to the circuit shown in Figure P 5.3-9 is the voltage source voltage, v_s. The output is the voltage v_o. The current source current, i_a, is used to adjust the relationship between the input and output. Design the circuit so that input and output are related by the equation $v_o = 2v_s + 9$.

Hint: Determine the required values of A and i_a.

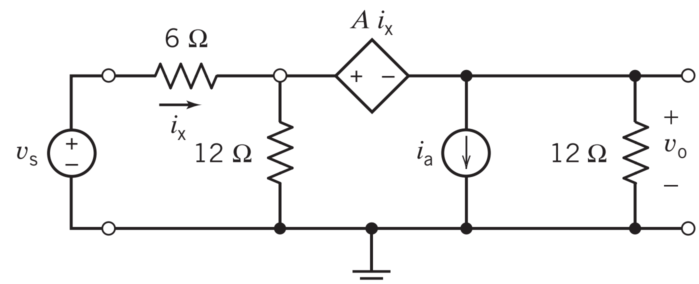


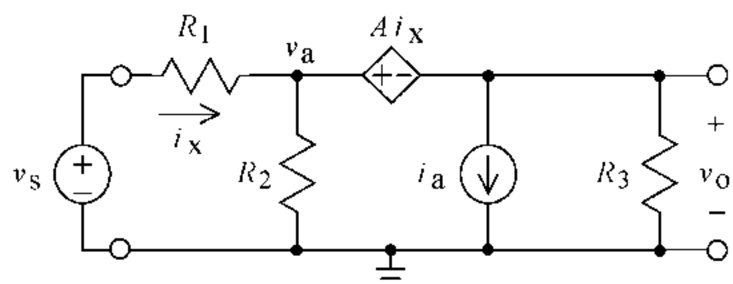
Figure P 5.3-9

Soluton:

$$i_x = \frac{v_s - v_a}{R_1}$$

$$v_a - v_o = A\,i_x = A\frac{v_s - v_a}{R_1}$$

$$v_a = \frac{R_1\,v_o + A\,v_s}{R_1 + A}$$



Apply KCL to the supernode corresponding to the CCVS to get

$$\frac{v_a - v_s}{R_1} + \frac{v_a}{R_2} + i_a + \frac{v_o}{R_3} = 0$$

$$\frac{R_1 + R_2}{R_1\,R_2}v_a - \frac{v_s}{R_1} + i_a + \frac{v_o}{R_3} = 0$$

$$\frac{R_1 + R_2}{R_1\,R_2}\left(\frac{R_1\,v_o + A\,v_s}{R_1 + A}\right) - \frac{v_s}{R_1} + i_a + \frac{v_o}{R_3} = 0$$

$$\left(\frac{R_1 + R_2}{R_2\left(R_1 + A\right)} + \frac{1}{R_3}\right)v_o + \left(\frac{\left(R_1 + R_2\right)A}{R_1\,R_2\left(R_1 + A\right)} - \frac{1}{R_1}\right)v_s + i_a = 0$$

$$\frac{R_3\left(R_1 + R_2\right) + R_2\left(R_1 + A\right)}{R_2\,R_3\left(R_1 + A\right)}v_o + \frac{A - R_2}{R_2\left(R_1 + A\right)}v_s + i_a = 0$$

$$v_o = \frac{R_3\left(R_2 - A\right)}{R_3\left(R_1 + R_2\right) + R_2\left(R_1 + A\right)} v_s - \frac{R_2\, R_3\left(R_1 + A\right)}{R_3\left(R_1 + R_2\right) + R_2\left(R_1 + A\right)} i_a$$

When $R_1 = 6\ \Omega$, $R_2 = 12\ \Omega$ and $R_3 = 12\ \Omega$

$$v_o = \frac{12 - A}{24 + A} v_s - \frac{12\left(6 + A\right)}{24 + A} i_a$$

Comparing this equation to $v_o = 2v_s + 9$, we requires

$$\frac{12 - A}{24 + A} = 2 \quad \Leftrightarrow \quad A = -12 \ \frac{\text{V}}{\text{A}}$$

Then $2v_s + 9 = v_o = 2v_s + 6i_a$ so we require

$$9 = 6i_a \quad \Rightarrow \quad i_a = 1.5 \text{ A}$$

(checked: LNAP 6/22/04)

P 5.3-10 The circuit shown in Figure P 5.3-10 has three inputs: v_1, v_2, and i_3. The output of the circuit is v_o. The output is related to the inputs by

$$v_o = av_1 + bv_2 + ci_3$$

where a, b, and c are constants. Determine the values of a, b, and c.

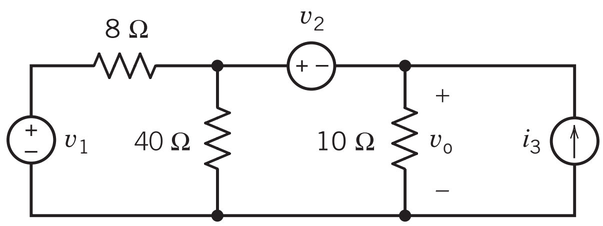
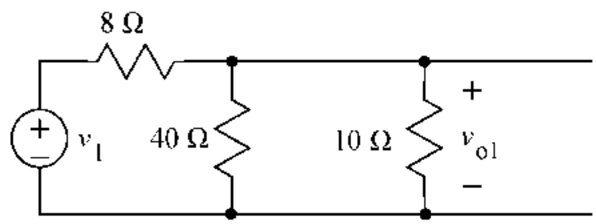


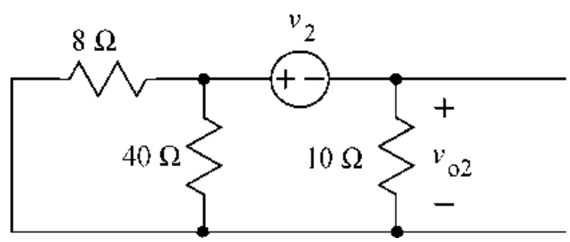
Figure P 5.3-10

Solution:

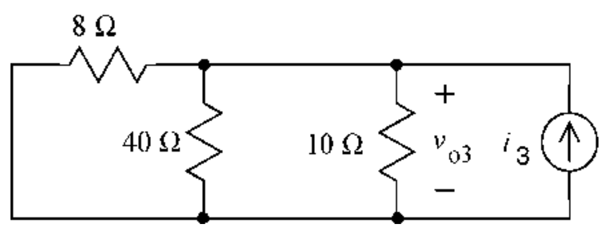
$$v_{o1} = \frac{40 \| 10}{8 + 40 \| 10} v_1 = \frac{1}{2} v_1 \quad \Rightarrow \quad a = \frac{1}{2}$$



$$v_{o2} = -\frac{10}{8 \| 40 + 10} v_1 = -\frac{3}{5} v_2 \quad \Rightarrow \quad b = -\frac{3}{5}$$



$$v_{o3} = \left(8 \| 10 \| 40\right) i_3 = 4 i_3 \quad \Rightarrow \quad c = 4$$



(checked: LNAP 6/22/04)

P 5.3-11 Determine the voltage $v_o(t)$ for the circuit shown in Figure P 5.3-11.

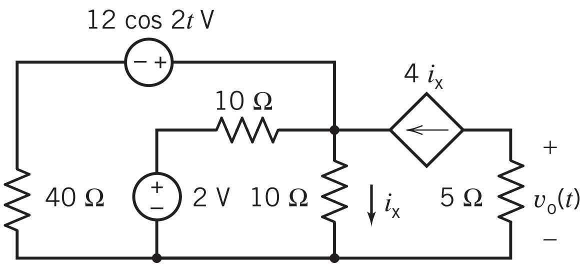
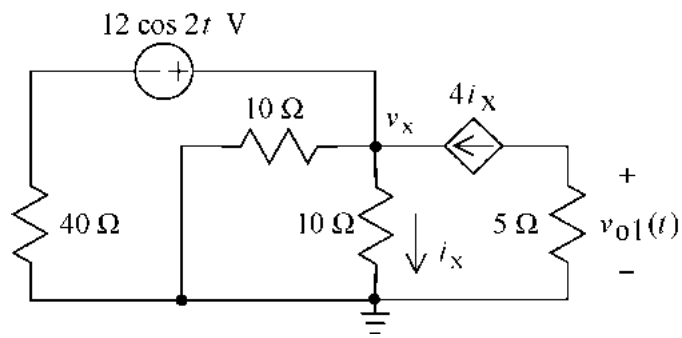


Figure P 5.3-11

Solution: Using superposition:

$$v_x = 10\, i_x$$



and

$$\frac{v_x - 12\cos 2t}{40} + \frac{v_x}{10} + \frac{v_x}{10} = 4\, i_x$$

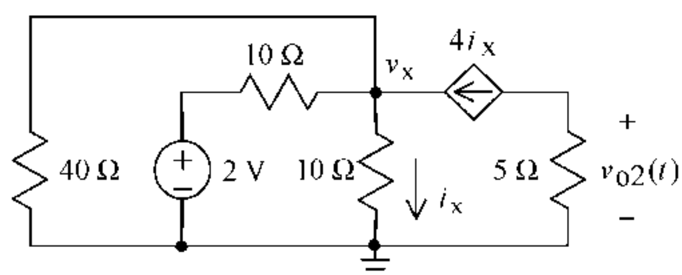
so

$$\frac{10\, i_x - 12\cos 2t}{40} = 2\, i_x \quad \Rightarrow \quad i_x = -\frac{12}{70}\cos 2t$$

Finally,

$$v_{o1} = -5\left(4\, i_x\right) = 3.429 \cos 2t \;\; \text{V}$$

$$v_x = 10\, i_x$$



and

$$\frac{v_x}{40} + \frac{v_x - 2}{10} + \frac{v_x}{10} = 4\, i_x$$

so

$$-0.2 = 1.75\, i_x \quad \Rightarrow \quad i_x = -0.11429 \text{ A}$$

Finally,

$$v_{o1} = -5\left(4\, i_x\right) = 2.286 \;\; \text{V}$$

$$v_o = v_{o1} + v_{o2} = 3.429 \cos 2t + 2.286 \;\; \text{V}$$

(checked: LNAP 6/22/04)

P 5.3-12 Determine the value of the voltage v_o in the circuit shown in Figure P 5.3-12.

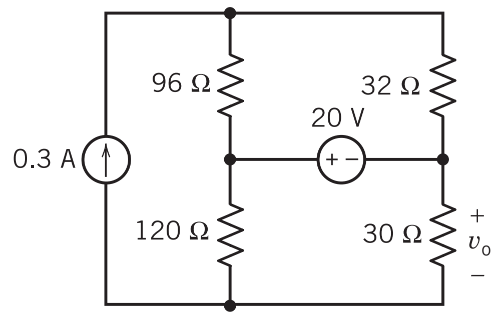
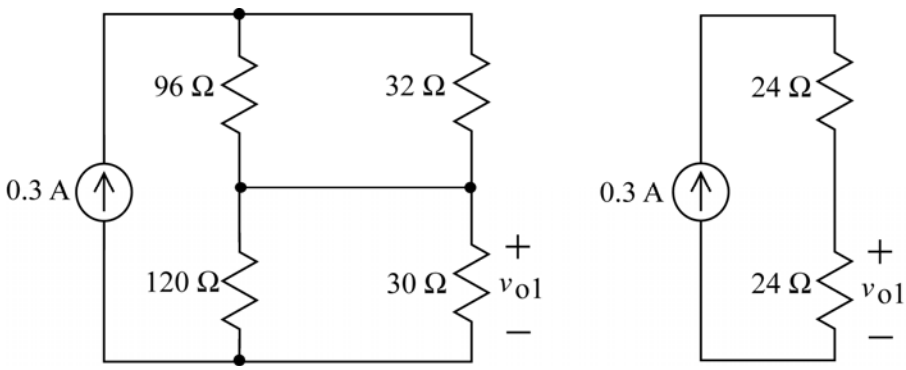
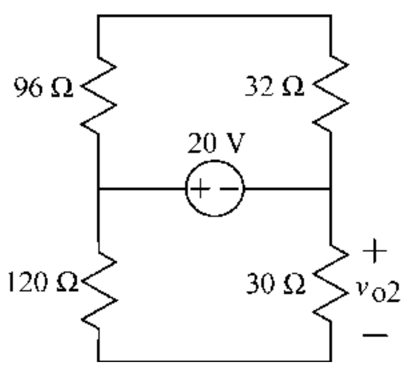


Figure P 5.3-12

Solution: Using superposition:





$$v_{o1} = 24(0.3) = 7.2 \text{ V}$$

$$v_{o2} = -\frac{30}{120+30} 20 = -4 \text{ V}$$

$$v_o = v_{o1} + v_{o2} = 3.2 \text{ V}$$

(checked: LNAP 5/24/04)

P 5.3-13 Determine the value of the voltage v_o in the circuit shown in Figure P 5.3-13.

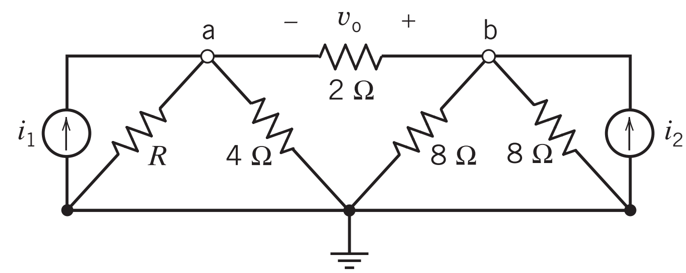


Figure P 5.3-13

Solution:
Using superposition

$$v_o = -2\left(\frac{R \| 4}{6+(R \| 4)}\right) i_1 + 2\left(\frac{4}{2+(R \| 4)+4}\right) i_2$$

Comparing to $v_o = -0.5\, i_1 + 4$, we require

$$-2\left(\frac{R \| 4}{6+(R \| 4)}\right) = -0.5 \quad \Rightarrow \quad 4(R \| 4) = 6 + (R \| 4) \quad \Rightarrow \quad R \| 4 = 2 \quad \Rightarrow \quad R = 4\ \Omega$$

and

$$2\left(\frac{4}{2+(R \| 4)+4}\right) i_2 = 4 \quad \Rightarrow \quad 2\left(\frac{4}{2+(4 \| 4)+4}\right) i_2 = 4 \quad \Rightarrow \quad i_2 = 4\ \text{A}$$

(checked LNAP 6/12/04)

P 5.3-14 Determine values of the current, i_a, and the resistance, R, for the circuit shown in Figure P 5.3-14.

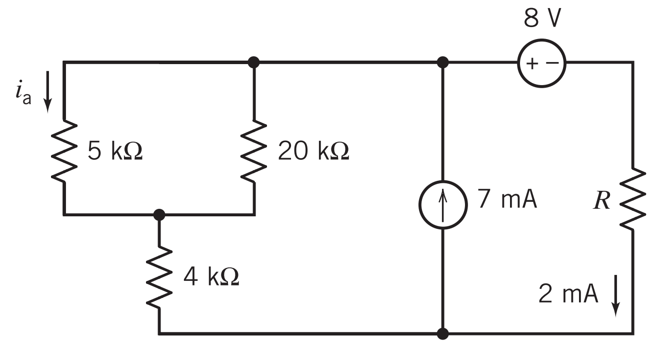


Figure P 5.3-14

Solution:

Use units of mA, kΩ and V.

$$4 + (5||20) = 8 \text{ k}\Omega$$

(a) Using superposition

$$2 = \left(\frac{8}{R+8}\right)7 - \frac{8}{R+8} \quad \Rightarrow \quad 2(R+8) = 48 \quad \Rightarrow \quad R = 16 \text{ k}\Omega$$

(b) Using superposition again

$$i_a = \left(\frac{5}{5+20}\right)\left[\left(\frac{16}{8+16}\right)7 + \frac{8}{8+16}\right] = \frac{4}{5}\left(\frac{2}{3}\times 7 + \frac{1}{3}\right) = 4 \text{ mA}$$

P 5.3-15 The circuit shown in Figure P 5.3-15 has three inputs: v_1, i_2, and v_3. The output of the circuit is the current i_o. The output of the circuit is related to the inputs by

$$i_1 = a v_o + b v_2 + c i_3$$

where a, b, and c are constants. Determine the values of a, b, and c.

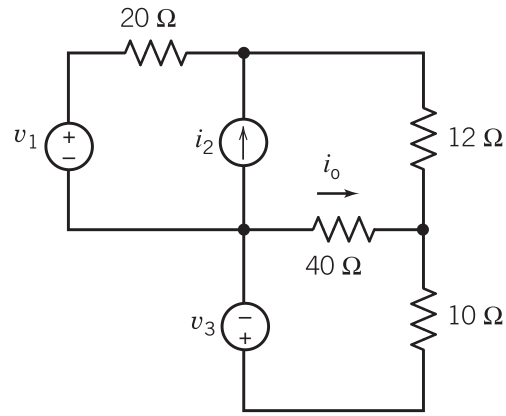


Figure P 5.3-15

Solution:

$$i_o = \left(-\frac{10}{10+40}\right)\left(\frac{v_1}{20+12+(40\|10)}\right) + \left(-\frac{10}{10+40}\right)\left(\frac{20}{20+\left[12+(40\|10)\right]}\right) i_2$$
$$+\left(-\frac{20+12}{40+(20+12)}\right)\left(\frac{v_3}{10+\left[40\|(20+12)\right]}\right)$$

$$i_o = \left(-\frac{1}{200}\right)v_1 + \left(-\frac{1}{10}\right)i_2 + \left(-\frac{1}{62.5}\right)v_3$$

So

$$a = -0.05,\ b = -0.1 \text{ and } c = -0.016$$

(checked: LNAP 6/19/04)

P 5.3-16 Using the superposition principle, find the value of the current measured by the ammeter in Figure P 5.3-16*a*.

Hint: Figure P 5.3-16*b* shows the circuit after the ideal ammeter has been replaced by the equivalent short circuit and a label has been added to indicate the current measured by the ammeter, i_m.

Answer:

$$i_m = \frac{25}{3+2} - \frac{3}{2+3}5 = 5 - 3 = 2 \text{ A}$$

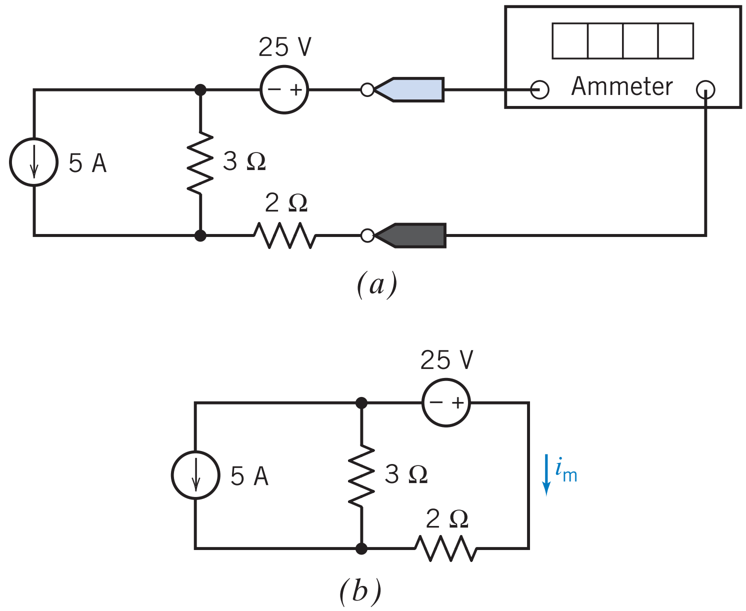


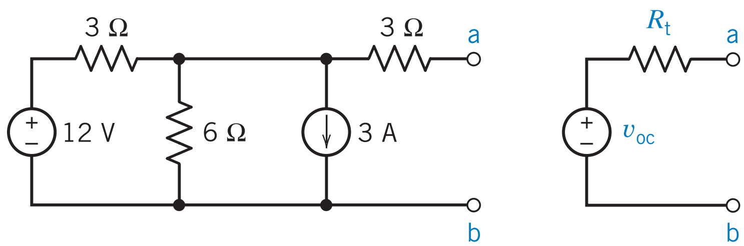
Figure P 5.3-16

Solution:

$$i_m = \frac{25}{3+2} - \frac{3}{2+3}(5) = 5 - 3 = 2 \text{ A}$$

Section 5-4: Thèvenin's Theorem

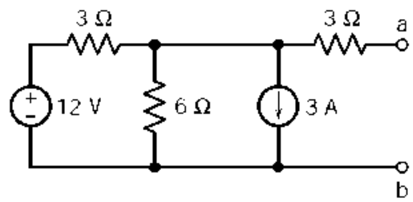
P 5.4-1 Determine values of R_t and v_{oc} that cause the circuit shown in Figure P 5.4-1*b* to be the Thévenin equivalent circuit of the circuit in Figure P 5.4-1*a*.
Hint: Use source transformations and equivalent resistances to reduce the circuit in Figure P 5.4-1*a* until it is the circuit in Figure P 5.4-1*b*.
Answer: $R_t = 5\ \Omega$ and $v_{oc} = 2$ V

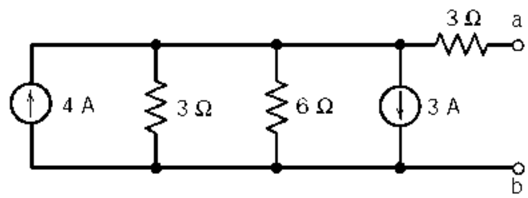


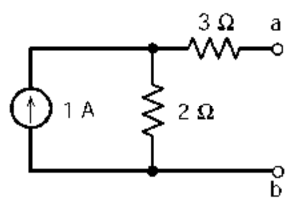
(*a*) (*b*)

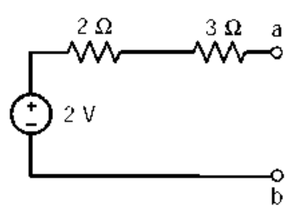
Figure P 5.4-1

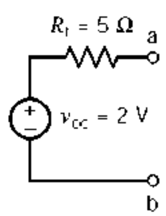
Solution:











(checked using LNAP 8/15/02)

P 5.4-2 The circuit shown in Figure P 5.4-2*b* is the Thévenin equivalent circuit of the circuit shown in Figure P 5.4-2*a*. Find the value of the open-circuit voltage, v_{oc}, and Thévenin resistance, R_t.

Answer: $v_{oc} = -12$ V and $R_t = 16\ \Omega$

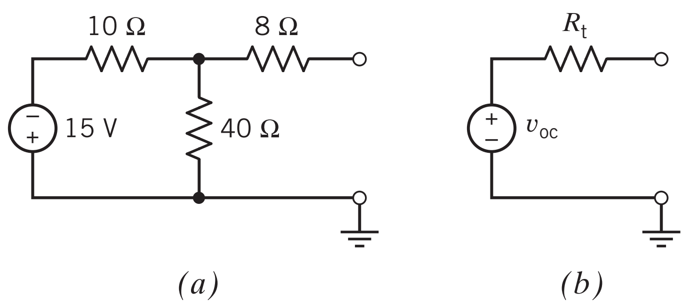
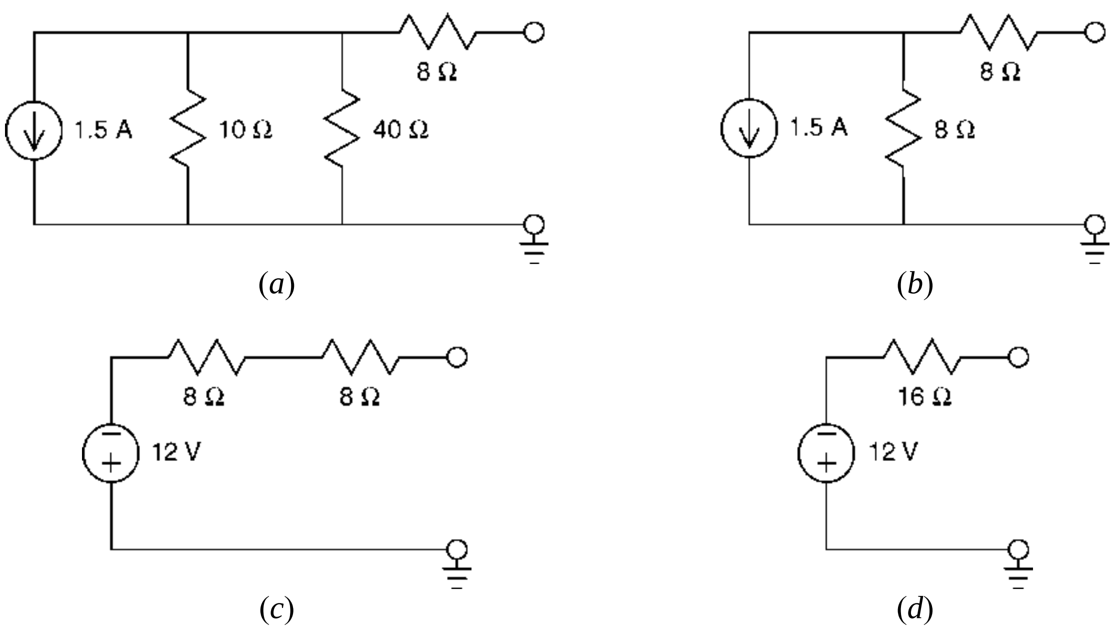


Figure P 5.4-2

Solution:

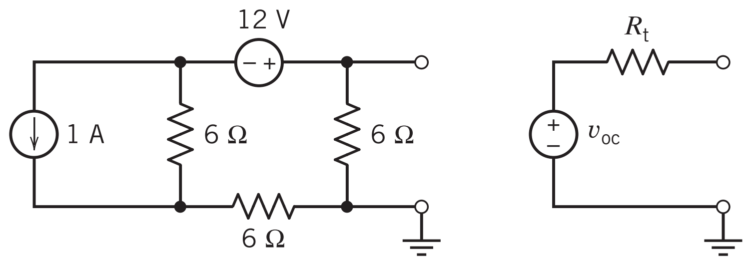
The circuit from Figure P5.4-2*a* can be reduced to its Thevenin equivalent circuit in four steps:



Comparing (*d*) to Figure P5.4-2*b* shows that the Thevenin resistance is $R_t = 16\ \Omega$ and the open circuit voltage, $v_{oc} = -12$ V.

P 5.4-3 The circuit shown in Figure P 5.4-3*b* is the Thévenin equivalent circuit of the circuit shown in Figure P 5.4-3*a*. Find the value of the open-circuit voltage, v_{oc}, and Thévenin resistance, R_t.

Answer: $v_{oc} = 2$ V and $R_t = 4\ \Omega$

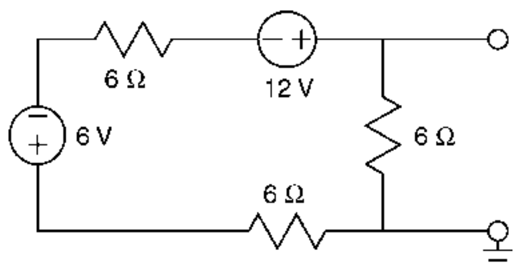


(*a*) (*b*)

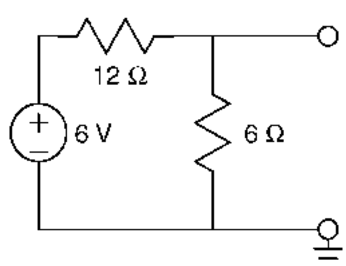
Figure P 5.4-3

Solution:

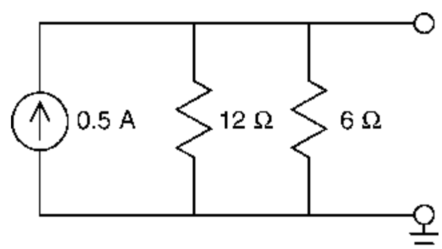
The circuit from Figure P5.4-3*a* can be reduced to its Thevenin equivalent circuit in five steps:



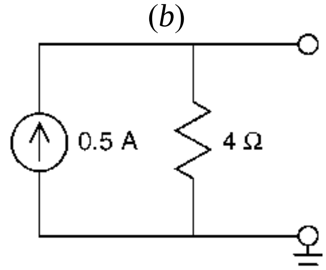
(*a*)



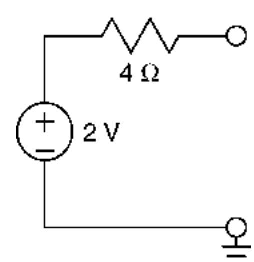
(*b*)



(*c*)



(*d*)



(*e*)

Comparing (*e*) to Figure P5.4-3*b* shows that the Thevenin resistance is $R_t = 4\ \Omega$ and the open circuit voltage, $v_{oc} = 2$ V.

(checked using LNAP 8/15/02)

P 5.4-4 Find the Thévenin equivalent circuit for the circuit shown in Figure P 5.4-4.

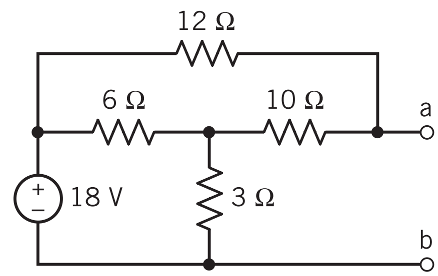
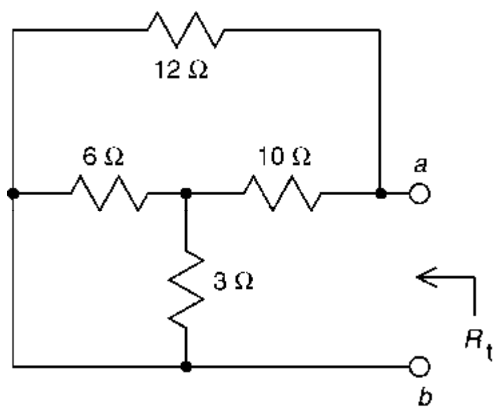
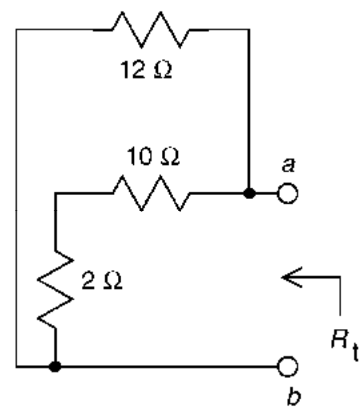


Figure P 5.4-4

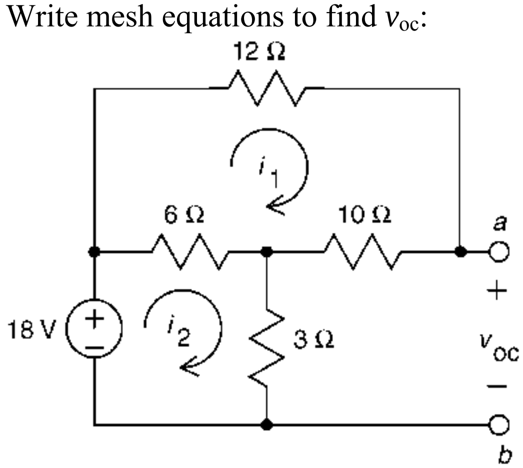
Find R_t:





$$R_t = \frac{12(10+2)}{12+(10+2)} = 6\ \Omega$$

Write mesh equations to find v_{oc}:



Mesh equations:

$$12\,i_1 + 10\,i_1 - 6\left(i_2 - i_1\right) = 0$$

$$6\left(i_2 - i_1\right) + 3\,i_2 - 18 = 0$$

$$28\,i_1 = 6\,i_2$$

$$9\,i_2 - 6\,i_1 = 18$$

$$36\,i_1 = 18 \quad \Rightarrow \quad i_1 = \frac{1}{2}\ \text{A}$$

$$i_2 = \frac{14}{3}\left(\frac{1}{2}\right) = \frac{7}{3}\ \text{A}$$

Finally, $$v_{oc} = 3\,i_2 + 10\,i_1 = 3\left(\frac{7}{3}\right) + 10\left(\frac{1}{2}\right) = 12\ \text{V}$$

(checked using LNAP 8/15/02)

P 5.4-5 Find the Thévenin equivalent circuit for the circuit shown in Figure P 5.4-5.

Answer: v_{oc} = –2 V and R_t = –8/3 Ω

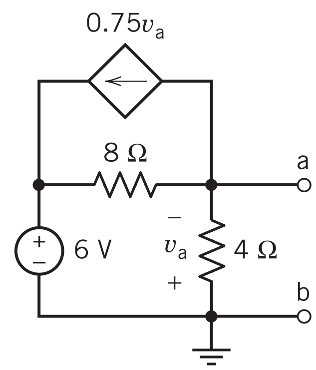
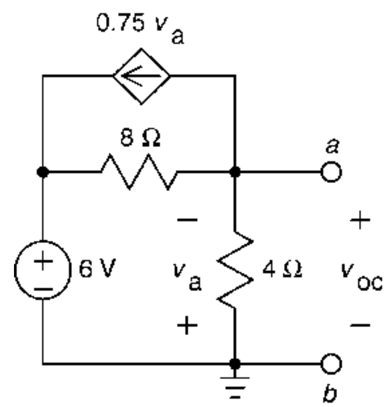


Figure P 5.4-4

Solution:
Find v_{oc}:

Notice that v_{oc} is the node voltage at node a. Express the controlling voltage of the dependent source as a function of the node voltage:



$$v_a = -v_{oc}$$

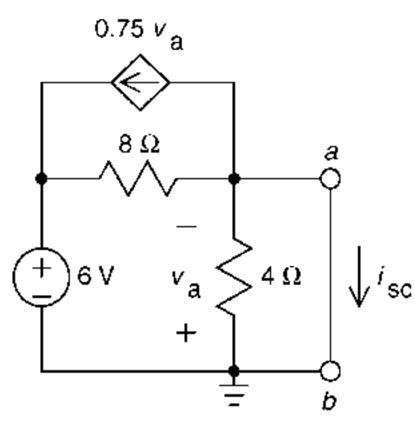
Apply KCL at node a:

$$-\left(\frac{6-v_{oc}}{8}\right)+\frac{v_{oc}}{4}+\left(-\frac{3}{4}v_{oc}\right)=0$$

$$-6+v_{oc}+2\,v_{oc}-6\,v_{oc}=0 \quad \Rightarrow \quad v_{oc}=-2 \text{ V}$$

Find R_t:

We'll find i_{sc} and use it to calculate R_t. Notice that the short circuit forces



$$v_a = 0$$

Apply KCL at node a:

$$-\left(\frac{6-0}{8}\right)+\frac{0}{4}+\left(-\frac{3}{4}0\right)+i_{sc}=0$$

$$i_{sc}=\frac{6}{8}=\frac{3}{4} \text{ A}$$

$$R_t=\frac{v_{oc}}{i_{sc}}=\frac{-2}{3/4}=-\frac{8}{3}\,\Omega$$

(checked using LNAP 8/15/02)

P 5.4-6 Find the Thévenin equivalent circuit for the circuit shown in Figure P 5.4-6.

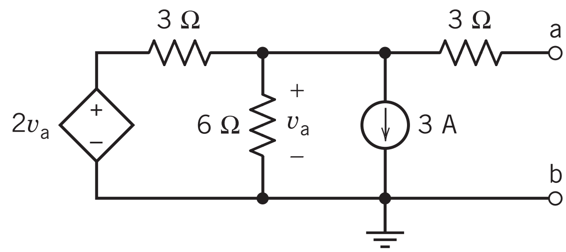
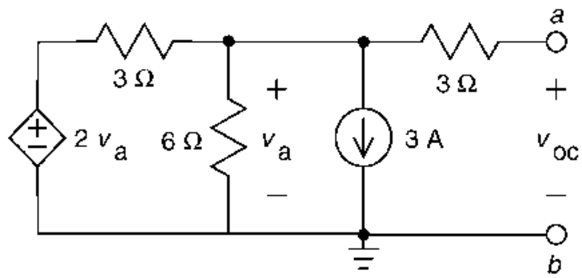


Figure P 5.4-6

Solution:

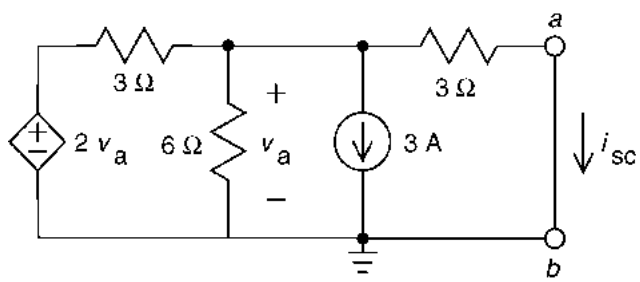
Find v_{oc}:



Apply KCL at the top, middle node: $\frac{2\,v_a - v_a}{3} = \frac{v_a}{6} + 3 + 0 \quad \Rightarrow \quad v_a = 18 \text{ V}$

The voltage across the right-hand 3 Ω resistor is zero so: $v_a = v_{oc} = 18$ V

Find i_{sc}:



Apply KCL at the top, middle node: $\frac{2\,v_a - v_a}{3} = \frac{v_a}{6} + 3 + \frac{v_a}{3} \quad \Rightarrow \quad v_a = -18 \text{ V}$

Apply Ohm's law to the right-hand 3 Ω resistor : $i_{sc} = \frac{v_a}{3} = \frac{-18}{3} = -6 \text{ V}$

Finally: $R_t = \frac{v_{oc}}{i_{sc}} = \frac{18}{-6} = -3\,\Omega$

(checked using LNAP 8/15/02)

P5.4-7 The equivalent circuit in Figure P5.4-7 is obtained by replacing part of the original circuit by its Thevenin equivalent circuit. The values of the parameters of the Thevenin equivalent circuit are

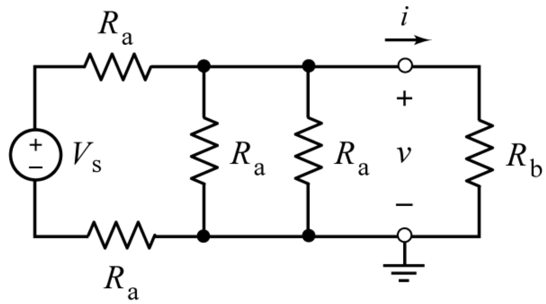
$$v_{oc} = 15 \text{ V and } R_t = 60\ \Omega$$

Determine the following:

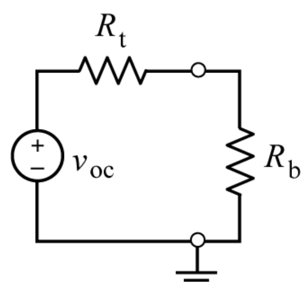
a.) The values of V_s and R_a. (Three resistors in the original circuit have equal resistance, R_a.)

b.) The value of R_b required to cause $i = 0.2$ A.

c.) The value of R_b required to cause $v = 12$ V.



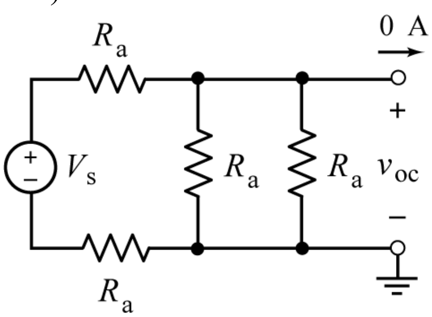
original circuit

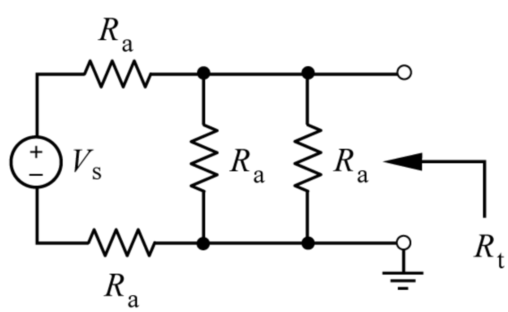


equivalent circuit

Figure P5.4-7

Solution: a.) From



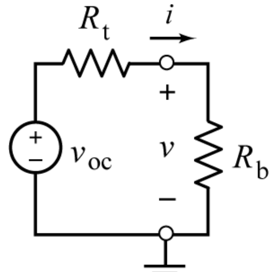


We see that $v_{oc} = \dfrac{V_s}{5}$ and $R_t = \dfrac{2}{5} R_a$. With the given values of v_{oc} and R_t we calculate

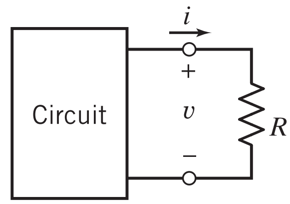
$$15 = \frac{V_s}{5} \quad \Rightarrow \quad V_s = 75 \text{ V} \quad \text{and} \quad 60 = \frac{2}{5} R_a \quad \Rightarrow \quad R_a = 150\ \Omega .$$

b.) $i = \dfrac{v_{oc}}{R_t + R_b} \quad \Rightarrow \quad 0.2 = \dfrac{15}{60 + R_b} \quad \Rightarrow \quad R_b = 15\ \Omega$

c.) $v = \dfrac{R_b}{R_t + R_b} v_{oc} \quad \Rightarrow \quad 12 = \dfrac{15\, R_b}{60 + R_b} \quad \Rightarrow \quad R_b = 240\ \Omega$



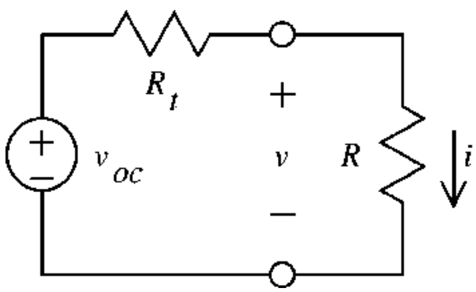
P 5.4-8 A resistor, R, was connected to a circuit box as shown in Figure P 5.4-8. The voltage, v, was measured. The resistance was changed, and the voltage was measured again. The results are shown in the table. Determine the Thévenin equivalent of the circuit within the box and predict the voltage, v, when $R = 8$ kΩ.



R	v
2 kΩ	6 V
4 kΩ	2 V

Figure P 5.4-8

Solution:



$$v = \frac{R}{R_t + R} v_{oc}$$

From the given data:

$$\left.\begin{array}{l} 6 = \dfrac{2000}{R_t + 2000} v_{oc} \\ 2 = \dfrac{4000}{R_t + 4000} v_{oc} \end{array}\right\} \Rightarrow \begin{cases} v_{oc} = 1.2 \text{ V} \\ R_t = -1600\ \Omega \end{cases}$$

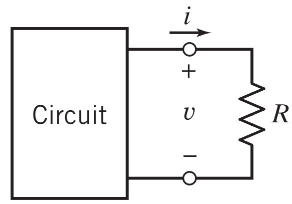
When $R = 8000\ \Omega$,

$$v = \frac{8000}{-1600 + 8000}(1.2) = 1.5 \text{ V}$$

P 5.4-9 A resistor, R, was connected to a circuit box as shown in Figure P 5.4-9. The current, i, was measured. The resistance was changed, and the current was measured again. The results are shown in the table.

(a) Specify the value of R required to cause i = 2 mA.

(b) Given that $R > 0$, determine the maximum possible value of the current i.

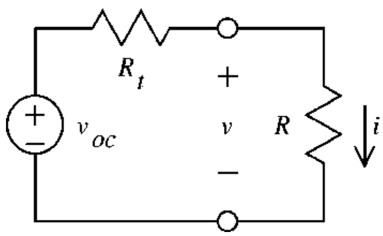


R	i
2 kΩ	4 mA
4 kΩ	3 mA

Figure P 5.4-9

Hint: Use the data in the table to represent the circuit by a Thévenin equivalent.

Solution:



$$i = \frac{v_{oc}}{R_t + R}$$

From the given data:

$$\left.\begin{array}{l} 0.004 = \dfrac{v_{oc}}{R_t + 2000} \\ 0.003 = \dfrac{v_{oc}}{R_t + 4000} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} v_{oc} = 24 \text{ V} \\ R_t = 4000\ \Omega \end{array}\right.$$

(a) When i = 0.002 A:

$$0.002 = \frac{24}{4000 + R} \quad \Rightarrow \quad R = 8000\ \Omega$$

(b) Maximum i occurs when R = 0:

$$\frac{24}{4000} = 0.006 = 6 \text{ mA} \quad \Rightarrow \quad i \le 6 \text{ mA}$$

P 5.4-10 For the circuit of Figure P 5.4-10, specify the resistance R that will cause current i_b to be 2 mA. The current i_a has units of amps.
Hint: Find the Thévenin equivalent circuit of the circuit connected to R.

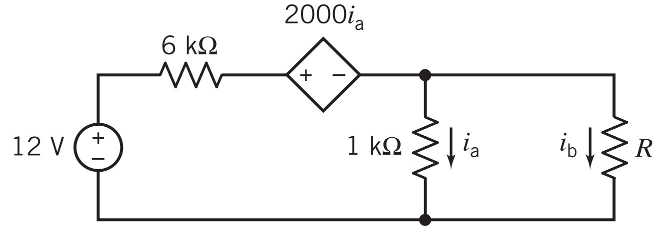
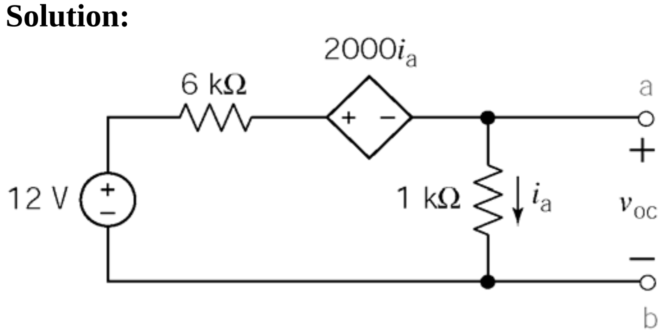
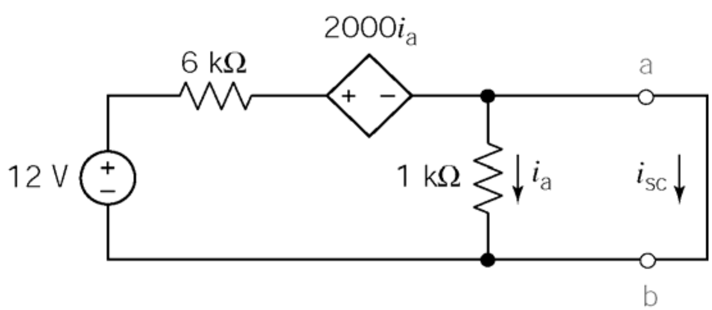


Figure P 5.4-10

Solution:



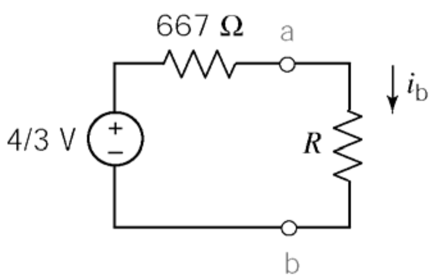
$$-12+6000\,i_a+2000\,i_a+1000\,i_a=0$$
$$i_a = 4/3000 \text{ A}$$
$$v_{oc}=1000\ i_a=\frac{4}{3}\text{ V}$$



$i_a = 0$ due to the short circuit

$$-12+6000\,i_{sc}=0\ \Rightarrow\ i_{sc}=2\ \text{mA}$$

$$R_t = \frac{v_{oc}}{i_{sc}}=\frac{\frac{4}{3}}{.002}=667\ \Omega$$



$$i_b=\frac{\frac{4}{3}}{667+R}$$

$i_b = 0.002$ A requires

$$R = \frac{\frac{4}{3}}{0.002}-667 = 0$$

(checked using LNAP 8/15/02)

P 5.4-11 For the circuit of Figure P 5.4-11, specify the value of the resistance R_L that will cause current i_L to be –2 A.

Answer: $R_L = 12\ \Omega$

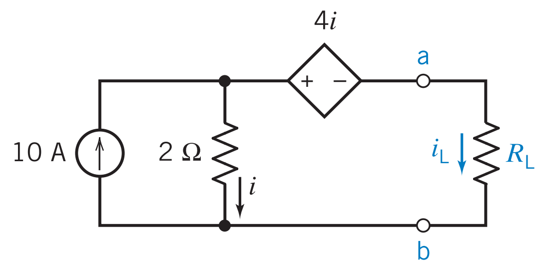
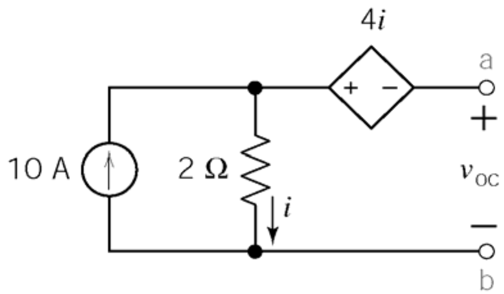


Figure P 5.4-11

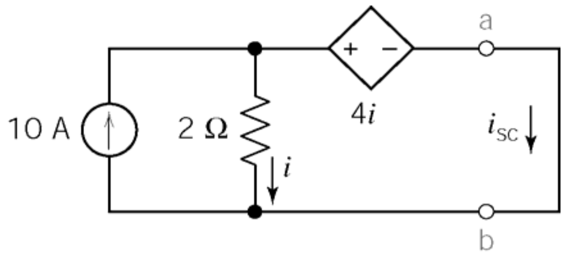
Solution:



$$10 = i + 0 \quad \Rightarrow \quad i = 10\ \text{A}$$

$$v_{oc} + 4\,i - 2\,i = 0$$

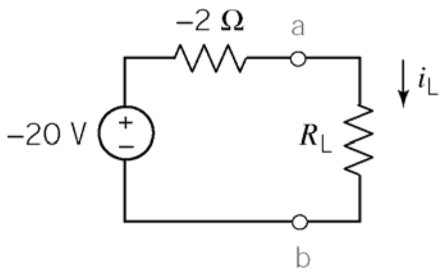
$$\Rightarrow \quad v_{oc} = -2\,i = \underline{-20\ \text{V}}$$



$$i + i_{sc} = 10 \quad \Rightarrow \quad i = 10 - i_{sc}$$

$$4\,i + 0 - 2\,i = 0 \quad \Rightarrow \quad i = 0 \quad \Rightarrow \quad i_{sc} = 10\ \text{A}$$

$$R_t = \frac{v_{oc}}{i_{sc}} = \frac{-20}{10} = -2\ \Omega$$



$$-2 = i_L = \frac{-20}{R_L - 2} \quad \Rightarrow \quad R_L = 12\ \Omega$$

(checked using LNAP 8/15/02)

P 5.4-12 The circuit shown in Figure P 5.4-12 contains an adjustable resistor. The resistance R can be set to any value in the range $0 \le R \le 100$ kΩ.

(a) Determine the maximum value of the current i_a that can be obtained by adjusting R. Determine the corresponding value of R.

(b) Determine the maximum value of the voltage v_a that can be obtained by adjusting R. Determine the corresponding value of R.

(c) Determine the maximum value of the power supplied to the adjustable resistor that can be obtained by adjusting R. Determine the corresponding value of R.

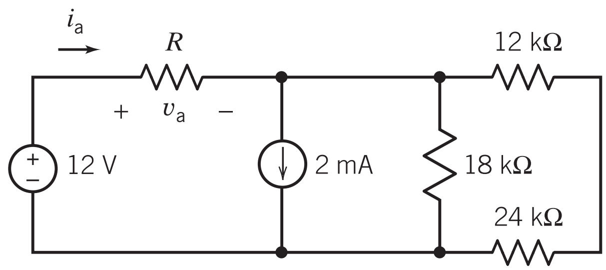
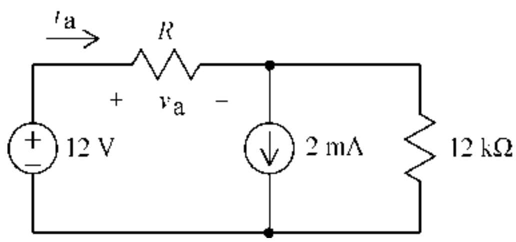
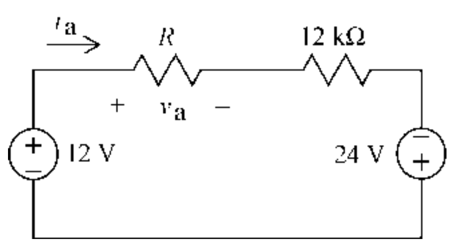


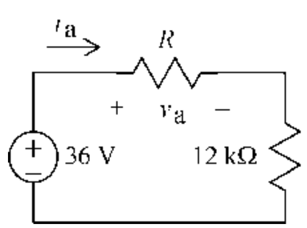
Figure P 5.4-12

Solution: Replace the part of the circuit that is connected to the variable resistor by its Thevenin equivalent circuit:



$$18\ \text{k}\Omega \,||\, (12\ \text{k}\Omega + 24\ \text{k}\Omega) = 18\ \text{k}\Omega \,||\, 36\ \text{k}\Omega = 12\ \text{k}\Omega$$





$$i_a = \frac{36}{R+12000} \quad \text{and} \quad v_a = \frac{R}{R+12000} 36$$

$$p = i_a\, v_a = \left(\frac{36}{R+12000}\right)^2 R$$

(a) $i_a = \dfrac{36}{0+12000} = 3$ mA when $R = 0\ \Omega$ (a short circuit).

(b) $v_a = \dfrac{10^5}{10^5+12000} 36 = 32.14$ V when R is as large as possible, i.e. $R = 100$ kΩ.

(c) Maximum power is delivered to the adjustable resistor when $R = R_t = 12\ \text{k}\Omega$. Then

$$p = i_a \, v_a = \left(\frac{36}{12000+12000} \right)^2 12000 = 0.027 = 27 \text{ mW}$$

(checked: LNAP 6/22/04)

P 5.4-13 The circuit shown in Figure P 5.4-13 consists of two parts, the source (to the left of the terminals) and the load. The load consists of a single adjustable resistor having resistance $0 \le R_L \le 20\ \Omega$. The resistance R is fixed, but unspecified. When $R_L = 4\ \Omega$, the load current is measured to be $i_o = 0.375$ A. When $R_L = 8\ \Omega$, the value of the load current is $i_o = 0.300$ A.

(a) Determine the value of the load current when $R_L = 10\ \Omega$.

(b) Determine the value of R.

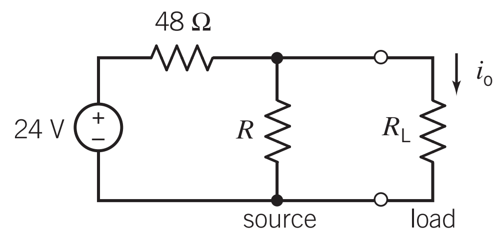
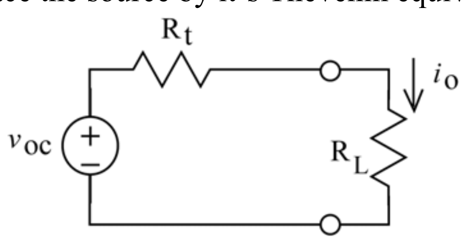


Figure P 5.4-13

Solution:

Replace the source by it's Thevenin equivalent circuit to get



$$i_o = \frac{v_{oc}}{R_t + R_L}$$

Using the given formation

$$\left.\begin{aligned} 0.375 &= \frac{v_{oc}}{R_t + 4} \\ 0.300 &= \frac{v_{oc}}{R_t + 8} \end{aligned}\right\} \quad \Rightarrow \quad 0.375(R_t + 4) = 0.300(R_t + 8)$$

So

$$R_t = \frac{(0.300)8 - (0.375)4}{0.075} = 12\ \Omega \text{ and } v_{oc} = 0.3(12 + 8) = 6\text{ V}$$

(a) When $R_L = 10\ \Omega$, $i_o = \frac{6}{12 + 10} = 0.27\overline{27}$ A.

(b) $12\ \Omega = R_t = 48 \parallel R \quad \Rightarrow \quad R = 16\ \Omega$.

(checked: LNAP 5/24/04)

P 5.4-14 The circuit shown in Figure P 5.4-14 contains an unspecified resistance, *R*. Determine the value of *R* in each of the following two ways.

(a) Write and solve mesh equations.
(b) Replace the part of the circuit connected to the resistor R by a Thévenin equivalent circuit. Analyze the resulting circuit.

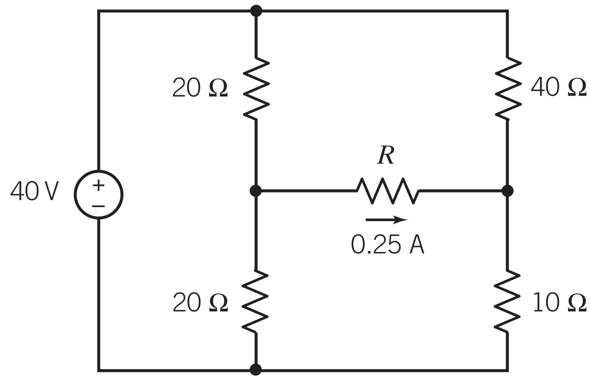
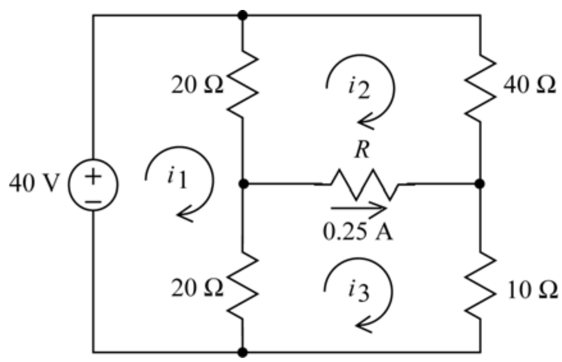


Figure P 5.4-14

Solution:
(a)



$$i_3 - i_2 = 0.25 \text{ A}$$

Apply KVL to mesh 1 to get

$$20(i_1 - i_2) + 20(i_1 - i_3) - 40 = 0$$

Apply KVL to the supermesh corresponding to the unspecified resistance to get

$$40i_2 + 10i_3 - 20(i_1 - i_3) - 20(i_1 - i_2) = 0$$

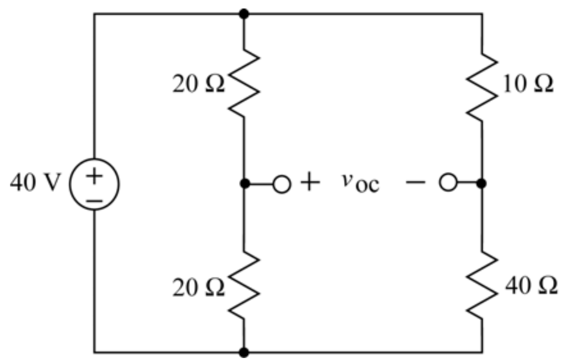
Solving, for example using MATLAB, gives

$$\begin{bmatrix} 0 & -1 & 1 \\ 40 & -20 & -20 \\ -40 & 60 & 30 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0.25 \\ 40 \\ 0 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 1.875 \\ 0.750 \\ 1.000 \end{bmatrix}$$

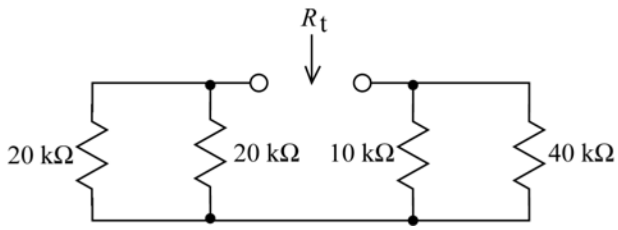
Apply KVL to mesh 2 to get

$$40i_2 + R(i_2 - i_3) - 20(i_1 - i_2) = 0 \quad \Rightarrow \quad R = \frac{20(i_1 - i_2) - 40i_2}{i_2 - i_3} = 30 \ \Omega$$

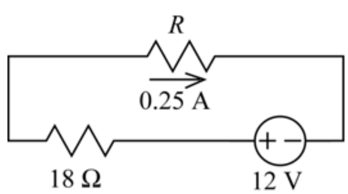
(b)



$$v_{oc} = \left(\frac{20}{20+20}\right)40 - \left(\frac{40}{10+40}\right)40 = -12 \text{ V}$$



$$R_t = 18 \ \Omega$$



$$0.25 = \frac{12}{18+R} \quad \Rightarrow \quad R = 30 \ \Omega$$

(checked: LNAP 5/25/04)

P 5.4-15 Consider the circuit shown in Figure P 5.4-15. Replace the part of the circuit to the left of terminals a–b by its Thévenin equivalent circuit. Determine the value of the current i_o.

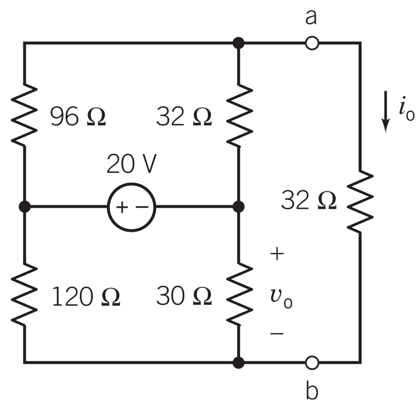
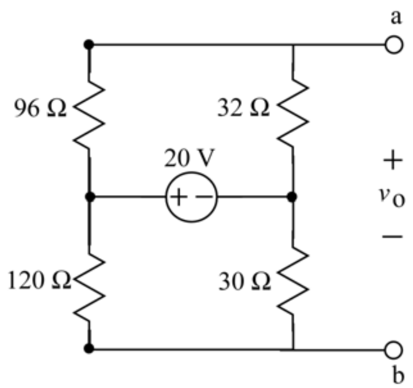


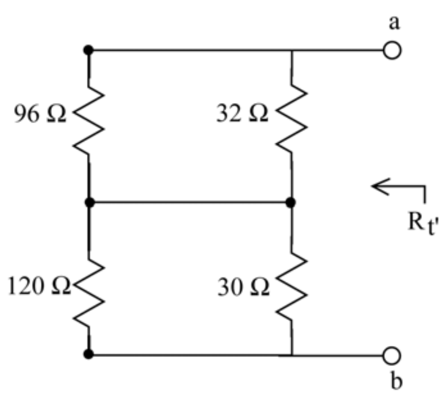
Figure P 5.4-15

Solution:
Find the Thevenin equivalent circuit for the part of the circuit to the left of the terminals a-b.



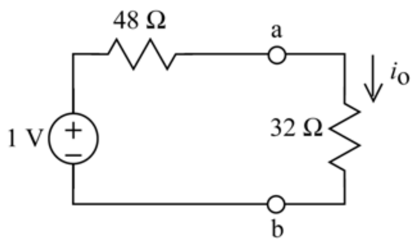
Using voltage division twice

$$v_{oc} = \frac{32}{32+96}20 - \frac{30}{120+30}20 = 5 - 4 = 1 \text{ V}$$



$$R_t = (96 \,||\, 32) + (120 \,||\, 30) = 24 + 24 = 48 \ \Omega$$

Replacing the part of the circuit to the left of terminals a-b by its Thevenin equivalent circuit gives



$$i_o = \frac{1}{48+32} = 0.0125 \text{ A} = 12.5 \text{ mA}$$

(checked: LNAP 5/24/04)

P 5.4-16 An ideal voltmeter is modeled as an open circuit. A more realistic model of a voltmeter is a large resistance. Figure P 5.4-16*a* shows a circuit with a voltmeter that measures the voltage v_m. In Figure P 5.4-16*b* the voltmeter is replaced by the model of an ideal voltmeter, an open circuit. The voltmeter measures v_{mi}, the ideal value of v_m.

As $R_m \to \infty$, the voltmeter becomes an ideal voltmeter and $v_m \to v_{mi}$. When $R_m < \infty$, the voltmeter is not ideal and $v_m > v_{mi}$. The difference between v_m and v_{mi} is a measurement error caused by the fact that the voltmeter is not ideal.

(a) Determine the value of v_{mi}.

(b) Express the measurement error that occurs when $R_m = 1000\ \Omega$ as a percentage of v_{mi}.

(c) Determine the minimum value of R_m required to ensure that the measurement error is smaller than 2 percent of v_{mi}.

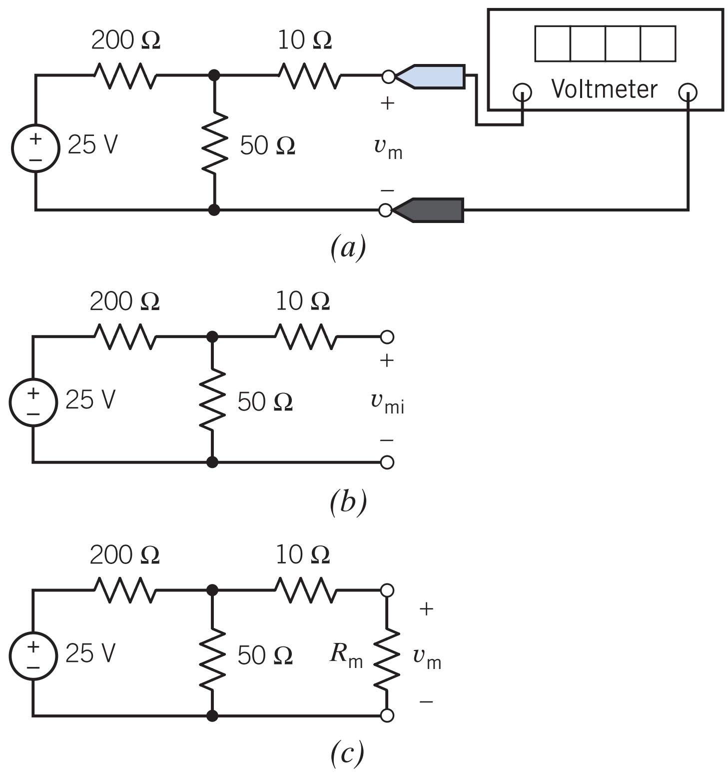
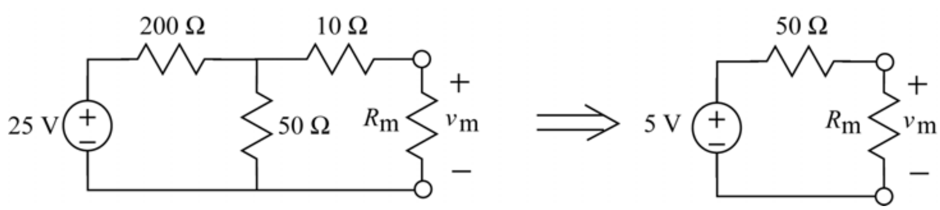


Figure P 5.4-16

Solution: Replace the circuit by its Thevenin equivalent circuit:



$$v_m = \left(\frac{R_m}{R_m + 50} \right) 5$$

(a)
$$v_{mi} = \lim_{R_m \to \infty} v_m = 5\text{ V}$$

(b) When $R_m = 1000\ \Omega,\ v_m = 4.763\text{ V}$ so

$$\%\text{ error} = \frac{5 - 4.762}{5} \times 100 = 4.76\%$$

(c) $$0.02 \geq \frac{5-\left(\frac{R_m}{R_m+50}\right)5}{5} \quad \Rightarrow \quad \frac{R_m}{R_m+50} \geq 0.98 \quad \Rightarrow \quad R_m \geq 2450\ \Omega$$

(checked: LNAP 6/16/04)

P5.4-17

Given that $0 \le R \le \infty$ in the circuit shown in Figure P5.4-17, consider these two observations:

Observation 1: When $R = 2\ \Omega$ then $v_R = 4$ V and $i_R = 2$ A.

Observation 1: When $R = 6\ \Omega$ then $v_R = 6$ V and $i_R = 1$ A.

Determine the following

a) The maximum value of i_R and the value of R that causes i_R to be maximal.

b) The maximum value of v_R and the value of R that causes v_R to be maximal.

c) The maximum value of $p_R = i_R v_R$ and the value of R that causes p_R to be maximal.

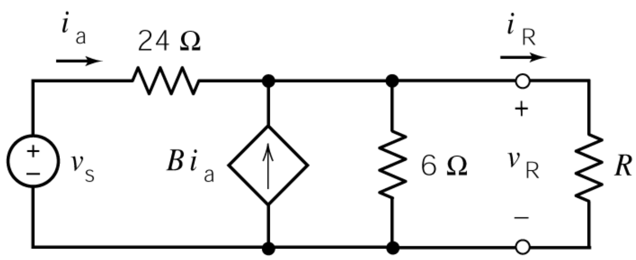
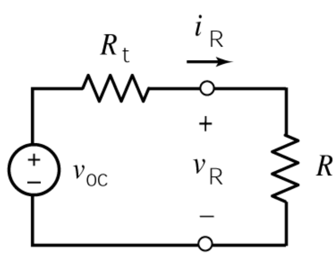


Figure P5.4-17

Solution:

We can replace the part of the circuit to the left of the terminals by its Thevenin equivalent circuit:



Using voltage division $v_R = \dfrac{R}{R+R_t} v_{oc}$ and using Ohm's law

$$i_R = \frac{v_{oc}}{R+R_t}.$$

By inspection , $v_R = \dfrac{R}{R+R_t} v_{oc} = \dfrac{v_{oc}}{1+\dfrac{R_t}{R}}$ will be maximum when $R = \infty$. The maximum value of v_R will be v_{oc}. Similarly,

$i_R = \dfrac{v_{oc}}{R+R_t}$ will be maximum when $R = 0$. The maximum value of i_R will be $\dfrac{v_{oc}}{R_t} = i_{sc}$.

The maximum power transfer theorem tells use that $p_R = i_R v_R$ will be maximum when $R = R_t$.

Then $p_R = i_R v_R = \left(\dfrac{v_{oc}}{R+R_t}\right)\left(\dfrac{R}{R+R_t} v_{oc}\right) = R\left(\dfrac{v_{oc}}{R+R_t}\right)^2$.

Let's substitute the given data into the equation $i_R = \frac{v_{oc}}{R + R_t}$.

When R = 2 Ω we get $2 = \frac{v_{oc}}{2 + R_t} \quad \Rightarrow \quad 4 + 2R_t = v_{oc}$. When R = 6 Ω we get

$1 = \frac{v_{oc}}{6 + R_t} \quad \Rightarrow \quad 6 + R_t = v_{oc}$.

So $6 + R_t = 4 + 2R_t \quad \Rightarrow \quad R_t = 2\ \Omega$ and $v_{oc} = 4 + 2R_t = 8\ \text{V}$. Also $i_{sc} = \frac{v_{oc}}{R_t} = \frac{8}{2} = 4\ \text{A}$.

P5.4-18 Consider the circuit shown in Figure P5.4-18. Determine

a) The value of v_R that occurs when $R = 9\ \Omega$.

b) The value of R that causes $v_R = 5.4$ V.

c) The value of R that causes $i_R = 300$ mA.

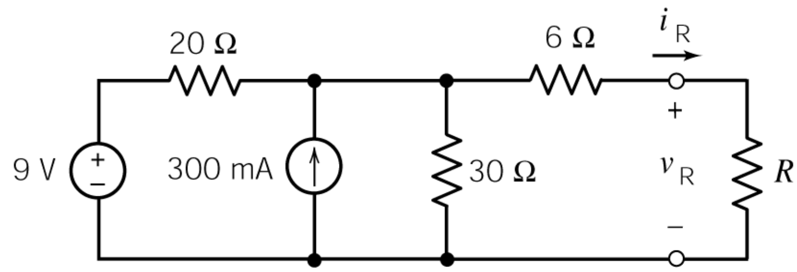
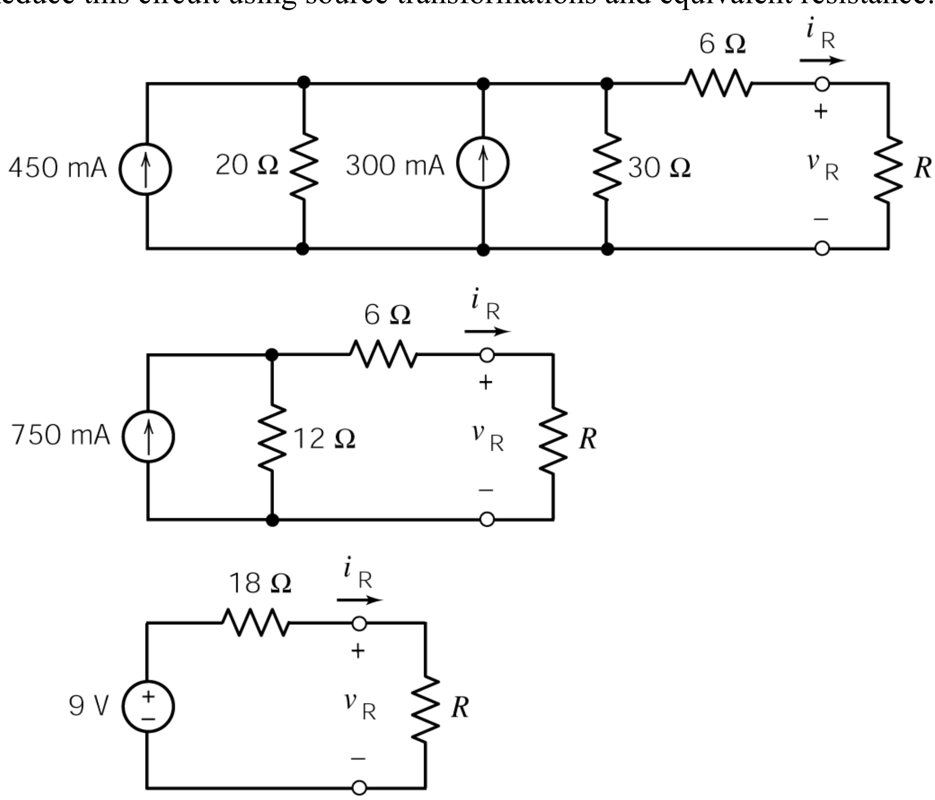


Figure P5.4-18

Solution: Reduce this circuit using source transformations and equivalent resistance:



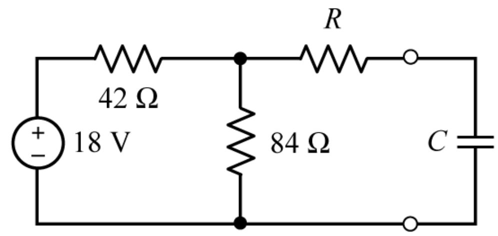
Now $v_R = \left(\frac{R}{R+18}\right) 9$ and $i_R = \frac{9}{R+18}$ so the questions can be easily answered:

a) When $R = 9\ \Omega$ then $v_R = 3$ V.

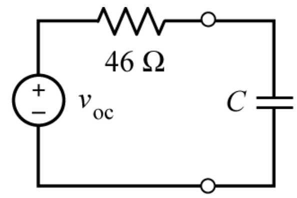
b) When $R = 27\ \Omega$ then $v_R = 5.4$ V.

c) When $R = 12\ \Omega$ then $i_R = 300$ mA.

P5.4-19 The circuit shown in Figure P5.4-19*a* can be reduced to the circuit shown in Figure P5.4-19*b* using source transformations and equivalent resistances. Determine the values of the source voltage v_{oc} and the resistance *R*.



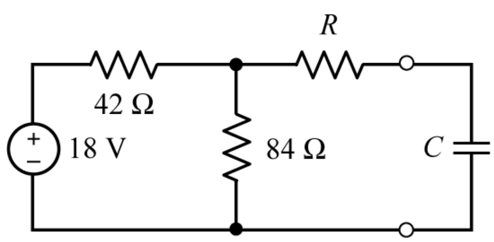
(*a*)

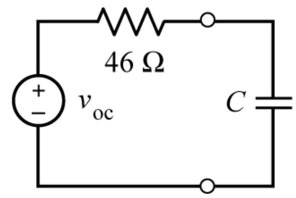


(*b*)

Figure P5.4-19

Solution





$$46 = R_t = R + (42 \,||\, 84) = R + 28 \quad \Rightarrow \quad R = 18\ \Omega$$

$$v_{oc} = \frac{84}{42+84}(18) = 12\ \text{V}$$

P5.4-20 The equivalent circuit in Figure P5.4-20 is obtained by replacing part of the original circuit by its Thevenin equivalent circuit. The values of the parameters of the Thevenin equivalent circuit are

$$v_{oc} = 15 \text{ V and } R_t = 60\ \Omega$$

Determine the following:

a.) The values of V_s and R_a. (Three resistors in the original circuit have equal resistance, R_a.)

b.) The value of R_b required to cause $i = 0.2$ A.

c.) The value of R_b required to cause $v = 5$ V.

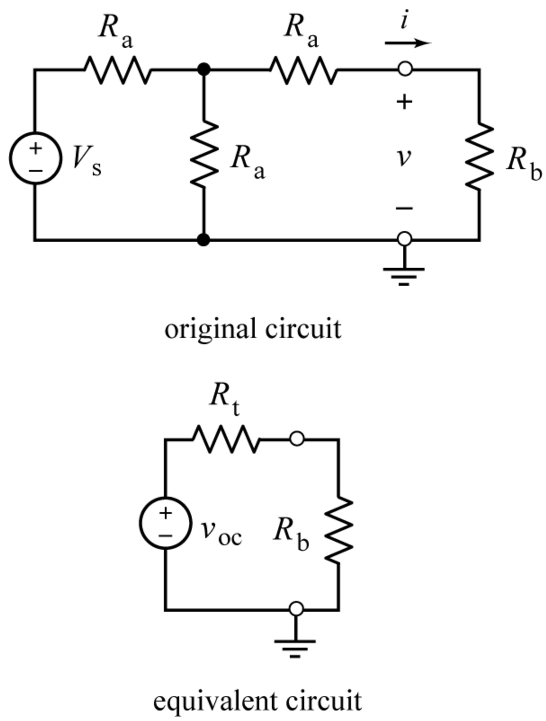
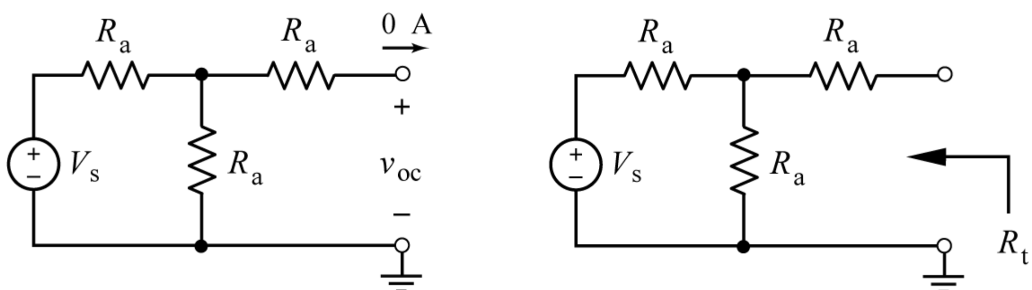


Figure P5.4-20

Solution

a.) From

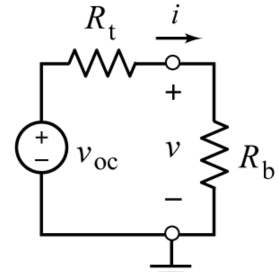


We see that $v_{oc} = \frac{V_s}{2}$ and $R_t = \frac{3}{2} R_a$. With the given values of v_{oc} and R_t we calculate

$$15 = \frac{V_s}{2} \quad \Rightarrow \quad V_s = 30 \text{ V} \quad \text{and} \quad 60 = \frac{3}{2} R_a \quad \Rightarrow \quad R_a = 40\ \Omega .$$

b.) $i = \dfrac{v_{oc}}{R_t + R_b} \quad \Rightarrow \quad 0.2 = \dfrac{15}{60 + R_b} \quad \Rightarrow \quad R_b = 15\ \Omega$

c.) $v = \dfrac{R_b}{R_t + R_b} v_{oc} \quad \Rightarrow \quad 5 = \dfrac{15\, R_b}{60 + R_b} \quad \Rightarrow \quad R_b = 30\ \Omega$



Section 5-5: Norton's Theorem

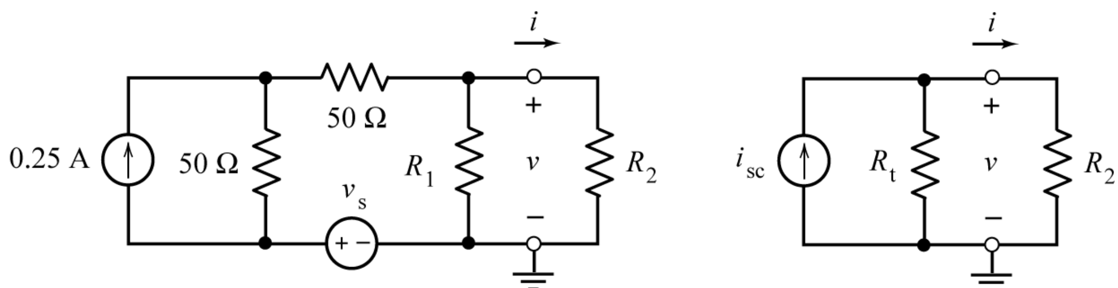
P5.5-1 The part of the circuit shown in Figure P5.3-1a to the left of the terminals can be reduced to its Norton equivalent circuit using source transformations and equivalent resistance. The resulting Norton equivalent circuit, shown in Figure P5.3-1b, will be characterized by the parameters:

$$i_{sc} = 0.5 \text{ A} \quad \text{and} \quad R_t = 20\ \Omega$$

a) Determine the values of v_s and R_1.

b) Given that $0 \le R_2 \le \infty$, determine the maximum values of the voltage, v, and of the power, $p = vi$.

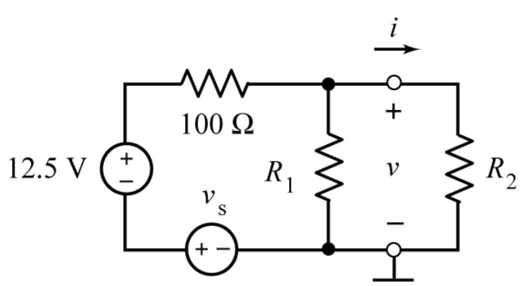
Answers: v_s = 37.5 V, R_1 = 25 Ω, max v = 10 V and max p = 1.25 W

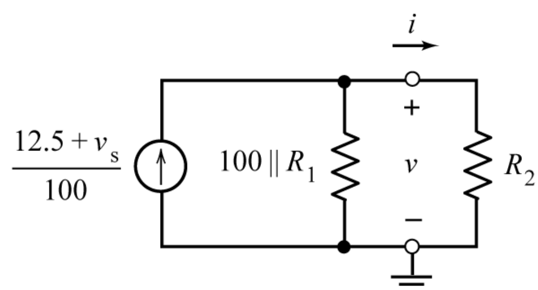


(*a*) (*b*)

Figure P5.5-1

Solution: Two source transformations reduce the circuit as follows:





Recognizing the parameters of the Norton equivalent circuit gives:

$$0.5 = i_{sc} = \frac{12.5 + v_s}{100} \quad \Rightarrow \quad v_s = 37.5 \text{ V} \quad \text{and} \quad 20 = R_t = 100 \,||\, R_1 = \frac{100\, R_1}{100 + R_1} \quad \Rightarrow \quad R_1 = 25\ \Omega$$

Next, the voltage across resistor R_2 is given by $v = i_{sc}\left(R_t \,||\, R_2\right) = \dfrac{R_t\, R_2\, i_{sc}}{R_t + R_2} = \dfrac{R_t\, i_{sc}}{\dfrac{R_t}{R_2} + 1}$ so this

voltage is maximum when $R_2 = \infty$ and max v = $R_t\, i_{sc}$ =10 V. The power $p = vi$ will be maximum when $R_2 = R_t = 20\ \Omega$. Then $v = \dfrac{R_t\, i_{sc}}{2} = \dfrac{20(0.5)}{2} = 5\ \text{V}$, $i = \dfrac{v}{R_2} = \dfrac{5}{20} = 0.25\ \text{A}$ and

$p = v\,i = 5(0.25) = 1.25\ \text{W}$.

P 5.5-2 Two black boxes are shown in Figure P 5.5-2. Box A contains the Thévenin equivalent of some linear circuit, and box B contains the Norton equivalent of the same circuit. With access to just the outsides of the boxes and their terminals, how can you determine which is which, using only one shorting wire?

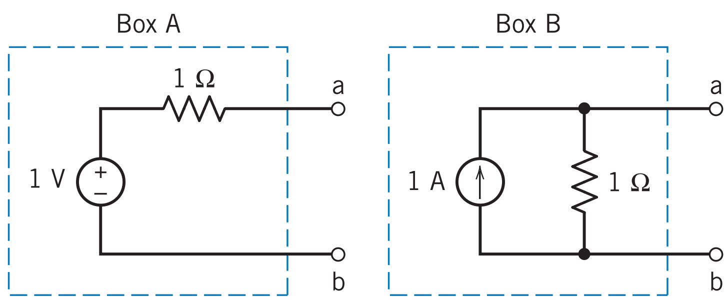
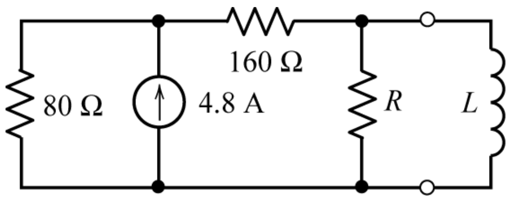


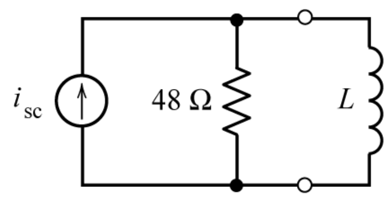
Figure P 5.5-2

Solution:

When the terminals of the boxes are open-circuited, no current flows in Box A, but the resistor in Box B dissipates 1 watt. Box B is therefore warmer than Box A. If you short the terminals of each box, the resistor in Box A will draw 1 amp and dissipate 1 watt. The resistor in Box B will be shorted, draw no current, and dissipate no power. Then Box A will warm up and Box B will cool off.



(*a*)

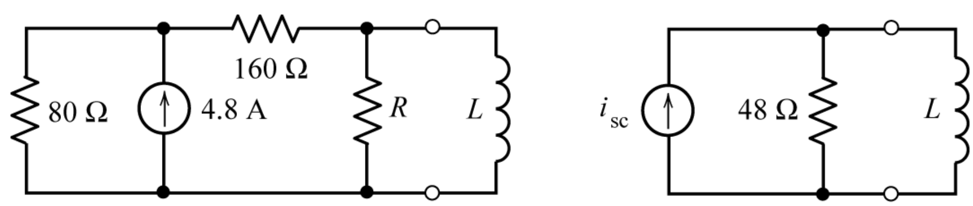


(*b*)

Figure P5.5-3

P5.5-3 The circuit shown in Figure P5.5-3*a* can be reduced to the circuit shown in Figure P5.5-3*b* using source transformations and equivalent resistances. Determine the values of the source current i_{sc} and the resistance *R*.

Solution:



$$48 = R_t = R \| (80+160) = \frac{240\,R}{R+240} \quad \Rightarrow \quad R = 60\ \Omega$$

$$i_{sc} = \frac{80}{80+160}(4.8) = 1.6\ \text{A}$$

P 5.5-4 Find the Norton equivalent circuit for the circuit shown in Figure P 5.5-4.

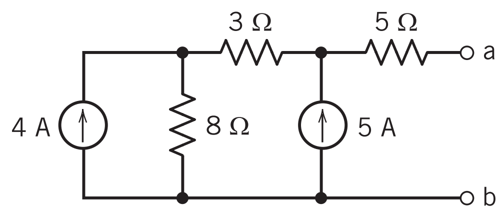
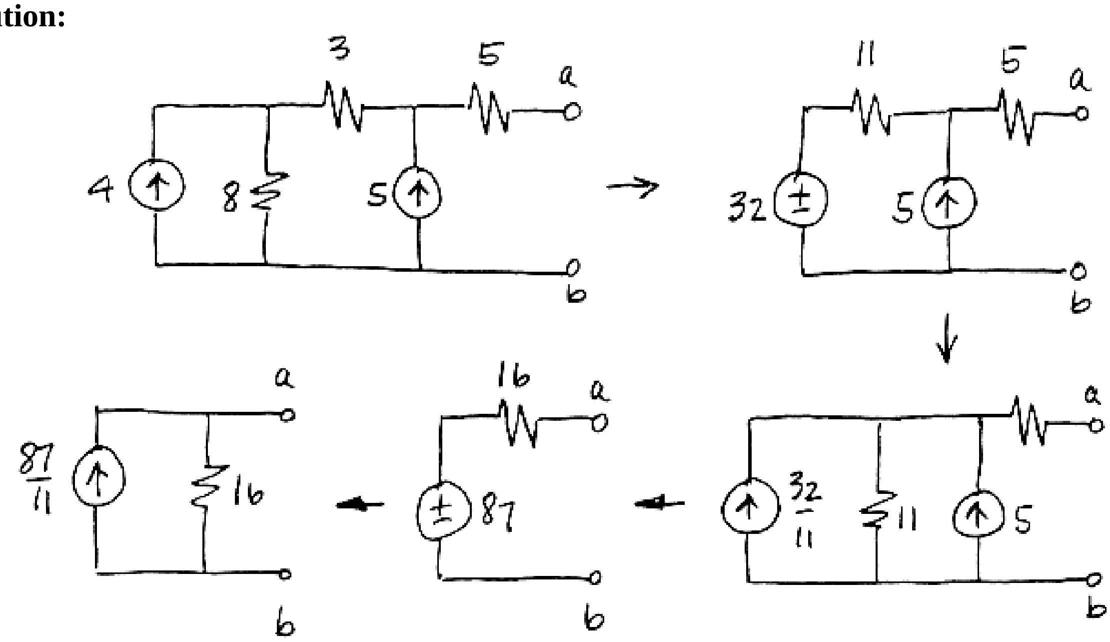


Figure P 5.5-4

Solution:



P 5.5-5 The circuit shown in Figure P 5.5-5*b* is the Norton equivalent circuit of the circuit shown in Figure P 5.5-5*a*. Find the value of the short-circuit current, i_{sc}, and Thévenin resistance, R_t.

Answer: i_{sc} = 1.13 A and R_t = 7.57 Ω

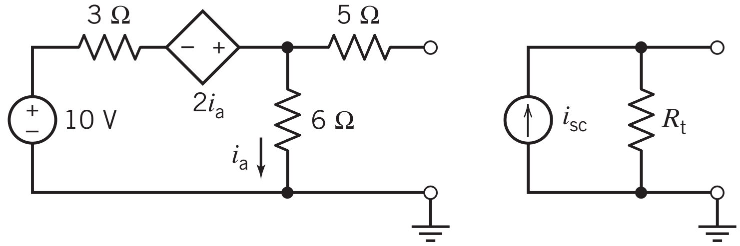


Figure P 5.5-5

Solution:

To determine the value of the short circuit current, i_{sc}, we connect a short circuit across the terminals of the circuit and then calculate the value of the current in that short circuit. Figure (a) shows the circuit from Figure 5.6-4*a* after adding the short circuit and labeling the short circuit current. Also, the meshes have been identified and labeled in anticipation of writing mesh equations. Let i_1 and i_2 denote the mesh currents in meshes 1 and 2, respectively.

In Figure (a), mesh current i_2 is equal to the current in the short circuit. Consequently, $i_2 = i_{sc}$. The controlling current of the CCVS is expressed in terms of the mesh currents as

$$i_a = i_1 - i_2 = i_1 - i_{sc}$$

Apply KVL to mesh 1 to get

$$3\,i_1 - 2\left(i_1 - i_2\right) + 6\left(i_1 - i_2\right) - 10 = 0 \quad \Rightarrow \quad 7\,i_1 - 4\,i_2 = 10 \qquad \textbf{(1)}$$

Apply KVL to mesh 2 to get

$$5\,i_2 - 6\left(i_1 - i_2\right) = 0 \quad \Rightarrow \quad -6\,i_1 + 11\,i_2 = 0 \quad \Rightarrow \quad i_1 = \frac{11}{6}\,i_2$$

Substituting into equation 1 gives

$$7\left(\frac{11}{6}\,i_2\right) - 4\,i_2 = 10 \quad \Rightarrow \quad i_2 = 1.13 \text{ A} \quad \Rightarrow \quad i_{sc} = 1.13 \text{ A}$$

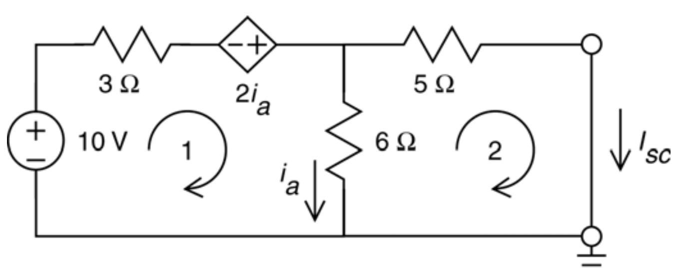


Figure (a) Calculating the short circuit current, i_{sc}, using mesh equations.

To determine the value of the Thevenin resistance, R_t, first replace the 10 V voltage source by a 0 V voltage source, i.e. a short circuit. Next, connect a current source across the terminals of the circuit and then label the voltage across that current source as shown in Figure (b). The Thevenin resistance will be calculated from the current and voltage of the current source as

$$R_t = \frac{v_T}{i_T}$$

In Figure (b), the meshes have been identified and labeled in anticipation of writing mesh equations. Let i_1 and i_2 denote the mesh currents in meshes 1 and 2, respectively.

In Figure (b), mesh current i_2 is equal to the negative of the current source current. Consequently, $i_2 = i_T$. The controlling current of the CCVS is expressed in terms of the mesh currents as

$$i_a = i_1 - i_2 = i_1 + i_T$$

Apply KVL to mesh 1 to get

$$3\,i_1 - 2\left(i_1 - i_2\right) + 6\left(i_1 - i_2\right) = 0 \quad \Rightarrow \quad 7\,i_1 - 4\,i_2 = 0 \quad \Rightarrow \quad i_1 = \frac{4}{7}i_2 \qquad \textbf{(2)}$$

Apply KVL to mesh 2 to get

$$5\,i_2 + v_T - 6\left(i_1 - i_2\right) = 0 \quad \Rightarrow \quad -6\,i_1 + 11\,i_2 = -v_T$$

Substituting for i_1 using equation 2 gives

$$-6\left(\frac{4}{7}i_2\right) + 11\,i_2 = -v_T \quad \Rightarrow \quad 7.57\,i_2 = -v_T$$

Finally,

$$R_t = \frac{v_T}{i_T} = \frac{-v_T}{-i_T} = \frac{-v_T}{i_2} = 7.57\ \Omega$$

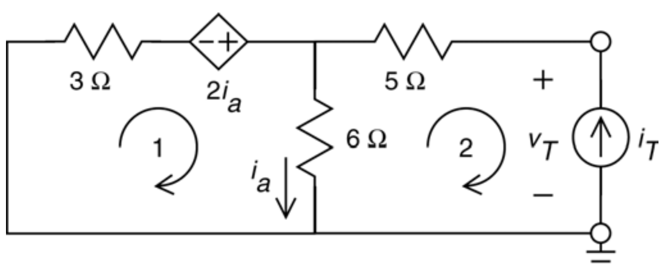


Figure (b) Calculating the Thevenin resistance, $R_t = \dfrac{v_T}{i_T}$, using mesh equations.

To determine the value of the open circuit voltage, v_{oc}, we connect an open circuit across the terminals of the circuit and then calculate the value of the voltage across that open circuit. Figure (c) shows the circuit from Figure 4.6-4*a* after adding the open circuit and labeling the open circuit voltage. Also, the meshes have been identified and labeled in anticipation of writing mesh equations. Let i_1 and i_2 denote the mesh currents in meshes 1 and 2, respectively.

In Figure (c), mesh current i_2 is equal to the current in the open circuit. Consequently, $i_2 = 0$ A. The controlling current of the CCVS is expressed in terms of the mesh currents as

$$i_a = i_1 - i_2 = i_1 - 0 = i_1$$

Apply KVL to mesh 1 to get

$$3\,i_1 - 2\left(i_1 - i_2\right) + 6\left(i_1 - i_2\right) - 10 = 0 \;\;\Rightarrow\;\; 3\,i_1 - 2\left(i_1 - 0\right) + 6\left(i_1 - 0\right) - 10 = 0$$

$$\Rightarrow\;\; i_1 = \frac{10}{7} = 1.43 \text{ A}$$

Apply KVL to mesh 2 to get

$$5\,i_2 + v_{oc} - 6\left(i_1 - i_2\right) = 0 \;\;\Rightarrow\;\; v_{oc} = 6\left(i_1\right) = 6\left(1.43\right) = 8.58 \text{ V}$$

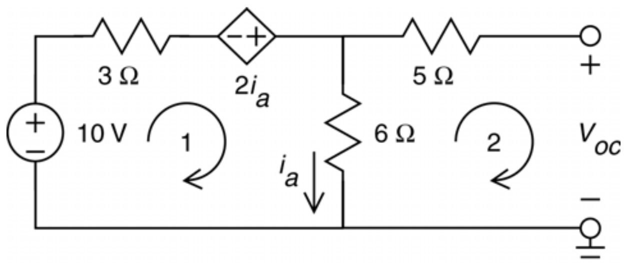


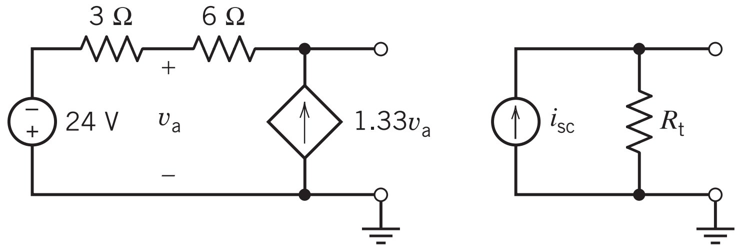
Figure (c) Calculating the open circuit voltage, v_{oc}, using mesh equations.

As a check, notice that $R_t\; i_{sc} = \left(7.57\right)\left(1.13\right) = 8.55 \approx v_{oc}$

(checked using LNAP 8/16/02)

P 5.5-6 The circuit shown in Figure P 5.5-6*b* is the Norton equivalent circuit of the circuit shown in Figure P 5.5-6*a*. Find the value of the short-circuit current, i_{sc}, and Thévenin resistance, R_t.

Answer: $i_{sc} = -24$ A and $R_t = -3\ \Omega$



(a) *(b)*

Figure P 5.5-6

Solution:

To determine the value of the short circuit current, I_{sc}, we connect a short circuit across the terminals of the circuit and then calculate the value of the current in that short circuit. Figure (a) shows the circuit from Figure 4.6-5a after adding the short circuit and labeling the short circuit current. Also, the nodes have been identified and labeled in anticipation of writing node equations. Let v_1, v_2 and v_3 denote the node voltages at nodes 1, 2 and 3, respectively.

In Figure (a), node voltage v_1 is equal to the negative of the voltage source voltage. Consequently, $v_1 = -24$ V. The voltage at node 3 is equal to the voltage across a short, $v_3 = 0$. The controlling voltage of the VCCS, v_a, is equal to the node voltage at node 2, i.e. $v_a = v_2$. The voltage at node 3 is equal to the voltage across a short, i.e. $v_3 = 0$.

Apply KCL at node 2 to get

$$\frac{v_1 - v_2}{3} = \frac{v_2 - v_3}{6} \Rightarrow 2\,v_1 + v_3 = 3\,v_2 \Rightarrow -48 = 3\,v_a \Rightarrow v_a = -16 \text{ V}$$

Apply KCL at node 3 to get

$$\frac{v_2 - v_3}{6} + \frac{4}{3} v_2 = i_{sc} \Rightarrow \frac{9}{6} v_a = i_{sc} \Rightarrow i_{sc} = \frac{9}{6}(-16) = -24 \text{ A}$$

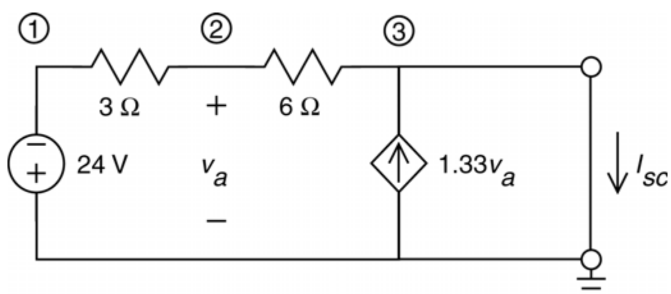


Figure (a) Calculating the short circuit current, I_{sc}, using mesh equations.

To determine the value of the Thevenin resistance, R_{th}, first replace the 24 V voltage source by a 0 V voltage source, i.e. a short circuit. Next, connect a current source circuit across the terminals of the circuit and then label the voltage across that current source as shown in Figure (b). The Thevenin resistance will be calculated from the current and voltage of the current source as

$$R_{th} = \frac{v_T}{i_T}$$

Also, the nodes have been identified and labeled in anticipation of writing node equations. Let v_1, v_2 and v_3 denote the node voltages at nodes 1, 2 and 3, respectively.

In Figure (b), node voltage v_1 is equal to the across a short circuit, i.e. $v_1 = 0$. The controlling voltage of the VCCS, v_a, is equal to the node voltage at node 2, i.e. $v_a = v_2$. The voltage at node 3 is equal to the voltage across the current source, i.e. $v_3 = v_T$.

Apply KCL at node 2 to get

$$\frac{v_1 - v_2}{3} = \frac{v_2 - v_3}{6} \quad \Rightarrow \quad 2\,v_1 + v_3 = 3\,v_2 \quad \Rightarrow \quad v_T = 3\,v_a$$

Apply KCL at node 3 to get

$$\frac{v_2 - v_3}{6} + \frac{4}{3}v_2 + i_T = 0 \quad \Rightarrow \quad 9\,v_2 - v_3 + 6\,i_T = 0$$

$$\Rightarrow \quad 9\,v_a - v_T + 6\,i_T = 0$$

$$\Rightarrow \quad 3\,v_T - v_T + 6\,i_T = 0 \quad \Rightarrow \quad 2\,v_T = -6\,i_T$$

Finally,

$$R_t = \frac{v_T}{i_T} = -3\,\Omega$$

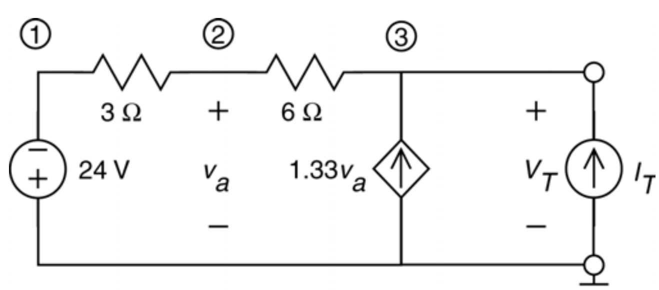


Figure (b) Calculating the Thevenin resistance, $R_{th} = \frac{v_T}{i_T}$, using mesh equations.

To determine the value of the open circuit voltage, v_{oc}, we connect an open circuit across the terminals of the circuit and then calculate the value of the voltage across that open circuit. Figure (c) shows the circuit from Figure P 4.6-5a after adding the open circuit and labeling the open circuit voltage. Also, the nodes have been identified and labeled in anticipation of writing node equations. Let v_1, v_2 and v_3 denote the node voltages at nodes 1, 2 and 3, respectively.

In Figure (c), node voltage v_1 is equal to the negative of the voltage source voltage. Consequently, $v_1 = -24$ V. The controlling voltage of the VCCS, v_a, is equal to the node voltage at node 2, i.e. $v_a = v_2$. The voltage at node 3 is equal to the open circuit voltage, i.e. $v_3 = v_{oc}$.

Apply KCL at node 2 to get

$$\frac{v_1 - v_2}{3} = \frac{v_2 - v_3}{6} \quad \Rightarrow \quad 2\,v_1 + v_3 = 3\,v_2 \quad \Rightarrow \quad -48 + v_{oc} = 3\,v_a$$

Apply KCL at node 3 to get

$$\frac{v_2 - v_3}{6} + \frac{4}{3} v_2 = 0 \quad \Rightarrow \quad 9\,v_2 - v_3 = 0 \quad \Rightarrow \quad 9\,v_a = v_{oc}$$

Combining these equations gives

$$3\left(-48 + v_{oc}\right) = 9\,v_a = v_{oc} \quad \Rightarrow \quad v_{oc} = 72 \text{ V}$$

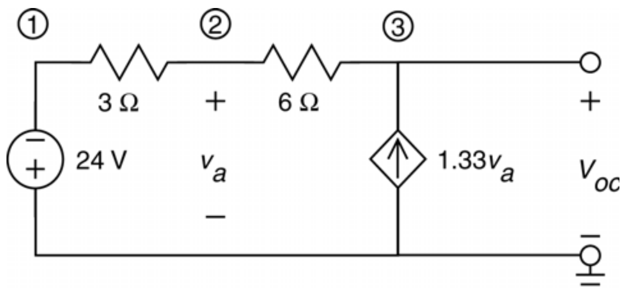


Figure (c) Calculating the open circuit voltage, v_{oc}, using node equations.

As a check, notice that

$$R_{th}\, I_{sc} = (-3)(-24) = 72 = V_{oc}$$

(checked using LNAP 8/16/02)

P 5.5-7 Determine the value of the resistance *R* in the circuit shown in Figure P 5.5-7 by each of the following methods:

(a) Replace the part of the circuit to the left of terminals a–b by its the Norton equivalent circuit. Use current division to determine the value of *R*.

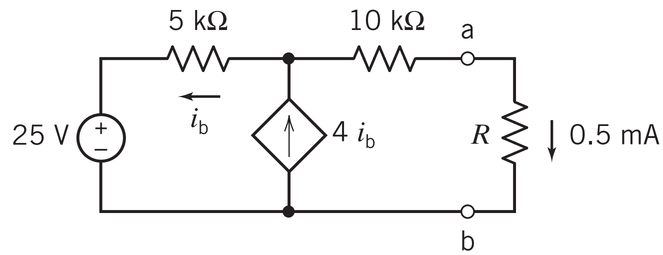
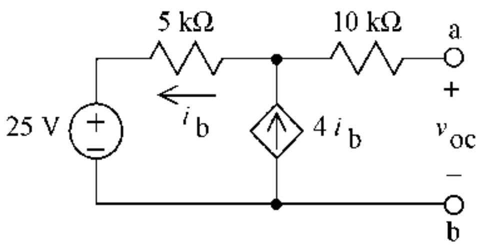


Figure P 5.5-7

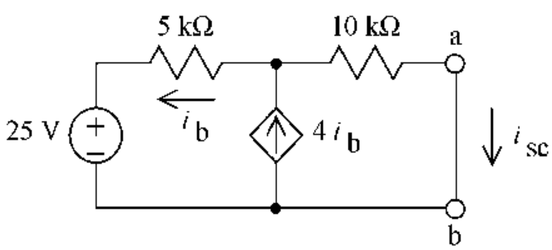
(b) Analyze the circuit shown Figure P 5.5-6 using mesh equations. Solve the mesh equations to determine the value of *R*.

Solution: (a) Replace the part of the circuit that is connected to the left of terminals a-b by its Norton equivalent circuit:



Apply KCL at the top node of the dependent source to see that $i_b = 0$ A . Then

$$v_{oc} = 25 + 5000(i_b) = 25 \text{ V}$$

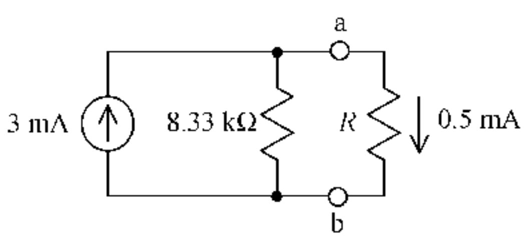


Apply KVL to the supermesh corresponding to the dependent source to get

$$-5000\, i_b + 10000(3\, i_b) - 25 = 0 \quad \Rightarrow \quad i_b = 1 \text{ mA}$$

Apply KCL to get

$$i_{sc} = 3\, i_b = 3 \text{ mA}$$



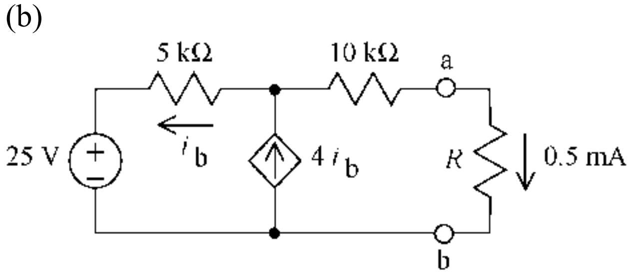
Then

$$R_t = \frac{v_{oc}}{i_{sc}} = 8.3\bar{3} \text{ k}\Omega$$

Current division gives

$$0.5 = \frac{8333}{R + 8333} 3 \quad \Rightarrow \quad R = 41.67 \text{ k}\Omega$$

(b)



Notice that i_b and 0.5 mA are the mesh currents.

Apply KCL at the top node of the dependent source to get

$$i_b + 0.5 \times 10^{-3} = 4\, i_b \quad \Rightarrow \quad i_b = \frac{1}{6} \text{ mA}$$

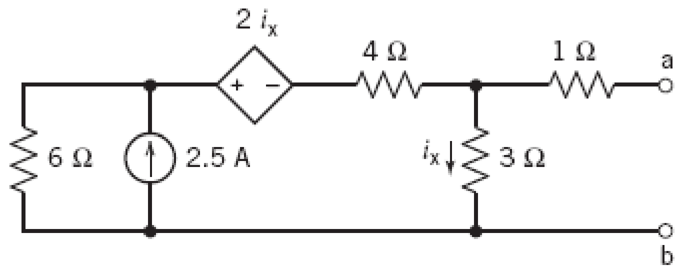
Apply KVL to the supermesh corresponding to the dependent source to get

$$-5000\, i_{\mathrm{b}} + (10000 + R)\left(0.5 \times 10^{-3}\right) - 25 = 0$$

$$-5000\left(\frac{1}{6} \times 10^{-3}\right) + (10000 + R)\left(0.5 \times 10^{-3}\right) = 25$$

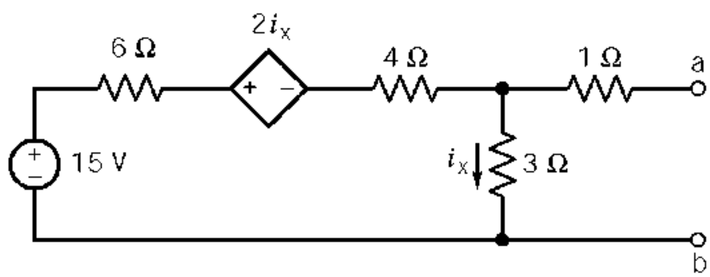
$$R = \frac{\frac{125}{6}}{0.5 \times 10^{-3}} = 41.67\ \mathrm{k}\Omega$$

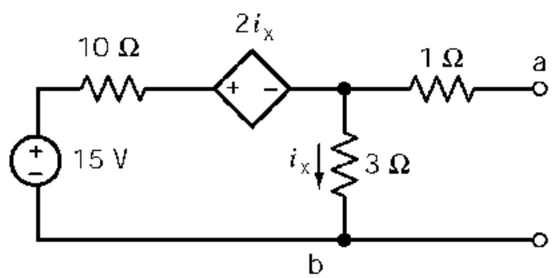
P5.5-8 Find the Norton equivalent circuit of this circuit:



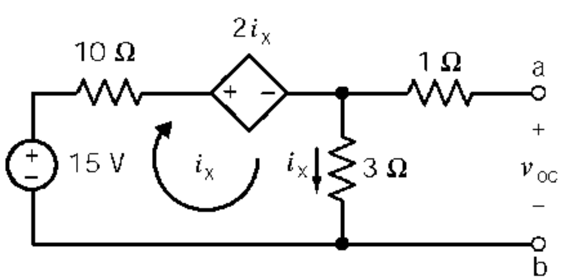
Solution

Simplify the circuit using a source transformation:





Identify the open circuit voltage and short circuit current.

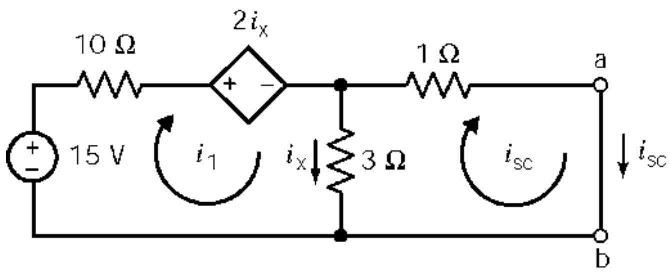


Apply KVL to the mesh to get:

$$(10+2+3)i_x - 15 = 0 \quad \Rightarrow \quad i_x = 1 \text{ A}$$

Then

$$v_{oc} = 3\, i_x = 3 \text{ V}$$



Express the controlling current of the CCVS in terms of the mesh currents:

$$i_x = i_1 - i_{sc}$$

The mesh equations are

$$10\, i_1 + 2\left(i_1 - i_{sc}\right) + 3\left(i_1 - i_{sc}\right) - 15 = 0 \quad \Rightarrow \quad 15\, i_1 - 5\, i_{sc} = 15$$

and

$$i_{sc} - 3\left(i_1 - i_{sc}\right) = 0 \quad \Rightarrow \quad i_1 = \frac{4}{3}\, i_{sc}$$

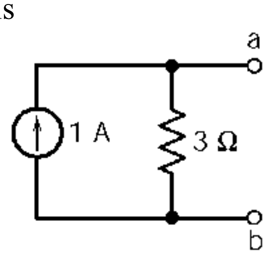
so

$$15\left(\frac{4}{3} i_{sc}\right) - 5\, i_{sc} = 15 \quad \Rightarrow \quad i_{sc} = 1\text{ A}$$

The Thevenin resistance is

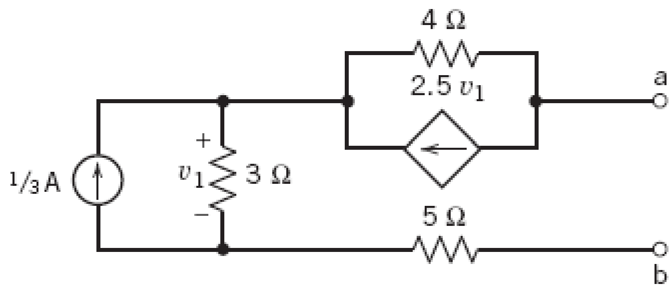
$$R_t = \frac{3}{1} = 3\ \Omega$$

Finally, the Norton equivalent circuit is



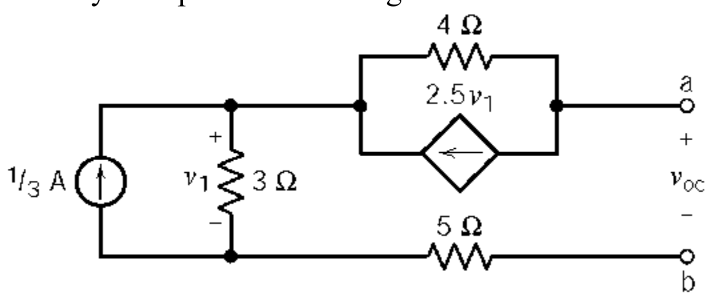
(checked: LNAP 6/21/04)

P5.5-9 Find the Norton equivalent circuit of this circuit:



Solution

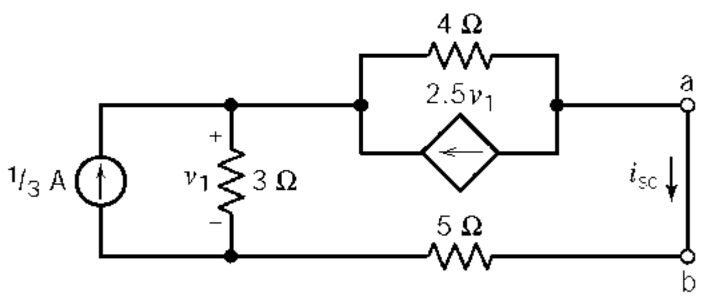
Identify the open circuit voltage and short circuit current.



$$v_1 = \left(\frac{1}{3}\right)3 = 1 \text{ V}$$

Then

$$v_{oc} = v_1 - 4\left(2.5\, v_1\right) = -9 \text{ V}$$



$$v_1 = 3\left(\frac{1}{3} - i_{sc}\right) = 1 - 3\, i_{sc}$$

$$4\left(2.5\, v_1 + i_{sc}\right) + 5 i_{sc} - v_1 = 0$$

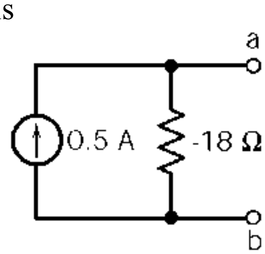
$$\Rightarrow \quad 9\, v_1 + 9\, i_{sc} = 0$$

$$9\left(1 - 3\, i_{sc}\right) + 9\, i_{sc} = 0 \quad \Rightarrow \quad i_{sc} = \frac{1}{2} \text{ A}$$

The Thevenin resistance is

$$R_t = \frac{-9}{0.5} = -18\ \Omega$$

Finally, the Norton equivalent circuit is



(checked: LNAP 6/21/04)

P 5.5-10 An ideal ammeter is modeled as a short circuit. A more realistic model of an ammeter is a small resistance. Figure P 5.5-10*a* shows a circuit with an ammeter that measures the current i_m. In Figure P 5.5-10*b* the ammeter is replaced by the model of an ideal ammeter, a short circuit. The ammeter measures i_{mi}, the ideal value of i_m.

As $R_m \rightarrow 0$, the ammeter becomes an ideal ammeter and $i_m \rightarrow i_{mi}$. When $R_m > 0$, the ammeter is not ideal and $i_m < i_{mi}$. The difference between i_m and i_{mi} is a measurement error caused by the fact that the ammeter is not ideal.

(a) Determine the value of i_{mi}.

(b) Express the measurement error that occurs when $R_m = 20\ \Omega$ as a percentage of i_{mi}.

(c) Determine the maximum value of R_m required to ensure that the measurement error is smaller than 2 percent of i_{mi}.

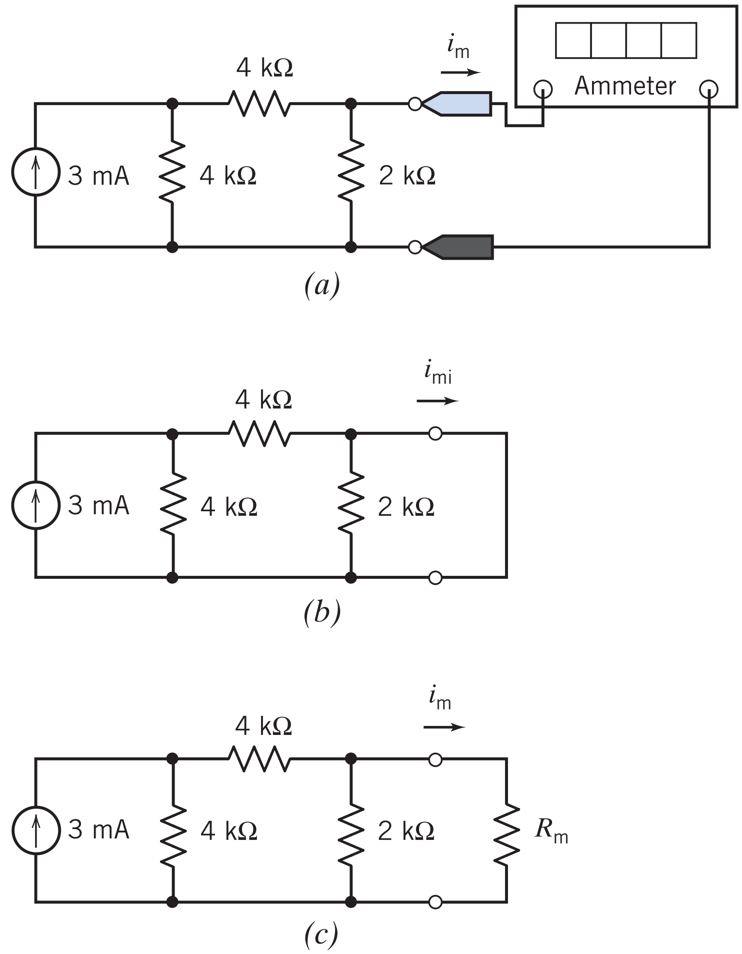
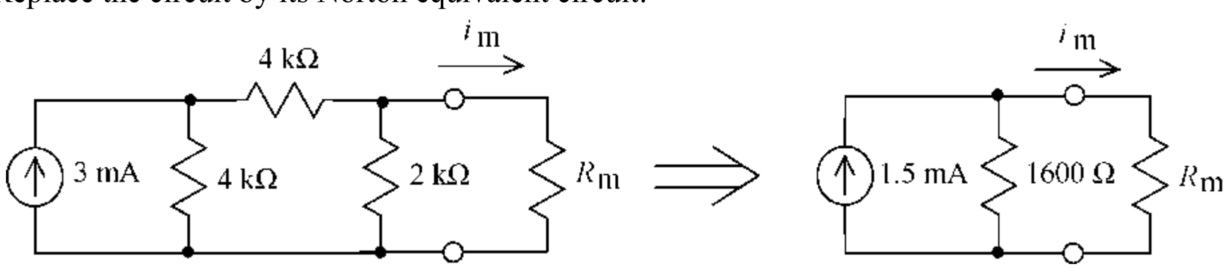


Figure P 5.5-10

Solution:

Replace the circuit by its Norton equivalent circuit:



$$i_m = \left(\frac{1600}{1600 + R_m}\right)\left(1.5 \times 10^{-3}\right)$$

(a)

$$i_{mi} = \lim_{R_m \to 0} i_m = 1.5 \text{ mA}$$

(b) When $R_m = 20\ \Omega$ then $i_m = 1.48$ mA so

$$\% \text{ error} = \frac{1.5 - 1.48}{1.5} \times 100 = 1.23\%$$

(c)

$$0.02 \ge \frac{0.015 - \left(\frac{1600}{1600 + R_m}\right)(0.015)}{0.015} \quad \Rightarrow \quad \frac{1600}{1600 + R_m} \ge 0.98 \quad \Rightarrow \quad R_m \le 32.65\ \Omega$$

(checked: LNAP 6/18/04)

P 5.5-11 Determine values of R_t and i_{sc} that cause the circuit shown in Figure P 5.5-11*b* to be the Norton equivalent circuit of the circuit in Figure P 5.5-11*a*.

Answer: $R_t = 3\ \Omega$ and $i_{sc} = -2$ A

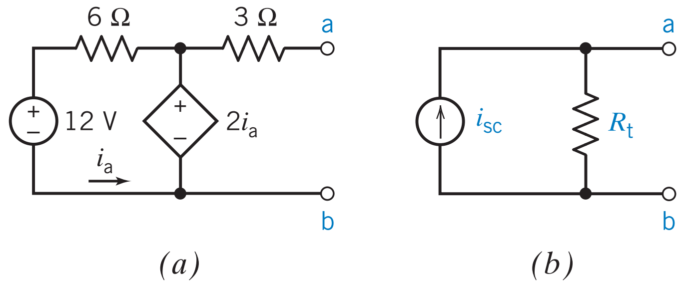
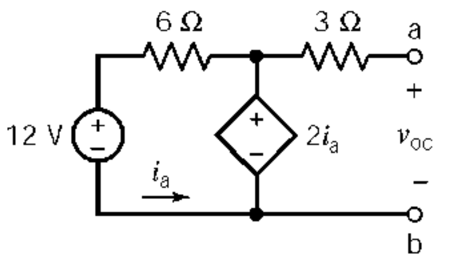


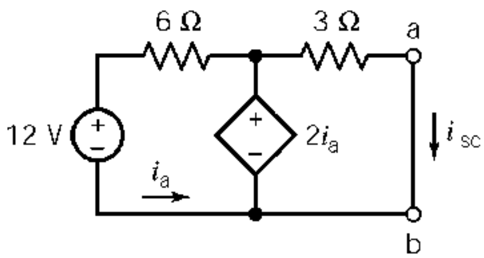
Figure P 5.5-11

Solution:



$$i_a = \frac{2\,i_a - 12}{6} \quad\Rightarrow\quad i_a = -3\text{ A}$$

$$v_{oc} = 2\,i_a = -6\text{ V}$$



$$12 + 6\,i_a = 2\,i_a \quad\Rightarrow\quad i_a = -3\text{ A}$$

$$3\,i_{sc} = 2\,i_a \quad\Rightarrow\quad i_{sc} = \frac{2}{3}(-3) = -2\text{ A}$$

$$R_t = \frac{-6}{-2} = 3\,\Omega$$

P 5.5-12 Use Norton's theorem to formulate a general expression for the current i in terms of the variable resistance R shown in Figure P 5.5-12.

Answer: $i = 20/(8 + R)$ A

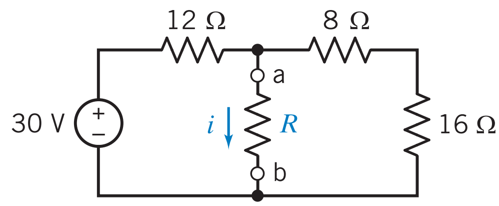
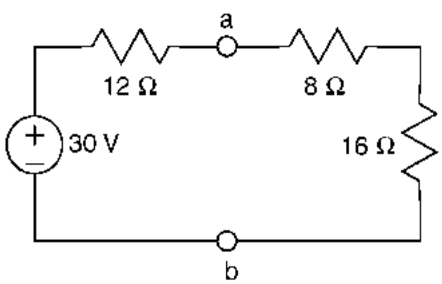


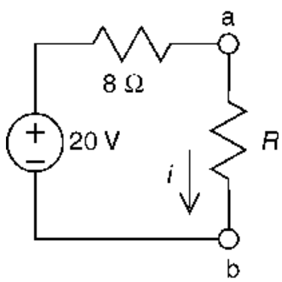
Figure P 5.5-12

Solution:



$$R_t = \frac{12 \times 24}{12+24} = \frac{12 \times 24}{36} = 8\ \Omega$$

$$v_{oc} = \frac{24}{12+24}(30) = 20\text{ V}$$



$$i = \frac{20}{8+R}$$

Section 5-6: Maximum Power Transfer

P 5.6-1 The circuit model for a photovoltaic cell is given in Figure P 5.6-1 (Edelson, 1992). The current i_s is proportional to the solar insolation (kW/m^2).

(a) Find the load resistance, R_L, for maximum power transfer.

(b) Find the maximum power transferred when i_s = 1 A.

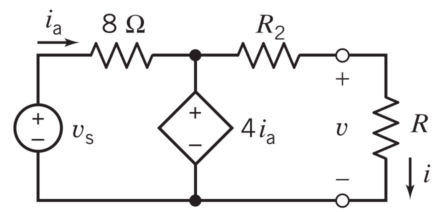
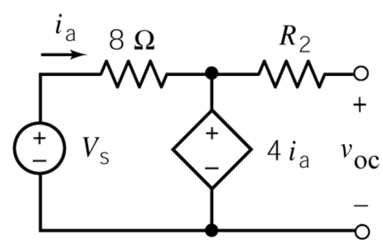


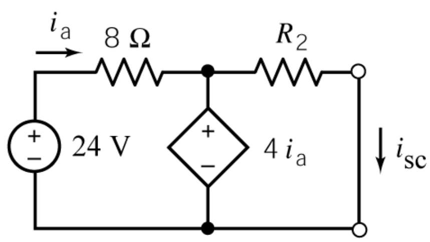
Figure P 5.6-1

Solution:

(a) The value of the current in R_2 is 0 A so $v_{oc} = 4i_a$. Then KVL gives



$$8i_a + 4i_a - V_s = 0 \quad \Rightarrow \quad V_s = 12i_a = 3(4i_a) = 3(v_{oc}) = 24 \text{ V}$$



Next, KVL gives

$$8i_a + 4i_a - 24 = 0 \quad \Rightarrow \quad i_a = 2 \text{ A}$$

and

$$4i_a = R_2\, i_{sc} \quad \Rightarrow \quad 4(2) = R_2(2) \quad \Rightarrow \quad R_2 = 4\ \Omega$$

(b) The power delivered to the resistor to the right of the terminals is maximized by setting R equal to the Thevenin resistance of the part of the circuit to the left of the terminals:

$$R = R_t = \frac{v_{oc}}{i_{sc}} = \frac{8}{2} = 4\ \Omega$$

Then

$$p_{max} = \frac{v_{oc}^{\ 2}}{4R_t} = \frac{8^2}{4(4)} = 4 \text{ W}$$

P 5.6-2 For the circuit in Figure P 5.6-2, (a) find R such that maximum power is dissipated in R and (b) calculate the value of maximum power.

Answer: $R = 60\ \Omega$ and $P_{max} = 54$ mW

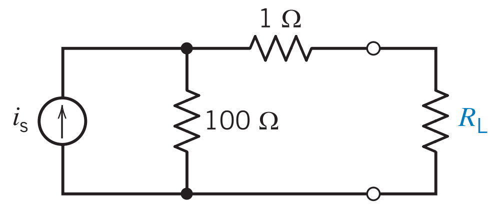
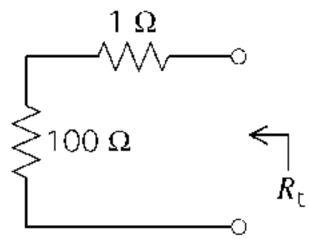


Figure P 5.6-2

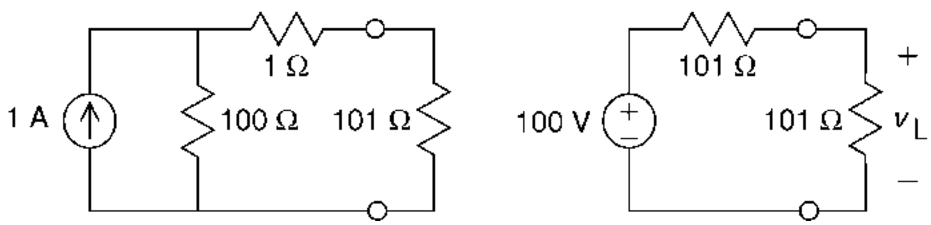
Solution:

a) For maximum power transfer, set R_L equal to the Thevenin resistance:

$$R_L = R_t = 100 + 1 = 101\,\Omega$$



b) To calculate the maximum power, first replace the circuit connected to R_L be its Thevenin equivalent circuit:



The voltage across R_L is $$v_L = \frac{101}{101+101}(100) = 50 \text{ V}$$

Then $$p_{max} = \frac{{v_L}^2}{R_L} = \frac{50^2}{101} = 24.75 \text{ W}$$

P 5.6-3 For the circuit in Figure P 5.6-3, prove that for R_s variable and R_L fixed, the power dissipated in R_L is maximum when $R_s = 0$.

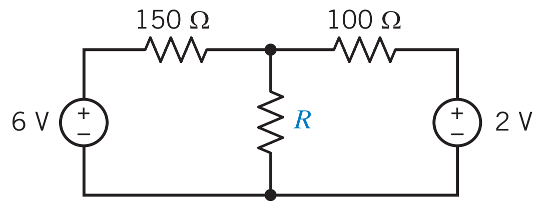
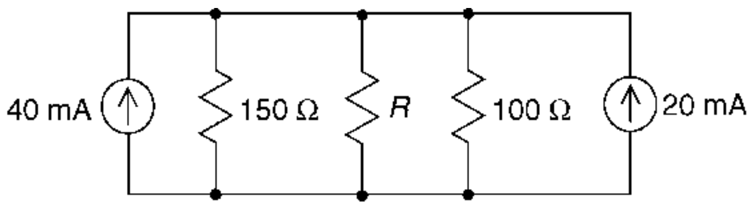
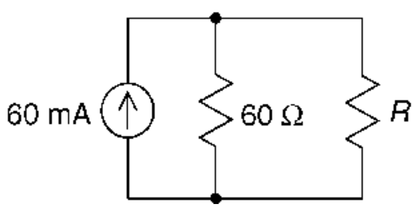


Figure P 5.6-3

Solution:
Reduce the circuit using source transformations:





Then (a) maximum power will be dissipated in resistor R when: $R = R_t = 60\ \Omega$ and (b) the value of that maximum power is

$$P_{max} = i_R^2(R) = (0.03)^2(60) = \underline{54 \text{ mW}}$$

P 5.6-4 Find the maximum power to the load R_L if the maximum power transfer condition is met for the circuit of Figure P 5.6-4.

Answer: max $p_L = 0.75$ W

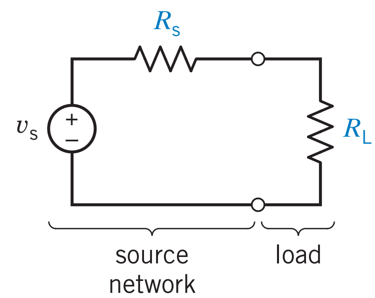
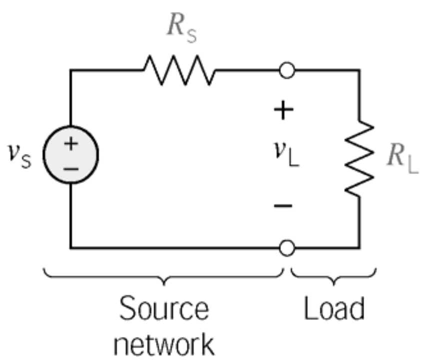


Figure P 5.6-4

Solution:



$$v_L = v_S \left[\frac{R_L}{R_S + R_L} \right]$$

$$\therefore p_L = \frac{v_L^2}{R_L} = \frac{v_S^2 R_L}{(R_S + R_L)^2}$$

By inspection, p_L is max when you reduce R_S to get the smallest denominator.

$\therefore$ set $R_S = 0$

P 5.6-5 Determine the maximum power that can be absorbed by a resistor, R, connected to terminals a–b of the circuit shown in Figure P 5.6-5. Specify the required value of R.

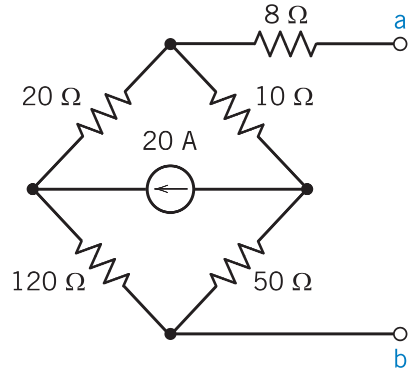
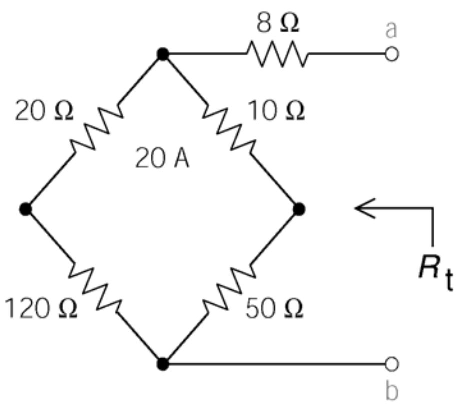
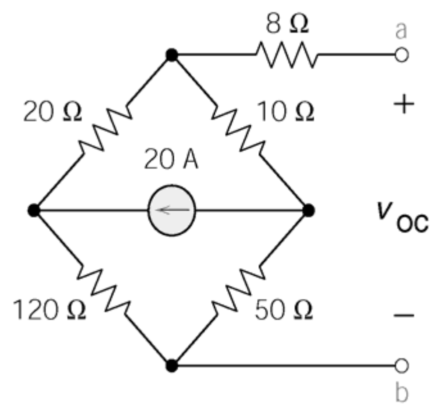


Figure P 5.6-5

Solution:





$$v_{oc} = \left[\frac{170}{170+30}(20)\right]10 - \left[\frac{30}{170+30}(20)\right]50$$

$$= \frac{170(20)(10)-30(20)(50)}{200} = \frac{4000}{200} = 20 \text{ V}$$

The required value of R is

$$R = R_t = 8 + \frac{(20+120)(10+50)}{(20+120)+(10+50)} = 50\,\Omega$$

The maximum power is given by

$$p_{\max} = \frac{{v_{oc}}^2}{4\,R_t} = \frac{20^2}{4(50)} = 2 \text{ W}$$

P 5.6-6 Figure P 5.6-6 shows a source connected to a load through an amplifier. The load can safely receive up to 15 W of power. Consider three cases:

(a) $A = 20$ V/V and $R_o = 10\ \Omega$. Determine the value of R_L that maximizes the power delivered to the load and the corresponding maximum load power.

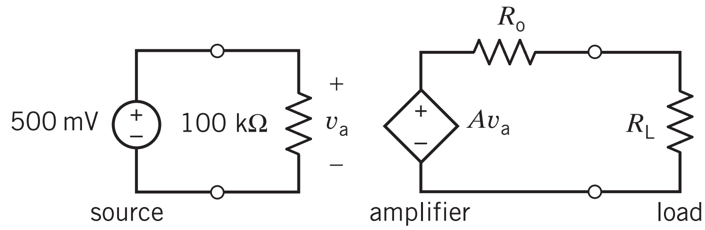
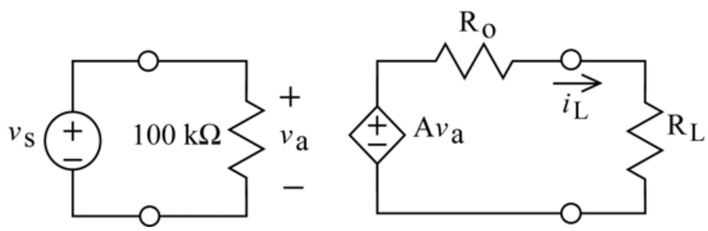


Figure P 5.6-6

(b) $A = 20$ V/V and $R_L = 8\ \Omega$. Determine the value of R_o that maximizes the power delivered to the load and the corresponding maximum load power.

(c) $R_o = 10\ \Omega$ and $R_L = 8\ \Omega$. Determine the value of A that maximizes the power delivered to the load and the corresponding maximum load power.

Solution:



$$i_L = \frac{A}{R_o + R_L} v_s$$

$$P_L = i_L^2 R_L = \frac{A^2 v_s^2 R_L}{\left(R_o + R_L\right)^2}$$

(a) $R_t = R_o$ so $R_L = R_o = 10\ \Omega$ maximizes the power delivered to the load. The corresponding load power is

$$P_L = \frac{20^2 \left(\frac{1}{2}\right)^2 10}{\left(10+10\right)^2} = 2.5 \text{ W}.$$

(b) $R_o = 0$ maximizes P_L (The numerator of P_L does not depend on R_o so P_L can be maximized by making the denominator as small as possible.) The corresponding load power is

$$P_L = \frac{A^2 v_s^{\,2} R_L}{R_L^{\,2}} = \frac{A^2 v_s^{\,2}}{R_L} = \frac{20^2\left(\frac{1}{2}\right)^2}{8} = 12.5\ \text{W}.$$

(c) P_L is proportional to A^2 so the load power continues to increase as A increases. The load can safely receive 15 W. This limit corresponds to

$$15 = \frac{A^2\left(\frac{1}{2}\right)^2 8}{(18)^2} \quad \Rightarrow \quad A = 36\sqrt{\frac{15}{8}} = 49.3\ \text{V}.$$

(checked: LNAP 6/9/04)

P 5.6-7 The circuit in Figure P 5.6-7 contains a variable resistance, *R*, implemented using a potentiometer. The resistance of the variable resistor varies over the range $0 \le R \le 1000\ \Omega$. The variable resistor can safely receive 1/4 W power. Determine the maximum power received by the variable resistor. Is the circuit safe?

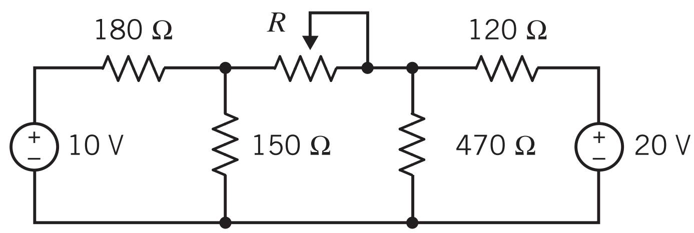
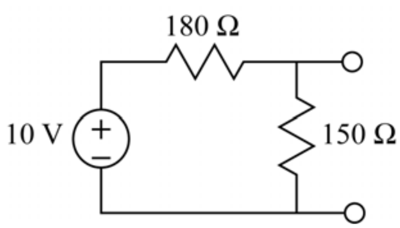


Figure P 5.6-7

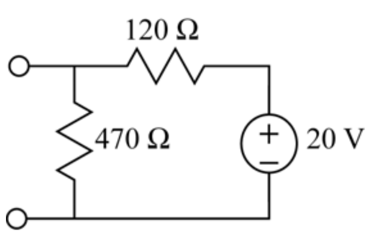
Solution:
Replace the part of the circuit connected to the variable resistor by its Thevenin equivalent circuit. First, replace the left part of the circuit by its Thevenin equivalent:



$$v_{oc1} = \left(\frac{150}{150+180}\right) 10 = 4.545\ \text{V}$$

$$R_{t1} = 180 \,||\, 150 = 81.8\ \Omega$$

Next, replace the right part of the circuit by its Thevenin equivalent:



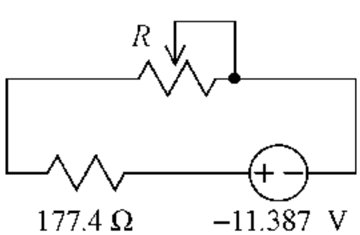
$$v_{oc2} = \left(\frac{470}{470+120}\right) 20 = 15.932\ \text{V}$$

$$R_{t2} = 120 \,||\, 470 = 95.6\ \Omega$$

Now, combine the two partial Thevenin equivalents:

$$v_{oc} = v_{oc1} - v_{oc2} = -10.387\ \text{V and } R_t = R_{t1} + R_{t2} = 177.4\ \Omega$$

So



The power received by the adjustable resistor will be maximum when $R = R_t = 177.4\ \Omega$. The maximum power received by the adjustable resistor will be $p = \dfrac{(-11.387)^2}{4(177.4\ \Omega)} = 0.183\ \text{W}$.

(checked LNAPDC 7/24/04)

P 5.6-8 For the circuit of Figure P 5.6-8, find the power delivered to the load when R_L is fixed and R_t may be varied between 1 Ω and 5 Ω. Select R_t so that maximum power is delivered to R_L.

Answer: 13.9 W

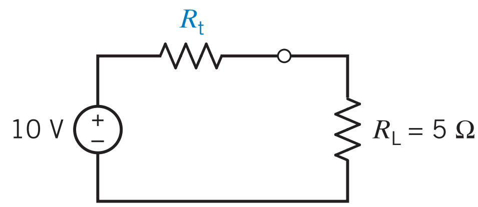


Figure P 5.6-8

Solution:

$$p = i\,v = \left(\frac{10}{R_t + R_L}\right)\left[\frac{R_L}{R_t + R_L}(10)\right] = \frac{100\,R_L}{\left(R_t + R_L\right)^2}$$

The power increases as R_t decreases so choose R_t = 1 Ω. Then

$$p_{\max} = i\,v = \frac{100\,(5)}{(1+5)^2} = 13.9\text{ W}$$

P 5.6-9 A resistive circuit was connected to a variable resistor, and the power delivered to the resistor was measured as shown in Figure P 5.6-9. Determine the Thévenin equivalent circuit.

Answer: $R_t = 20\ \Omega$ and $v_{oc} = 20$ V

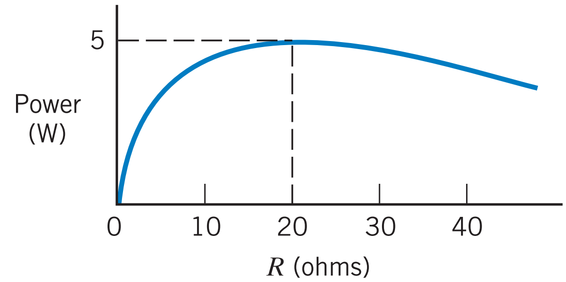


Figure P 5.6-9

Solution:

From the plot, the maximum power is 5 W when $R = 20\ \Omega$. Therefore:

$$R_t = 20\ \Omega$$

and

$$p_{\max} = \frac{v_{oc}^{\ 2}}{4\,R_t} \quad \Rightarrow \quad v_{oc} = \sqrt{p_{\max}\,4\,R_t} = \sqrt{5(4)20} = 20\ \text{V}$$

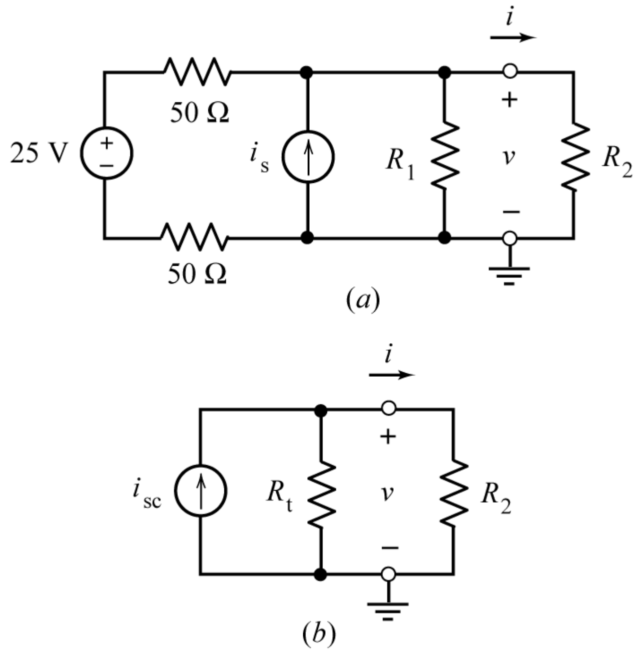


Figure P5.6-10

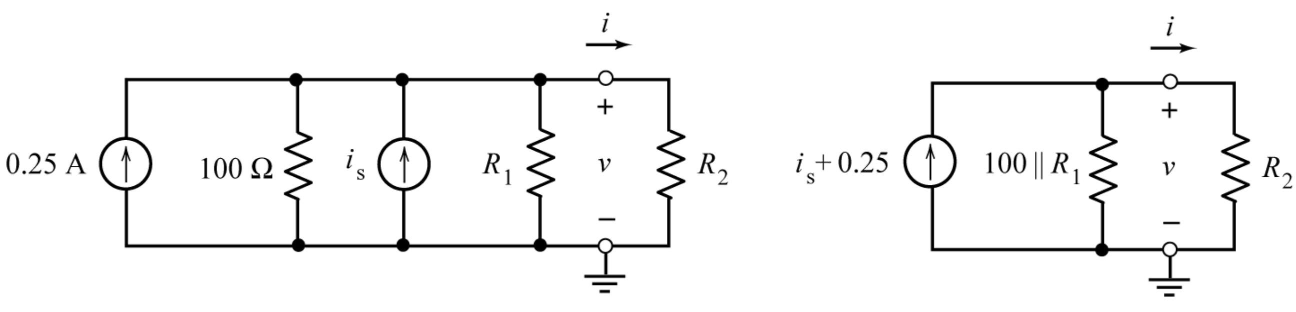
P5.6-10 The part circuit shown in Figure P5.6-10*a* to left of the terminals can be reduced to its Norton equivalent circuit using source transformations and equivalent resistance. The resulting Norton equivalent circuit, shown in Figure P5.6-10*b*, will be characterized by the parameters:

$$i_{sc} = 1.5 \text{ A} \quad \text{and} \quad R_t = 80\ \Omega$$

(*a*) Determine the values of i_s and R_1.

(*b*) Given that $0 \le R_2 \le \infty$, determine the maximum value of $p = vi$, the power delivered to R_2.

Solution: Two source transformations reduce the circuit as follows:



(*a*) Recognizing the parameters of the Norton equivalent circuit gives:

$$1.5 = i_{sc} = i_s + 0.25 \quad \Rightarrow \quad i_s = 1.25 \text{ A and } 80 = R_t = 100 \,||\, R_1 = \frac{100\,R_1}{100 + R_1} \quad \Rightarrow \quad R_1 = 400\ \Omega$$

(*b*) The maximum value of the power delivered to R_2 occurs when $R_2 = R_t = 80\ \Omega$. Then

$$i = \frac{1}{2} i_{sc} = 0.75 \text{ A} \quad \text{and} \quad p = \left(\frac{1}{2} i_{sc}\right)^2 R_t = \left(0.625^2\right) 80 = 45 \text{ W}$$

P5.6-11. Given that $0 \le R \le \infty$ in the circuit shown in Figure P5.6-12, determine (*a*) maximum value of i_a, (*b*) the maximum value of v_a, and (*c*) the maximum value of $p_a = i_a v_a$.

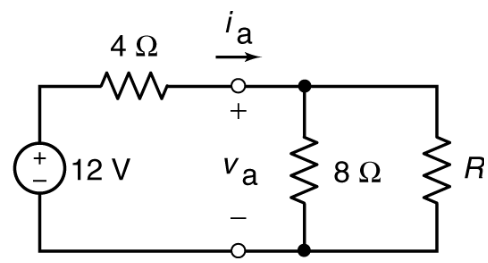
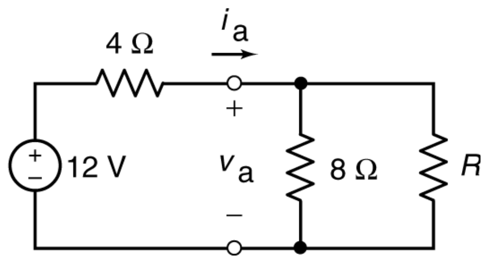
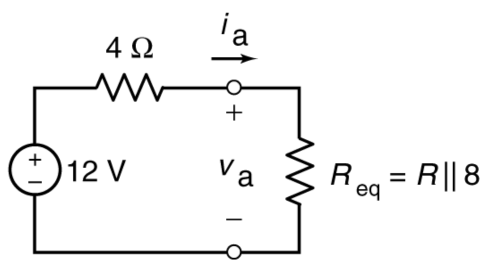


Figure P5.6-11

Solution:



Replace the parallel combination of resistor *R* and the 8 Ω resistor by an equivalent resistance.



Using voltage division

$$v_a = \frac{R_{eq}}{4 + R_{eq}}(12) = \frac{1}{\frac{4}{R_{eq}} + 1}(12)$$

Consequently, the maximum value of v_a corresponds to the is obtained by maximizing R_{eq}. The maximum of R_{eq} is obtained by maximizing *R*. Given that $0 \le R \le \infty$, the maximum value of R_{eq} is 8 Ω and the maximum value of v_a is

$$v_{a\,max} = \frac{1}{\frac{4}{8} + 1}(12) = 8 \text{ V}$$

Using Ohm's law

$$i_a = \frac{12}{4 + R_{eq}}$$

Consequently, the maximum value of i_a corresponds to the is obtained by minimizing R_{eq}. The minimum of R_{eq} is obtained by maximizing *R*. Given that $0 \le R \le \infty$, the minimum value of R_{eq} is 0 Ω and the maximum value of i_a is

$$i_{\text{a max}} = \frac{12}{4+0} = 3 \text{ A}$$

The maximum power theorem indicates that the maximum value of $p_a = i_a v_a$ occurs when R_{eq} = R_t. In this case, R_t = 4 Ω. We require R_{eq} = 4 Ω which is accomplished by making R = 8 Ω, an acceptable value since
$0 \le 8 \le \infty$. Then

$$p_a = \frac{\left(\frac{12}{2}\right)^2}{R_{eq}} = \frac{\left(\frac{12}{2}\right)^2}{4} = 9 \text{ W}$$

P5.6-12. Given that $0 \le R \le \infty$ in the circuit shown in Figure P5.6-12, determine value of R that maximizes the power $p_a = i_a v_a$ and the corresponding maximum value of p_a.

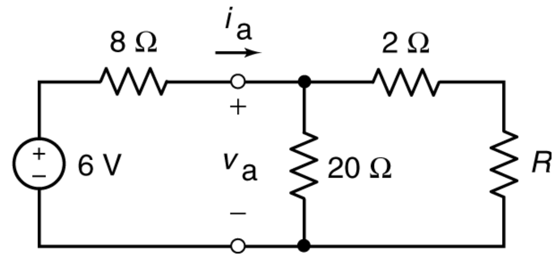
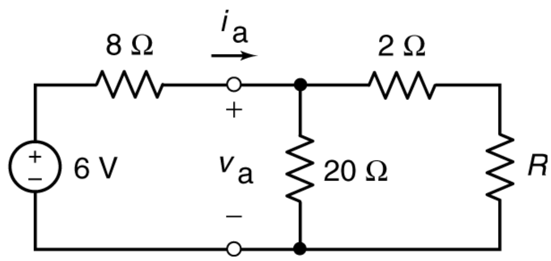
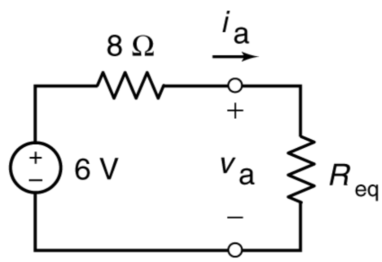


Figure P5.6-12

Solution:



Replace the combination of resistor R and the 20 Ω and 2 Ω resistors by an equivalent resistance.



The maximum power theorem indicates that the maximum value of $p_a = i_a v_a$ occurs when $R_{eq} = R_t$. In this case, $R_t = 8$ Ω. We require

$$8 = R_{eq} = \frac{20(R+2)}{20+(R+2)} = \frac{20R+40}{R+22}$$

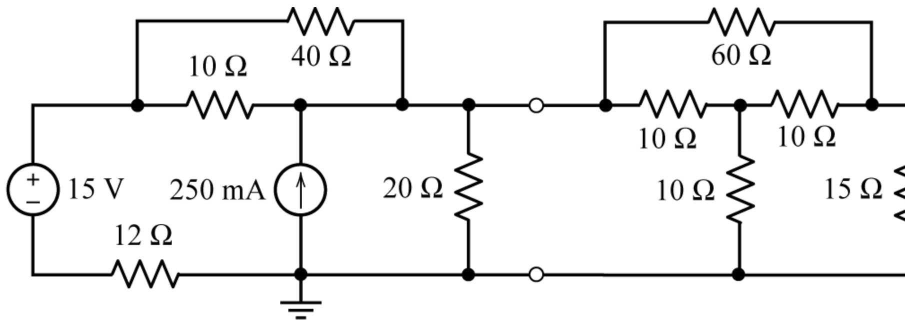
$$8(R+22) = 20R+40 \quad \Rightarrow \quad R = \frac{8(22)-40}{20-8} = 11.333 \ \Omega$$

This isn't a standard resistance value but it is an acceptable value for this problem since $0 \le 11.333 \le \infty$. Then

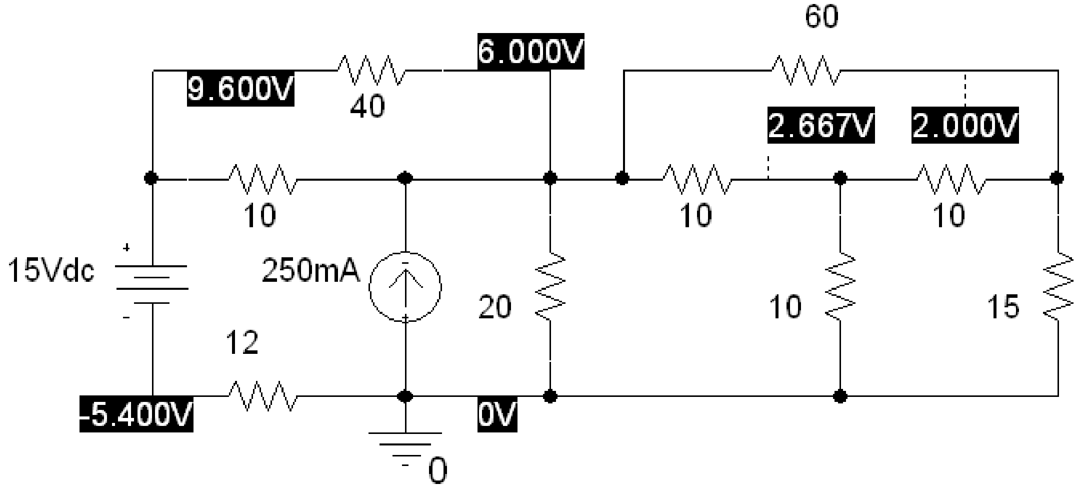
$$p_a = \frac{\left(\frac{6}{2}\right)^2}{R_{eq}} = \frac{\left(\frac{6}{2}\right)^2}{8} = 1.125 \text{ W}$$

Section 5.8 Using PSpice to Determine the Thevenin Equivalent Circuit

P5.8-1



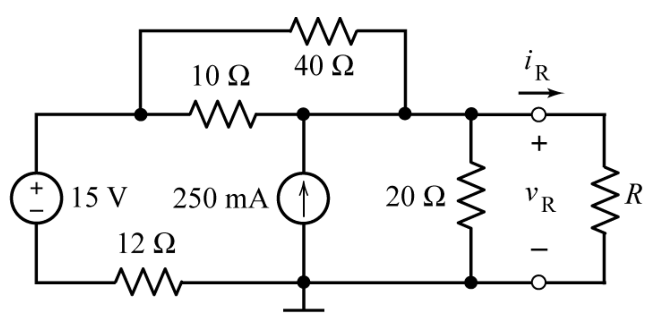
a) Here are the results of simulating the circuit in PSpice. The numbers shown in white on a black background are the node voltages.



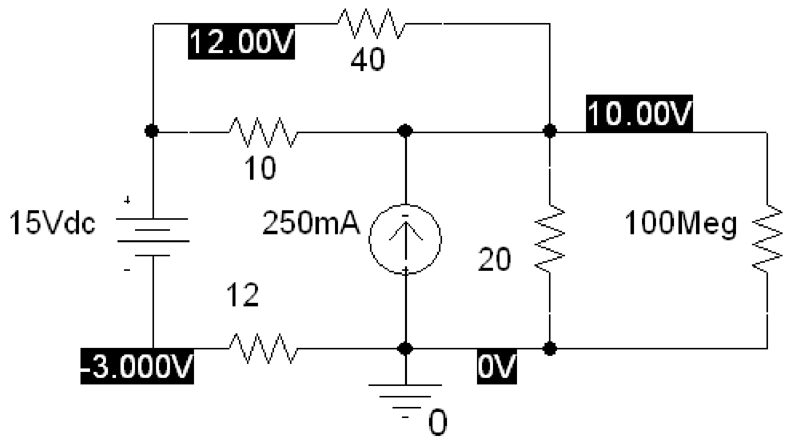
b) Add a resistor across the terminals of Circuit A. Then

$$v_{oc} = v_R \quad \text{when} \quad R \approx \infty$$

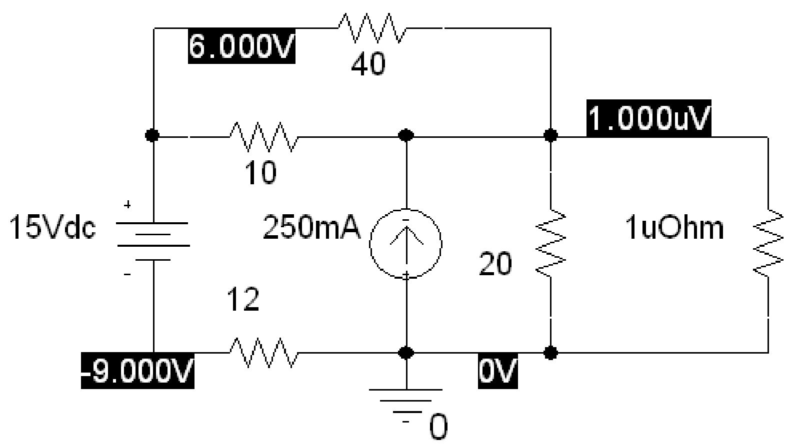
$$i_{sc} = \frac{v_R}{R} \quad \text{when} \quad R \approx 0$$



Here are the PSpice simulation results:



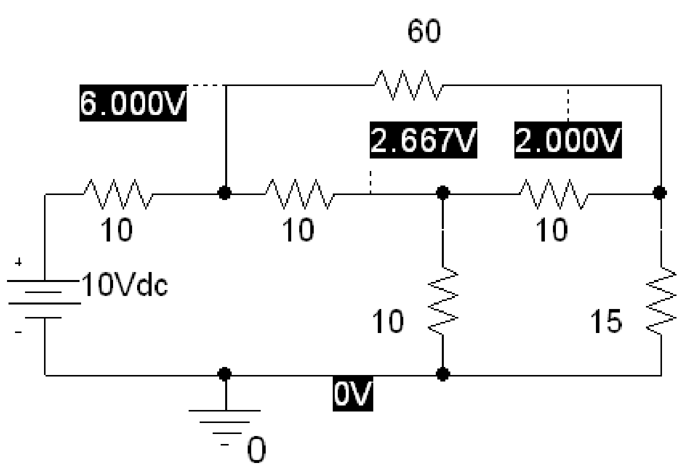
$$v_{oc} = 10 \text{ V}$$



$$i_{sc} = \frac{1\times10^{-6}}{1\times10^{-6}} = 1 \text{ A}$$

$$R_t = \frac{v_{oc}}{i_{sc}} = \frac{10}{1} = 10\ \Omega$$

c) Here is the result of simulation the circuit after replacing Circuit A by its Thevenin equivalent:



d) The node voltages of Circuit B are the same before and after replacing Circuit A by its Thevenin equivalent circuit.

Section 5-9 How Can We Check…?

P 5.9-1 For the circuit of Figure P 5.9-1, the current *i* has been measured for three different values of *R* and is listed in the table. Are the data consistent?

$R(\Omega)$	i(mA)
5000	16.5
500	43.8
0	97.2

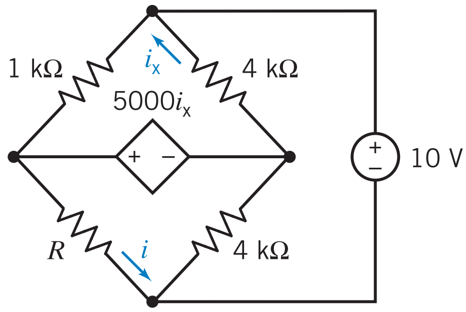
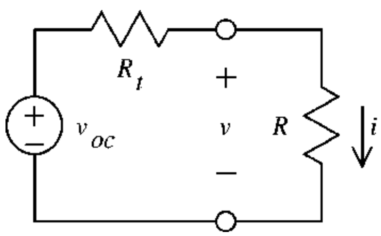


Figure P 5.9-1

Solution:



Use the data in the first two lines of the table to determine v_{oc} and R_t:

$$\left.\begin{aligned} 0.0972 &= \frac{v_{oc}}{R_t + 0} \\ 0.0438 &= \frac{v_{oc}}{R_t + 500} \end{aligned}\right\} \Rightarrow \begin{cases} v_{oc} = 39.9 \text{ V} \\ R_t = 410\,\Omega \end{cases}$$

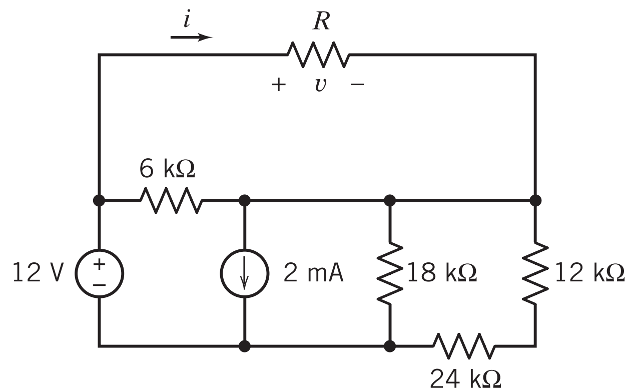
Now check the third line of the table. When R= 5000 Ω:

$$i = \frac{v_{oc}}{R_t + R} = \frac{39.9}{410 + 5000} = 7.37 \text{ mA}$$

which disagree with the data in the table.

<u>**The data is not consistent.**</u>

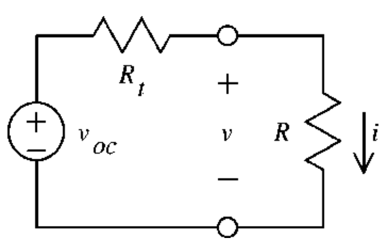
P 5.9-2 Your lab partner built the circuit shown in Figure P 5.9-2 and measured the current i and voltage v corresponding to several values of the resistance R. The results are shown in the table in Figure P 5.9-2. Your lab partner says that R_L = 8000 Ω is required to cause i = 1 mA. Do you agree? Justify your answer.



R	i	v
open	0 mA	12 V
10 kΩ	0.857 mA	8.57 V
short	3 mA	0 V

Figure P 5.9-2

Solution:



Use the data in the table to determine v_{oc} and i_{sc}:

$v_{oc} = 12$ V (line 1 of the table)

$i_{sc} = 3$ mA (line 3 of the table)

so $R_t = \dfrac{v_{oc}}{i_{sc}} = 4\ \text{k}\Omega$

Next, check line 2 of the table. When R = 10 kΩ:

$$i = \frac{v_{oc}}{R_t + R} = \frac{12}{10(10^3) + 5(10^3)} = 0.857\ \text{mA}$$

which agrees with the data in the table.

To cause i = 1 mA requires $0.001 = i = \dfrac{v_{oc}}{R_t + R} = \dfrac{12}{10(10^3) + R} \Rightarrow R = 8000\ \Omega$

I agree with my lab partner's claim that R = 8000 causes i = 1 mA.

P 5.9-3 In preparation for lab, your lab partner determined the Thévenin equivalent of the circuit connected to R_L in Figure P 5.9-3. She says that the Thévenin resistance is $R_t = \frac{6}{11}R$ and the open-circuit voltage is $v_{oc} = \frac{60}{11}$ V. In lab, you built the circuit using $R = 110\ \Omega$ and $R_L = 40\ \Omega$ and measured that $i = 54.5$ mA. Is this measurement consistent with the prelab calculations? Justify your answers.

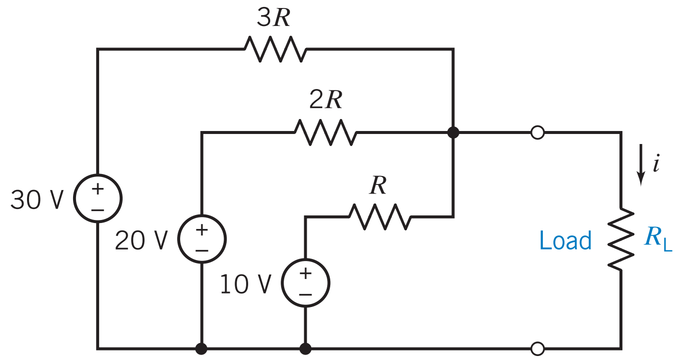
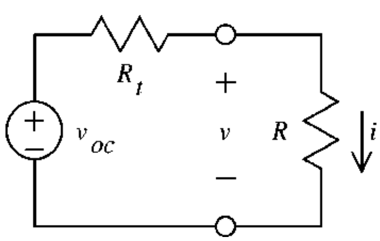


Figure P 5.9-3

Solution:



$$\frac{1}{R_t} = \frac{1}{R} + \frac{1}{2R} + \frac{1}{3R} = \frac{11}{6R} \quad \Rightarrow \quad R_t = \frac{6R}{11}$$

and

$$v_{oc} = \left(\frac{2/3}{3+2/3}\right)30 + \left(\frac{3/4}{2+3/4}\right)20 + \left(\frac{6/5}{1+6/5}\right)10 = \frac{180}{11}$$

so the prelab calculation isn't correct.

But then

$$i = \frac{v_{oc}}{R_t + R} = \frac{\frac{180}{11}}{\frac{6}{11}(110)+40} = \frac{\frac{180}{11}}{60+40} = 163 \text{ mA} \neq 54.5 \text{ mA}$$

so the measurement does not agree with the corrected prelab calculation.

P 5.9-4 Your lab partner claims that the current i in Figure P 5.9-4 will be no greater than 12.0 mA, regardless of the value of the resistance R. Do you agree?

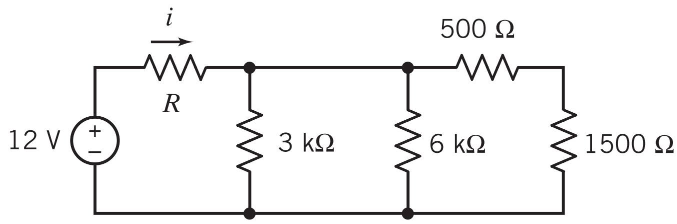


Figure P 5.9-4

Solution:

$$6000 \,||\, 3000 \,||\, (500+1500) = 2000 \,||\, 2000 = 1000\ \Omega$$

$$i = \frac{12}{R+1000} \le \frac{12}{1000} = 12\text{ mA}$$

How about that?! Your lab partner is right.

(checked using LNAP 6/21/05)

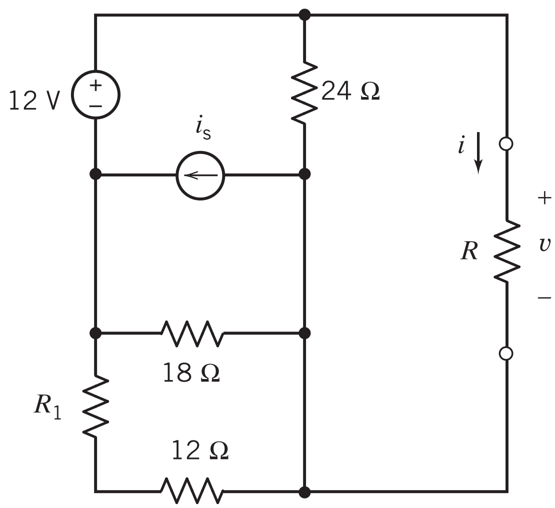
P 5.9-5 Figure P 5.9-5 shows a circuit and some corresponding data. Two resistances, R_1 and R, and the current source current are unspecified. The tabulated data provide values of the current, i, and voltage, v, corresponding to several values of the resistance R.

(a) Consider replacing the part of the circuit connected to the resistor R by a Thévenin equivalent circuit. Use the data in rows 2 and 3 of the table to find the values of R_t and v_{oc}, the Thévenin resistance and the open-circuit voltage.

(b) Use the results of part (a) to verify that the tabulated data are consistent.

(c) Fill in the blanks in the table.

(d) Determine the values of R_1 and i_s.



(a)

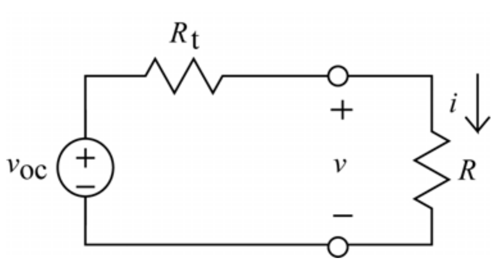
R, Ω	i, A	v, V
0	3	0
10	1.333	13.33
20	0.857	17.14
40	0.5	?
80	?	21.82

(b)

Figure P5.9-5

Solution:

(a)



KVL gives

$$v_{oc} = (R_t + R)i$$

from row 2

$$v_{oc} = (R_t + 10)(1.333)$$

from row 3

$$v_{oc} = (R_t + 20)(0.857)$$

So

$$(R_t + 10)(1.333) = (R_t + 20)(0.857)$$

$$28(R_t + 10) = 18(R_t + 20)$$

Solving gives

$$10R_t = 360 - 280 = 80 \quad \Rightarrow \quad R_t = 8\ \Omega$$

and

$$v_{oc} = (8 + 10)(1.333) = 24\ \text{V}$$

(b)

$$i = \frac{v_{oc}}{R_t + R} = \frac{24}{8+R} \quad \text{and} \quad v = \frac{R}{R+R_t} v_{oc} = \frac{24R}{R+8}$$

When $R = 0$, $i = 3$ A, and $v = 0$ V.

When $R = 40\ \Omega$, $i = \frac{1}{2}\ \text{A}$.

When $R = 80\ \Omega$, $v = \frac{24(80)}{88} = \frac{240}{11} = 21.82$.

These are the values given in the tabulated data so the data is consistent.

(c) When $R = 40\ \Omega$, $v = \frac{24(40)}{48} = 20\ \text{V}$.

When $R = 80\ \Omega$, $i = \frac{24}{88} = 0.2727\ \text{A}$.

(d) First

$$8 = R_t = 24 \,||\, 18 \,||\, (R_1 + 12) \quad \Rightarrow \quad R_1 = 24\ \Omega$$

the, using superposition,

$$24 = v_{oc} = \frac{24}{24 + (18 \,||\, (R_1 + 12))} 12 + (24 \,||\, 18(R_1 + 12)) i_s = 8 + 8 i_s \quad \Rightarrow \quad i_s = 2\ \text{A}$$

(checked using LNAP 6/21/05)

PSpice Problems

SP 5-1 The circuit in Figure SP 5.1 has three inputs: v_1, v_2, and i_3. The circuit has one output, v_o. The equation

$$v_o = av_1 + bv_2 + ci_3$$

expresses the output as a function of the inputs. The coefficients *a*, *b*, and *c* are real constants.

(a) Use PSpice, and the principle of superposition, to determine the values of *a*, *b*, and *c*.

(b) Suppose $v_1 = 10$ V, $v_2 = 8$ V, and we want the output to be $v_o = 7$ V. What is the required value of i_3?

Hint: The output is given by $v_o = a$ when $v_1 = 1$ V, $v_2 = 0$ V, and $i_3 = 0$ A.

Answer: (a) $v_o = 0.3333v_1 + 0.3333v_2 + 33.33i_3$, (b) $i_3 = 30$ mA

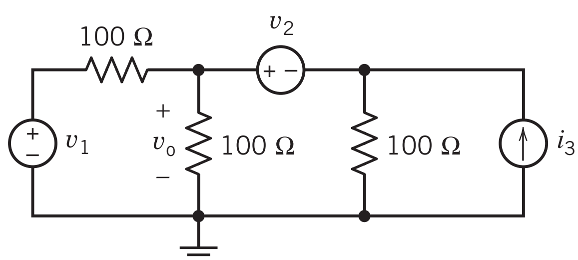
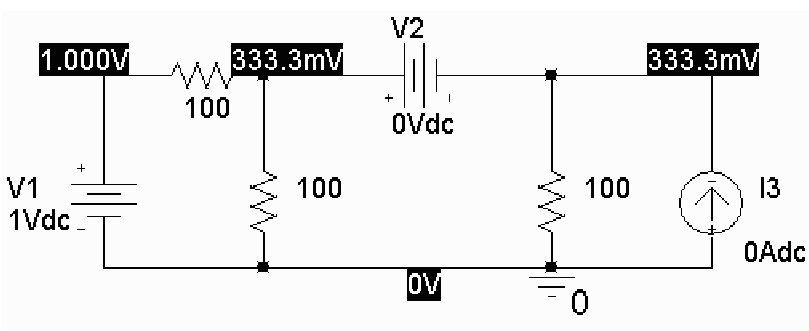
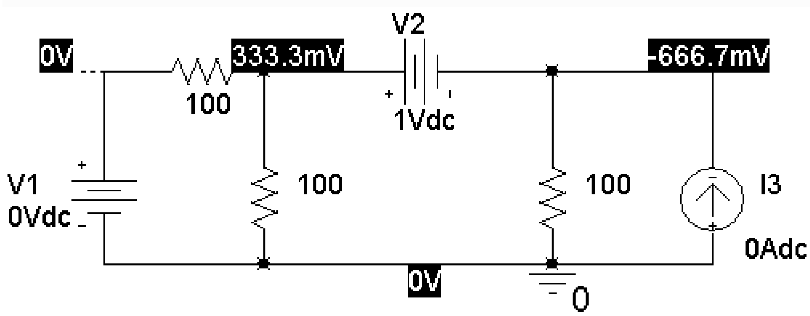


Figure SP 5.1

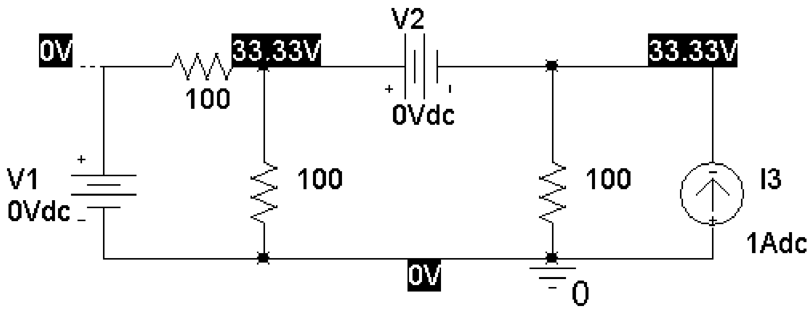
Solution:



a = 0.3333



b = 0.3333



c =33.33 V/A

(a) $$v_o = 0.3333\,v_1 + 0.3333\,v_2 + 33.33\,i_3$$

(b) $$7 = 0.3333\left(10\right) + 0.3333\left(8\right) + 33.33\,i_3 \quad \Rightarrow \quad i_3 = \frac{7 - \frac{18}{3}}{\frac{100}{3}} = \frac{3}{100} = 30 \text{ mA}$$

SP 5-2 The pair of terminals a–b partitions the circuit in Figure SP 5.2 into two parts. Denote the node voltages at nodes 1 and 2 as v_1 and v_2. Use PSpice to demonstrate that performing a source transformation on the part of the circuit to the left of the terminal does not change anything to the right of the terminals. In particular, show that the current, i_o, and the node voltages, v_1 and v_2, have the same values after the source transformation as before the source transformation.

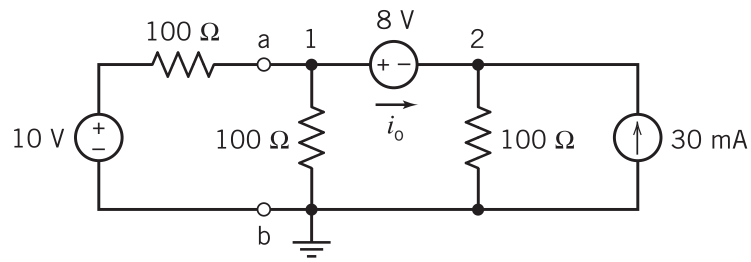
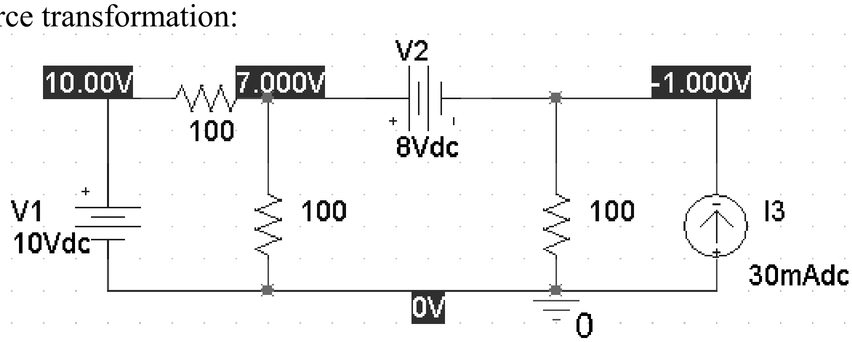


Figure SP 5.2

Solution:

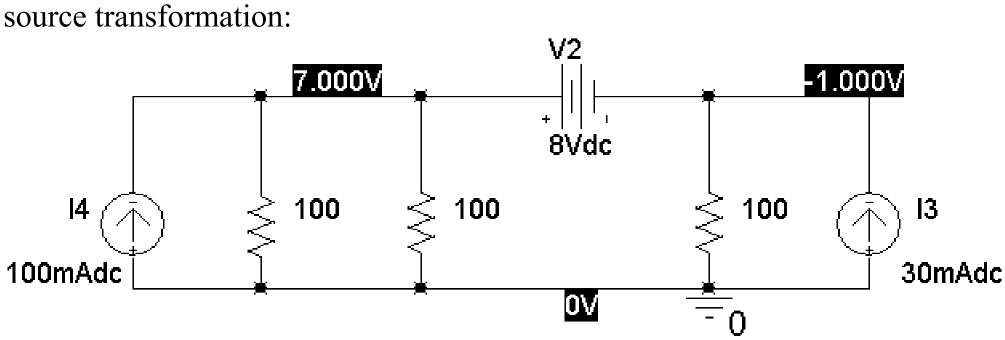
Before the source transformation:



VOLTAGE SOURCE CURRENTS

NAME	CURRENT
V_V1	-3.000E-02
V_V2	-4.000E-02

After the source transformation:



VOLTAGE SOURCE CURRENTS

NAME	CURRENT
V_V2	-4.000E-02

SP 5-3 Use PSpice to find the Thévenin equivalent circuit for the circuit shown in Figure SP 5.3.

Answer: $v_{oc} = -2$ V and $R_t = -8/3\ \Omega$

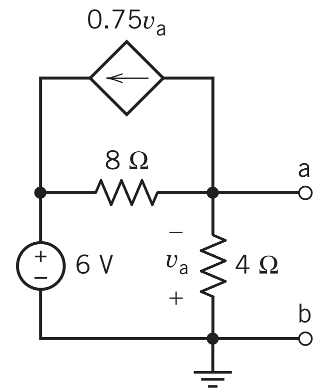
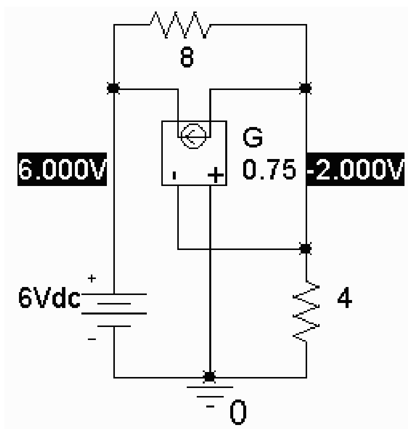
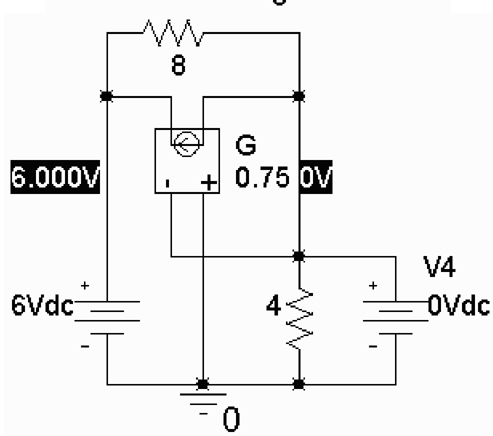


Figure SP 5.3

Solution:



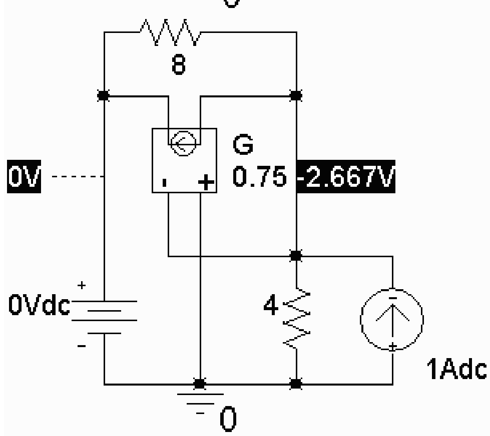
$v_{oc} = -2$ V



VOLTAGE SOURCE CURRENTS

NAME	CURRENT
V_V3	-7.500E-01
V_V4	7.500E-01

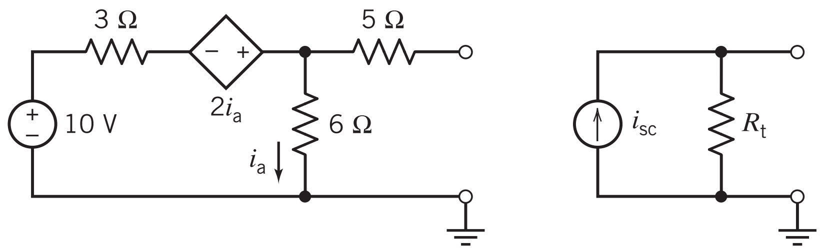
$i_{sc} = 0.75$ A



$R_t = -2.66\ \Omega$

SP 5-4 The circuit shown in Figure SP 5-4*b* is the Norton equivalent circuit of the circuit shown in Figure SP 5-4*a*. Find the value of the short-circuit current, i_{sc}, and Thévenin resistance, R_t.

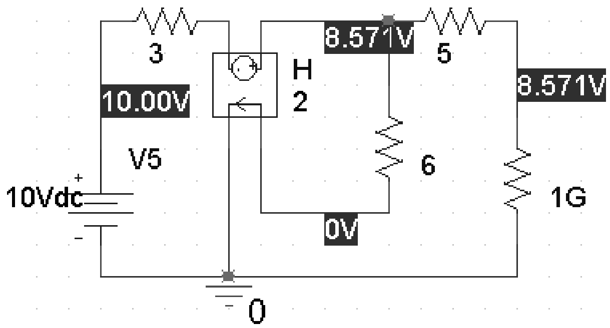
Answer: i_{sc} = 1.13 V and R_t = 7.57 Ω



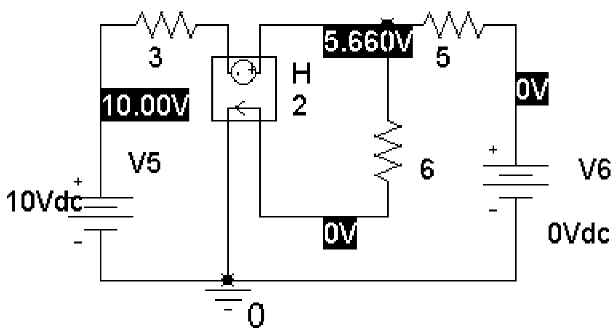
(*a*) (*b*)

Figure SP 5-4

Solution:



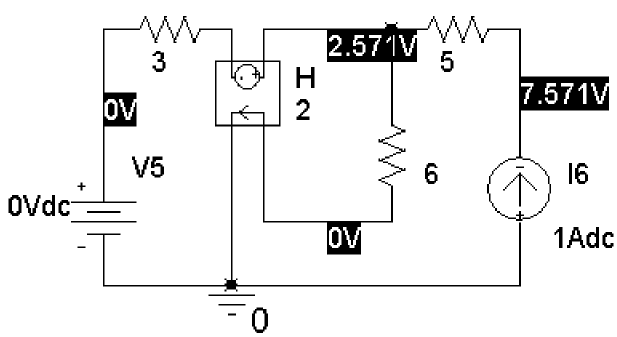
v_{oc} = 8.571 V



VOLTAGE SOURCE CURRENTS

NAME	CURRENT
V_V5	-2.075E+00
V_V6	1.132E+00
X_H1.VH_H1	9.434E-01

i_{sc} = 1.132 A



R_t = 7.571 Ω

Design Problems

DP 5-1 The circuit shown in Figure DP 5-1*a* has four unspecified circuit parameters: v_s, R_1, R_2, and R_3. To design this circuit, we must specify the values of these four parameters. The graph shown in Figure DP 5-1*b* describes a relationship between the current *i* and the voltage *v*.

Specify values of v_s, R_1, R_2, and R_3 that cause the current *i* and the voltage *v* in Figure DP 5-1*a* to satisfy the relationship described by the graph in Figure DP 5-1*b*.

First Hint: The equation representing the straight line in Figure DP 5-1*b* is

$$v = -R_t i + v_{oc}$$

That is, the slope of the line is equal to –1 times the Thévenin resistance and the "*v*-intercept" is equal to the open-circuit voltage.

Second Hint: There is more than one correct answer to this problem. Try setting $R_1 = R_2$.

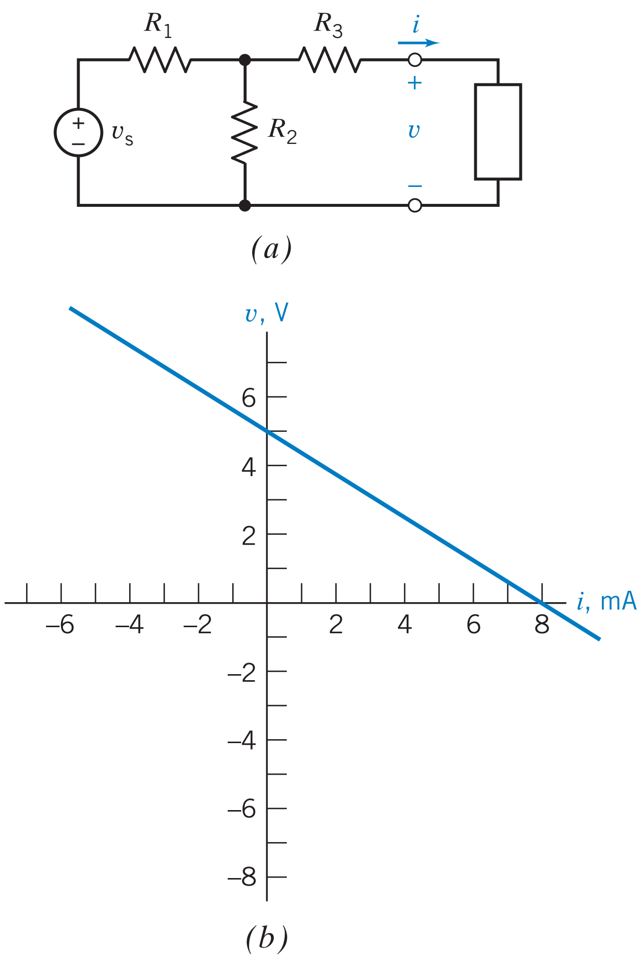


Figure DP 5-1

Solution:

The equation of representing the straight line in Figure DP 5-1b is $v = -R_t\, i + v_{oc}$. That is, the slope of the line is equal to -1 times the Thevenin resistance and the "*v* - intercept" is equal to the open circuit voltage. Therefore: $R_t = -\dfrac{0-5}{0.008-0} = 625\ \Omega$ and v_{oc} = 5 V.

Try $R_1 = R_2 = 1\ \text{k}\Omega$. ($R_1 \parallel R_2$ must be smaller than R_t = 625 Ω.) Then

$$5 = \frac{R_2}{R_1 + R_2} v_s = \frac{1}{2} v_s \quad \Rightarrow v_s = 10\ \text{V}$$

and

$$625 = R_3 + \frac{R_1 R_2}{R_1 + R_2} = R_3 + 500 \quad \Rightarrow \quad R_3 = 125\ \Omega$$

Now v_s, R_1, R_2 and R_3 have all been specified so the design is complete.

DP 5-2 The circuit shown in Figure DP 5.2*a* has four unspecified circuit parameters: i_s, R_1, R_2, and R_3. To design this circuit, we must specify the values of these four parameters. The graph shown in Figure DP 5-2*b* describes a relationship between the current *i* and the voltage *v*.

Specify values of i_s, R_1, R_2, and R_3 that cause the current *i* and the voltage *v* in Figure DP 5-2*a* to satisfy the relationship described by the graph in Figure DP 5-2*b*.

First Hint: Calculate the open-circuit voltage, v_{oc}, and the Thévenin resistance, R_t, of the part of the circuit to the left of the terminals in Figure DP 5-2*a*.

Second Hint: The equation representing the straight line in Figure DP 5-2*b* is

$$v = -R_t i + v_{oc}$$

That is, the slope of the line is equal to –1 times the Thévenin resistance and the "*v*-intercept" is equal to the open-circuit voltage.

Third Hint: There is more than one correct answer to this problem. Try setting both R_3 and $R_1 + R_2$ equal to twice the slope of the graph in Figure DP 5-2*b*.

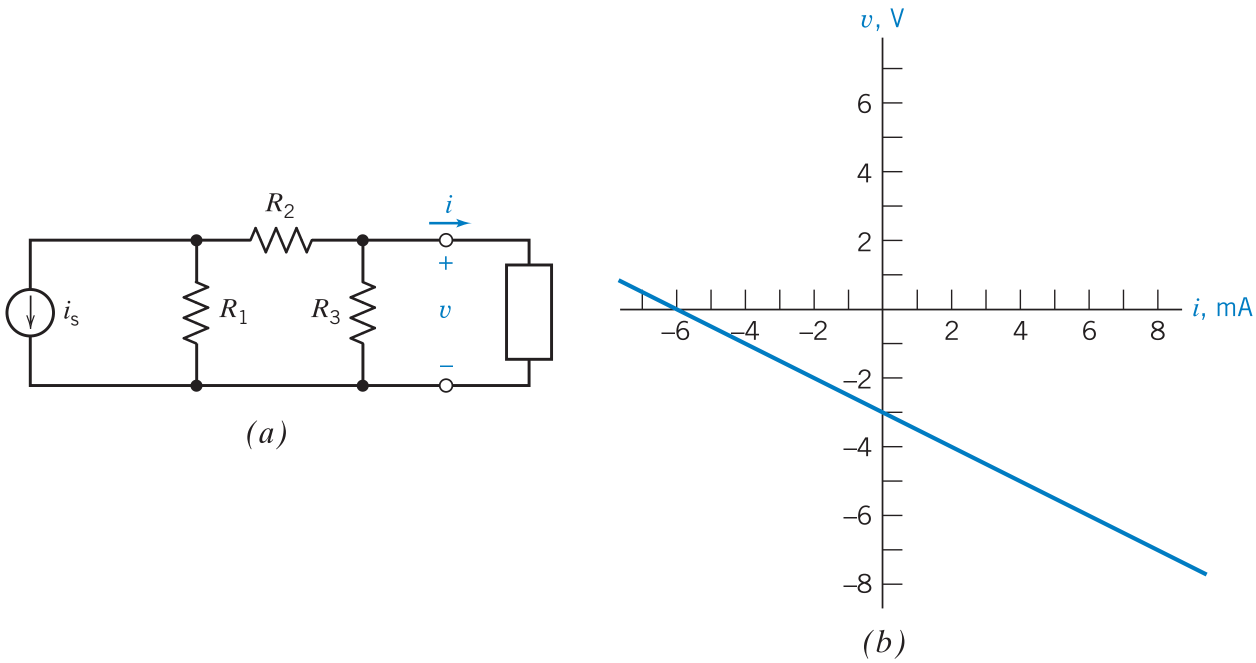


Figure DP 5-2

Solution:

The equation of representing the straight line in Figure DP 5-2b is $v = -R_t\, i + v_{oc}$. That is, the slope of the line is equal to -1 times the Thevenin resistance and the "*v* - intercept" is equal to the open circuit voltage. Therefore: $R_t = -\dfrac{0-(-3)}{-0.006-0} = 500\ \Omega$ and $v_{oc} = -3$ V.

From the circuit we calculate

$$R_t = \frac{R_3\left(R_1+R_2\right)}{R_1+R_2+R_3} \text{ and } v_{oc} = -\frac{R_1 R_3}{R_1+R_2+R_3} i_s$$

so

$$500\ \Omega = \frac{R_3\left(R_1 + R_2\right)}{R_1 + R_2 + R_3} \text{ and } -3\text{ V} = -\frac{R_1 R_3}{R_1 + R_2 + R_3} i_s$$

Try $R_3 = 1\text{k}\Omega$ and $R_1 + R_2 = 1\text{k}\Omega$. Then $R_t = 500\ \Omega$ and

$$-3 = -\frac{1000 R_1}{2000} i_s = -\frac{R_1}{2} i_s \quad \Rightarrow 6 = R_1\, i_s$$

This equation can be satisfied by taking R_1 = 600 Ω and i_s = 10 mA. Finally, R_2 = 1 kΩ - 400 Ω = 400 Ω. Now i_s, R_1, R_2 and R_3 have all been specified so the design is complete.

DP 5-3 The circuit shown in Figure DP 5-3*a* has four unspecified circuit parameters: v_s, R_1, R_2, and R_3. To design this circuit, we must specify the values of these four parameters. The graph shown in Figure DP 5-3*b* describes a relationship between the current *i* and the voltage *v*.

Is it possible to specify values of v_s, R_1, R_2, and R_3 that cause the current *i* and the voltage *v* in Figure DP 5-1*a* to satisfy the relationship described by the graph in Figure DP 5-3*b*? Justify your answer.

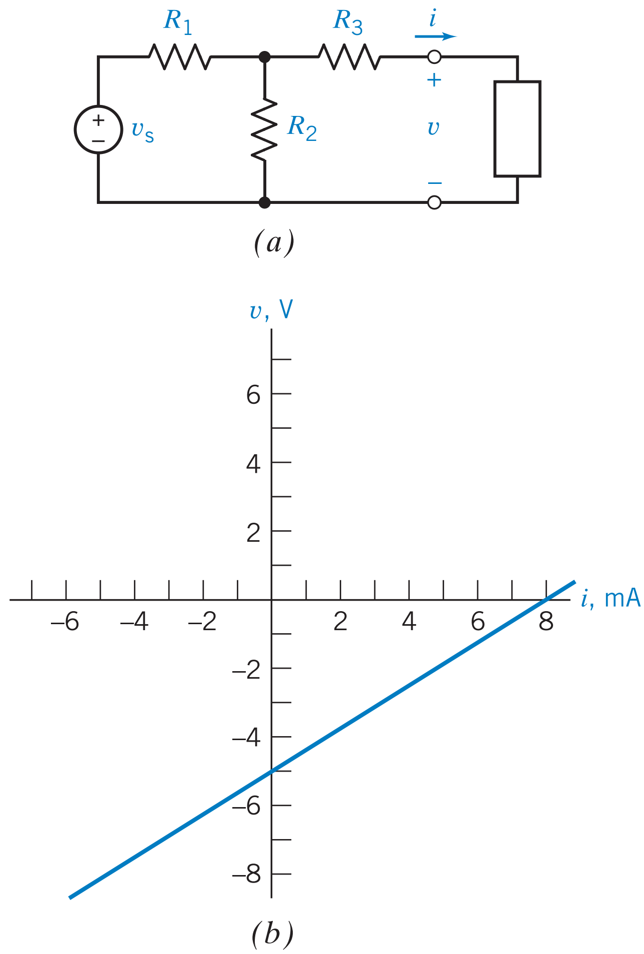


Figure DP 5-3

Solution:

The slope of the graph is positive so the Thevenin resistance is negative. This would require

$R_3 + \frac{R_1 R_2}{R_1 + R_2} < 0$, which is not possible since R_1, R_2 and R_3 will all be non-negative.

Is it not possible to specify values of v_s, R_1, R_2 and R_3 that cause the current *i* and the voltage *v* in Figure DP 5-3*a* to satisfy the relationship described by the graph in Figure DP 5-3*b*.

DP 5-4 The circuit shown in Figure DP 5-4*a* has four unspecified circuit parameters: v_s, R_1, R_2, and d, where d is the gain of the CCCS. To design this circuit, we must specify the values of these four parameters. The graph shown in Figure DP 5-4*b* describes a relationship between the current i and the voltage v.

Specify values of v_s, R_1, R_2, and d that cause the current i and the voltage v in Figure DP 5-4*a* to satisfy the relationship described by the graph in Figure DP 5-4*b*.

First Hint: The equation representing the straight line in Figure DP 5-4b is

$$v = -R_t i + v_{oc}$$

That is, the slope of the line is equal to –1 times the Thévenin resistance and the "v-intercept" is equal to the open-circuit voltage.

Second Hint: There is more than one correct answer to this problem. Try setting $R_1 = R_2$.

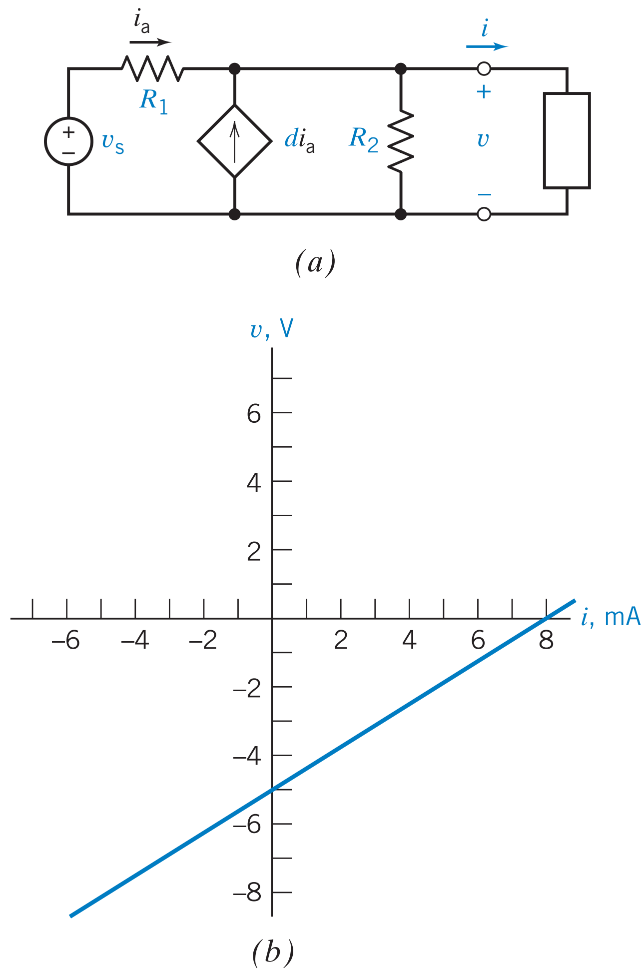


Figure DP 5-4

Solution:

The equation of representing the straight line in Figure DP 5-4b is $v = -R_t\, i + v_{oc}$. That is, the slope of the line is equal to the Thevenin impedance and the "v - intercept" is equal to the open circuit voltage. Therefore: $R_t = -\dfrac{-5-0}{0-0.008} = -625\,\Omega$ and $v_{oc} = -5$ V.

The open circuit voltage, v_{oc}, the short circuit current, i_{sc}, and the Thevenin resistance, R_t, of this circuit are given by

$$v_{oc} = \frac{R_2(d+1)}{R_1 + (d+1)R_2} v_s ,$$

$$i_{sc} = \frac{(d+1)}{R_1} v_s$$

and

$$R_t = \frac{R_1 R_2}{R_1 + (d+1) R_2}$$

Let $R_1 = R_2 = 1$ kΩ. Then

$$-625\ \Omega = R_t = \frac{1000}{d+2} \quad \Rightarrow \quad d = \frac{1000}{-625} - 2 = -3.6 \text{ A/A}$$

and

$$-5 = \frac{(d+1) v_s}{d+2} \quad \Rightarrow \quad v_s = \frac{-3.6+2}{-3.6+1}(-5) = -3.077 \text{ V}$$

Now v_s, R_1, R_2 and d have all been specified so the design is complete.

Chapter 6 The Operational Amplifier

Exercises

Exercise 6.6-1 Specify the values of R_1 and R_2 in Figure E 6.6-1 that are required to cause v_3 to be related to v_1 and v_2 by the equation $v_3 = (4)v_1 - (\frac{1}{5})v_2$.

Answer: R_1 = 10 kΩ and R_2 = 2.5 kΩ

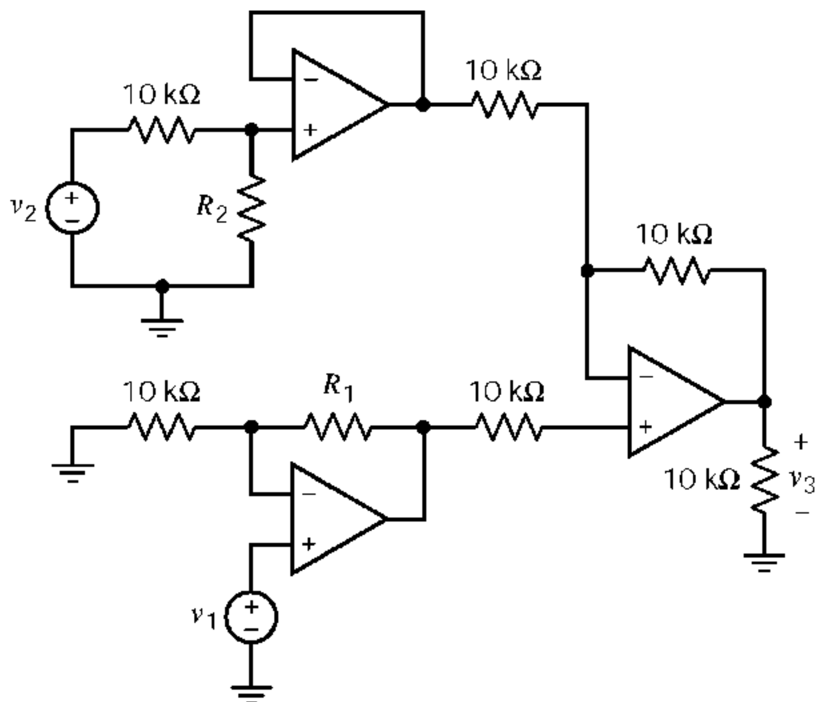


Figure E 6.6-1

Solution:

$$v_3 = \left(-\frac{10\times10^3}{10\times10^3}\right)\left(\frac{R_2}{R_2+10\times10^3}\right)v_2 + \left(1+\frac{10\times10^3}{10\times10^3}\right)\left(1+\frac{R_1}{10\times10^3}\right)v_1$$

$$= -\left(\frac{R_2}{R_2+10\times10^3}\right)v_2 + 2\left(1+\frac{R_1}{10\times10^3}\right)v_1$$

We require $v_3 = (4)v_1 - \left(\frac{1}{5}\right)v_2$, so

$$4 = 2\left(1+\frac{R_1}{10\times10^3}\right) \quad\Rightarrow\quad R_1 = 10\times10^3 = 10\ \text{k}\Omega$$

and

$$\frac{1}{5} = \frac{R_2}{R_2+10\times10^3} \quad\Rightarrow\quad R_2 + 10\times10^3 = 5\,R_2 \quad\Rightarrow\quad R_2 = 2.5\ \text{k}\Omega$$

Exercise 6.6-2 Specify the values of R_1 and R_2 in Figure E 6.6-1 that are required to cause v_3 to be related to v_1 and v_2 by the equation $v_3 = (6)v_1 - (\frac{4}{5})v_2$.

Answer: $R_1 = 20\ \text{k}\Omega$ and $R_2 = 40\ \text{k}\Omega$

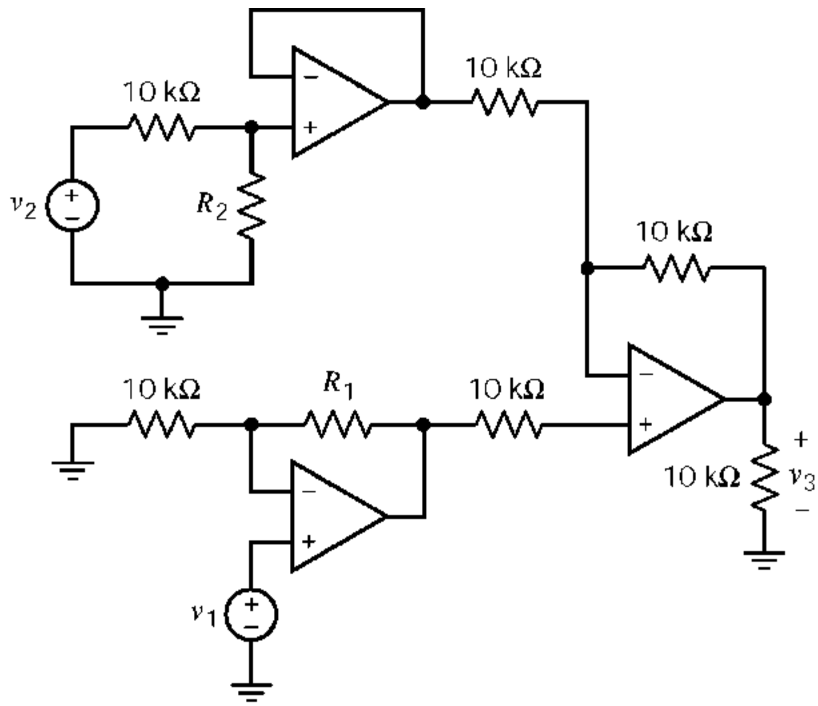


Figure E 6.6-1

Solution: As in Ex 6.7-1

$$v_3 = -\left(\frac{R_2}{R_2 + 10\times 10^3}\right) v_2 + 2\left(1 + \frac{R_1}{10\times 10^3}\right) v_1$$

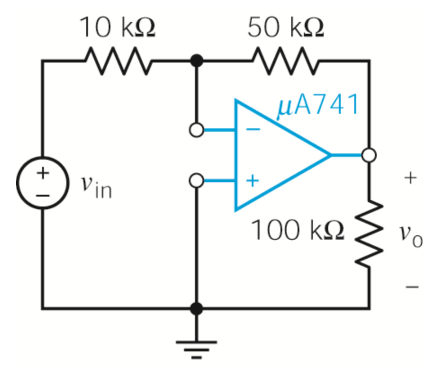
We require $v_3 = (6)v_1 - \left(\frac{4}{5}\right)v_2$, so

$$6 = 2\left(1 + \frac{R_1}{10\times 10^3}\right) \quad \Rightarrow \quad R_1 = 20\times 10^3 = 20\ \text{k}\Omega$$

and

$$\frac{4}{5} = \frac{R_2}{R_2 + 10\times 10^3} \quad \Rightarrow \quad 4\,R_2 + 40\times 10^3 = 5\,R_2 \quad \Rightarrow \quad R_2 = 40\ \text{k}\Omega$$

Exercise 6.7-1 The input offset voltage of a *typical* μA741 operational amplifier is 1 mV and the bias current is 80 nA. Suppose the operational amplifier in Figure 6.7-2*a* is a typical μA741. Show that the output offset voltage of the inverting amplifier will be at most 10 mV.



(*a*)

Figure 6.7-2*a*

Solution:

Analysis of the circuit in Section 6.7 showed that output offset voltage = $6\,v_{os} + (50\times10^3)\,i_{b1}$

For a μA741 op amp, $|v_{os}| \le 1$ mV and $|i_{b1}| \le 80$ nA so

$$|\text{output offset voltage}| = \left|6\,v_{os} + (50\times10^3)i_{b1}\right| \le 6\,(10^{-3})+(50.10^3)(80\times10^{-9}) = 10 \text{ mV}$$

Exercise 6.7-2 Suppose the 10-kΩ resistor in Figure 6.7-2*a* is changed to 2 kΩ and the 50-kΩ resistor is changed to 10 kΩ. (These changes will not change the gain of the inverting amplifier. It will still be – 5.) Show that the *maximum* output offset voltage is reduced to 35 mV. (Use i_b = 500 nA and v_{os} = 5 mV to calculate the *maximum* output offset voltage that could be caused by the μA741 amplifier.)

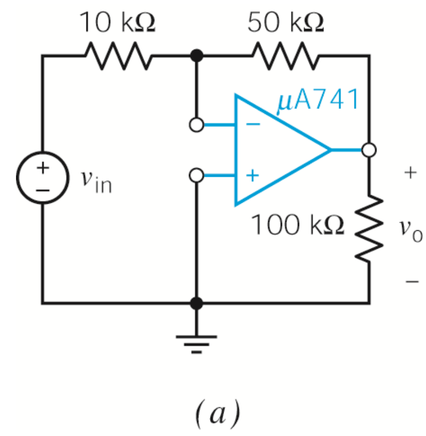
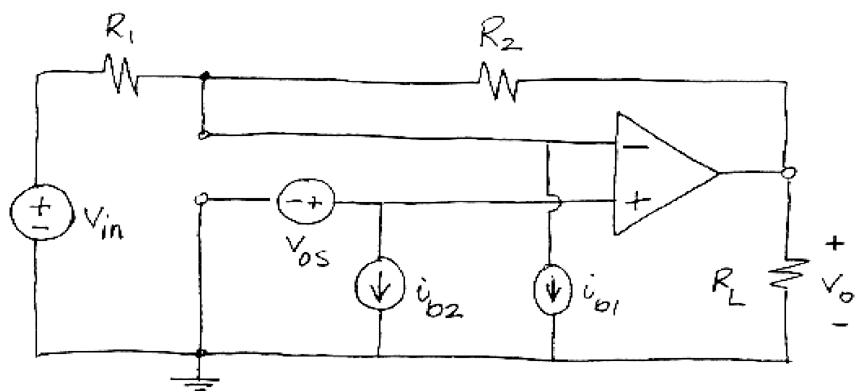


Figure 6.7-2*a*

Solution:

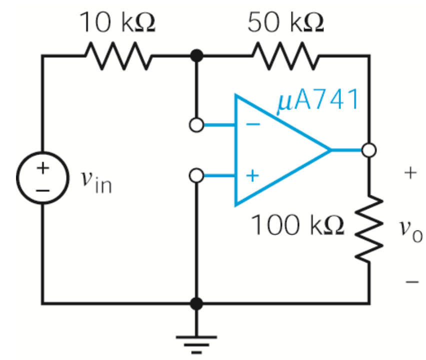


$$v_o = -\frac{R_2}{R_1} v_{in} + \left(1 + \frac{R_2}{R_1}\right) v_{os} + R_2 i_{b1}$$

When $R_2 = 10$ kΩ, $R_1 = 2$ kΩ, $|v_{os}| \le 5$ mV and $|i_{b1}| \le 500$ nA then

output offset voltage $\le 6\left(5\times10^{-3}\right) + \left(10\times10^{3}\right)\left(500.10^{-9}\right) \le 35\times10^{-3} = 35$ mV

Exercise 6.7-3 Suppose the μA741 operational amplifier in Figure 6.7-2*a* is replaced with a *typical* OPA101AM operational amplifier. Show that the output offset voltage of the inverting amplifier will be at most 0.6 mV.



(*a*)

Figure 6.7-2*a*

Solution:

Analysis of this circuit in Section 6.7 showed that output offset voltage = $6\, v_{os} + \left(50\times10^3\right) i_{b1}$

For a typical OPA1O1AM, $v_{os} = 0.1$ mV and $i_b = 0.012$ nA so

$$\left|\text{output offset voltage}\right| \le 6\left[0.1\times10^{-3}\right] + \left(50\times10^3\right)\left[0.012\times10^{-9}\right]$$
$$\le 0.6\times10^{-3} + 0.6\times10^{-6} \simeq 0.6\times10^{-3} = 0.6 \text{ mV}$$

Exercise 6.7-4

(a) Determine the voltage ratio v_o/v_s for the op amp circuit shown in Figure E 6.7-4

(b) Calculate v_o/v_s for a practical op amp with $A = 10^5$, $R_o = 100\ \Omega$, and $R_i = 500\ \text{k}\Omega$. The circuit resistors are $R_s = 10\ \text{k}\Omega$, $R_f = 50\ \text{k}\Omega$, and $R_a = 25\ \text{k}\Omega$.

Answer: (b) $v_o/v_s = -2$

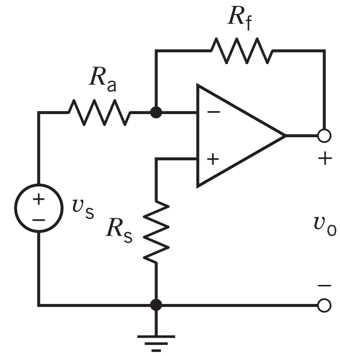
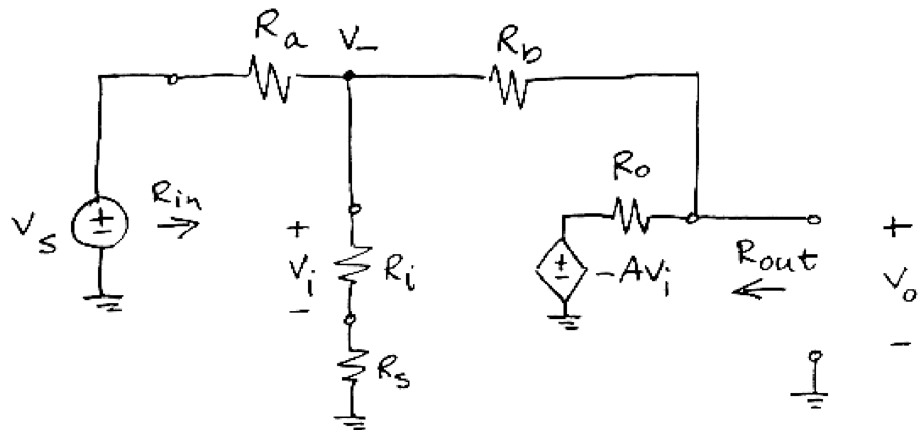


Figure E 6.7-4

Solution:



Writing node equations

$$\frac{v_- - v_s}{R_a} + \frac{v_- - v_o}{R_b} + \frac{v_-}{R_i + R_s} = 0$$

$$\frac{v_o - \left(-A \dfrac{R_i}{R_i + R_s} v_-\right)}{R_0} + \frac{v_o - v_-}{R_b} = 0$$

After some algebra

$$A_v = \frac{v_o}{v_s} = \frac{R_0\left(R_i + R_s\right) + AR_iR_f}{\left(R_f + R_0\right)\left(R_i + R_s\right) + R_a\left(R_f + R_0 + R_i + R_s\right) - AR_iR_a}$$

For the given values, $A_v = -2.00006$ V/V.

Section 6-3: The Ideal Operational Amplifier

P 6.3-1 Determine the value of voltage measured by the voltmeter in Figure P 6.3-1.

Answer: – 4 V

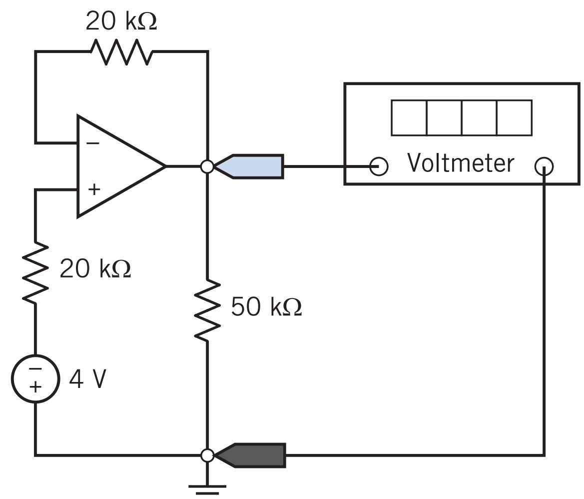
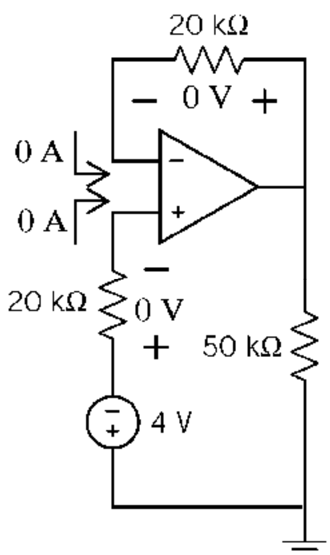
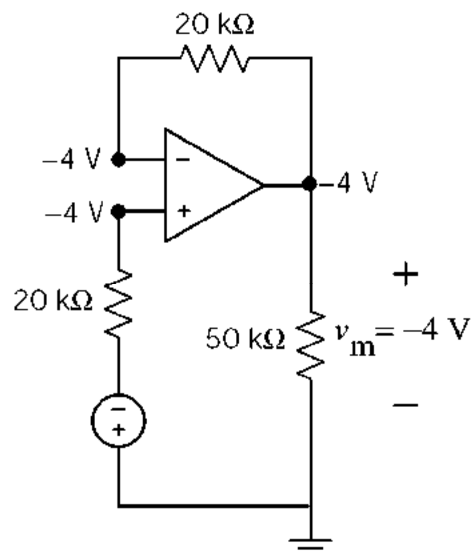


Figure P 6.3-1

Solution:





(checked using LNAP 8/16/02)

P 6.3-2 Find v_o and i_o for the circuit of Figure P 6.3-2.

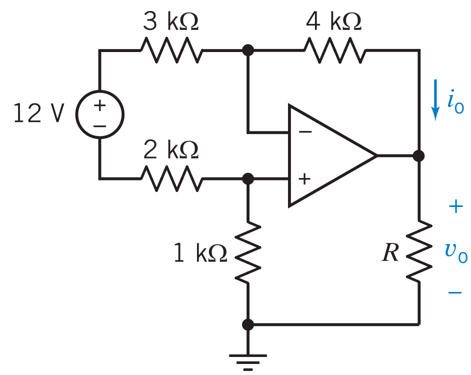
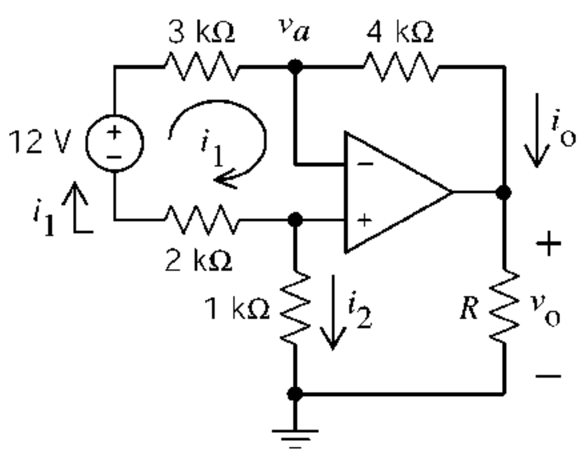


Figure P 6.3-2

Solution:



Apply KVL to loop 1:

$$-12 + 3000\, i_1 + 0 + 2000\, i_1 = 0$$

$$\Rightarrow i_1 = \frac{12}{5000} = 2.4 \text{ mA}$$

The currents into the inputs of an ideal op amp are zero so

$$i_o = i_1 = 2.4 \text{ mA}$$

$$i_2 = -i_1 = -2.4 \text{ mA}$$

$$v_a = i_2(1000) + 0 = -2.4 \text{ V}$$

Apply Ohm's law to the 4 kΩ resistor

$$v_o = v_a - i_o(4000)$$

$$= -2.4 - (2.4 \times 10^{-3})(4000) = -12 \text{ V}$$

(checked using LNAP 8/16/02)

P 6.3-3 Find v_o and i_o for the circuit of Figure P 6.3-3.
Answer: $v_o = -30$ V and $i_o = 3.5$ mA

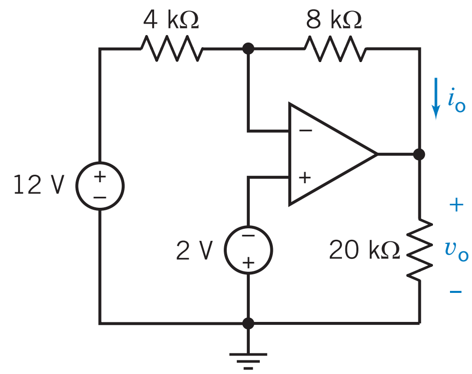
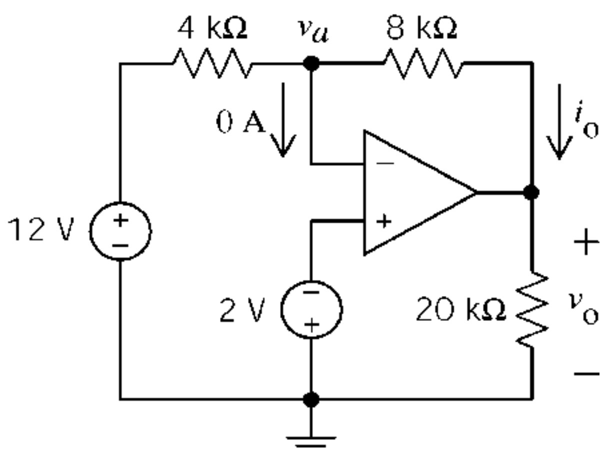


Figure P 6.3-3

Solution:



The voltages at the input nodes of an ideal op amp are equal so $v_a = -2$ V .

Apply KCL at node *a*:

$$\frac{v_o - (-2)}{8000} + \frac{12 - (-2)}{4000} = 0 \quad \Rightarrow v_o = -30 \text{ V}$$

Apply Ohm's law to the 8 kΩ resistor

$$i_o = \frac{-2 - v_o}{8000} = 3.5 \text{ mA}$$

(checked using LNAP 8/16/02)

P 6.3-4 Find v and i for the circuit of Figure P 6.3-4.

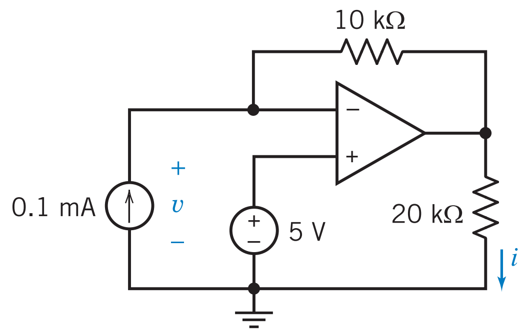
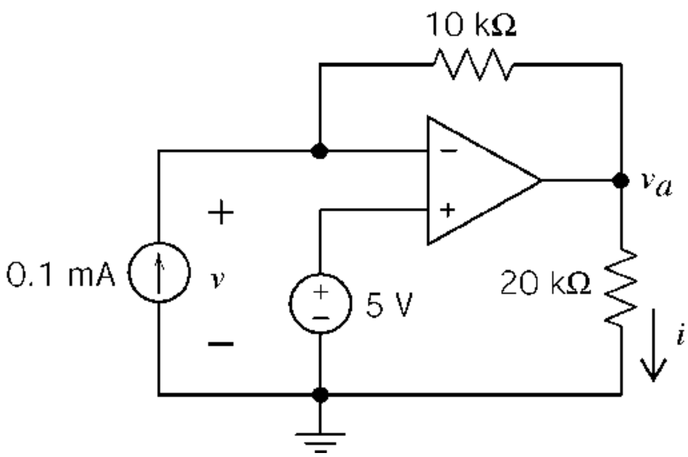


Figure P 6.3-4

Solution:



The voltages at the input nodes of an ideal op amp are equal so $v = 5\text{ V}$.

Apply KCL at the inverting input node of the op amp:

$$-\left(\frac{v_a - 5}{10000}\right) - 0.1\times10^{-3} - 0 = 0 \quad\Rightarrow\quad v_a = 4\text{ V}$$

Apply Ohm's law to the 20 kΩ resistor

$$i = \frac{v_a}{20000} = \frac{1}{5}\text{mA}$$

(checked using LNAP 8/16/02)

P 6.3-5 Find v_o and i_o for the circuit of Figure P 6.3-5.

Answer: $v_o = -15$ V and $i_o = 7.5$ mA

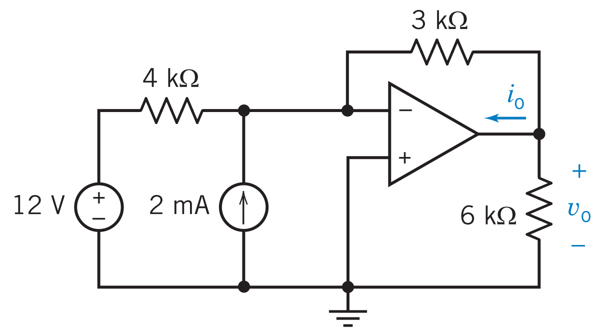


Figure P 6.3-5

Solution:

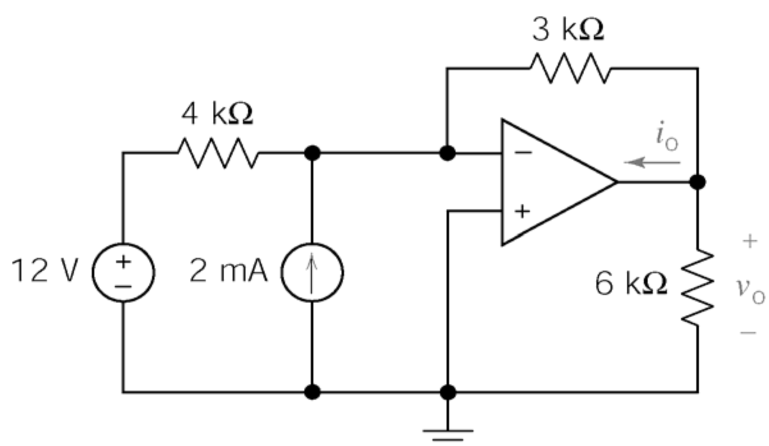
The voltages at the input nodes of an ideal op amp are equal, so $v_a = 0$ V. Apply KCL at node *a*:

$$-\left(\frac{v_o - 0}{3000}\right) - \left(\frac{12 - 0}{4000}\right) - 2 \cdot 10^{-3} = 0$$

$$\Rightarrow v_o \;=\; -15 \text{ V}$$

Apply KCL at the output node of the op amp:

$$i_o + \frac{v_o}{6000} + \frac{v_o}{3000} = 0 \Rightarrow i_o = 7.5 \text{ mA}$$



(checked using LNAP 8/16/02)

P 6.3-6 Determine the value of voltage measured by the voltmeter in Figure P 6.3-6.

Answer: 7.5 V

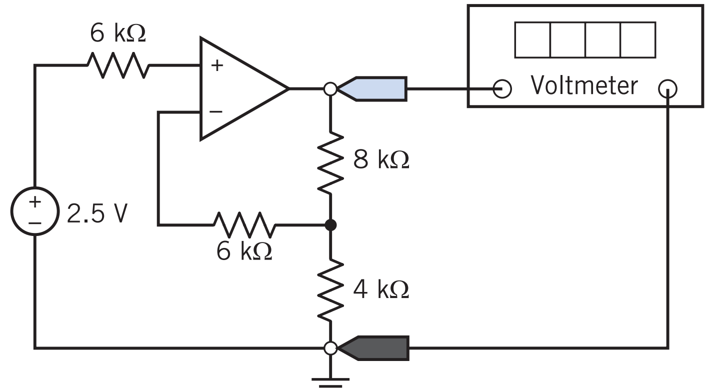


Figure P 6.3-6

Soluton:

The currents into the inputs of an ideal op amp are zero and the voltages at the input nodes of an ideal op amp are equal so $v_a = 2.5\text{ V}$.

Apply Ohm's law to the 4 kΩ resistor:

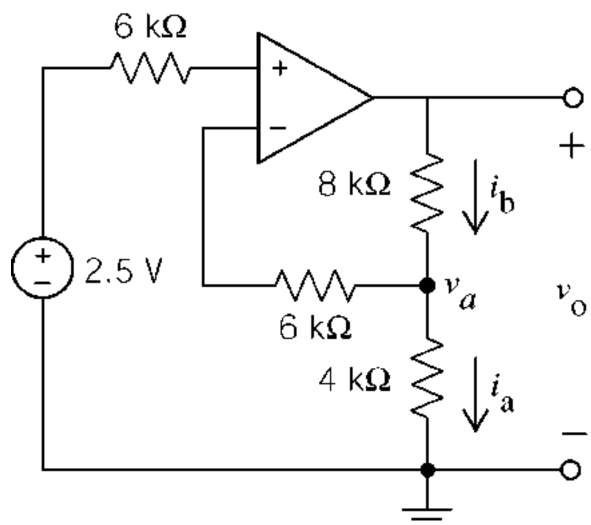
$$i_a = \frac{v_a}{4000} = \frac{2.5}{4000} = 0.625\text{ mA}$$

Apply KCL at node *a*:

$$i_b = i_a = 0.625\text{ mA}$$

Apply KVL:

$$v_o = 8000\, i_b + 4000\, i_a$$
$$= \left(12\times10^3\right)\left(0.625\times10^{-3}\right) = 7.5\text{ V}$$



(checked using LNAP 8/16/02)

P 6.3-7 Find v_o and i_o for the circuit of Figure P 6.3-7.

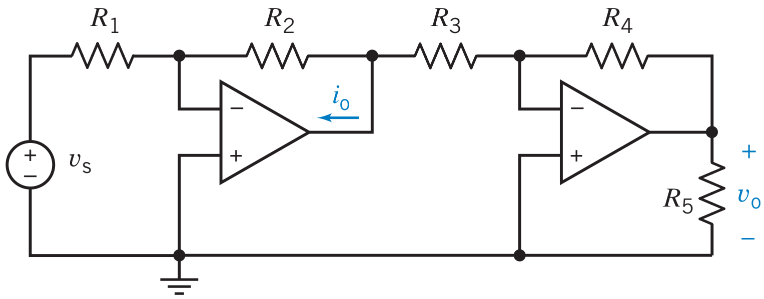
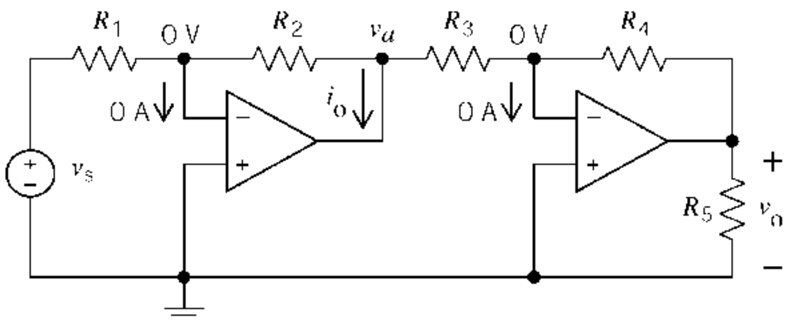


Figure P 6.3-7

Solution:

Label the circuit to account for the properties of the ideal op amps:



Apply KCL at the inverting node of the left op amp to get:

$$-\left(\frac{v_s - 0}{R_1}\right) - \left(\frac{v_a - 0}{R_2}\right) + 0 = 0 \;\Rightarrow\; v_a = -\frac{R_2}{R_1} v_s$$

Apply KCL at the output node of the left op amp to get:

$$i_o = \frac{0 - v_a}{R_2} + \frac{0 - v_a}{R_3} = -\frac{R_2 + R_3}{R_2 R_3} v_a = \left(\frac{R_2 + R_3}{R_1 R_3}\right) v_s$$

Apply KCL at the inverting node of the right op amp to get:

$$-\left(\frac{v_o - 0}{R_4}\right) - \left(\frac{v_a - 0}{R_3}\right) + 0 = 0 \;\Rightarrow\; v_o = -\frac{R_4}{R_3} v_a = \frac{R_2 R_4}{R_1 R_3} v_s$$

P 6.3-8 Determine the current i_o for the circuit shown in Figure P 6.3-8.

Answer: $i_o = 2.5$ mA

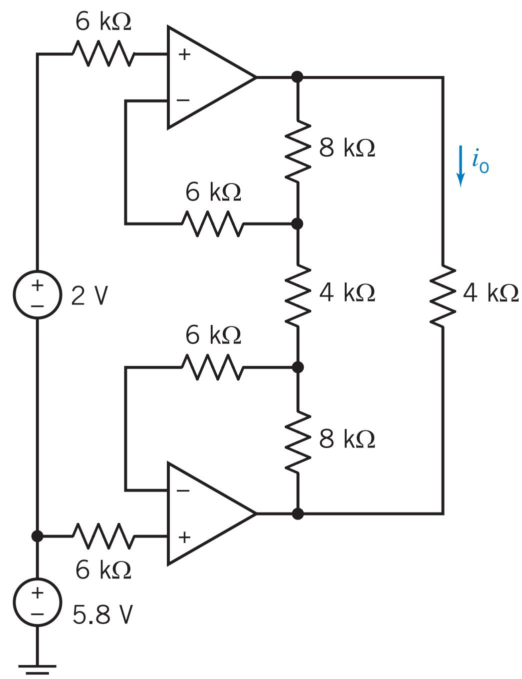


Figure P 6.3-8

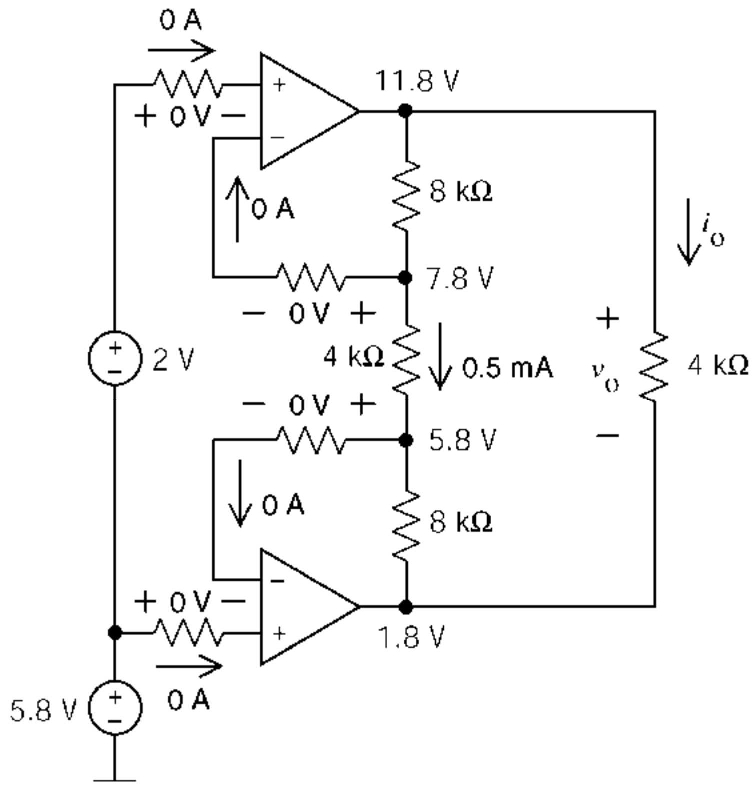
Solution:

The node voltages have been labeled using:

1. The currents into the inputs of an ideal op amp are zero and the voltages at the input nodes of an ideal op amp are equal.

2. KCL

3. Ohm's law



then $v_0 = 11.8 - 1.8 = 10$ V

and $i_o = \dfrac{10}{4000} = 2.5$ mA

(checked using LNAP 8/16/02)

P 6.3-9 Determine the current i_o for the circuit shown in Figure P 6.3-9.
Answer: i_o = 2.5 mA

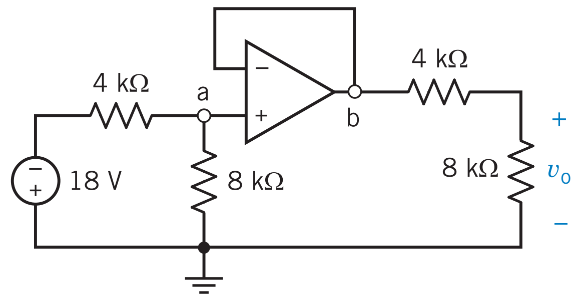


Figure P 6.3-9

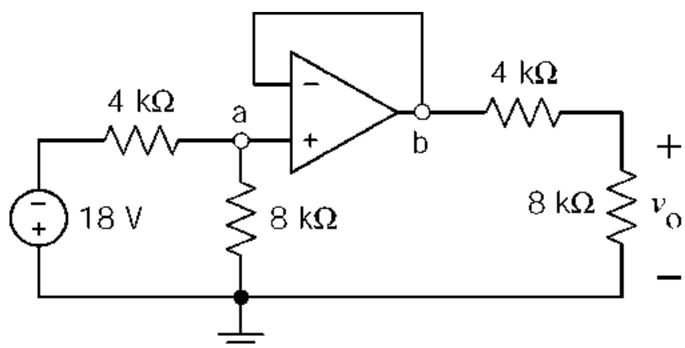
Solution:
Apply KCL at node a:

$$\frac{v_a - (-18)}{4000} + \frac{v_a}{8000} + 0 = 0 \quad \Rightarrow \quad v_a = -12 \text{ V}$$

The node voltages at the input nodes of ideal op amps are equal, so $v_b = v_a$.

Using voltage division:

$$v_o = \frac{8000}{4000 + 8000} v_b = -8 \text{ V}$$



(check using LNAP 8/16/02)

P 6.3-10 Determine the current i_o for the circuit shown in Figure P 6.3-10.

Answer: i_o = 2.5 mA

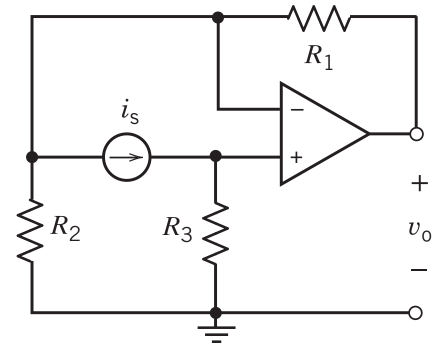
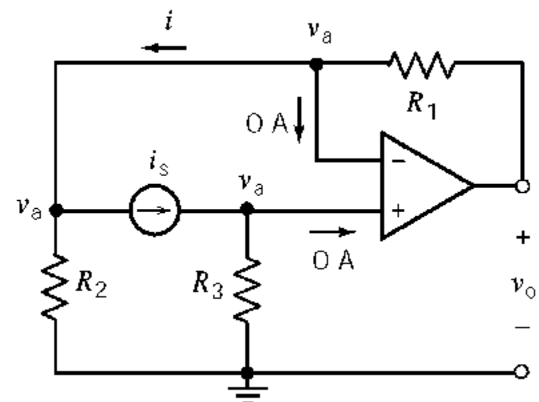


Figure P 6.3-10

Solution:

Label the circuit as shown. The current in resistor R_3 is i_s. Consequently:



$$v_a = i_s\ R_3$$

Apply KCL at the top node of R_2 to get

$$i = \frac{v_a}{R_2} + i_s = \left(1 + \frac{R_3}{R_2}\right) i_s$$

Using Ohm's law gives

$$\frac{v_o - v_a}{R_1} = i = \left(1 + \frac{R_3}{R_2}\right) i_s \quad \Rightarrow \quad v_o = \left(R_1 + R_3 + \frac{R_1\ R_3}{R_2}\right) i_s$$

We require

$$R_1 + R_3 + \frac{R_1\ R_3}{R_2} = 20$$

e.g. $R_1 = 5\ \text{k}\Omega$ and $R_2 = R_3 = 10\ \text{k}\Omega$.

(checked: LNAP 6/2/04)

P 6.3-11 Determine the voltage v_o for the circuit shown in Figure P 6.3-11.

Answer: $v_o = -8$ V

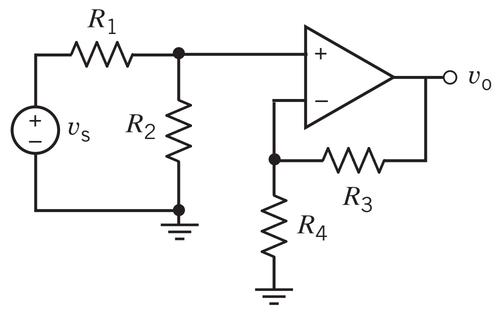
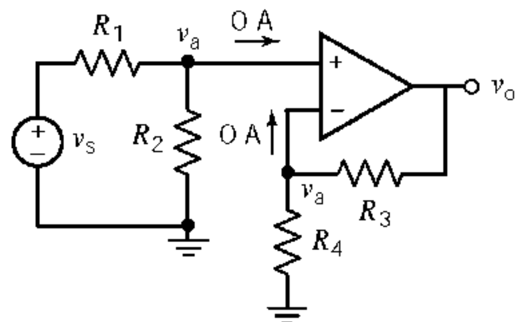


Figure P 6.3-11

Solution:
Label the circuit as shown. Apply KCL at the top node of R_2 to get



$$\frac{v_s - v_a}{R_1} = \frac{v_a}{R_2} + 0 \quad \Rightarrow \quad v_a = \left(\frac{R_2}{R_1 + R_2}\right) v_s$$

Apply KCL at the inverting node of the op amp to get

$$\frac{v_o - v_a}{R_3} = \frac{v_a}{R_4} + 0 \quad \Rightarrow \quad v_o = \left(\frac{R_3 + R_4}{R_4}\right) v_a = \left(\frac{R_3 + R_4}{R_4}\right)\left(\frac{R_2}{R_1 + R_2}\right) v_s = \frac{R_2 \left(R_3 + R_4\right)}{\left(R_1 + R_2\right) R_4} v_s$$

We require

$$\frac{R_2 \left(R_3 + R_4\right)}{\left(R_1 + R_2\right) R_4} = 5$$

e.g. $R_1 = R_2 = 10 \text{ k}\Omega, R_3 = 90 \text{ k}\Omega$ and $R_4 = 10 \text{ k}\Omega$.

(checked: LNAP 6/2/04)

P 6.3-12 The circuit shown in Figure P 6.3-12 has one input, i_s, and one output, v_o. Show that the output is proportional to the input. Design the circuit so that the gain is $\frac{v_o}{i_s} = 20\frac{\text{V}}{\text{mA}}$.

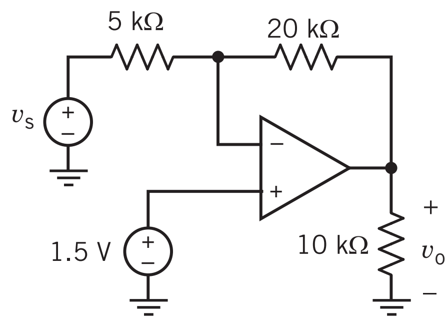
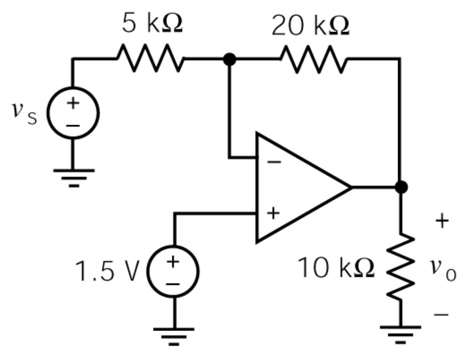


Figure P 6.3-12

Solution:

The node voltage at both input nodes of the op amp is 1.5 V. Apply KCL at the inverting input node of the op amp to get



$$\frac{v_s - 1.5}{5000} + \frac{v_o - 1.5}{20000} = 0 \quad \Rightarrow \quad 4(v_s - 1.5) + v_o - 1.5 = 0$$

$$\Rightarrow \quad v_o = -4(v_s - 1.5) + 1.5 = 0$$

$$\Rightarrow \quad v_o = -4v_s + 7.5$$

Comparing this equation to $v_o = m\, v_s + b$, we determine that m = –4 V/V and b = 75. V.

P 6.3-13 The circuit shown in Figure P 6.3-13 has one input, v_s, and one output, v_o. Show that the output is proportional to the input. Design the circuit so that $v_o = 5\, v_s$.

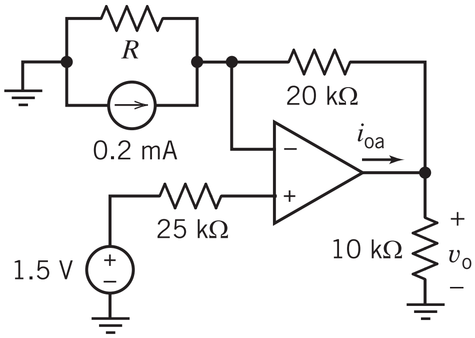
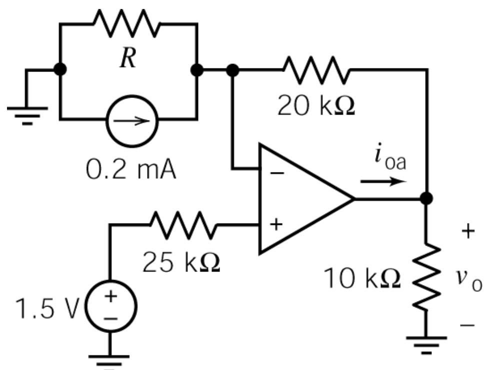


Figure P 6.3-13

Solution:

The output of this circuit is $v_o = 3.5$ V.

(a) The current in the 25 kΩ resistor is 0 A because this current is also the input current of an ideal op amp. Consequently, the voltage at the input nodes of the op amp is 1.5 V. Apply KCL at the inverting input of the op amp to get



$$\frac{1.5}{R} = 0.2 + \frac{3.5 - 1.5}{20} = 0.3$$

so

$$R = \frac{1.5}{0.3} = 5 \text{ k}\Omega$$

(b) The voltage source current is 0A so the voltage source supplies 0 W of power. The voltage across the current source is equal to the node voltage at the inverting input of the op amp, 1.5 V. Notice that this voltage and the given current source current do not adhere to the passive convention so (0.2)(1.5) = 0.3 mW is the power **supplied** by the current source.

(c) Apply KCL at the output node of the op amp to get

$$i_{oa} = \frac{3.5}{10} + \frac{3.5 - 1.5}{20} = 0.45 \text{ A}$$

The op amp supplies $p_{oa} = i_{oa} \times v_o = (0.45)(3.5) = 1.575$ W

P 6-3.14 Determine the values of the node voltages at nodes a, b, c, and d of the circuit shown in Figure P 6-3.14.

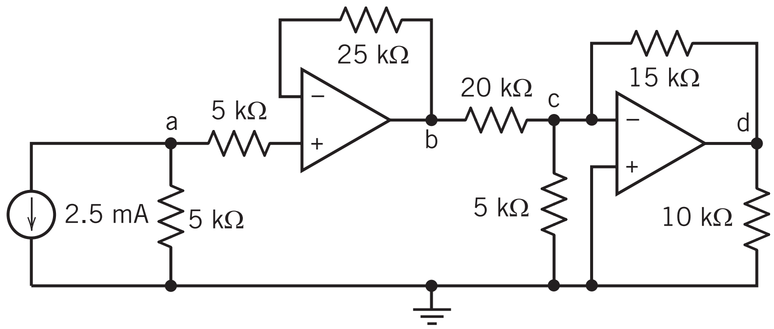
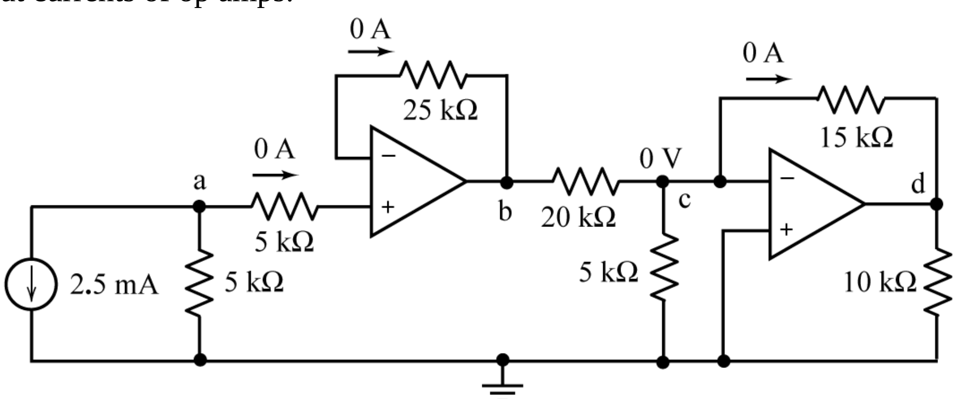


Figure P 6.3-14

Solution:
Label the circuit as shown, noticing that some resistor currents are zero because these currents are also input currents of op amps:



Now $v_a = -2.5(5) = -12.5\text{ V}$, $v_b = v_a = -12.5\text{ V}$, $v_c = 0\text{ V}$

Apply KCL to get

$$\frac{v_b - 0}{20} = 0 + \frac{0 - v_d}{15} \Rightarrow v_d = -\frac{15}{20}v_b = -\frac{3}{4}(-12.5) = 9.375\text{ V}$$

P 6-3.15 Determine the values of the node voltages at nodes a, b, c, and d of the circuit shown in Figure P 6-3.15.

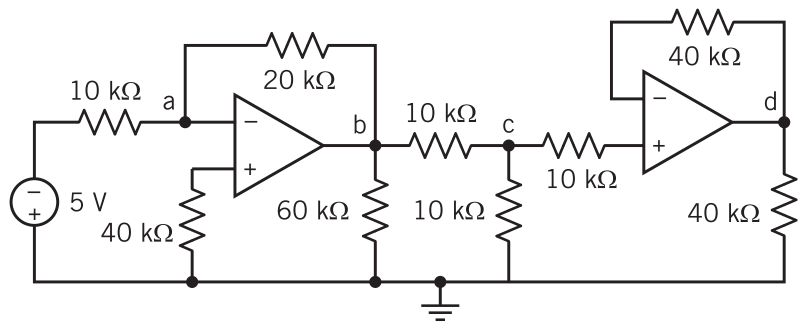
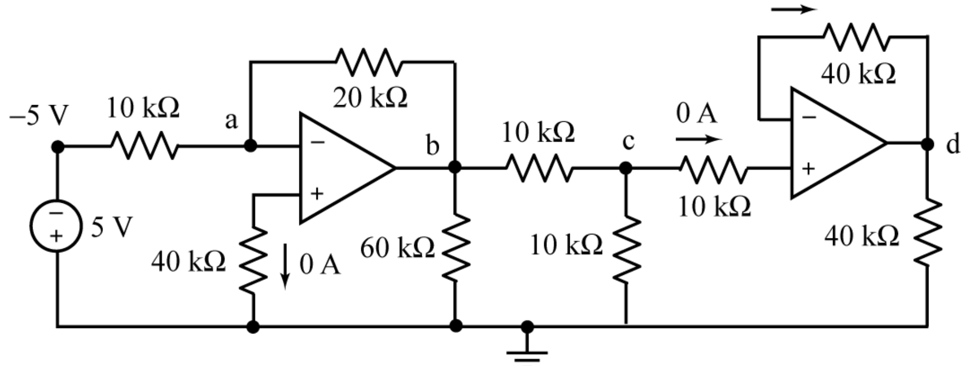


Figure P 6.3-15

Solution:
Label the circuit as shown, noticing that some resistor currents are zero because these currents are also input currents of op amps:



Now $v_a = 0$ V, $v_b = -\left(\frac{20}{10}\right)(-5) = 10$ V, $v_c = \frac{v_b}{2} = 5$ V and $v_d = v_c = 5$ V.

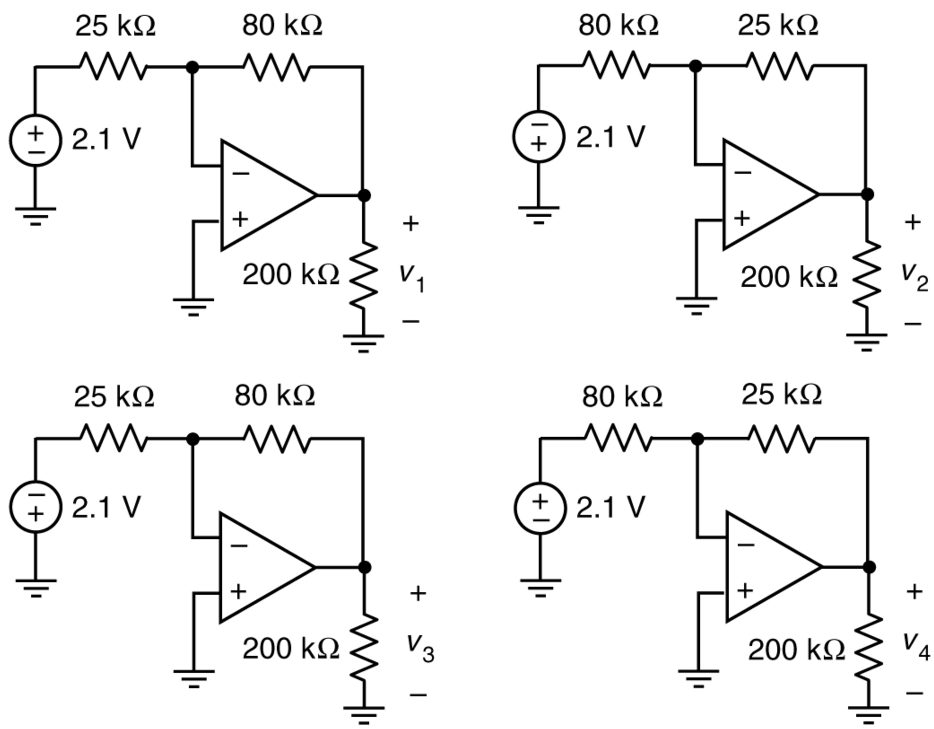


Figure P6.3-16

P6.3-16 . Figure P6.3-16 shows four similar circuits. The outputs of the circuits are the voltages v_1, v_2, v_3 and v_4. Determine the values of these four outputs.

Solution:

$v_1 = -\frac{80}{25}(2.1) = -6.72 \text{ V}$, $v_2 = -\frac{25}{80}(-2.1) = 0.65625 \text{ V}$, $v_3 = -\frac{80}{25}(-2.1) = 6.72 \text{ V}$ and

$v_4 = -\frac{25}{80}(2.1) = -0.65625 \text{ V}$

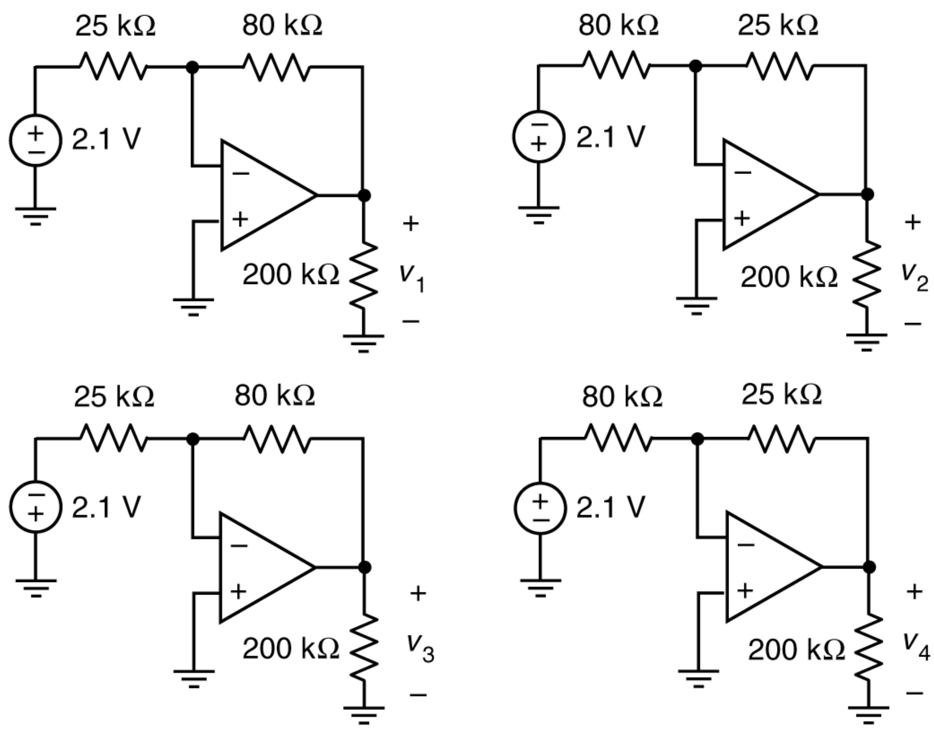


Figure P6.3-17

P6.3-17 . Figure P6.3-17 shows four similar circuits. The outputs of the circuits are the voltages v_1, v_2, v_3 and v_4. Determine the values of these four outputs.

Solution

$v_1 = \left(1+\frac{80}{25}\right)(2.1) = 8.82 \text{ V}$, $v_2 = \left(1+\frac{25}{80}\right)(-2.1) = -2.75625 \text{ V}$, $v_3 = \left(1+\frac{80}{25}\right)(-2.1) = -8.82 \text{ V}$ and $v_4 = \left(1+\frac{25}{80}\right)(2.1) = 2.75625 \text{ V}$.

Section 6-4: Nodal Analysis of Circuits Containing Ideal Operational Amplifiers

P 6.4-1 Determine the node voltages for the circuit shown in Figure P 6.4-1.

Answer: $v_a = 2$ V, $v_b = -0.25$ V, $v_c = -5$ V, $v_d = -2.5$ V, and $v_e = -0.25$ V

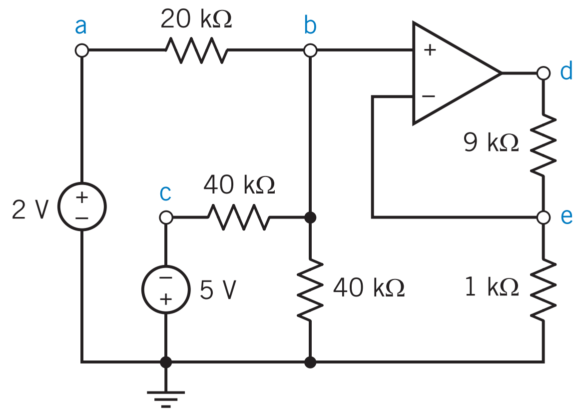
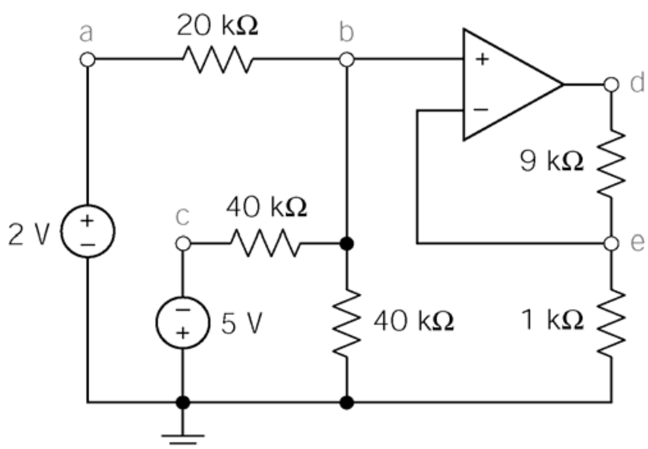


Figure P 6.4-1

Solution:



KCL at node b: $\dfrac{v_b - 2}{20000} + \dfrac{v_b}{40000} + \dfrac{v_b + 5}{40000} = 0 \quad \Rightarrow \quad v_b = -\dfrac{1}{4}$ V

The node voltages at the input nodes of an ideal op amp are equal so $v_e = v_b = -\dfrac{1}{4}$ V.

KCL at node e: $\dfrac{v_e}{1000} + \dfrac{v_e - v_d}{9000} = 0 \quad \Rightarrow \quad v_d = 10\, v_e = -\dfrac{10}{4}$ V

(checked using LNAP 8/16/02)

P 6.4-2 Find v_o and i_o for the circuit of Figure P 6.4-2.

Answer: $v_o = -4$ V and $i_o = 1.33$ mA

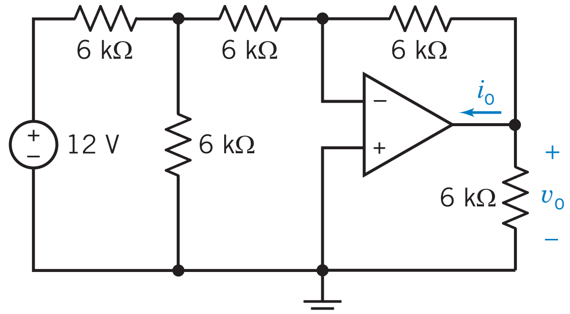
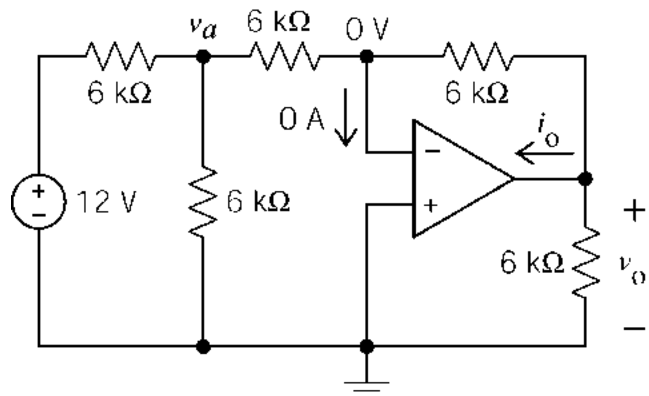


Figure P 6.4-2

Solution:

Apply KCL at node *a*:

$$0 = \frac{v_a - 12}{6000} + \frac{v_a}{6000} + \frac{v_a - 0}{6000} \quad \Rightarrow \quad v_a = 4 \text{ V}$$



Apply KCL at the inverting input of the op amp:

$$-\left(\frac{v_a - 0}{6000}\right) + 0 + \left(\frac{0 - v_o}{6000}\right) = 0$$

$$\Rightarrow \quad v_o = -v_a = -4 \text{ V}$$

Apply KCL at the output of the op amp:

$$i_o - \left(\frac{0 - v_o}{6000}\right) + \frac{v_o}{6000} = 0$$

$$\Rightarrow \quad i_o = -\frac{v_o}{3000} = 1.33 \text{ mA}$$

(checked using LNAP 8/16/02)

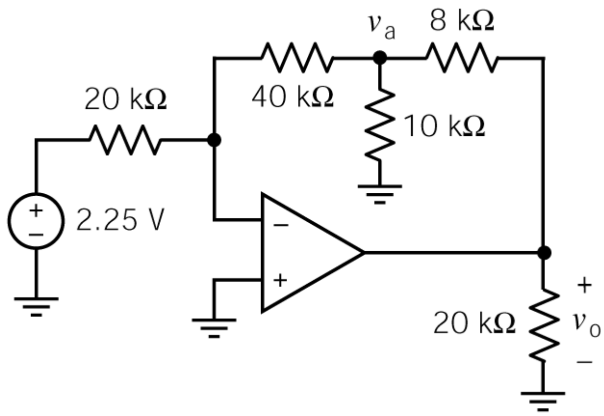
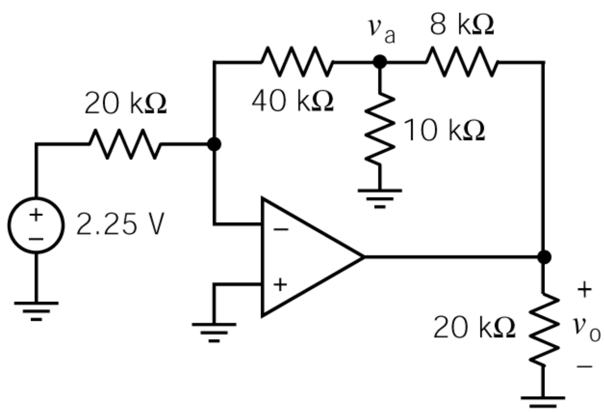


Figure P6.4-3

P6.4-3.

Determine the values of the node voltages, v_a and v_o, of the circuit shown in Figure P6.4-3.

Solution:



Writing node equations:

$$\frac{2.25}{20\times10^3}+\frac{v_a}{40\times10^3}=0 \quad\Rightarrow\quad v_a=-\left(40\times10^3\right)\frac{2.25}{20\times10^3}=-4.5\text{ V}$$

and

$$\frac{v_a}{40\times10^3}+\frac{v_a}{10\times10^3}+\frac{v_a-v_o}{8\times10^3}=0 \quad\Rightarrow\quad v_o=\left(\frac{8}{40}+\frac{8}{10}+\frac{8}{8}\right)v_a=2(-4.5)=-9\text{ V}$$

P 6.4-4 The output of the circuit shown in Figure P 6.4-4 is v_o. The inputs are v_1 and v_2. Express the output as a function of the inputs and the resistor resistances.

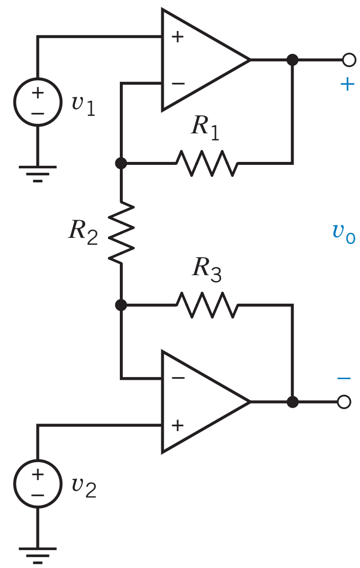
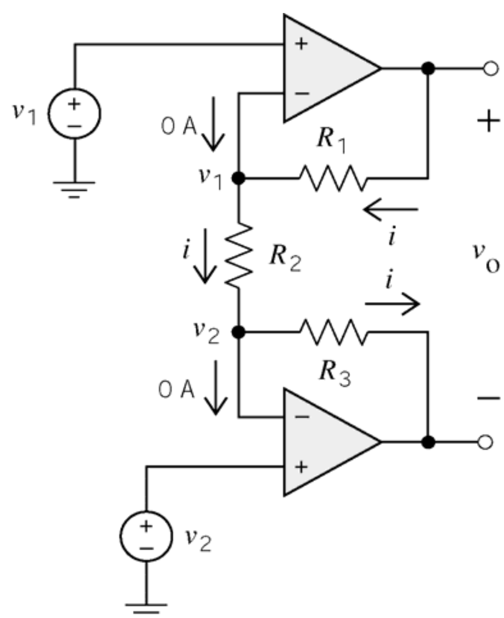


Figure P 6.4-4

Solution:



Ohm's law:

$$i = \frac{v_1 - v_2}{R_2}$$

KVL:

$$v_0 = \left(R_1 + R_2 + R_3\right) i = \frac{R_1 + R_2 + R_3}{R_2}\left(v_1 - v_2\right)$$

P 6.4-5 The outputs of the circuit shown in Figure P 6.4-5 are v_o and i_o. The inputs are v_1 and v_2. Express the outputs as functions of the inputs and the resistor resistances.

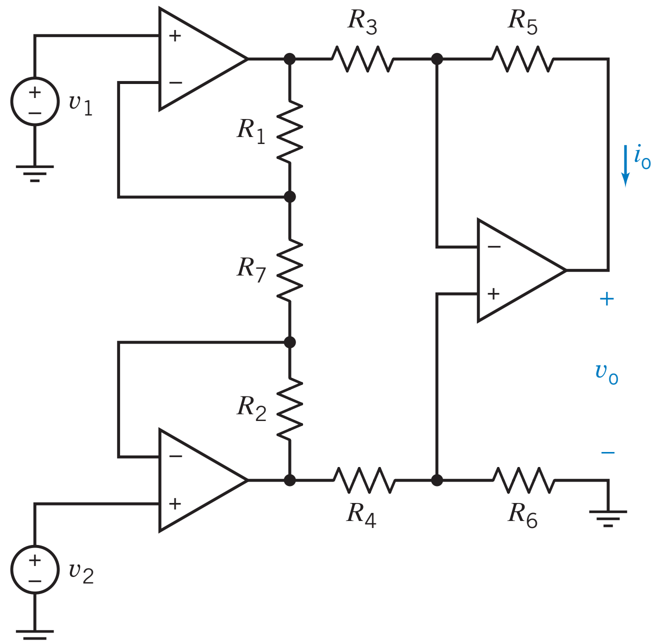
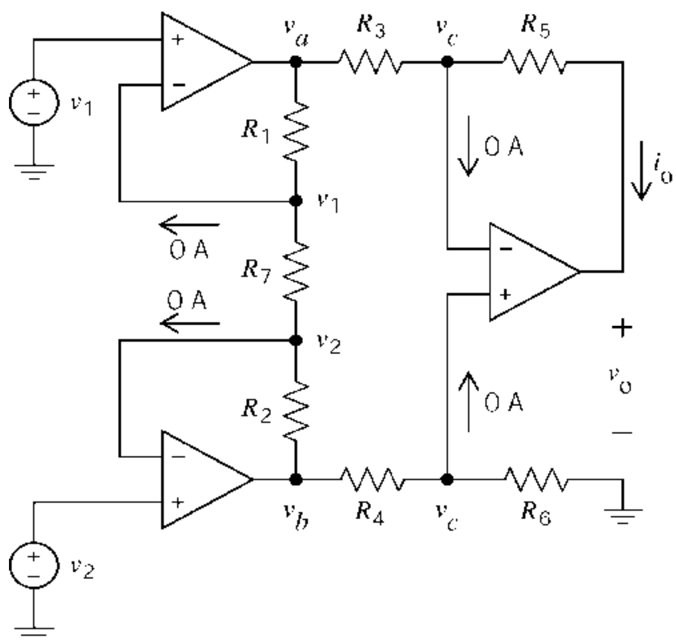


Figure P 6.4-5

Solution:



$$\frac{v_1 - v_a}{R_1} + \frac{v_1 - v_2}{R_7} + 0 = 0 \quad \Rightarrow \quad v_a = \left(1 + \frac{R_1}{R_7}\right) v_1 - \frac{R_1}{R_7} v_2$$

$$\frac{v_2 - v_b}{R_2} - \frac{v_1 - v_2}{R_7} + 0 = 0 \quad \Rightarrow v_b = \left(1 + \frac{R_2}{R_7}\right) v_2 - \frac{R_2}{R_7} v_1$$

$$-\left(\frac{v_b - v_c}{R_4}\right) + \frac{v_c - 0}{R_6} + 0 = 0 \Rightarrow v_c = \frac{R_6}{R_4 + R_6} v_b$$

$$-\left(\frac{v_a - v_c}{R_3}\right) + \left(\frac{v_c - v_0}{R_5}\right) + 0 = 0 \quad \Rightarrow v_0 = -\frac{R_5}{R_3} v_a + (1 + \frac{R_5}{R_3}) v_c$$

$$v_0 = \left[\frac{R_5 R_1}{R_3 R_7} + \frac{R_6\left(R_3 + R_5\right)}{R_3\left(R_4 + R_6\right)}(1 + \frac{R_2}{R_7})\right] v_2 - \left[\frac{R_5}{R_3}(1 + \frac{R_1}{R_7}) + \frac{R_6\left(R_3 + R_5\right) R_2}{R_3\left(R_4 + R_6\right) R_7}\right] v_1$$

$$i_0 = \frac{v_c - v_0}{R5} = \cdots$$

P 6.4-6 Determine the node voltages for the circuit shown in Figure P 6.4-6.
Answer: v_a = – 0.75 V, v_b = 0 V, and v_c = – 0.9375 V

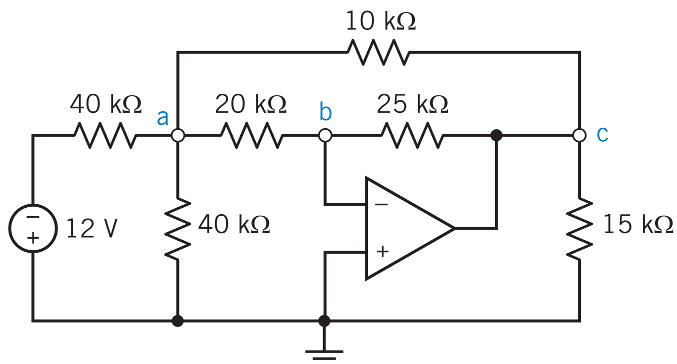
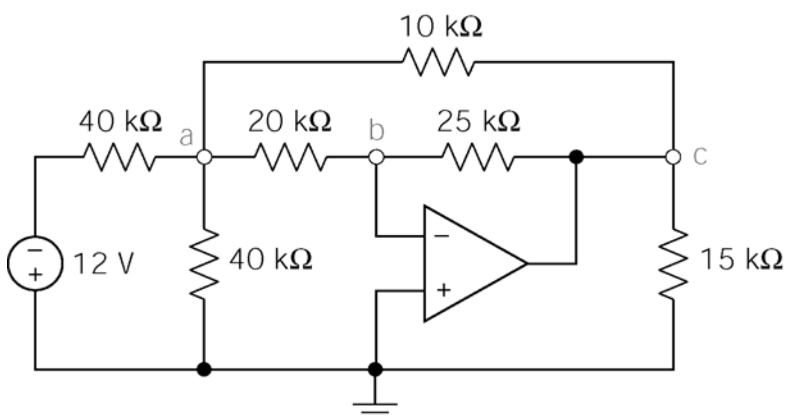


Figure P 6.4-6

Solution:



KCL at node b:

$$\frac{v_a}{20\times10^3}+\frac{v_c}{25\times10^3}=0 \quad\Rightarrow\quad v_c=-\frac{5}{4}v_a$$

KCL at node a:

$$\frac{v_a-(-12)}{40\times10^3}+\frac{v_a}{40\times10^3}+\frac{v_a+0}{20\times10^3}+\frac{v_a-\left(-\frac{5}{4}v_a\right)}{10\times10^3}=0 \quad\Rightarrow\quad v_a=-\frac{12}{13}\text{ V}$$

so

$$v_c=-\frac{5}{4}v_a=-\frac{15}{13}$$

(checked using LNAP 6/21/05)

P 6.4-7 Find v_o and i_o for the circuit shown in Figure P 6.4-7. Assume an ideal operational amplifier.

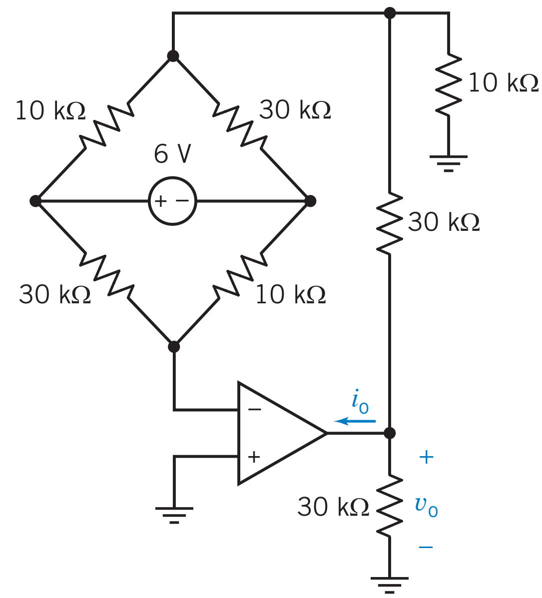
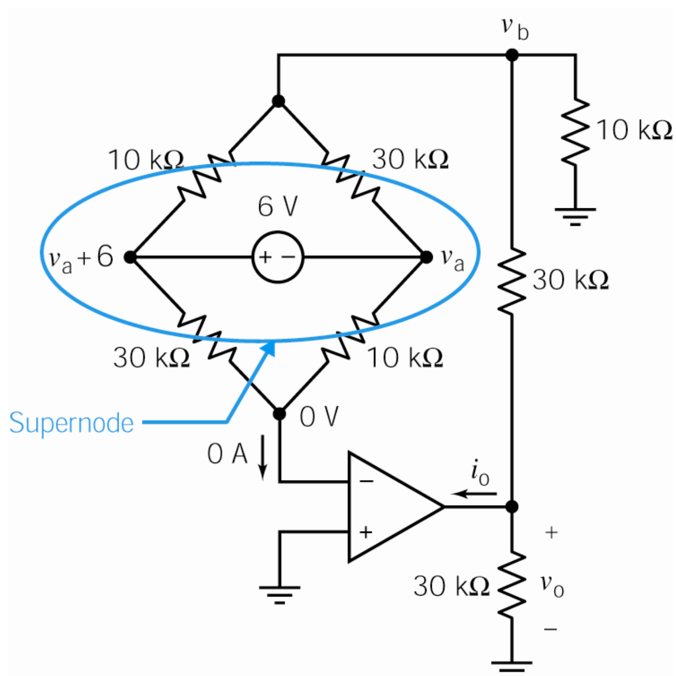


Figure P 6.4-7

Solution:

Label the circuit to account for the properties of the ideal op amp and to identify the supernode corresponding to the voltage source.



Apply KCL at the inverting input node of the op amp

$$-\left(\frac{v_a-0}{10000}\right)+0-\left(\frac{(v_a+6)-0}{30000}\right) = 0$$

or

$$v_a = -1.5 \text{ V}$$

Apply KCL to the super node corresponding the voltage source:

$$\frac{v_a-0}{10000}+\frac{v_a+6-0}{30000}+\frac{v_a-v_b}{30000}+\frac{(v_a+6)-v_b}{10000} = 0$$

Multiply both sides by 30000 to get

$$3v_a+(v_a+6)+(v_a-v_b)+3\left[(v_a+6)-v_b\right]=0$$

Solving gives

$$v_b = 2v_a + 6 = 3 \text{ V}$$

Apply KCL at node *b*:

$$\frac{v_b}{10000} + \frac{v_b - v_o}{30000} - \left(\frac{v_a - v_b}{30000}\right) - \left(\frac{(v_a + 6) - v_b}{10000}\right) = 0$$

Multiply both sides by 30000 to get

$$3v_b + (v_b - v_o) - (v_a - v_b) - 3\left[(v_a + 6) - v_b\right] = 0$$

Solving gives

$$v_o = 8v_b - 4v_a - 18 = 12 \text{ V}$$

Apply KCL at the output node of the op amp:

$$i_o + \frac{v_o}{30000} + \frac{v_o - v_b}{30000} = 0 \quad \Rightarrow \quad i_o = -0.7 \text{ mA}$$

P 6.4-8 Find v_o and i_o for the circuit shown in Figure P 6.4-8. Assume an ideal operational amplifier.

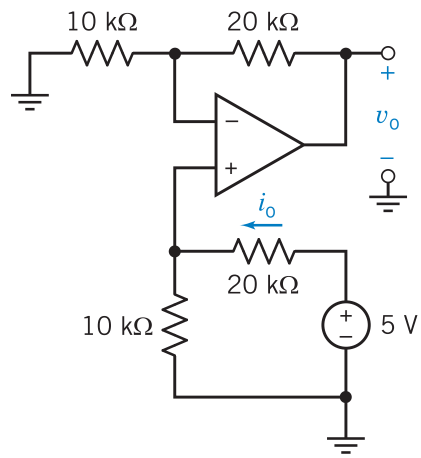


Figure P 6.4-8

Solution:

Apply KVL to the bottom mesh:

$$-i_0(10000) - i_0(20000) + 5 = 0$$

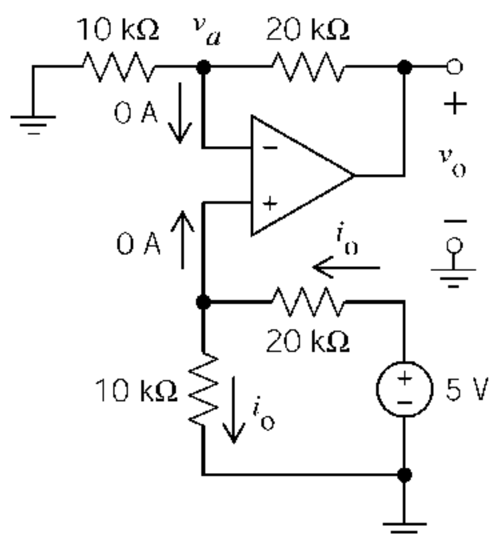
$$\Rightarrow \quad i_0 = \frac{1}{6} \text{ mA}$$

The node voltages at the input nodes of an ideal op amp are equal. Consequently

$$v_a = 10000\, i_0 = \frac{10}{6} \text{ V}$$

Apply KCL at node *a*:

$$\frac{v_a}{10000} + \frac{v_a - v_0}{20000} = 0 \quad \Rightarrow \quad v_0 = 3v_a = 5 \text{ V}$$



P 6.4-9 Determine the node voltages for the circuit shown in Figure P 6.4-9.

Answer: $v_a = -12$ V, $v_b = -4$ V, $v_c = -4$ V, $v_d = -4$ V, $v_e = -3.2$ V, $v_f = -4.8$ V, and $v_g = -3.2$

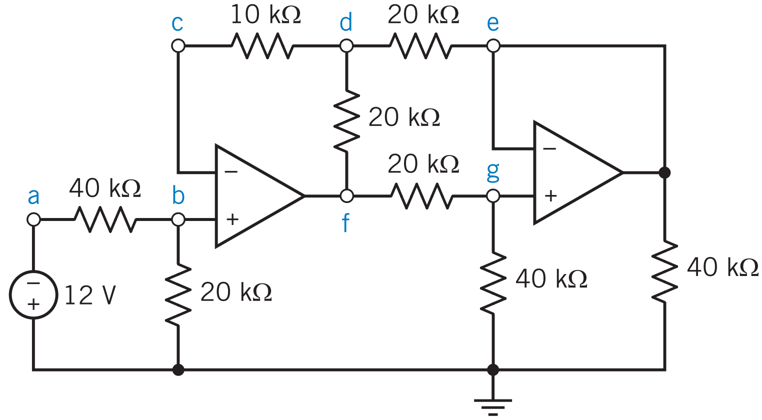
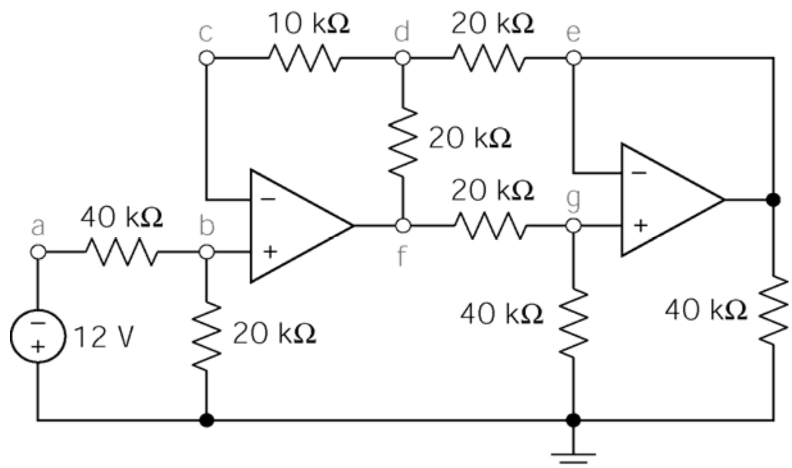


Figure P 6.4-9

Solution:



KCL at node b: $$\frac{v_b + 12}{40000} + \frac{v_b}{20000} = 0 \quad \Rightarrow \quad v_b = -4 \text{ V}$$

The node voltages at the input nodes of an ideal op amp are equal, so $v_c = v_b = -4$ V.

The input currents of an ideal op amp are zero, so $v_d = v_c + 0 \times 10^4 = -4$ V.

KCL at node g: $$-\left(\frac{v_f - v_g}{20 \times 10^3}\right) + \frac{v_g}{40 \times 10^3} = 0 \quad \Rightarrow \quad v_g = \frac{2}{3} v_f$$

The node voltages at the input nodes of an ideal op amp are equal, so $v_e = v_g = \frac{2}{3} v_f$.

KCL at node d: $$0 = \frac{v_d - v_f}{20 \times 10^3} + \frac{v_d - v_e}{20 \times 10^3} = \frac{v_d - v_f}{20 \times 10^3} + \frac{v_d - \frac{2}{3} v_f}{20 \times 10^3} \quad \Rightarrow \quad v_f = \frac{6}{5} v_d = -\frac{24}{5} \text{ V}$$

Finally, $v_e = v_g = \frac{2}{3} v_f = -\frac{16}{5}$ V.

P 6.4-10 The circuit shown in Figure P 6.4-10 includes a simple strain gauge. The resistor R changes its value by ΔR when it is twisted or bent. Derive a relation for the voltage gain v_o/v_s and show that it is proportional to the fractional change in R, namely $\Delta R/R_o$.

Answer: $v_o = \dfrac{R_o}{R_o + R_1}\dfrac{\Delta R}{R_o}$

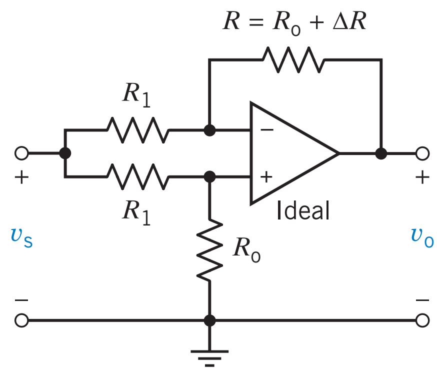


Figure P 6.4-10

Solution:

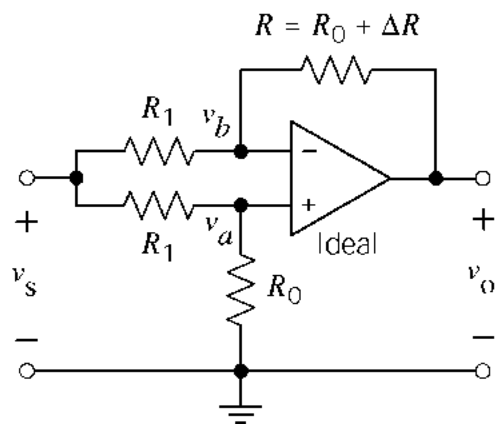
By voltage division (or by applying KCL at node a)

$$v_a = \frac{R_0}{R_1+R_0} v_s$$

Applying KCL at node b:

$$\frac{v_b - v_s}{R_1} + \frac{v_b - v_0}{R_0+\Delta R} = 0$$

$$\Rightarrow \quad \frac{R_0+\Delta R}{R_1}(v_b - v_s) + v_b = v_0$$



The node voltages at the input nodes of an ideal op amp are equal so $v_b = v_a$.

$$v_0 = \left[\left(\frac{R_0+\Delta R}{R_1}+1\right)\frac{R_0}{R_1+R_0} - \frac{R_0+\Delta R}{R_1}\right]v_s = -\frac{\Delta R}{R_1+R_0}v_s = \left(-v_s\frac{R_0}{R_1+R_0}\right)\frac{\Delta R}{R_0}$$

P 6.4-11 Find v_o for the circuit shown in Figure P 6.4-11. Assume an ideal operational amplifier.

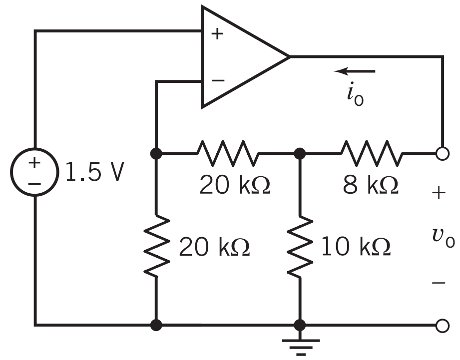
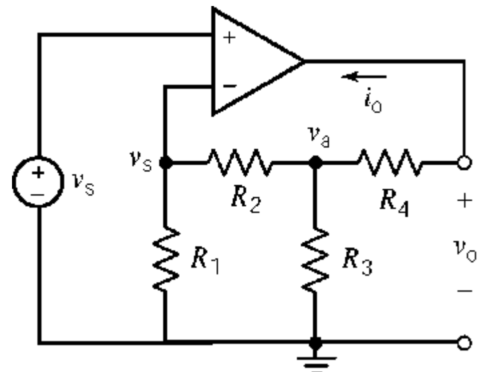


Figure P 6.4-11

Solution:
Node equations:

$$\frac{v_s}{R_1}+\frac{v_s-v_a}{R_2}=0 \quad\Rightarrow\quad v_a=\left(1+\frac{R_2}{R_1}\right)v_s$$



and

$$\frac{v_s-v_a}{R_2}=\frac{v_a}{R_3}+\frac{v_a-v_o}{R_4}$$

so

$$v_o=\left(1+\frac{R_4}{R_2}+\frac{R_4}{R_3}\right)v_a-\frac{R_4}{R_2}v_s$$

$$v_o=\left(1+\frac{R_4}{R_2}+\frac{R_4}{R_3}\right)\left(1+\frac{R_2}{R_1}\right)v_s-\frac{R_4}{R_2}v_s=\left(1+\frac{R_4}{R_3}+\frac{R_4}{R_1}+\frac{R_2}{R_1}+\frac{R_2\,R_4}{R_1\,R_3}\right)v_s$$

$$=\left(\frac{\left(R_1+R_2\right)\left(R_3+R_4\right)+R_3\,R_4}{R_1\,R_3}\right)v_s$$

with the given values:

$$v_o=\left(\frac{\left(20+20\right)\left(10+8\right)+10\times 8}{20\times 10}\right)v_s=\left(\frac{40\times 18+80}{200}\right)v_s=\left(4\right)v_s$$

(checked: LNAP 5/24/04)

P 6.4-12 The circuit shown in Figure P 6.4-12 has one output, v_o, and two inputs, v_1 and v_2. Show that when $\frac{R_3}{R_4} = \frac{R_6}{R_5}$, the output is proportional to the difference of the inputs, $v_1 - v_2$. Specify resistance values to cause $v_o = 5\,(v_1 - v_2)$.

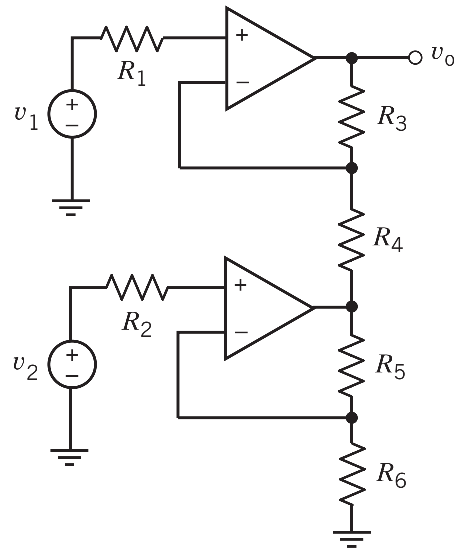
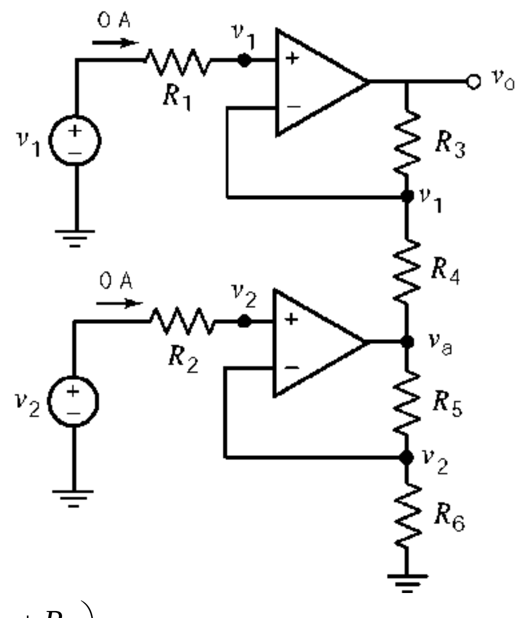


Figure P 6.4-12

Solution:

Notice that the currents in resistance R_1 and R_2 are both zero, as shown. Consequently, the voltages at the noninverting inputs of the op amps are v_1 and v_2, as shown. The voltages at the inverting inputs of the ideal op amps are also v_1 and v_2, as shown.



Apply KCL at the top node of R_6 to get

$$\frac{v_a - v_2}{R_5} = \frac{v_2}{R_6} \quad \Rightarrow \quad v_a = \left(\frac{R_5 + R_6}{R_6}\right) v_2$$

Apply KCL at the top node of R_4 to get

$$\frac{v_o - v_1}{R_3} = \frac{v_1 - v_a}{R_4} \quad \Rightarrow \quad v_o = \left(1 + \frac{R_3}{R_4}\right) v_1 - \left(\frac{R_3}{R_4}\right) v_a$$

$$v_o = \left(1 + \frac{R_3}{R_4}\right) v_1 - \left(\frac{R_3}{R_4}\right)\left(\frac{R_5 + R_6}{R_6}\right) v_2$$

When $\frac{R_3}{R_4} = \frac{R_6}{R_5}$

$$v_o = \left(1 + \frac{R_3}{R_4}\right) v_1 - \left(\frac{R_3}{R_4}\right)\left(1 + \frac{R_5}{R_6}\right) v_2 = \left(1 + \frac{R_3}{R_4}\right) v_1 - \left(\frac{R_3}{R_4}\right)\left(1 + \frac{R_4}{R_3}\right) v_2$$

$$= \left(1 + \frac{R_3}{R_4}\right)(v_1 - v_2)$$

so v_o is proportional to the difference of the inputs, $v_1 - v_2$, as required.

Next, to make $v_o = 5\left(v_1 - v_2\right)$, we choose R_3 and R_4 so that $5 = 1 + \frac{R_3}{R_4}$, e.g. $R_3 = 40$ kΩ and $R_4 = 10$ kΩ. Then with $\frac{R_3}{R_4} = \frac{R_6}{R_5}$ we have

$$R_1 = 50\text{ k}\Omega,\ R_2 = 50\text{ k}\Omega,\ R_3 = 40\text{ k}\Omega,\ R_4 = 10\text{ k}\Omega,\ R_5 = 10\text{ k}\Omega \text{ and } R_6 = 40\text{ k}\Omega.$$

(checked: LNAP 5/24/04)

P 6.4-13 The circuit shown in Figure P 6.4-13 has one output, v_o, and one input, v_i. Show that the output is proportional to the input. Specify resistance values to cause $v_o = 20\, v_i$.

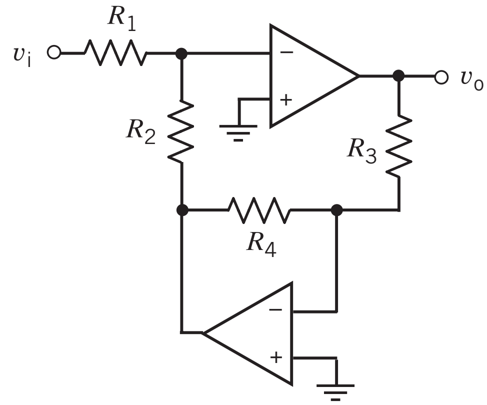
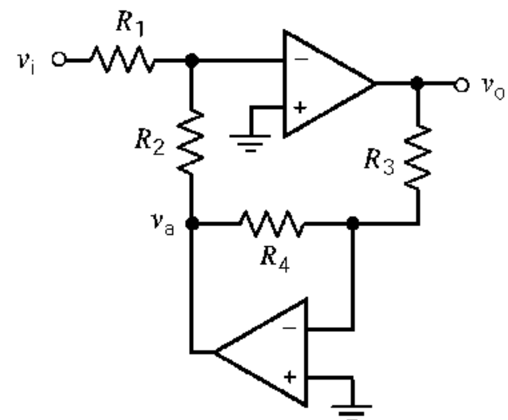


Figure P 6.4-13

Solution:

Write a node equation at the inverting input of the bottom op amp:



$$\frac{v_o}{R_3} + \frac{v_a}{R_4} = 0 \quad \Rightarrow \quad v_a = -\frac{R_4}{R_3} v_o$$

Write a node equation at the inverting input of the top op amp:

$$0 = \frac{v_i}{R_1} + \frac{v_a}{R_2} = \frac{v_i}{R_1} + \frac{-\frac{R_4}{R_3} v_o}{R_2} \quad \Rightarrow \quad v_o = \frac{R_2\, R_3}{R_1\, R_4} v_i$$

The output is proportional to the input and the constant of proportionality is $\frac{R_2\, R_3}{R_1\, R_4}$. We require $v_o = 20\, v_i$ so $\frac{R_2\, R_3}{R_1\, R_4} = 20$. For example, $R_1 = R_4 = 10$ kΩ, $R_2 = 40$ kΩ and $R_3 = 50$ kΩ.

P 6.4-14 The circuit shown in Figure P 6.4-14 has one input, v_s, and one output, v_o. Show that the output is proportional to the input. Design the circuit so that $v_o = 20v_s$.

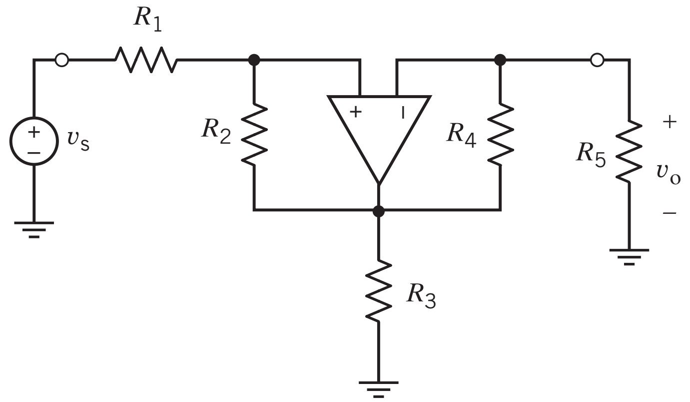
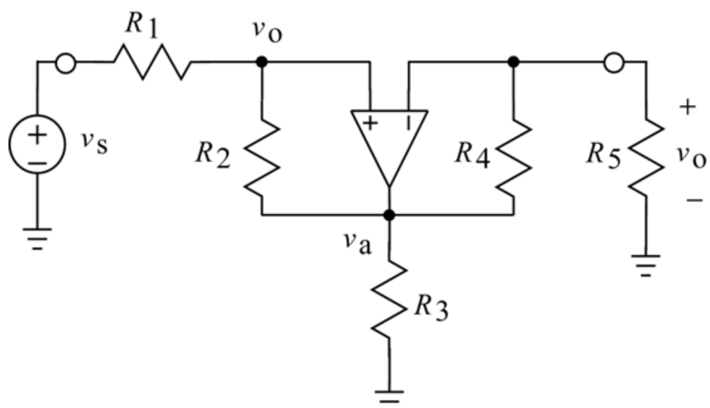


Figure P 6.4-14

Solution:



Represent this circuit by node equations.

$$\frac{v_o - v_a}{R_2} + \frac{v_o - v_s}{R_1} = 0 \quad \Rightarrow \quad R_2 v_s = (R_1 + R_2) v_o - R_1 v_a$$

$$\frac{v_o - v_a}{R_4} + \frac{v_o}{R_5} = 0 \quad \Rightarrow \quad v_a = \left(1 + \frac{R_4}{R_5}\right) v_o$$

So $$v_s = \left(1 + \frac{R_1}{R_2}\right) v_o - \left(\frac{R_1}{R_2}\right)\left(1 + \frac{R_4}{R_5}\right) v_o = \frac{(R_1 + R_2) R_5 - R_1 (R_4 R_5) v_o}{R_2 R_5} = \frac{R_2 R_5}{R_2 R_5 - R_1 R_4} v_s$$

Then $$20 = \frac{R_2 R_5}{R_2 R_5 \text{-} R_1 R_4} \quad \Rightarrow \quad \frac{19}{20} = \frac{R_1 R_4}{R_2 R_5}$$

For example $R_1 = 19\ \text{k}\Omega$, $R_4 = 10\ \text{k}\Omega$, $R_2 = 20\ \text{k}\Omega$, $R_5 = 10\ \text{k}\Omega$, $R_3 = 10\ \text{k}\Omega$.

(checked: LNAP 5/24/04)

P 6.4-15 The circuit shown in Figure P 6.4-15 has one input, v_s, and one output, v_o. The circuit contains seven resistors having equal resistance, R. Express the gain of the circuit, v_o/v_s, in terms of the resistance R.

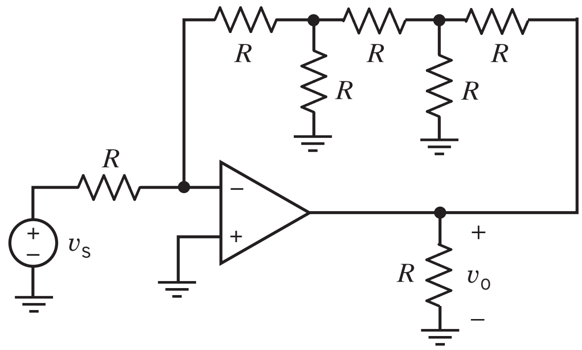
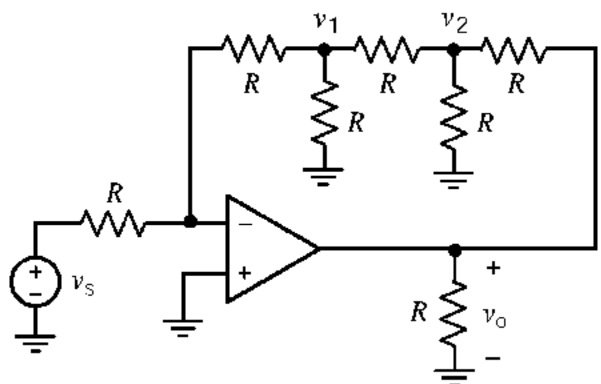


Figure P 6.4-15

Solution:



Writing node equations:

$$\frac{v_s}{R}+\frac{v_1}{R}=0 \quad \Rightarrow \quad v_1=-v_s$$

$$\frac{v_1}{R}+\frac{v_1}{R}+\frac{v_1-v_2}{R}=0 \quad \Rightarrow \quad v_2=3v_1=-3v_s$$

$$\frac{v_2-v_1}{R}+\frac{v_2}{R}+\frac{v_2-v_o}{R}=0 \quad \Rightarrow \quad v_o=3v_2-v_1=-8v_s$$

The gain of this circuit, $\frac{v_o}{v_s}=-8$, does not depend on R.

(checked: LNAP 6/21/04)

P 6.4-16 The circuit shown in Figure P 6.4-16 has one input, v_s, and one output, v_o. Express the gain, v_o/v_s, in terms of the resistances R_1, R_2, R_3, R_4, and R_5. Design the circuit so that $v_o = -20\ v_s$.

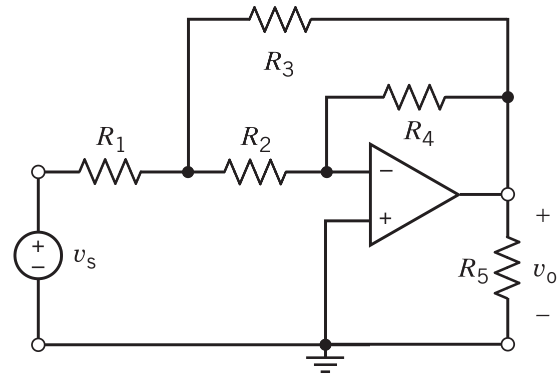
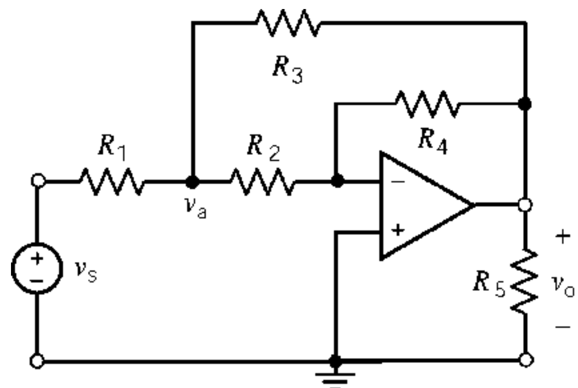


Figure P 6.4-16

Solution:
Represent this circuit by node equations.



$$\frac{v_s - v_a}{R_1} + \frac{v_o - v_a}{R_3} = \frac{v_a}{R_2}$$

and

$$\frac{v_a}{R_2} + \frac{v_o}{R_4} = 0 \quad \Rightarrow \quad v_o = -\frac{R_4}{R_2} v_a$$

So

$$\frac{v_s}{R_1} + \frac{v_o}{R_3} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_2}\right) v_a = \left(\frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_1 R_2 R_3}\right)\left(-\frac{R_2}{R_4} v_o\right)$$

$$\frac{v_s}{R_1} = -\left(\frac{1}{R_3} + \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_1 R_3 R_4}\right) v_o = -\left(\frac{R_1 R_4 + R_1 R_2 + R_1 R_3 + R_2 R_3}{R_1 R_3 R_4}\right) v_o$$

$$v_o = -\left(\frac{R_3 R_4}{R_1 R_4 + R_1 R_2 + R_1 R_3 + R_2 R_3}\right) v_s$$

We require
$$20 = \frac{R_3 R_4}{R_1 R_4 + R_1 R_2 + R_1 R_3 + R_2 R_3}$$

Try $R_1 = R_2 = R$ and $R_3 = R_4 = aR$

Then
$$20 = \frac{a^2}{3a+1}$$

So
$$a^2 - 60a - 20 = 0$$

$$a = \frac{+60 \pm \sqrt{3600 + 4(80)}}{2} = 60.332,\ -0.332$$

e.g.
$$R_1 = R_2 = 10\ \text{k}\Omega \ \text{ and } \ R_3 = R_4 = 603.32\ \text{k}\Omega$$

(checked: LNAP 6/9/04)

P 6.4-17 The circuit shown in Figure P 6.4-17 has one input, v_s, and one output, v_o. Express the gain of the circuit, v_o/v_s, in terms of the resistances R_1, R_2, R_3, R_4, R_5, and R_6. Design the circuit so that

$$v_o = -20\ v_s.$$

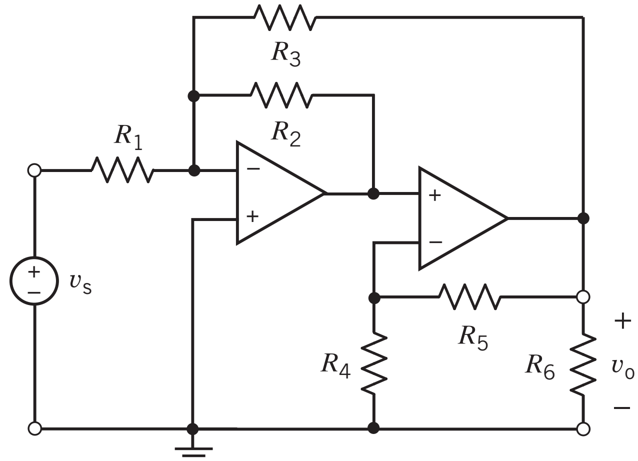
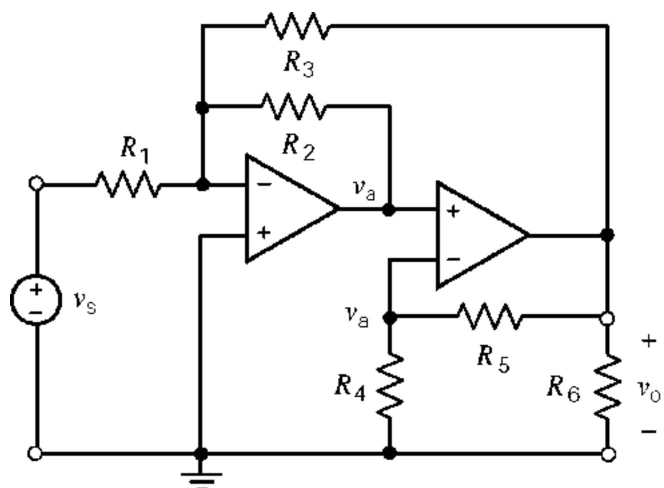


Figure P 6.4-17

Solution:

Represent this circuit by node equations.



$$\frac{v_s}{R_1}+\frac{v_a}{R_2}+\frac{v_o}{R_3}=0$$

and

$$\frac{v_a}{R_4}+\frac{v_a-v_o}{R_5}=0 \quad \Rightarrow \quad v_a=\left(\frac{R_4}{R_4+R_5}\right)v_o$$

So
$$\frac{v_s}{R_1}+\frac{v_o}{R_3}=-\frac{v_a}{R_2}=-\frac{R_4}{R_2\left(R_4+R_5\right)}v_o$$

$$\frac{v_s}{R_1}=-\left(\frac{1}{R_3}+\frac{R_4}{R_2\left(R_4 R_5\right)}\right)v_o=-\frac{R_2\left(R_4+R_5\right)+R_4R_3}{R_2R_3\left(R_4+R_5\right)}v_o$$

$$v_o=-\frac{R_2R_3\left(R_4+R_5\right)}{R_1\left(R_2R_4+R_2R_5+R_3R_4\right)}v_s$$

We require
$$20=\frac{R_2R_3\left(R_4+R_5\right)}{R_1\left(R_2R_4+R_2R_5+R_3R_4\right)}$$

Try
$$R_1=R_4=R_5=R \text{ and } R_2=R_3=aR$$

Then $$20 = \frac{2a^2 R^3}{3aR^3} = \frac{2}{3}a \quad \Rightarrow \quad a = 30$$

e.g. $$R_1 = R_4 = R_5 = 10\ \text{k}\Omega \ \text{ and } \ R_2 = R_3 = 300\ \text{k}\Omega$$

(checked: LNAP 6/10/04)

P 6.4-18 The circuit shown in Figure P 6.4-18 has one input, v_s, and one output, i_o. Express the gain of the circuit, i_o/v_s, in terms of the resistances R_1, R_2, R_3, and R_o. (This circuit contains a pair of resistors having resistance R_1 and another pair having resistance R_2.) Design the circuit so that $i_o = 0.02\ v_s$.

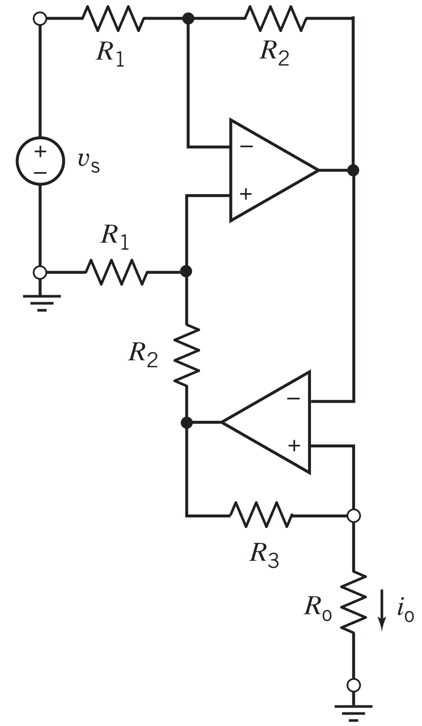
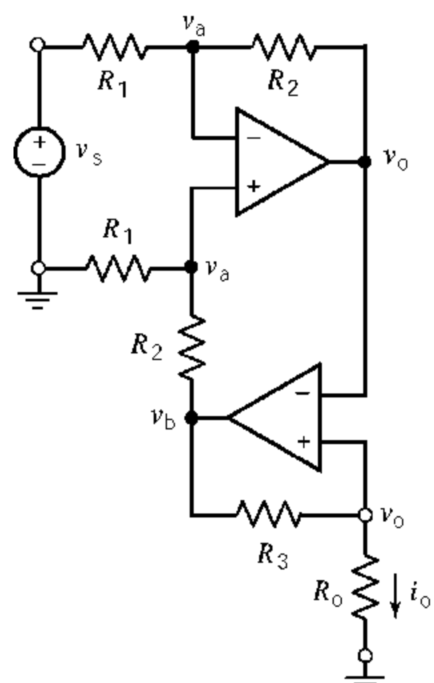


Figure P 6.4-18

Solution:

Label the node voltages as shown. Represent this circuit by node equations.



$$\frac{v_b - v_a}{R_2} = \frac{v_a}{R_1} \quad \Rightarrow \quad v_a = \frac{R_1}{R_1 + R_2} v_b$$

$$i_o + \frac{v_o - v_b}{R_3} = 0 \quad \Rightarrow \quad v_b = R_3 i_o + v_o$$

$$\frac{v_o - v_a}{R_2} + \frac{v_s - v_a}{R_1} = 0 \quad \Rightarrow \quad \frac{v_s}{R_1} = \left(\frac{R_1 + R_2}{R_1 R_2}\right) v_a - \frac{v_o}{R_2}$$

So

$$\frac{v_s}{R_1} = \left(\frac{R_1 + R_2}{R_1 R_2}\right)\left(\frac{R_1}{R_1 + R_2}\right)\left(R_3 i_o + v_o\right) - \frac{v_o}{R_2} = \frac{R_3}{R_2} i_o$$

$$\frac{i_o}{v_s} = \frac{R_2}{R_1 R_3}$$

We require $\dfrac{R_2}{R_1 R_3} = 0.02$, e.g. $R_2 = 8\ \text{k}\Omega,\ R_1 = R_3 = 20\ \text{k}\Omega$.

(checked: LNAP 6/21/04)

P 6.4-19 The circuit shown in Figure P 6.4-19 has one input, v_s, and one output, v_o. The circuit contains one unspecified resistance, *R*.

(a) Express the gain of the circuit, v_o/v_s, in terms of the resistance *R*.

(b) Determine the range of values of the gain that can be obtained by specifying a value for the resistance *R*.

(c) Design the circuit so that $v_o = -3\, v_s$.

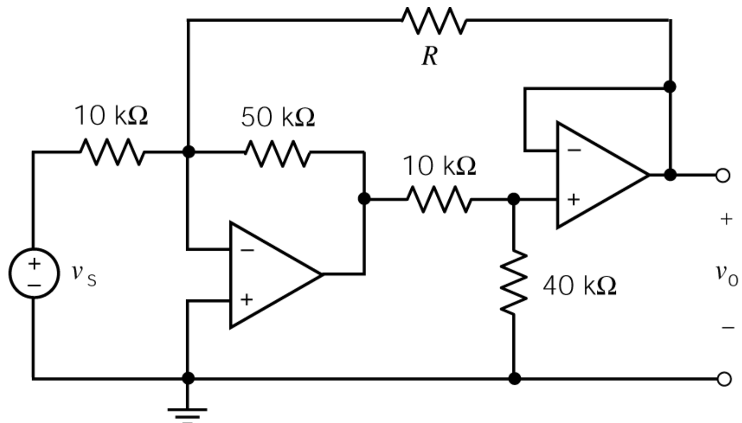


Figure P 6.4-19

Solution:

(a) Use units of volts, mA, and kΩ. Apply KCL at the inverting input of the left op amp to get

$$\frac{v_s}{10}+\frac{v_a}{50}+\frac{v_o}{R}=0 \quad \Rightarrow v_a=-\left(5v_s+\frac{50}{R}v_o\right)$$

$$v_o=\frac{4}{5}v_a=-4v_s-\frac{40}{R}v_o \quad \Rightarrow \quad \left(1+\frac{40}{R}\right)v_o=-4v_s$$

$$\frac{v_o}{v_s}=-\frac{4}{1+\dfrac{40}{R}}=-\frac{4R}{R+40}$$

(b) $$0\le R\le\infty \quad \Rightarrow \quad -4\le\frac{v_o}{v_s}\le 0$$

(c) We require $$-3=-\frac{4R}{R+40} \quad \Rightarrow \quad R=120\ \text{k}\Omega$$

(checked: LNAP 6/21/04)

P 6.4-20 The circuit shown in Figure P 6.4-20 has one input, v_s, and one output, v_o. The circuit contains one unspecified resistance, R.

(a) Express the gain of the circuit, v_o/v_s, in terms of the resistance R.

(b) Determine the range of values of the gain that can be obtained by specifying a value for the resistance R.

(c) Design the circuit so that $v_o = -5\, v_s$.

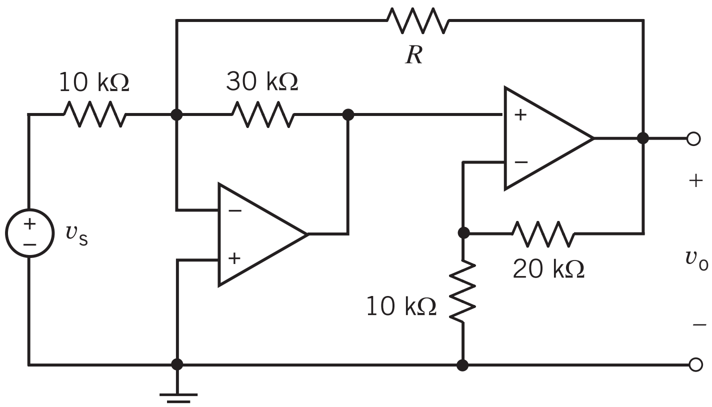


Figure P 6.4-20

Solution:

(a) Use units of V, mA and kΩ. Apply KCL at the inverting input of the left op amp to get

$$\frac{v_1}{10}+\frac{v_a}{30}+\frac{v_o}{R}=0 \quad\Rightarrow\quad v_a=-\left(3v_s+\frac{30}{R}v_o\right)$$

$$v_o=3v_a=-9v_s-\frac{90}{R}v_o \quad\Rightarrow\quad \left(1+\frac{90}{R}\right)v_o=-9v_s$$

$$\frac{v_o}{v_s}=-\frac{9}{1+\dfrac{90}{R}}=-\frac{9R}{R+90}$$

(b) $$0\le R\le\infty \quad\Rightarrow\quad -9\le\frac{v_o}{v_s}\le 0$$

(c) We require $$-5=\frac{-9R}{R+90} \quad\Rightarrow\quad R=112.5\ \text{k}\Omega$$

(checked: LNAP 7/8/04)

P 6.4-21 The circuit shown in Figure P 6.4-21 has three inputs: v_1, v_2, and v_3. The output of the circuit is v_o. The output is related to the inputs by

$$v_o = av_1 + bv_2 + cv_3$$

where *a*, *b*, and *c* are constants. Determine the values of *a*, *b*, and *c*.

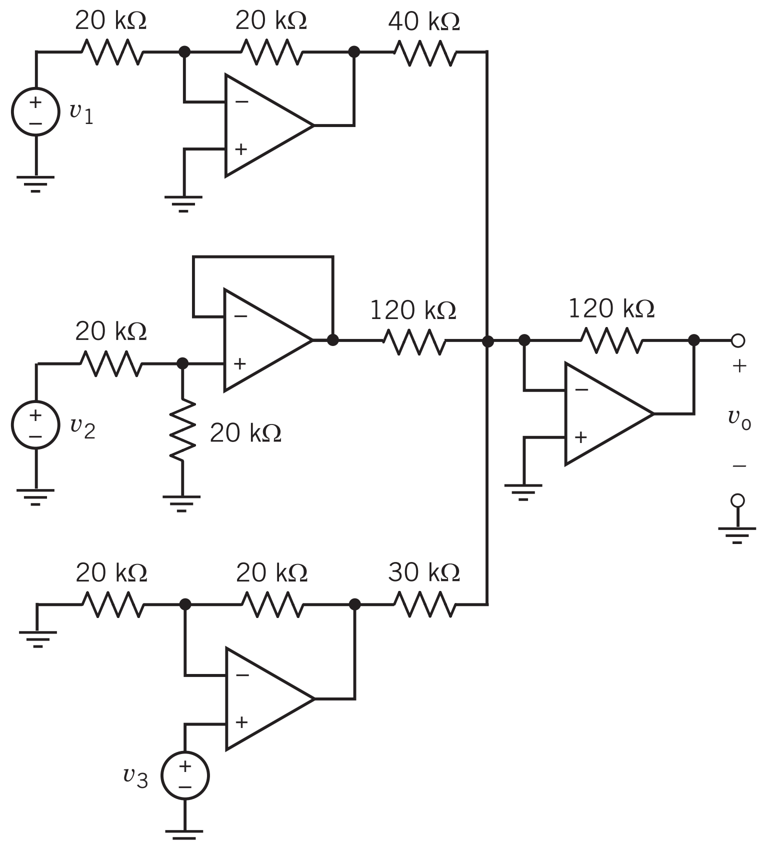
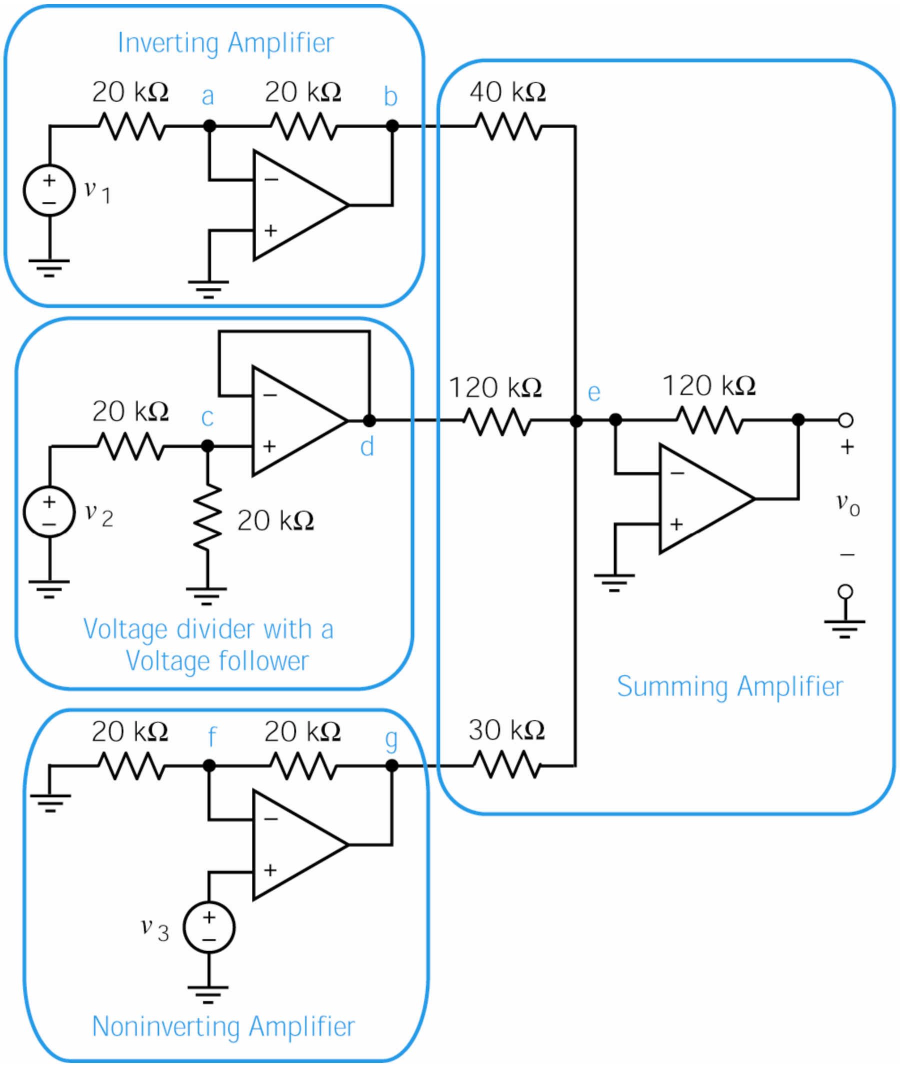


Figure P 6.4-21

Solution:

Label the node voltages and identify some standard op amp circuits:



Either by recognizing the inverting amplifier or by writing and solving the node equation corresponding to node a, we obtain

$$v_b = \left(-\frac{20}{20}\right) v_1 = -v_1$$

Either by recognizing the voltage divider and voltage follower or by writing and solving the node equation corresponding to node c, we obtain

$$v_d = \left(\frac{20}{20+20}\right) v_2 = \frac{1}{2} v_2$$

Either by recognizing the noninverting amplifier or by writing and solving the node equation corresponding to node f, we obtain

$$v_g = \left(1+\frac{20}{20}\right) v_3 = 3\, v_3$$

Either by recognizing the summing amplifier or by writing and solving the node equation corresponding to node e, we obtain

$$v_o = -\left[\left(\frac{120}{40}\right)v_b + \left(\frac{120}{120}\right)v_d + \left(\frac{120}{30}\right)v_g\right] = -\left[3v_b + v_d + 4v_g\right]$$

Substituting for v_b, v_d and v_g gives

$$v_o = -\left[3\left(-v_1\right) + \left(\frac{1}{2}v_2\right) + 4\left(3\,v_3\right)\right] = 3\,v_1 - 0.5\,v_2 - 12\,v_3$$

so

$$a = 3,\ \ b = -0.5 \text{ and } c = -12$$

(checked: LNAP 6/21/04)

P 6.4-22 The circuit shown in Figure P 6.4-22 has two inputs: v_1 and v_2. The output of the circuit is v_o. The output is related to the inputs by

$$v_o = av_1 + bv_2$$

where a and b are constants. Determine the values of a and b.

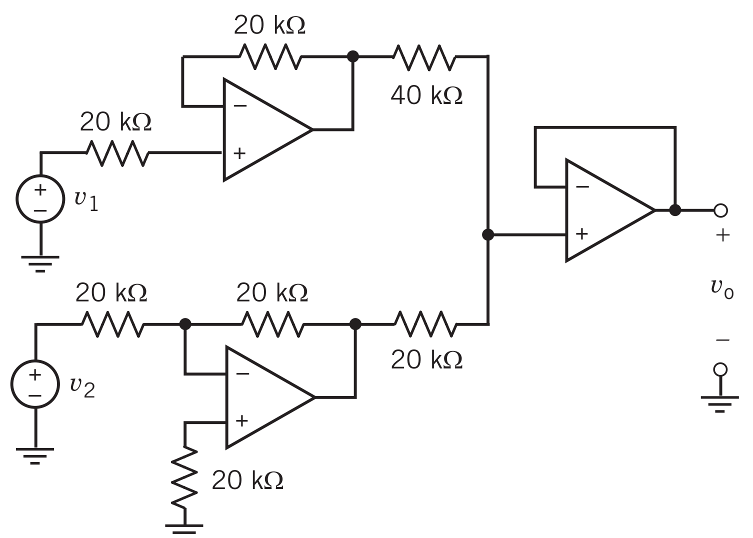
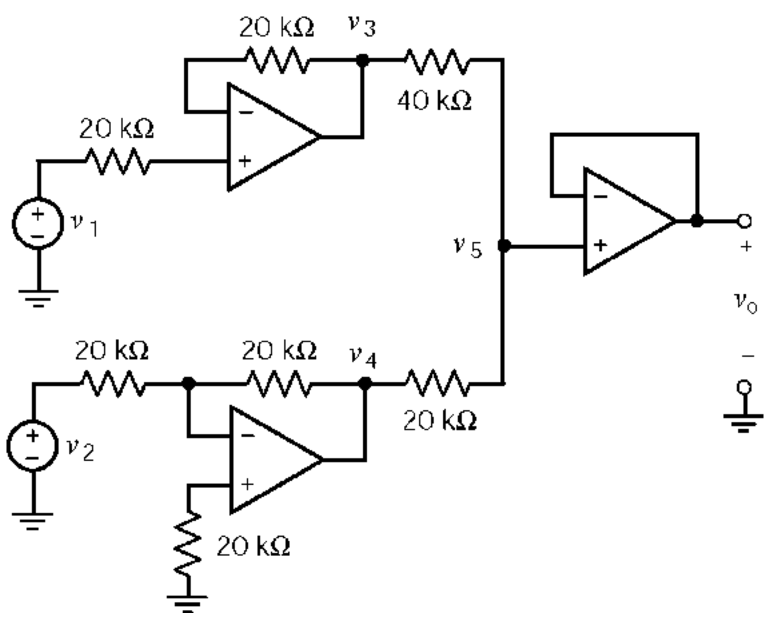


Figure P 6.4-22

Solution:



Label the node voltages as shown. Use units of V, mA and kΩ.

$$v_3 = v_1 \text{ and } v_4 = -v_2$$

$$\frac{v_5 - v_3}{40} + \frac{v_5 - v_4}{20} = 0 \qquad \Rightarrow \qquad v_5 = \frac{1}{3}\left(v_3 + 2v_2\right) = \frac{1}{3}v_1 - \frac{2}{3}v_2$$

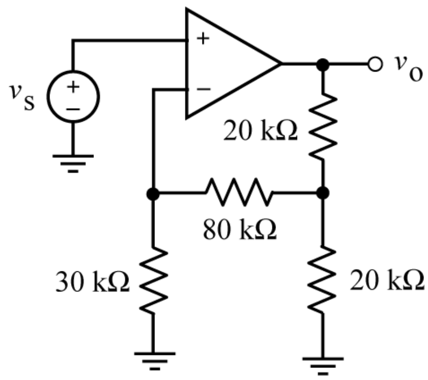
so

$$a = -\frac{1}{3} \text{ and } b = -\frac{2}{3}$$

(checked: LNAP 6/21/04)

P6.4-23

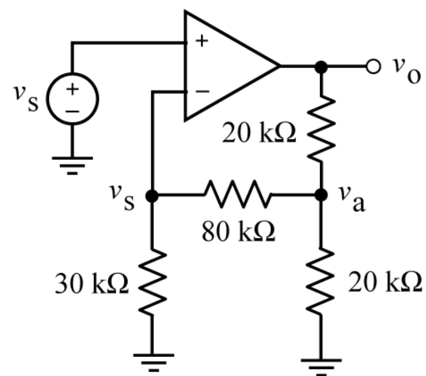
The input to the circuit shown in Figure P6.4-23 is the voltage source voltage v_s. The output is the node voltage v_o. The output is related to the input by the equation $v_o = k\, v_s$ where $k = \frac{v_o}{v_s}$ is called the gain of the circuit. Determine the value of the gain *k*.



FigureP6.4-23

Solution:

Label the node voltages as shown. Apply KCL at the inverting input node of the op amp to get



$$\frac{v_s}{30000} + \frac{v_s - v_a}{80000} = 0 \;\Rightarrow\; 11 v_s = 3 v_a \;\Rightarrow\; v_a = \frac{11}{3} v_s$$

Apply KCL at the right node of the 80 kΩ resistor to get

$$\frac{v_s - v_a}{80000} + \frac{v_o - v_a}{20000} = \frac{v_a}{20000} \;\Rightarrow\; v_s - 9 v_a + 4 v_o = 0$$

$$\Rightarrow\; v_s - 9\left(\frac{11}{3} v_s\right) + 4 v_o = 0$$

$$\Rightarrow\; v_s - 33 v_s + 4 v_o = 0$$

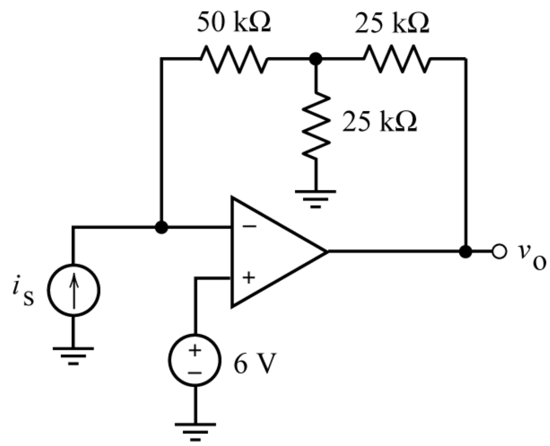
$$\Rightarrow\; 4 v_o = 32 v_s$$

$$\Rightarrow\; v_o = 8 v_s$$

Comparing this equation to $v_o = k\, v_s$, we determine that $k = 8$ V/V.

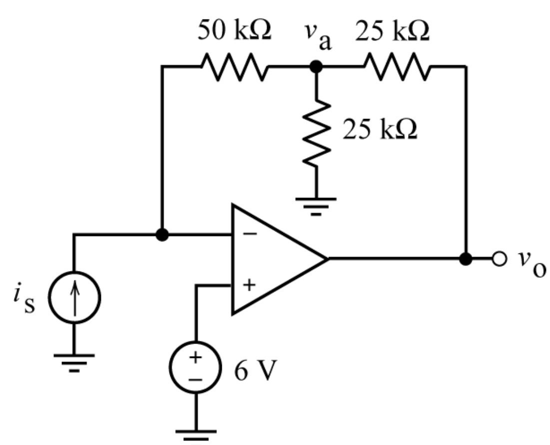
P6.4-24

The input to the circuit shown in Figure P6.4-23 is the voltage source voltage v_s. The output is the node voltage v_o. The output is related to the input by the equation $v_o = m\, i_s + b$ where *m* and *b* are constants. Determine the values of *m* and *b*.



FigureP6.4-24

Solution:



Label the node voltages as shown. Apply KCL at the inverting input node of the op amp to get

$$i_s = \frac{6 - v_a}{50000} = 0 \quad \Rightarrow \quad v_a = 6 - \left(50 \times 10^3\right) i_s$$

Apply KCL at the top node of the 25 kΩ resistor to get

$$i_s + \frac{v_o - v_a}{25000} = \frac{v_a}{25000}$$

Solving:

$$\left(25 \times 10^3\right) i_s - 2 v_a + v_o = 0$$

$$\left(25 \times 10^3\right) i_s - 2\left(6 - \left(50 \times 10^3\right) i_s\right) + v_o = 0$$

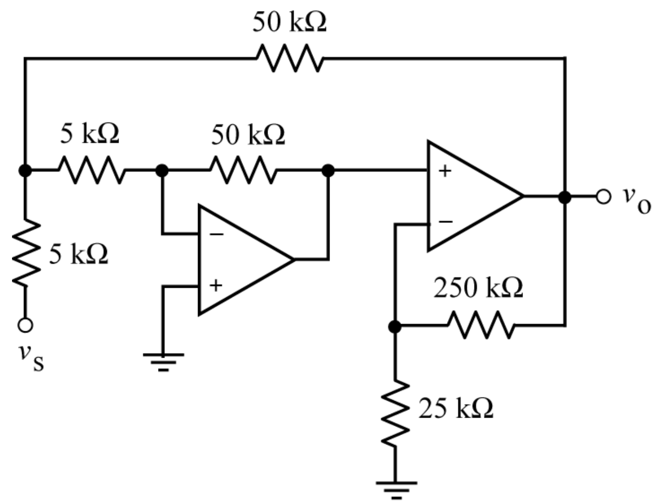
$$\left(125 \times 10^3\right) i_s - 12 + v_o = 0$$

$$v_o = -\left(125 \times 10^3\right) i_s + 12$$

Comparing this equation to $v_o = m\, i_s + b$, we get $m = -\left(125 \times 10^3\right)$ V/A and b =12 V.

P6.4-25

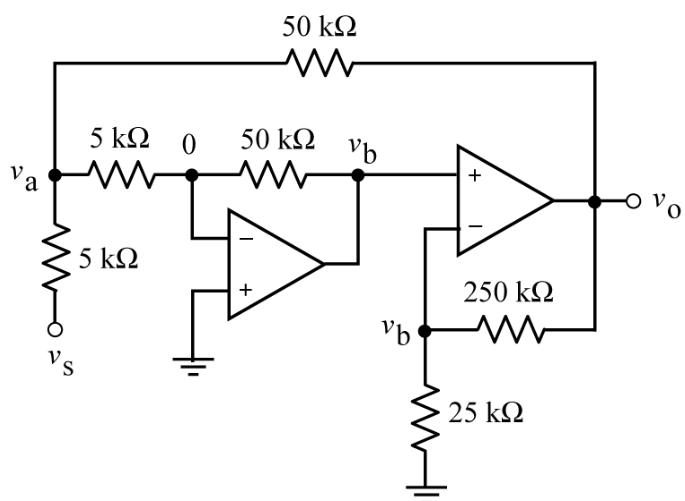
The input to the circuit shown in Figure P6.4-25 is the node voltage v_s. The output is the node voltage v_o. The output is related to the input by the equation $v_o = k\, v_s$ where $k = \frac{v_o}{v_s}$ is called the gain of the circuit. Determine the value of the gain *k*.



FigureP6.4-25

Solution:

Label the node voltages as shown. The node equations are



$$\frac{v_s - v_a}{5000} = \frac{v_a - 0}{5000} + \frac{v_a - v_o}{50000}$$

$$\frac{v_a - 0}{5000} = \frac{0 - v_b}{50000} \quad \Rightarrow \quad v_b = -10\, v_a$$

$$\frac{v_o - v_b}{5000} = \frac{v_b}{25000} \quad \Rightarrow \quad v_o = 11 v_b$$

Substituting $v_a = -\frac{v_o}{110}$ into the first node equation gives:

$$\frac{v_s + \frac{v_o}{110}}{5000} = \frac{-\frac{v_o}{110}}{5000} + \frac{-\frac{v_o}{110} - v_o}{50000} \quad \Rightarrow \quad 10\left(v_s + \frac{v_o}{110}\right) = 10\left(-\frac{v_o}{110}\right) - \frac{v_o}{110} - v_o$$

$$\Rightarrow \quad 10 v_s = -\frac{131}{110} v_o$$

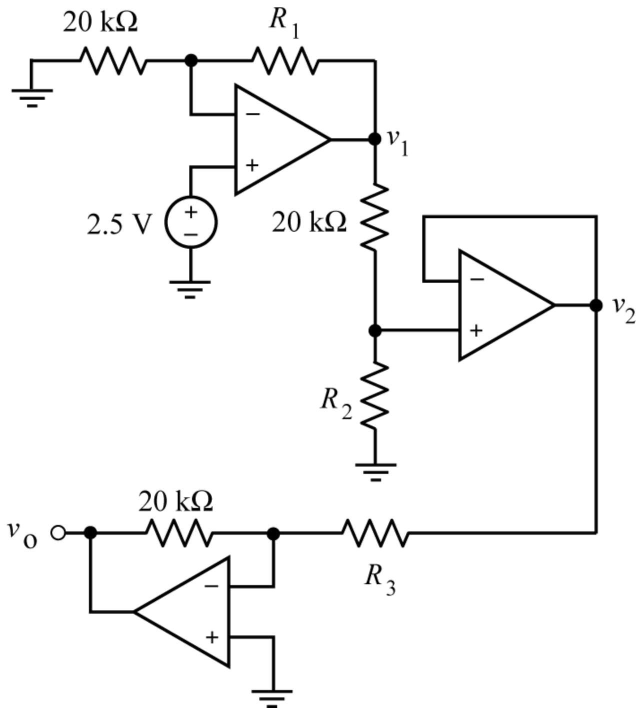
$$\Rightarrow \quad v_o = -\frac{1100}{131} v_s = -8.397 v_s$$

Comparing this equation to $v_o = k\, v_s$, we determine that $k = -8.397$ V/V.

P6.4-26
The values of the node voltages v_1, v_2 and v_o in Figure P6.4-26 are

v_1 = 6.25 V, v_2 = 3.75 V and v_o = −15 V.

Determine the value of the resistances R_1, R_2 and R_3:

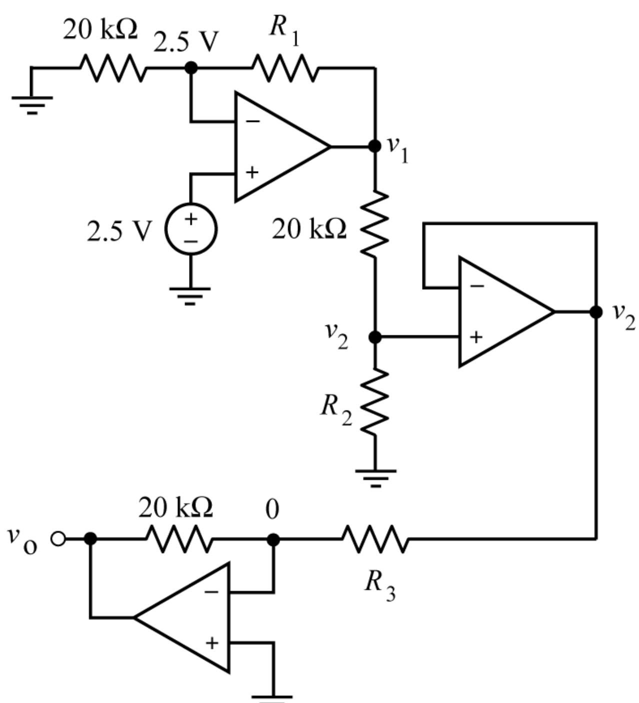


FigureP6.4-26

Solution:
Label the node voltages as shown. Using units of Volts, kΩ and mAmps, the node equations are:

$$\frac{2.5}{20}=\frac{v_1-2.5}{R_1},\quad \frac{v_1-v_2}{20}=\frac{v_2}{R_2}$$

And $$\frac{v_2-0}{R_3}+\frac{v_o-0}{20}=0$$



Using $v_1 = 6.25$ V gives

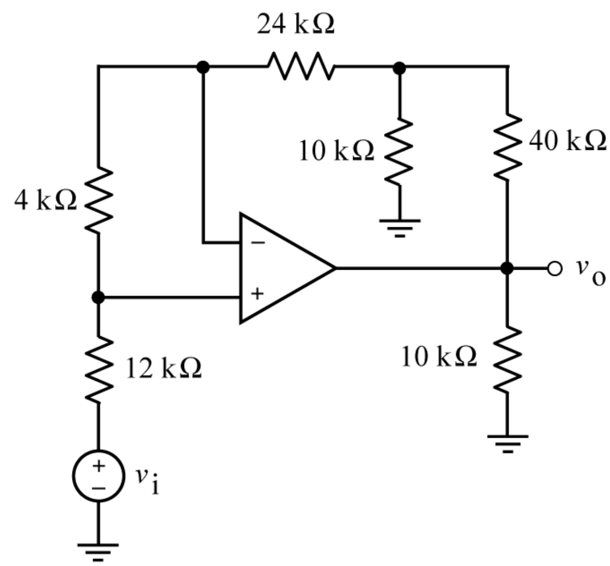
$$\frac{2.5}{20}=\frac{6.25-2.5}{R_1} \quad\Rightarrow\quad R_1 = 30\text{ k}\Omega$$

Using $v_1 = 6.25$ V and $v_2 = 3.75$ V gives

$$\frac{6.25-3.75}{20}=\frac{3.75}{R_2} \quad\Rightarrow\quad R_2 = 30\text{ k}\Omega$$

Using $v_2 = 3.75$ V and $v_o = -15$ V gives $$\frac{3.75-0}{R_3}+\frac{-15-0}{20}=0 \quad\Rightarrow\quad R_3 = 5\text{ k}\Omega$$

P6.4-27
The input to the circuit shown in Figure P6.4-27 is the voltage source voltage, v_i. The output is the node voltage, v_o. The output is related to the input by the equation $v_o = k\, v_i$ where $k = \dfrac{v_o}{v_i}$ is called the gain of the circuit. Determine the value of the gain *k*.



FigureP6.4-27

Solution:
Label the node voltages as shown. The node voltages at the input nodes of an ideal op amp are equal so

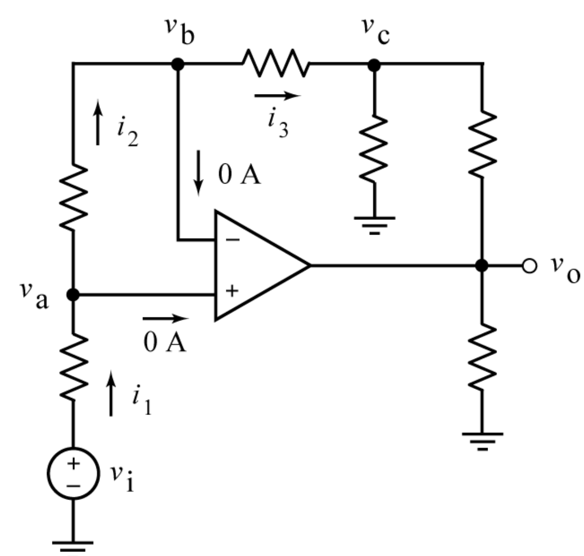
$$v_a = v_b$$

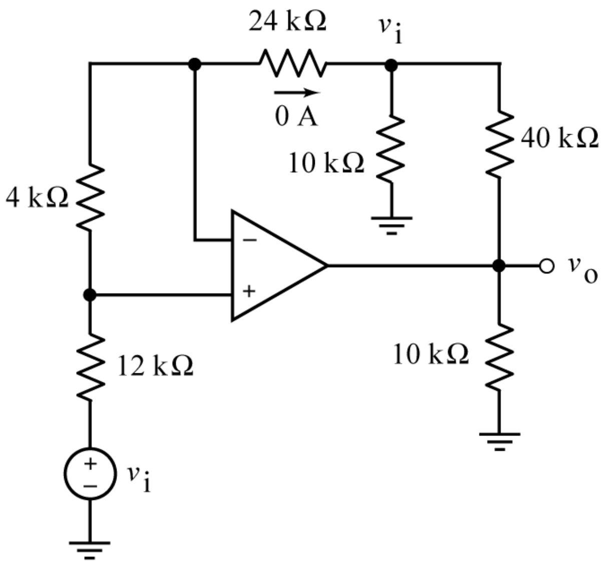
Consequently

$$i_2 = \frac{v_a - v_b}{R} = 0$$

The input currents of an ideal op amp are 0 A, so applying KCL at nodes a and b shows that $i_1 = 0$ and $i_3 = 0$. Consequently, the voltages across the corresponding resistor are 0 V. Finally

$$v_a = v_b = v_c = v_i$$





Write an node equation at the right node of the 24 kΩ resistor:

$$0 = \frac{v_i}{10} + \frac{v_i - v_o}{40} \quad \Rightarrow \quad v_o = 5\,v_i$$

Finally

$$k = \frac{v_o}{v_i} = 5$$

Section 6-5: Design Using Operational Amplifier

P 6.5-1 Design the operational amplifier circuit in Figure P 6.5-1 so that

$$v_{out} = r \cdot i_{in}$$

where $$r = 20 \frac{\text{V}}{\text{mA}}$$

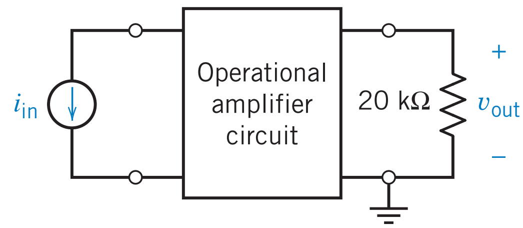
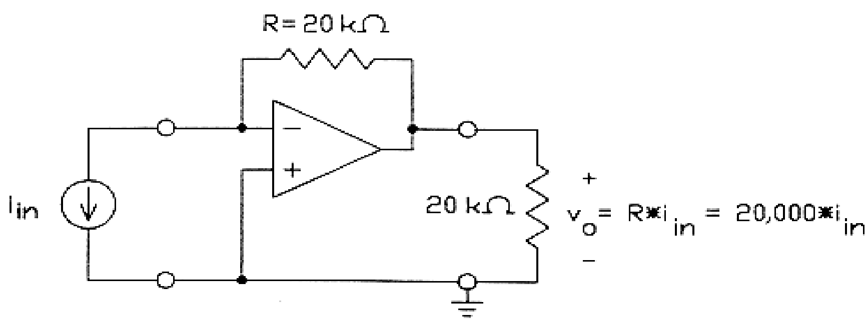


Figure P 6.5-1

Solution:

Use the current-to-voltage converter, entry (*g*) in Figure 6.6-1.



P 6.5-2 Design the operational amplifier circuit in Figure P 6.5-2 so that

$$i_{out} = g \cdot v_{in}$$

where $$g = 2 \frac{\text{mA}}{\text{V}}$$

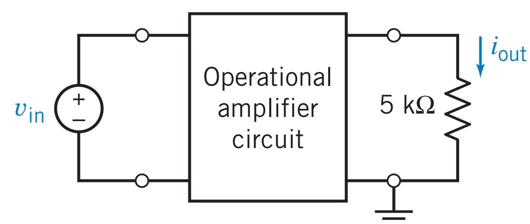
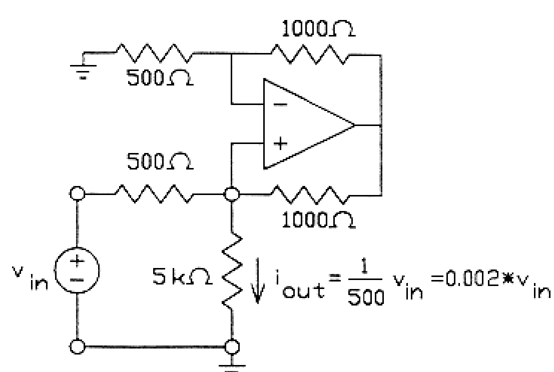


Figure P 6.5-2

Solution:

Use the voltage –controlled current source, entry (*i*) in Figure 6.6-1.



P 6.5-3 Design the operational amplifier circuit in Figure P 6.5-3 so that

$$v_{out} = 5 \cdot v_1 + 2 \cdot v_2$$

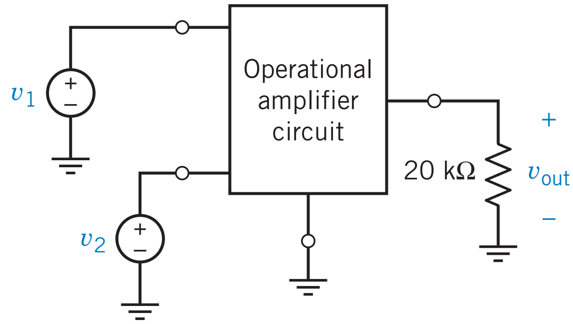
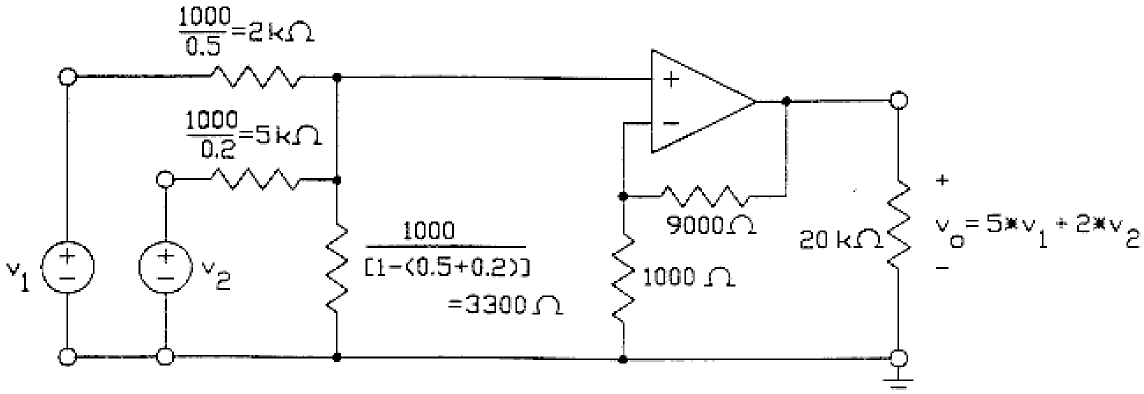


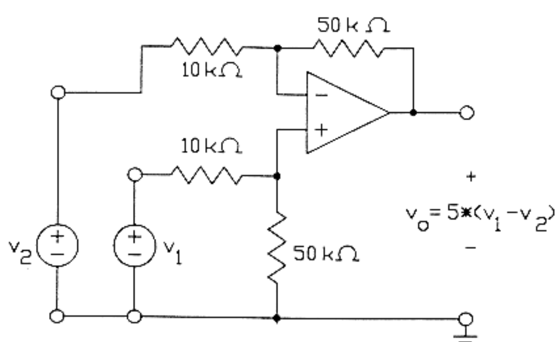
Figure P 6.5-3

Solution:
Use the noninverting summing amplifier, entry (*e*) in Figure 6.6-1.



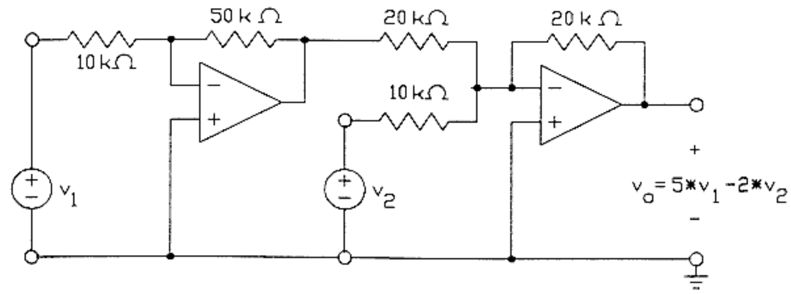
P 6.5-4 Design the operational amplifier circuit in Figure P 6.5-3 so that $v_{out} = 5 \cdot (v_1 - v_2)$.

Solution:
Use the difference amplifier, entry (*f*) in Figure 6.6-1.



P 6.5-5 Design the operational amplifier circuit in Figure P 6.5-3 so that $v_{out} = 5 \cdot v_1 - 2 \cdot v_2$

Solution:
Use the inverting amplifier and the summing amplifier, entries (*a*) and (*d*) in Figure 6.6-1.



P 6.5-6 The voltage divider shown in Figure P 6.5-6 has a gain of

$$\frac{v_{out}}{v_{in}} = \frac{-10\,\text{k}\Omega}{5\ \text{k}\Omega + (-10\ \text{k}\Omega)} = 2$$

Design an operational amplifier circuit to implement the – 10-kΩ resistor.

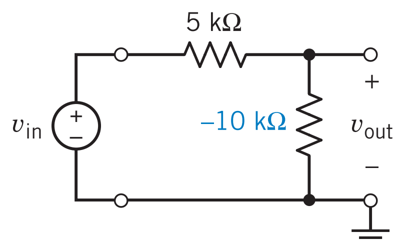
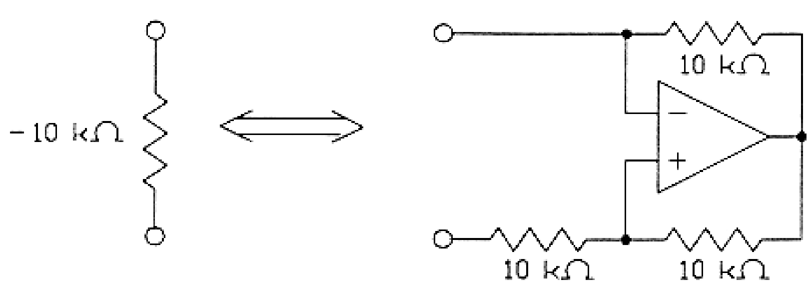


Figure P 6.5-6

Solution:
Use the negative resistance converter, entry (*h*) in Figure 6.6-1.



P 6.5-7 Design the operational amplifier circuit in Figure P 6.5-7 so that

$i_{in} = 0$ and $v_{out} = 3 \cdot v_{in}$

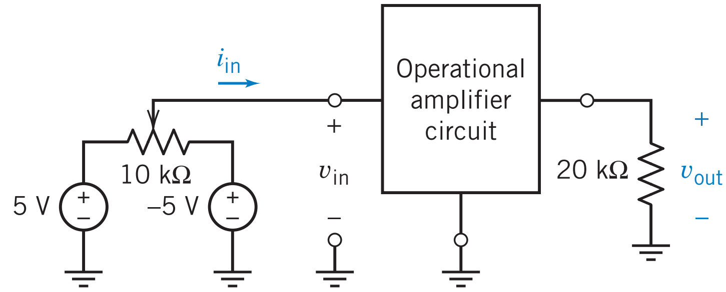
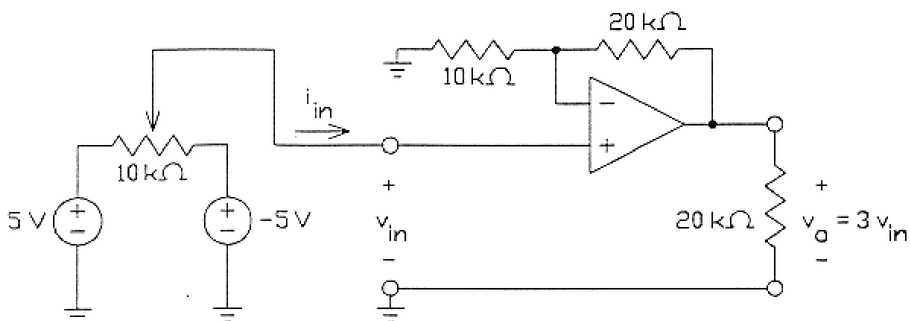


Figure P 6.5-7

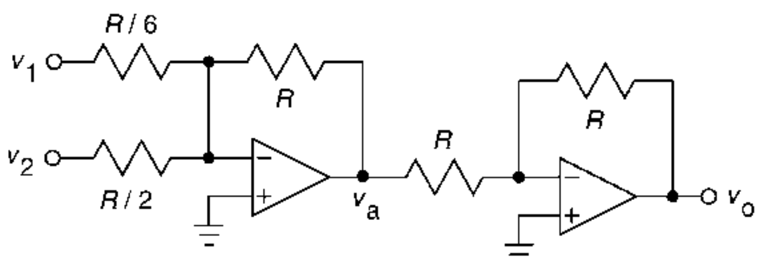
Solution:

Use the noninverting amplifier, entry (*b*) in Figure 6.6-1. Notice that the ideal op amp forces the current i_{in} to be zero.



P 6.5-8 Design an operational amplifier circuit with output $v_o = 6\, v_1 + 2\, v_2$, where v_1 and v_2 are input voltages.

Solution:



$$\left.\begin{array}{ll}\text{Summing Amplifier:} & v_a = -(6\, v_1 + 2\, v_2) \\ \text{Inverting Amplifier:} & v_o = -v_a\end{array}\right\} \Rightarrow v_o = 6\, v_1 + 2\, v_2$$

P 6.5-9 Determine the voltage v_o for the circuit shown in Figure P 6.5-9.

Hint: Use superposition.

Answer: $v_o = (-3)(3) + (4)(-4) + (4)(8) = 7$ V

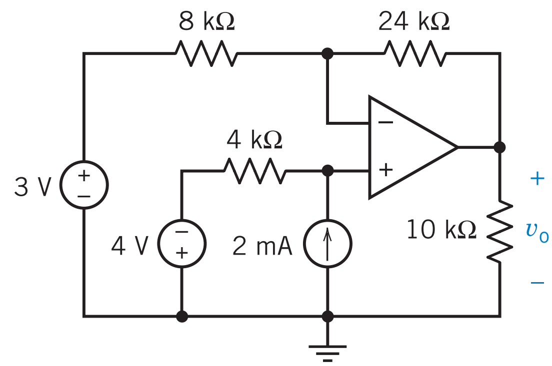
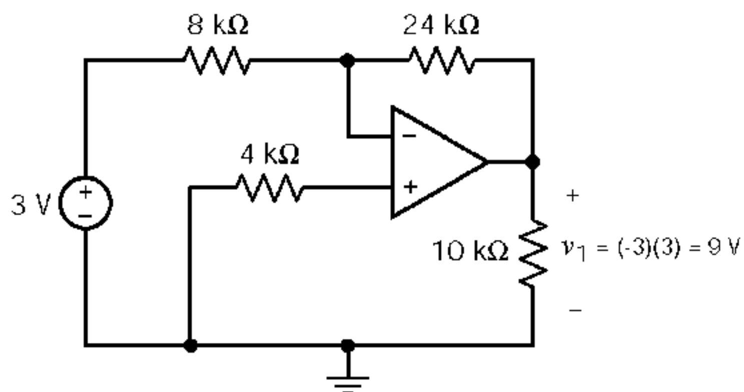
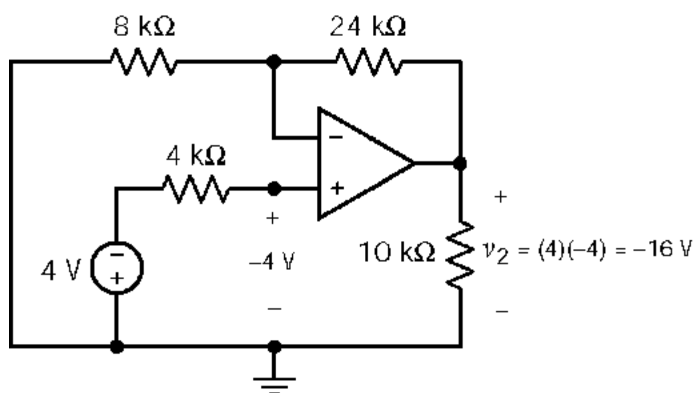
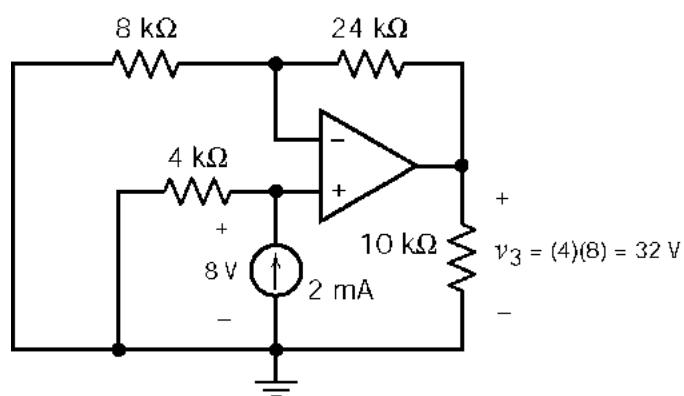


Figure P 6.5-9

Solution:







Using superposition, $v_o = v_1 + v_2 + v_3 = -9 - 16 + 32 = 7$ V

P 6.5-10 For the op amp circuit shown in Figure P 6.5-10, find and list all the possible voltage gains that can be achieved by connecting the resistor terminals to either the input or the output voltage terminals.

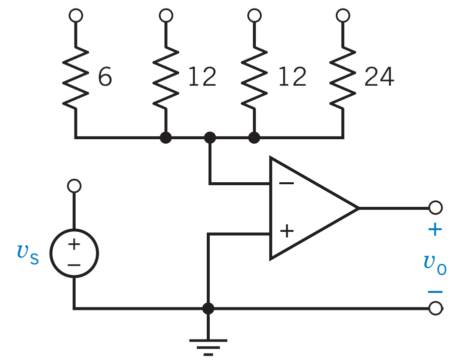


Figure P 6.5-10

Solution:

R_1	6	12	24	6||12	6||24
R_2	12||12||24	6||12||24	6||12||12	12||24	12||12
$-v_o/v_s$	0.8	0.286	0.125	2	1.25

R_1	12||12	12||24	6||12||12	6||12||24	12||12||24
R_2	6||24	6||12	24	12	6
$-v_o/v_s$	0.8	0.5	8	3.5	1.25

P 6.5-11 The circuit shown in Figure P 6.5-11 is called a Howland current source. It has one input, v_{in}, and one output, i_{out}. Show that when the resistances are chosen so that $R_2 R_3 = R_1 R_4$, the output is related to the input by the equation

$$i_{out} = \frac{v_{in}}{R_1}$$

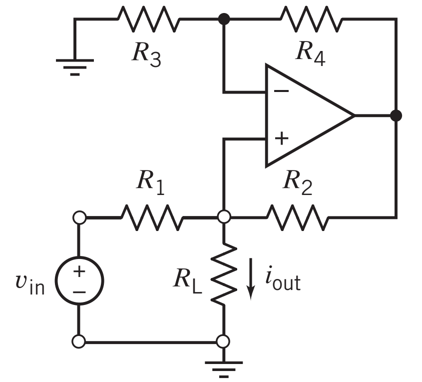
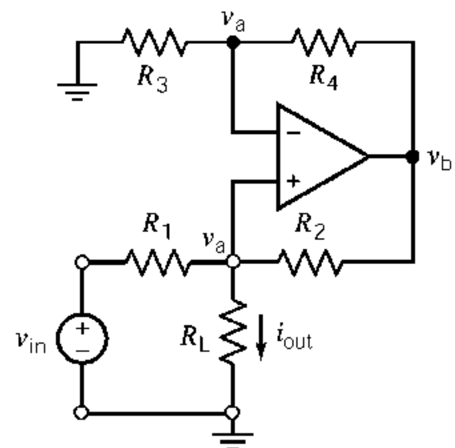


Figure P 6.5-11

Solution:

Label the node voltages as shown. Apply KCL at the inverting input of the op amp to get



$$\frac{v_a}{R_3} + \frac{v_a - v_b}{R_3} = 0 \quad \Rightarrow \quad v_b = \left(\frac{R_3 + R_4}{R_3}\right) v_a$$

Apply KCL at the noninverting input of the op amp to get

$$\frac{v_a - v_{in}}{R_1} + \frac{v_a - v_b}{R_2} + i_{out} = 0$$

Solving gives

$$v_a\left(\frac{R_1 + R_2}{R_1\, R_2}\right) - \frac{v_{in}}{R_1} - \frac{v_b}{R_2} + i_{out} = 0 \quad \Rightarrow \quad v_a\left(\frac{R_1 + R_2}{R_1\, R_2} - \frac{R_3 + R_4}{R_2\, R_3}\right) - \frac{v_{in}}{R_1} + i_{out} = 0$$

When $R_2 R_3 = R_1 R_4$ the quantity in parenthesis vanishes leaving

$$i_{out} = \frac{1}{R_1} v_{in}$$

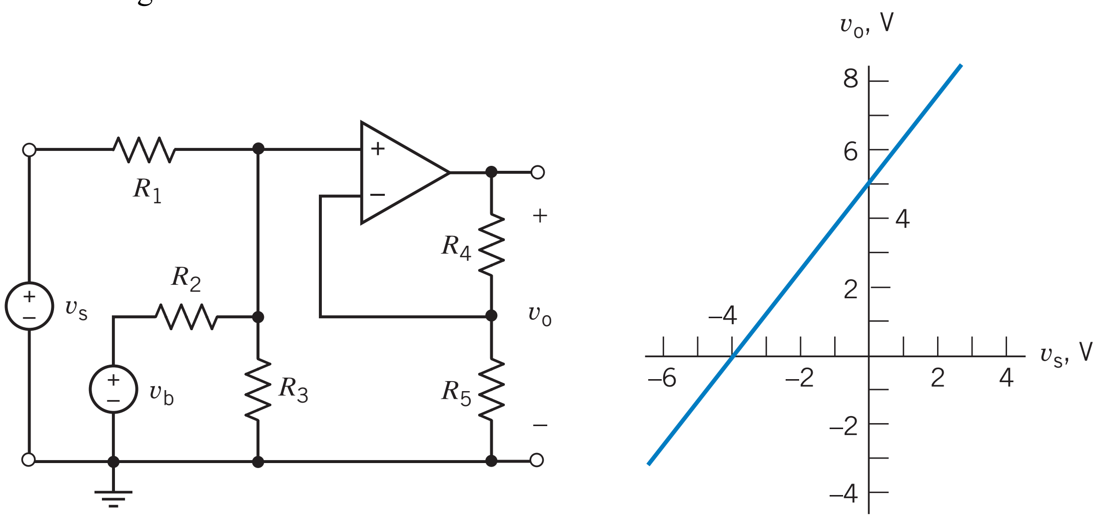
P 6.5-12 The input to the circuit shown in Figure P 6.5-12a is the voltage v_s. The output is the voltage v_o. The voltage v_b is used to adjust the relationship between the input and output.

(a) Show that the output of this circuit is related to the input by the equation

$$v_o = a v_s + b$$

where a and b are constants that depend on R_1, R_2, R_3, R_4, and v_b.

(b) Design the circuit so that its input and output have the relationship specified by the graph shown in Figure P 6.5-12*b*.

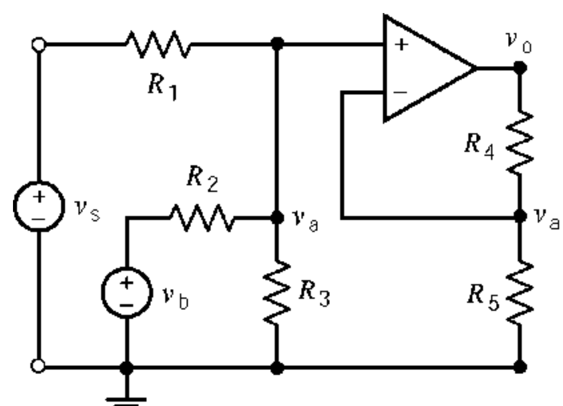


(a) *(b)*

Figure P 6.5-12

Solution:

(a) Label the node voltages as shown. The node equations are



$$\frac{v_s - v_a}{R_1} + \frac{v_b - v_a}{R_2} = \frac{v_a}{R_3}$$

and

$$\frac{v_a}{R_5} = \frac{v_o - v_a}{R_4} \quad \Rightarrow \quad v_a = \left(\frac{R_5}{R_4 + R_5} \right) v_o$$

Solving these equations gives

$$\frac{v_s}{R_1} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) v_a - \frac{v_b}{R_2} = \left(\frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_1 R_2 R_3} \times \frac{R_5}{R_4 + R_5} \right) v_o - \frac{v_b}{R_2}$$

So

$$v_o = \left(\frac{R_2 R_3}{R_1 R_2 + R_2 R_3 + R_1 R_3} \times \frac{R_4 + R_5}{R_5} \right) v_s + \frac{R_1 R_3}{R_1 R_2 + R_2 R_3 + R_1 R_3} \times \frac{R_4 + R_5}{R_5} \times v_b$$

So

$$a = \left(\frac{R_2 R_3}{R_1 R_2 + R_2 R_3 + R_1 R_3} \times \frac{R_4 + R_5}{R_5} \right) v_s \quad \text{and} \quad b = \frac{R_1 R_3}{R_1 R_2 + R_2 R_3 + R_1 R_3} \times \frac{R_4 + R_5}{R_5} \times v_b$$

(b) The equation of the straight line is

$$v_o = \frac{5}{4} v_s + 5$$

We require

$$\frac{R_2 R_3}{R_1 R_2 + R_2 R_3 + R_1 R_3} \times \frac{R_4 + R_5}{R_5} = \frac{5}{4}$$

For example, let $R_1 = R_2 = R_3 = 10\ \text{k}\Omega$, $R_4 = 55\ \text{k}\Omega$ and $R_5 = 20\ \text{k}\Omega$. Next we require

$$5 = \frac{R_1 R_3}{R_1 R_2 + R_2 R_3 + R_1 R_3} \times \frac{R_4 + R_5}{R_5} \times v_b = \frac{5}{4} v_b$$

i.e.

$$v_b = 4\ \text{V}$$

(checked: LNAP 6/20/04)

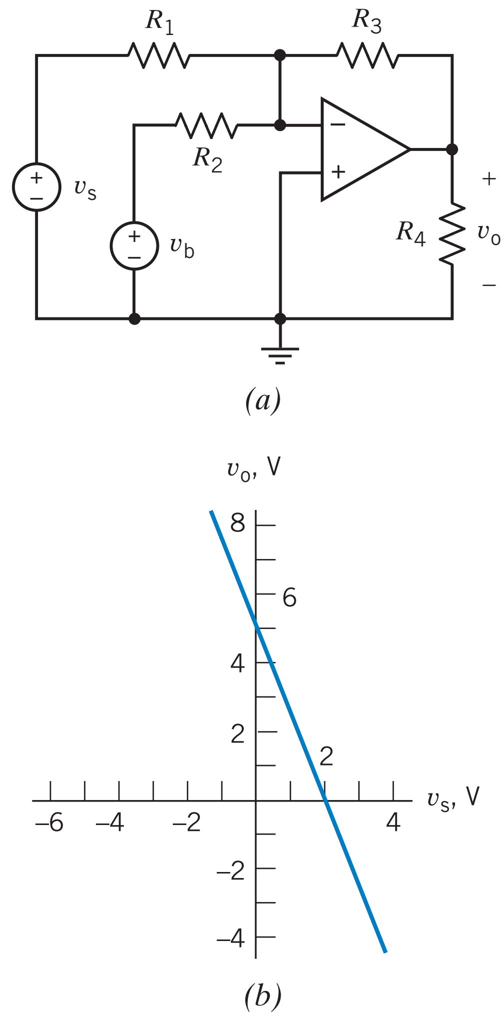
P 6.5-13 The input to the circuit shown in Figure P 6.5-13a is the voltage v_s. The output is the voltage v_o. The voltage v_b is used to adjust the relationship between the input and output.

(a) Show that the output of this circuit is related to the input by the equation

$$v_o = a v_s + b$$

where a and b are constants that depend on R_1, R_2, R_3, R_4, and v_b.

(b) Design the circuit so that its input and output have the relationship specified by the graph shown in Figure P 6.5-13*b*.



P 6.5-13

Solution:

(a) Apply KCL at the inverting input of the op amp to get:

$$\frac{v_s}{R_1} + \frac{v_b}{R_2} + \frac{v_o}{R_3} = 0 \quad \Rightarrow \quad v_o = \left(-\frac{R_3}{R_1}\right) v_s - \frac{R_3}{R_2} v_b$$

so

$$a = -\frac{R_3}{R_1} \quad \text{and} \quad b = -\frac{R_3}{R_2} v_b$$

(b) The equation of the straight line is $v_o = -\frac{5}{2} v_s + 5$

We require $-\frac{5}{2} = -\frac{R_3}{R_1}$ e.g. $R_1 = 20\ \text{k}\Omega$ and $R_3 = 50\ \text{k}\Omega$.

Next, we require $5 = -\frac{R_3 v_b}{R_2}$ e.g. $R_2 = R_3 = 50\ \text{k}\Omega$ and $v_b = -5\ \text{V}$.

P6.5-14 The input to the circuit shown in Figure P6.5-14 is the voltage source voltage v_s. The output is the node voltage v_o. the output is related to the input by the equation $v_o = m\, v_s + b$ where *m* and *b* are constants. (a) Specify values of R_3 and v_a that cause the output to be related to the input by the equation $v_o = 4\, v_s + 7$. (b) Determine the values of m and b when $R_3 = 20\ \text{k}\Omega$ and $v_a = 2.5\ \text{V}$.

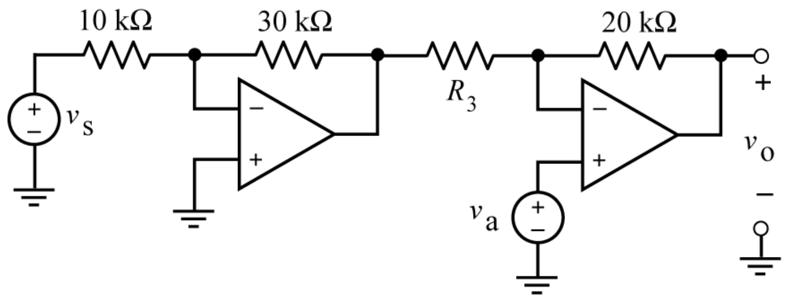
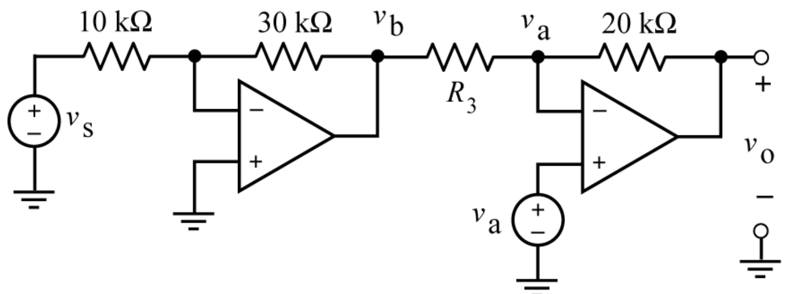


Figure P6.5-14

Solution: Label the node voltages:



Recognizing an inverting amplifier and a noninverting amplifier we write $v_b = \left(-\dfrac{30}{10}\right) v_s$.

Applying KCL at the inverting input node of the right op amp gives

$$\frac{v_b - v_a}{R_3} = \frac{v_a - v_o}{20} \Rightarrow v_o = \left(1+\frac{20}{R_3}\right) v_a - \left(\frac{20}{R_3}\right) v_b = \left(1+\frac{20}{R_3}\right) v_a - \left(\frac{20}{R_3}\right)\left(-\frac{30}{10}\right) v_s$$

$$\Rightarrow v_o = \left(1+\frac{20}{R_3}\right) v_a + \left(\frac{60}{R_3}\right) v_s$$

(a) We require $\left(\dfrac{60}{R_3}\right) = 4$ and $\left(1+\dfrac{20}{R_3}\right) v_a = 7\ \text{V}$ so $R_3 = 15\ \text{k}\Omega$ and then $v_a = 3\ \text{V}$.

(b) $m = \left(\dfrac{60}{R_3}\right) = \left(\dfrac{60}{20}\right) = 3\ \dfrac{\text{V}}{\text{V}}$ and $b = \left(1+\dfrac{20}{R_3}\right) v_a = \left(1+\dfrac{20}{20}\right) 2.5 = 5\ \text{V}$.

P6.5-15. The circuit shown in Figure 6.5-15 uses a potentiometer to implement a variable resistor having a resistance R that varies over the range

$$0 \le R \le 200\ \text{k}\Omega$$

The gain of this circuit is $G = \dfrac{v_o}{v_s}$. Varying the resistance R over it's range causes the value of the gain G to vary over the range

$$G_{min} \le \frac{v_o}{v_s} \le G_{max}$$

Determine the minimum and maximum values of the gain, G_{min} and G_{max}.

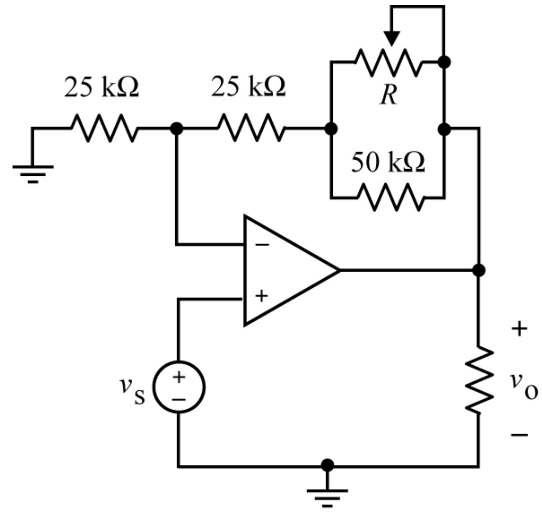


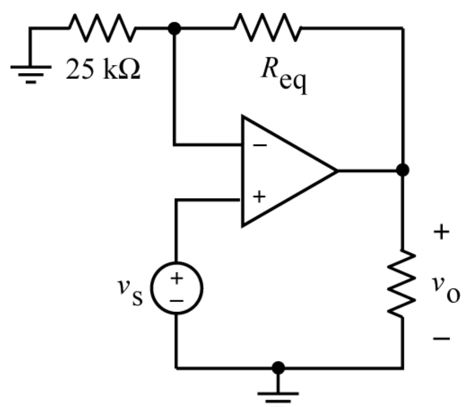
Figure P6.5-15

Solution:

Let

$$R_{eq} = 25 + R \,||\, 50$$

Then, recognizing this circuit as a noninverting amplifier, we get



$$G = 1 + \frac{R_{eq}}{25} = 1 + \frac{25 + R \,||\, 50}{25}$$

G_{min} corresponds to $R = 0$:

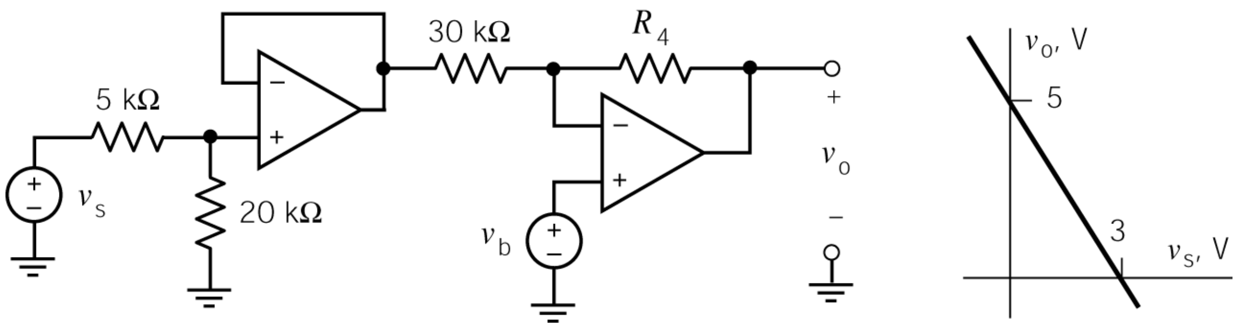
$$G_{min} = 1 + \frac{25 + 0 \,||\, 50}{25} = 1 + \frac{25 + 0}{25} = 2\ \text{V/V}$$

G_{max} corresponds to $R = 200\ \text{k}\Omega$:

$$G_{max} = 1 + \frac{25 + 200 \,||\, 50}{25} = 1 + \frac{25 + 40}{25} = 3.6\ \text{V/V}$$

P6.5-16 The input to the circuit shown in Figure P6.5-16a is the voltage, v_s. The output is the voltage v_o. The voltage v_b is used to adjust the relationship between the input and output. Determine values of R_4 and v_b that cause the circuit input and output have the relationship specified by the graph shown in Figure P6.5-21b.

Answers: $v_b =$ 1.62 V and $R_4 =$ 62.5 kΩ.



(a) (b)

Figure P6.5-16

Solution: Recognize the voltage divider, voltage follower and noninverting amplifier to write

$$v_o = \left(\frac{20\times10^3}{20\times10^3+5\times10^3}\right)\left(-\frac{R_4}{30\times10^3}\right)v_s + \left(1+\frac{R_4}{30\times10^3}\right)v_b = \left(-\frac{2R_4}{75\times10^3}\right)v_s + \left(1+\frac{R_4}{30\times10^3}\right)v_b$$

(Alternately, this equation can be obtained by writing two node equations: one at the noninverting node of the left op amp and the other at the inverting node of the right op amp.)

The equation of the straight line is $$v_o = -\frac{5}{3}v_s + 5$$

Comparing coefficients gives

$$-\frac{2R_4}{75\times10^3} = -\frac{5}{3} \Rightarrow R_4 = \frac{5}{3}\times\frac{75\times10^3}{2} = 62.5\times10^3 = 62.5\ \text{k}\Omega$$

and

$$5 = \left(1+\frac{R_4}{30\times10^3}\right)v_b = \left(1+\frac{62.5\times10^3}{30\times10^3}\right)v_b = 3.08333\,v_b \Rightarrow v_b = \frac{5}{3.08333} = 1.62\ \text{V}$$

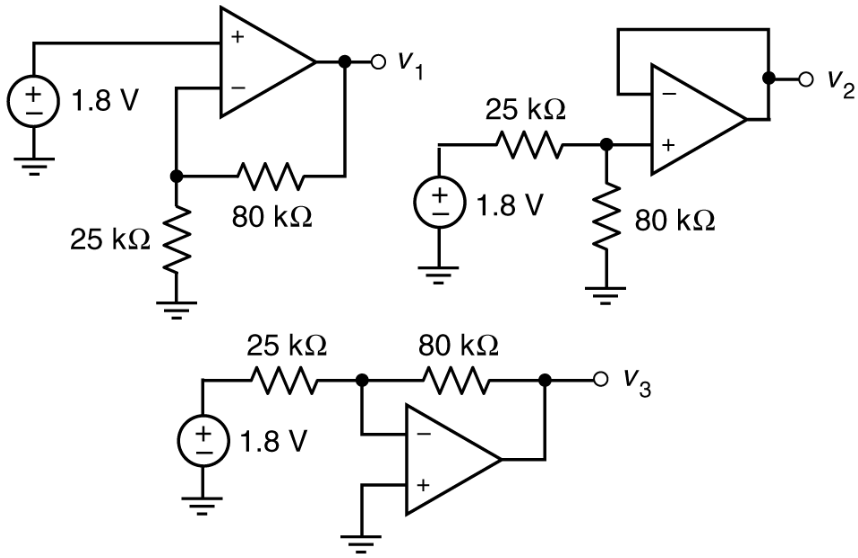


Figure P6.5-17

P6.5-17 Figure P6.5-17 shows three similar circuits. The outputs of the circuits are the voltages v_1, v_2 and v_3. Determine the values of these three outputs.

Solution:

$$v_1 = \left(1+\frac{80}{25}\right)(1.8) = 7.56 \text{ V}, \quad v_2 = \frac{80}{80+25}(1.8) = 1.3714 \text{ V} \text{ and } v_3 = -\frac{80}{25}(1.8) = -5.76 \text{ V}$$

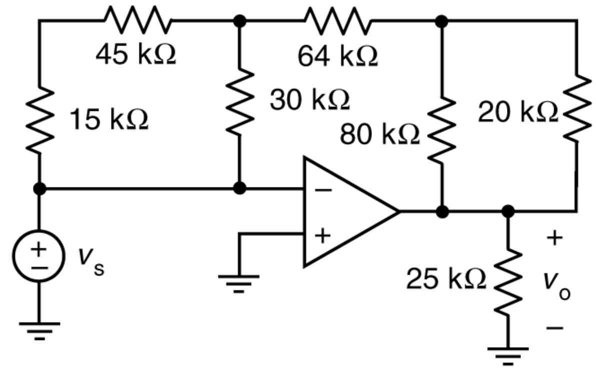
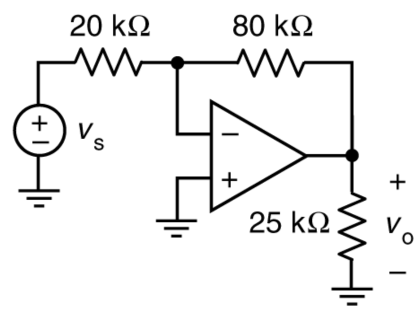


Figure P6.5-18

P6.5-18 . The input to the circuit shown in Figure P6.5-18 is the source voltage v_s. The output is the voltage across the 25 kΩ resistor, v_o. The output is related to the input by the equation $v_o = (g)\, v_i$ where g is the gain of the circuit. Determine the value of g.

Solution:

Using equivalent resistance we can draw the circuit as shown.



$$20 = (15+45)\,||\,30 \;\text{ and }\; 80 = 64 + (80\,||\,20)$$

Recognizing this circuit as an inverting amplifier, we write

$$g = \frac{v_o}{v_s} = -\frac{80}{20} = -4 \;\frac{\text{V}}{\text{V}}$$

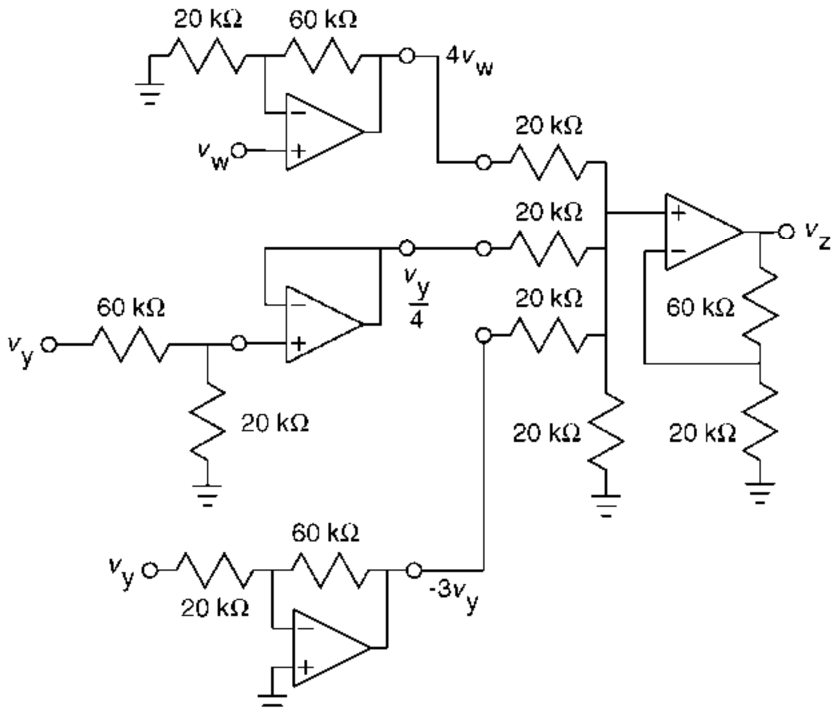
Section 6-6: Operational Amplifier Circuits and Linear Algebraic Equations

P 6.6-1 Design a circuit to implement the equation

$$z = 4w + \frac{x}{4} - 3y$$

The circuit should have one output, corresponding to *z*, and three inputs, corresponding to *w*, *x*, and *y*.

Solution:

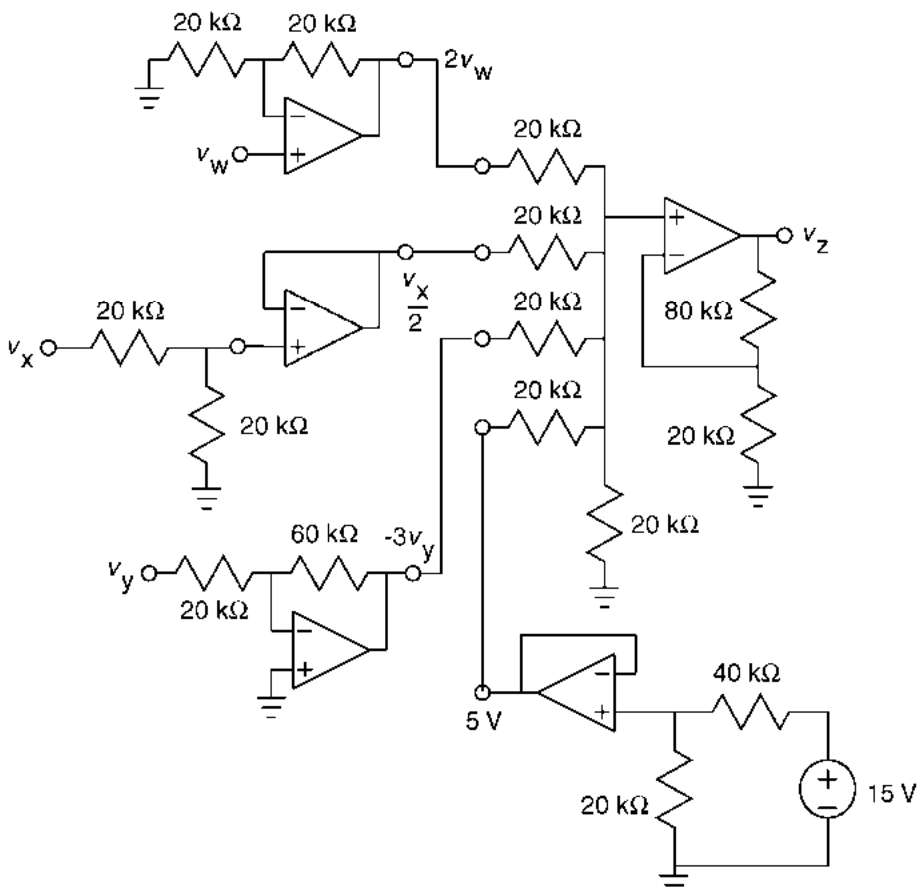


P 6.6-2 Design a circuit to implement the equation

$$0 = 4w + x + 10 - (6y + 2z)$$

The output of the circuit should correspond to *z*.

Solution:



Section 6-7: Characteristics of the Practical Operational Amplifier

P 6.7-1 Consider the inverting amplifier shown in Figure P 6.7-1. The operational amplifier is a typical OP-07E (Table 6.7-1). Use the offsets model of the operational amplifier to calculate the output offset voltage. (Recall that the input, v_{in}, is set to zero when calculating the output offset voltage.)

Answer: 0.45 mV

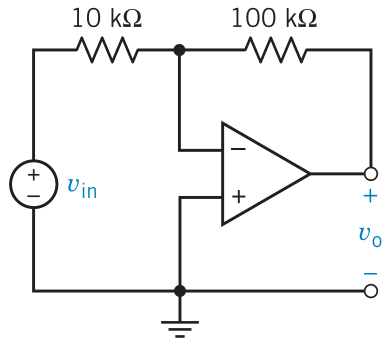
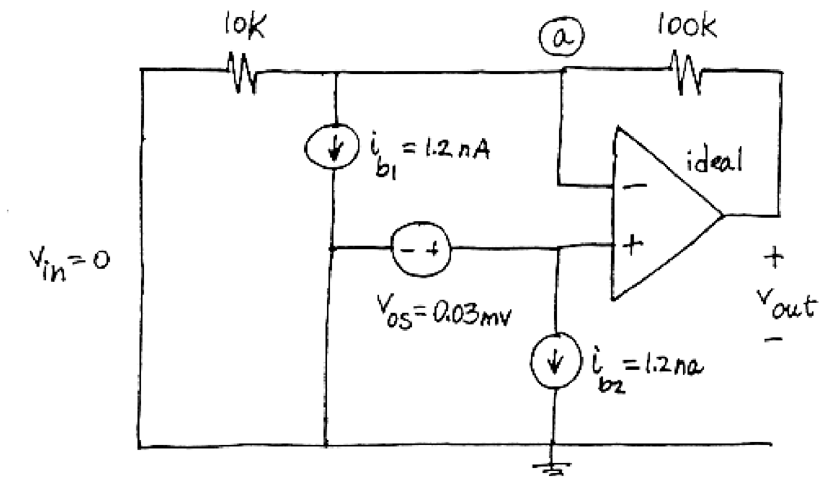


Figure P 6.7-1

Solution:



The node equation at node *a* is: $\frac{v_{out} - v_{os}}{100\times 10^3} = \frac{v_{os}}{10\times 10^3} + i_{b1}$

Solving for v_{out}:

$$v_{out} = \left(1 + \frac{100\times 10^3}{10\times 10^3}\right) v_{os} + \left(100\times 10^3\right) i_{b1} = 11 v_{os} + \left(100\times 10^3\right) i_{b1}$$

$$= 11\left(0.03\times 10^{-3}\right) + \left(100\times 10^3\right)\left(1.2\times 10^{-9}\right) = 0.45 \text{ mV}$$

P 6.7-2 Consider the noninverting amplifier shown in Figure P 6.7-2. The operational amplifier is a typical LF351 (Table 6.7-1). Use the offsets model of the operational amplifier to calculate the output offset voltage. (Recall that the input, v_{in}, is set to zero when calculating the output offset voltage.)

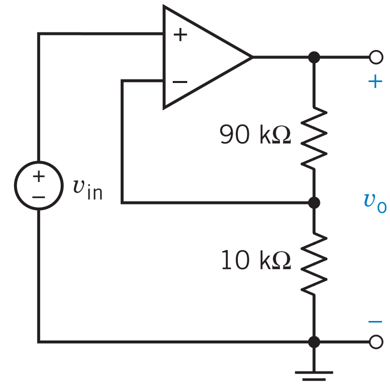
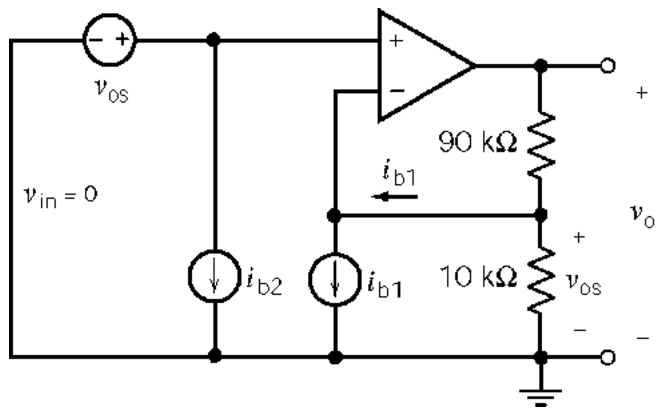


Figure P 6.7-2

Solution:



The node equation at node *a* is: $$\frac{v_{os}}{10000} + i_{b1} = \frac{v_0 - v_{os}}{90000}$$

Solving for v_o:

$$v_o = \left(1 + \frac{90\times10^3}{10\times10^3}\right) v_{os} + \left(90\times10^3\right) i_{b1} = 10\, v_{os} + \left(90\times10^3\right) i_{b1}$$

$$= 10(5\times10^{-3}) + \left(90\times10^3\right)(.05\times10^{-9}) = 50.0045\times10^{-3} \simeq \underline{50 \text{ mV}}$$

P 6.7-3 Consider the inverting amplifier shown in Figure P 6.7-3. Use the finite gain model of the operational amplifier (Figure 6.7-1*c*) to calculate the gain of the inverting amplifier. Show that

$$\frac{v_o}{v_{in}} = \frac{R_{in}(R_o - AR_2)}{(R_1 + R_{in})(R_o - R_2) + R_1 R_{in}(1 + A)}$$

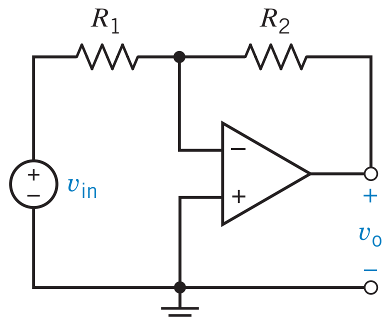
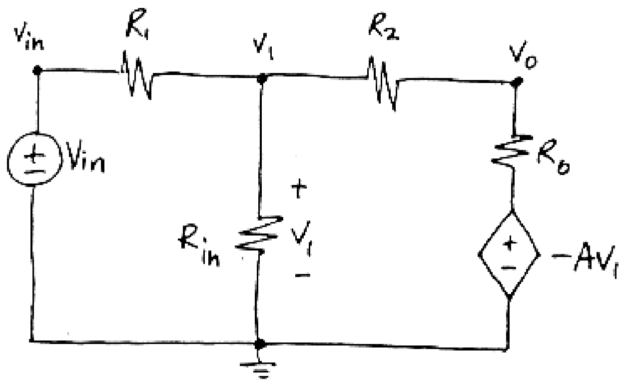


Figure P 6.7-3

Solution:



$$\left.\begin{aligned} \frac{v_1 - v_{in}}{R_1} + \frac{v_1}{R_{in}} + \frac{v_1 - v_0}{R_2} &= 0 \\ \frac{v_0 + Av_1}{R_0} + \frac{v_0 - v_1}{R_2} &= 0 \end{aligned}\right\} \Rightarrow \frac{v_0}{v_{in}} = \frac{R_{in}(R_0 - AR_2)}{(R_1 + R_{in})(R_0 + R_2) + R_1 R_{in}(1 + A)}$$

P 6.7-4 Consider the inverting amplifier shown in Figure P 6.7-3. Suppose the operational amplifier is ideal, $R_1 = 5\ \text{k}\Omega$, and $R_2 = 50\ \text{k}\Omega$. The gain of the inverting amplifier will be

$$\frac{v_o}{v_{in}} = -10$$

Use the results of Problem P 6.7-3P 6.7-3 to find the gain of the inverting amplifier in each of the following cases:

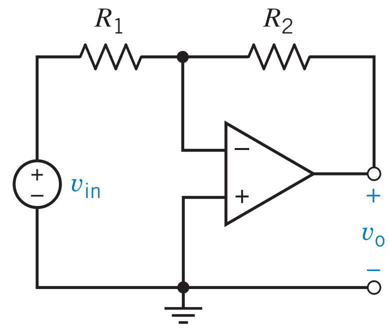


Figure P 6.7-3

(a) The operational amplifier is ideal, but 2 percent resistors are used and

$$R_1 = 5.1\ \text{k}\Omega \text{ and } R_2 = 49\ \text{k}\Omega.$$

(b) The operational amplifier is represented using the finite gain model with

$$A = 200{,}000,\ R_i = 2\ \text{M}\Omega, \text{ and } R_o = 75\ \Omega;\ R_1 = 5\ \text{k}\Omega \text{ and } R_2 = 50\ \text{k}\Omega.$$

(c) The operational amplifier is represented using the finite gain model with

$$A = 200{,}000,\ R_i = 2\ \text{M}\Omega, \text{ and } R_o = 75\ \Omega;\ R_1 = 5.1\ \text{k}\Omega \text{ and } R_2 = 49\ \text{k}\Omega.$$

Solution:

a)
$$\frac{v_0}{v_{in}} = -\frac{R_2}{R_1} = -\frac{49\times10^3}{5.1\times10^3} = \underline{-9.6078}$$

b)
$$\frac{v_0}{v_{in}} = \frac{\left(2\times10^6\right)\left(75-\left(200{,}000\right)\left(50\times10^3\right)\right)}{(5\times10^3+2\times10^6)(75+50\times10^3)+(5\times10^3)(2\times10^6)(1+200{,}000)} = \underline{-9.9957}$$

c)
$$\frac{v_0}{v_{in}} = \frac{2\times10^6(75-(200{,}000)(49\times10^3))}{(5.1\times10^3+2\times10^6)(75+49\times10^3)+(5.1\times10^3)(2\times10^6)(1+200{,}000)} = \underline{-9.6037}$$

P 6.7-5 The circuit in Figure P 6.7-5 is called a difference amplifier and is used for instrumentation circuits. The output of a measuring element is represented by the common mode signal v_{cm} and the differential signal $(v_n + v_p)$. Using an ideal operational amplifier, show that

$$v_o = -\frac{R_4}{R_1}(v_n + v_p)$$

When $$\frac{R_4}{R_1} = \frac{R_3}{R_2}$$

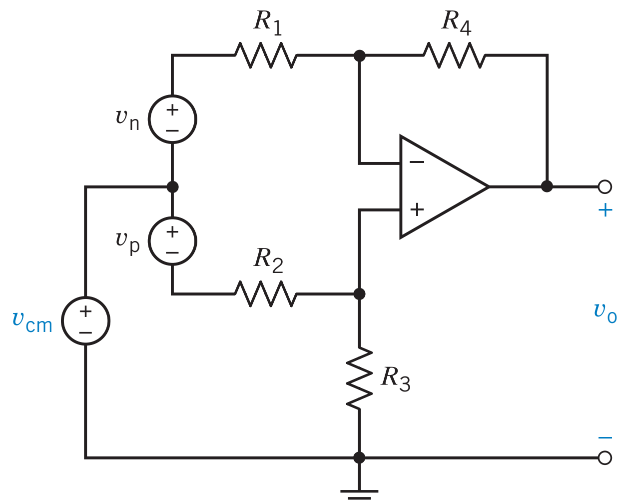
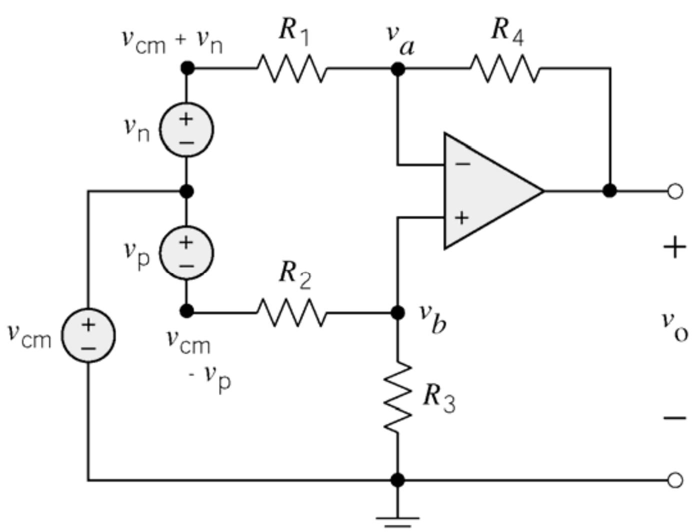


Figure P 6.7-5

Solution:



Apply KCL at node *b*:

$$v_b = \frac{R_3}{R_2 + R_3}(v_{cm} - v_p)$$

Apply KCL at node *a*:

$$\frac{v_a - v_0}{R_4} + \frac{v_a - (v_{cm} + v_n)}{R_1} = 0$$

The voltages at the input nodes of an ideal op amp are equal so $v_a = v_b$.

$$v_0 = -\frac{R_4}{R_1}(v_{cm} + v_n) + \frac{R_4 + R_1}{R_1} v_a$$

$$v_0 = -\frac{R_4}{R_1}(v_{cm} + v_n) + \frac{(R_4 + R_1)R_3}{R_1(R_2 + R_3)}(v_{cm} - v_p)$$

when $\frac{R_4}{R_1} = \frac{R_3}{R_2}$ then $\frac{(R_4 + R_1)R_3}{R_1(R_2 + R_3)} = \frac{\frac{R_4}{R_1} + 1}{\frac{R_3}{R_2} + 1} \times \frac{R_3}{R_2} = \frac{R_4}{R_1}$

so

$$v_0 = -\frac{R_4}{R_1}(v_{cm} + v_n) + \frac{R_4}{R_1}(v_{cm} - v_p) = -\frac{R_4}{R_1}(v_n + v_p)$$

Section 6-10 How Can We Check…?

P 6.10-1 Analysis of the circuit in Figure P 6.10-1 shows that $i_o = -1$ mA and $v_o = 7$ V. Is this analysis correct?

Hint: Is KCL satisfied at the output node of the op amp?

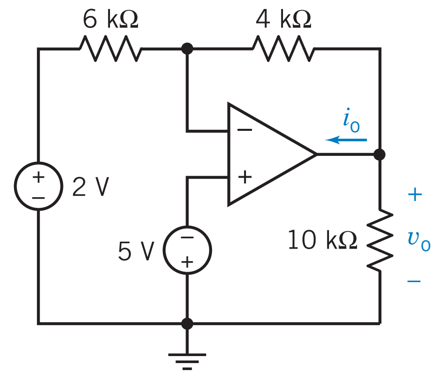
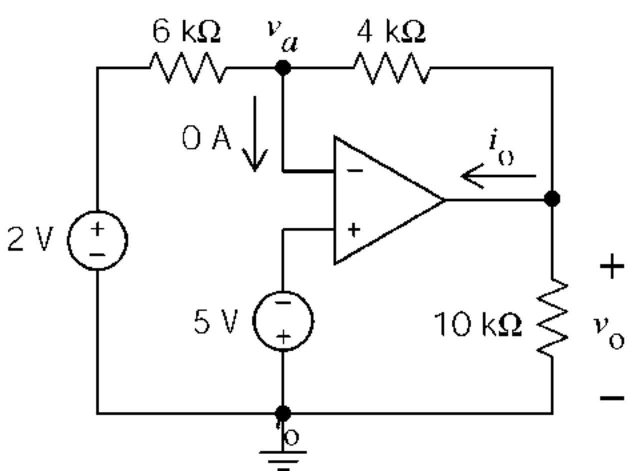


Figure P 6.10-1

Solution:



Apply KCL at the output node of the op amp

$$i_o = \frac{v_o}{10000} + \frac{v_o - (-5)}{4000} = 0$$

Try the given values: $i_o = -1$ mA and $v_o = 7$ V

$$-1\times10^{-3} \neq 3.7\times10^{-3} = \frac{7}{10000} + \frac{7-(-5)}{4000}$$

KCL is not satisfied. These cannot be the correct values of i_o and v_o.

P 6.10-2 Your lab partner measured the output voltage of the circuit shown in Figure P 6.10-2 to be $v_o = 9.6$ V. Is this the correct output voltage for this circuit?

Hint: Ask your lab partner to check the polarity of the voltage that he or she measured.

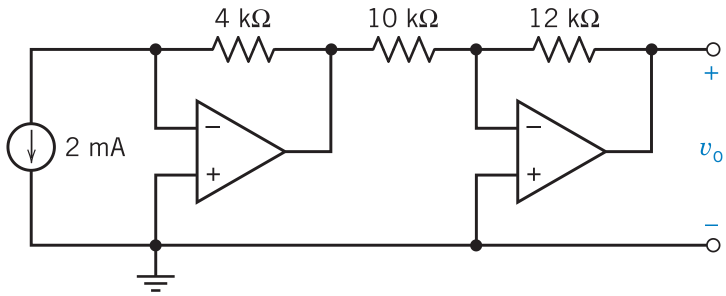
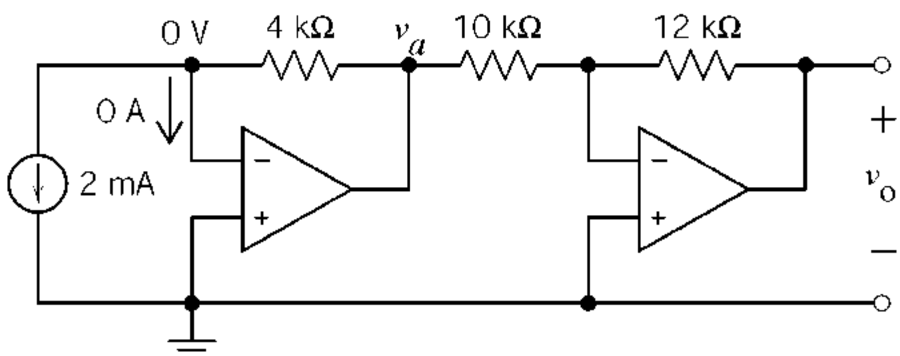


Figure P 6.10-2

Solution:



$$v_a = \left(4\times10^3\right)\left(2\times10^{-3}\right) = 8 \text{ V}$$

$$v_o = -\frac{12\times10^3}{10\times10^3} v_a = -1.2\,(8) = -9.6 \text{ V}$$

So $v_o = -9.6$ V instead of 9.6 V.

P 6.10-3 Nodal analysis of the circuit shown in Figure P 6.10-3 indicates that $v_o = -12$ V. Is this analysis correct?

Hint: Redraw the circuit to identify an inverting amplifier and a noninverting amplifier.

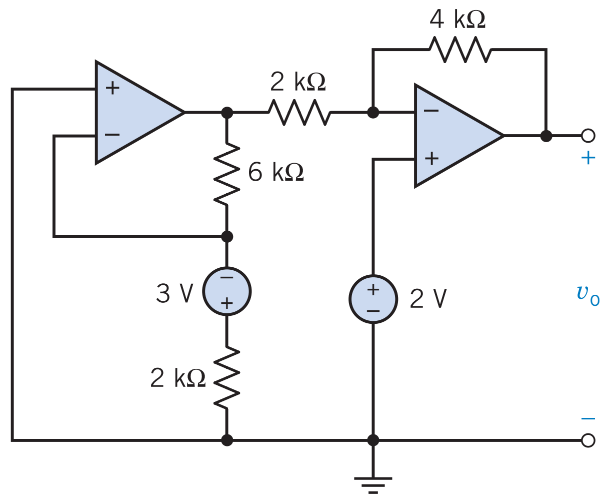
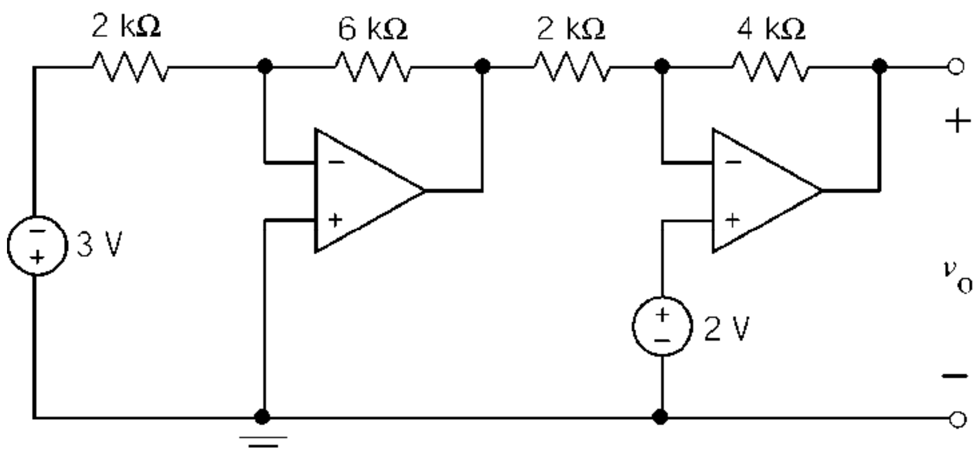


Figure P 6.10-3

Solution: First, redraw the circuit as:



Then using superposition, and recognizing of the inverting and noninverting amplifiers:

$$v_o = \left(-\frac{6}{2}\right)\left(-\frac{4}{2}\right)(-3) + \left(1+\frac{4}{2}\right)(2) = -18 + 6 = -12 \text{ V}$$

The given answer is correct.

P 6.10-4 Computer analysis of the circuit in Figure P 6.10-4 indicates that the node voltages are $v_a = -5$ V, $v_b = 0$ V, $v_c = 2$ V, $v_d = 5$ V, $v_e = 2$ V, $v_f = 2$ V, and $v_g = 11$ V. Is this analysis correct? Justify your answer.
Hint: Verify that the resistor currents indicated by these node voltages satisfy KCL at nodes b, c, d, and f.

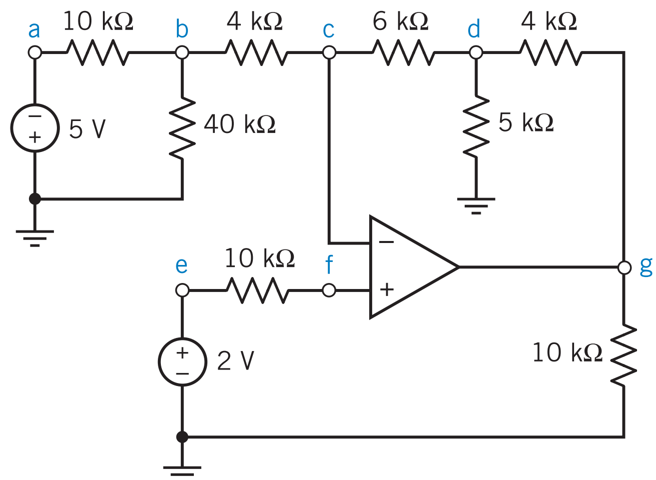


Figure P 6.10-4

Solution:

First notice that $v_e = v_f = v_c$ is required by the ideal op amp. (There is zero current into the input lead of an ideal op amp so there is zero current in the 10 kΩ connected between nodes e and f, hence zero volts across this resistor. Also, the node voltages at the input nodes of an ideal op amp are equal.)

The given voltages satisfy all the node equations at nodes b, c and d:

node b:
$$\frac{0-(-5)}{10000} + \frac{0}{40000} + \frac{0-2}{4000} = 0$$

node c:
$$\frac{0-2}{4000} = \frac{2-5}{6000} + 0$$

node d:
$$\frac{2-5}{6000} = \frac{5}{5000} + \frac{5-11}{4000}$$

Therefore, the analysis is correct.

P 6.10-5 Computer analysis of the noninverting summing amplifier shown in Figure P 6.10-5 indicates that the node voltages are $v_a = 2$ V, $v_b = -0.25$ V, $v_c = -5$ V, $v_d = -2.5$ V, and $v_e = -0.25$ V.

(a) Is this analysis correct?

(b) Does this analysis verify that the circuit is a noninverting summing amplifier? Justify your answers.

1st Hint: Verify that the resistor currents indicated by these node voltages satisfy KCL at nodes b and e.

2nd Hint: Compare to Figure 6.5-1*e* to see that $R_a = 10$ kΩ and $R_b = 1$ kΩ. Determine K_1, K_2, and K_4 from the resistance values. Verify that $v_d = K_4(K_1 v_a + K_2 v_c)$.

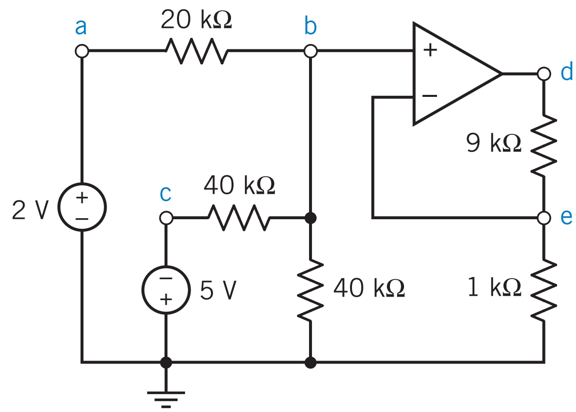


Figure P 6.10-5

Solution:

The given voltages satisfy the node equations at nodes b and e:

node b:

$$\frac{-.25-2}{20000}+\frac{-.25}{40000}+\frac{-.25-(-5)}{40000}=0$$

node e:

$$\frac{-2.5-(-0.25)}{9000}\neq\frac{-0.25}{1000}+0$$

Therefore, the analysis is not correct.

Notice that

$$\frac{-2.5-(+0.25)}{9000}=\frac{+0.25}{1000}+0$$

So it appears that $v_e = +0.25$ V instead of $v_e = -0.25$ V.

Also, the circuit is an noninverting summer with $R_a = 10$ kΩ and $R_b = 1$ kΩ, $K_1 = 1/2$, $K_2 = 1/4$ and $K_4 = 9$. The given node voltages satisfy the equation

$$-2.5 = v_d = K_4\left(K_1 v_a + K_2 v_c\right) = 10\left(\frac{1}{2}(2)+\frac{1}{4}(-5)\right)$$

None-the-less, the analysis is not correct.

PSpice Problems

SP 6-1 The circuit in Figure SP 6-1 has three inputs: v_w, v_x, and v_y. The circuit has one output, v_z. The equation

$$v_z = av_w + bv_x + cv_y$$

expresses the output as a function of the inputs. The coefficients *a, b*, and *c* are real constants.

(a) Use PSpice and the principle of superposition to determine the values of *a, b*, and *c*.

(b) Suppose $v_w = 2$ V, $v_x = x$, $v_y = y$ and we want the output to be $v_z = z$. Express *z* as a function of *x* and *y*.

Hint: The output is given by $v_z = a$ when $v_w = 1$ V, $v_x = 0$ V, and $v_y = 0$ V.

Answer: (a) $v_z = v_w + 4\,v_x - 5\,v_y$ (b) $z = 4\,x - 5\,y + 2$

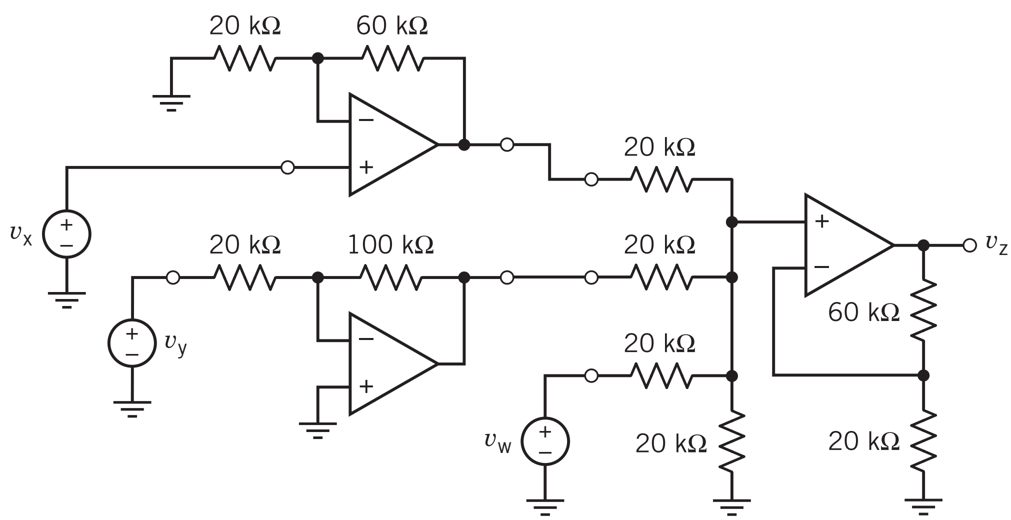


Figure SP 6-1

Solution:

(*a*) $$v_z = a\,v_w + b\,v_x + c\,v_y$$

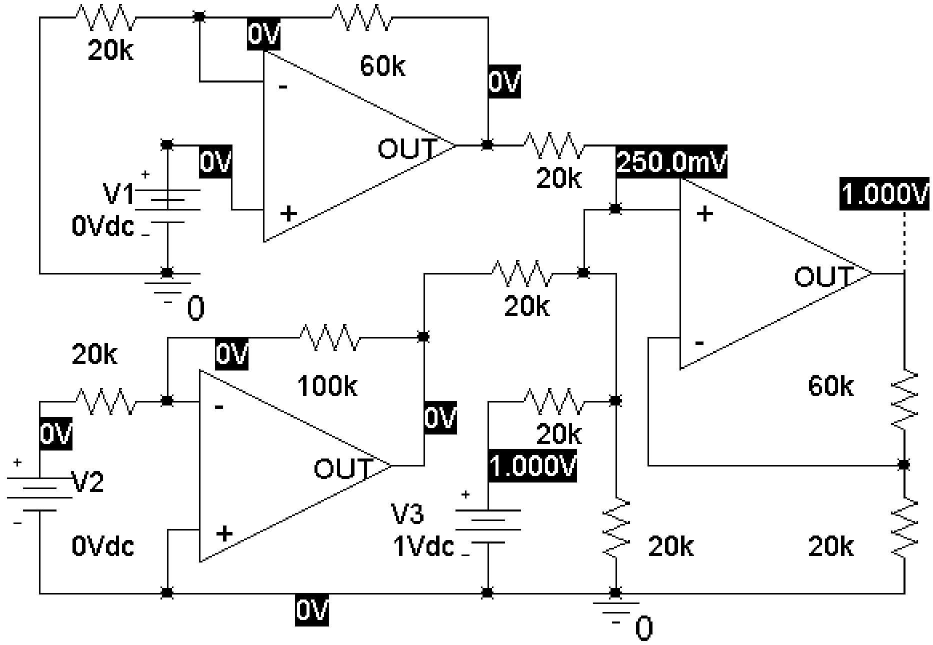
The following three PSpice simulations show

$$1\text{ V} = v_z = a \text{ when } v_w = 1\text{ V},\ v_x = 0\text{ V and } v_y = 0\text{ V}$$

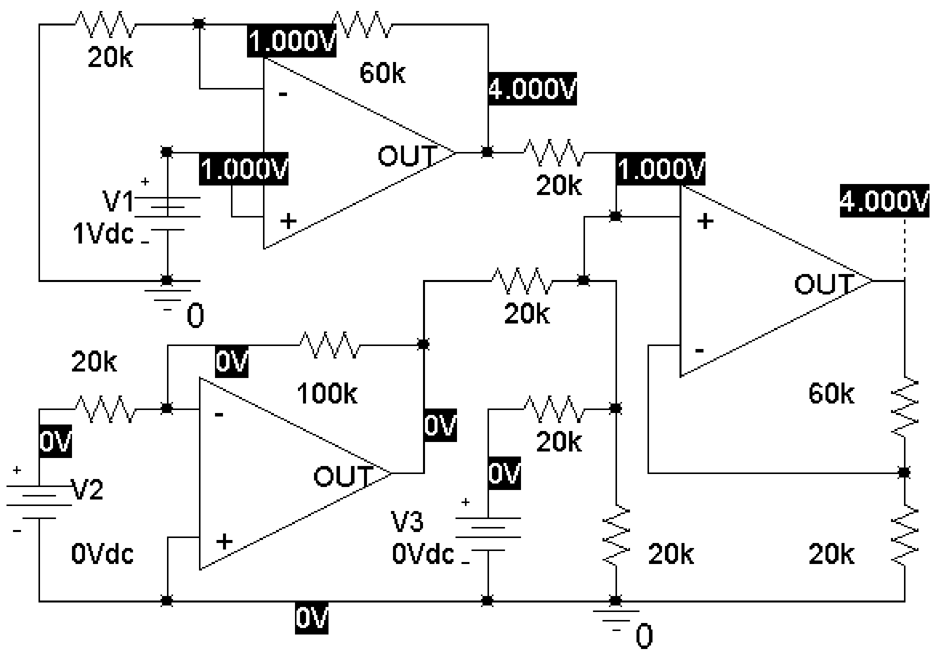
$$4\text{ V} = v_z = b \text{ when } v_w = 0\text{ V},\ v_x = 1\text{ V and } v_y = 0\text{ V}$$

$$-5\text{ V} = v_z = c \text{ when } v_w = 0\text{ V},\ v_x = 0\text{ V and } v_y = 1\text{ V}$$

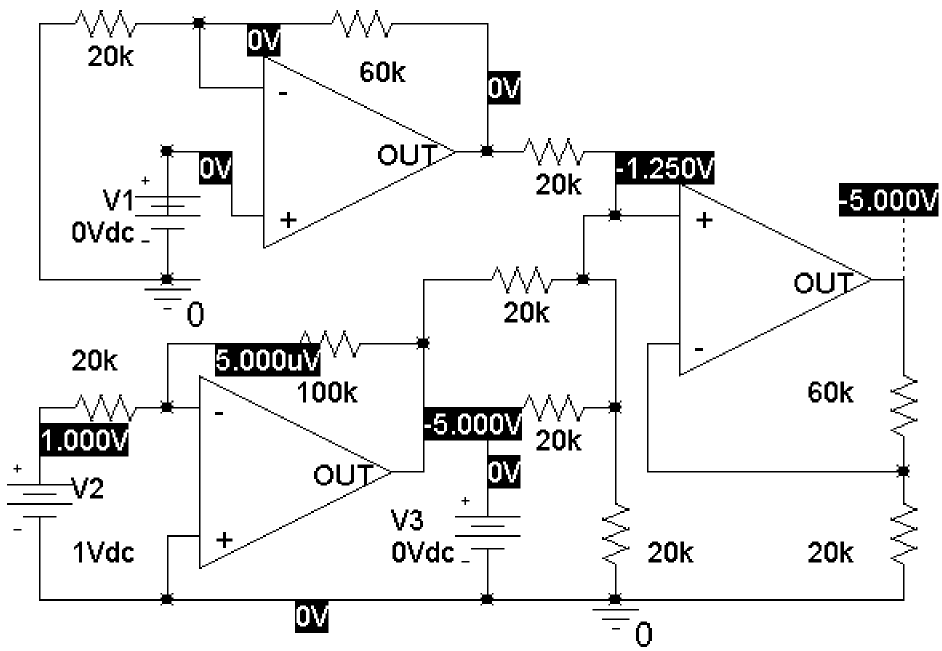
1 V = v_z = *a* when v_w= 1 V, v_x = 0 V and v_y = 0 V:



4 V = v_z = *b* when v_w= 0 V, v_x = 1 V and v_y = 0 V:



-5 V = v_z = c when v_w= 0 V, v_x = 0 V and v_y = 1 V:



Therefore $$v_z = v_w + 4\,v_x - 5\,v_y$$

(b) When v_w= 2 V: $$v_z = 4\,v_x - 5\,v_y + 2$$

SP 6-2 The input to the circuit in Figure SP 6-2 is v_s, and the output is v_o.

(a) Use superposition to express v_o as a function of v_s.

(b) Use the DC Sweep feature of PSpice to plot v_o as a function of v_s.

(c) Verify that the results of parts (a) and (b) agree with each other.

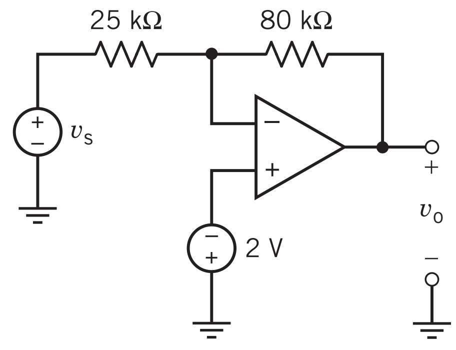
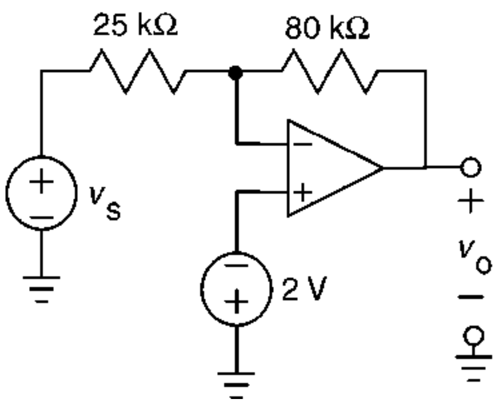


Figure SP 6-2

Solution:



a) Using superposition and recognizing the inverting and noninverting amplifiers:

$$v_o = -\frac{80}{25} v_s + \left(1 + \frac{80}{25}\right)(-2) = -3.2\, v_s - 8.4$$

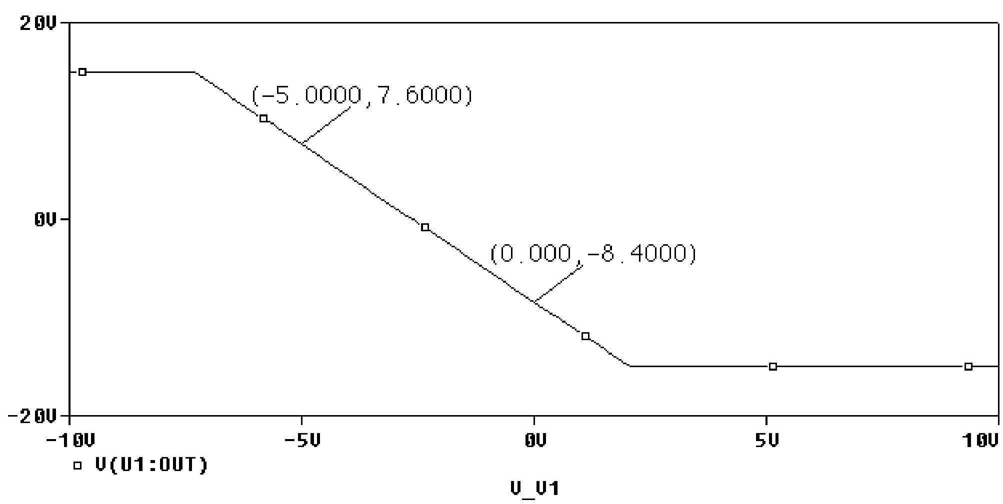
b) Using the DC Sweep feature of PSpice produces the plot show below. Two points have been labeled in anticipation of c).

c) Notice that the equation predicts

$$(-3.2)(-5) - 8.4 = 7.6$$

and
$$(-3.2)(0) - 8.4 = -8.4$$

Both agree with labeled points on the plot.



SP 6-3 A circuit with its nodes identified is shown in Figure SP 6-3. Determine v_{34}, v_{23}, v_{50}, and i_o.

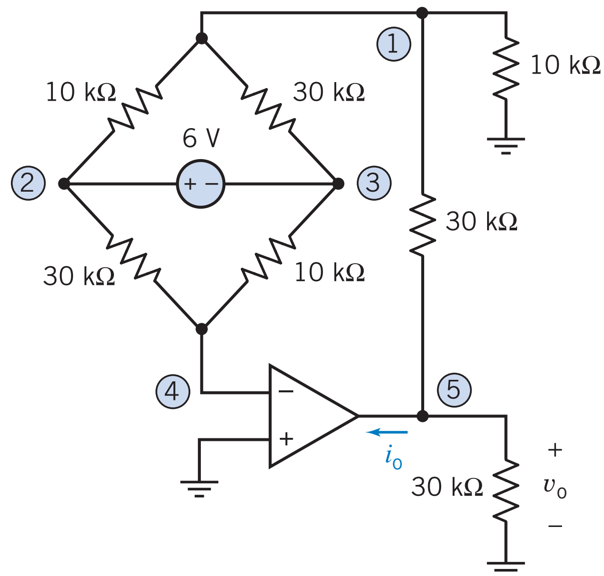
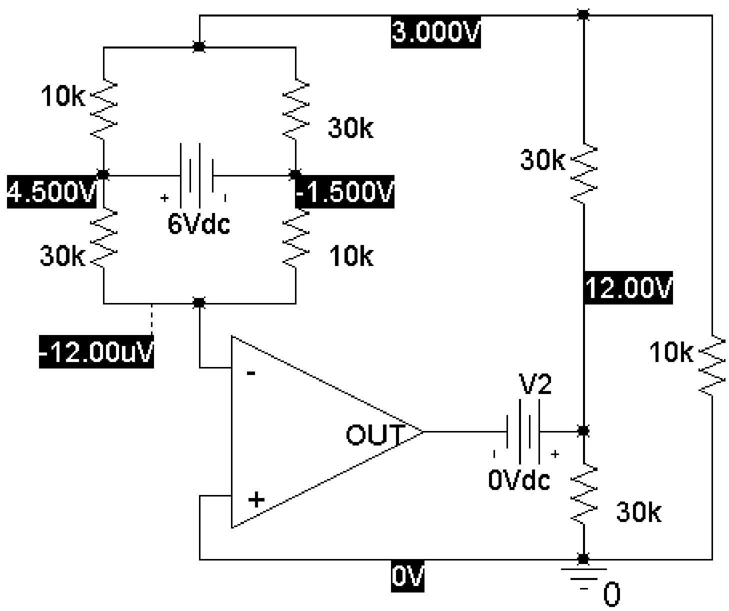


Figure SP 6-3

Solution:



VOLTAGE SOURCE CURRENTS

NAME	CURRENT
V_V1	-3.000E-04
V_V2	-7.000E-04

$$v_{34} = -1.5 - -12\times10^{-6} \cong -1.5 \text{ V}$$

$$v_{23} = 4.5 - (-1.5) = 6 \text{ V}$$

$$v_{50} = 12 - 0 = 12 \text{ V}$$

$$i_o = -7\times10^{-4} = -0.7 \text{ mA}$$

SP 6-4 Use PSpice to analyze the VCCS shown in Figure SP 6-4. Consider two cases:

(a) The operational amplifier is ideal.

(b) The operational amplifier is a typical μA741 represented by the offsets and finite gain model.

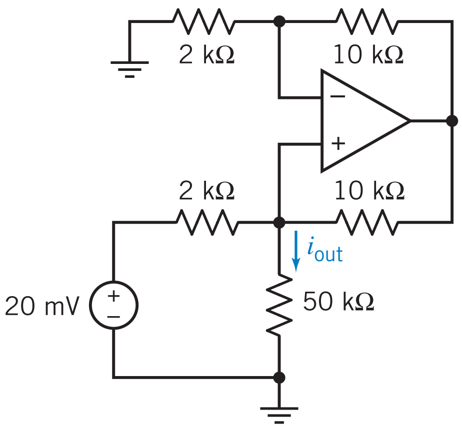


Figure SP 6-4

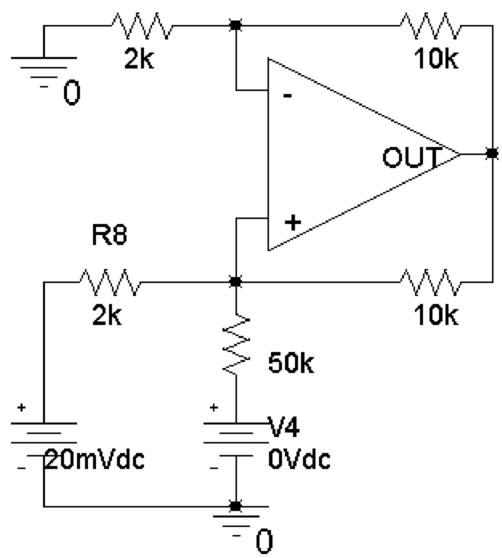
Solution:

V4 is a short circuit used to measure i_o.

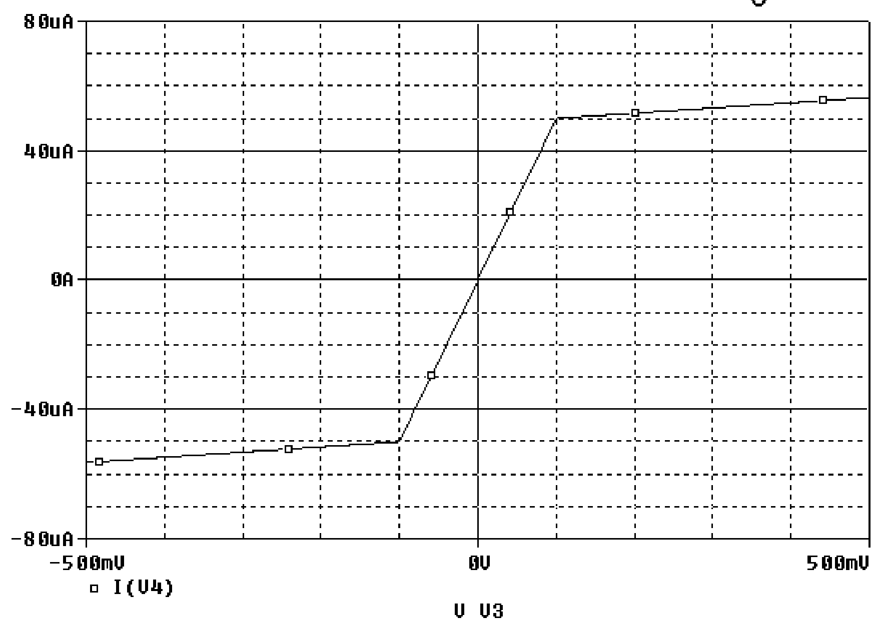
The input of the VCCS is the voltage of the left-hand voltage source. (The nominal value of the input is 20 mV.) The output of the VCCS is i_o.

A plot of the output of the VCCS versus the input is shown below.

The gain of the VCVS is



$$gain = \frac{50\times10^{-6} - \left(-50\times10^{-6}\right)}{100\times10^{-3} - \left(-100\times10^{-3}\right)} = \frac{1}{2}\times10^{-3}\ \frac{\text{A}}{\text{V}}$$



Design Problems

DP 6-1 Design the operational amplifier circuit in Figure DP 6-1 so that

$$i_{out} = \frac{1}{4} \cdot i_{in}$$

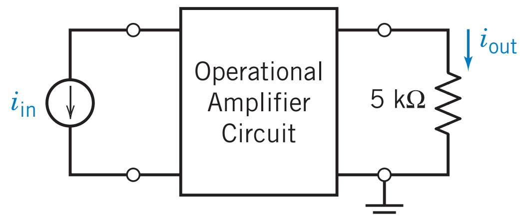
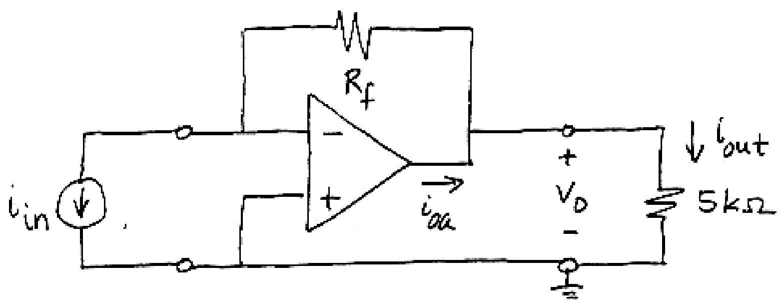


Figure DP 6-1

Solution:

From Figure 6.6-1*g*, this circuit



is described by $v_o = R_f \, i_{in}$. Since $i_{out} = \frac{v_o}{5000}$, we require $\frac{1}{4} = \frac{i_{out}}{i_{in}} = \frac{R_f}{5000}$, or R_f = 1250 Ω

Notice that $i_{oa} = i_{in} + \frac{(1250)\, i_{in}}{5000} = \frac{5}{4} i_{in}$. To avoid current saturation requires $\frac{5}{4} i_{in} < i_{sat}$ or $i_{in} < \frac{4}{5} i_{sat}$. For example, if i_{sat} = 2 mA, then i_{in} < 1.6 mA is required to avoid current saturation.

DP 6-2 Figure DP 6-2*a* shows a circuit that has one input, v_i, and one output, v_o. Figure DP 6-2*b* shows a graph that specifies a relationship between v_o and v_i. Design a circuit having input, v_i, and output, v_o, that have the relationship specified by the graph in Figure DP 6-2*b*.

Hint: A constant input is required. Assume that a 5-V source is available.

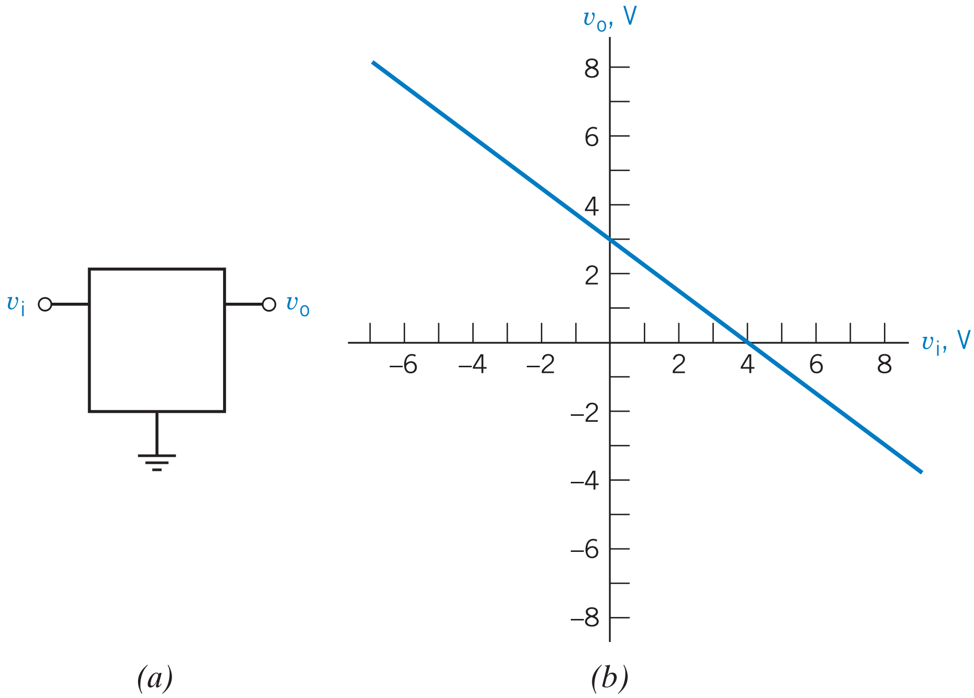
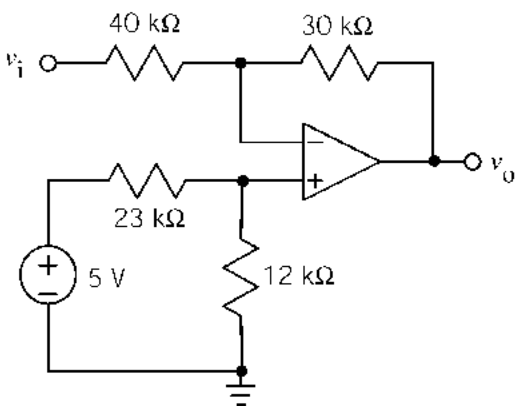


Figure DP 6-2

Solution:

$$v_o = -\frac{3}{4}v_i + 3 = -\frac{3}{4}v_i + \left(1+\frac{3}{4}\right)\left(\frac{12}{35}\right)5 = -\frac{3}{4}v_i + \left(1+\frac{3}{4}\right)\left(\frac{12}{12+23}\right)5$$



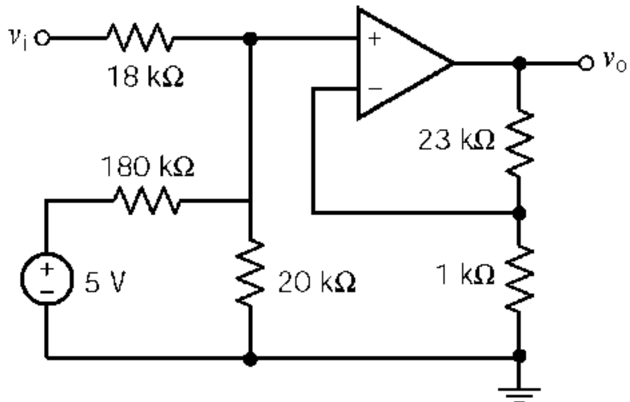
DP 6-3 Design a circuit having input, v_i, and output, v_o, that are related by the equations

(a) $v_o = 12v_i + 6$, (b) $v_o = 12v_i - 6$, (c) $v_o = -12v_i + 6$, and (d) $v_o = -12v_i - 6$.

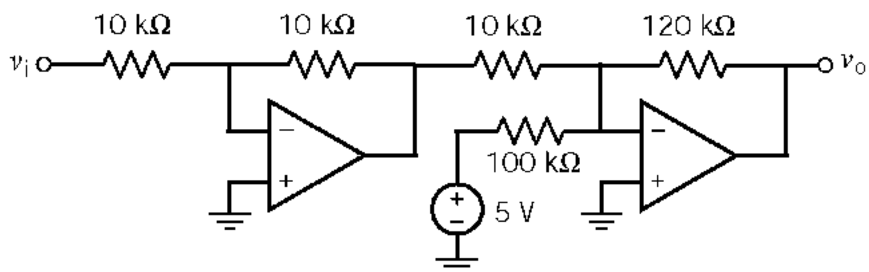
Hint: A constant input is required. Assume that a 5-V source is available.

Solution:

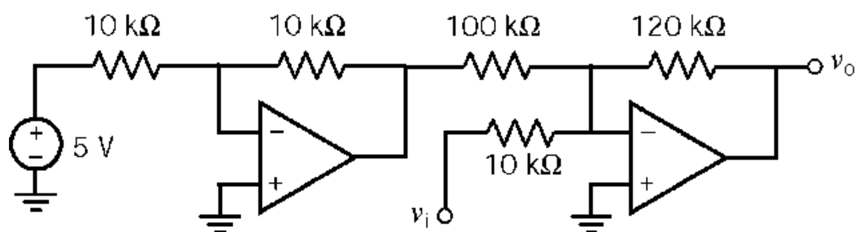
(a) $12v_i + 6 = 24\left(\frac{1}{2}v_i + \frac{1}{20}(5)\right) \Rightarrow K_4 = 24$, $K_1 = \frac{1}{2}$, and $K_2 = \frac{1}{20}$. Take R_a = 18 kΩ and R_b = 1 kΩ to get



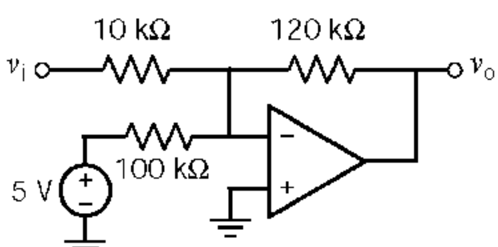
(b)



(c)



(d)

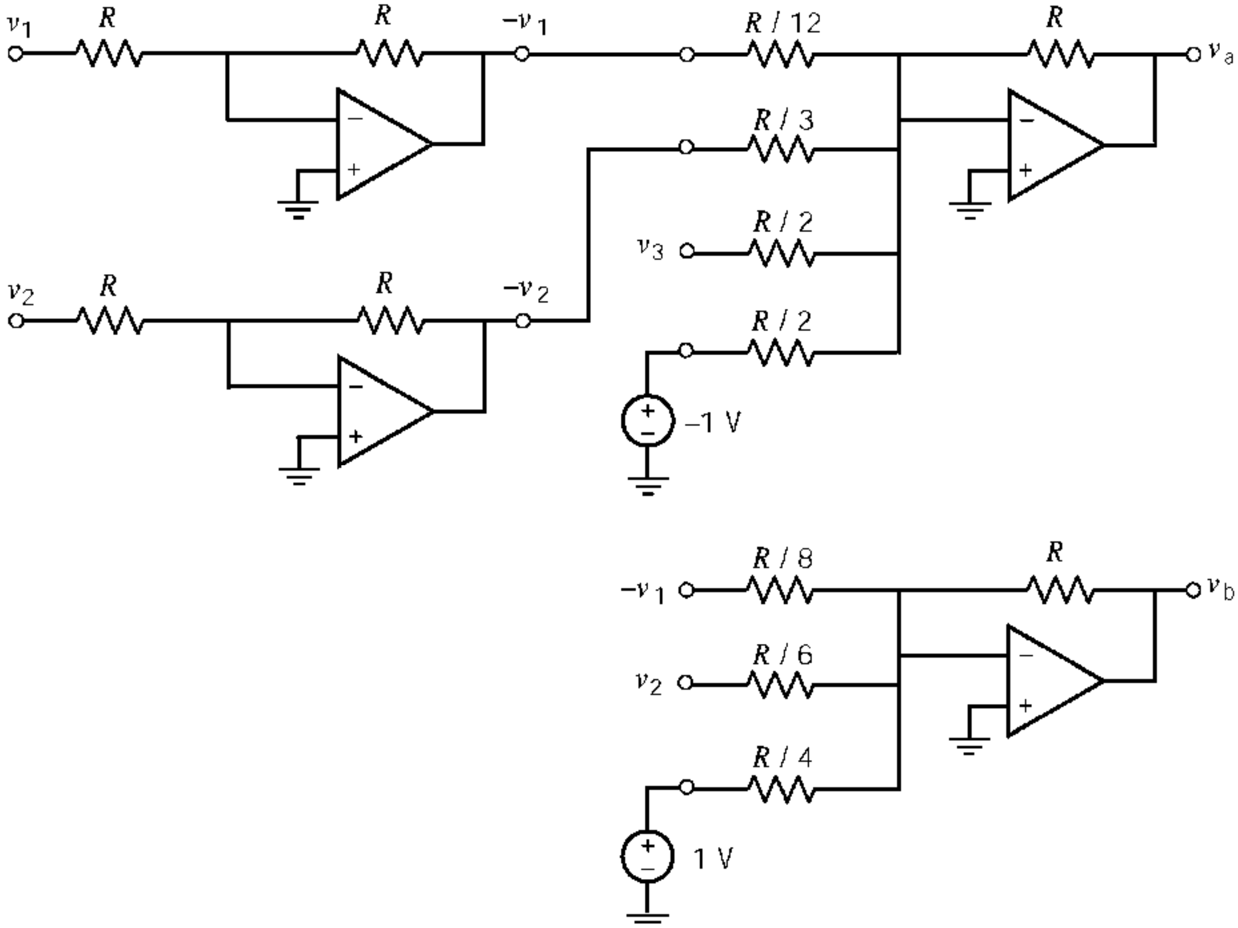


DP 6-4 Design a circuit having three inputs, v_1, v_2, v_3, and two outputs, v_a, v_b, that are related by the equation

$$\begin{bmatrix} v_a \\ v_b \end{bmatrix} = \begin{bmatrix} 12 & 3 & -2 \\ 8 & -6 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} + \begin{bmatrix} 2 \\ -4 \end{bmatrix}$$

Hint: A constant input is required. Assume that a 5-V source is available.

Solution:



DP 6-5 A microphone has an unloaded voltage v_s = 20 mV, as shown in Figure DP 6-5*a*. An op amp is available as shown in Figure DP 6-5*b*. It is desired to provide an output voltage of 4 V. Design an inverting circuit and a noninverting circuit and contrast the input resistance at terminals x–y seen by the microphone. Which configuration would you recommend in order to achieve good performance in spite of changes in the microphone resistance R_s?

Hint: We plan to connect terminal a to terminal x and terminal b to terminal y, or visa versa.

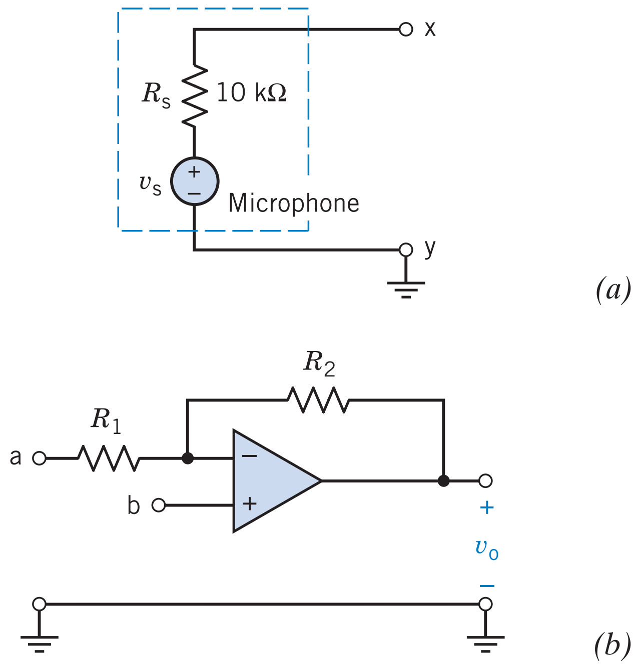
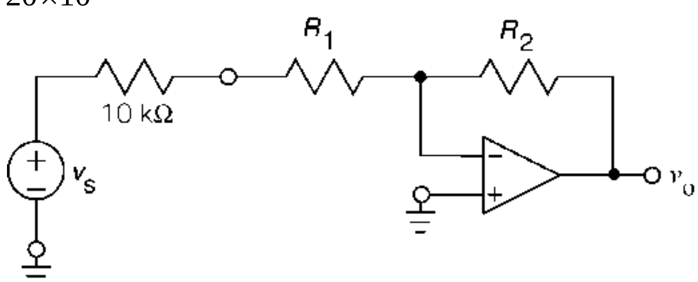


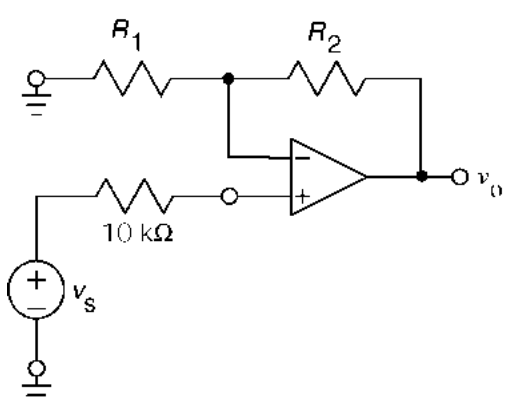
Figure DP 6-5

Solution:

We require a gain of $\dfrac{4}{20\times10^{-3}} = 200$. Using an inverting amplifier:



Here we have $200 = \left| -\dfrac{R_2}{10\times10^3 + R_1} \right|$. For example, let R_1 = 0 and R_2 = 1 MΩ. Next, using the noninverting amplifier:

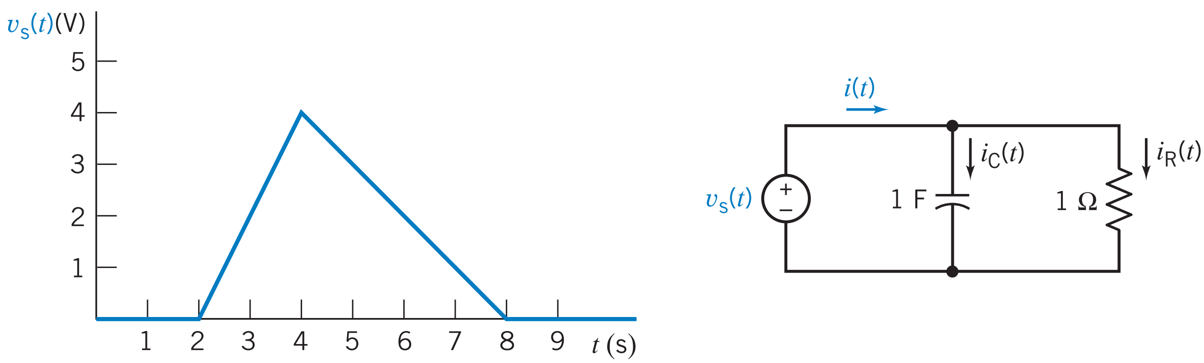


Here we have $200 = 1 + \frac{R_2}{R_1}$. For example, let R_1 = 1 kΩ and R_2 = 199 kΩ.

The gain of the inverting amplifier circuit does not depend on the resistance of the microphone. Consequently, the gain does not change when the microphone resistance changes.

Chapter 7 Exercises

Exercise 7.2-1 Determine the current *i(t)* for t > 0 for the circuit of Figure E 7.2-1*b* when $v_s(t)$ is the voltage shown in Figure E 7.2-1*a*.
Hint: Determine $i_C(t)$ and $i_R(t)$ separately, then use KCL.



(*a*) (*b*)

Figure E 7.2-1

Answer: $$v(t) = \begin{cases} 2t-2 & 2<t<4 \\ 7-t & 4<t<8 \\ 0 & \text{otherwise} \end{cases}$$

Solution:

$$i_C(t) = 1\frac{d}{dt}v_s(t) = \begin{cases} 2 & 2<t<4 \\ -1 & 4<t<8 \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad i_R(t) = 1\,v_s(t) = \begin{cases} 2t-4 & 2<t<4 \\ 8-t & 4<t<8 \\ 0 & \text{otherwise} \end{cases}$$

so

$$i(t) = i_C(t) + i_R(t) = \begin{cases} 2t-2 & 2<t<4 \\ 7-t & 4<t<8 \\ 0 & \text{otherwise} \end{cases}$$

Exercise 7.3-1 A 200-μF capacitor has been charged to 100 V. Find the energy stored by the capacitor. Find the capacitor voltage at $t = 0^+$ if $v(0^-) = 100$ V.
Answer: $w(1) = 1$ J and $v(0^+) = 100$ V

Solution:

$$W = \frac{Cv^2}{2} = \frac{1}{2}\left(2\times10^{-4}\right)\left(100\right)^2 = \underline{1\text{ J}}$$

$$v_c\left(0^+\right) = v_c\left(0^-\right) = \underline{100\text{ V}}$$

Exercise 7.3-2 A constant current $i = 2$ A flows into a capacitor of 100μF after a switch is closed at $t = 0$. The voltage of the capacitor was equal to zero at $t = 0^-$. Find the energy stored at **(a)** $t = 1$ s and **(b)** $t = 100$ s.
Answer: $w(1) = 20$ kJ and $w(100) = 200$ MJ

Solution:

(a)

$$W(t) = W(0) + \int_0^t vi\, dt$$

First, $W(0) = 0$ since $v(0) = 0$

Next, $v(t) = v(0) + \frac{1}{C}\int_0^t i\, dt = 10^4 \int_0^t 2\, dt = \underline{2\times10^4 t}$

$$\therefore\ W(t) = \int_0^t \left(2\times10^4\right)t\,(2)dt = 2\times10^4 t^2$$

$$\text{W}(1\text{s}) = 2\times10^4\text{ J} = \underline{20\text{ kJ}}$$

(b)

$$\text{W}(100\text{s}) = 2\times10^4\left(100\right)^2 = 2\times10^8\text{ J} = \underline{200\text{ MJ}}$$

Exercise 7.4-1 Find the equivalent capacitance for the circuit of Figure E 7.4-1

Answer: C_{eq} = 4 mF

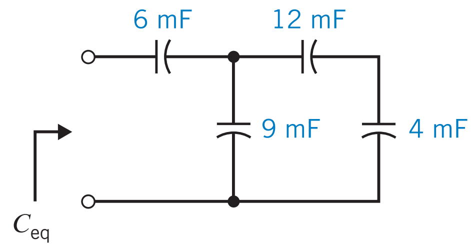
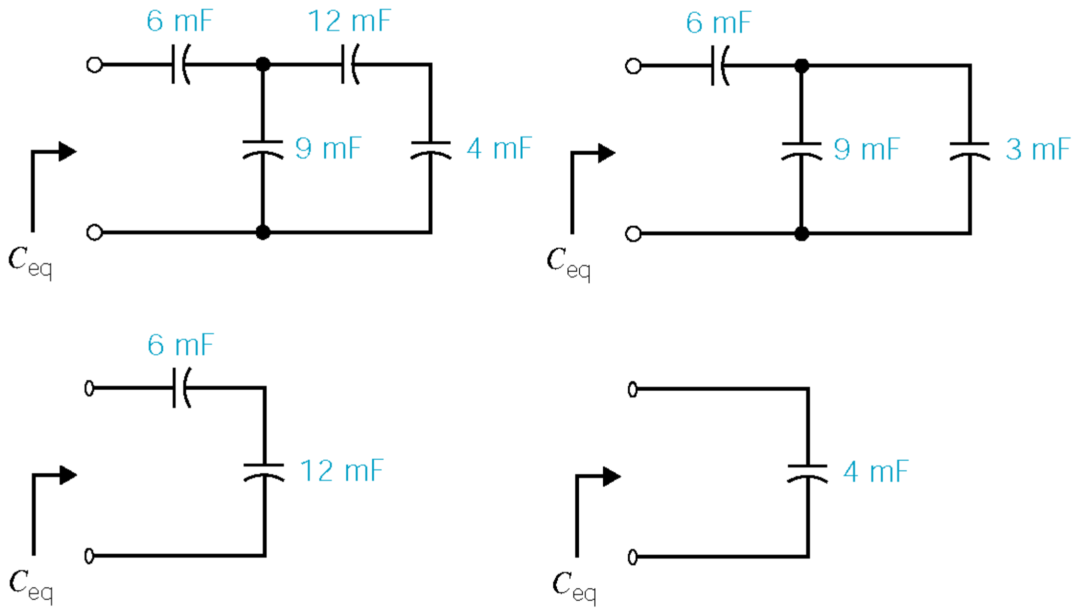


Figure E 7.4-1

Solution:



Exercise 7.4-2 Determine the equivalent capacitance C_{eq} for the circuit shown in Figure E 7.4-2.

Answer: 10/19 mF

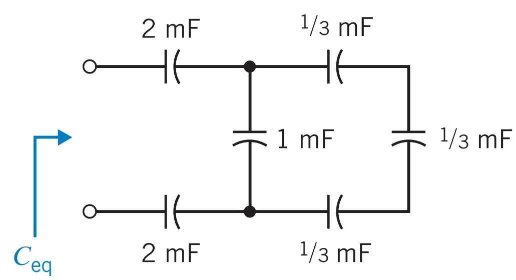
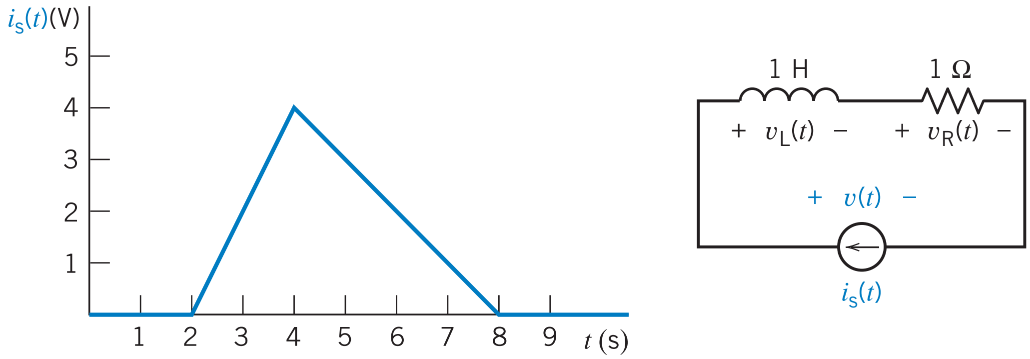


Figure E 7.4-2

Solution:

$$C_{eq1} = \frac{1}{\frac{1}{1/3}+\frac{1}{1/3}+\frac{1}{1/3}} = \frac{1}{9}, \quad C_{eq2} = 1 + C_{eq1} = \frac{10}{9}, \quad C_{eq} = \frac{1}{\frac{1}{2}+\frac{1}{2}+\frac{1}{C_{eq2}}} = \frac{1}{\frac{1}{2}+\frac{1}{2}+\frac{9}{10}} = \frac{1}{\frac{19}{10}} = \frac{10}{19} \text{ mF}$$

Exercise 7.5-1 Determine the voltage $v(t)$ for $t > 0$ for the circuit of Figure E 7.5-1*b* when $i_s(t)$ is the current shown in Figure E 7.5-1*a*.



(*a*) (*b*)

Figure E 7.5-1*b*

Hint: Determine $v_L(t)$ and $v_R(t)$ separately, then use KVL.

Answer:

$$v(t) = \begin{cases} 2t-2 & 2<t<4 \\ 7-t & 4<t<8 \\ 0 & \text{otherwise} \end{cases}$$

Solution:

$$v_L(t) = 1\frac{d}{dt}i_s(t) = \begin{cases} 2 & 2<t<4 \\ -1 & 4<t<8 \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad v_R(t) = 1\,i_s(t) = \begin{cases} 2t-4 & 2<t<4 \\ 8-t & 4<t<8 \\ 0 & \text{otherwise} \end{cases}$$

so

$$v(t) = v_L(t) + v_R(t) = \begin{cases} 2t-2 & 2<t<4 \\ 7-t & 4<t<8 \\ 0 & \text{otherwise} \end{cases}$$

Exercise 7.7-1 Find the equivalent inductance of the circuit of Figure E 7.7-1.
Answer: L_{eq} = 14 mH

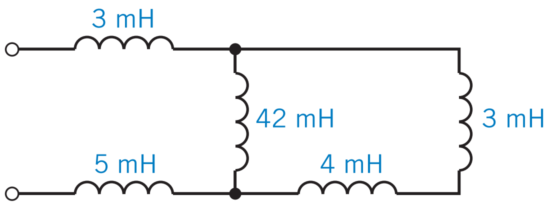
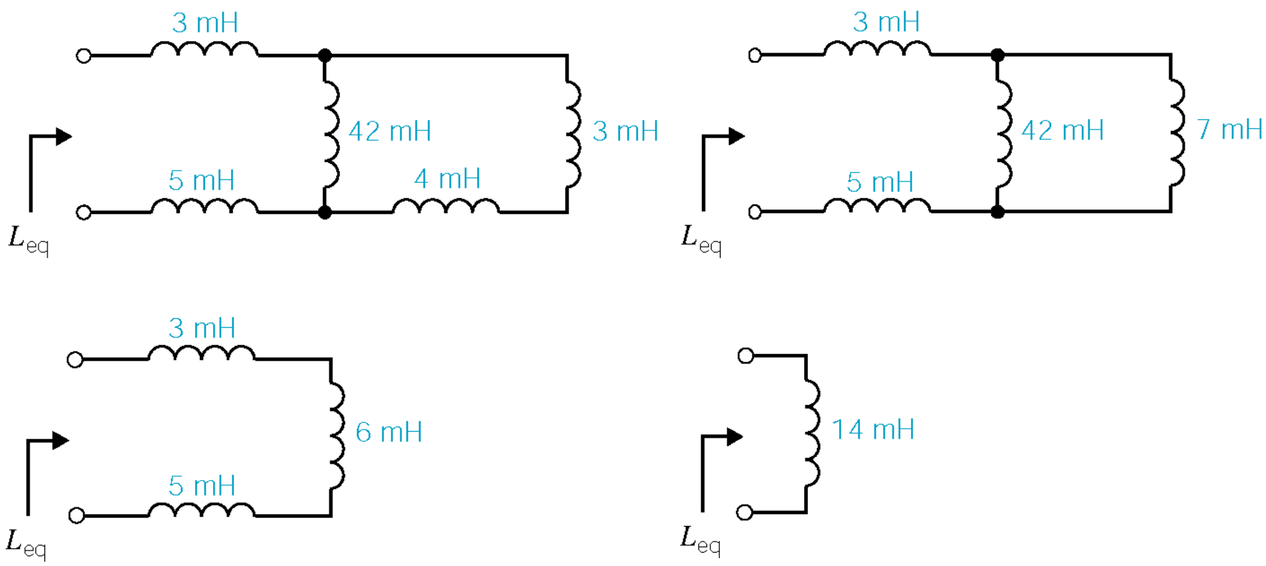


Figure E 7.7-1

Solution:



Exercise 7.7-2 Find the equivalent inductance of the circuit of Figure E 7.7-2.

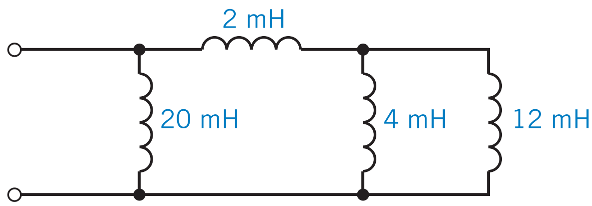
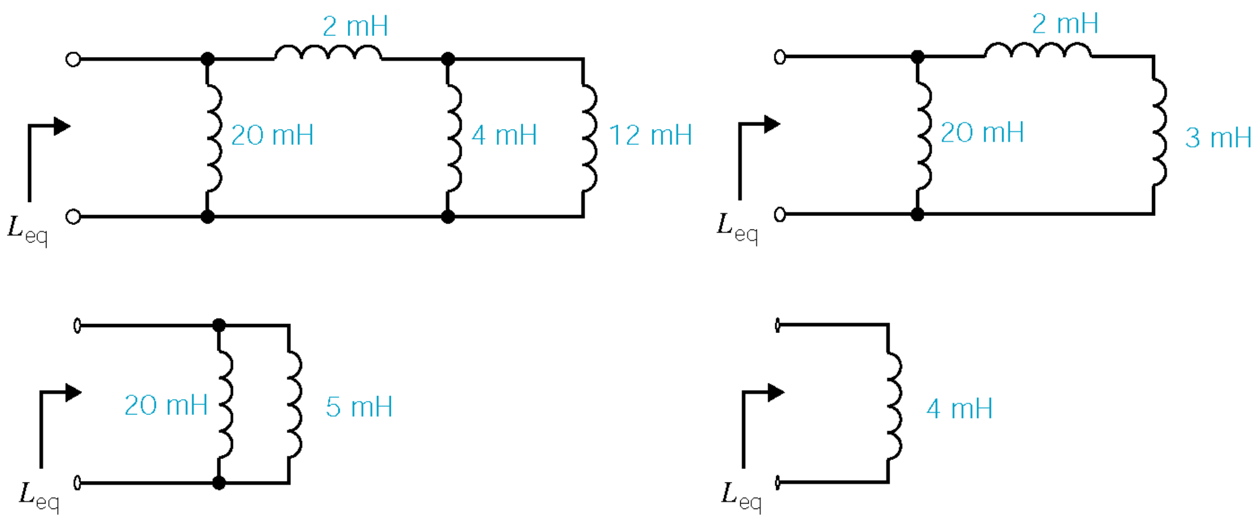


Figure E 7.7-2

Ex. 7.7-2



Section 7-2: Capacitors

P 7.2-1 A 15-μF capacitor has a voltage of 5 V across it at t = 0. If a constant current of 25 mA flows through the capacitor, how long will it take for the capacitor to charge up to 150 μC?
Answer: t = 3 ms

Solution: $$v(t) = v(0)+\frac{1}{C}\int_0^t i(\tau)\,d\tau \quad and \quad q = Cv$$

In our case, the current is constant so $\int_0^t i(\tau)\,d\tau$.

$$\therefore\ Cv(t) = Cv(0)+i\,t$$

$$\therefore\ t=\frac{q-Cv(0)}{i}=\frac{150\times10^{-6}-(15\times10^{-6})(5)}{25\times10^{-3}}\ \underline{= 3\text{ ms}}$$

P 7.2-2 The voltage, $v(t)$, across a capacitor and current, $i(t)$, in that capacitor adhere to the passive convention. Determine the current, $i(t)$, when the capacitance is C = 0.125 F and the voltage is $v(t)$ = 12 cos(2t + 30°) V.

Hint: $$\frac{d}{dt}A\cos(\omega t+\theta)=-A\sin(\omega t+\theta)\cdot\frac{d}{dt}(\omega t+\theta)$$
$$=-A\omega\sin(\omega t+\theta)$$
$$=A\omega\cos\left(\omega t+\left(\theta+\frac{\pi}{2}\right)\right)$$

Answer: $i(t)$ = 3 cos (2t + 120°) A

Solution:

$$i(t)=C\frac{d}{dt}v(t)=\frac{1}{8}\frac{d}{dt}12\cos(2t+30^\circ)=\frac{1}{8}(12)(-2)\sin(2t+30^\circ)=3\cos(2t+120^\circ)\text{ A}$$

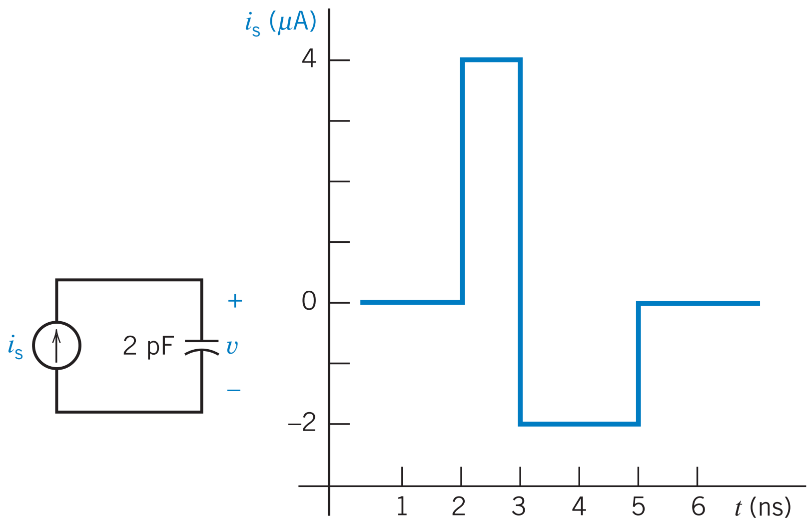
P 7.2-3 The voltage, $v(t)$, across a capacitor and current, $i(t)$, in that capacitor adhere to the passive convention. Determine the capacitance when the voltage is $v(t)$ = 12 cos(500t–45°) V and the current is $i(t)$ = 3 cos (500t + 45°) mA.
Answer: C = 0.5 μF

Solution:

$$(3\times10^{-3})\cos(500t+45^\circ)=C\frac{d}{dt}\,12\cos(500t-45^\circ)=C(12)(-500)\sin(500t-45^\circ)$$
$$=C(6000)\cos(500t+45^\circ)$$

so $$C=\frac{3\times10^{-3}}{6\times10^{3}}=\frac{1}{2}\times10^{-6}=\frac{1}{2}\ \mu\text{F}$$

P 7.2-4 Determine $v(t)$ for the circuit shown in Figure P 7.2-4*a* when the $i_s(t)$ is as shown in Figure P 7.2-4*b* and $v_0(0^-) = -1$ mV.



(a) *(b)*

Figure P 7.2-4

Solution:

$$v(t) = \frac{1}{C}\int_0^t i(\tau)\, d\tau + v(0) = \frac{1}{2\times10^{-12}}\int_0^t i(\tau)\, d\tau - 10^{-3}$$

$0 < t < 2\times10^{-9}$

$$i_s(t) = 0 \quad \Rightarrow \quad v(t) = \frac{1}{2\times10^{-12}}\int_0^t 0\, d\tau - 10^{-3} = -10^{-3}$$

$2\times10^{-9} < t < 3\times10^{-9}$ $\quad i_s(t) = 4\times10^{-6}$ A

$$\Rightarrow \quad v(t) = \frac{1}{2\times10^{-12}}\int_{2\text{ns}}^t \left(4\times10^{-6}\right) d\tau - 10^{-3} = -5\times10^{-3} + \left(2\times10^6\right) t$$

In particular, $v\left(3\times10^{-9}\right) = -5\times10^{-3} + \left(2\times10^6\right)\left(3\times10^{-9}\right) = 10^{-3}$

$3\times10^{-9} < t < 5\times10^{-9}$ $\quad i_s(t) = -2\times10^{-6}$ A

$$\Rightarrow \quad v(t) = \frac{1}{2\times10^{-12}}\int_{3\text{ns}}^t \left(-2\times10^{-6}\right) d\tau + 10^{-3} = 4\times10^{-3} - \left(10^6\right) t$$

In particular, $v\left(5\times10^{-9}\right) = 4\times10^{-3} - \left(10^6\right)\left(5\times10^{-9}\right) = -10^{-3}$ V

$5\times10^{-9} < t$

$$i_s(t) = 0 \quad \Rightarrow \quad v(t) = \frac{1}{2\times10^{-12}}\int_{5\text{ns}}^t 0\, d\tau - 10^{-3} = -10^{-3} \text{ V}$$

P 7.2-5 The voltage, $v(t)$, and current, $i(t)$, of a 1-F capacitor adhere to the passive convention. Also, $v(0) = 0$ V and $i(0) = 0$ A. (a) Determine $v(t)$ when $i(t) = x(t)$, where $x(t)$ is shown in Figure P 7.2-5 and $i(t)$ has units of A. (b) Determine $i(t)$ when $v(t) = x(t)$, where $x(t)$ is shown in Figure P 7.2-5 and $v(t)$ has units of V.
Hint: $x(t) = 4t - 4$ when $1 < t < 2$, and $x(t) = -4t + 12$ when $2 < t < 3$.

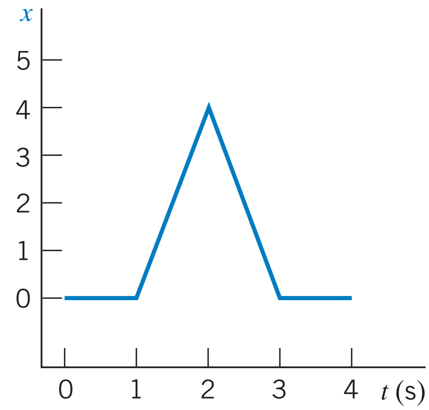


Figure P 7.2-5

Solution:

(b)

$$i(t) = C\frac{d}{dt}v(t) = \begin{cases} 0 & 0 < t < 1 \\ 4 & 1 < t < 2 \\ -4 & 2 < t < 3 \\ 0 & 3 < t \end{cases}$$

(a)

$$v(t) = \frac{1}{C}\int_0^t i(\tau)\,d\tau + v(0) = \int_0^t i(\tau)\,d\tau$$

For $0 < t < 1$, $i(t) = 0$ A so $v(t) = \int_0^t 0\,d\tau + 0 = 0$ V

For $1 < t < 2$, $i(t) = (4t - 4)$ A so

$$v(t) = \int_1^t (4\tau - 4)\,d\tau + 0 = \left(2\tau^2 - 4\tau\right)\Big|_1^t = 2t^2 - 4t + 2 \text{ V}$$

$v(2) = 2\left(2^2\right) - 4(2) + 2 = 2$ V. For $2 < t < 3$, $i(t) = (-4t + 12)$ A so

$$v(t) = \int_2^t (-4\tau + 12)\,d\tau + 2 = \left(-2\tau^2 + 12\tau\right)\Big|_1^t + 2 = \left(-2t^2 + 12t - 14\right) \text{ V}$$

$$v(3) = -2\left(3^2\right) + 12(3) - 14 = 4 \text{ V}$$

For $3 < t$, $i(t) = 0$ A so $v(t) = \int_0^t 0\,d\tau + 4 = 4$ V

P 7.2-6 The voltage, $v(t)$, and current, $i(t)$, of a 0.5-F capacitor adhere to the passive convention. Also, $v(0)$ = 0 V and $i(0)$ = 0 A. (a) Determine $v(t)$ when $i(t) = x(t)$, where $x(t)$ is shown in Figure P 7.2-6 and $i(t)$ has units of A. (b) Determine $i(t)$ when $v(t) = x(t)$, where $x(t)$ is shown in Figure P 7.2-6 and $v(t)$ has units of V.

Hint: $x(t) = 0.2t - 0.4$ when $2 < t < 6$.

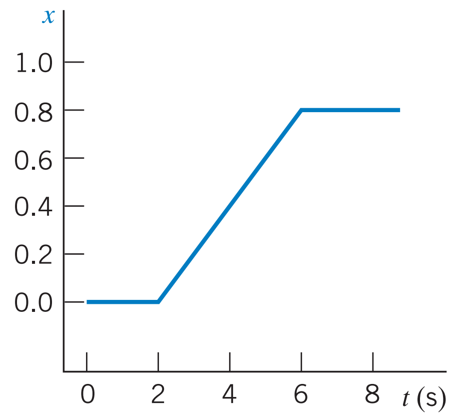


Figure P 7.2-6

Solution:

(a)

$$i(t) = C\frac{d}{dt}v(t) = \begin{cases} 0 & 0 < t < 2 \\ 0.1 & 2 < t < 6 \\ 0 & 6 < t \end{cases}$$

(b)

$$v(t) = \frac{1}{C}\int_0^t i(\tau)\,d\tau + v(0) = 2\int_0^t i(\tau)\,d\tau$$

For $0 < t < 2$, $i(t) = 0$ A so $v(t) = 2\int_0^t 0\,d\tau + 0 = 0$ V

For $2 < t < 6$, $i(t) = 0.2\,t - 0.4$ V so

$$v(t) = 2\int_1^t (0.2\tau - 0.4)\,d\tau + 0 = \left(0.2\tau^2 - 0.8\tau\right)\Big|_2^t = 0.2t^2 - 0.8t + 0.8 \text{ V}$$

$$v(6) = 0.2\left(6^2\right) - 0.8\left(6\right) + 0.8 = 3.2 \text{ V}.$$

For $6 < t$, $i(t) = 0.8$ A so $v(t) = 2\int_6^t 0.8\,d\tau + 3.2 = 1.6t - 6.4$ V

P 7.2-7 The voltage across a 40-μF capacitor is 25 V at $t_0 = 0$. If the current through the capacitor as a function of time is given by $i(t) = 6e^{-6t}$ mA for $t < 0$, find $v(t)$ for $t > 0$.
Answer: $v(t) = 50 - 25e^{-6t}$ V

Solution:

$$v(t) = v(0) + \frac{1}{C}\int_0^t i(\tau)\,d\tau = 25 + 2.5\times10^4 \int_0^t \left(6\times10^{-3}\right) e^{-6\tau}\,d\tau$$

$$= 25 + 150\int_0^t e^{-6\tau}\,d\tau$$

$$= 25 + 150\left[-\frac{1}{6}e^{-6\tau}\right]_0^t = \underline{50 - 25e^{-6t}\ \text{V}}$$

P 7.2-8 Find i for the circuit of Figure P 7.2-8 if $v = 5(1 - 2e^{-2t})$ V.

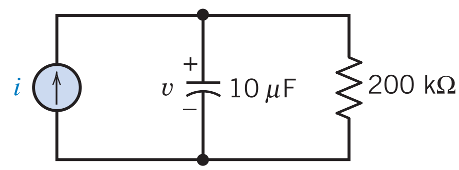
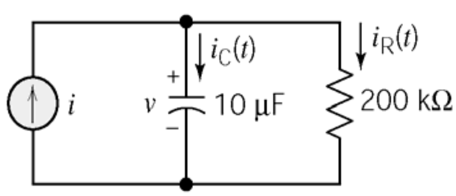


Figure P 7.2-8

Solution:



$$i_R = \frac{v}{200\times10^3} = \frac{1}{40}\left(1-2e^{-2t}\right)\times10^{-3} = 25\left(1-2e^{-2t}\right)\ \mu\text{A}$$

$$i_C = C\frac{dv}{dt} = \left(10\times10^{-6}\right)\left(-2\right)\left(-10\,e^{-2t}\right) = 200\,e^{-2t}\ \mu\text{A}$$

$$i = i_R + i_C = 200\,e^{-2t} + 25 - 50\,e^{-2t}$$

$$= \underline{25 + 150e^{-2t}\ \ \mu\text{A}}$$

P 7.2-9 Determine $v(t)$ for $t \ge 0$ for the circuit of Figure P 7.2-9*a* when $i_s(t)$ is the current shown in Figure P 7.2-9*b* and $v(0) = 1$ V.

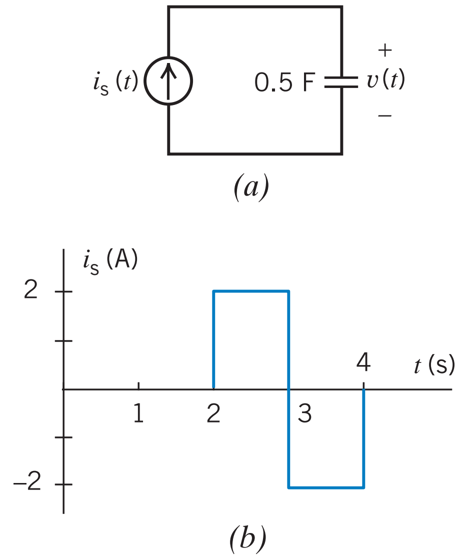


Figure P 7.2-9

Solution:

$$v(t) = 2\int_0^t i(t)\, dt + 1$$

$$= 1 \qquad \text{for} \qquad 0 \le t \le 2$$

$$= 2\int_2^t 2\, dt + 1 = 4(t-2) + 1 = 4t - 7 \qquad \text{for} \qquad 2 \le t \le 3$$

$$= 2\int_2^3 2\, dt + 2\int_3^t -2\, dt + 1 = 4 - 4(t-3) + 1 = -4t + 17 \qquad \text{for} \qquad 3 \le t \le 4$$

$$= 2\int_2^3 2\, dt + 2\int_3^4 -2\, dt + 1 = 1 \qquad \text{for} \qquad t \ge 4$$

In summary

$$v(t) = \begin{cases} 1 & 0 \le t \le 2 \\ 4t - 7 & 2 \le t \le 3 \\ -4t + 17 & 3 \le t \le 4 \\ 1 & 4 \le t \end{cases}$$

P 7.2-10 Determine $v(t)$ for $t \ge 0$ for the circuit of Figure P 7.2-10*a* when $v(0) = -4$ V and $i_s(t)$ is the current shown in Figure P 7.2-10*b*.

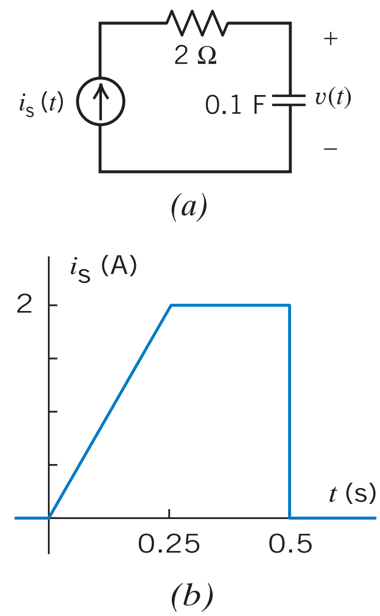


Figure P 7.2-10

Solution:

$$v(t) = v(0) + \frac{1}{C}\int_0^t i_s(t)\ dt = -4 + 10\int_0^t i_s(t)\ dt$$

For $0 \le t \le 0.25$ $\left(i_s(t) = 8t \text{ for } 0 \le t \le 0.25\right)$

$$v(t) = -4 + 10\int_0^t 8\tau\ d\tau = -4 + 80\left(\frac{\tau^2}{2}\right)\Bigg|_0^t = -4 + 40t^2$$

For example $v(0) = -4,\ v\left(\frac{1}{8}\right) = -3.375,\ v\left(\frac{1}{4}\right) = -1.5$

For $0.25 \le t \le 0.5$

$$v(t) = -1.5 + 10\int_{0.25}^t 2\ d\tau = -1.5 + 20(t - 0.25) = 20t - 6.5$$

For example $v(0.25) = -1.5,\ v(0.5) = 3.5$

For $0.5 \le t$ $\quad v(t) = 3.5 + 10\int_{0.5}^t 0\ d\tau = 3.5$

In summary
$$v(t) = \begin{cases} -4 + 40t^2 & 0 \le t \le 0.25 \text{ s} \\ 20t - 6.5 & 0.25 \le t \le 0.5 \text{ s} \\ 3.5 & t \ge 0.5 \text{ s} \end{cases}$$

P 7.2-11 Determine $i(t)$ for $t \geq 0$ for the circuit of Figure P 7.2-11*a* when $v_s(t)$ is the voltage shown in Figure P 7.2-11*b*.

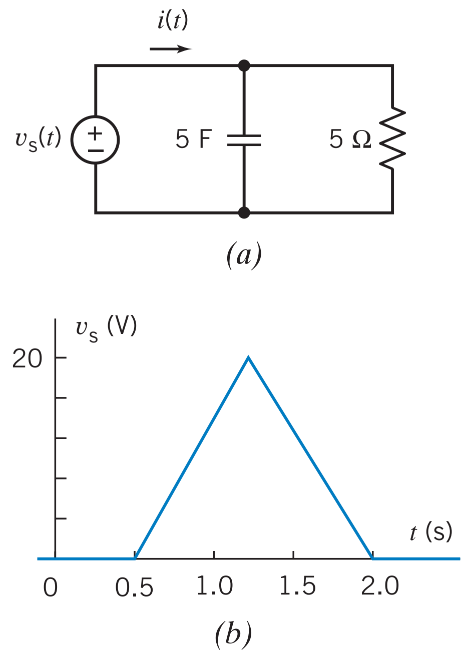


Figure P 7.2-11

Solution:
Representing $v_s(t)$ using equations of the straight line segments gives

$$v_s(t) = \begin{cases} 0 & t \leq 0.5 \\ 40t - 20 & 0.5 \leq t \leq 1.0 \\ -20t + 40 & 1.0 \leq t \leq 2.0 \\ 0 & 2.0 \leq t \end{cases}$$

Use KCL to get

$$i(t) = \frac{1}{2}\frac{d}{dt}v_s(t) + \frac{v_s(t)}{5} = \begin{cases} 0 & t \leq 0.5 \\ 20 + \dfrac{40t - 20}{5} & 0.5 \leq t \leq 1.0 \\ -10 + \dfrac{40 - 20t}{5} & 1.0 \leq t \leq 2.0 \\ 0 & t \geq 2.0 \end{cases}$$

$$i(t) = \begin{cases} 0 & t \leq 0.5 \\ 8t - 16 & 0.5 \leq t \leq 1.0 \\ -4t - 2 & 1.0 \leq t \leq 2.0 \\ 0 & t \geq 2.0 \end{cases}$$

P 7.2-12 The capacitor voltage in the circuit shown in Figure P 7.2-12 is given by

$$v(t) = 12 - 10e^{-2t}\ \text{V} \quad \text{for} \quad t \geq 0$$

Determine $i(t)$ for $t > 0$.

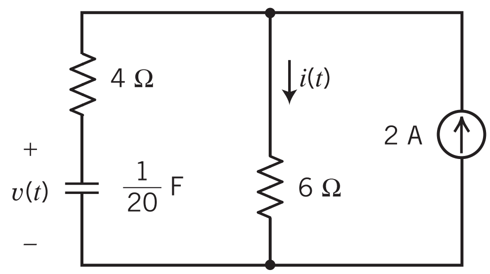
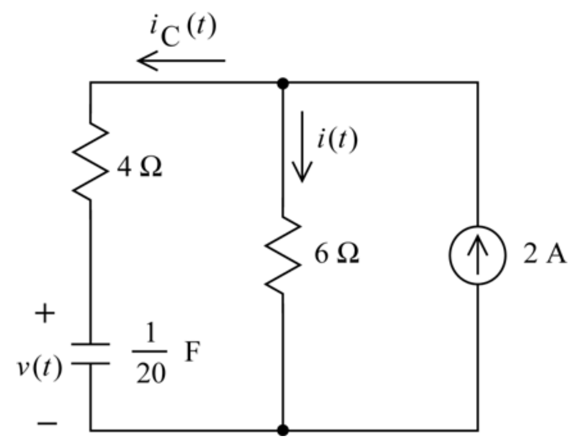


Figure P 7.2-12

Solution:



$$i_C(t) = \frac{1}{20}\frac{d}{dt}v(t)$$
$$= \frac{1}{20}\left(+20e^{-2t}\right)$$
$$= e^{-2t}\ \text{A} \quad \text{for}\ t > 0$$

Apply KCL to get

$$i(t) = 2 - i_C(t) = 2 - e^{-2t}\ \text{A} \quad \text{for}\ t > 0$$

(checked: LNAP 6/25/04)

P 7.2-13 The capacitor voltage in the circuit shown in Figure P 7.2-13 is given by

$$v(t) = 2.4 + 5.6e^{-5t} \text{ V} \quad \text{for} \quad t \ge 0$$

Determine $i(t)$ for $t > 0$.

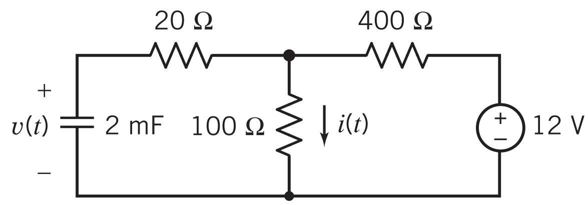
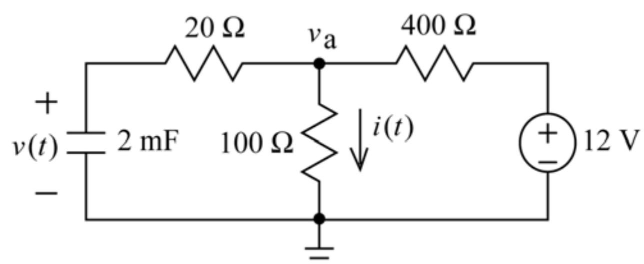


Figure P 7.2-13

Solution:
We'll write and solve a node equation. Label the node voltages as shown. Apply KCL at node *a* to get



$$\frac{v(t) - v_a}{20} = \frac{v_a}{100} + \frac{v_a - 12}{400} \quad \Rightarrow \quad v_a = \frac{20v(t) + 12}{25}$$

So
$$v_a = 2.4 + 4.48e^{-5t} \text{ V} \quad \text{for } t > 0$$

Then
$$i(t) = \frac{v_a}{100} = 24 + 44.8e^{-5t} \text{ mA} \quad \text{for } t > 0$$

(checked: LNAP 6/25/04)

P 7.2-14 The capacitor voltage in the circuit shown in Figure P 7.2-14 is given by

$$v(t) = 10 - 8e^{-5t} \text{ V} \quad \text{for} \quad t \ge 0$$

Determine $i(t)$ for $t > 0$.

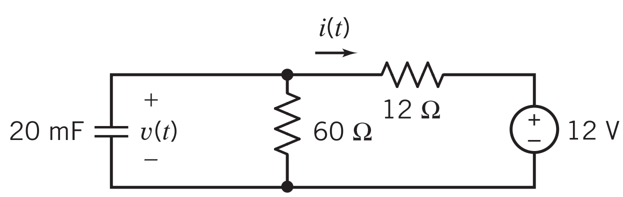


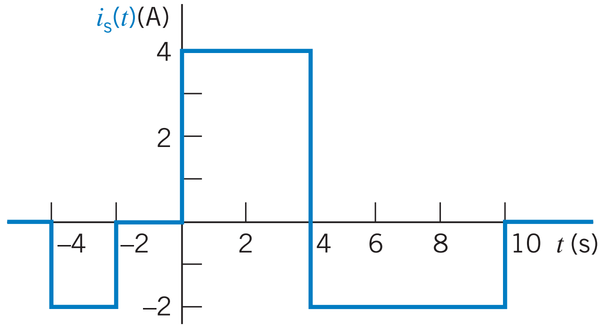
Figure P 7.2-14

Solution: Apply KVL to the outside loop to get

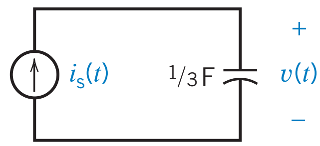
$$12i(t) + 12 - v(t) = 0 \quad \Rightarrow \quad i(t) = \frac{12 - 10 - 8e^{-5t}}{12} = -\frac{1}{6} - \frac{2}{3}e^{-5t} \text{ A} \quad \text{for } t > 0$$

(checked: LNAP 6/25/04)

P 7.2-15 Determine the voltage $v(t)$ for $t > 0$ for the circuit of Figure P 7.2-15*b* when $i_s(t)$ is the current shown in Figure P 7.2-15*a*. The capacitor voltage at time $t = 0$ is $v(0) = -12$ V.



(a)



(b)

Figure P 7.2-15

Solution:

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i_s(\tau)\,d\tau + v(t_0) = \frac{1}{\frac{1}{3}}\int_{0}^{t} i_s(\tau)\,d\tau - 12$$

$$v(t) = 3\int_{0}^{t} 4\,d\tau - 12 = 12\,t - 12 \quad \text{for} \quad 0 < t < 4$$

In particular, $v(4) = 36$ V.

$$v(t) = 3\int_{4}^{t} (-2)\,d\tau + 36 = 60 - 6t \quad \text{for} \quad 4 < t < 10$$

In particular, $v(10) = 0$ V.

$$v(t) = 3\int_{10}^{t} 0\,d\tau + 0 = 0 \quad \text{for} \quad 10 < t$$

P7.2-16 The input to the circuit shown in Figure P7.2-16 is the current

$$i(t)=3.75\,e^{-1.2t}\ \text{ A}\quad \text{for } t>0$$

The output is the capacitor voltage

$$v(t)=4-1.25\,e^{-1.2t}\ \text{ V}\quad \text{for } t>0$$

Find the value of the capacitance, *C*.

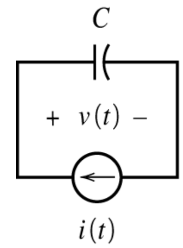


Figure P7.2-16

Solution: The capacitor voltage is related to the capacitor current by

$$v(t)=\frac{1}{C}\int_0^t i(\tau)\,d\tau+v(0)$$

That is:

$$4-1.25\,e^{-1.2t}=\frac{1}{C}\int_0^t 3.75\,e^{-1.2\tau}\,d\tau+v(0)=\frac{3.75}{C(-1.2)}e^{-1.2\tau}\Big|_0^t+v(0)=\frac{-3.125}{C}\left(e^{-1.2t}-1\right)+v(0)$$

Equating the coefficients of $e^{-1.2t}$ gives

$$12.5=\frac{3.125}{C}\quad\Rightarrow\quad C=\frac{3.125}{12.5}=0.25=250\text{ mF}$$

P7.2-17 The input to the circuit shown in Figure P7.2-17 is the current

$$i(t)=3\,e^{-25t}\ \text{ A}\quad \text{for } t>0$$

The initial capacitor voltage is $v_C(0)=-2$ V. Determine the current source voltage, $v(t)$, for $t>0$.

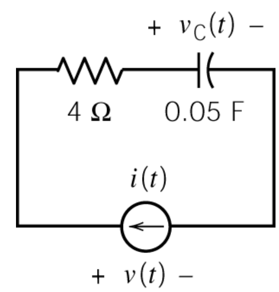


Figure P7.2-17

Solution: Apply KVL to the mesh to get

$$v(t)=4\,i(t)+v_C(t)=4\,i(t)+\left[\frac{1}{0.05}\int_0^t i(\tau)\,d\tau+v(0)\right]$$

That is,

$$v(t)=4\left(3\,e^{-25t}\right)+\frac{1}{0.05}\int_0^t 3\,e^{-25\tau}\,d\tau-2=12\,e^{-25t}+\frac{3}{0.05(-25)}\left(e^{-25t}-1\right)-2$$

$$=12\,e^{-25t}-2.4\left(e^{-25t}-1\right)-2=9.6\,e^{-25t}+0.4\ \text{ V}\quad \text{for } t>0$$

P7.2-18 The input to the circuit shown in Figure P7.2-18 is the current

$$i(t) = 3e^{-25t} \text{ A} \quad \text{for } t > 0$$

The output is the voltage

$$v(t) = 9.6e^{-25t} + 0.4 \text{ V} \quad \text{for } t > 0$$

The initial capacitor voltage is $v_C(0) = -2$ V. Determine the values of the capacitance, *C*, and resistance, *R*.

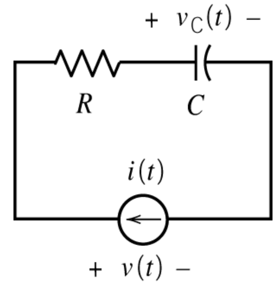


Figure P7.2-18

Solution: Apply KVL to the mesh to get

$$v(t) = R\,i(t) + v_C(t) = R\,i(t) + \left[\frac{1}{C}\int_0^t i(\tau)\,d\tau + v(0)\right]$$

That is

$$9.6e^{-25t} + 0.4 = R\left(3e^{-25t}\right) + \left[\frac{1}{C}\int_0^t 3e^{-25\tau}\,d\tau - 2\right]$$

$$= 3Re^{-25t} + \frac{3}{C(-25)}\left(e^{-25t} - 1\right) - 2 = 3\left(R - \frac{1}{25C}\right)e^{-25t} + \frac{3}{25C} - 2$$

Equating coefficients gives

$$0.4 = \frac{3}{25C} - 2 \quad \Rightarrow \quad C = 0.05 = 50 \text{ mF}$$

and

$$9.6 = 3\left(R - \frac{1}{25C}\right) = 3\left(R - \frac{1}{25(0.05)}\right) = 3(R - 0.8) \quad R = 4\ \Omega$$

P7.2-19 The input to the circuit shown in Figure P7.2-19 is the voltage

$$v(t) = 8 + 5e^{-10t} \text{ V} \quad \text{for } t > 0$$

Determine the current, $i(t)$ for $t > 0$.

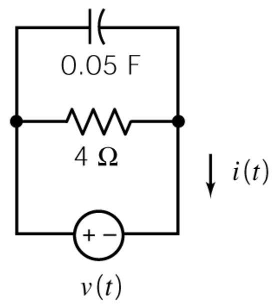


Figure P7.2-19

Solution: Apply KCL at either node to get

$$\begin{aligned} i(t) &= \frac{v(t)}{4} + 0.05\frac{d}{dt}v(t) = \frac{8+5e^{-10t}}{4} + 0.05\frac{d}{dt}\left(8+5e^{-10t}\right) \\ &= 2 + 1.25e^{-10t} + 0.05(5)(-10)e^{-10t} \\ &= 2 - 1.25e^{-10t} \text{ A} \quad \text{for } t > 0 \end{aligned}$$

P7.2-20 The input to the circuit shown in Figure P7.2-20 is the voltage:

$$v(t) = 3 + 4e^{-2t} \text{ A} \quad \text{for } t > 0$$

The output is the current: $i(t) = 0.3 - 1.6e^{-2t}$ V for $t > 0$

Determine the values of the resistance and capacitance

Answer: $R =$ 10 Ω and $C =$ 0.25 F.

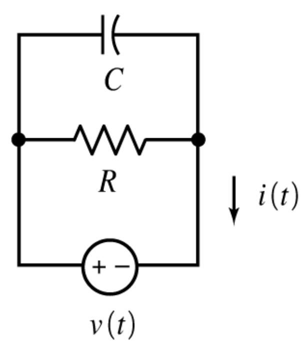


Figure P7.2-20

Solution: Apply KCL at either node to get

$$\begin{aligned} 0.3 - 1.6e^{-2t} &= \frac{3+4e^{-2t}}{R} + C\frac{d}{dt}\left(3+4e^{-2t}\right) \\ &= \frac{3+4e^{-2t}}{R} + (-2)4Ce^{-2t} = \frac{3}{R} + \left(\frac{4}{R} - 8C\right)e^{-2t} \end{aligned}$$

Equating coefficients:

$$0.3 = \frac{3}{R} \Rightarrow R = 10\ \Omega \quad \text{and} \quad -1.6 = \frac{4}{10} - 8C \Rightarrow C = 0.25 \text{ F}$$

P7.2-21 Consider the capacitor shown in Figure P7.2-21. The current and voltage are given by

$$i(t)=\begin{cases}0.5 & 0<t<0.5\\ 2 & 0.5<t<1.5\\ 0 & t>1.5\end{cases} \quad \text{and} \quad v(t)=\begin{cases}2t+8.6 & 0\le t\le 0.5\\ a\,t+b & 0.5\le t\le 1.5\\ c & t\ge 1.5\end{cases}$$

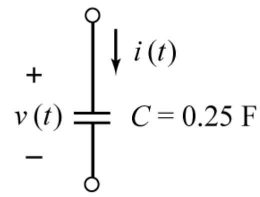


Figure P7.2-21

where *a*, *b* and *c* are real constants. (The current is given in Amps, the voltage in Volts and the time in seconds.) Determine the values of *a*, *b* and *c*.

Answer: $a =$ 8 V/s, $b =$ 5.6 V and $c =$ 17.6 V

Solution: At $t = 0.5$ s

$$v(0.5)=2(0.5)+8.6=9.6 \text{ V}$$

For $0.5\le t\le 1.5$

$$v(t)=\frac{1}{0.25}\int_{0.5}^{t}2\,d\tau+9.6=8\,\tau\Big|_{0.5}^{t}+9.6=8(t-0.5)+9.6=8t+5.6 \text{ V}$$

At $t = 1.5$ s

$$v(1.5)=8(1.5)+5.6=17.6 \text{ V}$$

For $t\ge 1.5$

$$v(t)=\frac{1}{0.25}\int_{1.5}^{t}0\,d\tau+17.6=17.6$$

Checks:

At $t = 1.0$ s $$i(t)=\frac{1}{4}\frac{d}{dt}v(t)=\frac{1}{4}\frac{d}{dt}(8t+5.6)=\frac{1}{4}(8)=2 \text{ A} \quad \surd$$

At $t = 0.5$ s $$v(0.5)=8(0.5)+5.6=9.6 \text{ V} \quad \surd$$

P7.2-22 At time $t = 0$, the voltage across the capacitor shown in Figure P7.2-22 is $v(0) = -20$ V. Determine the values of the capacitor voltage at times 1 ms, 3 ms and 7 ms.

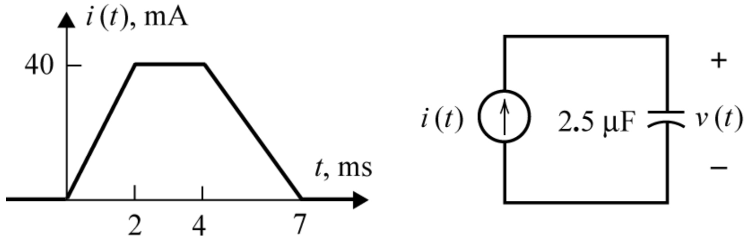


Figure P7.2-22

Solution:

$$v(t) = v(0) + \frac{1}{C}\int_0^t i(\tau)\, d\tau = v(0) + \frac{\text{"area under the curve"}}{C} = -20 + \frac{\text{"area under the curve"}}{2.5\times10^{-6}}$$

$$v(0.001) = -20 + \frac{\frac{1}{2}\left(20\times10^{-3}\right)\left(1\times10^{-3}\right)}{2.5\times10^{-6}} = -20 + \frac{10}{2.5} = -16 \text{ V}$$

(When calculating the value of $v(0.001)$, "area under the curve" indicates the area under the graph of $i(t)$ versus t corresponding to the time interval 0 to 1 ms = 0.001 s.)

$$v(0.003) = -20 + \frac{\frac{1}{2}\left(40\times10^{-3}\right)\left(2\times10^{-3}\right) + \left(40\times10^{-3}\right)\left(1\times10^{-3}\right)}{2.5\times10^{-6}} = -20 + \frac{40+40}{2.5} = 12 \text{ V}$$

(When calculating the value of $v(0.003)$, "area under the curve" indicates the area under the graph of $i(t)$ versus t corresponding to the time interval 0 to 3 ms = 0.003 s.)

$$v(0.007) = -20 + \frac{\frac{1}{2}\left(40\times10^{-3}\right)\left(2\times10^{-3}\right) + \left(40\times10^{-3}\right)\left(2\times10^{-3}\right) + \frac{1}{2}\left(40\times10^{-3}\right)\left(3\times10^{-3}\right)}{2.5\times10^{-6}} = 52 \text{ V}$$

(When calculating the value of $v(0.007)$, "area under the curve" indicates the area under the graph of $i(t)$ versus t corresponding to the time interval 0 to 7 ms = 0.007 s.)

Section 7-3: Energy Storage in a Capacitor

P 7.3-1 The current, *i*, through a capacitor is shown in Figure P 7.3-1. When $v(0) = 0$ and $C = 0.5$ F, determine and plot $v(t)$, $p(t)$, and $w(t)$ for $0 \text{ s} < t < 6$ s.

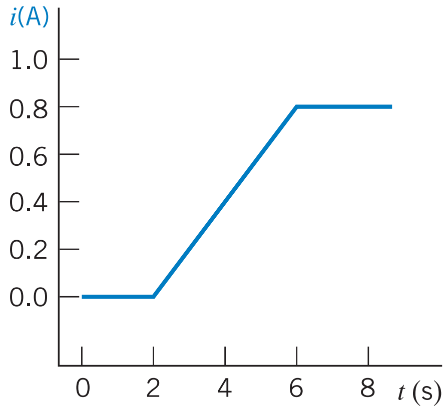


Figure P 7.3-1

Solution:

Given
$$i(t) = \begin{cases} 0 & t < 2 \\ 0.2(t-2) & 2 < t < 6 \\ 0.8 & t > 6 \end{cases}$$

The capacitor voltage is given by

$$v(t) = \frac{1}{0.5}\int_0^t i(\tau)\,d\tau + v(0) = 2\int_0^t i(\tau)\,d\tau + v(0)$$

For $t < 2$
$$v(t) = 2\int_0^t 0\,d\tau + 0 = 0$$

In particular, $v(2) = 0$. For $2 < t < 6$

$$v(t) = 2\int_2^t 2(\tau - 2)\,d\tau + 0 = \left(0.2\tau^2 - 0.8\tau\right)\Big|_2^t = \left(0.2t^2 - 0.8t + 0.8\right) \text{ V} = 0.2\left(t^2 - 4t + 4\right) \text{ V}$$

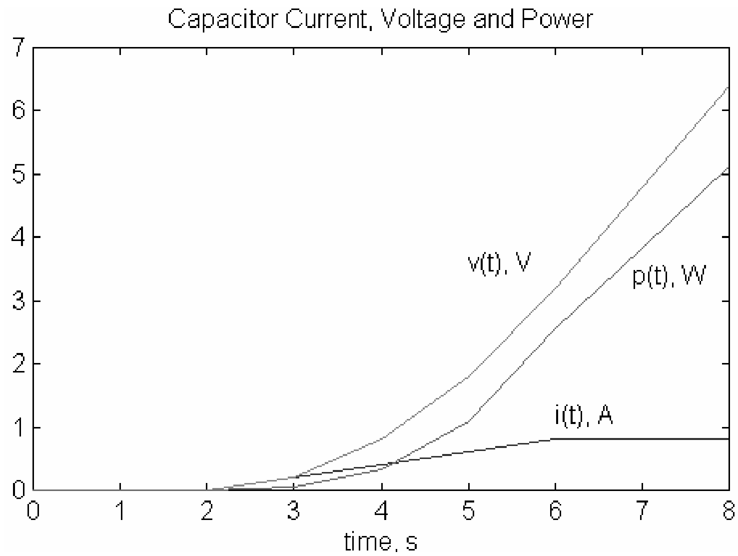
In particular, $v(6) = 3.2$ V. For $6 < t$

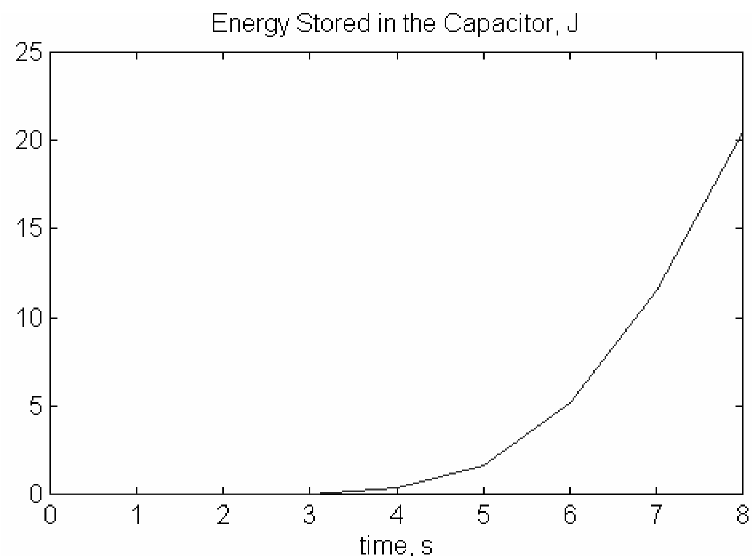
$$v(t) = 2\int_6^t 0.8\,d\tau + 3.2 = 1.6\tau\Big|_2^t + 3.2 = (1.6t - 6.4) \text{ V} = 1.6(t-4) \text{ V}$$

Now the power and energy are calculated as

$$p(t) = v(t)\,i(t) = \begin{cases} 0 & t < 2 \\ 0.04(t-2)^2 & 2 < t < 6 \\ 1.28(t-4) & 6 < t \end{cases}$$

and $$\mathsf{W}(t)=\int_0^t p(\tau)\,d\tau=\begin{cases} 0 & t<2 \\ 0.01(t-2)^4 & 2<t<6 \\ 0.8(t-4)^2-0.64 & 6<t \end{cases}$$





These plots were produced using three MATLAB scripts:

capvol.m

```
function v = CapVol(t)
   if t<2
      v = 0;
   elseif t<6
      v = 0.2*t*t - .8*t +.8;
   else
      v = 1.6*t - 6.4;
   end
```

capcur.m

```
function i = CapCur(t)
   if t<2
      i=0;
   elseif t<6
      i=.2*t - .4;
   else
      i =.8;
   end
```

c7s4p1.m

```
t=0:1:8;
for k=1:1:length(t)
  i(k)=CapCur(k-1);
  v(k)=CapVol(k-1);
  p(k)=i(k)*v(k);
  w(k)=0.5*v(k)*v(k);
end

 plot(t,i,t,v,t,p)
 text(5,3.6,'v(t), V')
 text(6,1.2,'i(t), A')
 text(6.9,3.4,'p(t), W')
 title('Capacitor Current, Voltage and Power')
 xlabel('time, s')

% plot(t,w)
% title('Energy Stored in the Capacitor, J')
% xlabel('time, s')
```

P 7.3-2 In a pulse power circuit the voltage of a 10-μF capacitor is zero for $t < 0$ and

$$v = 5(1 - e^{-4000t})\text{ V} \quad t \geq 0$$

Determine the capacitor current and the energy stored in the capacitor at $t = 0$ ms and $t = 10$ ms.

Solution:

$$i_c = C\frac{dv}{dt} = \left(10\times10^{-6}\right)\left(-5\right)\left(-4000\right)e^{-4000t} = \underline{0.2e^{-4000t}\text{ A}} \Rightarrow \begin{cases} \underline{i_c(0) = 0.2\ A} \\ \underline{i_c(10ms) = 8.5\times10^{-19}\ A} \end{cases}$$

$$W(t) = \frac{1}{2}Cv^2(t) \text{ and } v(0) = 5 - 5e^0 = 0 \Rightarrow \underline{W(0) = 0}$$

$$v\left(10\times10^{-3}\right) = 5 - 5e^{-40} = 5 - 21.2\times10^{-18} \cong 5 \Rightarrow \underline{W(10) = 1.25\times10^{-4}\text{ J}}$$

P 7.3-3 If $v_c(t)$ is given by the waveform shown in Figure P 7.3-3, sketch the capacitor current for $-1\text{ s} \leq t \leq 2$ s. Sketch the power and the energy for the capacitor over the same time interval when $C = 1$ mF.

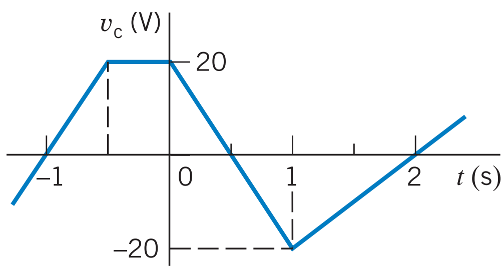
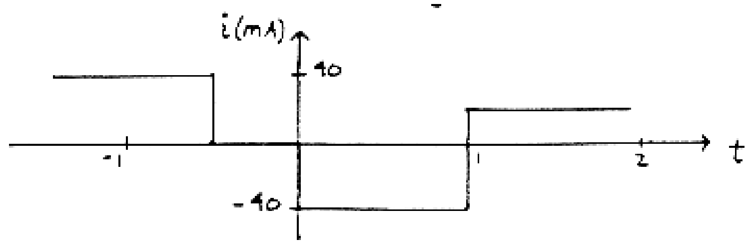
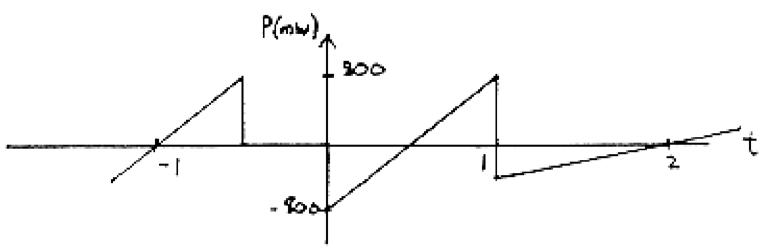


Figure P 7.3-3

Solution:





$$i(t) = C\frac{dv_c}{dt} \text{ so read off slope of } v_c(t) \text{ to get } i(t)$$

$$p(t) = v_c(t)\, i(t) \text{ so multiply } v_c(t) \;\&\; i(t) \text{ curves to get } p(t)$$

P 7.3-4 The current through a 2-μF capacitor is 50 cos(10t + π/6) μA for all time. The average voltage across the capacitor is zero. What is the maximum value of the energy stored in the capacitor? What is the first nonnegative value of t at which the maximum energy is stored?

Solution:

$$v_c(t)=v_c(0)+\frac{1}{C}\int_0^t i\,d\tau=v_c(0)+\frac{1}{2}\int_0^t 50\cos\left(10t+\frac{\pi}{6}\right)d\tau=\left[v_c(0)-\frac{5}{2}\sin\frac{\pi}{6}\right]+\frac{5}{2}\sin\left(10t+\frac{\pi}{6}\right)$$

Now since $v_c(t)_{ave}=0\Rightarrow v_c(0)-\frac{5}{2}\sin\frac{\pi}{6}=0\Rightarrow v_c(t)=\frac{5}{2}\sin\left(10t+\frac{\pi}{6}\right)$ V

$$\therefore W_{\max}=\frac{1}{2}C\,v_{c_{\max}}^2=\frac{(2\times10^{-6})(2.5)^2}{2}=\underline{6.25\ \mu\text{J}}$$

First non-negative t for max energy occurs when: $10t+\frac{\pi}{6}=\frac{\pi}{2}\Rightarrow \underline{t=\frac{\pi}{30}=0.1047\ s}$

P 7.3-5 A capacitor is used in the electronic flash unit of a camera. A small battery with a constant voltage of 6 V is used to charge a capacitor with a constant current of 10 μA. How long does it take to charge the capacitor when C = 10 μF? What is the stored energy?

Solution:

Max. charge on capacitor $=C\,v=(10\times10^{-6})\ (6)=60\ \mu$C

$$\Delta t=\frac{\Delta q}{i}=\frac{60\times10^{-6}}{10\times10^{-6}}=\underline{6\text{ sec}}\text{ to charge}$$

$$\text{stored energy}=W=\frac{1}{2}C\,v^2=\frac{1}{2}(10\times10^{-6})\ (6)^2=\underline{180\ \mu\text{J}}$$

P 7.3-6 The initial capacitor voltage of the circuit shown in Figure P 7.3-6 is $v_c(0^-) = 3$ V. Determine (a) the voltage $v(t)$ and (b) the energy stored in the capacitor at $t = 0.2$ s and $t = 0.8$ s when

$$i(t) = \begin{cases} 3e^{5t} \text{ A} & 0 < t < 1 \\ 0 & t \geq 1\text{s} \end{cases}$$

Answer:

(a) $18e^{5t}$ V, $0 \leq t < 1$

(b) $w(0.2) = 6.65$ J and $w(0.8) = 2.68$ kJ

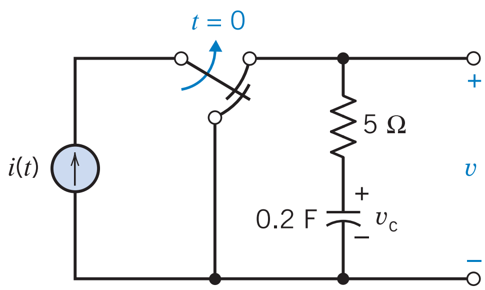


Figure P 7.3-6

Solution:

We have $v(0^+) = v(0^-) = 3$ V

$$v_c(t) = \frac{1}{C}\int_0^t i(t)\, dt + v_c(0) = 5\int_0^t 3\, e^{5t}\ dt + 3 = 3\left(e^{5t} - 1\right) + 3 = 3\, e^{5t} \text{ V}, \quad 0<t<1$$

a) $$v(t) = v_R(t) + v_c(t) = 5\, i(t) + v_c(t) = 15\, e^{5t} + 3\, e^{5t} = \underline{18\, e^{5t} \text{ V}}, \quad 0 < t < 1$$

b) $$\text{W}(t) = \frac{1}{2} C v_c^2(t) = \frac{1}{2} \times 0.2\left(3e^{5t}\right)^2 = 0.9e^{10t} \text{ J} \quad \Rightarrow \quad \begin{cases} \text{W}(t)\big|_{t=0.2s} = \underline{6.65 \text{ J}} \\ \text{W}(t)\big|_{t=0.8s} = \underline{2.68 \text{ kJ}} \end{cases}$$

P7.3-7. (a) Determine the energy stored in the capacitor in the circuit shown in Figure P7.3-7 when the switch is closed and the circuit is at steady state. (b) Determine the energy stored in the capacitor when the switch is open and the circuit is at steady state.

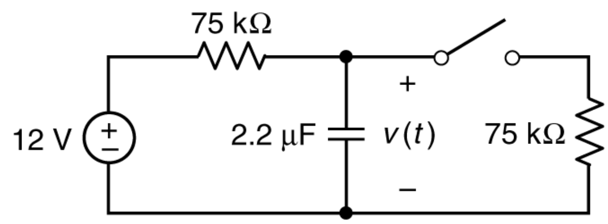


Figure P7.3-7

Solution:

The capacitor acts like an open circuit when this circuit is at steady state.

(a) When the switch is closed and the circuit is at steady state, $v(t)=6\text{ V}$. The energy stored by the capacitor is $W=\frac{1}{2}\left(2.2\times10^{-9}\right)\left(6^2\right)=39.6\ \ \mu\text{J}$.

(b) When the switch is closed and the circuit is at steady state, $v(t)=12\text{ V}$. The energy stored by the capacitor is $W=\frac{1}{2}\left(2.2\times10^{-9}\right)\left(12^2\right)=158.4\ \ \mu\text{J}$.

Section 7-4: Series and Parallel Capacitors

P 7.4-1 Find the current $i(t)$ for the circuit of Figure P 7.4-1.
Answer: $i(t) = -1.2 \sin 100t$ mA

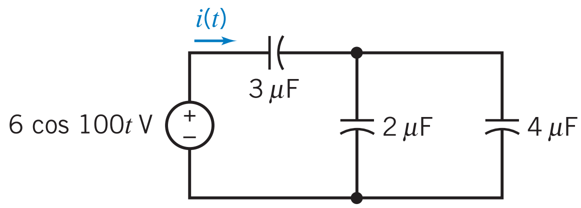


Figure P 7.4-1

Solution:

$$2\mu\text{F} \parallel 4\mu\text{F} = 6\mu\text{F}$$

$$6\mu\text{F in series with } 3\mu\text{F} = \frac{6\mu\text{F}\cdot 3\mu\text{F}}{6\mu\text{F}+3\mu\text{F}} = 2\mu\text{F}$$

$$i(t) = 2\ \mu\text{F}\ \frac{d}{dt}(6\cos 100t) = (2\times10^{-6})\ (6)\ (100)\ \ (-\sin 100t)\ \text{A} = \underline{-1.2\ \sin 100t\ \ \text{mA}}$$

P 7.4-2 Find the current $i(t)$ for the circuit of Figure P 7.4-2.
Answer: $i(t) = -1.5e^{-250t}$ mA

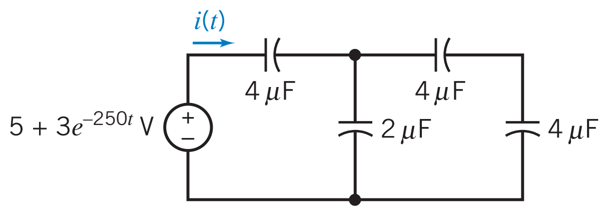


Figure P 7.4-2

Solution:

$$4\ \mu\text{F in series with } 4\ \mu\text{F} = \frac{4\ \mu\text{F}\times 4\ \mu\text{F}}{4\ \mu\text{F}+4\ \mu\text{F}} = 2\ \mu\text{F}$$

$$2\ \mu\text{F} \parallel 2\ \mu\text{F} = 4\ \mu\text{F}$$

$$4\ \mu\text{F in series with } 4\ \mu\text{F} = 2\ \mu\text{F}$$

$$i(t) = (2\times10^{-6})\frac{d}{dt}(5+3\ e^{-250t}) = (2\times10^{-6})\ (0+3(-250)\ e^{-250t})\ \ \text{A} = \underline{-1.5\ e^{-250t}\ \ \ \text{mA}}$$

P 7.4-3 The circuit of Figure P 7.4-3 contains five identical capacitors. Find the value of the capacitance C.

Answer: $C = 10\ \mu\text{F}$

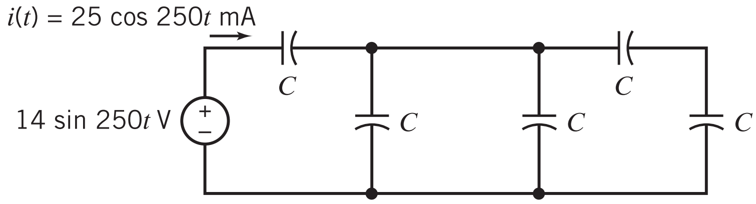


Figure P 7.4-3

Solution:

$$C \text{ in series with } C = \frac{C \cdot C}{C+C} = \frac{C}{2}$$

$$C \parallel C \parallel \frac{C}{2} = \frac{5}{2}C$$

$$C \text{ in series with } \frac{5}{2}C = \frac{C \cdot \frac{5}{2}C}{C + \frac{5}{2}C} = \frac{5}{7}C$$

$$(25\times10^{-3})\cos 250t = \left(\frac{5}{7}C\right)\frac{d}{dt}(14\sin 250t) = \left(\frac{5}{7}C\right)(14)(250)\cos 250t$$

$$\text{so } 25\times10^{-3} = 2500\,C \Rightarrow C = 10\times10^{-6} = 10\ \mu\text{F}$$

P7.4-4 The circuit shown in Figure P 7.4-4 contains seven capacitors, each having capacitance *C*. The source voltage is given by

$$v(t) = 4\cos(3t)\,\text{V}$$

Find the current *i*(*t*) when *C* = 1 F.

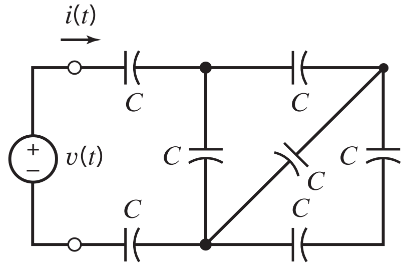
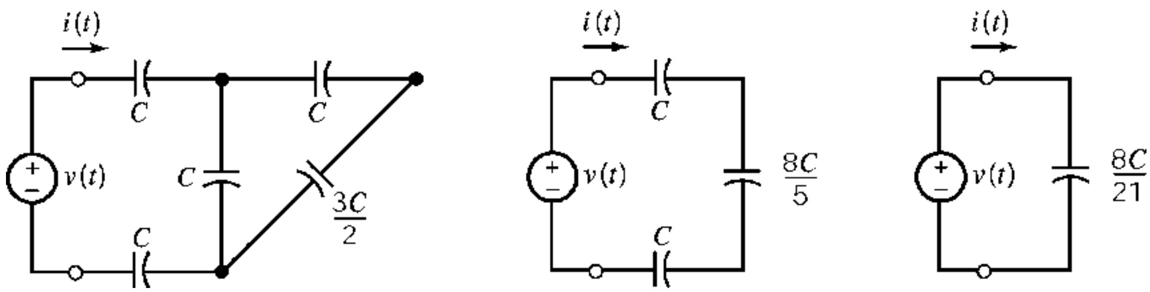


Figure P 7.4-4

Solution: Replacing series and parallel capacitors by equivalent capacitors, the circuit can be reduced as follows:



Then

$$i(t) = \frac{8C}{21}\frac{d}{dt}v(t) = \frac{8C}{21}\frac{d}{dt}4\cos(3t) = \frac{8\times 1}{21}\left[-12\sin(3t)\right] = -\frac{32}{7}\sin(3t) \ \ \text{V}$$

(Checked using LNAP 6/25/04)

P7.4-5 Determine the value of the capacitance C in the circuit shown in Figure P 7.4-5, given that $C_{eq} = 8$ F.
Answer: $C = 20$ F

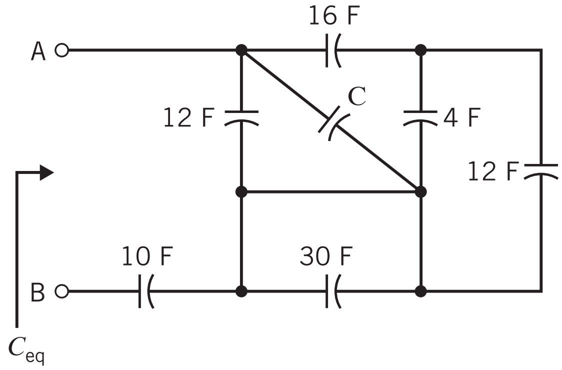
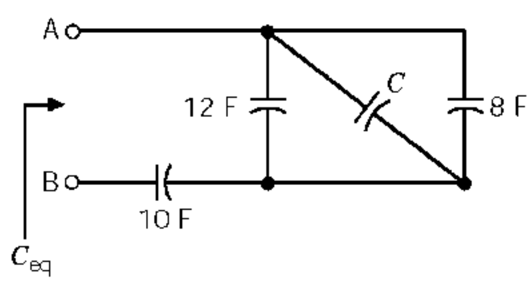


Figure P 7.4-5

Solution: The 16 F capacitor is in series with a parallel combination of 4 F and 12 F capacitors. The capacitance of the equivalent capacitor is

$$\frac{16(4+12)}{16+(4+12)} = 8 \text{ F}$$

The 30 F capacitor is in parallel with a short circuit, which is equivalent to a short circuit. After making these simplifications, we have



Then

$$8 = C_{eq} = \frac{10(12+C+8)}{10+(12+C+8)} \quad \Rightarrow \quad C = 20 \text{ F}$$

(Checked using LNAP 6/26/04)

P 7.4-6 Determine the value of the equivalent capacitance, C_{eq}, in the circuit shown in Figure P 7.4-6.
Answer: $C_{eq} = 10$ F

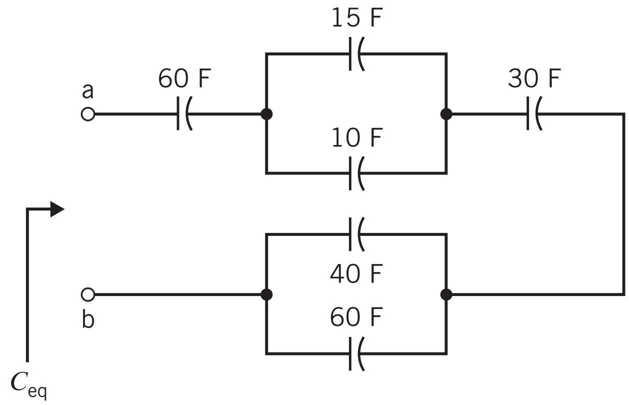


Figure P 7.4-6.

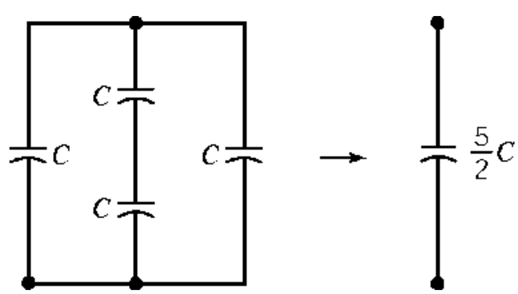
Solution:

$$C_{eq} = \frac{1}{\frac{1}{60} + \frac{1}{15+10} + \frac{1}{30} + \frac{1}{40+60}} = 10 \text{ F}$$

(Checked using LNAP 6/26/04)

P 7.4-7 The circuit shown in Figure P 7.4-7 consists of nine capacitors having equal capacitance, C. Determine the value of the capacitance C, given that $C_{eq} = 50$ mF.
Answer: $C = 90$ mF

Solution: First



Then

$$50 = C_{eq} = \frac{1}{\frac{1}{C} + \frac{2}{5\,C} + \frac{2}{5\,C}} \quad \Rightarrow \quad C = 90 \text{ mF}$$

(Checked using LNAP 6/26/04)

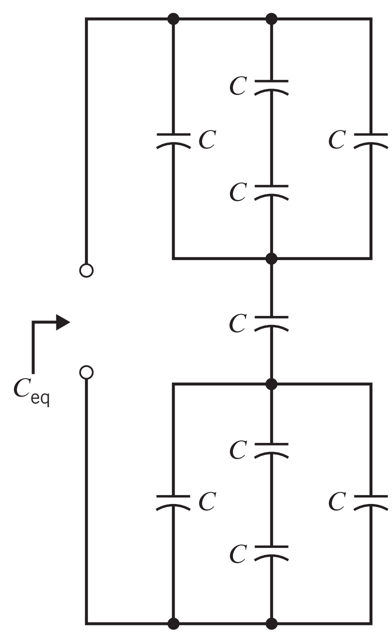


Figure P 7.4-7

P 7.4-8 The circuit shown in Figure P 7.4-8 is at steady state before the switch opens at time $t = 0$.

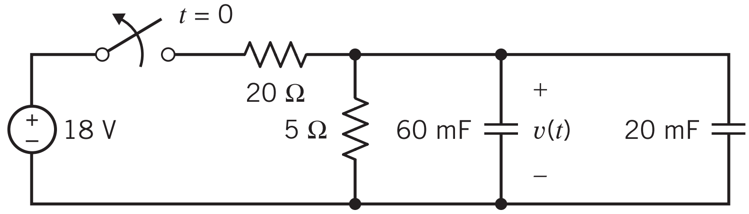


Figure P 7.4-8

The voltage $v(t)$ is given by

$$v(t) = \begin{cases} 3.6\text{ V} & \text{for } t \le 0 \\ 3.6e^{-2.5t} & \text{for } t \ge 0 \end{cases}$$

(a) Determine the energy stored by each capacitor before the switch opens.
(b) Determine the energy stored by each capacitor 1 s after the switch opens.
The parallel capacitors can be replaced by an equivalent capacitor.
(c) Determine the energy stored by the equivalent capacitor before the switch opens.
(d) Determine the energy stored by the equivalent capacitor 1 s after the switch opens.

Solution:

(a) The energy stored in the 60 mF capacitor is $w_1 = \frac{1}{2}(0.060)3.6^2 = 0.3888\text{ W}$ and the energy stored in the 20 mF capacitor is $w_2 = \frac{1}{2}(0.020)3.6^2 = 0.1296\text{ J}$.

(b) One second after the switch opens, the voltage across the capacitors is $3.6e^{-2.5} = 0.2955\text{ V}$. Then $w_1 = 2.620$ mJ and $w_2 = 0.873$ mJ.

Next $C_{eq} = 0.06 + 0.02 = 80$ mF.

(c) $w_{eq} = \frac{1}{2}(0.08)3.6^2 = 0.5184\text{ J} = w_1 + w_2$

(d) $w_{eq} = \frac{1}{2}(0.08)(0.2955)^2 = 3.493\text{ mJ} = w_1 + w_2$

P 7.4-9 The circuit shown in Figure P 7.4-9 is at steady state before the switch closes. The capacitor voltages are both zero before the switch closes ($v_1(0) = v_2(0) = 0$).

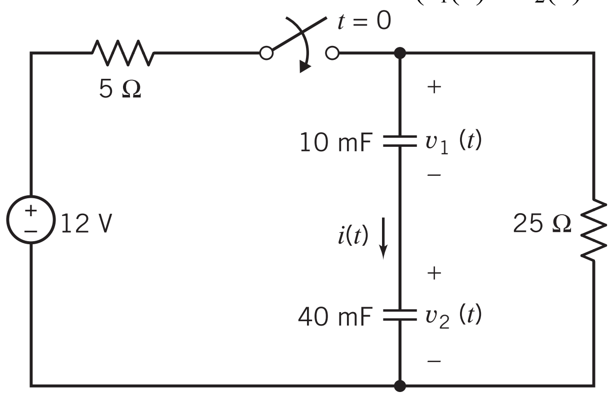


Figure P 7.4-9

The current $i(t)$ is given by

$$i(t)=\begin{cases} 0\text{ A} & \text{for } t<0 \\ 2.4e^{-30t}\text{ A} & \text{for } t>0 \end{cases}$$

(a) Determine the capacitor voltages, $v_1(t)$ and $v_2(t)$, for $t \ge 0$.
(b) Determine the energy stored by each capacitor 20 ms after the switch closes.

The series capacitors can be replaced by an equivalent capacitor.

(c) Determine the voltage across the equivalent capacitor, + on top, for $t \ge 0$.
(d) Determine the energy stored by the equivalent capacitor 20 ms after the switch closes.

Solution:

(a)
$$v_1(t)=\frac{1}{0.01}\int_0^t 2.4e^{-30\tau}d\tau+0=\frac{240}{-30}\left(e^{-30t}-1\right)=+8\left(1-e^{-30\tau}\right)\text{ V}\quad\text{for } t\ge 0$$
$$v_2(t)=\frac{1}{0.04}\int_0^t 2.4e^{-30\tau}d\tau+0=2\left(1-e^{-30t}\right)\text{ V}\quad\text{for } t\ge 0$$

(b) When t = 20 ms, $v_1(0.02)=8\left(1-e^{-0.6}\right)=3.610\text{ V}$ and $v_2(0.02)=2\left(1-e^{-0.6}\right)=0.902\text{ V}$ so the energy stored by the 10 mF capacitor is $w_1=\frac{1}{2}(0.01)3.610^2=65.2\text{ mJ}$ and the energy stored by the 40 mF capacitor is $w_2=\frac{1}{2}(0.04)0.902^2=16.3\text{ mJ}$.

Next $C_{eq}=\frac{10\times 40}{10+40}=8\text{ mF}$

(c)
$$v(t)=\frac{1}{0.08}\int 2.4e^{-30\tau}d\tau=10\left(1-e^{-30t}\right)\text{ V}\quad\text{for } t\ge 0$$

(d) When t = 20 ms, $v(0.02)=10\left(1-e^{-0.6}\right)=4.512\text{ V}$ so the energy stored by the equivalent capacitor is $w=\frac{1}{2}(0.008)4.512^2=81.4\text{ mJ}=w_1+w_2$.

P 7.4-10 Find the relationship for the division of current between two parallel capacitors as shown in Figure P 7.4-10.

Answer: $i_n = iC_n/(C_1 + C_2)$, $n = 1, 2$

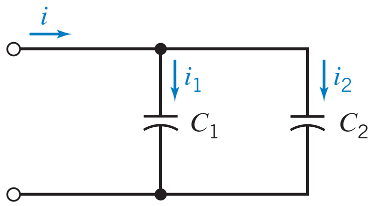
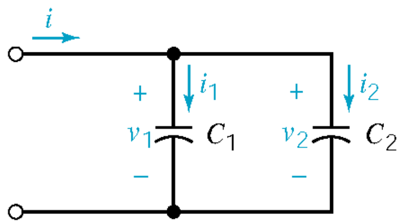


Figure P 7.4-10

Solution:



$$v_1 = v_2 \Rightarrow \frac{dv_1}{dt} = \frac{dv_2}{dt} \Rightarrow \frac{i_1}{C_1} = \frac{i_2}{C_2} \Rightarrow \underline{i_1 = \frac{C_1}{C_2} i_2}$$

$$\text{KCL:}\quad i = i_1 + i_2 = \left(\frac{C_1}{C_2} + 1\right) i_2 \Rightarrow \underline{i_2 = \frac{C_2}{C_1 + C_2} i}$$

Section 7-5: Inductors

P 7.5-1 Nikola Tesla (1857–1943) was an American electrical engineer who experimented with electric induction. Tesla built a large coil with a very large inductance. The coil was connected to a source current

$$i_s = 100 \sin 400t \text{ A}$$

so that the inductor current $i_L = i_s$. Find the voltage across the inductor and explain the discharge in the air shown in the figure. Assume that $L = 200$ H and the average discharge distance is 2 m. Note that the dielectric strength of air is 3×10^6 V/m.

Solution:

Find max. voltage across coil: $v(t) = L\dfrac{di}{dt} = 200\ \left[100(400)\cos\ 400t\right]$ V

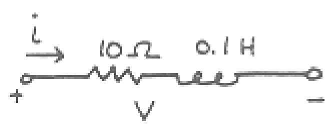
$\therefore\ v_{max} = 8\times10^6$ V thus have a field of $\dfrac{8\times10^6}{2}\ \text{V/m} = 4\times10^6\ \text{V/m}$

which exceeds dielectric strength in air of 3×10^6 V/m

$\therefore$ We get a discharge as the air is ionized.

P 7.5-2 The model of an electric motor consists of a series combination of a resistor and inductor. A current $i(t) = 4te^{-t}$ A flows through the series combination of a 10-Ω resistor and 0.1-H inductor. Find the voltage across the combination.
Answer: $v(t) = 0.4e^{-t} + 39.6te^{-t}$ V

Solution:



$$v = L\frac{di}{dt} + R\ i = (.1)\ (4e^{-t} - 4te^{-t}) + 10(4te^{-t}) = \underline{0.4e^{-t} + 39.6te^{-t}\ \text{V}}$$

P 7.5-3 The voltage, $v(t)$, and current, $i(t)$, of a 1-H inductor adhere to the passive convention. Also, $v(0) = 0$ V and $i(0) = 0$ A.

(a) Determine $v(t)$ when $i(t) = x(t)$, where $x(t)$ is shown in Figure P 7.5-3 and $i(t)$ has units of A.

(b) Determine $i(t)$ when $v(t) = x(t)$, where $x(t)$ is shown in Figure P 7.5-3 and $v(t)$ has units of V.

Hint: $x(t) = 4t - 4$ when $1 < t < 2$, and $x(t) = -4t + 12$ when $2 < t < 3$.

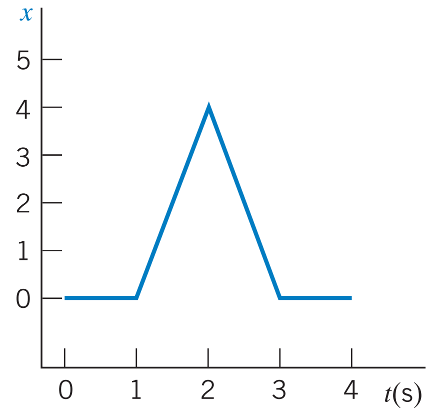


Figure P 7.5-3

Solution:

(a)

$$v(t) = L\frac{d}{dt}i(t) = \begin{cases} 0 & 0 < t < 1 \\ 4 & 1 < t < 2 \\ -4 & 2 < t < 3 \\ 0 & 3 < t \end{cases}$$

(b)

$$i(t) = \frac{1}{L}\int_0^t v(\tau)\,d\tau + i(0) = \int_0^t v(\tau)\,d\tau$$

For $0 < t < 1$, $v(t) = 0$ V so $i(t) = \int_0^t 0\,d\tau + 0 = 0$ A

For $1 < t < 2$, $v(t) = (4t - 4)$ V so

$$i(t) = \int_0^t (4\tau - 4)\,d\tau + 0 = \left(2\tau^2 - 4\tau\right)\Big|_1^t = 2t^2 - 4t + 2 \text{ A}$$

$$i(2) = 4\left(2^2\right) - 4(2) + 2 = 2 \text{ A}$$

For $2 < t < 3$, $v(t) = -4t + 12$ V so

$$i(t) = \int_2^t (-4\tau + 12)\,d\tau + 2 = \left(-2\tau^2 + 12\tau\right)\Big|_2^t + 2 = \left(-2t^2 + 12t - 14\right) \text{ A}$$

$$i(3) = -2\left(3^2\right) + 12(3) - 14 = 4 \text{ A}$$

For $3 < t$, $v(t) = 0$ V so $i(t) = \int_3^t 0\,d\tau + 4 = 4$ A

P 7.5-4 The voltage, $v(t)$, across an inductor and current, $i(t)$, in that inductor adhere to the passive convention. Determine the voltage, $v(t)$, when the inductance is L = 250 mH and the current is $i(t)$ = 120 sin (500t – 30°) mA.

Hint: $\frac{d}{dt} A\sin(\omega t+\theta) = A\cos(\omega t+\theta)\cdot\frac{d}{dt}(\omega t+\theta) = A\omega\cos(\omega t+\theta) = A\omega\sin\left(\omega t+\left(\theta+\frac{\pi}{2}\right)\right)$

Answer: $v(t)$ = 15 sin(500t + 60°) V

Solution:

$$v(t) = (250\times10^{-3})\frac{d}{dt}(120\times10^{-3})\sin(500t-30^{\circ}) = (0.25)(0.12)(500)\cos(500t-30^{\circ})$$

$$=15\cos(500t-30^{\circ})$$

P 7.5-5 Determine $i_L(t)$ for $t > 0$ when $i_L(0) = -2\ \mu A$ for the circuit of Figure P 7.5-5*a* when $v_s(t)$ is as shown in Figure P 7.5-5*b*.

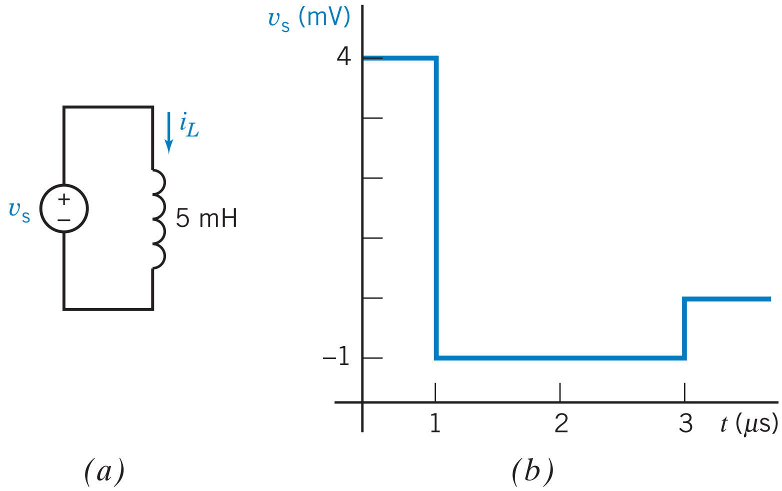


Figure P 7.5-5

Solution:

$$i_L(t) = \frac{1}{5\times10^{-3}} \int_0^t v_s(\tau)\, d\tau - 2\times10^{-6}$$

for $0 < t < 1\ \mu s$ $\quad v_s(t) = 4\ \text{mV}$

$$i_L(t) = \frac{1}{5\times10^{-3}} \int_0^t 4\times10^{-3}\, d\tau - 2\times10^{-6} = \left(\frac{4\times10^{-3}}{5\times10^{-3}}\right) t - 2\times10^{-6} = 0.8 - 2\times10^{-6}\ \text{A}$$

$$i_L(1\mu s) = \left(\frac{4\times10^{-3}}{5\times10^{-3}}\left(1\times10^{-6}\right)\right) - 2\times10^{-6} = -\frac{6}{5}\times10^{-6}\ \text{A} = -1.2\ \text{A}$$

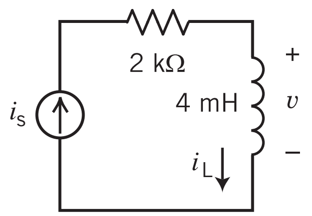
for $1\mu s < t < 3\ \mu s$ $\quad v_s(t) = -1\ \text{mV}$

$$i_L(t) = \frac{1}{5\times10^{-3}} \int_{1\mu s}^t \left(-1\times10^{-3}\right) d\tau - \frac{6}{5}\times10^{-6} = -\frac{1\times10^{-3}}{5\times10^{-3}}(t - 1\times10^{-6}) - \frac{6}{5}\times10^{-6} = \left(-0.2t - 10^{-6}\right)\ \text{A}$$

$$i_L(3\mu s) = \left(-\frac{1\times10^{-3}}{5\times10^{-3}} + 3\times10^{-6}\right) - 1\times10^{-6} = -1.6\ \mu\text{A}$$

for $3\mu s < t$ $\quad v_s(t) = 0$ so $i_L(t)$ remains $-1.6\ \mu\text{A}$

P 7.5-6 Determine $v(t)$ for $t > 0$ for the circuit of Figure P 7.5-6a when $i_L(0) = 0$ and i_s is as shown in Figure P 7.5-6b.



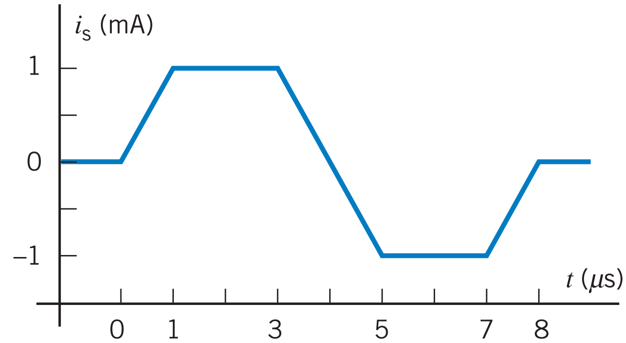


Figure P 7.5-6

Solution:

In general $$v(t) = \left(2\times10^3\right) i_s(t) + \left(4\times10^{-3}\right)\frac{d}{dt} i_s(t)$$

For 0<t<1 μs $i_s(t) = (1)\left(\frac{1\times10^{-3}}{1\times10^{-6}}\right)t = 10^3\, t \Rightarrow \frac{d}{dt} i_s(t) = 1\times10^3$. Consequently

$$v(t) = (2\times10^3)\left(1\times10^3\right) t + 4\times10^{-3}\left(1\times10^3\right) = \left(2\times10^6\, t + 4\right) \text{ V}$$

For 1μs <t< 3μs $i_s(t) = 1$ mA $\Rightarrow \frac{d}{dt} i_s(t) = 0$. Consequently

$$v(t) = (2\times10^3)\left(1\times10^{-3}\right) + \left(4\times10^{-3}\right)\times 0 = 2 \text{ V}$$

For 3μs< t < 5μs $i_s(t) = 4\times10^{-3} - \left(\frac{1\times10^{-3}}{1\times10^{-6}}\right)t \Rightarrow \frac{d}{dt} i_s(t) = -\frac{1\times10^{-3}}{1\times10^{-6}} = -10^3$. Consequently

$$v(t) = \left(2\times10^3\right)\left(4\times10^{-3} - 10^3\, t\right) + 4\times10^{-3}\left(-10^3\right) = 4 - \left(2\times10^6\right) t$$

When 5μs <t< 7μs $i_s(t) = -1\times10^{-3}$ and $\frac{d}{dt} i_s(t) = 0$. Consequently

$$v(t) = \left(2\times10^3\right)\left(10^{-3}\right) = -2 \text{ V}$$

When 7μs< t < 8μs $i_s(t) = \left(\frac{1\times10^{-3}}{1\times10^{-6}}\right)t - 8\times10^{-3} \Rightarrow \frac{d}{dt} i_s(t) = 1\times10^3$

$$v(t) = \left(2\times10^3\right)\left(10^3\, t - 8\times10^{-3}\right) + \left(4\times10^{-3}\right)\left(10^3\right) = -12 + \left(2\times10^6\right) t$$

When 8μs < t, then $i_s(t) = 0 \Rightarrow \frac{d}{dt} i_s(t) = 0$. Consequently $v(t) = 0$.

P 7.5-7 The voltage, $v(t)$, and current, $i(t)$, of a 0.5-H inductor adhere to the passive convention. Also, $v(0) = 0$ V and $i(0) = 0$ A.

(a) Determine $v(t)$ when $i(t) = x(t)$, where $x(t)$ is shown in Figure P 7.5-7 and $i(t)$ has units of A.

(b) Determine $i(t)$ when $v(t) = x(t)$, where $x(t)$ is shown in Figure P 7.5-7 and $v(t)$ has units of V.

Hint: $x(t) = 0.2t - 0.4$ when $2 < t < 6$.

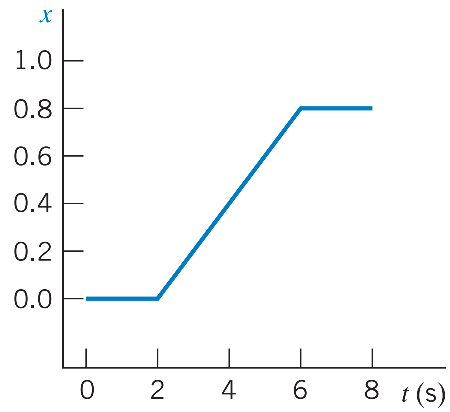


Figure P 7.5-7

Solution:

(a)

$$v(t) = L\frac{d}{dt}i(t) = \begin{cases} 0 & 0 < t < 2 \\ 0.1 & 2 < t < 6 \\ 0 & 6 < t \end{cases}$$

(b)

$$i(t) = \frac{1}{L}\int_0^t v(\tau)\,d\tau + i(0) = 2\int_0^t v(\tau)\,d\tau$$

For $0 < t < 2$, $v(t) = 0$ V so $i(t) = 2\int_0^t 0\,d\tau + 0 = 0$ A

For $2 < t < 6$, $v(t) = 0.2\,t - 0.4$ V so

$$i(t) = 2\int_2^t (0.2\tau - 0.4)\,d\tau + 0 = \left(0.2\tau^2 - 0.8\tau\right)\Big|_2^t = 0.2t^2 - 0.8t + 0.8 \text{ A}$$

$$i(6) = 0.2\left(6^2\right) - 0.8(6) + 0.8 = 3.2 \text{ A}.$$

For $6 < t$, $v(t) = 0.8$ V so

$$i(t) = 2\int_6^t 0.8\,d\tau + 3.2 = (1.6t - 6.4) \text{ A}$$

P 7.5-8 Determine $i(t)$ for $t \geq 0$ for the current of Figure P 7.5-8*a* when $i(0) = 25$ mA and $v_s(t)$ is the voltage shown in Figure P 7.5-8*b*.

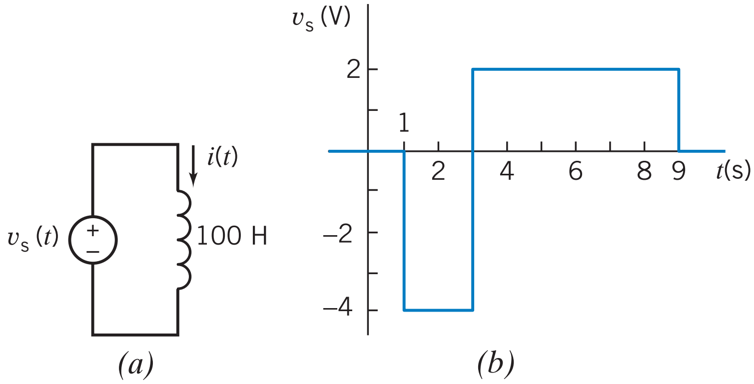


Figure P 7.5-8

Solution:

$$i(t) = \frac{1}{100}\int_0^t 0 \, dt + 0.025 = 0.025 \qquad \text{for} \qquad 0 < t < 1$$

so $i(1) = 0.025$

$$i(t) = \frac{1}{100}\int_1^t -4 \, d\tau + 0.025 = \frac{-4(t-1)}{100} \qquad \text{for} \qquad 1 < t < 3$$

so $i(3) = -0.055$

$$i(t) = \frac{1}{100}\int_3^t 2 \, d\tau - 0.055 = \frac{2(t-3)}{100} - 0.055 \qquad \text{for} \qquad 3 < t < 9$$

so $i(9) = 0.065$

$$i(t) = \frac{1}{100}\int_9^t 0 \, d\tau + 0.065 = 0.065 \qquad \text{for} \qquad t > 9$$

P 7.5-9 Determine $i(t)$ for $t \geq 0$ for the current of Figure P 7.5-9*a* when $i(0) = -2$ A and $v_s(t)$ is the voltage shown in Figure P 7.5-9*b*.

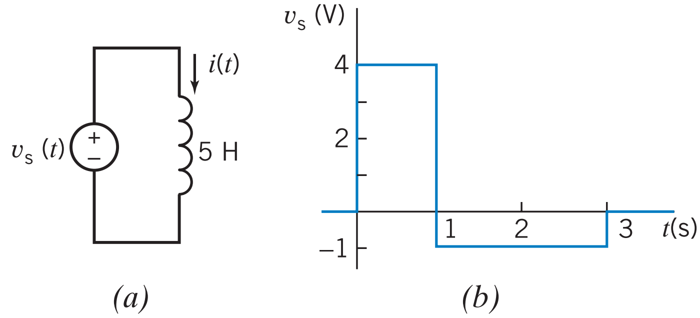


Figure P 7.5-9

Solution:

$$i(t) = i(0) + \frac{1}{L}\int_0^t v_s(\tau)\, d\tau = -2 + \frac{1}{5}\int_0^t v_s(\tau)\, d\tau$$

For $0 \leq t \leq 1$ s

$$i(t) = -2 + \frac{1}{5}\int_0^t 4\, d\tau = -2 + \frac{4}{5}\tau\Big|_0^t = -2 + \frac{4}{5}t$$

For example $i(0) = -2,\; i(1) = -\frac{6}{5}$ and $i\left(\frac{1}{2}\right) = -\frac{8}{5}$

For $1 \leq t \leq 3$ s

$$i(t) = -2 + \frac{1}{5}\int_0^1 4\, dt + \frac{1}{5}\int_1^t -1\, d\tau = -2 + \frac{4}{5} - \frac{1}{5}\tau\Big|_0^t = -\frac{6}{5} - \frac{1}{5}(t-1) = -1 - \frac{t}{5}$$

For example $i(1) = -\frac{6}{5},\; i(2) = -\frac{7}{5},\; i(3) = -\frac{8}{5}$

Notice that $$i(t) = i(1) + \frac{1}{5}\int_1^t -1\, d\tau = -\frac{6}{5} - \frac{1}{5}(t-1) = -1 - \frac{t}{5}$$

For $3 \leq t$

$$i(t) = i(0) + \frac{1}{5}\int_0^1 4\, d\tau + \frac{1}{5}\int_1^3 -1\, d\tau + \frac{1}{5}\int_3^t 0\, d\tau = -\frac{8}{5}$$

In summary

$$i(t) = \begin{cases} -2 + 0.8t & 0 \leq t \leq 1 \\ -1 - 0.2t & 1 \leq t \leq 3 \\ -1.6 & 3 \leq t \end{cases}$$

P 7.5-10 Determine $i(t)$ for $t \geq 0$ for the current of Figure P 7.5-10*a* when $i(0) = 1$ A and $v_s(t)$ is the voltage shown in Figure P 7.5-10*b*.

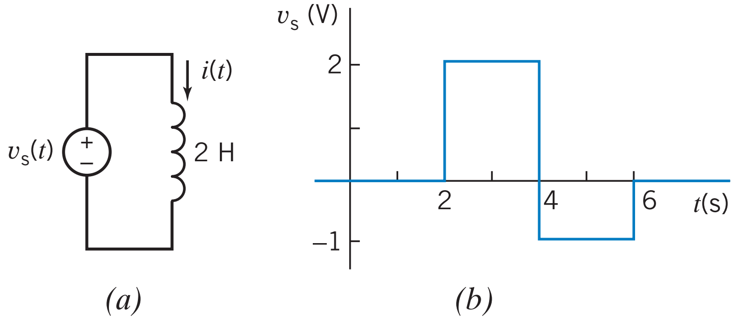


Figure P 7.5-10

Solution:

$$i(t)=\frac{1}{2}\int_0^t v(t)\,dt+1=\begin{cases} 1 & t\leq 2 \\ \int_2^t d\tau+1=(t-2)+1=t-1 & 2\leq t\leq 4 \\ \frac{1}{2}\int_4^t d\tau+3=-\frac{1}{2}t+5 & 4\leq t\leq 6 \\ 2 & 6\leq t \end{cases}$$

P 7.5-11 Determine $i(t)$ for $t \geq 0$ for the circuit of Figure P 7.5-11*a* when $i(0) = 25$ mA and $v_s(t)$ is the voltage shown in Figure P 7.5-11*b*.

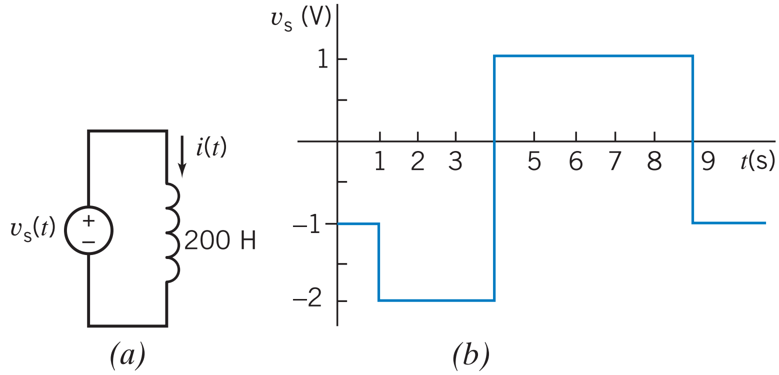


Figure P 7.5-11

Solution:

$$i(t) = \frac{1}{200}\int_0^t -d\tau + 0.025 = \frac{-t}{200} + 0.025 \qquad \text{for} \qquad 0 < t < 1$$

$$i(t) = \frac{1}{200}\int_1^t -2\ d\tau + 0.02 = \frac{-2(t-1)}{200} + 0.02 \qquad \text{for} \qquad 1 < \mathrm{t} < 4$$

$$i(t) = \frac{1}{200}\int_4^t d\tau - 0.01 = \frac{t-4}{200} - 0.01 \qquad \text{for} \qquad 4 < \mathrm{t} < 9$$

$$i(t) = 0.015 = 15 \text{ mA} \qquad t < 9$$

P 7.5-12 The inductor current in the circuit shown in Figure P 7.5-12 is given by

$$i(t) = 6 + 4e^{-8t} \text{ A} \quad \text{for } t \geq 0$$

Determine $v(t)$ for $t > 0$.

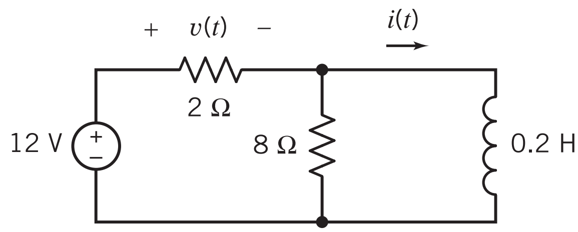
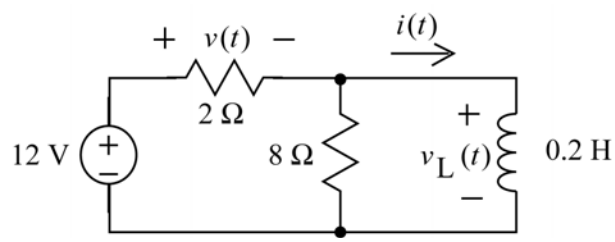


Figure P 7.5-12

Solution:

$$v_L(t) = 0.2\frac{d}{dt}i(t)$$
$$= -6.4e^{-8t} \text{ V} \quad \text{for } t > 0$$



Use KVL to get

$$v(t) = 12 - \left(-6.4e^{-8t}\right) = 12 + 6.4e^{-8t} \text{ V} \quad \text{for } t > 0$$

(checked: LNAP 6/25/04)

P 7.5-13 The inductor current in the circuit shown in Figure P 7.5-13 is given by

$$i(t) = 5 - 3e^{-4t} \text{ A} \quad \text{for } t \geq 0$$

Determine $v(t)$ for $t > 0$.

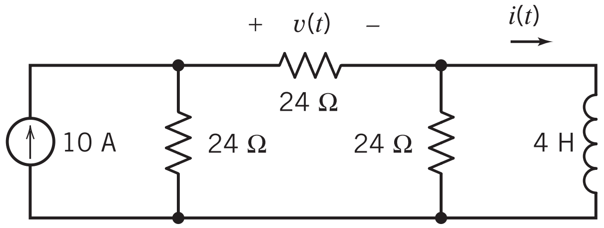
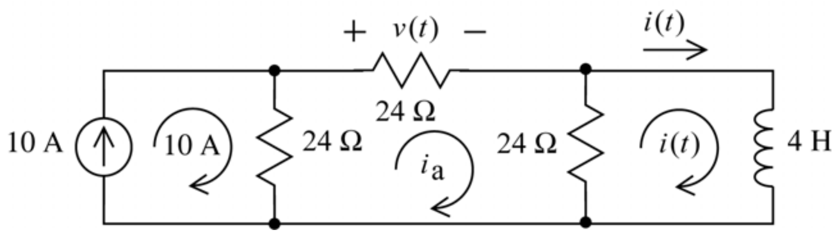


Figure P 7.5-13

Solution:



We'll write and solve a mesh equation. Label the meshes as shown. Apply KVL to the center mesh to get

$$24i_a + 24\left(i_a - i(t)\right) + 24\left(i_a - 10\right) = 0 \quad \Rightarrow \quad i_a = \frac{i(t)+10}{3} = 5 - e^{-4t} \text{ A for } t > 0$$

Then
$$v(t) = 24i_a = 120 - 24e^{-4t} \text{ V for } t > 0$$

(checked: LNAP 6/25/04)

P 7.5-14 The inductor current in the circuit shown in Figure P 7.5-14 is given by

$$i(t) = 3 + 2e^{-3t} \text{ A} \quad \text{for } t \geq 0$$

Determine $v(t)$ for $t > 0$.

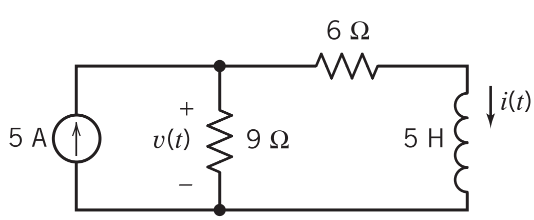


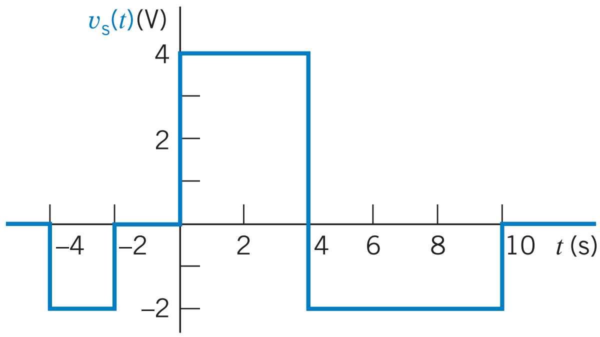
Figure P 7.5-14

Solution: Apply KVL to get

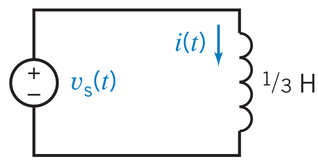
$$v(t) = 6i(t) + 5\frac{d}{dt}i(t) = 6\left(3 + 2e^{-3t}\right) + 5\frac{d}{dt}\left(3 + 2e^{-3t}\right) = 18\left(1 - e^{-3t}\right) \text{ V for } t > 0$$

(checked: LNAP 6/25/04)

P 7.5-15 Determine the current $i(t)$ for $t > 0$ for the circuit of Figure P 7.5-15b when $v_s(t)$ is the voltage shown in Figure P 7.5-15a. The inductor current at time $t = 0$ is $i(0) = -12$ A.



(a)



(b)

Figure P 7.5-15

Solution:

$$i(t) = \frac{1}{L}\int_{t_0}^{t} v_s(\tau)\,d\tau + i(t_0) = \frac{1}{\frac{1}{3}}\int_{0}^{t} v_s(\tau)\,d\tau - 12$$

$$i(t) = 3\int_{0}^{t} 4\,d\tau - 12 = 12\,t - 12 \quad \text{for} \quad 0 < t < 4$$ In particular, $i(4) = 36$ A.

$$i(t) = 3\int_{4}^{t} (-2)\,d\tau + 36 = 60 - 6t \quad \text{for} \quad 4 < t < 10$$ In particular, $i(10) = 0$ A.

$$i(t) = 3\int_{10}^{t} 0\,d\tau + 0 = 0 \quad \text{for} \quad 10 < t$$

P7.5-16 The input to the circuit shown in Figure P7.5-16 is the voltage

$$v(t) = 15\,e^{-4t} \text{ V} \quad \text{for } t > 0$$

The initial current in the inductor is $i(0) = 2$ A . Determine the inductor current, $i(t)$, for $t > 0$.

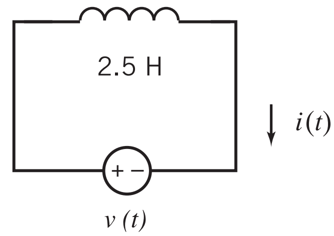


Figure P7.5-16

Solution: The inductor current is related to the inductor voltage by

$$i(t) = \frac{1}{L}\int_0^t v(\tau)\,d\tau + i(0)$$

That is

$$i(t) = \frac{1}{2.5}\int_0^t 15\,e^{-4\tau}\,d\tau + 2 = \frac{15}{2.5(-4)}e^{-4\tau}\Big|_0^t + 2 = -1.5\left(e^{-4t} - 1\right) + 2 = 3.5 - 1.5\,e^{-4t} \text{ A} \quad \text{for } t > 0$$

P7.5-17 The input to the circuit shown in Figure P7.5-17 is the voltage

$$v(t) = 4\,e^{-20t} \text{ V} \quad \text{for } t > 0$$

The output is the current

$$i(t) = -1.2\,e^{-20t} - 1.5 \text{ A} \quad \text{for } t > 0$$

The initial inductor current is $i_L(0) = -3.5$ A . Determine the values of the inductance, *L*, and resistance, *R*.

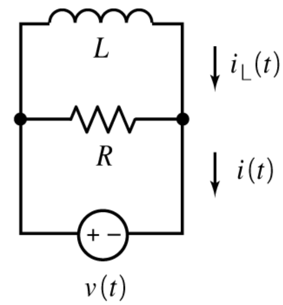


Figure P7.5-17

Solution: Apply KCL at either node to get

$$i(t) = \frac{v(t)}{R} + i_L(t) = \frac{v(t)}{R} + \left[\frac{1}{L}\int_0^t v(\tau)\,d\tau + i(0)\right]$$

That is

$$-1.2\,e^{-20t} - 1.5 = \frac{4\,e^{-20t}}{R} + \frac{1}{L}\int_0^t 4\,e^{-20\tau}\,d\tau - 3.5 = \frac{4\,e^{-20t}}{R} + \frac{4}{L(-20)}\left(e^{-20t} - 1\right) - 3.5$$

$$= \left(\frac{4}{R} - \frac{1}{5L}\right)e^{-20t} + \frac{1}{5L} - 3.5$$

Equating coefficients gives

$$-1.5 = \frac{1}{5L} - 3.5 \quad \Rightarrow \quad L = 0.1 \text{ H}$$

and

$$-1.2 = \frac{4}{R} - \frac{1}{5L} = \frac{4}{R} - \frac{1}{5(0.1)} = \frac{4}{R} - 2 \quad \Rightarrow \quad R = 5\ \Omega$$

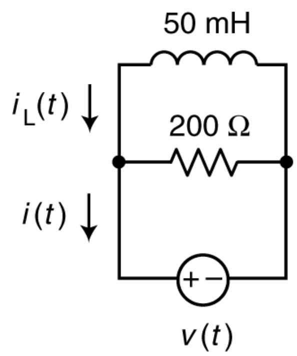
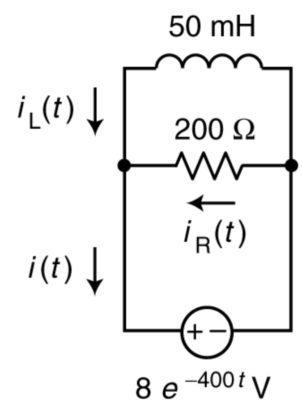


Figure P7.5-18

P7.5-18 . The source voltage the circuit shown in Figure P7.5-18 is $v(t) = 8\ e^{-400t}$ V after time t = 0. The initial inductor current is $i_L(0) = 210$ mA . Determine the source current i (t) for $t > 0$.

Answer: $i(t) = 360\,e^{-400t} - 190$ mA for $t > 0$

Solution:



Label the resistor current as shown. The resistor, inductor and voltage source are connected in parallel so the voltage across each is $v(t) = 2.5\ e^{-400t}$ V . Notice that the labeled voltage and current of both the resistor and inductor do not adhere to the passive convention.

The resistor current is $i_R(t) = -\dfrac{8\ e^{-400t}}{200} = -40\ e^{-400t}$ mA

The inductor current is $i_L(t) = 0.21 + \dfrac{1}{0.05}\displaystyle\int_0^t -8\ e^{-400\tau}\, d\tau$ A

$$i_L(t) = 0.21 + \frac{-8}{0.05(-400)}\int_0^t e^{-400\tau}\, d\tau \text{ A}$$

$$= 0.21 + 0.4\left(e^{-400t} - 1\right) \text{ A}$$

$$= 400\,e^{-400t} - 190 \text{ mA}$$

Using KCL $\qquad i(t) = 360\,e^{-400t} - 190$ mA for $t > 0$

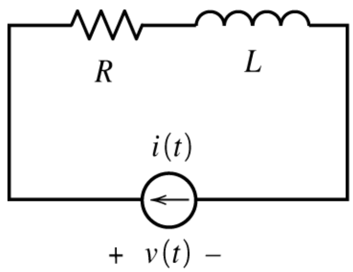


Figure P7.5-19

P7.5-19 . The input to the circuit shown in Figure P7.5-19 is the current

$$i(t) = 5 + 2e^{-7t} \ \text{A} \quad \text{for } t > 0$$

The output is the voltage: $v(t) = 75 - 82e^{-7t} \ \text{V} \quad \text{for } t > 0$

Determine the values of the resistance and inductance.

Solution:

$$v(t) = 75 - 82e^{-7t} = R\left(5 + 2e^{-7t}\right) + L\frac{d}{dt}\left(5 + 2e^{-7t}\right)$$
$$= R\left(5 + 2e^{-7t}\right) + L\left((-7)2e^{-7t}\right) = 5R + (2R - 14L)e^{-7t}$$

Equating coefficients gives $75 = 5R \quad \Rightarrow \quad R = 15\ \Omega$ and

and $-82 = 2R - 14L = 30 - 14L \quad \Rightarrow \quad L = \dfrac{82 + 30}{14} = 8\ \text{H}$

P7.5-20 Consider the inductor shown in Figure P7.5-20. The current and voltage are given by

$$i(t)=\begin{cases}5t-4.6 & 0\le t\le 0.2\\ a\,t+b & 0.2\le t\le 0.5\\ c & t\ge 0.5\end{cases} \quad \text{and} \quad v(t)=\begin{cases}12.5 & 0<t<0.2\\ 25 & 0.2<t<0.5\\ 0 & t>0.5\end{cases}$$

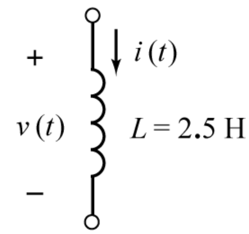


Figure P7.5-20

where *a*, *b* and *c* are real constants. (The current is given in Amps, the voltage in Volts and the time in seconds.) Determine the values of *a*, *b* and *c*.

Answer: *a* = 10 A/s, *b* = –5.6 A and *c* = –0.6 A

Solution: At *t* = 0.2 s

$$i(0.2)=5(0.2)-4.6=-3.6 \text{ A}$$

For $0.2\le t\le 0.5$

$$i(t)=\frac{1}{2.5}\int_{0.2}^{t}25\,d\tau-3.6=10\,\tau\,\Big|_{0.2}^{t}-3.6=10(t-0.2)-3.6=10t-5.6 \text{ A}$$

At *t* = 0.5 s

$$i(0.5)=10(0.5)-5.6=-0.6 \text{ A}$$

For $t\ge 0.5$

$$i(t)=\frac{1}{2.5}\int_{0.5}^{t}0\,d\tau-0.6=-0.6$$

Checks:

At *t* = 0.2 s $\quad i(0.2)=10(0.2)-5.6=-3.6 \text{ A} \quad \surd$

For $0.2\le t\le 0.5$ $\quad v(t)=2.5\frac{d}{dt}i(t)=2.5\frac{d}{dt}(10t-5.6)=2.5(10)=25 \text{ V} \quad \surd$

$$-0.6-(-3.6)=i(0.5)-i(0.2)=\frac{1}{2.5}\int_{0.2}^{0.5}25\,d\tau=10(0.5-0.2)=3 \text{ A} \quad \surd$$

P7.5-21 At time $t = 0$, the current in the inductor shown in Figure P7.5-21 is $i(0) = 45$ mA. Determine the values of the inductor current at times 1 ms, 4 ms and 6 ms.

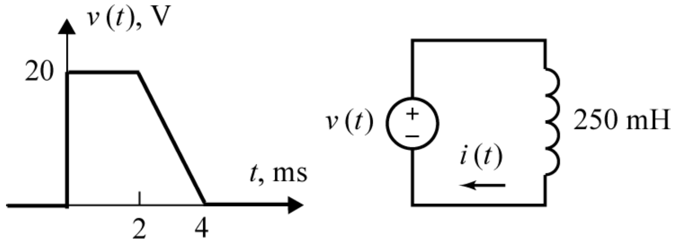


Figure P7.5-21

Solution:

$$i(t) = i(0) + \frac{1}{L}\int_0^t v(\tau)\, d\tau = i(0) + \frac{\text{"area under the curve"}}{L} = 0.045 + \frac{\text{"area under the curve"}}{0.250}$$

$$i(0.001) = 0.045 + \frac{20(0.001)}{0.250} = 0.125 \text{ A} = 125 \text{ mA},$$

$$i(0.004) = 0.045 + \frac{20(0.002) + \frac{1}{2}20(0.002)}{0.250} = 0.285 \text{ A} = 285 \text{ mA}$$

$$i(0.006) = 0.045 + \frac{20(0.002) + \frac{1}{2}20(0.002) + 0}{0.250} = 0.285 \text{ A} = 285 \text{ mA}$$

P7.5-22 One of the three elements shown in Figure P7.5-22 is a resistor, one is a capacitor and one is an inductor. Given

$$i(t) = 0.25\cos(2t) \text{ A},$$

and $\quad v_a(t) = -10\sin(2t)$ V, $v_b(t) = 10\sin(2t)$ V and $v_c(t) = 10\cos(2t)$ V

Determine the resistance of the resistor, the capacitance of the capacitor and the inductance of the inductor. (We require positive values of resistor, capacitance and inductance.)
Answers: resistance = 64Ω, capacitance = 0.0125 F and inductance = 20 H

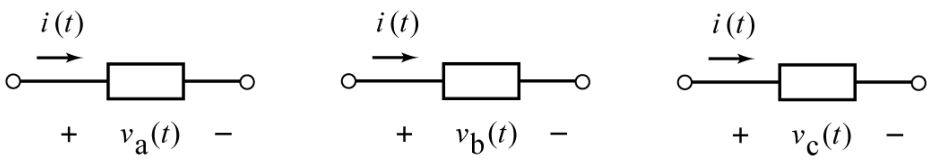


Figure P7.5-22

Solution:

First, $$\frac{d}{dt}i(t) = \frac{d}{dt}\left(0.25\cos(2t)\right) = -(0.25)(2)\sin(2t) = -0.5\sin(2t)$$

The voltage of an inductor is proportional to the derivative of the current. The constant of proportionality is the inductance. We see that $v_a(t)$ is proportional to $\frac{d}{dt}i(t)$ and the constant of proportionality is positive. Consequently, element a is the inductor. Then

$$L = \frac{v_a(t)}{\frac{d}{dt}i(t)} = \frac{-10\sin(2t)}{-0.5\sin(2t)} = 20 \text{ H}$$

Next $$\int_{-\infty}^{t} i(\tau)\,d\tau = \int_{-\infty}^{t} 0.25\cos(2\tau)\,d\tau = \frac{0.25\sin(2\tau)}{2} = 0.125\sin(2\tau)$$

The voltage of a capacitor is proportional to the integral of the current. The constant of proportionality is the reciprocal of the capacitance. We see that $v_b(t)$ is proportional to $\int_{-\infty}^{t} i(\tau)\,d\tau$ and the constant of proportionality is positive. Consequently, element b is the capacitor. Then

$$\frac{1}{C} = \frac{v_b(t)}{\int_{-\infty}^{t} i(\tau)\,d\tau} = \frac{10\sin(2t)}{0.125\sin(2t)} = 80 \quad \Rightarrow \quad C = \frac{1}{80} = 0.0125 \text{ F}$$

Finally, the voltage of element c is proportional to the current and the constant of proportionality is positive. Consequently, element c is the resistor and $R = \frac{v_c(t)}{i(t)} = 40\ \Omega$.

P7.5-23 One of the three elements shown in Figure P7.5-23 is a resistor, one is a capacitor and one is an inductor. Given

$$v(t) = 24\cos(5t) \text{ V},$$

and $i_a(t) = 3\cos(5t)$ A, $i_b(t) = 12\sin(5t)$ A and $i_c(t) = -1.8\sin(5t)$ A

Determine the resistance of the resistor, the capacitance of the capacitor and the inductance of the inductor. (We require positive values of resistor, capacitance and inductance.)

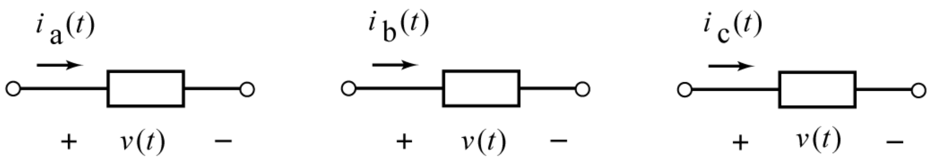


Figure P7.5-23

Solution:

First, the current of element a is proportional to the voltage and the constant of proportionality is positive. Consequently, element a is the resistor and $R = \frac{v(t)}{i_a(t)} = \frac{24\cos(5t)}{3\cos(5t)} = 8\ \Omega$.

Next
$$\frac{d}{dt}v(t) = \frac{d}{dt}\left(24\cos(5t)\right) = -(24)(5)\sin(5t) = -120\sin(5t)$$

The current of a capacitor is proportional to the derivative of the voltage. The constant of proportionality is the capacitance. We see that $i_c(t)$ is proportional to $\frac{d}{dt}v(t)$ and the constant of proportionality is positive. Consequently, element c is the capacitor. Then

$$C = \frac{i_c(t)}{\frac{d}{dt}i(t)} = \frac{-1.8\sin(5t)}{-120\sin(5t)} = 0.015 \text{ F} = 15 \text{ mF}$$

Next
$$\int_{-\infty}^{t} v(\tau)\,d\tau = \int_{-\infty}^{t} 24\cos(5\tau)\,d\tau = \frac{24\sin(5\tau)}{5} = 4.8\sin(5\tau)$$

The voltage of an inductor is proportional to the integral of the current. The constant of proportionality is the reciprocal of the inductance. We see that $i_b(t)$ is proportional to $\int_{-\infty}^{t} v(\tau)\,d\tau$ and the constant of proportionality is positive. Consequently, element b is the inductor. Then

$$\frac{1}{L} = \frac{i_b(t)}{\int_{-\infty}^{t} v(\tau)\,d\tau} = \frac{12\sin(5t)}{4.8\sin(5t)} = 2.5 \quad \Rightarrow \quad L = \frac{1}{2.5} = 0.4 \text{ H}$$

Section 7-6: Energy Storage in an Inductor

P 7.6-1 The current, $i(t)$, in a 100-mH inductor connected in a telephone circuit changes according to

$$i(t)=\begin{cases}0 & t\le 0\\ 4t & 0\le t\le 1\\ 4 & t\ge 1\end{cases}$$

where the units of time are seconds and the units of current are amperes. Determine the power, $p(t)$, absorbed by the inductor and the energy, $w(t)$, stored in the inductor.

Answer: $p(t)=\begin{cases}0 & t\le 0\\ 1.6t & 0<t<1\\ 0 & t\ge 1\end{cases}$ and $w(t)=\begin{cases}0 & t\le 0\\ 0.8t^2 & 0<t<1\\ 0.8 & t\ge 1\end{cases}$

The units of $p(t)$ are W and the units of $w(t)$ are J.

Solution:

$$v(t)=100\times 10^{-3}\,\frac{d}{dt}\,i(t)=\begin{cases}0 & \mathrm{t}<0\\ 0.4 & 0\le\mathrm{t}\le 1\\ 0 & \mathrm{t}>1\end{cases}$$

$$p(t)=v(t)\,i(t)=\begin{cases}0 & t<0\\ 1.6t & 0\le t\le 1\\ 0 & t>1\end{cases}$$

$$w(t)=\int_0^t p(\tau)\,d\tau=\begin{cases}0 & t<0\\ 0.8t^2 & 0<t<1\\ 0.8 & t>1\end{cases}$$

P 7.6-2 The current, $i(t)$, in a 5-H inductor is

$$i(t)=\begin{cases}0 & t\le 0\\ 4\sin 2t & t\ge 0\end{cases}$$

where the units of time are s and the units of current are A. Determine the power, $p(t)$, absorbed by the inductor and the energy, $w(t)$, stored in the inductor.

Hint: 2 (cos A)(sin B) = sin (A + B) + sin (A – B)

Solution:

$$p(t)=v(t)\ i(t)=\left[5\frac{d}{dt}(4\sin 2t)\right](4\sin 2t)$$
$$= 5\ (8\cos 2t)\ (4\sin 2t)$$
$$= 80\ [2\cos 2t\ \ \sin 2t]$$
$$= 80\ [\sin(2t+2t)\ +\ \sin(2t-2t)] = 80\ \sin 4t\ \text{W}$$

$$\text{W}(t)=\int_0^t p(\tau)\ d\tau\ = 80\int_0^t \sin 4\tau\ \text{d}\tau = -\ \frac{80}{4}\ [\cos 4\tau\ |_0^t] = 20\ (1-\cos 4t)$$

P 7.6-3 The voltage, $v(t)$, across a 25-mH inductor used in a fusion power experiment is

$$v(t)=\begin{cases}0 & t\le 0\\ 6\cos 100t & t\ge 0\end{cases}$$

where the units of time are s and the units of voltage are V. The current in this inductor is zero before the voltage changes at $t = 0$. Determine the power, $p(t)$, absorbed by the inductor and the energy, $w(t)$, stored in the inductor. ***Hint:*** 2(cos A)(sin B) = sin(A + B) + sin(A – B)

Answer: $p(t) = 7.2 \sin 200t$ W and $w(t) = 3.6[1 - \cos 200t]$ mJ

Solution:

$$i(t)=\frac{1}{25\times 10^{-3}}\int_0^t 6\cos 100\tau\ d\tau + 0 = \frac{6}{(25\times 10^{-3})(100)}\ [\sin 100\tau\ |_0^t] = 2.4\sin 100t$$

$$p(t)=v(t)\ i(t)=(6\cos 100\,t)(2.4\sin 100t)=7.2\ [\ 2(\cos 100\,t)(\sin 100\,t)\ ]$$
$$=7.2\ [\sin 200\,t+\sin 0]=7.2\sin 200\,t$$

$$w(t)=\int_0^t p(\tau)\ d\tau = 7.2\int_0^t \sin\ 200\tau\, d\tau = -\frac{7.2}{200}\left[\cos\ 200\tau|_0^t\right]$$
$$=0.036[1-\cos 200t]\ \text{J} =\ 36\ [1-\cos 200t\ ]\ \text{mJ}$$

P 7.6-4 The current in an inductor, $L = 1/4$ H, is $i = 4te^{-t}$ A for $t \geq 0$ and $i = 0$ for $t < 0$. Find the voltage, power, and energy in this inductor.

Partial Answer: $w = 2t^2 e^{-2t}$ J

Solution:

$$v = L\frac{di}{dt} = \left(\frac{1}{4}\right)\frac{d}{dt}\left(4\,t\,e^{-t}\right) = \underline{\left(1-t\right)e^{-t}\ \text{V}}$$

$$P = vi = \left[\ \left(1-t\right)e^{-t}\ \right]\left(4\,t\,e^{-t}\right) = \underline{4\,t\left(1-t\right)e^{-2t}\ \text{W}}$$

$$W = \frac{1}{2}Li^2 = \frac{1}{2}\left(\frac{1}{4}\right)\left(4\,t\,e^{-t}\right)^2 = \underline{2\,t^2\,e^{-2t}\ \text{J}}$$

P 7.6-5 The current through the inductor of a television tube deflection circuit is shown in Figure P 7.6-5 when $L = 1/2$ H. Find the voltage, power, and energy in the inductor.

Partial Answer: $p = 2t$ for $0 \le t < 1$

$= 2(t-2)$ for $1 < t < 2$

$= 0$ for other t

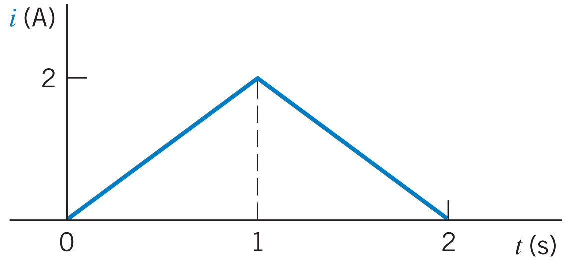


Figure P 7.6-5

Solution:

$$v(t) = L\frac{di}{dt} = \frac{1}{2}\frac{di}{dt} \quad \text{and} \quad i(t) = \begin{cases} 0 & t<0 \\ 2t & 0<t<1 \\ -2(t-2) & 1<t<2 \\ 0 & t>2 \end{cases} \Rightarrow v(t) = \begin{cases} 0 & t<0 \\ 1 & 0<t<1 \\ -1 & 1<t<2 \\ 0 & t>2 \end{cases}$$

$$p(t) = v(t)\,i(t) = \begin{cases} 0 & t<0 \\ 2\,t & 0<t<1 \\ 2(t-2) & 1<t<2 \\ 0 & t>2 \end{cases}$$

$$W(t) = W(t_0) + \int_{t_0}^{t} p(t)\,dt$$

$$i(t) = 0 \text{ for } t<0 \Rightarrow p(t) = 0 \text{ for } t<0 \Rightarrow W(t_0) = 0$$

$$0<t<1:\ W(t) = \int_0^t 2\,t\,dt = t^2$$

$$1<t<2:\ W(t) = W(1) + \int_1^t 2(t-2)\,dt = t^2 - 4t + 4$$

$$t>2:\ W(t) = W(2) = 0$$

Section 7-7: Series and Parallel Inductors

P 7.7-1 Find the current $i(t)$ for the circuit of Figure P 7.7-1.
Answer: $i(t) = 15 \sin 100t$ mA

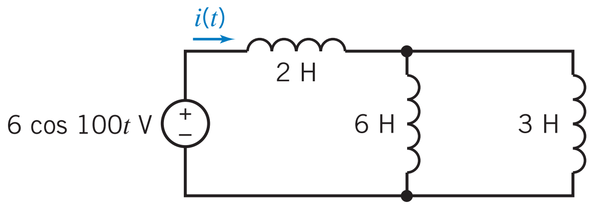


Figure P 7.7-1

Solution:

$$6\text{ H} \parallel 3\text{ H} = \frac{6\times 3}{6+3} = 2\text{ H} \quad \text{and} \quad 2\text{ H} + 2\text{ H} = 4\text{ H}$$

$$i(t) = \frac{1}{4}\int_0^t 6\cos 100\tau \; d\tau = \frac{6}{4\times 100}\left[\sin 100\tau\big|_0^t\right] = 0.015\sin 100t \text{ A} = 15\sin 100t \text{ mA}$$

P 7.7-2 Find the voltage $v(t)$ for the circuit of Figure P 7.7-2.
Answer: $v(t) = -6e^{-250t}$ mV

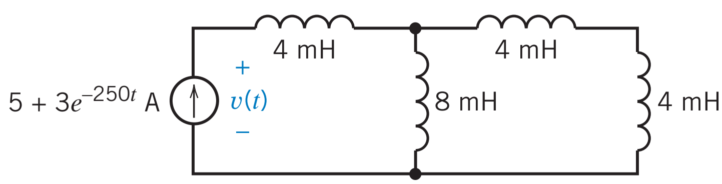


Figure P 7.7-2

Soluton:

$$4\text{ mH} + 4\text{ mH} = 8\text{ mH} \quad , \quad 8\text{mH} \parallel 8\text{mH} = \frac{\left(8\times 10^{-3}\right)\times\left(8\times 10^{-3}\right)}{8\times 10^{-3}+8\times 10^{-3}} = 4\text{ mH}$$

and $4\text{ mH} + 4\text{ mH} = 8\text{ mH}$

$$v(t) = (8\times 10^{-3})\,\frac{d}{dt}\,(5+3e^{-250t}) = (8\times 10^{-3})(0+3(-250)\,e^{-250t}) = -6\,e^{-250t}\text{ V}$$

P 7.7-3 The circuit of Figure P 7.7-3 contains four identical inductors. Find the value of the inductance L.
Answer: $L = 2.86$ H

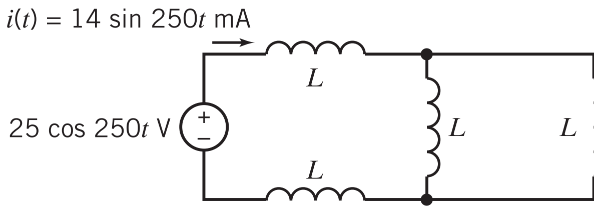


Figure P 7.7-3

Solution:

$$L \parallel L = \frac{L \cdot L}{L+L} = \frac{L}{2} \quad \text{and} \quad L + L + \frac{L}{2} = \frac{5}{2} L$$

$$25\cos 250t = \left(\frac{5}{2} L\right) \frac{d}{dt}\left(\left(14\times10^{-3}\right)\sin 250t\right) = \left(\frac{5}{2} L\right)(14\times10^{-3})(250)\cos 250t$$

$$\text{so } L = \frac{25}{\frac{5}{2}(14\times10^{-3})(250)} = 2.86 \text{ H}$$

P 7.7-4 The circuit shown in Figure P 7.7-4 contains seven inductors, each having inductance L. The source voltage is given by

$$v(t) = 4\cos(3t) \text{ V}$$

Find the current $i(t)$ when $L = 4$ H.

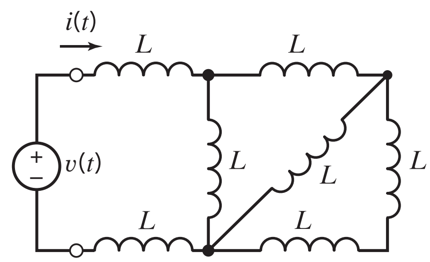


Figure P 7.7-4

Solution: The equivalent inductance is: $\dfrac{\left(\dfrac{L\times 2L}{L+2L}+L\right)\times L}{\left(\dfrac{L\times 2L}{L+2L}+L\right)+L} + 2L = \dfrac{21}{8}L$

Then $$i(t) = \frac{1}{\frac{21}{8}L}\int_{-\infty}^{t} 4\cos(3\tau)\,d\tau = \frac{8}{21\times 4}\times\frac{4}{3}\sin(3t) = 127\sin(3t) \text{ mA}$$

(Checked using LNAP 6/26/04)

P 7.7-5 Determine the value of the inductance *L* in the circuit shown in Figure P 7.7-5, given that L_{eq} = 18 H.
Answer: L = 20 H

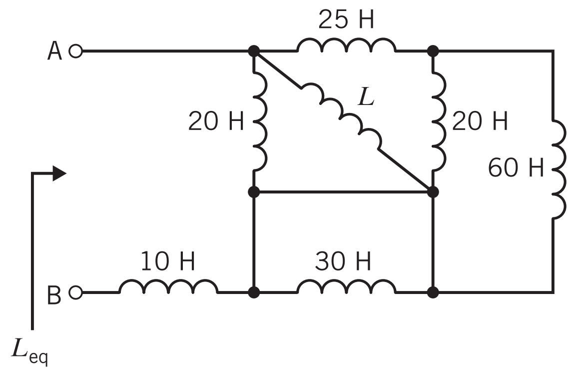
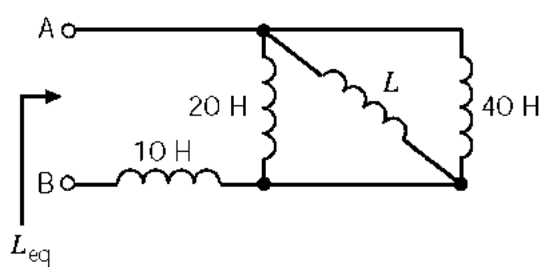


Figure P 7.7-5

Solution: The 25 H inductor is in series with a parallel combination of 20 H and 60 H inductors. The inductance of the equivalent inductor is

$$25+\frac{60\times 20}{60+20}=40 \text{ H}$$

The 30 H inductor is in parallel with a short circuit, which is equivalent to a short circuit. After making these simplifications, we have



Then

$$18=L_{eq}=10+\frac{1}{\frac{1}{20}+\frac{1}{L}+\frac{1}{40}} \quad \Rightarrow \quad \frac{1}{20}+\frac{1}{L}+\frac{1}{40}=\frac{1}{8} \quad \Rightarrow \quad L=20 \text{ H}$$

(Checked using LNAP 6/26/04)

P 7.7-6 Determine the value of the equivalent inductance, L_{eq}, for the circuit shown in Figure P 7.7-6.
Answer: $L_{eq} = 120$ H

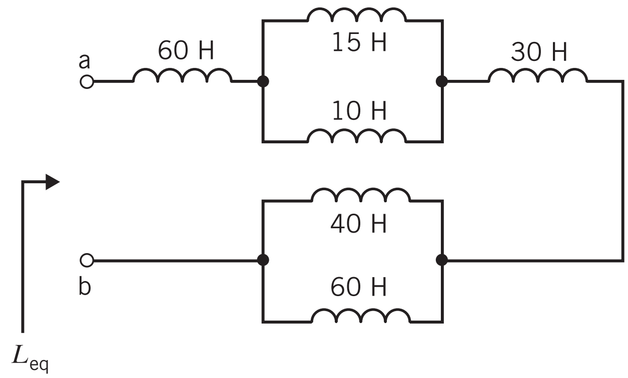


Figure P 7.7-6

Solution:

$$L_{eq} = 60 + \frac{15 \times 10}{15+10} + 30 + \frac{40 \times 60}{40+60} = 60+6+30+24 = 120 \text{ H}$$

(Checked using LNAP 6/26/04)

P 7.7-7 The circuit shown in Figure P 7.7-7 consists of 10 inductors having equal inductance, L. Determine the value of the inductance L, given that L_{eq} = 12 mH.
Answer: L = 35 mH

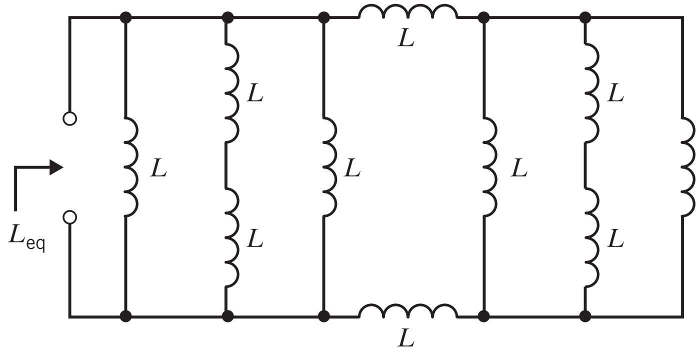
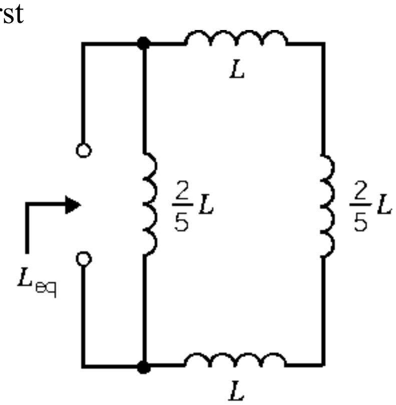


Figure P 7.7-7

Solution:
First



Then

$$12 = L_{eq} = \frac{\left(\frac{2}{5}L\right)\times\left(\frac{2}{5}L+2L\right)}{\left(\frac{2}{5}L\right)+\left(\frac{2}{5}L+2L\right)} = \frac{12}{35}L \quad \Rightarrow \quad L = 35 \text{ mH}$$

(Checked using LNAP 6/26/04)

P 7.7-8 The circuit shown in Figure P 7.7-8 is at steady state before the switch closes. The inductor currents are both zero before the switch closes ($i_1(0) = i_2(0) = 0$).

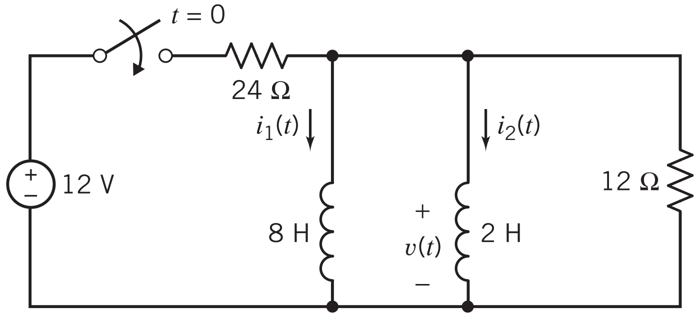


Figure P 7.7-8

The voltage $v(t)$ is given by

$$v(t) = \begin{cases} 0\text{ V} & \text{for } t < 0 \\ 4e^{-5t}\text{ V} & \text{for } t > 0 \end{cases}$$

(a) Determine the inductor currents, $i_1(t)$ and $i_2(t)$, for $t \geq 0$.
(b) Determine the energy stored by each inductor 200 ms after the switch closes.
The parallel inductors can be replaced by an equivalent inductor.
(c) Determine the current in the equivalent inductor, directed downward, for $t \geq 0$.
(d) Determine the energy stored by the equivalent inductor 200 ms after the switch closes.

Solution:

(a)
$$i_1(t) = \frac{1}{8}\int_0^t 4e^{-5\tau}d\tau + 0 = \frac{1}{-10}\left(e^{-5t} - 1\right) = 0.1\left(1 - e^{-5t}\right) \text{ A} \quad \text{for } t \geq 0$$

$$i_2(t) = \frac{1}{2}\int_0^t 4e^{-5\tau}d\tau = 0.4\left(1 - e^{-5t}\right) \text{ A} \quad \text{for } t \geq 0$$

(b) When $t = 0.25$, $i_1(0.2) = 0.1\left(1 - e^{-1}\right) = 63.2 \text{ mA}$ and $i_2(0.2) = 0.4\left(1 - e^{-1}\right) = 252.8 \text{ mA}$ so the energy stored by the 8 H inductor is $w_1 = \frac{1}{2}(8)0.0632^2 = 16.0 \text{ mJ}$ and the energy stored by the 2 H indictor is $w_2 = 63.9 \text{ mJ}$.

(c)
$$L_{eq} = \frac{8 \cdot 2}{8 + 2} = 1.6 \text{ H}$$

$$i(t) = \frac{1}{1.6}\int_0^t 4e^{-5\tau}d\tau = 0.5\left(1 - e^{-5t}\right) \text{ A} \quad \text{for } t \geq 0$$

(d) When $t = 0.2$ s, $i(0.2) = 0.5\left(1 - e^{-1}\right) = 316 \text{ mA}$ so the energy stored by the equivalent inductor is $w = \frac{1}{2}(1.6)0.316^2 = 79.9 \text{ mJ} = w_1 + w_2$.

P 7.7-9 The circuit shown in Figure P 7.7-9 is at steady state before the switch opens at time $t = 0$. The current $i(t)$ is given by

$$i(t) = \begin{cases} 0.8\text{ A} & \text{for } t \le 0 \\ 0.8e^{-2t}\text{ A} & \text{for } t \ge 0 \end{cases}$$

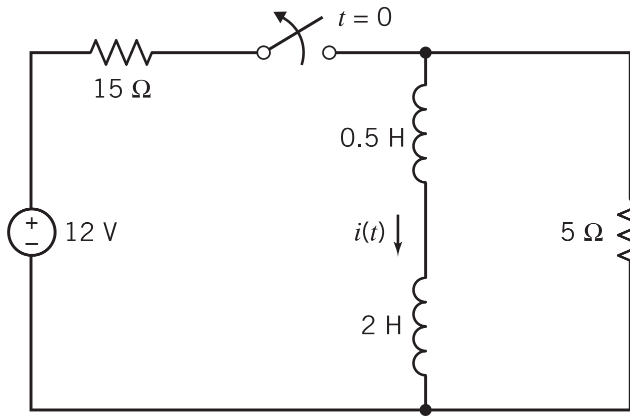


Figure P 7.7-9

(a) Determine the energy stored by each inductor before the switch opens.
(b) Determine the energy stored by each inductor 200 ms after the switch opens.
The series inductors can be replaced by an equivalent inductor.
(c) Determine the energy stored by the equivalent inductor before the switch opens.
(d) Determine the energy stored by the equivalent inductor 200 ms after the switch opens.

Solution:

(a) The energy stored by the 0.5 H inductor is $w_1 = \frac{1}{2}(0.5)\left(0.8^2\right) = 0.16\text{ J}$ and the energy stored by the 2 H inductor is $w_2 = \frac{1}{2}(2)\left(0.8^2\right) = 0.64\text{ J}$.

(b) 200 ms after the switch opens the current in the inductors is $0.8e^{-0.4} = 0.536\text{ A}$. Then $w_1 = \frac{1}{2}(0.5)\left(0.536^2\right) = 71.8\text{ mJ}$ and $w_2 = \frac{1}{2}(2)\left(0.535^2\right) = 287.3\text{ mJ}$.

Next, $L_{eq} = 2 + 0.5 = 2.5\text{ H}$.

(c) $w_{eq} = \frac{1}{2}(2.5)\left(0.8^2\right) = 0.8\text{ J} = w_1 + w_2$

(d) $w_{eq} = \frac{1}{2}(2.5)\left(0.536^2\right) = 359.12\text{ mJ} = w_1 + w_2$

P 7.7-10 Determine the current ratio i_1/i for the circuit shown in Figure P 7.7-10. Assume that the initial currents are zero at t_0.

Answer: $\dfrac{i_1}{i} = \dfrac{L_1}{L_1 + L_2}$

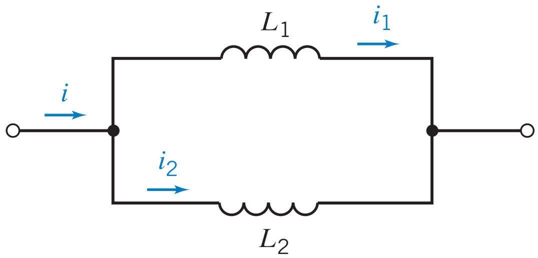


Figure P 7.7-10

Solution:

$$i_1 = \frac{1}{L_1}\int_{t_0}^{t} v\,dt + i_1(t_0), \quad i_2 = \frac{1}{L_2}\int_{t_0}^{t} v\,dt + i_2(t_0) \qquad \text{but } i_1(t_0) = 0 \text{ and } i_2(t_0) = 0$$

$$i = i_1 + i_2 = \frac{1}{L_1}\int_{t_0}^{t} v\,dt + \int_{t_0}^{t} v\,dt = \left(\frac{1}{L_1} + \frac{1}{L_2}\right)\int_{t_0}^{t} v\,dt = \frac{1}{L_P}\int_{t_0}^{t} v\,dt$$

$$\therefore \frac{i_1}{i} = \frac{\frac{1}{L_1}\int_{t_0}^{t} v\,dt}{\frac{1}{L_P}\int_{t_0}^{t} v\,dt} = \frac{\frac{1}{L_1}}{\frac{1}{L_1} + \frac{1}{L_2}} = \underline{\frac{L_2}{L_1 + L_2}}$$

P7.7-11

Consider the combination of circuit elements shown in Figure P7.7-11.

(a) Suppose element A is a 20 μF capacitor, element B is a 5 μF capacitor and element C is a 20 μF capacitor. Determine the equivalent capacitance.

(b) Suppose element A is a 50 mH inductor, element B is a 30 mH inductor and element C is a 20 mH inductor. Determine the equivalent inductance.

(c) Suppose element A is a 9 kΩ resistor, element B is a 6 kΩ resistor and element C is a 10 kΩ resistor. Determine the equivalent resistance.

Answers: (a) C_{eq} = 20 μF, (b) L_{eq} = 16 mH, (c) R_{eq} = 6 kΩ

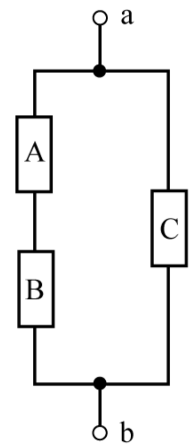


Figure P7.7-11

Solution:

(a) $C_{eq} = \frac{20(5)}{20+5} + 20 = 24\ \mu F$ (b) $L_{eq} = \frac{(50+30)(20)}{(50+30)+20} = 16\ mH$ (c) $R_{eq} = \frac{(9+6)(10)}{(9+6)+10} = 6\ k\Omega$

P7.7-12

Consider the combination of circuit elements shown in Figure P7.7-12.

(a) Suppose element A is a 8 μF capacitor, element B is a 16 μF capacitor and element C is a 12 μF capacitor. Determine the equivalent capacitance.

(b) Suppose element A is a 20 mH inductor, element B is a 5 mH inductor and element C is a 8 mH inductor. Determine the equivalent inductance.

(c) Suppose element A is a 20 kΩ resistor, element B is a 30 kΩ resistor and element C is a 16 kΩ resistor. Determine the equivalent resistance.

Answers: (a) C_{eq} = 8 μF, (b) L_{eq} = 12 mH, (c) R_{eq} = 28 kΩ

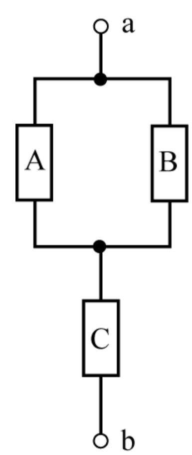


Figure P7.7-11

Solution:

(a) $C_{eq} = \frac{(8+16)(12)}{(8+16)+12} = 8\ \mu F$ (b) $L_{eq} = \frac{(20)(5)}{20+5} + 8 = 12\ mH$ (c) $R_{eq} = \frac{(20)(30)}{20+30} + 16 = 28\ k\Omega$

Section 7-8: Initial Conditions of Switched Circuits

P 7.8-1 The switch in Figure P 7.8-1 has been open for a long time before closing at time $t = 0$. Find $v_c(0^+)$ and $i_L(0^+)$, the values of the capacitor voltage and inductor current immediately after the switch closes. Let $v_c(\infty)$ and $i_L(\infty)$ denote the values of the capacitor voltage and inductor current after the switch has been closed for a long time. Find $v_c(\infty)$ and $i_L(\infty)$.

Answer: $v_c(0^+) = 12$ V, $i_L(0^+) = 0$, $v_c(\infty) = 4$ V, and $i_L(\infty) = 1$ mA

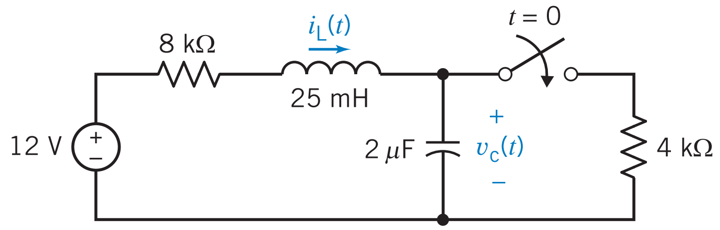
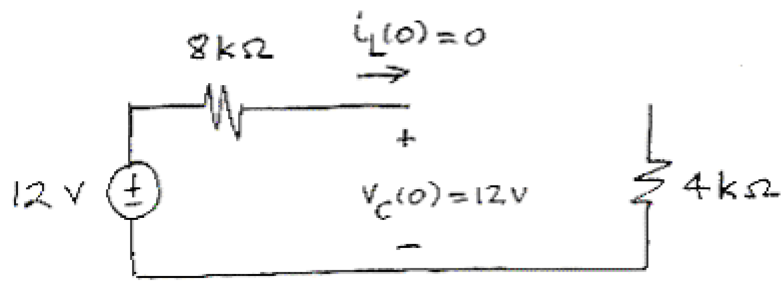


Figure P 7.8-1

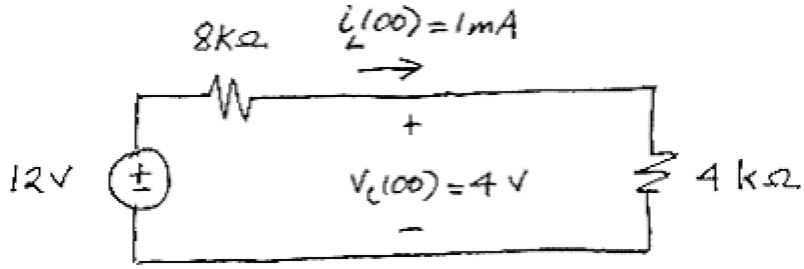
Solution:



Then

$$i_L\left(0^+\right) = i_L\left(0^-\right) = 0 \quad \text{and} \quad v_C\left(0^+\right) = v_C\left(0^-\right) = 12 \text{ V}$$

Next



P 7.8-2 The switch in Figure P 7.8-2 has been open for a long time before closing at time $t = 0$. Find $v_c(0^+)$ and $i_L(0^+)$, the values of the capacitor voltage and inductor current immediately after the switch closes. Let $v_c(\infty)$ and $i_L(\infty)$ denote the values of the capacitor voltage and inductor current after the switch has been closed for a long time. Find $v_c(\infty)$ and $i_L(\infty)$.

Answer: $v_c(0^+) = 6$ V, $i_L(0^+) = 1$ mA, $v_c(\infty) = 3$ V, and $i_L(\infty) = 1.5$ mA

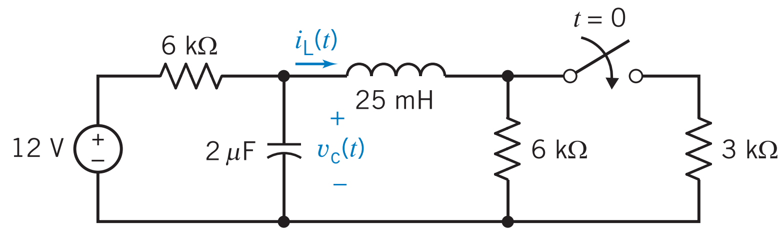
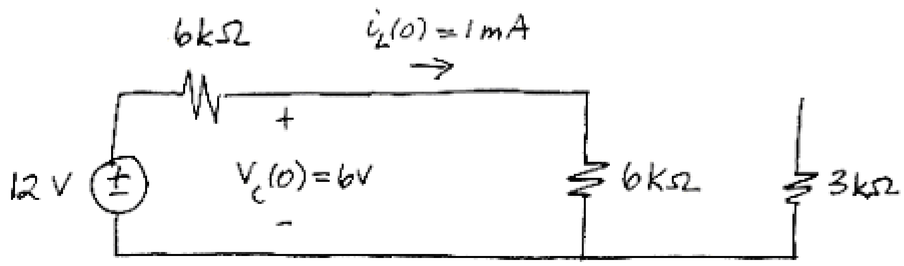


Figure P7.8-2

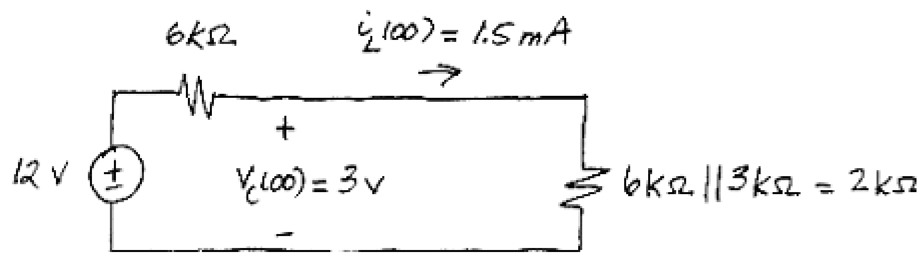
Solution:



Then

$$i_L(0^+) = i_L(0^-) = 1 \text{ mA} \quad \text{and} \quad v_C(0^+) = v_C(0^-) = 6 \text{ V}$$

Next



P 7.8-3 The switch in Figure P 7.8-3 has been open for a long time before closing at time $t = 0$. Find $v_c(0^+)$ and $i_L(0^+)$, the values of the capacitor voltage and inductor current immediately after the switch closes. Let $v_c(\infty)$ and $i_L(\infty)$ denote the values of the capacitor voltage and inductor current after the switch has been closed for a long time. Find $v_c(\infty)$ and $i_L(\infty)$.

Answer: $v_c(0^+) = 0$ V, $i_L(0^+) = 0$, $v_c(\infty) = 8$ V, and $i_L(\infty) = 0.5$ mA

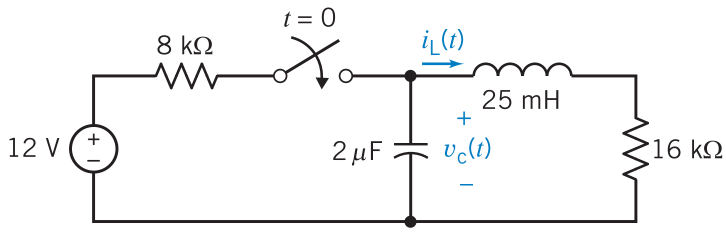
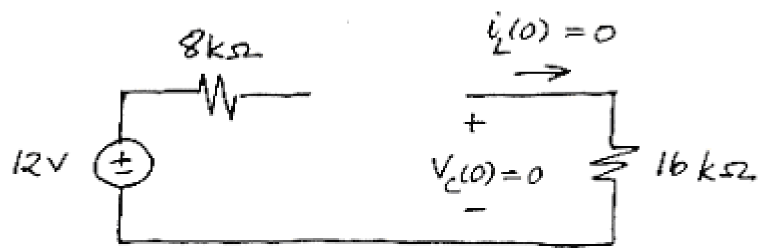


Figure P 7.8-3

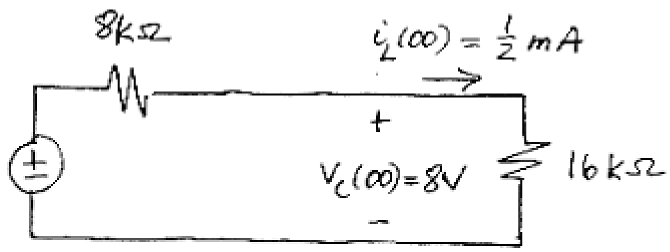
Solution:



Then

$$i_L(0^+) = i_L(0^-) = 0 \quad \text{and} \quad v_C(0^+) = v_C(0^-) = 0 \text{ V}$$

Next



P7.8-4 . The switch in the circuit shown in Figure P7.8-4 has been closed for a long time before it opens at time $t = 0$. Determine the values of $v_R(0-)$ and $v_L(0-)$, the voltage across the 4 Ω resistor and the inductor immediately before the switch opens and the values of $v_R(0+)$ and $v_L(0+)$, the voltage across the 4 Ω resistor and the inductor immediately after the switch opens.

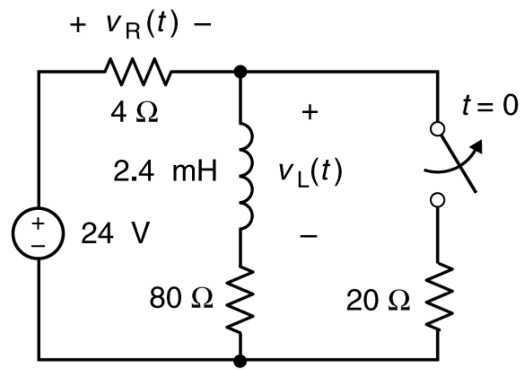
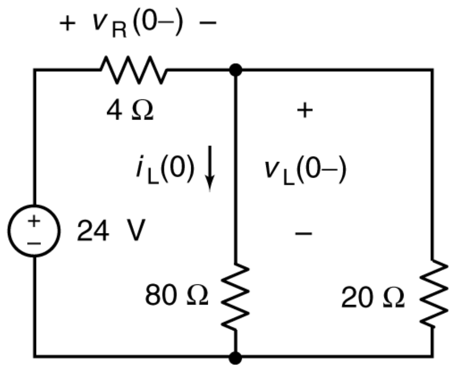


Figure P7.8-4

Solution:

The circuit is at steady state immediately before the switch opens. We have



The inductor acts like a short circuit so $v_L(0-) = 0$.

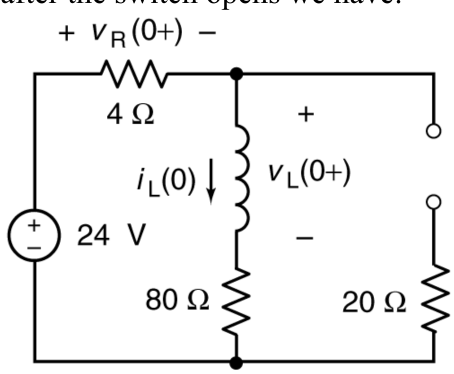
Noticing that the 80 Ω and 20 Ω are connected in parallel and using voltage division:

$$v_R(0-) = \frac{4}{4+(80\,||\,20)}(24) = \frac{4}{4+16}(24) = 4.8 \text{ V}$$

Using current division:

$$i_L(0) = \left(\frac{20}{80+20}\right)\frac{24}{4+(80\,||\,20)} = \frac{1}{5}\left(\frac{24}{4+16}\right) = 0.24 \text{ A}$$

The inductor current does not change instantaneously so $i_L(0+) = i_L(0-) \triangleq i_L(0)$. Immediately after the switch opens we have:



$$v_R(0+) = 4\,i_L(0) = 4(0.24) = 0.96 \text{ V}$$

Using KVL:

$$v_R(0+) + v_L(0+) + 80\,i_L(0) - 24 = 0$$
$$0.96 + v_L(0+) + 80(0.24) - 24 = 0$$
$$v_L(0+) = 3.84 \text{ V}$$

$$v(t) = 75 - 82e^{-7t} = R\left(5+2e^{-7t}\right) + L\frac{d}{dt}\left(5+2e^{-7t}\right)$$
$$= R\left(5+2e^{-7t}\right) + L\left((-7)2e^{-7t}\right) = 5R + (2R - 14L)e^{-7t}$$

Equation coefficients gives $\quad 75 = 5R \;\Rightarrow\; R = 15\ \Omega$ and

and $\quad -82 = 2R - 14L = 30 - 14L \;\Rightarrow\; L = \dfrac{82+30}{14} = 8 \text{ H}$

P7.8-5 . The switch in the circuit shown in Figure P7.8-5 has been open for a long time before it closes at time $t = 0$. Determine the values of $i_R(0-)$ and $i_C(0-)$, the current in one of the 20 Ω resistors and in the capacitor immediately before the switch closes and the values of $i_R(0+)$ and $i_C(0+)$, the current in that 20 Ω resistor and in the capacitor immediately after the switch closes.

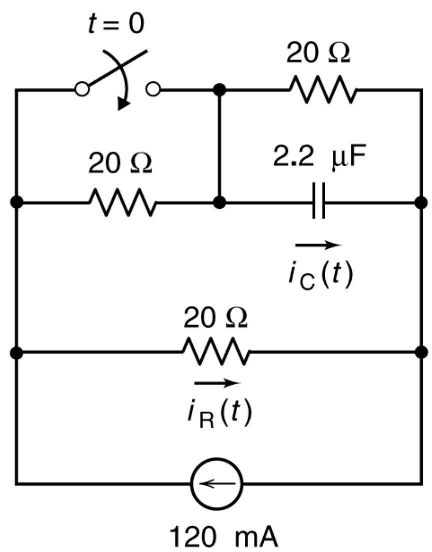
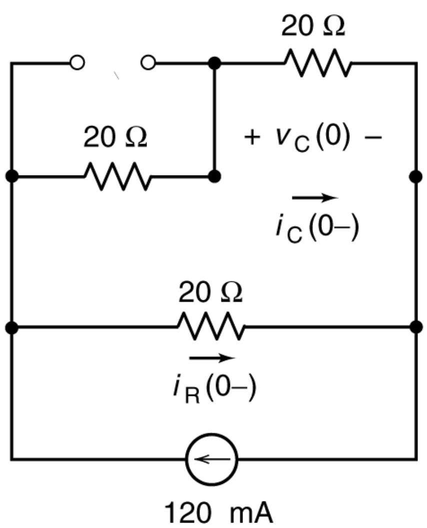


Figure P7.8-5

Solution:

The circuit is at steady state immediately before the switch closes. We have



The capacitor acts like an open circuit so $i_C(0-) = 0$.

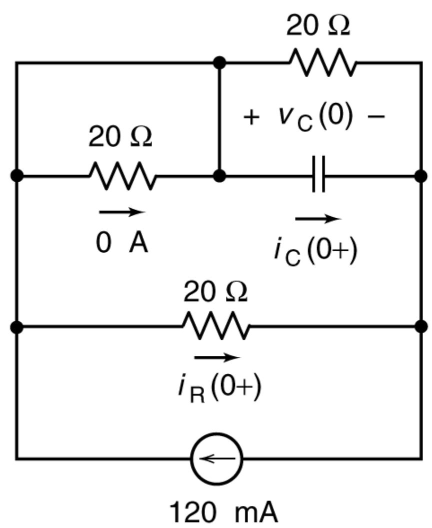
Noticing that two 20 Ω are connected in series and using current division:

$$i_R(0-) = \frac{(20+20)}{20+(20+20)}(120) = \frac{2}{3}(120) = 80 \text{ mA}$$

Using current division and Ohm's law:

$$v_C(0) = \left[\frac{20}{20+(20+20)}(120)\right](20) = 0.8 \text{ V}$$

The capacitor does not change instantaneously so $v_C(0+) = v_C(0-) \triangleq v_C(0)$. Immediately after the switch closes we have:



Applying KVL to the loop consisting of the closed switch, the capacitor and a 20 Ω resistor gives

$$0 + v_C(0) - 20\, i_R(0+) = 0$$
$$0 + 0.8 = 20\, i_R(0+)$$
$$i_R(0+) = 40 \text{ mA}$$

Applying KCL at the node at the right side of the circuit gives:

$$\frac{v_C(0+)}{20} + i_C(0+) + i_R(0+) = 0.120$$
$$\frac{0.8}{20} + i_C(0+) + 0.04 = 0.120$$
$$i_C(0+) = 0.04 = 40 \text{ mA}$$

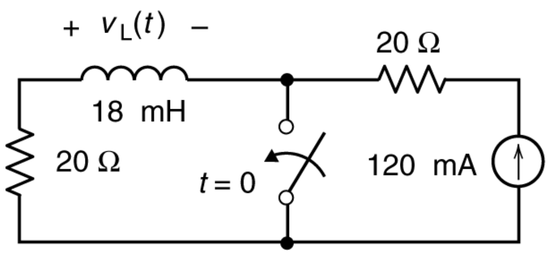
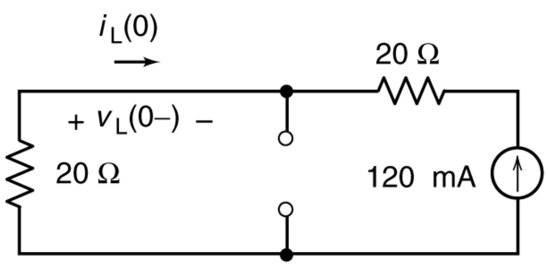


Figure P7.8-6

P7.8-6. The switch in the circuit shown in Figure P7.8-6 has been open for a long time before it closes at time t = 0. Determine the values of v_L(0-), the voltage across the inductor immediately before the switch closes and v_L(0+), the voltage across the inductor immediately after the switch closes.

Solution:

The circuit is at steady state immediately before the switch closes. We have

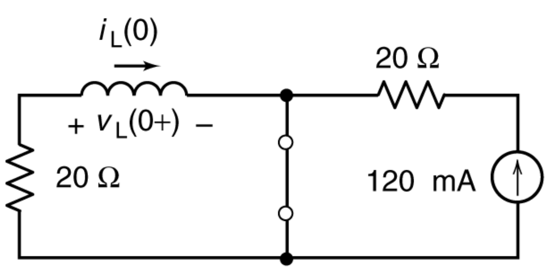


The inductor acts like a short circuit so $v_L(0-)=0$.

The inductor current is the negative of the current source current:

$$i_L(0)=-120\text{ mA}$$

The inductor current does not change instantaneously so $i_L(0+)=i_L(0-)\triangleq i_L(0)$. Immediately after the switch closes we have:



Applying KVL to the left mesh gives:

$$v_L(0+)+20\,i_L(0)=0$$
$$v_L(0+)+20(-0.12)=0$$
$$v_L(0+)=2.4$$

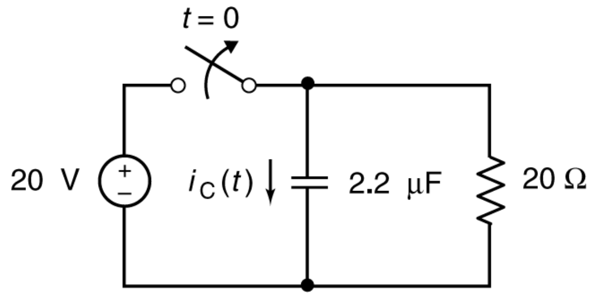
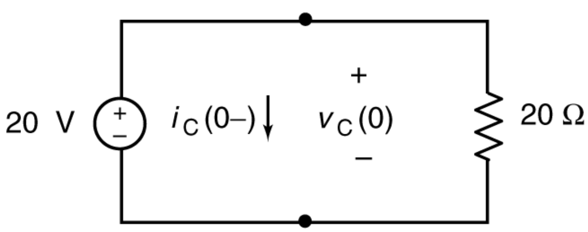


Figure P7.8-7

P7.8-7. The switch in the circuit shown in Figure P7.8-7 has been closed for a long time before it opens at time $t = 0$. Determine the values of $i_C(0\text{-})$, the current in the capacitor immediately before the switch opens and $i_C(0+)$, the current in the capacitor immediately after the switch opens.

Solution:

The circuit is at steady state immediately before the switch opens. We have

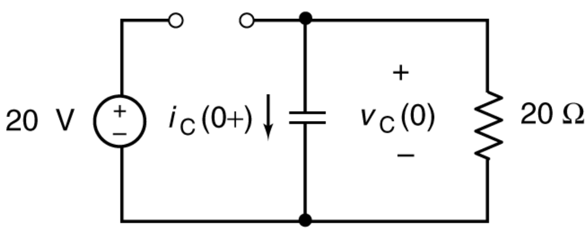


The capacitor acts like an open circuit so $i_C(0-)=0$.

The capacitor voltage is equal to the voltage source voltage:

$$v_C(0)=20\text{ V}$$

The capacitor does not change instantaneously so $v_C(0+)=v_C(0-)\triangleq v_C(0)$. Immediately after the switch opens we have:



Applying KCL at the top node of the capacitor, we see that:

$$i_C(0+)+\frac{v_C(0)}{20}=0$$

$$i_C(0+)=-\frac{v_C(0)}{20}=-1\text{ A}$$

P 7.8-8 The circuit shown in Figure P 7.8-8 is at steady state when the switch opens at time $t = 0$. Determine $v_1(0-)$, $v_1(0+)$, $i_2(0-)$, $i_2(0+)$, $i_3(0-)$, $i_3(0+)$, $v_4(0-)$, and $v_4(0+)$.

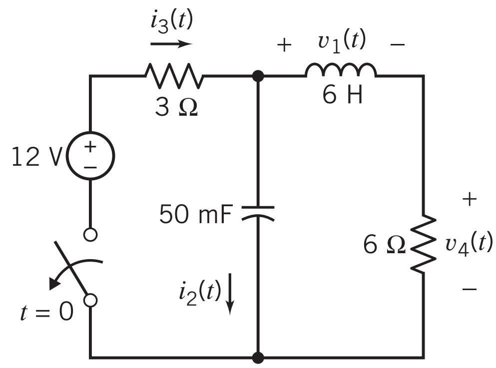
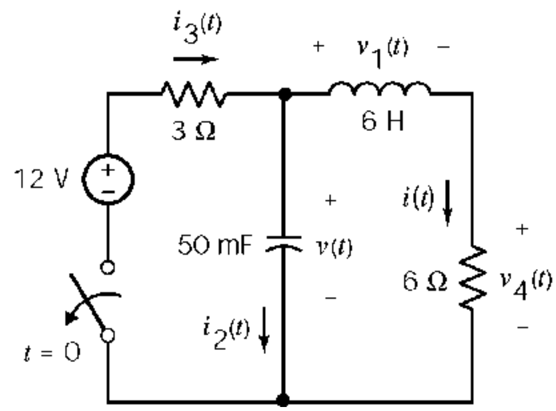


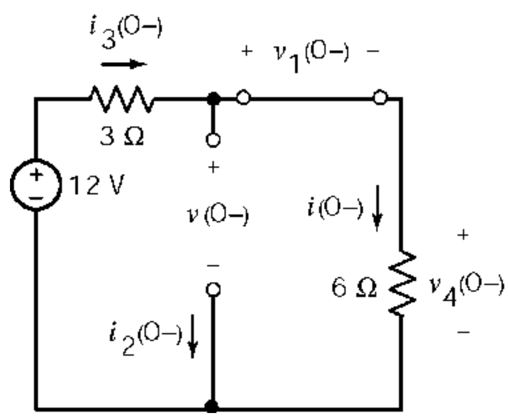
Figure P 7.8-8

Solution: The capacitor voltage and inductor current don't change instantaneously and so are the keys to solving this problem.

Label the capacitor voltage and inductor current as shown.



Before $t = 0$, with the switch closed and the circuit at steady state, the inductor acts like a short circuit and the capacitor acts like an open circuit.



$$i_3(0-) = i(0-) = \frac{12}{9} = 1.33 \text{ A}$$

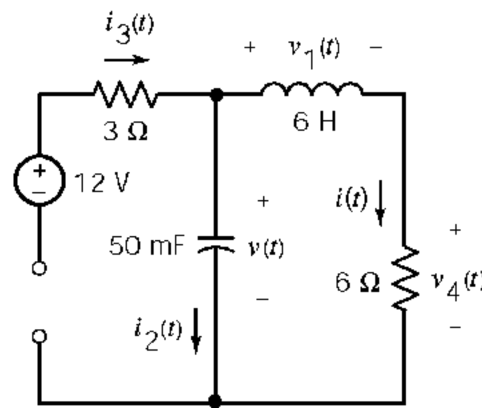
$$v_4(0-) = v(0-) = 6i(0-) = 8 \text{ V}$$

$$v_1(0-) = 0 \text{ V and } i_2(0-) = 0 \text{ A}$$

The capacitor voltage and inductor current don't change instantaneously so

$$v(0+) = v(0-) = 8 \text{ V} \text{ and } i(0+) = i(0-) = 1.33 \text{ A}$$

After the switch opens the circuit looks like this:



From KCL:

$$i_3(t) = 0 \text{ A and } i_2(t) = -i(t)$$

From KVL:

$$v_1(t) + 6\, i(t) = v(t)$$

From Ohm's Law:

$$v_4(t) = 6\, i(t)$$

At $t = 0+$

$$i_3(0+) = 0 \text{ A and } i_2(0+) = -i(0+) = -1.33 \text{ A}$$

$$v_1(0+) = v(0+) - 6\, i(0+) = 8 - 6(1.333) = 0 \text{ V}$$

$$v_4(0+) = 6\, i(0+) = 8 \text{ V}$$

P 7.8-9 The circuit shown in Figure P 7.8-9 is at steady state when the switch opens at time $t = 0$. Determine $v_1(0–)$, $v_1(0+)$, $i_2(0–)$, and $i_2(0+)$.

Hint: Modeling the open switch as an open circuit leads us to conclude that the inductor current changes instantaneously, which would require an infinite voltage. We can use a more accurate model of the open switch, a large resistance, to avoid the infinite voltage.

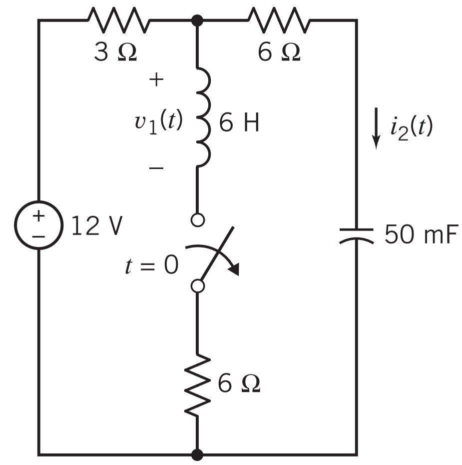
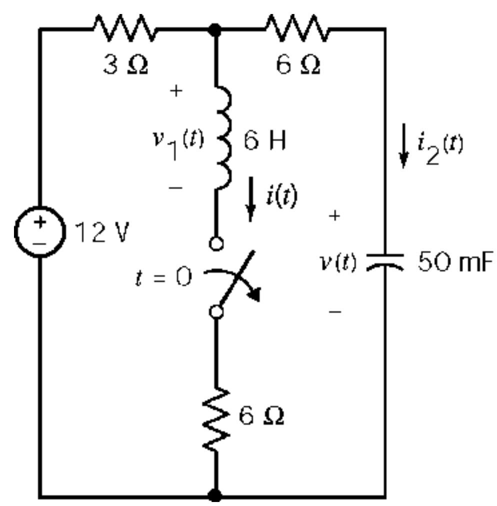


Figure P 7.8-9

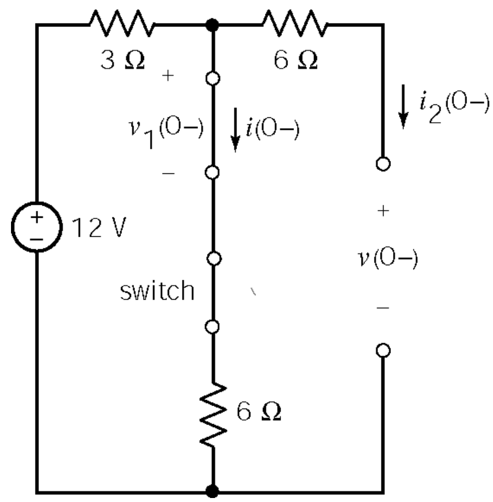
Solution:

The capacitor voltage and inductor current don't change instantaneously and so are the keys to solving this problem.

Label the capacitor voltage and inductor current as shown.



Before $t = 0$, with the switch closed and the circuit at steady state, the inductor acts like a short circuit and the capacitor acts like an open circuit.



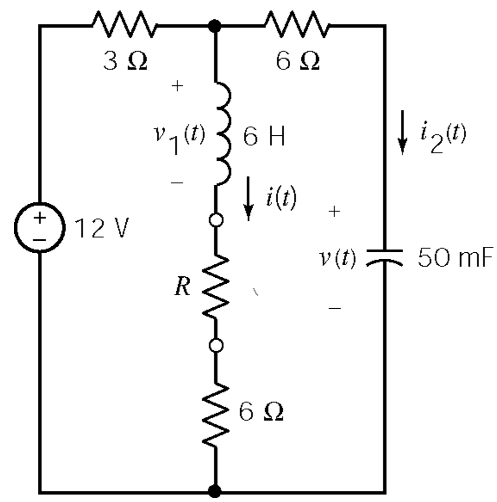
$$i_2(0-) = 0$$

$$i(0-) = \frac{12}{9} = 1.333 \text{ A}$$

$$v_1(0-) = 0 \text{ V}$$

$$v(0-) = 6\,i(0-) = 8 \text{ V}$$

After the switch opens we model the open switch as a large resistance, R.



From KVL:

$$12 = 3\big(i(t) + i_2(t)\big) + v_1(t) + (R+6)\,i(t)$$

and

$$v_1(t) + (R+6)\,i(t) = 6\,i_2(t) + v(t)$$

The capacitor voltage and inductor current don't change instantaneously so

$$v(0+) = v(0-) = 8 \text{ V}$$

and

$$i(0+) = i(0-) = 1.333 \text{ A}$$

At $t = 0+$

$$\left.\begin{array}{r} 12 = 3\big(i(0+) + i_2(0+)\big) + v_1(0+) + (R+6)\,i(0+) \\ v_1(0+) + (R+6)\,i(0+) = 6\,i_2(0+) + v(0+) \end{array}\right\} \Rightarrow \; v_1(0+) = \frac{4}{3}R \text{ and } i_2(0+) = 0$$

As expected $\lim\limits_{R\to\infty} v_1(0+) = \infty$.

P 7.8-10 The circuit shown in Figure P 7.8-10 is at steady state when the switch closes at time $t = 0$. Determine $v_1(0-)$, $v_1(0+)$, $i_2(0-)$, and $i_2(0+)$.

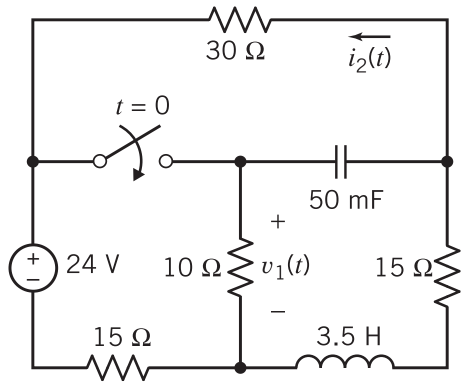
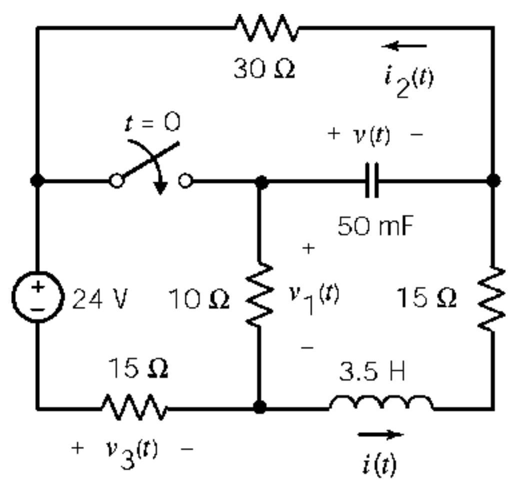


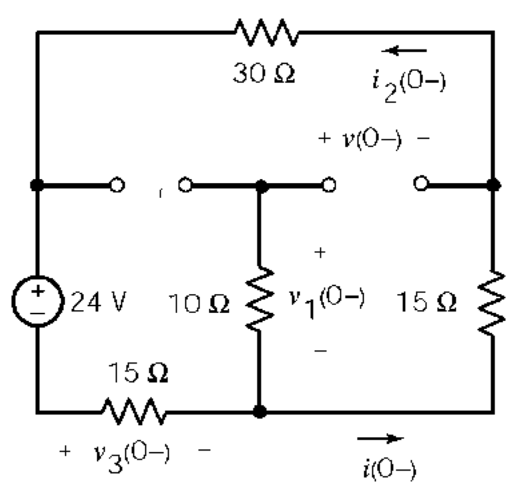
Figure P 7.8-10

Solution:



The capacitor voltage and inductor current don't change instantaneously and so are the keys to solving this problem.

Label the capacitor voltage and inductor current as shown.



Before $t = 0$, with the switch open and the circuit at steady state, the inductor acts like a short circuit and the capacitor acts like an open circuit.

$$i_2(0-) = i(0-) = \frac{24}{60} = -0.4 \text{ A}$$

$$v_1(0-) = 0 \text{ V}$$

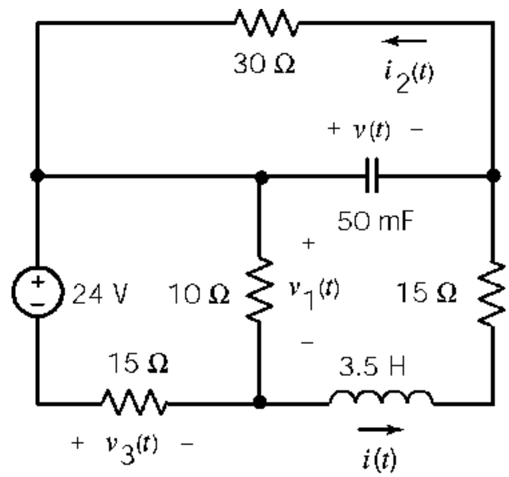
$$v(0-) - 15\,i(0-) = v_1(0-) \quad \Rightarrow \quad v(0-) = -6 \text{ V}$$

$$v_3(0-) = 15\,i(0-) = -6 \text{ V}$$

The capacitor voltage and inductor current don't change instantaneously so

$$v(0+) = v(0-) = -6 \text{ V} \;\text{ and }\; i(0+) = i(0-) = -0.4 \text{ A}$$

After the switch closes the circuit looks like this:



From Ohm's Law:

$$i_2(t) = -\frac{v(t)}{30}$$

From KVL:

$$v_1(t) = v_3(t) + 24$$

From KCL:

$$\frac{v_1(t)}{10} + \frac{v_3(t)}{15} = i(t)$$

At $t = 0+$

$$i_2(0+) = -\frac{v(0+)}{30} = 0.2 \text{ A}$$

$$\left.\begin{array}{l} v_1(0+) = v_3(0+) + 24 \\ \dfrac{v_1(0+)}{10} + \dfrac{v_3(0+)}{15} = i(0+) \end{array}\right\} \Rightarrow \; v_1(0+) = 7.2 \text{ V and } v_3(0+) = -16.8 \text{ V}$$

P7.8-11

The circuit shown in Figure 7.8-11 has reached steady state before the switch opens at time $t = 0$. Determine the values of $i_L(t)$, $v_C(t)$ and $v_R(t)$ immediately before the switch opens and the value of $v_R(t)$ immediately after the switch opens.

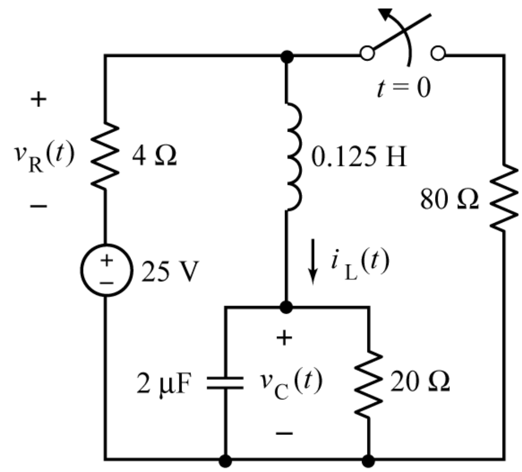
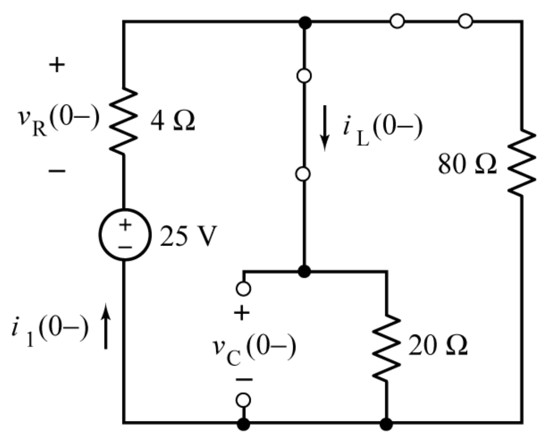


Figure 7.8-11

Answers: $i_L(0-) = 1.25$ A, $v_C(0-) = 20$ V, $v_R(0-) = -5$ V and $v_R(0+) = -4$ V

Solution: Because

- This **circuit has reached steady state** before the switch opens at time $t = 0$.
- The only source is a **constant** voltage **source**.



At t=0–, t**he capacitor acts like an open circuit** and **the inductor acts like a short circuit**. From the circuit

$$i_1(0-) = \frac{25}{4 + (20 \| 80)} = \frac{25}{4+16} = 1.25 \text{ A},$$

$$i_L(0-) = \left(\frac{80}{20+80}\right) i_1(0-) = 1 \text{ A},$$

$$v_C(0-) = 20\, i_L(0-) = 20 \text{ V}$$

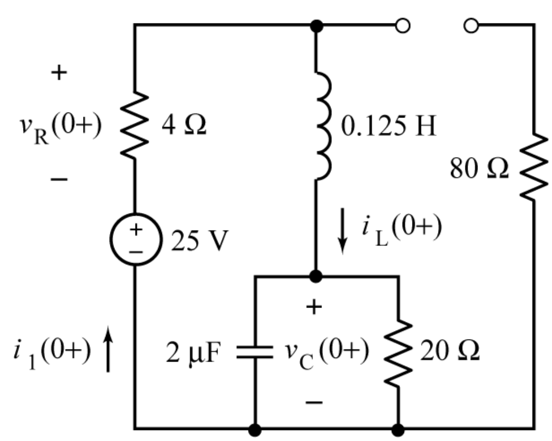
and

$$v_R(0-) = -4\, i_1(0-) = -5 \text{ V}$$

The **capacitor voltage and inductor current don't change instantaneously** so

$$v_C(0+) = v_C(0-) = 20 \text{ V and}$$

$$i_L(0+) = i_L(0-) = 1 \text{ A}$$



Apply KCL at the top node to see that

$$i_1(0+) = i_L(0+) = 1 \text{ A}$$

From Ohm's law

$$v_R(0+) = -4\, i_1(0+) = -4 \text{ V}$$

(Notice that the resistor voltage did change instantaneously.)

P7.8-12. The circuit shown in Figure 7.8-12 has reached steady state before the switch closes at time $t = 0$.

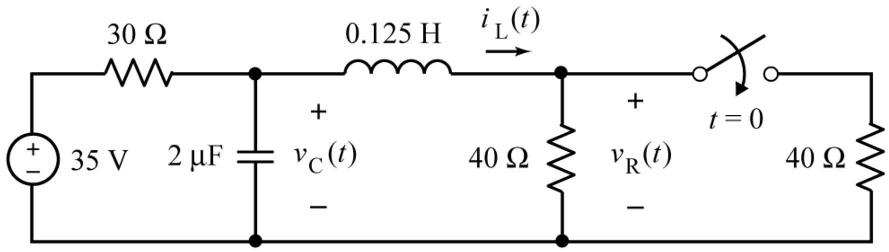


Figure 7.8-12

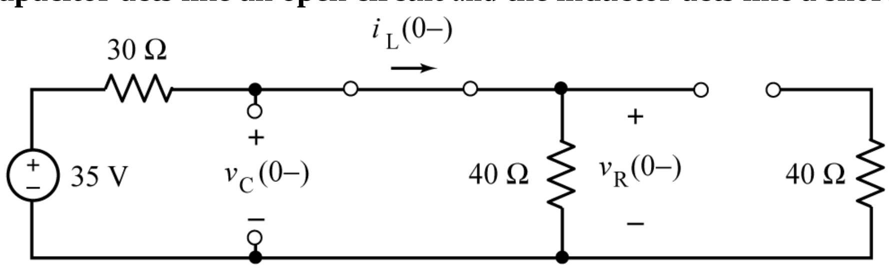
(a) Determine the values of $i_L(t)$, $v_C(t)$ and $v_R(t)$ immediately before the switch closes.

(b) Determine the value of $v_R(t)$ immediately after the switch closes.

Solution: Because

- This **circuit has reached steady state** before the switch closes at time $t = 0$.
- The only source is a **constant** voltage **source**.

At t=0–, t**he capacitor acts like an open circuit** and **the inductor acts like a short circuit**.

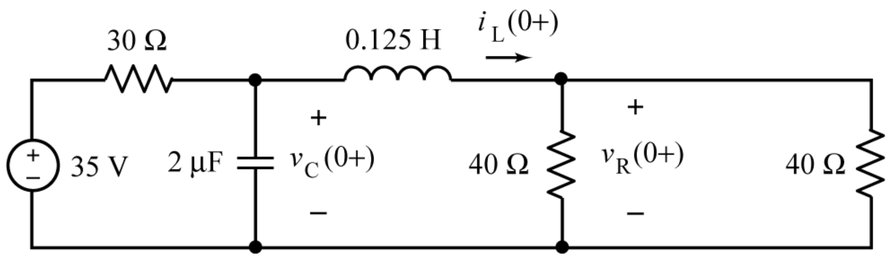


From the circuit $\quad i_L(0-) = \dfrac{35}{30+40} = 0.5 \text{ A}, \quad v_R(0-) = 40\, i_L(0-) = 20 \text{ V},$

And
$$v_C(0-) = v_R(0-) = 20 \text{ V}$$

The **capacitor voltage and inductor current don't change instantaneously** so

$$v_C(0+) = v_C(0-) = 20 \text{ V} \text{ and } i_L(0+) = i_L(0-) = 0.5 \text{ A}$$



$$v_R(0+) = 40\left(\frac{40}{40+40}\right) i_L(0+) = 10 \text{ V}$$

(Notice that the resistor voltage did change instantaneously.)

P7.8-13

The circuit shown in Figure 7.8-12 has reached steady state before the switch opens at time $t = 0$. Determine the values of $i_L(t)$, $v_C(t)$ and $v_R(t)$ immediately before the switch opens and the value of $v_R(t)$ immediately after the switch opens.

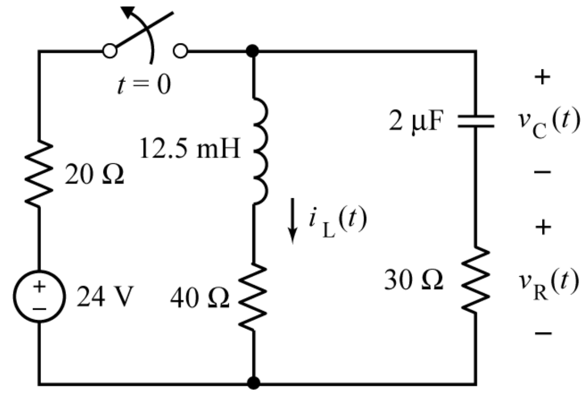


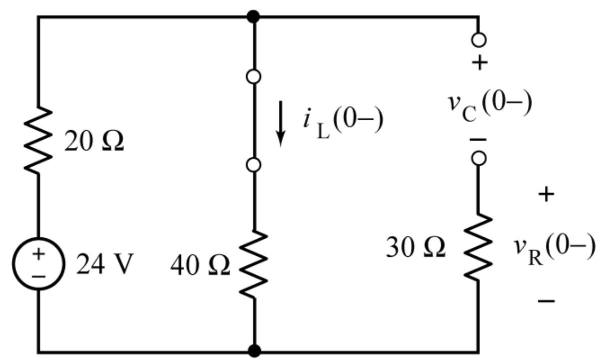
Figure 7.8-13

Answer: $i_L(0-) = 0.4$__A, $v_C(0-) = 16$ V, $v_R(0-) = 0$ V and $v_R(0+) = -12$ V

Solution: Because

- This **circuit has reached steady state** before the switch opens at time $t = 0$.
- The only source is a **constant** voltage **source**.

At t=0–, t**he capacitor acts like an open circuit** and **the inductor acts like a short circuit**.



The current in the 30 Ω resistor is zero so $v_R(0-) = 0$ V . Next

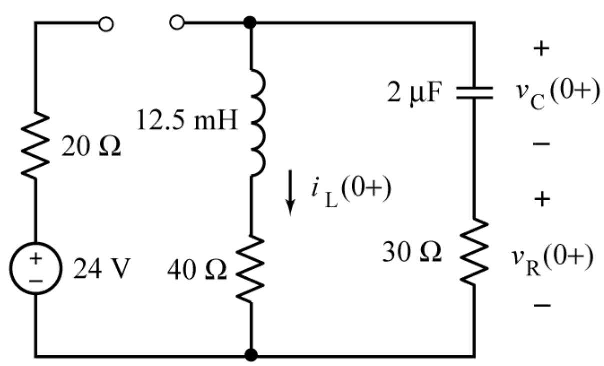
$$i_1(0-) = \frac{24}{20+40} = 0.4 \text{ A and}$$

$$v_C(0-) = 40\, i_L(0-) = 16 \text{ V}$$

The **capacitor voltage and inductor current don't change instantaneously** so

$$v_C(0+) = v_C(0-) = 16 \text{ V and}$$

$$i_L(0+) = i_L(0-) = 0.4 \text{ A}$$



Apply KCL at the bottom node and then Ohm's law to get

$$v_R(0+) = -30\, i_L(0+) = -12 \text{ V}$$

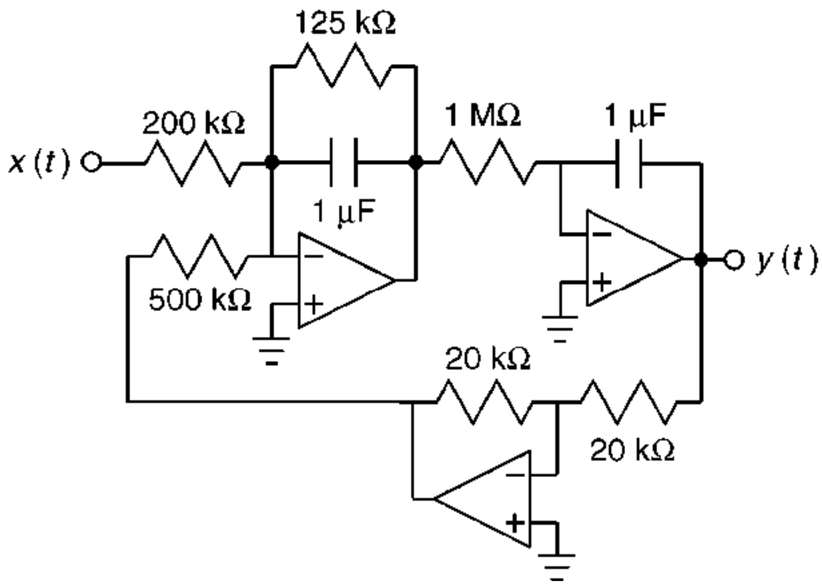
(Notice that the resistor voltage did change instantaneously.)

Section 7-9: Operational amplifier Circuits and Linear Differential Equations

P 7.9-1 Design a circuit with one input, $x(t)$, and one output, $y(t)$, that are related by this differential equation:

$$\frac{1}{2}\frac{d^2}{dt^2}y(t)+4\frac{d}{dt}y(t)+y(t)=\frac{5}{2}x(t)$$

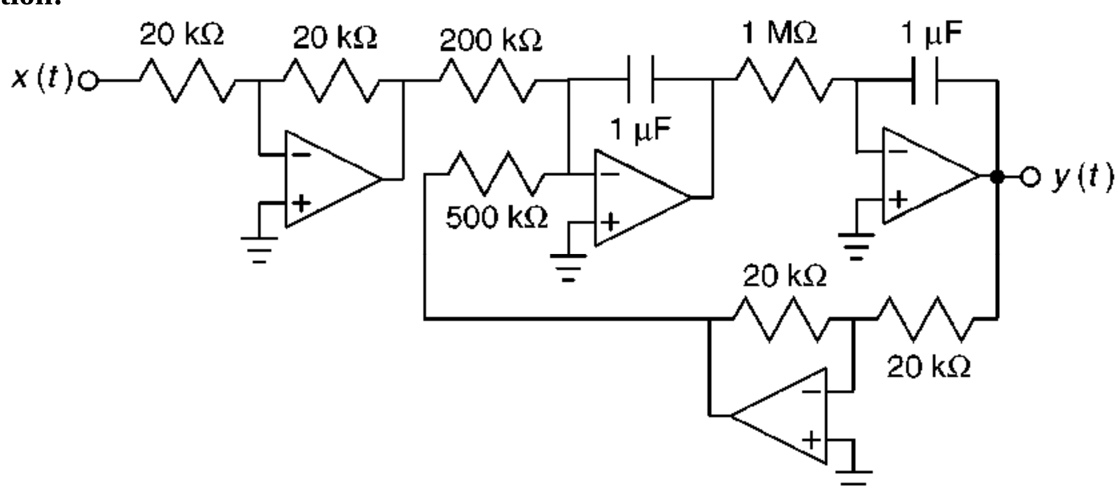
Solution:



P 7.9-2 Design a circuit with one input, $x(t)$, and one output, $y(t)$, that are related by this differential equation:

$$\frac{1}{2}\frac{d^2}{dt^2}y(t)+y(t)=-\frac{5}{2}x(t)$$

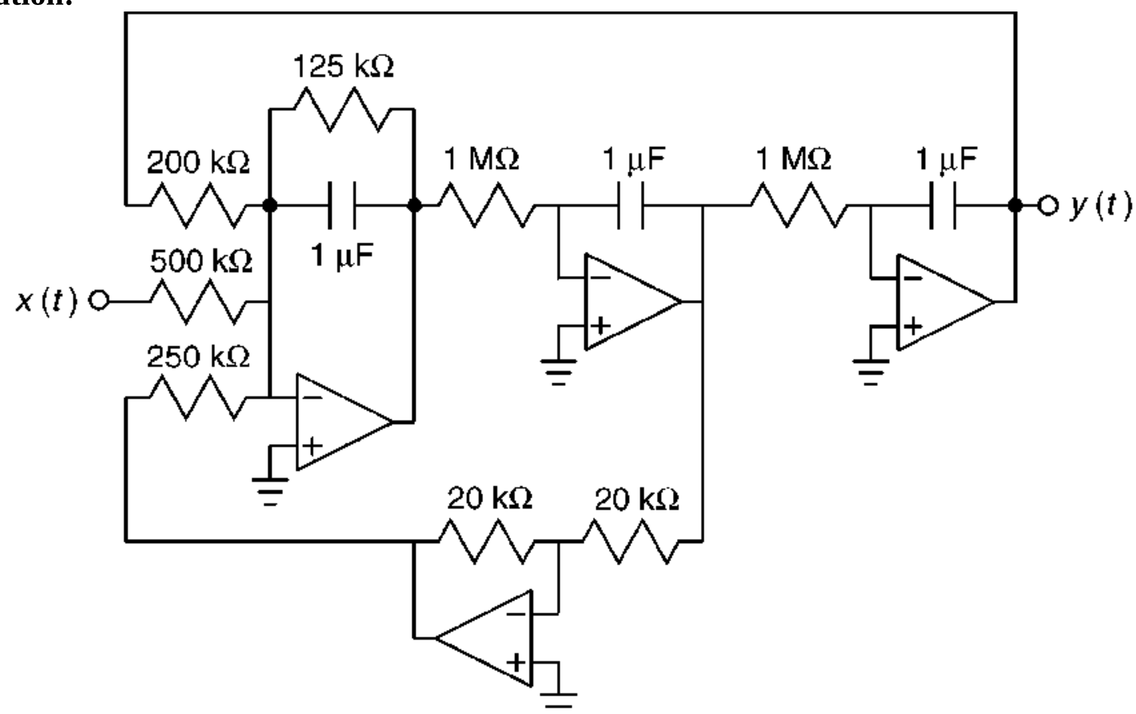
Solution:



P 7.9-3 Design a circuit with one input, $x(t)$, and one output, $y(t)$, that are related by this differential equation:

$$2\frac{d^3}{dt^3}y(t)+16\frac{d^2}{dt^2}y(t)+8\frac{d}{dt}y(t)+10y(t)=-4x(t)$$

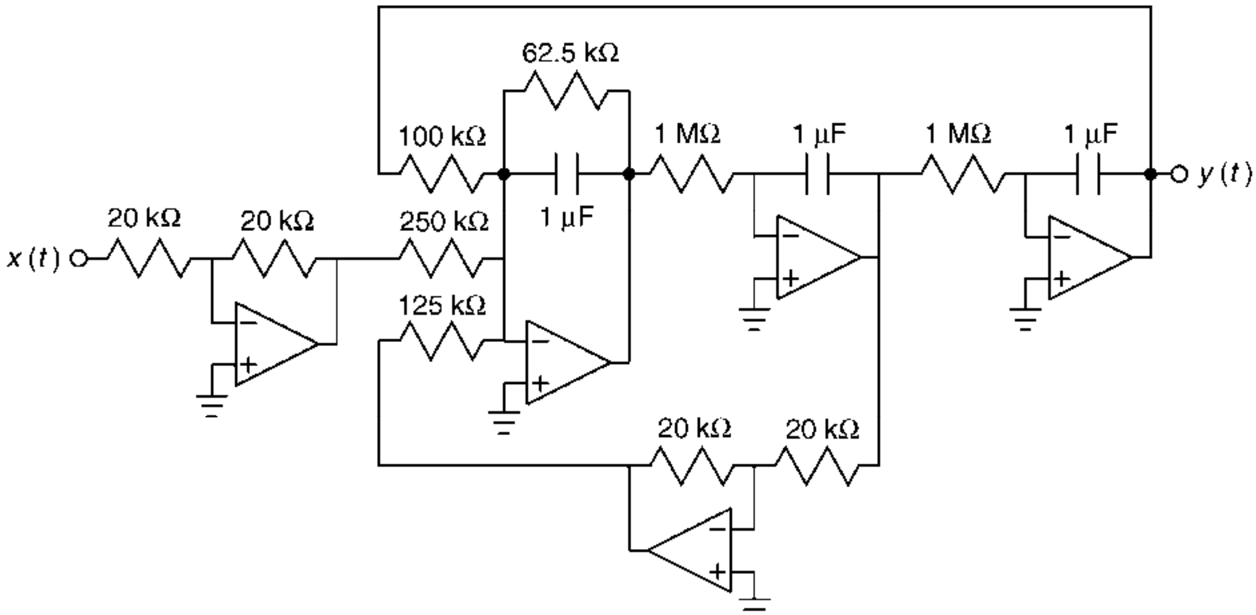
Solution:



P 7.9-4 Design a circuit with one input, $x(t)$, and one output, $y(t)$, that are related by this differential equation:

$$\frac{d^3}{dt^3}y(t)+16\frac{d^2}{dt^2}y(t)+8\frac{d}{dt}y(t)+10y(t)=4x(t)$$

Solution:



Section 7.11 How Can We Check…?

P 7.11-1 A homework solution indicates that the current and voltage of a 100-H inductor are

$$i(t)=\begin{cases} 0.025 & t<1 \\ -\dfrac{t}{25}+0.065 & 1<t<3 \\ \dfrac{t}{50}-0.115 & 3<t<9 \\ 0.065 & t<9 \end{cases} \quad \text{and} \quad v(t)=\begin{cases} 0 & t<1 \\ -4 & 1<t<3 \\ 2 & 3<t<9 \\ 0 & t>9 \end{cases}$$

where the units of current are A, the units of voltage are V, and the units of time are s. Verify that the inductor current does not change instantaneously.

Solution: We need to check the values of the inductor current at the ends of the intervals.

$$\text{at } t=1 \qquad 0.025 \stackrel{?}{=} -\frac{1}{25}+0.065=0.025 \quad (\text{Yes!})$$

$$\text{at } t=3 \qquad -\frac{3}{25}+0.065 \stackrel{?}{=} \frac{3}{50}-0.115$$

$$-0.055=-0.055 \quad (\text{Yes!})$$

$$\text{at } t=9 \qquad \frac{9}{50}-0.115 \stackrel{?}{=} 0.065$$

$$0.065=0.065 \quad (\text{Yes!})$$

The given equations for the inductor current describe a current that is continuous; as must be the case since the given inductor voltage is bounded.

P 7.11-2 A homework solution indicates that the current and voltage of a 100-H inductor are

$$i(t)=\begin{cases} -\dfrac{t}{200}+0.025 & t<1 \\ -\dfrac{t}{100}+0.03 & 1<t<4 \\ \dfrac{t}{100}-0.03 & 4<t<9 \\ 0.015 & t<9 \end{cases} \quad \text{and} \quad v(t)=\begin{cases} -1 & t<1 \\ -2 & 1<t<4 \\ 1 & 4<t<9 \\ 0 & t>9 \end{cases}$$

where the units of current are A, the units of voltage are V, and the units of time are s. Is this homework solution correct? Justify your answer.

P7.11-2 We need to check the values of the inductor current at the ends of the intervals.

$$\text{at } t=1 \quad -\frac{1}{200} + 0.025 \overset{?}{=} -\frac{1}{100} + 0.03 \quad (\text{Yes!})$$

$$\text{at } t=4 \quad -\frac{4}{100} + 0.03 \overset{?}{=} \frac{4}{100} - 0.03 \quad (\text{No!})$$

The equation for the inductor current indicates that this current changes instantaneously at t = 4s. This equation cannot be correct.

Design Problems

DP 7-1 Consider a single-circuit element, that is, a single resistor, capacitor, or inductor. The voltage, $v(t)$, and current, $i(t)$, of the circuit element adhere to the passive convention. Consider the following cases:

(a) $v(t) = 4 + 2e^{-3t}$ V and $i(t) = -3e^{-3t}$ A for $t > 0$

(b) $v(t) = -3e^{-3t}$ V and $i(t) = 4 + 2e^{-3t}$ A for $t > 0$

(c) $v(t) = 4 + 2e^{-3t}$ V and $i(t) = 2 + e^{-3t}$ A for $t > 0$

For each case, specify the circuit element to be a capacitor, resistor, or inductor and give the value of its capacitance, resistance, or inductance.

Solution:

a) $\frac{d}{dt}v(t) = -6\,e^{-3t}$ is proportional to $i(t)$ so the element is a capacitor. $C = \frac{i(t)}{\frac{d}{dt}v(t)} = 0.5\text{ F}$.

b) $\frac{d}{dt}i(t) = -6\,e^{-3t}$ is proportional to $v(t)$ so the element is an inductor. $L = \frac{v(t)}{\frac{d}{dt}i(t)} = 0.5\text{ H}$.

c) $v(t)$ is proportional to $i(t)$ so the element is a resistor. $R = \frac{v(t)}{i(t)} = 2\ \Omega$.

DP 7-2 Figure DP 7.2 shows a voltage source and unspecified circuit elements. Each circuit element is a single resistor, capacitor, or inductor. Consider the following cases:

(a) $i(t) = 1.131 \cos(2t + 45°)$ A

(b) $i(t) = 1.131 \cos(2t - 45°)$ A

For each case, specify each circuit element to be a capacitor, resistor, or inductor and give the value of its capacitance, resistance, or inductance.
Hint: $\cos(\theta + \phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$

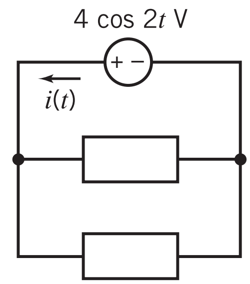
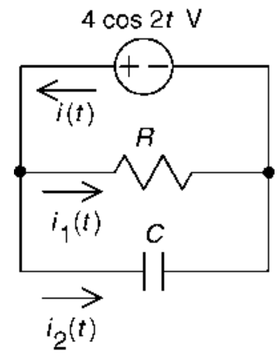


Figure DP 7.2

Solution: (a)

$$1.131\cos(2t+45°) = 1.131\left[\cos(45°)\cos(2t) - \sin(45°)\sin(2t)\right]$$
$$= 0.8\cos 2t - 0.8\sin 2t$$



The first term is proportional to the voltage. Associate it with the resistor. The noticing that

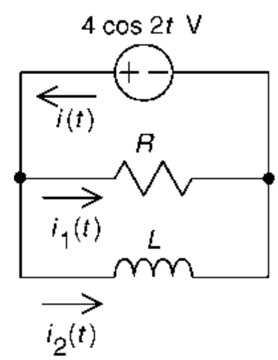
$$\int_{-\infty}^{t} v(\tau)\,d\tau = \int_{-\infty}^{t} 4\cos 2t\,d\tau = 2\sin 2t$$
$$\frac{d}{dt}v(t) = \frac{d}{dt}4\cos 2t = -8\sin 2t$$

associate the second term with a capacitor to get the minus sign. Then

$$R = \frac{4\cos 2t}{i_1(t)} = \frac{4\cos 2t}{0.8\cos 2t} = 5\,\Omega \text{ and}$$
$$C = \frac{i_2(t)}{\frac{d}{dt}4\cos 2t} = \frac{-0.8\sin 2t}{-8\sin 2t} = 0.1\text{ F}$$

(b)

$$1.131\cos(2t-45°) = 1.131\left[\cos(-45°)\cos(2t) - \sin(-45°)\sin(2t)\right]$$
$$= 0.8\cos 2t + 0.8\sin 2t$$



The first term is proportional to the voltage. Associate it with the resistor. Then noticing that

$$\int_{-\infty}^{t} v(\tau)\,d\tau = \int_{-\infty}^{t} 4\cos 2t\,d\tau = 2\sin 2t$$
$$\frac{d}{dt}v(t) = \frac{d}{dt}4\cos 2t = -8\sin 2t$$

associate the second term with an inductor to get the plus sign. Then

$$R = \frac{4\cos 2t}{i_1(t)} = \frac{4\cos 2t}{0.8\cos 2t} = 5\,\Omega \text{ and}$$
$$L = \frac{\int_{-\infty}^{t} 4\cos 2t\,d\tau}{i_2(t)} = \frac{2\sin 2t}{0.8\sin 2t} = 2.5\text{ H}$$

DP 7-3 Figure DP 7.3 shows a voltage source and unspecified circuit elements. Each circuit element is a single resistor, capacitor, or inductor. Consider the following cases:

(a) $v(t) = 11.31 \cos(2t + 45°)$ V

(b) $v(t) = 11.31 \cos(2t - 45°)$ V

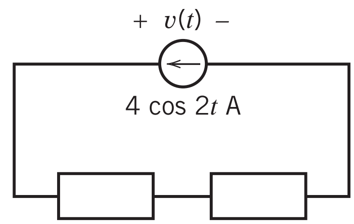


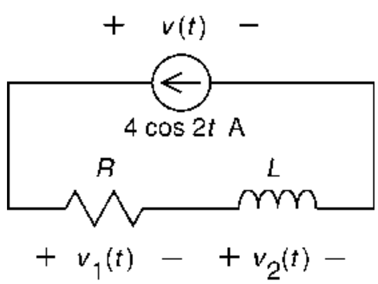
Figure DP 7.3

For each case, specify each circuit element to be a capacitor, resistor, or inductor and give the value of its capacitance, resistance, or inductance.

Hint: $\cos(\theta + \phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$

Solution:

a)



$$11.31\cos(2t+45°) = 11.31\left[\cos(45°)\cos(2t) - \sin(45°)\sin(2t)\right]$$
$$= 8\cos 2t - 8\sin 2t$$

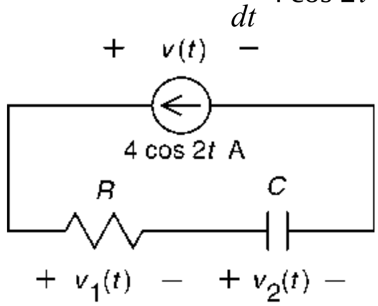
The first term is proportional to the voltage. Associate it with the resistor. The noticing that

$$\int_{-\infty}^{t} i(\tau)\,d\tau = \int_{-\infty}^{t} 4\cos 2t\,d\tau = 2\sin 2t$$
$$\frac{d}{dt}i(t) = \frac{d}{dt}4\cos 2t = -8\sin 2t$$

associate the second term with an inductor to get the minus sign. Then

$$R = \frac{v_1(t)}{4\cos 2t} = \frac{8\cos 2t}{4\cos 2t} = 2\,\Omega \text{ and } L = \frac{v_2(t)}{\frac{d}{dt}4\cos 2t} = \frac{-8\sin 2t}{-8\sin 2t} = 1\text{ H}$$

b)



$$11.31\cos(2t+45°) = 11.31\left[\cos(-45°)\cos(2t) - \sin(-45°)\sin(2t)\right]$$
$$= 8\cos 2t + 8\sin 2t$$

The first term is proportional to the voltage. Associate it with the resistor. The noticing that

$$\int_{-\infty}^{t} i(\tau)\,d\tau = \int_{-\infty}^{t} 4\cos 2t\,d\tau = 2\sin 2t$$

$$\frac{d}{dt} i(t) = \frac{d}{dt} 4\cos 2t = -8\sin 2t$$

associate the second term with a capacitor to get the minus sign. Then

$$R = \frac{v_1(t)}{4\cos 2t} = \frac{8\cos 2t}{4\cos 2t} = 2\ \Omega \ \text{ and } \ C = \frac{\int_{-\infty}^{t} 4\cos 2t\,d\tau}{v_2(t)} = \frac{2\sin 2t}{8\sin 2t} = 0.25\ \text{F}$$

DP 7-4 A high-speed flash unit for sports photography requires a flash voltage $v(0^+) =$ 3 V and

$$\left.\frac{dv(t)}{dt}\right|_{t=0} = 24 \text{ V/s}$$

The flash unit uses the circuit shown in Figure DP 7.4. Switch 1 has been closed a long time, and switch 2 has been open a long time at $t = 0$. Actually, the long time in this case is 3 s. Determine the required battery voltage, V_B, when $C = 1/8$ F.

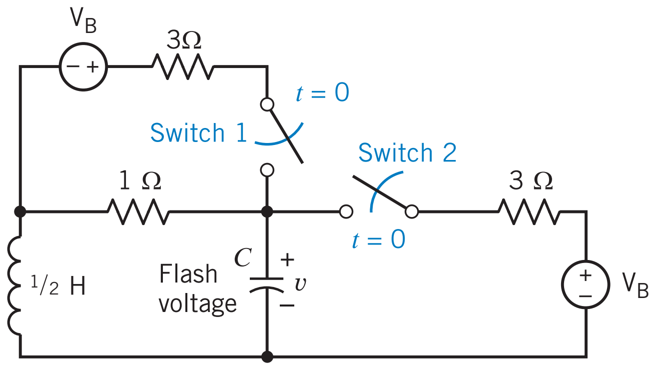
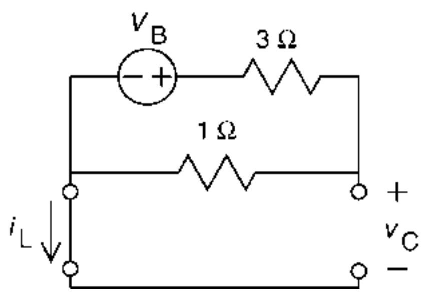


Figure DP 7.4

Solution:

at $t=0^-$

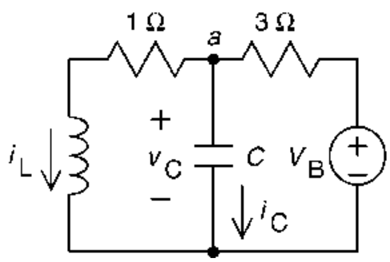


$$i_L(0^-) = 0$$

By voltage division: $v_C(0^-) = \frac{V_B}{4}$

We require $v_C(0^-) = 3$ V so $V_B = 12$ V

at $t=0^+$



Now we will check $\left.\frac{dv_C}{dt}\right|_{t=0^+}$

First: $i_L(0^+) = i_L(0^-) = 0$

and $v_C(0^+) = v_C(0^-) = 3$ V

Apply KCL at node a: $$i_L(0^+) + i_C(0^+) = \frac{V_B - v_C(0^+)}{3}$$

$$0 + i_C(0^+) = \frac{12-3}{3} \Rightarrow i_C(0^+) = 3 \text{ A}$$

Finally $$\left.\frac{dv_C}{dt}\right|_{t=0^+} = \frac{i_C(0^+)}{C} = \frac{3}{0.125} = 24\ \frac{\text{V}}{\text{s}}$$

as required.

DP 7-5 For the circuit shown in Figure DP 7.5, select a value of *R* so that the energy stored in the inductor is equal to the energy stored in the capacitor at steady state.

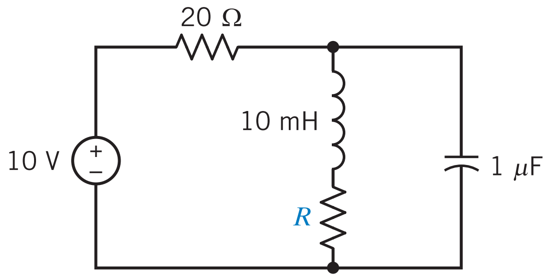


Figure DP 7.5

Solution: We require $\frac{1}{2} L\, i_L^2 = \frac{1}{2} C\, v_C^2$ where i_L and v_c are the steady-state inductor current and capacitor voltage. At steady state, $i_L = \frac{v_C}{R}$. Then

$$L\left(\frac{v_C}{R}\right)^2 = C\, v_C^2 \quad \Rightarrow \quad C = \frac{L}{R^2} \quad \Rightarrow \quad R = \sqrt{\frac{L}{C}} = \sqrt{\frac{10^{-2}}{10^{-6}}} = \sqrt{10^4} = 10^2\ \Omega$$

Chapter 8 Exercises

Exercise 8.3-1 The circuit shown in Figure E 8.3-1 is at steady state before the switch closes at time $t = 0$. Determine the capacitor voltage, $v(t)$, for $t \geq 0$.
Answer: $v(t) = 2 + e^{-2.5t}$ V for $t > 0$

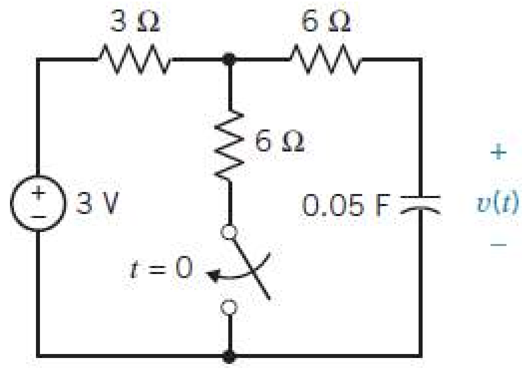
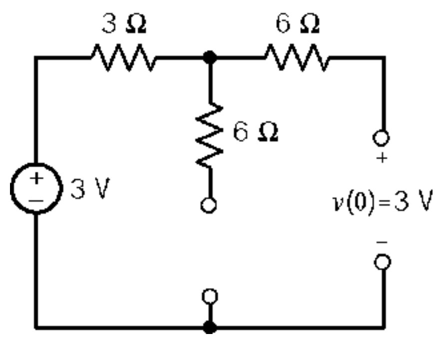
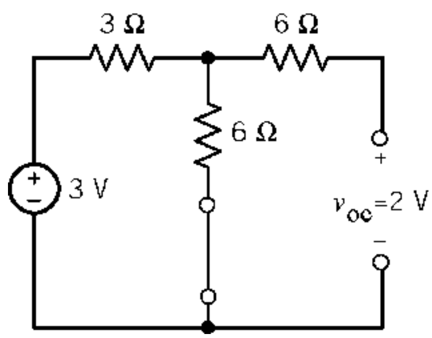


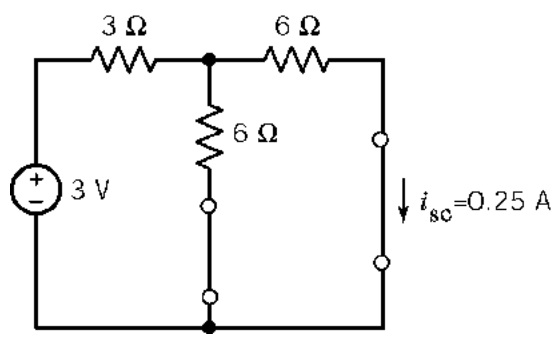
Figure E 8.3-1

Solution: Before the switch closes:



After the switch closes:





Therefore $$R_t = \frac{2}{0.25} = 8\ \Omega \quad \text{so} \quad \tau = 8(0.05) = 0.4\ \text{s}.$$

Finally, $$v(t) = v_{oc} + (v(0) - v_{oc})\, e^{-t/\tau} = 2 + e^{-2.5t}\ \text{V} \quad \text{for } t > 0$$

Exercise 8.3-2 The circuit shown in Figure E 8.3-2 is at steady state before the switch closes at time $t = 0$. Determine the inductor current, $i(t)$, for $t > 0$.

Answer: $i(t) = \frac{1}{4} + \frac{1}{12} e^{-1.33t}$ A for $t > 0$

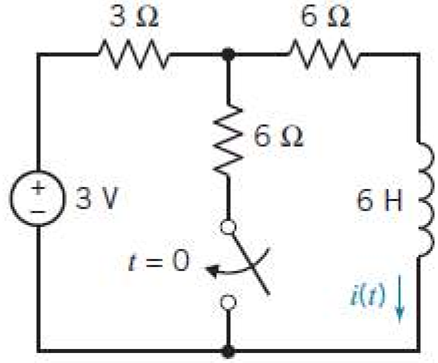
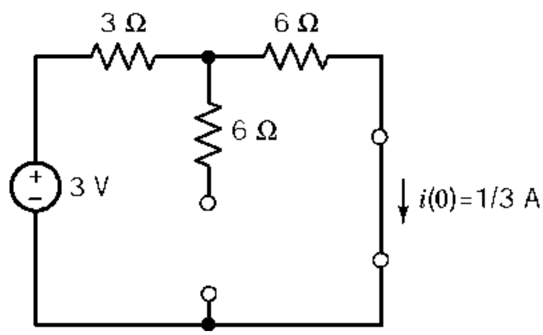
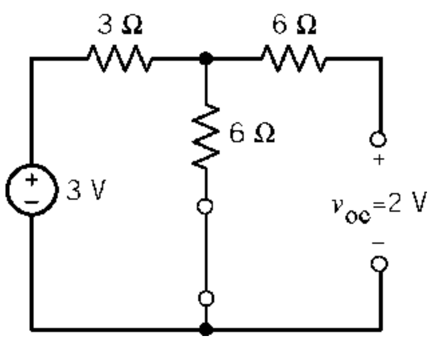


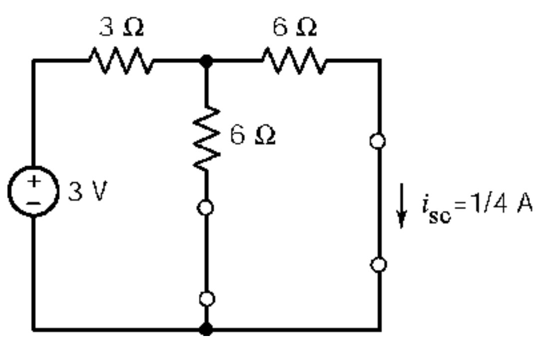
Figure E 8.3-2

Solution: Before the switch closes:



After the switch closes:

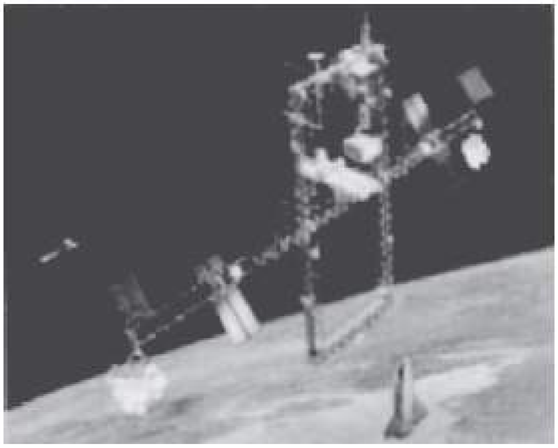




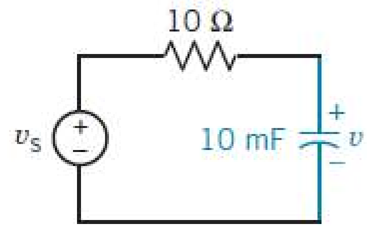
Therefore $$R_t = \frac{2}{0.25} = 8\,\Omega \quad \text{so} \quad \tau = \frac{6}{8} = 0.75 \text{ s}.$$

Finally, $$i(t) = i_{sc} + (i(0) - i_{sc})\, e^{-t/\tau} = \frac{1}{4} + \frac{1}{12} e^{-1.33t} \text{ A} \quad \text{for } t > 0$$

Exercise 8.7-1 The electrical power plant for the orbiting space station shown in Figure E 8.7-1*a* uses photovoltaic cells to store energy in batteries. The charging circuit is modeled by the circuit shown in Figure E 8.7-1*b*, where $v_s = 10 \sin 20t$ V. If $v(0^-) = 0$, find $v(t)$ for $t > 0$.
Answer: $v = 4\,e^{-10t} - 4 \cos 20t + 2 \sin 20t$ V



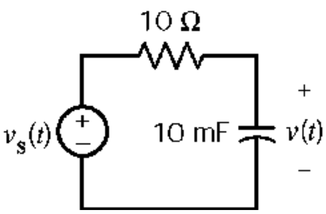
(a)



(b)

Figure E 8.7-1

Solution:



$$v_s(t) = 10 \sin 20t \ \text{ V}$$

Apply KVL:

$$-10 \sin 20t + 10\left(.01 \frac{d\,v(t)}{dt}\right) + v(t) \; = \; 0$$

$$\Rightarrow \quad \frac{d\,v(t)}{dt} + 10\,v(t) \; = \; 100 \sin 20t$$

Natural Response: $v_n(t) = Ae^{-t/\tau}$ where $\tau = R_t\,C \quad \therefore \; v_n(t) = Ae^{-10t}$

Forced Response: try $v_f(t) = B_1 \cos 20t \; + \; B_2 \sin \; 20\,t$

Plugging $v_f(t)$ into the differential equation and equating like terms yields:

$$-20\,B_1 \sin 20t \; + 20\,B_2 \cos 20t + 10\,B_1 \cos 20t \; + \; 10\,B_2 \sin 20t = 100 \sin 20t$$

$$20\,B_2 + 10\,B_1 = 0 \quad \text{and} \quad -20\,B_1 + \; 10\,B_2 = 100$$

$$B_1 = -4 \quad \text{and} \quad B_2 = 2$$

Complete Response: $v(t) \; = \; v_n(t) \; + \; v_f(t) = \; Ae^{-10t} - 4 \cos 20t + 2 \; \sin \; 20t$

Now $v(0^-) = v(0^+) = 0 = \; A - 4 \quad \Rightarrow \quad A = 4$

$$\underline{v(t) \; = \; 4\,e^{-10t} \; -4 \cos \; 20t \; + \; 2 \sin \; 20\,t \; \text{V}}$$

Section 8.3: The Response of a First Order Circuit to a Constant Input

P 8.3-1 The circuit shown in Figure P 8.3-1 is at steady state before the switch closes at time $t = 0$. The input to the circuit is the voltage of the voltage source, 12 V. The output of this circuit is the voltage across the capacitor, $v(t)$. Determine $v(t)$ for $t > 0$.

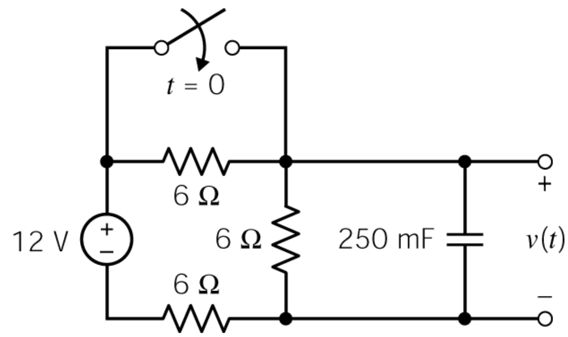
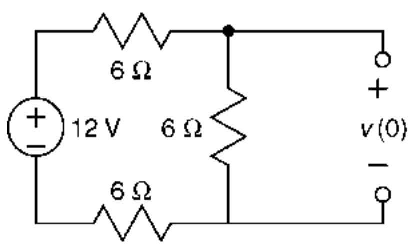


Figure P 8.3-1

Answer: $v(t) = 6 - 2e^{-1.33t}$ V for $t > 0$

Solution:

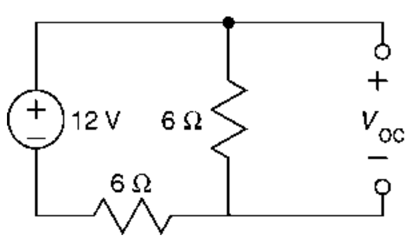


Here is the circuit before $t = 0$, when the switch is open and the circuit is at steady state. The open switch is modeled as an open circuit.

A capacitor in a steady-state dc circuit acts like an open circuit, so an open circuit replaces the capacitor. The voltage across that open circuit is the initial capacitor voltage, $v(0)$.

By voltage division

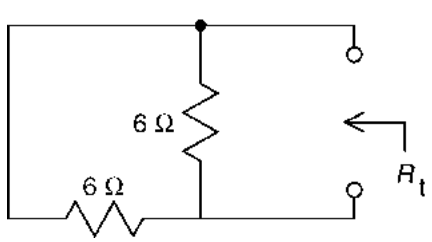
$$v(0) = \frac{6}{6+6+6}(12) = 4 \text{ V}$$



Next, consider the circuit after the switch closes. The closed switch is modeled as a short circuit.

We need to find the Thevenin equivalent of the part of the circuit connected to the capacitor. Here's the circuit used to calculate the open circuit voltage, V_{oc}.

$$V_{oc} = \frac{6}{6+6}(12) = 6 \text{ V}$$



Here is the circuit that is used to determine R_t. A short circuit has replaced the closed switch. Independent sources are set to zero when calculating R_t, so the voltage source has been replaced by a short circuit.

$$R_t = \frac{(6)(6)}{6+6} = 3 \ \Omega$$

Then $\tau = R_t\, C = 3(0.25) = 0.75$ s

Finally, $$v(t) = V_{oc} + \left(v(0) - V_{oc}\right) e^{-t/\tau} = 6 - 2\, e^{-1.33\, t} \text{ V} \quad \text{for } t > 0$$

P 8.3-2 The circuit shown in Figure P 8.3-2 is at steady state before the switch opens at time $t = 0$. The input to the circuit is the voltage of the voltage source, 12 V. The output of this circuit is the current in the inductor, $i(t)$. Determine $i(t)$ for $t > 0$.

Answer: $i(t) = 1 + e^{-0.5t}$ A for $t > 0$

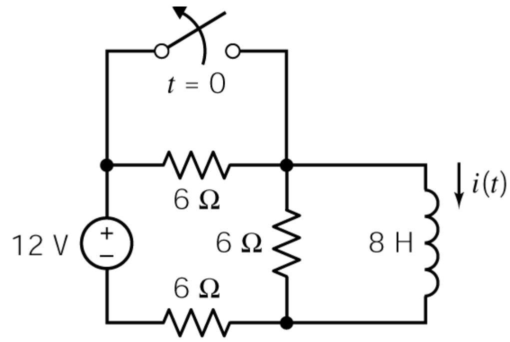
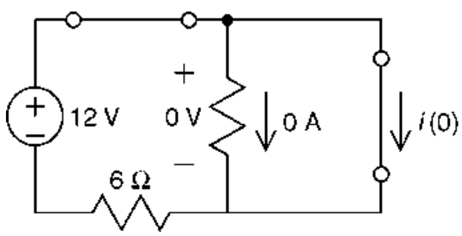


Figure P 8.3-2

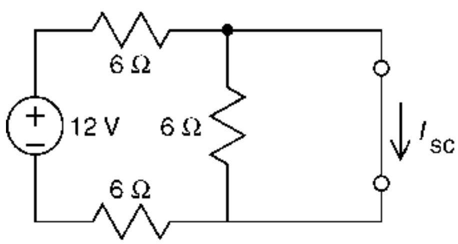
Solution:



Here is the circuit before $t = 0$, when the switch is closed and the circuit is at steady state. The closed switch is modeled as a short circuit.

An inductor in a steady-state dc circuit acts like an short circuit, so a short circuit replaces the inductor. The current in that short circuit is the initial inductor current, $i(0)$.

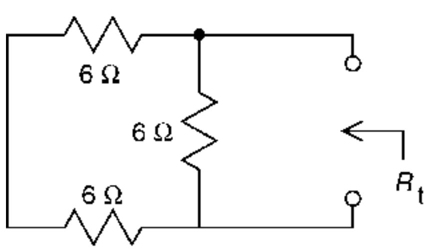
$$i(0) = \frac{12}{6} = 2 \text{ A}$$



Next, consider the circuit after the switch opens. The open switch is modeled as an open circuit.

We need to find the Norton equivalent of the part of the circuit connected to the inductor. Here's the circuit used to calculate the short circuit current, I_{sc}.

$$I_{sc} = \frac{12}{6+6} = 1 \text{ A}$$



Here is the circuit that is used to determine R_t. An open circuit has replaced the open switch. Independent sources are set to zero when calculating R_t, so the voltage source has been replaced by an short circuit.

$$R_t = 6 \,||\, (6+6) = \frac{(6+6)(6)}{(6+6)+6} = 4 \text{ } \Omega$$

Then $$\tau = \frac{L}{R_t} = \frac{8}{4} = 2 \text{ s}$$

Finally, $$i(t) = I_{sc} + \left(i(0) - I_{sc}\right) e^{-t/\tau} = 1 + e^{-0.5\,t} \text{ A} \quad \text{for } t > 0$$

P 8.3-3 The circuit shown in Figure P 8.3-3 is at steady state before the switch closes at time $t = 0$. Determine the capacitor voltage, $v(t)$, for $t > 0$.

Answer: $v(t) = -6 + 18e^{-6.67t}$ V for $t > 0$

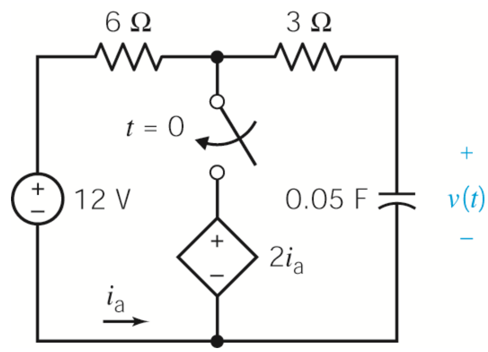
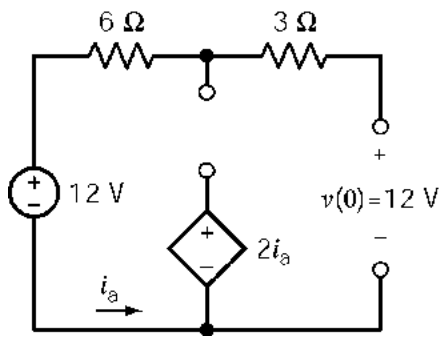
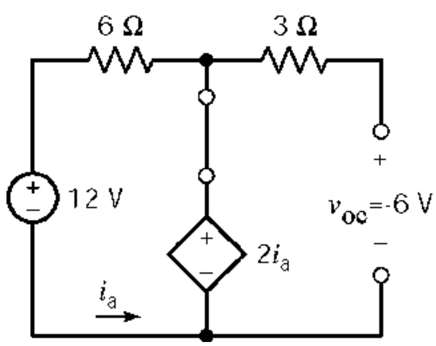


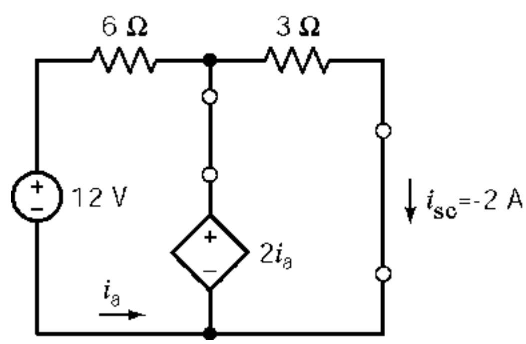
Figure P 8.3-3

Solution: Before the switch closes:



After the switch closes:





Therefore $R_t = \dfrac{-6}{-2} = 3\,\Omega$ so $\tau = 3(0.05) = 0.15$ s.

Finally, $v(t) = v_{oc} + (v(0) - v_{oc})\, e^{-t/\tau} = -6 + 18\, e^{-6.67t}$ V for $t > 0$

P 8.3-4 The circuit shown in Figure P 8.3-4 is at steady state before the switch closes at time $t = 0$. Determine the inductor current, $i(t)$, for $t > 0$.

Answer: $i(t) = -2 + \frac{10}{3} e^{-0.5t}$ A for $t > 0$

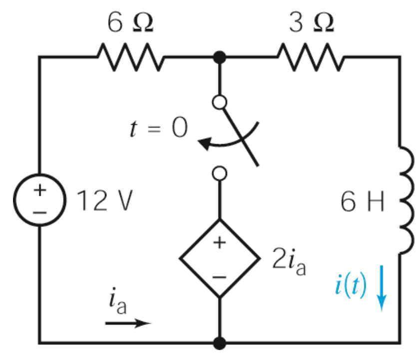
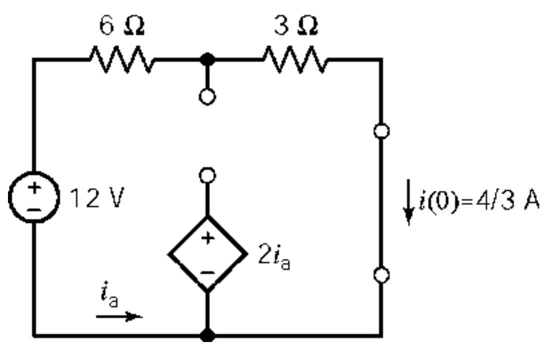
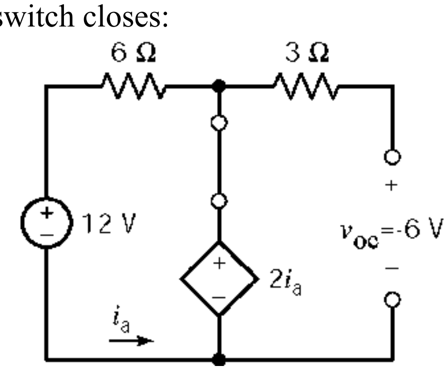


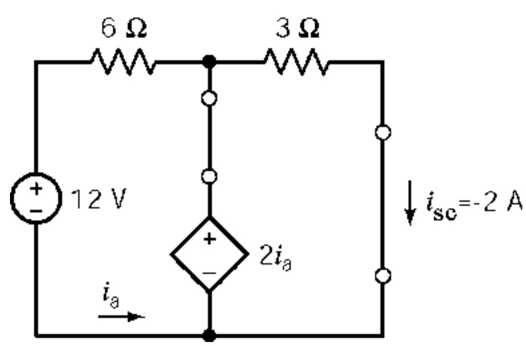
Figure P 8.3-4

Solution: Before the switch closes:



After the switch closes:





Therefore $R_t = \frac{-6}{-2} = 3\ \Omega$ so $\tau = \frac{6}{3} = 2$ s.

Finally, $i(t) = i_{sc} + (i(0) - i_{sc})\, e^{-\frac{t}{\tau}} = -2 + \frac{10}{3} e^{-0.5t}$ A for $t > 0$

P 8.3-5 The circuit shown in Figure P 8.3-5 is at steady state before the switch opens at time $t = 0$. Determine the voltage, $v_o(t)$, for $t > 0$.

Answer: $v_o(t) = 10 - 5e^{-12.5t}$ V for $t > 0$

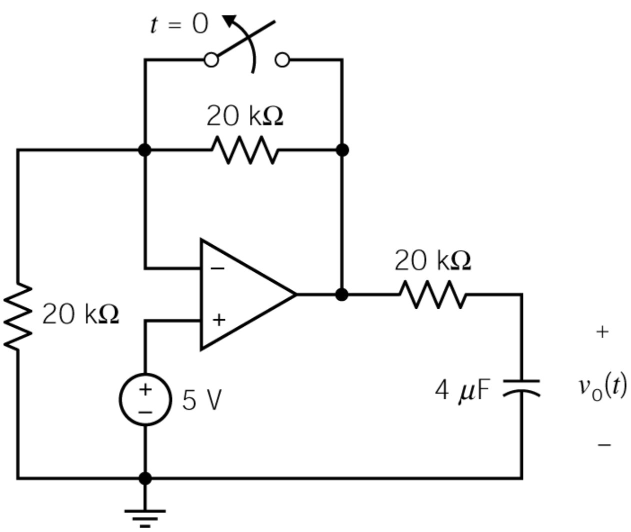
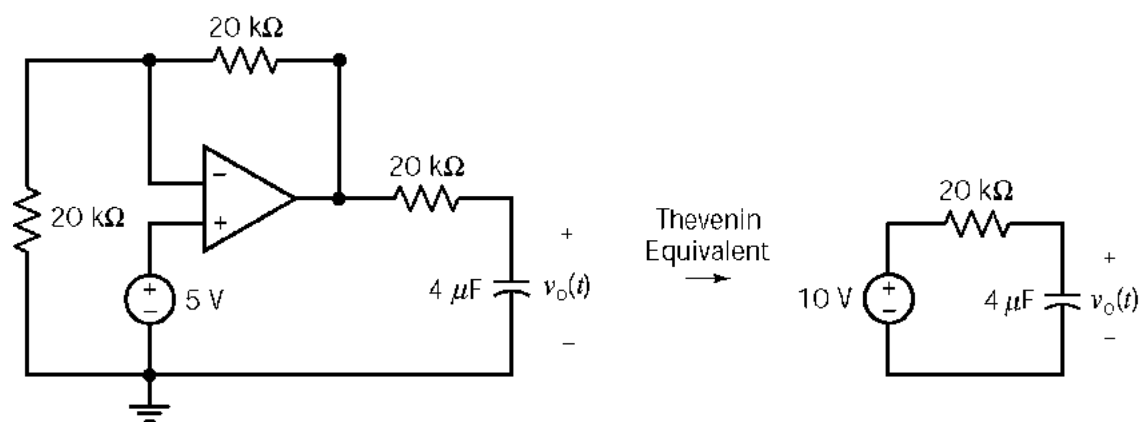


Figure P 8.3-5

Solution: Before the switch opens, $v_o(t) = 5\text{ V} \Rightarrow v_o(0) = 5\text{ V}$. After the switch opens the part of the circuit connected to the capacitor can be replaced by it's Thevenin equivalent circuit to get:



Therefore $\tau = \left(20\times10^{3}\right)\left(4\times10^{-6}\right) = 0.08\text{ s}$.

Next, $v_C(t) = v_{oc} + (v(0) - v_{oc})\, e^{-\frac{t}{\tau}} = 10 - 5\, e^{-12.5t}$ V for $t > 0$

Finally, $v_0(t) = v_C(t) = 10 - 5\, e^{-12.5t}$ V for $t > 0$

P 8.3-6 The circuit shown in Figure P 8.3-6 is at steady state before the switch opens at time $t = 0$. Determine the voltage, $v_o(t)$, for $t > 0$.

Answer: $v_o(t) = 5e^{-4000t}$ V for $t > 0$

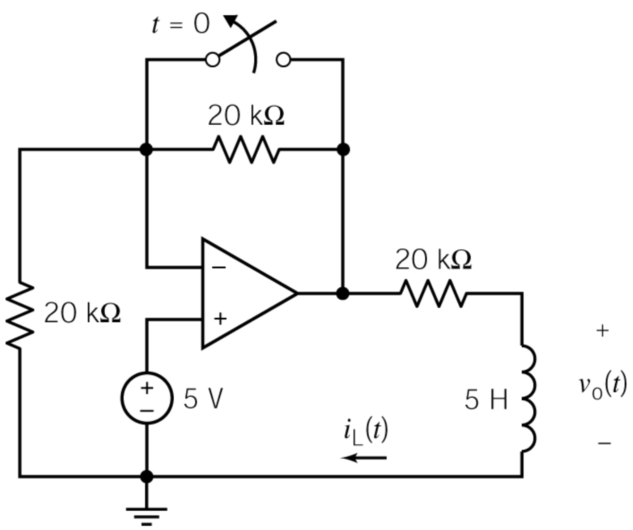
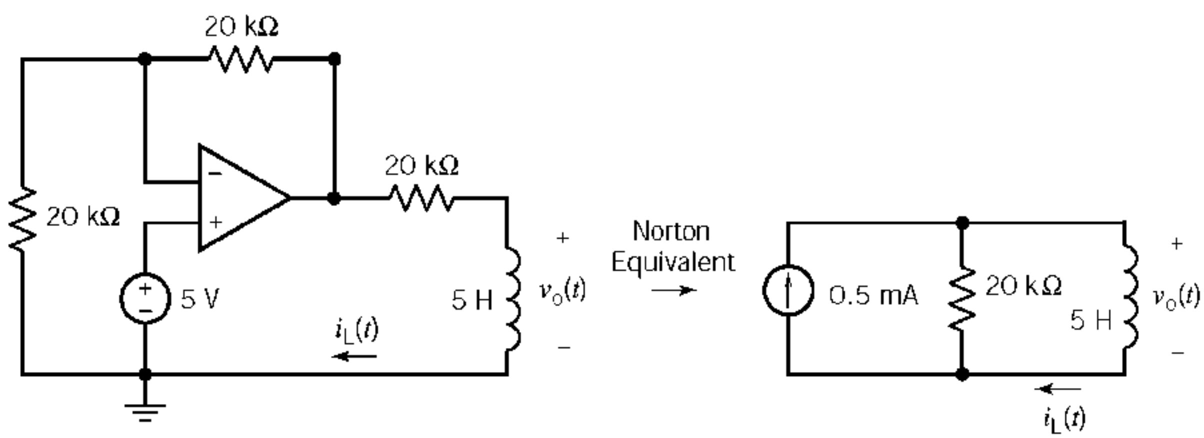


Figure P 8.3-6

Solution: Before the switch opens, $i_o(t) = \dfrac{5}{20 \times 10^3} = 0.25$ mA $\Rightarrow i_o(0) = 0.25$ mA. After the switch opens the part of the circuit connected to the inductor can be replaced by it's Norton equivalent circuit to get:



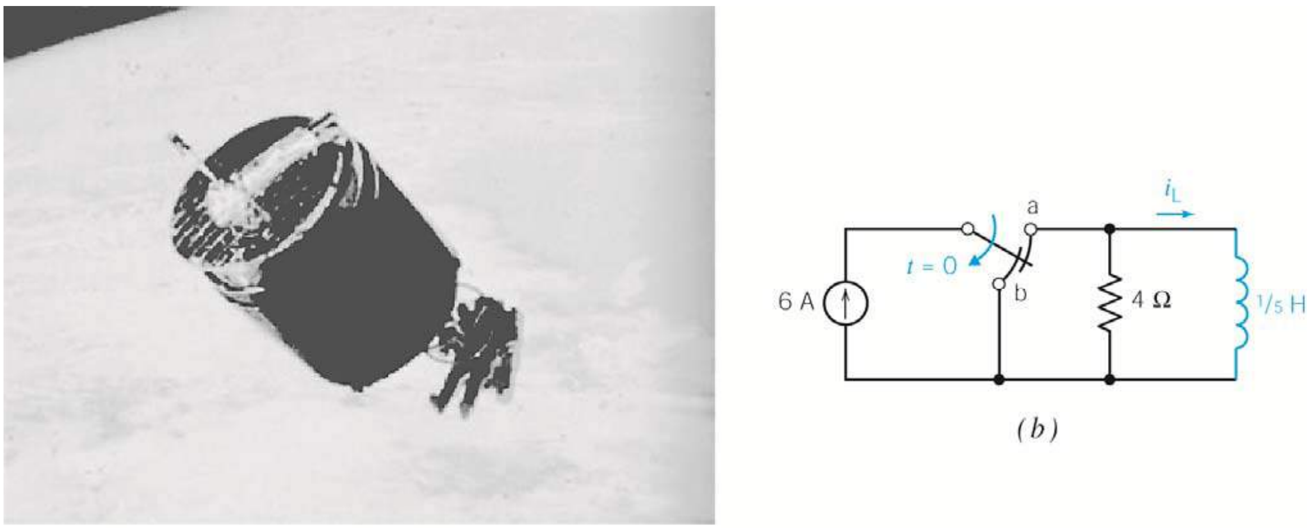
Therefore $\tau = \dfrac{5}{20 \times 10^3} = 0.25$ ms.

Next, $i_L(t) = i_{sc} + (i_L(0) - i_{sc})\, e^{-\frac{t}{\tau}} = 0.5 - 0.25 e^{-4000t}$ mA for $t > 0$

Finally, $v_o(t) = 5 \dfrac{d}{dt} i_L(t) = 5\, e^{-4000t}$ V for $t > 0$

P 8.3-7 Figure P 8.3-7*a* shows astronaut Dale Gardner using the manned maneuvering unit to dock with the spinning *Westar VI* satellite on November 14, 1984. Gardner used a large tool called the apogee capture device (ACD) to stabilize the satellite and capture it for recovery, as shown in Figure P 8.3-7*a*. The ACD can be modeled by the circuit of Figure P 8.3-7*b*. Find the inductor current i_L for $t > 0$.

Answer: $i_L(t) = 6e^{-20t}$ A

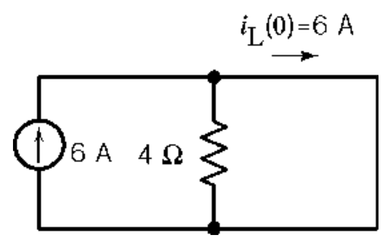


(*a*) (*b*)

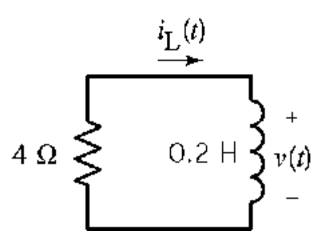
Figure P 8.3-7

Solution: At $t = 0^-$ (steady-state)

Since the input to this circuit is constant, the inductor will act like a short circuit when the circuit is at steady-state:



for t > 0



$$\underline{i_L(t) = i_L(0)\, e^{-(R/L)\,t} = 6\, e^{-20t} \text{ A}}$$

P 8.3-8 The circuit shown in Figure P 8.3-8 is at steady state before the switch opens at time $t = 0$. The input to the circuit is the voltage of the voltage source, V_s. This voltage source is a dc voltage source; that is, V_s is a constant. The output of this circuit is the voltage across the capacitor, $v_o(t)$. The output voltage is given by

$$v_o(t) = 2 + 8e^{-0.5t}\text{ V} \quad \text{for} \quad t > 0$$

Determine the values of the input voltage, V_s, the capacitance, C, and the resistance, R.

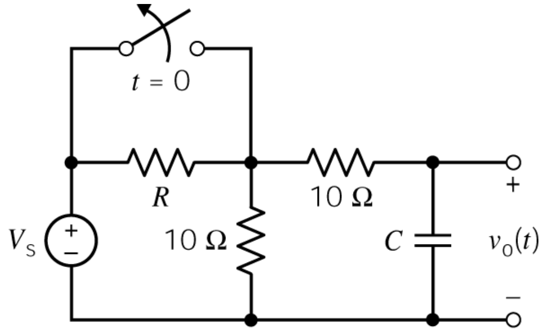
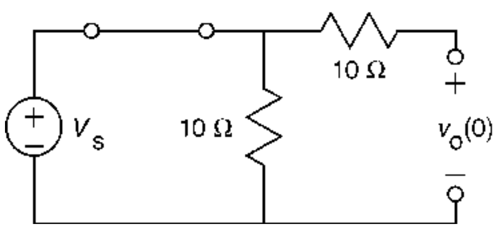


Figure P 8.3-8

Solution: Before the switch opens, the circuit will be at steady state. Because the only input to this circuit is the constant voltage of the voltage source, all of the element currents and voltages, including the capacitor voltage, will have constant values. Opening the switch disturbs the circuit. Eventually the disturbance dies out and the circuit is again at steady state. All the element currents and voltages will again have constant values, but probably different constant values than they had before the switch opened.



Here is the circuit before $t = 0$, when the switch is closed and the circuit is at steady state. The closed switch is modeled as a short circuit. The combination of resistor and a short circuit connected is equivalent to a short circuit. Consequently, a short circuit replaces the switch and the resistor R. A capacitor in a steady-state dc circuit acts like an open circuit, so an open circuit replaces the capacitor. The voltage across that open circuit is the capacitor voltage, $v_o(t)$.

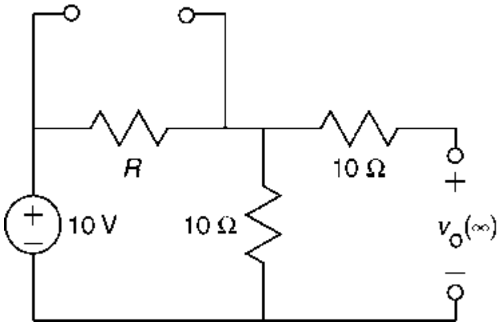
Because the circuit is at steady state, the value of the capacitor voltage will be constant. This constant is the value of the capacitor voltage just before the switch opens. In the absence of unbounded currents, the voltage of a capacitor must be continuous. The value of the capacitor voltage immediately after the switch opens is equal to the value immediately before the switch opens. This value is called the initial condition of the capacitor and has been labeled as $v_o(0)$. There is no current in the horizontal resistor due to the open circuit. Consequently, $v_o(0)$ is equal to the voltage across the vertical resistor, which is equal to the voltage source voltage. Therefore

$$v_o(0) = V_s$$

The value of $v_o(0)$ can also be obtained by setting $t = 0$ in the equation for $v_o(t)$. Doing so gives

$$v_o(0) = 2 + 8\,e^0 = 10 \text{ V}$$

Consequently,
$$V_s = 10 \text{ V}$$



Next, consider the circuit after the switch opens. Eventually (certainly as $t \to \infty$) the circuit will again be at steady state. Here is the circuit at $t = \infty$, when the switch is open and the circuit is at steady state. The open switch is modeled as an open circuit. A capacitor in a steady-state dc circuit acts like an open circuit, so an open circuit replaces the capacitor. The voltage across that open circuit is the steady-state capacitor voltage, $v_o(\infty)$. There is no current in the horizontal resistor and $v_o(\infty)$ is equal to the voltage across the vertical resistor. Using voltage division,

$$v_o(\infty) = \frac{10}{R+10}(10)$$

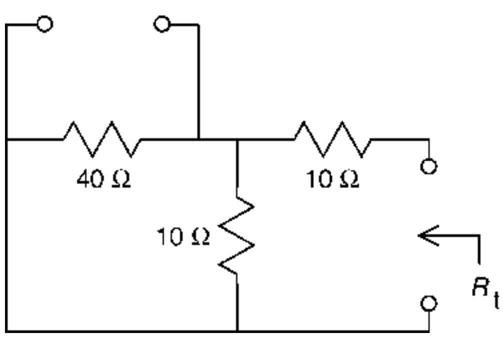
The value of $v_o(\infty)$ can also be obtained by setting $t = \infty$ in the equation for $v_o(t)$. Doing so gives

$$v_o(\infty) = 2 + 8\,e^{-\infty} = 2 \text{ V}$$

Consequently,

$$2 = \frac{10}{R+10}(10) \quad \Rightarrow \quad 2\,R + 20 = 100 \quad \Rightarrow \quad R = 40\,\Omega$$

Finally, the exponential part of $v_o(t)$ is known to be of the form $e^{-t/\tau}$ where $\tau = R_t\,C$ and R_t is the Thevenin resistance of the part of the circuit connected to the capacitor.



Here is the circuit that is used to determine R_t. An open circuit has replaced the open switch. Independent sources are set to zero when calculating R_t, so the voltage source has been replaced by a short circuit.

$$R_t = 10 + \frac{(40)(10)}{40+10} = 18 \ \Omega$$

so

$$\tau = R_t\,C = 18\,C$$

From the equation for $v_o(t)$

$$-0.5\,t = -\frac{t}{\tau} \quad \Rightarrow \quad \tau = 2 \text{ s}$$

Consequently,

$$2 = 18\,C \quad \Rightarrow \quad C = 0.111 = 111 \text{ mF}$$

P 8.3-9 The circuit shown in Figure P 8.3-9 is at steady state before the switch closes at time $t = 0$. The input to the circuit is the voltage of the voltage source, 24 V. The output of this circuit, the voltage across the 3-Ω resistor, is given by

$$v_o(t) = 6 - 3e^{-0.35t} \text{ V} \quad \text{when} \quad t > 0$$

Determine the value of the inductance, L, and of the resistances, R_1 and R_2.

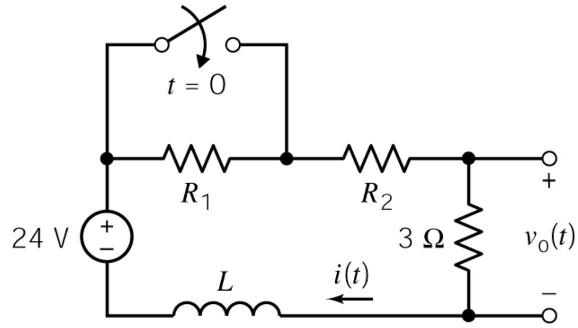


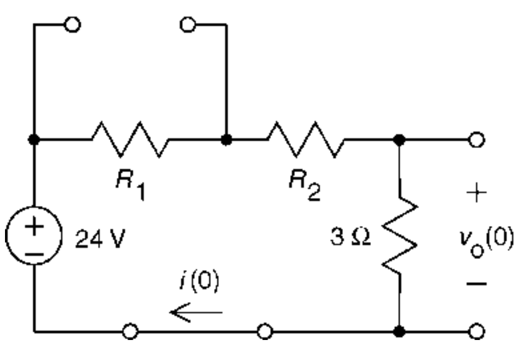
Figure P 8.3-9

Solution: Before the switch closes, the circuit will be at steady state. Because the only input to this circuit is the constant voltage of the voltage source, all of the element currents and voltages, including the inductor current, will have constant values. Closing the switch disturbs the circuit by shorting out the resistor R_1. Eventually the disturbance dies out and the circuit is again at steady state. All the element currents and voltages will again have constant values, but probably different constant values than they had before the switch closed.

The inductor current is equal to the current in the 3 Ω resistor. Consequently,

$$i(t) = \frac{v_o(t)}{3} = \frac{6 - 3\,e^{-0.35\,t}}{3} = 2 - e^{-0.35\,t} \text{ A when } t > 0$$

In the absence of unbounded voltages, the current in any inductor is continuous. Consequently, the value of the inductor current immediately before $t = 0$ is equal to the value immediately after $t = 0$.



Here is the circuit before $t = 0$, when the switch is open and the circuit is at steady state. The open switch is modeled as an open circuit. An inductor in a steady-state dc circuit acts like a short circuit, so a short circuit replaces the inductor. The current in that short circuit is the steady state inductor current, $i(0)$. Apply KVL to the loop to get

$$R_1\, i(0) + R_2\, i(0) + 3\, i(0) - 24 = 0$$

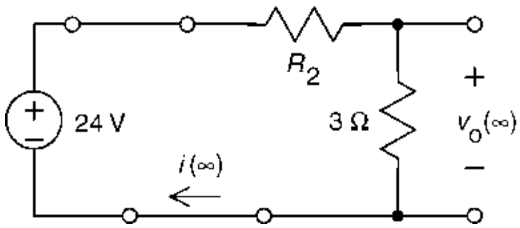
$$\Rightarrow \quad i(0) = \frac{24}{R_1 + R_2 + 3}$$

The value of $i(0)$ can also be obtained by setting $t = 0$ in the equation for $i(t)$. Do so gives

$$i(0) = 2 - e^0 = 1 \text{ A}$$

Consequently,

$$1 = \frac{24}{R_1 + R_2 + 3} \quad \Rightarrow \quad R_1 + R_2 = 21$$



Next, consider the circuit after the switch closes. Here is the circuit at $t = \infty$, when the switch is closed and the circuit is at steady state. The closed switch is modeled as a short circuit. The combination of resistor and a short circuit connected is equivalent to a short circuit. Consequently, a short circuit replaces the switch and the resistor R_1.

An inductor in a steady-state dc circuit acts like a short circuit, so a short circuit replaces the inductor. The current in that short circuit is the steady state inductor current, $i(\infty)$. Apply KVL to the loop to get

$$R_2\, i(\infty) + 3\, i(\infty) - 24 = 0 \quad \Rightarrow \quad i(\infty) = \frac{24}{R_2 + 3}$$

The value of $i(\infty)$ can also be obtained by setting $t = \infty$ in the equation for $i(t)$. Doing so gives

$$i(\infty) = 2 - e^{-\infty} = 2 \text{ A}$$

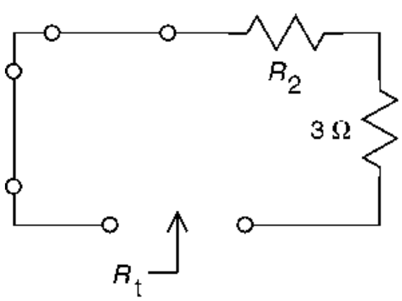
Consequently

$$2 = \frac{24}{R_2 + 3} \quad \Rightarrow \quad R_2 = 9 \;\; \Omega$$

Then

$$R_1 = 12 \;\; \Omega$$

Finally, the exponential part of $i(t)$ is known to be of the form $e^{-t/\tau}$ where $\tau = \frac{L}{R_t}$ and R_t is the Thevenin resistance of the part of the circuit that is connected to the inductor.



Here is shows the circuit that is used to determine R_t. A short circuit has replaced combination of resistor R_1 and the closed switch. Independent sources are set to zero when calculating R_t, so the voltage source has been replaced by an short circuit.

$$R_t = R_2 + 3 = 9 + 3 = 12 \;\; \Omega$$

so

$$\tau = \frac{L}{R_t} = \frac{L}{12}$$

From the equation for $i(t)$

$$-0.35\, t = -\frac{t}{\tau} \quad \Rightarrow \quad \tau = 2.857 \text{ s}$$

Consequently,

$$2.857 = \frac{L}{12} \quad \Rightarrow \quad L = 34.28 \;\; \text{H}$$

P 8.3-10 A security alarm for an office building door is modeled by the circuit of Figure P 8.3-10. The switch represents the door interlock, and *v* is the alarm indicator voltage. Find *v*(*t*) for $t > 0$ for the circuit of Figure P 8.3-10. The switch has been closed for a long time at $t = 0^-$.

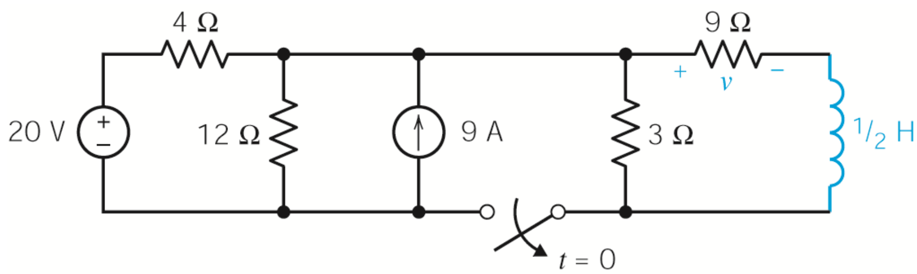
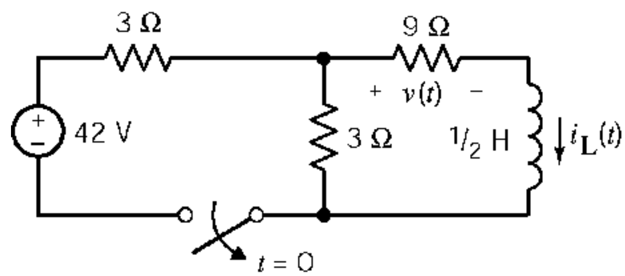
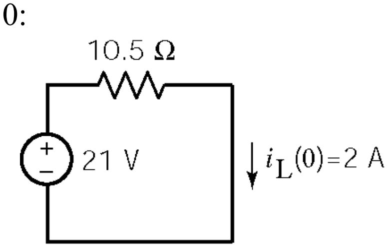


Figure P 8.3-10

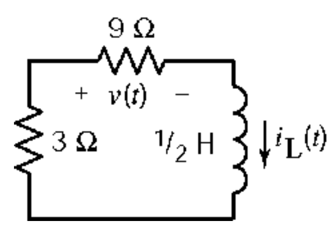
Solution: First, use source transformations to obtain the equivalent circuit



for $t < 0$:



for $t > 0$:



So $i_L(0) = 2$ A, $I_{sc} = 0$, $R_t = 3+9=12\ \Omega$, $\tau = \dfrac{L}{R_t} = \dfrac{\frac{1}{2}}{12} = \dfrac{1}{24}$ s

and $i_L(t) = 2e^{-24t} \quad t > 0$

Finally $v(t) = 9\, i_L(t) = 18\, e^{-24t} \quad t>0$

P 8.3-11 The voltage $v(t)$ in the circuit shown in Figure P 8.3-11 is given by

$$v(t) = 8 + 4e^{-2t} \text{ V} \quad \text{for } t > 0$$

Determine the values of R_1, R_2, and C.

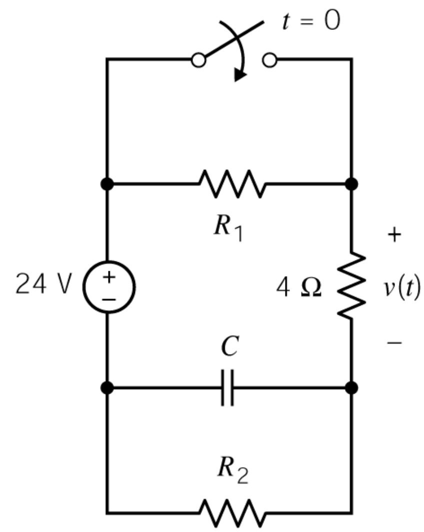
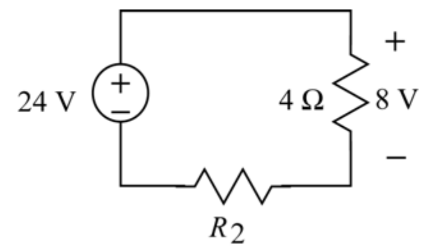


Figure P 8.3-11

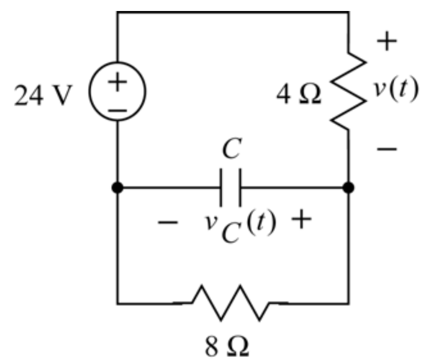
Solution: As $t \to \infty$ the circuit reaches steady state and the capacitor acts like an open circuit. Also, from the given equation, $v(t) \to 8 \text{ V}$, as labeled on the drawing to the right, then



$$8 = \frac{4}{R_2 + 4} 24 \quad \Rightarrow \quad R_2 = 8 \ \Omega$$

After $t = 0$

$$v_C(t) = 24 - v(t) = 16 - 4e^{-2t}$$



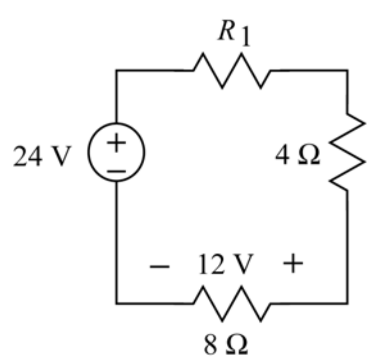
Immediately after $t = 0$

$$v_C(0+) = 16 - 4 = 12 \text{ V}$$

The capacitor voltage cannot change instantaneously so

$$v(0-) = 12 \text{ V}$$

The circuit is at steady state just before the switch closes so the capacitor acts like an open circuit. Then



$$12 = \frac{8}{R_1 + 4 + 8} 24 \quad \Rightarrow \quad R_1 = 4 \ \Omega$$

After $t = 0$ the Thevenin resistance seen by the capacitor is

$$R_t = 8 \,||\, 4 = \frac{8}{3} \ \Omega$$

so

$$2 = \frac{1}{\frac{8}{3}C} \quad \Rightarrow \quad C = \frac{3}{16} \text{ F}$$

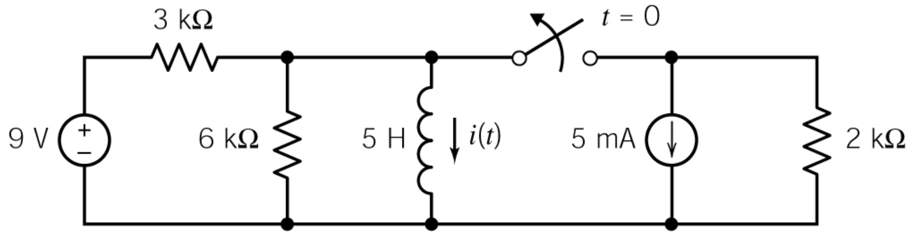
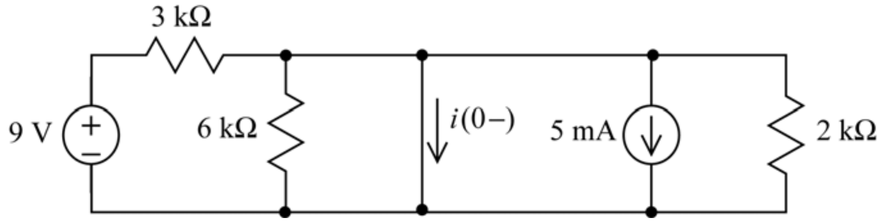


Figure P 8.3-12

P 8.3-12 The circuit shown in Figure P 8.3-12 is at steady state when the switch opens at time $t = 0$. Determine $i(t)$ for $t \geq 0$.

Solution: Before t = 0, with the switch closed and the circuit at steady state, the inductor acts like a short circuit so we have

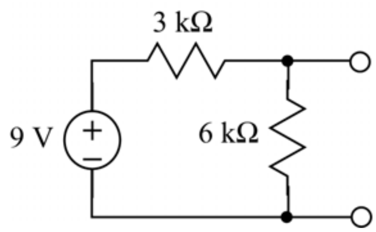


Using superposition

$$i(0-) = \frac{9}{3000} - 5\times10^{-3} = -2 \text{ mA}$$

The inductor current is continuous so $i(0+) = i(0-) = -2$ mA.

After t = 0, the switch is open. Determine the Norton equivalent circuit for the part of the circuit connected to the inductor:



$$i_{sc} = \frac{9}{3000} = 3 \text{ mA}$$
$$R_t = 3000 \,||\, 6000 = 2000\ \Omega$$

The time constant is given by $\tau = \frac{L}{R_t} = \frac{5}{2000} = 0.0025$ so $\frac{1}{\tau} = 400$.

The inductor current is given by

$$i_L(t) = \left(i(0+) - i_{sc}\right)e^{-t/\tau} + i_{sc} = (-0.002 - 0.003)e^{-400t} + 0.003 = 3 - 5e^{-400t} \text{ mA} \quad \text{for } t \geq 0$$

(checked: LNAP 6/29/04)

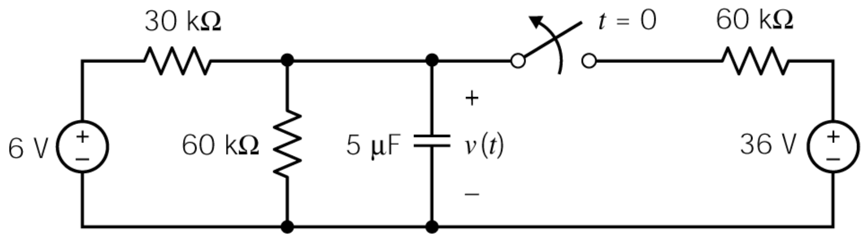
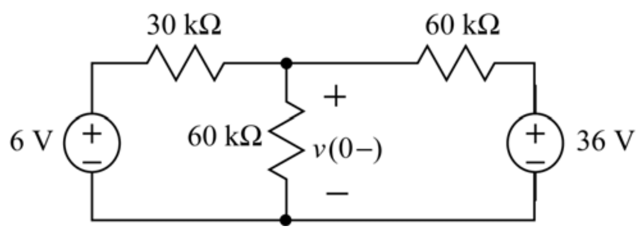


Figure P 8.3-13

P 8.3-13 The circuit shown in Figure P 8.3-13 is at steady state when the switch opens at time $t = 0$. Determine $v(t)$ for $t \ge 0$.

Solution: Before $t = 0$, with the switch closed and the circuit at the steady state, the capacitor acts like an open circuit so we have

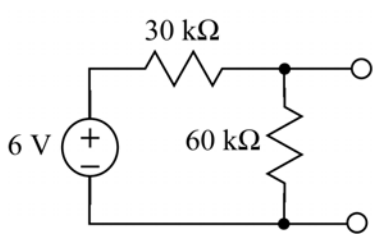


Using superposition

$$v(0-) = \frac{60 \parallel 60}{30 + (60 \parallel 60)} 6 + \frac{60 \parallel 30}{60 + (60 \parallel 30)} 36 = \left(\frac{1}{2}\right) 6 + \left(\frac{1}{4}\right) 36 = 12 \text{ V}$$

The capacitor voltage is continuous so $v(0+) = v(0-) = 12 \text{ V}$.

After $t = 0$ the switch is open. Determine the Thevenin equivalent circuit for the part of the circuit connected to the capacitor:



$$v_{oc} = \frac{60}{60 + 30} 6 = 4 \text{ V}$$

$$R_t = 30 \parallel 60 = 20 \text{ k}\Omega$$

The time constant is $\tau = R_t C = (20 \times 10^3)(5 \times 10^{-6}) = 0.1 \text{ s}$ so $\frac{1}{\tau} = 10 \ \frac{1}{\text{s}}$.

The capacitor voltage is given by

$$v(t) = (v(0+) - v_{oc}) e^{-t/\tau} + v_{oc} = (12 - 4) e^{-10t} + 4 = 4 + 8 e^{-10t} \text{ V} \quad \text{for } t \ge 0$$

(checked: LNAP 6/29/04)

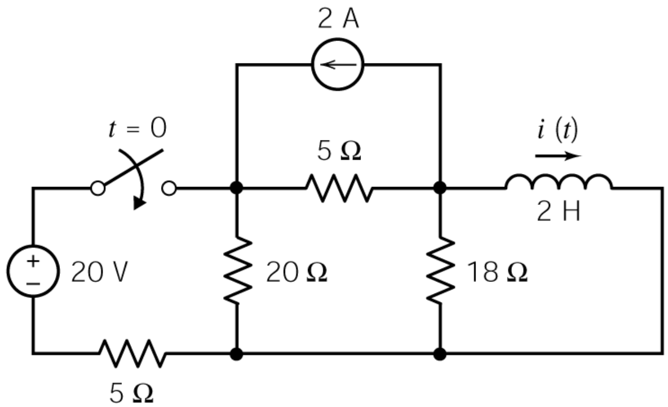
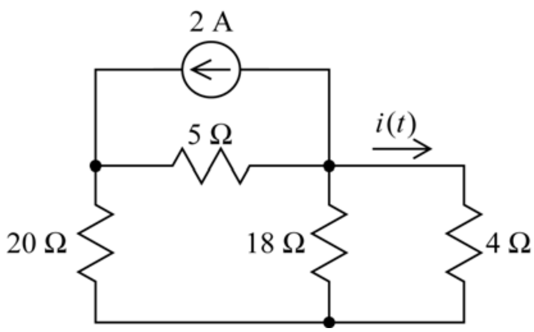


Figure P 8.3-14

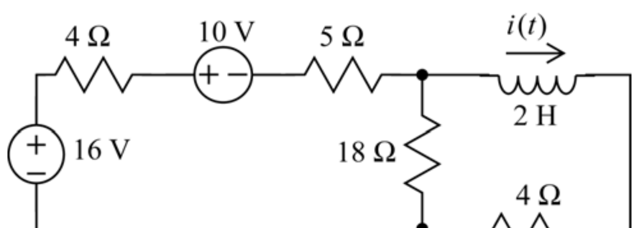
P 8.3-14 The circuit shown in Figure P 8.3-14 is at steady state when the switch closes at time $t = 0$. Determine $i(t)$ for $t \geq 0$.

Solution: Before $t = 0$, with the switch open and the circuit at steady state, the inductor acts like a short circuit so we have

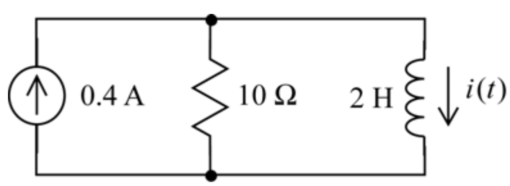


$$i(t) = -\frac{18}{4+18}\left[\frac{5}{5+20+(18\,||\,4)}2\right] = 0.29 \text{ A}$$

After $t = 0$, we can replace the part of the circuit connected to the inductor by its Norton equivalent circuit. First, performing a couple of source transformations reduces the circuit to



Next, replace the series voltage sources by an equivalent voltage source, replace the series resistors by an equivalent resistor and do a couple of source transformations to get



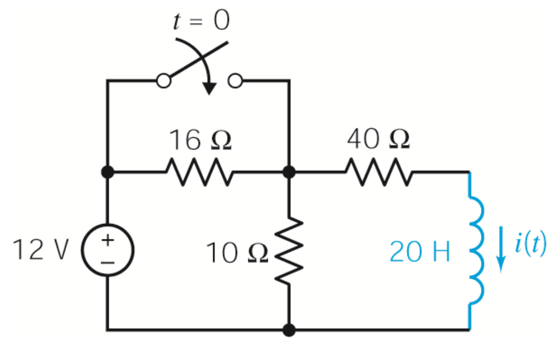
so $$\tau = \frac{2}{10} = 0.25 \quad \Rightarrow \quad \frac{1}{\tau} = 5\frac{1}{\text{s}}$$

The current is given by $$i(t) = [0.29 - 0.4]e^{-5t} + 0.4 = 0.4 - 0.11e^{-5t} \text{ A} \quad \text{for } t \geq 0$$

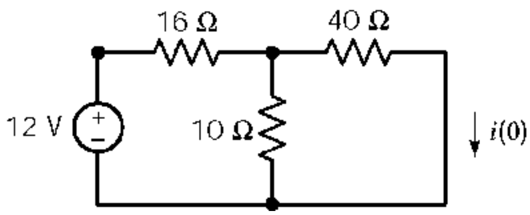
P 8.3-15 The circuit in Figure P 8.3-15 is at steady state before the switch closes. Find the inductor current after the switch closes.

Hint: $i(0) = 0.1$ A, $I_{sc} = 0.3$ A, $R_t = 40\ \Omega$

Answer: $i(t) = 0.3 - 0.2e^{-2t}$ A $t \geq 0$

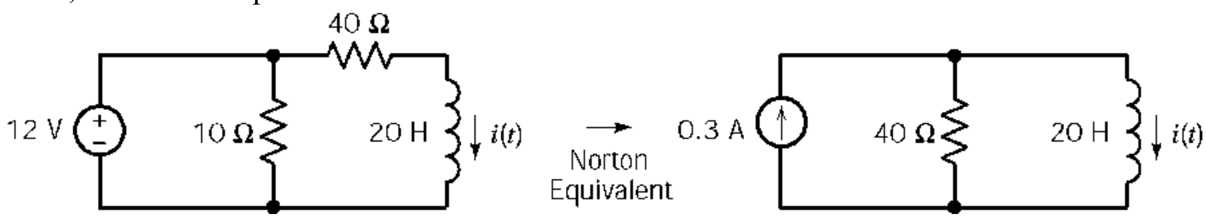


Solution: At steady-state, immediately before $t = 0$:



$$i(0) = \left(\frac{10}{10+40}\right)\left(\frac{12}{16+40\|10}\right) = 0.1\text{ A}$$

After $t = 0$, the Norton equivalent of the circuit connected to the inductor is found to be



$$\text{so } I_{sc} = 0.3\text{ A},\ R_t = 40\ \Omega,\ \tau = \frac{L}{R_t} = \frac{20}{40} = \frac{1}{2}\text{ s}$$

$$\text{Finally:} \quad i(t) = (0.1-0.3)e^{-2t} + 0.3 = 0.3-0.2e^{-2t}\text{ A}$$

P8.3-16. Consider the circuit shown in Figure P8.3-16.

a) Determine the time constant, τ, and the steady state capacitor voltage when the switch is **open**.

b) Determine the time constant, τ, and the steady state capacitor voltage when the switch is **closed**.

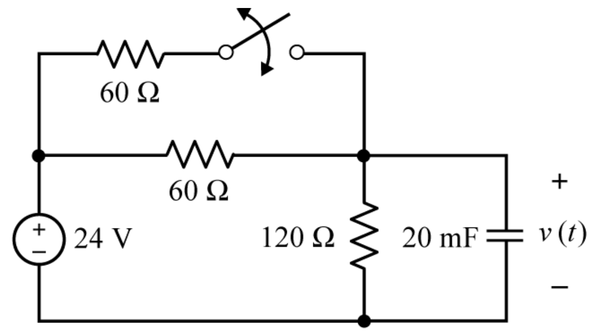


Figure P8.3-16.

Solution:
Replace the part of the circuit connected to the capacitor by its Thevenin equivalent circuit.

a.) When the switch is open, $v_{oc} = \left(\dfrac{120}{60+120}\right)24 = 16 \text{ V}$ and $R_t = 120 \,||\, 60 = 40\ \Omega$. The steady state capacitor voltages is v_{oc} =16 V. The time constant is $\tau = (40)(0.02) = 0.8$ s.

b.) When the switch is closed, $v_{oc} = \left(\dfrac{120}{30+120}\right)24 = 19.2 \text{ V}$ and $R_t = 120 \,||\, 60 \,||\, 60 = 24\ \Omega$. The steady state capacitor voltages is v_{oc} =19.2 V. The time constant is $\tau = (24)(0.02) = 0.48$ s.

LNAPTR, 11/1/07

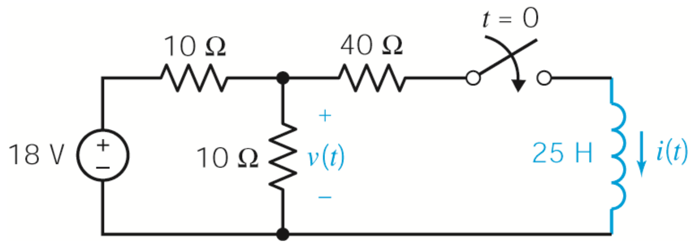
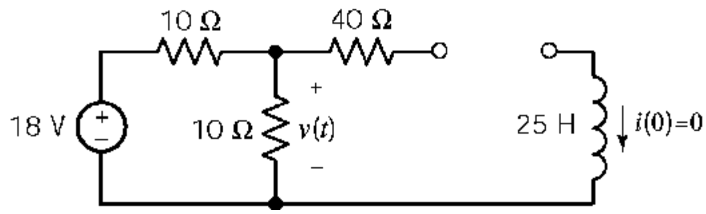


Figure P 8.3-17

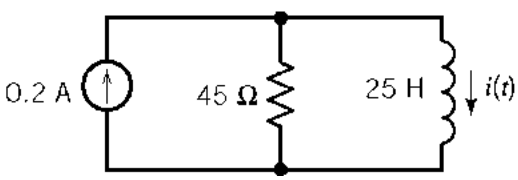
P 8.3-17 The circuit shown in Figure P 8.3-17 is at steady state before the switch closes. The response of the circuit is the voltage $v(t)$. Find $v(t)$ for $t > 0$.
Hint: After the switch closes, the inductor current is $i(t) = 0.2(1 - e^{-1.8t})$ A

Answer: $v(t) = 8 + e^{-1.8t}$ V

Solution: Immediately before $t = 0,\ i(0) = 0.$



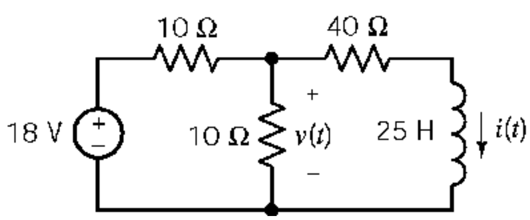
After $t = 0$, replace the circuit connected to the inductor by its Norton equivalent to calculate the inductor current:



$$I_{sc} = 0.2 \text{ A},\ R_t = 45\ \Omega,\ \tau = \frac{L}{R_{th}} = \frac{25}{45} = \frac{5}{9}$$

$$\text{So } i(t) = 0.2\,(1 - e^{-1.8t}) \text{ A}$$

Now that we have the inductor current, we can calculate $v(t)$:



$$\begin{aligned} v(t) &= 40\ i(t) + 25\frac{d}{dt}i(t) \\ &= 8(1 - e^{-1.8t}) + 5(1.8)e^{-1.8t} \\ &= 8 + e^{-1.8t} \text{ V} \quad \text{for } t > 0 \end{aligned}$$

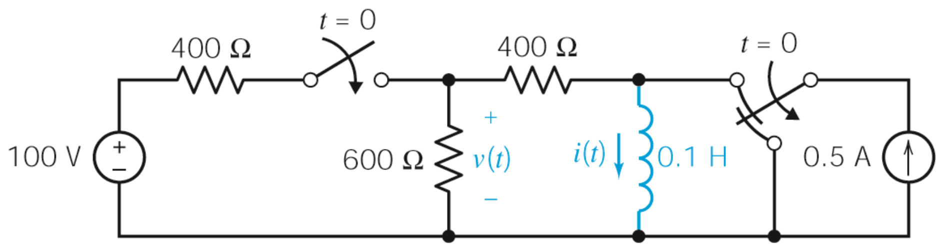
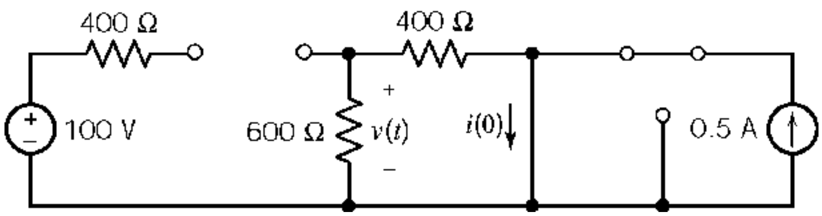


Figure P 8.3-18

P 8.3-18 The circuit shown in Figure P 8.3-18 is at steady state before the switch closes. The response of the circuit is the voltage $v(t)$. Find $v(t)$ for $t > 0$.

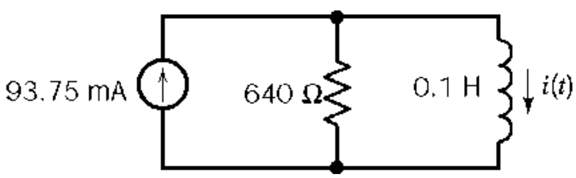
Answer: $v(t) = 37.5 - 97.5e^{-6400t}$ V

Solution: At steady-state, immediately before t = 0



so $i(0) = 0.5$ A.

After t > 0: Replace the circuit connected to the inductor by its Norton equivalent to get

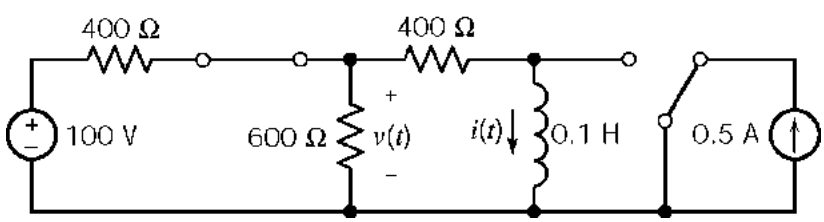


$$I_{sc} = 93.75 \text{ mA}, R_t = 640\ \Omega,$$

$$\tau = \frac{L}{R_t} = \frac{.1}{640} = \frac{1}{6400} \text{ s}$$

$$i\ (t) = 406.25\ e^{-6400t} + 93.75 \text{ mA}$$

Finally:



$$v(t) = 400\ i(t) + \ 0.1\ \frac{d}{dt}\ i(t) = 400\ (.40625e^{-6400t} + .09375) + 0.1\,(-6400)\,(0.40625e^{-6400t})$$

$$= \ 37.5 - 97.5e^{-6400t} \text{ V}$$

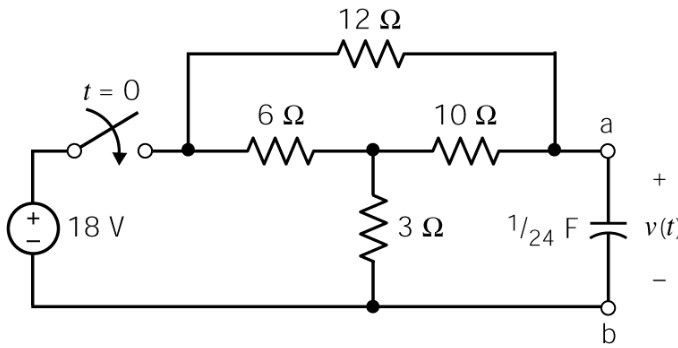


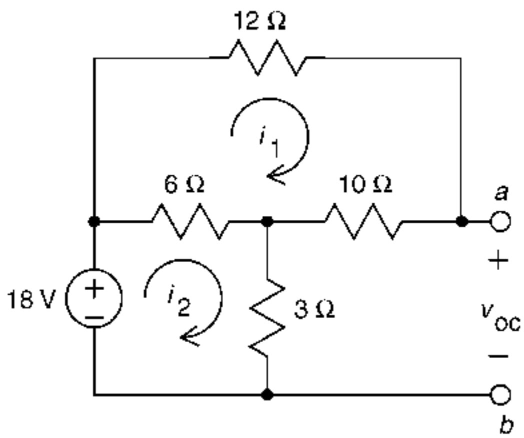
Figure P 8.3-19

P 8.3-19 The circuit shown in Figure P 8.3-19 is at steady state before the switch closes. Find $v(t)$ for $t \geq 0$.

Solution: Before the switch closes $v(t) = 0$ so $v(0+) = v(0-) = 0$ V.

For $t > 0$, we find the Thevenin equivalent circuit for the part of the circuit connected to the capacitor, i.e. the part of the circuit to the left of the terminals $a - b$.

Write mesh equations to find v_{oc}:



Mesh equations:

$$12\, i_1 + 10\, i_1 - 6\left(i_2 - i_1\right) = 0$$
$$6\left(i_2 - i_1\right) + 3\, i_2 - 18 = 0$$

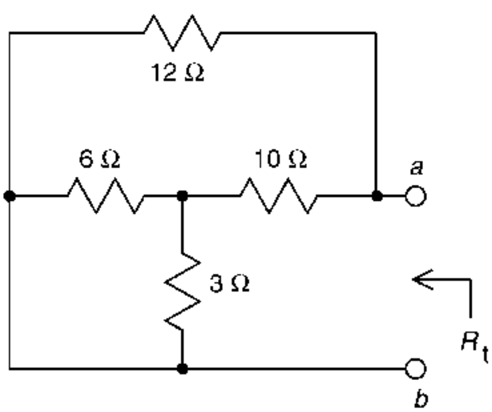
$$28\, i_1 = 6\, i_2$$
$$9\, i_2 - 6\, i_1 = 18$$

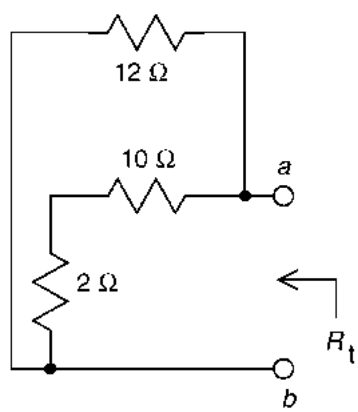
$$36\, i_1 = 18 \quad \Rightarrow \quad i_1 = \frac{1}{2}\text{ A}$$

$$i_2 = \frac{14}{3}\left(\frac{1}{2}\right) = \frac{7}{3}\text{ A}$$

Using KVL, $$v_{oc} = 3\, i_2 + 10\, i_1 = 3\left(\frac{7}{3}\right) + 10\left(\frac{1}{2}\right) = 12\text{ V}$$

Find R_t:





$$R_t = \frac{12(10+2)}{12+(10+2)} = 6\ \Omega$$

Then $$\tau = R_t C = 6\left(\frac{1}{24}\right) = \frac{1}{4}\ \text{s} \quad \Rightarrow \quad \frac{1}{\tau} = 4\ \frac{1}{\text{s}}$$

and $$v(t) = \left(v(0+) - v_{\text{oc}}\right)e^{-t/\tau} + v_{\text{oc}} = (0-12)e^{-4t} + 12 = 12\left(1-e^{-4t}\right)\ \text{V} \quad \text{for } t \ge 0$$

(checked: LNAP 7/15/04)

P 8.3-20 The circuit shown in Figure P 8.3-20 is at steady state before the switch closes. Determine $i(t)$ for $t \geq 0$.

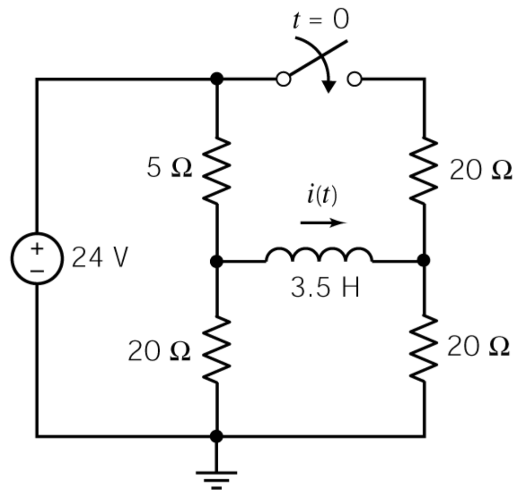
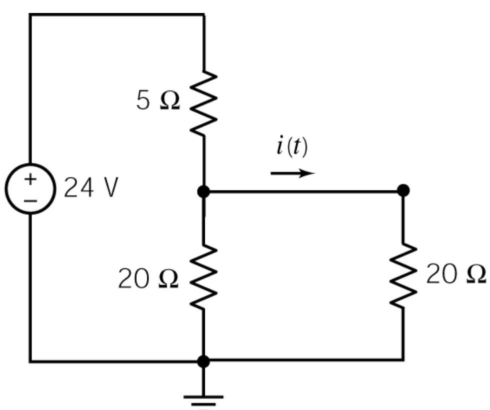


Figure P 8.3-20

Solution: Before the switch closes the circuit is at steady state so the inductor acts like a short circuit. We have

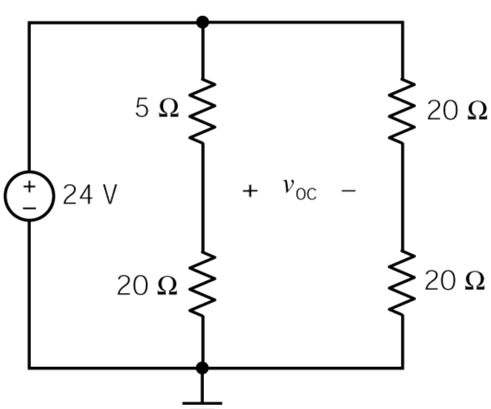


so

$$i(t) = \frac{1}{2}\left(\frac{24}{5+(20\,\|\,20)}\right) = 0.8 \text{ A}$$

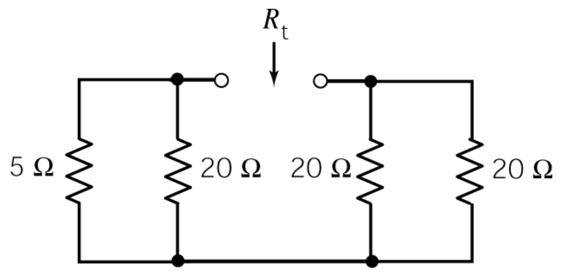
$$i(0+) = i(0-) = 0.8 \text{ A}$$

After the switch closes, find the Thevenin equivalent circuit for the part of the circuit connected to the inductor.



Using voltage division twice

$$v_{oc} = \left(\frac{20}{25} - \frac{1}{2}\right)24 = 7.2 \text{ V}$$



$$R_t = (5 \| 20) + (20 \| 20) = 14\ \Omega$$

$$i_{sc} = \frac{v_{oc}}{R_t} = \frac{7.2}{14} = 0.514\ \text{A}$$

Then

$$\tau = \frac{L}{R_t} = \frac{3.5}{14} = \frac{1}{4}\ \text{s} \quad \Rightarrow \quad \frac{1}{\tau} = 4\ \frac{1}{\text{s}}$$

and

$$i(t) = \left(i(0+) - i_{sc}\right)e^{-t/\tau} + i_{sc} = (0.8 - 0.514)e^{-4t} + 0.514 = 0.286e^{-4t} + 0.514\ \text{A} \quad \text{for } t \geq 0$$

(checked: LNAP 7/15/04)

P8.3-21 The circuit in Figure P8.3-21 at steady state before the switch closes. Determine an equation that represents the capacitor voltage after the switch closes.

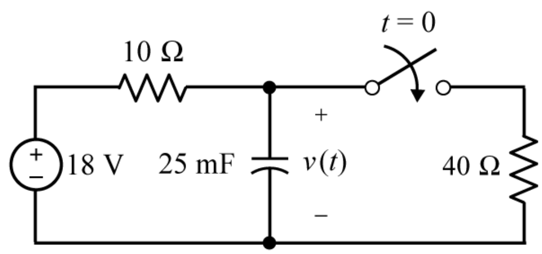


Figure P8.3-21

Solution:

For $t < 0$, the switch is open and the capacitor acts like an open circuit because the circuit is at steady state. Consequently, the current in the 10 Ω resistor is 0A and so the voltage across this resistor is 0 V. KVL gives $v(t) = 18$ V. Immediately before the switch opens we have $v(0-) = 18$ V. The capacitor voltage does not change instantaneously so $v(0+) = v(0-) = 18$ V.

For $t > 0$, the Thevenin equivalent of the part of the circuit connected to the capacitor is characterized by $R_t = 10 \| 40 = 8\ \Omega$ and, using voltage division, $v_{oc} = \frac{40}{10+40}(18) = 14.4$ V.

$A = v_{oc} = 14.4$ V, $B = v(0+) - v_{oc} = 18 - 14.4 = 3.6$ V and $a = \frac{1}{\tau} = \frac{1}{R_t C} = \frac{1}{8(0.025)} = 5\ \frac{1}{\text{s}}$

LNAPTR, 11/3/07

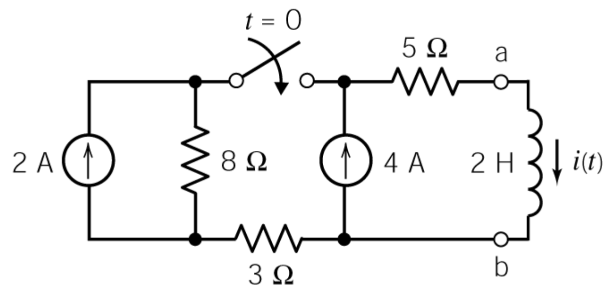
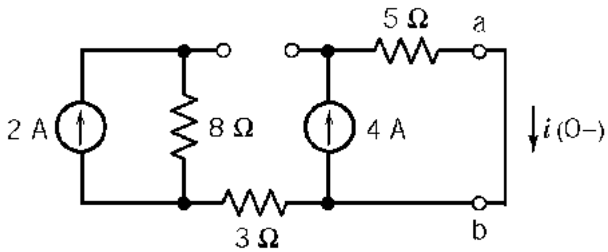


Figure P 8.3-22

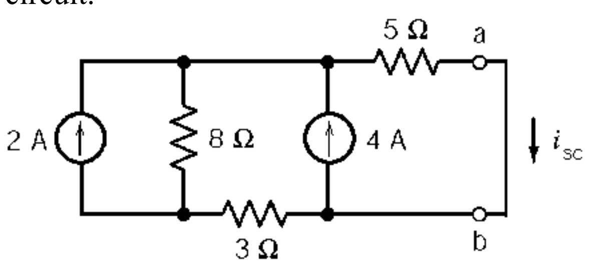
P 8.3-22 The circuit shown in Figure P 8.3-22 is at steady state when the switch closes at time $t = 0$. Determine $i(t)$ for $t \geq 0$.

Solution: Before $t = 0$, with the switch open and the circuit at steady state, the inductor acts like a short circuit so we have



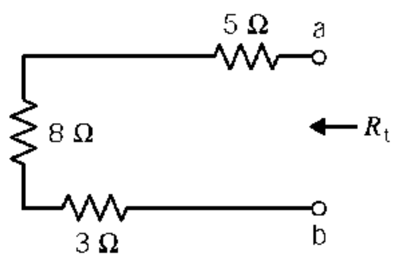
$$i(0+) = i(0-) = 4 \text{ A}$$

After $t = 0$, we can replace the part of the circuit connected to the inductor by its Norton equivalent circuit.



Using superposition, the short circuit current is given by

$$i_{sc} = \left(\frac{8}{8+(5+3)} \right) 2 + \left(\frac{3+8}{(3+8)+5} \right) 4 = 3.75 \text{ A}$$



$$R_t = 8+3+5 = 16 \ \Omega$$

so

$$\tau = \frac{2}{16} = 0.125 \text{ s} \quad \Rightarrow \quad \frac{1}{\tau} = 8 \ \frac{1}{\text{s}}$$

The inductor current is given by

$$i_L(t) = \left(i(0+) - i_{sc} \right) e^{-t/\tau} + i_{sc} = (4-3.75) e^{-8t} + 3.75 = 3.75 - 0.25 \, e^{-8t} \text{ A} \quad \text{for } t \geq 0$$

(checked: LNAP 7/15/04)

P8.3-23 The circuit in Figure P8.3-23 at steady state before the switch closes. Determine an equation that represents the inductor current after the switch closes.

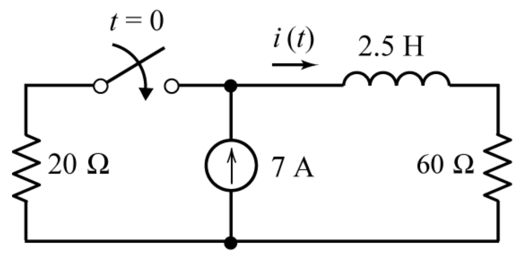


Figure P8.3-23

Solution:

For $t < 0$, the switch is open and the inductor acts like short circuit because the circuit is at steady state. Consequently, the current in inductor just before the switch closes is $i(0-) = 7 \text{ Amps}$. The inductor current does not change instantaneously so $i(0+) = i(0-) = 7 \text{ Amps}$.

For $t > 0$, the Thevenin equivalent of the part of the circuit connected to the inductor is characterized by $R_t = 20 + 60 = 80\ \Omega$ and, using current division, $i_{sc} = \frac{20}{20+60}(7) = 1.75 \text{ Amps}$.

$A = i_{sc} = 1.75 \text{ Amps}$, $B = i(0+) - i_{sc} = 7 - 1.75 = 5.25 \text{ Amps}$ and $a = \frac{1}{\tau} = \frac{R_t}{L} = \frac{80}{2.5} = 32\ \frac{1}{s}$

LNAPTR, 11/3/07

P8.3-24 Consider the circuit shown in Figure 8.3-24a and corresponding plot of the inductor current shown in Figure 8.3-24b. Determine the values of L, R_1 and R_2.

Answer: $L = 4.8$ H, $R_1 = 200\ \Omega$ and $R_2 = 300\ \Omega$.

Hint: Use the plot to determine values of D, E, F and a such that the inductor current can be represented as

$$i(t) = \begin{cases} D & \text{for } t \le 0 \\ E + F\,e^{-at} & \text{for } t \ge 0 \end{cases}$$

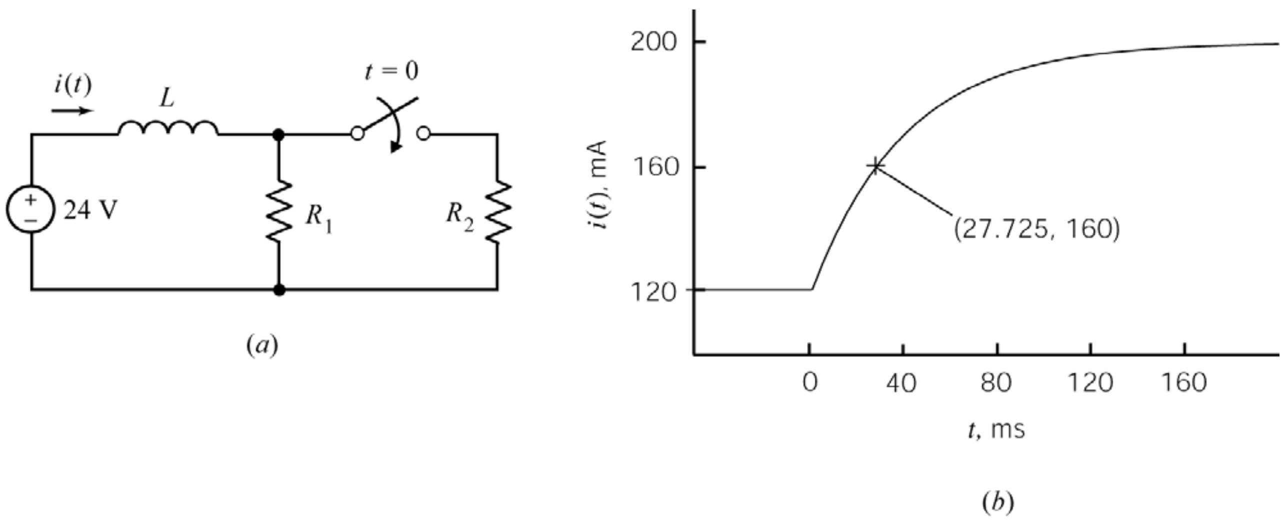


Figure 8.3-24

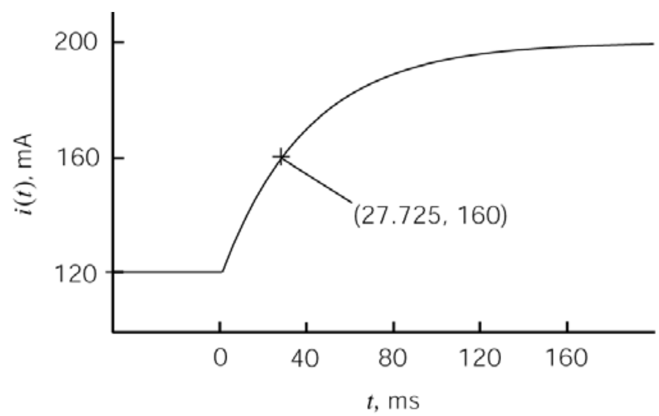
Solution: From the plot

$$D = i(t) \text{ for } t < 0 = 120 \text{ mA} = 0.12 \text{ A},$$

$$E + F = i(0+) = 120 \text{ mA} = 0.12 \text{ A}$$

and

$$E = \lim_{t\to\infty} i(t) = 200 \text{ mA} = 0.2 \text{ A}.$$

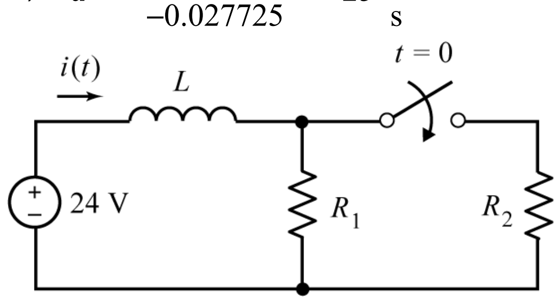


The point labeled on the plot indicates that $i(t) = 160$ mA when $t = 27.725$ ms $= 0.027725$ s. Consequently

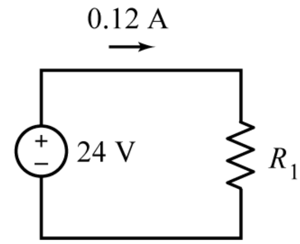
$$160 = 200 - 80\,e^{-a(0.027725)} \quad \Rightarrow \quad a = \frac{\ln\left(\dfrac{160-200}{80}\right)}{-0.027725} = 25\ \frac{1}{\text{s}}$$

Then

$$i(t) = \begin{cases} 120 \text{ mA} & \text{for } t \le 0 \\ 200 - 80\,e^{-25t} \text{ mA} & \text{for } t \ge 0 \end{cases}$$

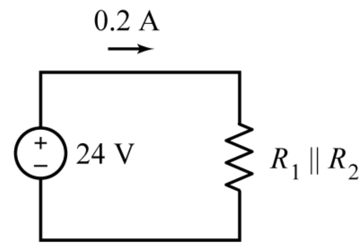


When $t < 0$, the circuit is at steady state so the inductor acts like a short circuit.



$$R_1 = \frac{24}{0.12} = 200\ \Omega$$

As $t \to \infty$, the circuit is again at steady state so the inductor acts like a short circuit.



$$R_1 \,||\, R_2 = \frac{24}{0.2} = 120\ \Omega$$

$$120 = 200 \,||\, R_2 \quad \Rightarrow \quad R_2 = 300\ \Omega$$

Next, the inductance can be determined using the time constant:

$$25 = a = \frac{1}{\tau} = \frac{R_1 \,||\, R_2}{L} = \frac{120}{L} \quad \Rightarrow \quad L = \frac{120}{25} = 4.8\ \text{H}$$

P8.3-25 Consider the circuit shown in Figure P8.3-25*a* and corresponding plot of the voltage across the 40 Ω resistor shown in Figure P8.3-25*b*. Determine the values of *L* and R_2.

Answer: $L = 8$ H and $R_2 = 10\ \Omega$.

***Hint*:** Use the plot to determine values of *D*, *E*, *F* and a such that the voltage can be represented as

$$v(t)=\begin{cases} D & \text{for } t<0 \\ E+F\,e^{-at} & \text{for } t>0 \end{cases}$$

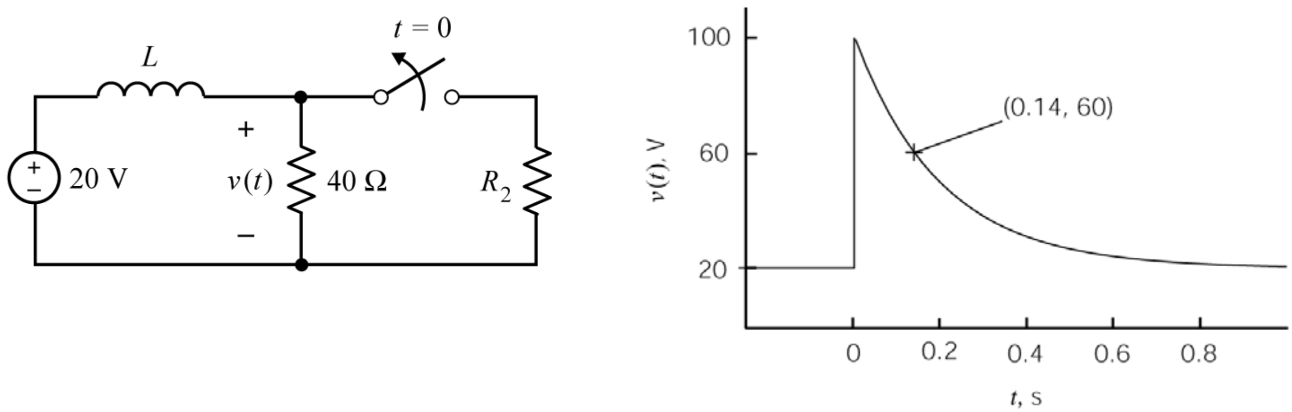


Figure P8.3-25

Solution: From the plot $D=$ $v(t)$ for $t < 0$ =20 V, $E + F = v(0+) = 100$ V and $E=\lim_{t\to\infty} v(t)=20\text{ V}$. The point labeled on the plot indicates that $v(t) = 60$ V when $t =$ 0.14 s. Consequently

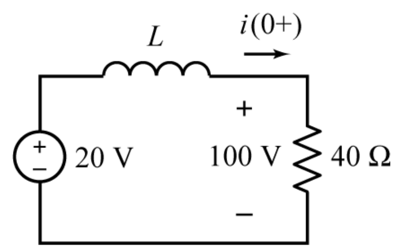
$$60=20+80\,e^{-a(0.14)} \quad\Rightarrow\quad a=\frac{\ln\left(\dfrac{60-20}{80}\right)}{-0.14}=5\ \frac{1}{\text{s}}$$

Then

$$v(t)=\begin{cases} 20\text{ V} & \text{for } t\le 0 \\ 20+80\,e^{-5t}\text{ V} & \text{for } t\ge 0 \end{cases}$$

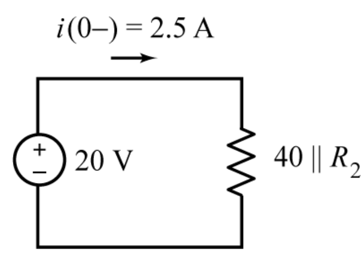
At $t = 0+$,

$$i(0+)=\frac{100}{40}=2.5\text{ A}$$



When $t < 0$, the circuit is at steady state so the inductor acts like a short circuit.

$$40\,\|\,R_2=\frac{20}{2.5}=8\ \Omega \Rightarrow \quad R_2=10\ \Omega$$



Next, the inductance can be determined using the time constant:

$$5=a=\frac{1}{\tau}=\frac{40}{L} \quad\Rightarrow\quad L=\frac{40}{5}=8\text{ H}$$

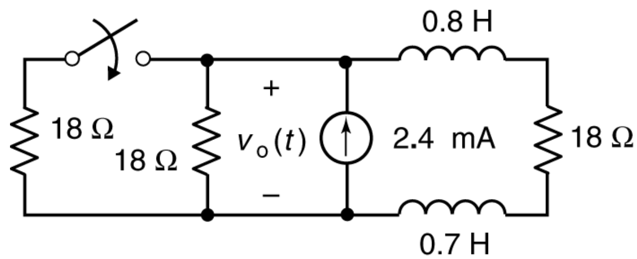
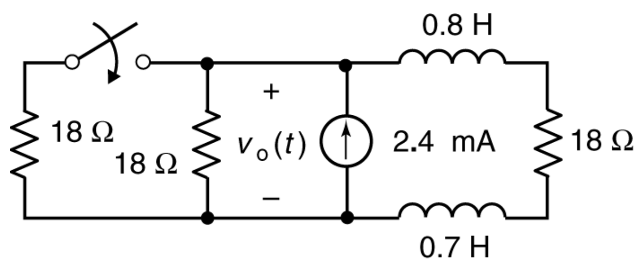


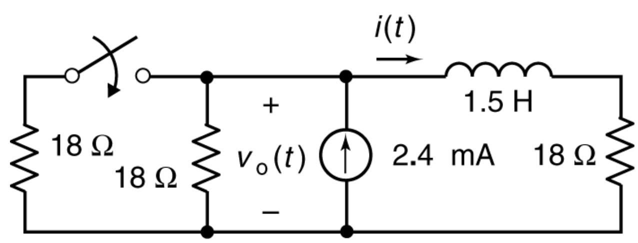
Figure P8.3-26

P8.3-26 Determine $v_o(t)$ for $t > 0$ for the circuit shown in Figure P8.3-26.

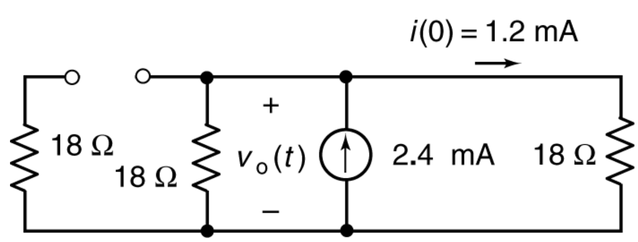
Solution:



Replace the series inductors with an equivalent inductor and label the current in the inductor:



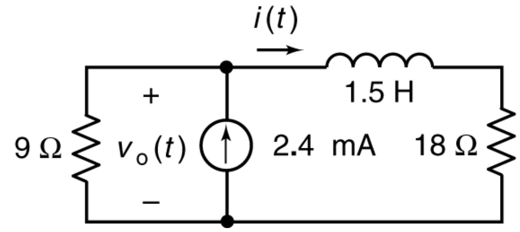
We will determine the inductor current, $i(t)$,first and then use it to determine $v_o(t)$. Determine the initial condition, $i(o)$, by considering the circuit when $t < 0$ and the circuit is at steady state. Since an inductor in a dc circuit acts like a short circuit , we have



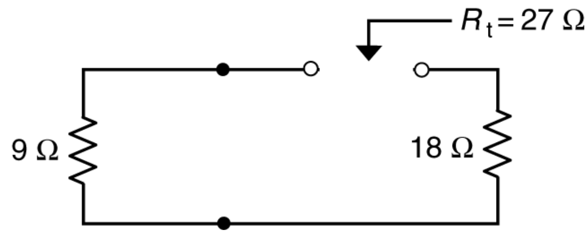
Using current division, we have

$$i(0) = \left(\frac{18}{18+18}\right) 2.4 = 1.2 \text{ mA}$$

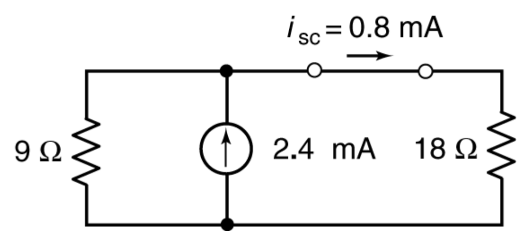
Next, consider the circuit when $t > 0$ and the circuit is not at steady state:



To find the Norton equivalent of the part of the circuit connected to the inductor we determine both the Thevenin resistance and the short circuit current:



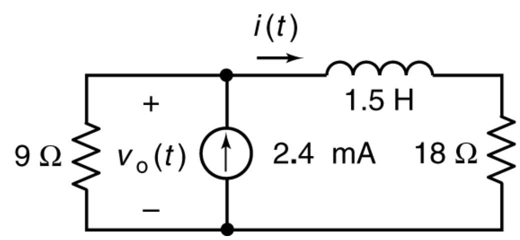
and



The time constant is: $\tau = \dfrac{L}{R_t} = \dfrac{1.5}{27} = \dfrac{1}{18} = 0.0556$ second

The inductor current is given by

$$i(t) = \left(i(0) - i_{sc}\right)e^{-t/\tau} + i_{sc} = (1.2 - 0.8)e^{-18t} - 0.8 \quad \text{mA for } t \ge 0$$



Using KCL $\dfrac{v_o(t)}{9} + \left[(1.2 - 0.8)e^{-18t} - 0.8\right] \times 10^{-3} = 2.4 \times 10^{-3}$

Finally $v_o(t) = 14.4 - 3.6e^{-18t}$ mV for $t > 0$

P8.3-27

The circuit shown in Figure P8.3-27 is at steady state before the switch closes at time $t = 0$. After the switch closes, the inductor current is given by

$$i(t) = 0.6 - 0.2\,e^{-5t} \text{ A} \quad \text{for } t \geq 0$$

Determine the values of R_1, R_2 and L.

Answers: $R_1 = 20\ \Omega$, $R_2 = 10\ \Omega$ and $L = 5$ H

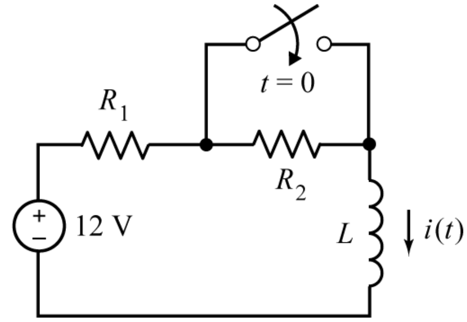
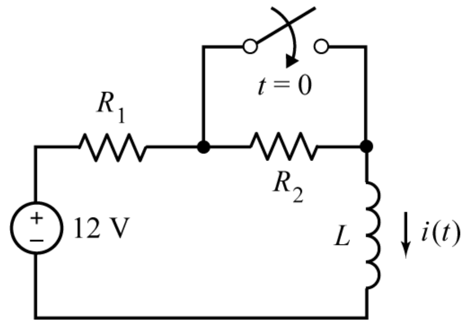


Figure P8.3-27

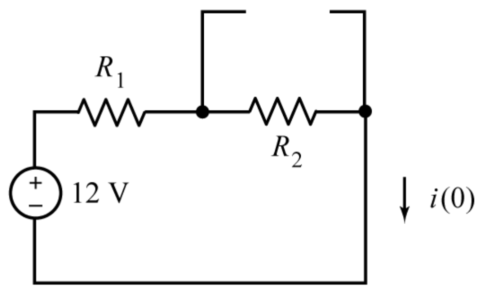
Solution:



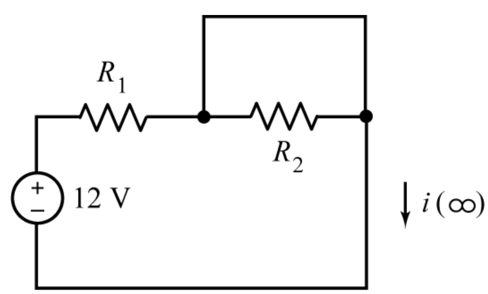
The steady state current before the switch closes is equal to $i(0) = 0.6 - 0.2\,e^{-5(0)} = 0.4$ A .

The inductor will act like a short circuit when this circuit is at steady state so

$$0.4 = i(0) = \frac{12}{R_1 + R_2} \quad \Rightarrow \quad R_1 + R_2 = 30\ \Omega$$



After the switch has been open for a long time, the circuit will again be at steady state. The steady state inductor current will be $i(\infty) = 0.6 - 0.2\,e^{-5(\infty)} = 0.6$ A



The inductor will act like a short circuit when this circuit is at steady state so

$$0.6 = i(\infty) = \frac{12}{R_1} \quad \Rightarrow \quad R_1 = 20\ \Omega$$

Then $R_2 = 10\ \Omega$.

After the switch is closed, the Thevenin resistance of the part of the circuit connected to the inductor is $R_t = R_1$. Then

$$5 = \frac{1}{\tau} = \frac{R_t}{L} = \frac{R_1}{L} = \frac{20}{L} \quad \Rightarrow \quad L = 4 \text{ H}$$

P8.3-28 After time $t = 0$, a given circuit is represented by the circuit diagram shown in FigureP8.3-28.

a.) Suppose that the inductor current is

$$i(t) = 21.6 + 28.4e^{-4t} \text{ mA} \quad \text{for } t \geq 0$$

Determine the values of R_1 and R_3.

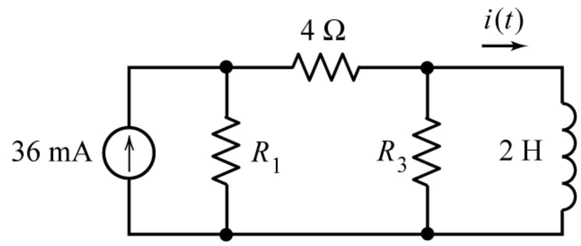


Figure P8.3-28

b.) Suppose instead that $R_1 = 16\ \Omega$, $R_3 = 20\ \Omega$ and the initial condition is $i(0) = 10$ mA. Determine the inductor current for $t \geq 0$.

Solution: The inductor current is given by $i(t) = i_{sc} + (i(0) - i_{sc})e^{-at}$ for $t \geq 0$ where $a = \frac{1}{\tau} = \frac{R_t}{L}$.

a. Comparing this to the given equation gives $21.6 = i_{sc} = \frac{R_1}{R_1 + 4}(36) \Rightarrow R_1 = 6\ \Omega$ and

$4 = \frac{R_t}{2} \Rightarrow R_t = 8\ \Omega$. Next $8 = R_t = (R_1 + 4) \| R_3 = 10 \| R_3 \Rightarrow R_3 = 40\ \Omega$.

b. $R_t = (16 + 4) \| 20 = 10\ \Omega$ so $a = \frac{1}{\tau} = \frac{10}{2} = 5$ s. also $i_{sc} = \frac{16}{16+4}(36) = 28.8$ mA. Then

$i(t) = i_{sc} + (i(0) - i_{sc})e^{-at} = 28.8 + (10 - 28.8)e^{-5t} = 28.2 - 18.8e^{-5t}$.

P8.3-29 Consider the circuit shown in Figure P8.3-29.

a.) Determine the time constant, τ, and the steady state capacitor voltage, $v(\infty)$, when the switch is **open**.

b.) Determine the time constant, τ, and the steady state capacitor voltage, $v(\infty)$, when the switch is **closed**.

Answers: a.) $\tau = 3$ s and $v(\infty) = 24$ V; b.) $\tau = 2.25$ s and $v(\infty) = 2$ V;

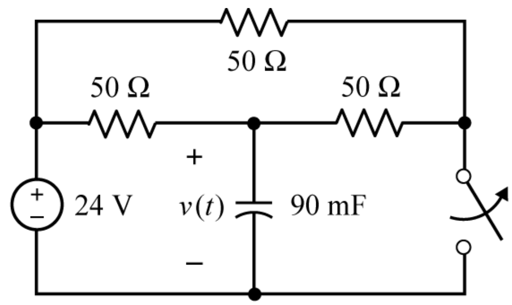
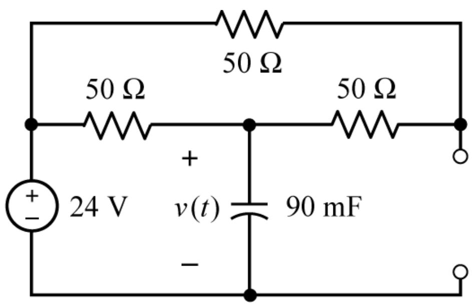
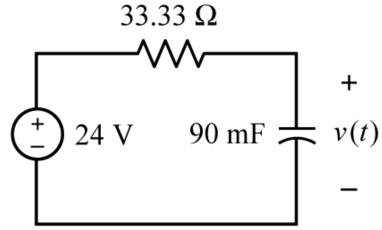


Figure P8.3-29

Solution: a.) When the switch is open we have



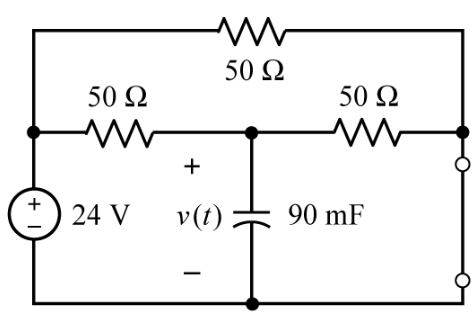


After replacing series and parallel resistors by equivalent resistors, the part of the circuit connected to the capacitor is a Thevenin equivalent circuit with $R_t = 33.33\ \Omega$. The time constant is

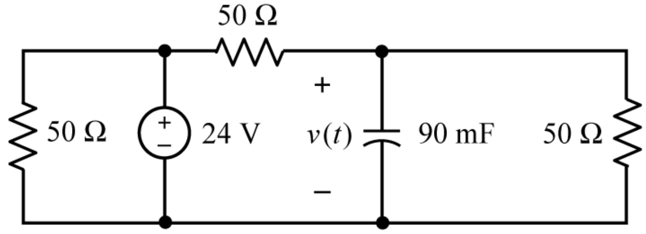
$\tau = R_t\, C = 33.33(0.090) = 3$ s.

Since the input is constant, the capacitor acts like an open circuit when the circuit is at steady state. Consequently, there is zero current in the 33.33 Ω resistor and KVL gives $v(\infty) = 24$ V.

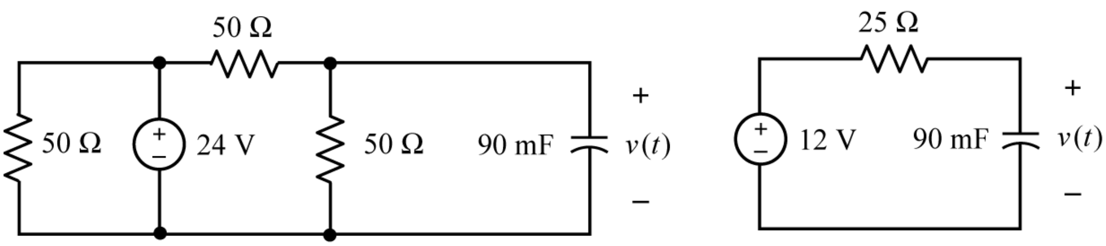
b.) When the switch is closed we have



This circuit can be redrawn as



Now we find the Thevenin equivalent of the part of the circuit connected to the capacitor:



So $R_t = 25\ \Omega$ and

$$\tau = R_t C = 25(0.090) = 2.25 \text{ s}$$

Since the input is constant, the capacitor acts like an open circuit when the circuit is at steady state. Consequently, there is zero current in the 25 Ω resistor and KVL gives $v(\infty) = 12$ V.

Section 8-4: Sequential Switching

P 8.4-1 The circuit shown in Figure P 8.4-1 is at steady state before the switch closes at time $t = 0$. The switch remains closed for 1.5 s and then opens. Determine the capacitor voltage, $v(t)$, for $t > 0$.

Hint: Determine $v(t)$ when the switch is closed. Evaluate $v(t)$ at time $t = 1.5$ s to get $v(1.5)$. Use $v(1.5)$ as the initial condition to determine $v(t)$ after the switch opens again.

Answer: $v(t) = \begin{cases} 5+5e^{-0.5t} \text{ V} & \text{for } 0 < t < 1.5 \text{ s} \\ 10-2.64e^{-2.5(t-1.5)} \text{ V} & \text{for } 1.5 \text{ s} < t \end{cases}$

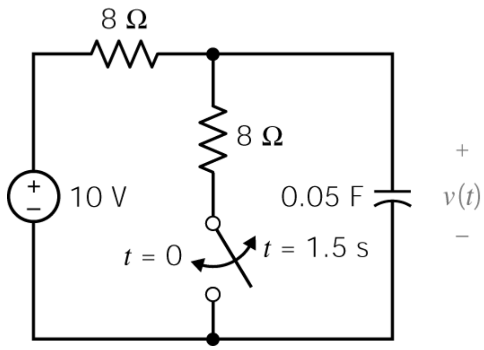
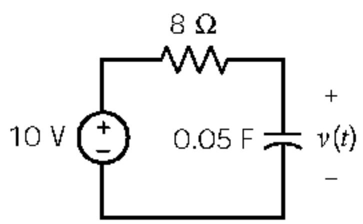


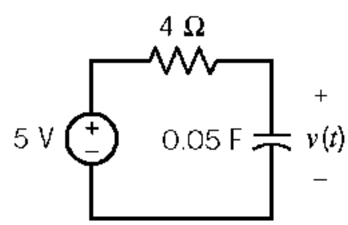
Figure P 8.4-1

Solution:

Replace the part of the circuit connected to the capacitor by its Thevenin equivalent circuit to get:



$t < 0$ and $t > 1.5$ s



$0 < t < 1.5$ s

Before the switch closes at t = 0 the circuit is at steady state so $v(0) = 10$ V. For $0 < t < 1.5$s, $v_{oc} = 5$ V and $R_t = 4\ \Omega$ so $\tau = 4 \times 0.05 = 0.2$ s . Therefore

$$v(t) = v_{oc} + (v(0) - v_{oc})\, e^{-t/\tau} = 5 + 5e^{-5t} \text{ V} \quad \text{for } 0 < t < 1.5 \text{ s}$$

At t =1.5 s, $v(1.5) = 5 + 5e^{-0.05(1.5)} = 5$ V . For 1.5 s < t, $v_{oc} = 10$ V and $R_t = 8\ \Omega$ so $\tau = 8 \times 0.05 = 0.4$ s . Therefore

$$v(t) = v_{oc} + (v(1.5) - v_{oc})\, e^{-(t-1.5)/\tau} = 10 - 5\, e^{-2.5(t-1.5)} \text{ V} \quad \text{for } 1.5 \text{ s} < t$$

Finally

$$v(t) = \begin{cases} 5 + 5\, e^{-5t} \text{ V} & \text{for } 0 < t < 1.5 \text{ s} \\ 10 - 5\, e^{-2.5(t-1.5)} \text{ V} & \text{for } 1.5 \text{ s} < t \end{cases}$$

P 8.4-2 The circuit shown in Figure P 8.4-2 is at steady state before the switch closes at time t = 0. The switch remains closed for 1.5 s and then opens. Determine the inductor current, $i(t)$, for $t > 0$.

Answer: $v(t) = \begin{cases} 2+e^{-0.5t} \text{ A} & \text{for } 0<1<1.5 \text{ s} \\ 3-0.53e^{-0..667(t-1.5)} \text{ A} & \text{for } 1.5 \text{ s}<t \end{cases}$

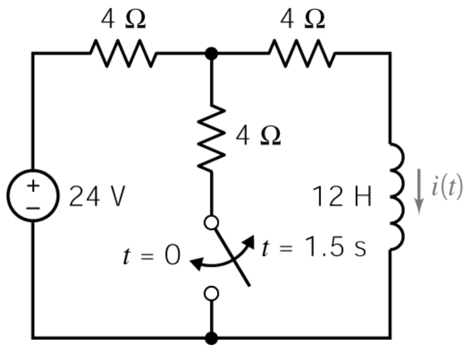
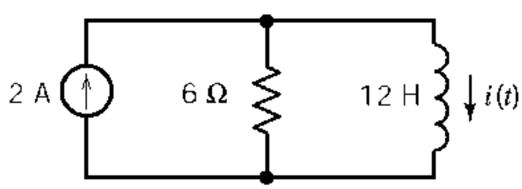


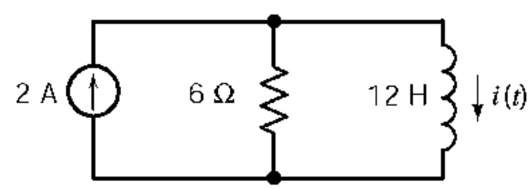
Figure P 8.4-2

Solution:

Replace the part of the circuit connected to the inductor by its Norton equivalent circuit to get:



$t < 0$ and $t > 1.5$ s



$0 < t < 1.5$ s

Before the switch closes at $t = 0$ the circuit is at steady state so $i(0) = 3$ A. For $0 < t < 1.5$s, $i_{sc} = 2$ A and $R_t = 6\ \Omega$ so $\tau = \frac{12}{6} = 2$ s. Therefore

$$i(t) = i_{sc} + (i(0) - i_{sc})\, e^{-t/\tau} = 2 + e^{-0.5t} \text{ A} \quad \text{for } 0 < t < 1.5 \text{ s}$$

At t =1.5 s, $i(1.5) = 2 + e^{-0.5(1.5)} = 2.47$ A. For 1.5 s < t, $i_{sc} =$ 3 A and $R_t = 8\ \Omega$ so $\tau = \frac{12}{8} = 1.5$ s.

Therefore

$$i(t) = i_{sc} + (i(1.5) - i_{sc})\, e^{-(t-1.5)/\tau} = 3 - 0.53\, e^{-0.667(t-1.5)} \text{ V} \quad \text{for } 1.5 \text{ s} < t$$

Finally

$$i(t) = \begin{cases} 2 + e^{-0.5\,t} \text{ A} & \text{for } 0 < t < 1.5 \text{ s} \\ 3 - 0.53\, e^{-0.667(t-1.5)} \text{ A} & \text{for } 1.5 \text{ s} < t \end{cases}$$

P 8.4-3 Cardiac pacemakers are used by people to maintain regular heart rhythm when they have a damaged heart. The circuit of a pacemaker can be represented as shown in Figure P 8.4-3. The resistance of the wires, R, can be neglected since $R < 1$ mΩ. The heart's load resistance, R_L, is 1 kΩ. The first switch is activated at $t = t_0$, and the second switch is activated at $t_1 = t_0 + 10$ ms. This cycle is repeated every second. Find $v(t)$ for $t_0 \le t \le 1$. Note that it is easiest to consider $t_0 =$ 0 for this calculation. The cycle repeats by switch 1 returning to position a and switch 2 returning to its open position.
Hint: Use $q = Cv$ to determine $v(0^-)$ for the 100-μF capacitor.

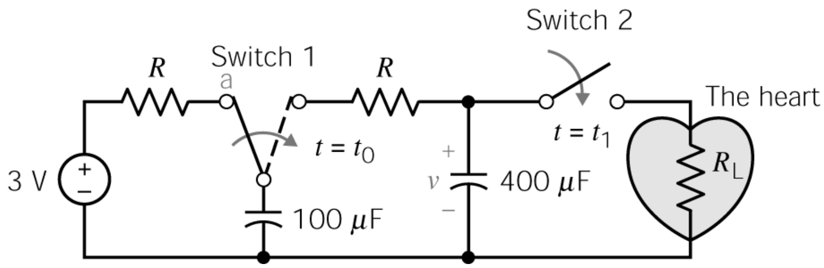
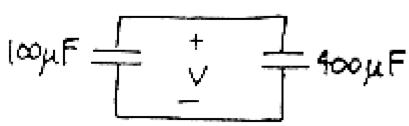


Figure P 8.4-3

Solution:
At $t = 0^-$: Assume that the circuit has reached steady state so that the voltage across the 100 μF capacitor is 3 V. The charge stored by the capacitor is

$$q\left(0^-\right) = \left(100\times10^{-6}\right)\left(3\right) = 300\times10^{-6} \text{ C}$$

0 < t < 10ms: With R negligibly small, the circuit reaches steady state almost immediately (i.e. at $t = 0^+$). The voltage across the parallel capacitors is determined by considering charge conservation:

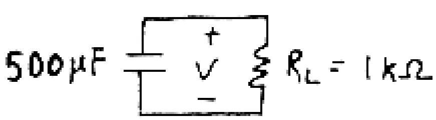


$$q\left(0^+\right) = (100\ \mu\text{F})\ v\left(0^+\right) + (400\ \mu\text{F})\ v\left(0^+\right)$$

$$v\left(0^+\right) = \frac{q\left(0^+\right)}{100\times10^{-6} + 400\times10^{-6}} = \frac{q\left(0^-\right)}{500\times10^{-6}} = \frac{300\times10^{-6}}{500\times10^{-6}}$$

$$\underline{v\left(0^+\right) = 0.6 \text{ V}}$$

10 ms < t < 1 s: Combine 100 μF & 400 μF in parallel to obtain



$$v(t) = v\left(0^+\right)\ e^{-(t-.01)/RC}$$

$$= 0.6e^{-(t-.01)/(10^3)\ (5x10^{-4})}$$

$$\underline{v(t) = 0.6\ e^{-2(t-.01)} \text{ V}}$$

P 8.4-4 An electronic flash on a camera uses the circuit shown in Figure P 8.4-4. Harold E. Edgerton invented the electronic flash in 1930. A capacitor builds a steady-state voltage and then discharges it as the shutter switch is pressed. The discharge produces a very brief light discharge. Determine the elapsed time t_1 to reduce the capacitor voltage to one-half of its initial voltage. Find the current, $i(t)$, at $t = t_1$.

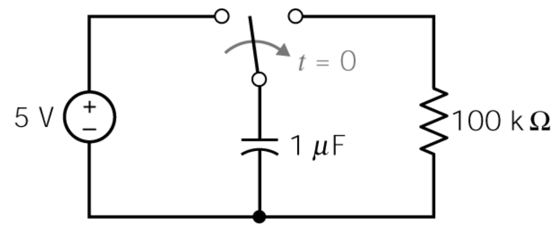
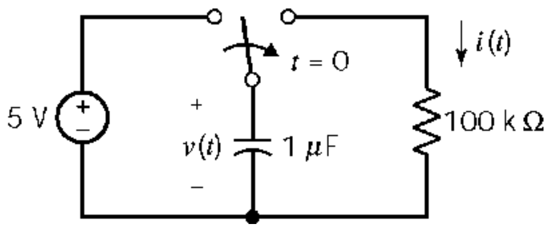


Figure P 8.4-4

Solution:



$$v(0) = 5 \text{ V}, \quad v(\infty) = 0 \quad \text{and} \quad \tau = 10^5 \times 10^{-6} = 0.1 \text{ s}$$

$$\therefore \quad v(t) = 5\, e^{-10\,t} \text{ V} \quad \text{for } t > 0$$

$$2.5 = 5\, e^{-10\,t_1} \qquad \underline{t_1 = 0.0693 \text{ s}}$$

$$i(t_1) = \frac{v(t_1)}{100 \times 10^3} = \frac{2.5}{100 \times 10^3} = \underline{25 \ \mu\text{A}}$$

P 8.4-5 The circuit shown in Figure P 8.4-5 is at steady state before the switch opens at $t = 0$. The switch remains open for 0.5 second and then closes. Determine $v(t)$ for $t \geq 0$.

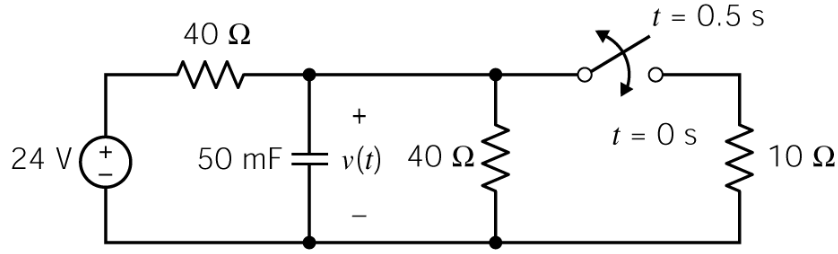
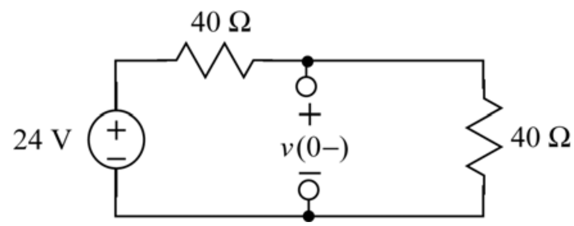


Figure P 8.4-5

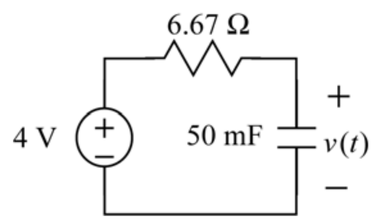
Solution:

The circuit is at steady state before the switch closes. The capacitor acts like an open circuit. The initial condition is

$$v(0+) = v(0-) = \left(\frac{40}{40+40}\right) 24 = 12 \text{ V}$$



After the switch closes, replace the part of the circuit connected to the capacitor by its Thevenin equivalent circuit.



Recognize that $R_t = 6.67\ \Omega$ and $v_{oc} = 4$ V

The time constant is

$$\tau = R_t C = (6.67)(0.05) = 0.335 \text{ s} \quad \Rightarrow \quad \frac{1}{\tau} = 2.988 \simeq 3\ \frac{1}{\text{s}}$$

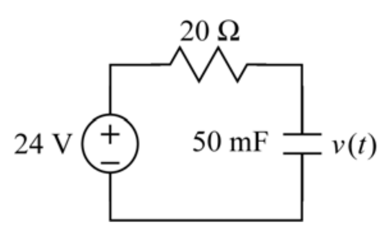
The capacitor voltage is

$$v(t) = \left(v(0+) - v_{oc}\right) e^{-t/\tau} + v_{oc} = (12-4)e^{-3t} + 4 = 4 + 8e^{-3t} \text{ V} \quad \text{for } 0 \geq t \geq 0.5 \text{ s}$$

When the switch opens again at time $t = 0.5$ the capacitor voltage is

$$v(0.5+) = v(0.5-) = 4 + 8e^{-3(0.5)} = 5.785 \text{ V}$$

After time $t = 0.5$ s, replace the part of the circuit connected to the capacitor by its Thevenin equivalent circuit.



Recognize that $R_t = 20\ \Omega$ and $v_{oc} = 12$ V

The time constant is

$$\tau = R_t C = 20(0.05) = 1 \quad \Rightarrow \quad \frac{1}{\tau} = 1 \ \frac{1}{\text{s}}$$

The capacitor voltage is

$$v(t) = \left(v(0.5+) - v_{oc}\right) e^{-(t-0.5)/\tau} + v_{oc} = (5.785 - 12) e^{-10(t-0.5)} + 12$$
$$= 12 - 6.215 e^{-10(t-0.5)} \text{ V} \quad \text{for } t \geq 0.5 \text{ s}$$

so

$$v(t) = \begin{cases} 12 \text{ V} & \text{for } t \geq 0 \\ 4 + 8e^{-3t} \text{ V} & \text{for } 0 \leq t \leq 0.5 \text{ s} \\ 12 - 6.215 e^{-(t-0.5)} \text{ V} & \text{for } t \geq 0.5 \text{ s} \end{cases}$$

Section 8.5 Stability of First-Order Circuits

P 8.5-1 The circuit in Figure P 8.5-1 contains a current-controlled voltage source. What restriction must be placed on the gain, R, of this dependent source in order to guarantee stability?
Answer: $R < 400\ \Omega$

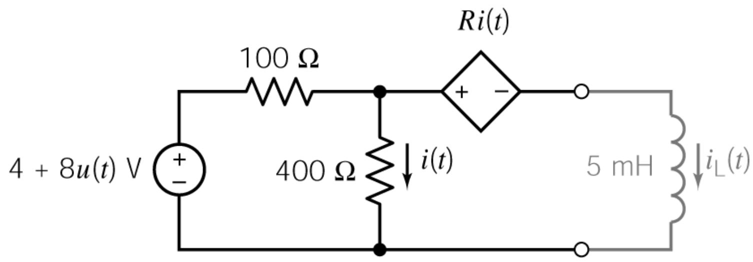
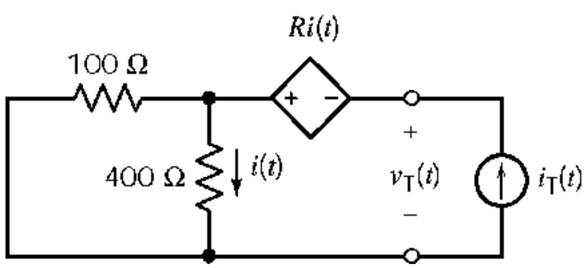


Figure P 8.5-1

Solution:
This circuit will be stable if the Thèvenin equivalent resistance of the circuit connected to the inductor is positive. The Thèvenin equivalent resistance of the circuit connected to the inductor is calculated as



$$\left.\begin{array}{l} i(t) = \dfrac{100}{100+400}\, i_T \\ v_T = 400\, i(t) - R\, i(t) \end{array}\right\} \Rightarrow R_t = \frac{v_T}{i_T} = \frac{(400-R)\,100}{100+400}$$

<u>The circuit is stable when $R < 400\ \Omega$.</u>

P 8.5-2 The circuit in Figure P 8.5-2 contains a current-controlled current source. What restriction must be placed on the gain, *B*, of this dependent source in order to guarantee stability?

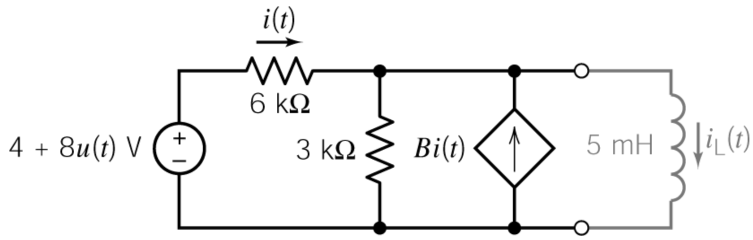
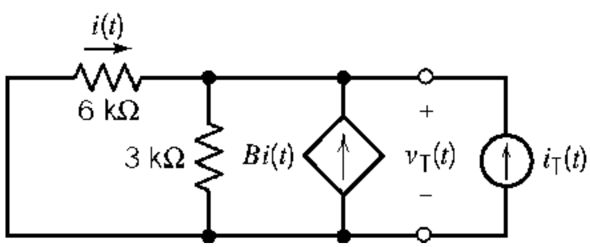


Figure P 8.5-2

Solution:

The Thèvenin equivalent resistance of the circuit connected to the inductor is calculated as



Ohm's law: $i(t) = -\dfrac{v_T(t)}{6000}$

KCL: $i(t) + B\,i(t) + i_T(t) = \dfrac{v_T(t)}{3000}$

$$\therefore i_T(t) = -(B+1)\left(-\frac{v_T(t)}{6000}\right) + \frac{v_T(t)}{3000}$$

$$= \frac{(B+3)v_T(t)}{6000}$$

$$R_t = \frac{v_T(t)}{i_T(t)} = \frac{6000}{B+3}$$

<u>The circuit is stable when $B > -3$ A/A.</u>

Section 8.6 The Unit Step Source

P 8.6-1 The input to the circuit shown in Figure P 8.6-1 is the voltage of the voltage source, $v_s(t)$. The output is the voltage across the capacitor, $v_o(t)$. Determine the output of this circuit when the input is $v_s(t) = 8 - 15\, u(t)$ V.

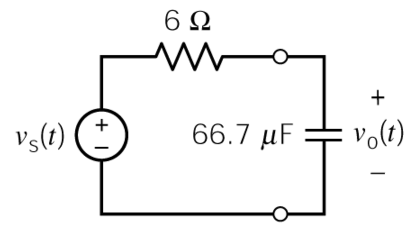


Figure P 8.6-1

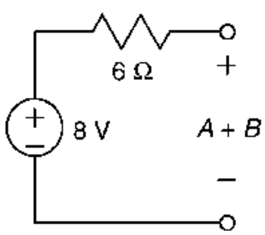
Solution:

The value of the input is one constant, 8 V, before time $t = 0$ and a different constant, –7 V, after time $t = 0$. The response of the first order circuit to the change in the value of the input will be

$$v_o(t) = A + B\, e^{-a\,t} \quad \text{for } t > 0$$

where the values of the three constants A, B and a are to be determined.

The values of A and B are determined from the steady state responses of this circuit before and after the input changes value.



The steady-state circuit for t < 0.

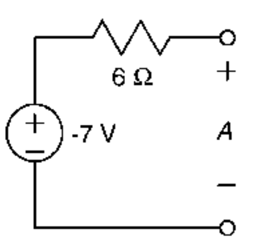
Capacitors act like open circuits when the input is constant and the circuit is at steady state. Consequently, the capacitor is replaced by an open circuit.

The value of the capacitor voltage at time t = 0, will be equal to the steady state capacitor voltage before the input changes. At time $t = 0$ the output voltage is

$$v_o(0) = A + B\, e^{-a(0)} = A + B$$

Consequently, the capacitor voltage is labeled as $A + B$. Analysis of the circuit gives

$$A + B = 8 \text{ V}$$



The steady-state circuit for t > 0.

Capacitors act like open circuits when the input is constant and the circuit is at steady state. Consequently, the capacitor is replaced by an open circuit.

The value of the capacitor voltage at time t = ∞, will be equal to the steady state capacitor voltage after the input changes. At time $t = \infty$ the output voltage is

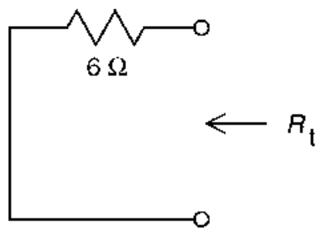
$$v_o(\infty) = A + B\, e^{-a(\infty)} = A$$

Consequently, the capacitor voltage is labeled as A.
Analysis of the circuit gives

Therefore $\quad A = \text{-7 V}$ and $B = 15$ V

The value of the constant a is determined from the time constant, τ, which is in turn calculated from the values of the capacitance C and of the Thevenin resistance, R_t, of the circuit connected to the capacitor.

$$\frac{1}{a} = \tau = R_t\, C$$



Here is the circuit used to calculate R_t.

$$R_t = 6\ \Omega$$

Therefore

$$a = \frac{1}{(6)\left(66.7\times 10^{-3}\right)} = 2.5\ \frac{1}{\text{s}}$$

(The time constant is $\tau = (6)\left(66.7\times 10^{-3}\right) = 0.4$ s .)

Putting it all together:

$$v_o(t) = \begin{cases} 8\ \text{V} & \text{for } t \le 0 \\ -7 + 15\, e^{-2.5\,t}\ \text{V} & \text{for } t \ge 0 \end{cases}$$

P 8.6-2 The input to the circuit shown in Figure P 8.6-2 is the voltage of the voltage source, $v_s(t)$. The output is the voltage across the capacitor, $v_o(t)$. Determine the output of this circuit when the input is $v_s(t) = 3 + 3\,u(t)$ V.

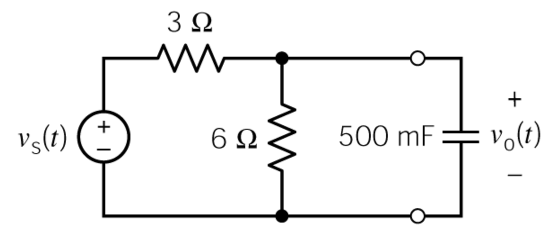


Figure P 8.6-2

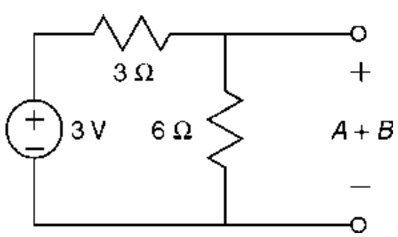
Solution:

The value of the input is one constant, 3 V, before time $t = 0$ and a different constant, 6 V, after time $t = 0$. The response of the first order circuit to the change in the value of the input will be

$$v_o(t) = A + B\,e^{-a\,t} \quad \text{for } t > 0$$

where the values of the three constants A, B and a are to be determined.

The values of A and B are determined from the steady state responses of this circuit before and after the input changes value.



The steady-state circuit for $t < 0$.

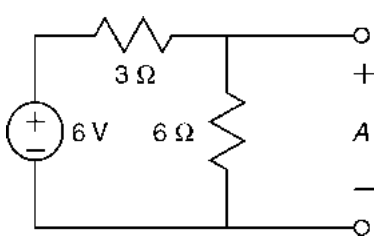
Capacitors act like open circuits when the input is constant and the circuit is at steady state. Consequently, the capacitor is replaced by an open circuit.

The value of the capacitor voltage at time t = 0, will be equal to the steady state capacitor voltage before the input changes. At time $t = 0$ the output voltage is

$$v_o(0) = A + B\,e^{-a(0)} = A + B$$

Consequently, the capacitor voltage is labeled as $A + B$. Analysis of the circuit gives

$$A + B = \frac{6}{3+6}(3) = 2 \text{ V}$$



The steady-state circuit for $t > 0$.

Capacitors act like open circuits when the input is constant and the circuit is at steady state. Consequently, the capacitor is replaced by an open circuit.

The value of the capacitor voltage at time t = ∞, will be equal to the steady state capacitor voltage after the input changes. At time $t = \infty$ the output voltage is

$$v_o(\infty) = A + B\,e^{-a(\infty)} = A$$

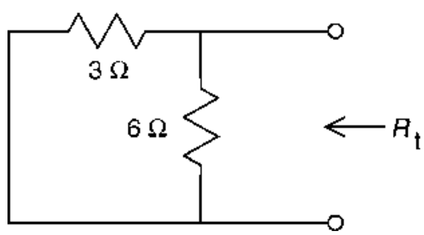
Consequently, the capacitor voltage is labeled as A. Analysis of the circuit gives

$$A = \frac{6}{3+6}(6) = 4 \text{ V}$$

Therefore $B = -2$ V

The value of the constant a is determined from the time constant, τ, which is in turn calculated from the values of the capacitance C and of the Thevenin resistance, R_t, of the circuit connected to the capacitor.

$$\frac{1}{a} = \tau = R_t\, C$$



Here is the circuit used to calculate R_t.

$$R_t = \frac{(3)(6)}{3+6} = 2\ \Omega$$

Therefore

$$a = \frac{1}{(2)(.5)} = 1\ \frac{1}{\text{s}}$$

(The time constant is $\tau = (2)(0.5) = 1$ s .)

Putting it all together:

$$v_o(t) = \begin{cases} 2\ \text{V} & \text{for } t \le 0 \\ 4 - 2\,e^{-t}\ \text{V} & \text{for } t \ge 0 \end{cases}$$

P 8.6-3 The input to the circuit shown in Figure P 8.6-3 is the voltage of the voltage source, $v_s(t)$. The output is the current across the inductor, $i_o(t)$. Determine the output of this circuit when the input is $v_s(t) = -7 + 13\,u(t)$ V.

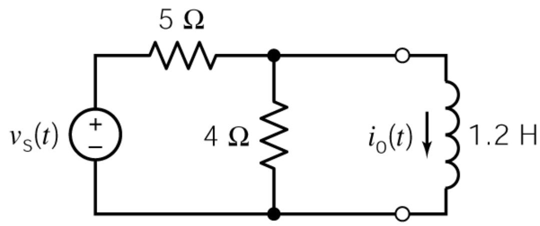


Figure P 8.6-3

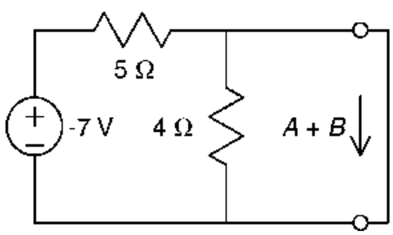
Solution:

The value of the input is one constant, −7 V, before time $t = 0$ and a different constant, 6 V, after time $t = 0$. The response of the first order circuit to the change in the value of the input will be

$$v_o(t) = A + B\,e^{-a\,t} \quad \text{for } t > 0$$

where the values of the three constants A, B and a are to be determined.

The values of A and B are determined from the steady state responses of this circuit before and after the input changes value.



The steady-state circuit for $t < 0$.

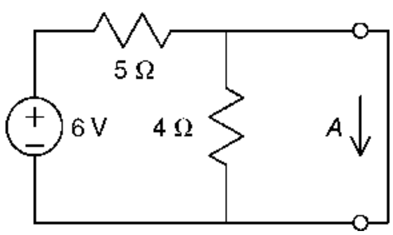
Inductors act like short circuits when the input is constant and the circuit is at steady state. Consequently, the inductor is replaced by a short circuit.

The value of the inductor current at time $t = 0$, will be equal to the steady state inductor current before the input changes. At time $t = 0$ the output current is

$$i_o(0) = A + B\,e^{-a(0)} = A + B$$

Consequently, the inductor current is labeled as $A + B$. Analysis of the circuit gives

$$A + B = \frac{-7}{5} = -1.4 \text{ A}$$



The steady-state circuit for $t > 0$.

Inductors act like short circuits when the input is constant and the circuit is at steady state. Consequently, the inductor is replaced by a short circuit.

The value of the inductor current at time $t = \infty$, will be equal to the steady state inductor current after the input changes. At time $t = \infty$ the output current is

$$i_o(\infty) = A + B\,e^{-a(\infty)} = A$$

Consequently, the inductor current is labeled as A. Analysis of the circuit gives

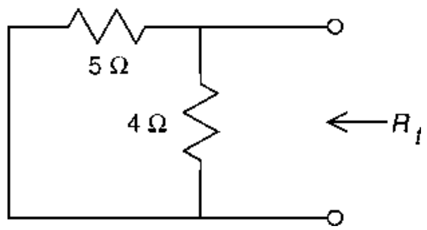
$$A = \frac{6}{5} = 1.2 \text{ A}$$

Therefore

$$B = -2.6 \text{ V}$$

The value of the constant a is determined from the time constant, τ, which is in turn calculated from the values of the inductance L and of the Thevenin resistance, R_t, of the circuit connected to the inductor.

$$\frac{1}{a} = \tau = \frac{L}{R_t}$$



Here is the circuit used to calculate R_t.

$$R_t = \frac{(5)(4)}{5+4} = 2.22 \ \Omega$$

Therefore $$a = \frac{2.22}{1.2} = 1.85 \ \frac{1}{\text{s}}$$

(The time constant is $\tau = \dfrac{1.2}{2.22} = 0.54$ s.)

Putting it all together:

$$i_o(t) = \begin{cases} -1.4 \text{ A} & \text{for } t \le 0 \\ 1.2 - 2.6\, e^{-1.85t} \text{ A} & \text{for } t \ge 0 \end{cases}$$

P8.6-4 Determine $v_o(t)$ for $t > 0$ for the circuit shown in Figure P8.6-4.

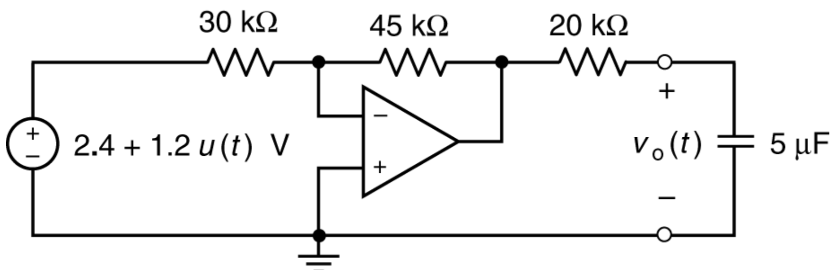
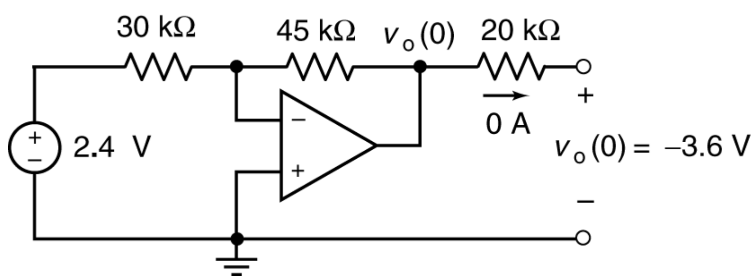


Figure P8.6-4

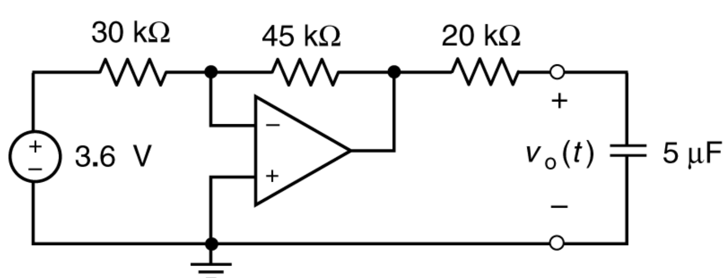
Solution: Determine the initial condition, $v_o(o)$, by considering the circuit when $t < 0$ and the circuit is at steady state. Since a capacitor in a dc circuit acts like a open circuit , we have



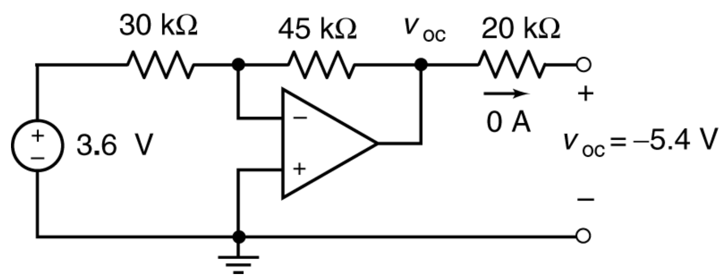
Recognizing the inverting amplifier, we have

$$v_o(0) = \left(-\frac{45}{30}\right)2.4 = -3.6 \text{ V}$$

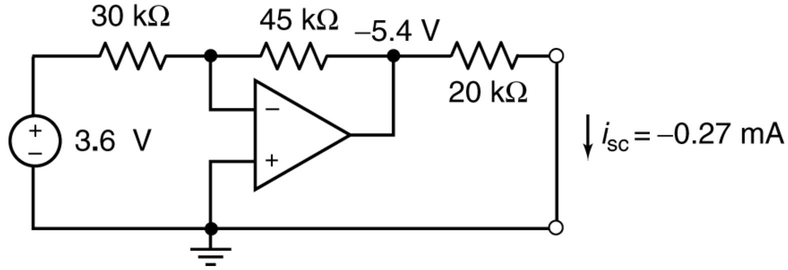
Next, consider the circuit when $t > 0$ and the circuit is not at steady state:



To find the Thevenin equivalent of the part of the circuit connected to the capacitor we determine both the open circuit voltage and the short circuit current:



and



Now we calculate the Thevenin resistance:

$$R_t = \frac{v_{oc}}{i_{sc}} = \frac{-5.4}{-0.27 \times 10^{-3}} = 20 \text{ k}\Omega$$

and the time constant: $\tau = R_t\, C = \left(20 \times 10^3\right)\left(5 \times 10^{-6}\right) = 0.1$ second

The capacitor voltage is given by

$$v_o(t) = \left(v_o(0) - v_{oc}\right)e^{-t/\tau} + v_{oc} = \left(-3.6 - (-5.4)\right)e^{-10t} - 5.4 = 1.8\,e^{-10t} - 5.4 \text{ V} \quad \text{for } t \ge 0$$

P 8.6-5 The initial voltage of the capacitor of the circuit shown in Figure P 8.6-5 is zero. Determine the voltage $v(t)$ when the source is a pulse, described by

$$v_s = \begin{cases} 0 & t < 1 \text{ s} \\ 4\text{ V} & 1 < t < 2 \text{ s} \\ 0 & t > 2 \text{ s} \end{cases}$$

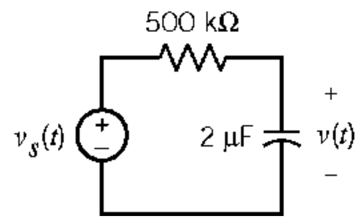


Figure P 8.6-5

Solution

$$\tau = R\,C = \left(5\times10^{5}\right)\left(2\times10^{-6}\right) = 1 \text{ s}$$

Assume that the circuit is at steady state at t = 1⁻. Then

$$v(t) = 4 - 4\,e^{-(t-1)} \text{ V} \quad \text{for } 1 \le t \le 2$$

so

$$v(2) = 4 - 4\,e^{-(2-1)} = 2.53 \text{ V}$$

and

$$v(t) = 2.53\,e^{-(t-2)} \text{ V} \quad \text{for } t \ge 2$$

Finally

$$v(t) = \begin{cases} 0 & t \le 1 \\ 4 - 4e^{-(t-1)} & 1 \le t \le 2 \\ 2.53e^{-(t-2)} & t \ge 2 \end{cases}$$

P 8.6-6 Studies of an artificial insect are being used to understand the nervous system of animals. A model neuron in the nervous system of the artificial insect is shown in Figure P 8.6-6. A series of pulses, called synapses, is required. The switch generates a pulse by opening at $t = 0$ and closing at $t = 0.5$ s. Assume that the circuit is in steady state and that $v(0^-) = 10$ V. Determine the voltage $v(t)$ for $0 < t < 2$ s.

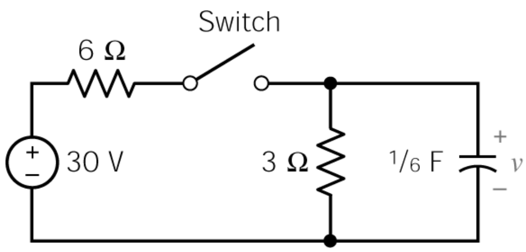
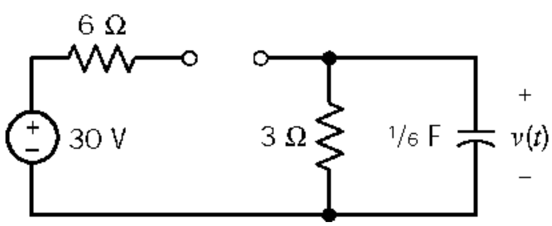


Figure P 8.6-6

Solution:

The capacitor voltage is $v(0^-) = 10$ V immediately before the switch opens at $t = 0$.

For 0 < t < 0.5 s the switch is open:

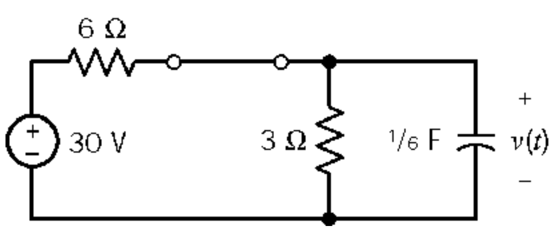


$$v(0) = 10 \text{ V}, \; v(\infty) = 0 \text{ V}, \; \tau = 3 \times \frac{1}{6} = \frac{1}{2} \text{ s}$$

so $v(t) = 10\, e^{-2t}$ V

In particular, $v(0.5) = 10\, e^{-2(0.5)} = 3.679$ V

For t > 0.5 s the switch is closed:



$$v(0) = 3.679 \text{ V}, \; v(\infty) = 10 \text{ V}, R_t = 6 \,||\, 3 = 2 \;\Omega,$$

$$\tau = 2 \times \frac{1}{6} = \frac{1}{3} \text{ s}$$

so

$$v(t) = 10 + (3.679 - 10) e^{-3(t-0.5)} \text{ V}$$
$$= 10 - 6.321\, e^{-3(t-0.5)} \text{ V}$$

P8.6-7 Determine the voltage $v_o(t)$ in the circuit shown in Figure P8.6-7.

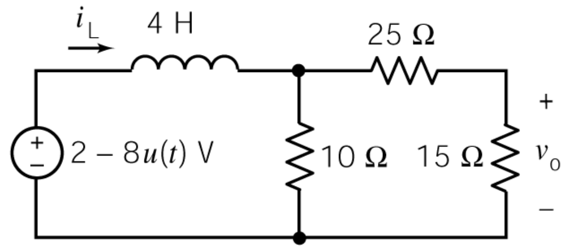


Figure P8.6-7

Solution: This is a first order circuit containing an inductor. First, determine $i_L(t)$.

Consider the circuit for time *t* < 0.

Step 1: Determine the initial inductor current.

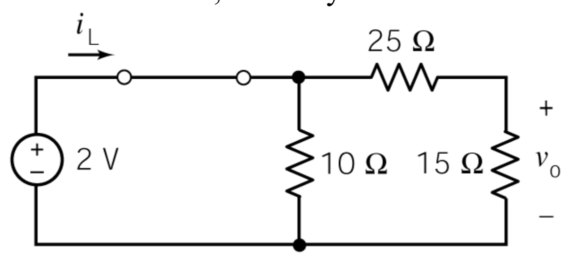
The circuit will be at steady state before the source voltage changes abruptly at time $t = 0$.

The source voltage will be 2 V, a constant.

The inductor will act like a short circuit.

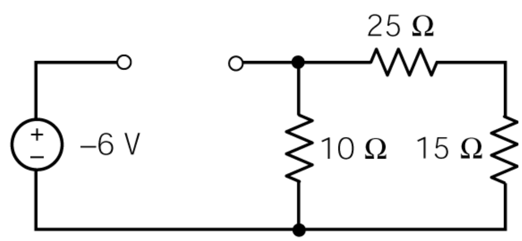
$$i_L(0) = \frac{2}{10 \,||\, (25+15)} = \frac{2}{8} = 0.25 \text{ A}$$

$t < 0$, at steady state:



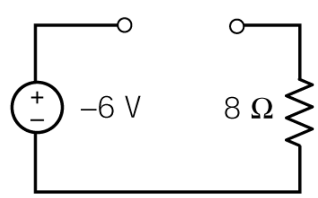
Consider the circuit for time *t* > 0.

Step 2. The circuit will not be at steady state immediately after the source voltage changes abruptly at time $t = 0$. Determine the Norton equivalent circuit for the part of the circuit connected to the inductor.



Replacing the resistors by an equivalent resistor, we recognize

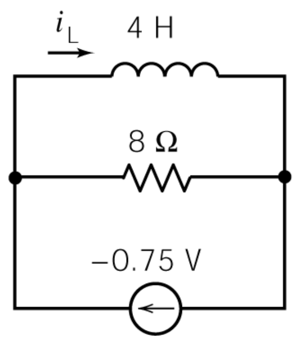
$$v_{oc} = -6 \text{ V} \quad \text{and} \quad R_t = 8\ \Omega$$



Consequently

$$i_{sc} = \frac{-6}{8} = -0.75 \text{ A}$$

Step 3. The time constant of a first order circuit containing an inductor is given by



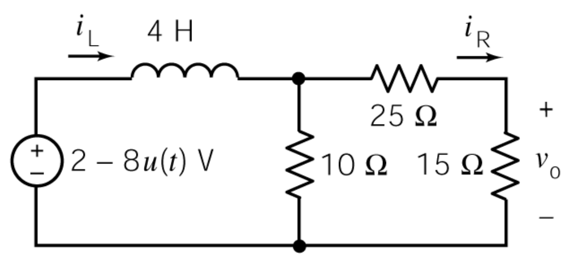
$$\tau = \frac{L}{R_t}$$

Consequently $\qquad \tau = \frac{L}{R_t} = \frac{4}{8} = 0.5 \text{ s} \;\text{ and }\; a = \frac{1}{\tau} = 2 \;\frac{1}{\text{s}}$

Step 4. The inductor current is given by:

$$i_L(t) = i_{sc} + \left(i(0) - i_{sc}\right)e^{-at} = -0.75 + \left(0.25 - (-0.75)\right)e^{-2t} = -0.75 + e^{-2t} \quad \text{for } t \geq 0$$

Step 5. Express the output voltage as a function of the source voltage and the inductor current.



Using current division:

$$i_R = \frac{10}{10 + (25 + 15)} i_L = 0.2\, i_L$$

Then Ohm's law gives

$$v_o = 15\, i_R = 3\, i_L$$

Step 6. The output voltage is given by

$$v_o(t) = -2.25 + 3e^{-2t} \quad \text{for } t \geq 0$$

P 8.6-8 Determine $v_c(t)$ for $t > 0$ for the circuit of Figure P 8.6-8.

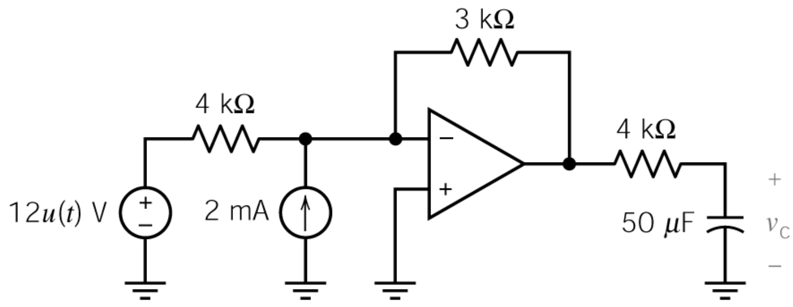
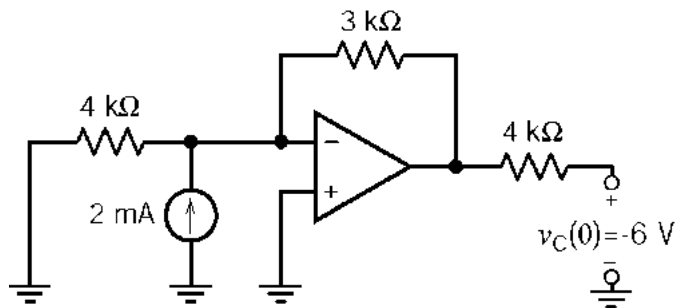
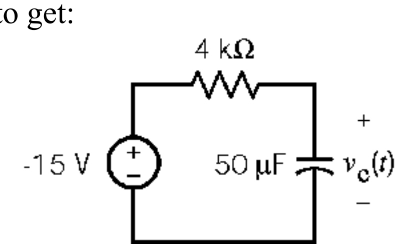


Figure P 8.6-8

Solution:
For t < 0, the circuit is:



After t = 0, replace the part of the circuit connected to the capacitor by its Thevenin equivalent circuit to get:



$$v_c(t) = -15 + \left(-6-(-15)\right) e^{-t/(4000\times 0.00005)}$$
$$= -15 + 9\, e^{-5t} \text{ V}$$

P 8.6-9 The voltage source voltage in the circuit shown in Figure P 8.6-9 is

$$v_s(t) = 7 - 14u \quad (t) \text{ V}$$

Determine $v(t)$ for $t > 0$.

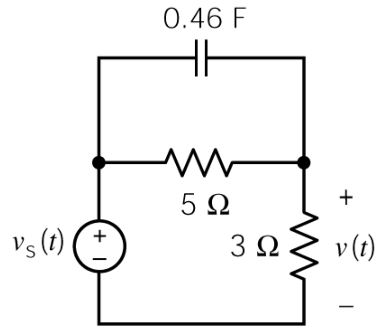
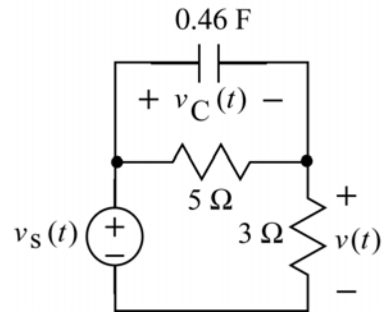


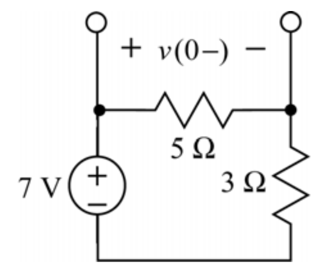
Figure P 8.6-9

Solution:

The input changes abruptly at time $t = 0$. The voltage $v(t)$ may not be continuous at $t = 0$, but the capacitor voltage, $v_C(t)$ will be continuous. We will find $v_C(t)$ first and then use KVL to find $v(t)$.

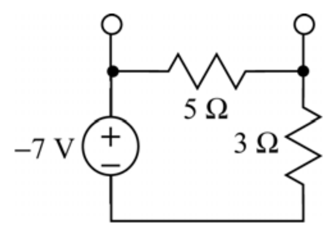


The circuit will be at steady state before $t = 0$ so the capacitor will act like an open circuit.



$$v(0+) = v(0-) = \frac{5}{5+3}7 = 4.375 \text{ V}$$

After $t = 0$, we replace the part of the circuit connected to the capacitor by its Thevenin equivalent circuit.



$$v_{oc} = \frac{5}{8}(7-14) = \frac{5}{8}(-7) = -4.375 \text{ V}$$

$$R_t = 5 \,||\, 3 = 1.875 \text{ } \Omega$$

The time constant is $$\tau = R_t C = 0.8625 \text{ } s$$

$$\frac{1}{\tau} = 1.16 \ \frac{1}{s}$$

So $$v_C(t) = \left[4.375 - (-4.375)\right]e^{-1.16t} + (-4.375) = -4.375 + 8.75e^{-1.16t} \text{ V} \quad \text{for } t \geq 0$$

Using KVL

$$v(t) = v_s(t) - v_C(t) = -7 - \left[-4.375 + 8.75e^{-1.16t}\right] = -2.625 - 8.75e^{-1.16t} \text{ V} \quad \text{for } t > 0$$

P 8.6-10 Determine the voltage $v(t)$ for $t \geq 0$ for the circuit shown in Figure P 8.6-10.

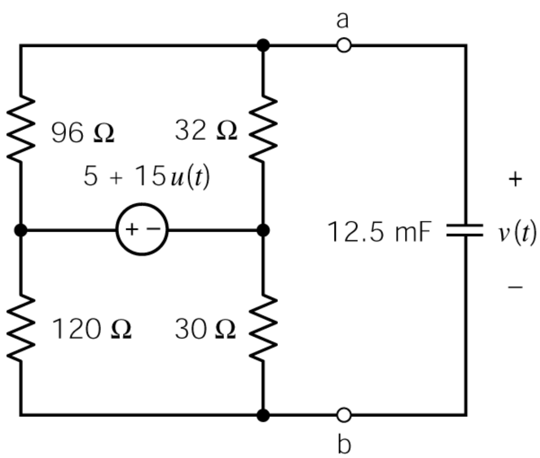
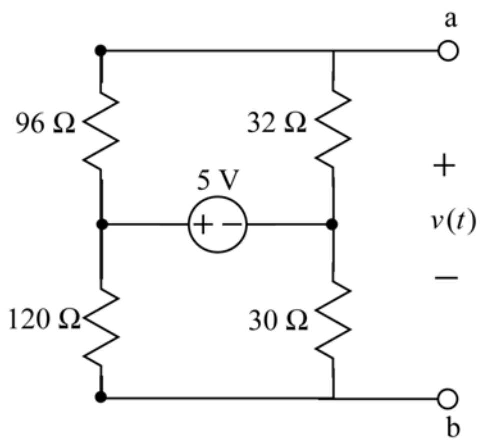


Figure P 8.6-10

Solution:
For $t < 0$



Using voltage division twice

$$v(t) = \frac{32}{32+96}5 - \frac{30}{120+30}5 = 0.25 \text{ V}$$

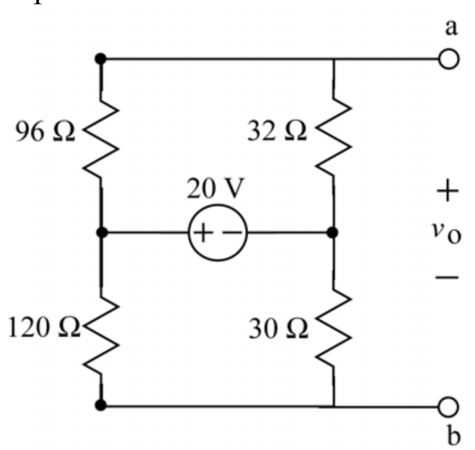
so

$$v(0-) = 0.25 \text{ V}$$

and

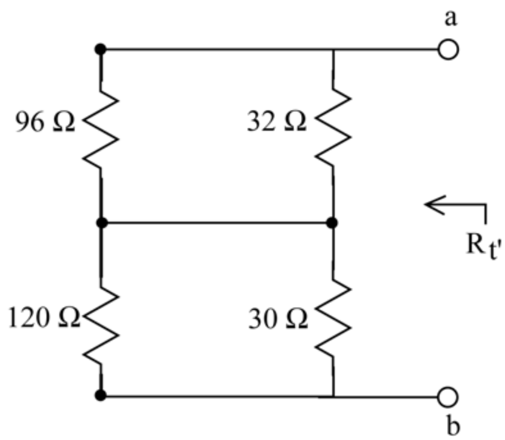
$$v(0+) = v(0-) = 0.25 \text{ V}$$

For $t > 0$, find the Thevenin equivalent circuit for the part of the circuit connected to the capacitor.



Using voltage division twice

$$v_{oc} = \frac{32}{32+96}20 - \frac{30}{120+30}20 = 5 - 4 = 1 \text{ V}$$



$$R_t = (96 \parallel 32) + (120 \parallel 30) = 24 + 24 = 48\ \Omega$$

then

$$\tau = 48 \times 0.0125 = 0.6\ \text{s}$$

so

$$\frac{1}{\tau} = 1.67\ \frac{1}{\text{s}}$$

Now

$$v(t) = [0.25 - 1]e^{-1.67t} + 1 = 1 - 0.75e^{-1.67t}\ \ \text{V} \quad \text{for } t \geq 0$$

(checked: LNAP 7/1/04)

P 8.6-11 The voltage source voltage in the circuit shown in Figure P 8.6-11 is

$$v_s(t) = 5 + 20u\,(t)$$

Determine $i(t)$ for $t \geq 0$.

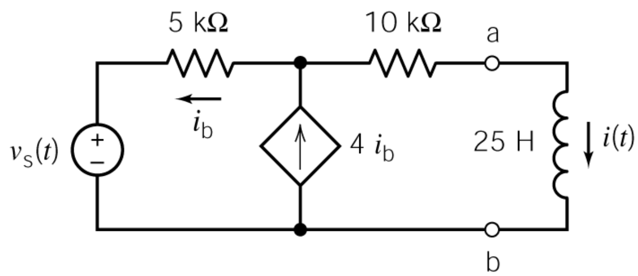
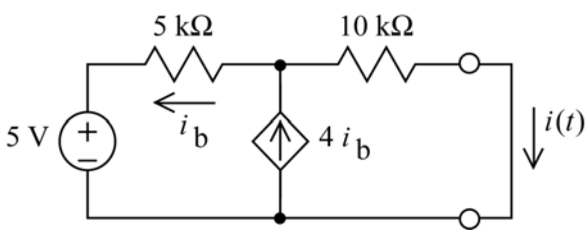


Figure P 8.6-11

Solution:

For $t > 0$ the circuit is at steady state so the inductor acts like a short circuit:



Apply KVL to the supermesh corresponding to the dependent source to get

$$-5000 i_b + 1000\left(3 i_b\right) - 5 = 0 \quad \Rightarrow \quad i_b = 0.2 \text{ mA}$$

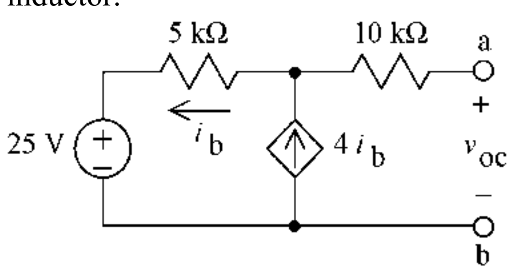
Apply KVL to get

$$i\left(t\right) = 3 i_b = 0.6 \text{ mA}$$

so $$i\left(0-\right) = 0.6 \text{ mA}$$

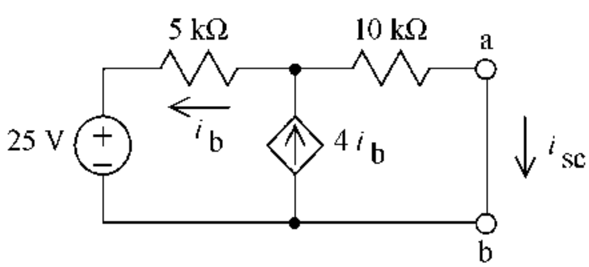
and $$i\left(0+\right) = i\left(0-\right) = 0.6 \text{ mA}$$

For t > 0, find the Norton equivalent circuit for the part of the circuit that is connected to the inductor.



Apply KCL at the top node of the dependent source to see that $i_b = 0$ A . Then

$$v_{oc} = 25 - 5000\left(i_b\right) = 25 \text{ V}$$



Apply KVL to the supermesh corresponding to the dependent source to get

$$-5000\, i_b + 10000\left(3\, i_b\right) - 25 = 0 \quad \Rightarrow \quad i_b = 1 \text{ mA}$$

Apply KCL to get

$$i_{sc} = 3\, i_b = 3 \text{ mA}$$

Then $$R_t = \frac{v_{oc}}{i_{sc}} = 8.3\bar{3} \text{ k}\Omega$$

Then $$\tau = \frac{25}{8333} = 3 \text{ ms}$$

So $$\frac{1}{\tau} = 333 \ \frac{1}{\text{s}}$$

Now
$$i(t) = \left[0.6 - 3\right] e^{-333t} + 3 = 3 - 2.4e^{-333t} \ \text{ mA} \quad \text{for } \ t \ge 0$$

(checked: LNAP 7/2/04)

P 8.6-12 The voltage source voltage in the circuit shown in Figure P 8.6-12 is

$$v_s(t) = 12 - 6u(t) \text{ V}$$

Determine $v(t)$ for $t \ge 0$.

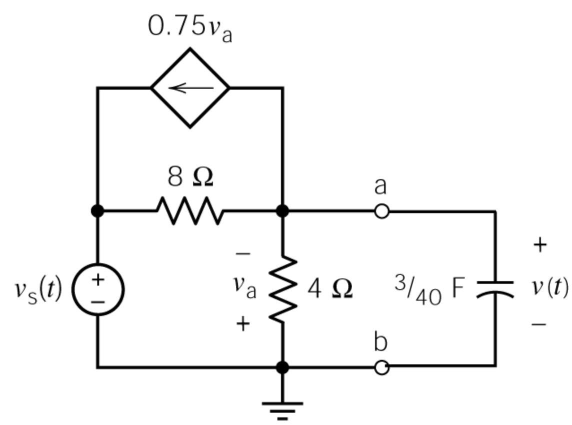
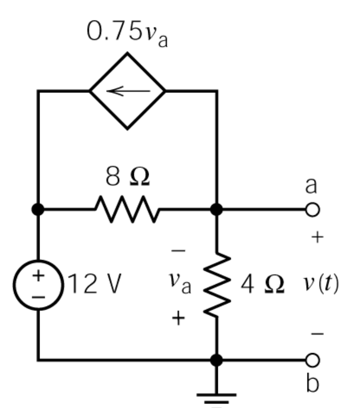


Figure P 8.6-12

Solution:
For $t > 0$, the circuit is at steady state so the capacitor acts like an open circuit. We have the following situation.

Notice that $v(t)$ is the node voltage at node a. Express the controlling voltage of the dependent source as a function of the node voltage:

$$v_a = -v(t)$$



Apply KCL at node a:

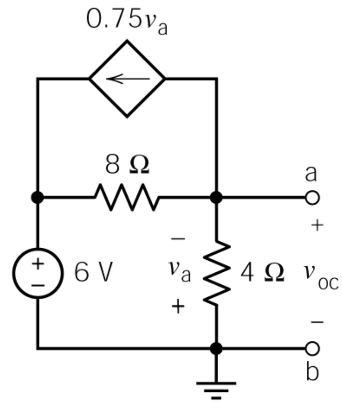
$$-\left(\frac{12 - v(t)}{8}\right) + \frac{v(t)}{4} + \left(-\frac{3}{4} v(t)\right) = 0$$

$$-12 + v(t) + 2\,v(t) - 6\,v(t) = 0 \quad \Rightarrow \quad v(t) = -4 \text{ V}$$

So $\quad v(0+) = v(0-) = -4$ V

For $t > 0$, we find the Thevenin equivalent circuit for the part of the circuit connected to the capacitor, i.e. the part of the circuit to the left of terminals $a - b$.

Notice that v_{oc} is the node voltage at node a. Express the controlling voltage of the dependent source as a function of the node voltage:



$$v_a = -v_{oc}$$

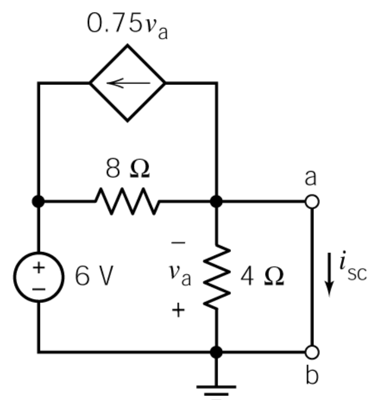
Apply KCL at node a:

$$-\left(\frac{6-v_{oc}}{8}\right)+\frac{v_{oc}}{4}+\left(-\frac{3}{4}v_{oc}\right)=0$$

$$-6+v_{oc}+2\,v_{oc}-6\,v_{oc}=0 \quad\Rightarrow\quad v_{oc}=-2 \text{ V}$$

Find R_t:

We'll find i_{sc} and use it to calculate R_t. Notice that the short circuit forces



$$v_a = 0$$

Apply KCL at node a:

$$-\left(\frac{6-0}{8}\right)+\frac{0}{4}+\left(-\frac{3}{4}0\right)+i_{sc}=0$$

$$i_{sc}=\frac{6}{8}=\frac{3}{4} \text{ A}$$

$$R_t=\frac{v_{oc}}{i_{sc}}=\frac{-2}{3/4}=-\frac{8}{3}\,\Omega$$

Then

$$\tau = R_t C=\left(-\frac{8}{3}\right)\left(\frac{3}{40}\right)=-\frac{1}{5} \text{ s} \quad\Rightarrow\quad \frac{1}{\tau}=-5\ \frac{1}{\text{s}}$$

and

$$v(t)=\left(v(0+)-v_{oc}\right)e^{-t/\tau}+v_{oc}=\left(-4-(-2)\right)e^{5t}+(-2)=-2\left(1+e^{5t}\right) \text{ V} \quad \text{for } t\ge 0$$

Notice that $v(t)$ grows exponentially as t increases.

(checked: LNAP 7/8/04)

P 8.6-13 Determine $i(t)$ for $t \geq 0$ for the circuit shown in Figure P 8.6-13.

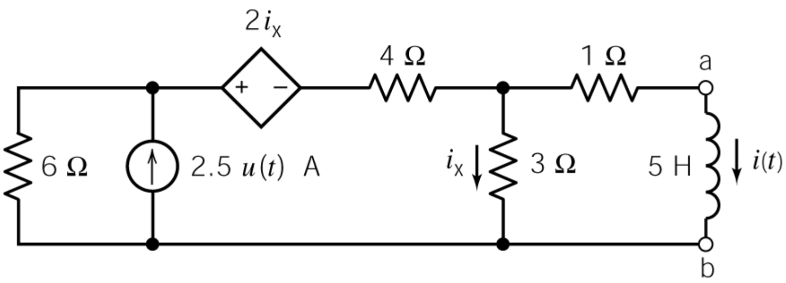
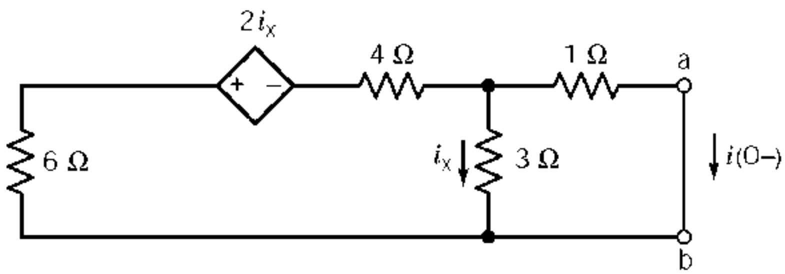


Figure P 8.6-13

Solution: When $t < 0$ and the circuit is at steady state, the inductor acts like a short circuit.



The mesh equations are

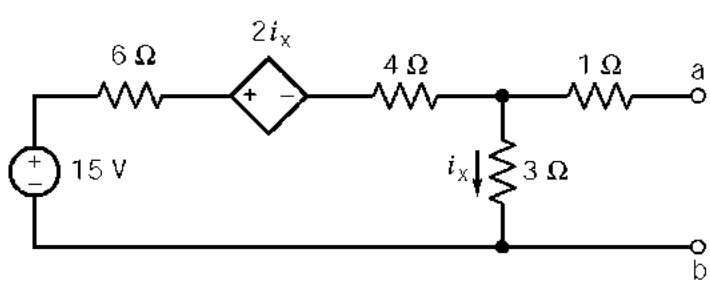
$$2i_x + 4\left(i_x + i(0-)\right) + 3i_x + 6\left(i_x + i(0-)\right) = 0$$

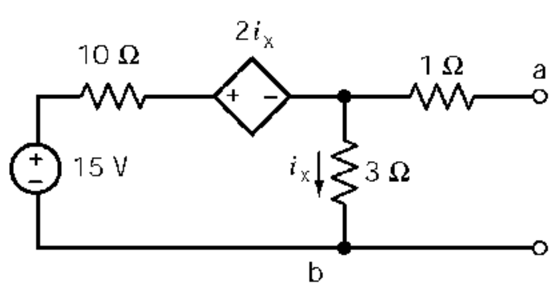
$$1i(0-) - 3i_x = 0$$

so

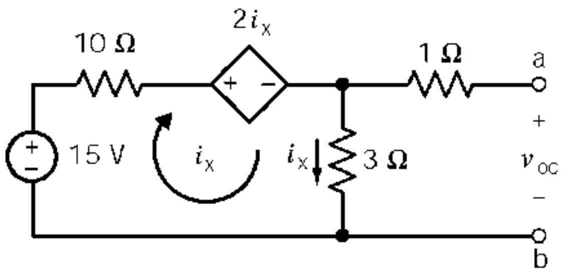
$$i(0+) = i(0-) = 0$$

For $t \geq 0$, we find the Norton equivalent circuit for the part of the circuit connected to the inductor. First, simplify the circuit using a source transformation:





Identify the open circuit voltage and short circuit current.

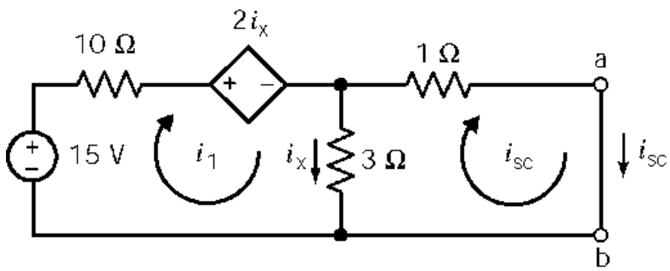


Apply KVL to the mesh to get:

$$(10+2+3)i_x - 15 = 0 \quad \Rightarrow \quad i_x = 1 \text{ A}$$

Then

$$v_{oc} = 3\,i_x = 3 \text{ V}$$



Express the controlling current of the CCVS in terms of the mesh currents:

$$i_x = i_1 - i_{sc}$$

The mesh equations are

$$10\,i_1 + 2\left(i_1 - i_{sc}\right) + 3\left(i_1 - i_{sc}\right) - 15 = 0 \quad \Rightarrow \quad 15\,i_1 - 5\,i_{sc} = 15$$

and

$$i_{sc} - 3\left(i_1 - i_{sc}\right) = 0 \quad \Rightarrow \quad i_1 = \frac{4}{3}\,i_{sc}$$

so

$$15\left(\frac{4}{3}i_{sc}\right) - 5\,i_{sc} = 15 \quad \Rightarrow \quad i_{sc} = 1 \text{ A}$$

The Thevenin resistance is

$$R_t = \frac{3}{1} = 3\ \Omega$$

The time constant is given by $\tau = \dfrac{L}{R_t} = \dfrac{5}{3} = 1.67$ s so $\dfrac{1}{\tau} = 0.6\ \dfrac{1}{\text{s}}$.

The inductor current is given by

$$i_L(t) = \left(i(0+) - i_{sc}\right)e^{-t/\tau} + i_{sc} = (0-1)e^{-0.6t} + 1 = 1 - e^{-0.6t} \text{ A} \quad \text{for } t \geq 0$$

P 8.6-14 Determine $i(t)$ for $t \geq 0$ for the circuit shown in Figure P 8.6-14.

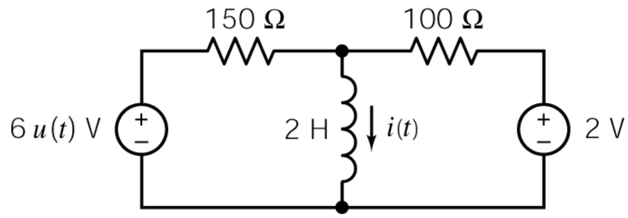
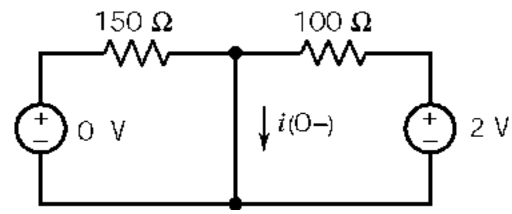


Figure P 8.6-14

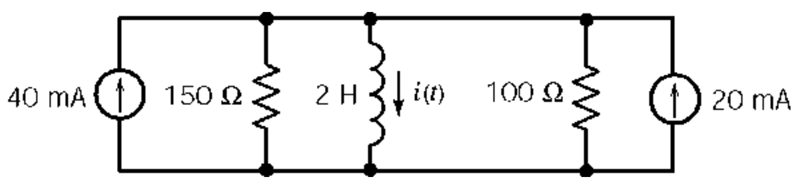
Solution:

When $t < 0$ and the circuit is at steady state, the inductor acts like a short circuit. The initial condition is

$$i(0+) = i(0-) = \frac{0}{150} + \frac{2}{100} = 0.02 \text{ A}$$



For $t \geq 0$, we find the Norton equivalent circuit for the part of the circuit connected to the inductor. First, simplify the circuit using source transformations:



$$i_{sc} = 20 + 40 = 60 \text{ mA}$$

$$R_t = 100 \| 150 = \frac{100 \times 150}{100 + 150} = 60 \ \Omega$$

The time constant is given by $\tau = \frac{L}{R_t} = \frac{2}{60} = 0.0333 \text{ s}$ so $\frac{1}{\tau} = 30 \ \frac{1}{\text{s}}$.

The inductor current is given by

$$i_L(t) = \left(i(0+) - i_{sc}\right) e^{-t/\tau} + i_{sc} = (20 - 60) e^{-30t} + 60 = 60 - 40\, e^{-30t} \text{ mA} \quad \text{for } t \geq 0$$

(checked: LNAP 7/15/04)

P 8.6-15 Determine $v(t)$ for $t \geq 0$ for the circuit shown in Figure P 8.6-15.

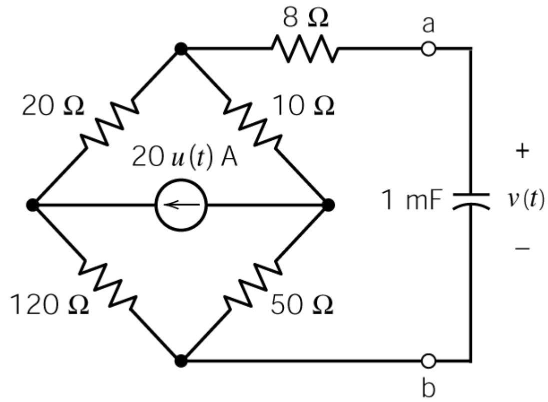
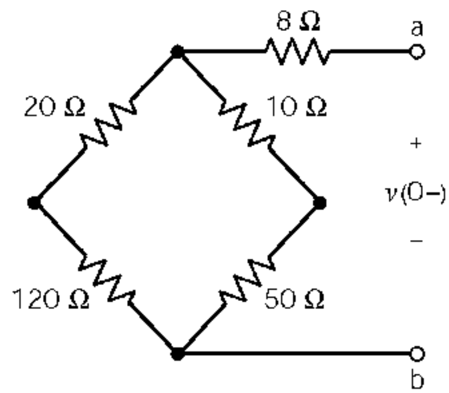


Figure P 8.6-15

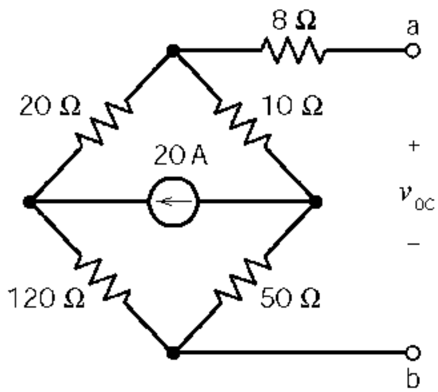
Solution:

When $t < 0$ and the circuit is at steady state, the capacitor acts like an open circuit. The 0 A current source also acts like an open circuit. The initial condition is

$$v(0+) = v(0-) = 0 \text{ V}$$



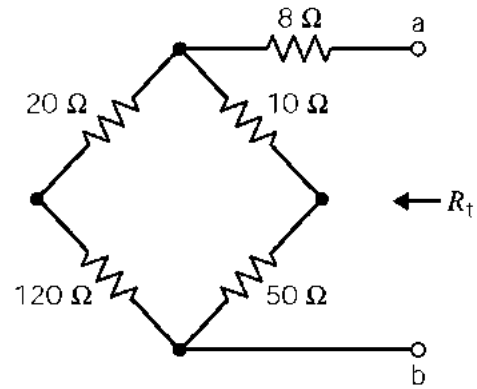
For $t \geq 0$, we find the Thevenin equivalent circuit for the part of the circuit connected to the capacitor.



$$v_{oc} = \left[\frac{170}{170+30}(20)\right]10 - \left[\frac{30}{170+30}(20)\right]50$$

$$= \frac{170(20)(10) - 30(20)(50)}{200} = \frac{4000}{200} = 20 \text{ V}$$

$$R_t = 8 + \frac{(20+120)(10+50)}{(20+120)+(10+50)} = 50\ \Omega$$



The time constant is $\tau = R_t C = (50)(10^{-3}) = 0.05\ \text{s}$ s so $\frac{1}{\tau} = 20\ \frac{1}{\text{s}}$.

The capacitor voltage is given by

$$v(t) = \left(v(0+) - v_{oc}\right)e^{-t/\tau} + v_{oc} = (0-20)e^{-20t} + 20 = 20\left(1 - e^{-20t}\right)\ \text{V} \quad \text{for } t \geq 0$$

P 8.6-16 Determine $v(t)$ for $t \geq 0$ for the circuit shown in Figure P 8.6-16.

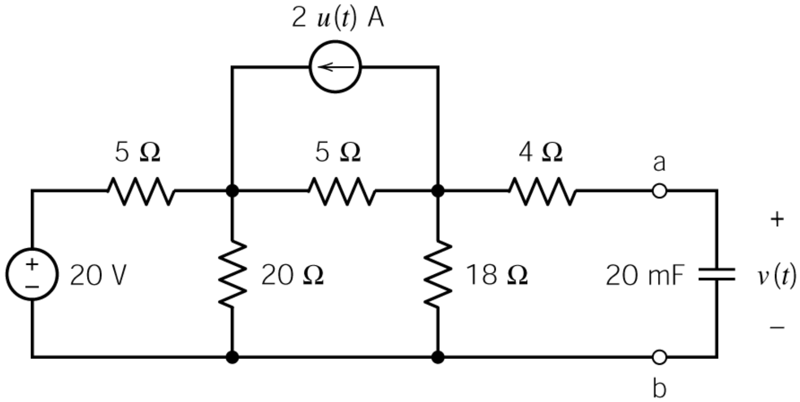
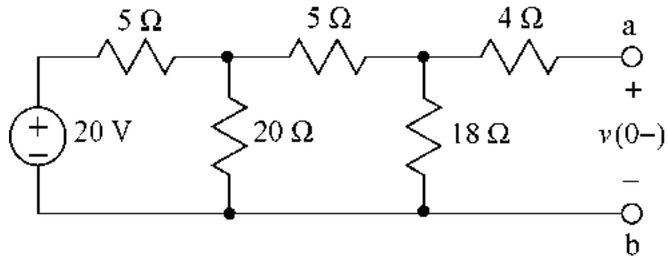


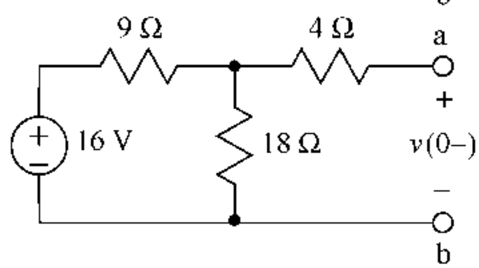
Figure P 8.6-16

Solution:

When $t < 0$ and the circuit is at steady state, the capacitor acts like an open circuit. The 0 A current source also acts like an open circuit.

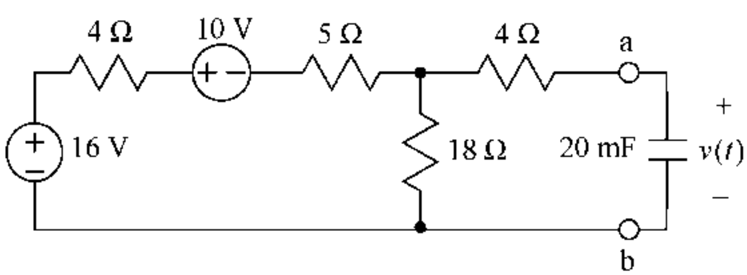


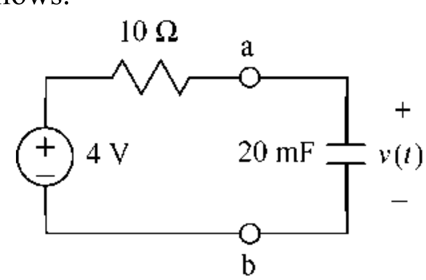
After a couple of source transformations, the initial condition is calculated as



$$v(0+) = v(0-) = \frac{18}{9+18} 16 = 10.667 \text{ V}$$

For $t \geq 0$, we find the Thevenin equivalent circuit for the part of the circuit connected to the capacitor. Using source transformations, reduce the circuit as follows.





Now recognize $R_t = 10\ \Omega$ and $v_{oc} = 4$ V.

The time constant is $\tau = R_t C = (10)(20\times10^{-3}) = 0.2$ s so $\frac{1}{\tau} = 5\ \frac{1}{\text{s}}$.

The capacitor voltage is given by

$$v(t) = \left(v(0+) - v_{oc}\right)e^{-t/\tau} + v_{oc} = (10.667 - 4)e^{-5t} + 4 = 4 + 6.667\, e^{-5t} \text{ V} \quad \text{for } t \geq 0$$

(checked: LNAP 7/15/04)

P 8.6-17 Determine $i(t)$ for $t \geq 0$ for the circuit shown in Figure P 8.6-17.

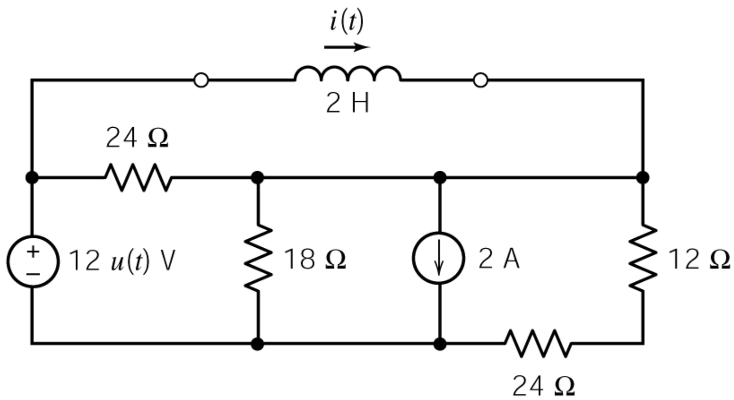
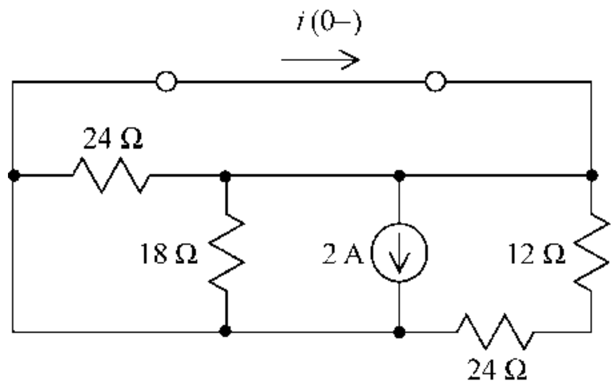


Figure P 8.6-17

Solution:

When $t < 0$ and the circuit is at steady state, the inductor acts like a short circuit. The 0 V voltage source also acts like a short circuit.



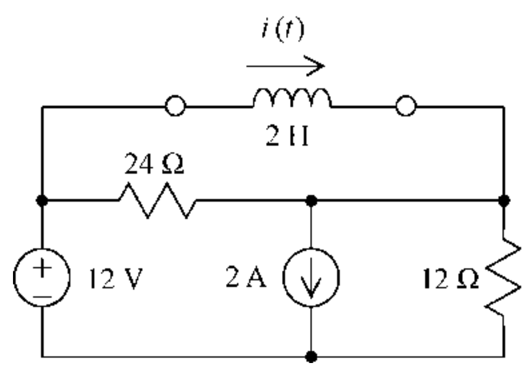
After a replacing series and parallel resistors by equivalent resistors, the equivalent resistors, current source and short circuit are all connected in parallel. Consequently

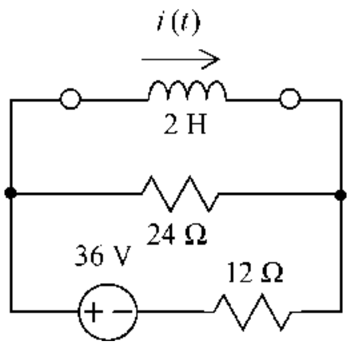
$$i(0+) = i(0-) = 2 \text{ A}$$

For $t \geq 0$, we find the Thevenin equivalent circuit for the part of the circuit connected to the capacitor.

Replace series and parallel resistors by an equivalent resistor.

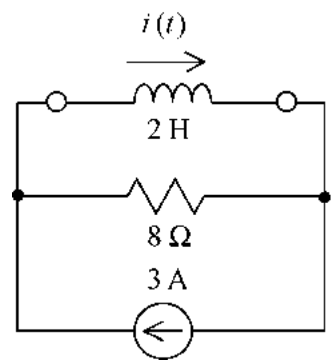
$$18 \,||\, (12+24) = 12 \; \Omega$$





Do a source transformation, then replace series voltage sources by an equivalent voltage source.

Do two more source transformations.



Now recognize $R_t = 8\ \Omega$ and $i_{sc} = 3$ A .

The time constant is given by

$$\tau = \frac{L}{R_t} = \frac{2}{8} = 0.25 \text{ s so } \frac{1}{\tau} = 4\ \frac{1}{\text{s}}.$$

The inductor current is given by

$$i_L(t) = \left(i(0+) - i_{sc}\right)e^{-t/\tau} + i_{sc} = (2-3)e^{-4t} + 3 = 3 - e^{-4t} \quad \text{A} \quad \text{for } t \geq 0$$

(checked: LNAP 7/15/04)

P 8.6-18 The voltage source voltage in the circuit shown in Figure P 8.6-18 is

$$v_s(t) = 8 + 12u(t) \text{ V}$$

Determine $v(t)$ for $t \geq 0$.

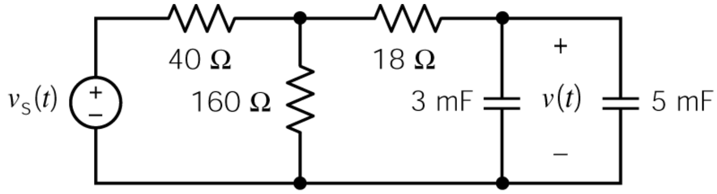
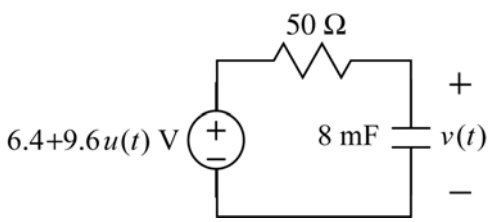


Figure P 8.6-18

Solution: Simplify the circuit by replacing the parallel capacitors by an equivalent capacitor and by doing a couple of source transformations:



For $t < 0$ the circuit is at steady state so the capacitor acts like an open circuit. The voltage source voltage is 6.4 V, so

$$v(0+) = v(0-) = 6.4 \text{ V}$$

For $t > 0$ we find the Thevenin equivalent circuit of the part of the circuit connected to the capacitor. In this case we recognize the v_{oc} = 16 V and R_t = 50 Ω.

The time constant is

$$\tau = R_t C = (50)(8\times10^{-3}) = 0.4 \text{ s} \quad \Rightarrow \quad \frac{1}{\tau} = 2.5 \ \frac{1}{\text{s}}$$

Then

$$v(t) = (v(0+) - v_{oc})e^{\frac{-t}{\tau}} + v_{oc} = (6.4-16)e^{-2.5t} + 16 = 16 - 9.6e^{-2.5t} \text{ V} \quad \text{for } t \geq 0$$

(checked: LNAP 7/12/04)

P8.6-19 Determine the current $i_o(t)$ in the circuit shown in Figure P8.6-19.

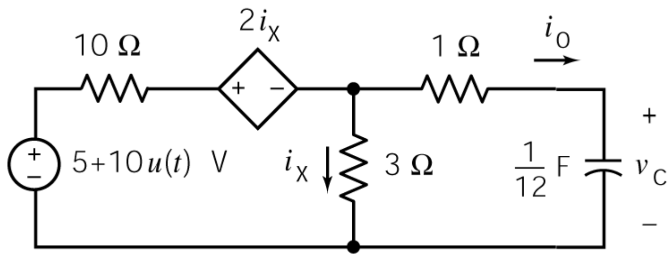


Figure P8.6-19

Solution: This is a first order circuit containing a capacitor. First, determine $v_C(t)$.

Consider the circuit for time *t* < 0.

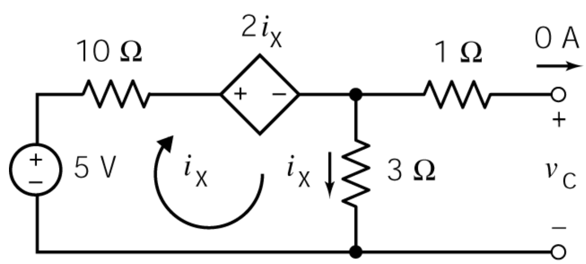
Step 1: Determine the initial capacitor voltage.

The circuit will be at steady state before the source voltage changes abruptly at time $t = 0$.

The source voltage will be 5 V, a constant.

The capacitor will act like an open circuit.

$t < 0$, at steady state:



Apply KVL to the mesh to get:

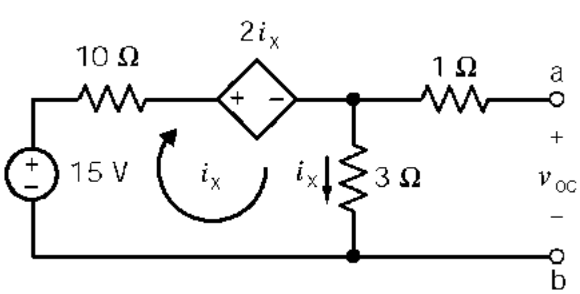
$$(10+2+3)i_x - 5 = 0 \quad \Rightarrow \quad i_x = \frac{1}{3} \text{ A}$$

Then

$$v_C(0) = 3\,i_x = 1 \text{ V}$$

Consider the circuit for time *t* > 0.

Step 2. The circuit will not be at steady state immediately after the source voltage changes abruptly at time $t = 0$. Determine the Thevenin equivalent circuit for the part of the circuit connected to the capacitor. First, determine the open circuit voltage, v_{oc}:



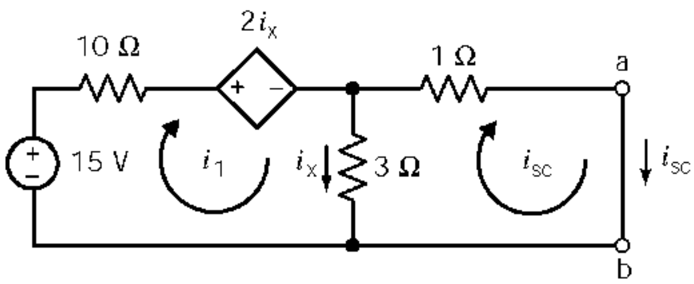
Apply KVL to the mesh to get:

$$(10+2+3)i_x - 15 = 0 \quad \Rightarrow \quad i_x = 1 \text{ A}$$

Then

$$v_{oc} = 3\,i_x = 3 \text{ V}$$

Next, determine the short circuit current, i_{sc}:



Express the controlling current of the CCVS in terms of the mesh currents:

$$i_x = i_1 - i_{sc}$$

The mesh equations are

$$10\, i_1 + 2\left(i_1 - i_{sc}\right) + 3\left(i_1 - i_{sc}\right) - 15 = 0 \quad \Rightarrow \quad 15\, i_1 - 5\, i_{sc} = 15$$

And $$i_{sc} - 3\left(i_1 - i_{sc}\right) = 0 \quad \Rightarrow \quad i_1 = \frac{4}{3}\, i_{sc}$$

so $$15\left(\frac{4}{3} i_{sc}\right) - 5\, i_{sc} = 15 \quad \Rightarrow \quad i_{sc} = 1 \text{ A}$$

The Thevenin resistance is

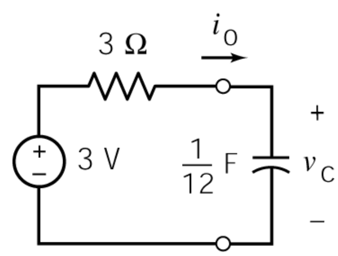
$$R_t = \frac{3}{1} = 3\ \Omega$$

Step 3. The time constant of a first order circuit containing an capacitor is given by

$$\tau = R_t\, C$$

Consequently

$$\tau = R_t\, C = 3\left(\frac{1}{12}\right) = 0.25 \text{ s and } a = \frac{1}{\tau} = 4\ \frac{1}{\text{s}}$$



Step 4. The capacitor voltage is given by:

$$v_C(t) = v_{oc} + \left(v_C(0) - v_{oc}\right)e^{-at} = 3 + (1-3)e^{-4t} = 3 - 2\,e^{-4t} \quad \text{for } t \ge 0$$

Step 5. Express the output current as a function of the source voltage and the capacitor voltage.

$$i_o(t) = C\frac{d}{dt}v_C(t) = \frac{1}{12}\frac{d}{dt}v_C(t)$$

Step 6. The output current is given by

$$i_o(t) = \frac{1}{12}\frac{d}{dt}\left(3 - 2\,e^{-4t}\right) = \frac{1}{12}(-2)(-4)e^{-4t} = \frac{2}{3}e^{-4t} \quad \text{for } t \ge 0$$

P 8.6-20 The voltage source voltage in the circuit shown in Figure P 8.6-20 is

$$v_s(t) = 25u(t) - 10 \text{ V}$$

Determine $i(t)$ for $t \geq 0$.

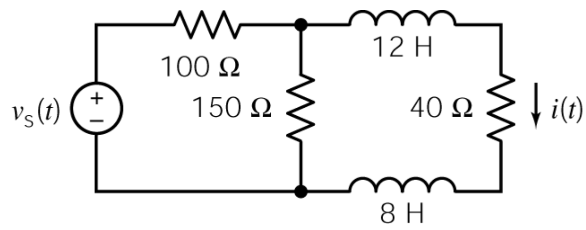
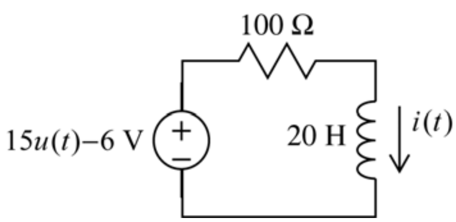


Figure P 8.6-20

Solution: Simplify the circuit by replacing the series inductors by an equivalent inductor. Then, after a couple of source transformations, we have



For $t < 0$ the circuit is at steady state and so the inductor acts like a short circuit. The voltage source voltage is −6 V so

$$i(0+) = i(0-) = -60 \text{ mA}$$

For $t > 0$ we find the Norton equivalent circuit for the part of the circuit connected to the inductor. In this case we recognize v_{oc} = 9V and R_t = 100 Ω so i_{sc} = 90 mA.

The time constant is $$\tau = \frac{L}{R_t} = \frac{20}{100} = 0.2 \text{ s} \quad \Rightarrow \quad \frac{1}{\tau} = 5 \ \frac{1}{\text{s}}$$

Then

$$i(t) = \left(i(0+) - i_{sc}\right)e^{-t/\tau} + i_{sc} = (-60 - 90)e^{-5t} + 90 = 90 - 150e^{-5t} \text{ mA} \quad \text{for } t \geq 0$$

(checked: LNAP 7/12/04)

P 8.6-21 The voltage source voltage in the circuit shown in Figure P 8.6-21 is

$$v_s(t) = 30 - 24u(t) \text{ V}$$

Determine $i(t)$ for $t \geq 0$.

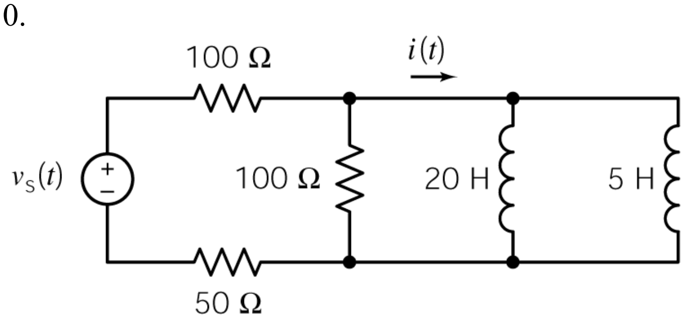
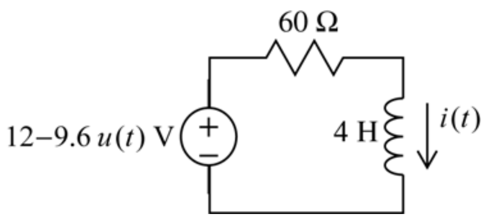


Figure P 8.6-21

Solution: Simplify the circuit by replacing the parallel inductors by an equivalent inductor. Then, after doing a couple of source transformations, we have



For $t < 0$ the circuit is at steady state and the inductor acts like a short circuit. The voltage source voltage is 12 V so

$$i(0+) = i(0-) = 0.2 \text{ A}$$

For $t > 0$ we find the Norton equivalent circuit for the part of the circuit connected to the inductor. In this case, we recognize $v_{oc} = 2.4$ V and $R_t = 60\ \Omega$ so $i_{sc} = 0.04$ A.

The time constant is

$$\tau = \frac{L}{R_t} = \frac{4}{60} = \frac{1}{15} \text{ s} \quad \Rightarrow \quad \frac{1}{\tau} = 15 \ \frac{1}{\text{s}}$$

Then

$$i(t) = \left(i(0+) - i_{sc}\right)e^{-t/\tau} + i_{sc} = (0.2 - 0.04)e^{-15t} + 0.04 = 40 + 160e^{-15t} \text{ mA}$$

(checked: LNAP 7/13/04)

P 8.6-22 The voltage source voltage in the circuit shown in Figure P 8.6-22 is

$$v_s(t) = 10 + 40u(t) \text{ V}$$

Determine $v(t)$ for $t \geq 0$.

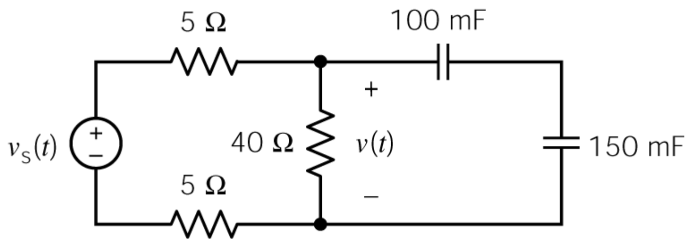
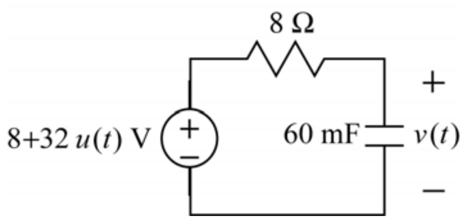


Figure P 8.6-22

Solution: Simplify the circuit by replacing the series capacitors by an equivalent capacitor. Then, after doing some source transformations, we have



For $t < 0$ the circuit is at steady state so the capacitor acts like an open circuit. The voltage source voltage is 8 V so

$$v(0+) = v(0-) = 8 \text{ V}$$

For $t > 0$ we find the Thevenin equivalent circuit of the part of the circuit connected to the capacitor. In this case we recognize v_{oc} = 40 V and R_t = 8 Ω.

The time constant is

$$\tau = R_t C = (8)(60 \times 10^{-3}) = 0.48 \quad \Rightarrow \quad \frac{1}{\tau} = 2.08 \ \frac{1}{\text{s}}$$

Then

$$v(t) = (v(0+) - v_{oc})e^{-t/\tau} + v_{oc} = (8 - 40)e^{-2.08t} + 40 = 40 - 32e^{-2.08t} \text{ V} \quad \text{for } t \geq 0$$

(checked: LNAP 7/13/04)

P 8.6-23 Determine $v(t)$ for $t > 0$ for the circuit shown in Figure P 8.6-23.

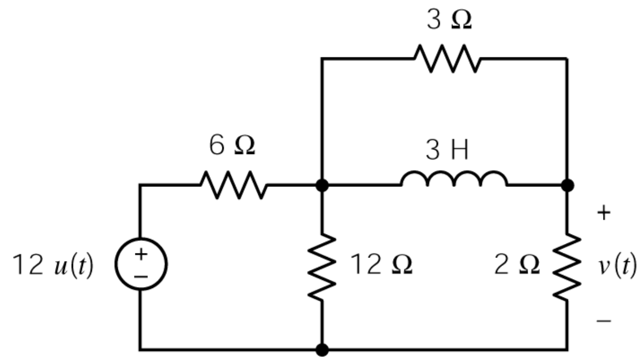
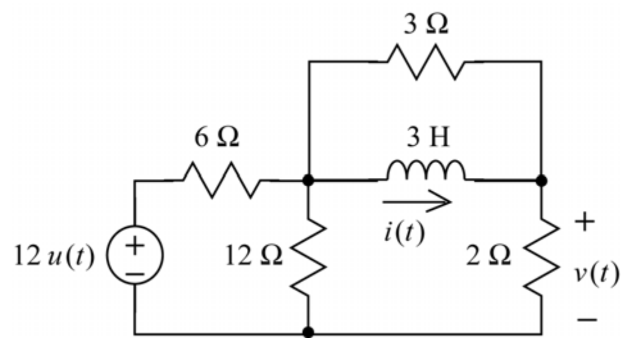
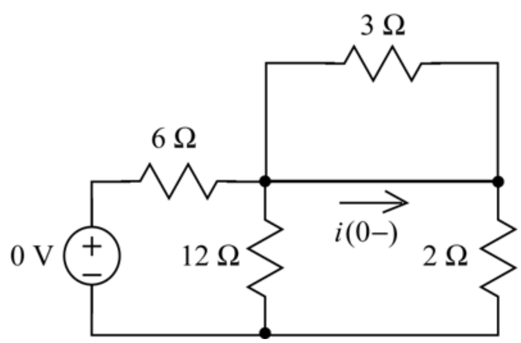


Figure P 8.6-23

Solution: The resistor voltage, $v(t)$, may not be continuous at time $t = 0$. The inductor will be continuous. We will find the inductor current first and then find $v(t)$. Label the inductor current as $i(t)$.

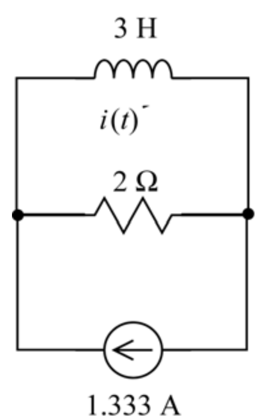




For $t < 0$ the circuit is at steady state and the inductor acts like a short circuit. The initial condition is

$$i(0+) = i(0-) = 0 \text{ A}$$

For $t > 0$ use source transformations to simplify the part of the circuit connected to the inductor until is a Norton equivalent circuit.



Recognize that

$$R_t = 2 \ \Omega \quad \text{and} \quad i_{sc} = 1.333 \text{ A}$$

The time constant is
$$\tau = \frac{L}{R_t} = \frac{3}{2} \quad \Rightarrow \quad \frac{1}{\tau} = 0.667 \ \frac{1}{\text{s}}$$

Then $$i(t) = \left(i(0+) - i_{\text{sc}}\right)e^{-t/\tau} + i_{\text{sc}} = 1.333\left(1 - e^{-0.667t}\right) \text{ A} \quad \text{for } t \ge 0$$

Returning to the original circuit we see that

$$\frac{v(t)}{2} = i(t) + \frac{3\frac{d}{dt}i(t)}{3} = i(t) + \frac{d}{dt}i(t)$$
$$= 1.333\left(1 - e^{-0.667t}\right) + (-0.667)(1.333)\left(-e^{-0.667t}\right) = 1.333 - 0.4439e^{-0.667}$$

Finally $$v(t) = 2.667 - 0.889e^{0.667t} \text{ V} \quad \text{for } t > 0$$

(checked: LNAP 7/14/04)

P 8.6-24 The input to the circuit shown in Figure P 8.6-24 is the current source current

$$i_s(t) = 2 + 4u(t) \text{ A}$$

The output is the voltage $v(t)$. Determine $v(t)$ for $t > 0$.

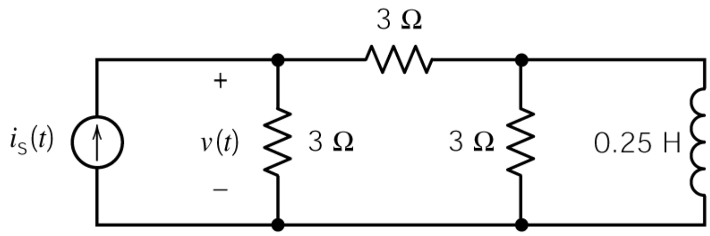
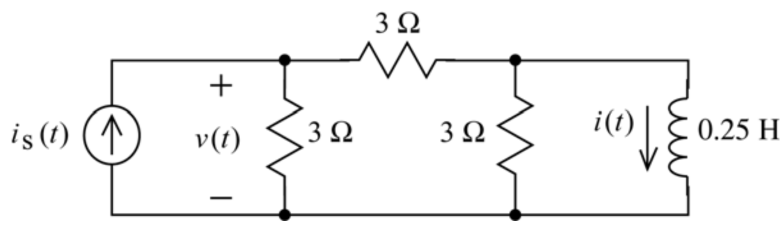


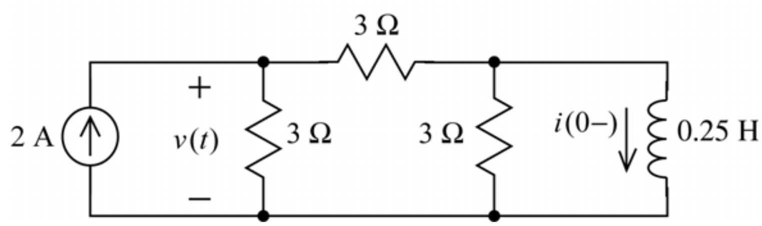
Figure P 8.6-24

Solution:
Label the inductor current, $i(t)$. We will find $i(t)$ first, then find $v(t)$.



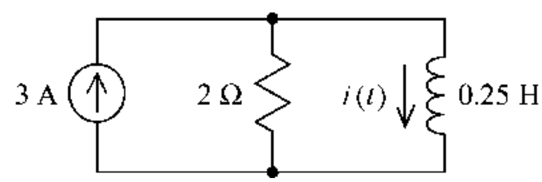
For $t < 0$ the circuit is at steady state and the inductor acts like a short circuit. The initial condition is

$$i(0+) = i(0-) = \left(\frac{3}{3+3}\right) 2 = 1 \text{ A}$$



For $t > 0$ use source transformations to simplify the part of the circuit connected to the inductor until it is a Norton equivalent circuit.

Recognize that $R_t = 2\ \Omega$ and $i_{sc} = 3$ A



The time constant is $\tau = \dfrac{L}{R_t} = \dfrac{0.25}{2} = 0.125 \text{ s} \quad \Rightarrow \quad \dfrac{1}{\tau} = 8 \ \dfrac{1}{\text{s}}$

Then $i(t) = \left(i(0+) - i_{sc}\right)e^{-t/\tau} + i_{sc} = (1-3)e^{-8t} + 3 = 3 - 2e^{-8t} \text{ A} \quad \text{for } t \ge 0$

Returning to the original circuit

$$v(t) = 3\left(i(t) + \frac{0.25\frac{d}{dt}i(t)}{3} \right) + 0.25\frac{d}{dt}i(t) = 3i(t) + 0.5\frac{d}{dt}i(t) = 3\left(3 - 2e^{-8t}\right) + 0.5\left(16e^{-8t}\right)$$

$$= 9 + 2\,e^{-8t} \text{ V} \quad \text{for } t > 0$$

(checked: LNAP 7/26/04)

P 8.6-25 The input to the circuit shown in Figure P 8.6-25 is the voltage source voltage

$$v_s = 6 + 6u(t)$$

The output is the voltage $v_o(t)$. Determine $v_o(t)$ for $t > 0$.

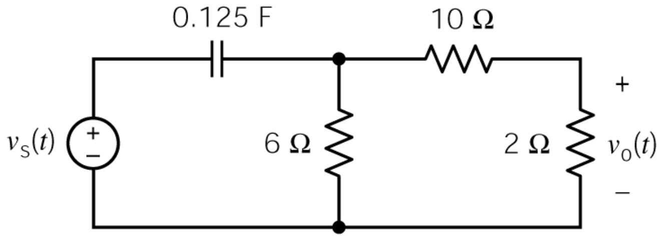
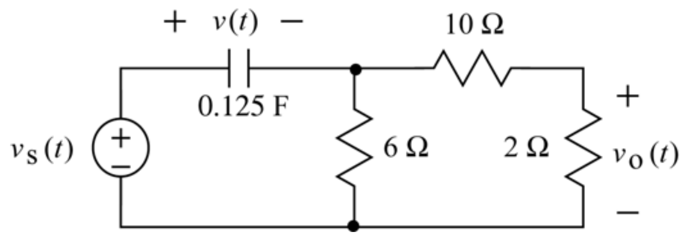


Figure P 8.6-25

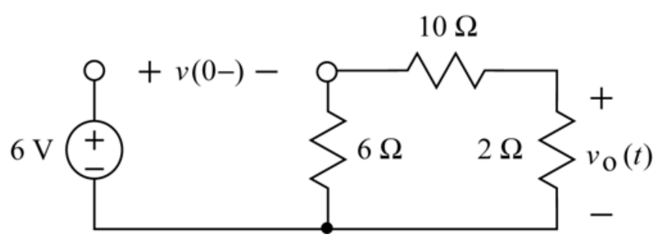
Solution:

Label the capacitor voltage, $v(t)$. W will find $v(t)$ first then find $v_o(t)$.

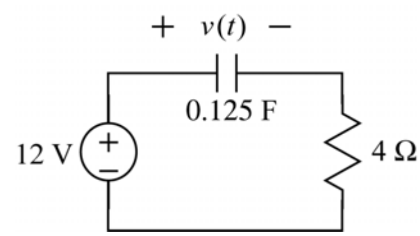


For $t < 0$ the circuit is at steady state and the capacitor acts like an open circuit. The initial condition is

$$v(0+) = v(0-) = 6 \text{ V}$$



For $t > 0$ we replace series and then parallel resistors by equivalent resistors in order to replace the part of the circuit connected to the capacitor by its Thevenin equivalent circuit.



We recognize $R_t = 4\ \Omega$ and $v_{oc} = 12$ V

The time constant is $\tau = R_t C = 4(0.125) = 0.5$ s $\Rightarrow$ $\frac{1}{\tau} = 2\ \frac{1}{s}$

The capacitor voltage is given by

$$v(t) = \left(v(0+) - v_{oc}\right)e^{\frac{-t}{\tau}} + v_{oc} = (6-12)e^{-2t} + 12 = 12 - 6e^{-2t} \text{ V} \quad \text{for } t \geq 0$$

Returning to the original circuit and applying KCL we see

$$0.125\frac{d}{dt}v(t) = \frac{12 - v(t)}{6} + \frac{v_o(t)}{2}$$

so $$v_o(t) = 0.25\frac{d}{dt}v(t) - 4 + \frac{v(t)}{3} = 0.25\left(12e^{-2t}\right) - 4 + 4 - 2e^{-2t} = e^{-2t} \text{ V} \quad \text{for } t > 0$$

(checked: LNAP 7/26/04)

P 8.6-26 Determine $v(t)$ for $t > 0$ for the circuit shown in Figure P 8.6-26.

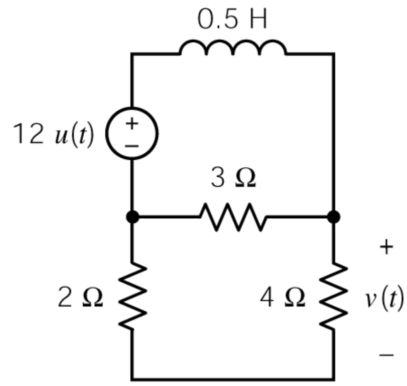
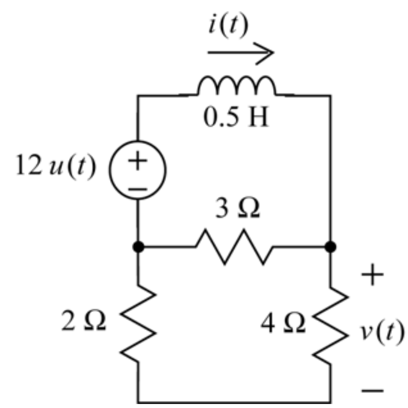


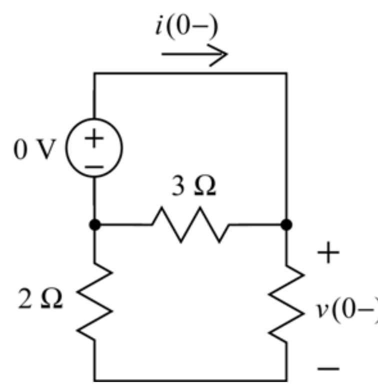
Figure P 8.6-26

Solution:
Label the inductor current as $i(t)$. We will find $i(t)$ first then use it to find $v(t)$.



For $t < 0$ the circuit is at steady state and the inductor acts like a short circuit. The initial condition is

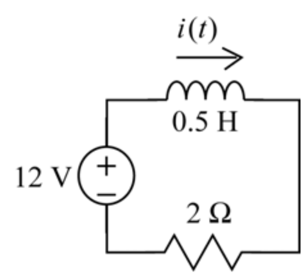
$$i(0+) = i(0-) = 0 \text{ A}$$



For $t > 0$ we replace series and parallel resistors by equivalent resistors. Then the part of the circuit connected to the inductor will be a Thevenin equivalent circuit.

We recognize $\quad R_t = 2\ \Omega \quad$ and $\quad v_{oc} = 12$ V

so $$i_{sc} = \frac{v_{oc}}{R_t} = 6 \text{ A}$$



The time constant is $\tau = \frac{L}{R_t} = \frac{0.5}{2} = 0.25 \quad \Rightarrow \quad \frac{1}{\tau} = 4 \ \frac{1}{\text{s}}$

The inductor current is given by

$$i(t) = \left(i(0+) - i_{sc}\right)e^{-t/\tau} + i_{sc} = (0-6)e^{-4t} + 6 = 6\left(1-e^{-4t}\right) \text{ A for } t \ge 0$$

Returning to the original circuit and applying KCL we see

$$i(t) + \frac{0.5\frac{d}{dt}i(t) - 12}{3} = \frac{v(t)}{4}$$

so

$$v(t) = 4i(t) + \frac{2}{3}\frac{d}{dt}i(t) - 16 = 24\left(1-e^{-4t}\right) + \left(\frac{2}{3}\right)\left(24e^{-4t}\right) - 16 = 8 - 8e^{-4t} \text{ V for } t > 0$$

(checked: LNAP 7/26/04)

P 8.6-27 When the input to the circuit shown in Figure P 8.6-27 is the voltage source voltage

$$v_s(t) = 3 - u(t) \text{ V}$$

the output is the voltage

$$v_o(t) = 10 + 5\, e^{-50t} \text{ V} \quad \text{for} \quad t \ge 0$$

Determine the values of R_1 and R_2.

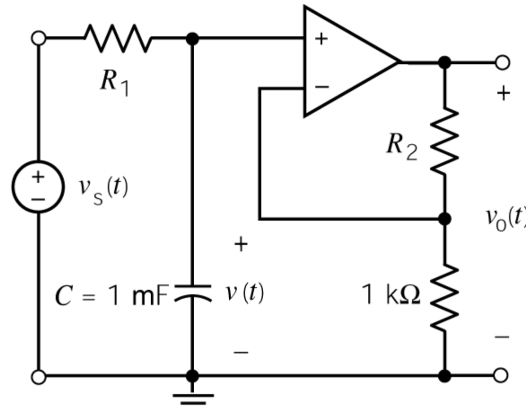


Figure P 8.6-27

Solution: Apply KCL at the inverting input of the op amp to get

$$\frac{v_o(t) - v(t)}{R_2} = \frac{v(t)}{1000} \quad \Rightarrow \quad v_o(t) = \left(1 + \frac{R_2}{1000}\right) v(t)$$

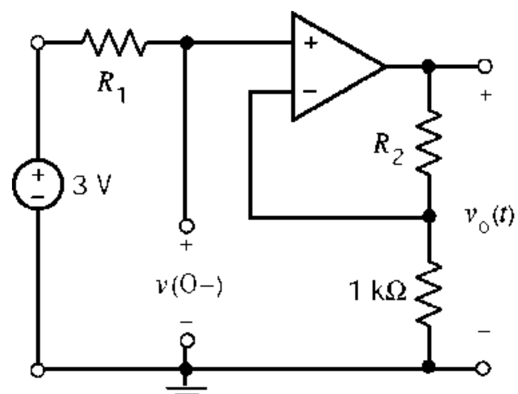
We will determine the capacitor voltage first and then use it to determine the output voltage.

When $t < 0$ and the circuit is at steady state, the capacitor acts like an open circuit. Apply KCL at the noninverting input of the op amp to get

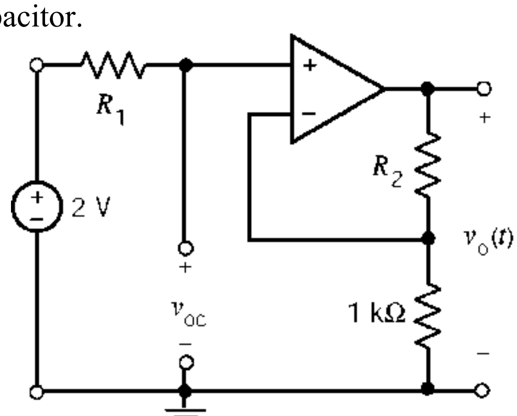
$$\frac{3 - v(0-)}{R_1} = 0 \quad \Rightarrow \quad v(0-) = 3 \text{ V}$$

The initial condition is

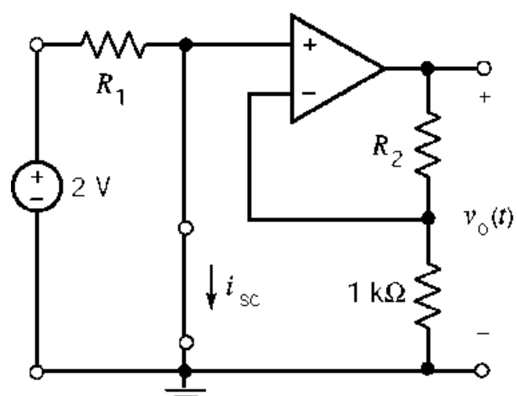
$$v(0+) = v(0-) = 3 \text{ V}$$



For $t \ge 0$, we find the Thevenin equivalent circuit for the part of the circuit connected to the capacitor.



$$\frac{2 - v_{oc}}{R_1} = 0 \quad \Rightarrow \quad v_{oc} = 2 \text{ V}$$



$$\frac{2}{R_1} = i_{sc} \quad \Rightarrow \quad R_t = \frac{v_{oc}}{i_{sc}} = R_1$$

The time constant is $\tau = R_t C = R_t\left(10^{-6}\right)$. From the given equation for $v_o(t)$, $\frac{1}{\tau} = 50\ \frac{1}{s}$, so

$$R_t\left(10^{-6}\right) = \frac{1}{50} \quad \Rightarrow \quad R_1 = R_t = \frac{10^6}{50} = 20\ \text{k}\Omega$$

The capacitor voltage is given by

$$v(t) = \left(v(0) - v_{oc}\right)e^{-t/\tau} + v_{oc} = (3-2)e^{-50t} + 2 = 2 + e^{-50t}\ \text{V} \quad \text{for } t \ge 0$$

So
$$v_o(t) = 5\,v(t) \quad \Rightarrow \quad 5 = 1 + \frac{R_2}{1000} \quad \Rightarrow \quad R_2 = 4\ \text{k}\Omega$$

(checked LNAPTR 7/31/04)

P8.6-28
The time constant of a particular circuit is τ = 0.25 s. In response to a step input, a capacitor voltage changes from –2.5 V to 4.2 V. How long did it take for the capacitor voltage to increase from –2.0 V to +2.0 V?

Solution:
The capacitor voltage can be represented by the equation $v(t) = A + Be^{-4t}$ for t ≥ 0. Given that $A + B = v(0) = -2.5$ V and $A = v(\infty) = 4.2$ V we determine $v(t) = 4.2 - 6.7\,e^{-4t}$.

Let t_1 be the time at which $v(t_1) = -2.0$ V. Then $t_1 = \frac{\ln\left(\frac{-2-4.2}{-6.7}\right)}{-4} = 0.01939$ s.

Let t_2 be the time at which $v(t_2) = 2.0$ V. Then $t_2 = \frac{\ln\left(\frac{2-4.2}{-6.7}\right)}{-4} = 0.27841$ s.

The transition requires 0.27841 – 0.01939 = 0.25902 s.

Section 8.7 The Response of a First-Order Circuit to a Nonconstant Source

P 8.7-1 Find $v_c(t)$ for $t > 0$ for the circuit shown in Figure P 8.7-1 when $v_1 = 8e^{-5t}u(t)$ V. Assume the circuit is in steady state at $t = 0^-$.
Answer: $v_c(t) = 4e^{-9t} + 18e^{-5t}$ V

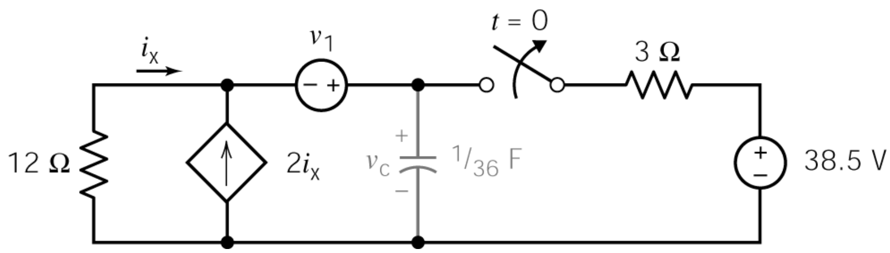
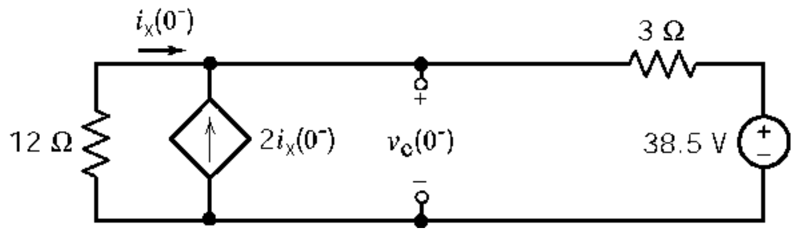


Figure P 8.7-1

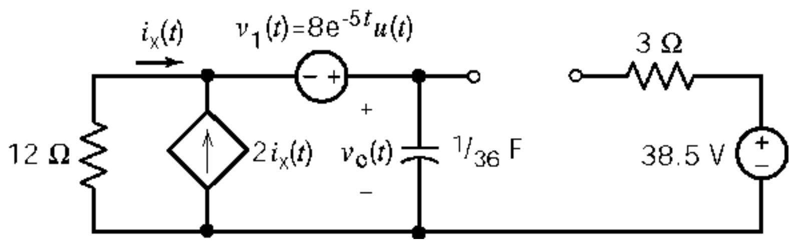
Solution: Assume that the circuit is at steady state before $t = 0$:



$$\text{KVL}: \ 12i_x + 3(3\,i_x) + 38.5 = 0 \quad \Rightarrow \quad i_x = -1.8\bar{3}\text{ A}$$

$$\text{Then } \underline{v_c(0^-) = -12\,i_x = 22\text{ V} = v_c(0^+)}$$

After $t = 0$:



$$\text{KVL}: \ 12i_x(t) - 8e^{-5t} + v_c(t) = 0$$

$$\text{KCL}: -i_x(t) - 2i_x(t) + (1/36)\frac{dv_c(t)}{dt} = 0 \quad \Rightarrow \quad i_x(t) = \frac{1}{108}\frac{dv_c(t)}{dt}$$

$$\therefore \quad 12\left[\frac{1}{108}\frac{dv_c(t)}{dt}\right] - 8e^{-5t} + v_c(t) = 0$$

$$\frac{dv_c(t)}{dt} + 9v_c(t) = 72e^{-5t} \quad \Rightarrow \quad v_{cn}(t) = Ae^{-9t}$$

Try $v_{cf}(t) = Be^{-5t}$ & substitute into the differential equation $\Rightarrow B = 18$

$$\therefore\ v_C(t) = Ae^{-9t} + 18\,e^{-5t}$$

$$v_C(0) = 22 = A + 18 \Rightarrow A = 4$$

$$\therefore v_C(t) = 4e^{-9t} + 18e^{-5t}\ \text{V}$$

P 8.7-2 Find $v(t)$ for $t > 0$ for the circuit shown in Figure P 8.7-2. Assume steady state at $t = 0^-$.
Answer: $v(t) = 20e^{-10t/3} - 12e^{-2t}$ V

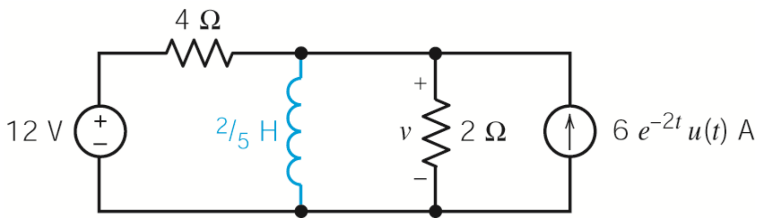
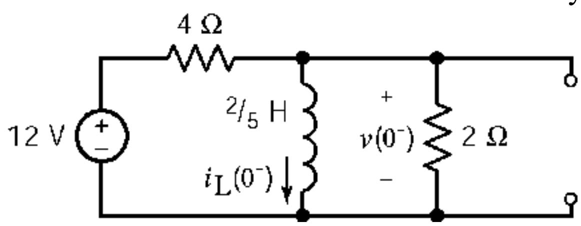


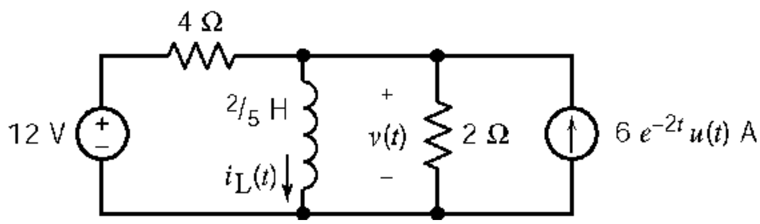
Figure P 8.7-2

Solution: Assume that the circuit is at steady state before $t = 0$:



$$i_L(0^+) = i_L(0^-) = \frac{12}{4} = 3\ \text{A}$$

After $t = 0$:



$$\text{KCL}: \frac{v(t)-12}{4} + i_L(t) + \frac{v(t)}{2} = 6\,e^{-2t}$$

$$\text{also}: v(t) = (2/5)\,\frac{di_L(t)}{dt}$$

$$i_L(t) + \frac{3}{4}\left[(2/5)\,\frac{di_L(t)}{dt}\right] = 3 + 6\,e^{-2t}$$

$$\frac{di_L(t)}{dt} + \frac{10}{3} i_L(t) = 10 + 20\,e^{-2t}$$

$\therefore i_n(t) = Ae^{-(10/3)t}$, try $i_f(t) = B + Ce^{-2t}$, substitute into the differential equation,

and then equating like terms $\Rightarrow B=3,\ C=15 \Rightarrow i_f(t)=3+15e^{-2t}$

$\therefore i_L(t)=i_n(t)+i_f(t) = Ae^{-(10/3)t} + 3 + 15e^{-2t}$, $\quad i_L(0) = 3 = A + 3 + 15 \Rightarrow A = -15$

$$\therefore\ i_L(t) = -15e^{-(10/3)t} + 3 + 15e^{-2t}$$

Finally, $v(t) = (2/5)\,\dfrac{di_L}{dt} = 20\,e^{-(10/3)t} - 12\,e^{-2t}$ V

P 8.7-3 Find $v_c(t)$ for $t > 0$ for the circuit shown in Figure P 8.7-3 when $i_s = [2 \cos 2t]\, u(t)$ mA.

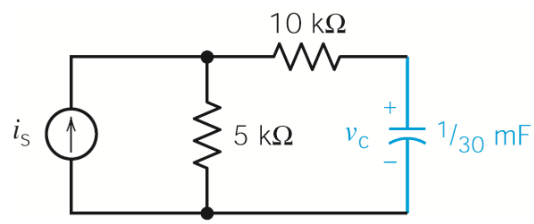
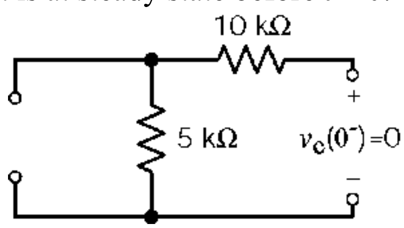
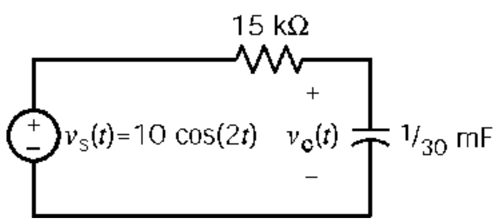


Figure P 8.7-3

Solution: Assume that the circuit is at steady state before $t = 0$:



Replace the circuit connected to the capacitor by its Thevenin equivalent (after t=0) to get:



KVL: $-10\cos 2t + 15\left(\frac{1}{30}\frac{dv_c(t)}{dt}\right) + v_c(t) = 0 \quad \Rightarrow \quad \frac{dv_c(t)}{dt} + 2v_c(t) = 20\cos 2t$

$v_n(t) = Ae^{-2t}$, Try $v_f(t) = B\cos 2t + C\sin 2t$ & substitute into the differential equation to get

$B = C = 5 \quad \Rightarrow \quad v_f(t) = 5\cos 2t + 5\sin 2t. \quad \therefore \; v_c(t) = v_n(t) + v_f(t) = Ae^{-2t} + 5\cos 2t + 5\sin 2t$

Now $v_c(0) = 0 = A + 5 \Rightarrow A = -5 \quad \Rightarrow \quad \underline{v_c(t) = -5e^{-2t} + 5\cos 2t + 5\sin 2t \text{ V}}$

P 8.7-4 Many have witnessed the use of an electrical megaphone for amplification of speech to a crowd. A model of a microphone and speaker is shown in Figure P 8.7-4*a*, and the circuit model is shown in Figure P 8.7-4*b*. Find $v(t)$ for

$$v_s = 10(\sin 100t)u(t)$$

which could represent a person whistling or singing a pure tone.

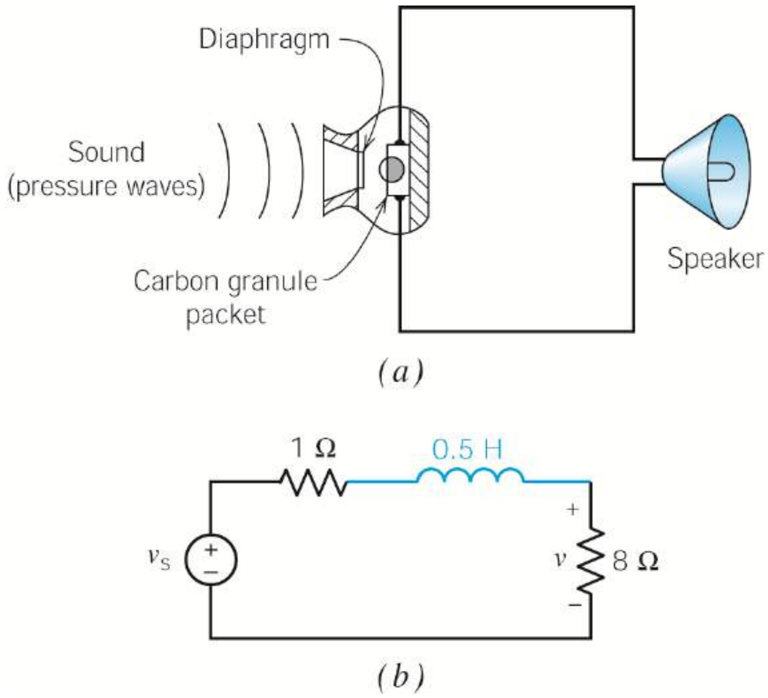
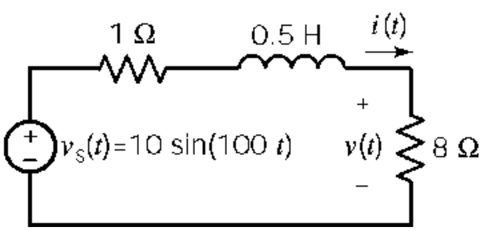


Figure P 8.7-4

Solution: Assume that the circuit is at steady state before $t = 0$. There are no sources in the circuit so $i(0) = 0$ A. After $t = 0$, we have:



$$\text{KVL :} -10\sin 100t + i(t) + 5\frac{di(t)}{dt} + v(t) = 0$$

$$\text{Ohm's law : } i(t) = \frac{v(t)}{8}$$

$$\therefore \quad \underline{\frac{dv(t)}{dt} + 18\, v(t) = 160\sin 100t}$$

$\therefore \; v_n(t) = Ae^{-18t}$, try $v_f(t) = B\cos 100t + C\sin 100t$, substitute into the differential equation and equate like terms $\Rightarrow \; B = -1.55 \;\; \& \;\; C = 0.279 \Rightarrow \; v_f(t) = -1.55\cos 100t + 0.279\sin 100t$

$\therefore \; v(t) = v_n(t) + v_f(t) = Ae^{-18t} - 1.55\cos 100t + 0.279\sin 100t$

$v(0) = 8\, i(0) = 0 \quad \Rightarrow \quad v(0) = 0 = A - 1.55 \Rightarrow A = 1.55$

so $\underline{v(t) = 1.55e^{-18t} - 1.55\cos 100t + 0.279\sin 100t \;\; \text{V}}$

P 8.7-5 A lossy integrator is shown in Figure P 8.7-5. The lossless capacitor of the ideal integrator circuit has been replaced with a model for the lossy capacitor, namely, a lossless capacitor in parallel with a 1-kΩ resistor. If

$$v_s = 15e^{-2t}u(t) \text{ V}$$

and

$$v_o(0) = 10 \text{ V},$$

find $v_o(t)$ for $t > 0$.

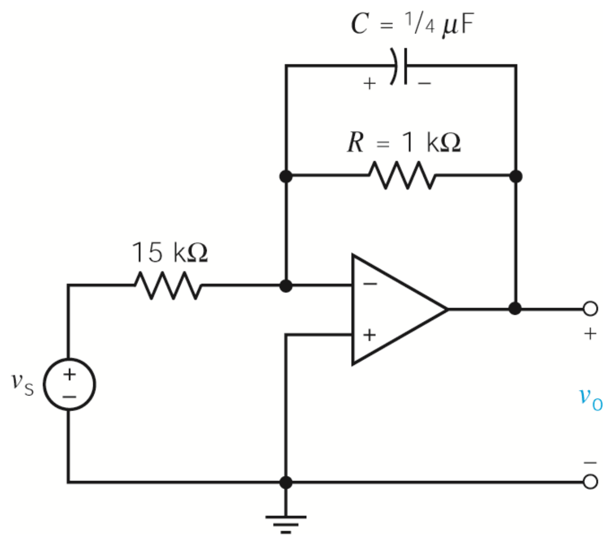


Figure P 8.7-5

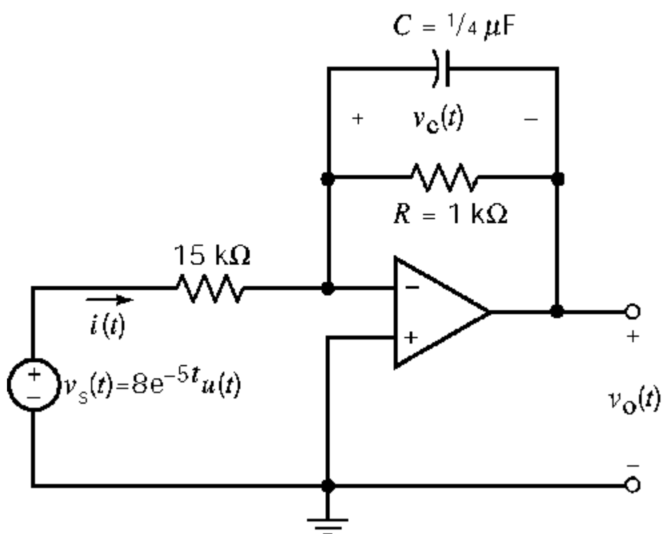
Solution: Assume that the circuit is at steady state before $t = 0$.

$$v_o(t) = -v_c(t)$$

$$v_C(0^+) = v_C(0^-) = -10 \text{ V}$$

After $t = 0$, we have

$$i(t) = \frac{v_s(t)}{15000} = \frac{8\,e^{-5t}}{15000} = 0.533\,e^{-5t} \text{ mA}$$



The circuit is represented by the differential equation: $i(t) = C\frac{dv_C(t)}{dt} + \frac{v_C(t)}{R}$. Then

$$\left(0.533\times10^{-3}\right)e^{-5t} = \left(0.25\times10^{-6}\right)\frac{dv_c(t)}{dt} + \left(10^{-3}\right)v_c(t) \quad\Rightarrow\quad \frac{dv_c(t)}{dt} + 4000\,v_c(t) = 4000\,e^{-5t}$$

Then $v_n(t) = Ae^{-4000t}$. Try $v_f(t) = Be^{-5t}$. Substitute into the differential equation to get

$$\frac{d\left(Be^{-5t}\right)}{dt} + 4000\left(Be^{-5t}\right) = 4000\,e^{-5t} \quad\Rightarrow\quad B = \frac{4000}{-3995} = -1.00125 \cong -1$$

$$v_C(t) = v_f(t) + v_n(t) = e^{-5t} + Ae^{-4000t}$$

$$v_C(0) = -10 = 1 + A \Rightarrow A = -11 \Rightarrow v_C(t) = 1\,e^{-2t} - 11\,e^{-4000t} \text{ V}$$

Finally $$\underline{v_o(t) = -\,v_C(t) = 11e^{-4000t} - 1e^{-5t} \text{ V}, \text{ t} \geq 0}$$

P 8.7-6 Determine *v*(*t*) for the circuit shown in Figure P 8.7-6.

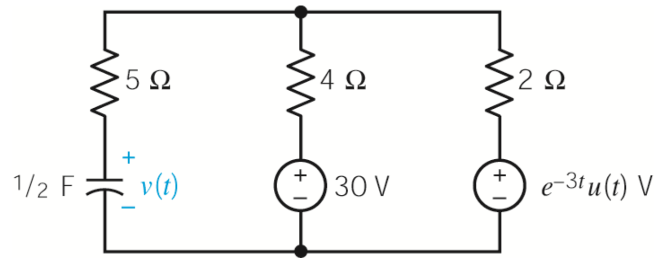
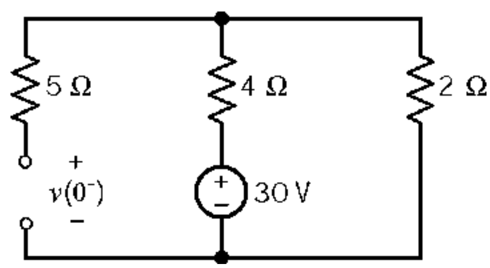


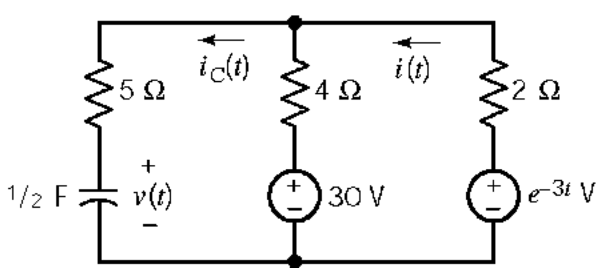
Figure P 8.7-6

Solution: Assume that the circuit is at steady state before $t = 0$.

$$v(0^+) = v(0^-) = \frac{2}{4+2}30 = 10 \text{ V}$$



After t = 0 we have



$$\text{KVL}: \ \frac{5}{2}\frac{d\,v(t)}{dt} + v(t) + 4\left(\frac{1}{2}\frac{d\,v(t)}{dt} - i\right) = 30$$

$$2\,i(t) + 4\left(i(t) - \frac{1}{2}\frac{d\,v(t)}{dt}\right) + 30 = e^{-3t}$$

The circuit is represented by the differential equation

$$\frac{d\,v(t)}{dt} + \frac{6}{19}v(t) = \frac{6}{19}\left(10 + \frac{2}{3}e^{-3t}\right)$$

Take $v_n(t) = Ae^{-(6/19)t}$. Try , $v_f(t) = B + Ce^{-3t}$, substitute into the differential equation to get

$$-3Ce^{-3t} + \frac{6}{19}(B + Ce^{-3t}) = \frac{60}{19} + \frac{4}{19}e^{-3t}$$

Equate coefficients to get

$$B = 10 \ , \ C = -\frac{4}{51} \quad \Rightarrow \quad v_f(t) = \frac{4}{51}e^{-3t} + Ae^{-(6/19)t}$$

Then

$$v(t) = v_n(t) + v_f(t) = 10 - \frac{4}{51}e^{-3t} + Ae^{-(6/19)t}$$

Finally

$$v_c(0^+) = 10 \text{ V}, \quad \Rightarrow \quad 10 = 10 - \frac{4}{51} + A \quad \Rightarrow \quad A = \frac{4}{51}$$

$$\therefore \ v_c(t) = 10 + \frac{4}{51}\left(e^{-(6/19)t} - e^{-3t}\right) \text{ V}$$

P 8.7-7 Determine $v(t)$ for the circuit shown in Figure P 8.7-7a when v_s varies as shown in Figure P 8.7-7b. The initial capacitor voltage is $v_c(0) = 0$.

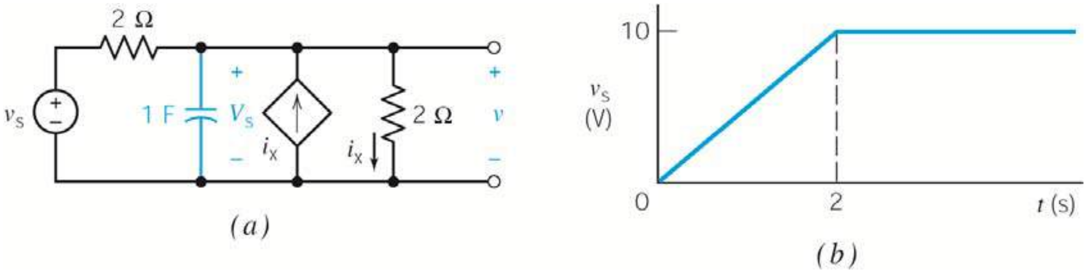
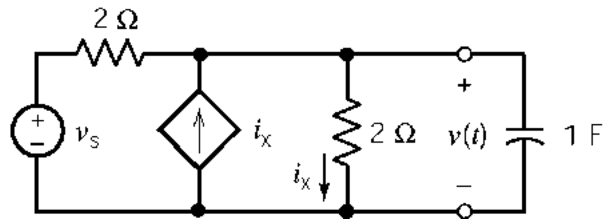


Figure P 8.7-7

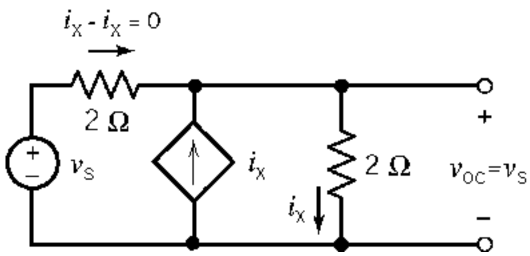
Solution: We are given $v(0) = 0$. From part b of the figure:

$$v_s(t) = \begin{cases} 5t & 0 \le t \le 2 \text{ s} \\ 10 & t > 2 \text{ s} \end{cases}$$

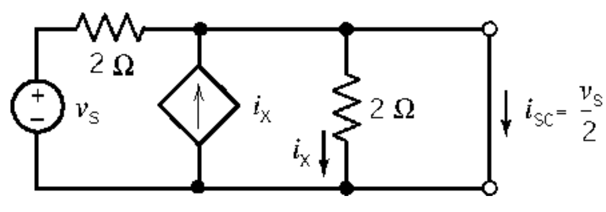
Find the Thevenin equivalent of the part of the circuit that is connected to the capacitor:



The open circuit voltage:

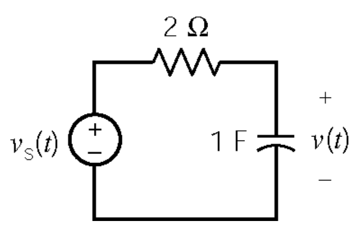


The short circuit current:



(i_x=0 because of the short across the right 2 Ω resistor)

Replace the part of the circuit connected to the capacitor by its Thevenin equivalent:



$$2\frac{dv(t)}{dt} + v(t) - v_s(t) = 0$$

$$\frac{dv(t)}{dt} + \frac{v(t)}{2} = \frac{v_s(t)}{2}$$

$$v_n(t) = Ae^{-0.5\,t}$$

For $0 < t < 2$ s, $v_s(t) = 5\,t$. Try $v_f(t) = B + C\,t$. Substituting into the differential equation and equating coefficients gives $B = -10$ and C =5. Therefore $v(t) = 5t - 10 + A\,e^{-t/2}$. Using $v(0) = 0$, we determine that A =10. Consequently, $\underline{v(t) = 5t + 10(e^{-t/2} - 1)}$.

At $t = 2$ s, $v(2) = 10e^{-1} = 3.68$.

Next, for $t > 2$ s, $v_s(t) = 10$ V. Try $v_f(t) = B$. Substituting into the differential equation and equating coefficients gives $B = 10$. Therefore $v(t) = 10 + Ae^{-(t-2)/2}$. Using $v(2) = 3.68$, we determine that $A = -6.32$. Consequently, $\underline{v(t) = 10 - 6.32\,e^{-(t-2)/2}}$.

P 8.7-8 The electron beam, which is used to "draw" signals on an oscilloscope, is moved across the face of a cathode-ray tube (CRT) by a force exerted on electrons in the beam. The basic system is shown in Figure P 8.7-8*a*. The force is created from a time-varying, ramp-type voltage applied across the vertical or the horizontal plates. As an example, consider the simple circuit of Figure P 8.7-8*b* for horizontal deflection where the capacitance between the plates is *C*.

Derive an expression for the voltage across the capacitance. If $v(t) = kt$ and $R_s = 625$ kΩ, $k = 1000$, and $C = 2000$ pF, compute v_c as a function of time. Sketch $v(t)$ and $v_c(t)$ on the same graph for time less than 10 ms. Does the voltage across the plates track the input voltage?

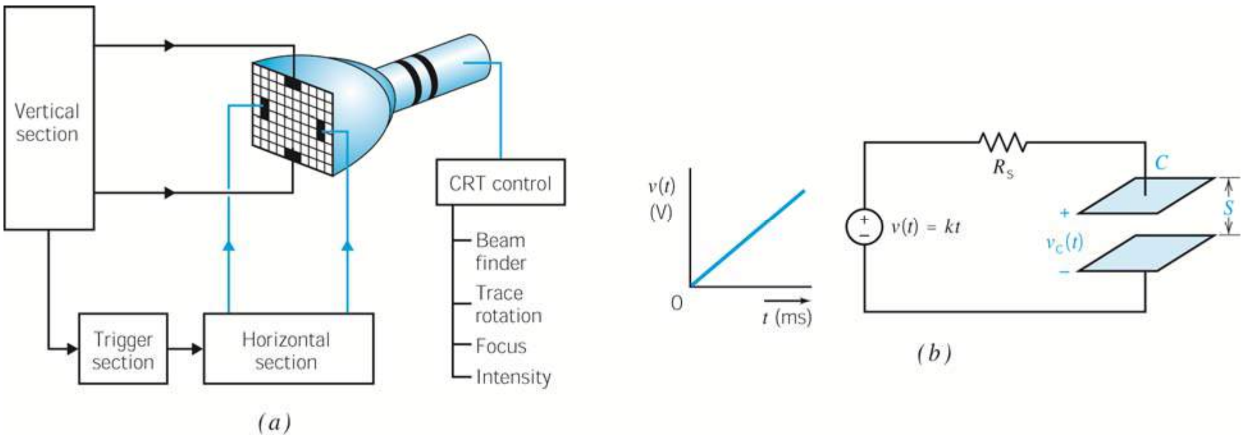
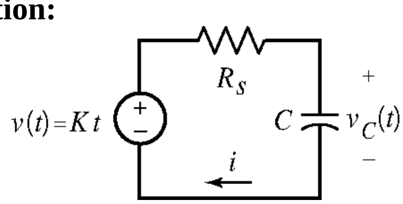


Figure P 8.7-8

Solution:



$$\text{KVL: } -kt + R_s\left[C\frac{dv_C(t)}{dt}\right] + v_C(t) = 0$$

$$\Rightarrow \frac{dv_C(t)}{dt} + \frac{1}{R_sC}v_C(t) = \frac{k}{R_sC}t$$

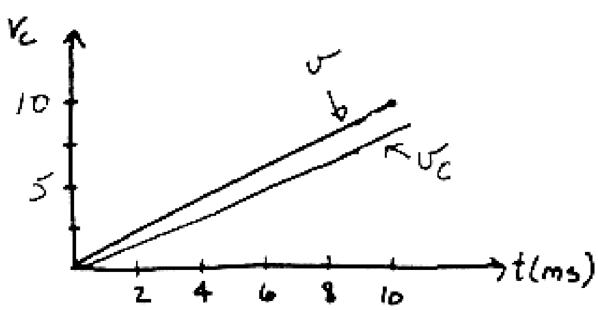
$v_c(t) = v_n(t) + v_f(t)$, where $v_c(t) = Ae^{-t/R_sC}$. Try $v_f(t) = B_0 + B_1 t$

& plug into D.E. $\Rightarrow B_1 + \frac{1}{R_sC}[B_0 + B_1 t] = \frac{k}{R_sC}t$ thus $B_0 = -kR_sC$, $B_1 = k$.

Now we have $v_c(t) = Ae^{-t/R_sC} + k(t - R_sC)$. Use $v_c(0) = 0$ to get $0 = A - kR_sC \Rightarrow A = kR_sC$.

$\therefore v_c(t) = k[t - R_sC(1 - e^{-t/R_sC})]$. Plugging in k=1000, R_s= 625 kΩ & C=2000 pF get

$\underline{v_c(t) = 1000[t - 1.25\times10^{-3}(1 - e^{-800\,t})]}$



$v(t)$ and $v_C(t)$ track well on a millisecond time scale.

Section 8.10 How Can We Check…?

P 8.10-1 Figure P 8.10-1 shows the transient response of a first-order circuit. This transient response was obtained using the computer program PSpice. A point on this transient response has been labeled. The label indicates a time and the capacitor voltage at that time. Placing the circuit diagram on the plot suggests that the plot corresponds to the circuit. Verify that the plot does indeed represent the voltage of the capacitor in this circuit.

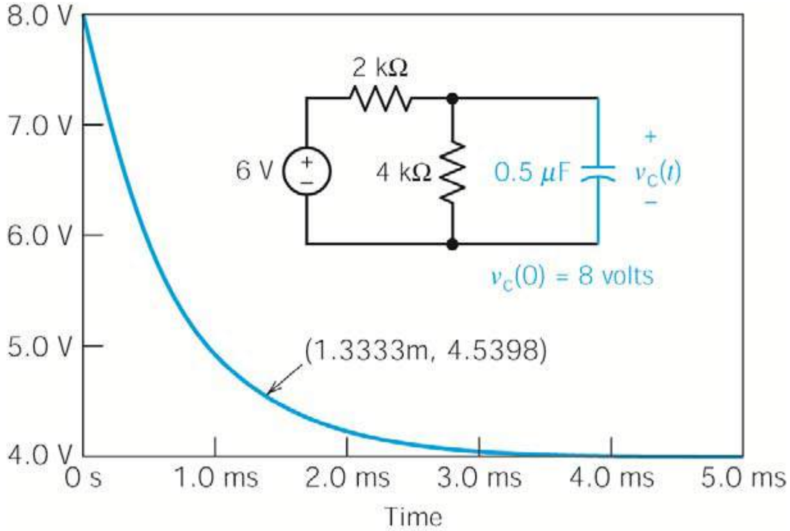


Figure P 8.10-1

Solution: First look at the circuit. The initial capacitor voltage is $v_c(0) = 8$ V. The steady-state capacitor voltage is $v_c = 4$ V.

We expect an exponential transition from 8 volts to 4 volts. That's consistent with the plot.

Next, let's check the shape of the exponential transition. The Thevenin resistance of the part of the circuit connected to the capacitor is $R_t = \dfrac{(2000)(4000)}{2000+4000} = \dfrac{4}{3}$ kΩ so the time constant is $\tau = R_t\, C = \left(\dfrac{4}{3}\times 10^3\right)\left(0.5\times 10^{-6}\right) = \dfrac{2}{3}$ ms . Thus the capacitor voltage is

$$v_c(t) = 4\, e^{-t/0.67} + 4 \text{ V}$$

where t has units of ms. To check the point labeled on the plot, let $t_1 = 1.33$ ms. Then

$$v_c(t_1) = 4\, e^{-\left(\frac{1.33}{.67}\right)} + 4 = 4.541 \simeq 4.5398 \text{ V}$$

So the plot is correct.

P 8.10-2 Figure P 8.10-2 shows the transient response of a first-order circuit. This transient response was obtained using the computer program PSpice. A point on this transient response has been labeled. The label indicates a time and the inductor current at that time. Placing the circuit diagram on the plot suggests that the plot corresponds to the circuit. Verify that the plot does indeed represent the current of the inductor in this circuit.

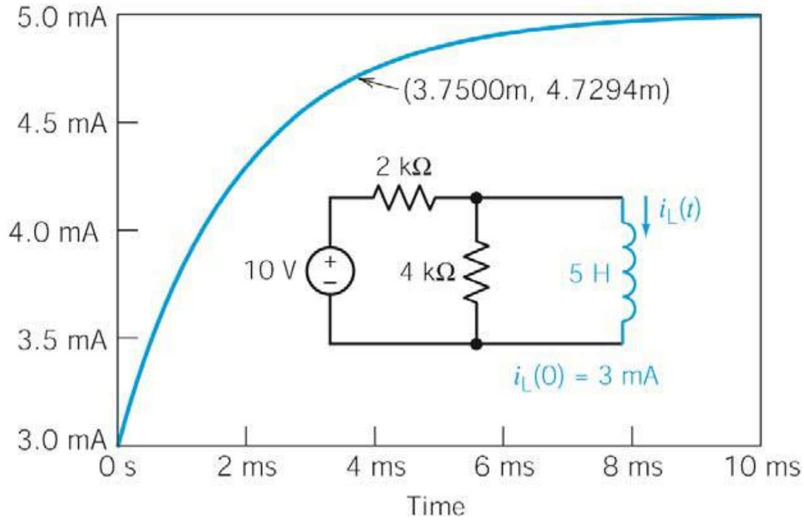


Figure P 8.10-2

Solution: The initial and steady-state inductor currents shown on the plot agree with the values obtained from the circuit.

Next, let's check the shape of the exponential transition. The Thevenin resistance of the part of the circuit connected to the inductor is $R_t = \frac{(2000)(4000)}{2000+4000} = \frac{4}{3}$ kΩ so the time constant is

$\tau = \frac{L}{R_t} = \frac{5}{\frac{4}{3}\times 10^3} = \frac{15}{4}$ ms. Thus inductor current is

$$i_L(t) - 2\,e^{-t/3.75} + 5 \text{ mA}$$

where t has units of ms. To check the point labeled on the plot, let t_1 = 3.75 ms. Then

$$i_L(t_1) = -2\,e^{-\left(\frac{3.75}{3.75}\right)} + 5 = \text{ 4.264 mA} \neq 4.7294 \text{ mA}$$

so the plot does not correspond to this circuit.

P 8.10-3 Figure P 8.10-3 shows the transient response of a first-order circuit. This transient response was obtained using the computer program PSpice. A point on this transient response has been labeled. The label indicates a time and the inductor current at that time. Placing the circuit diagram on the plot suggests that the plot corresponds to the circuit. Specify that value of the inductance, *L*, required to cause the current of the inductor in this circuit to be accurately represented by this plot.

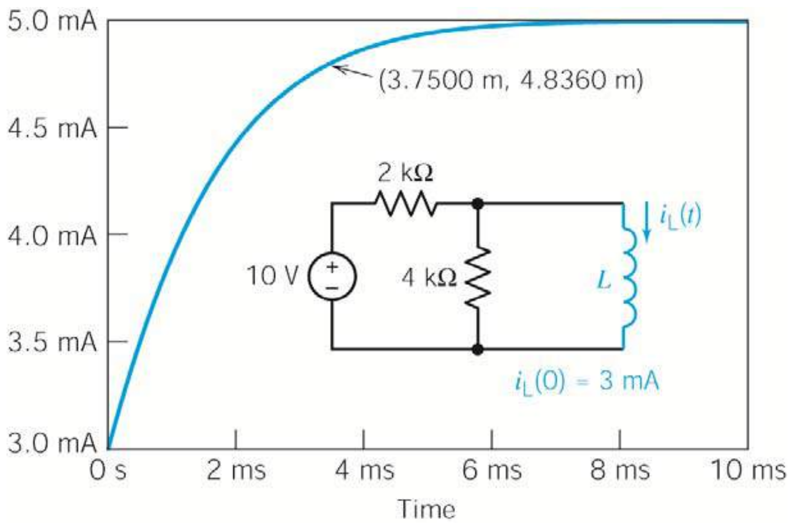
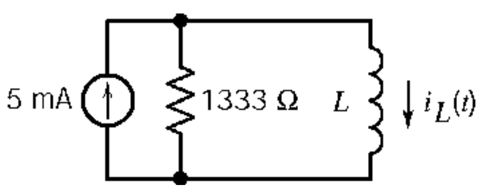


Figure P 8.10-3

Solution: Notice that the steady-state inductor current does not depend on the inductance, *L*. The initial and steady-state inductor currents shown on the plot agree with the values obtained from the circuit.

After $t = 0$



So $I_{sc} = 5$ mA and $\tau = \dfrac{L}{1333}$

The inductor current is given by $i_L(t) = -2e^{-1333t/L} + 5$ mA, where t has units of seconds and L has units of Henries. Let $t_1 = 3.75$ ms, then

$$4.836 = i_L(t_1) = -2\, e^{-(1333)\cdot(0.00375)/L} + 5 = -2e^{-5/L} + 5$$

so

$$\frac{4.836-5}{-2} = e^{-5/L}$$

and

$$L = \frac{-5}{\ln\left(\dfrac{4.836-5}{-2}\right)} = 2 \text{ H}$$

is the required inductance.

P 8.10-4 Figure P 8.10-4 shows the transient response of a first-order circuit. This transient response was obtained using the computer program PSpice. A point on this transient response has been labeled. The label indicates a time and the capacitor voltage at that time. Assume that this circuit has reached steady state before time t = 0. Placing the circuit diagram on the plot suggests that the plot corresponds to the circuit. Specify values of A, B, R_1, R_2, and C that cause the voltage across the capacitor in this circuit to be accurately represented by this plot.

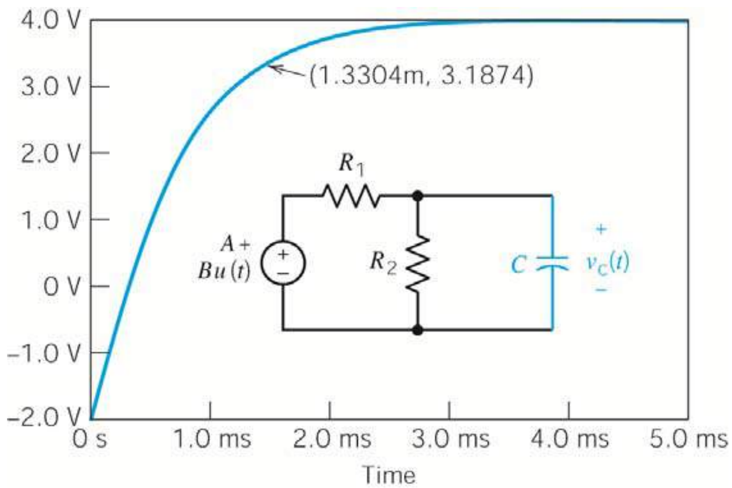
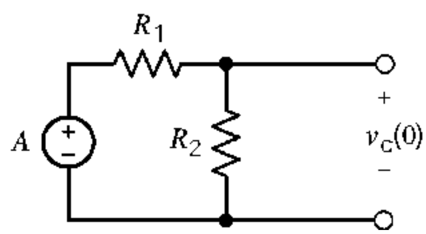
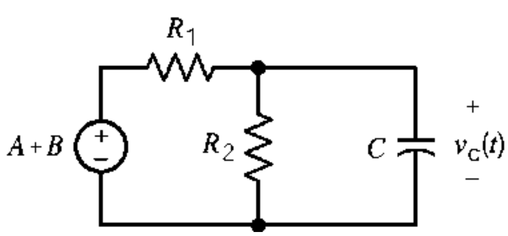


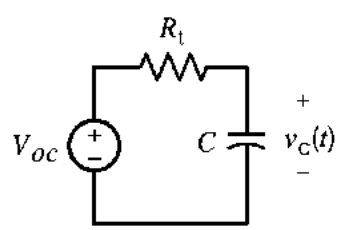
Figure P 8.10-4

Solution: First consider the circuit. When t < 0 and the circuit is at steady-state:



For t > 0





So $$V_{oc} = \frac{R_2}{R_1+R_2}(A+B)\ ,\quad R_t = \frac{R_1 R_2}{R_1+R_2} \quad \text{and} \quad \tau = \frac{R_1 R_2 C}{R_1+R_2}$$

Next, consider the plot. The initial capacitor voltage is (v_c (0)=) –2 and the steady-state capacitor voltage is (V_{oc} =) 4 V, so

$$v_C(t) = -6e^{-t/\tau} + 4$$

At t_1 = 1.333 ms $$3.1874 = v_C(t_1) = -6\, e^{-0.001333/\tau} + 4$$

so $$\tau = \frac{-0.001333}{\ln\left(\frac{-4+3.1874}{-6}\right)} = 0.67 \text{ ms}$$

Combining the information obtained from the circuit with the information obtained from the plot gives

$$\frac{R_2}{R_1+R_2} A = -2, \quad \frac{R_2}{R_1+R_2}(A+B) = 4, \quad \frac{R_1 R_2 C}{R_1+R_2} = 0.67 \text{ ms}$$

There are many ways that A, B, R_1, R_2, and C can be chosen to satisfy these equations. Here is one convenient way. Pick R_1 = 3000 and R_2 = 6000. Then

$$\frac{2A}{3} = -2 \quad \Rightarrow \quad A = -3$$

$$\frac{2(A+B)}{3} = 4 \Rightarrow B-3 = 6 \Rightarrow B = 9$$

$$2000 \cdot C = \frac{2}{3} \text{ ms} \quad \Rightarrow \quad \frac{1}{3}\mu\text{F} = C$$

Spice Problems

SP 8-1 The input to the circuit shown in Figure SP 8.1 is the voltage of the voltage source, $v_i(t)$. The output is the voltage across the capacitor, $v_o(t)$. The input is the pulse signal specified graphically by the plot. Use PSpice to plot the output, $v_o(t)$, as a function of t.
Hint: Represent the voltage source using the PSpice part named VPULSE.

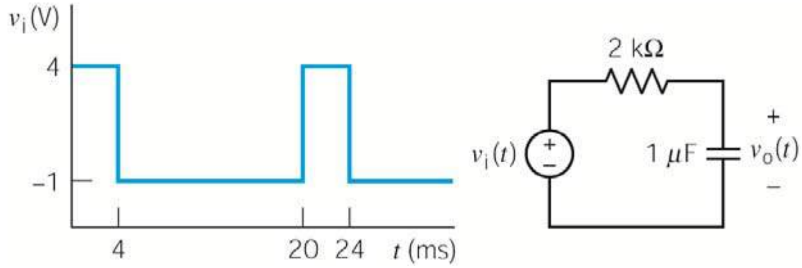
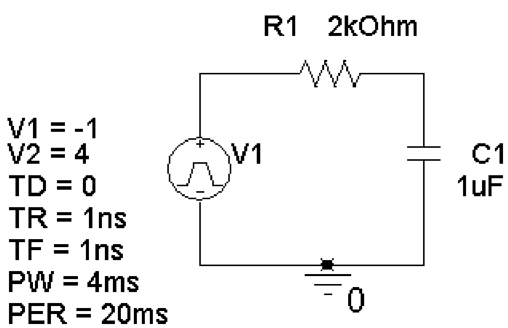
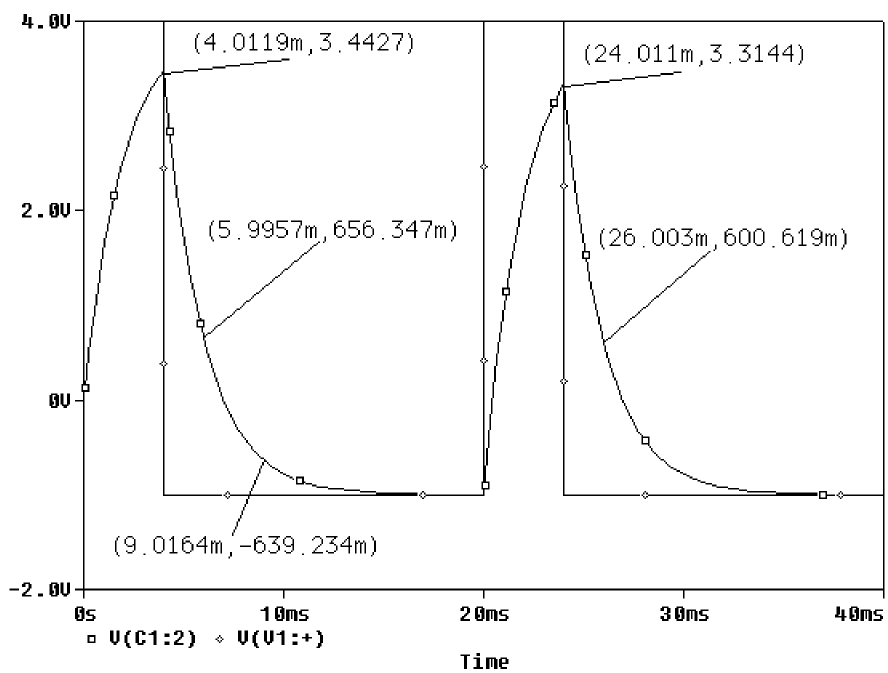


Figure SP 8.1

Solution:





SP 8-2 The input to the circuit shown in Figure SP 8.2 is the voltage of the voltage source, $v_i(t)$. The output is the current in the inductor, $i_o(t)$. The input is the pulse signal specified graphically by the plot. Use PSpice to plot the output, $i_o(t)$, as a function of t.
Hint: Represent the voltage source using the PSpice part named VPULSE.

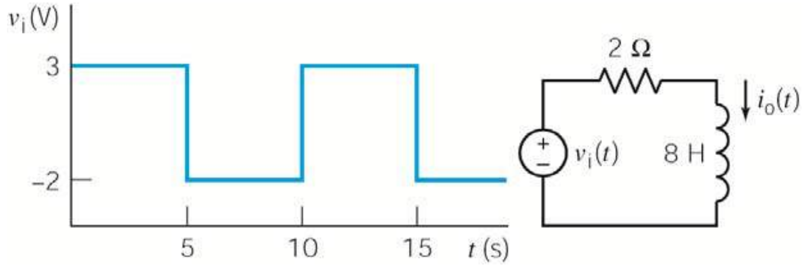
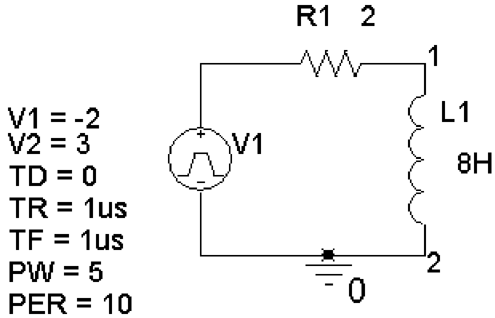
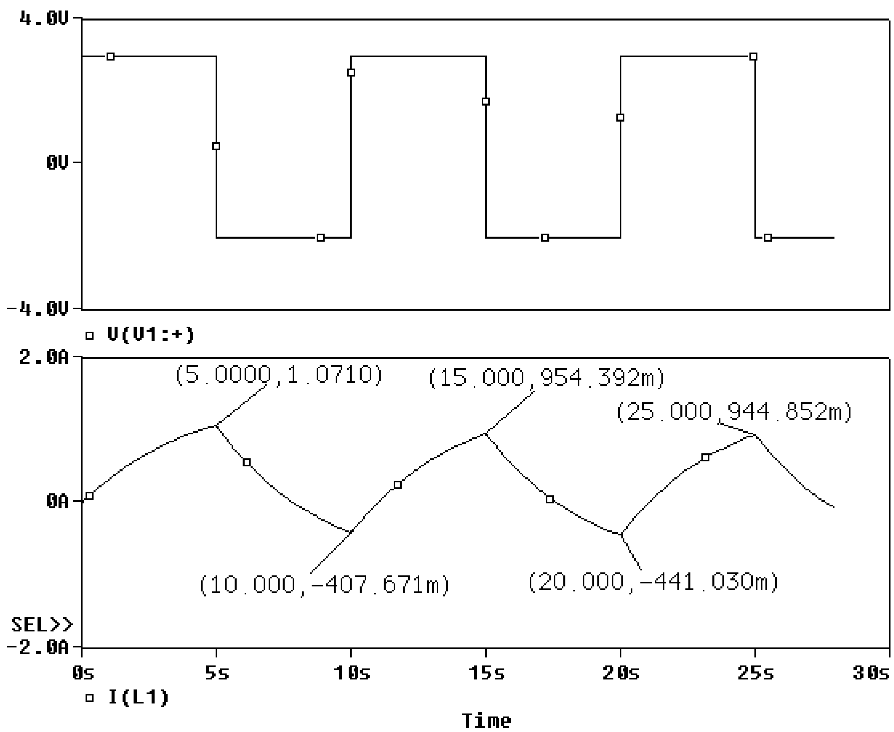


Figure SP 8.2

Solution:





SP 8-3 The circuit shown in Figure SP 8.3 is at steady state before the switch closes at time $t = 0$. The input to the circuit is the voltage of the voltage source, 12 V. The output of this circuit is the voltage across the capacitor, $v(t)$. Use PSpice to plot the output, $v(t)$, as a function of t. Use the plot to obtain an analytic representation of $v(t)$ for $t > 0$.

Hint: We expect $v(t) = A + Be^{-t/\tau}$ for $t > 0$, where A, B, and τ are constants to be determined.

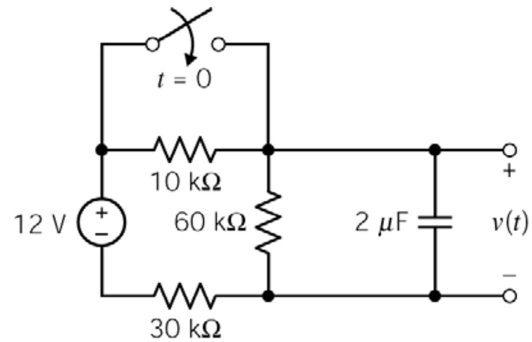
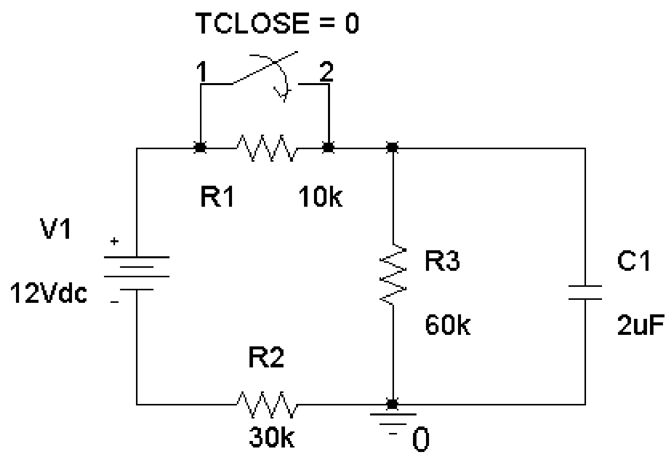
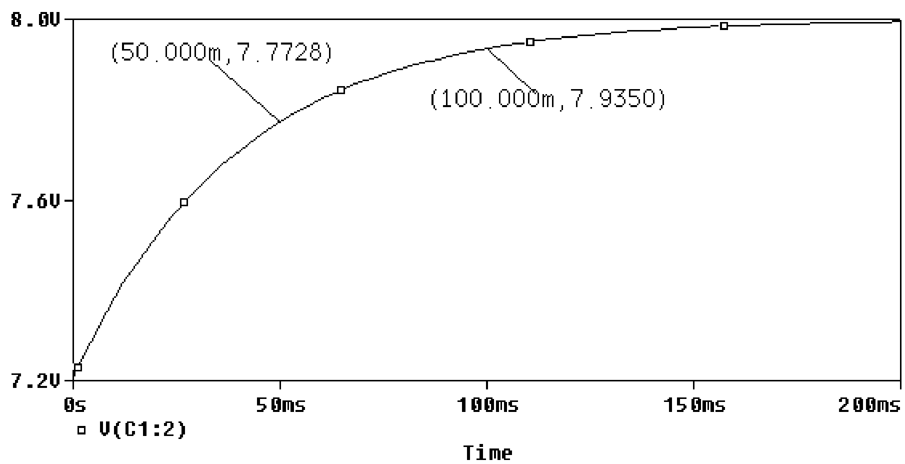


Figure SP 8.3

Solution:





$$v(t) = A + B\,e^{-t/\tau} \quad \text{for } t > 0$$

$$\left.\begin{array}{l} 7.2 = v(0) = A + B\,e^{0} \quad \Rightarrow \quad 7.2 = A + B \\ 8.0 = v(\infty) = A + B\,e^{-\infty} \quad \Rightarrow \quad A = 8.0 \text{ V} \end{array}\right\} \Rightarrow \quad B = -0.8 \text{ V}$$

$$7.7728 = v(0.05) = 8 - 0.8\, e^{-0.05/\tau} \quad \Rightarrow \quad -\frac{0.05}{\tau} = \ln\left(\frac{8-7.7728}{0.8}\right) = -1.25878$$

$$\Rightarrow \quad \tau = \frac{0.05}{1.25878} = 39.72 \text{ ms}$$

Therefore

$$v(t) = 8 - 0.8\, e^{-t/0.03972} \text{ V} \quad \text{for } t > 0$$

SP 8-4 The circuit shown in Figure SP 8.4 is at steady state before the switch closes at time $t = 0$. The input to the circuit is the current of the current source, 4 mA. The output of this circuit is the current in the inductor, $i(t)$. Use PSpice to plot the output, $i(t)$, as a function of t. Use the plot to obtain an analytic representation of $i(t)$ for $t > 0$.

Hint: We expect $i(t) = A + Be^{-t/\tau}$ for $t > 0$, where A, B, and τ are constants to be determined.

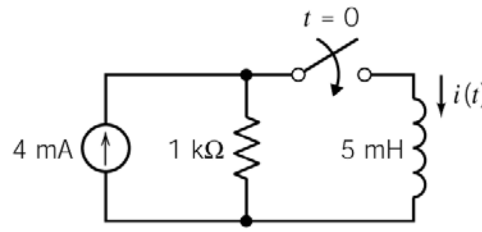
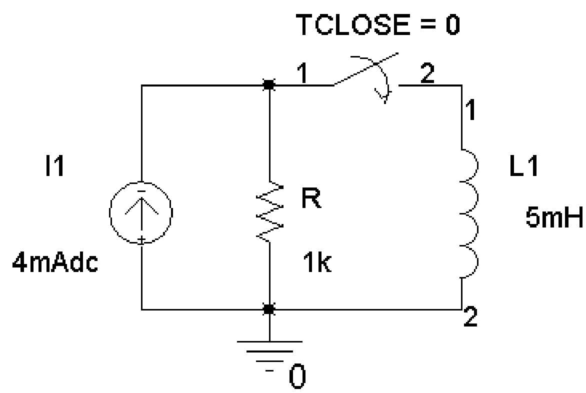
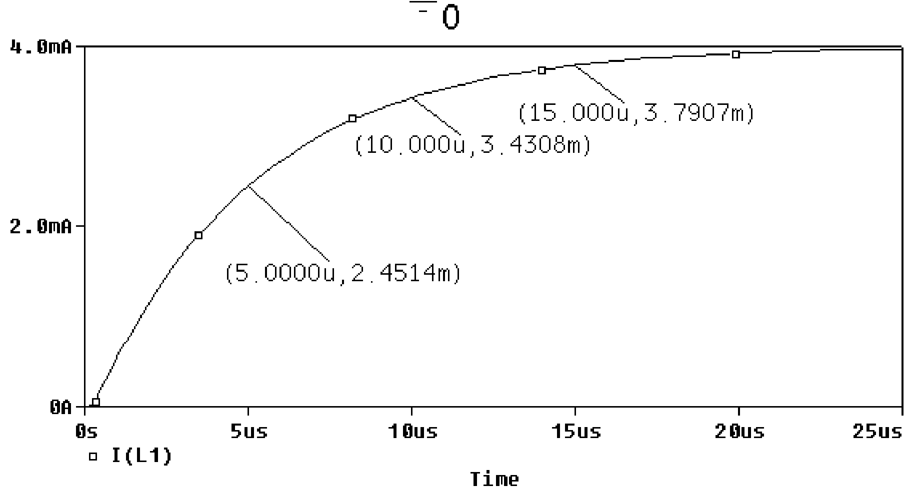


Figure SP 8.4

Solution:





$$i(t) = A + B\,e^{-t/\tau} \quad \text{for } t > 0$$

$$\left.\begin{array}{l} 0 = i(0) = A + B\,e^{0} \quad \Rightarrow \quad 0 = A + B \\ 4\times10^{-3} = i(\infty) = A + B\,e^{-\infty} \quad \Rightarrow \quad A = 4\times10^{-3} \text{ A} \end{array}\right\} \quad \Rightarrow \quad B = -4\times10^{-3} \text{ A}$$

$$2.4514\times10^{-3} = v(5\times10^{-6}) = \left(4\times10^{-3}\right) - \left(4\times10^{-3}\right) e^{-\left(5\times10^{-6}\right)/\tau}$$

$$\Rightarrow \quad -\frac{5\times10^{-6}}{\tau} = \ln\left(\frac{\left(4-2.4514\right)\times10^{-3}}{4\times10^{-3}}\right) = -0.94894$$

$$\Rightarrow \quad \tau = \frac{5\times10^{-6}}{0.94894} = 5.269 \ \mu\text{s}$$

Therefore

$$i(t) = 4 - 4\,e^{-t/5.269\times10^{-6}} \text{ mA} \quad \text{for } t > 0$$

DESIGN PROBLEMS

DP 8-1 Design the circuit in Figure DP 8.1 so that $v(t)$ makes the transition from $v(t) = 6$ V to $v(t) = 10$ V in 10 ms after the switch is closed. Assume that the circuit is at steady state before the switch is closed. Also assume that the transition will be complete after 5 time constants.

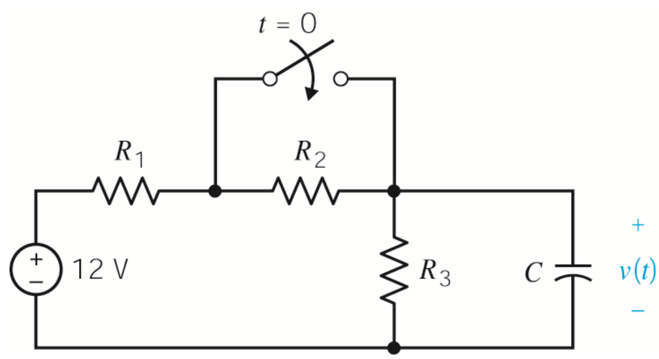


Figure DP 8.1

Solution:

Steady-state response when the switch is open: $6 = \dfrac{R_3}{R_1+R_2+R_3}12 \quad \Rightarrow \quad R_1 + R_2 = R_3$.

Steady-state response when the switch is closed: $10 = \dfrac{R_3}{R_1+R_3}12 \quad \Rightarrow \quad R_1 = \dfrac{R_3}{5}$.

$10 \text{ ms} = 5\,\tau = \left(R_1 \,||\, R_3\right)C = \dfrac{R_3}{6}C$

Let $C = 1$ μF. Then $R_3 = 60$ kΩ, $R_1 = 30$ kΩ and $R_2 = 30$ kΩ.

DP 8-2 Design the circuit in Figure DP 8.2 so that $i(t)$ makes the transition from $i(t) = 1$ mA to $i(t) = 4$ mA in 10 ms after the switch is closed. Assume that the circuit is at steady state before the switch is closed. Also assume that the transition will be complete after 5 time constants.

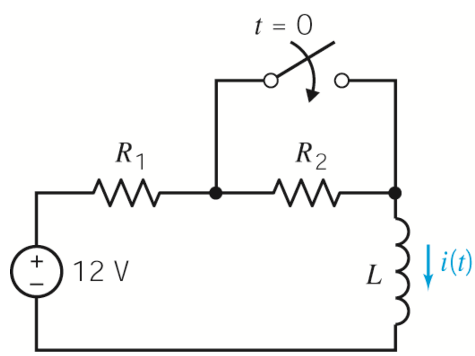


Figure DP 8.2

Solution:

steady state response when the switch is open: $0.001 = \dfrac{12}{R_1+R_2} \quad \Rightarrow \quad R_1 + R_2 = 12 \text{ k}\Omega$.

steady state response when the switch is closed: $0.004 = \dfrac{12}{R_1} \quad \Rightarrow \quad R_1 = 3 \text{ k}\Omega$.

Therefore, $R_2 = 9$ kΩ.

$$10 \text{ ms} = 5\,\tau = 5\left(\frac{L}{R_1+R_2}\right) = \frac{L}{2400} \quad \Rightarrow \quad L = 240 \text{ H}$$

DP 8-3 The switch in Figure DP 8.3 closes at time 0, $2\Delta t$, $4\Delta t$, …$2k\Delta t$ and opens at times Δt, $3\Delta t$, $5\Delta t$….$(2k + 1)\Delta t$. When the switch closes, $v(t)$ makes the transition from $v(t) = 0$ V to $v(t) = 5$ V. Conversely, when the switch opens, $v(t)$ makes the transition from $v(t) = 5$ V to $v(t) = 0$ V. Suppose we require that $\Delta t = 5\tau$ so that one transition is complete before the next one begins. (a) Determine the value of C required so that $\Delta t = 1$ μs. (b) How large must Δt be when $C = 2$ μF?

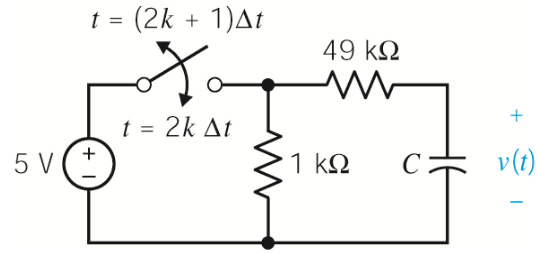


Figure DP 8.3

Answer: (a) $C = 4$ pF; (b) $\Delta t = 0.5$s

Solution:

$R_t = 50$ kΩ when the switch is open and $R_t = 49$ kΩ ≈ 50 kΩ when the switch is closed so use $R_t = 50$ kΩ.

(a) $\Delta t = 5\,R_t\,C \quad \Rightarrow \quad C = \dfrac{10^{-6}}{5\left(50\times 10^3\right)} = 4 \text{ pF}$

(b) $\Delta t = 5\left(50\times 10^3\right)\left(2\times 10^{-6}\right) = 0.5 \text{ s}$

DP 8-4 The switch in Figure DP 8.3 closes at time 0, $2\Delta t$, $4\Delta t$, …$2k\Delta t$ and opens at times Δt, $3\Delta t$, $5\Delta t$….$(2k + 1)\Delta t$. When the switch closes, $v(t)$ makes the transition from $v(t) = 0$ V to $v(t) = 5$ V. Conversely, when the switch opens, $v(t)$ makes the transition from $v(t) = 5$ V to $v(t) = 0$ V. Suppose we require that one transition be 95 percent complete before the next one begins. (a) Determine the value of C required so that $\Delta t = 1$ μs. (b) How large must Δt be when $C = 2$ μF?

Hint: Show that $\Delta t = -\,\tau \ln(1 - k)$ is required for the transition to be 100 k percent complete.

Answer: (a) $C = 6.67$ pF; (b) $\Delta t = 0.3$ s

Solution: $R_t = 50$ kΩ when the switch is open and $R_t = 49$ kΩ ≈ 50 kΩ when the switch is closed so use $R_t = 50$ kΩ.

When the switch is open: $\quad 5\,e^{-\Delta t\,/\,\tau} = (1-k)\,5 \quad \Rightarrow \quad \ln\left(1-k\right) = -\dfrac{\Delta t}{\tau} \quad \Rightarrow \quad \Delta t = -\tau \ln\left(1-k\right)$

When the switch is open: $\quad 5 - 5\,e^{-\Delta t\,/\,\tau} = k\,5 \quad \Rightarrow \quad \Delta t = -\tau \ln\left(1-k\right)$

(a) $C = \dfrac{10^{-6}}{-\ln\left(1-.95\right)\left(50\times 10^3\right)} = 6.67 \text{ pF}$

(b) $\Delta t = -\ln(1-.95)\left(50\times 10^3\right)\left(2\times 10^{-6}\right) = 0.3 \text{ s}$

DP 8-5 A laser trigger circuit is shown in Figure DP 8.5. In order to trigger the laser, we require 60 mA < $|i|$ < 180 mA for $0 < t < 200\ \mu$s. Determine a suitable value for R_1 and R_2.

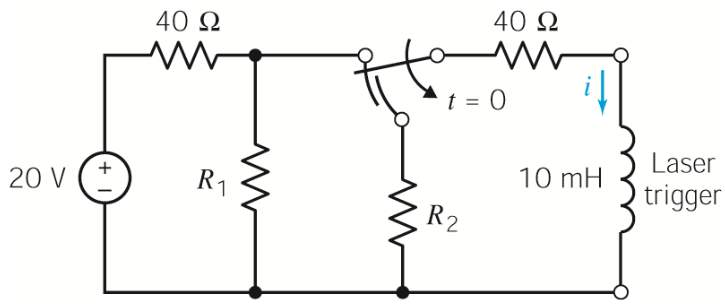
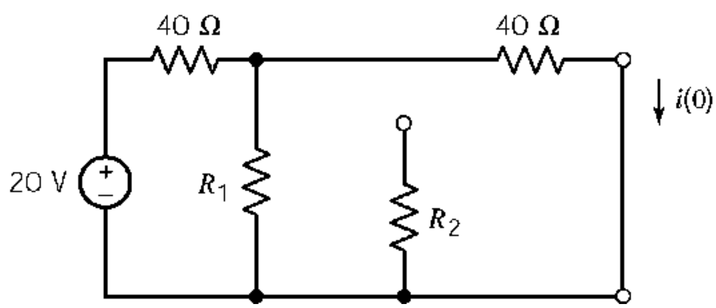


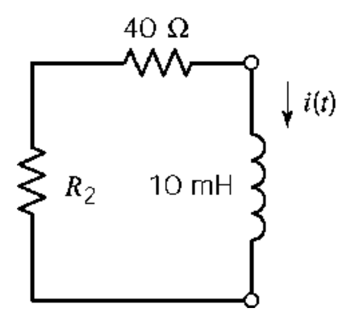
Figure DP 8.5

Solution:



$$i(0) = \frac{20}{40 + \frac{40R_1}{40+R_1}} \times \frac{R_1}{R_1+40}$$

For $t > 0$:



$$i(t) = i(0)\, e^{-t/\tau} \quad \text{where } \tau = \frac{L}{R_t} = \frac{10^{-2}}{40+R_2}$$

At $t < 200\mu s$ we need $i(t) > 60$ mA and $i(t) < 180$ mA

First let's find a value of R_2 to cause $i(0) < 180$ mA.

Try $\underline{R_2 = 40\ \Omega}$. Then $i(0) = \frac{1}{6}$ A = 166.7 mA so $i(t) = 0.1667\ e^{-t/\tau}$.

Next, we find a value of R_2 to cause $i(0.0002) > 60$ mA.

Try $\underline{R_2 = 10\Omega}$, then $\tau = \frac{10^{-2}}{50} = 0.2 \text{ ms} = \frac{1}{5000}$ s.

$i(0.0002) = 166.7\times10^{-3} e^{-5000\times0.0002} = 166.7\times10^{-3} e^{-1} = 61.3$ mA

DP 8-6 Fuses are used to open a circuit when excessive current flows (Wright, 1990). One fuse is designed to open when the power absorbed by R exceeds 10 W for 0.5 s. Consider the circuit shown in Figure DP 8.6. The input is given by

$$v_s = A[u(t) - u(t - 0.75)] \text{ V}.$$

Assume that $i_L(0^-) = 0$. Determine the largest value of A that will not cause the fuse to open.

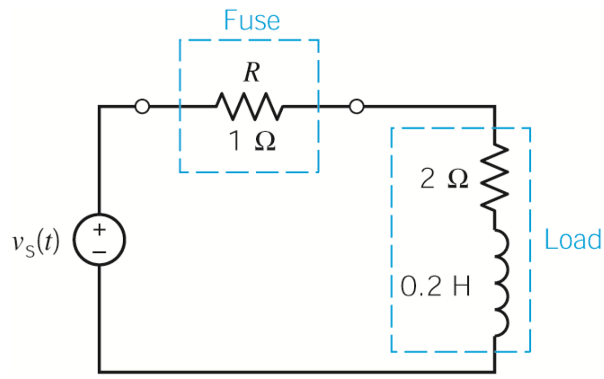
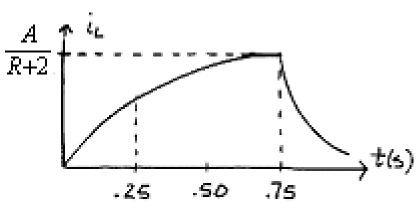


Figure DP 8.6

Solution:

The current waveform will look like this:



We only need to consider the rise time:

$$i_L(t) = \frac{V_s}{R+2}(1-e^{-t/\tau}) = \frac{A}{R+2}(1-e^{-t/\tau})$$

where

$$\tau = \frac{L}{R_t} = \frac{0.2}{3} = \frac{1}{15} \text{ s}$$

$$\therefore i_L(t) = \frac{A}{3}(1-e^{-15t})$$

Now find A so that $i_L^2 R_{fuse} \geq 10$ W during $0.25 \leq t \leq 0.75$ s

$\therefore$ we want $[i_L^2(0.25)]R_{fuse} = 10$ W $\Rightarrow \frac{A^2}{9}(1-e^{-15(.25)})^2(1)=10 \Rightarrow \underline{A = 9.715 \text{ V}}$

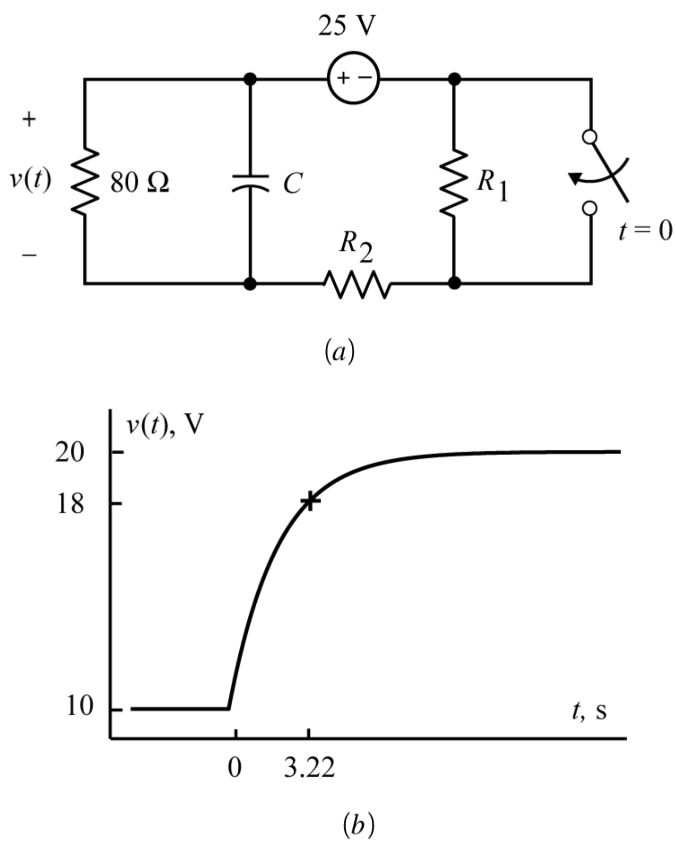


Figure DP 8-7

DP 8-7 Design the circuit in Figure DP 8-7(a) to have the response in Figure DP 8-7 (b) by specifying the values of C, R_1 and R_2.

Solution:

The voltage $v(t)$ is represented by an equation of the form $v(t) = \begin{cases} D & \text{for } t < 0 \\ E + F\,e^{-at} & \text{for } t > 0 \end{cases}$

where D, E, F and a are unknown constants. The constants D, E and F are described by

$$D = v(t) \text{ when } t < 0, \quad E = \lim_{t\to\infty} v(t), \quad E + F = \lim_{t\to 0+} v(t)$$

From the plot, we see that

$$D = 10,\ E = 20,\ \text{and}\ E + F = 10\ \text{V}$$

Consequently,

$$v(t) = \begin{cases} 10 & \text{for } t < 0 \\ 20 - 10\,e^{-at} & \text{for } t > 0 \end{cases}$$

To determine the value of a, we pick a time when the circuit is not at steady state. One such point is labeled on the plot. We see $v(3.22) = 18\ \text{V}$, that is, the value of the voltage is 18 volts at time 3.22 seconds. Substituting these into the equation for $v(t)$ gives

$$18 = 20 - 10e^{-a(3.22)} \quad \Rightarrow \quad a = \frac{\ln(0.2)}{-3.22} = 0.5$$

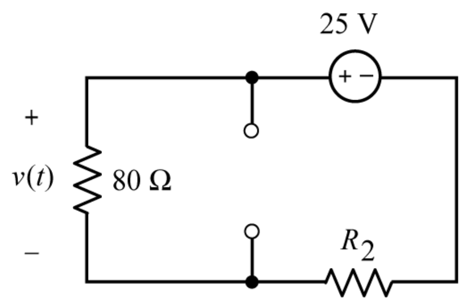
Consequently

$$v(t) = \begin{cases} 10 & \text{for } t < 0 \\ 20 - 10\, e^{-0.5t} & \text{for } t > 0 \end{cases}$$

Now let's turn our attention to the circuit. When the circuit is at steady state, the capacitor acts like an open circuit.

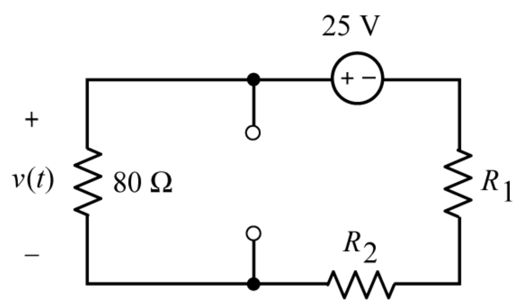
After $t = 0$, the switch is closed and the steady state voltage is determined from the plot to be $v(t) = E = 20$ V. On the right we see the circuit that results from replacing the capacitor by an open circuit and the switch by a short circuit. Using voltage division gives

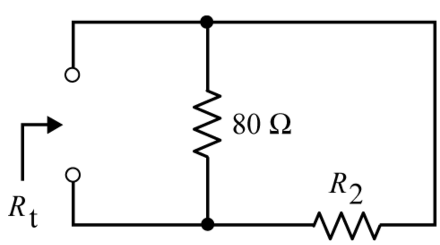
$$20 = \frac{80}{80 + R_2}(25) \quad \Rightarrow \quad R_2 = 20\ \Omega$$



Before $t = 0$, the switch is open and the steady state voltage is determined from the plot to be $v(t) = E + F = 10$ V. On the right we see the circuit that results from replacing the capacitor by an open circuit and the switch by an open circuit. Using voltage division gives

$$10 = \frac{80}{80 + R_1 + R_2}(25) = \frac{80}{100 + R_1}(25) \quad \Rightarrow \quad R_1 = 100\ \Omega$$





Recalling that $a = 0.5$ from the plot, consider the time constant $2 = \frac{1}{a} = \tau = C\,R_t$. After $t = 0$, the Thevenin resistance of the part of the circuit connected to the capacitor is $R_t = 80 \,||\, R_2 = 80 \,||\, 20 = 16\ \Omega$.

Then $C = \frac{2}{R_t} = \frac{2}{16} = 0.125\ \text{F}$.

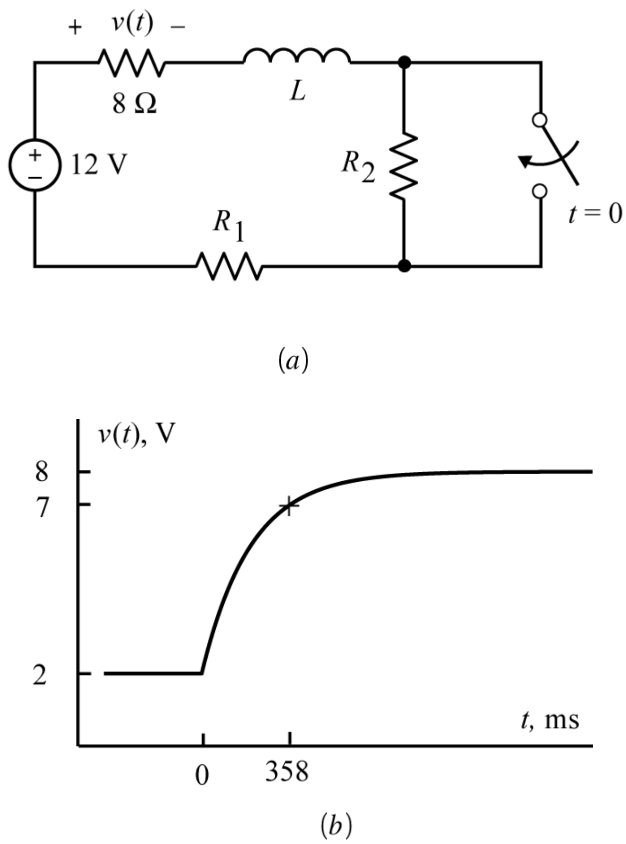


Figure DP 8-8

DP 8-8 Design the circuit in Figure DP 8-8(*a*) to have the response in Figure DP 8-8 (*b*) by specifying the values of *L*, R_1 and R_2.

Solution:

The voltage *v*(*t*) is represented by an equation of the form $v(t) = \begin{cases} D & \text{for } t < 0 \\ E + F\,e^{-at} & \text{for } t > 0 \end{cases}$

where *D*, *E, F* and *a* are unknown constants. The constants *D*, *E* and *F* are described by

$$D = v(t) \;\text{ when } t < 0, \quad E = \lim_{t\to\infty} v(t), \quad E + F = \lim_{t\to 0+} v(t)$$

From the plot, we see that

$$D = 2,\; E = 8,\; \text{and}\; E + F = 2\;\text{ V}$$

Consequently,

$$v(t) = \begin{cases} 2 & \text{for } t < 0 \\ 8 - 6\,e^{-at} & \text{for } t > 0 \end{cases}$$

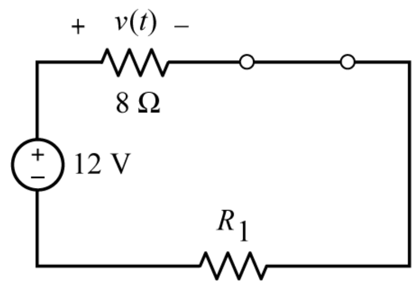
To determine the value of a, we pick a time when the circuit is not at steady state. One such point is labeled on the plot. We see $v(0.358) = 7$ V, that is, the value of the voltage is 7 volts at time 0.358 seconds. Substituting these into the equation for $v(t)$ gives

$$7 = 8 - 6e^{-a(0.358)} \quad \Rightarrow \quad a = \frac{\ln\left(\frac{1}{6}\right)}{-0.358} = 5$$

Consequently

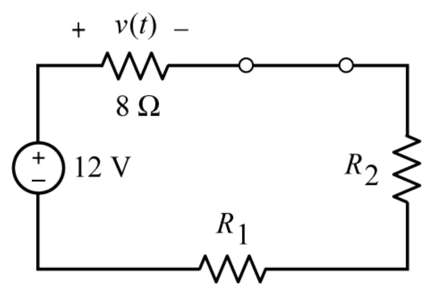
$$v(t) = \begin{cases} 2 & \text{for } t < 0 \\ 8 - 6\,e^{-5t} & \text{for } t > 0 \end{cases}$$

Now let's turn our attention to the circuit. When the circuit is at steady state, the inductor acts like a short circuit.



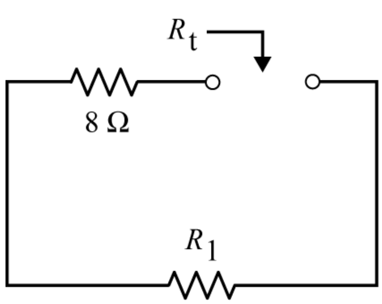
After t = 0, the switch is closed and the steady state voltage is determined from the plot to be $v(t) = E = 8$ V. On the right we see the circuit that results from replacing the inductor by a short circuit and the switch by a short circuit. Using voltage division gives

$$8 = \frac{8}{8 + R_1}(12) \quad \Rightarrow \quad R_1 = 4\ \Omega$$



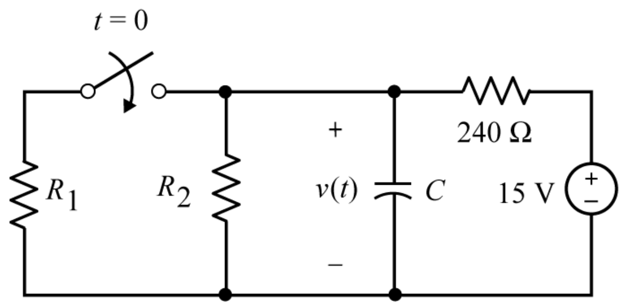
Before t = 0, the switch is open and the steady state voltage is determined from the plot to be $v(t) = E + F = 2$ V. On the right we see the circuit that results from replacing the inductor by a short circuit and the switch by an open circuit. Using voltage division gives

$$2 = \frac{8}{8 + R_1 + R_2}(12) = \frac{8}{12 + R_2}(12) \quad \Rightarrow \quad R_2 = 36\ \Omega$$

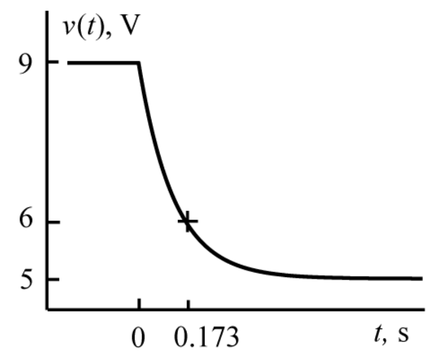


Recalling that a = 5 from the plot, consider the time constant $0.2 = \frac{1}{a} = \tau = \frac{L}{R_t}$. After t = 0, the Thevenin resistance of the part of the circuit connected to the capacitor is $R_t = 8 + R_2 = 8 + 4 = 12\ \Omega$.

Then $L = 0.2\,R_t = 2.4$ H.



(a)



(b)

Figure DP 8-9

DP 8-9 Design the circuit in Figure DP 8-9(*a*) to have the response in Figure DP 8-9 (*b*) by specifying the values of *C*, R_1 and R_2.

Solution:

The voltage *v*(*t*) is represented by an equation of the form $v(t) = \begin{cases} D & \text{for } t < 0 \\ E + F\, e^{-at} & \text{for } t > 0 \end{cases}$

where *D*, *E, F* and *a* are unknown constants. The constants *D*, *E* and *F* are described by

$$D = v(t) \text{ when } t < 0, \quad E = \lim_{t \to \infty} v(t), \quad E + F = \lim_{t \to 0+} v(t)$$

From the plot, we see that $D = 9,\; E = 5, \text{ and } E + F = 9 \text{ V}$

Consequently, $$v(t) = \begin{cases} 9 & \text{for } t < 0 \\ 5 + 4\, e^{-at} & \text{for } t > 0 \end{cases}$$

To determine the value of a , we pick a time when the circuit is not at steady state. One such point is labeled on the plot. We see $v(0.173) = 6 \text{ V}$, that is, the value of the voltage is 6 volts at time 0.173 seconds. Substituting these into the equation for $v(t)$ gives

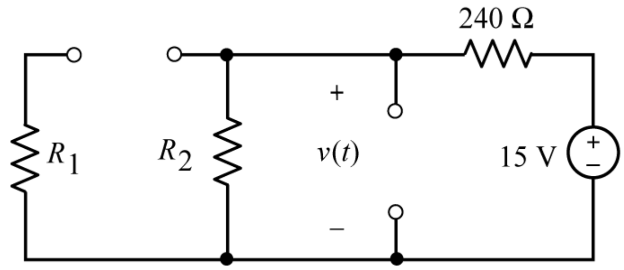
$$6 = 5 + 4e^{-a(0.173)} \quad \Rightarrow \quad a = \frac{\ln(0.25)}{-0.173} = 8$$

Consequently

$$v(t) = \begin{cases} 9 & \text{for } t < 0 \\ 5 + 4e^{-8t} & \text{for } t > 0 \end{cases}$$

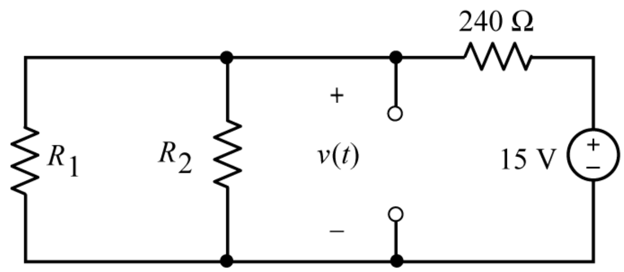
Now let's turn our attention to the circuit. When the circuit is at steady state, the capacitor acts like an open circuit.

Before $t = 0$, the switch is open and the steady state voltage is determined from the plot to be $v(t) = E + F = 9$ V. On the right we see the circuit that results from replacing the capacitor by an open circuit and the switch by an open circuit. Using voltage division gives



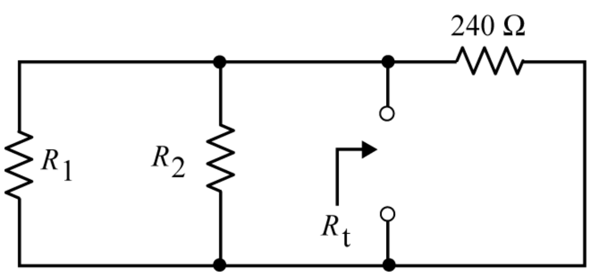
$$9 = \frac{R_2}{240 + R_2}(15) \quad \Rightarrow \quad R_2 = 360\ \Omega$$

After $t = 0$, the switch is closed and the steady state voltage is determined from the plot to be $v(t) = E = 5$ V. On the right we see the circuit that results from replacing the capacitor by an open circuit and the switch by a short circuit. Using voltage division gives



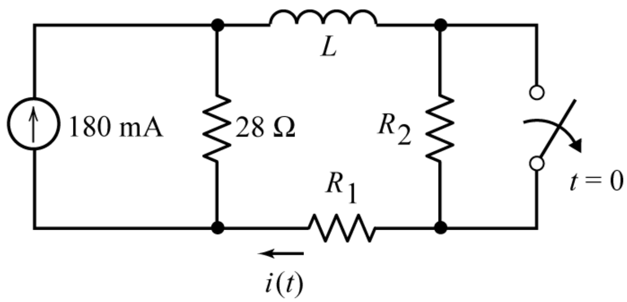
$$5 = \frac{R_1 \| R_2}{240 + R_1 \| R_2}(15) \quad \Rightarrow \quad R_1 \| R_2 = 120\ \Omega$$

$$R_1 \| R_2 = R_1 \| 360 = 120\ \Omega \quad \Rightarrow \quad R_1 = 180\ \Omega$$

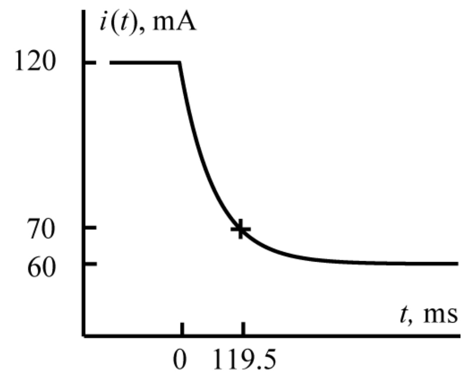


Recalling that $a = 8$ from the plot, consider the time constant $0.125 = \frac{1}{a} = \tau = C\,R_t$. After $t = 0$, the Thevenin resistance of the part of the circuit connected to the capacitor is $R_t = 240 \| R_1 \| R_2 = 240 \| 180 \| 360 = 80\ \Omega$. Then

$$C = \frac{0.125}{R_t} = \frac{0.125}{80} = 0.0015625\ \text{F} = 1.5625\ \text{mF}.$$



(*a*)



(*b*)

Figure DP 8-10

DP 8-10 Design the circuit in Figure DP 8-10(*a*) to have the response in Figure DP 8-10 (*b*) by specifying the values of L, R_1 and R_2.

Solution:

The current $i(t)$ is represented by an equation of the form $i(t) = \begin{cases} D & \text{for } t < 0 \\ E + F\,e^{-at} & \text{for } t > 0 \end{cases}$

where D, E, F and a are unknown constants. The constants D, E and F are described by

$$D = i(t) \text{ when } t < 0, \quad E = \lim_{t \to \infty} i(t), \quad E + F = \lim_{t \to 0+} i(t)$$

From the plot, we see that

$$D = 120, \; E = 60, \text{ and } E + F = 120 \text{ mA}$$

Consequently,

$$i(t) = \begin{cases} 120 \text{ mA} & \text{for } t < 0 \\ 60 + 60\,e^{-at} \text{ mA} & \text{for } t > 0 \end{cases}$$

To determine the value of a, we pick a time when the circuit is not at steady state. One such point is labeled on the plot. We see $i(0.1195) = 70$ mA, that is, the value of the current is 70 mA at time 0.1195 seconds. Substituting these into the equation for $i(t)$ gives

$$70 = 60 + 60e^{-a(0.1195)} \quad \Rightarrow \quad a = \frac{\ln\left(\frac{1}{6}\right)}{-0.1195} = 15$$

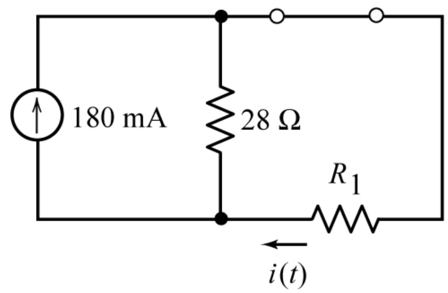
Consequently

$$i(t) = \begin{cases} 120 \text{ mA} & \text{for } t < 0 \\ 60 + 60\,e^{-15t} \text{ mA} & \text{for } t > 0 \end{cases}$$

Now let's turn our attention to the circuit. When the circuit is at steady state, the inductor acts like a short circuit.

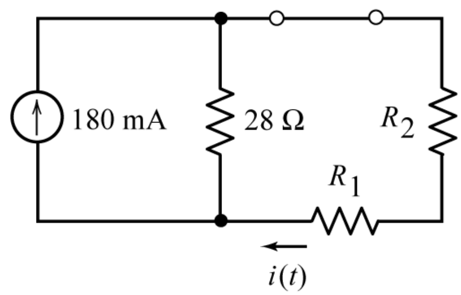
Before $t = 0$, the switch is closed and the steady state current is determined from the plot to be $i(t) = E + F = 120$ mA. On the right we see the circuit that results from replacing the inductor by a short circuit and the switch by a short circuit. Using current division gives

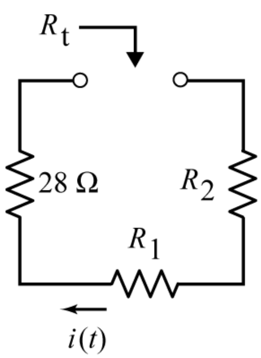
$$120 = \frac{28}{28 + R_1}(180) \quad \Rightarrow \quad R_1 = 14\ \Omega$$



After $t = 0$, the switch is open and the steady state voltage is determined from the plot to be $v(t) = E + F = 2$ V. On the right we see the circuit that results from replacing the inductor by a short circuit and the switch by an open circuit. Using current division gives

$$60 = \frac{28}{28 + R_1 + R_2}(180) = \frac{28}{28 + 14 + R_2}(180) \quad \Rightarrow \quad R_2 = 42\ \Omega$$





Recalling that $a = 15$ from the plot, consider the time constant $\frac{1}{15} = \frac{1}{a} = \tau = \frac{L}{R_t}$. After $t = 0$, the Thevenin resistance of the part of the circuit connected to the capacitor is $R_t = 28 + R_1 + R_2 = 28 + 14 + 42 = 84\ \Omega$.

Then $L = \frac{R_t}{15} = \frac{84}{15} = 5.6$ H .

Chapter 9 - Complete Response of Circuits with Two Energy Storage Elements

Exercises

Exercise 9.2-1 Find the second-order differential equation for the circuit shown in Figure E 9.2-1 in terms of i using the direct method.

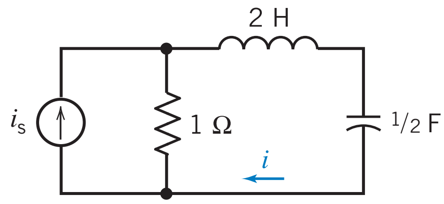
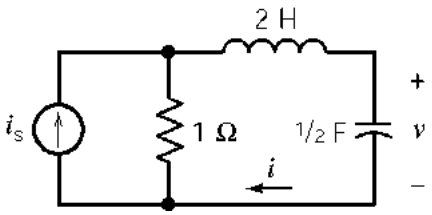


Figure E 9.2-1

Answer: $\dfrac{d^2i}{dt^2}+\dfrac{1}{2}\dfrac{di}{dt}+i=\dfrac{1}{2}\dfrac{di_s}{dt}$

Solution:



$$\text{KVL a: } 2\,{}^{di}\!/_{dt} + v + 1(i-i_s) = 0$$

$$\Rightarrow v = -2\,{}^{di}\!/_{dt} - i + i_s$$

$$i = {}^{1}\!/_{2}\,{}^{dv}\!/_{dt} = {}^{1}\!/_{2}\,{}^{d}\!/_{dt}\left(-2\,{}^{di}\!/_{dt}-i+i_s\right)$$

$$= {}^{1}\!/_{2}\,{}^{di_s}\!/_{dt} - {}^{1}\!/_{2}\,{}^{di}\!/_{dt} - \frac{d^2i}{dt^2}$$

$$\therefore \frac{d^2i}{dt^2} + \frac{1}{2}\frac{di}{dt} + i = \frac{1}{2}\frac{di_s}{dt}$$

Exercise 9.2-2 Find the second-order differential equation for the circuit shown in Figure E 9.2-2 in terms of v using the operator method.

Answer: $\frac{d^2v}{dt^2} + 2\frac{di}{dt} + 2v = 2\frac{di_s}{dt}$

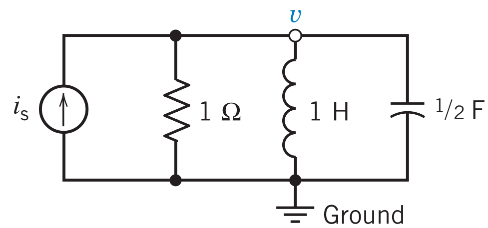
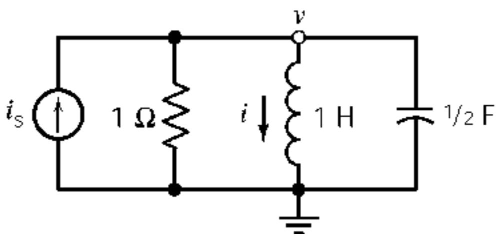


Figure E 9.2-2

Solution:



KCL at v : using $s = d/dt$

$$\frac{v}{1} + i + \frac{1}{2}sv = i_s \quad (1)$$

also $v = si$ (2) Solving for i in (1) & plugging into (2)

yields $s^2v + 2sv + 2v = 2si_s$ *or* $\frac{d^2v}{dt} + 2\frac{dv}{dt} + 2v = 2\frac{di_s}{dt}$

Exercise 9.3-1 Find the characteristic equation and the natural frequencies for the circuit shown in Figure E 9.3-1.

Answer: $s^2 + 7s + 10 = 0$; $s_1 = -2$, $s_2 = -5$

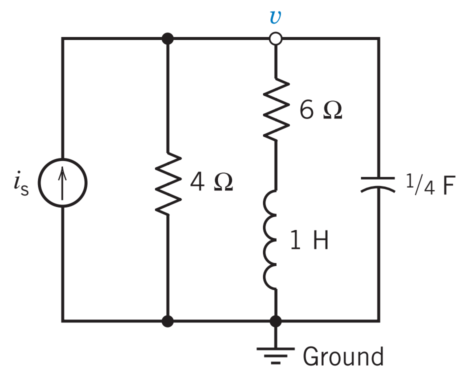
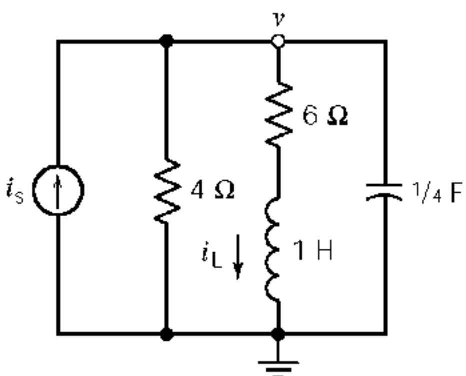


Figure E 9.3-1

Solution:



KCL at the top node : $\dfrac{v}{4} + i_L + \frac{1}{4}\,\frac{dv}{dt} = i_s$ (1)

KVL for the right mesh : $v = 6i_L + \frac{di_L}{dt}$ (2)

Plugging (2) into (1) yields

$$\frac{d^2 i_L}{dt^2} + 7\frac{di_L}{dt} + 10 i_L = 4 i_s$$

$\therefore$ characteristic equation $\Rightarrow$ $\underline{s^2 + 7s + 10 = 0}$ & natural frequencies $\Rightarrow$ $\underline{s = -2,\ -5}$

Exercise 9.4-1 Find the natural response of the *RLC* circuit of Figure 9.4-1 when $R = 6\ \Omega$, $L = 7$ H, and $C = 1/42$ F. The initial conditions are $v(0) = 0$ and $i(0) = 10$ A.

Answer: $v_n(t) = -84(e^{-t} - e^{-6t})$ V

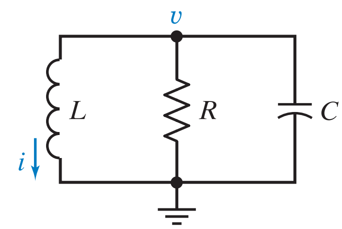


Figure 9.4-1

Solution:

This is a parallel *RLC* circuit with

$$\alpha = \frac{1}{2RC} = \frac{1}{2(6)(1/42)} = 7/2 \quad \text{and} \quad \omega_o^2 = \frac{1}{LC} = \frac{1}{(7)(1/42)} = 6$$

The roots of the characteristic equation are

$$s = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -7/2 \pm \sqrt{\left(7/2\right)^2 - 6} = -1,\ -6$$

so the natural response is

$$v_n(t) = A_1 e^{-t} + A_2 e^{-6t}$$

Need $v_n(0)$ and $\left.\frac{dv_n}{dt}\right|_{t=0}$ to evaluate A_1 & A_2. We are given $v_n(0) = 0$.

Apply KCL at the top node to get

$$i(t) + \frac{v(t)}{6} + \frac{1}{42}\frac{d}{dt}v(t) = 0 \quad \Rightarrow \quad \frac{d}{dt}v(t) = -\left(7\,v(t) + 42\,i(t)\right)$$

At time t=0,

$$\left.\frac{d}{dt}v(t)\right|_{t=0} = -\left(7\,v(0) + 42\,i(0)\right) = -\left(7\times 0 + 42\times 10\right) = -420\ \frac{\text{V}}{\text{s}}$$

So

$$\left.\begin{array}{l} v_n(0) = 0 = A_1 + A_2 \\ \left.\frac{dv_n}{dt}\right|_{t=0} = -420 = -A_1 - 6A_2 \end{array}\right\} A_1 = -84\ ,\ A_2 = 84$$

and

$$\underline{v_n(t) = -84e^{-t} + 84e^{-6t}\ \text{V}}$$

Exercise 9.5-1 A parallel *RLC* circuit has $R = 10\ \Omega$, $C = 1$ mF, $L = 0.4$ H, $v(0) = 8$ V, and $i(0) = 0$. Find the natural response $v_n(t)$ for $t < 0$.

Answer: $v_n(t) = e^{-50t}(8 - 400t)$ V

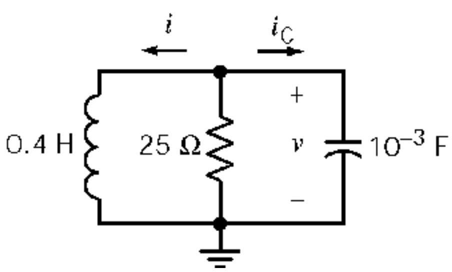
Solution:

For parallel RLC

$$\alpha = \frac{1}{2RC} = \frac{1}{2(10)(10^{-3})} = 50,\ \ \omega_o^2 = \frac{1}{LC} = \frac{1}{(0.4)(10^{-3})} = 2500$$

$$\therefore s = -50 \pm \sqrt{(50)^2 - 2500} = -50,\ -50$$

$$\therefore v_n(t) = A_1 e^{-50t} + A_2 t e^{-50t}$$



with $i(0^+) = 0\ \ \&\ \ v(0^+) = 8$ V

$$i_C(0^+) = \frac{-v(0^+)}{10\Omega} = -0.8\ \text{V}$$

$$\left.\frac{dv}{dt}\right|_{t=0^+} = \frac{i_C(0^+)}{C} = -800\ \frac{\text{V}}{\text{s}}$$

At $\underline{t = 0^+}$

$$v_n(0) = 8 = A_1 \Rightarrow v_n(t) = 8e^{-50t} + A_2 t e^{-50t}$$

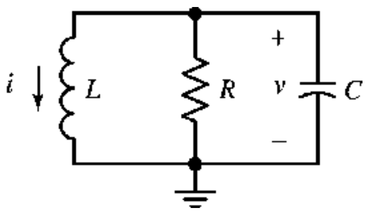
$$\frac{dv(0)}{dt} = -800 = -400 + A_2 \Rightarrow A_2 = -400$$

$$\therefore \underline{v_n(t) = 8e^{-50t} - 400t\, e^{-50t}\ \text{V}}$$

Exercise 9.6-1 A parallel *RLC* circuit has $R = 62.5\ \Omega$, $L = 10$ mH, $C = 1\ \mu$F, $v(0) = 10$ V, and $i(0) = 80$ mA. Find the natural response $v_n(t)$ for $t > 0$.

Answer: $v_n(t) = e^{-8000t}\,[10 \cos 6000t - 26.7 \sin 6000t]$ V

Solution:

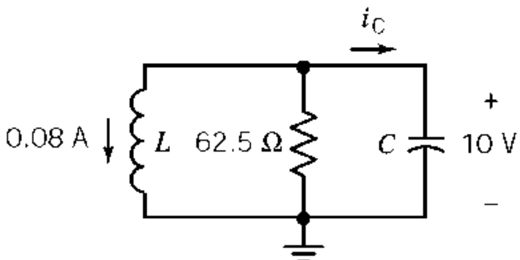


$$\alpha = \frac{1}{2RC} = \frac{1}{2(62.5)(10^{-6})} = 8000$$

$$\omega_0^2 = \frac{1}{LC} = \frac{1}{(.01)(10^{-6})} = 10^8$$

$$\therefore\ s = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -8000 \pm \sqrt{(8000)^2 - 10^8} = -8000 \pm j\ 6000$$

$$\therefore\ v_n(t) = e^{-8000t}[A_1 \cos 6000\,t + A_2 \sin 6000t\,]$$



$$\text{KCL at top} : 0.08 + \frac{10}{62.5} + i_C = 0$$

$$\Rightarrow i_C(0^+) = -0.24\ \text{A}$$

$$\therefore\ \frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C} = -2.4\times 10^{+5}\ \ \text{V/s}$$

$$v_n(0) = 10 = A_1$$

$$\frac{dv_n(0)}{dt} = -2.4\times 10^5 = 6000A_2 - 8000(10) \Rightarrow A_2 = -26.7$$

$$\therefore\ \underline{v_n(t) = e^{-8000t}\ [10\ \cos\ 6000\ t - 26.7\ \sin\ 6000t\ ]\ \text{V}}$$

Exercise 9.7-1 A circuit is described for $t > 0$ by the equation

$$\frac{d^2 i}{dt^2} + 9\frac{di}{di} + 20i = 6i_s$$

where $i_s = 6 + 2t$ A. Find the forced response i_f for $t > 0$.

Answer: $i_f = 1.53 + 0.6t$ A

Solution:

$$i'' + 9i' + 20i = 36 + 12t$$

Try $i_f = A + Bt$ & plug into above

$$0 + 9B + 20(A + Bt) = 36 + 12t$$

Equating the constant coefficients and the coefficients of t gives

$$20Bt = 12t \Rightarrow B = 0.6 \quad \text{and} \quad 9B + 20A = 36 \Rightarrow A = 1.53$$

$$\therefore \underline{i_f = 1.53 + 0.6t} \text{ A}$$

Exercise 9.9-1 Find $v_2(t)$ for $t > 0$ for the circuit of Figure E 9.9-1. Assume there is no initial stored energy.

Answer: $v_2(t) = -15e^{-2t} + 6e^{-4t} - e^{-6t} + 10$ V

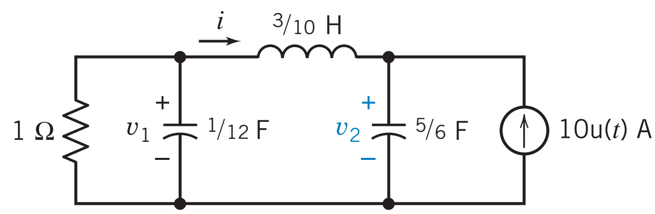
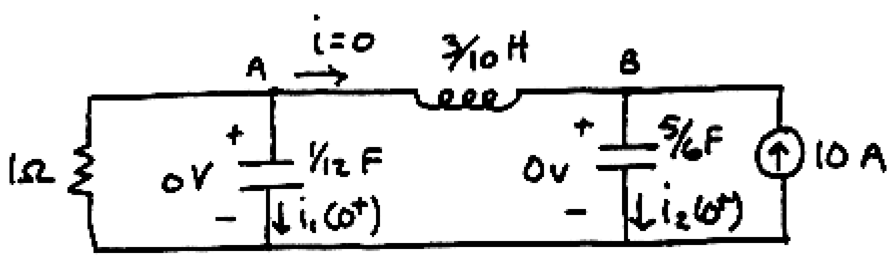


Figure E 9.9-1

Solution:

no initial stored energy $\Rightarrow v_1(0^+) = v_2(0^+) = i(0^+) = 0$

$\underline{t = 0^+}$

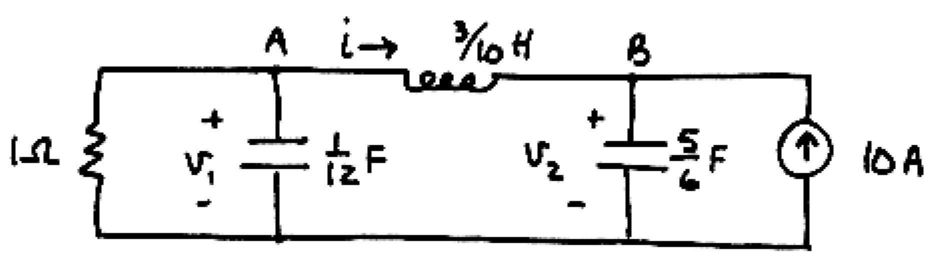


$$\text{KVL}: -0 + \tfrac{3}{10}\frac{di(0^+)}{dt} + 0 = 0 \quad \Rightarrow \quad \frac{di(0^+)}{dt} = 0$$

$$\text{KCL at A}: \frac{0\text{V}}{1\Omega} + i_1(0^+) + 0 = 0 \quad \Rightarrow \quad \frac{dv_1(0^+)}{dt} = 0$$

$$\text{KCL at B}: -0 + i_2(0^+) - 10 = 0 \Rightarrow i_2(0^+) = 5/6\frac{dv_2(0^+)}{dt} = 10 \quad \Rightarrow \quad \frac{dv_2(0^+)}{dt} = 12\text{ V/s}$$

$\underline{t > 0}$



$$\text{KCL at A}: \frac{v_1}{1} + \frac{1}{12}v_1' + i = 0 \qquad (1)$$

$$\text{KCL at B}: -i + (5/6)v_2' = 10 \qquad (2)$$

$$\text{KVL}: -v_1 + (3/10)i' + v_2 = 0 \qquad (3)$$

Eliminating i from (1) & (3) yields

$$v_1 + \frac{1}{12}v'_1 + (5/6)v_2' - 10 = 0 \qquad (4)$$

$$-v_1 + \frac{3}{10}\left(\frac{5}{6}v_2''\right) + v_2 = 0 \qquad (5)$$

From (5)

$$v_1 = v_2 + \frac{1}{4}v_2'' \quad \Rightarrow \quad v_1' = v_2'' + (1/4)v_2''$$

Now substituting into (4) yields

$$v_2' + \frac{1}{4}v_2'' + \frac{1}{12}\left(v_2' + \frac{1}{4}v_2'''\right) + \frac{5}{6}v_2' = 10$$

$$\underline{v_2''' + 12v_2'' + 44v_2' + 48v_2 = 480}$$

Natural Response: $v_{2n}: s^3 + 12s^2 + 44s + 48 = 0 \quad \Rightarrow s = -2, -4, -6$

$\therefore \quad v_{2n} = A_1e^{-2t} + A_2e^{-4t} + A_3e^{-6t}$

Forced Response: v_{2f} : try $v_{2f} = B$ and plug into Diff. Eq. $\Rightarrow B = 10$

Complete Response: $v_2(t) = A_1e^{-2t} + A_2e^{-4t} + A_3e^{-6t} + 10$

Recall $v_2(0^+) = 0$, $\dfrac{dv_2(0^+)}{dt} = 12$ V/s, then from (5) $\dfrac{d^2v_2(0^+)}{dt^2} = 4[v_1(0^+) - v_2(0^+)] = 0$.

$$v_2(0^+) = 0 = A_1 + A_2 + A_3 + 10 \qquad (6)$$

$$\frac{dv_2(0^+)}{dt} = 12 = -2A_1 - 4A_2 - 6A_3 \qquad (7)$$

$$\frac{d^2v_2(0^+)}{dt^2} = 0 = 4A_1 + 16A_2 + 36A_3 \qquad (8)$$

Solving (6)-(80 simultaneously gives $A_1 = -15$, $A_2 = 6$, $A_3 = -1$

Then

$$\underline{v_2(t) = -15e^{-2t} + 6e^{-4t} - e^{-6t} + 10 \text{ V}}$$

Exercise 9.10-1 A parallel *RLC* circuit has $L = 0.1$ H and $C = 100$ mF. Determine the roots of the characteristic equation and plot them on the *s*-plane when (a) $R = 0.4\ \Omega$ and (b) $R = 1.0\ \Omega$.

Answer: (a) $s = -5, -20$ (Figure E 9.10-1)

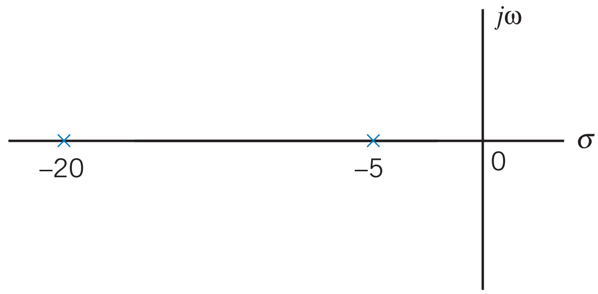


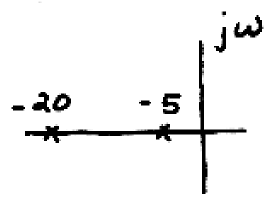
Figure E 9.10-1

Solution:

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0 \quad \text{and} \quad L = 0.1,\ C = 0.1 \quad \Rightarrow \quad s^2 + \frac{10}{R}s + 100 = 0$$

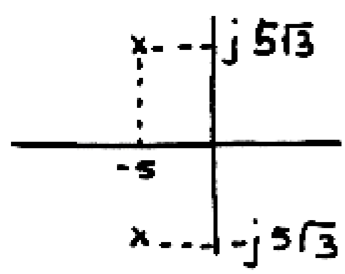
a)

$$R = 0.4\ \Omega \quad \Rightarrow \quad s^2 + 25s + 100 = 0$$
$$s = -5,\ -20$$



b)

$$R = 1\Omega \quad \Rightarrow \quad s^2 + 10s + 100 = 0$$
$$s = -5 \pm j5\sqrt{3}$$



Section 9-2: Differential Equations for Circuits with Two Energy Storage Elements

P 9.2-1 Find the differential equation for the circuit shown in Figure P 9.2-1 using the direct method.

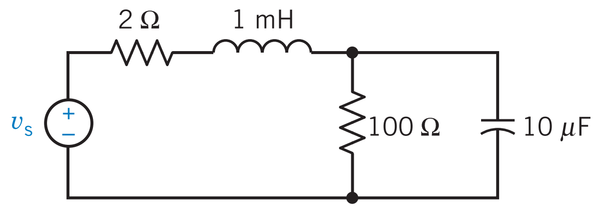
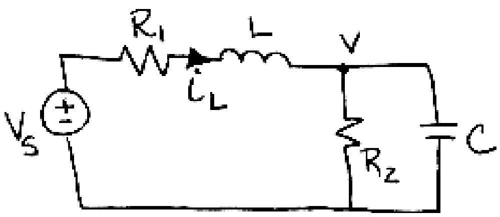


Figure P 9.2-1

Solution:



$$\text{KCL:}\quad i_L = \frac{v}{R_2} + C\frac{dv}{dt}$$

$$\text{KVL:}\quad V_s = R_1 i_L + L\frac{di_L}{dt} + v$$

$$v_s = R_1\left[\frac{v}{R_2} + C\frac{dv}{dt}\right] + \frac{L}{R_2}\frac{dv}{dt} + LC\frac{d^2v}{dt^2} + v$$

$$v_s = \left[\frac{R_1}{R_2} + 1\right]v + \left[R_1C + \frac{L}{R_2}\right]\frac{dv}{dt} + [LC]\frac{d^2v}{dt^2}$$

$$R_1 = 2\Omega,\ R_2 = 100\Omega,\ L = 1\text{mH},\ C = 10\mu\text{F}$$

$$v_s = 1.02v + .00003\frac{dv}{dt} + 1\times10^{-8}\frac{d^2v}{dt^2}$$

$$\underline{1\times10^8 v_s = 1.02\times10^8 v + 3000\frac{dv}{dt} + \frac{d^2v}{dt^2}}$$

P 9.2-2 Find the differential equation for the circuit shown in Figure P 9.2-2 using the operator method.

Answer:

$$\frac{d^2}{dt^2}i_L(t)+11{,}000\frac{d}{dt}i_L(t)+1.1\times10^8 i_L(t)=10^8 i_s(t)$$

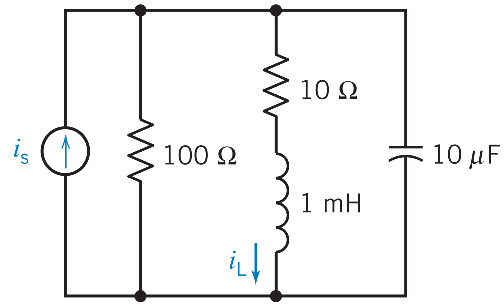
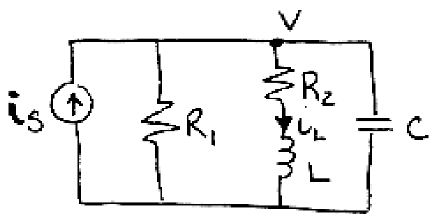


Figure P 9.2-2

Solution:



$$\text{KCL: } i_s = \frac{v}{R_1}+i_L+Csv$$

$$\text{KVL: } v = R_2 i_L + Lsi_L$$

Solving Cramer's rule for i_L:

$$i_L = \frac{i_s}{\dfrac{R_2}{R_1}+\dfrac{Ls}{R_1}+R_2Cs+LCs^2+1}$$

$$\left[1+\frac{R_2}{R_1}\right]i_L+\left[\frac{L}{R_1}+R_2C\right]si_L+\left[LC\right]s^2 i_L = i_s$$

$$R_1 = 100\Omega,\ R_2 = 10\Omega,\ L = 1\text{mH},\ C = 10\mu\text{F}$$

$$1.1i_L+.00011si_L+1\times10^{-8}s^2i_L = i_s$$

$$\underline{1.1\times10^8 i_L+11000si_L+s^2i_L = 1\times10^8 i_s}$$

P 9.2-3 Find the differential equation for $i_L(t)$ for $t > 0$ for the circuit of Figure P 9.2-3.

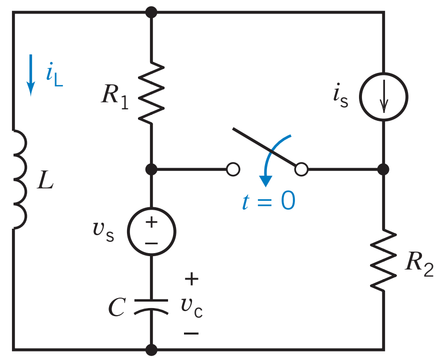
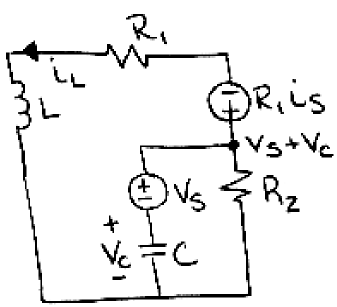


Figure P 9.2-3

Solution:

t > 0



$$\text{KCL: } i_L + C\frac{dv_c}{dt} + \frac{v_s + v_c}{R_2} = 0$$

$$\text{KVL: } R_1 i_s + R_1 i_L + L\frac{di_L}{dt} - v_c - v_s = 0$$

Solving for i_L :

$$\frac{d^2 i_L}{dt^2} + \left[\frac{R_1}{L} + \frac{1}{R_2 C}\right]\frac{di_L}{dt} + \left[\frac{R_1}{LR_2 C} + \frac{1}{LC}\right] i_L = \frac{-R_1}{LCR_2} i_s - \frac{R_1}{L}\frac{di_s}{dt} + \frac{1}{L}\frac{dv_s}{dt}$$

P 9.2-4 The input to the circuit shown in Figure P 9.2-4 is the voltage of the voltage source, V_s. The output is the inductor current $i(t)$. Represent the circuit by a second-order differential equation that shows how the output of this circuit is related to the input, for $t > 0$.

Hint: Use the direct method.

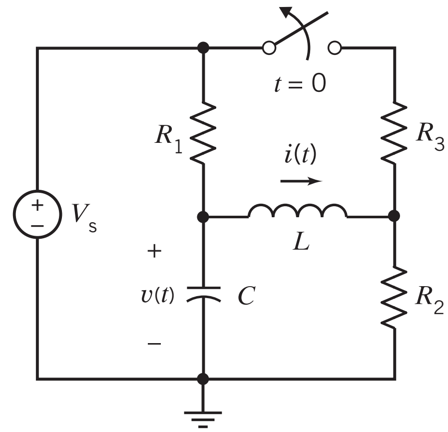


Figure P 9.2-4

Solution:

After the switch opens, apply KCL and KVL to get

$$R_1\left(i(t)+C\frac{d}{dt}v(t)\right)+v(t)=V_s$$

Apply KVL to get

$$v(t)=L\frac{d}{dt}i(t)+R_2\,i(t)$$

Substituting $v(t)$ into the first equation gives

$$R_1\left(i(t)+C\frac{d}{dt}\left(L\frac{d}{dt}i(t)+R_2\,i(t)\right)\right)+L\frac{d}{dt}i(t)+R_2\,i(t)=V_s$$

then

$$R_1\,C\,L\frac{d^2}{dt^2}i(t)+\left(R_1\,C\,R_2+L\right)\frac{d}{dt}i(t)+\left(R_1+R_2\right)i(t)=V_s$$

Dividing by $R_1\,C\,L$:

$$\frac{d^2}{dt^2}i(t)+\left(\frac{R_1\,C\,R_2+L}{R_1\,C\,L}\right)\frac{d}{dt}i(t)+\left(\frac{R_1+R_2}{R_1\,C\,L}\right)i(t)=\frac{V_s}{R_1\,C\,L}$$

P 9.2-5 The input to the circuit shown in Figure P 9.2-5 is the voltage of the voltage source, v_s. The output is the capacitor voltage $v(t)$. Represent the circuit by a second-order differential equation that shows how the output of this circuit is related to the input, for $t > 0$.
Hint: Use the direct method.

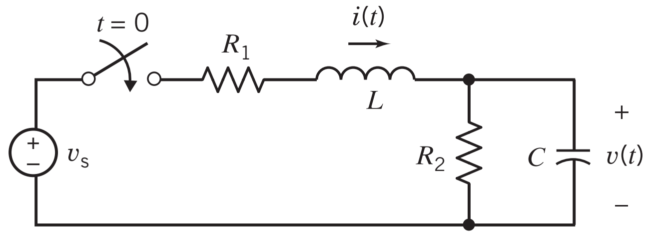


Figure P 9.2-5

Solution:
After the switch closes, use KCL to get

$$i(t) = \frac{v(t)}{R_2} + C\frac{d}{dt}v(t)$$

Use KVL to get

$$v_s = R_1\, i(t) + L\frac{d}{dt}i(t) + v(t)$$

Substitute to get

$$\begin{aligned} v_s &= \frac{R_1}{R_2}v(t) + R_1C\frac{d}{dt}v(t) + \frac{L}{R_2}\frac{d}{dt}v(t) + CL\frac{d^2}{dt^2}v(t) + v(t) \\ &= CL\frac{d^2}{dt^2}v(t) + \left(R_1C + \frac{L}{R_2}\right)\frac{d}{dt}v(t) + \frac{R_1+R_2}{R_2CL}v(t) \end{aligned}$$

Finally,

$$\frac{v_s}{CL} = \frac{d^2}{dt^2}v(t) + \left(\frac{R_1}{L} + \frac{1}{R_2C}\right)\frac{d}{dt}v(t) + \frac{R_1+R_2}{R_2CL}v(t)$$

P 9.2-6 The input to the circuit shown in Figure P 9.2-6 is the voltage of the voltage source, v_s. The output is the inductor current $i(t)$. Represent the circuit by a second-order differential equation that shows how the output of this circuit is related to the input, for $t > 0$.

Hint: Use the direct method.

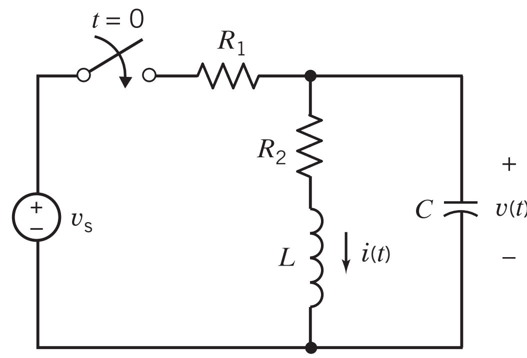


Figure P 9.2-6

Solution:

After the switch closes use KVL to get

$$R_2\, i(t) + L\frac{d}{dt}i(t) = v(t)$$

Use KCL and KVL to get

$$v_s = R_1\left(i(t) + C\frac{d}{dt}v(t)\right) + v(t)$$

Substitute to get

$$\begin{aligned} v_s &= R_1 i(t) + R_1 C R_2 \frac{d}{dt}i(t) + R_1 C L\frac{d^2}{dt^2}i(t) + R_2 i(t) + L\frac{d}{dt}i(t) \\ &= R_1 C L\frac{d^2}{dt^2}i(t) + \left(R_1 R_2 C + L\right)\frac{d}{dt}i(t) + \left(R_1 + R_2\right)i(t) \end{aligned}$$

Finally

$$\frac{v_s}{R_1 C L} = \frac{d^2}{dt^2}i(t) + \left(\frac{R_2}{L} + \frac{1}{R_1 C}\right)\frac{d}{dt}i(t) + \frac{R_1 + R_2}{R_1 C L}i(t)$$

P 9.2-7 The input to the circuit shown in Figure P 9.2-7 is the voltage of the voltage source, v_s. The output is the inductor current $i_2(t)$. Represent the circuit by a second-order differential equation that shows how the output of this circuit is related to the input, for $t > 0$.
Hint: Use the operator method.

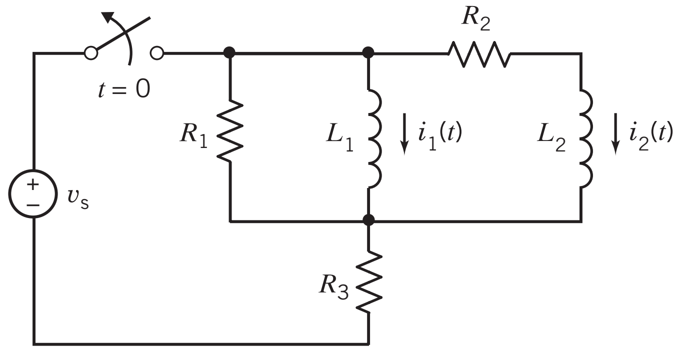


Figure P 9.2-7

Solution:
After the switch opens, KVL gives

$$L_1 \frac{d}{dt} i_1(t) = R_2\, i_2(t) + L_2 \frac{d}{dt} i_2(t)$$

KVL and KCL give

$$L_1 \frac{d}{dt} i_1(t) + R_1\left(i_1(t) + i_2(t)\right) = 0$$

Use the operator method to get

$$L_1 s\, i_1 = R_2\, i_2 + L_2 s\, i_2$$

$$L_1 s\, i_1 + R_1\left(i_1 + i_2\right) = 0$$

$$L_1 s^2 i_1 + R_1 s\, i_1 + R_1 s\, i_2 = 0$$

$$s\left(R_2 i_2 + L_2 s\, i_2\right) + \frac{R_1}{L_1}\left(R_2 i_2 + L_2 s\, i_2\right) + R_1 s\, i_2 = 0$$

$$L_2 s^2\, i_2 + \left(R_2 + R_1 \frac{L_2}{L_1} + R_1\right) s\, i_2 + \frac{R_1 R_2}{L_1} i_2 = 0$$

$$s^2 i_2 + \left(\frac{R_2}{L_2} + \frac{R_1}{L_2} + \frac{R_1}{L_1}\right) s\, i_2 + \frac{R_1 R_2}{L_1 L_2} i_2 = 0$$

so

$$\frac{d^2}{dt^2} i_2(t) + \left(\frac{R_2}{L_2} + \frac{R_1}{L_2} + \frac{R_1}{L_1}\right)\frac{d}{dt} i_2(t) + \frac{R_1 R_2}{L_1 L_2} i_2(t) = 0$$

P 9.2-8 The input to the circuit shown in Figure P 9.2-8 is the voltage of the voltage source, v_s. The output is the capacitor voltage $v_2(t)$. Represent the circuit by a second-order differential equation that shows how the output of this circuit is related to the input, for $t > 0$.

Hint: Use the operator method.

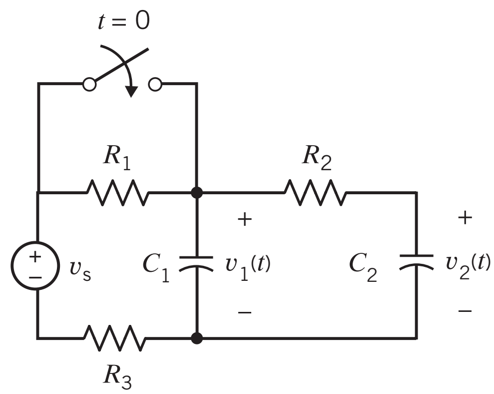


Figure P 9.2-8

Solution:

After the switch closes, KVL and KCL give

$$v_1(t)+R_3\left(C_1\frac{d}{dt}v_1(t)+C_2\frac{d}{dt}v_2(t)\right)=v_s$$

KVL gives

$$v_1(t)=R_2C_2\frac{d}{dt}v_2(t)+v_2(t)$$

Using the operator method

$$v_1+R_3\left(C_1sv_1+C_2sv_2\right)=v_s$$

$$v_1=R_2C_2sv_2+v_2$$

so

$$v_1=\left(1+R_2C_2s\right)v_2$$

$$\left(1+R_2C_2s\right)v_2+R_3C_1s\left(1+R_2C_2s\right)v_2+R_3C_2sv_2=v_s$$

Then

$$R_2R_3C_1C_2s^2v_2+\left(R_2C_2+R_3C_1+R_3C_2\right)sv_2+v_2=v_s$$

$$s^2v_2+\frac{R_2C_2+R_3C_1+R_3C_2}{R_2R_3C_1C_2}sv_2+\frac{1}{R_3R_2C_1C_2}v_2=\frac{v_s}{R_2R_3C_1C_2}$$

$$s^2v_2+\left(\frac{1}{R_3C_1}+\frac{1}{R_2C_2}+\frac{1}{R_2C_1}\right)sv_2+\frac{1}{R_2R_3C_1C_2}v_2=\frac{v_s}{R_2R_3C_1C_2}$$

so

$$\frac{v_s}{R_2R_3C_1C_2}=\frac{d^2}{dt^2}v_2(t)+\left(\frac{1}{R_3C_1}+\frac{1}{R_2C_2}+\frac{1}{R_2C_1}\right)\frac{d}{dt}v(t)+\frac{1}{R_2R_3C_1C_2}v(t)$$

P 9.2-9 The input to the circuit shown in Figure P 9.2-9 is the voltage of the voltage source, v_s. The output is the capacitor voltage $v(t)$. Represent the circuit by a second-order differential equation that shows how the output of this circuit is related to the input, for $t > 0$.

Hint: Use the direct method.

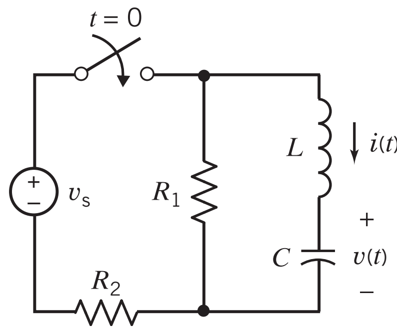
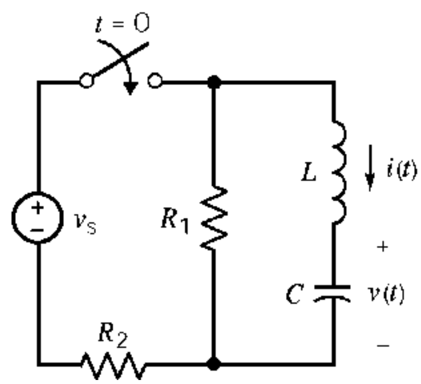


Figure P 9.2-9

Solution:

After the switch closes

$$i(t) = C\frac{d}{dt}v(t)$$



KCL and KVL give

$$v_s = R_2\left(i(t) + \frac{1}{R_1}\left(L\frac{d}{dt}i(t) + v(t)\right)\right) + L\frac{d}{dt}i(t) + v(t)$$

Substituting gives

$$\begin{aligned} v_s &= \left(1 + \frac{R_2}{R_1}\right)LC\frac{d^2}{dt^2}v(t) + R_2C\frac{d}{dt}v(t) + \left(1 + \frac{R_2}{R_1}\right)v(t) \\ &= \left(1 + \frac{R_2}{R_1}\right)LC\frac{d^2}{dt^2}v(t) + R_2C\frac{d}{dt}v(t) + \left(1 + \frac{R_2}{R_1}\right)v(t) \end{aligned}$$

Finally

$$\frac{R_1 v_s}{LC(R_1 + R_2)} = \frac{d^2}{dt}v(t) + \frac{R_1R_2}{L(R_1 + R_2)}\frac{d}{dt}v(t) + \frac{1}{LC}v(t)$$

P 9.2-10 The input to the circuit shown in Figure P 9.2-10 is the voltage of the voltage source, v_s. The output is the capacitor voltage $v(t)$. Represent the circuit by a second-order differential equation that shows how the output of this circuit is related to the input, for $t > 0$.
Hint: Find a Thévenin equivalent circuit.

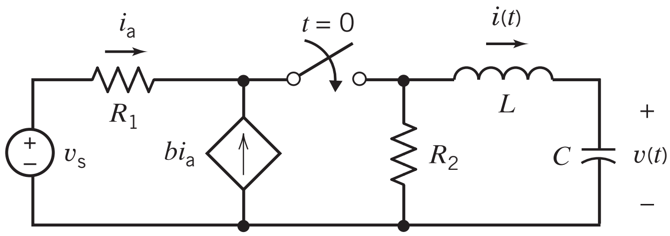
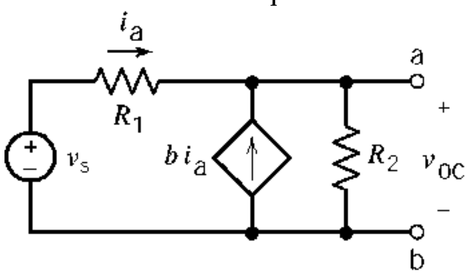
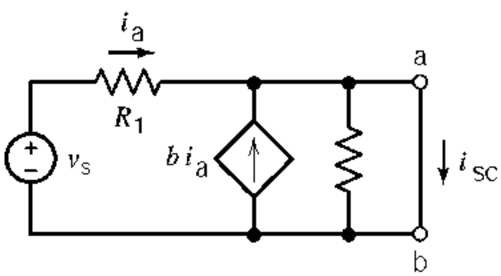


Figure P 9.2-10

Solution:
Find the Thevenin equivalent circuit for the part of the circuit to the left of the inductor.

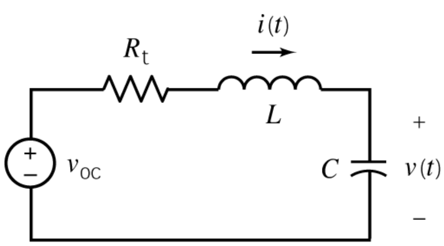


$$\left.\begin{array}{l} i_a = \dfrac{v_s - v_{oc}}{R_1} \\ i_a + b\,i_a = \dfrac{v_{oc}}{R_2} \end{array}\right\} \Rightarrow \quad v_{oc} = \frac{v_s\,R_2\,(1+b)}{R_1 + R_2\,(1+b)}$$



$$i_{sc} = i_a\,(1+b) = \frac{v_s}{R_1}(1+b)$$

$$R_t = \frac{v_{oc}}{i_{sc}} = \frac{\dfrac{v_s\,R_2\,(1+b)}{R_1 + R_2\,(1+b)}}{\dfrac{v_s}{R_1}(1+b)} = \frac{R_1\,R_2}{R_1 + R_2\,(1+b)}$$



$$R_t\,i(t) + L\,\frac{d\,i(t)}{d\,t} + v(t) - v_{oc} = 0$$

$$i(t) = C\,\frac{d\,v(t)}{d\,t}$$

$$R_t\,C\,\frac{d\,v(t)}{d\,t} + LC\,\frac{d^2\,v(t)}{d\,t^2} + v(t) = v_{oc} \quad \Rightarrow \quad \frac{d^2\,v(t)}{d\,t^2} + \frac{R_t}{L}\,\frac{d\,v(t)}{d\,t} + \frac{1}{LC}v(t) = \frac{v(t)}{LC}$$

Finally,

$$\frac{d^2\,v(t)}{d\,t^2} + \frac{R_1\,R_2}{L\big(R_1 + R_2\,(1+b)\big)}\,\frac{d\,v(t)}{d\,t} + \frac{1}{LC}v(t) = \frac{v(t)}{LC}$$

P 9.2-11 The input to the circuit shown in Figure P 9.2-11 is the voltage of the voltage source, $v_s(t)$. The output is the voltage $v_2(t)$. Derive the second-order differential equation that shows how the output of this circuit is related to the input.

Hint: Use the direct method.

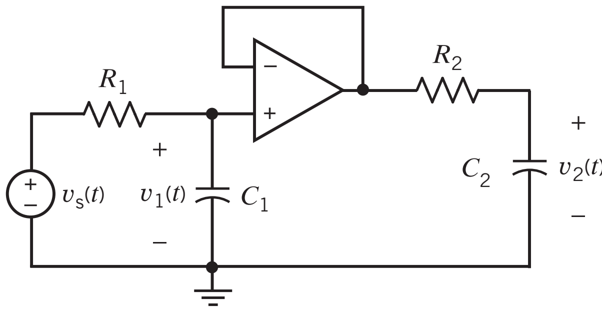


Figure P 9.2-11

Solution:

KCL gives

$$\frac{v_s(t)-v_1(t)}{R_1}=C_1\frac{d}{dt}v_1(t) \quad \Rightarrow \quad v_s(t)=R_1C_1\frac{d}{dt}v_1(t)+v_1(t)$$

and

$$\frac{v_1(t)-v_2(t)}{R_2}=C_2\frac{d}{dt}v_2(t) \quad \Rightarrow \quad v_1(t)=R_2C_2\frac{d}{dt}v_2(t)+v_2(t)$$

Substituting gives

$$v_s(t)=R_1C_1\frac{d}{dt}\left[R_2C_2\frac{d}{dt}v_2(t)+v_2(t)\right]+R_2C_2\frac{d}{dt}v_2(t)+v_2(t)$$

so

$$\frac{1}{R_1R_2C_1C_2}v_s(t)=\frac{d^2}{dt^2}v_2(t)+\left(\frac{1}{R_1C_1}+\frac{1}{R_2C_2}\right)v_2(t)+\frac{1}{R_1R_2C_1C_2}v_2(t)$$

P 9.2-12 The input to the circuit shown in Figure P 9.2-12 is the voltage of the voltage source, $v_s(t)$. The output is the voltage $v_o(t)$. Derive the second-order differential equation that shows how the output of this circuit is related to the input.

Hint: Use the operator method.

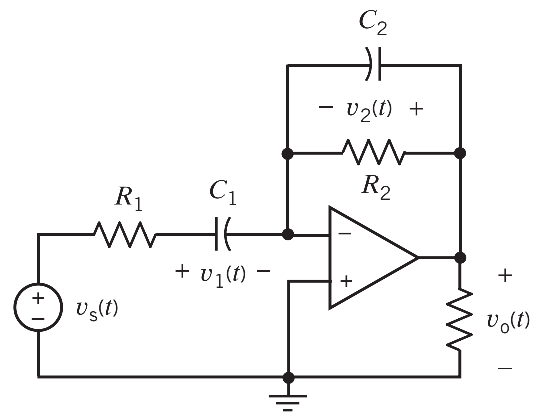


Figure P 9.2-12

Solution:

KVL gives
$$v_s(t) = R_1 C_1 \frac{d}{dt} v_1(t) + v_1(t)$$

KCL gives
$$C_1 \frac{d}{dt} v_1(t) + C_2 \frac{d}{dt} v_2(t) + \frac{v_2(t)}{R_2} = 0$$

KVL gives
$$v_o(t) = v_2(t)$$

Using the operator method
$$v_s = R_1 C_1 s v_1 + v_1$$
$$C_1 s v_1 + C_2 s v_2 + \frac{v_2}{R_2} = 0$$

Solving
$$v_1 = -\left(\frac{C_2}{C_1} v_2 + \frac{1}{R_2 C_1 s} v_2\right)$$
$$s v_s = (s R_1 C_1 + 1)\left(\frac{C_2}{C_1} s + \frac{1}{R_2 C_1}\right) v_o$$
$$\frac{1}{R_1 C_2} s v_s = s^2 v_o + \left(\frac{1}{R_1 C_1} + \frac{1}{R_2 C_2}\right) s v_o + \frac{1}{R_1 R_2 C_1 C_2} v_o$$

The corresponding differential equation is
$$\frac{1}{R_1 C_2} \frac{d}{dt} v_s(t) = \frac{d^2}{dt^2} v_o(t) + \left(\frac{1}{R_1 C_1} + \frac{1}{R_2 C_2}\right) \frac{d}{dt} v_o(t) + \frac{1}{R_1 R_2 C_1 C_2} v_o(t)$$

P 9.2-13 The input to the circuit shown in Figure P 9.2-13 is the voltage of the voltage source, $v_s(t)$. The output is the voltage $v_o(t)$. Derive the second-order differential equation that shows how the output of this circuit is related to the input.

Hint: Use the direct method.

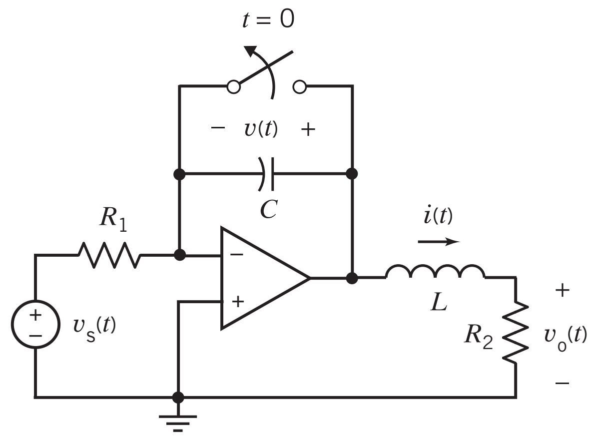


Figure P 9.2-13

Solution:

After the switch opens, KCL gives

$$\frac{v_s(t)}{R_1}+C\frac{d}{dt}v(t)=0$$

KVL gives

$$v(t)-v_o(t)=L\frac{d}{dt}i(t)$$

and Ohm's law gives

$$v_o(t)=R_2\,i(t)$$

so

$$\frac{d}{dt}v(t)=-\frac{1}{R_1C}v_s(t)$$

and

$$\frac{d}{dt}v(t)-\frac{d}{dt}v_o(t)=L\frac{d^2}{dt^2}i(t)$$

Then

$$-\frac{1}{R_1C}v_s(t)=\frac{d}{dt}v(t)=L\frac{d^2}{dt^2}i(t)+R_2\frac{d}{dt}i(t)$$

or

$$-\frac{1}{R_1CL}v_s(t)=\frac{d^2}{dt^2}i(t)+\frac{R_2}{L}\frac{d}{dt}i(t)$$

P 9.2-14 The input to the circuit shown in Figure P 9.2-14 is the voltage of the voltage source, $v_s(t)$. The output is the voltage $v_2(t)$. Derive the second-order differential equation that shows how the output of this circuit is related to the input.

Hint: Use the direct method.

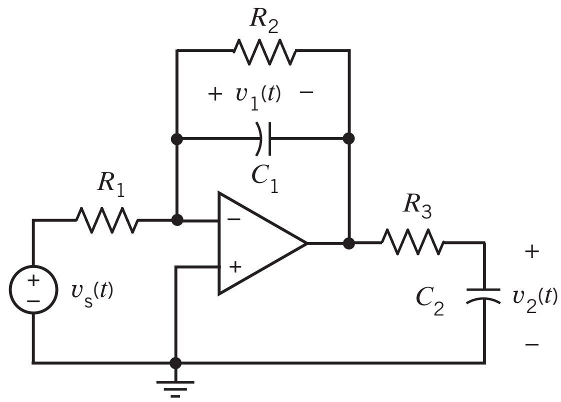


Figure P 9.2-14

Solution:

KCL gives

$$\frac{v_s(t)}{R_1} = \frac{v_1(t)}{R_2} + C_1 \frac{d}{dt} v_1(t)$$

and

$$\frac{v_2(t) + v_1(t)}{R_3} + C_2 \frac{d}{dt} v_2(t) = 0$$

so

$$v_1(t) + R_2 C_1 \frac{d}{dt} v_1(t) = \frac{R_2}{R_1} v_s(t)$$

and

$$v_1(t) = -\left(v_2(t) + R_3 C_2 \frac{d}{dt} v_2(t) \right)$$

Substituting gives

$$\left[v_2(t) + R_3 C_2 \frac{d}{dt} v_2(t) + R_2 C_1 \frac{d}{dt}\left[v_2(t) + R_3 C_2 \frac{d}{dt} v_2(t) \right] \right] = -\frac{R_2}{R_1} v_s(t)$$

or

$$\frac{d^2}{dt^2} v_2(t) + \left(\frac{1}{R_2 C_1} + \frac{1}{R_3 C_2} \right) \frac{d}{dt} v_2(t) + \frac{1}{R_2 R_3 C_1 C_2} v_2(t) = -\frac{1}{R_1 R_3 C_1 C_2} v_s(t)$$

P 9.2-15 Find the second-order differential equation for i_2 for the circuit of Figure P 9.2-15 using the operator method. Recall that the operator for the integral is $1/s$.

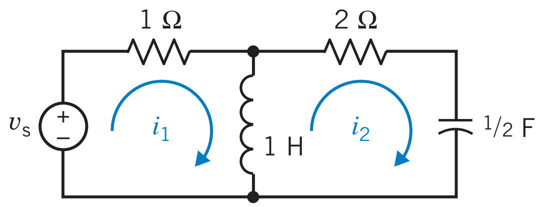
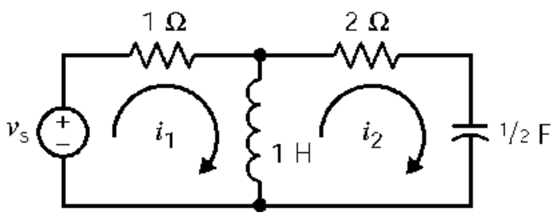


Figure P 9.2-15

Answer: $3\frac{d^2i_2}{dt^2}+4\frac{di_2}{dt}+2i_2=\frac{d^2v_s}{dt^2}$

Solution:



Apply KVL to the left mesh : $\underline{i_1 + s(i_1-i_2) = v_s}$ (1)

where $s = d/dt$

Apply KVL to the right mesh : $2i_2 + 2\left(\frac{1}{s}\right)i_2 + s(i_2-i_1) = 0$

$$\Rightarrow \underline{i_1 = 2\left(\frac{1}{s}\right)i_2 + 2\left(\frac{1}{s^2}\right)i_2 + i_2} \qquad (2)$$

Plugging (2) into (1) yields

$$3s^2i_2 + 4si_2 + 2i_2 = s^2v_s \quad or \quad \underline{3\frac{d^2i_2}{dt^2} + 4\frac{di_2}{dt} + 2i_2 = \frac{d^2v_s}{dt^2}}$$

Section 9-3: Solution of the Second Order Differential Equation - The Natural Response

P 9.3-1 Find the characteristic equation and its roots for the circuit of Figure P 9.2-2.

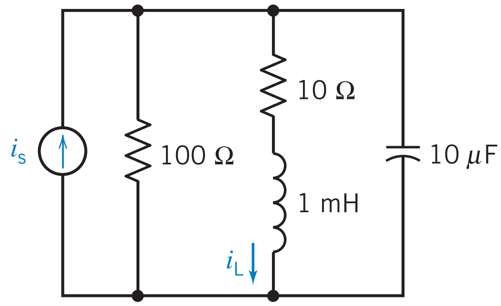


Figure P 9.2-2

Solution:
From Problem P 9.2-2 the characteristic equation is

$$\underline{1.1\times10^{8}+11000s+s^{2}\ =\ 0}\quad\Rightarrow\quad s_1,\ s_2=\frac{-11000\pm\sqrt{(11000)^{2}-4(1.1\times10^{8})}}{2}\ =\ \underline{-5500\pm j8930}$$

P 9.3-2 Find the characteristic equation and its roots for the circuit of Figure P 9.3-2.

Answer: $s^2 + 400s + 3\times10^4 = 0$
roots: $s = -300, -100$

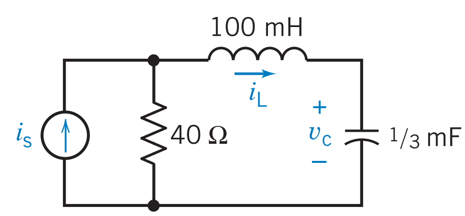
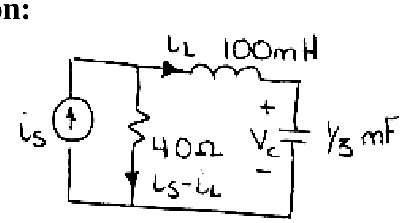


Figure P 9.3-2

Solution:



$$\text{KVL: } 40\,(i_s - i_L) = 100\times10^{-3}\,\frac{di_L}{dt}+v_c$$

$$i_L\ =\ i_c\ =\ \left(\frac{1}{3}\times10^{-3}\right)\frac{dv_c}{dt}$$

$$i_L\ =\ \frac{40}{3}\times10^{-3}\,\frac{di_s}{dt}-\frac{40}{3}\times10^{-3}\,\frac{di_L}{dt}-\frac{100}{3}\times10^{-6}\,\frac{d^{2}i_L}{dt^{2}}$$

$$\frac{d^{2}i_L}{dt^{2}}+400\frac{di_L}{dt}+30000i_L\ =\ 400\frac{di_s}{dt}$$

$$\underline{s^{2}+400s+30000\ =\ 0}\quad\Rightarrow\quad(s+100)(s+300)\ =\ 0\quad\Rightarrow\quad\underline{s_1=-100,\quad s_2=-300}$$

P 9.3-3 Find the characteristic equation and its roots for the circuit shown in Figure P 9.3-3.

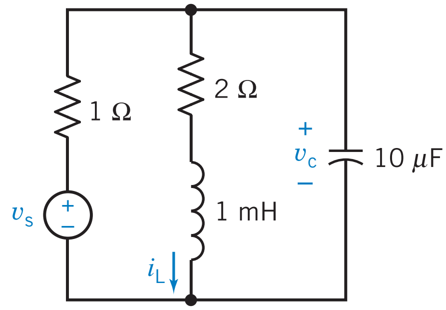
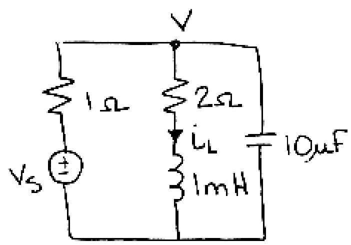


Figure P 9.3-3

Solution:



$$\text{KCL:}\quad \frac{v - v_s}{1} + i_L + 10\times10^{-6}\frac{dv}{dt} = 0$$

$$\text{KVL:}\quad v = 2i_L + 10^{-3}\frac{di_L}{dt}$$

$$0 = 2i_L + 10^{-3}\frac{di_L}{dt} - v_s + i_L + 10\times10^{-6}\cdot 2\frac{di_L}{dt} + 10\times10^{-6}\times10^{-3}\frac{d^2i_L}{dt}$$

$$v_s = 3i_L + .00102\frac{di_L}{dt} + 1\times10^{-8}\frac{d^2i_L}{dt^2}$$

$$\frac{d^2i_L}{dt} + 102000\frac{di_L}{dt} + 3\times10^{-8}i_L = 1\times10^8\ v_s$$

$$\underline{s^2 + 102000s + 3\times10^8 = 0}, \quad \therefore \underline{s_1 = 3031,\ s_2 = -98969}$$

P 9.3-4 German automaker Volkswagen, in its bid to make more efficient cars, has come up with an auto whose engine saves energy by shutting itself off at stoplights. The "stop–start" system springs from a campaign to develop cars in all its world markets that use less fuel and pollute less than vehicles now on the road. The stop–start transmission control has a mechanism that senses when the car does not need fuel: coasting downhill and idling at an intersection. The engine shuts off, but a small starter flywheel keeps turning so that power can be quickly restored when the driver touches the accelerator.

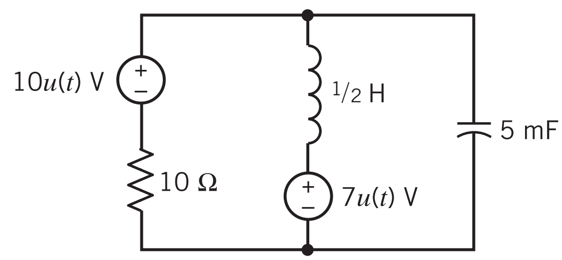


Figure P 9.3-4

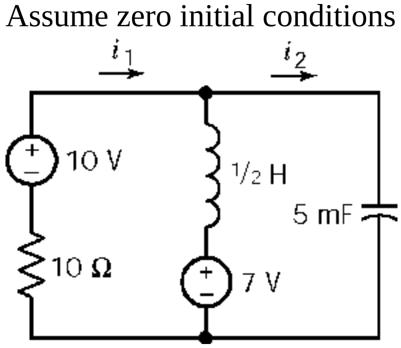
A model of the stop–start circuit is shown in Figure P 9.3-4. Determine the characteristic equation and the natural frequencies for the circuit.

Answer: $s^2 + 20s + 400 = 0$

$s = -10 \pm j17.3$

Solution:

Assume zero initial conditions



$$\text{loop 1}: 10i_1 + \frac{1}{2}\frac{di_1}{dt} - \frac{1}{2}\frac{di_2}{dt} = 10-7$$

$$\text{loop 2}: -\frac{1}{2}\frac{di_1}{dt} + \frac{1}{2}\frac{di_2}{dt} + 200\int i_2 dt = 7$$

$$\text{determinant}: \begin{bmatrix} \left(10 + \frac{1}{2}s\right) & -\frac{1}{2}s \\ -\frac{1}{2}s & \left(\frac{1}{2}s + \frac{200}{s}\right) \end{bmatrix}$$

$$s^2 + 20s + 400 = 0, \quad \therefore \; s = -10 \pm j\,17.3$$

Section 9.4: Natural Response of the Unforced Parallel RLC Circuit

P 9.4-1 Determine $v(t)$ for the circuit of Figure P 9.4-1 when $L = 1$ H and $v_s = 0$ for $t \geq 0$. The initial conditions are $v(0) = 6$ V and $dv/dt(0) = -3000$ V/s.

Answer: $v(t) = -2e^{-100t} + 8e^{-400t}$ V

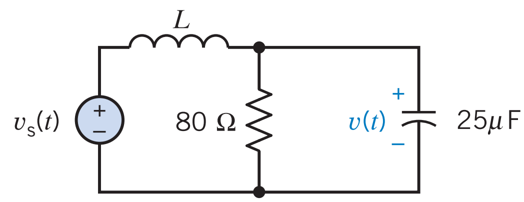


Figure P 9.4-1

Solution:

$$v(0) = 6, \quad \frac{dv(0)}{dt} = -3000$$

Using operators, the node equation is: $Csv + \frac{v}{R} + \frac{(v - v_s)}{sL} = 0 \quad or \quad \left(LCs^2 + \frac{L}{R}s + 1\right) v = v_s$

So the characteristic equation is: $s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$

$$\Rightarrow s_{1,2} = -250 \pm \sqrt{250^2 - 40,000} = -100, \ -400$$

So $v(t) = Ae^{-100t} + Be^{-400t}$

$$v(0) = 6 = A + B$$

$$\left.\frac{dv(0)}{dt} = -3000 = -100A - 400B\right\} \quad \begin{aligned} A &= -2 \\ B &= 8 \end{aligned}$$

$$\therefore \underline{v(t) = -2e^{-100t} + 8e^{-400t}} \qquad t>0$$

P 9.4-2 An *RLC* circuit is shown in Figure P 9.4-2, where $v(0) = 2$ V. The switch has been open for a long time before closing at $t = 0$. Determine and plot $v(t)$.

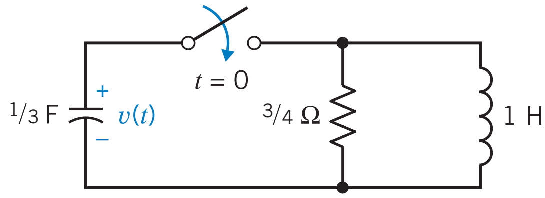


Figure P 9.4-2

Solution:

$$v(0) = 2, \quad i(0) = 0$$

Characteristic equation $s^2 + \frac{1}{RC} s + \frac{1}{LC} = 0 \Rightarrow s^2 + 4s + 3 = 0 \Rightarrow s = -1, \ -3$

$$v(t) = Ae^{-t} + Be^{-3t}$$

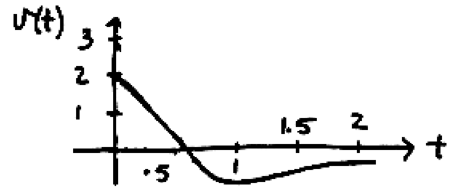
Use eq. 9.5−12 $\Rightarrow s_1 A + s_2 B = -\frac{v(0)}{RC} - \frac{i(0)}{C}$

$$-1A - 3B = -\frac{2}{1/4} - 0 = -8 \quad (1)$$

also have $v(0) = 2 = A + B \quad (2)$

From (1) & (2) get $A = -1, \ B = 3$

$\therefore \ \underline{v(t) = -e^{-t} + 3e^{-3t} \ \text{V}}$



P 9.4-3 Determine $i_1(t)$ and $i_2(t)$ for the circuit of Figure P 9.4-3 when $i_1(0) = i_2(0) = 11$ A.

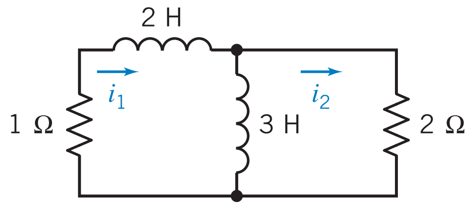


Figure P 9.4-3

Solution:

$$\text{KVL}: i_1 + 5\frac{di_1}{dt} - 3\frac{di_2}{dt} = 0 \qquad (1)$$

$$\text{KVL}: -3\frac{di_1}{dt} + 3\frac{di_2}{dt} + 2i_2 = 0 \qquad (2)$$

in operator form

$$\left.\begin{array}{l}(1+5s)i_1 + (-3s)i_2 = 0\\ (-3s)i_1 + (3s+2)i_2 = 0\end{array}\right\} \text{ thus } \Delta = (1+5s)(3s+2) - 9s^2 = 6s^2 + 13s + 2 = 0 \Rightarrow s = -1/6, -2$$

$$\text{Thus } i_1(t) = Ae^{-t/6} + Be^{-2t}$$

$$i_2(t) = Ce^{-t/6} + De^{-2t}$$

Now $i_1(0) = 11 = A + B;\quad i_2(0) = 11 = C + D$

from (1) & (2) get

$$\frac{di_1(0)}{dt} = -\frac{33}{2} = -\frac{A}{6} - 2B\ ;\quad \frac{di_2(0)}{dt} = -\frac{143}{6} = -\frac{C}{6} - 20$$

which yields $A = 3,\ B = 8,\ C = -1,\ 0 = 12$

$$\underline{i_1(t) = 3e^{-t/6} + 8e^{-2t}\ A} \quad \& \quad \underline{i_2(t) = -e^{-t/6} + 12e^{-2t}\ A}$$

P 9.4-4 The circuit shown in Figure P 9.4-4 contains a switch that is sometimes open and sometimes closed. Determine the damping factor, α, the resonant frequency, ω_0, and the damped resonant frequency, ω_d, of the circuit when (a) the switch is open and (b) the switch is closed.

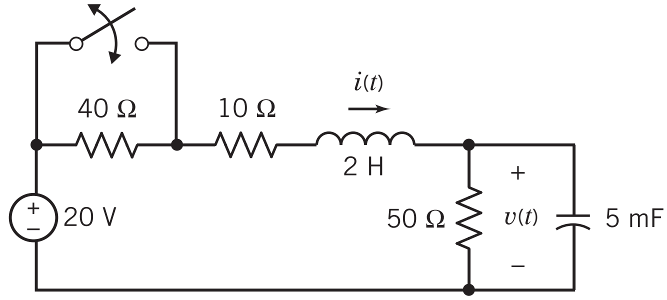
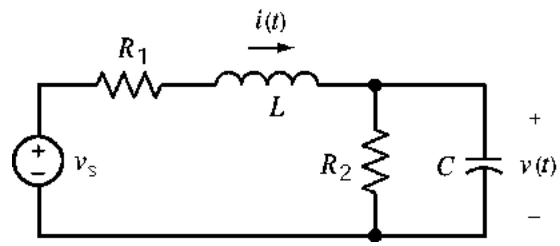


Figure P 9.4-4

Solution:

Represent this circuit by a differential equation. (R_1 = 50 Ω when the switch is open and R_1 = 10 Ω when the switch is closed.)



Use KCL to get

$$i(t)=\frac{v(t)}{R_2}+C\frac{d}{dt}v(t)$$

Use KVL to get

$$v_s=R_1\,i(t)+L\frac{d}{dt}i(t)+v(t)$$

Substitute to get

$$v_s=\frac{R_1}{R_2}v(t)+R_1C\frac{d}{dt}v(t)+\frac{L}{R_2}\frac{d}{dt}v(t)+CL\frac{d^2}{dt^2}v(t)+v(t)$$

$$=CL\frac{d^2}{dt^2}v(t)+\left(R_1C+\frac{L}{R_2}\right)\frac{d}{dt}v(t)+\frac{R_1+R_2}{R_2}v(t)$$

Finally,

$$\frac{v_s}{CL}=\frac{d^2}{dt^2}v(t)+\left(\frac{R_1}{L}+\frac{1}{R_2C}\right)\frac{d}{dt}v(t)+\frac{R_1+R_2}{R_2CL}v(t)$$

Compare to

$$\frac{d^2}{dt^2}v(t)+2\alpha\frac{d}{dt}v(t)+\omega_0^{\,2}\,v(t)=f(t)$$

to get

$$2\alpha=\frac{R_1}{L}+\frac{1}{R_2C}\quad\text{and}\quad\omega_0^{\,2}=\frac{R_1+R_2}{R_2CL}$$

(a) When the switch is open $\alpha = 14.5$, ω_0 = 14.14 rad/s and $\omega_d = j3.2$ (the circuit is overdamped).

(b) When the switch is closed $\alpha = 4.5$, ω_0 = 10.954 rad/s and ω_d = 9.987 (the circuit is underdamped).

P 9.4-5 The circuit shown in Figure P 9.4-5 is used to detect smokers in airplanes who surreptitiously light up before they can take a single puff. The sensor activates the switch, and the change in the voltage $v(t)$ activates a light at the flight attendant's station. Determine the natural response $v(t)$.

Answer: $v(t) = -1.16e^{-2.7t} + 1.16e^{-37.3t}$ V

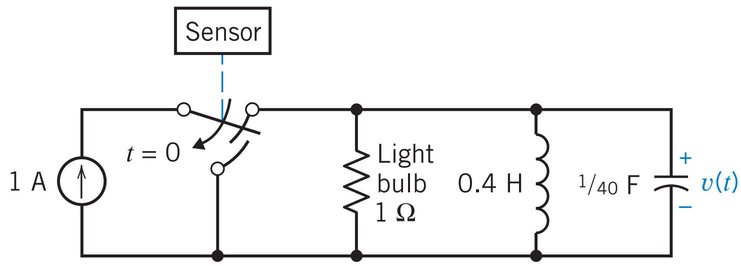
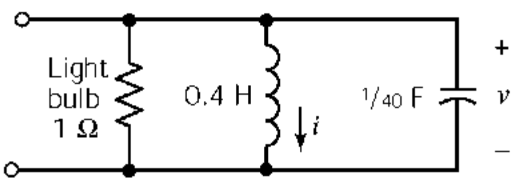


Figure P 9.4-5

Solution:



$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$$

$$s^2 + 40s + 100 = 0$$

$$s = -2.7\ ,\ \ -37.3$$

The initial conditions are $v(0) = 0$, $i(0)\ 1\ \text{A}$.

$$v_n = A_1 e^{-2.7t} + A_2 e^{-37.3t}\ ,\ \ v(0) = 0 = A_1 + A_2 \qquad (1)$$

KCL at $t = 0^+$ yields : $\dfrac{v(0^+)}{1} + i(0^+) + \dfrac{1}{40}\dfrac{dv(0^+)}{dt} = 0$

$$\therefore \frac{dv(0^+)}{dt} = -40v(0^+) - 40i(0^+) = -40(1) = -2.7A_1 - 37.3A_2 \qquad (2)$$

from (1) and (2) $\Rightarrow A_1 = -1.16\ ,\ A_2 = 1.16$

So $v(t) = v_n(t) = -1.16e^{-2.7t} + 1.16e^{-37.3t}$

Section 9.5: Natural Response of the Critically Damped Unforced Parallel RLC Circuit

P 9.5-1 Find $v_c(t)$ for $t > 0$ for the circuit shown in Figure P 9.5-1.

Answer: $v_c(t) = (3 + 6000t)e^{-2000t}$ V

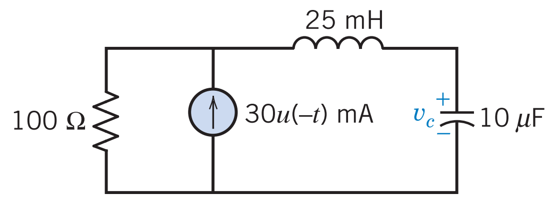
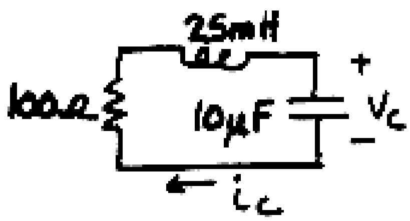


Figure P 9.5-1

Solution:

$\underline{t>0}$

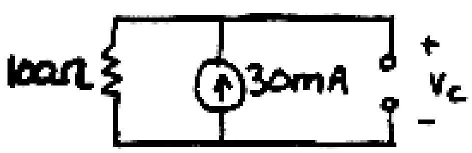


$$\text{KVL a: } 100 i_c + .025\frac{di_c}{dt} + v_c = 0,\ i_c = 10^{-5}\frac{dv_c}{dt}$$

$$\therefore\ \frac{d^2 v_c}{dt^2} + 4000\frac{dv_c}{dt} + 4\times10^6 v_c = 0$$

$$s^2 + 4000s + 4\times10^6 = 0 \Rightarrow s = -2000,\ -2000 \quad \therefore\ v_c(t) = A_1 e^{-2000t} + A_2 t e^{-2000t}$$

$\underline{t=0^-}$ (Steady – State)



$$i_L = i_c(0^-) = 0 = i_c(0^+) \Rightarrow \frac{dv_c(0^+)}{dt} = 0$$

$$v_c(0^-) = 3\text{ V} = v_c(0^+)$$

so $v_c(0^+) = 3 = A_1$

$$\frac{dv_c(0^+)}{dt} = 0 = -2000A_1 + A_2 \Rightarrow A_2 = 6000$$

$$\underline{v_c(t) = (3+6000t)e^{-2000t}\text{ V}}$$

P 9.5-2 Find $v_c(t)$ for $t > 0$ for the circuit of Figure P 9.5-2. Assume steady-state conditions exist at $t = 0^-$.

Answer: $v_c(t) = -8te^{-2t}$ V

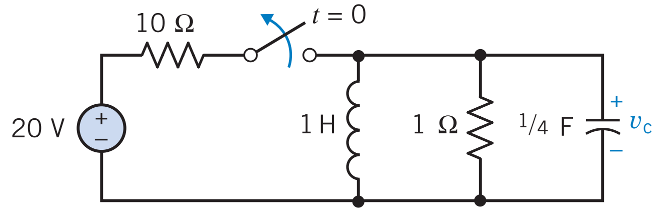
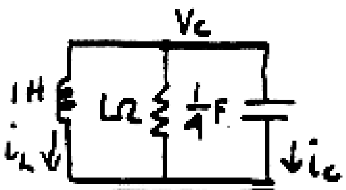


Figure P 9.5-2

Solution:

$\underline{t > 0}$

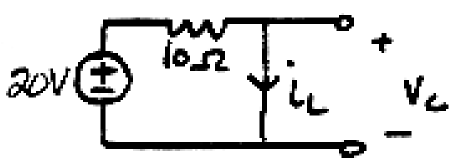


KCL at v_c : $\int_{-\infty}^{t} v_c\, dt + v_c + \left(\frac{1}{4}\right)\frac{dv_c}{dt} = 0$

$$\Rightarrow \frac{d^2 v_c}{dt} + 4\frac{dv_c}{dt} + 4v_c = 0$$

$$s^2 + 4s + 4 = 0, \quad s = -2, \ -2 \quad \Rightarrow \quad v_c(t) = A_1 e^{-2t} + A_2\, t\, e^{-2t}$$

$\underline{t = 0^-}$ (Steady-State)



$$v_c(0^-) = 0 = v_c(0^+) \quad \& \quad i_L(0^-) = \frac{20\,V}{10\Omega} = 2\,A = i_L(0^+)$$

Since $v_c(0^+) = 0$ then $i_c(0^+) = -i_L(0^+) = -2\,A$

$$\therefore \frac{dv_c(0^+)}{dt} = \frac{i_c(0^+)}{1/4} = -8\,\mathrm{V/s}$$

So $v_c(0^+) = 0 = A_1$

$$\frac{dv_c(0^+)}{dt} = -8 = A_2$$

$$\therefore v_c(t) = -8te^{-2t}\,V$$

P 9.5-3 Police often use stun guns to incapacitate potentially dangerous felons. The hand-held device provides a series of high-voltage, low-current pulses. The power of the pulses is far below lethal levels, but it is enough to cause muscles to contract and put the person out of action. The device provides a pulse of up to 50,000 V, and a current of 1 mA flows through an arc. A model of the circuit for one period is shown in Figure P 9.5-3. Find $v(t)$ for $0 < t < 1$ ms. The resistor R represents the spark gap. Select C so that the response is critically damped.

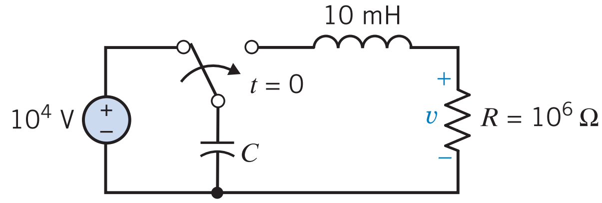


Figure P 9.5-3

Solution:

Assume steady−state at $t = 0^-$ $\therefore v_c(0^-) = 10^4$ V & $i_L(0^-) = 0$

$\underline{t>0}$

$$\text{KVL a:} \quad -v_c + .01\frac{di_L}{dt} + 10^6 i_L = 0 \qquad (1)$$

$$\text{Also} \quad : i_L = -C\,{dv_c}/{dt} = -C\left[.01\frac{d^2 i_L}{dt^2} + 10^6\frac{di_L}{dt}\right] \qquad (2)$$

$$\therefore \quad 0.01C\frac{d^2 i_L}{dt^2} + 10^6 C\frac{di_L}{dt} + i_L = 0$$

$$\text{Characteristic eq.} \Rightarrow 0.01C\,s^2 + 10^6 s + 1 = 0 \Rightarrow s = \frac{-10^6 C \pm \sqrt{(10^6 C)^2 - 4(.01C)}}{2(.01C)}$$

for critically damped: $10^{12}C^2 - .04C = 0$

$$\Rightarrow C = 0.04 \text{ pF} \quad \therefore s = -5\times10^7, -5\times10^7$$

So $i_L(t) = A_1 e^{-5\times10^7 t} + A_2 t e^{-5\times10^7 t}$

Now from (1) $\Rightarrow \dfrac{di_L}{dt}(0^+) = 100\left[v_c(0^+) - 10^6 i_L(0^+)\right] = 10^6\ {A}/{s}$

So $i_L(0) = 0 = A_1$ and $\dfrac{di_L(0)}{dt} = 10^6 = A_2 \quad \therefore i_L(t) = 10^6 t e^{-5\times10^7 t}$ A

Now $\underline{v(t) = 10^6 i_L(t) = 10^{12} t e^{-5\times10^7 t}\text{ V}}$

P 9.5-4 Reconsider Problem P 9.4-1 when $L = 640$ mH and the other parameters and conditions remain the same.

Answer: $v(t) = (6 - 1500t)e^{-250t}$ V

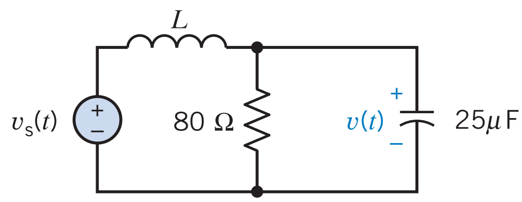


Figure P 9.4-1

Solution:

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0 \quad \text{with} \quad \frac{1}{RC} = 500 \quad \text{and} \quad \frac{1}{LC} = 62.5\times10^3 \quad \text{yields } s = -250, -250$$

$$v(t) = Ae^{-250t} + Bte^{-250t}$$

$$v(0) = 6 = A$$

$$\frac{dv(0)}{dt} = -3000 = -250A + B \Rightarrow B = -1500$$

$$\therefore \underline{v(t) = 6e^{-250t} - 1500te^{-250t}}$$

P 9.5-5 An automobile ignition uses an electromagnetic trigger. The *RLC* trigger circuit shown in Figure P 9.5-5 has a step input of 6 V, and $v(0) = 2$ V and $i(0) = 0$. The resistance R must be selected from $2\ \Omega < R < 7\ \Omega$ so that the current $i(t)$ exceeds 0.6 A for greater than 0.5 s in order to activate the trigger. A critically damped response $i(t)$ is required to avoid oscillations in the trigger current. Select R and determine and plot $i(t)$.

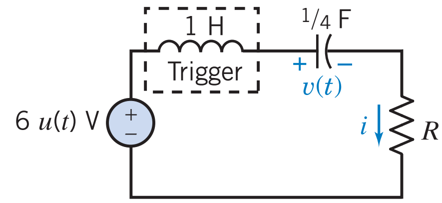


Figure P 9.5-5

Solution:

$$\text{KVL}: \frac{di}{dt} + Ri + \underbrace{2 + 4\int_0^t i\,dt}_{v(t)} = 6 \qquad (1)$$

taking the derivative with respect to t: $\dfrac{d^2 i}{dt^2} + R\dfrac{di}{dt} + 4i = 0$

Characteristic equataion: $s^2 + Rs + 4 = 0$

Let $R = 4$ for critical damping $\Rightarrow\ (s+2)^2 = 0$

So $i(t) = Ate^{-2t} + Be^{-2t}$

$i(0) = 0\ \Rightarrow\ B = 0$

from (1) $\dfrac{di(0)}{dt} = 4 - R(i(0)) = 4 - R(0) = 4 = A$

$\therefore\ \underline{i(t) = 4te^{-2t}\text{A}}$

Section 9-6: Natural Response of an Underdamped Unforced Parallel RLC Circuit

P 9.6-1 A communication system from a space station uses short pulses to control a robot operating in space. The transmitter circuit is modeled in Figure P 9.6-1. Find the output voltage $v_c(t)$ for $t > 0$. Assume steady-state conditions at $t = 0^-$.

Answer:

$v_c(t) = e^{-400t}\,[3 \cos 300t + 4 \sin 300t]$ V

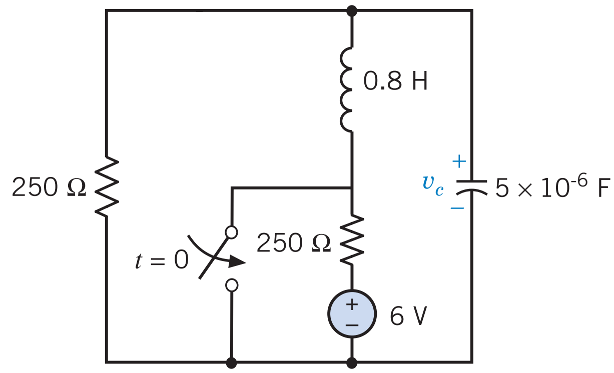
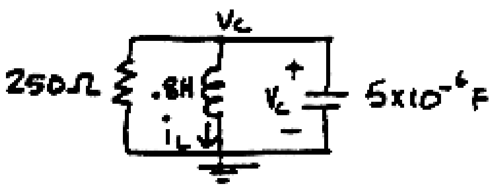


Figure P 9.6-1

Solution:

t > 0



KCL at v_c : $\quad \dfrac{v_c}{250} + i_L + 5\times10^{-6}\dfrac{dv_c}{dt} = 0 \qquad (1)$

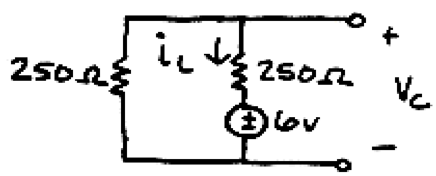
also : $\quad v_c = 0.8\dfrac{di_L}{dt} \qquad (2)$

Solving for i_L in (1) & plugging into (2)

$$\frac{d^2v_c}{dt^2} + 800\frac{dv_c}{dt} + 2.5\times10^5 v_c = 0 \quad \Rightarrow s^2 + 800s + 250{,}000 = 0,\ \ s = -400 \pm j\,300$$

$$\therefore\ v_c(t) = e^{-400t}\left[A_1 \cos 300t + A_2 \sin 300t\right]$$

t = 0^- (Steady – State)



$$i_L(0^-) = \frac{-6\text{ V}}{500\ \Omega} = -6/500\ \text{A} = i_L(0^+)$$

$$v_c(0^-) = 250\left(-6/500\right) + 6 = 3\text{ V} = v_c(0^+)$$

$$\text{Now from (1):}\quad \frac{dv_c(0^+)}{dt} = -\ 2\times10^5 i_L(0^+) - 800 v_c(0^+) = 0$$

$$\text{So } v_c(0^+) = 3 = A_1$$

$$\frac{dv_c(0^+)}{dt} = 0 = -400A_1 + 300A_2 \ \Rightarrow A_2 = 4$$

$$\therefore\ v_c(t) = e^{-400t}\left[3\cos 300t + 4\sin\ 300t\right]\ V$$

P 9.6-2 The switch of the circuit shown in Figure P 9.6-2 is opened at $t = 0$. Determine and plot $v(t)$ when $C = 1/4$ F. Assume steady state at $t = 0^-$.

Answer: $v(t) = -4e^{-2t} \sin 2t$ V

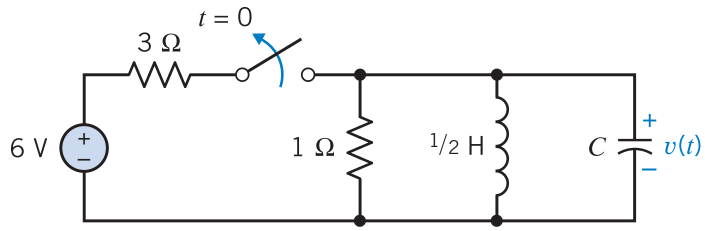
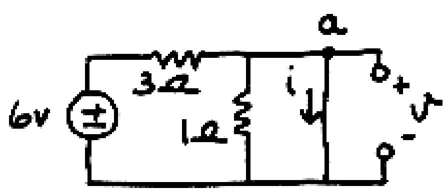


Figure P 9.6-2

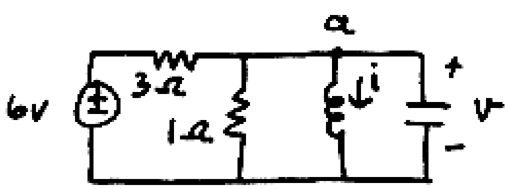
Solution:

$\underline{t = 0^-}$



$$i(0) = 2 \text{ A}$$
$$v(0) = 0$$

$\underline{t = 0^-}$



KCL at node a:

$$\frac{v}{1} + C\frac{dv}{dt} + \frac{1}{L}\int_0^t v\,dt + i(0) = 0 \quad (1)$$

in operator form have $v + Csv + \dfrac{1}{Ls}v + i(0) = 0$ or $\left(s^2 + \dfrac{1}{C}s + \dfrac{1}{LC}\right)v = 0$

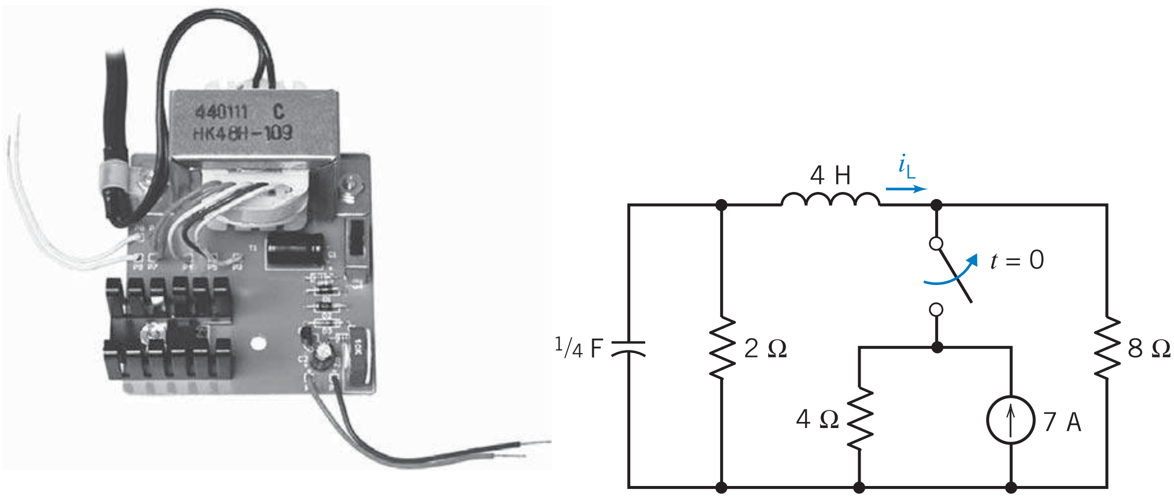
with $s^2 + 4s + 8 = 0 \quad \Rightarrow \quad s = -2 \pm j2$

$$v(t) = e^{-2t}\left[B_1 \cos 2t + B_2 \sin 2t\right]$$
$$v(0) = 0 = B_1$$

From (1), $\dfrac{dv(0)}{dt} = \dfrac{1}{C}\left[-i(0) - v(0)\right] = -4[2] = -8 = 2B_2 \;\; or \;\; B_2 = -4$

So $\underline{v(t) = -4e^{-2t} \sin 2t \text{ V}}$

P 9.6-3 A 240-W power supply circuit is shown in Figure P 9.6-3*a*. This circuit employs a large inductor and a large capacitor. The model of the circuit is shown in Figure P 9.6-3*b*. Find $i_L(t)$ for $t > 0$ for the circuit of Figure P 9.6-3*b*. Assume steady-state conditions exist at $t = 0^-$.
Answer: $i_L(t) = e^{-2t}(-4\cos t + 2\sin t)$ A

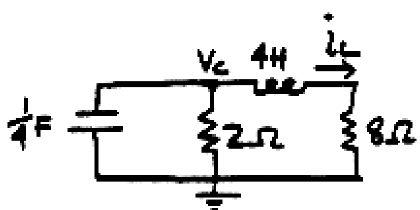


(a) *(b)*

Figure P 9.6-3

Solution:

$\underline{t > 0}$



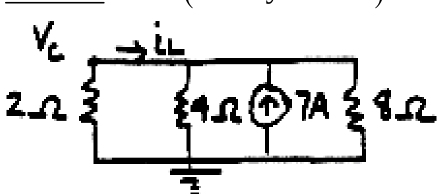
$$\text{KCL at } v_c: \quad \frac{1}{4}\frac{dv_c}{dt} + \frac{v_c}{2} + i_L = 0 \qquad (1)$$

$$\text{KVL}: \quad v_c = \frac{4 di_L}{dt} + 8 i_L \qquad (2)$$

$$(2) \text{ into } (1) \text{ yields } \frac{d^2 i_L}{dt^2} + 4\frac{di_L}{dt} + 5 i_L = 0 \Rightarrow s^2 + 4s + 5 = 0 \Rightarrow s = -2 \pm i$$

$$\therefore i_L(t) = e^{-2t}\left[A_1 \cos t + A_2 \sin t\right]$$

$\underline{t = 0^-}$ (Steady – State)



$$\frac{v_c(0^-)}{2} = 7\left(\frac{4\|8}{4\|8+2}\right)$$

$$\Rightarrow v_c(0^-) = 8 \text{ V} = v_c(0^+)$$

$$i_L(0^-) = \frac{-8 \text{ V}}{2\ \Omega} = -4 \text{ A} = i_L(0^+)$$

$$\therefore \text{ from } (2) \quad \frac{di_L(0^+)}{dt} = \frac{v_c(0^+)}{4} - 2i_L(0^+) = \frac{8 \text{ V}}{4} - 2(-4) = 10 \ \frac{\text{A}}{\text{s}}$$

$$\text{So } i_L(0^+) = -4 = A_1 \quad \text{and} \quad \frac{di_L(0^+)}{dt} = 10 = -2A_1 + A_2 \Rightarrow A_2 = 2$$

$$\therefore \underline{i_L(t) = e^{-2t}\left[-4 \cos t + 2 \sin t\right] \text{ A}}$$

P 9.6-4 The natural response of a parallel *RLC* circuit is measured and plotted as shown in Figure P 9.6-4. Using this chart, determine an expression for $v(t)$.

Hint: Notice that $v(t)$ = 260 mV at t = 5 ms and that $v(t)$ = – 200 mV at t = 7.5 ms. Also, notice that the time between the first and third zero-crossings is 5 ms.

Answer:

$v(t) = 544e^{-276t}\sin 1257t$ V

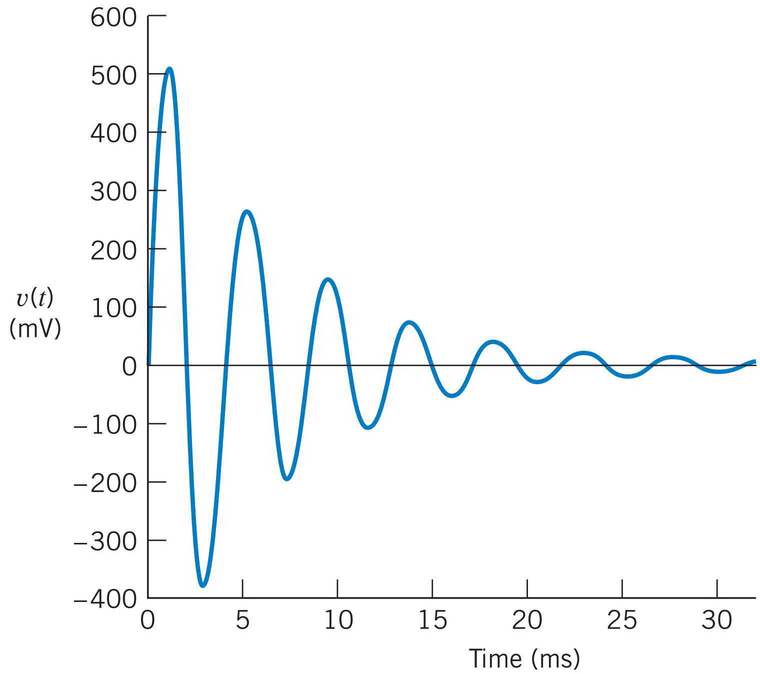


Figure P 9.6-4

Solution:

The response is underdamped so

$$\therefore\ v(t) = e^{-\alpha t}\left[k_1\cos\ \omega t + k_2\sin\omega t\ \right] + k_3$$

$$v(\infty) = 0\ \Rightarrow\ k_3 = 0,\ \ v(0) = 0\ \Rightarrow\ k_1 = \ 0$$

$$\therefore\ v(t) = \ k_2 e^{-\alpha t}\sin\omega t$$

From Fig. P 9.6-4

$$t\ \approx 5\text{ms}\ \leftrightarrow\ v \approx 260\text{mV}\quad(\text{max})$$

$$t\ \approx 7.5\text{ms} \leftrightarrow\ v \approx -200\ \text{mV}\ (\text{min})$$

$\therefore$ distance between adjacent maxima is $\approx\ \omega\ =\ \dfrac{2\pi}{T}\ =\ 1257\ \text{rad}/\text{s}$

so

$$0.26\ =\ k_2 e^{-\alpha\ (.005)}\sin\ \left(1257\ (.005)\right)\qquad (1)$$

$$-0.2\ =\ k_2 e^{-\alpha}\ (.0075)\sin\left(\ 1257\ (.0075)\right)\qquad (2)$$

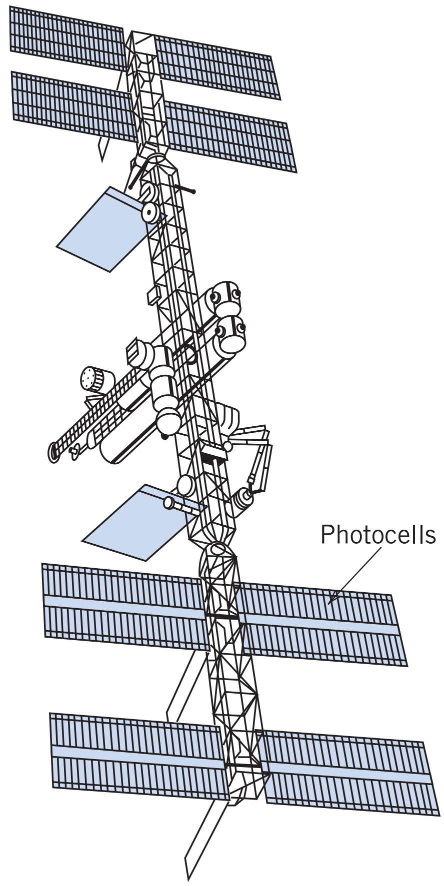
Dividing (1) by (2) gives

$$-\ 1.3\ =\ e^{\alpha\ (0.0025)}\left(\frac{\sin\ \ (6.29\ rad)}{\sin\ \ (9.43\ rad)}\right)\ \Rightarrow\ \ e^{0.0025\ \alpha}\ =\ 1.95\ \Rightarrow\ \alpha = 267$$

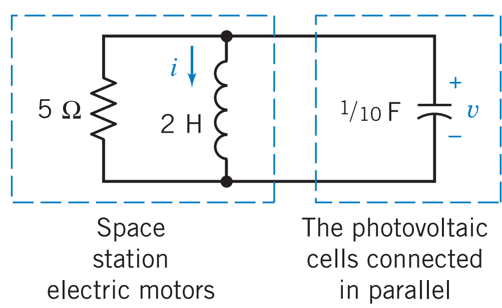
From (1) $k_2 = 544$ so

$$\underline{v(t)\ =\ 544e^{-267t}\sin 1257t}\qquad (\text{approx. answer})$$

P 9.6-5 The photovoltaic cells of the proposed space station shown in Figure P 9.6-5*a* provide the voltage $v(t)$ of the circuit shown in Figure P 9.6-5*b*. The space station passes behind the shadow of earth (at $t = 0$) with $v(0) = 2$ V and $i(0) = 1/10$ A. Determine and sketch $v(t)$ for $t > 0$.



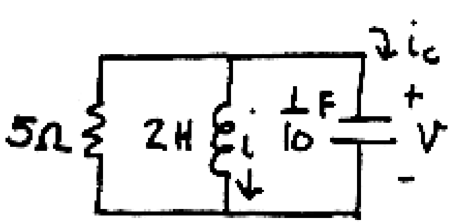
(a)



(b)

Figure P 9.6-5

Solution:



$v(0) = 2\ \text{V}$ and $i(0) = 1/10$ A

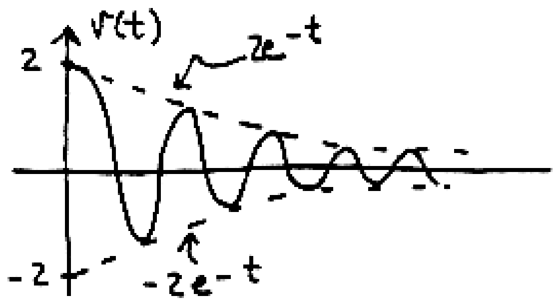
Char. eq. $\Rightarrow \quad s^2 + \dfrac{1}{RC}s + \dfrac{1}{LC} = 0$ or $s^2 + 2s + 5 = 0$ thus the roots are $s = -1 \pm j2$

So have $v(t) = e^{-t}\left[B_1 \cos 2t + B_2 \sin 2t\right]$ now $v(0^+) = \underline{2 = B_1}$

Need $\dfrac{dv(0^+)}{dt} = \dfrac{1}{C} i_c(0^+)$. KCL yields $i_c(0^+) = -\dfrac{v(0^+)}{5} - i(0^+) = -\dfrac{1}{2}\ \dfrac{\text{V}}{\text{s}}$

So $\dfrac{dv(0^+)}{dt} = 10\left(-\dfrac{1}{2}\right) = -B_1 + 2B_2 \Rightarrow \underline{B_2 = -3/2}$

Finally, we have $\underline{v(t) = 2e^{-t}\cos 2t - \dfrac{3}{2}e^{-t}\sin 2t \text{ V}}\quad t > 0$



Section 9-7: Forced Response of an RLC Circuit

P 9.7-1 Determine the forced response for the inductor current i_f when (a) $i_s = 1$ A, (b) $i_s = 0.5t$ A, and (c) $i_s = 2e^{-250t}$ A for the circuit of Figure P 9.7-1.

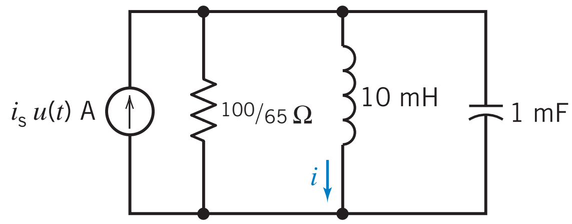
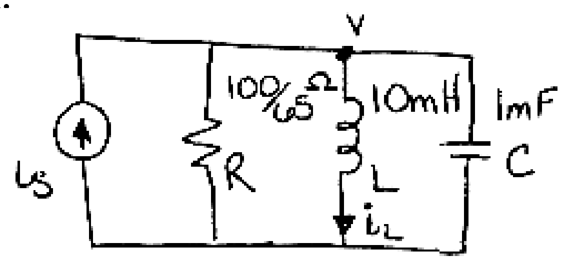


Figure P 9.7-1

Solution:



$$\text{KCL}: i_s = \frac{v}{R} + i_L + C\frac{dv}{dt}$$

$$\text{KVL}: v = L\frac{di_L}{dt}$$

$$i_s = \frac{L}{R}\frac{di_L}{dt} + i_L + LC\frac{d^2 i_L}{dt^2}$$

(a)

$i_s = 1\,u(t) \quad \therefore \quad \text{assume } i_f = A$

Let $i_L = i_f = A$ in $\dfrac{d^2 i_L}{dt^2} + \dfrac{1}{RC}\dfrac{di_L}{dt} + \dfrac{1}{LC}i_L = i_s$

to get: $0 + 0 + A\dfrac{1}{(.01)(1\times10^{-3})} = 1 \;\Rightarrow\; A = \underline{1\times10^{-5} = i_f}$

(b)

$i_s = 0.5t\,u(t) \quad \therefore \quad \text{assume } i_f = At + B$

$$0 + A\frac{65}{(100)\ (.001)} + (At + B)\frac{1}{(.01)(.001)} = 0.5t$$

$\Rightarrow \quad 650A + 100000B = 0 \quad$ and $\quad 100000At = 0.5t$

$A = 5\times10^{-6}$

$B = 3.25\times10^{-8}$

$\underline{i_f = 5\times10^{-6}t - 3.25\times10^{-8}\,A}$

(c)

$i_s = 2e^{-250t}$ Assumming $i_f = Ae^{-250t}$ does not work

because i_f cannot have the same form as i_s $\therefore$ we choose $i_f = Bte^{-250t}$

$$\frac{Be^{-250t}}{RC} + \frac{-250Bte^{-250t}}{RC} + \frac{Bte^{-250t}}{LC} = 2e^{-250t}$$

$150\,B = 2$

$B = 0.0133$

$\underline{i_f = 0.0133\,te^{-250t}\text{ A}}$

P 9.7-2 Determine the forced response for the capacitor voltage, v_f, for the circuit of Figure P 9.7-2 when

(a) $v_s = 2$ V, (b) $v_s = 0.2t$ V, and (c) $v_s = 1e^{-30t}$ V.

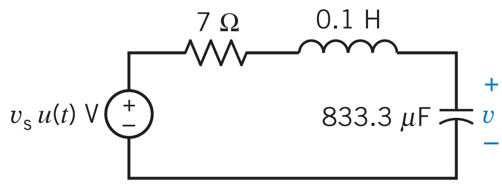


Figure P 9.7-2

Solution:

Represent the circuit by the differential equation: $\frac{d^2v}{dt}+\frac{R}{L}\frac{dv}{dt}+\frac{1}{LC}v=v_s$

(a) $v_s = 2 \quad \therefore$ assume $v_f = A$

Then $0+0+12000A=2$ so $A = \underline{1/6000 = v_f}$

(b) $v_s = 0.2t \quad \therefore$ assume $v_f = At+B$

$70A+12000At+12000B = 0.2t \quad \Rightarrow \quad 70A+12000\,B = 0$ and $12000At=0.2t$

$$A=\frac{1}{60000}\;,\quad B=\frac{70A}{12000} \quad \Rightarrow \quad B=350$$

$$\therefore v_f = \underline{\frac{t}{60000}+350\text{ V}}$$

(c) $v_s = e^{-30t} \quad \therefore$ assume $v_f = Ae^{-30t}$

$$900A-2100Ae^{-30t}+12000Ae^{-30t} = e^{-30t} \quad \Rightarrow \quad 10800Ae^{-30t} = e^{-30t} \quad \Rightarrow \quad A = \frac{1}{10800}$$

$$v_f = \underline{\frac{e^{-30t}}{10800}\text{ V}}$$

P 9.7-3 A circuit is described for $t > 0$ by the equation

$$\frac{d^2v}{dt^2}+5\frac{dv}{dt}+6v=v_s$$

Find the forced response v_f for $t > 0$ when (a) $v_s = 8$ V, (b) $v_s = 3e^{-4t}$ V, and (c) $v_s = 2e^{-2t}$ V.

Answer: (a) $v_f = 8/6$ V (b) $v_f = \frac{3}{2}\ e^{-4t}$ V (c) $v_f = 2te^{-2t}$ V

Solution:

(a)

The differential equation is $\frac{d^2v}{dt^2}+5\frac{dv}{dt}+6v=8$ so we try $v_f = B$.

Substituting $v_f = B$ into the differential equation gives $6B=8$ $\therefore\ \underline{v_f = 8/6\ \text{V}}$

(b)

The differential equation is $\frac{d^2v}{dt^2}+5\frac{dv}{dt}+6v=3e^{-4t}$ so we try $v_f = Be^{-4t}$.

Substituting $v_f = Be^{-4t}$ into the differential equation gives

$(-4)^2 B + 5(-4)B+6B=3 \Rightarrow B=3/2$

$\therefore\ \underline{v_f = 3/2e^{-4t}}$

(c)

The differential equation is $\frac{d^2v}{dt^2}+5\frac{dv}{dt}+6v=2e^{-2t}$ so we try $v_f = Bte^{-2t}$

(since -2 is a natural frequency). Substituting $v_f = Bte^{-2t}$ into the differential equation gives

$\Rightarrow (4t-4)B+5B(1-2t)+6Bt=2 \Rightarrow B=2$

$\therefore\ \underline{v_f = 2te^{-2t}}$

Section 9-8: Complete Response of an RLC Circuit

P 9.8-1 Determine $i(t)$ for $t > 0$ for the circuit shown in Figure P 9.8-1.

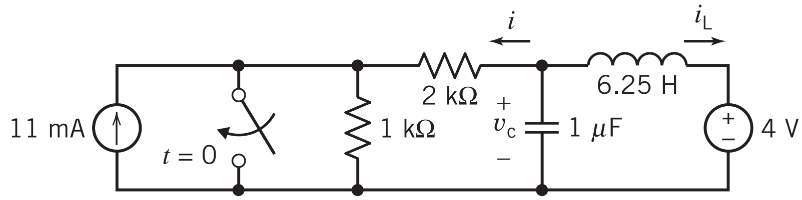
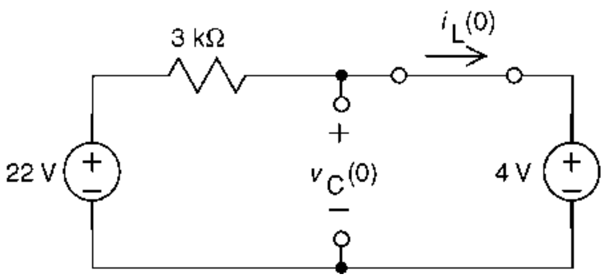


Figure P 9.8-1

Solution:

First, find the steady state response for t < 0, when the switch is open. Both inputs are constant so the capacitor will act like an open circuit at steady state, and the inductor will act like a short circuit. After a source transformation at the left of the circuit:



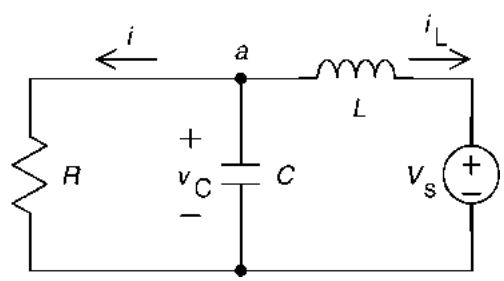
$$i_L(0) = \frac{22-4}{3000} = 6 \text{ mA}$$

and

$$v_C(0) = 4 \text{ V}$$

After the switch closes

Apply KCL at node a:



$$\frac{v_C}{R} + C\frac{d}{dt}v_C + i_L = 0$$

Apply KVL to the right mesh:

$$L\frac{d}{dt}i_L + V_s - v_C = 0 \quad \Rightarrow \quad v_C = L\frac{d}{dt}i_L + V_s$$

After some algebra:

$$\frac{d^2}{dt^2}i_L + \frac{1}{RC}\frac{d}{dt}i_L + \frac{1}{LC}i_L = -\frac{V_s}{RLC} \quad \Rightarrow \quad \frac{d^2}{dt^2}i_L + \left(10^3\right)\frac{d}{dt}i_L + \left(\frac{4}{25}\times10^6\right)i_L = -\frac{16}{25}\times10^3$$

The characteristic equation is

$$s^2 + \left(10^3\right)s + \left(\frac{4}{25}\times10^6\right) = 0 \quad \Rightarrow \quad s_{1,2} = -200, -800 \text{ rad/s}$$

After the switch closes the steady-state inductor current is $i_L(\infty)$ = -4 mA so

$$i_L(t) = -0.004 + A_1\, e^{-200\,t} + A_2\, e^{-800\,t}$$

$$v_C(t) = \left(\frac{4}{25}\right)\frac{d}{dt}i_L(t) + 4 = \frac{4}{25}\left[(-200)\,A_1\, e^{-200\,t} + (-800)\,A_2\, e^{-800\,t}\right] + 4$$
$$= (-32)\,A_1\, e^{-200\,t} + (-128)\,A_2\, e^{-800\,t} + 4$$

Let $t = 0$ and use the initial conditions:

$$0.006 = -0.004 + A_1 + A_2 \quad \Rightarrow \quad 0.01 = A_1 + A_2$$

$$4 = (-32)\,A_1 + (-128)\,A_2 + 4 \quad \Rightarrow \quad A_1 = (-4)\,A_2$$

So A_1 = 8.01 and A_2 = 2.00 and

$$i_L(t) = -0.004 + 8.01 e^{-200\,t} + 2.00\, e^{-800\,t} \ \text{ A}$$

$$v_C(t) = \left(-10^4\right) e^{-200\,t} + \left(10^4\right) e^{-800\,t} + 4 \ \text{ V}$$

$$i(t) = \frac{v_C(t)}{1000} = (-10)\, e^{-200\,t} + (10)\, e^{-800\,t} + 0.004 \ \text{ A}$$

P 9.8-2 Determine $i(t)$ for $t > 0$ for the circuit shown in Figure P 9.8-2.

Hint: Show that $1 = \frac{d^2}{dt^2}i(t)+5\frac{d}{dt}i(t)+5i(t)$ for $t > 0$

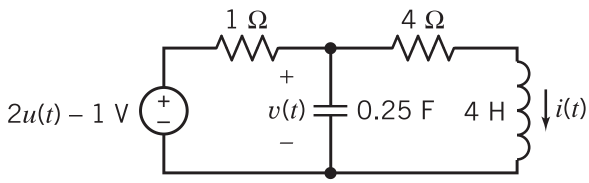
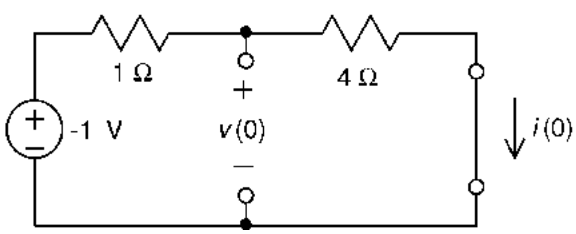


Figure P 9.8-2

Answer: $i(t) = 0.2 + 0.246\, e^{-3.62t} - 0.646\, e^{-1.38t}$ A for $t > 0$.

Solution:
First, find the steady state response for t < 0. The input is constant so the capacitor will act like an open circuit at steady state, and the inductor will act like a short circuit.

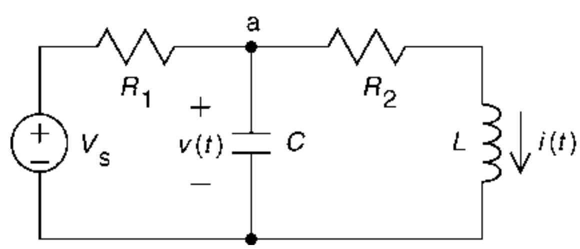


$$i(0) = \frac{-1}{1+4} = 0.2 \text{ A}$$

and

$$v(0) = \frac{4}{1+4}(-1) = -0.8 \text{ V}$$

For $t > 0$
Apply KCL at node a:



$$\frac{v - V_s}{R_1} + C\frac{d}{dt}v + i = 0$$

Apply KVL to the right mesh:

$$R_2\, i + L\frac{d}{dt}i - v = 0 \quad\Rightarrow\quad v = R_2\, i + L\frac{d}{dt}i_L$$

After some algebra:

$$\frac{d^2}{dt^2}i + \frac{L + R_1R_2C}{R_1L\,C}\frac{d}{dt}i + \frac{R_1+R_2}{R_1L\,C}i = \frac{V_s}{R_1L\,C} \quad\Rightarrow\quad \frac{d^2}{dt^2}i + 5\frac{d}{dt}i + 5\,i = 1$$

The forced response will be a constant, i_f= B so $1 = \frac{d^2}{dt^2}B + 5\frac{d}{dt}B + 5B \quad\Rightarrow\quad B = 0.2 \text{ A}$.

To find the natural response, consider the characteristic equation:

$$0 = s^2 + 5\,s + 5 = (s + 3.62)(s + 1.38)$$

The natural response is

$$i_n = A_1 e^{-3.62t} + A_2 e^{-1.38t}$$

so

$$i(t) = A_1 e^{-3.62t} + A_2 e^{-1.38t} + 0.2$$

Then

$$v(t) = \left(4\, i(t) + 4 \frac{d}{dt} i(t)\right) = -10.48 A_1 e^{-3.62t} - 1.52 A_2 e^{-1.38t} + 0.8$$

At t=0+

$$-0.2 = i(0+) = A_1 + A_2 + 0.2$$
$$-0.8 = v(0+) = -10.48 A_1 - 1.52 A_2 + 0.8$$

so $A_1 = 0.246$ and $A_2 = -0.646$. Finally

$$i(t) = 0.2 + 0.246\, e^{-3.62\,t} - 0.646\, e^{-1.38\,t} \text{ A}$$

P 9.8-3 Determine $v_1(t)$ for $t > 0$ for the circuit shown in Figure P 9.8-3.

Answer: $v_1(t) = 10 + e^{-2.4\times10^4}t - 6e^{-4\times10^3 t}$ V for $t > 0$

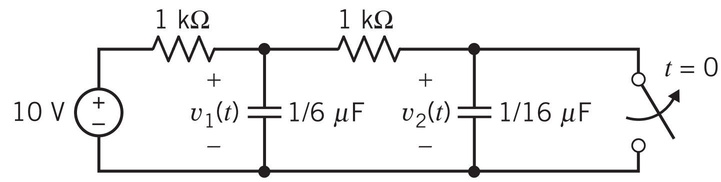
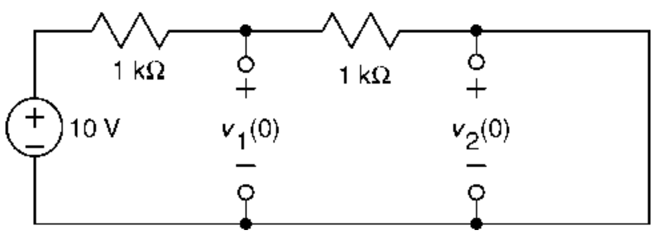


Figure P 9.8-3

Solution:

First, find the steady state response for t < 0. The input is constant so the capacitors will act like an open circuits at steady state.



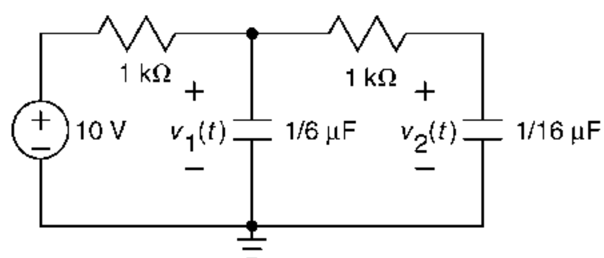
$$v_1(0) = \frac{1000}{1000+1000}(10) = 5 \text{ V}$$

and

$$v_2(0) = 0 \text{ V}$$

For $t > 0$,

Node equations:



$$\frac{v_1 - 10}{1000} + \left(\frac{1}{6}\times10^{-6}\right)\frac{d}{dt}v_1 + \frac{v_1 - v_2}{1000} = 0$$

$$\Rightarrow 2\,v_1 + \left(\frac{1}{6}\times10^{-3}\right)\frac{d}{dt}v_1 - 10 = v_2$$

$$\frac{v_1 - v_2}{1000} = \left(\frac{1}{16}\times10^{-6}\right)\frac{d}{dt}v_2$$

$$\Rightarrow \quad v_1 - v_2 = \left(\frac{1}{16}\times10^{-3}\right)\frac{d}{dt}v_2$$

After some algebra:

$$\frac{d^2}{dt^2}v_1 + \left(2.8\times10^4\right)\frac{d}{dt}v_1 + \left(9.6\times10^7\right)v_1 = 9.6\times10^8$$

The forced response will be a constant, $v_f = B$ so

$$\frac{d^2}{dt^2}B + \left(2.8\times10^4\right)\frac{d}{dt}B + \left(9.6\times10^7\right)B = 9.6\times10^8 \quad \Rightarrow \quad B = 10 \text{ V}.$$

To find the natural response, consider the characteristic equation:

$$s^2 + \left(2.8\times10^4\right)s + \left(9.6\times10^7\right) = 0 \quad \Rightarrow \quad s_{1,2} = -4\times10^3, -2.4\times10^4$$

The natural response is

$$v_{\text{n}} = A_1\, e^{-4\times10^3 t} + A_2\, e^{-2.4\times10^4 t}$$

so

$$v_1(t) = A_1\, e^{-4\times10^3 t} + A_2\, e^{-2.4\times10^4 t} + 10$$

At $t = 0$

$$5 = v_1(0) = A_1\, e^{-4\times10^3 (0)} + A_2\, e^{-2.4\times10^4 (0)} + 10 = A_1 + A_2 + 10 \qquad (1)$$

Next

$$2\, v_1 + \left(\frac{1}{6}\times10^{-3}\right)\frac{d}{dt}v_1 - 10 = v_2 \quad \Rightarrow \quad \frac{d}{dt}v_1 = 12000 v_1 + 6000\, v_2 - 6\times10^4$$

At $t = 0$

$$\frac{d}{dt}v_1(0) = 12000 v_1(0) + 6000\, v_2(0) - 6\times10^4 = 12000(5) + 6000(0) - 6\times10^4 = 0$$

so

$$\frac{d}{dt}v_1(t) = A_1\left(-4\times10^3\right)e^{-4\times10^3 t} + A_2\left(-2.4\times10^4\right)e^{-2.4\times10^4 t}$$

At $t = 0+$

$$0 = \frac{d}{dt}v_1(0) = A_1\left(-4\times10^3\right)e^{-4\times10^3 (0)} + A_2\left(-2.4\times10^4\right)e^{-2.4\times10^4 (0)} = A_1\left(-4\times10^3\right) + A_2\left(-2.4\times10^4\right)$$

so $A_1 = -6$ and $A_2 = 1$. Finally

$$v_1(t) = 10 + e^{-2.4\times10^4 t} - 6\, e^{-4\times10^3 t} \text{ V for } t > 0$$

P 9.8-4 Find $v(t)$ for $t > 0$ for the circuit shown in Figure P 9.8-4 when $v(0) = 1$ V and $i_L(0) = 0$.

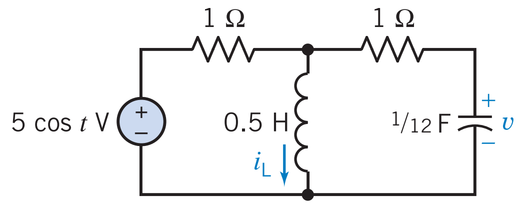


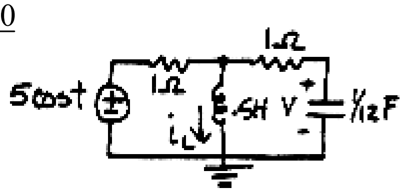
Figure P 9.8-4

Answer:

$$v = 25e^{-3t} - \frac{1}{17}\left[429e^{-4t} - 21\cos t + 33\ \sin t\right]\text{V}$$

Solution:

t > 0



$$\text{KCL at top node : }\left(0.5\frac{di_L}{dt} - 5\cos t\right) + i_L + \tfrac{1}{12}\tfrac{dv}{dt} = 0 \quad (1)$$

$$\text{KVL at right loop : } 0.5\frac{di_L}{dt} = \tfrac{1}{12}\frac{dv}{dt} + v \quad (2)$$

$$\tfrac{d}{dt}\text{ of }(1) \Rightarrow 0.5\frac{d^2 i_L}{dt^2} + \frac{di_L}{dt} + \frac{1}{12}\frac{d^2 v}{dt^2} = -5\sin t \ (3)$$

$$\tfrac{d}{dt}\text{ of }(2) \Rightarrow 0.5\frac{d^2 i_L}{dt^2} = \tfrac{1}{12}\frac{d^2 v}{dt^2} + \frac{dv}{dt} \quad (4)$$

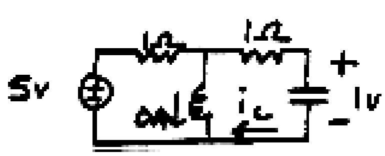
Solving for $\frac{d^2 i_L}{dt^2}$ in (4) and $\frac{di_L}{dt}$ in (2) & plugging into (3)

$$\frac{d^2 v}{dt^2} + 7\frac{dv}{dt} + 12v = -30\sin t \quad \Rightarrow s^2 + 7s + 12 = 0 \Rightarrow s = -3,\ -4$$

$$\text{so } v(t) = A_1 e^{-3t} + A_2 e^{-4t} + v_f$$

Try $v_f = B_1 \cos t + B_2 \sin t$ & plug into D.E., equating like terms yields $B_1 = \tfrac{21}{17}$, $B_2 = -\tfrac{33}{17}$

$t = 0^+$



$$i_c\left(0^+\right) = \frac{5-1}{1\,1} = 2A \quad \therefore \quad \frac{dv(0^+)}{dt} = \frac{2}{\tfrac{1}{12}} = 24\,\tfrac{\text{V}}{\text{s}}$$

$$\text{So } v(0^+) = 1 = A_1 + A_2 + \tfrac{21}{17}$$

$$\frac{dv(0^+)}{dt} = 24 = -3A_1 - 4A_2 - \tfrac{33}{17}$$

$$A_1 = 25, \quad A_2 = -\tfrac{429}{17}$$

$$\therefore\ v(t) = 25e^{-3t} - \tfrac{1}{17}\left(429e^{-4t} - 21\cos t + 33\sin t\right)\ \text{V}$$

P 9.8-5 Find $v(t)$ for $t > 0$ for the circuit of Figure P 9.8-5.

Answer: $v(t) = [-16e^{-t} + 16e^{-3t} + 8]u(t) + [16e^{-(t-2)} - 16e^{-3(t-2)} - 8]u(t-2)$ V

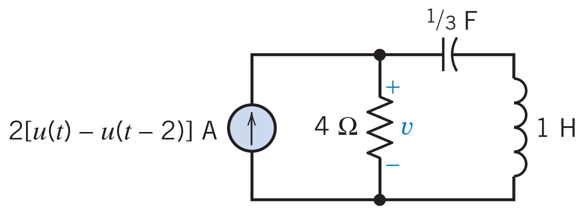
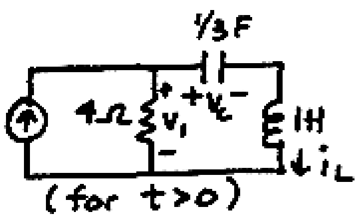


Figure P 9.8-5

Solution: Use superposition – first consider the $2u(t)$ source.



KVL at right mesh : $v_c + s i_L + 4(i_L - 2) = 0$ (1)

also : $i_L = (1/3)s v_c \Rightarrow v_c = (3/s)\, i_L$ (2)

Plugging (2) into (1) yields $(s^2 + 4s + 3)\, i_L = 0$, roots : $s = -1, -3$

So $i_L(t) = A_1 e^{-t} + A_2 e^{-3t}$

$\underline{t = 0^-}$ $\Rightarrow$ circuit is dead $\therefore v_c(0) = i_L(0) = 0$

Now from (1) $\dfrac{di_L(0^+)}{dt} = 8 - 4i_L(0^+) - v_C(0^+) = 8$ A/s

$$\left.\begin{aligned} &\text{So } i_L(0) = 0 = A_1 + A_2 \\ &\frac{di_L(0)}{dt} = 8 = -A_1 - 3A_2 \end{aligned}\right\} A_1 = 4,\ A_2 = -4$$

$\therefore i_L(t) = 4e^{-t} - 4e^{-3t}$

$\therefore$ $\underline{v_1(t) = 8 - 4\, i_L(t) = 8 - 16e^{-t} + 16e^{-3t}}$ V

Now for $2u(t-2)$ source, just take above expression and replace $t \to t-2$ and flip signs

$\therefore v_2(t) = -8 + 16e^{-(t-2)} - 16\, e^{-3(t-2)}$ V

$\therefore v(t) = v_1(t) + v_2(t)$

$$\underline{v(t) = \left[8 - 16e^{-t} + 16e^{-3t}\right] u(t) + \left[-8 + 16e^{-(t-2)} - 16\, e^{-3(t-2)}\right] u\,(t-2)\ \text{V}}$$

P 9.8-6 An experimental space station power supply system is modeled by the circuit shown in Figure P 9.8-6. Find $v(t)$ for $t > 0$. Assume steady-state conditions at $t = 0^{-}$.

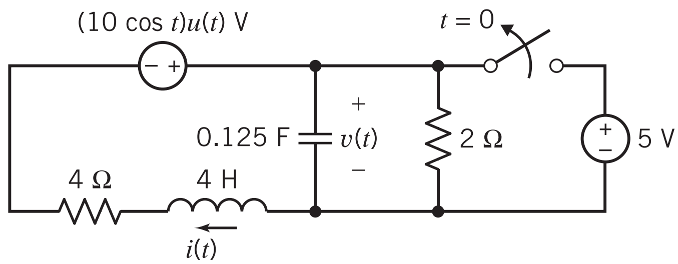
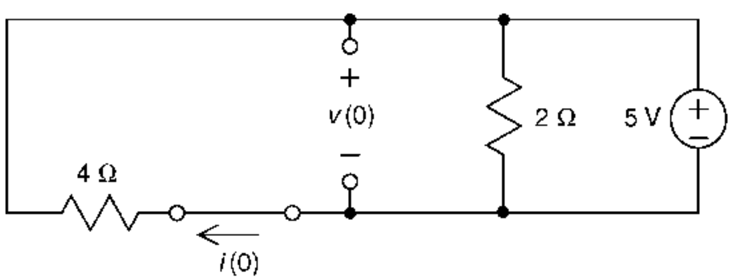


Figure P 9.8-6

Solution:
First, find the steady state response for t < 0, when the switch is closed. The input is constant so the capacitor will act like an open circuit at steady state, and the inductor will act like a short circuit.

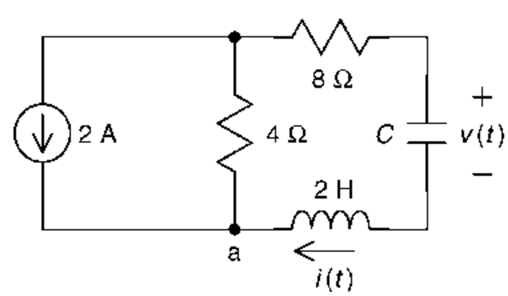


and

$$i(0) = -\frac{5}{4} = -1.25 \text{ mA}$$

$$v(0) = 5 \text{ V}$$

After the switch closes



Apply KCL at node a:

$$\frac{v}{2} + 0.125\frac{d}{dt}v = i$$

Apply KVL to the right mesh:

$$-10\cos t + v + 4\frac{d}{dt}i + 4\,i = 0$$

After some algebra: $$\frac{d^2}{dt^2}v + 5\frac{d}{dt}v + 6\,v = 20\cos t$$

The characteristic equation is

$$s^2 + 5\,s + 6 = 0 \quad \Rightarrow \quad s_{1,2} = -2, -3 \text{ rad/s}$$

Try $$v_f = A\cos t + B\sin t$$

$$\frac{d^2}{dt^2}\left(A\cos t + B\sin t\right) + 5\frac{d}{dt}\left(A\cos t + B\sin t\right) + 6\left(A\cos t + B\sin t\right) = 20\cos t$$

$$\left(-A\cos t - B\sin t\right) + 5\left(-A\sin t + B\cos t\right) + 6\left(A\cos t + B\sin t\right) = 20\cos t$$

$$\left(-A+5\,B+6\,A\right)\cos t+\left(-B-5\,A+6\,B\right)\sin t=20\cos t$$

So A =2 and B =2. Then $$v_{\mathrm{f}}=2\cos t+2\sin t$$

$$v(t)=2\cos t+2\sin t+A_1\,e^{-2t}+A_2\,e^{-3t}$$

Next $$\frac{v(t)}{2}+0.125\frac{d}{dt}v(t)=i(t)\quad\Rightarrow\quad\frac{d}{dt}v(t)=8\,i(t)-4\,v(t)$$

$$\frac{d}{dt}v(0)=8\,i(0)-4\,v(0)=8\left(-\frac{5}{4}\right)-4\,(5)=-30\quad\frac{\mathrm{V}}{s}$$

Let $t = 0$ and use the initial conditions:

$$5=v(0)=2\cos 0+2\sin 0+A_1\,e^{-0}+A_2\,e^{-0}=2+A_1+A_2$$

$$\frac{d}{dt}v(t)=-2\sin t+2\cos t-2\,A_1\,e^{-2t}-3\,A_2\,e^{-3t}$$

$$-30=\frac{d}{dt}v(0)=-2\sin 0+2\cos 0-2\,A_1\,e^{-0}-3\,A_2\,e^{-0}=2-2\,A_1-3\,A_2$$

So A_1 = -23 and A_2 = 26 and

$$v(t)=2\cos t+2\sin t-23e^{-2t}+26e^{-3t}$$

P 9.8-7 Find $v_c(t)$ for $t > 0$ in the circuit of Figure P 9.8-7 when (a) $C = 1/18$ F, (b) $C = 1/10$ F, and (c) $C = 1/20$ F.

Answer:

(a) $v_c(t) = 8e^{-3t} + 24te^{-3t} - 8$ V

(b) $v_c(t) = 10e^{-t} - 2e^{-5t} - 8$ V

(c) $v_c(t) = e^{-3t}(8 \cos t + 24 \sin t) - 8$ V

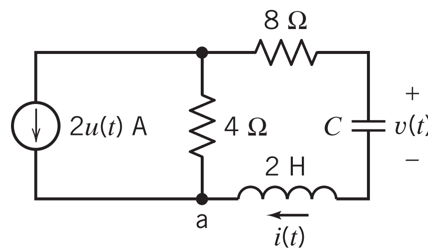
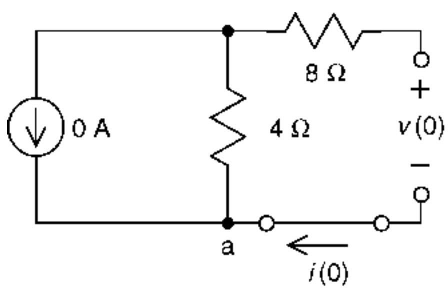


Figure P 9.8-7

Solution:

First, find the steady state response for t < 0, when the switch is closed. The input is constant so the capacitor will act like an open circuit at steady state, and the inductor will act like a short circuit.



$$i(0) = 0 \text{ A}$$

and

$$v(0) = 0 \text{ V}$$

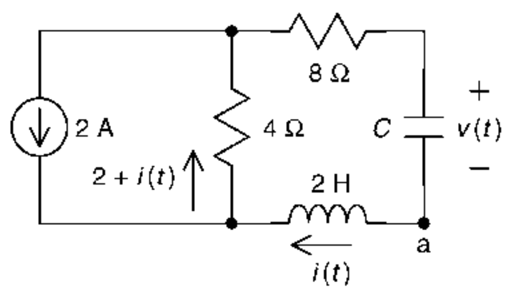
After the switch closes

Apply KCL at node a: $\quad C\frac{d}{dt}v = i$

Apply KVL to the right mesh:

$$8\,i + v + 2\frac{d}{dt}i + 4\left(2 + i\right) = 0$$

$$12\,i + v + 2\frac{d}{dt}i = -8$$



After some algebra: $\quad \frac{d^2}{dt^2}v + (6)\frac{d}{dt}v + \left(\frac{1}{2\,C}\right)v = -\frac{4}{C}$

The forced response will be a constant, $v_f = B$ so

$$\frac{d^2}{dt^2}B + (6)\frac{d}{dt}B + \left(\frac{1}{2\,C}\right)B = -\frac{4}{C} \quad \Rightarrow \quad B = -8 \text{ V}$$

(a)

When $C = 1/18$ F the differential equation is $\frac{d^2}{dt^2}v + (6)\frac{d}{dt}v + (9)v = -72$

The characteristic equation is $s^2 + 6\,s + 9 = 0 \quad \Rightarrow \quad s_{1,2} = -3, -3$

Then $v(t) = \left(A_1 + A_2\,t\right)e^{-3t} - 8$.

Using the initial conditions:

$$0 = v(0) = \left(A_1 + A_2(0)\right)e^0 - 8 \quad \Rightarrow \quad A_1 = 8$$

$$0 = \frac{d}{dt}v(0) = -3\left(A_1 + A_2(0)\right)e^0 + A_2\,e^0 \quad \Rightarrow \quad A_2 = 24$$

So

$$v(t) = (8 + 24t)e^{-3t} - 8 \text{ V} \quad \text{for } t > 0$$

(b)

When C = 1/10 F the differential equation is $\frac{d^2}{dt^2}v + (6)\frac{d}{dt}v + (5)v = -40$

The characteristic equation is $s^2 + 6\,s + 5 = 0 \quad \Rightarrow \quad s_{1,2} = -1, -5$

Then $v(t) = A_1\,e^{-t} + A_2\,e^{-5\,t} - 8$.

Using the initial conditions:

$$\left.\begin{aligned} 0 = v(0) = A_1\,e^0 + A_2\,e^0 - 8 \quad &\Rightarrow \quad A_1 + A_2 = 8 \\ 0 = \frac{d}{dt}v(0) = -A_1\,e^0 - 5\,A_2\,e^0 \quad &\Rightarrow \quad -A_1 - 5A_2 = 0 \end{aligned}\right\} \quad \Rightarrow \quad A_1 = 10 \text{ and } A_2 = -2$$

So

$$v(t) = 10e^{-t} - 2e^{-5\,t} - 8 \text{ V} \quad \text{for } t > 0$$

(c)

When C = 1/20 F the differential equation is $\frac{d^2}{dt^2}v + (6)\frac{d}{dt}v + (10)v = -80$

The characteristic equation is $s^2 + 6\,s + 10 = 0 \quad \Rightarrow \quad s_{1,2} = -3 \pm j$

Then $v(t) = e^{-3\,t}\left(A_1 \cos t + A_2 \sin t\right) - 8$.

Using the initial conditions:

$$0 = v(0) = e^0\left(A_1 \cos 0 + A_2 \sin 0\right) - 8 \quad \Rightarrow \quad A_1 = 8$$

$$0 = \frac{d}{dt}v(0) = -3e^0\left(A_1 \cos 0 + A_2 \sin 0\right) + e^0\left(-A_1 \sin 0 + A_2 \cos 0\right) \quad \Rightarrow \quad A_2 = 24$$

So

$$v(t) = e^{-3\,t}\left(8\cos t + 24 \sin t\right) - 8 \text{ V} \quad \text{for } t > 0$$

P 9.8-8 Find $v_c(t)$ for $t > 0$ for the circuit shown in Figure P 9.8-8.

Hint:

$$2=\frac{d^2}{dt^2}v_c(t)+6\frac{d}{dt}v_c(t)+2v_c(t) \text{ for } t>0$$

Answer:

$v_c(t) = 0.123e^{-5.65t} + 0.877e^{-0.35t} + 1$ V for $t > 0$.

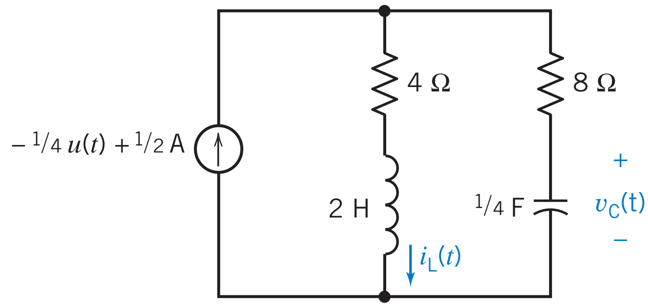
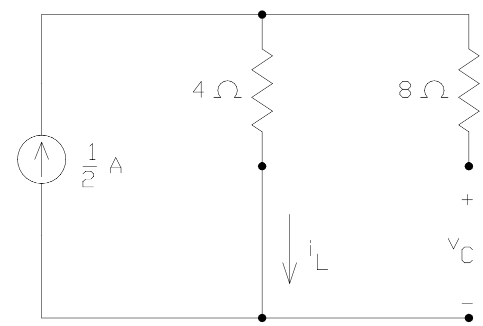


Figure P 9.8-8

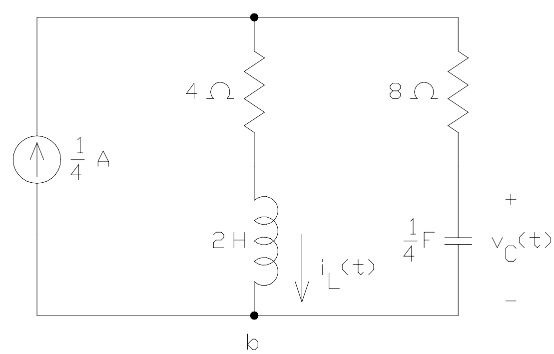
Solution:

The circuit will be at steady state for t<0:

so $i_L(0+) = i_L(0-) = 0.5$ A and $v_C(0+) = v_C(0-) = 2$ V.



For t>0:



Apply KCL at node b to get:

$$\frac{1}{4}=i_L(t)+\frac{1}{4}\frac{d}{dt}v_C(t) \quad\Rightarrow\quad i_L(t)=\frac{1}{4}-\frac{1}{4}\frac{d}{dt}v_C(t)$$

Apply KVL to the right-most mesh to get:

$$4\,i_L(t)+2\frac{d}{dt}i_L(t)=8\left(\frac{1}{4}\frac{d}{dt}v_C(t)\right)+v_C(t)$$

Use the substitution method to get

$$4\left(\frac{1}{4}-\frac{1}{4}\frac{d}{dt}v_C(t)\right)+2\frac{d}{dt}\left(\frac{1}{4}-\frac{1}{4}\frac{d}{dt}v_C(t)\right)=8\left(\frac{1}{4}\frac{d}{dt}v_C(t)\right)+v_c(t)$$

or

$$2=\frac{d^2}{dt^2}v_C(t)+6\frac{d}{dt}v_C(t)+2v_C(t)$$

The forced response will be a constant, v_C= B so $2=\frac{d^2}{dt^2}B+6\frac{d}{dt}B+2B \quad\Rightarrow\quad B=1\text{ V}$.

To find the natural response, consider the characteristic equation:

$$0=s^2+6s+2=(s+5.65)(s+0.35)$$

The natural response is

$$v_n = A_1 e^{-5.65t} + A_2 e^{-0.35t}$$

so

$$v_C(t) = A_1 e^{-5.65t} + A_2 e^{-0.35t} + 1$$

Then

$$i_L(t) = \frac{1}{4} + \frac{1}{4}\frac{d}{dt}v_C(t) = \frac{1}{4} + 1.41A_1 e^{-5.65t} + 0.0875A_2 e^{-0.35t}$$

At t=0+

$$2 = v_C(0+) = A_1 + A_2 + 1$$

$$\frac{1}{2} = i_L(0+) = \frac{1}{4} + 1.41A_1 + 0.0875A_2$$

so $A_1 = 0.123$ and $A_2 = 0.877$. Finally

$$v_C(t) = 0.123\, e^{-5.65t} + 0.877\, e^{-0.35t} + 1 \text{ V}$$

P 9.8-9 In Figure P 9.8-9, determine the inductor current $i(t)$ when $i_s = 5u(t)$ A. Assume that $i(0) = 0$, $v_c(0) = 0$.

Answer: $i(t) = 5 + e^{-2t}\,[-5\cos 5t - 2\sin 5t]$ A

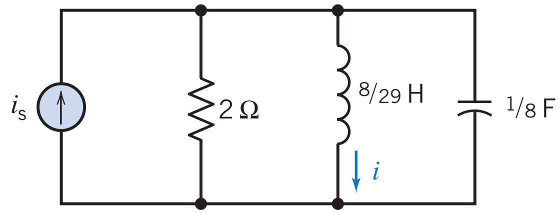
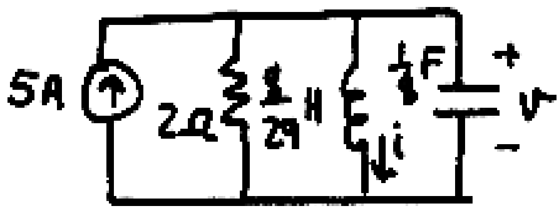


Figure P 9.8-9

Solution:



$$\text{KCL}: \quad C\frac{dv}{dt} + i + \frac{v}{2} = i_s$$

$$LC\frac{d^2i}{dt} + i + \left(\frac{L}{2}\right)\frac{di}{dt} = 5u(t)$$

$$\frac{1}{29}\frac{d^2i}{dt} + i + \left(\frac{4}{29}\right)\frac{di}{dt} = 5u(t)$$

$$\frac{d^2i}{dt} + 4\frac{di}{dt} + i = 145u(t)$$

Characteristic eqn: $s^2 + 4s + 29 = 0 \quad \Rightarrow \quad$ roots : $s = -2 \pm j5$

$\therefore \; i_n = e^{-2t}\left[A\cos 5t + B\sin 5t\right]$ and $i_f = {}^{145}\!/_{29} = 5$

So $i(t) = 5 + e^{-2t}\left[A\cos 5t + B\sin 5t\right]$

Now $i\,(0) = 0 = A + 5 \;\Rightarrow\; A = -5$

$$\frac{di(0)}{dt} = 0 = -2A + 5B \;\Rightarrow B = -2$$

P 9.8-10 Railroads widely use automatic identification of railcars. When a train passes a tracking station, a wheel detector activates a radio-frequency module. The module's antenna, as shown in Figure P 9.8-10*a*, transmits and receives a signal that bounces off a transponder on the locomotive. A trackside processor turns the received signal into useful information consisting of the train's location, speed, and direction of travel. The railroad uses this information to schedule locomotives, trains, crews, and equipment more efficiently.

One proposed transponder circuit is shown in Figure P 9.8-10*b* with a large transponder coil of $L = 5$ H. Determine $i(t)$ and $v(t)$. The received signal is

$$i_s = 9 + 3e^{-2t}u(t) \text{ A.}$$

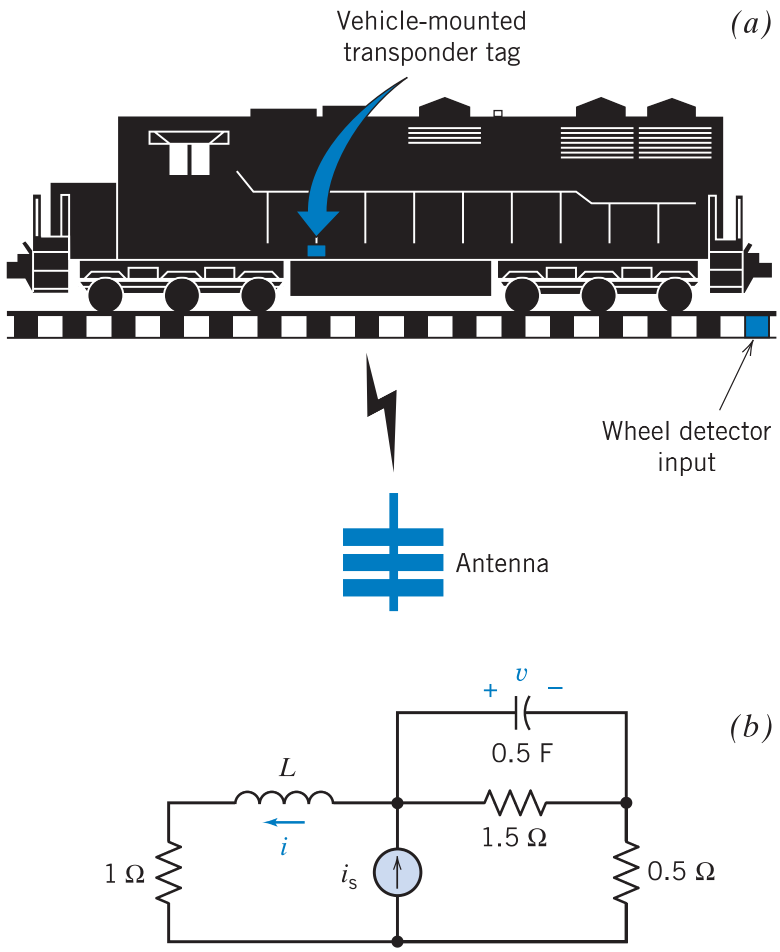
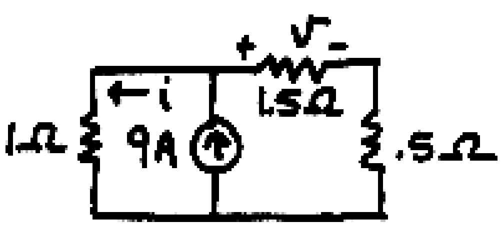


Figure P 9.8-10

Solution:

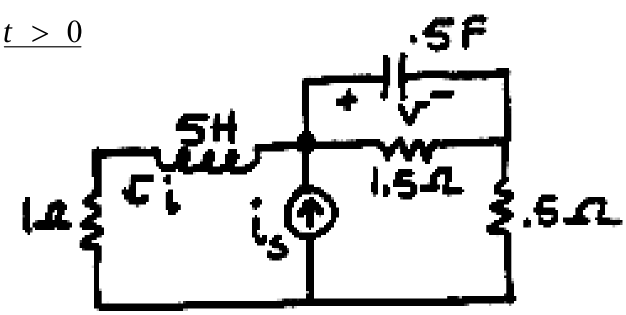
<u>$t = 0^-$</u>



$$i(0^-) = \frac{2}{2+1} \times 9 = 6 \text{ A} = i(0^+)$$

$$\& \quad v(0^-) = \frac{1}{2+1} \times 9 \times 1.5 = 4.5 \text{ V} = v(0^+)$$

<u>$t > 0$</u>



KCL at middle node: $i + 0.5\,\dfrac{dv}{dt} + \dfrac{v}{1.5} = i_s$ (1)

KVL : $v + (0.5\,\dfrac{dv}{dt} + \dfrac{v}{1.5})\,(0.5) = \dfrac{5di}{dt} + i$ (2)

Solving for i in (1) and plugging into (2) yields

$$\frac{d^2v}{dt^2} + \left(\frac{49}{30}\right)\frac{dv}{dt} + \left(\frac{4}{5}\right)v = \left(\frac{2}{5}\right)i_s + 2\frac{d_{is}}{dt} \quad \text{where } i_s = 9 + 3e^{-2t}\,\text{A}$$

So the characteristic equation is $s^2 + \dfrac{49}{30}s + \dfrac{4}{5} = 0$ and its roots are $s = -0.817 \pm j0.365$

$v_n(t) = e^{-.817t}\left[A_1\cos(0.365t) + A_2\sin(0.365t)\right]$

Try $v_f(t) = B_0 + B_1e^{-2t}$ and substitute $v_f(t)$ into the differential equation and equate like terms

to get $B_0 = 4.5,\ B_1 = -7.04$

Then $v(t) = e^{-.817t}\left[A_1\cos(0.365t) + A_2\sin(0.365t)\right] + 4.5 - 7.04e^{-2t}$

Now using the initial conditions gives $v(0) = 4.5 = A_1 + 4.5 - 7.04 \Rightarrow A_1 = 7.04$

and $\dfrac{dv(0)}{dt} = 2i_s(0) - 2i(0) - \dfrac{4}{3}v(0) = 2(9+3) - 2(6) - \dfrac{4}{3}(4.5) = 6$

$\therefore\ 6 = -0.817A_1 + 0.365\,A_2 + 14.08 \Rightarrow A_2 = -22.82$

so $i(t) = i_s(t) - \dfrac{v(t)}{1.5} - 0.5\dfrac{dv(t)}{dt}$

$i(t) = \underline{e^{-0.817t}\left[2.37\cos(0.365t) + 7.14\sin(0.365t)\right] + 6 + 0.65e^{-2t}\ \text{A}}$

P 9.8-11 Determine $v(t)$ for $t > 0$ for the circuit shown in Figure P 9.8-11.

Answer:

$v_c(t) = 0.75\, e^{-4t} - 6.75\, e^{-36t} + 16$ V for $t > 0$

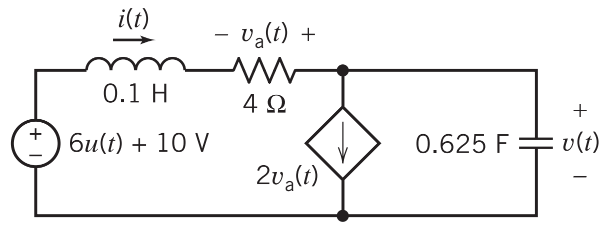
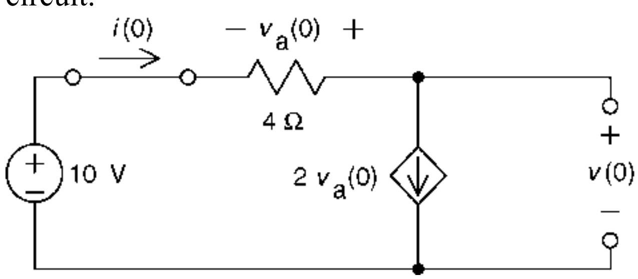


Figure P 9.8-11

Solution:

First, find the steady state response for t < 0, when the switch is closed. The input is constant so the capacitor will act like an open circuit at steady state, and the inductor will act like a short circuit.

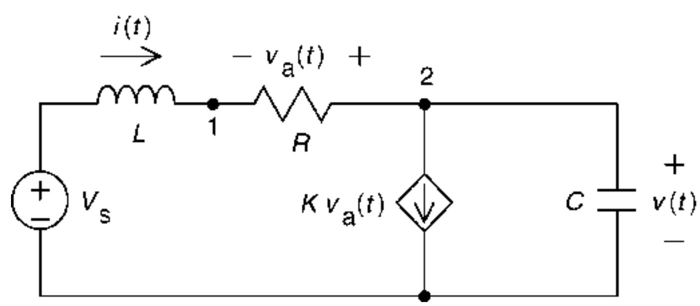


$$v_a(0) = -4\,i(0)$$

$$i(0) = 2(-4\,i(0)) \Rightarrow i(0) = 0 \text{ A}$$

and $\quad v(0) = 10 \text{ V}$

For $t > 0$



Apply KCL at node 2:

$$\frac{v_a}{R} + K\,v_a + C\,\frac{d}{dt}v = 0$$

KCL at node 1 and Ohm's Law:

$$v_a = -R\,i$$

so

$$\frac{d}{dt}v = \frac{1+K\,R}{C\,R}\,i$$

Apply KVL to the outside loop: $\quad L\,\frac{d}{dt}i + R\,i + v - V_s = 0$

After some algebra:

$$\frac{d^2}{dt^2}v + \frac{R}{L}\frac{d}{dt}v + \frac{1+K\,R}{L\,C}v = \frac{1+K\,R}{L\,C}V_s \quad \Rightarrow \quad \frac{d^2}{dt^2}v + 40\,\frac{d}{dt}v + 144\,v = 2304$$

The forced response will be a constant, $v_f = B$ so

$$\frac{d^2}{dt^2}B + (40)\frac{d}{dt}B + (144)B = 2304 \quad \Rightarrow \quad B = 16 \text{ V}$$

The characteristic equation is $s^2 + 40\,s + 144 = 0 \quad \Rightarrow \quad s_{1,2} = -4, -36$.

Then
$$v(t) = A_1\,e^{-4t} + A_2\,e^{-36\,t} + 16\,.$$

Using the initial conditions:

$$\left.\begin{aligned} 10 = v(0) = A_1\,e^0 + A_2\,e^0 + 16 \quad &\Rightarrow \quad A_1 + A_2 = -6 \\ 0 = \frac{d}{dt}v(0) = -4\,A_1\,e^0 - 36\,A_2\,e^0 \quad &\Rightarrow \quad -4\,A_1 - 36\,A_2 = 0 \end{aligned}\right\} \quad \Rightarrow \quad A_1 = 0.75 \text{ and } A_2 = -6.75$$

So

$$v(t) = 0.75\,e^{-4\,t} - 6.75\,e^{-36\,t} + 16 \text{ V} \quad \text{for } t > 0$$

P 9.8-12 The circuit shown in Figure P 9.8-12 is at steady state before the switch opens. The inductor current is given to be

$i(t) = 240 + 193e^{-6.25t}\cos(9.27t - 102°)$ mA for $t \geq 0$

Determine the values of R_1, R_3, C, and L.

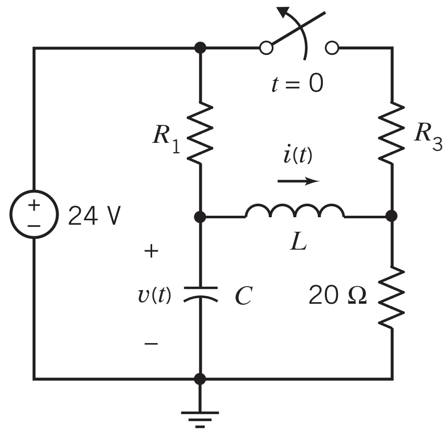
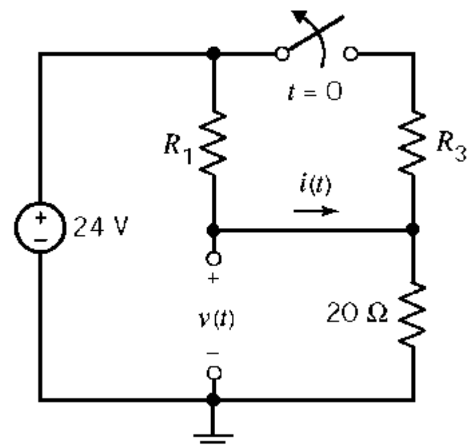


Figure P 9.8-12

Solution:

Two steady state responses are of interest, before and after the switch opens. At steady state, the capacitor acts like an open circuit and the inductor acts like a short circuit.



For $t > 0$, the switch is open. At steady state, inductor current is $i(\infty) = \dfrac{24}{R_1 + 20}$. From the given equation,

$i(\infty) = \lim_{t\to\infty} i(t) = 0.24$. Thus,

$$0.24 = \frac{24}{R_1 + 20} \quad \Rightarrow \quad R_1 = 80\ \Omega.$$

For $t < 0$, the switch is closed and the circuit is at steady state.

$$\frac{24}{(80 \,\|\, R_3) + 20} = 0.24 + 0.193\cos(-102°) = 0.2$$

Consequently, $R_3 = 80\ \ \Omega$

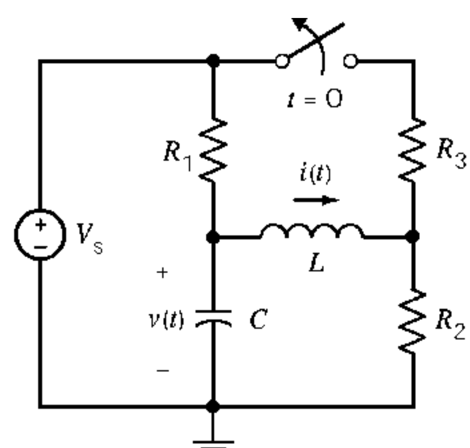
After the switch opens, apply KCL and KVL to get

$$R_1\left(i(t) + C\frac{d}{dt}v(t)\right) + v(t) = V_s$$

Apply KVL to get

$$v(t) = L\frac{d}{dt}i(t) + R_2\, i(t)$$

Substituting $v(t)$ into the first equation gives



$$R_1\left(i(t)+C\frac{d}{dt}\left(L\frac{d}{dt}i(t)+R_2\,i(t)\right)\right)+L\frac{d}{dt}i(t)+R_2\,i(t)=V_s$$

then

$$R_1\,C\,L\frac{d^2}{dt^2}i(t)+\left(R_1\,C\,R_2+L\right)\frac{d}{dt}i(t)+\left(R_1+R_2\right)i(t)=V_s$$

Dividing by $R_1\,C\,L$:

$$\frac{d^2}{dt^2}i(t)+\left(\frac{R_1\,C\,R_2+L}{R_1\,C\,L}\right)\frac{d}{dt}i(t)+\left(\frac{R_1+R_2}{R_1\,C\,L}\right)i(t)=\frac{V_s}{R_1\,C\,L}$$

Compare to

$$\frac{d^2}{dt^2}i(t)+2\alpha\frac{d}{dt}i(t)+{\omega_0}^2\,i(t)=f(t)$$

to get

$$2\alpha=\frac{R_1\,C\,R_2+L}{R_1\,C\,L},\quad {\omega_0}^2=\frac{R_1+R_2}{R_1\,C\,L}\quad\text{and}\quad f(t)=\frac{V_s}{R_1\,C\,L}$$

From the given equation, we have $\alpha=6.25$ and $\omega_d=9.27$ rad/s . Consequently, $\omega_0=\sqrt{{\omega_d}^2+\alpha^2}=11.18$ rad/s . Next

$$12.5=\frac{R_1\,C\,R_2+L}{R_1\,C\,L}=\frac{20}{L}+\frac{1}{80C}\quad\text{and}\quad 125=\frac{R_1+R_2}{R_1\,C\,L}=\frac{1.25}{C\,L}\quad\Rightarrow\quad 100=\frac{1}{C\,L}$$

So

$$12.5=\frac{20}{\frac{1}{100C}}+\frac{1}{80C}\quad\Rightarrow\quad 0=2000C^2-12.5C+0.0125\quad\Rightarrow\quad C=1.25,\ 5\ \text{mF}$$

The corresponding values of the inductance are $L=8,\ 2$ H .

There are two solutions:

$$R_1=80\ \Omega,\ R_3=80\ \Omega,\ C=1.25\text{ mF and }L=8\text{ H}$$

and

$$R_1=80\ \Omega,\ R_3=80\ \Omega,\ C=5\text{ mF and }L=2\text{ H}$$

We have used the initial condition $i(0)=0.2$ A but we have not yet used the initial condition

$$v(t)=L\frac{d}{dt}i(t)+R_2\,i(t)\quad\Rightarrow\quad\frac{d}{dt}i(0)=\frac{v(0)}{L}-\frac{R_2\,i(0)}{L}=\frac{8}{L}-\frac{4}{L}=\frac{4}{L}$$

from the given equation,

$$i(t) = 0.24 + e^{-6.25t}\left(-0.04\cos(9.27t) + 0.1888\sin(9.27t)\right) \text{ A} \quad \text{for } t \geq 0$$

$$\frac{d}{dt}i(t) = (-6.25)e^{-6.25t}\left(-0.04\cos(9.27t) + 0.1888\sin(9.27t)\right)$$
$$+(9.27)e^{-6.25t}\left(0.04\sin(9.27t) + 0.1888\cos(9.27t)\right) \quad \text{for } t \geq 0$$

$$\frac{d}{dt}i(0) = (-6.25)(-0.04) + (9.27)(1.888) = 2$$

Consequently,

$$2 = \frac{d}{dt}i(0) = \frac{4}{L} \quad \Rightarrow \quad L = 2 \text{ H}$$

and we choose

$$R_1 = 80\ \Omega,\ R_3 = 80\ \Omega,\ C = 5 \text{ mF and } L = 2 \text{ H}$$

P 9.8-13 The circuit shown in Figure P 9.8-13 is at steady state before the switch opens. Determine the inductor current, $i(t)$, for $t > 0$.

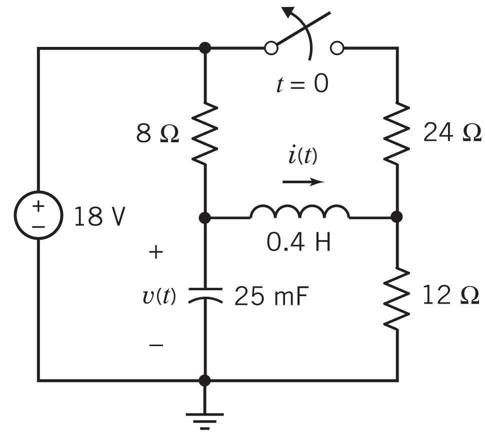
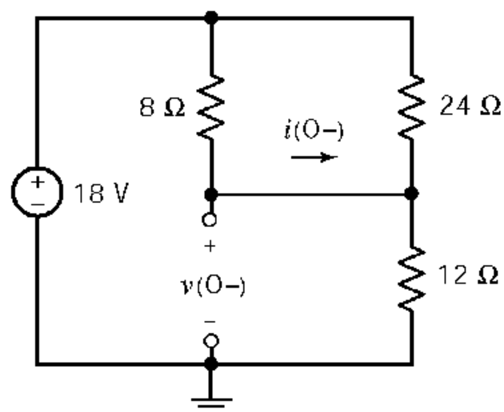


Figure P 9.8-13

Solution:
First, we find the initial conditions;

For $t < 0$, the switch is closed and the circuit is at steady state. At steady state, the capacitor acts like an open circuit and the inductor acts like a short circuit.

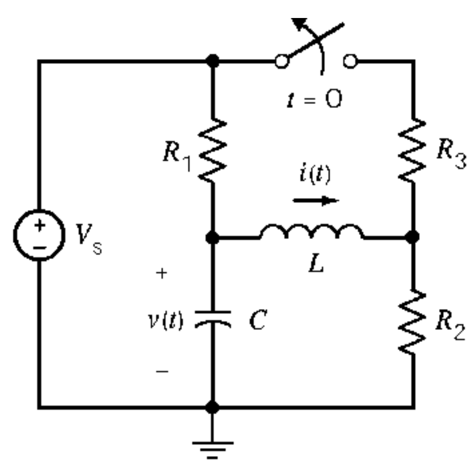


$$v(0-) = \frac{12}{(8 \| 24) + 12} \times 18 = 12 \text{ V}$$

and

$$i(0-) = \frac{24}{8+24} \times \frac{18}{(8 \| 24) + 12} = 0.75 \text{ A}$$

Next, represent the circuit by a differential equation.



After the switch opens, apply KCL and KVL to get

$$R_1\left(i(t) + C\frac{d}{dt}v(t)\right) + v(t) = V_s$$

Apply KVL to get

$$v(t) = L\frac{d}{dt}i(t) + R_2\, i(t)$$

Substituting $v(t)$ into the first equation gives

$$R_1\left(i(t) + C\frac{d}{dt}\left(L\frac{d}{dt}i(t) + R_2\, i(t)\right)\right) + L\frac{d}{dt}i(t) + R_2\, i(t) = V_s$$

then

$$R_1 C L \frac{d^2}{dt^2} i(t) + \left(R_1 C R_2 + L\right) \frac{d}{dt} i(t) + \left(R_1 + R_2\right) i(t) = V_s$$

Dividing by $R_1 C L$:

$$\frac{d^2}{dt^2} i(t) + \left(\frac{R_1 C R_2 + L}{R_1 C L}\right) \frac{d}{dt} i(t) + \left(\frac{R_1 + R_2}{R_1 C L}\right) i(t) = \frac{V_s}{R_1 C L}$$

Compare to

$$\frac{d^2}{dt^2} i(t) + 2\alpha \frac{d}{dt} i(t) + \omega_0^2 \, i(t) = f(t)$$

to get

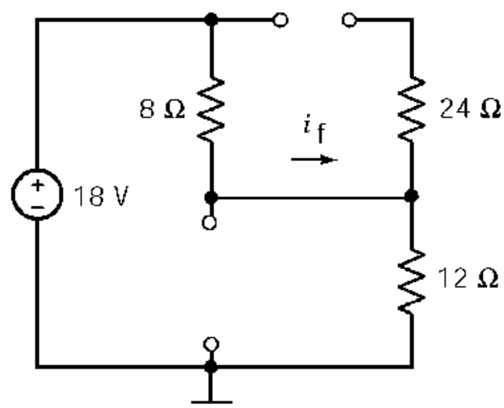
$$2\alpha = \frac{R_1 C R_2 + L}{R_1 C L}, \quad \omega_0^2 = \frac{R_1 + R_2}{R_1 C L} \quad \text{and} \quad f(t) = \frac{V_s}{R_1 C L}$$

With the given element values, we have $\alpha = 17.5$ and $\omega_0^2 = 250$. Consequently, the roots of the characteristic equation are $s_1 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -25$ and $s_2 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -10$. The natural response is

$$i_n(t) = A_1 e^{-10t} + A_2 e^{-25t}$$

Next, determine the forced response.

The steady state response after the switch opens will be used as the forced response. At steady state, the capacitor acts like an open circuit and the inductor acts like a short circuit.



$$i_f = \frac{18}{8+12} = 0.9 \text{ A}$$

So

$$i(t) = i_n(t) + i_f(t) = A_1 e^{-10t} + A_2 e^{-25t} + 0.9$$

It remains to evaluate A_1 and A_2 using the initial conditions. At $t = 0$ we have

$$0.75 = i(0) = A_1 + A_2 + 0.9$$

The other initial condition comes from

$$\frac{d}{dt} i(t) = \frac{v(t)}{L} - \frac{R_2}{L} i(t) \quad \Rightarrow \quad \frac{d}{dt} i(0) = \frac{12}{0.4} - \frac{12}{0.4} \times 0.75 = 7.5$$

then

$$7.5 = \frac{d}{dt} i(0) = -10\, A_1 - 25\, A_2$$

Solving these equations gives $A_1 = 0.25$ and $A_2 = -0.4$ so

$$i(t) = 0.25\, e^{-10t} - 0.4\, e^{-25t} + 0.9 \;\; \text{A} \quad \text{for } t > 0$$

(checked using LNAPTR 7/21/04)

***P 9.8-14** The circuit shown in Figure P 9.8-14 is at steady state before the switch closes. Determine the capacitor voltage, $v(t)$, for $t > 0$.

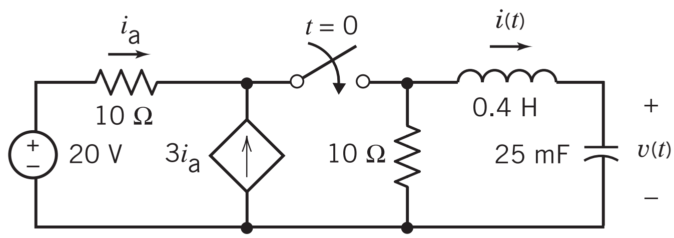
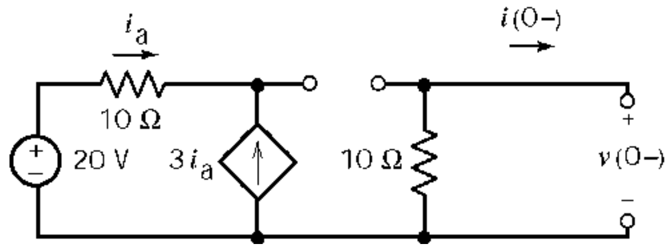


Figure P 9.8-14

Solution:

First, we find the initial conditions;

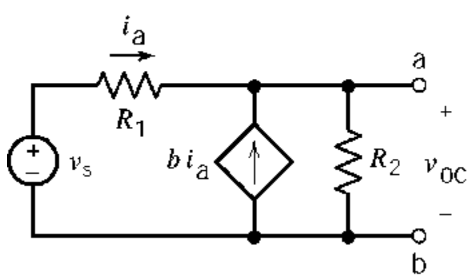
For $t < 0$, the switch is open and the circuit is at steady state. At steady state, the capacitor acts like an open circuit and the inductor acts like a short circuit.



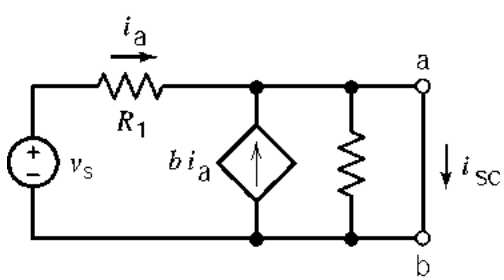
$$v(0-)=0 \text{ V and } i(0-)=0 \text{ A}$$

also $$\frac{d}{dt}v(0)=\frac{i(0)}{0.025}=0$$

Next, represent the circuit after the switch closes by a differential equation. To do so, we find the Thevenin equivalent circuit for the part of the circuit to the left of the inductor.



$$\left.\begin{aligned} i_a &= \frac{v_s - v_{oc}}{R_1} \\ i_a + b\,i_a &= \frac{v_{oc}}{R_2} \end{aligned}\right\} \Rightarrow v_{oc} = \frac{v_s R_2 (1+b)}{R_1 + R_2(1+b)}$$



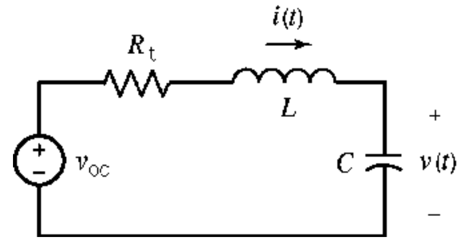
$$i_{sc} = i_a(1+b) = \frac{v_s}{R_1}(1+b)$$

$$R_t = \frac{v_{oc}}{i_{sc}} = \frac{\dfrac{v_s R_2(1+b)}{R_1 + R_2(1+b)}}{\dfrac{v_s}{R_1}(1+b)} = \frac{R_1 R_2}{R_1 + R_2(1+b)}$$

With the given values, $v_{oc} = 16$ V and $R_t = 2\ \Omega$.

After the switch closes, apply KVL to get

$$R_t\, i(t) + L\frac{d}{dt}i(t) + v(t) = v_{oc}$$



Apply KCL to get $i(t) = C\frac{d}{dt}v(t)$

Substituting $i(t)$ into the first equation gives

$$\frac{d^2}{dt^2}v(t) + \left(\frac{R}{L}\right)\frac{d}{dt}v(t) + \left(\frac{1}{CL}\right)v(t) = \frac{v_{oc}}{CL}$$

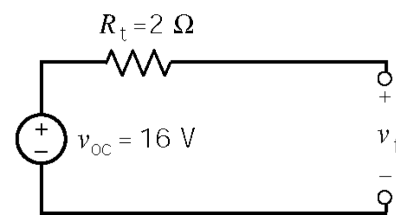
Compare to
$$\frac{d^2}{dt^2}v(t) + 2\alpha\frac{d}{dt}v(t) + {\omega_0}^2\, v(t) = f(t)$$

to get
$$2\alpha = \frac{R_t}{L},\quad {\omega_0}^2 = \frac{1}{CL} \quad \text{and} \quad f(t) = \frac{v_{oc}}{CL}$$

With the given element values, we have $\alpha = 2.5$ and ${\omega_0}^2 = 100$. Consequently, the roots of the characteristic equation are $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - {\omega_0}^2} = -2.5 \pm j9.682$ and the circuit is underdamped. The damped resonant frequency is $\omega_d = \sqrt{{\omega_0}^2 - \alpha^2} = 9.682 \text{ rad/s}$. The natural response is

$$v_n(t) = e^{-2.5t}\left(A_1\cos 9.682t + A_2 \sin 9.682t\right)$$

Next, determine the forced response.



The steady state response after the switch closes will be used as the forced response. At steady state, the capacitor acts like an open circuit and the inductor acts like a short circuit.

$$v_f = v_{oc} = 16 \text{ V}$$

so
$$v(t) = 16 + e^{-2.5t}\left(A_1\cos 9.682t + A_2 \sin 9.682t\right)$$

It remains to evaluate A_1 and A_2 using the initial conditions. At $t = 0$ we have

$$0 = v(0) = 16 + A_1 \quad \Rightarrow \quad A_1 = -16$$

And
$$0 = \frac{d}{dt}v(0) = -2.5\, A_1 + 9.682\, A_2 \quad \Rightarrow \quad A_2 = -\frac{2.5 \times 16}{9.682} = -4.131$$

Finally,
$$v(t) = 16 + e^{-2.5t}\left(-16\cos 9.682t - 4.131\sin 9.682t\right)$$
$$= 16 + 16.525\, e^{-2.5t}\cos\left(9.682t + 165.5°\right) \text{ V} \quad \text{for } t \ge 0$$

(checked using LNAPTR 7/22/04)

P 9.8-15 The circuit shown in Figure P 9.8-15 is at steady state before the switch closes. Determine the capacitor voltage, $v(t)$, for $t > 0$.

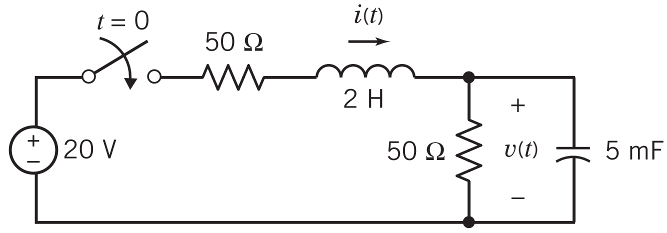
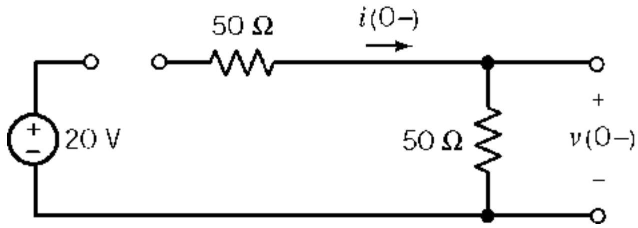


Figure P 9.8-15

Solution:
First, we find the initial conditions;

For $t < 0$, the switch is open and the circuit is at steady state. At steady state, the capacitor acts like an open circuit and the inductor acts like a short circuit.



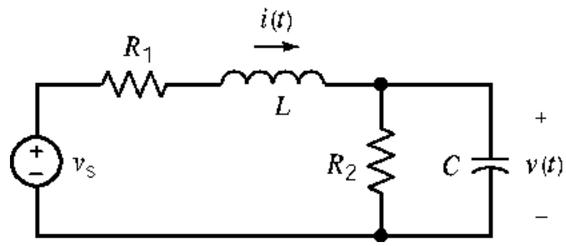
$$v(0-) = 0 \text{ V and } i(0-) = 0 \text{ A}$$

also

$$\frac{d}{dt}v(0) = \frac{i(0)}{0.005} - \frac{v(0)}{50 \times 0.005} = 0$$

Next, represent the circuit after the switch closes by a differential equation.

After the switch closes, use KCL to get



$$i(t) = \frac{v(t)}{R_2} + C\frac{d}{dt}v(t)$$

Use KVL to get

$$v_s = R_1\, i(t) + L\frac{d}{dt}i(t) + v(t)$$

Substitute to get

$$v_s = \frac{R_1}{R_2}v(t) + R_1 C\frac{d}{dt}v(t) + \frac{L}{R_2}\frac{d}{dt}v(t) + CL\frac{d^2}{dt^2}v(t) + v(t)$$

$$= CL\frac{d^2}{dt^2}v(t) + \left(R_1 C + \frac{L}{R_2}\right)\frac{d}{dt}v(t) + \frac{R_1 + R_2}{R_2}v(t)$$

Finally,

$$\frac{v_s}{CL} = \frac{d^2}{dt^2}v(t) + \left(\frac{R_1}{L} + \frac{1}{R_2 C}\right)\frac{d}{dt}v(t) + \frac{R_1 + R_2}{R_2 CL}v(t)$$

Compare to

$$\frac{d^2}{dt^2}i(t) + 2\alpha\frac{d}{dt}i(t) + \omega_0^{\,2}\, i(t) = f(t)$$

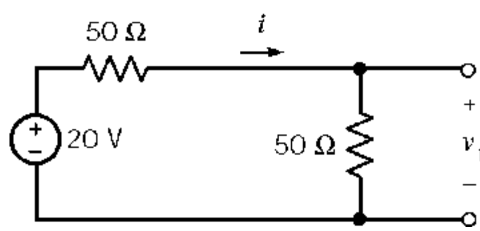
to get

$$2\alpha = \frac{R_1}{L} + \frac{1}{R_2 C}, \quad \omega_0^{\,2} = \frac{R_1 + R_2}{R_2 CL} \quad \text{and} \quad f(t) = \frac{v_s}{CL}$$

With the given element values, we have $\alpha = 14.5$ and $\omega_0^{\,2} = 200$. Consequently, the roots of the characteristic equation are $s_1 = -11.3$ and $s_2 = -17.7$ so the circuit is overdamped. The natural response is

$$v_n(t) = A_1 e^{-11.3t} + A_2 e^{-17.7t}$$

Next, determine the forced response.



The steady state response after the switch opens will be used as the forced response. At steady state, the capacitor acts like an open circuit and the inductor acts like a short circuit.

$$v_f = \frac{1}{2}v_s = 10 \text{ V}$$

So

$$v_n(t) = 10 + A_1 e^{-11.3t} + A_2 e^{-17.7t}$$

It remains to evaluate A_1 and A_2 using the initial conditions. At $t = 0$ we have

$$0 = v(0) = 10 + A_1 + A_2$$

and

$$0 = \frac{d}{dt}v(0) = -11.3\, A_1 - 17.7\, A_2$$

Solving these equations gives

$$A_1 = -27.6 \text{ and } A_2 = 17.6$$

Finally,

$$v(t) = 10 - 27.6e^{-11.3t} + 17.6e^{-17.7t}$$

(checked using LNAPTR 7/26/04)

P 9.8-16 The circuit shown in Figure P 9.8-16 is at steady state before the switch closes. Determine the inductor current, $i(t)$, for $t > 0$.

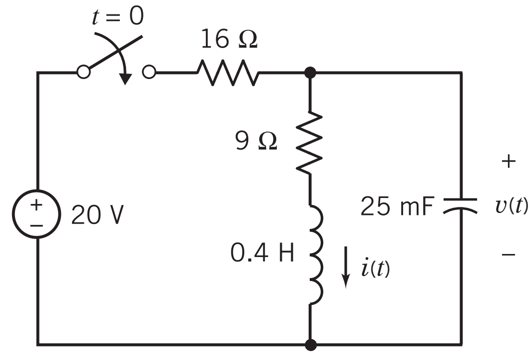
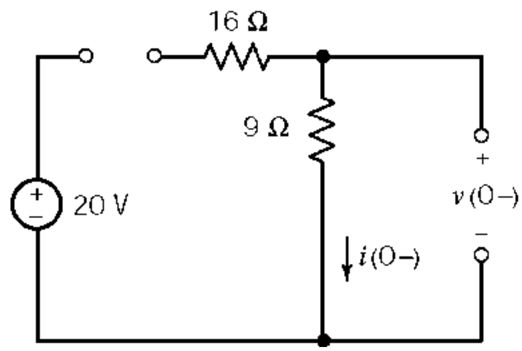


Figure P 9.8-16

Solution:

First, we find the initial conditions;



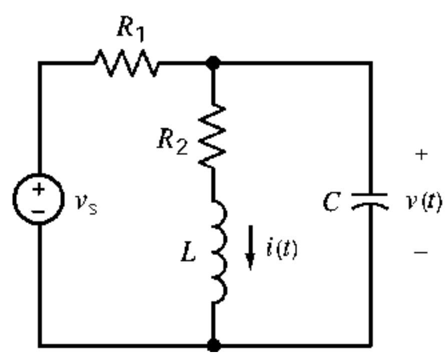
For $t < 0$, the switch is closed and the circuit is at steady state. At steady state, the capacitor acts like an open circuit and the inductor acts like a short circuit.

$$v(0-) = 0 \text{ V and } i(0-) = 0 \text{ A}$$

Also

$$9\, i(0) + 0.4 \frac{d}{dt} i(0) = v(0) \quad \Rightarrow \quad \frac{d}{dt} i(0) = 0$$

Next, represent the circuit by a differential equation.



After the switch closes use KVL to get

$$R_2\, i(t) + L \frac{d}{dt} i(t) = v(t)$$

Use KCL and KVL to get

$$v_s = R_1 \left(i(t) + C \frac{d}{dt} v(t) \right) + v(t)$$

Substitute to get

$$\begin{aligned} v_s &= R_1 i(t) + R_1 C R_2 \frac{d}{dt} i(t) + R_1 C\, L \frac{d^2}{dt^2} i(t) + R_2 i(t) + L \frac{d}{dt} i(t) \\ &= R_1 C L \frac{d^2}{dt^2} i(t) + \left(R_1 R_2 C + L \right) \frac{d}{dt} i(t) + \left(R_1 + R_2 \right) i(t) \end{aligned}$$

then

$$\frac{v_s}{R_1 C L} = \frac{d^2}{dt^2} i(t) + \left(\frac{R_2}{L} + \frac{1}{R_1 C} \right) \frac{d}{dt} i(t) + \frac{R_1 + R_2}{R_1 C L} i(t)$$

Compare to

$$\frac{d^2}{dt^2}i(t)+2\alpha\frac{d}{dt}i(t)+\omega_0^{\,2}\,i(t)=f(t)$$

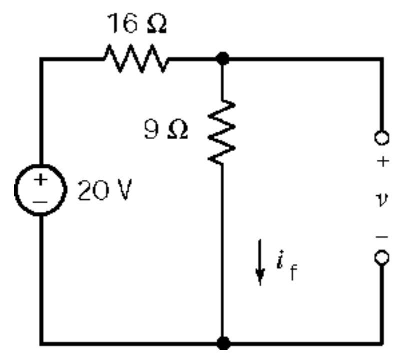
to get

$$2\alpha=\frac{R_2}{L}+\frac{1}{R_1C},\quad \omega_0^{\,2}=\frac{R_1+R_2}{R_1\,C\,L}\quad\text{and}\quad f(t)=\frac{V_s}{R_1\,C\,L}$$

With the given element values, we have $\alpha=12.5$ and $\omega_0^{\,2}=156.25$. Consequently, the roots of the characteristic equation are $s_{1,2}=-\alpha\pm\sqrt{\alpha^2-\omega_0^{\,2}}=-12.5,\ -12.5$ so the circuit is critically damped. The natural response is

$$i_n(t)=\left(A_1+A_2\,t\right)e^{-12.5t}$$

Next, determine the forced response.



The steady state response after the switch opens will be used as the forced response. At steady state, the capacitor acts like an open circuit and the inductor acts like a short circuit.

$$i_f=\frac{20}{16+9}=0.8\ \text{A}$$

So

$$i(t)=i_n(t)+i_f(t)=\left(A_1+A_2\,t\right)e^{-12.5t}+0.8$$

It remains to evaluate A_1 and A_2 using the initial conditions. At $t = 0$ we have

$$0=i(0)=A_1+0.8\quad\Rightarrow\quad A_1=-0.8$$

And

$$0=\frac{d}{dt}i(0)=-12.5\,A_1-A_2\quad\Rightarrow\quad A_2=10$$

Thus

$$i(t)=\left(-0.8+10t\right)e^{-12.5t}+0.8\quad\text{for } t>0$$

(checked using LNAPTR 7/27/04)

P 9.8-17 The circuit shown in Figure P 9.8-17 is at steady state before the switch opens. Determine the inductor current, $i_2(t)$, for $t > 0$.

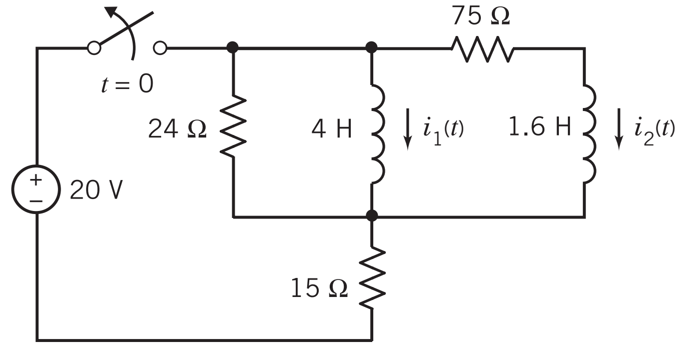
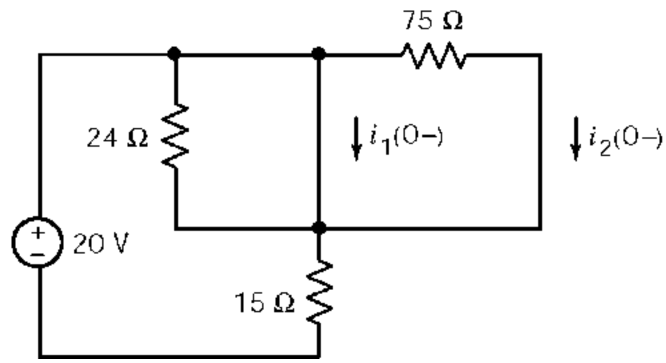


Figure P 9.8-17

Solution:

First, we find the initial conditions;

For $t < 0$, the switch is closed and the circuit is at steady state. At steady state, the inductors act like short circuits.



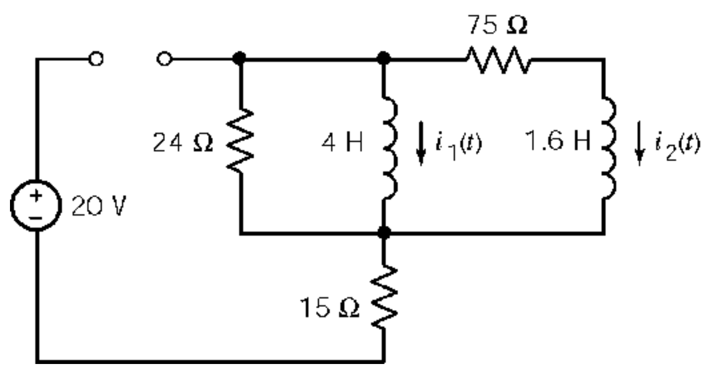
$$i_1(0-) = \frac{20}{15} = 1.333 \text{ A}$$

and

$$i_2(0-) = 0 \text{ A}$$

Next, represent the circuit by a differential equation.

After the switch opens, KVL gives



$$L_1 \frac{d}{dt} i_1(t) = R_2\, i_2(t) + L_2 \frac{d}{dt} i_2(t)$$

KVL and KCL give

$$L_1 \frac{d}{dt} i_1(t) + R_1 \left(i_1(t) + i_2(t) \right) = 0$$

Use the operator method to get

$$L_1 s\, i_1 = R_2\, i_2 + L_2 s\, i_2$$

$$L_1 s\, i_1 + R_1 \left(i_1 + i_2 \right) = 0$$

$$L_1 s^2 i_1 + R_1 s i_1 + R_1 s i_2 = 0$$

$$s\left(R_2 i_2 + L_2 s i_2\right) + \frac{R_1}{L_1}\left(R_2 i_2 + L_2 s i_2\right) + R_1 s i_2 = 0$$

$$L_2 s^2 i_2 + \left(R_2 + R_1 \frac{L_2}{L_1} + R_1\right) s i_2 + \frac{R_1 R_2}{L_1} i_2 = 0$$

$$s^2 i_2 + \left(\frac{R_2}{L_2} + \frac{R_1}{L_2} + \frac{R_1}{L_1}\right) s i_2 + \frac{R_1 R_2}{L_1 L_2} i_2 = 0$$

so

$$\frac{d^2}{dt^2} i_2(t) + \left(\frac{R_2}{L_2} + \frac{R_1}{L_2} + \frac{R_1}{L_1}\right)\frac{d}{dt} i_2(t) + \frac{R_1 R_2}{L_1 L_2} i_2(t) = 0$$

Compare to

$$\frac{d^2}{dt^2} i(t) + 2\alpha \frac{d}{dt} i(t) + \omega_0^2 \, i(t) = f(t)$$

to get

$$2\alpha = \frac{R_2}{L_2} + \frac{R_1}{L_2} + \frac{R_1}{L_1}, \quad \omega_0^2 = \frac{R_1 R_2}{L_1 L_2} \quad \text{and} \quad f(t) = 0$$

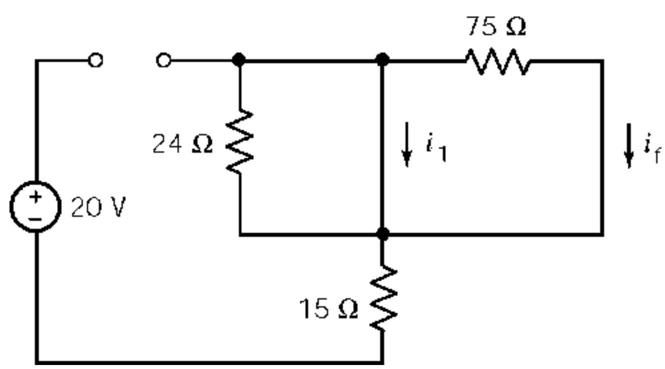
With the given element values, we have $\alpha = 33.9$ and $\omega_0^2 = 281.25$. Consequently, the roots of the characteristic equation are $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -4.4, \; -63.4$ so the circuit is overdamped. The natural response is

$$i_n(t) = A_1 e^{-4.4t} + A_2 e^{-63.4t}$$

Next, determine the forced response.

The steady state response after the switch opens will be used as the forced response. At steady state the inductors act like short circuits.

$$i_f = 0 \text{ A}$$



So

$$i_2(t) = i_n(t) + i_f(t) = A_1 e^{-4.4t} + A_2 e^{-63.4t}$$

It remains to evaluate A_1 and A_2 using the initial conditions. At $t = 0$ we have

$$0 = i_2(0) = A_1 + A_2$$

$$L_2 \frac{d}{dt} i_2(0) + R_2\, i_2(0) + R_1\, i_1(0) + R_1\, i_2(0) \quad \Rightarrow \quad \frac{d}{dt} i_2(0) = -20$$

and

$$-20 = \frac{d}{dt} i(0) = -4.4\ A_1 - 63.4\ A_2$$

Solving these equations gives $A_1 = -0.339$ and $A_2 = 0.339$ so

$$i_2(t) = -0.339\, e^{-4.4t} + 0.339\, e^{-63.4t} \quad \text{for } t \geq 0$$

(checked using LNAPTR 7/27/04)

P 9.8-18 The circuit shown in Figure P 9.8-18 is at steady state before the switch closes. Determine the capacitor voltage, $v(t)$, for $t > 0$.

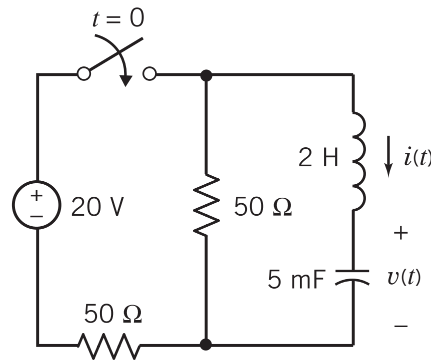
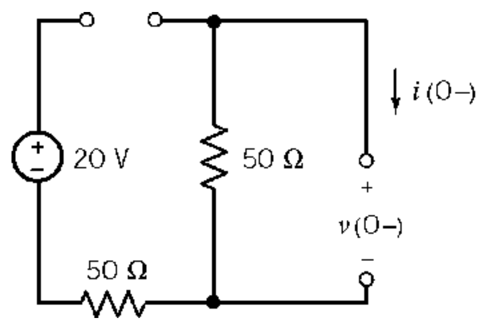


Figure P 9.8-18

Solution:

First, we find the initial conditions;

For $t < 0$, the switch is open and the circuit is at steady state. At steady state, the capacitor acts like an open circuit and the inductor acts like a short circuit.



$$v(0-) = 0 \text{ V and } i(0-) = 0 \text{ A}$$

also

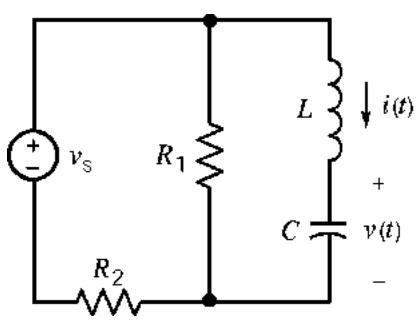
$$\frac{d}{dt}v(0) = \frac{i(0)}{0.005} = 0$$

Next, represent the circuit after the switch closes by a differential equation.

After the switch closes

$$i(t) = C\frac{d}{dt}v(t)$$

KCL and KVL give



$$v_s = R_2\left(i(t) + \frac{1}{R_1}\left(L\frac{d}{dt}i(t) + v(t)\right)\right) + L\frac{d}{dt}i(t) + v(t)$$

Substituting gives

$$v_s = \frac{R_2}{R_1}LC\frac{d^2}{dt^2}v(t) + R_2C\frac{d}{dt}v(t) + \left(1 + \frac{R_2}{R_1}\right)v(t) = \left(1 + \frac{R_2}{R_1}\right)LC\frac{d^2}{dt^2}v(t) + R_2C\frac{d}{dt}v(t) + \left(1 + \frac{R_2}{R_1}\right)v(t)$$

So the differential equation is

$$\frac{R_1 v_s}{LC\left(R_1+R_2\right)} = \frac{d^2}{dt}v(t) + \frac{R_1 R_2}{L\left(R_1+R_2\right)}\frac{d}{dt}v(t) + \frac{1}{LC}v(t)$$

Compare to

$$\frac{d^2}{dt^2}i(t) + 2\alpha\frac{d}{dt}i(t) + \omega_0^{\,2}\, i(t) = f(t)$$

to get

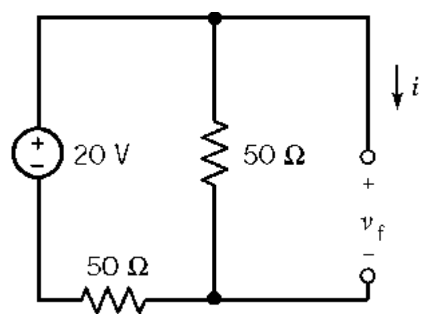
$$2\alpha = \frac{R_1 R_2}{L\left(R_1+R_2\right)}, \quad \omega_0^{\,2} = \frac{1}{CL} \quad \text{and} \quad f(t) = \frac{R_1 v_s}{LC\left(R_1+R_2\right)}$$

With the given element values, we have $\alpha = 6.25$ and $\omega_0^{\,2} = 100$. Consequently, the roots of the characteristic equation are $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^{\,2}} = -6.25 \pm j\,7.806$ and the circuit is underdamped. The damped resonant frequency is $\omega_d = \sqrt{\omega_0^{\,2} - \alpha^2} = 7.806$ rad/s . The natural response is

$$v_n(t) = e^{-6.25t}\left(A_1 \cos 7.806t + A_2 \sin 7.806t\right)$$

Next, determine the forced response.

The steady state response after the switch opens will be used as the forced response. At steady state, the capacitor acts like an open circuit and the inductor acts like a short circuit.



$$v_f = \frac{50}{50+50} \times 20 = 10 \text{ V}$$

So

$$v(t) = 10 + e^{-6.25t}\left(A_1 \cos 7.806t + A_2 \sin 7.806t\right)$$

It remains to evaluate A_1 and A_2 using the initial conditions. At $t = 0$ we have

$$0 = v(0) = 10 + A_1 \quad \Rightarrow \quad A_1 = -10$$

And

$$0 = \frac{d}{dt}v(0) = -6.25\, A_1 + 7.806\, A_2 \quad \Rightarrow \quad A_2 = -\frac{6.25 \times 10}{7.806} = -8.006$$

Finally,

$$v(t) = 10 + e^{-6.25t}\left(-10 \cos 7.806t - 8.006 \sin 7.806t\right)$$
$$= 10 + 12.81\, e^{-6.25t} \cos\left(7.806t + 141.3°\right) \text{ V} \quad \text{for } t \ge 0$$

(checked using LNAPTR 7/26/04)

P 9.8-19 Find the differential equation for $v_c(t)$ in the circuit of Figure P 9.8-19 using the direct method. Find $v_c(t)$ for time $t > 0$ for each of the following sets of component values:

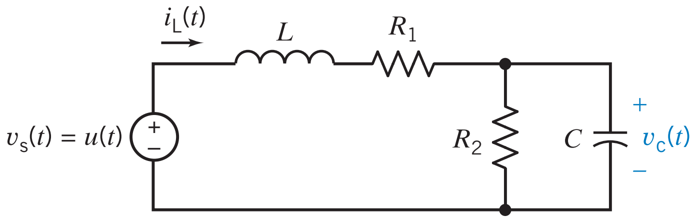


Figure P 9.8-19

(a) $C = 1$ F, $L = 0.25$ H, $R_1 = R_2 = 1.309\ \Omega$
(b) $C = 1$ F, $L = 1$ H, $R_1 = 3\ \Omega$, $R_2 = 1\ \Omega$
(c) $C = 0.125$ F, $L = 0.5$ H, $R_1 = 1\ \Omega$, $R_2 = 4\ \Omega$

Answer:

(a) $v_c(t) = \frac{1}{2} - e^{-2t} + \frac{1}{2}e^{-4t}$ V; (b) $v_c(t) = \frac{1}{4} - (\frac{1}{4} + \frac{1}{2}t)e^{-2t}$ V;
(c) $v_c(t) = 0.8 - e^{-2t}(0.8 \cos 4t + 0.4 \sin 4t)$ V

Solution:
When the circuit reaches steady state after t = 0, the capacitor acts like an open circuit and the inductor acts like a short circuit. Under these conditions

$$v_C(\infty) = \frac{R_2}{R_1 + R_2} 1$$

Next, represent the circuit by a 2nd order differential equation:

KCL at the top node of R_2 gives: $$\frac{v_C(t)}{R_2} + C\frac{d}{dt}v_C(t) = i_L(t)$$

KVL around the outside loop gives: $$v_s(t) = L\frac{d}{dt}i_L(t) + R_1\, i_L(t) + v_C(t)$$

Use the substitution method to get

$$\begin{aligned} v_s(t) &= L\frac{d}{dt}\left(\frac{v_C(t)}{R_2} + C\frac{d}{dt}v_C(t)\right) + R_1\left(\frac{v_C(t)}{R_2} + C\frac{d}{dt}v_C(t)\right) + v_C(t) \\ &= LC\frac{d^2}{dt^2}v_C(t) + \left(\frac{L}{R_2} + R_1 C\right)\frac{d}{dt}v_C(t) + \left(1 + \frac{R_1}{R_2}\right)v_C(t) \end{aligned}$$

(a) $C = 1$ F, $L = 0.25$ H, $R_1 = R_2 = 1.309\ \Omega$
Use the steady state response as the forced response:

$$v_f = v_C(\infty) = \frac{R_2}{R_1 + R_2} 1 = \frac{1}{2}$$

The characteristic equation is

$$s^2+\left(\frac{1}{R_2\,C}+\frac{R_1}{L}\right)s+\left(\frac{1+\dfrac{R_1}{R_2}}{LC}\right)=s^2+6s+8=(s+2)(s+4)$$

so the natural response is

$$v_n = A_1\,e^{-2t}+A_2\,e^{-4t}\text{ V}$$

The complete response is

$$v_c(t)=\frac{1}{2}+A_1\,e^{-2t}+A_2\,e^{-4t}\text{ V}$$

$$i_L(t)=\frac{v_C(t)}{1.309}+\frac{d}{dt}v_C(t)=-1.236\,A_1\,e^{-2t}-3.236\,A_2\,e^{-4t}+0.3819$$

At t = 0+

$$0=v_c(0+)=A_1+A_2+0.5$$
$$0=i_L(0+)=-1.236\,A_1-3.236\,A_2+0.3819$$

Solving these equations gives A_1 = -1 and A_2 = 0.5, so

$$v_c(t)=\frac{1}{2}-e^{-2t}+\frac{1}{2}e^{-4t}\text{ V}$$

(b) C = 1 F, L = 1 H, R_1 = 3 Ω, R_2 = 1 Ω

Use the steady state response as the forced response:

$$v_f=v_C(\infty)=\frac{R_2}{R_1+R_2}1=\frac{1}{4}$$

The characteristic equation is

$$s^2+\left(\frac{1}{R_2\,C}+\frac{R_1}{L}\right)s+\left(\frac{1+\dfrac{R_1}{R_2}}{LC}\right)=s^2+4s+4=(s+2)^2$$

so the natural response is

$$v_f=(A_1+A_2\,t)\,e^{-2t}\text{ V}$$

The complete response is

$$v_c(t)=\frac{1}{4}+(A_1+A_2\,t)\,e^{-2t}\text{ V}$$

$$i_L(t)=v_C(t)+\frac{d}{dt}v_C(t)=\frac{1}{4}+\left((A_2-A_1)-A_2\,t\right)e^{-2t}$$

At t = 0+

$$0 = v_c(0+) = A_1 + \frac{1}{4}$$

$$0 = i_L(0+) = \frac{1}{4} + A_2 - A_1$$

Solving these equations gives A_1 = -0.25 and A_2 = -0.5, so

$$v_c(t) = \frac{1}{4} - \left(\frac{1}{4} + \frac{1}{2}t\right)e^{-2t} \text{ V}$$

(c) *C* = 0.125 F, *L* = 0.5 H, *R*₁ = 1 Ω, *R*₂ = 4 Ω

Use the steady state response as the forced response:

$$v_f = v_C(\infty) = \frac{R_2}{R_1 + R_2} 1 = \frac{4}{5}$$

The characteristic equation is

$$s^2 + \left(\frac{1}{R_2 C} + \frac{R_1}{L}\right)s + \left(\frac{1 + \frac{R_1}{R_2}}{LC}\right) = s^2 + 4s + 20 = (s + 2 - j4)(s + 2 + j4)$$

so the natural response is

$$v_f = e^{-2t}\left(A_1 \cos 4t + A_2 \sin 4t\right) \text{ V}$$

The complete response is

$$v_c(t) = 0.8 + e^{-2t}\left(A_1 \cos 4t + A_2 \sin 4t\right) \text{ V}$$

$$i_L(t) = \frac{v_C(t)}{4} + \frac{1}{8}\frac{d}{dt}v_C(t) = 0.2 + \frac{A_2}{2}e^{-2t}\cos 4t - \frac{A_1}{2}e^{-2t}\sin 4t$$

At t = 0+

$$0 = v_c(0+) = 0.8 + A_1$$

$$0 = i_L(0+) = 0.2 + \frac{A_2}{2}$$

Solving these equations gives A_1 = -0.8 and A_2 = -0.4, so

$$v_c(t) = 0.8 - e^{-2t}\left(0.8 \cos 4t + 0.4 \sin 4t\right) \text{ V}$$

P 9.8-20 Find the differential equation for $v_o(t)$ in the circuit of Figure P 9.8-20 using the direct method. Find $v_o(t)$ for time $t > 0$ for each of the following sets of component values:

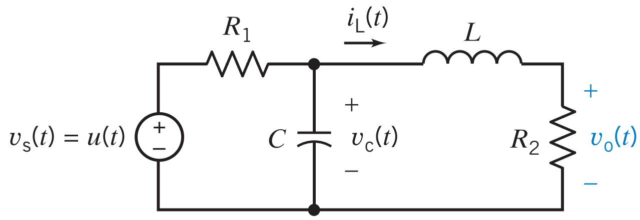


Figure P 9.8-20

(a) $C = 1$ F, $L = 0.25$ H, $R_1 = R_2 = 1.309\ \Omega$
(b) $C = 1$ F, $L = 1$ H, $R_1 = 1\ \Omega$, $R_2 = 3\ \Omega$
(c) $C = 0.125$ F, $L = 0.5$ H, $R_1 = 4\ \Omega$, $R_2 = 1\ \Omega$

Answer:

(a) $v_o(t) = \frac{1}{2} - e^{-2t} + \frac{1}{2} e - 4t$ V

(b) $v_o(t) = \frac{3}{4} - (\frac{3}{4} + \frac{3}{2} t)e^{-2t}$ V

(c) $v_o(t) = 0.2 - e^{-2t}(0.2 \cos 4t + 0.1 \sin 4t)$ V

Solution:

When the circuit reaches steady state after $t = 0$, the capacitor acts like an open circuit and the inductor acts like a short circuit. Under these conditions

$$v_C(\infty) = \frac{R_2}{R_1 + R_2} 1, \quad i_L(\infty) = \frac{1}{R_1 + R_2} \quad and \quad v_o(\infty) = \frac{R_2}{R_1 + R_2} 1$$

Next, represent the circuit by a 2nd order differential equation:

KVL around the right-hand mesh gives: $v_C(t) = L \frac{d}{dt} i_L(t) + R_2 \, i_L(t)$

KCL at the top node of the capacitor gives: $\frac{v_s(t) - v_C(t)}{R_1} - C \frac{d}{dt} v_C(t) = i_L(t)$

Use the substitution method to get

$$v_s(t) = R_1 C \frac{d}{dt}\left(L \frac{d}{dt} i_L(t) + R_2 \, i_L(t) \right) + \left(L \frac{d}{dt} i_L(t) + R_2 \, i_L(t) \right) + R_1 \, i_L(t)$$

$$= R_1 LC \frac{d^2}{dt^2} i_L(t) + \left(L + R_1 R_2 C \right) \frac{d}{dt} i_L(t) + \left(R_1 + R_2 \right) i_L(t)$$

Using $i_L(t) = \frac{v_o(t)}{R_2}$ gives

$$v_s(t)=\frac{R_1}{R_2}LC\frac{d^2}{dt^2}v_o(t)+\left(\frac{L}{R_2}+R_1C\right)\frac{d}{dt}v_o(t)+\left(\frac{R_1+R_2}{R_2}\right)v_o(t)$$

(a) C = 1 F, L = 0.25 H, $R_1 = R_2$ = 1.309 Ω

Use the steady state response as the forced response:

$$v_f=v_o(\infty)=\frac{R_2}{R_1+R_2}1=\frac{1}{2}$$

The characteristic equation is

$$s^2+\left(\frac{1}{R_1C}+\frac{R_2}{L}\right)s+\left(\frac{1+\frac{R_2}{R_1}}{LC}\right)=s^2+6s+8=(s+2)(s+4)$$

so the natural response is

$$v_n=A_1\,e^{-2t}+A_2\,e^{-4t}\ \text{V}$$

The complete response is

$$v_o(t)=\frac{1}{2}+A_1\,e^{-2t}+A_2\,e^{-4t}\ \text{V}$$

$$i_L(t)=\frac{v_o(t)}{1.309}=\frac{1}{2.618}+\frac{A_1}{1.309}\,e^{-2t}+\frac{A_2}{1.309}\,e^{-4t}\ \text{V}$$

$$v_C(t)=1.309\,i_L(t)+\frac{1}{4}\frac{d}{dt}i_L(t)=\frac{1}{2}+0.6167\,A_1\,e^{-2t}+0.2361\,A_2\,e^{-4t}$$

At t = 0+

$$0=i_L(0+)=\frac{1}{2.618}+\frac{A_1}{1.309}+\frac{A_2}{1.309}$$

$$0=v_C(0+)=\frac{1}{2}+0.6167\,A_1+0.2361\,A_2$$

Solving these equations gives A_1 = -1 and A_2 = 0.5, so

$$v_o(t)=\frac{1}{2}-e^{-2t}+\frac{1}{2}e^{-4t}\ \text{V}$$

(b) C = 1 F, L = 1 H, R_1 = 1 Ω, R_2 = 3 Ω

Use the steady state response as the forced response:

$$v_f=v_o(\infty)=\frac{R_2}{R_1+R_2}1=\frac{3}{4}$$

The characteristic equation is

$$s^2+\left(\frac{1}{R_1 C}+\frac{R_2}{L}\right)s+\left(\frac{1+\frac{R_2}{R_1}}{LC}\right)=s^2+4s+4=\left(s+2\right)^2$$

so the natural response is

$$v_f=\left(A_1+A_2\,t\right)e^{-2t}\text{ V}$$

The complete response is

$$v_o\left(t\right)=\frac{3}{4}+\left(A_1+A_2\,t\right)e^{-2t}\text{ V}$$

$$i_L\left(t\right)=\frac{v_o\left(t\right)}{3}=\frac{1}{4}+\left(\frac{A_1}{3}+\frac{A_2}{3}t\right)e^{-2t}\text{ V}$$

$$v_C\left(t\right)=3\,i_L\left(t\right)+\frac{d}{dt}i_L\left(t\right)=\frac{3}{4}+\left(\left(\frac{A_1}{3}+\frac{A_2}{3}\right)+\frac{A_2}{3}t\right)e^{-2t}$$

At t = 0+

$$0=i_L\left(0+\right)=\frac{A_1}{3}+\frac{1}{4}$$

$$0=v_C\left(0+\right)=\frac{3}{4}+\frac{A_1}{3}+\frac{A_2}{3}$$

Solving these equations gives A_1 = -0.75 and A_2 = -1.5, so

$$v_o\left(t\right)=\frac{3}{4}-\left(\frac{3}{4}+\frac{3}{2}t\right)e^{-2t}\text{ V}$$

(c) *C* = 0.125 F, *L* = 0.5 H, R_1 = 4 Ω, R_2 = 1 Ω

Use the steady state response as the forced response:

$$v_f=v_o\left(\infty\right)=\frac{R_2}{R_1+R_2}1=\frac{1}{5}$$

The characteristic equation is

$$s^2+\left(\frac{1}{R_1 C}+\frac{R_2}{L}\right)s+\left(\frac{1+\frac{R_2}{R_1}}{LC}\right)=s^2+4s+20=\left(s+2-j4\right)\left(s+2+j4\right)$$

so the natural response is

$$v_f=e^{-2t}\left(A_1\cos 4t+A_2\sin 4t\right)\text{ V}$$

The complete response is

$$v_o\left(t\right)=0.2+e^{-2t}\left(A_1\cos 4t+A_2\sin 4t\right)\text{ V}$$

$$i_L\left(t\right)=\frac{v_o\left(t\right)}{1}=0.2+e^{-2t}\left(A_1\cos 4t+A_2\sin 4t\right)\text{ V}$$

$$v_C\left(t\right)=i_L\left(t\right)+\frac{1}{2}\frac{d}{dt}i_L\left(t\right)=0.2+2\,A_2\,e^{-2t}\cos 4t-2\,A_1\,e^{-2t}\sin 4t$$

At t = 0+

$$0=i_L\left(0+\right)=0.2+A_1$$
$$0=v_C\left(0+\right)=0.2+2\,A_2$$

Solving these equations gives A_1 = -0.8 and A_2 = -0.4, so

$$v_c\left(t\right)=0.2-e^{-2t}\left(0.2\cos 4t+0.1\sin 4t\right)\text{ V}$$

Section 9-9: State Variable Approach to Circuit Analysis

P 9.9-1 Find $v(t)$ for $t > 0$ using the state variable method of Section 9.9 when $C = 1/5$ F in the circuit of Figure P 9.9-1. Sketch the response for $v(t)$ for $0 < t < 10$ s.

Answer: $v(t) = -25e^{-t} + e^{-5t} + 24$ V

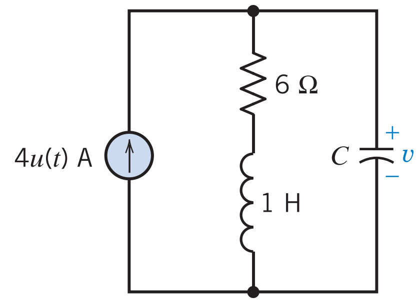
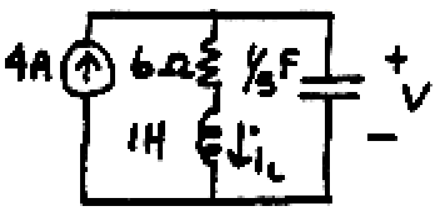


Figure P 9.9-1

Solution:

$\underline{t=0^-}$ circuit is source free $\therefore\ i_L(0) = 0\ \&\ v(0) = 0$

$\underline{t>0}$



KCL at top node: $i_L + \left(\frac{1}{5}\right) {dv}/{dt} = 4$ (1)

KVL at right loop: $(v-1)\ {di_L}/{dt} - 6i_L = 0$

Solving for i_1 in (1) & plugging into (2) $\Rightarrow \dfrac{d^2v}{dt^2} + 6\dfrac{dv}{dt} + 5v = 120$

The characteristic equation is: $s^2 + 6s + 5 = 0$,

The roots of the characteristic equation are $s = -1, -5$

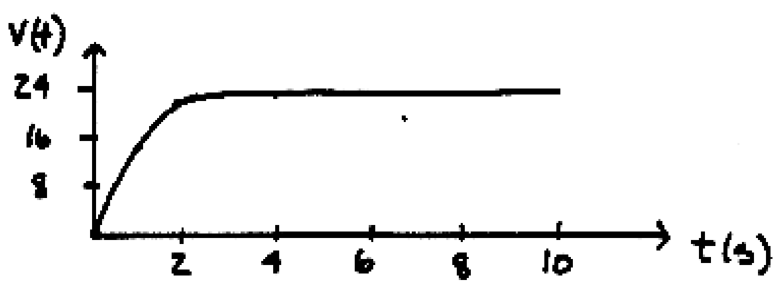
$\therefore$ Tthe natural response is: $v_n(t) = A_1 e^{-t} + A_2 e^{-5t}$

Try $v_f = B$ & plug into D.E. $\Rightarrow B = 24 = v_f$

From (1) $\dfrac{dv(0)}{dt} = 20 - 5i_L(0) = 20\ {V}/{s}$

So $\left.\begin{aligned} v(0) = 0 &= A_1 + A_2 + 24 \\ \frac{dv(0)}{dt} = 20 &= -A_1 - 5A_2 \end{aligned}\right\}$ $A_1 = -25,\quad A_2 = 1$

$\therefore\ \underline{v(t) = -25e^{-t} + e^{-5t} + 24\text{ V}}$



P 9.9-2 Repeat Problem P9.9-1when $C = 1/10$ F. Sketch the response for $v(t)$ for $0 < t < 3$ s.

Answer: $v(t) = e^{-3t}(-24\cos t - 32\sin t) + 24$ V

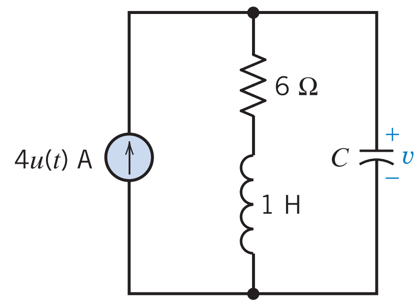
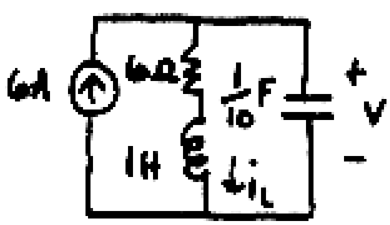


Figure P 9.9-1

Solution:

At $\underline{t = 0^-}$ the circuit is source free $\therefore\ i_L(0) = 0,\ \&\ v(0) = 0$

At $\underline{t > 0}$



KCL at top node: $i_L = 4 - \left(\frac{1}{10}\right)\frac{dv}{dt}$ (1)

KVL at right node: $v - \frac{di_L}{dt} - 6i_L = 0$ (2)

(1) into (2) yields $\frac{d^2v}{dt^2} + 6\frac{dv}{dt} + 10v = 240$

$\Rightarrow s^2 + 6s + 10 = 0,\ s = -3 \pm j \quad \therefore\ v_n(t) = e^{-3t}\left[A_1\cos t + A_2\sin t\right]$

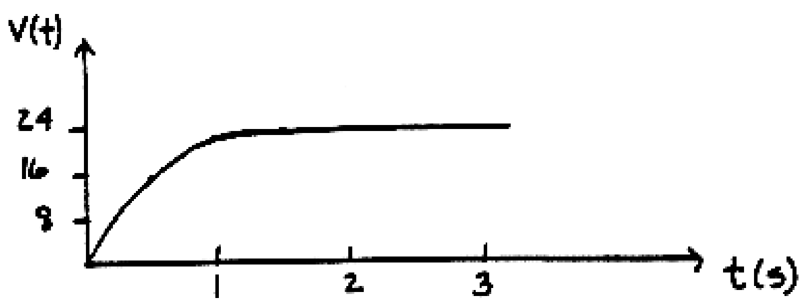
Try $v_f = B$ & plug into D.E. $\Rightarrow v_f = B = 24$

From (1) $\frac{dv(0)}{dt} = 40 - 10\,i_L(0) = 40\ \text{V/s}$

So $v(0) = 0 = A_1 + 24 \Rightarrow A_1 = -24$ & $\frac{dv(0)}{dt} = 40 = -3A_1 + A_2$

$\Rightarrow A_2 = -32$

$\underline{\therefore\ v(t) = e^{-3t}\left[-24\cos t - 32\sin t\right] + 24\ \text{V}}$



P 9.9-3 Determine the current $i(t)$ and the voltage $v(t)$ for the circuit of Figure P 9.9-3.

Answer: $i(t) = (3.08e^{-2.57t} - 0.08e^{-97.4t} - 6)$ A

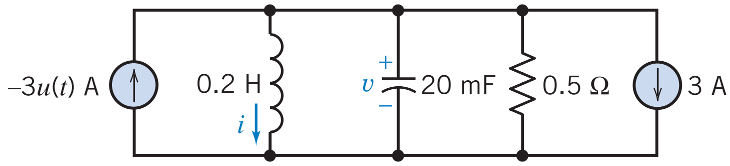
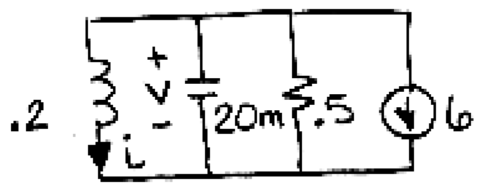


Figure P 9.9-3

Solution:

$$i(0) = -3,\ v(0) = 0$$

$$t > 0$$



$$\text{KCL: } i + C\frac{dv}{dt} + \frac{v}{R} + 6 = 0$$

$$\text{KVL: } v = L\frac{di}{dt}$$

$$\frac{d^2i}{dt^2} + 100\frac{di}{dt} + 250i = -1500$$

$$s = -2.57,\ -97.4$$

$$i_f(t) = \frac{-1500}{250} = -6$$

$$i(t) = A_1 e^{-2.57t} + A_2\, e^{-97.4t} - 6$$

$$\left.\begin{aligned} i(0) &= A_1 + A_2 - 6 = -3 \\ \frac{di(0)}{dt} &= 0 = -2.57A_1 - 97.4A_2 \end{aligned}\right\} \quad \begin{aligned} A_1 &= 3.081 \\ A_2 &= -0.081 \end{aligned}$$

$$\underline{i(t) = 3.081\, e^{-2.57t} - 0.081 e^{-97.4t} - 6 \quad \text{A}}$$

$$\underline{v(t) = 0.2\frac{di}{dt} = -1.58e^{-2.57t} + 1.58e^{-97.4t} \quad \text{V}}$$

P 9.9-4 Clean-air laws are pushing the auto industry toward the development of electric cars. One proposed vehicle using an ac motor is shown in Figure P 9.9-4*a*. The motor-controller circuit is shown in Figure P 9.9-4*b* with L = 100 mH and C = 10 mF. Using the state equation approach, determine $i(t)$ and $v(t)$ where $i(t)$ is the motor-control current. The initial conditions are $v(0)$ = 10 V and $i(0)$ = 0.

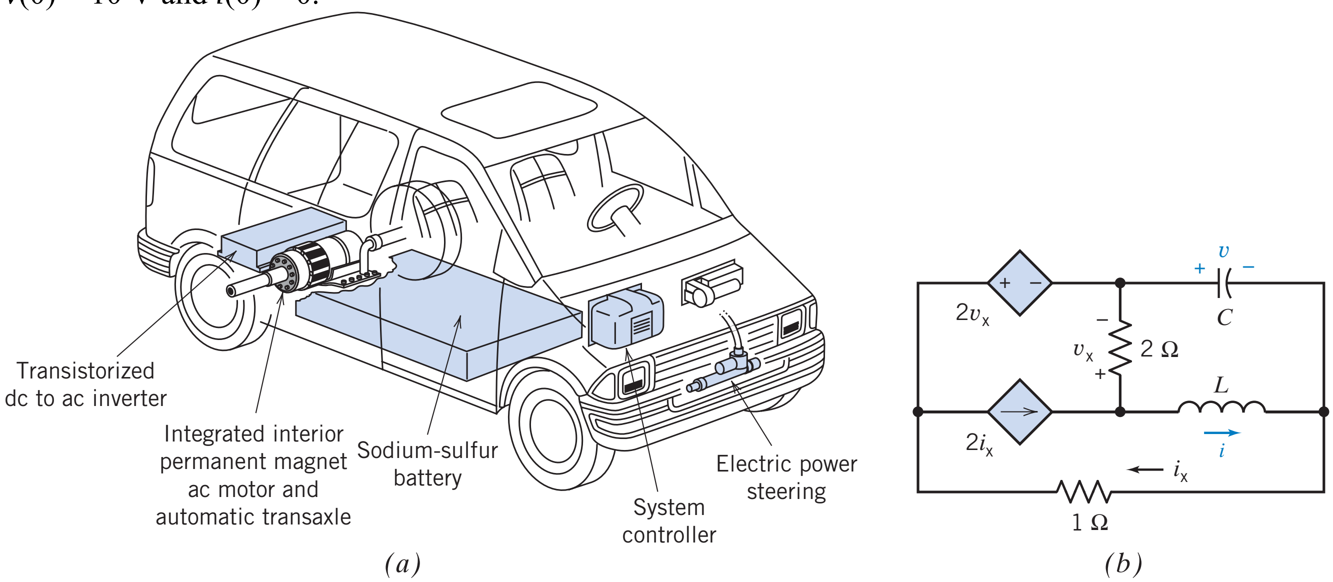


Figure P 9.9-4

Apply KCL to the supernode corresponding to the VCVS to get

$$i_x + \frac{v_x}{2} = 2\,i_x + C\frac{dv}{dt} \quad\Rightarrow\quad -i_x + \frac{v_x}{2} = 0.01\frac{dv}{dt} \tag{1}$$

Apply KCL to the right node of the CCCS to get $i + \dfrac{v_x}{2} = 2\,i_x$. (2)

Apply KVL to the mesh consisting of the 2-Ω resistor, inductor and capacitor to get

$$v_x + v - L\frac{di}{dt} = 0 \quad\Rightarrow\quad v_x + v = 0.1\frac{di}{dt} \tag{3}$$

Apply KVL to the outside loop to get $v + i_x + 2\,v_x = 0$ (4)

Combine equations (2) and (4) to get

$$i_x = \frac{4}{9}\,i - \frac{1}{9}\,v \quad\text{and}\quad v_x = -\frac{2}{9}\,i - \frac{4}{9}\,v \tag{5}$$

Use (5) to eliminate i_x and v_x from (1) and (3) to get

$$0.01\frac{dv}{dt} = -\frac{5}{9}\,i - \frac{1}{9}\,v \quad\text{and}\quad 0.1\frac{di}{dt} = -\frac{2}{9}\,i + \frac{5}{9}\,v \tag{6}$$

Use operators to write

$$s\,v = -\frac{500}{9}\,i - \frac{100}{9}\,v \quad\text{and}\quad s\,i = -\frac{20}{9}\,i + \frac{50}{9}\,v \tag{7}$$

The characteristic equation is : $s^2 + 13.33s + 333.33 = 0 \quad\Rightarrow\quad s_1,\ s_2 = -6.67 \pm j\,17$

The natural response is $v(t)=e^{-6.67t}\left[A\cos(17t)+B\sin(17t)\right]$ and there is no forced because there is no forcing function. The constants A and B are evaluated using the initial conditions:

$$v(0)=10=A \text{ and } \frac{dv(0)}{dt}=-111=-6.67A+17\,B \quad\Rightarrow\quad B=-2.6$$

Then
$$v(t)=e^{-6.67t}\left[10\cos(17t)-2.6\sin(17t)\right] \text{ V}$$

Similarly, the natural response is $i(t)=e^{-6.67t}\left[A\cos(17t)+B\sin(17t)\right]$ and again there is no forced because there is no forcing function. The constants A and B are evaluated using the initial conditions:

$$i(0)=0=A \text{ and } \frac{di(0)}{dt}=55.6=-6.67A+17\,B \quad\Rightarrow\quad B=-3.27$$

Then
$$i(t)=e^{-6.67t}\left[3.27\sin(17t)\right] \text{ V}$$

P 9.9-5 Studies of an artificial insect are being used to understand the nervous system of animals. A model neuron in the nervous system of the artificial insect is shown in Figure P 9.9-5. The input signal, v_s, is used to generate a series of pulses, called synapses. The switch generates a pulse by opening at $t = 0$ and closing at $t = 0.5$ s. Assume that the circuit is at steady state and that $v(0^-) = 10$ V.

Determine the voltage $v(t)$ for $0 < t < 2$ s.

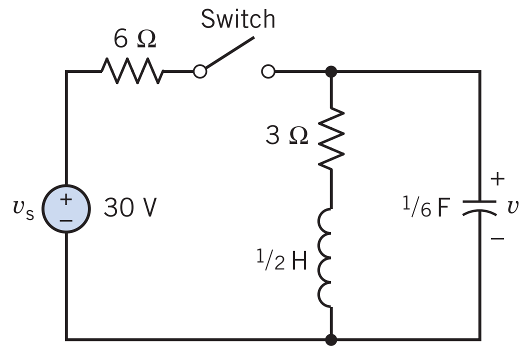
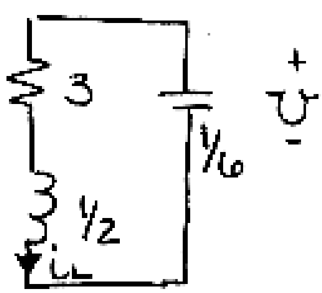


Figure P 9.9-5

Solution

First consider $t < 0$: $\quad v(0) = 10 \text{ V}, \; i_L(0) = \dfrac{10}{3} \text{ A}$

Next consider $0 < t < 0.5\,\text{s}$



$$\alpha = \frac{R}{L} = \frac{3}{2} \quad \text{and} \quad \omega_0^2 = LC = \frac{1}{12}$$

$$s = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} \;\Rightarrow\; s_1 = -.028 \text{ and } s_2 = -2.97$$

The natural response is $v(t) = Ae^{-0.028t} + Be^{-2.97t}$ and the forced response is $v_f = 0$.

The constants are evaluated using the initial conditions:

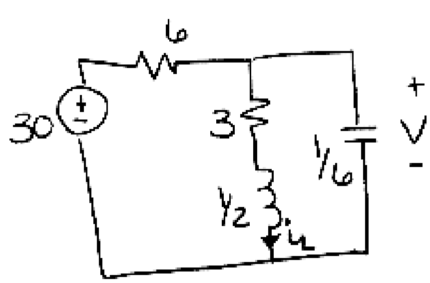
$$\left.\begin{aligned} v(0) &= 10 = A + B \\ \frac{dv(0)}{dt} &= 20 = -0.028A - 2.97B \end{aligned}\right\} \begin{aligned} A &= 16.89 \\ B &= -6.89 \end{aligned}$$

so $v(t) = 16.89\, e^{-0.028t} - 6.89e^{-2.97t}$

Similarly $i(t) = -.079\text{e}^{-0.028t} + 3.41\text{e}^{-2.97\text{t}}$

At $t = 0.5$ s, $v(0.5) = 15.1$ V and $i(0.5) = 0.7$ A

For $t > 0.55$ s:



KCL: $\dfrac{v - 30}{6} + i_L + \dfrac{1}{6}\dfrac{dv}{dt} = 0$

KVL: $v = 3i_L + \dfrac{1}{2}\dfrac{di_L}{dt}$

Characteristic equation: $0 = s^2 - 7s - 18 \Rightarrow s = -1, 9$

$v_f = 10$ V

$v(t) = Ae^{9t} + Be^{-t} + 10$

$$\left.\begin{aligned} v(0.5) &= 15.1 = 90A + 0.61B + 10 \\ \frac{dv(0.5)}{dt} &= 10.7 = 810A - 0.61B \end{aligned}\right\} \quad \begin{aligned} A &= 17.6 \times 10^{-3} \\ B &= 5.77 \end{aligned}$$

t	$v(t)$
0	$16.89e^{-0.28\tau} - 6.89e^{-2.97\tau}$ V
$\rightarrow .5$	
.5	$17.6 \times 10^{-3} e^{9t} + 5.77e^{-t} + 10$ V
$\rightarrow 2$	

Section 9-10: Roots in the Complex Plane

P 9.10-1 For the circuit of Figure P 9.10-1, determine the roots of the characteristic equation, and plot the roots on the *s*-plane.

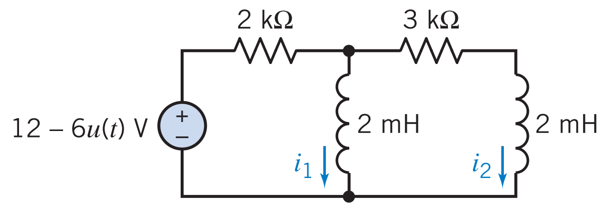


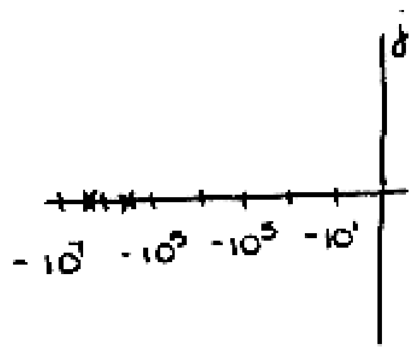
Figure P 9.10-1

Solution:

$$s^2 + 3.5\times10^6 s + 1.5\times10^{12} = 0$$

$$s_1 = -5\times10^5$$

$$s_2 = -3\times10^6$$



P 9.10-2 For the circuit of Figure P 9.6-1, determine the roots of the characteristic equation and plot the roots on the *s*-plane.

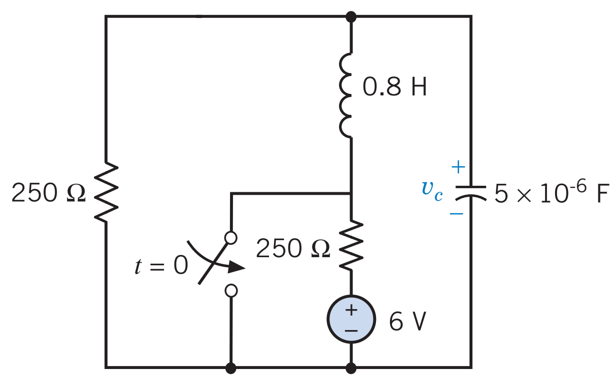
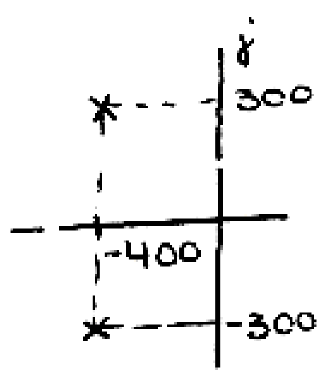


Figure P 9.6-1

Solution:

$$s^2 + 800s + 250000 = 0$$

$$s = 400 \pm j\,300$$



P 9.10-3 For the circuit of Figure P 9.10-3, determine the roots of the characteristic equation and plot the roots on the *s*-plane.

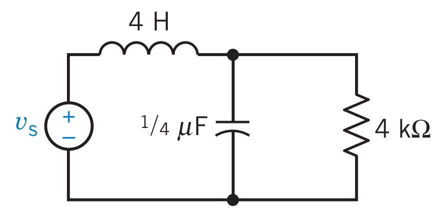
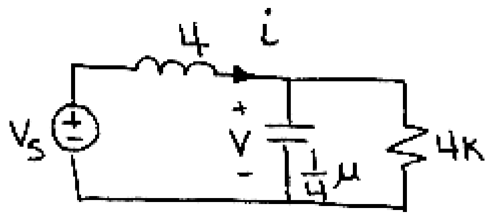


Figure P 9.10-3

Solution:

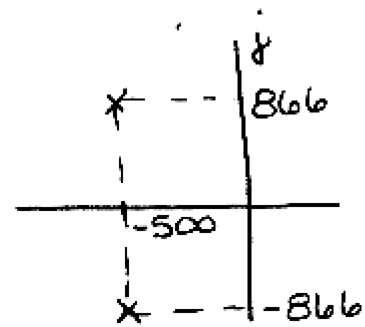


KCL: $i = \frac{1}{4} \times 10^{-6} \frac{dv}{dt} + \frac{v}{4000}$

KVL: $v_S = 4 \frac{di}{dt} + v$

Characteristic equation: $s^2 + 1\times10^3 s + 1\times10^6 = 0$

$s = -500 \pm j\,866$



P 9.10-4 An *RLC* circuit is shown in Figure P 9.10-4.

(a) Obtain the two-node voltage equations using operators.
(b) Obtain the characteristic equation for the circuit.
(c) Show the location of the roots of the characteristic equation in the *s*-plane.
(d) Determine $v(t)$ for $t > 0$.

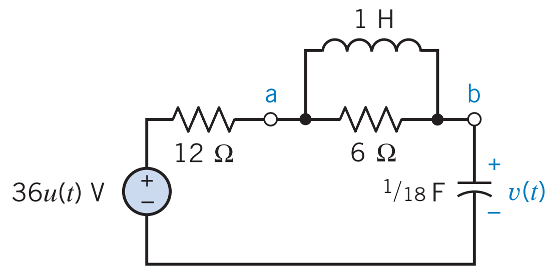
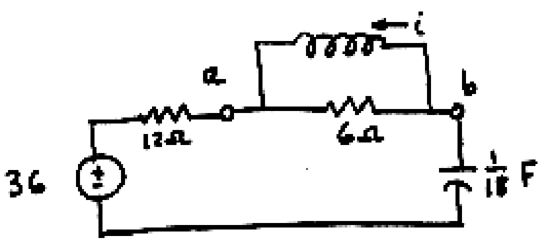


Figure P 9.10-4

Solution:



at $t = 0$ the initial conditions are $v(0) = v_b(0) = 0$,

$$i(0) = 0 \quad \text{and} \quad C\frac{dv_b}{dt} + \frac{v_b - v_a}{6} + i = 0 \qquad (1)$$

$t = 0$

$$\text{Node a:} \quad \frac{v_a(0)-36}{12} - i(0) + \frac{v_a(0)-v_b(0)}{6} = 0 \quad \text{then} \quad v_a(0) + 2v_a(0) = 36 \text{ so } v_a(0) = 12 \text{ V}$$

$t \geq 0$

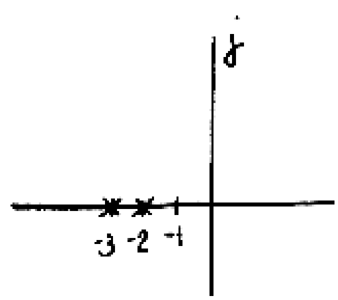
$$\text{Node a:} \quad \frac{v_a - v_s}{12} + \frac{1}{L}\int (v_a - v_b)\, dt + \frac{v_a - v_b}{6} = 0$$

$$\text{Node b:} \quad C\frac{dv_b}{dt} + \frac{v_b - v_a}{6} + \frac{1}{L}\int (v_a - v_b) dt = 0$$

$$\text{Using operators} \left(\frac{1}{12} + \frac{1}{6} + \frac{1}{s}\right) v_a + \left(-\frac{1}{6} - \frac{1}{s}\right) v_b = \frac{v_s}{12}$$

$$\left(-\frac{1}{6} - \frac{1}{s}\right) v_a + \left(\frac{1}{18}s + \frac{1}{6} + \frac{1}{s}\right) v_b = 0$$

$$\text{Cramers rule:} \quad (s^2 + 5s + 6)v_b = (s + 6)v_s$$



$$\text{Then} \quad v_b = 36 + A_1 e^{-2t} + A_2 e^{-3t}$$

$$v_b(0) = 36 + A_1 + A_2 \qquad (2)$$

$$\text{need} \quad \frac{dv_b}{dt}(0) = -2A_1 - 3A_2$$

$$\text{Use 1 above:} \quad C\frac{dv_b(0)}{dt} = \frac{1}{18}(-2A_1 - 3A_2) = \frac{v_a(0) - v_b(0)}{6} - i(0) = \frac{12}{6} = 2 \qquad (3)$$

Use (2) and (3) to get

$A_1 = -72$ and $A_2 = 36$ so $\underline{v(t) = v_b(t) = 36 - 72e^{-2t} + 36e^{-3t}}$, $t \geq 0$

Section 9-11 How Can We Check…?

P 9.11-1 Figure P 9.11-1*a* shows an *RLC* circuit. The voltage, $v_s(t)$, of the voltage source is the square wave shown in Figure P 9.11-1*a*. Figure P 9.11-1*c* shows a plot of the inductor current, $i(t)$, which was obtained by simulating this circuit using PSpice. Verify that the plot of $i(t)$ is correct.

Answer: The plot is correct.

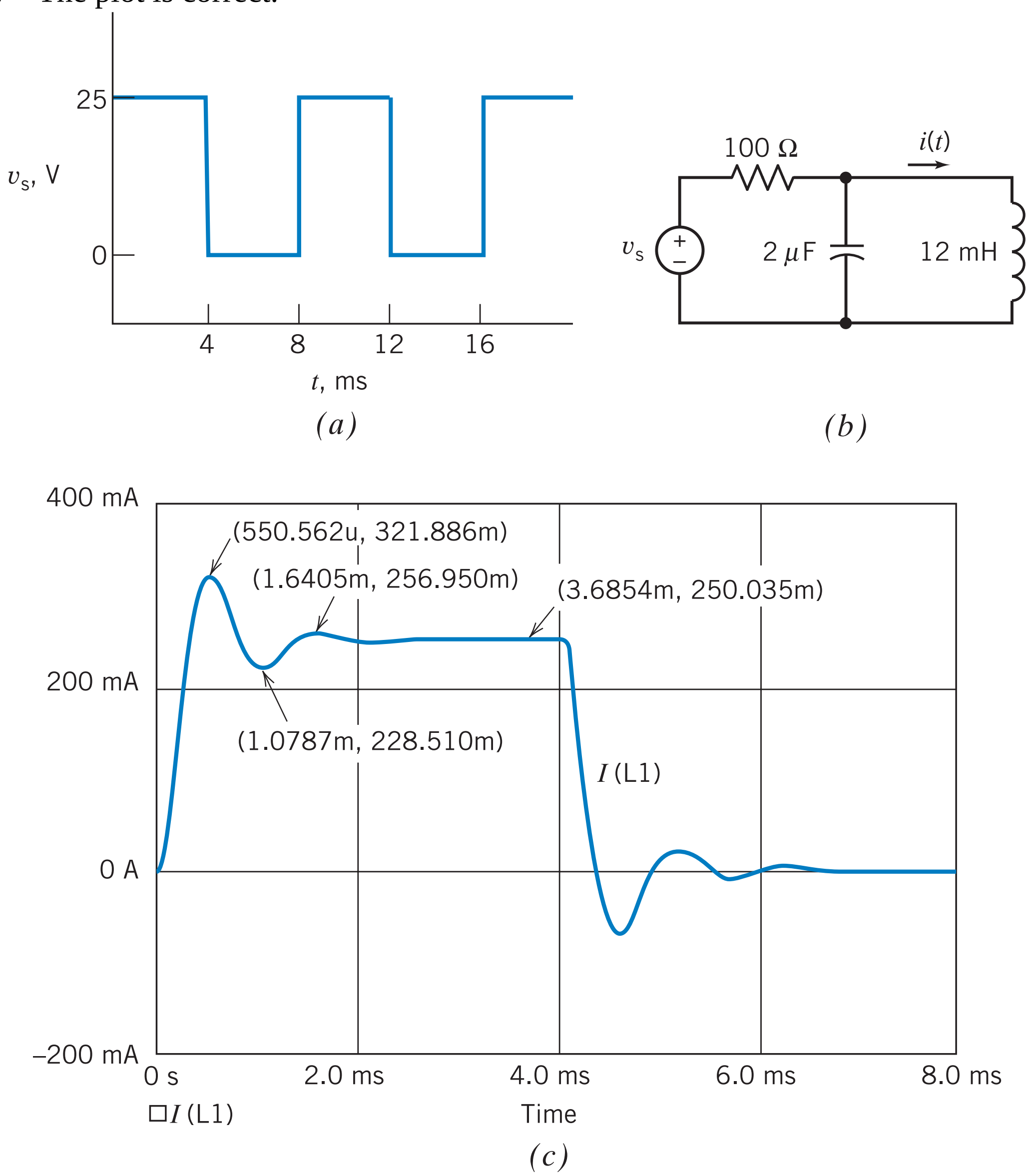


Figure P 9.11-1*a*

Solution:

This problem is similar to the verification example in this chapter. First, check the steady-state inductor current

$$i(t) = \frac{v_s}{100} = \frac{25}{100} = 250 \text{ mA}$$

This agrees with the value of 250.035 mA shown on the plot. Next, the plot shows an underdamped response. That requires

$$12\cdot 10^{-3} \ = \ L \ < \ 4R^2C \ = \ 4(100)^2 \ (2\cdot 10^{-6}) \ = \ 8\cdot 10^{-2}$$

This inequality is satisfied, which also agrees with the plot. The damped resonant frequency is given by

$$\omega_{\mathrm{d}} = \sqrt{\frac{1}{\mathrm{LC}} - \left(\frac{1}{2\mathrm{RC}}\right)^2} = \sqrt{\frac{1}{\left(2\cdot 10^{-6}\right)\left(12\cdot 10^{-3}\right)} - \left(\frac{1}{2(100)\,(2\cdot 10^{-6})}\right)^2} = 5.95\cdot 10^3$$

The plot indicates a maxima at 550.6μs and a minima at 1078.7μs. The period of the damped oscillation is

$$T_d = 2\ (1078.7\mu\mathrm{s} - 550.6\mu\mathrm{s}) = 1056.2\mu\mathrm{s}$$

Finally, check that $5.95\cdot 10^3 \ = \ \omega_d \ = \ \dfrac{2\pi}{T_d} = \dfrac{2\pi}{1056.2\cdot 10^{-6}} = 5.949\cdot 10^3$

The value of ω_d determined from the plot agrees with the value obtained from the circuit.

The plot is correct

P 9.11-2 Figure P 9.11-2*b* shows an *RLC* circuit. The voltage, $v_s(t)$, of the voltage source is the square wave shown in Figure P 9.11-2*a*. Figure P 9.11-2*c* shows a plot of the inductor current, $i(t)$, which was obtained by simulating this circuit using PSpice. Verify that the plot of $i(t)$ is correct.

Answer: The plot is not correct.

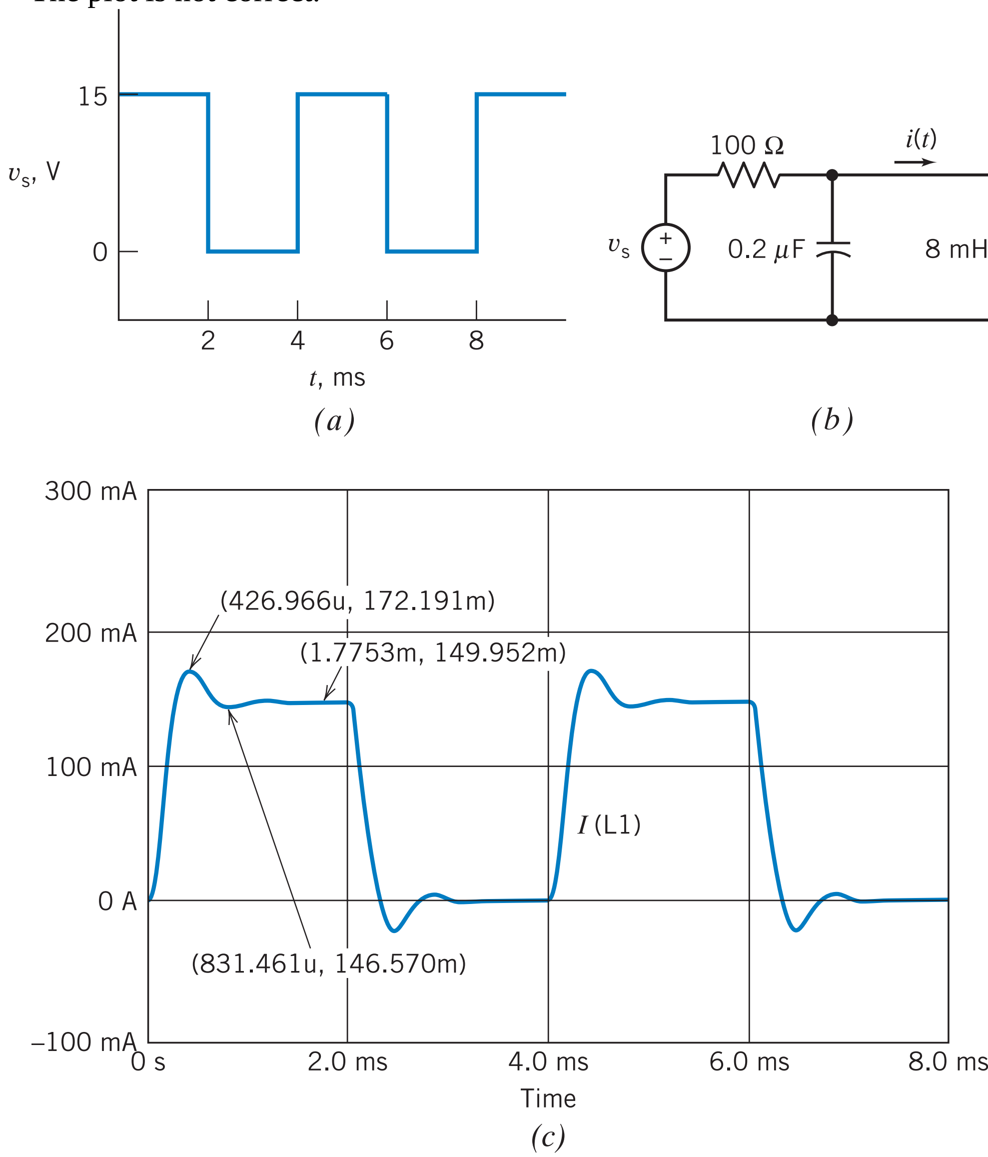


Figure P 9.11-2

Solution:

This problem is similar to the verification example in this chapter. First, check the steady-state inductor current.

$$i(t) = \frac{v_s}{100} = \frac{15}{100} = 150 \text{ mA}$$

This agrees with the value of 149.952 mA shown on the plot. Next, the plot shows an under damped response. This requires

$$8 \cdot 10^{-3} = L < 4R^2C = 4\,(100)^2\,(0.2 \cdot 10^{-6}) = 8 \cdot 10^{-3}$$

This inequality is not satisfied. The values in the circuit would produce a critically damped, not underdamped, response. This plot is not correct.

PSpice Problems

SP 9-1 The input to the circuit shown in Figure SP 9-1 is the voltage of the voltage source, $v_i(t)$. The output is the voltage across the capacitor, $v_o(t)$. The input is the pulse signal specified graphically by the plot. Use PSpice to plot the output, $v_o(t)$, as a function of t for each of the following cases:

(a) $C = 1$ F, $L = 0.25$ H, $R_1 = R_2 = 1.309\ \Omega$
(b) $C = 1$ F, $L = 1$ H, $R_1 = 3\ \Omega$, $R_2 = 1\ \Omega$
(c) $C = 0.125$ F, $L = 0.5$ H, $R_1 = 1\ \Omega$, $R_2 = 4\ \Omega$

Plot the output for these three cases on the same axis.
Hint: Represent the voltage source using the PSpice part named VPULSE.

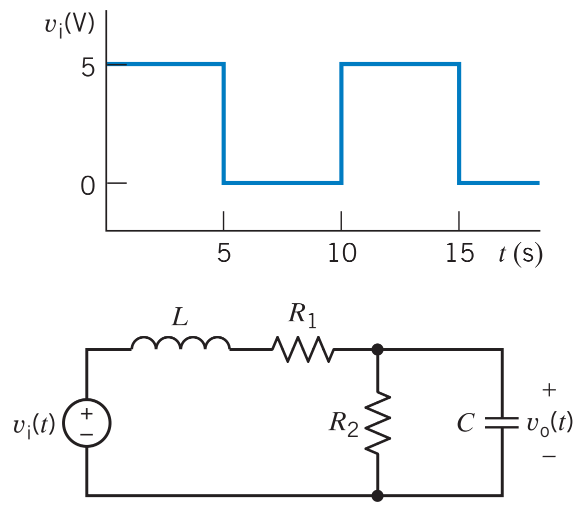
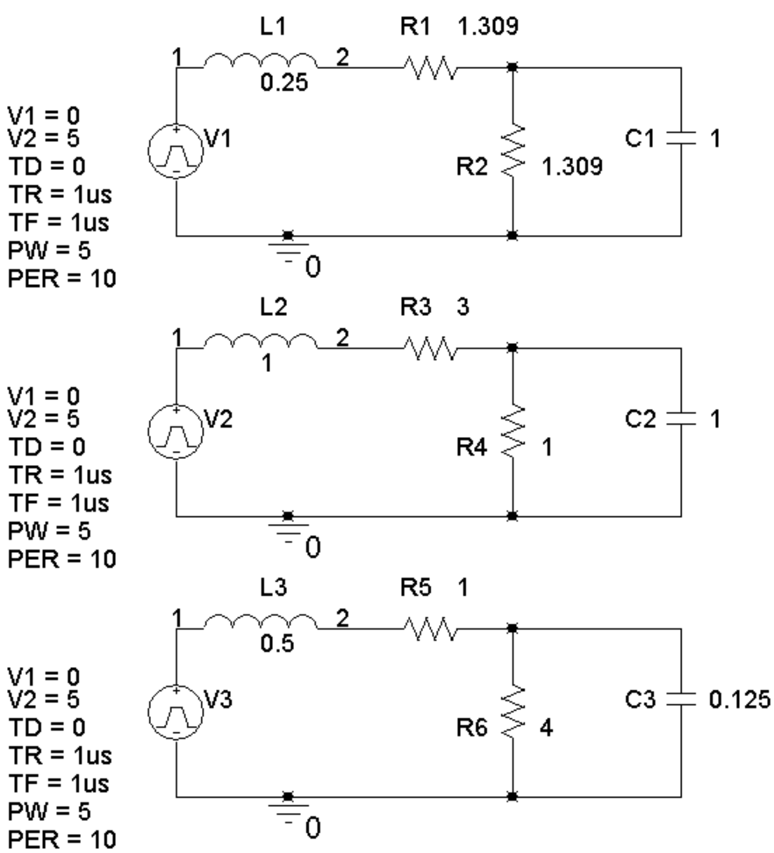
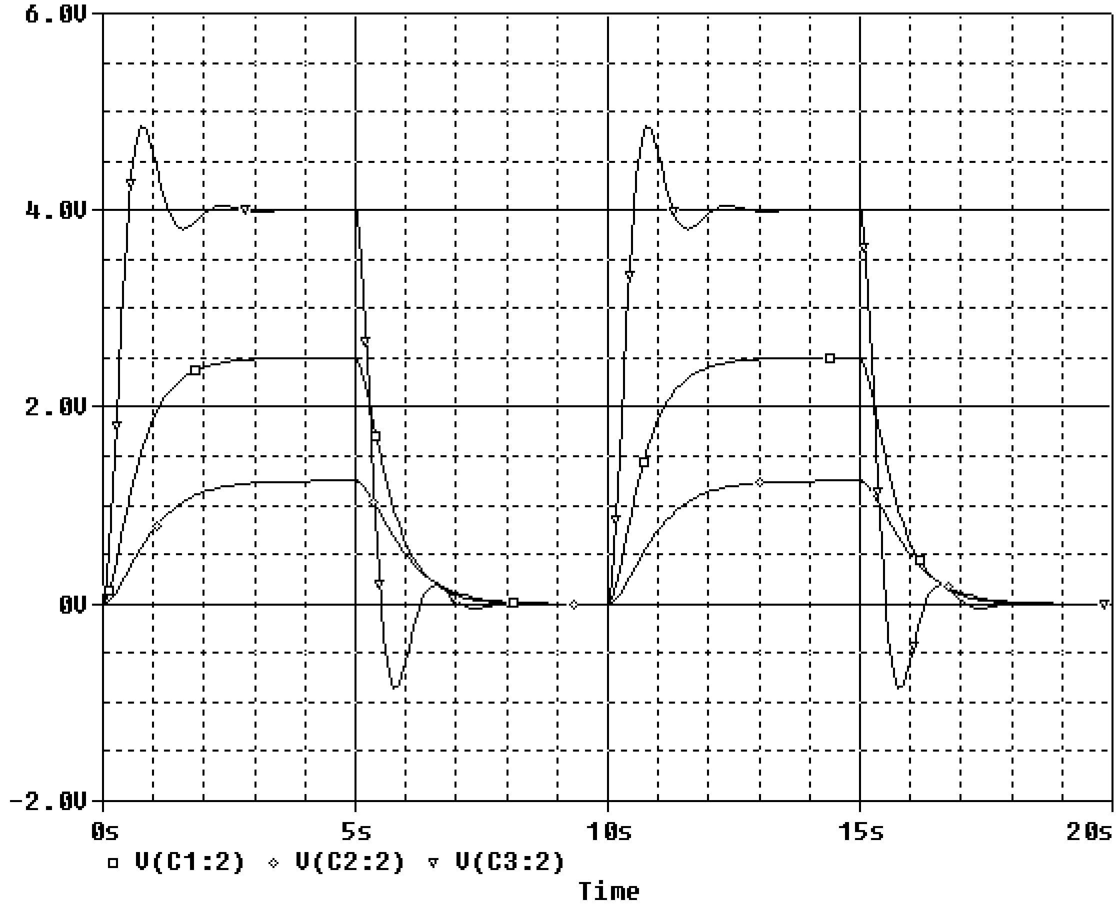


Figure SP 9-1

Solution:

Make three copies of the circuit: one for each set of parameter values. (Cut and paste, but be sure to edit the labels of the parts so, for example, there is only one R1.)





V(C1:2), V(C2:2) and V(C3:2) are the capacitor voltages, listed from top to bottom.

SP 9-2 The input to the circuit shown in Figure SP 9-2 is the voltage of the voltage source, $v_i(t)$. The output is the voltage, $v_o(t)$, across resistor R_2. The input is the pulse signal specified graphically by the plot. Use PSpice to plot the output, $v_o(t)$, as a function of t for each of the following cases:

(a) $C = 1$ F, $L = 0.25$ H, $R_1 = R_2 = 1.309\ \Omega$
(b) $C = 1$ F, $L = 1$ H, $R_1 = 3\ \Omega$, $R_2 = 1\ \Omega$
(c) $C = 0.125$ F, $L = 0.5$ H, $R_1 = 1\ \Omega$, $R_2 = 4\ \Omega$

Plot the output for these three cases on the same axis.
Hint: Represent the voltage source using the PSpice part named VPULSE.

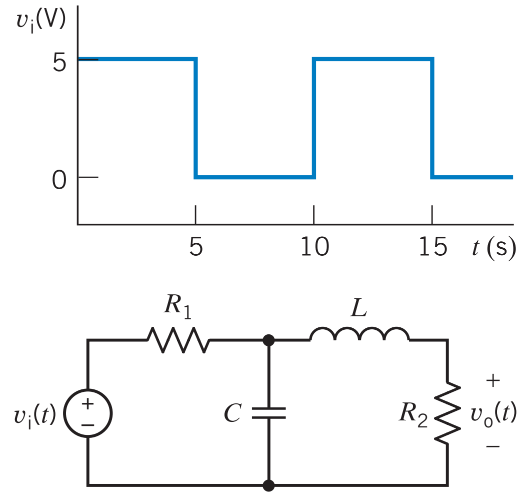
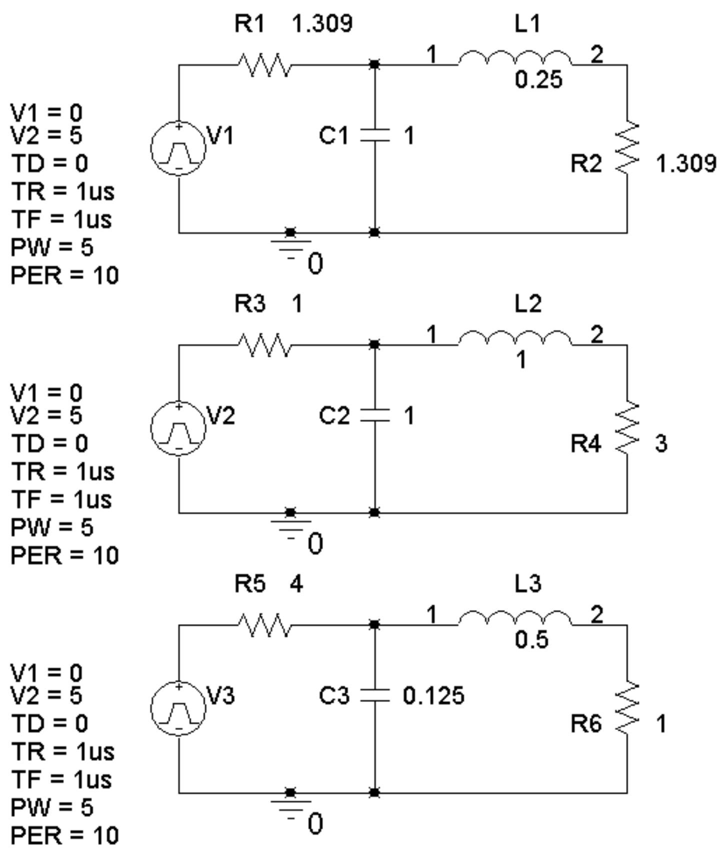
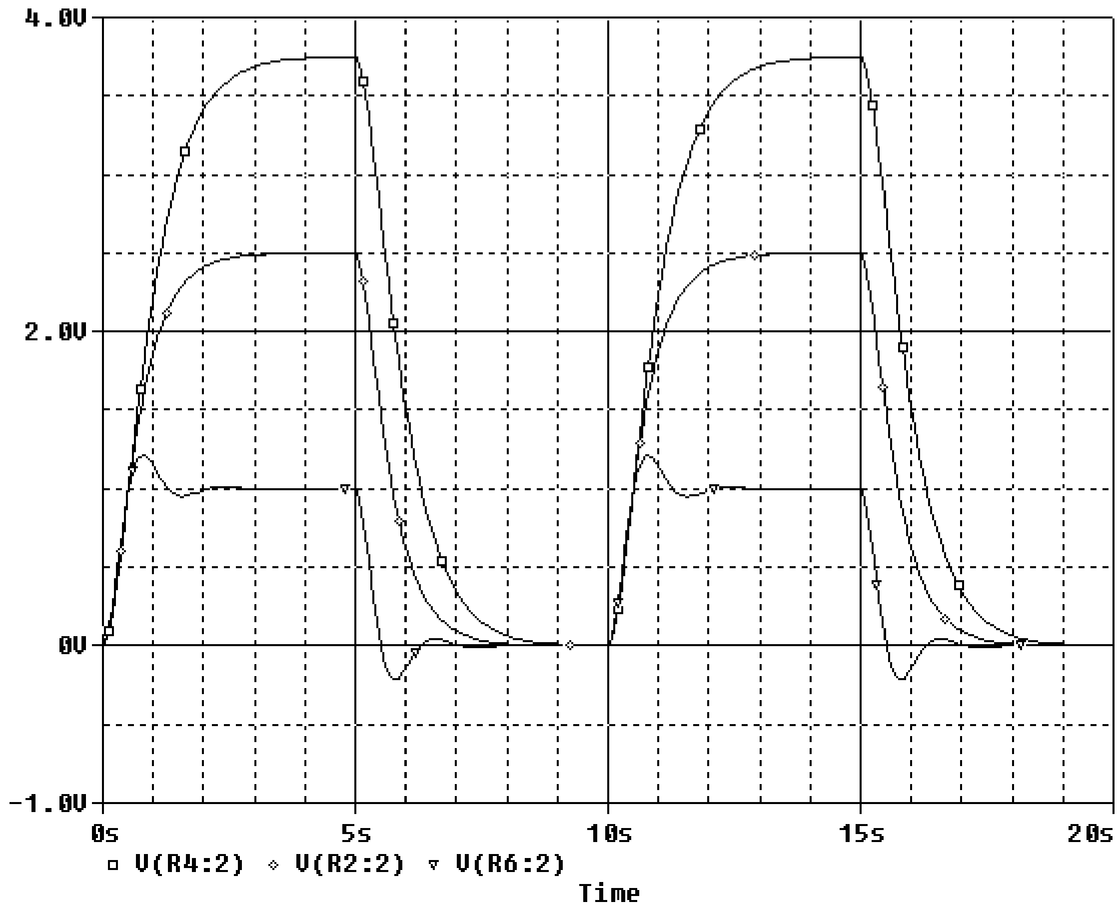


Figure SP 9-2

Solution:

Make three copies of the circuit: one for each set of parameter values. (Cut and paste, but be sure to edit the labels of the parts so, for example, there is only one R1.)



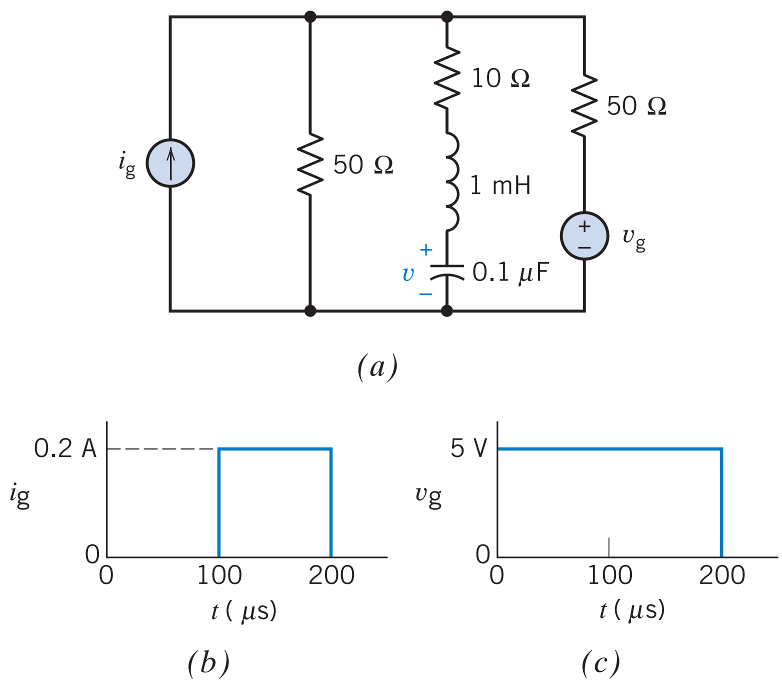


V(R2:2), V(R4:2) and V(R6:2) are the output voltages, listed from top to bottom.

SP 9-3 Determine and plot the capacitor voltage

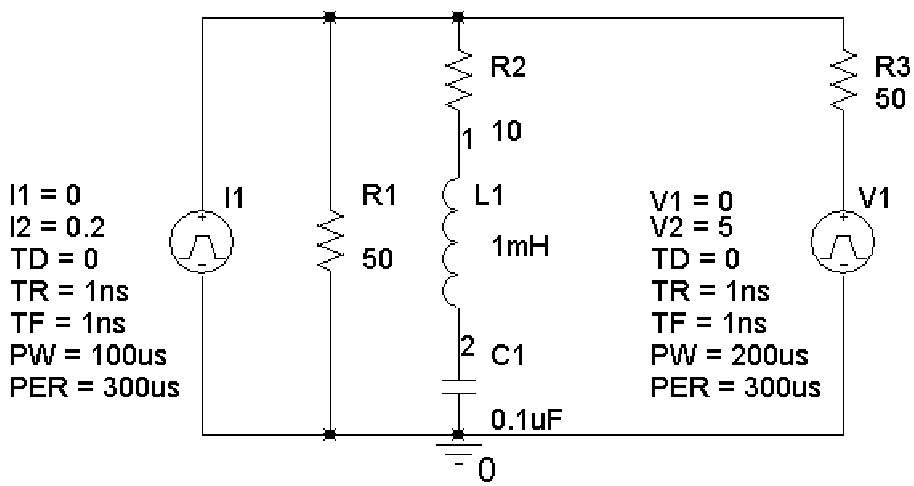
$$v(t) \text{ for } 0 < t < 300\ \mu s$$

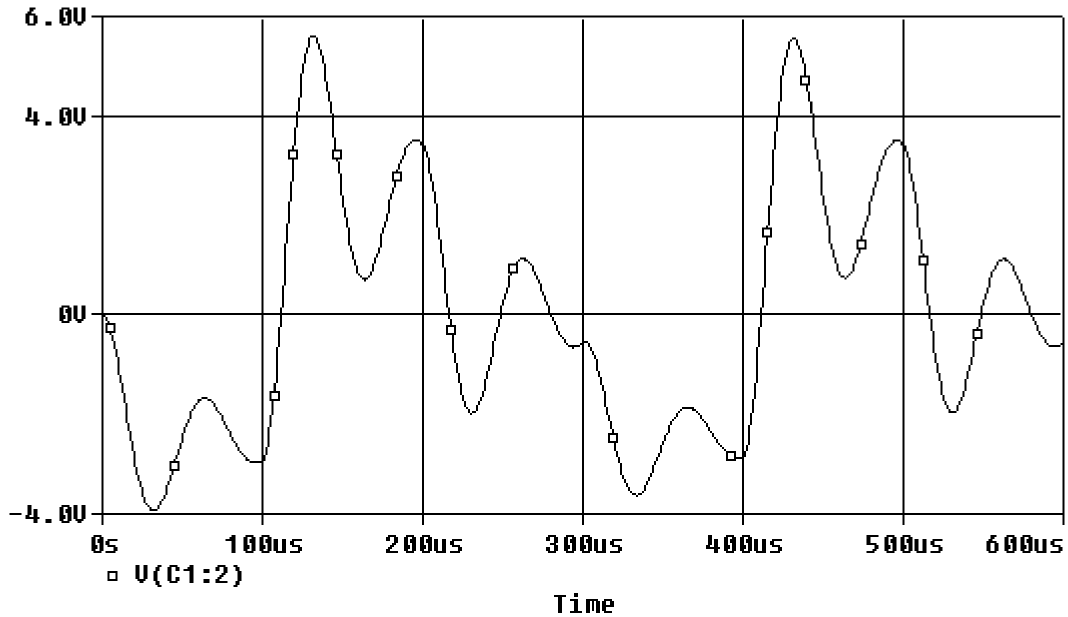
for the circuit shown in Figure SP 9-3*a*. The sources are pulses as shown in Figures SP 9-3*b*, *c*.



Figures SP 9-3

Solution:




6.0V
4.0V
0V
-4.0V
0s
100us
200us
300us
400us
500us
600us
V(C1:2)
Time

SP 9-4 Determine and plot $v(t)$ for the circuit of Figure SP 9-4 when $v_s(t) = 5u(t)$ V. Plot $v(t)$ for $0 < t < 0.25$ s.

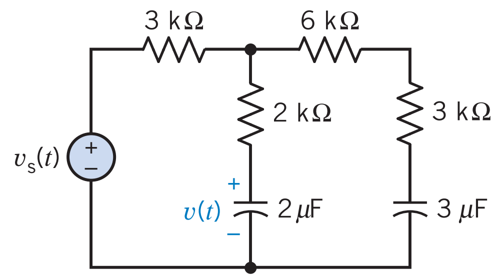
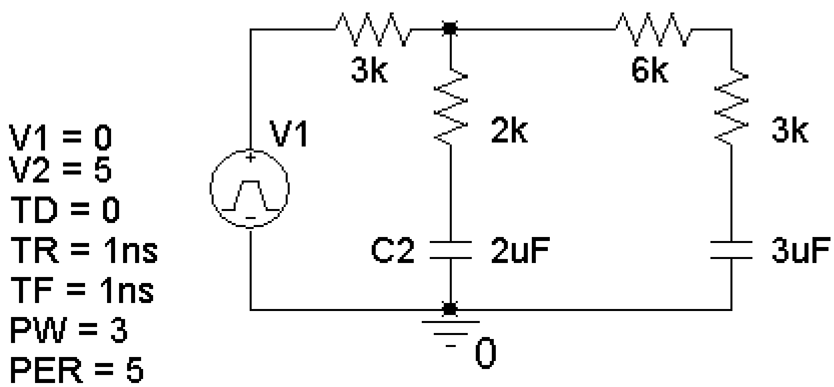
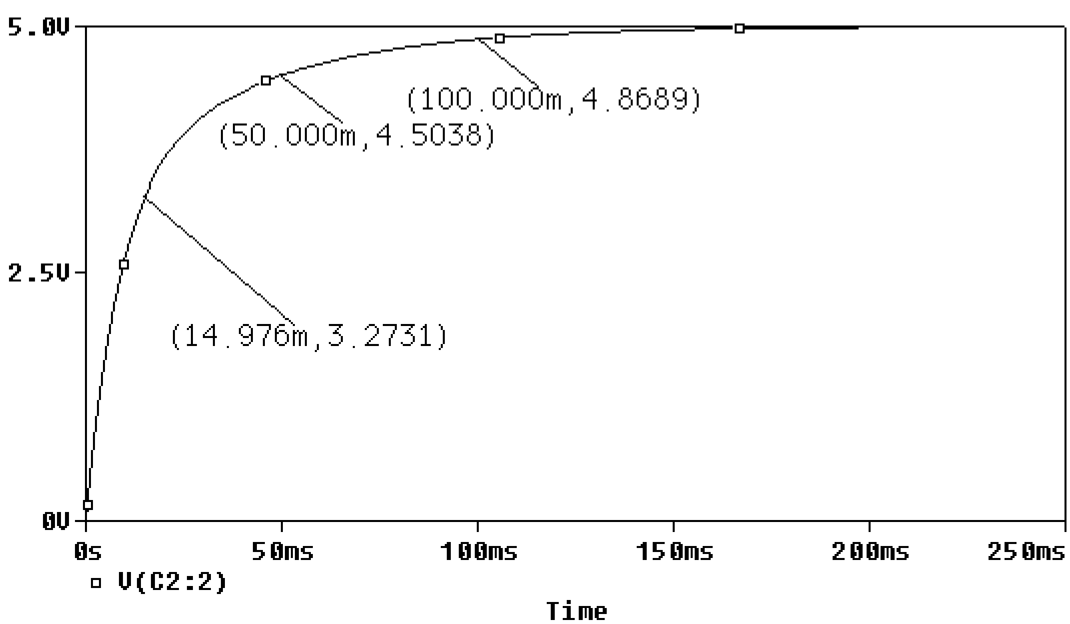


Figure SP 9-4

Solution:





Design Problems

DP 9-1 Design the circuit shown in Figure DP 9-1 so that

$$v_c(t) = \frac{1}{2} + A_1 e^{-2t} + A_2 e^{-4t} \text{ V for } t > 0$$

Determine the values of the unspecified constants, A_1 and A_2.

Hint: The circuit is overdamped, and the natural frequencies are 2 and 4 rad/sec.

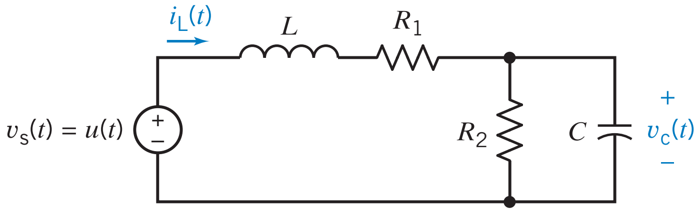


Figure DP 9-1

Solution:

When the circuit reaches steady state after $t = 0$, the capacitor acts like an open circuit and the inductor acts like a short circuit. Under these conditions

$$v_C(\infty) = \frac{R_2}{R_1 + R_2} 1$$

The specifications require that $v_C(\infty) = \frac{1}{2}$ so

$$\frac{1}{2} = \frac{R_2}{R_1 + R_2} \quad \Rightarrow \quad R_1 = R_2$$

Next, represent the circuit by a 2nd order differential equation:

KCL at the top node of R_2 gives: $$\frac{v_C(t)}{R_2} + C\frac{d}{dt}v_C(t) = i_L(t)$$

KVL around the outside loop gives: $$v_s(t) = L\frac{d}{dt}i_L(t) + R_1 i_L(t) + v_C(t)$$

Use the substitution method to get

$$\begin{aligned} v_s(t) &= L\frac{d}{dt}\left(\frac{v_C(t)}{R_2} + C\frac{d}{dt}v_C(t)\right) + R_1\left(\frac{v_C(t)}{R_2} + C\frac{d}{dt}v_C(t)\right) + v_C(t) \\ &= LC\frac{d^2}{dt^2}v_C(t) + \left(\frac{L}{R_2} + R_1 C\right)\frac{d}{dt}v_C(t) + \left(1 + \frac{R_1}{R_2}\right)v_C(t) \end{aligned}$$

The characteristic equation is

$$s^2+\left(\frac{1}{R_2\,C}+\frac{R_1}{L}\right)s+\left(\frac{1+\frac{R_1}{R_2}}{LC}\right)=s^2+6s+8=(s+2)(s+4)$$

Equating coefficients of like powers of s:

$$\frac{1}{R_2\,C}+\frac{R_1}{L}=6 \text{ and } \frac{1+\frac{R_1}{R_2}}{LC}=8$$

Using $R_1=R_2=R$ gives

$$\frac{1}{R\,C}+\frac{R}{L}=6 \text{ and } \frac{1}{LC}=4$$

These equations do not have a unique solution. Try C = 1 F. Then $L=\frac{1}{4}\text{ H}$ and

$$\frac{1}{R}+4R=6 \quad\Rightarrow\quad R^2-\frac{3}{2}R+\frac{1}{4}=0 \quad\Rightarrow\quad R=1.309\,\Omega \text{ or } R=0.191\,\Omega$$

Pick R = 1.309 Ω. Then

$$v_c(t)=\frac{1}{2}+A_1\,e^{-2t}+A_2\,e^{-4t}\text{ V}$$

$$i_L(t)=\frac{v_C(t)}{1.309}+\frac{d}{dt}v_C(t)=-1.236\,A_1\,e^{-2t}-3.236\,A_2\,e^{-4t}+0.3819$$

At t = 0+

$$0=v_c(0+)=A_1+A_2+0.5$$
$$0=i_L(0+)=-1.236\,A_1-3.236\,A_2+0.3819$$

Solving these equations gives A_1 = -1 and A_2 = 0.5, so

$$v_c(t)=\frac{1}{2}-e^{-2t}+\frac{1}{2}e^{-4t}\text{ V}$$

DP 9-2 Design the circuit shown in Figure DP 9-1 so that

$$v_c(t) = \frac{1}{4} + (A_1 + A_2 t)e^{-2t} \text{ V for } t > 0$$

Determine the values of the unspecified constants, A_1 and A_2.

Hint: The circuit is critically damped, and the natural frequencies are both 2 rad/sec.

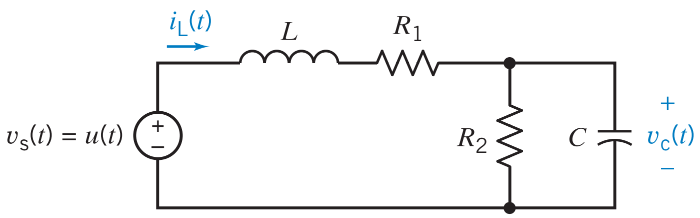


Figure DP 9-1

Solution:

When the circuit reaches steady state after t = 0, the capacitor acts like an open circuit and the inductor acts like a short circuit. Under these conditions

$$v_C(\infty) = \frac{R_2}{R_1 + R_2} 1$$

The specifications require that $v_C(\infty) = \frac{1}{4}$ so

$$\frac{1}{4} = \frac{R_2}{R_1 + R_2} \quad \Rightarrow \quad 3R_2 = R_1$$

Next, represent the circuit by a 2nd order differential equation:

KCL at the top node of R_2 gives: $\frac{v_C(t)}{R_2} + C\frac{d}{dt}v_C(t) = i_L(t)$

KVL around the outside loop gives: $v_s(t) = L\frac{d}{dt}i_L(t) + R_1 i_L(t) + v_C(t)$

Use the substitution method to get

$$\begin{aligned} v_s(t) &= L\frac{d}{dt}\left(\frac{v_C(t)}{R_2} + C\frac{d}{dt}v_C(t)\right) + R_1\left(\frac{v_C(t)}{R_2} + C\frac{d}{dt}v_C(t)\right) + v_C(t) \\ &= LC\frac{d^2}{dt^2}v_C(t) + \left(\frac{L}{R_2} + R_1 C\right)\frac{d}{dt}v_C(t) + \left(1 + \frac{R_1}{R_2}\right)v_C(t) \end{aligned}$$

The characteristic equation is

$$s^2 + \left(\frac{1}{R_2 C} + \frac{R_1}{L}\right)s + \left(\frac{1 + \frac{R_1}{R_2}}{LC}\right) = s^2 + 4s + 4 = (s+2)^2$$

Equating coefficients of like powers of s:

$$\frac{1}{R_2 C}+\frac{R_1}{L}=4 \text{ and } \frac{1+\frac{R_1}{R_2}}{LC}=4$$

Using $R_2 = R$ and $R_1 = 3R$ gives

$$\frac{1}{R C}+\frac{3R}{L}=4 \text{ and } \frac{1}{LC}=1$$

These equations do not have a unique solution. Try C = 1 F. Then L = 1 H and

$$\frac{1}{R}+3R=4 \quad \Rightarrow \quad R^2-\frac{4}{3}R+\frac{1}{3}=0 \quad \Rightarrow \quad R=1\,\Omega \text{ or } R=\frac{1}{3}\,\Omega$$

Pick R = 1 Ω. Then $R_1 = 3\,\Omega$ and $R_2 = 1\,\Omega$.

$$v_c(t)=\frac{1}{4}+\left(A_1+A_2\,t\right)e^{-2t} \text{ V}$$

$$i_L(t)=v_C(t)+\frac{d}{dt}v_C(t)=\frac{1}{4}+\left(\left(A_2-A_1\right)-A_2\,t\right)e^{-2t}$$

At t = 0+

$$0=v_c(0+)=A_1+\frac{1}{4}$$

$$0=i_L(0+)=\frac{1}{4}+A_2-A_1$$

Solving these equations gives A_1 = -0.25 and A_2 = -0.5, so

$$v_c(t)=\frac{1}{4}-\left(\frac{1}{4}+\frac{1}{2}t\right)e^{-2t} \text{ V}$$

DP 9-3 Design the circuit shown in Figure DP 9-1 so that

$$v_c(t) = 0.8 + e^{-2t}(A_1 \cos 4t + A_2 \sin 4t) \text{ V for } t > 0$$

Determine the values of the unspecified constants, A_1 and A_2.

Hint: The circuit is underdamped and the damped resonant frequency is 4 rad/sec and the damping coefficient is 2.

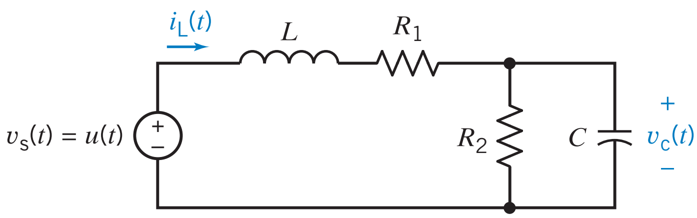


Figure DP 9-1

Solution:

When the circuit reaches steady state after t = 0, the capacitor acts like an open circuit and the inductor acts like a short circuit. Under these conditions

$$v_C(\infty) = \frac{R_2}{R_1 + R_2} 1$$

The specifications require that $v_C(\infty) = \frac{4}{5}$ so

$$\frac{4}{5} = \frac{R_2}{R_1 + R_2} \quad \Rightarrow \quad 4R_1 = R_2$$

Next, represent the circuit by a 2nd order differential equation:

KCL at the top node of R_2 gives: $\frac{v_C(t)}{R_2} + C\frac{d}{dt}v_C(t) = i_L(t)$

KVL around the outside loop gives: $v_s(t) = L\frac{d}{dt}i_L(t) + R_1 i_L(t) + v_C(t)$

Use the substitution method to get

$$v_s(t) = L\frac{d}{dt}\left(\frac{v_C(t)}{R_2} + C\frac{d}{dt}v_C(t)\right) + R_1\left(\frac{v_C(t)}{R_2} + C\frac{d}{dt}v_C(t)\right) + v_C(t)$$

$$= LC\frac{d^2}{dt^2}v_C(t) + \left(\frac{L}{R_2} + R_1 C\right)\frac{d}{dt}v_C(t) + \left(1 + \frac{R_1}{R_2}\right)v_C(t)$$

The characteristic equation is

$$s^2 + \left(\frac{1}{R_2 C} + \frac{R_1}{L}\right)s + \left(\frac{1 + \frac{R_1}{R_2}}{LC}\right) = s^2 + 4s + 20 = (s + 2 - j4)(s + 2 + j4)$$

Equating coefficients of like powers of s:

$$\frac{1}{R_2\,C}+\frac{R_1}{L}=4 \text{ and } \frac{1+\dfrac{R_1}{R_2}}{LC}=20$$

Using $R_1=R$ and $R_2=4\,R$ gives

$$\frac{1}{4R\,C}+\frac{R}{L}=4 \text{ and } \frac{1}{LC}=16$$

These equations do not have a unique solution. Try $C=\frac{1}{8}$ F. Then $L=\frac{1}{2}$ H and

$$\frac{2}{R}+2\,R=4 \quad \Rightarrow \quad R^2-2R+2=0 \quad \Rightarrow \quad R=1\,\Omega$$

Then $R_1=1\,\Omega$ and $R_2=4\,\Omega$. Next

$$v_c(t)=0.8+e^{-2t}\left(A_1\cos 4t+A_2\sin 4t\right) \text{ V}$$

$$i_L(t)=\frac{v_C(t)}{4}+\frac{1}{8}\frac{d}{dt}v_C(t)=0.2+\frac{A_2}{2}e^{-2t}\cos 4t-\frac{A_1}{2}e^{-2t}\sin 4t$$

At t = 0+

$$0=v_c(0+)=0.8+A_1$$

$$0=i_L(0+)=0.2+\frac{A_2}{2}$$

Solving these equations gives A_1 = -0.8 and A_2 = -0.4, so

$$v_c(t)=0.8-e^{-2t}\left(0.8\cos 4t+0.4\sin 4t\right) \text{ V}$$

DP 9-4 Show that the circuit shown in Figure DP 9-1 cannot be designed so that

$$v_c(t) = 0.5 + e^{-2t}(A_1 \cos 4t + A_2 \sin 4t) \text{ V for } t > 0$$

Hint: Show that such a design would require $1/RC + 10RC = 4$ where $R = R_1 = R_2$. Next, show that $1/RC + 10\,RC = 4$ would require the value of RC to be complex.

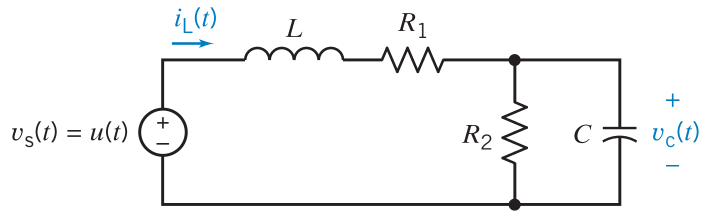


Figure DP 9-1

Solution:

When the circuit reaches steady state after t = 0, the capacitor acts like an open circuit and the inductor acts like a short circuit. Under these conditions

$$v_C(\infty) = \frac{R_2}{R_1 + R_2} 1$$

The specifications require that $v_C(\infty) = \dfrac{1}{2}$ so

$$\frac{1}{2} = \frac{R_2}{R_1 + R_2} \quad \Rightarrow \quad R_1 = R_2$$

Next, represent the circuit by a 2nd order differential equation:

KCL at the top node of R_2 gives: $$\frac{v_C(t)}{R_2} + C\frac{d}{dt}v_C(t) = i_L(t)$$

KVL around the outside loop gives: $$v_s(t) = L\frac{d}{dt}i_L(t) + R_1\, i_L(t) + v_C(t)$$

Use the substitution method to get

$$\begin{aligned} v_s(t) &= L\frac{d}{dt}\left(\frac{v_C(t)}{R_2} + C\frac{d}{dt}v_C(t)\right) + R_1\left(\frac{v_C(t)}{R_2} + C\frac{d}{dt}v_C(t)\right) + v_C(t) \\ &= LC\frac{d^2}{dt^2}v_C(t) + \left(\frac{L}{R_2} + R_1 C\right)\frac{d}{dt}v_C(t) + \left(1 + \frac{R_1}{R_2}\right)v_C(t) \end{aligned}$$

The characteristic equation is

$$s^2 + \left(\frac{1}{R_2 C} + \frac{R_1}{L}\right)s + \left(\frac{1 + \frac{R_1}{R_2}}{LC}\right) = s^2 + 4s + 20 = (s + 2 - j4)(s + 2 + j4)$$

Equating coefficients of like powers of s:

$$\frac{1}{R_2 C}+\frac{R_1}{L}=4 \quad \text{and} \quad \frac{1+\frac{R_1}{R_2}}{LC}=20$$

Using $R_1 = R_2 = R$ gives

$$\frac{1}{R C}+\frac{R}{L}=4 \quad \text{and} \quad \frac{1}{LC}=10$$

Substituting $L=\frac{1}{10C}$ into the first equation gives

$$(RC)^2-\frac{4}{10}(RC)+\frac{1}{10}=0 \quad \Rightarrow \quad RC=\frac{0.4\pm\sqrt{0.4^2-4(0.1)}}{2}$$

Since *RC* cannot have a complex value, the specification cannot be satisfied.

DP 9-5 Design the circuit shown in Figure DP 9-5 so that

$$v_o(t) = \frac{1}{2} + A_1 e^{-2t} + A_2 e^{-4t} \text{ V for } t > 0$$

Determine the values of the unspecified constants, A_1 and A_2.
Hint: The circuit is overdamped, and the natural frequencies are 2 and 4 rad/sec.

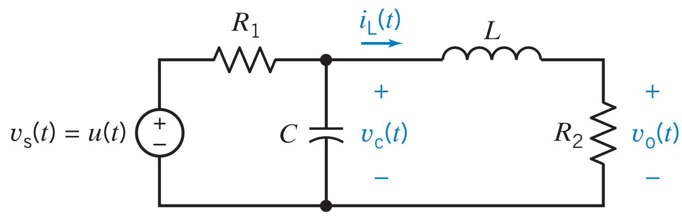


Figure DP 9-5

Solution:

When the circuit reaches steady state after t = 0, the capacitor acts like an open circuit and the inductor acts like a short circuit. Under these conditions

$$v_C(\infty) = \frac{R_2}{R_1+R_2}1, \quad i_L(\infty) = \frac{1}{R_1+R_2} \quad \textit{and} \quad v_o(\infty) = \frac{R_2}{R_1+R_2}1$$

The specifications require that $v_o(\infty) = \frac{1}{2}$ so

$$\frac{1}{2} = \frac{R_2}{R_1+R_2} \quad \Rightarrow \quad R_1 = R_2$$

Next, represent the circuit by a 2nd order differential equation:

KVL around the right-hand mesh gives: $v_C(t) = L\frac{d}{dt}i_L(t) + R_2\, i_L(t)$

KCL at the top node of the capacitor gives: $\frac{v_s(t) - v_C(t)}{R_1} - C\frac{d}{dt}v_C(t) = i_L(t)$

Use the substitution method to get

$$\begin{aligned} v_s(t) &= R_1 C\frac{d}{dt}\left(L\frac{d}{dt}i_L(t) + R_2\, i_L(t)\right) + \left(L\frac{d}{dt}i_L(t) + R_2\, i_L(t)\right) + R_1\, i_L(t) \\ &= R_1 LC\frac{d^2}{dt^2}i_L(t) + \left(L + R_1 R_2 C\right)\frac{d}{dt}i_L(t) + \left(R_1 + R_2\right)i_L(t) \end{aligned}$$

Using $v_o(t) = \frac{i_L(t)}{R_2}$ gives

$$v_o(t) = \frac{R_1}{R_2}LC\frac{d^2}{dt^2}i_L(t) + \left(\frac{L}{R_2} + R_1 C\right)\frac{d}{dt}i_L(t) + \left(\frac{R_1+R_2}{R_2}\right)i_L(t)$$

The characteristic equation is

$$s^2+\left(\frac{1}{R_1 C}+\frac{R_2}{L}\right)s+\left(\frac{1+\frac{R_2}{R_1}}{LC}\right)=s^2+6s+8=(s+2)(s+4)$$

Equating coefficients of like powers of s:

$$\frac{1}{R_1 C}+\frac{R_2}{L}=6 \text{ and } \frac{1+\frac{R_2}{R_1}}{LC}=8$$

Using $R_1=R_2=R$ gives

$$\frac{1}{R\,C}+\frac{R}{L}=6 \text{ and } \frac{1}{LC}=4$$

These equations do not have a unique solution. Try C = 1 F. Then $L=\frac{1}{4}$ H and

$$\frac{1}{R}+4R=6 \quad\Rightarrow\quad R^2-\frac{3}{2}R+\frac{1}{4}=0 \quad\Rightarrow\quad R=1.309\,\Omega \text{ or } R=0.191\,\Omega$$

Pick R = 1.309 Ω. Then

$$v_o(t)=\frac{1}{2}+A_1\,e^{-2t}+A_2\,e^{-4t} \text{ V}$$

$$i_L(t)=\frac{v_o(t)}{1.309}=\frac{1}{2.618}+\frac{A_1}{1.309}\,e^{-2t}+\frac{A_2}{1.309}\,e^{-4t} \text{ V}$$

$$v_C(t)=1.309\,i_L(t)+\frac{1}{4}\frac{d}{dt}i_L(t)=\frac{1}{2}+0.6167\,A_1\,e^{-2t}+0.2361\,A_2\,e^{-4t}$$

At t = 0+

$$0=i_L(0+)=\frac{1}{2.618}+\frac{A_1}{1.309}+\frac{A_2}{1.309}$$

$$0=v_C(0+)=\frac{1}{2}+0.6167\,A_1+0.2361\,A_2$$

Solving these equations gives A_1 = -1 and A_2 = 0.5, so

$$v_o(t)=\frac{1}{2}-e^{-2t}+\frac{1}{2}e^{-4t} \text{ V}$$

DP 9-6 Design the circuit shown in Figure DP 9-5 so that

$$v_o(t) = \frac{3}{4} + (A_1 + A_2 t)e^{-2t} \text{ V for } t > 0$$

Determine the values of the unspecified constants, A_1 and A_2.
Hint: The circuit is critically damped, and the natural frequencies are both 2 rad/sec.

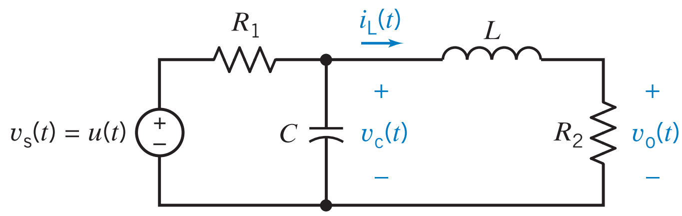


Figure DP 9-5

Solution:

When the circuit reaches steady state after t = 0, the capacitor acts like an open circuit and the inductor acts like a short circuit. Under these conditions

$$v_C(\infty) = \frac{R_2}{R_1 + R_2} 1, \quad i_L(\infty) = \frac{1}{R_1 + R_2} \quad \text{and} \quad v_o(\infty) = \frac{R_2}{R_1 + R_2} 1$$

The specifications require that $v_o(\infty) = \frac{3}{4}$ so

$$\frac{3}{4} = \frac{R_2}{R_1 + R_2} \quad \Rightarrow \quad 3R_1 = R_2$$

Next, represent the circuit by a 2nd order differential equation:

KVL around the right-hand mesh gives: $v_C(t) = L\frac{d}{dt} i_L(t) + R_2\, i_L(t)$

KCL at the top node of the capacitor gives: $\dfrac{v_s(t) - v_C(t)}{R_1} - C\dfrac{d}{dt} v_C(t) = i_L(t)$

Use the substitution method to get

$$v_s(t) = R_1 C \frac{d}{dt}\left(L\frac{d}{dt} i_L(t) + R_2\, i_L(t)\right) + \left(L\frac{d}{dt} i_L(t) + R_2\, i_L(t)\right) + R_1\, i_L(t)$$

$$= R_1 LC \frac{d^2}{dt^2} i_L(t) + \left(L + R_1 R_2 C\right)\frac{d}{dt} i_L(t) + \left(R_1 + R_2\right) i_L(t)$$

Using $v_o(t) = \dfrac{i_L(t)}{R_2}$ gives

$$v_o(t) = \frac{R_1}{R_2} LC \frac{d^2}{dt^2} i_L(t) + \left(\frac{L}{R_2} + R_1 C\right)\frac{d}{dt} i_L(t) + \left(\frac{R_1 + R_2}{R_2}\right) i_L(t)$$

The characteristic equation is

$$s^2+\left(\frac{1}{R_1 C}+\frac{R_2}{L}\right)s+\left(\frac{1+\frac{R_2}{R_1}}{LC}\right)=s^2+4s+4=(s+2)^2$$

Equating coefficients of like powers of s:

$$\frac{1}{R_1 C}+\frac{R_2}{L}=4 \text{ and } \frac{1+\frac{R_2}{R_1}}{LC}=4$$

Using $R_1=R$ and $R_2=3R$ gives

$$\frac{1}{RC}+\frac{3R}{L}=4 \text{ and } \frac{1}{LC}=1$$

These equations do not have a unique solution. Try C = 1 F. Then L = 1 H and

$$\frac{1}{R}+3R=4 \quad \Rightarrow \quad R^2-\frac{4}{3}R+\frac{1}{3}=0 \quad \Rightarrow \quad R=1\,\Omega \text{ or } R=\frac{1}{3}\,\Omega$$

Pick R = 1 Ω. Then $R_1=1\,\Omega$ and $R_2=3\,\Omega$.

$$v_o(t)=\frac{3}{4}+\left(A_1+A_2 t\right)e^{-2t}\ \text{V}$$

$$i_L(t)=\frac{v_o(t)}{3}=\frac{1}{4}+\left(\frac{A_1}{3}+\frac{A_2}{3}t\right)e^{-2t}\ \text{V}$$

$$v_C(t)=3i_L(t)+\frac{d}{dt}i_L(t)=\frac{3}{4}+\left(\left(\frac{A_1}{3}+\frac{A_2}{3}\right)+\frac{A_2}{3}t\right)e^{-2t}$$

At t = 0+

$$0=i_L(0+)=\frac{A_1}{3}+\frac{1}{4}$$

$$0=v_C(0+)=\frac{3}{4}+\frac{A_1}{3}+\frac{A_2}{3}$$

Solving these equations gives A_1 = -0.75 and A_2 = -1.5, so

$$v_o(t)=\frac{3}{4}-\left(\frac{3}{4}+\frac{3}{2}t\right)e^{-2t}\ \text{V}$$

DP 9-7 Design the circuit shown in Figure DP 9-5 so that

$$v_c(t) = 0.2 + e^{-2t}(A_1 \cos 4t + A_2 \sin 4t) \text{ V for } t > 0$$

Determine the values of the unspecified constants, A_1 and A_2.

Hint: The circuit is underdamped, the damped resonant frequency is 4 rad/sec, and the damping coefficient is 2.

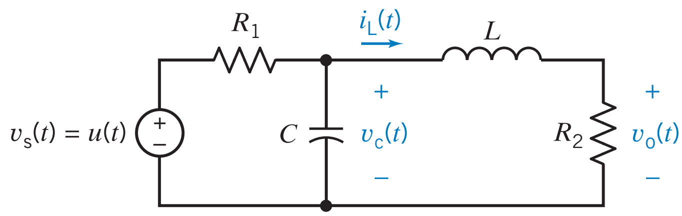


Figure DP 9-5

Solution:

When the circuit reaches steady state after t = 0, the capacitor acts like an open circuit and the inductor acts like a short circuit. Under these conditions

$$v_C(\infty) = \frac{R_2}{R_1 + R_2} 1, \quad i_L(\infty) = \frac{1}{R_1 + R_2} \quad \textit{and} \quad v_o(\infty) = \frac{R_2}{R_1 + R_2} 1$$

The specifications require that $v_o(\infty) = \frac{1}{5}$ so

$$\frac{1}{5} = \frac{R_2}{R_1 + R_2} \quad \Rightarrow \quad R_1 = 4 R_2$$

Next, represent the circuit by a 2nd order differential equation:

KVL around the right-hand mesh gives: $v_C(t) = L \frac{d}{dt} i_L(t) + R_2 \, i_L(t)$

KCL at the top node of the capacitor gives: $\frac{v_s(t) - v_C(t)}{R_1} - C \frac{d}{dt} v_C(t) = i_L(t)$

Use the substitution method to get

$$v_s(t) = R_1 C \frac{d}{dt}\left(L \frac{d}{dt} i_L(t) + R_2 \, i_L(t) \right) + \left(L \frac{d}{dt} i_L(t) + R_2 \, i_L(t) \right) + R_1 \, i_L(t)$$

$$= R_1 L C \frac{d^2}{dt^2} i_L(t) + \left(L + R_1 R_2 C \right) \frac{d}{dt} i_L(t) + \left(R_1 + R_2 \right) i_L(t)$$

Using $v_o(t) = \frac{i_L(t)}{R_2}$ gives

$$v_o(t) = \frac{R_1}{R_2} L C \frac{d^2}{dt^2} i_L(t) + \left(\frac{L}{R_2} + R_1 C \right) \frac{d}{dt} i_L(t) + \left(\frac{R_1 + R_2}{R_2} \right) i_L(t)$$

The characteristic equation is

$$s^2+\left(\frac{1}{R_1 C}+\frac{R_2}{L}\right)s+\left(\frac{1+\frac{R_2}{R_1}}{LC}\right)=s^2+4s+20=(s+2-j4)(s+2+j4)$$

Equating coefficients of like powers of s:

$$\frac{1}{R_1 C}+\frac{R_2}{L}=4 \text{ and } \frac{1+\frac{R_2}{R_1}}{LC}=20$$

Using $R_2 = R$ and $R_1 = 4R$ gives

$$\frac{1}{4R\,C}+\frac{R}{L}=4 \text{ and } \frac{1}{LC}=16$$

These equations do not have a unique solution. Try $C=\frac{1}{8}$ F. Then $L=\frac{1}{2}$ H and

$$\frac{2}{R}+2R=4 \quad \Rightarrow \quad R^2-2R+2=0 \quad \Rightarrow \quad R=1\,\Omega$$

Then $R_1 = 4\,\Omega$ and $R_2 = 1\,\Omega$. Next

$$v_o(t)=0.2+e^{-2t}\left(A_1\cos 4t+A_2\sin 4t\right)\text{ V}$$

$$i_L(t)=\frac{v_o(t)}{1}=0.2+e^{-2t}\left(A_1\cos 4t+A_2\sin 4t\right)\text{ V}$$

$$v_C(t)=i_L(t)+\frac{1}{2}\frac{d}{dt}i_L(t)=0.2+2A_2\,e^{-2t}\cos 4t-2A_1\,e^{-2t}\sin 4t$$

At t = 0+

$$0=i_L(0+)=0.2+A_1$$

$$0=v_C(0+)=0.2+2A_2$$

Solving these equations gives A_1 = -0.8 and A_2 = -0.4, so

$$v_c(t)=0.2-e^{-2t}\left(0.2\cos 4t+0.1\sin 4t\right)\text{ V}$$

DP 9-8 Show that the circuit shown in Figure DP 9-5 cannot be designed so that

$$v_c(t) = 0.5 + e^{-2t}(A_1 \cos 4t + A_2 \sin 4t)\text{ V for } t > 0$$

Hint: Show that such a design would require $1/RC + 10RC = 4$ where $R = R_1 = R_2$. Next, show that $1/RC + 10\,RC = 4$ would require the value of RC to be complex.

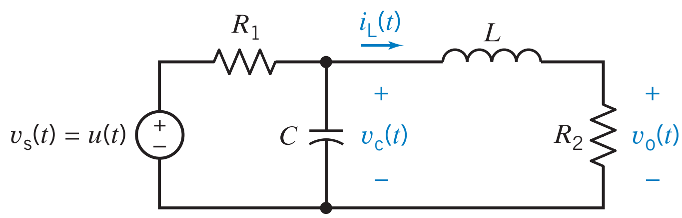


Figure DP 9-5

Solution:

When the circuit reaches steady state after t = 0, the capacitor acts like an open circuit and the inductor acts like a short circuit. Under these conditions

$$v_C(\infty) = \frac{R_2}{R_1+R_2}1, \quad i_L(\infty) = \frac{1}{R_1+R_2} \quad \textit{and} \quad v_o(\infty) = \frac{R_2}{R_1+R_2}1$$

The specifications require that $v_C(\infty) = \frac{1}{2}$ so

$$\frac{1}{2} = \frac{R_2}{R_1+R_2} \quad \Rightarrow \quad R_1 = R_2$$

Next, represent the circuit by a 2nd order differential equation:

KVL around the right-hand mesh gives: $v_C(t) = L\frac{d}{dt}i_L(t) + R_2\, i_L(t)$

KCL at the top node of the capacitor gives: $\dfrac{v_s(t) - v_C(t)}{R_1} - C\frac{d}{dt}v_C(t) = i_L(t)$

Use the substitution method to get

$$v_s(t) = R_1 C\frac{d}{dt}\left(L\frac{d}{dt}i_L(t) + R_2\, i_L(t)\right) + \left(L\frac{d}{dt}i_L(t) + R_2\, i_L(t)\right) + R_1\, i_L(t)$$

$$= R_1 LC\frac{d^2}{dt^2}i_L(t) + \left(L + R_1 R_2 C\right)\frac{d}{dt}i_L(t) + \left(R_1 + R_2\right)i_L(t)$$

Using $v_o(t) = \dfrac{i_L(t)}{R_2}$ gives

$$v_o(t) = \frac{R_1}{R_2}LC\frac{d^2}{dt^2}i_L(t) + \left(\frac{L}{R_2} + R_1 C\right)\frac{d}{dt}i_L(t) + \left(\frac{R_1+R_2}{R_2}\right)i_L(t)$$

The characteristic equation is

$$s^2+\left(\frac{1}{R_1\,C}+\frac{R_2}{L}\right)s+\left(\frac{1+\frac{R_2}{R_1}}{LC}\right)=s^2+4s+20=(s+2-j4)(s+2+j4)$$

Equating coefficients of like powers of s:

$$\frac{1}{R_1\,C}+\frac{R_2}{L}=4 \text{ and } \frac{1+\frac{R_2}{R_1}}{LC}=20$$

Using $R_1=R_2=R$ gives

$$\frac{1}{R\,C}+\frac{R}{L}=4 \text{ and } \frac{1}{LC}=10$$

Substituting $L=\frac{1}{10\,C}$ into the first equation gives

$$(RC)^2-\frac{4}{10}(RC)+\frac{1}{10}=0 \quad\Rightarrow\quad RC=\frac{0.4\pm\sqrt{0.4^2-4(0.1)}}{2}$$

Since *RC* cannot have a complex value, the specification cannot be satisfied.

DP 9-9 A fluorescent light uses cathodes (coiled tungsten filaments coated with an electron-emitting substance) at each end that send current through mercury vapors sealed in the tube. Ultraviolet radiation is produced as electrons from the cathodes knock mercury electrons out of their natural orbits. Some of the displaced electrons settle back into orbit, throwing off the excess energy absorbed in the collision. Almost all of this energy is in the form of ultraviolet radiation. The ultraviolet rays, which are invisible, strike a phosphor coating on the inside of the tube. The rays energize the electrons in the phosphor atoms, and the atoms emit white light. The conversion of one kind of light into another is known as fluorescence.

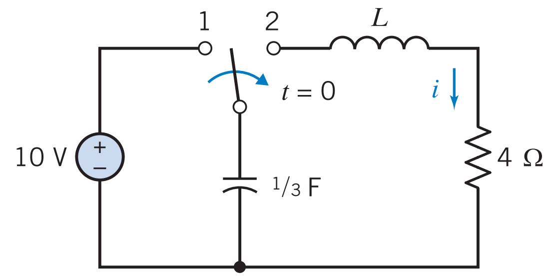
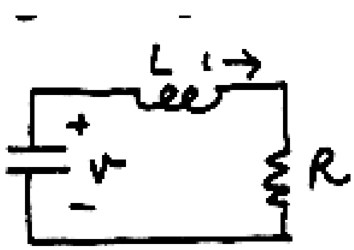


Figure DP 9-9

One form of a fluorescent lamp is represented by the *RLC* circuit shown in Figure DP 9-9. Select L so that the current $i(t)$ reaches a maximum at approximately t = 0.5 s. Determine the maximum value of $i(t)$. Assume that the switch was in position 1 for a long time before switching to position 2 at t = 0.

Hint: Use PSpice to plot the response for several values of L.

Solution:



Characteristic equation: $s^2+\frac{R}{L}s+\frac{1}{LC}=0$

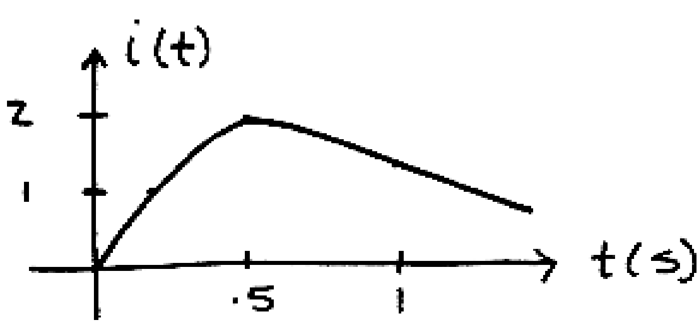
Select L so fast response and i achieve maximum at t = 0.5 s

$$s^2+\frac{4}{L}s+\frac{3}{L}=0 \quad \text{Try } L = 1\text{ H} \quad \Rightarrow s^2+4s+3=0 \text{ or } s=-3,-1$$

$$i(t) = A_1 e^{-t} + A_2 e^{-3t}$$

$$\left.\begin{aligned} i(0)&=0=A_1+A_2 \\ \frac{di(0)}{dt}&=\frac{v(0)}{L}=\frac{10}{1}=-A_1-3A_2 \end{aligned}\right\} A_1=5,\ A_2=-5$$

$$i(t)=5e^{-t}-5e^{-3t} \qquad at\ t=0.5\text{ s}\ \ i\simeq 1.92$$



Section 10.2 Sinusoidal Sources

P10.2-1 Given the sinusoids $v_1(t) = 8\cos(250t + 15°)$ V and $v_2(t) = 6\cos(250t - 45°)$ V, determine the time by which $v_2(t)$ is advanced or delayed with respect to $v_1(t)$.

Solution:

The period of both sinusoids is $T = \frac{2\pi}{250} = 25.1327 \text{ ms}$

The difference in the phase angles is

$$\theta_2 - \theta_1 = -45° - 15° = -60°$$

The delay time is $t_d = \frac{-60(25.1327)}{2\pi} = -4.188 \text{ ms}$

(The minus sign indicates a delay.) The voltage $v_2(t)$ is delayed by 4.188 ms with respect to $v_1(t)$.

P10.2-2 Given the sinusoids $v_1(t) = 8\cos(100t - 54°)$ V and $v_2(t) = 8\cos(100t - 102°)$ V, determine the time by which $v_2(t)$ is advanced or delayed with respect to $v_1(t)$.

Solution:

The period of both sinusoids is $T = \frac{2\pi}{100} = 62.8319 \text{ ms}$

The difference in the phase angles is

$$\theta_2 - \theta_1 = -102° - (-54°) = -48°$$

The delay time is $t_d = \frac{-48°(62.8319)}{360°} = -8.3776 \text{ ms}$

(The minus sign indicates a delay.) The voltage $v_2(t)$ is delayed by 48.3776 ms with respect to $v_1(t)$.

P10.2-3 A sinusoidal current is given as

$$i(t) = 125\cos(5000\pi t - 135°) \text{ mA}$$

Determine the period, *T*, and the time, t_1, at which the first positive peak occurs.

Answer: *T* = 0.4 ms and t_1 = 0.15 ms.

Solution: The frequency is 5000 π rad/s or 2500 Hertz so the period is

$$T = \frac{1}{2500} = 0.0004 = 0.4 \text{ ms}$$

Converting the angle from degrees to radians, we get $-135°\left(\frac{\pi}{180°}\right) = -0.75\pi$ radians. The current sinusoid is shifted from $125\cos(5000\pi t)$ mA by $\frac{-0.75\pi}{5000\pi} = -15$ ms. The minus sign indicates a delay. A positive peak occurs at $t_1 = 0.15$ ms. Since 15 ms is less than the period of $i(t_1)$, the positive peak at $t_1 = 0.15$ ms is the first positive peak.

P10.2-4 Express the voltage shown in Figure P10.2-7 in the general form

$$v(t) = A\cos(\omega t + \theta) \text{ V}$$

where $A \geq 0$ and $-180° < \theta \leq 180°$.

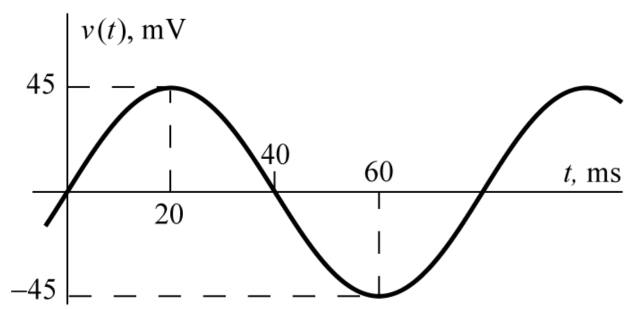


Figure P10.2-4

Solution: The amplitude is *A* = 45 mv and the period is given by $\frac{T}{2} = 60 - 20 = 40$ ms so the period is *T* = 80 ms. The frequency is given by $\omega = \frac{2\pi}{80\times10^{-3}} = 78.54$ rad/s. Noticing that *v*(*t*) is 0 at time 0 and is increasing at time 0, we can write

$$v(t) = 45\sin(78.54t) = 45\cos(78.54t - 90°) \text{ mV}$$

P 10.2-5 Figure P 10.2-5 shows a sinusoidal voltage, $v(t)$, plotted as a function of time, t. Represent $v(t)$ by a function of the form $A\cos(\omega + \theta)$.

Answer: $v(t) = 18\cos(393t - 27°)$

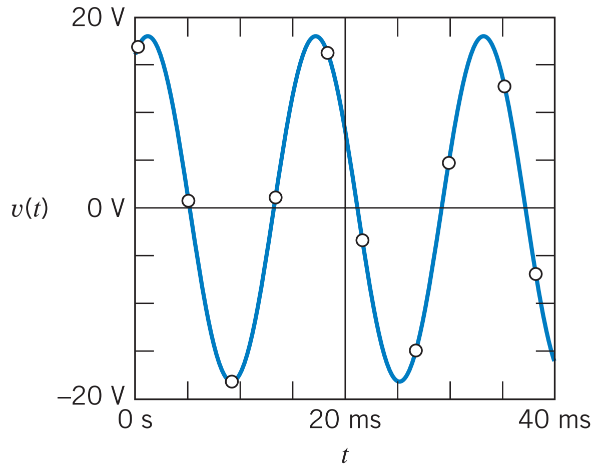
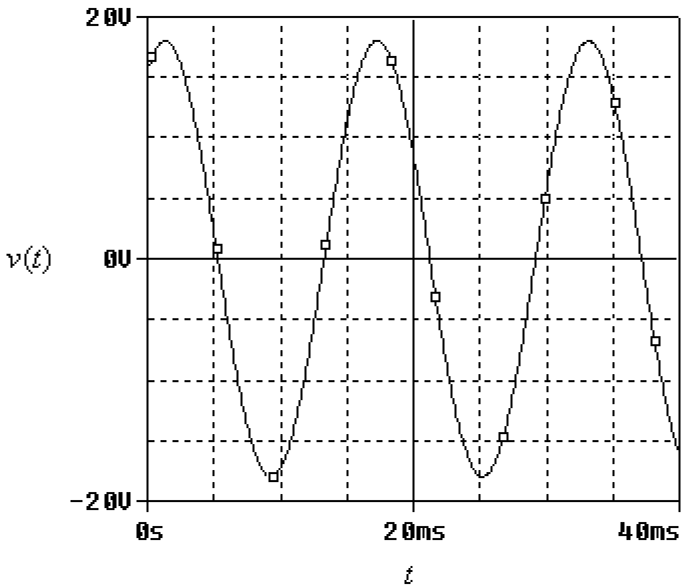


Figure P 10.2-5

Solution:



$$A = 18 \text{ V}$$

$$T = 18 - 2 = 16 \text{ ms}$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.016} = 393 \ \text{ rad/s}$$

$$\theta = -\cos^{-1}\left(\frac{16}{18}\right) = -27°$$

$$v(t) = 18\cos(393\,t - 27°) \ \text{ V}$$

P 10.2-6 Figure P 10.2-6 shows a sinusoidal voltage, $v(t)$, plotted as a function of time, t. Represent $v(t)$ by a function of the form $A\cos(\omega t+\theta)$.

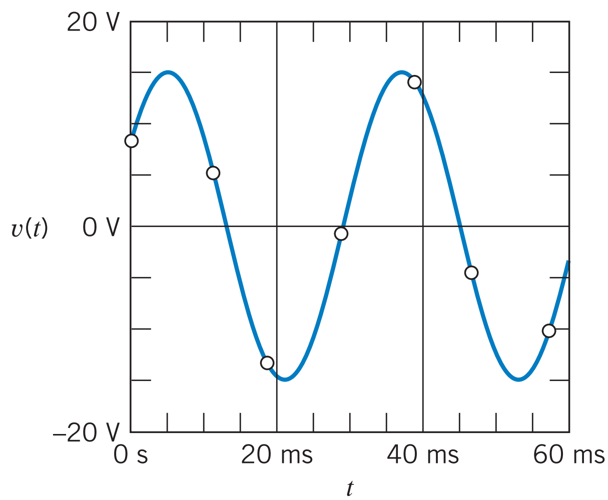
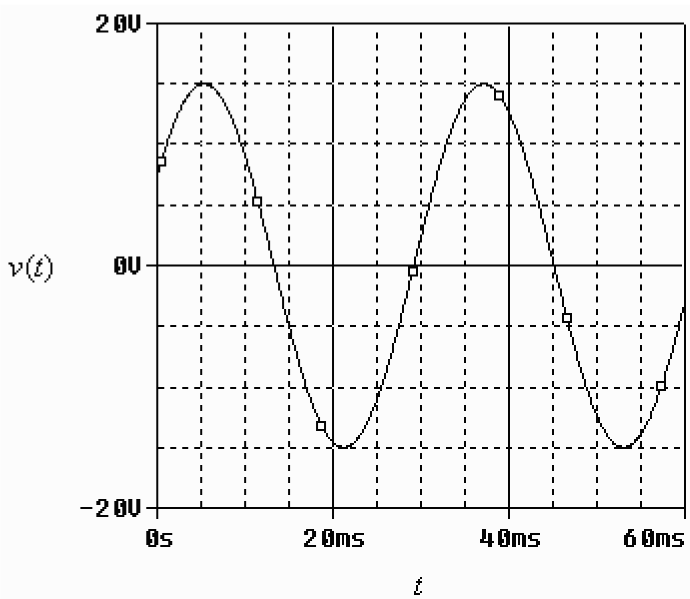


Figure P 10.2-6

Solution:



$$A = 15 \text{ V}$$

$$T = 43 - 11 = 32 \text{ ms}$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.032} = 196 \text{ rad/s}$$

$$\theta = -\cos^{-1}\left(\frac{8}{15}\right) = -58°$$

$$v(t) = 15\cos(196\,t - 58°) \text{ V}$$

Section 10.3 Phasors and Sinusoids

P10.3-1 Express the current

$$i(t) = 2\cos(6t + 120°) + 2\sin(6t - 60°) \text{ mA}$$

In the general form

$$i(t) = A\cos(\omega t + \theta) \text{ mA}$$

where $A \geq 0$ and $-180° < \theta \leq 180°$.

Solution:

$$\begin{aligned} i(t) &= 2\cos(6t + 120°) + 2\sin(6t - 60°) \text{ mA} \\ &= 2\cos(6t + 120°) + 2\cos(6t - 60° - 90°) \text{ mA} \end{aligned}$$

Representing the sinusoids using phasors gives:

$$\begin{aligned} \mathbf{I}(\omega) &= 2\angle 120° + 4\angle -150° = (-1 + j1.732) + (-3.464 - j2) \\ &= -4.464 - j0.268 = 4.472\angle 183.4° = 4.472\angle -176.6° \text{ mA} \end{aligned}$$

The corresponding sinusoid is:

$$i(t) = 4.472\cos(6t - 176.6°) \text{ mA}$$

P10.3-2 Express the voltage

$$v(t) = 5\sqrt{2}\cos(8t) + 2\sin(8t + 45°) \text{ V}$$

In the general form

$$v(t) = A\cos(\omega t + \theta) \text{ V}$$

where $A \geq 0$ and $-180° < \theta \leq 180°$.

Solution:

$$\begin{aligned} v(t) &= 5\sqrt{2}\cos(8t) + 2\sin(8t + 45°) \\ &= 5\sqrt{2}\cos(8t) + 2\cos(8t + 45° - 90°) = 5\sqrt{2}\cos(8t) + 2\cos(8t - 45°) \text{ V} \end{aligned}$$

Representing the sinusoids using phasors gives:

$$\begin{aligned} \mathbf{V}(\omega) &= 7.0711 + 10\angle -45° = 7.0711 + (7.0711 - j7.0711) \\ &= 14.1422 - j7.0711 = 15.811\angle -26.6° \text{ V} \end{aligned}$$

The corresponding sinusoid is:

$$v(t) = 15.811\cos(8t - 26.6°) \text{ V}$$

P 10.3-3 Determine the polar form of the quantity

$$\frac{(25\angle 36.9°)(80\angle -53.1°)}{(4+j8)+(6-j8)}$$

Answer: $200\angle -16.2°$

Solution:

$$\frac{(25\angle 36.9°)(80\angle -53.1°)}{(4+j8)+(6-j8)} = \frac{25\cdot 80\angle(36.9°-53.1°)}{(4+6)+j(8-8)} = \frac{2000\angle -16.2°}{10} = 200\angle -16.2°$$

P 10.3-4 Determine the polar and rectangular form of the expression

$$5\angle +81.87°\left(4-j3+\frac{3\sqrt{2}\angle -45°}{7-j1}\right)$$

Answer: $88.162\angle 30.127° = 76.2520 + j44.2506$

Solution:

$$\begin{aligned}
8\angle 42°\left(8-j3+\frac{40\angle -45°}{7-j12}\right) &= 8\angle 42°\left(8-j3+\frac{40\angle -45°}{13.892\angle -59.744°}\right) \\
&= 8\angle 42°(8-j3+2.8793\angle 14.744°) \\
&= 8\angle 42°(8-j3+2.7845+j0.7328) \\
&= 8\angle 42°(10.7845-j2.2672) \\
&= 8\angle 42°(11.02\angle -11.873°) = 88.162\angle 30.127° = 76.2520+j44.2506
\end{aligned}$$

P 10.3-5 Determine the polar and rectangular form of the expression

$$\frac{(60\angle 120°)(-16+j12+20\angle 15°)}{5\angle -75°}$$

Solution:

$$\frac{(60\angle 120°)(-16+j12+20\angle 15°)}{5\angle -75°}=\frac{(60\angle 120°)(-16+j12+19.3185+j5.1764)}{5\angle -75°}$$

$$=\frac{(60\angle 120°)(3.3185+j17.1764)}{5\angle -75°}$$

$$=\frac{(60\angle 120°)(17.494\angle 79.065°)}{5\angle -75°}$$

$$=\frac{1049.6\angle -160.93°}{5\angle -75°}=139.95\angle 109.07°=45.714+j132.28$$

P10.3-6 The circuit shown in Figure 10.3-6 is at steady state. The input currents are

$$i_1(t)=10\cos(25t)\ \text{mA and } i_3(t)=10\cos(25t+135°)\ \text{mA}$$

Determine the voltage $v_2(t)$.

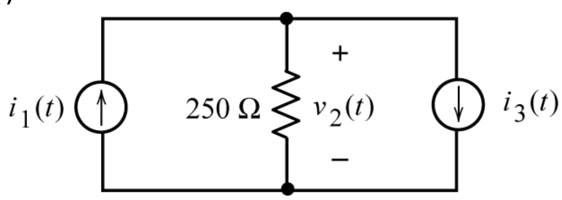
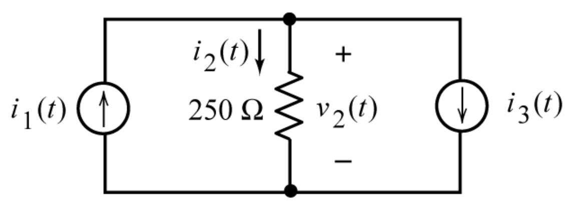


Figure 10.3-6

Solution:



Using first Ohm's law and then KCL

$$v_2(t)=250\,i_2(t)$$

and $$i_2(t)=i_1(t)-i_3(t)=10\cos(25t)-10\cos(25t+135°)\ \text{mA}$$

Using phasors

$$\mathbf{I}_2(\omega)=\mathbf{I}_1(\omega)-\mathbf{I}_3(\omega)=10-10\angle 135=10-(-7.071+j7.071)\ \text{mA}$$

$$=17.071-j7.071=18.478\angle -22.5°\ \text{mA}$$

The corresponding sinusoid is $i_2(t)=18.478\cos(25t-22.5°)\ \text{mA}$

Finally $$v_2(t)=250\,i_2(t)=4.6195\cos(25t-22.5°)\ \text{V}$$

P10.3-7 The circuit shown in Figure 10.3-7 is at steady state. The inputs to this circuit are the current source current

$$i_1(t) = 0.12\cos(100t + 45°) \text{ A}$$

and the voltage source voltage

$$v_2(t) = 24\cos(100t - 60°) \text{ V}$$

Determine the current $i_2(t)$.

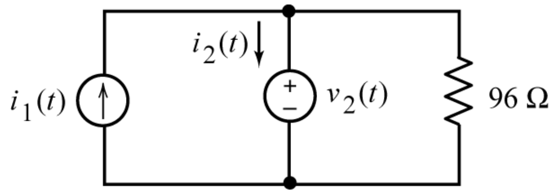


Figure P10.3-7

Solution: Using Ohm's and Kirchhoff's laws

$$i_2(t) = i_1(t) - \frac{v_2(t)}{96} = 0.12\cos(100t + 45°) - \frac{24\cos(100t - 60°)}{96}$$
$$= 0.12\cos(100t + 45°) - 0.25\cos(100t - 60°)$$

Using phasors

$$\mathbf{I}_2(\omega) = 0.12\angle 45° - 0.25\angle 60° = (0.0849 + j\,0.0849) - (0.1250 - j\,0.2165)$$
$$= -0.0401 + j\,0.3014 = 0.3040\angle 97.6° \text{ A}$$

The corresponding sinusoid is

$$i_2(t) = 0.3040\cos(100t + 97.6°) \text{ A}$$

Checked using LNAPAC 8/16/11

P 10.3-8 Given that

$i_1(t)$ = 30 cos (4t + 45°) mA

and $i_2(t)$ = – 40 cos (4t) mA

Determine $v(t)$ for the circuit shown in Figure P 10.3-8.

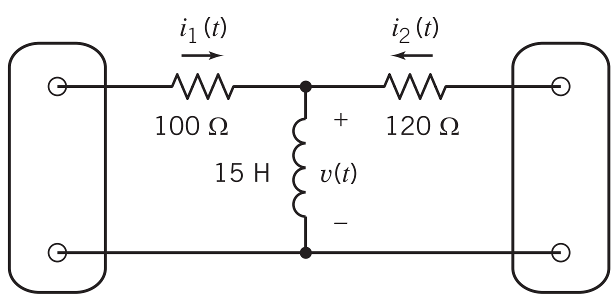


Figure P 10.3-8

Solution:

$$\mathbf{V} = j(4)(15)(\mathbf{I}_1 + \mathbf{I}_2) = j60(0.03\angle 45° - 0.04\angle 0°) = j60(0.0212 + j0.0212 - 0.04)$$
$$= -1.273 - j1.127$$
$$= 1.7\angle -138.5° \text{ V}$$

So $$v(t) = 1.7\cos(4t - 138.5°) \text{ V}$$

(checked: LNAP 8/7/04)

P 10.3-9 For the circuit shown in Figure P 10.3-9, find (a) the impedances $\mathbf{Z}_1$ and $\mathbf{Z}_2$ in polar form, (b) the total combined impedance in polar form, and (c) the steady-state current $i(t)$.

Answer:

(a) $\mathbf{Z}_1 = 5\angle 53.1°;\ \mathbf{Z}_2 = 8\sqrt{2}\angle -45°$

(b) $\mathbf{Z}_1 + \mathbf{Z}_2 = 11.7\angle -20°$

(c) $i(t) = (8.55)\cos(1250t + 20°)$ A

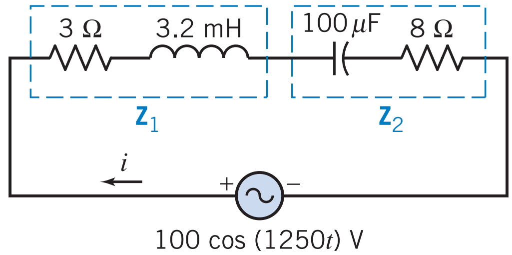
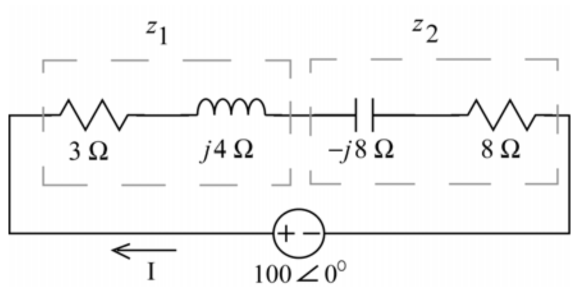


Figure P 10.3-9

Solution:



(a) $\underline{\mathbf{Z}_1 = 3 + j4 = 5\angle 53.1°\ \Omega}$ and $\underline{\mathbf{Z}_2 = 8 - j8 = 8\sqrt{2}\angle -45°\ \Omega}$

(b) Total impedance $= \mathbf{Z}_1 + \mathbf{Z}_2 = 3 + j4 + 8 - j8 = 11 - j4 = \underline{11.7\angle -20.0°\ \Omega}$

(c) $$\mathbf{I} = \frac{100\angle 0°}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{100}{11.7\angle -20°} = \frac{100}{11.7}\angle 20.0° \Rightarrow \underline{i(t) = 8.55\cos(1250t + 20.0°)\ \text{A}}$$

P 10.3-10 The circuit shown in Figure P 10.3-10 is at steady state. The voltages $v_s(t)$ and $v_2(t)$ are given by

$$v_s(t) = 7.68 \cos(2t + 47°) \text{ V}$$

and

$$v_2(t) = 1.59 \cos(2t + 125°) \text{ V}$$

Find the steady-state voltage $v_1(t)$.

Answer: $v_1(t) = 7.51 \cos(2t + 35°)$ V

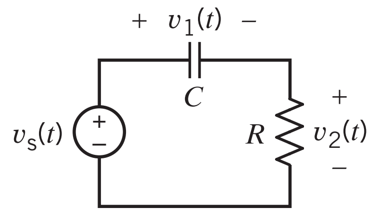


Figure P 10.3-10

Solution:

$$\begin{aligned}\mathbf{V}_1(\omega) = \mathbf{V}_s(\omega) - \mathbf{V}_2(\omega) &= 7.68\angle 47° - 1.59\angle 125° \\ &= (5.23 + j5.62) - (-0.91 + 1.30) \\ &= (5.23 + 0.91) + j(5.62 - 1.30) \\ &= 6.14 + j4.32 \\ &= 7.51\angle 35°\end{aligned}$$

$$v_1(t) = 7.51 \cos(2t + 35°) \text{ V}$$

P 10.3-11 The circuit shown in Figure P 10.3-11 is at steady state. The currents $i_1(t)$ and $i_2(t)$ are given by

$$i_1(t) = 744 \cos(2t - 118°) \text{ mA}$$

and

$$i_2(t) = 540.5 \cos(2t + 100°) \text{ mA}$$

Find the steady-state current $i(t)$.

Answer: $i(t) = 460 \cos(2t + 196°)$ mA

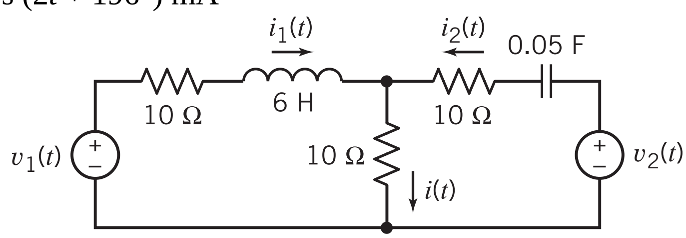


Figure P 10.3-11

Solution:

$$\begin{aligned}\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = 0.744\angle -118° + 0.5405\angle 100 &= (-0.349 - j0.657) + (-0.094 + j0.532) \\ &= (-0.349 - 0.094) + j(-0.657 + 0.532) \\ &= -0.443 - j0.125 \\ &= 0.460\angle 196°\end{aligned}$$

$$i(t) = 460 \cos(2t + 196°) \text{ mA}$$

P 10.3-12 Determine $i(t)$ of the *RLC* circuit shown in Figure P 10.3-12 when

$$v_s = 2\cos(4t + 30°)\ \text{V}.$$

Answer: $i(t) = 0.185\cos(4t - 26.3°)$ A

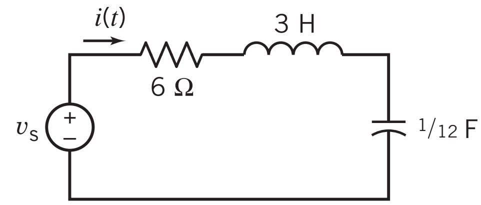
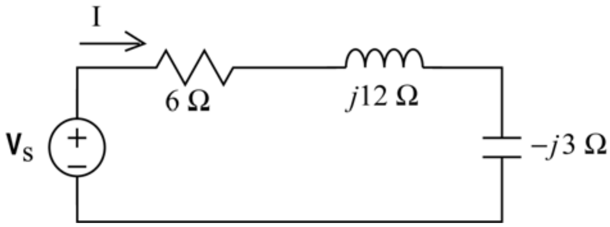


Figure P 10.3-12

Solution:



$$\mathbf{V}_s = 2\angle 30°\ \text{V}$$

and $$\mathbf{I} = \frac{2\angle 30°}{6 + j12 + 3/j} = 0.185\angle -26.3°\ \text{A}$$

$$\underline{i(t) = 0.185\cos(4t - 26.3°)\ \text{A}}$$

Section 10.4 Impedances

P10.4-1 Figure P10.4-1*a* shows a circuit represented in the time domain. Figure P10.4-1*b* shows the same circuit represented in the frequency domain, using phasors and impedances. $\mathbf{Z}_R$, $\mathbf{Z}_C$, $\mathbf{Z}_{L1}$, and $\mathbf{Z}_{L2}$ are the impedances corresponding to the resistor, capacitor, and two inductors in Figure P10.4-1*a*. $\mathbf{V}_s$ is the phasor corresponding to the voltage of the voltage source. Determine $\mathbf{Z}_R$, $\mathbf{Z}_C$, $\mathbf{Z}_{L1}$, $\mathbf{Z}_{L2}$, and $\mathbf{V}_s$.

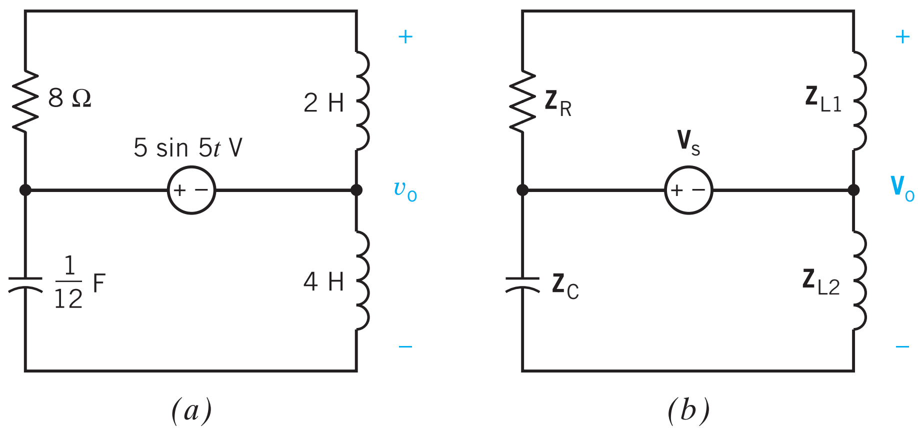


Figure P10.4-1

Hint: $5 \sin 5t = 5 \cos(5t - 90°)$

Answer: $\mathbf{Z}_R = 8\Omega, \mathbf{Z}_C = \dfrac{1}{j5\left(\dfrac{1}{12}\right)} = \dfrac{2.4}{j} = \dfrac{j2.4}{j \times j} = -j2.4\Omega, \mathbf{Z}_{L1} = j5(2) = j10\Omega,$

$\mathbf{Z}_{L2} = j5(4) = j20\ \Omega$, and $\mathbf{V}_s = 5\ \angle{-90°}$ V

Solution: $\mathbf{Z}_R = 8\ \Omega,\ \mathbf{Z}_C = \dfrac{1}{j\,5\dfrac{1}{12}} = \dfrac{2.4}{j} = \dfrac{j\,2.4}{j \times j} = -j\,2.4\ \ \Omega,\ \mathbf{Z}_{L1} = j\,5\ (2) = j\ 10\ \Omega,$

$$\mathbf{Z}_{L2} = j\ 5\ (4) = j\ 20\ \Omega \text{ and } \mathbf{V}_S = 5\ \angle{-90°}\ \text{V}.$$

P10.4-2 Figure P10.4-2*a* shows a circuit represented in the time domain. Figure P10.4-2*b* shows the same circuit represented in the frequency domain, using phasors and impedances. $\mathbf{Z}_R$, $\mathbf{Z}_C$, $\mathbf{Z}_{L1}$, and $\mathbf{Z}_{L2}$ are the impedances corresponding to the resistor, capacitor, and two inductors in Figure P10.4-2*a*. $\mathbf{I}_s$ is the phasor corresponding to the current of the current source. Determine $\mathbf{Z}_R$, $\mathbf{Z}_C$, $\mathbf{Z}_{L1}$, $\mathbf{Z}_{L2}$, and $\mathbf{I}_s$.

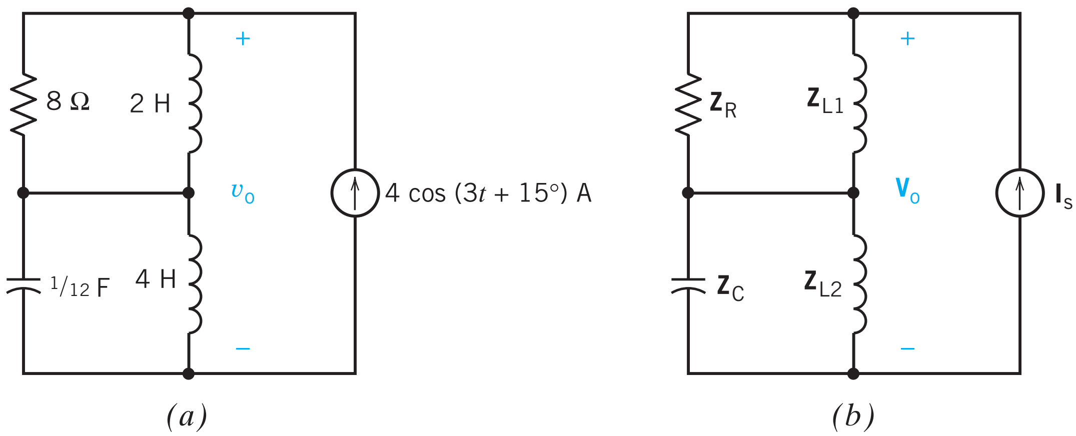


Figure P10.4-2

Answer: $\mathbf{Z}_R = 8\Omega, \mathbf{Z}_C = \dfrac{1}{j3\left(\dfrac{1}{12}\right)} = \dfrac{4}{j} = \dfrac{j4}{j \times j} = -j4\Omega, \mathbf{Z}_{L1} = j3(2) = j6\Omega,$

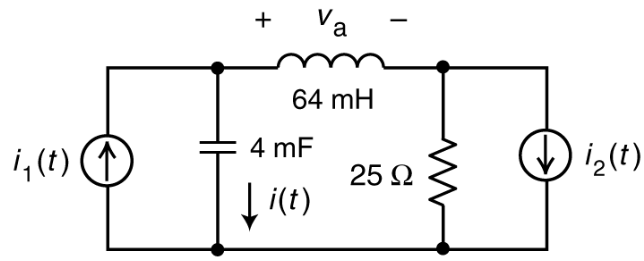
$\mathbf{Z}_{L2} = j3(4) = j12\ \Omega$, and $\mathbf{I}_s = 4\ \angle 15°$ A

Solution:

$$\mathbf{Z}_R = 8\ \Omega,\ \mathbf{Z}_C = \frac{1}{j3\frac{1}{12}} = \frac{4}{j} = \frac{j4}{j \times j} = -j4\ \Omega,\ \mathbf{Z}_{L1} = j\,3\,(2) = j\,6\ \Omega,$$

$$\mathbf{Z}_{L2} = j\,3\,(4) = j\,12\ \Omega \text{ and } \mathbf{I}_S = 4\ \angle 15°\text{ A.}$$

P10.4-3 Represent the circuit shown in Figure P10.4-3 in the frequency domain using impedances and phasors.

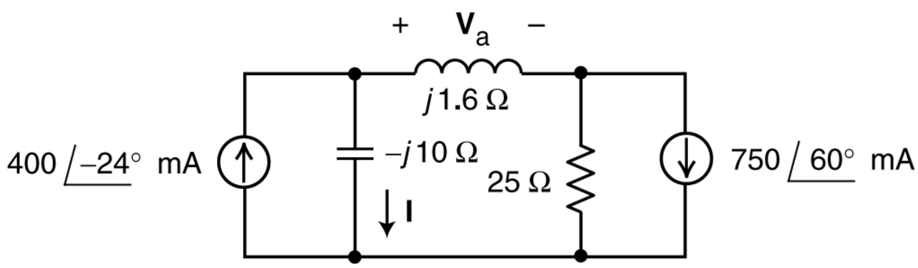


$$i_1(t) = 400\cos(25t - 24°)\ \text{mA}$$

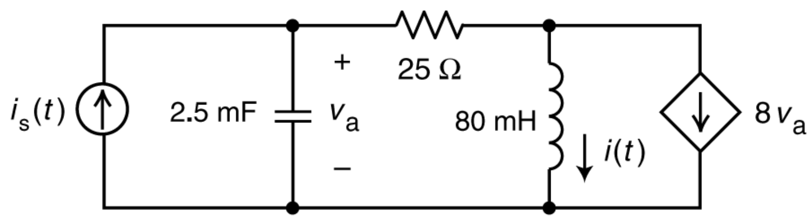
$$i_2(t) = 750\cos(25t + 60°)\ \text{mA}$$

Figure P10.4-3

Solution:



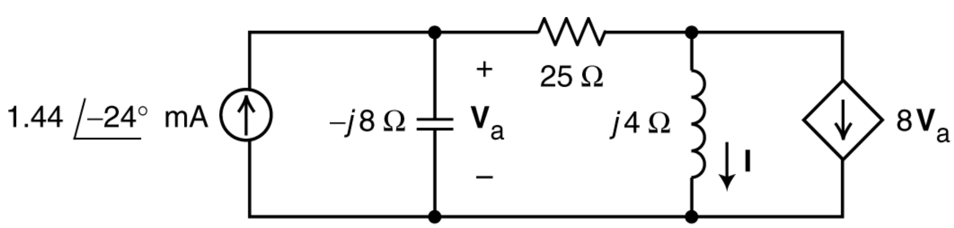
P10.4-4 Represent the circuit shown in Figure P10.4-4 in the frequency domain using impedances and phasors.



$$i_s(t) = 1.44\cos(50t - 24°)\ \text{mA}$$

Figure P10.4-4

Solution:



P10.4-5 Determine the current, *i*(*t*), for the circuit shown in Figure P10.4-5.

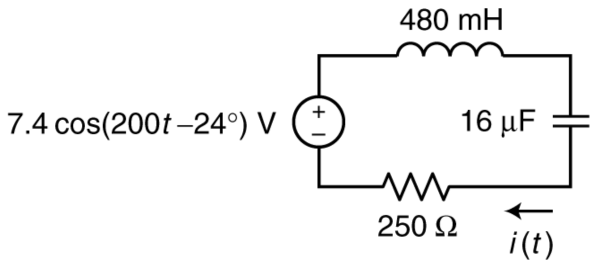
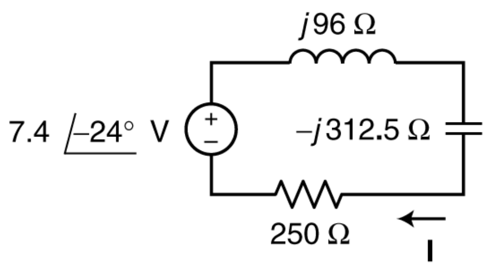


Figure P10.4-5

Solution: Represent the circuit in the frequency domain using phasors and impedances:



Using KVL: $j96\,\mathbf{I} - j312.5\,\mathbf{I} + 250\,\mathbf{I} + 7.4\angle -24° = 0$

Solving: $\mathbf{I} = \dfrac{7.4\angle -24°}{j96 - j312.5 + 250} = \dfrac{7.4\angle -24°}{-j216.5 + 250} = \dfrac{7.4\angle -24°}{330.715\angle -40.9°} = 0.0224\angle 16.9 \text{ A}$

In the time domain: $i(t) = 22.4\cos(200t + 16.9)$ mA

(Checked using LNAP)

P10.4-6 The input to the circuit shown in Figure P10.4-6 is the current

$$i(t) = 120\cos(4000t) \text{ mA}$$

Determine the voltage, $v(t)$, across the 40 Ω resistor.

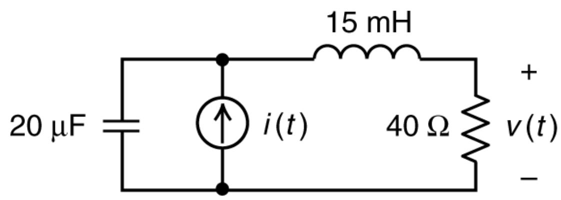
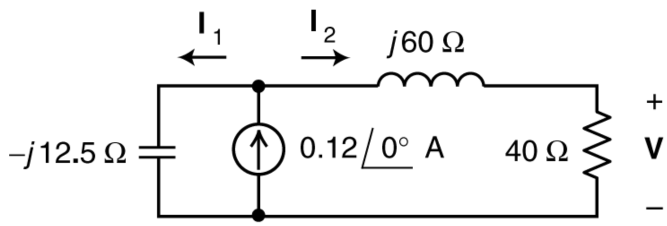


Figure P10.4-6

Solution: Represent the circuit in the frequency domain using phasors and impedances:



Using KCL $$0.120\angle 0° = \mathbf{I}_1 + \mathbf{I}_2$$

Using KVL $$j60\,\mathbf{I}_2 + 40\,\mathbf{I}_2 - (-j12.5)\mathbf{I}_1 = 0$$

$$j60\,\mathbf{I}_2 + 40\,\mathbf{I}_2 - (-j12.5)\left(0.12\angle 0° - \mathbf{I}_2\right) = 0$$

$$j60\,\mathbf{I}_2 + 40\,\mathbf{I}_2 + (-j12.5)\mathbf{I}_2 = (-j12.5)(0.12\angle 0°)$$

$$\mathbf{I}_2 = \frac{1.5\angle -90°}{j60 + 40 + (-j12.5)} = \frac{1.5\angle -90°}{40 + j47.5} = \frac{1.5\angle -90°}{62.1\angle 50°} = 0.024155\angle -140° \text{ A}$$

$$\mathbf{V} = 40\,\mathbf{I}_2 = 0.9662\angle -140° \text{ V}$$

In the time domain $$v(t) = 0.9662\cos(4000t - 140°) \text{ V}$$

(Checked using LNAP)

P10.4-7 The input to the circuit shown in Figure P10.4-7 is the current

$$i(t) = 82\cos(10000t) \ \ \mu\text{A}$$

Determine the voltage, $v(t)$, across the 50 kΩ resistor.

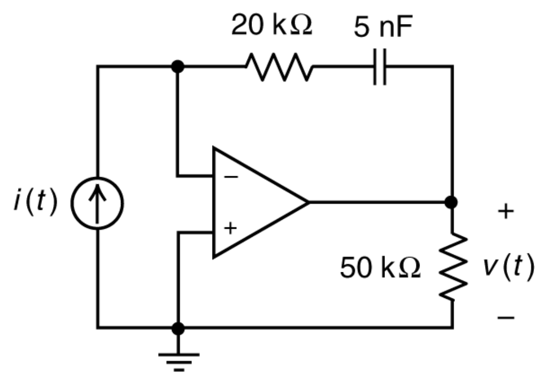
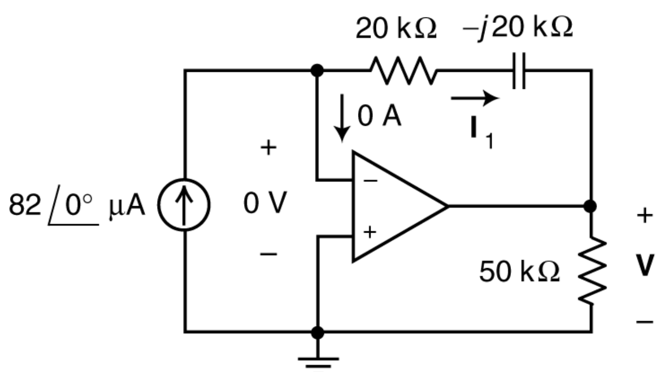


Figure P10.4-7

Solution: Represent the circuit in the frequency domain using phasors and impedances:



Using KCL $$82\times10^{-6}\angle0° = 0 + \mathbf{I}_1$$

Using KVL $$\left(20\times10^{3}\right)\mathbf{I}_1 + \left(-j\,20\times10^{3}\right)\mathbf{I}_1 + \mathbf{V} = 0$$

$$\mathbf{V} = -\left(20\times10^{3} - j\,20\times10^{3}\right)\left(82\times10^{-6}\right) = 2.3193\angle135° \ \ \text{V}$$

In the time domain $$v(t) = 2.3193\cos(10000t + 135°) \ \ \text{V}$$

(Checked using LNAP)

P 10.4-8 Each of the following pairs of element voltage and element current adheres to the passive convention. Indicate whether the element is capacitive, inductive, or resistive and find the element value.

(a) $v(t) = 15 \cos(400t + 30°)$; $i = 3 \sin(400t + 30°)$

(b) $v(t) = 8 \sin(900t + 50°)$; $i = 2 \sin(900t + 140°)$

(c) $v(t) = 20 \cos(250t + 60°)$; $i = 5 \sin(250t + 150°)$

Answer: (a) $L = 12.5$ mH

(b) $C = 277.77\ \mu$F

(c) $R = 4\ \Omega$

Solution:

(a) $v = 15\cos(400t + 30°)$ V

$i = 3\sin(400t+30°) = 3\cos(400t - 60°)$ V

v leads i by 90° $\Rightarrow$ element is an <u>inductor</u>

$$\left|\mathbf{Z}_L\right| = \frac{v_{peak}}{i_{peak}} = \frac{15}{3} = 5 = \omega L = 400L \Rightarrow \underline{L = 0.0125 \text{ H} = 12.5 \text{ mH}}$$

(b) i leads v by 90° $\Rightarrow$ the element is a <u>capacitor</u>

$$\left|\mathbf{Z}_c\right| = \frac{v_{peak}}{i_{peak}} = \frac{8}{2} = 4 = \frac{1}{\omega C} = \frac{1}{900C} \Rightarrow \underline{C = 277.77\ \mu\text{F}}$$

(c) $v = 20\cos(250t + 60°)$ V

$i = 5\sin(250t+150°) = 5\cos(250t+60°)$ A

Since v & i are in phase $\Rightarrow$ element is a <u>resistor</u>

$$\therefore R = \frac{v_{peak}}{i_{peak}} = \frac{20}{5} = \underline{4\ \Omega}$$

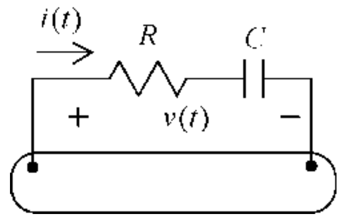


Figure P10.4-9

P10.4-9 This voltage and current for the circuit shown in Figure P10.4-9 are given by

$$v(t) = 20\cos(20t + 15°) \text{ V} \quad \text{and} \quad i(t) = 1.49\cos(20t + 63°) \text{ A}$$

Determine the values of the resistance, *R*, and capacitance, *C*.

Solution:

$$R - j\frac{1}{20C} = \mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{20\angle 15°}{1.49\angle 63°} = \frac{20}{1.49}\angle(15° - 63°) = 13.42\angle -48° = 8.98 - j9.97\ \Omega$$

Equating real and imaginary parts gives $R = 9\ \Omega$ and $C = \frac{1}{20 \times 9.97} = 5$ mF .

P10.4-10

Figure P10.4-10 shows an ac circuit represented in both the time domain and the frequency domain. Determine the values of A, B, a and b.

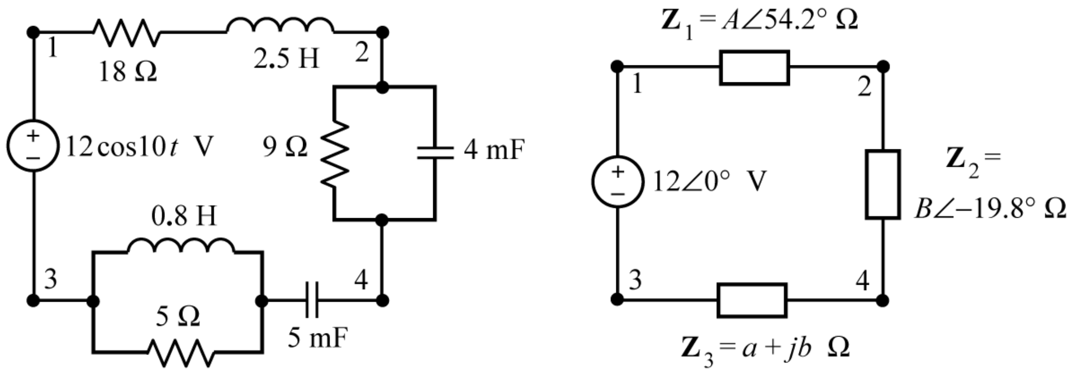


Figure P10.4-10

Solution:

The impedance between nodes a and b is given by

$$18+j(10)(2.5)=18+j25=30.8\angle 54.2°$$

To find the impedance between nodes b and c we first find the impedance of the capacitor:

$$-j\frac{1}{(10)(0.004)}=-j\frac{1}{0.04}=-j25$$

then

$$\frac{9(-j25)}{9-j25}=\frac{-j225}{26.57\angle -70.2°}=\frac{225\angle -90°}{26.57\angle -70.2°}=8.47\angle -19.8°\ \Omega$$

The impedance between nodes c and d is given by

$$\frac{(5)(j(10)(0.88))}{5+j(10)(0.8)}-j\frac{1}{(10)(0.005)}=\frac{j40}{5+j8}-j\frac{1}{0.05}=\frac{j40}{5+j8}\left(\frac{5-j8}{5-j8}\right)-j20$$

$$=\frac{320+j200}{25+64}-j20$$

$$=3.60+j2.25-j20=3.60-j17.75\ \Omega$$

So

$A=$ 30.8 V, $B=8.47$ Ω, , a = 3.57 Ω and $b=-17.75$ Ω.

P 10.4-11 Represent the circuit shown in Figure P 10.4-11 in the frequency domain using impedances and phasors.

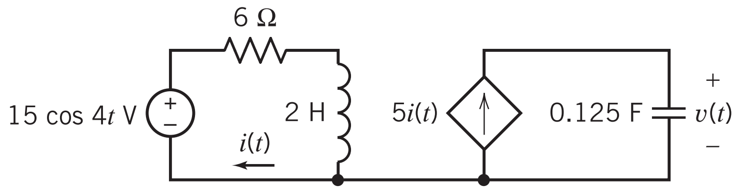
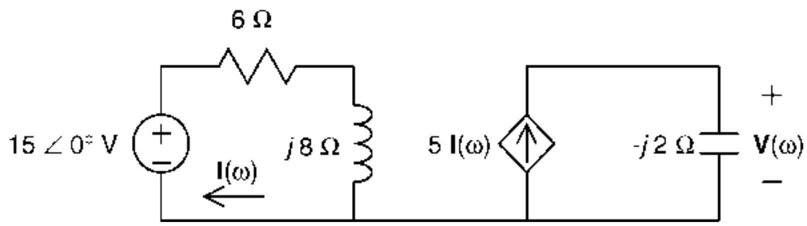


Figure P 10.4-11

Solution:



P 10.4-12 Represent the circuit shown in Figure P 10.4-12 in the frequency domain using impedances and phasors.

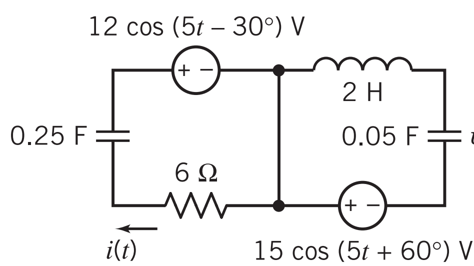
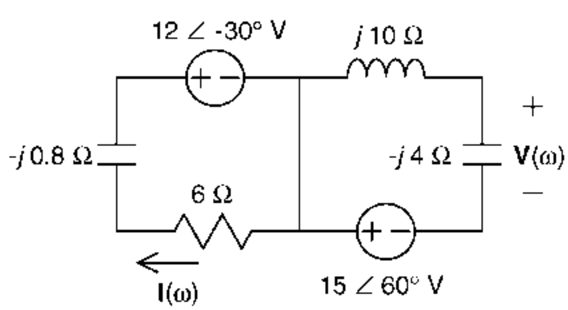


Figure P 10.4-12

Solution:



P 10.4-13 Find R and L of the circuit of Figure P 10.4-13 when $v(t) = 10 \cos(\omega t + 40°)$ V; $i(t) = 2 \cos(\omega t + 15°)$ mA, and $\omega = 2 \times 10^6$ rad/s.

Answer: $R = 4.532$ kω, $L = 1.057$ mH

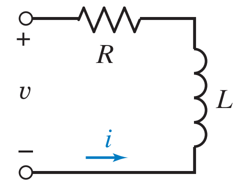


Figure P 10.4-13

Solution:

$$\mathbf{Z} = \frac{\mathbf{V}}{-\mathbf{I}} = \frac{10 \angle 40°}{-2\times 10^{-3} \angle -165°} = -5000\angle 205°\Omega = 4532 + j\,2113 = R + j\omega L$$

so $\underline{R = 4532\ \Omega}$ and $L = \dfrac{2113}{\omega} = \dfrac{2113}{2\times 10^6} = \underline{1.057 \text{ mH}}$

Section 10.5 Series and Parallel Impedances

P10.5-1 Determine the steady state voltage $v(t)$ in the circuit shown in Figure P10.5-1.

Answer: $v(t) = 32\cos(250t - 57.9°)$ V

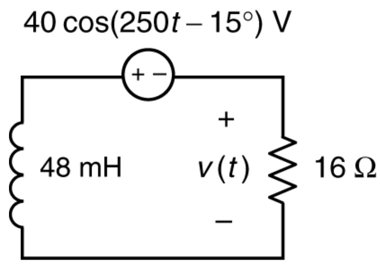
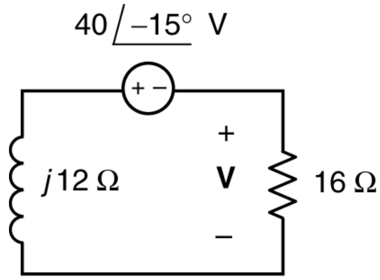


Figure P10.5-1

Solution: Represent the circuits in the frequency domain using phasors and impedances:



Using voltage division: $\mathbf{V} = -\frac{16}{16+j12}(40\angle -15°) = \frac{16\angle 180°}{20\angle 36.9°}(40\angle -15°) = 32\angle 128.1°$ V

In the time domain $v(t) = 32\cos(250t - 57.9°)$ V

P10.5-2 Determine the voltage $v(t)$ in the circuit shown in Figure P10.5-2.

Answer: $v(t) = 14.57\cos(800t + 111.7°)$ V

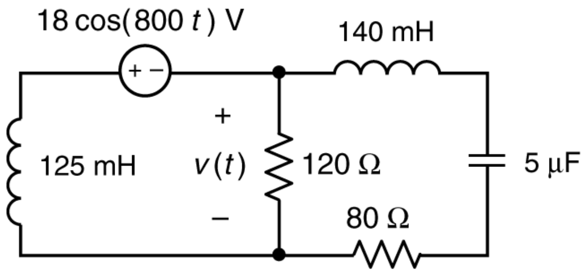
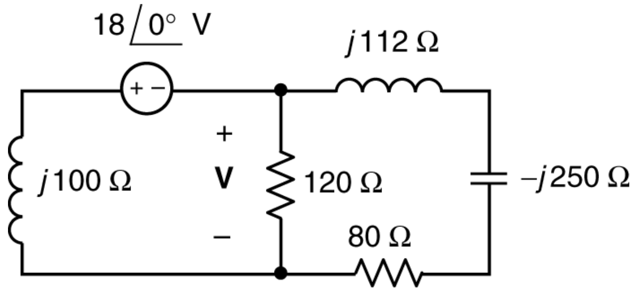


Figure P10.5-2

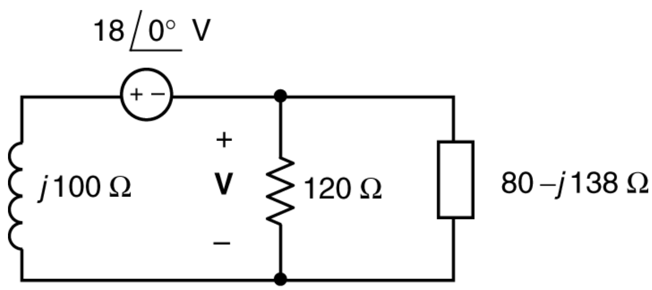
Solution:

Represent the circuit in the frequency domain:



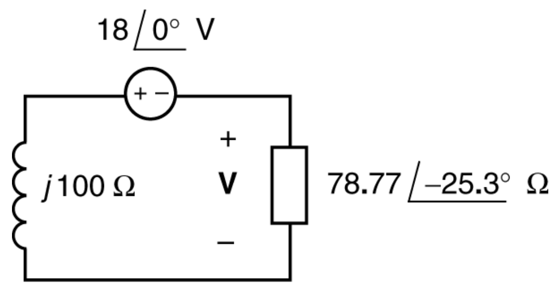
Replace the series impedances at the right of the circuit by an equivalent impedance

$$\mathbf{Z}_S = j112 + (-j250) + 80 = 80 - j138\ \Omega$$



Replace the parallel impedances at right of the circuit by an equivalent impedance

$$\mathbf{Z}_P = \frac{(80 - j138)120}{80 - j138 + 120} = \frac{(80 - j138)120}{200 - j138} = \frac{(159.51\angle -59.9)120}{242.99\angle -34.6} = 78.77\angle 25.3°\ \Omega$$



Using voltage division

$$\mathbf{V} = -\frac{78.77\angle -25.3°}{j100 + 78.77\angle -25.3°} 18\angle 0° = -\frac{78.77\angle -25.3°}{97.325\angle 42.97°} 18\angle 0° = 14.57\angle 111.73°\ \text{V}$$

In the time domain $\qquad v(t) = 14.57\cos(800t + 111.7°)$ V

(checked using LNAP)

P10.5-3 Determine the voltage *v*(*t*) in the circuit shown in Figure P10.5-3.

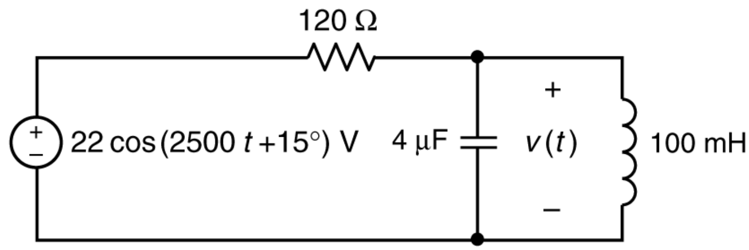
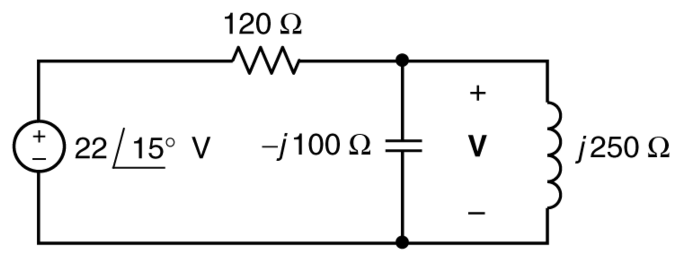


Figure P10.5-3

Answer: $v(t) = 14.1\cos(2500t - 35.2°)$ V

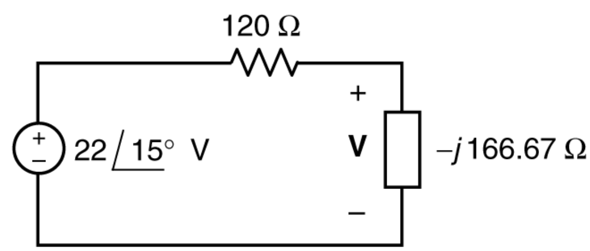
Solution:

Represent the circuit in the frequency domain:



Replace the parallel impedances at right of the circuit by an equivalent impedance:

$$\frac{(-j100)(j250)}{-j100 + j250} = -j166.67\ \Omega$$



Using voltage division

$$\mathbf{V} = -\frac{-j166.67}{200 - j166.67} 22\angle 15° = -\frac{166.67\angle -90°}{260.3\angle -39.8°} 22\angle 15° = 14.1\angle -35.2°\ \text{V}$$

In the time domain $v(t) = 14.1\cos(2500t - 35.2°)$ V

P10.5-4 The input to the circuit shown in Figure P10.5-4 is the current $i_s(t) = 48\cos(25t)$ mA. Determine the current $i_1(t)$.

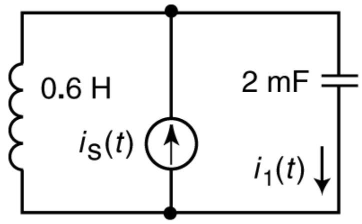
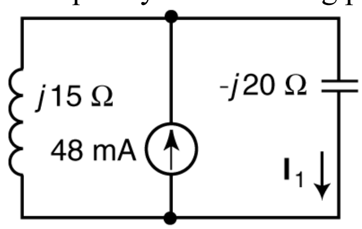


Figure P10.5-4

Answer: $i_1(t) = 144\cos(25t + 180°)$ mA

Solution: Represent the circuits in the frequency domain using phasors and impedances:



Using current division

$$\mathbf{I}_1 = \frac{j15}{j15 - j20}(48\angle 0°) = \frac{15}{-5}(48\angle 0°) = 144\angle 180° \text{ mA}$$

In the time domain $\qquad i_1(t) = 144\cos(25t + 180°)$ mA

P10.5-5 The input to the circuit shown in Figure P10.5-5 is the current $i_s(t) = 48\cos(25t)$ mA. Determine the current $i_2(t)$.

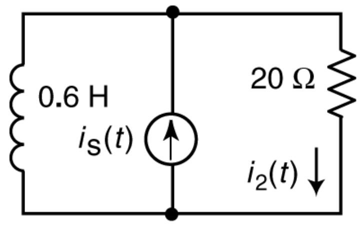
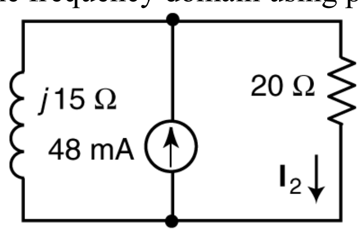


Figure P10.5-5

Solution: Represent the circuits in the frequency domain using phasors and impedances:



Using current division

$$\mathbf{I}_2(\omega) = \frac{j15}{20 + j15}(48\angle 0°) = \frac{15\angle 90°}{25\angle 36.9°}(48\angle 0°) = 28.8\angle 53.1° \text{ mA}$$

In the time domain $\qquad i_2(t) = 28.8\cos(25t + 53.1°)$ mA

P10.5-6 The input to the circuit shown in Figure P10.5-6 is the current $i_s(t) = 48\cos(25t)$ mA. Determine the current $i_3(t)$.

Answer: $i_3(t) = 16.85\cos(25t + 69.4°)$ mA

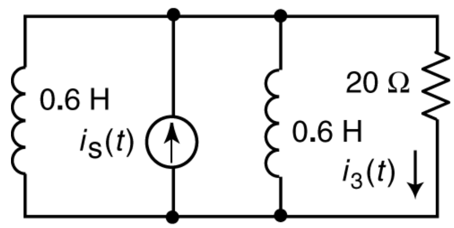
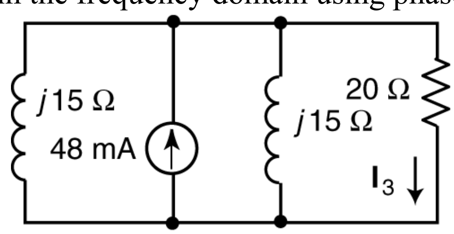


Figure P10.5-6

Solution: Represent the circuits in the frequency domain using phasors and impedances:



Using current division

$$\mathbf{I}_3 = \frac{j15 \,||\, j15}{(j15 \,||\, j15) + 20}(48\angle 0°) = \frac{j7.5}{j7.5 + 20}(48\angle 0°) = \frac{7.5\angle 90°}{21.36\angle 69.56°}(48\angle 0°) = 16.8539\angle 69.44° \text{ V}$$

In the time domain $\quad i_3(t) = 16.85\cos(25t + 69.4°)$ mA

P10.5-7 Figure P10.5-7 shows a circuit represented in the frequency domain. Determine the voltage phasor $\mathbf{V}_1$.

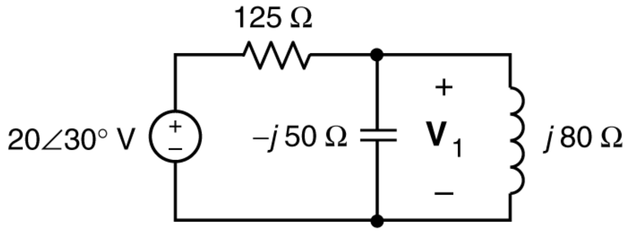


Figure P10.5-7

***Answer*:** $\mathbf{V}_1 = 14.59\angle -13.15°$ V

Solution:

$$\mathbf{V}_1 = \frac{j80 \,\|\, -j50}{125 + \left(j80 \,\|\, -j50\right)}\left(20\angle 30°\right) = \frac{-j133.33}{125 - j133.33}\left(20\angle 30°\right) = \frac{133.33\angle -90°}{182.8\angle -46.85°}\left(20\angle 30°\right)$$

$$= 14.59\angle -13.15° \text{ V}$$

P10.5-8 Figure P10.5-8 shows a circuit represented in the frequency domain. Determine the current phasor $\mathbf{I}_2$.

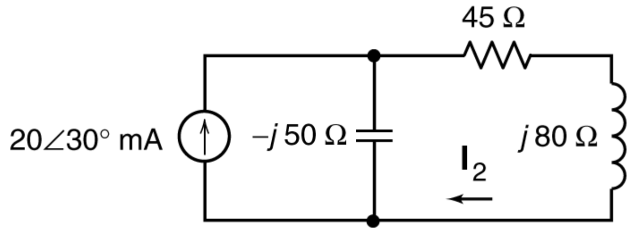


Figure P10.5-8

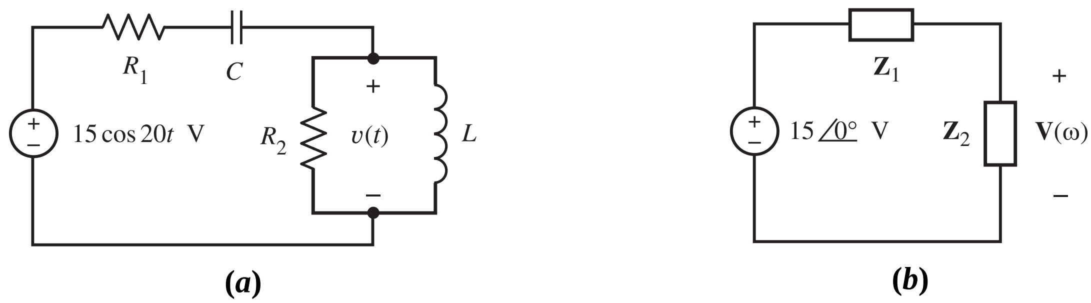
***Answer*:** $\mathbf{I}_2 = 18.48\angle -93.7°$ mA

Solution:

$$\mathbf{I}_2 = \frac{-j\,50}{-j\,50+45+j\,80}\left(20\angle 30°\right) = \frac{-j\,50}{45+j\,30}\left(20\angle 30°\right)$$

$$= \frac{50\angle -90}{54.08\angle 33.7}\left(20\angle 30°\right) = 18.49\angle -93.7° \text{ mA}$$

P10.5-9 Here's an ac circuit represented both in the time domain and frequency domain:



Suppose $\mathbf{Z}_1 = 15.3\angle -24.1°\ \Omega$ and $\mathbf{Z}_2 = 14.4\angle 53.1°\ \Omega$

Determine the node voltage $v(t)$ and the values of R_1, R_2, L and C.

Solution:

Consider $\mathbf{Z}_1$:

$$R_1 - j\frac{1}{20C} = 15.3\angle -24.1° = 14 - j6.25 \quad \Rightarrow \quad R_1 = 14\ \Omega \quad \text{and} \quad C = \frac{1}{20(6.25)} = 0.008\ \text{F} = 8\ \text{mF}$$

Next consider $\mathbf{Z}_2$:

$$\frac{1}{\dfrac{1}{R_2} + \dfrac{1}{j20L}} = 14.4\angle 53.1° \quad \Rightarrow \quad \frac{1}{R_2} + \frac{1}{j20L} = \frac{1}{14.4\angle 53.1°} = \frac{1}{14.4}\angle 53.1 = 0.05556 - j0.04167$$

Equating coefficients gives

$$R_2 = \frac{1}{0.05556} = 18\ \Omega \quad \text{and} \quad L = \frac{1}{20(0.04167)} = 1.2\ \text{H}$$

Next, consider the voltage divider:

$$A\angle 31.5° = \frac{14.4\angle 36.9°}{15.3\angle -24.1° + 14.4\angle 36.9°}(15\angle 0°) = \frac{(15)(14.4)\angle 36.9°}{(14 - j6.25)(11.52 + j8.64)}$$

$$= \frac{216\angle 36.9°}{25.52 + j2.39}$$

$$= \frac{216\angle 36.9°}{25.63\angle 5.4°} = 8.43\angle 31.5°\ \text{V}$$

In the time domain, $v(t) = 8.43\cos(20t + 31.5°)\ \text{V}$.

P 10.5-10 Find **Z** and **Y** for the circuit of Figure P 10.5-10 operating at 10 kHz.

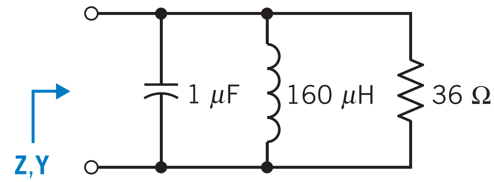


Figure P 10.5-10

Solution:

$$\omega = 2\pi f = 2\pi(10\times 10^3) \; = \; 62832 \;\; \text{rad/sec}$$

$$\mathbf{Z}_\text{R} \; = \; R \; = \; 36\;\Omega \quad \Leftrightarrow \quad \mathbf{Y}_\text{R} = \frac{1}{\mathbf{Z}_\text{R}} = \frac{1}{36} \; = \; 0.0278\text{ S}$$

$$\mathbf{Z}_\text{L} \; = j\omega L \; = j(62830)(160\times 10^{-6}) = j10.053 \; \approx \; j10\;\Omega \quad \Leftrightarrow \quad \mathbf{Y}_\text{L} \; = \; \frac{1}{\mathbf{Z}_\text{L}} \; = \; -0.1j \;\; \text{S}$$

$$\mathbf{Z}_\text{C} = \frac{-j}{\omega C} = \frac{-j}{(62830)(1\times 10^{-6})} = -j15.915 \approx -j\;16\;\Omega \quad \Leftrightarrow \quad \mathbf{Y}_\text{C} = \frac{1}{\mathbf{Z}_\text{C}} = \; j0.0625 \;\; \text{S}$$

$$\mathbf{Y}_\text{eq} = \mathbf{Y}_\text{R} + \mathbf{Y}_\text{L} + \mathbf{Y}_\text{C} = \; 0.0278 - \text{j}0.0375 = 0.0467 \;\angle -53.4° \;\; \text{S}$$

$$\mathbf{Z}_\text{eq} = \frac{1}{\mathbf{Y}_\text{eq}} = 21.43\angle 53.4° = \; \underline{12.75 \; + j17.22\;\Omega}$$

P 10.5-11 For the circuit of Figure P 10.5-11, find the value of C required so that $\mathbf{Z} = 590.7\Omega$ when $f = 1$ MHz.

Answer: $C = 0.27$ nF

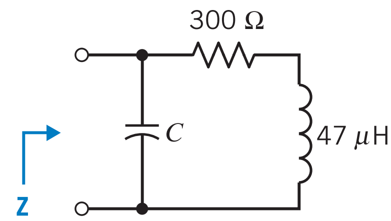


Figure P 10.5-11

Solution:

$$\mathbf{Z}_{\mathrm{L}} = j\omega L = j\ (6.28\times10^{6})(47\times10^{-6}) = j\ 295\ \Omega$$

$$\mathbf{Z}_{\mathrm{eq}} = \mathbf{Z}_{\mathrm{c}} \parallel (\mathbf{Z}_{\mathrm{R}} + \mathbf{Z}_{\mathrm{L}}) = \frac{\left(\dfrac{1}{j\omega C}\right)(300+j295)}{\dfrac{1}{j\omega C}+300+j295} = 590.7\ \Omega$$

$$590.7 = \frac{300+300j}{1+300j\omega C-300\omega C} \Rightarrow 590.7-(590.7)(295\omega C)+j(590.7)(300\omega C) = 300+j295$$

Equating imaginary terms $\left(\omega = 2\pi f = 6.28\times10^{6}\ \text{rad/sec}\right)$

$$(590.7)(300\omega C) = 295 \Rightarrow \underline{C = 0.27\ \text{nF}}$$

P 10.5-12 Determine the impedance **Z** for the circuit shown in Figure P 10.5-12.

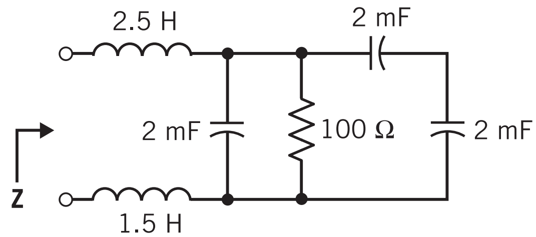
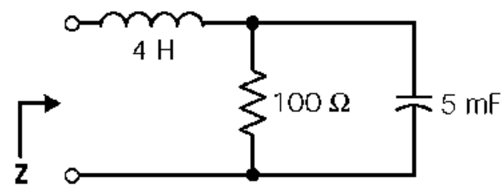


Figure P 10.5-12

Solution:

Replace series and parallel capacitors by an equivalent capacitor and series inductors by an equivalent inductor:



Then

$$\mathbf{Z} = j\omega 4 + \frac{100 \dfrac{1}{j\omega\left(5\times10^{-3}\right)}}{100 + \dfrac{1}{j\omega\left(5\times10^{-3}\right)}} = j\omega 4 + \frac{100\left(-j\dfrac{200}{\omega}\right)}{100 + \left(-j\dfrac{200}{\omega}\right)} = j\omega 4 + \frac{-j\dfrac{200}{\omega}}{1 - j\dfrac{2}{\omega}} \times \frac{1 + j\dfrac{2}{\omega}}{1 + j\dfrac{2}{\omega}}$$

$$\mathbf{Z} = j\omega 4 + 100\,\frac{\dfrac{4}{\omega^2} - j\dfrac{2}{\omega}}{1 + \dfrac{4}{\omega^2}} = j\omega 4 + 100\,\frac{4 - j2\omega}{4+\omega^2} = \frac{400}{4+\omega^2} + j\left(4\,\omega - \frac{200\,\omega}{4+\omega^2}\right)$$

P 10.5-13 The big toy from the hit movie *Big* is a child's musical fantasy come true—a sidewalk-sized piano. Like a hopscotch grid, this once-hot Christmas toy invites anyone who passes to jump on, move about, and make music. The developer of the "toy" piano used a tone synthesizer and stereo speakers as shown in Figure P 10.5-13 (Gardner, 1988). Determine the current $i(t)$ for a tone at 796 Hz when $C = 10\ \mu$F.

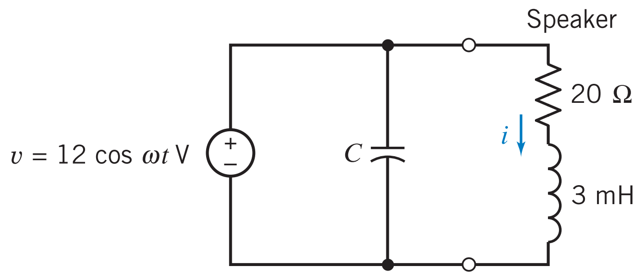
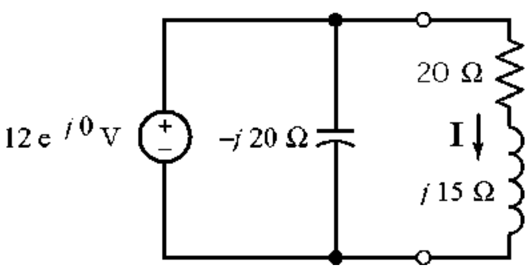


Figure P 10.5-13

Solution:



$$j(2\pi \cdot 796)(3 \cdot 10^{-3}) = j15\ \Omega$$

$$\mathbf{I} = \frac{12}{20 + j15} = 0.48\ \angle -37°\ \text{A}$$

$$i(t) = 0.48 \cos(2\pi \cdot 796 t - 37°)\ \text{A}$$

P 10.5-14 Determine $i(t)$, $v(t)$, and L for the circuit shown in Figure P 10.5-14.

Answer: $i(t) = 1.34 \cos(2t - 87°)$ A, $v(t) = 7.29 \cos(2t - 24°)$ V, and $L = 4$ H

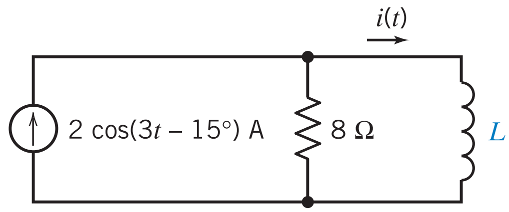


Figure P 10.5-14

Solution:

$$\mathbf{Z}_1 = R = 8\ \Omega,\quad \mathbf{Z}_2 = j3L,\quad \mathbf{I} = B\angle -51.87° \text{ and } \mathbf{I}_s = 2\angle -15°\text{ A}$$

$$\frac{\mathbf{I}}{\mathbf{I}_s} = \frac{B\angle -51.87°}{2\angle -15°} = \frac{\mathbf{Z}_1}{\mathbf{Z}_1+\mathbf{Z}_2} = \frac{8}{8+j3L} = \frac{8\angle 0°}{\sqrt{8^2+(3L)^2}\ \angle \tan^{-1}\left(\frac{3L}{8}\right)}$$

Equate the magnitudes and the angles.

$$\text{angles: } +36.87 = +\tan^{-1}\left(\frac{3L}{8}\right) \Rightarrow \underline{L=2\text{ H}}$$

$$\text{magnitudes: } \frac{8}{\sqrt{64+9L^2}} = \frac{B}{2} \Rightarrow \underline{B=1.6}$$

P 10.5-15 Spinal cord injuries result in paralysis of the lower body and can cause loss of bladder control. Numerous electrical devices have been proposed to replace the normal nerve pathway stimulus for bladder control. Figure P 10.5-15 shows the model of a bladder control system where $v_s = 20\cos\omega t$ V and $\omega = 100$ rad/s. Find the steady-state voltage across the 10-Ω load resistor.

Answer: $v(t) = 10\sqrt{2}\cos(100t + 45°)$ V

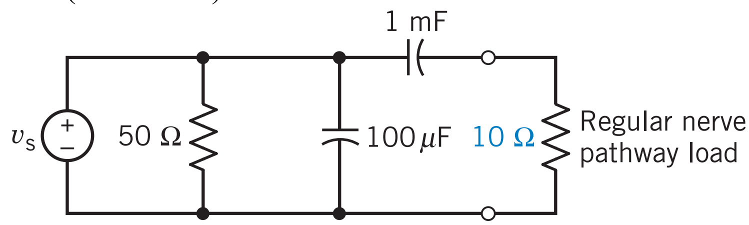
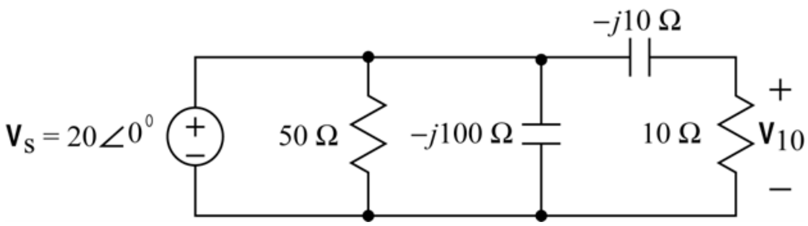


Figure P 10.5-15

Solution:



$$\mathbf{V}_{10} = \mathbf{V}_s\left(\frac{10}{10 - j10}\right)$$

$$= 20\angle 0°\left(\frac{10}{10\sqrt{2}\angle -45°}\right)$$

$$= 10\sqrt{2}\angle 45°$$

$$v_{10}(t) = 10\sqrt{2}\cos(100t + 45°)\ \text{V}$$

P 10.5-16 There are 500 to 1000 deaths each year in the United States from electric shock. If a person makes a good contact with his hands, the circuit can be represented by Figure P 10.5-16, where v_s = 160 cos ωt V and $\omega = 2\pi f$. Find the steady-state current i flowing through the body when (a) f = 60 Hz and (b) f = 400 Hz.

Answer: (a) $i(t) = 0.53 \cos(120\pi t + 5.9°)$

(b) $i(t) = 0.625 \cos(800\pi t + 59.9°)$ A

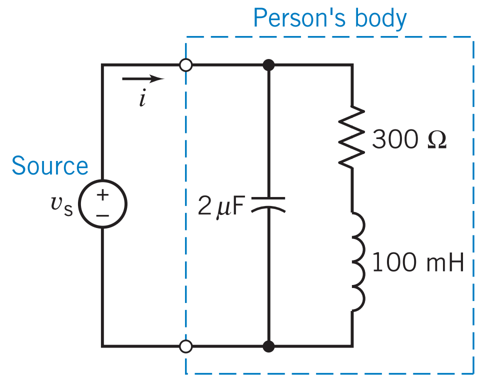
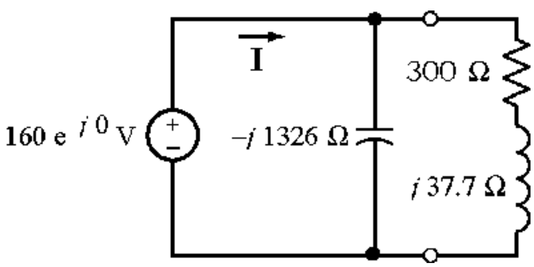


Figure P 10.5-16

Solution:

(a)

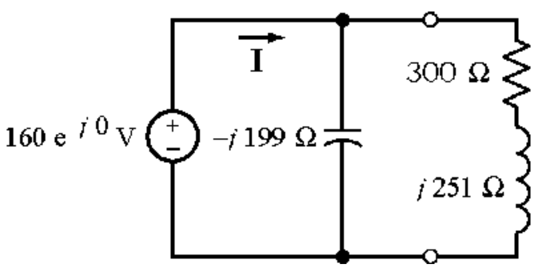


$$\mathbf{I} = \frac{160\angle 0°}{\dfrac{(-j1326)\,(300+j37.7)}{-j1326 + 300 + j\,37.7}} = \frac{160\angle 0°}{303\angle -5.9°}$$

$$= 0.53\angle 5.9° \text{ A}$$

$\underline{i(t) = 0.53\cos(120\pi t + 5.9°) \text{ A}}$

(b)

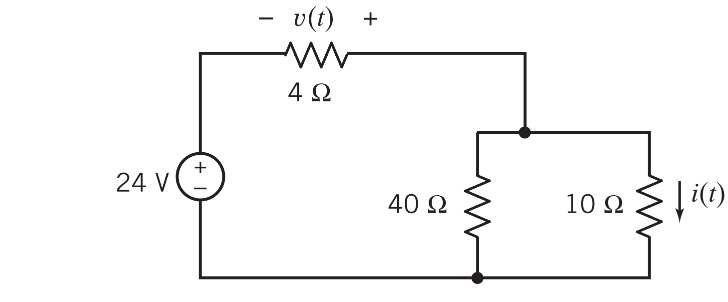


$$\mathbf{I} = \frac{160\angle 0°}{\dfrac{(-j199)(300+j251)}{-j199+300+j251}} = \frac{160\angle 0°}{256\angle -59.9°}$$

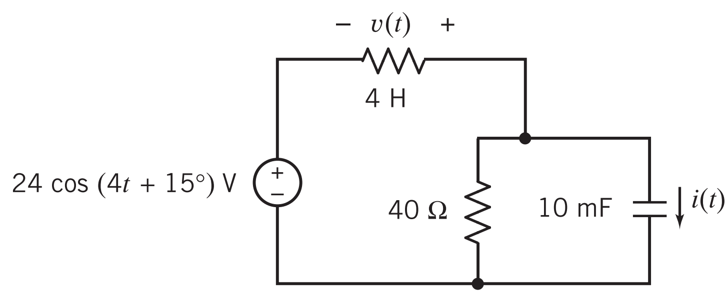
$$= 0.625\angle 59.9° \text{ A}$$

$\underline{i(t) = 0.625\ \cos\ (800\pi t + 59.9°) \text{ A}}$

P 10.5-17 Determine the steady-state voltage, $v(t)$, and current, $i(t)$, for each of the circuits shown in Figure P 10.5-17.



(a)



(b)

Figure P 10.5-17

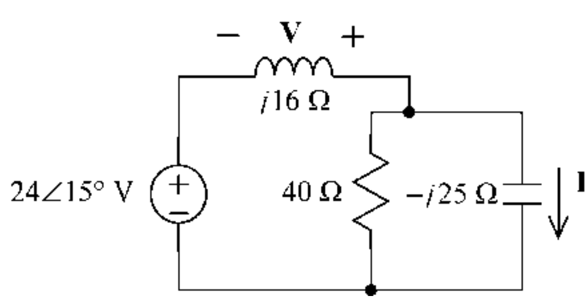
Solution:

(a)

$$v(t) = -\frac{4}{4+(40\,||\,10)} \times 24 = -8 \text{ V}$$

$$i(t) = \frac{40}{40+10} \times \frac{24}{4+(40\,||\,10)} = \frac{8}{5} = 1.6 \text{ A}$$

(b) Represent the circuit in the frequency domain using impedances and phasors.



$$\mathbf{V} = -\frac{j16}{j16+\left(40 \,\|\, -j25\right)} \times 24\angle 15° = \frac{\left(16\angle -90°\right)\left(24\angle 15°\right)}{j16+\dfrac{40\left(-j25\right)}{40-j25}} = 33.66\angle -65° \text{ V}$$

$$\mathbf{I} = \frac{40}{40-j25} \times \frac{24\angle 15°}{j16+\dfrac{40\left(-j25\right)}{40-j25}} = 1.78\angle 57° \text{ A}$$

so

$$v(t) = 33.66\cos\left(4t-65°\right) \text{ V}$$

and

$$i(t) = 1.78\cos\left(4t+57°\right) \text{ A}$$

(checked: LNAP 8/1/04)

P 10.5-18 Determine the steady-state current, $i(t)$, for the circuit shown in Figure P 10.5-18.

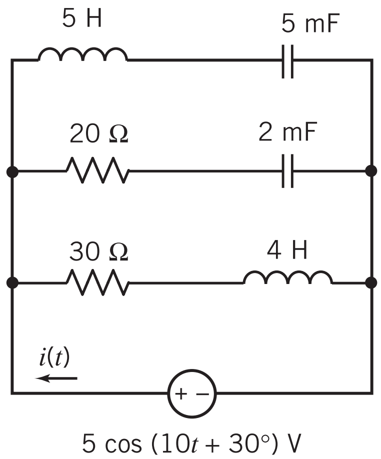


Figure P 10.5-18

Solution:

$$\mathbf{I} = \frac{5\angle 30^\circ}{30 + j40} + \frac{5\angle 30^\circ}{20 - j50} + \frac{5\angle 30^\circ}{j50 - j20} = 0.100\angle -23.1^\circ + 0.0923\angle 98.2^\circ + 0.1667\angle -60^\circ$$

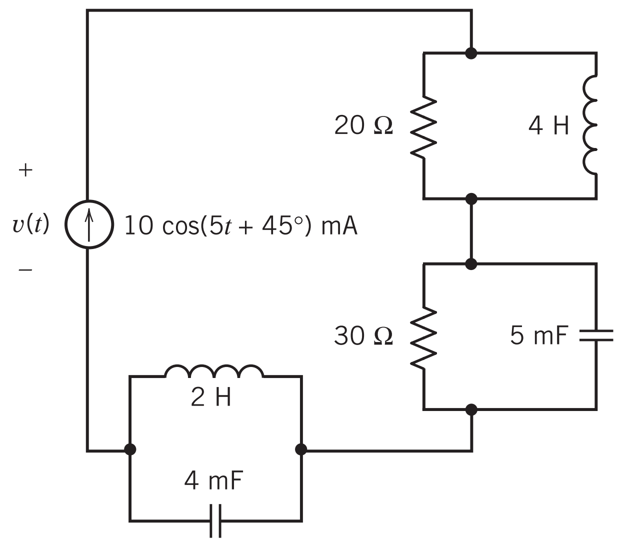
$$= 0.186\angle -29.5^\circ \text{ A}$$

so

$$i(t) = 0.186\cos(10t - 29.5^\circ) \text{ A}$$

(checked: LNAP 8/1/04)

P10.5-19 Determine the steady state voltage, $v(t)$, for this circuit:



Solution:

$$\mathbf{V} = 0.01\angle 45° \left[\left(20 \parallel j20\right) + \left(30 \parallel (-j40)\right) + \left(j10 \parallel (-j50)\right) \right]$$

$$= 0.01\angle 45° \left[\frac{20(j20)}{20+j20} + \frac{30(-j40)}{30-j40} + \frac{j10(-j50)}{j10-j50} \right]$$

$$= 0.01\angle 45° \left[14.14\angle 45° + 24\angle -36.9° + 12.5\angle 90° \right]$$

$$= 0.01\angle 45° \left[10 + j10 + 19.2 - j14.4 + j12.5 \right]$$

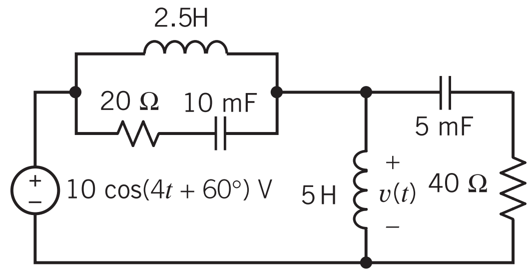
$$= 0.303\angle 60.5° \text{ V}$$

so

$$v(t) = 0.303\cos(5t + 60.5°) \text{ V}$$

(checked: LNAP 8/1/04)

P10.5-20 Determine the steady state voltage, $v(t)$, for this circuit:



Solution:
Let

$$\mathbf{Z}_1 = \left(20 - j\frac{1}{4(0.01)}\right) \| \; j10 = \frac{(20 - j25)\, j10}{20 - j25 + j10} = \frac{250 - j200}{20 - j15} = 12.81\angle 75.5°\ \Omega$$

and

$$\mathbf{Z}_2 = j20 \,\|\left(-j\frac{1}{4(0.005)} + 40\right) = \frac{j20(40 - j50)}{j20 + 40 - j50} = \frac{1000 + j800}{40 - j30} = 25.61\angle 75.5°\ \Omega$$

Then

$$\mathbf{V} = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} \times 10\angle 60° = \frac{25.61\angle 75.5°}{12.81\angle 75.5° + 25.6\angle 75.5°} \times 10\angle 60° = 6.67\angle 60°\ \text{V}$$

so

$$v(t) = 6.67\cos(4t + 60°)\ \text{V}$$

(checked: LNAP 8/1/04)

P 10.5-21 The input to the circuit shown in Figure P 10.5-21 is the current source current

$$i_s(t) = 25\cos(10t + 15°)\text{ mA}$$

The output is the current $i_1(t)$. Determine the steady-state response, $i_1(t)$.

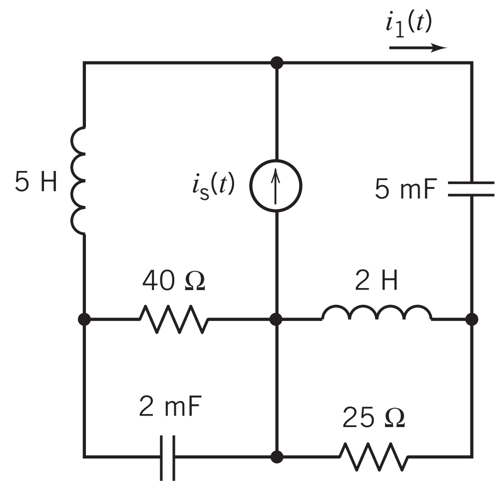


Figure P 10.5-21

Solution:

Represent the circuit in the frequency domain using impedances and phasors. Let

$$\mathbf{Z}_1 = j50 + \left(40 \,||\, \frac{1}{j10\times 2\times 10^{-3}}\right) = j50 + \frac{40(-j50)}{40-j50} = 39.0\angle 51.3°\ \Omega$$

and

$$\mathbf{Z}_2 = -j\frac{1}{10\left(5\times 10^{-3}\right)} + j20 \,||\, 25 = -j20 + \frac{j20(25)}{25+j20} = 12.5\angle -38.7\ \Omega$$

$\mathbf{Z}_1$ and $\mathbf{Z}_2$ are connected in parallel. Current division gives

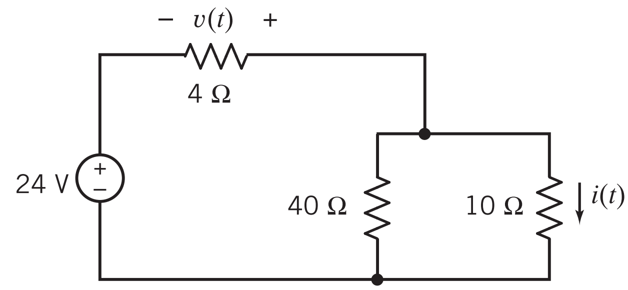
$$\mathbf{I}_1 = \frac{\mathbf{Z}_1}{\mathbf{Z}_1+\mathbf{Z}_2}\times 0.025\angle 15° = 0.024\angle 32.7°\text{ A}$$

so

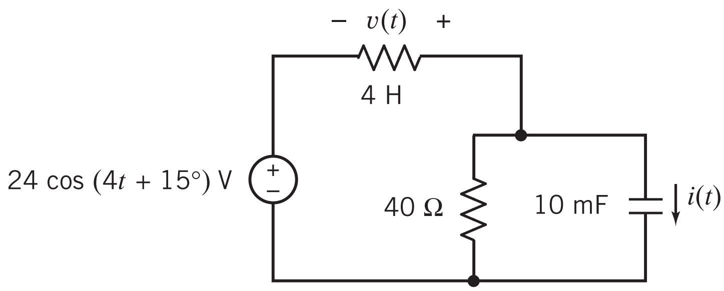
$$i_1(t) = 0.024\cos\left(10t + 32.7°\right)\text{ A}$$

(checked: LNAP 8/1/04)

P10.5-22 Determine the steady state voltage, $v(t)$, and current $i(t)$ for each of these circuits:



(a)



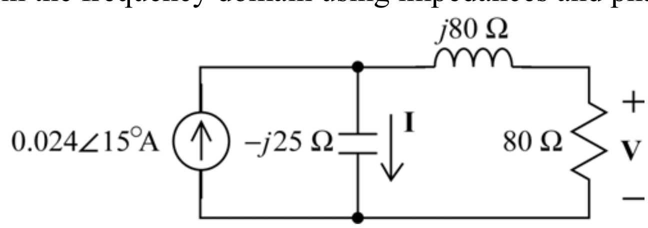
(b)

Solution:

(a)
$$i(t) = \frac{80+80}{40+(80+80)} 0.024 = 19.2 \text{ mA}$$

$$v(t) = \frac{80}{80+80} \times \left(40 \,||\, (80+80)\right) 0.024 = \frac{1}{2}(32)(0.024) = 0.384 \text{ V}$$

(b) Represent the circuit in the frequency domain using impedances and phasors.



$$\mathbf{I} = \frac{80+j80}{-j25+(80+j80)} \times 0.024\angle 15° = 0.028\angle 25.5° \text{ A}$$

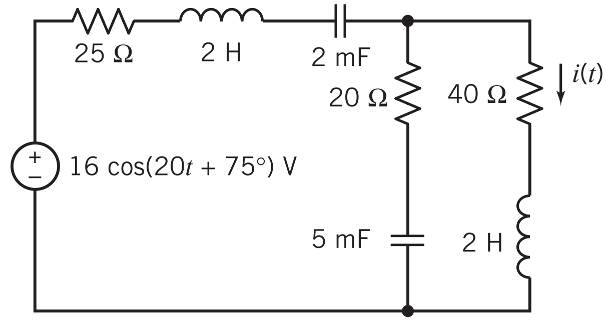
$$\mathbf{V} = \frac{80}{80+j80} \times \left[-j25 \,||\, (80+j80)\right] \times 0.024\angle 15° = 0.494\angle -109.5° \text{ V}$$

So
$$i(t) = 28\cos(10t + 25.5°) \text{ mA}$$

and
$$v(t) = 0.494\cos(10t - 109.5°) \text{ V}$$

(checked: LNAP 8/1/04)

P10.5-23 Determine the steady state current $i(t)$ for this circuit:



Solution:
Represent the circuit in the frequency domain using impedances and phasors. Let

$$\mathbf{Z}_1 = 25 + j(20)2 + \frac{1}{j(20)(0.002)} = 25 + j15 = 29.2\angle 31°\ \Omega$$

$$\mathbf{Z}_2 = 20 + \frac{1}{j(20)(0.005)} = 20 - j10 = 22.36\angle -26.6°\ \Omega$$

$$\mathbf{Z}_3 = 40 + j(20)2 = 40 + j40 = 56.57\angle 45°\ \Omega$$

and let

$$\mathbf{Z}_\text{p} = \mathbf{Z}_2 \parallel \mathbf{Z}_3 = 18.86\angle -8° = 18.67 - j2.67\ \Omega$$

Then

$$\mathbf{I} = \frac{16\angle 75°}{\mathbf{Z}_1 + \mathbf{Z}_\text{p}} \times \frac{\mathbf{Z}_2}{\mathbf{Z}_2 + \mathbf{Z}_3} = 0.118\angle 6.1°\ \text{A}$$

so

$$i(t) = 0.118\cos(20t + 6.1°)\ \text{A}$$

(checked: LNAP 8/2/04)

P 10.5-24 When the switch in the circuit shown in Figure P 10.5-24 is open and the circuit is at steady state, the capacitor voltage is

$$v(t) = 14.14 \cos(100t - 45°) \text{ V}$$

When the switch is closed and the circuit is at steady state, the capacitor voltage is

$$v(t) = 17.89 \cos(100t - 26.6°) \text{ V}$$

Determine the values of the resistances R_1 and R_2.

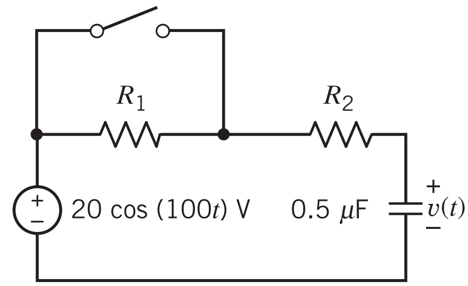


Figure P 10.5-24

Solution:

Represent the circuit in the frequency domain using phasors and impedances. The impedance capacitor is $\frac{1}{j(100)(0.5\times10^{-6})} = -j20{,}000$. When the switch is closed

$$17.89\angle -26.6° = \mathbf{V} = \frac{-j20{,}000}{R_2 - j20{,}000} \times 20\angle 0°$$

Equating angels gives

$$-26.6° = -90° - \tan^{-1}\left(\frac{-20{,}000}{R_2}\right) \quad \Rightarrow \quad R_2 = \frac{-20{,}000}{\tan(-63.4)} = 10015\ \Omega$$

When the switch is open

$$14.14\angle -45° = \mathbf{V} = \frac{-j20{,}000}{R_1 + R_2 - j20{,}000} \times 20\angle 0°$$

Equating angles gives

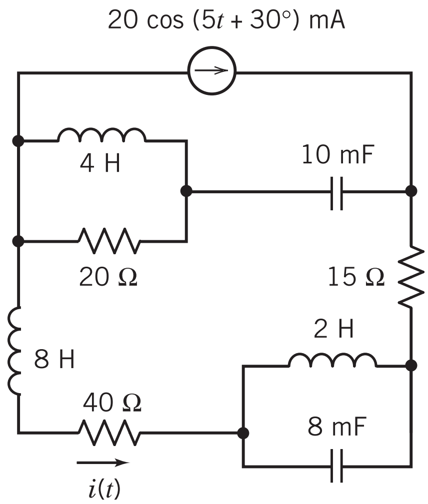
$$-45° = -90° - \tan^{-1}\left(\frac{-20{,}000}{R_1 + R_2}\right) \quad \Rightarrow \quad R_1 + R_2 = \frac{-20{,}000}{\tan(-45°)} = 20{,}000$$

So

$$R_1 = 20{,}000 - 10015 = 9985\ \Omega$$

(checked: LNAP 8/2/04)

P10.5-25 Determine the steady state current $i(t)$ for this circuit:



Solution:
Represent the circuit in the frequency domain using phasors and impedances. Let

$$\mathbf{Z}_1 = (j20 \,\|\, 20) + \frac{1}{j0.05} = 10 - j10 = 14.14\angle -45° \ \Omega$$

$$\mathbf{Z}_2 = j40 + 40 + \left(j10 \,\|\, \frac{1}{j0.04} \right) + 15 = 55 + j56.67 = 79\angle 46.3° \ \Omega$$

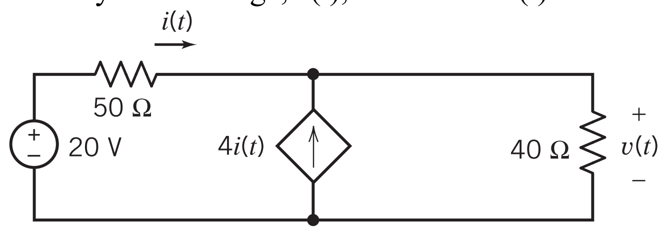
$$\mathbf{I} = -\frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2} \times 20\angle 30° = 3.535\angle 129.3° \text{ mA}$$

so

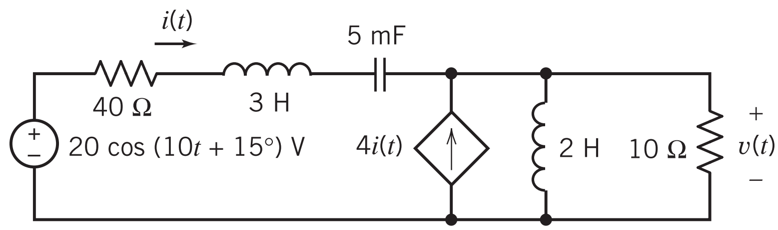
$$i(t) = 3.535\cos(5t + 129.3°) \text{ mA}$$

(checked: LNAP 8/2/04)

P10.5-26 Determine the steady state voltage, $v(t)$, and current $i(t)$ for each of these circuits:



(a)



(b)

Solution:

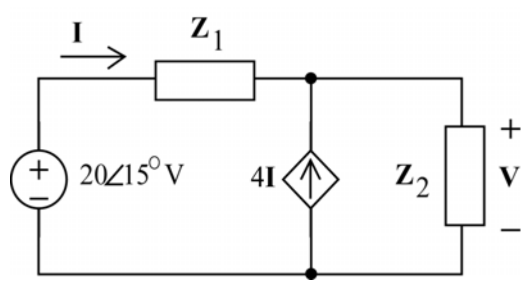
(*a*) Using KCL and then KVL gives

$$20 = 50\,i(t) + 40\big(5\,i(t)\big) \quad \Rightarrow \quad i(t) = \frac{20}{250} = 80 \text{ mA}$$

Then

$$v(t) = 40\big(5i(t)\big) = 200(0.08) = 16 \text{ V}$$

(*b*) Represent the circuit in the frequency domain using phasors and impedances.



Where

$$\mathbf{Z}_1 = 40 + j(10)3 + \frac{1}{j(10)(0.005)} = 40 + j10 = 41.23\angle 26.6° \ \Omega$$

And

$$\mathbf{Z}_2 = j(10)2 \,||\, 10 = 8 + j4 = 8.944\angle 26.6° \ \Omega$$

Using KCL and then KVL gives

$$20\angle 15° = \mathbf{Z}_1\mathbf{I} + 5\mathbf{Z}_2\mathbf{I} \quad \Rightarrow \quad \mathbf{I} = 0.234\angle -5.6° \text{ A}$$

Then

$$\mathbf{V} = \mathbf{Z}_2(5\mathbf{I}) = 10.47\angle 21° \text{ A}$$

so

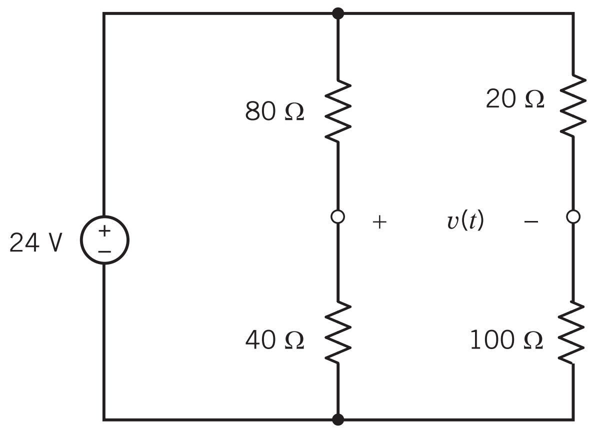
$$i(t) = 0.234\cos(10t - 5.6°) \text{ A}$$

and

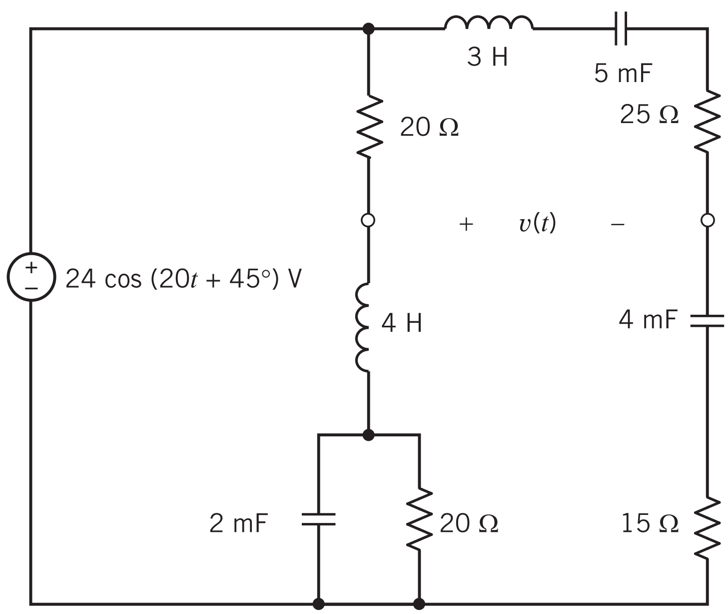
$$v(t) = 10.47\cos(10t + 21°) \text{ V}$$

(checked: 8/3/04)

P10.5-27 Determine the steady state voltage, $v(t)$, for each of these circuits:



(a)



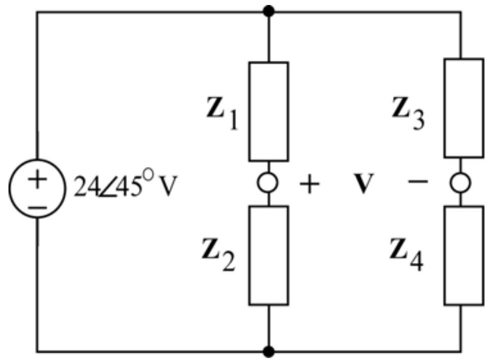
(b)

Solution:

(a) Using voltage division twice

$$v(t) = \frac{40}{40+80} \times 24 - \frac{100}{20+100} \times 24 = -12 \text{ V}$$

(b) Represent the circuit in the frequency domain using phasors and impedances.



Where

$$\mathbf{Z}_1 = 20\ \Omega$$

$$\mathbf{Z}_2 = j(20)4 + \left(\frac{1}{j(20)(0.002)} \parallel 20\right) = 12.2 + j70.2 = 71.30\angle 80.2°\ \Omega$$

$$\mathbf{Z}_3 = j(20)3 + \frac{1}{j(20)(0.005)} + 25 = 25 + j50 = 55.90\angle 63.4°\ \Omega$$

$$\mathbf{Z}_4 = \frac{1}{j(20)(0.004)} + 15 = 15 - j12.5 = 19.53\angle -39.8°\ \Omega$$

Using voltage division twice

$$\mathbf{V} = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} \times 24\angle 45° - \frac{\mathbf{Z}_4}{\mathbf{Z}_3 + \mathbf{Z}_4} \times 24\angle 45° = 24.8\angle 80°\ \text{V}$$

so

$$v(t) = 24.8\cos(20t + 80°)\ \text{V}$$

(Checked using LNAP 10/5/04)

P 10.5-28 The input to the circuit shown in Figure P 10.5-28 is the voltage of the voltage source

$$v_s(t) = 5\cos(2t + 45°)\text{ V}$$

The output is the inductor voltage, $v(t)$. Determine the steady-state output voltage.

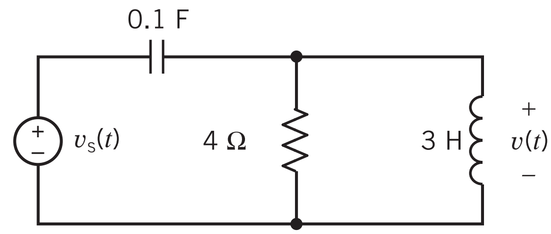
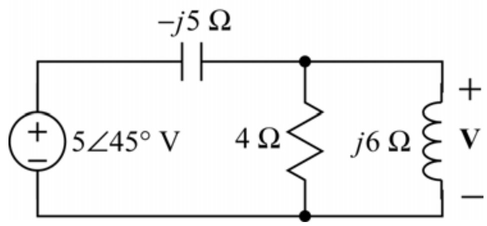


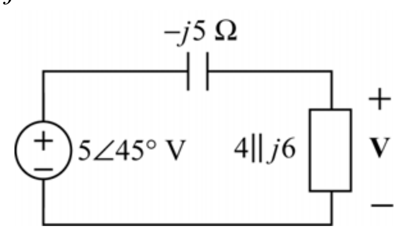
Figure P 10.5-28

Solution:

Represent the circuit in the frequency domain using phasors and impedances.



$$4 \,||\, j6 = \frac{4(j6)}{4+j6} = \frac{24\angle 90°}{7.2\angle 56°} = 3.33\angle 34° = 2.76 + j1.86\ \Omega$$



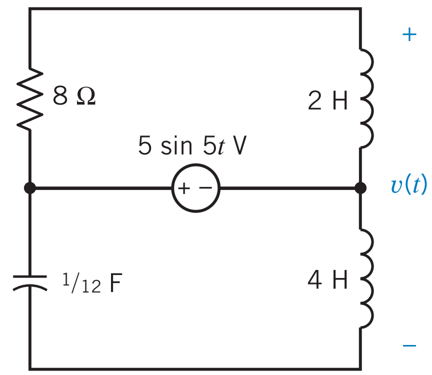
Using voltage division

$$\mathbf{V} = \frac{3.33\angle 34°}{-j5 + 2.76 + j1.86} \times 5\angle 45° = \frac{3.33\angle 34°}{2.76 - j3.14} \times 5\angle 45° = \frac{3.33\angle 34°}{4.18\angle -48°} \times 5\angle 45° = 3.98\angle 127°\text{ V}$$

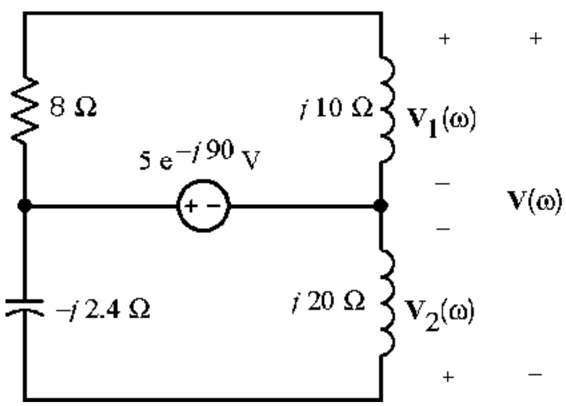
The corresponding voltage in the time domain is

$$v(t) = 3.98\cos(2t + 127°)\text{ V}$$

P10.5-29 Determine the steady state voltage, $v(t)$, for this circuit:



Solution:



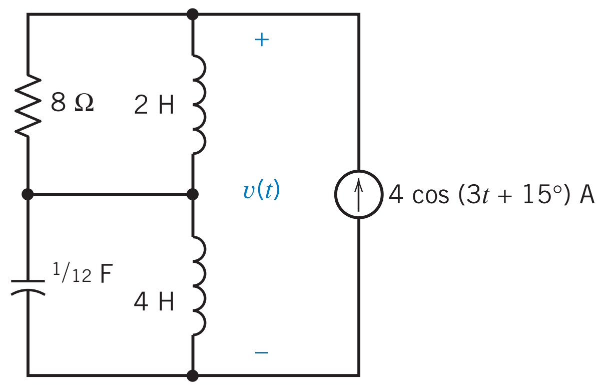
$$\mathbf{V}_1(\omega) = \frac{j10}{8+j10} 5e^{-j90} = 3.9e^{-j51} \text{ V}$$

$$\mathbf{V}_2(\omega) = \frac{j20}{j20-j2.4} 5e^{-j90} = 5.68e^{-j90} \text{ V}$$

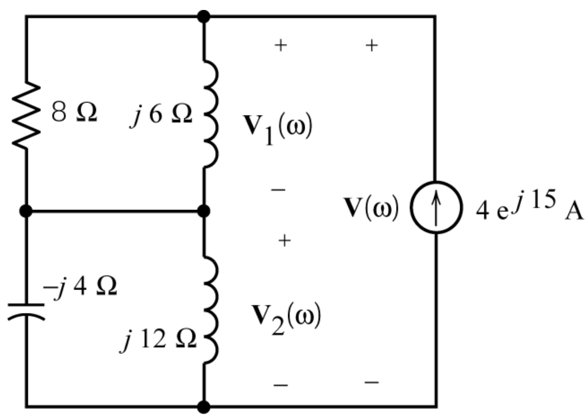
$$\mathbf{V}(\omega) = \mathbf{V}_1(\omega) - \mathbf{V}_2(\omega) = 3.9e^{-j51} - 5.68e^{-j90}$$
$$= 3.58e^{j47} \text{ V}$$

Answer: $v(t) = 3.58\cos(5t + 47.2°)$ V

P10.5-30 Determine the steady state voltage, $v(t)$, for this circuit:



Solution:



$$\mathbf{V}_1(\omega) = \frac{8(j6)}{8+j6} 4\, e^{j15} = 19.2\, e^{j68} \text{ V}$$

$$\mathbf{V}_2(\omega) = \frac{j12(-j4)}{j12-j4} 4\, e^{j15} = 24\, e^{-j75} \text{ V}$$

$$\mathbf{V}(\omega) = \mathbf{V}_1(\omega) + \mathbf{V}_2(\omega) = 14.4\, e^{-j22} \text{ V}$$

Answer: $v(t) = 14.4\cos(3t - 22°)$ V

P10.5-31

The input to the circuit in Figure P10.5-31 is the voltage source voltage, $v_s(t)$. The output is the voltage $v_o(t)$. When the input is $v_s(t) = 8\cos(40t)$ V, the output is $v_o(t) = 2.5\cos(40t + 14°)$ V. Determine the values of the resistances R_1 and R_2.

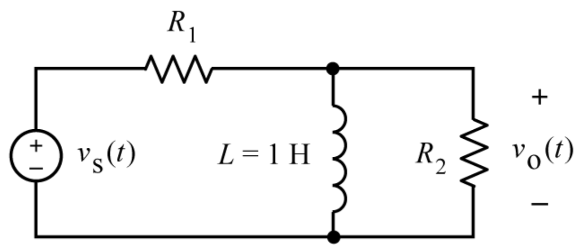
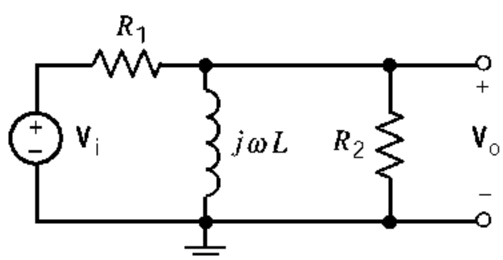


Figure P10.5-31

Solution:



Using voltage division in the frequency domain:

$$\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{\dfrac{j\omega L R_2}{R_2 + j\omega L}}{R_1 + \dfrac{j\omega L R_2}{R_2 + j\omega L}} = \frac{j\omega \dfrac{L}{R_1}}{1 + j\omega \dfrac{L}{R_p}}$$

where $R_p = \dfrac{R_1 R_2}{R_1 + R_2}$

Representing the given input and output in the frequency domain:

$$\frac{2.5\angle 14°}{8\angle 0°} = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{\omega \dfrac{L}{R_1}}{\sqrt{1 + \left(\omega \dfrac{L}{R_p}\right)^2}}\, e^{j\left(90 - \tan^{-1}\omega \frac{L}{R_p}\right)}$$

In this case the angle of $\dfrac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$ is specified to be 14° so $\dfrac{L}{R_p} = \dfrac{L(R_1 + R_2)}{R_1 R_2} = \dfrac{\tan(90° - 14°)}{40} = 0.1$

and the magnitude of $\dfrac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$ is specified to be $\dfrac{2.5}{8}$ so $\dfrac{40\dfrac{L}{R_1}}{\sqrt{1+16}} = \dfrac{2.5}{8} \quad \Rightarrow \quad \dfrac{L}{R_1} = 0.0322$. One set of values that satisfies these two equations is $L = 1\text{ H},\ R_1 = 31\,\Omega,\ R_2 = 14.76\,\Omega$.

Section 10.6 Mesh and Node Equations

P10.6-1 The input to the circuit shown in Figure P10.6-1 is the voltage

$$v_s(t) = 48\cos(2500t + 45°) \text{ V}$$

Write and solve node equations to determine the steady state output voltage $v_o(t)$.

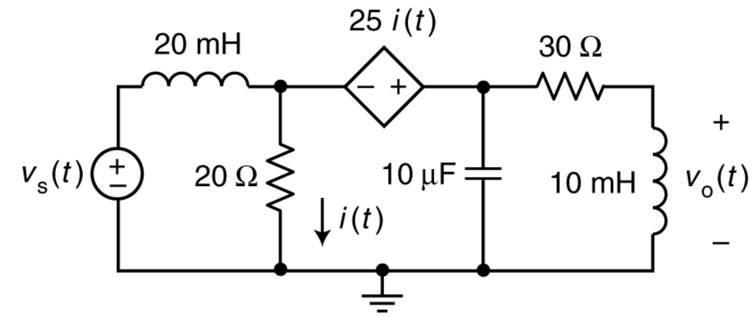
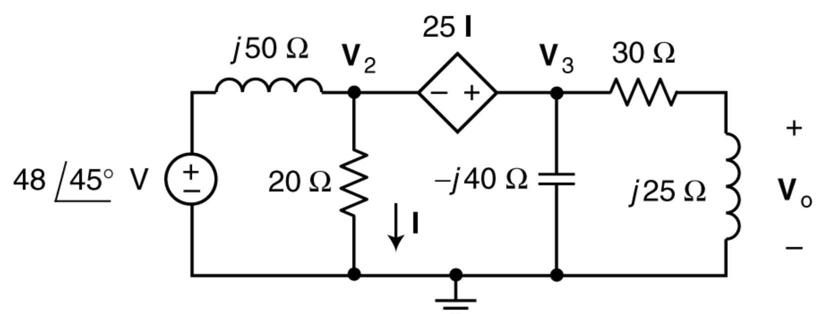


Figure P10.6-1

Solution: Represent the circuit in the frequency domain as



The node voltages are $48\angle 45° = \mathbf{V}_1$, $\mathbf{V}_2$, $\mathbf{V}_3$ and $\mathbf{V}_o$. Express the dependent source voltage in terms of the node voltages:

$$\mathbf{V}_3 - \mathbf{V}_2 = 25\mathbf{I} = 25\left(\frac{\mathbf{V}_2}{20}\right) \quad \Rightarrow \quad \mathbf{V}_3 = 2.25\,\mathbf{V}_2$$

Apply KCL to the supernode corresponding to the CCVS to get

$$\frac{48\angle 45° - \mathbf{V}_2}{j50} = \frac{\mathbf{V}_2}{20} + \frac{\mathbf{V}_3 - \mathbf{V}_o}{30} + \frac{\mathbf{V}_3}{-j40}$$

$$\frac{48\angle 45°}{j50} = \frac{\mathbf{V}_2}{j50} + \frac{\mathbf{V}_2}{20} + \frac{\mathbf{V}_3 - \mathbf{V}_o}{30} + \frac{\mathbf{V}_3}{-j40}$$

$$\frac{48\angle 45°}{j50} = \left(\frac{1}{j50} + \frac{1}{20}\right)\mathbf{V}_2 + \left(\frac{1}{30} + \frac{1}{-j40}\right)\mathbf{V}_3 - \frac{1}{30}\mathbf{V}_o$$

$$\frac{48\angle 45^\circ}{j50} = \left(\frac{1}{j50}+\frac{1}{20}\right)\mathbf{V}_2 + \left(\frac{1}{30}+\frac{1}{-j40}\right)2.25\mathbf{V}_2 - \frac{1}{30}\mathbf{V}_o$$

$$\frac{48\angle 45^\circ}{j50} = \left(\frac{1}{j50}+\frac{1}{20}+\frac{2.25}{30}+\frac{2.25}{-j40}\right)\mathbf{V}_2 - \frac{1}{30}\mathbf{V}_o$$

Apply KCL at the right node of the 30 Ω resistor to get

$$\frac{\mathbf{V}_3-\mathbf{V}_o}{30} = \frac{\mathbf{V}_o}{j25} \quad \Rightarrow \quad 0 = \left(-\frac{1}{30}\right)2.25\,\mathbf{V}_2 + \left(\frac{1}{30}+\frac{1}{j25}\right)\mathbf{V}_o$$

In matrix form

$$\begin{bmatrix} \frac{1}{j50}+\frac{1}{20}+\frac{2.25}{30}+\frac{2.25}{-j40} & -\frac{1}{30} \\ -\frac{2.25}{30} & \frac{1}{30}+\frac{1}{j25} \end{bmatrix} \begin{bmatrix} \mathbf{V}_2 \\ \mathbf{V}_o \end{bmatrix} = \begin{bmatrix} \frac{48\angle 45^\circ}{j50} \\ 0 \end{bmatrix}$$

Solving, perhaps using MATLAB,

$$\begin{bmatrix} \mathbf{V}_2 \\ \mathbf{V}_o \end{bmatrix} = \begin{bmatrix} 10.18\angle -44.6^\circ \\ 14.67\angle 5.6^\circ \end{bmatrix} \text{ V}$$

P10.6-2 Figure P10.6-2 shows an ac circuit represented in the frequency domain. Determine the values of the phasor node voltages, $\mathbf{V}_b$ and $\mathbf{V}_c$.

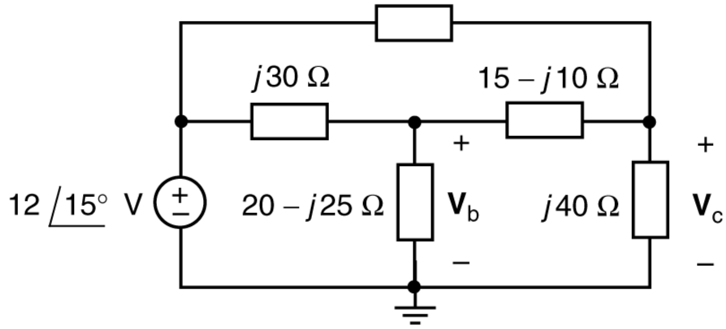


Figure P10.6-2

Solution:

Writing Node equations:

$$\frac{12\angle 45° - \mathbf{V}_b}{j30} = \frac{\mathbf{V}_b}{20-j25} + \frac{\mathbf{V}_b - \mathbf{V}_c}{15-j30}$$

$$\frac{\mathbf{V}_b - \mathbf{V}_c}{15-j30} + \frac{12\angle 45° - \mathbf{V}_c}{40+j20} = \frac{\mathbf{V}_c}{j40}$$

Rearranging:

$$\frac{12\angle 45°}{j30} = \left(\frac{1}{j30} + \frac{1}{20-j25} + \frac{1}{15-j30}\right)\mathbf{V}_b - \left(\frac{1}{15-j30}\right)\mathbf{V}_c$$

$$\frac{12\angle 45°}{40+j20} = -\left(\frac{1}{15-j30}\right)\mathbf{V}_b + \left(\frac{1}{15-j30} + \frac{1}{40+j20} + \frac{1}{j40}\right)\mathbf{V}_c$$

In matrix from:

$$\begin{bmatrix} \frac{1}{j30} + \frac{1}{20-j25} + \frac{1}{15-j30} & -\frac{1}{15-j30} \\ -\frac{1}{15-j30} & \frac{1}{15-j30} + \frac{1}{40+j20} + \frac{1}{j40} \end{bmatrix} \begin{bmatrix} \mathbf{V}_b \\ \mathbf{V}_c \end{bmatrix} = \begin{bmatrix} \frac{12\angle 45°}{j30} \\ \frac{12\angle 45°}{40+j20} \end{bmatrix}$$

Solving using MATLAB:

$$\mathbf{V}_b = 7.69\angle -19.8° \text{ and } \mathbf{V}_c = 10.18\angle 7.7° \text{ V}$$

Checked using LNAPAC

P10.6-3 Figure P10.6-3 shows an ac circuit represented in the frequency domain. Determine the value of the phasor node voltage **V**.

Answer: $\mathbf{V} = 71.0346\angle -39.627°$ V

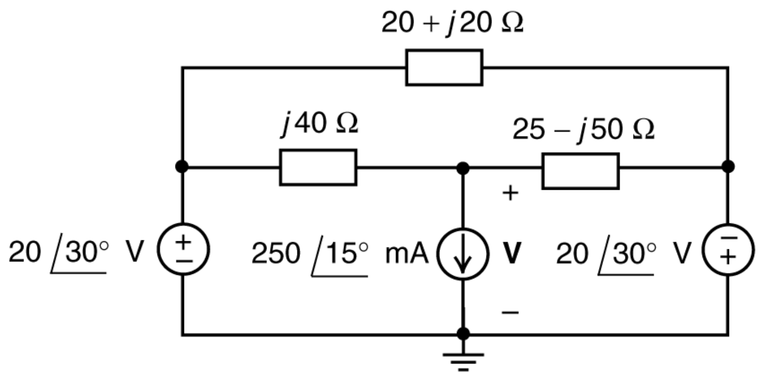


Figure P10.6-3

Solution:

Writing a node equation: $\dfrac{20\angle 30° - \mathbf{V}}{j40} + \dfrac{-20\angle 30° - \mathbf{V}}{25 - j50} = 0.25\angle 15°$

Rearranging $\left(\dfrac{1}{j40} + \dfrac{1}{25 - j50}\right)\mathbf{V} = \dfrac{20\angle 30°}{j40} + \dfrac{-20\angle 30°}{25 - j50} - 0.25\angle 15°$

Solving $\left(0.012042\angle -48.366°\right)\mathbf{V} = 0.85537\angle 87.993°$

Finally $\mathbf{V} = \dfrac{0.85537\angle 87.993°}{0.012042\angle -48.366°} = 71.0346\angle -39.627°$ V

P10.6-4 Figure P10.6-4 shows an ac circuit represented in the frequency domain. Determine the values of the phasor mesh currents.

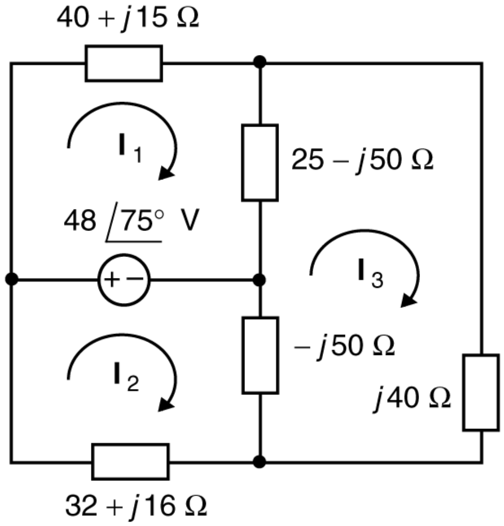


Figure P10.6-4

Solution:

Mesh 1:

$$(40+j15)\mathbf{I}_1+(25-j50)(\mathbf{I}_1-\mathbf{I}_3)-48\angle 75^\circ=0$$

$$(65-j35)\mathbf{I}_1-(25-j50)\mathbf{I}_3=48\angle 75^\circ$$

Mesh 2:

$$48\angle 75^\circ+(-j50)(\mathbf{I}_2-\mathbf{I}_3)+(32+j16)\mathbf{I}_2=0$$

$$(32-j34)\mathbf{I}_2+j50\mathbf{I}_3=-48\angle 75^\circ$$

Mesh 3:

$$j40\mathbf{I}_3-(-j50)(\mathbf{I}_2-\mathbf{I}_3)-(25-j50)(\mathbf{I}_1-\mathbf{I}_3)=0$$

$$(-25+j50)\mathbf{I}_1+j50\mathbf{I}_2+(25-j160)\mathbf{I}_3=0$$

In matrix form:

$$\begin{bmatrix} 65-j35 & 0 & -25+j50 \\ 0 & 32-j34 & +j50 \\ -25+j50 & +j50 & 25-j60 \end{bmatrix}\begin{bmatrix}\mathbf{I}_1\\ \mathbf{I}_2\\ \mathbf{I}_3\end{bmatrix}=\begin{bmatrix}48\angle 75^\circ\\ -48\angle 75^\circ\\ 0\end{bmatrix}$$

Solving using MATLAB:

$$\mathbf{I}_1=0.794\angle 111^\circ,\ \ \mathbf{I}_2=0.790\angle -61.7^\circ \text{ and } \mathbf{I}_3=0.229\angle 176^\circ \text{ A}$$

P 10.6-5 A commercial airliner has sensing devices to indicate to the cockpit crew that each door and baggage hatch is closed. A device called a search coil magnetometer, also known as a proximity sensor, provides a signal indicative of the proximity of metal or other conducting material to an inductive sense coil. The inductance of the sense coil changes as the metal gets closer to the sense coil. The sense coil inductance is compared to a reference coil inductance with a circuit called a balanced inductance bridge (see Figure P 10.6-5). In the inductance bridge, a signal indicative of proximity is observed between terminals a and b by subtracting the voltage at b, v_b, from the voltage at a, v_a (Lenz, 1990).

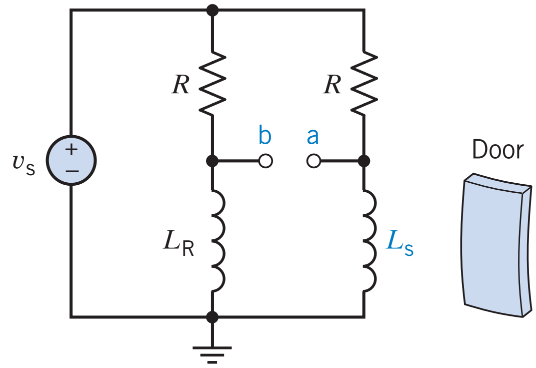
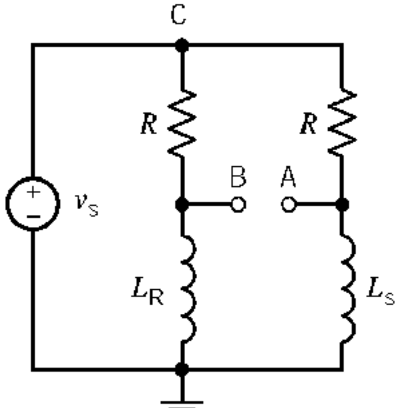


Figure P 10.6-5

The bridge circuit is excited by a sinusoidal voltage source $v_s = \sin(800\,\pi t)$ V. The two resistors, $R = 100\ \omega$, are of equal resistance. When the door is open (no metal is present), the sense coil inductance, L_S, is equal to the reference coil inductance, $L_R = 40$ mH. In this case, what is the magnitude of the signal $\mathbf{V}_a - \mathbf{V}_b$?

When the airliner door is completely closed, $L_S = 60$ mH. With the door closed, what is the phasor representation of the signal $\mathbf{V}_a - \mathbf{V}_b$?

Solution:



$$v_s = \sin\ (2\pi \cdot 400\,t)\ \ \text{V}$$

$$R = 100\ \Omega$$

$$L_R = 40\ \text{mH}$$

$$L_S = \begin{cases} 40\ \text{mH} & \text{door opened} \\ 60\ \text{mH} & \text{door closed} \end{cases}$$

With the door open $\left|\mathbf{V}_A - \mathbf{V}_B\right| = 0$ since the bridge circuit is balanced.

With the door closed $\mathbf{Z}_{L_R} = j(800\pi)(0.04) = j100.5\ \Omega$ and $\mathbf{Z}_{L_S} = j(800\pi)(0.06) = j150.8\ \Omega$.

The node equations are:

KCL at node B: $\frac{\mathbf{V}_\text{B} - \mathbf{V}_\text{C}}{R} + \frac{\mathbf{V}_\text{B}}{\mathbf{Z}_{\text{L}_\text{R}}} = 0 \quad \Rightarrow \mathbf{V}_\text{B} = \frac{j100.5}{j100.5+100}\mathbf{V}_\text{C}$

KCL at node A : $\frac{\mathbf{V}_\text{A} - \mathbf{V}_\text{C}}{R} + \frac{\mathbf{V}_\text{A}}{\mathbf{Z}_{\text{L}_\text{S}}} = 0$

Since $\mathbf{V}_\text{C} = |\mathbf{V}_\text{s}| = 1$ V $\quad \mathbf{V}_\text{B} = 0.709\angle 44.86°$ V and $\mathbf{V}_\text{A} = 0.833\angle 33.55$ V

Therefore

$$\mathbf{V}_\text{A} - \mathbf{V}_\text{B} = 0.833\angle 33.55° - 0.709\angle 44.86° = (0.694 + j.460) - (0.503 + j0.500) = 0.191 - j0.040$$
$$= 0.195\angle -11.83° \text{ V}$$

P 10.6-6 Using a tiny diamond-studded burr operating at 190,000 rpm, cardiologists can remove life-threatening plaque deposits is coronary arteries. The procedure is fast, uncomplicated, and relatively painless (McCarty, 1991). The Rotablator, an angioplasty system, consists of an advancer/catheter, a guide wire, a console, and a power source. The advancer/catheter contains a tiny turbine that drives the flexible shaft that rotates the catheter burr. The model of the operational and control circuit is shown in Figure P 10.6-6. Determine $v(t)$, the voltage that drives the tip, when $v_s = \sqrt{2}\ \cos(40t - 135°)$ V.

Answer: $v(t) = 1.414 \cos(40t + 135°)$ V

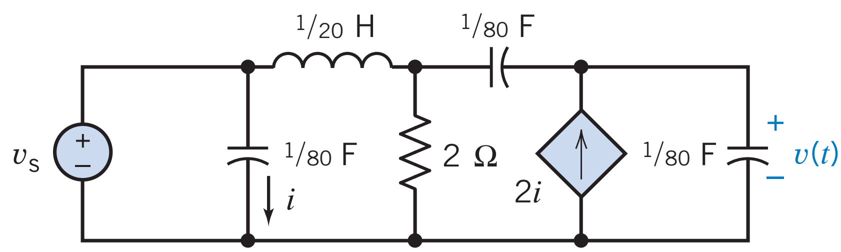
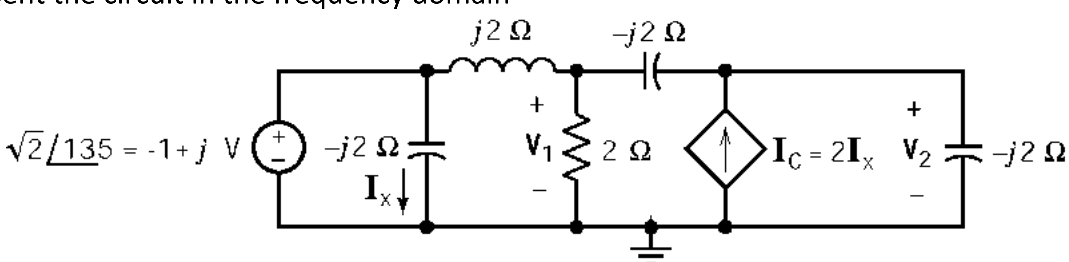


Figure P 10.6-6

Solution:

Represent the circuit in the frequency domain



The node equations are:

$$\frac{\mathbf{V}_1-(-1+j)}{j2}+\frac{\mathbf{V}_1}{2}+\frac{\mathbf{V}_1-\mathbf{V}_2}{-j2}=0$$

$$\frac{\mathbf{V}_2-\mathbf{V}_1}{-j2}+\frac{\mathbf{V}_2}{-j2}-\mathbf{I}_C = 0$$

Also, expressing the controlling signal of the dependent source in terms of the node voltages yields

$$\mathbf{I}_x=\frac{-1+j}{-2j} \Rightarrow \mathbf{I}_C=2\mathbf{I}_x = 2\left[\frac{-1+j}{-2j}\right]=-1-j \text{ A}$$

Solving these equations yields

$$\mathbf{V}_2=\frac{-3-j}{1+j2}=\sqrt{2}\ \angle -135° \text{ V} \Rightarrow v(t)=v_2(t)=\sqrt{2}\ \cos\ (40t-135°) \text{ V}$$

(checked: LNAP 7/19/04)

P 10.6-7 For the circuit of Figure P 10.6-7, it is known that

$v_2(t) = 0.7571 \cos(2t + 66.7°)$ V

$v_3(t) = 0.6064 \cos(2t - 69.8°)$ V

Determine $i_1(t)$.

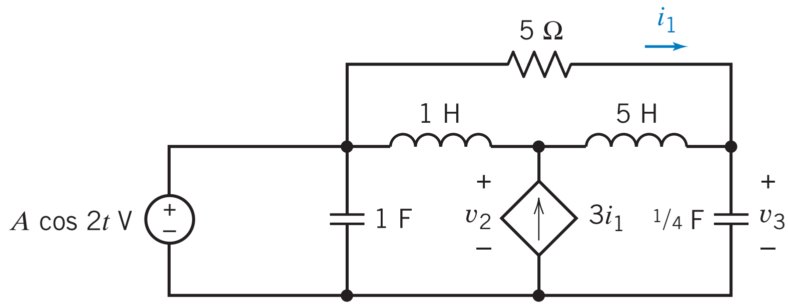
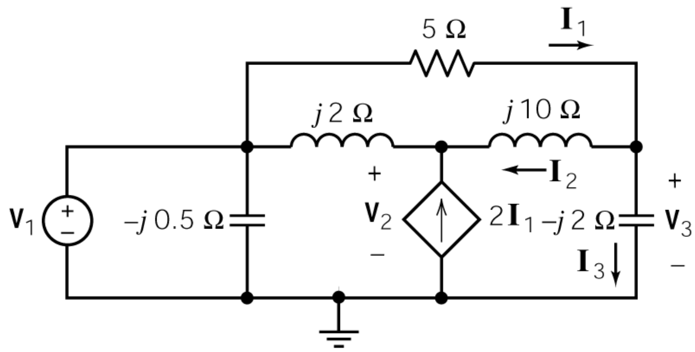


Figure P 10.6-7

Solution:



$$\mathbf{V}_2 = 0.7571\angle 66.7° \text{ V}$$
$$\mathbf{V}_3 = 0.6064\angle -69.8° \text{ V}$$

$$\left.\begin{array}{l} \mathbf{I}_1 = \mathbf{I}_2 + \mathbf{I}_3 \\ \mathbf{I}_2 = \dfrac{\mathbf{V}_3 - \mathbf{V}_2}{j10} \\ \mathbf{I}_3 = \dfrac{\mathbf{V}_3}{-j2} \end{array}\right\} \text{ yields } \left\{\begin{array}{l} \mathbf{I}_3 = 0.3032\angle 20.2° \text{ A} \\ \mathbf{I}_2 = 0.1267\angle -184° \text{ A} \\ \mathbf{I}_1 = 0.195\angle 36° \text{ A} \end{array}\right.$$

therefore

$$i_1(t) = 0.195 \cos(2t + 36°) \text{ A}$$

(checked: MATLAB 7/18/04)

P10.6-8 The input to the circuit shown in Figure P10.6-8 is the voltage

$$v_s = 25\cos(40t + 45°)\ \text{V}$$

Determine the mesh currents i_1 and i_2 and the voltage v_o.

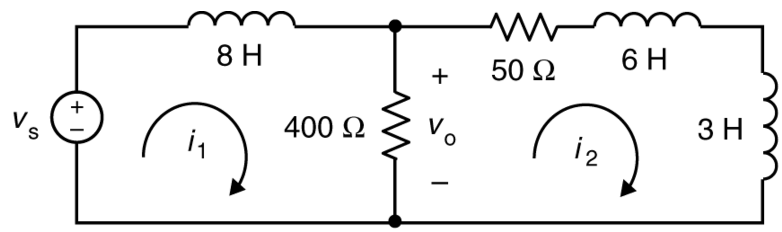
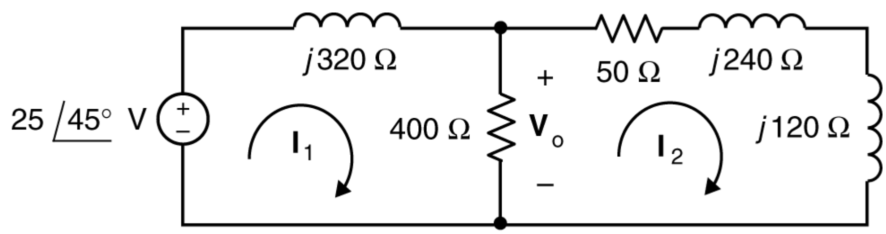


Figure P10.6-8

Solution

Represent the circuit in the frequency domain:



Apply KVL to mesh 1: $j320\mathbf{I}_1 + 400(\mathbf{I}_1 - \mathbf{I}_2) - 25\angle 45° = 0$

Apply KVL to mesh 2: $50\mathbf{I}_2 + j240\mathbf{I}_2 + j120\mathbf{I}_2 - 400(\mathbf{I}_1 - \mathbf{I}_2) = 0$

In matrix form:
$$\begin{bmatrix} 400 + j320 & -400 \\ -400 & 450 + j360 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 25\angle 45° \\ 0 \end{bmatrix}$$

Solving using MATLAB:
$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 47.5\angle -24.6° \\ 33.0\angle -63.3° \end{bmatrix}\ \text{mA}$$

Using Ohm's Law $\mathbf{V}_o = 400(\mathbf{I}_1 - \mathbf{I}_2) = 12\angle 18.8°\ \text{V}$

In the time domain $i_1(t) = 47.5\cos(40t - 24.6°)\ \text{mA}$, $i_2(t) = 33\cos(40t - 63.3°)\ \text{mA}$

and $v_o(t) = 12\cos(40t + 18.8°)\ \text{V}$

P10.6-9 The input to the circuit shown in Figure P10.6-9 is the voltage

$$v_s(t) = 42\cos(800t + 60°)\text{ mV}$$

Determine the output voltage $v_o(t)$.

Answer: $v_o(t) = 823.5\cos(800t - 55.6°)$ mV

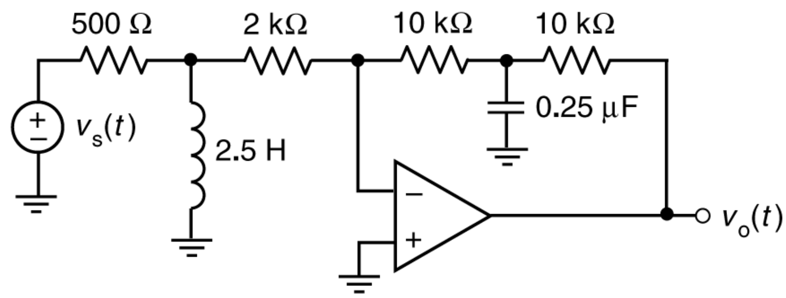
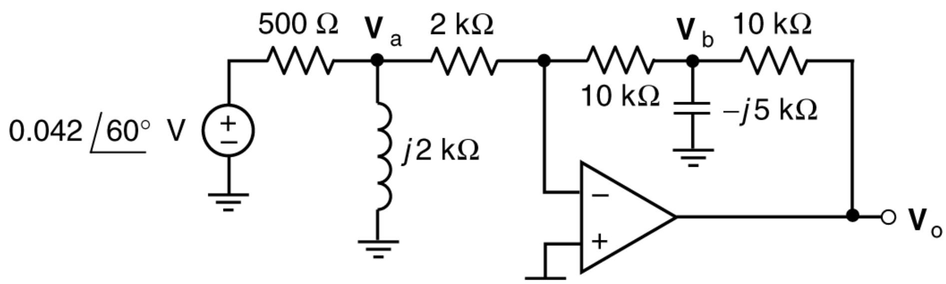


Figure P10.6-9

Solution

Represent the circuit in the frequency domain:



Apply KCL at the top node of the inductor, node a:

$$\frac{0.042\angle 60° - \mathbf{V}_a}{500} = \frac{\mathbf{V}_a}{j2000} + \frac{\mathbf{V}_a}{2000} \quad\Rightarrow\quad \mathbf{V}_a = \frac{4}{5-j}(0.042\angle 60°)$$

Apply KCL at the inverting input node of the op amp:

$$\frac{\mathbf{V}_a}{2000} + \frac{\mathbf{V}_b}{10{,}000} = 0 \quad\Rightarrow\quad \mathbf{V}_b = -5\mathbf{V}_a$$

Apply KCL at the top node of the capacitor, node b:

$$\frac{\mathbf{V}_b}{10{,}000} + \frac{\mathbf{V}_b}{-j5000} + \frac{\mathbf{V}_b - \mathbf{V}_o}{20{,}000} = 0 \quad\Rightarrow\quad \mathbf{V}_o = (3+j4)\mathbf{V}_b$$

Combining these results we get:

$$\mathbf{V}_o = (3+j4)(-5)\frac{4}{5-j}(0.042\angle 60°) = \frac{(5\angle 53.1°)(20\angle -180°)}{5.1\angle -11.3°}(0.042\angle 60°) = 0.8235\angle -55.6°$$

In the time domain $\qquad v_o(t) = 832.5\cos(800t - 55.6°)\text{ mV}$

P 10.6-10 The idea of using an induction coil in a lamp isn't new, but applying it in a commercially available product is. An induction coil in a bulb induces a high-frequency energy flow in mercury vapor to produce light. The lamp uses about the same amount of energy as a fluorescent bulb but lasts six times longer, with 60 times the life of a conventional incandescent bulb. The circuit model of the bulb and its associated circuit are shown in Figure P 10.6-10.

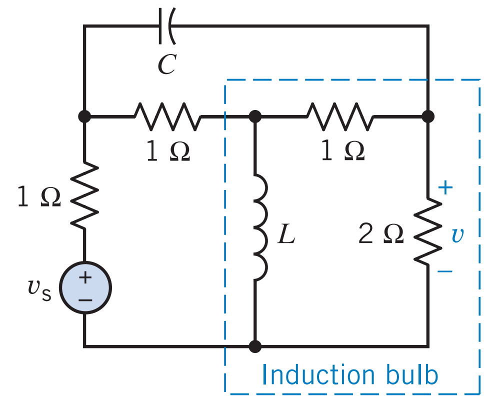


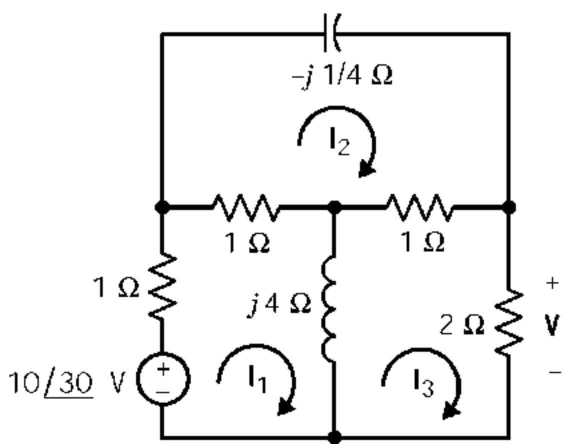
Figure P 10.6-10

Determine the voltage $v(t)$ across the 2-Ω resistor when $C = 40\ \mu\text{F}$, $L = 40\ \mu\text{H}$, $v_s = 10\cos(\omega_0 t + 30°)$, and $\omega_0 = 10^5$ rad/s.

Answer: $v(t) = 6.45\cos(10^5 t + 44°)$ V

Solution:

Represent the circuit in the frequency domain:



The mesh equations are:

$$\begin{bmatrix} (2+j4) & -1 & -j4 \\ -1 & (2+1/j4) & -1 \\ -j4 & -1 & (3+j4) \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 10\angle 30° \\ 0 \\ 0 \end{bmatrix}$$

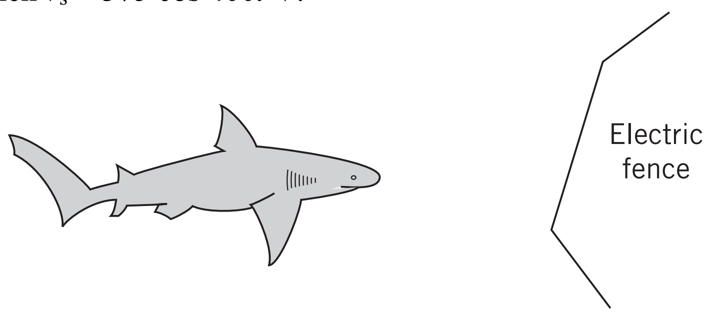
Using Cramer's rule yields

$$\mathbf{I}_3 = \frac{2+j8}{12+j\,22.5}(10\angle 30°) = 3.225\angle 44°\ \text{A}$$

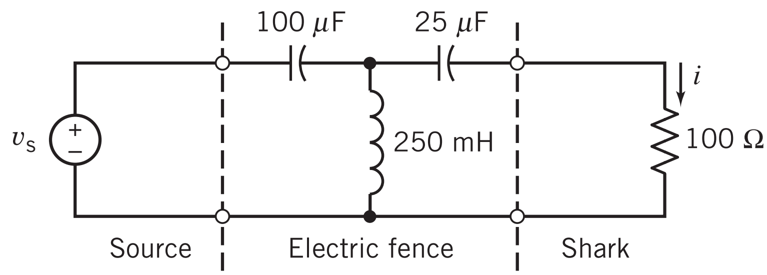
Then $\mathbf{V} = 2\,\mathbf{I}_3 = 2(3.225\angle 44°) = 6.45\angle 44°\ \text{V} \Rightarrow v(t) = 6.45\cos(10^5 t + 44°)\ \text{V}$

(checked: LNAP 7/19/04)

P 10.6-11 The development of coastal hotels in various parts of the world is a rapidly growing enterprise. The need for environmentally acceptable shark protection is manifest where these developments take place alongside shark-infested waters (Smith, 1991). One concept is to use an electrified line submerged in the water in order to deter the sharks, as shown in Figure P 10.6-11*a*. The circuit model of the electric fence is shown in Figure P 10.6-11*b*, where the shark is represented by an equivalent resistance of 100Ω. Determine the current flowing through the shark's body, $i(t)$, when $v_s = 375 \cos 400t$ V.



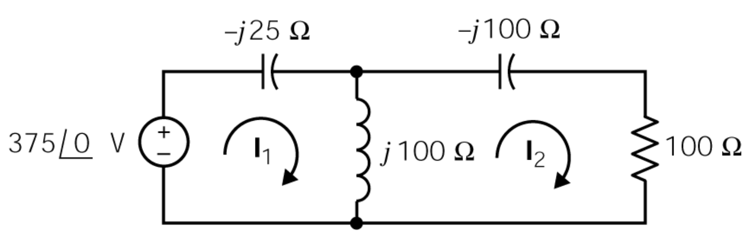
(a)



(b)

Figure P 10.6-11

Solution: Represent the circuit in the frequency domain:



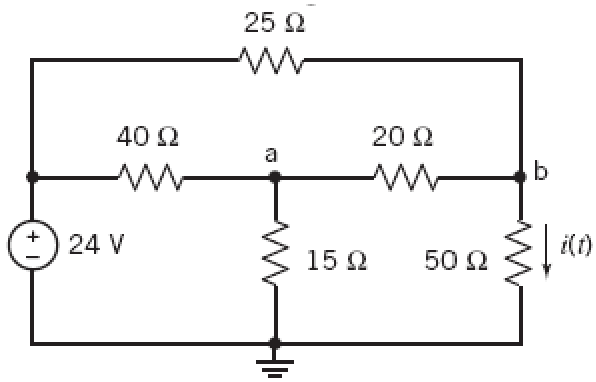
Mesh Equations:

$$j\,75\mathbf{I}_1 - j100\mathbf{I}_2 = 375$$
$$-j100\mathbf{I}_1 + (100 + j100)\mathbf{I}_2 = 0$$

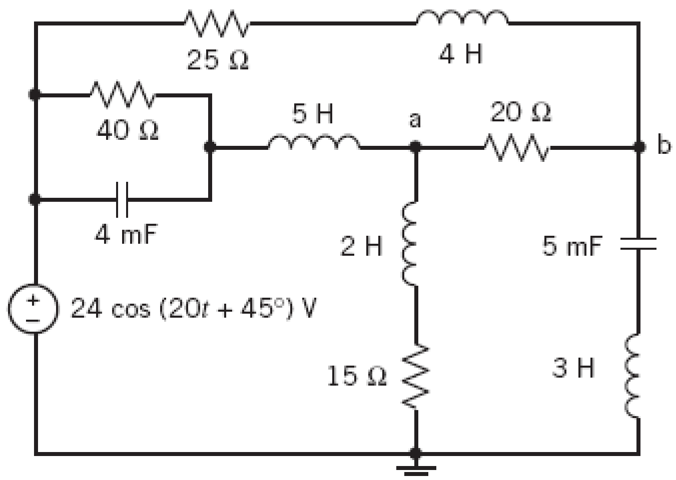
Solving for $\mathbf{I}_2$ yields $\mathbf{I}_2 = 4.5 + j1.5 \;=\; 3\angle 53.1°$ A $\quad\Rightarrow\quad i_2(t) 3\cos(400t + 53.1°)$ A

(checked: LNAP 7/19/04)

P 10.6-12 Determine the node voltage at nodes a and b in each of these circuits:



(a)



(b)

Solution

(a)

The node equations are

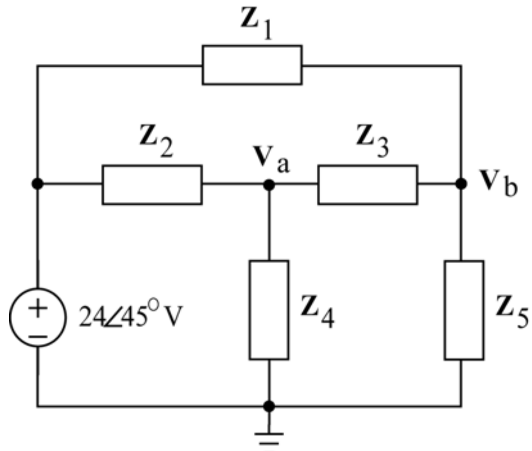
$$\frac{24-v_a}{40}=\frac{v_a-v_b}{20}+\frac{v_a}{15}$$
$$\frac{24-v_b}{25}+\frac{v_a-v_b}{20}=\frac{v_b}{50}$$

or

$$\begin{bmatrix} \frac{1}{40}+\frac{1}{20}+\frac{1}{15} & -\frac{1}{20} \\ -\frac{1}{20} & \frac{1}{25}+\frac{1}{20}+\frac{1}{50} \end{bmatrix}\begin{bmatrix} v_a \\ v_b \end{bmatrix}=\begin{bmatrix} \frac{24}{40} \\ \frac{24}{25} \end{bmatrix}$$

Solving using MATLAB gives $v_a = 8.713\text{ V}$ and $v_b = 12.69\text{ V}$

(b) Use phasors and impedances to represent the circuit in the frequency domain as



where

$$\mathbf{Z}_1 = 25 + j(20)4 = 25 + j80 = 83.82\angle 72.7°\ \Omega$$

$$\mathbf{Z}_2 = \left(40\,||\,\frac{1}{j(20)(0.004)}\right) + j(20)5 = 3.56 + j88.6 = 88.68\angle 87.7°\ \Omega$$

$$\mathbf{Z}_3 = 20\ \Omega$$

$$\mathbf{Z}_4 = 15 + j(20)2 = 15 + j40 = 42.72\angle 69.4°$$

$$\mathbf{Z}_5 = j(20)3 + \frac{1}{j(20)(0.005)} = j50 = 50\angle 90°\ \Omega$$

The node equations are

$$\frac{24\angle 45° - \mathbf{V}_a}{\mathbf{Z}_2} = \frac{\mathbf{V}_a}{\mathbf{Z}_4} + \frac{\mathbf{V}_a - \mathbf{V}_b}{\mathbf{Z}_3}$$

$$\frac{24\angle 45° - \mathbf{V}_b}{\mathbf{Z}_1} + \frac{\mathbf{V}_a - \mathbf{V}_b}{\mathbf{Z}_3} = \frac{\mathbf{V}_b}{\mathbf{Z}_5}$$

$$\begin{bmatrix} \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3} + \frac{1}{\mathbf{Z}_4} & -\frac{1}{\mathbf{Z}_3} \\ -\frac{1}{\mathbf{Z}_3} & \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_3} + \frac{1}{\mathbf{Z}_5} \end{bmatrix} \begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_b \end{bmatrix} = \begin{bmatrix} \frac{24\angle 45°}{\mathbf{Z}_2} \\ \frac{24\angle 45°}{\mathbf{Z}_1} \end{bmatrix}$$

Solving using MATLAB gives

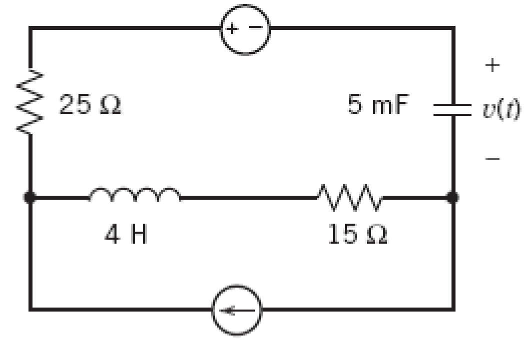
$$\mathbf{V}_a = 7.89\angle 44.0°$$
$$\mathbf{V}_b = 8.45\angle 45.1°$$

So

$$v_a(t) = 7.89\cos(20t + 44°)\ \text{V}$$
$$v_a(t) = 8.45\cos(20t + 45.1)\ \text{V}$$

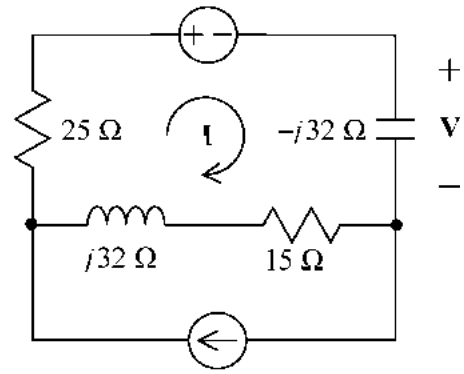
(checked: LNAP 8/3/04)

P 10.6-13 Determine the voltage *v*(*t*):



Solution:

Represent the circuit in the frequency domain using impedances and phasors



The mesh currents are **I** and $0.05\angle -30°$ A . Apply KVL to the top mesh to get

$$15\angle 45° + (-j25)\mathbf{I} + (15 + j32)(\mathbf{I} - 0.05\angle -30°) + 25\mathbf{I} = 0$$

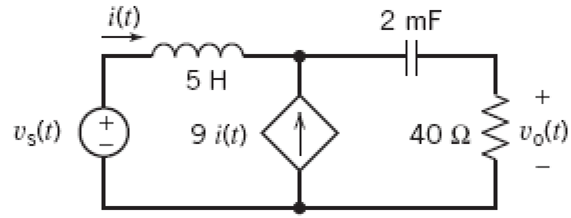
So $$\mathbf{I} = \frac{-15\angle 45° + (15 + j32)(0.05\angle -30°)}{25 - j25 + 15 + j32} = 0.3266\angle -143.6° = -0.2629 - j0.1939 \text{ A}$$

Then $$\mathbf{V} = (-j25)\mathbf{I} = 8.166\angle 126.4° = -4.8475 + j6.5715 \text{ V}$$

So $$v(t) = 8.166\cos(8t + 126.4°) \text{ V}$$

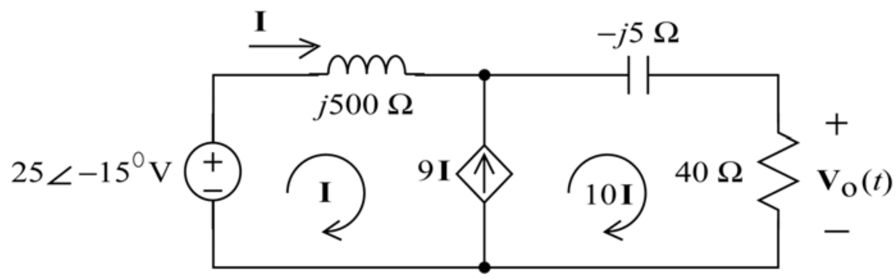
(checked: LNAP 8/3/04)

P 10.6-14 Determine the voltage $v_o(t)$ when $v_s(t)$ = 25 cos (100t-15°) V.



Solution:

Represent the circuit in the frequency domain using impedances and phasors.



The mesh currents are **I** and 10**I**. Apply KVL to the supermesh corresponding to the dependant current source to get

$$(j500)\mathbf{I}+(-j5)(10\mathbf{I})+40(10\mathbf{I})-25\angle-15°=0$$

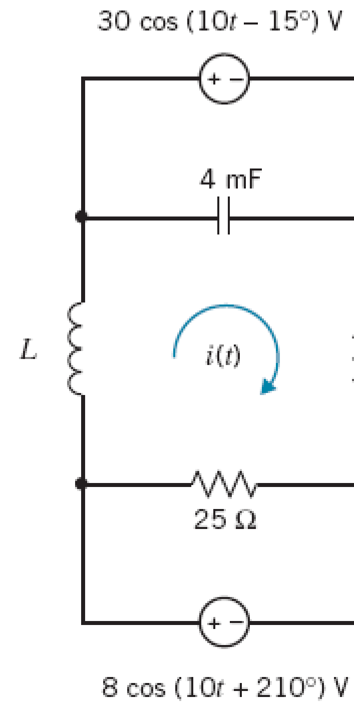
So $$\mathbf{I}=\frac{25\angle-15°}{400+j450}=0.04152\angle-63.37°\text{ A}$$

The output voltage is $$\mathbf{V}=40(10\mathbf{I})=16.61\angle-63.37°\text{ V}$$

So $$v(t)=16.61\cos(100t-63.37°)\text{ V}$$

(checked: LNAP 8/3/04)

P 10.6-15 Determine the mesh current is when $i(t)$ = 0.8394 cos (10t-138.5°) A. Determine the values of L and R.



Solution:

Represent the circuit in the frequency domain using phasors and impedances. Apply KVL to the center mesh to get

$$0.8394\angle 138.5° = \mathbf{I} = \frac{8\angle 210° - 30\angle -15°}{R + j10L} \quad \Rightarrow \quad R + j10L = 35 + j25 = 35 + j(10)2.5$$

So $$R = 35\ \Omega \quad \text{and} \quad L = 2.5\ \text{H}$$

(checked: LNAP 8/3/04)

P 10.6-16 The circuit shown in Figure P 10.6-16 has two inputs:

$$v_1(t) = 50\cos(20t - 75°) \text{ V}$$

$$v_2(t) = 35\cos(20t + 110°) \text{ V}$$

When the circuit is at steady state, the node voltage is

$$v(t) = 21.25\cos(20t - 168.8°) \text{ V}$$

Determine the values of *R* and *L*.

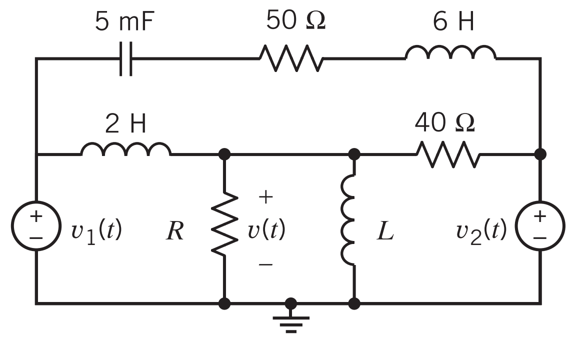


Figure P 10.6-16

Solution:

Represent the circuit in the frequency domain using phasors and impedances. Apply KCL at the top node of *R* and *L* to get

$$\frac{(50\angle -75°) - \mathbf{V}}{j40} + \frac{35\angle 100° - \mathbf{V}}{40} = \frac{\mathbf{V}}{R \,||\, j\omega L}$$

$$\Rightarrow \quad \frac{50\angle -75°}{40\angle 90°} + \frac{35\angle 110°}{40} = \left(\frac{1}{j40} + \frac{1}{40} + \frac{1}{R} - j\frac{1}{20L}\right)\mathbf{V}$$

Using the given equation for *v*(*t*) we get

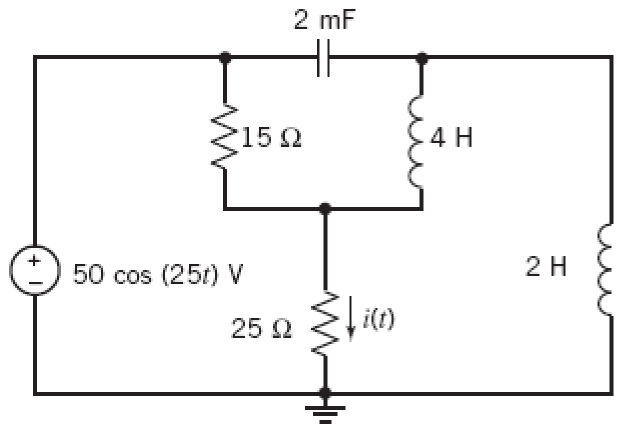
$$21.25\angle -168.8° = \mathbf{V} = \frac{1.587\angle 161.7°}{0.025(1-j) + \frac{1}{R} - j\frac{1}{20L}}$$

Then
$$\frac{1}{R} - j\frac{1}{20L} = \frac{1.587\angle 161.7°}{21.25\angle -168.8°} - 0.025(1-j) = 0.04 - j0.01176$$

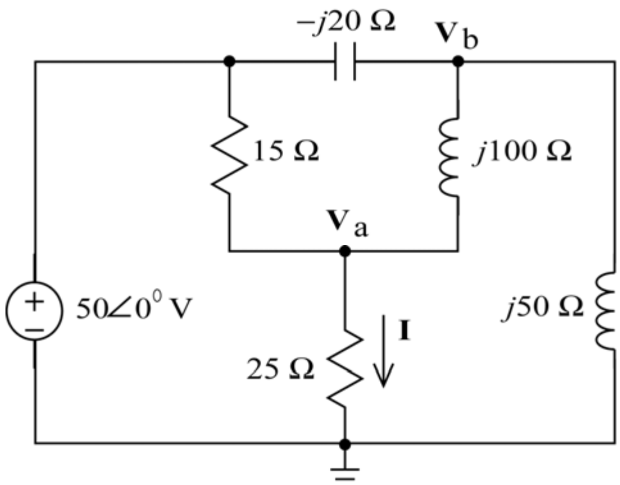
Finally
$$R = \frac{1}{0.04} = 25\ \Omega \quad \text{and} \quad L = \frac{1}{20(0.01176)} = 4.25 \text{ H}$$

(checked: LNAP 8/3/04)

P 10.6-17 Determine the steady state current *i*(*t*):



Solution: Represent the circuit in the frequency domain using phasors and impedances.



The node equations are

$$\frac{50\angle 0° - \mathbf{V}_a}{15} + \frac{\mathbf{V}_b - \mathbf{V}_a}{j100} = \frac{\mathbf{V}_a}{25}$$

$$\frac{50\angle 0° - \mathbf{V}_b}{-j20} = \frac{\mathbf{V}_b - \mathbf{V}_a}{j100} + \frac{\mathbf{V}_b}{j50}$$

or

$$\begin{bmatrix} \frac{1}{15}+\frac{1}{j100}+\frac{1}{25} & -\frac{1}{j100} \\ -\frac{1}{j100} & \frac{1}{j50}+\frac{1}{j100}+\frac{1}{-j20} \end{bmatrix} \begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_b \end{bmatrix} = \begin{bmatrix} \frac{50\angle 0°}{15} \\ \frac{50\angle 0°}{-j20} \end{bmatrix}$$

$$\begin{bmatrix} 0.1067 - j0.010 & j0.010 \\ j0.010 & j0.020 \end{bmatrix} \begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_b \end{bmatrix} = \begin{bmatrix} 3.333 \\ j2.5 \end{bmatrix}$$

Solving, e.g. using MATLAB, gives

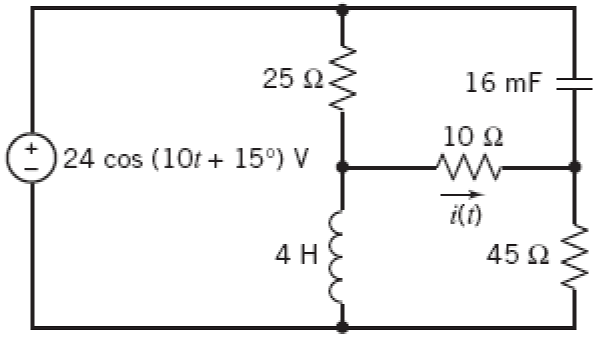
$$\mathbf{V}_a = 33.05\angle -12.6° \text{ V} \quad \text{and} \quad \mathbf{V}_b = 108.9\angle 1.9° \text{ V}$$

Then
$$\mathbf{I} = \frac{\mathbf{V}_a}{25} = 1.322\angle -12.6° \text{ A}$$

So
$$i(t) = 1.322\cos(25t - 12.6°) \text{ A}$$

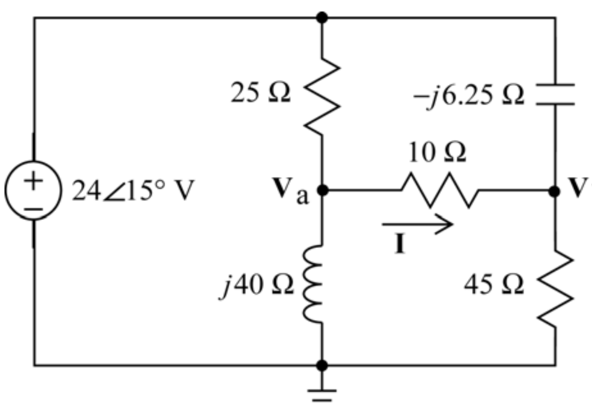
(checked: LNAP 8/3/04)

P 10.6-18 Determine the steady state current *i*(*t*):



Solution:

Represent the circuit in the frequency domain using phasors and impedances. Label the node voltages.



The node equations are

$$\frac{24\angle 15° - \mathbf{V}_a}{25} = \frac{\mathbf{V}_a}{j40} + \frac{\mathbf{V}_a - \mathbf{V}_b}{10}$$

$$\frac{24\angle 15° - \mathbf{V}_b}{-j6.25} + \frac{\mathbf{V}_a - \mathbf{V}_b}{10} = \frac{\mathbf{V}_b}{45}$$

or

$$\begin{bmatrix} \frac{1}{25} - j\frac{1}{40} + \frac{1}{10} & -\frac{1}{10} \\ -\frac{1}{10} & j\frac{1}{6.25} + \frac{1}{45} + \frac{1}{10} \end{bmatrix} \begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_b \end{bmatrix} = \begin{bmatrix} \frac{24\angle 15°}{25} \\ \frac{24\angle 15°}{6.25\angle -90°} \end{bmatrix}$$

$$\begin{bmatrix} 0.140 - j0.025 & -0.10 \\ -0.10 & 0.1222 + j0.160 \end{bmatrix} \begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_b \end{bmatrix} = \begin{bmatrix} 0.960\angle 15° \\ 3.840\angle 105° \end{bmatrix}$$

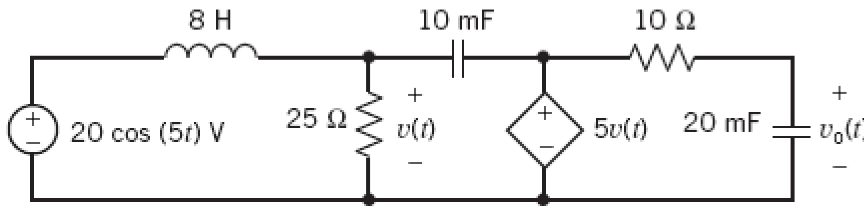
Solving gives $\mathbf{V}_a = 24.67\angle 32.6° \text{ V}$ and $\mathbf{V}_b = 25.59\angle 25.2° \text{ V}$

Then $$\mathbf{I} = \frac{\mathbf{V}_a - \mathbf{V}_b}{10} = 0.3347\angle 134.9° \text{ A}$$

So $$i(t) = 0.3347\cos(10t + 134.9°) \text{ A}$$

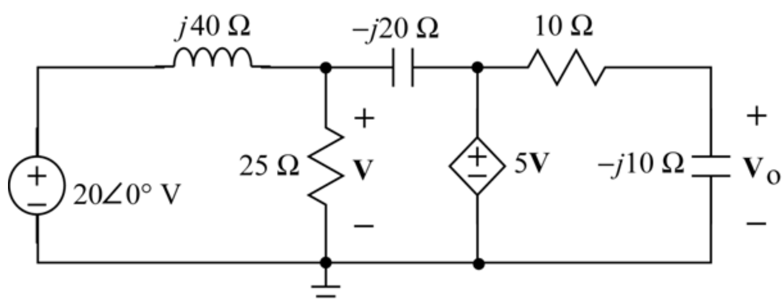
(checked: LANP 8/4/04)

P 10.6-19 Determine the steady state voltage $v_o(t)$:



Solution:

Represent the circuit in the frequency domain using phasors and impedances.



The node equations are

$$\frac{20\angle 0° - \mathbf{V}}{j40} = \frac{\mathbf{V}}{25} + \frac{\mathbf{V} - 5\mathbf{V}}{-j20}$$

$$\frac{5\mathbf{V} - \mathbf{V}_o}{10} = \frac{\mathbf{V}_o}{-j10}$$

$$\begin{bmatrix} \frac{1}{25} - j\frac{1}{5} - j\frac{1}{40} & 0 \\ -\frac{1}{2} & \frac{1}{10} + j\frac{1}{10} \end{bmatrix} \begin{bmatrix} \mathbf{V} \\ \mathbf{V}_o \end{bmatrix} = \begin{bmatrix} -j0.5 \\ 0 \end{bmatrix}$$

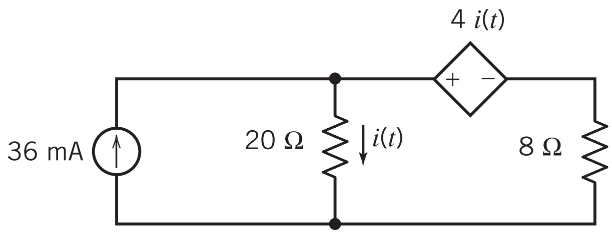
$$\begin{bmatrix} 0.04 - j0.225 & 0 \\ -0.50 & 0.10 + j0.10 \end{bmatrix} \begin{bmatrix} \mathbf{V} \\ \mathbf{V}_o \end{bmatrix} = \begin{bmatrix} -j0.5 \\ 0 \end{bmatrix}$$

Solving gives $\mathbf{V} = 2.188\angle -10.1° \text{ V}$ and $\mathbf{V}_o = 7.736\angle -55.1° \text{ V}$

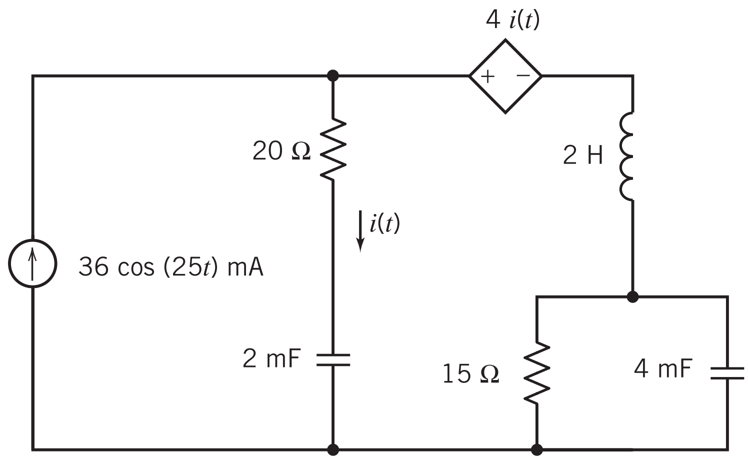
So $v_o(t) = 7.736\cos(5t - 55.1°) \text{ V}$

(checked: LNAP 8/4/04)

P 10.6-20 Determine the steady state current *i*(*t*) in each of these circuits:



(a)



(b)

Solution:

(a) Use KVL to see that the voltage across the 8 Ω resistor is $20i(t)-4i(t)=16i(t)$.

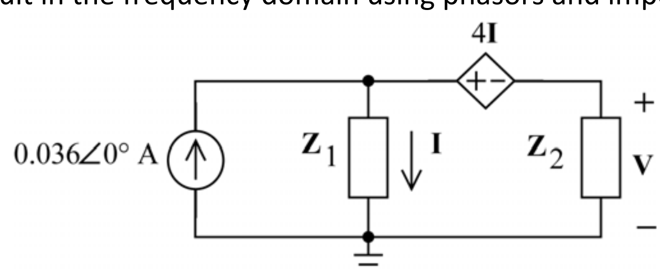
Apply KCL to the supernode corresponding to the dependent voltage source to get

$$0.036 = i(t) + \frac{16i(t)}{8} = 3i(t)$$

so

$$i(t) = 12 \text{ mA}$$

(b) Represent the circuit in the frequency domain using phasors and impedances.



where

$$\mathbf{Z}_1 = 20 + \frac{1}{j(25)(0.002)} = 20 - j20\ \Omega$$

$$\mathbf{Z}_2 = j50 + \left(15 \,||\, \frac{1}{j(25)(0.004)}\right) = 43.3\angle 83.9°\ \Omega$$

Use KVL to get

$$\mathbf{V} = \mathbf{Z}_1\mathbf{I} - 4\mathbf{I} = (\mathbf{Z}_1 - 4)\mathbf{I}$$

Then apply KCL to the supernode corresponding to the dependent source to get

$$0.036\angle 0° = \mathbf{I} + \frac{(\mathbf{Z}_1 - 4)\mathbf{I}}{\mathbf{Z}_2} = \left(\frac{\mathbf{Z}_1 + \mathbf{Z}_2 - 4}{\mathbf{Z}_2}\right)\mathbf{I}$$

so

$$\mathbf{I} = \frac{\mathbf{Z}_2(0.036\angle 0°)}{\mathbf{Z}_1 + \mathbf{Z}_2 - 4} = 50.4\angle 35.7°\ \text{mA}$$

so

$$i(t) = 50.4\cos(25t + 35.7°)\ \text{mA}$$

(checked: LNAP 8/4/04)

10.6-21 The input to the circuit show in Figure 10.9-24 is the current

$$i_s(t) = 50\cos(200t) \text{ mA}$$

Determine the steady-state mesh current i_2.

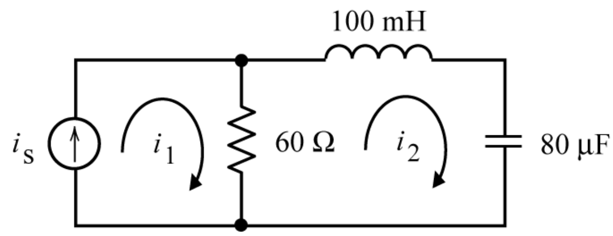
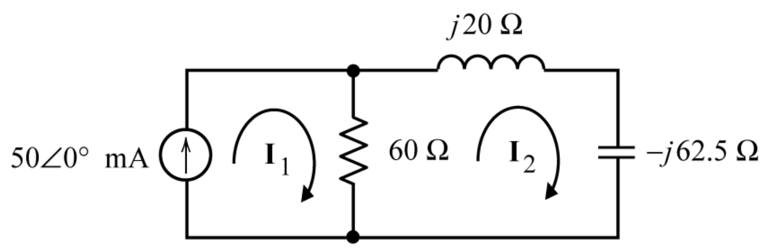


Figure P10.6-21

Solution:

Represent the circuit in the frequency domain using phasors and impedances:



Apply KVL to the right mesh to get:

$$(j20 - j62.5)\mathbf{I}_2 + 60(\mathbf{I}_2 - 0.050\angle 0^\circ) = 0 \quad \Rightarrow \quad \mathbf{I}_2 = \frac{3\angle 0^\circ}{60 - j42.5} = 0.0408\angle 35.3^\circ \text{ A}$$

In the time domain

$$i_2(t) = 40.8\cos(200t + 35.3^\circ) \text{ mA}$$

MATLAB, 11/20/09

10.6-22 The input to the circuit show in Figure 10.9-24 is the current

$$i_s(t) = 80\cos(250t) \text{ mA}$$

The steady-state mesh current in the right mesh is

$$i_s(t) = 66.56\cos(250t + 33.7°) \text{ mA}$$

Determine the value of the resistance *R*.

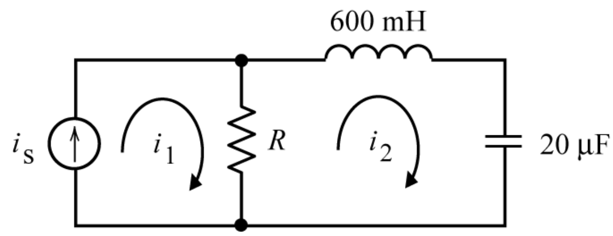


Figure P10.6-22

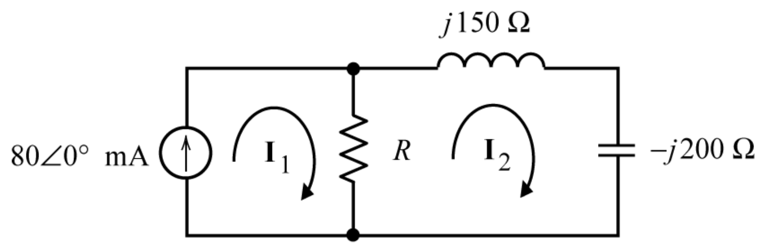
Solution:

Represent the circuit in the frequency domain using phasors and impedances. The mesh currents are

$$\mathbf{I}_1 = 0.080\angle 0° \text{ A}$$

and

$$\mathbf{I}_2 = 0.06656\angle 33.7° \text{ A}$$



Apply KVL to the right to get

$$(j150 - j200)(0.06656\angle 33.7°) + R(0.06656\angle 33.7° - 0.080\angle 0°) = 0$$

$$(-j50)(0.06656\angle 33.7°) + R(0.044376\angle 123.7°) = 0$$

$$R = \frac{(50\angle 90°)(0.06656\angle 33.7°)}{0.044376\angle 123.7°} = 74.9955 \simeq 75\ \Omega$$

MATLAB, 11/20/09

P10.6-23

This circuit shown in Figure P10.6-23 is at steady state. The voltage source voltages are given by

$$v_1(t) = 12 \cos(2t - 90°) \text{ V} \quad \text{and} \quad v_2(t) = 5 \cos(2t + 90°) \text{ V}$$

The currents are given by

$$i_1(t) = 744 \cos(2t - 118°) \text{ mA}, \quad i_2(t) = 540.5 \cos(2t + 100°) \text{ mA}$$

Determine the values of R_1, R_2, L and C.

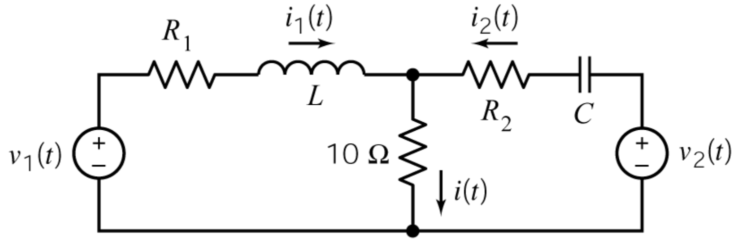
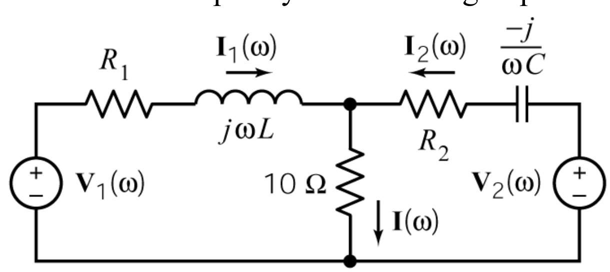


Figure P10.6-23

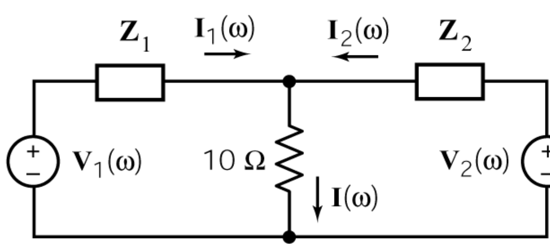
Solution: Represent the circuit in the frequency domain using impedances and phasors:



$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = 0.744\angle -118° + 0.5405\angle 100 = (-0.349 - j\,0.657) + (-0.094 + j0.532)$$
$$= (-0.349 - 0.094) + j(-0.657 + 0.532)$$
$$= -0.443 - j\,0.125$$
$$= 0.460\angle -164°$$

In the time domain $\qquad i(t) = 460 \cos(2t - 164°) \text{ mA}$

Replacing series impedances by equivalent impedances gives



$$\mathbf{Z}_1 = R_1 + j\,\omega\,L$$

and

$$\mathbf{Z}_2 = R_2 - j\frac{1}{\omega\,C}$$

From KVL

$$\mathbf{Z}_1\mathbf{I}_1+10\mathbf{I}-\mathbf{V}_1=0 \quad\Rightarrow\quad \mathbf{Z}_1=\frac{\mathbf{V}_1-10\mathbf{I}}{\mathbf{I}_1}=\frac{12\angle-90°-10\left(0.460\angle-164°\right)}{0.744\angle-118°}$$

$$=\frac{-j12-10\left(-0.443-j0.125\right)}{0.744\angle-118°}$$

$$=\frac{4.43-j10.75}{0.744\angle-118°}=\frac{11.63\angle-67.6°}{0.744\angle-118°}$$

$$=15.63\angle 50.4°$$

$$=10+j12\ \Omega$$

and

$$-\mathbf{Z}_2\mathbf{I}_2+\mathbf{V}_2-10\mathbf{I}=0 \quad\Rightarrow\quad \mathbf{Z}_2=\frac{\mathbf{V}_2-10\mathbf{I}}{\mathbf{I}_2}=\frac{5\angle 90°-10\left(0.460\angle-164°\right)}{0.5405\angle 100°}$$

$$=\frac{j5-10\left(-0.443-j0.125\right)}{0.5405\angle 100°}$$

$$=\frac{4.43+j6.25}{0.5405\angle 100°}=\frac{7.66\angle 54.7°}{0.5405\angle 100°}$$

$$=14.14\angle-55.3°$$

$$=10-j10\ \Omega$$

Next $\quad 10+j12=R_1+j\omega L=R_1+j2L \quad\Rightarrow\quad R_1=10\ \Omega \text{ and } L=\frac{12}{2}=6\text{ H}$

and $\quad 10-j10=R_2-j\frac{1}{\omega C}=R_2-j\frac{1}{2C} \quad\Rightarrow\quad R_2=10\ \Omega \text{ and } C=\frac{1}{2(10)}=0.05\text{ F}$

10.7 Thevenin and Norton Equivalent Circuits

P10.7-1 Determine the Thevenin equivalent circuit of the circuit shown in Figure P10.7-1 when (*a*) ω = 1000 rad/s, (*b*) ω = 2000 rad/s and (*c*) ω = 4000 rad/s.

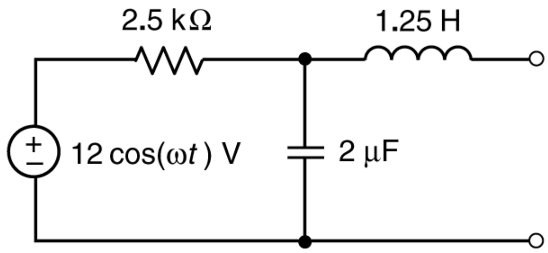
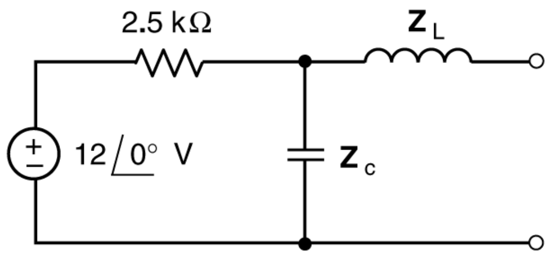
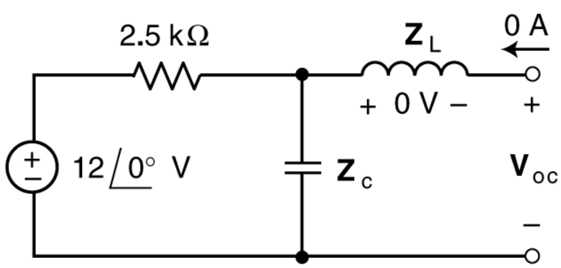


Figure P10.7-1

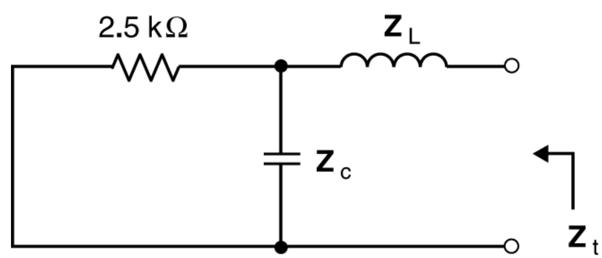
Solution: Represent the circuit in the frequency domain as



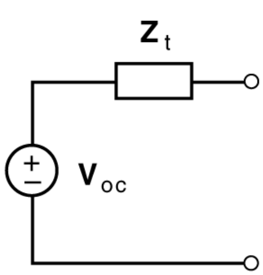
Determine the open circuit voltage and Thevenin impedance:



$$\mathbf{V}_{oc} = \frac{\mathbf{Z}_C}{2500 + \mathbf{Z}_C}(12\angle 0°)$$



$$\mathbf{Z}_t = \frac{2500\,\mathbf{Z}_C}{2500 + \mathbf{Z}_C} + \mathbf{Z}_L$$



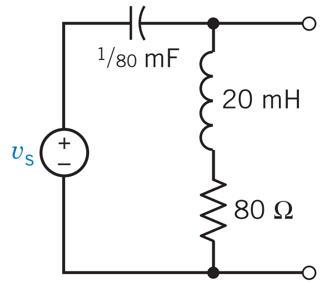
a. $\mathbf{V}_{oc} = 2.3534\angle -78.69°\text{ V}$ and $\mathbf{Z}_t = 775.22\angle 82.875°\ \Omega$

b. $\mathbf{V}_{oc} = 1.194\angle -84.29°\text{ V}$ and $\mathbf{Z}_t = 2252.6\angle 89.37°\ \Omega$

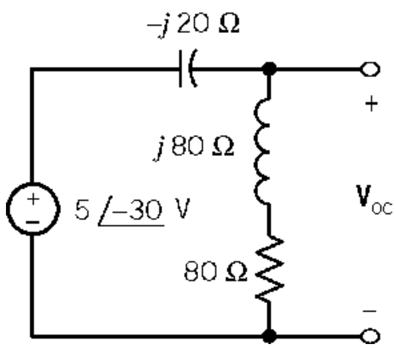
c. $\mathbf{V}_{oc} = 0.59925\angle -87.14°\text{ V}$ and $\mathbf{Z}_t = 4875.3\angle 89.93°\ \Omega$

The Thevenin Equivalent Circuit changes whenever the input frequency changes.

P10.7-2 Determine the Thevenin equivalent of this circuit when $v_s(t) = 5\cos(4000t - 30°)$ V.



Solution:

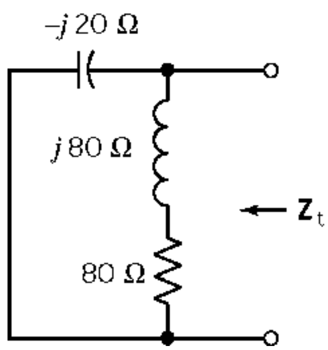


Find $\mathbf{V}_{oc}$:

$$\mathbf{V}_{oc} = (5\angle -30°)\left(\frac{80 + j80}{80 + j80 - j20}\right)$$

$$= (5\angle -30°)\left(\frac{80\sqrt{2}\angle -45°}{100\angle 36.90°}\right)$$

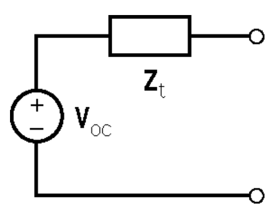
$$= 4\sqrt{2}\angle -21.9° \text{ V}$$



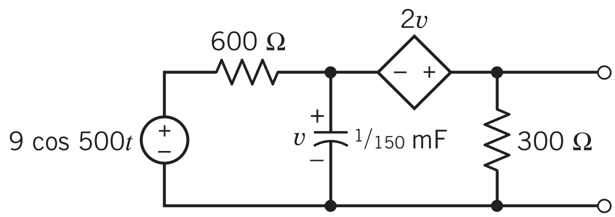
Find $\mathbf{Z}_t$:

$$\mathbf{Z}_t = \frac{(-j20)(80 + j80)}{-j20 + 80 + j80} = 23\angle -81.9° \ \Omega$$

The Thevenin equivalent is

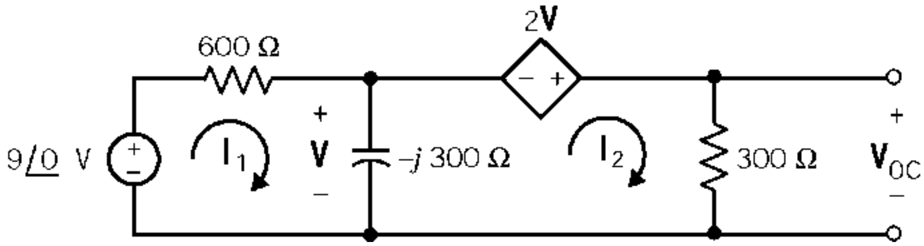


P10.7-3 Determine the Thevenin equivalent of this circuit



Solution:

First, determine $\mathbf{V}_{oc}$:



The mesh equations are

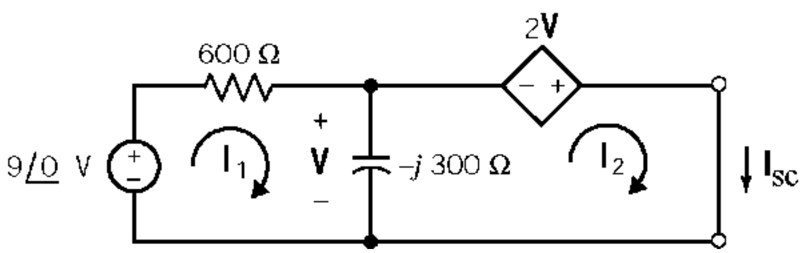
$$600\,\mathbf{I}_1 - j300(\mathbf{I}_1 - \mathbf{I}_2) = 9 \quad \Rightarrow \quad (600 - j300)\,\mathbf{I}_1 + j300\,\mathbf{I}_2 = 9\angle 0°$$

$$-2\,\mathbf{V} + 300\,\mathbf{I}_2 - j300(\mathbf{I}_1 - \mathbf{I}_2) = 0 \quad \text{and} \quad \mathbf{V} = j300(\mathbf{I}_1 - \mathbf{I}_2) \quad \Rightarrow \quad j3\,\mathbf{I}_1 + (1 - j3)\,\mathbf{I}_2 = 0$$

Using Cramer's rule: $\mathbf{I}_2 = 0.0124\angle -16° \text{ A}$

Then $\mathbf{V}_{oc} = 300\,\mathbf{I}_2 = 3.71\angle -16° \text{ V}$

Next, determine $\mathbf{I}_{sc}$:

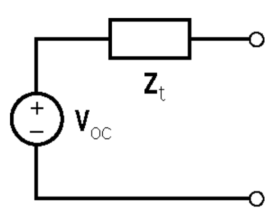


$$-2\,\mathbf{V} - \mathbf{V} = 0 \quad \Rightarrow \quad \mathbf{V} = 0 \quad \Rightarrow \quad \mathbf{I}_{sc} = \frac{9\angle 0°}{600} = 0.015\angle 0° \text{ A}$$

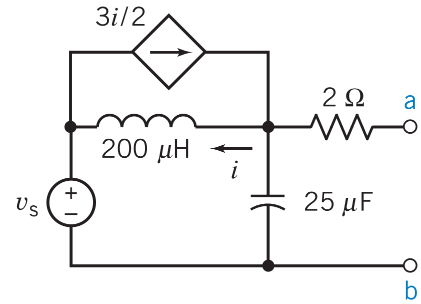
The Thevenin impedance is

$$\mathbf{Z}_T = \frac{\mathbf{V}_{oc}}{\mathbf{I}_{sc}} = \frac{3.545\angle -16°}{0.015\angle 0°} = 247\angle -16° \ \Omega$$

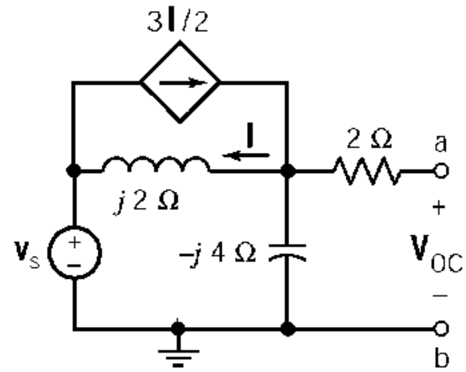
The Thevenin equivalent is



P10.7-4 Determine the Thevenin equivalent of this circuit when $v_s(t)$ = 10 cos (10,000t–53.1°) V.



Solution:



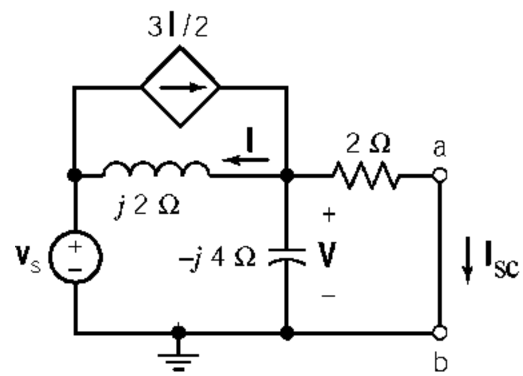
First, determine $\mathbf{V}_{oc}$:

The node equation is:

$$\frac{\mathbf{V}_{oc}}{-j4}+\frac{\mathbf{V}_{oc}-(6+j8)}{j2}-\frac{3}{2}\left(\frac{\mathbf{V}_{oc}-(6+j8)}{j2}\right)=0$$

$$\mathbf{V}_{oc}=3+j4=5\angle 53.1° \text{ V}$$

$$\mathbf{V}_s=10\angle 53°=6+j8 \text{ V}$$



Next, determine $\mathbf{I}_{sc}$:

The node equation is:

$$\frac{\mathbf{V}}{2}+\frac{\mathbf{V}}{-j4}+\frac{\mathbf{V}-(6+j8)}{j2}-\frac{3}{2}\left[\frac{\mathbf{V}-(6+j8)}{j2}\right]=0$$

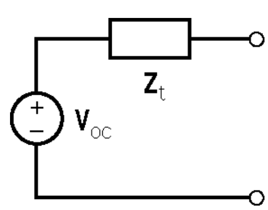
$$\mathbf{V}=\frac{3+j4}{1-j}$$

$$\mathbf{I}_{sc}=\frac{\mathbf{V}}{2}=\frac{3+j4}{2-j2}$$

$$\mathbf{V}_s=10\angle 53°=6+j8 \text{ V}$$

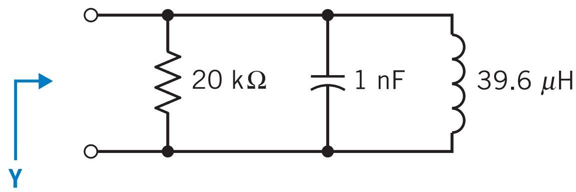
The Thevenin impedance is $\mathbf{Z}_T=\dfrac{\mathbf{V}_{oc}}{\mathbf{I}_{sc}}=3+j4\left(\dfrac{2-j2}{3+j4}\right)=2-j2 \ \Omega$

The Thevenin equivalent is



(checked: LNAP 7/18/04)

P10.7-5 Determine the frequency at which Y is a pure conductance



Solution:

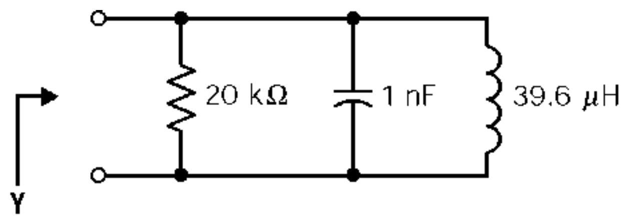
$$\mathbf{Y} = G + \mathbf{Y}_L + \mathbf{Y}_C$$

$$\mathbf{Y} = G \text{ when } \mathbf{Y}_L + \mathbf{Y}_C = 0 \text{ or } \frac{1}{j\omega L} + j\omega C = 0$$

$$\omega_O = \frac{1}{\sqrt{LC}},\ f_O = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{39.6\times10^{-15}}}$$

$$= 0.07998\times10^{7}\ \text{Hz}=800\ \text{kHz}$$

(80 on the dial of the radio)



P 10.7-6 Consider the circuit of Figure P 10.7-6, where we wish to determine the current **I**. Use a series of source transformations to reduce the part of the circuit connected to the 2-Ω resistor to a Norton equivalent circuit, and then find the current in the 2-Ω resistor by current division.

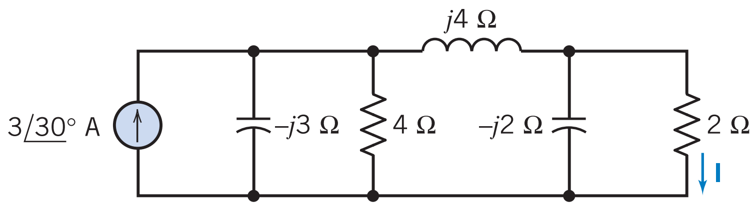
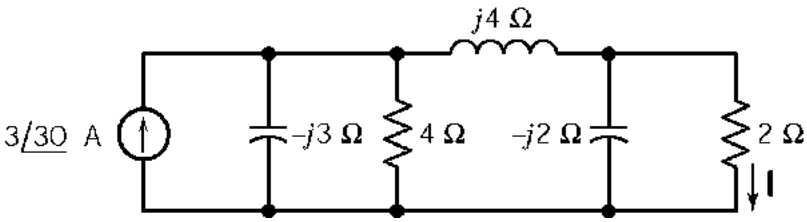
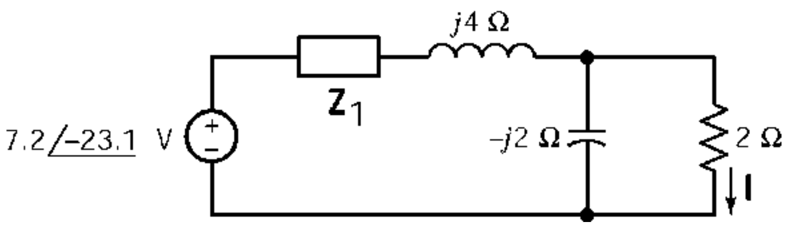


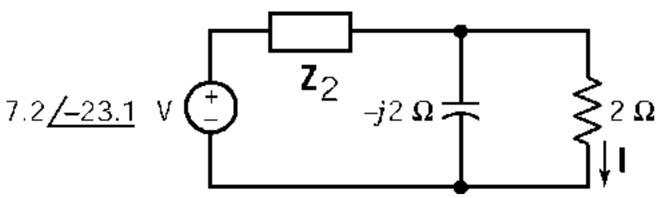
Figure P 10.7-6

Solution:

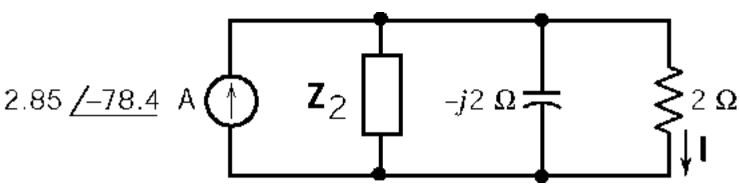




$$\mathbf{Z}_1 = \frac{(-j3)(4)}{-j3+4} = 2.4\angle -53.1^\circ\ \Omega$$
$$= 1.44 - j1.92\ \Omega$$



$$\mathbf{Z}_2 = \mathbf{Z}_1 + j4$$
$$= 1.44 + j2.08$$
$$= 2.53\angle 55.3^\circ\ \Omega$$



$$\mathbf{Z}_3 = 3.51\angle -37.9^\circ\ \Omega$$
$$= 2.77 - j2.16\ \Omega$$

$$\mathbf{I} = (2.85\angle -78.4^\circ)\left(\frac{3.51\angle -37.9^\circ}{2.77 - j2.16 + 2}\right) = (2.85\angle -78.4^\circ)\frac{(3.51\angle -37.9^\circ)}{(5.24\angle -24.4^\circ)} = 1.9\angle -92^\circ\ \text{A}$$

(checked: LNAP 7/18/04)

P 10.7-7 For the circuit of Figure P 10.7-7, determine the current **I** using a series of source transformations.

The source has $\omega = 25 \times 10^3$ rad/s.

Answer: $i(t) = 4\cos(25{,}000t - 44°)$ mA

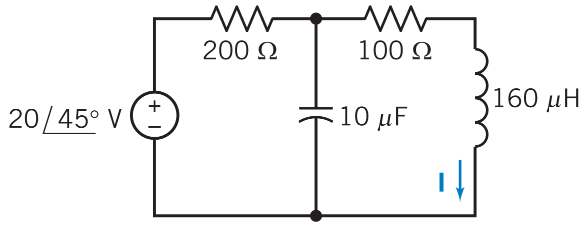
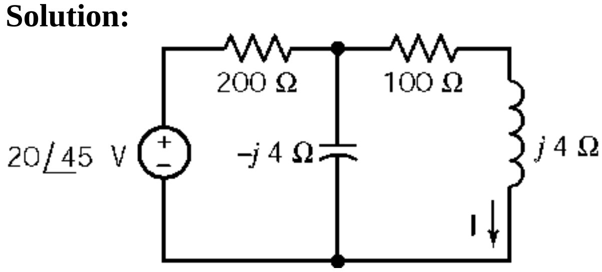
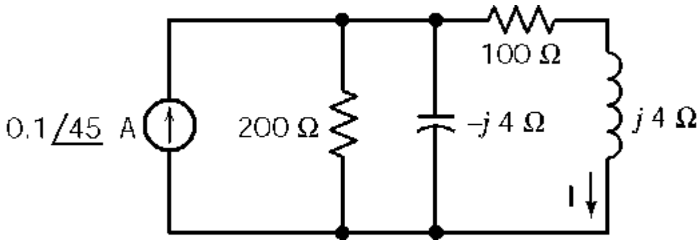
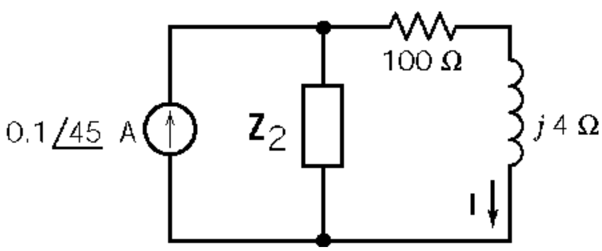


Figure P 10.7-7

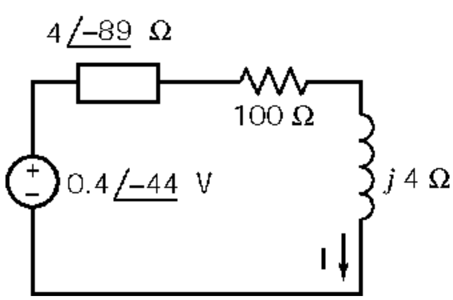
Solution:







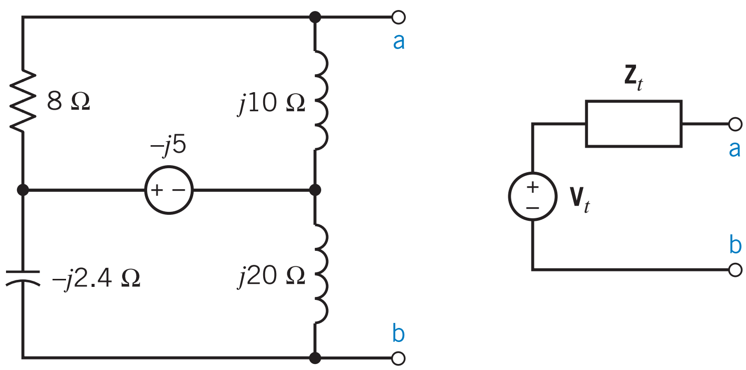
$$\mathbf{Z}_2 = \frac{(200)(-j4)}{200-j4} = 4\angle -88.8°\ \Omega$$



$$\mathbf{I} = \frac{0.4\angle -44°}{-4j+100+j4} = 4\angle -44°\ \text{ mA}$$

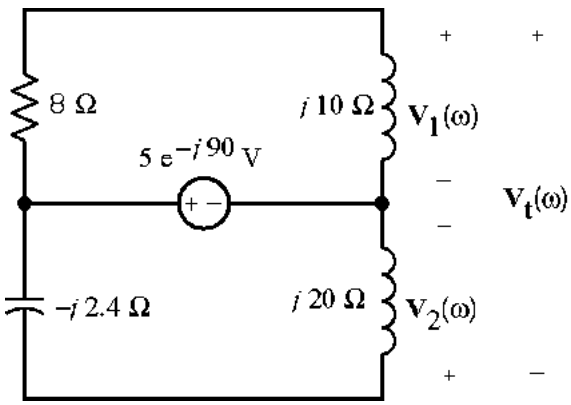
$$i(t) = 4\cos(25000t - 44°)\ \text{mA}$$

P10.7-8 Determine the Thevenin equivalent of the circuit in (*a*):



(*a*) (*b*)

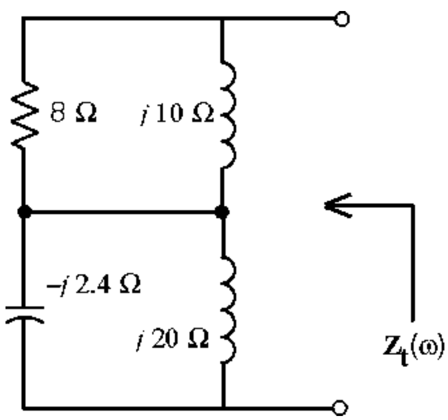
Solution:



$$\mathbf{V}_1 = \frac{j10}{8+j10}\,5e^{-j90} = 3.9\,e^{-j51}$$

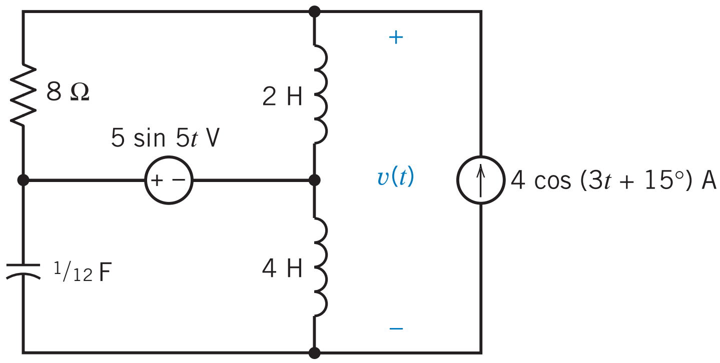
$$\mathbf{V}_2 = \frac{j20}{j20-j2.4}\,5e^{-j90} = 5.68\,e^{-j90}$$

$$\mathbf{V}_t = \mathbf{V}_1 - \mathbf{V}_2 = 3.9\,e^{-j51} - 5.68\,e^{-j90}$$
$$= 3.58\,e^{j47}$$

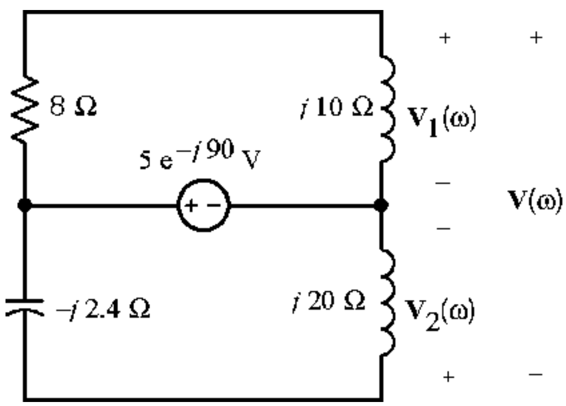


$$\mathbf{Z}_t = \frac{8(j10)}{8+j10} + \frac{-j2.4(j20)}{-j2.4+j20} = 4.9 + j\,1.2$$

P10.7-9 Determine the voltage *v*(*t*) for this circuit:



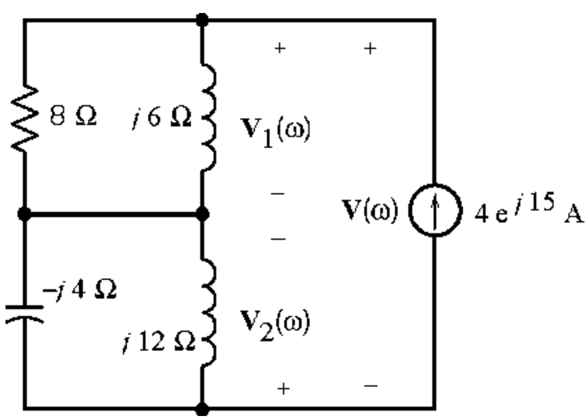
Solution:



$$\mathbf{V}_1(\omega) = \frac{j10}{8+j10} 5\,e^{-j90} = 3.9\,e^{-j51}$$

$$\mathbf{V}_2(\omega) = \frac{j20}{j20-j2.4} 5\,e^{-j90} = 5.68\,e^{-j90}$$

$$\mathbf{V}(\omega) = \mathbf{V}_1(\omega) - \mathbf{V}_2(\omega) = 3.9\,e^{-j51} - 5.68\,e^{-j90}$$
$$= 3.58\,e^{j47}$$



$$\mathbf{V}_1(\omega) = \frac{8(j6)}{8+j6} 4\,e^{j15} = 19.2\,e^{j68}$$

$$\mathbf{V}_2(\omega) = \frac{j12(-j4)}{j12-j4} 4\,e^{j15} = 24\,e^{-j75}$$

$$\mathbf{V}(\omega) = \mathbf{V}_1(\omega) + \mathbf{V}_2(\omega) = 14.4\,e^{-j22}$$

Using superposition: *v*(*t*) = 3.58 cos (5*t* + 47°) + 14.4 cos (3*t* - 22°) V.

10.8 Superposition

P10.8-1 Determine the steady state current $i(t)$ in the circuit shown in Figure P10.8-1 when the voltage source voltages are

$$v_{s1}(t) = 12 \cos(2500\,t) \text{ V} \quad \text{and} \quad v_{s2}(t) = 12 \cos(4000\,t) \text{ V}$$

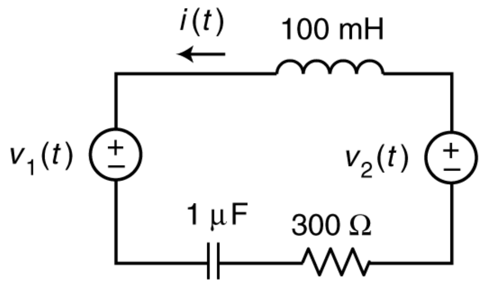
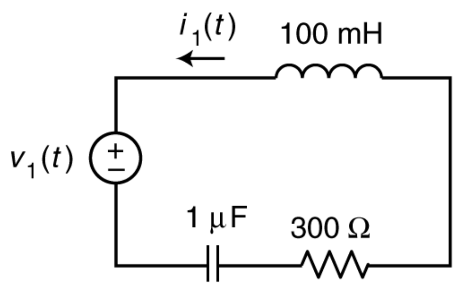
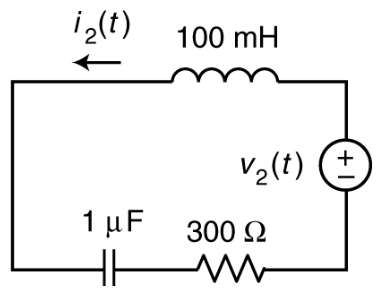


Figure 10.8-1

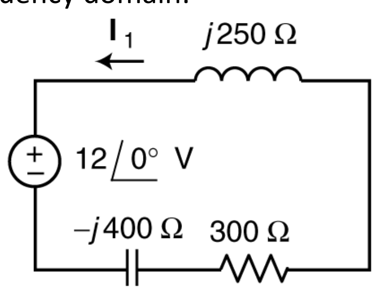
Solution:

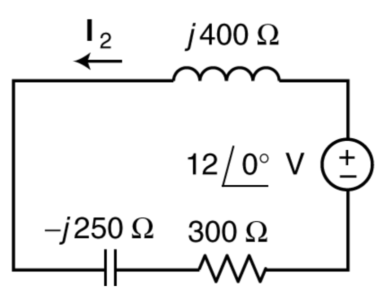
Use superposition *in the time domain*. These circuits can be used to find the part of i_o caused by v_{s1} and the part of i_o caused by v_{s2}.





In the frequency domain:





$$\mathbf{I}_1 = \frac{12\angle 0°}{300 + j\,250 - j400} = 0.03578\angle 26.6° \text{ V}$$

$$\mathbf{I}_2 = \frac{12\angle 0°}{300 - j\,250 + j400} = 0.03578\angle -26.6° \text{ V}$$

In the time domain

$$i_{o1}(t) = 35.78\cos(2500t + 26.6°) \text{ mA and } i_{o2}(t) = 35.78\cos(4000t - 26.6°) \text{ mA}$$

and

$$i_o(t) = i_{o1}(t) + i_{o2}(t) = 35.78\cos(2500t + 26.6°) + 35.78\cos(4000t - 26.6°) \text{ mA V}$$

P10.8-2 Determine the steady state voltage *v*(*t*) in the circuit shown in Figure P10.8-2 when the current source current is (*a*) 400 rad/s and (*b*) 200 rad/s.

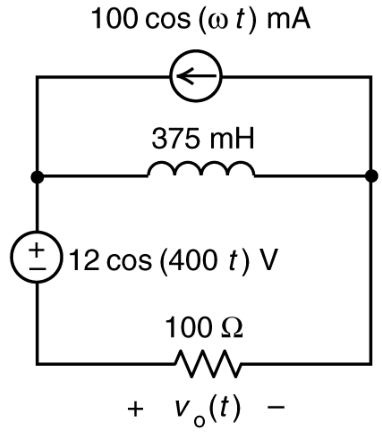
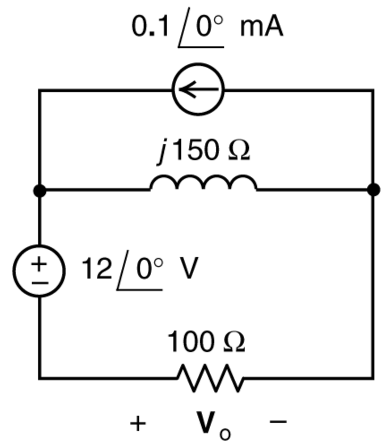


Figure 10.8-2

Solution:

(*a*) Represent the circuit in the frequency domain as



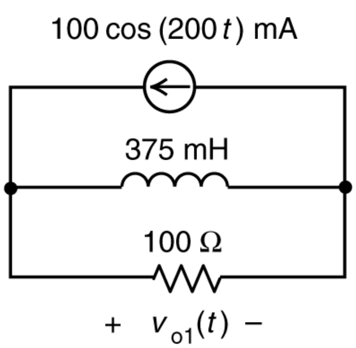
Use superposition *in the frequency domain* to write

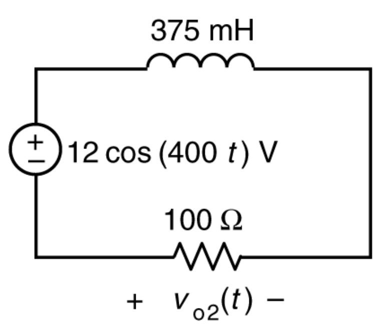
$$\mathbf{V}_o = -\frac{100}{100+j150}(12\angle 0°)+100\frac{j150}{100+j150}(0.1\angle 0°)$$

$$=\frac{-1200+j1500}{100+j150}=10.66\angle 72.35°$$

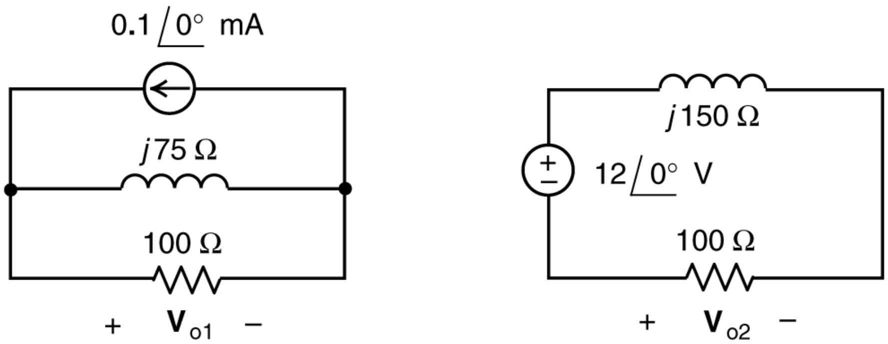
In the time domain $v_o(t)=10.66\cos(400t+72.35)\ \text{V}$

(*b*) Use superposition *in the time domain*. These circuits can be used to find the part of v_o caused by the current source and the part of v_o caused by the voltage source.





In the frequency domain:



$$\mathbf{V}_{o1} = 100\frac{j75}{100+j75}(0.1\angle 0°) = 6\angle 53.1° \text{ V}$$

$$\mathbf{V}_{o2} = -\frac{100}{100+j150}(12\angle 0°) = 6.656\angle 123.7° \text{ V}$$

In the time domain

$$v_{o1}(t) = 6\cos(200t + 53.1°) \text{ V} \text{ and } v_{o2}(t) = 6.656\cos(400t + 123.7°) \text{ V}$$

and

$$v_o(t) = v_{o1}(t) + v_{o2}(t) = 6\cos(200t + 53.1°) + 6.656\cos(400t + 123.7°) \text{ V}$$

P10.8-3 Determine the steady state current *i*(*t*) in the circuit shown in Figure P10.8-3 when the voltage source voltage is

$$v_s(t) = 8 + 8\cos(400t - 135°)\text{ V}$$

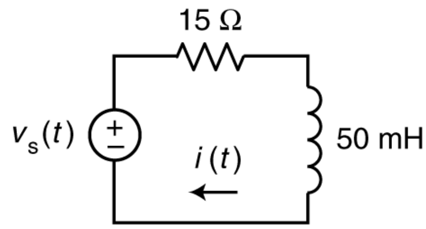
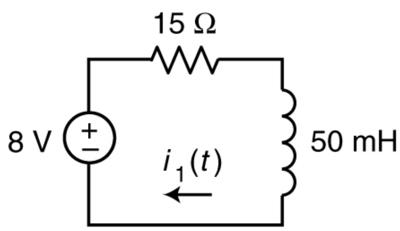
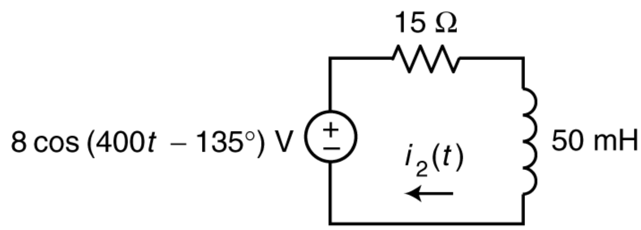


Figure 10.8-3

Solution:

Use superposition *in the time domain*:

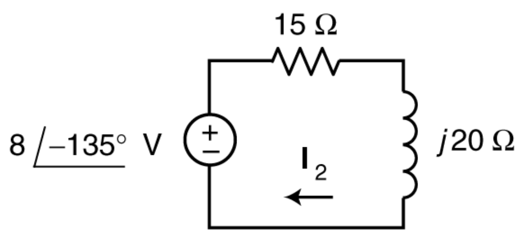




An inductor in a dc circuit acts like a short circuit so:

$$i_1(t) = \frac{8}{15} = 0.533\text{ A}$$

Represent the right circuit the frequency domain:



$$\mathbf{I}_2 = \frac{8\angle -135°}{15 + j20} = 0.32\angle -188°\text{ A}$$

In the time domain $i_2(t) = 0.32\cos(400t - 188°)\text{ A}$

and

$$i(t) = i_1(t) + i_2(t) = 0.533 + 0.32\cos(400t - 188°)\text{ A}$$

P10.8-4 Determine the steady state current $i(t)$ in the circuit shown in Figure P10.8-4 when the voltage source voltages are

$$v_{s1}(t) = 10 \cos(800t + 30°) \text{ V} \quad \text{and} \quad v_{s2}(t) = 15 \sin(200t - 30°) \text{ V}$$

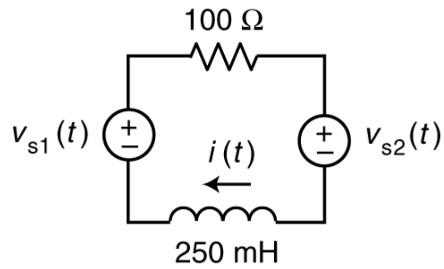
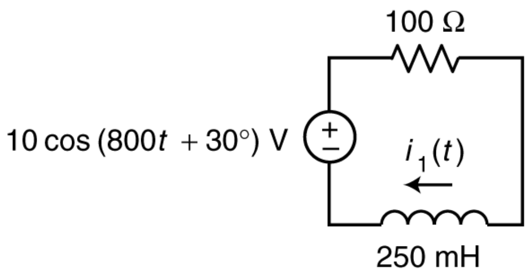
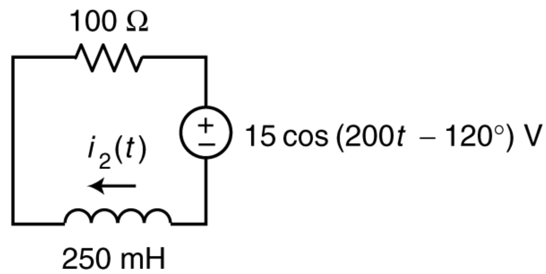


Figure 10.8-4

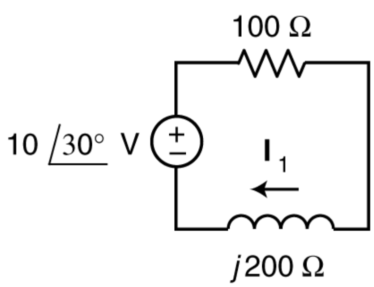
Solution:

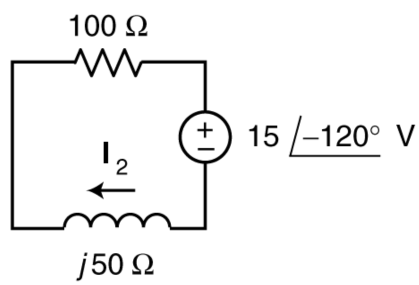
Use superposition *in the time domain*:





Represent these circuits the frequency domain:





$$\mathbf{I}_1 = \frac{10\angle 30°}{100 + j200} = 0.0447\angle -33.4° \text{ A} \quad \text{and} \quad \mathbf{I}_2 = -\frac{15\angle -120°}{100 + j50} = 0.1342\angle 33.4° \text{ A}$$

In the time domain

$$i(t) = i_1(t) + i_2(t) = 44.7\cos(800t - 33.4°) + 134.2\cos(200t + 33.4°) \text{ mA}$$

P 10.8-5 The input to the circuit shown in Figure P 10.8-5 is the current source current

$i_s(t) = 36 \cos(25t) + 48 \cos(50t + 45°)$ mA

Determine the steady-state current, $i(t)$.

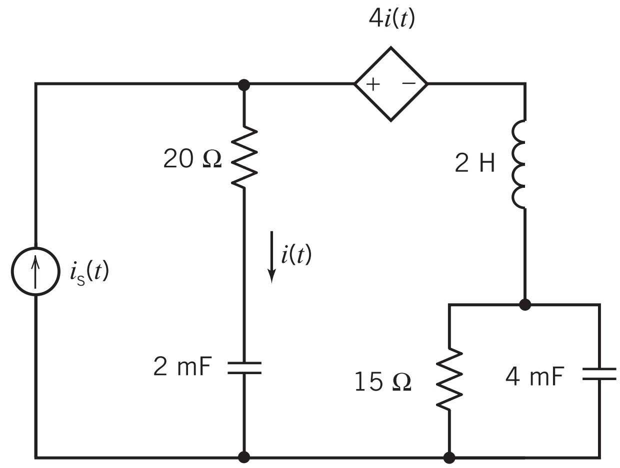


Figure P 10.8-5

Solution:

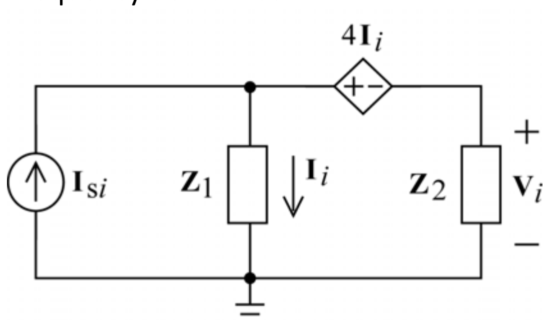
Use superposition in the time domain. Let

$$i_{s1}(t) = 36\cos(25t) \text{ mA} \quad \text{and} \quad i_{s2}(t) = 48\cos(50t + 45°) \text{ mA}$$

We will find the response to each of these inputs separately. Let $i_i(t)$ denote the response to $i_{si}(t)$ for i = 1,2. The sum of the two responses will be $i(t)$, i.e.

$$i(t) = i_1(t) + i_2(t)$$

Represent the circuit in the frequency domain as



Use KVL to get

$$\mathbf{V}_i = \mathbf{Z}_i \mathbf{I}_i - 4\mathbf{I}_i$$

Apply KCL to the supernode corresponding to the dependent voltage source.

$$\mathbf{I}_{si} = \mathbf{I}_i + \frac{\mathbf{V}_i}{\mathbf{Z}_2} = \frac{\mathbf{Z}_1 + \mathbf{Z}_2 - 4}{\mathbf{Z}_2} \mathbf{I}_i$$

or

$$\mathbf{I}_i = \frac{\mathbf{Z}_2 \mathbf{I}_{si}}{\mathbf{Z}_1 + \mathbf{Z}_2 - 4}$$

Consider the case i = 1 : $i_{s1}(t)$ = 26cos(25t) mA.

Here ω = 25 rad/s and

$$\mathbf{I}_{si} = 36\angle 0° \text{ mA}$$

$$\mathbf{Z}_1 = 20 + \frac{1}{j(25)(0.002)} = 20 - j20\ \Omega$$

$$\mathbf{Z}_2 = j50 + \left(15 \,||\, \frac{1}{j(25)(0.004)}\right) = 43.3\angle 83.9°\ \Omega$$

and

$$\mathbf{I}_1 = 50.4\angle 35.7° \text{ mA}$$

so

$$i(t) = 50.4\cos(25t + 35.7°) \text{ mA}$$

Next consider i = 2 : i_{s2} = 48cos(50t + 45°) mA.

Here ω = 50 rad/s and

$$\mathbf{I}_{s2} = 48\angle 45° \text{ mA}$$

$$\mathbf{Z}_1 = 20 + \frac{1}{j(50)(0.002)} = 20 - j10\ \Omega$$

$$\mathbf{Z}_2 = j100 + \left(15 \,||\, \frac{1}{j(50)(0.004)}\right) = 95.5\angle 89.1°\ \Omega$$

(Notice that $\mathbf{Z}_1$ and $\mathbf{Z}_2$ change when ω changes.)

$$\mathbf{I}_2 = 52.5\angle 55.7° \text{ mA}$$

so

$$i_2(t) = 52.5\cos(50t + 55.7°) \text{ mA}$$

Finally, using superposition <u>in the time</u> domain gives

$$i(t) = 50.4\cos(25t + 35.7°) + 52.5\cos(50t + 55.7°) \text{ mA}$$

(checked: LNAP 8/7/04)

P 10.8-6 The inputs to the circuit shown in Figure P 10.8-6 are

$v_{s1}(t) = 30 \cos (20t + 70°)$ V

and $v_{s2}(t) = 18 \cos (10t - 15°)$ V

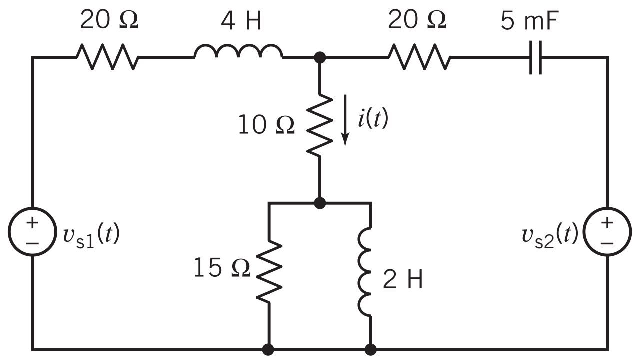
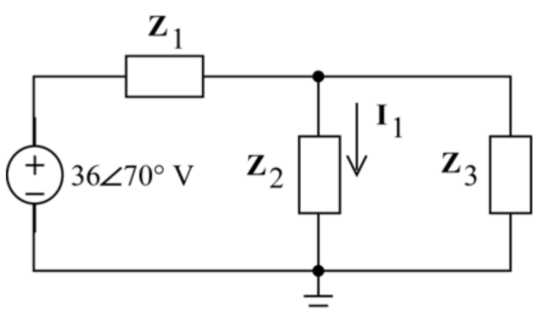


Figure P 10.8-6

Solution:

Use superposition in the time domain. Let $i_1(t)$ be the part of $i(t)$ due to $v_{s1}(t)$ and $i_2(t)$ be the part of $i(t)$ due to $v_{s2}(t)$. To determine $i_1(t)$, set $v_{s2}(t) = 0$. Represent the resulting circuit in the frequency domain to get



where

$$\mathbf{Z}_1 = 20 + j80 = 82.46\angle 76°\ \Omega$$

$$\mathbf{Z}_2 = 10 + (j40 \,||\, 15) = 23.15 + j4.93 = 23.67\angle 12°\ \Omega$$

$$\mathbf{Z}_3 = 20 + \frac{1}{j(20)(0.005)} = 20 - j10 = 22.36\angle -26.6°\ \Omega$$

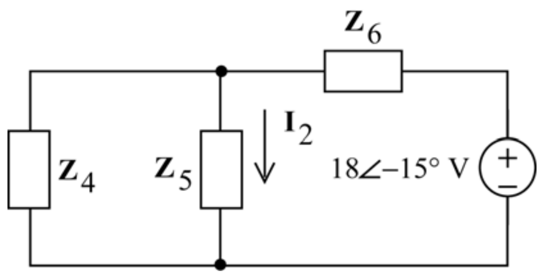
Next, using Ohm's law and current division gives

$$\mathbf{I}_1 = \frac{30\angle 70°}{\mathbf{Z}_1 + (\mathbf{Z}_2 \,||\, \mathbf{Z}_3)} \times \frac{\mathbf{Z}_3}{\mathbf{Z}_2 + \mathbf{Z}_3} = \frac{\mathbf{Z}_3(30\angle 70°)}{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_1\mathbf{Z}_3} = 0.182\angle -17.6°\ \text{A}$$

so

$$i(t) = 0.182 \cos(20t - 17.6°)\ \text{A}$$

To determine $i_2(t)$, set $v_{s1}(t) = 0$. Represent the resulting circuit in the frequency domain to get



where

$$\mathbf{Z}_4 = 20 + j40 = 44.72\angle 63.4°\ \Omega$$

$$\mathbf{Z}_5 = 10 + \left(j20 \,||\, 15\right) = 19.6 + j7.2 = 20.88\angle 20.2°\ \Omega$$

$$\mathbf{Z}_6 = 20 + \frac{1}{j(10)(0.005)} = 20 - j20 = 28.28\angle -45°\ \Omega$$

Next, using Ohm's law and current division gives

$$\mathbf{I}_2 = \frac{18\angle -15°}{\mathbf{Z}_6 + \left(\mathbf{Z}_4 \,||\, \mathbf{Z}_5\right)} \times \frac{\mathbf{Z}_4}{\mathbf{Z}_4 + \mathbf{Z}_5} = \frac{\mathbf{Z}_1\left(18\angle -15°\right)}{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_1\mathbf{Z}_3} = 0.377\angle 18°\ \text{A}$$

so

$$i_2(t) = 0.377\cos\left(10t + 18°\right)\ \text{A}$$

Using superposition,

$$i(t) = i_1(t) + i_2(t) = 0.182\cos\left(20t - 17.6°\right) + 0.377\cos\left(10t + 18°\right)\ \text{A}$$

(checked: LNAP 8/8/04)

P 10.8-7 The input to the circuit shown in Figure P 10.8-7 is the voltage source voltage

$$v_s(t) = 5 + 30\cos(100t) \text{ V}$$

Determine the steady-state current, $i(t)$.

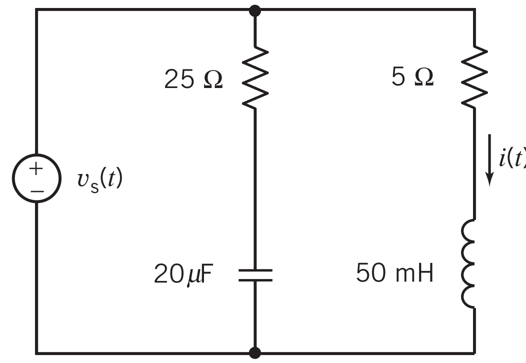


Figure P 10.8-7

Solution:

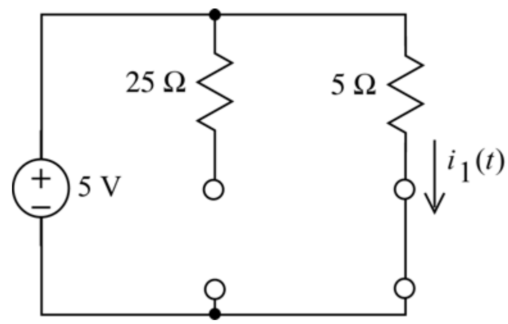
Use superposition in the time domain. Let $v_{s1}(t)$ = 5 V and $v_{s2}(t)$ = 30cos(100t) V.

Find the steady state response to $v_{s1}(t)$.

When the input is constant and the circuit is at steady state, the capacitor acts like an open circuit and the inductor acts like a short circuit.

So

$$i_1(t) = \frac{5}{5} = 1 \text{ A}$$



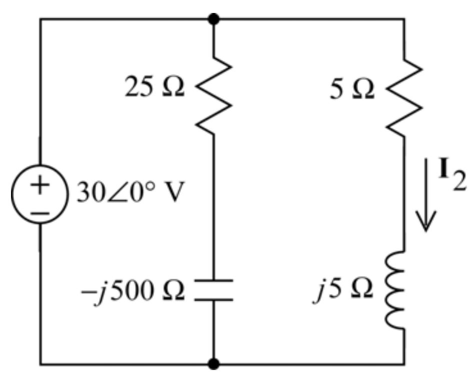
Find the steady state response to $v_{s2}(t)$.

Represent the circuit in the frequency domain using impedances and phasors.

$$\mathbf{I}_2 = \frac{30\angle 0°}{5 + j5} = 4.243\angle -45 \text{ A}$$

So

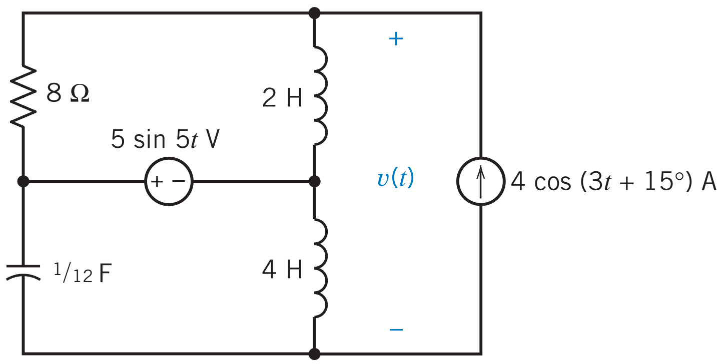
$$i_2(t) = 4.243\cos(100t - 45°) \text{ A}$$



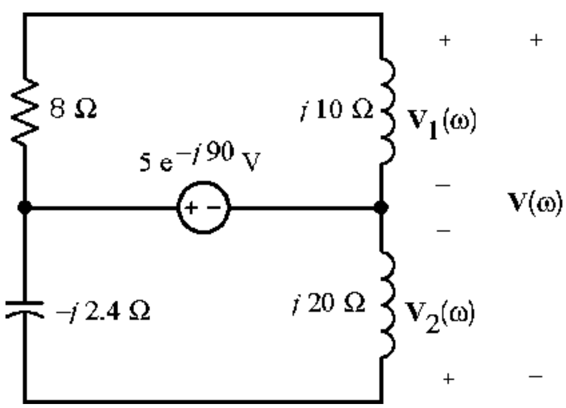
Using superposition

$$i(t) = i_1(t) + i_2(t) = 1 + 4.243\cos(100t - 45°)$$

P10.8-8 Determine the voltage *v*(*t*) for the circuit



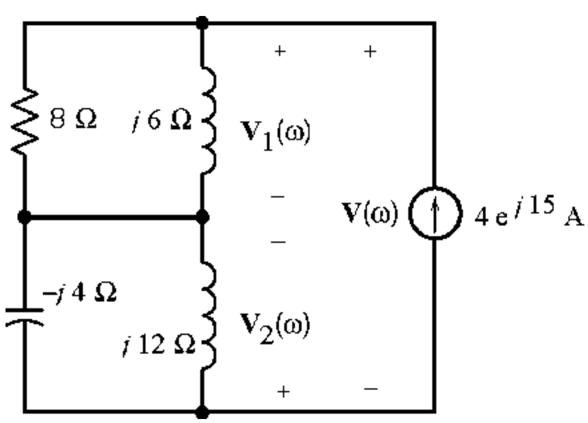
Solution:



$$\mathbf{V}_1(\omega)=\frac{j10}{8+j10}5e^{-j90}=3.9e^{-j51}$$

$$\mathbf{V}_2(\omega)=\frac{j20}{j20-j2.4}5e^{-j90}=5.68e^{-j90}$$

$$\mathbf{V}(\omega)=\mathbf{V}_1(\omega)-\mathbf{V}_2(\omega)=3.9e^{-j51}-5.68e^{-j90}$$
$$=3.58e^{j47}$$



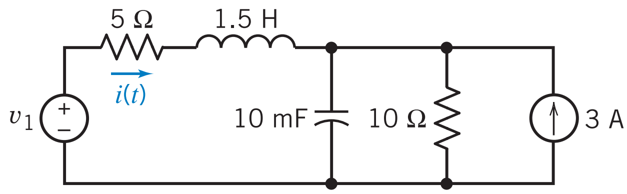
$$\mathbf{V}_1(\omega)=\frac{8(j6)}{8+j6}4e^{j15}=19.2e^{j68}$$

$$\mathbf{V}_2(\omega)=\frac{j12(-j4)}{j12-j4}4e^{j15}=24e^{-j75}$$

$$\mathbf{V}(\omega)=\mathbf{V}_1(\omega)+\mathbf{V}_2(\omega)=14.4e^{-j22}$$

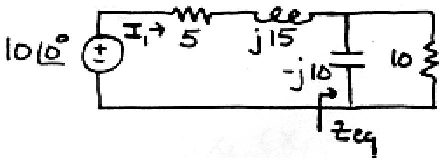
Using superposition: *v*(*t*) = 3.58 cos (5*t* + 47°) + 14.4 cos (3*t* - 22°) V.

P10.8-9 Determine the current $i(t)$ for this circuit when $v_1(t)$=10cos(10t) V



Solution:

Use superposition. First, find the response to the voltage source acting alone:



$$\mathbf{Z}_{eq} = \frac{-j10 \cdot 10}{10-j10} = 5(1-j)\ \Omega$$

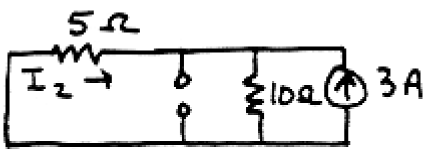
Replacing the parallel elements by the equivalent impedance. The write a mesh equation :

$$-10+5\,\mathbf{I}_1 + j15\,\mathbf{I}_1 + 5(1-j)\,\mathbf{I}_1 = 0 \quad \Rightarrow \quad \mathbf{I}_1 = \frac{10}{10+j10} = 0.707\angle -45°\ \text{A}$$

Therefore:

$$i_1(t) = 0.707\cos(10t-45^{\circ})\ \text{A}$$

Next, find the response to the dc current source acting alone:



Current division: $I_2 = -\frac{10}{15} \times 3 = -2\ \text{A}$

Using superposition:

$$i(t) = 0.707\cos(10t-45°)-2\ \ \text{A}$$

Section 10-9: Phasor Diagrams

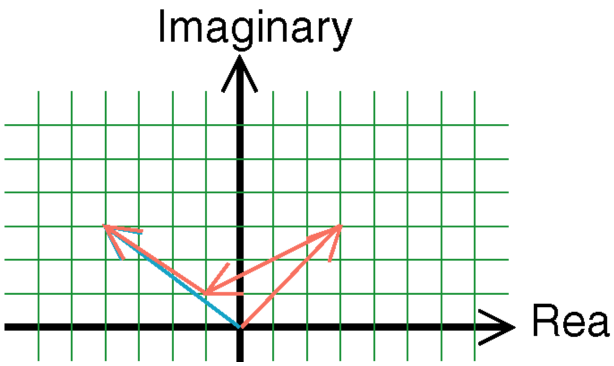
P 10.9-1 Using a phasor diagram, determine **V** when

$$\mathbf{V} = \mathbf{V}_1 - \mathbf{V}_2 + \mathbf{V}_3^* \text{ and } \mathbf{V}_1 = 3 + j3,\ \mathbf{V}_2 = 4 + j2, \text{ and } \mathbf{V}_3 = -3 - j2. \text{ (Units are volts.)}$$

Answer: **V** = 5 $\angle$143.1° V

.

Solution:



$$\mathbf{V} = \mathbf{V}_1 - \mathbf{V}_2 + \mathbf{V}_3^* = (3+j3) - (4+j2) + (-3-j2)^* = -4 + j3$$

P 10.9-2 Consider the series *RLC* circuit of Figure P 10.9-2 when

$$R = 10\ \Omega,\ L = 1\text{ mH},\ C = 100\ \mu\text{F, and } \omega = 10^3\text{ rad/s.}$$

Find **I** and plot the phasor diagram

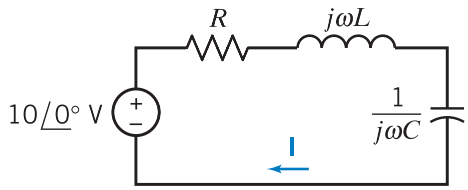
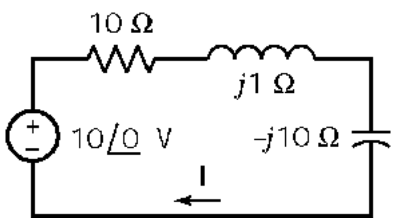


Figure P 10.9-2

Solution:



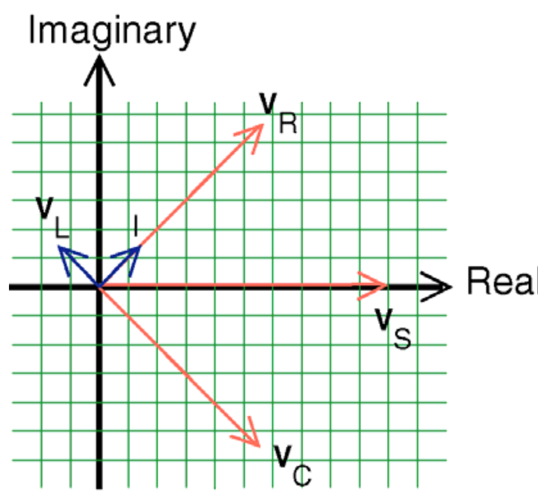
$$\mathbf{I} = \frac{10\angle 0°}{10 + j1 - j10} = 0.74\angle 42°\text{ A}$$

$$\mathbf{V}_R = R\,\mathbf{I} = 7.4\angle 42°\text{ V}$$

$$\mathbf{V}_L = \mathbf{Z}_L\mathbf{I} = (1\angle 90°)(0.74\angle 42°) = 0.74\angle 132°\text{ V}$$

$$\mathbf{V}_C = \mathbf{Z}_C\mathbf{I} = (10\angle -90°)(0.74\angle 42°) = 7.4\angle -48°\text{ V}$$

$$\mathbf{V}_S = 10\angle 0°\text{ V}$$



10.10 Op Amps in AC Circuits

P10.10-1 The input to the circuit shown in Figure P10.10-1 is the voltage

$$v_s(t) = 2.4\cos(500t)\ \text{V}.$$

Determine the output voltage $v_o(t)$.

Answer: $v_o(t) = 6.788\cos(500t + 135°)\ \text{V}$

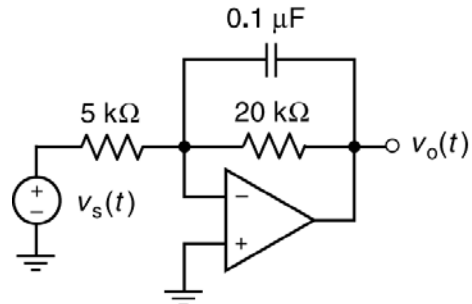
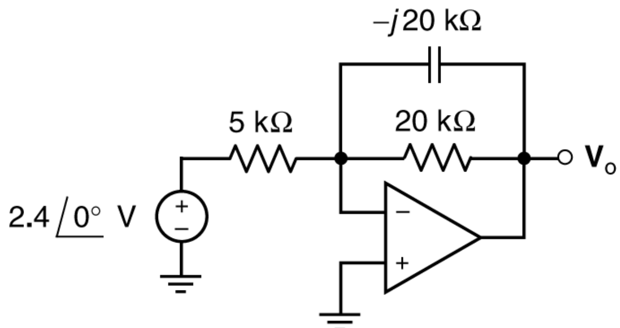


Figure P10.10-1

Solution:

Represent the circuit in the frequency domain as



Recognizing this circuit as an inverting amplifier, we can write

$$\mathbf{V}_o = \left(-\frac{20 \,\|\, -j\,20}{5}\right)(2.4\angle 0) = \left((1\angle 180°)\frac{14.14\angle -45°}{5}\right)(2.4\angle 0) = 6.788\angle 135°\ \text{V}$$

In the time domain $\quad v_o(t) = 6.788\cos(500t + 135°)\ \text{V}$

(Checked using LNAPAC 3/15/12)

P10.10-2 The input of the circuit shown in Figure P10.10-2 is the voltage

$$v_s(t) = 1.2\cos(400t + 20°)\ \text{V}.$$

Determine the output voltage $v_o(t)$.

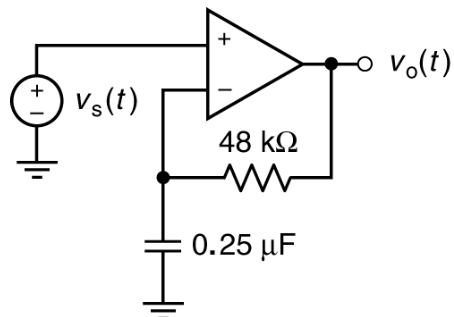
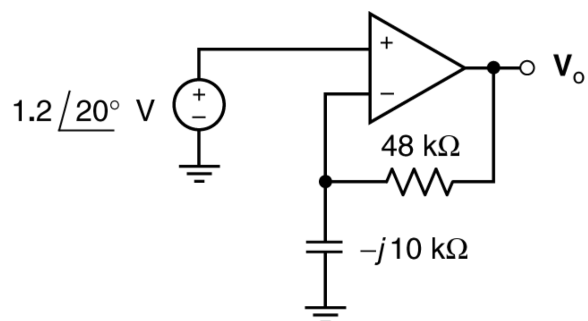


Figure P10.10-2

Solution:

Represent the circuit in the frequency domain as



Recognizing this circuit as a noninverting amplifier, we can write

$$\mathbf{V}_0 = \left(1 + \frac{48}{-j10}\right)(1.2\angle 20°) = (1 + j4.8)(1.2\angle 20°) = 5.88\angle 98°\ \text{V}$$

In the time domain $\quad v_o(t) = 5.88\cos(400t + 98°)\ \text{V}$

(Checked using LNAPAC 3/15/12)

P10.10-3 The input of the circuit shown in Figure P10.10-3 is the voltage

$$v_s(t) = 3.2\cos(200t)\ \text{V}.$$

Determine the output voltage $v_o(t)$.

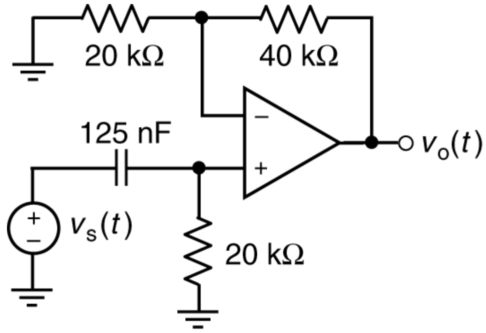
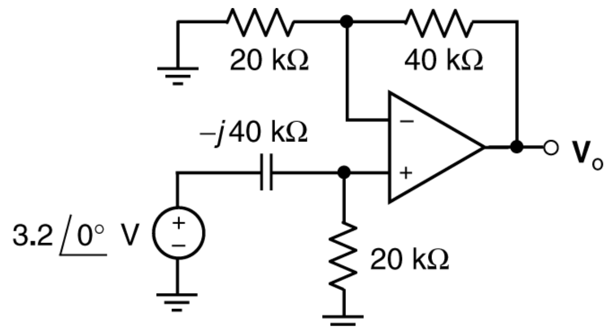


Figure P10.10-3

Solution:

Represent the circuit in the frequency domain as



Recognizing this circuit as a voltage divider followed by a noninverting amplifier, we can write

$$\mathbf{V}_o = \left(1+\frac{40}{20}\right)\left(\frac{20}{-j\,40+20}\right)(3.2\angle 0°) = \left(\frac{20}{44.72\angle -63.4°}\right)(9.6\angle 0°) = 4.293\angle 63.4°\ \text{V}$$

In the time domain $$v_o(t) = 4.293\cos(200t + 63.4°)\ \text{V}$$

(Checked using LNAPAC 3/15/12)

P10.10-4 The input of the circuit shown in Figure P10.10-4 is the voltage

$$v_s(t) = 1.2\cos(2000t)\ \text{V}.$$

Determine the output voltage $v_o(t)$.

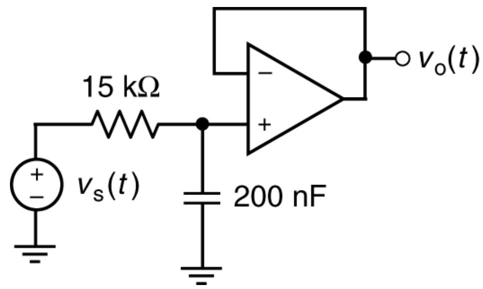
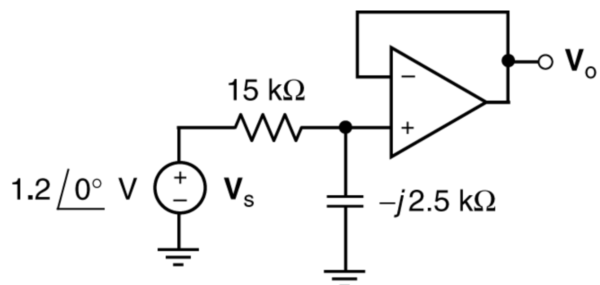


Figure P10.10-4

Solution:
Represent the circuit in the frequency domain as



Recognizing this circuit as a voltage divider followed by a voltage follower, we can write

$$\mathbf{V}_o = \left(\frac{-j2.5}{15 - j2.5}\right)(1.2\angle 0°) = \left(\frac{2.5\angle -90°}{15.2\angle -9.46°}\right)(1.2\angle 0°) = 0.1974\angle -80.54°\ \text{V}$$

In the time domain $$v_o(t) = 0.1974\cos(400t - 80.54°)\ \text{V}$$

(Checked using LNAPAC 3/15/12)

P10.10-5 The input of the circuit shown in Figure P10.10-5 is the voltage

$$v_s(t) = 1.2\cos(2000t) \text{ V}.$$

Determine the output voltage $v_o(t)$.

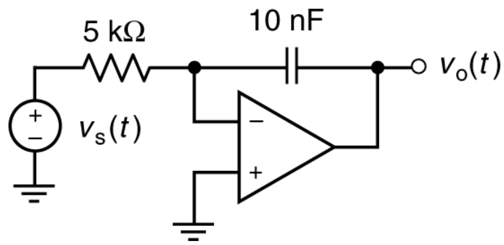
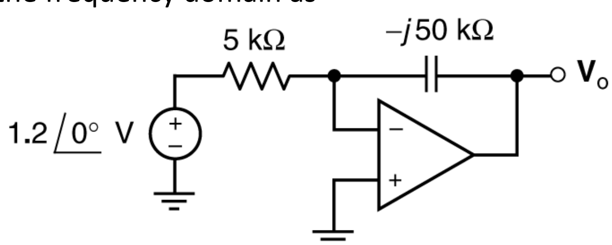


Figure P10.10-5

Solution:
Represent the circuit in the frequency domain as



Recognizing this circuit as a inverting amplifier, we can write

$$\mathbf{V}_o = \left(-\frac{-j50}{5}\right)(1.2\angle 0) = 12\angle 90° \text{ V}$$

In the time domain $$v_o(t) = 12\cos(400t + 90°) \text{ V}$$

(Checked using LNAPAC 3/15/12)

P 10.10-6 Determine the ratio $\mathbf{V}_o/\mathbf{V}_s$ for the circuit shown in Figure P 10.10-6.

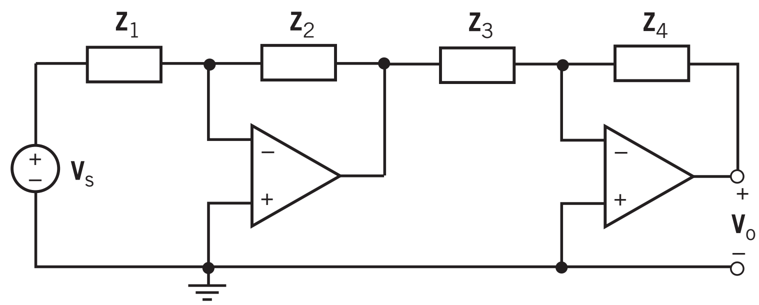
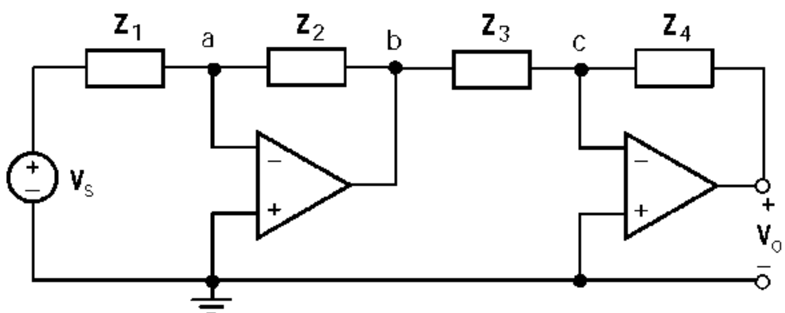


Figure P 10.10-6

Solution:

Label the nodes:



The ideal op amps force $\mathbf{V}_a = 0$ and $\mathbf{V}_c = 0$.

Apply KCL at node a to get

$$\mathbf{V}_b = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{V}_s$$

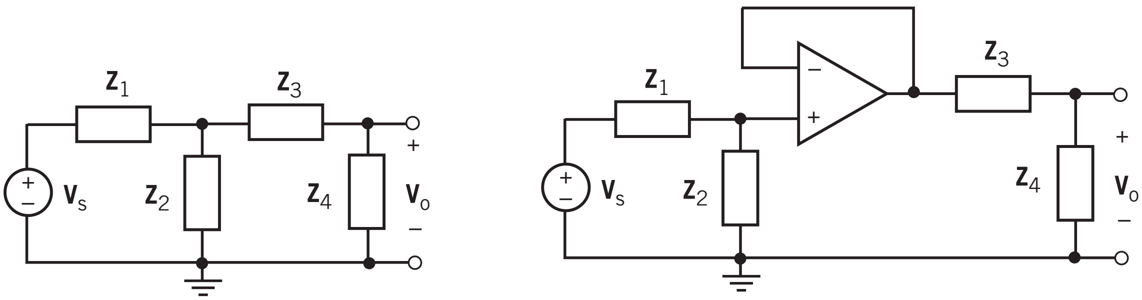
Apply KCL at node c to get

$$\mathbf{V}_o = \frac{\mathbf{Z}_4}{\mathbf{Z}_3 + \mathbf{Z}_4}\mathbf{V}_b$$

Therefore

$$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{\mathbf{Z}_4}{\mathbf{Z}_3 + \mathbf{Z}_4} \times \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}$$

P 10.10-7 Determine the ratio $\mathbf{V}_o/\mathbf{V}_s$ for both of the circuits shown in Figure P 10.10-7.

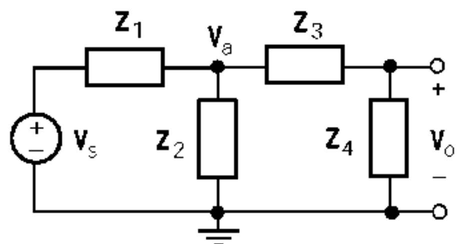


(a) *(b)*

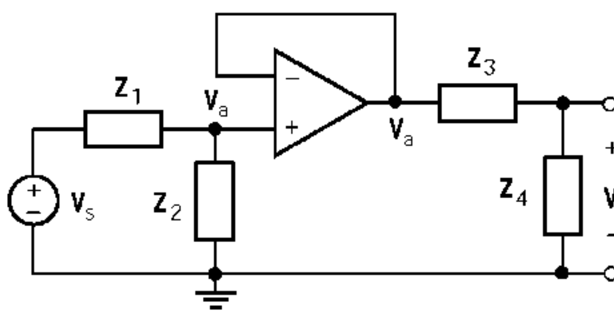
Figure P 10.10-7

Solution:

Label a node voltage as $\mathbf{V}_a$ in each of the circuits.



(a)



(b)

In both circuits, we can apply KCL at the node between $\mathbf{Z}_3$ and $\mathbf{Z}_4$ to get

$$\mathbf{V}_o = \frac{\mathbf{Z}_4}{\mathbf{Z}_3+\mathbf{Z}_4}\mathbf{V}_a$$

In (*a*)

$$\mathbf{V}_a = \frac{\mathbf{Z}_2 \| \left(\mathbf{Z}_3+\mathbf{Z}_4\right)}{\mathbf{Z}_1+\mathbf{Z}_2 \| \left(\mathbf{Z}_3+\mathbf{Z}_4\right)}\mathbf{V}_s$$

$$= \frac{\mathbf{Z}_2\left(\mathbf{Z}_3+\mathbf{Z}_4\right)}{\mathbf{Z}_1\left(\mathbf{Z}_2+\mathbf{Z}_3+\mathbf{Z}_4\right)+\mathbf{Z}_2\left(\mathbf{Z}_3+\mathbf{Z}_4\right)}\mathbf{V}_s$$

so

$$\frac{\mathbf{V}_a}{\mathbf{V}_s} = \frac{\mathbf{Z}_2\,\mathbf{Z}_4}{\mathbf{Z}_1\left(\mathbf{Z}_2+\mathbf{Z}_3+\mathbf{Z}_4\right)+\mathbf{Z}_2\left(\mathbf{Z}_3+\mathbf{Z}_4\right)}$$

In (*b*)

$$\mathbf{V}_a = \frac{\mathbf{Z}_2}{\mathbf{Z}_1+\mathbf{Z}_2}\mathbf{V}_s$$

so

$$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{\mathbf{Z}_4}{\mathbf{Z}_3+\mathbf{Z}_4}\times\frac{\mathbf{Z}_2}{\mathbf{Z}_1+\mathbf{Z}_2}$$

P 10.10-8 Determine the ratio $\mathbf{V}_o/\mathbf{V}_s$ for the circuit shown in Figure P 10.10-8.

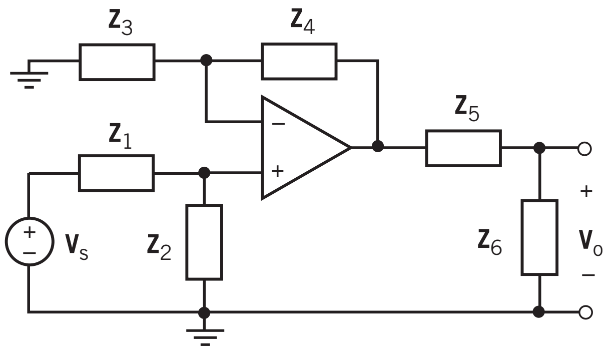
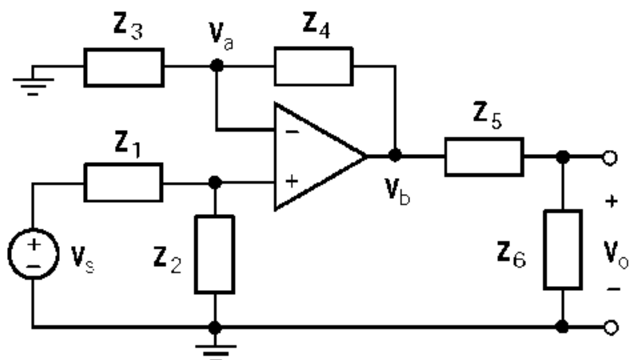


Figure P 10.10-8

Solution:

Label the node voltages $\mathbf{V}_a$ and $\mathbf{V}_b$ as shown:



Apply KCL at the node between $\mathbf{Z}_1$ and $\mathbf{Z}_2$ to get

$$\mathbf{V}_a = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} \mathbf{V}_s$$

Apply KCL at the node between $\mathbf{Z}_1$ and $\mathbf{Z}_2$ to get

$$\mathbf{V}_b = \frac{\mathbf{Z}_3 + \mathbf{Z}_4}{\mathbf{Z}_3} \mathbf{V}_a$$

Apply KCL at the node between $\mathbf{Z}_5$ and $\mathbf{Z}_6$ to get

$$\mathbf{V}_o = \frac{\mathbf{Z}_6}{\mathbf{Z}_5 + \mathbf{Z}_6} \mathbf{V}_b$$

so

$$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{\mathbf{Z}_6}{\mathbf{Z}_5 + \mathbf{Z}_6} \times \frac{\mathbf{Z}_3 + \mathbf{Z}_4}{\mathbf{Z}_3} \times \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}$$

P 10.10-9 When the input to the circuit shown in Figure P 10.10-9 is the voltage source voltage

$$v_s(t) = 2\cos(1000t)\ \text{V}$$

the output is the voltage

$$v_o(t) = 5\cos(1000t - 71.6°)\ \text{V}$$

Determine the values of the resistances R_1 and R_2.

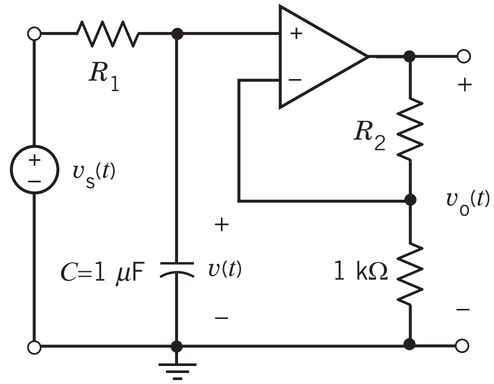


Figure P 10.10-9

Solution:

The network function of the circuit is

$$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \left(1+\frac{R_2}{1000}\right)\frac{\frac{1}{j\omega C}}{R_1+\frac{1}{j\omega C}} = \frac{1+\frac{R_2}{1000}}{1+j\omega C R_1} = \frac{1+\frac{R_2}{1000}}{1+j10^{-3}R_1}$$

Converting the given input and output sinusoids to phasors gives

$$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{5\angle 71.6°}{2}$$

Consequently

$$\frac{5\angle 71.6°}{2} = \frac{1+\frac{R_2}{1000}}{1+j10^{-3}R_1}$$

Equating angles gives

$$71.6° = -\tan^{-1}\left(10^{-3}R_1\right) \quad\Rightarrow\quad R_1 = \tan\left(71.6°\right)\times 10^3 = 3006\ \Omega$$

Equating magnitudes gives

$$\frac{5}{2} = \frac{1+\frac{R_2}{1000}}{\sqrt{1+\left(10^{-3}R_1\right)^2}} = \frac{1+\frac{R_2}{1000}}{\sqrt{1+\left(10^{-3}\times 3006\right)^2}} \quad\Rightarrow\quad R_2 = \left(\frac{5}{2}\sqrt{10}-1\right)\times 10^3 = 6906\ \Omega$$

P 10.10-10 When the input to the circuit shown in Figure P 10.10-10 is the voltage source voltage

$$v_s(t) = 4\cos(100t)\ \text{V}$$

the output is the voltage

$$v_o(t) = 8\cos(100t + 135°)\ \text{V}$$

Determine the values of C and R.

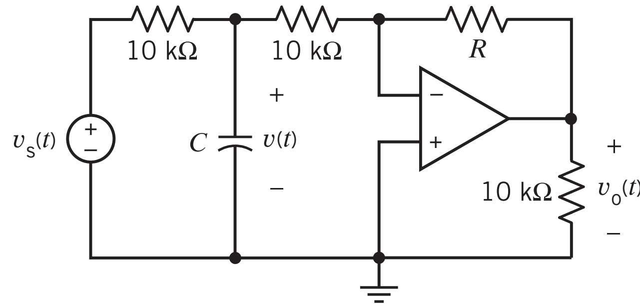
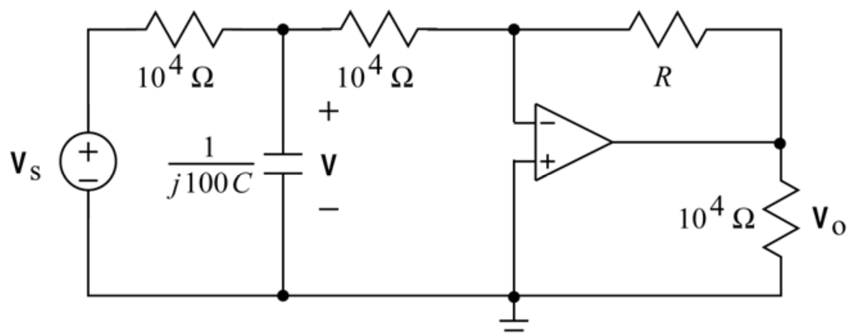


Figure P 10.10-10

Solution:

Represent the circuit in the frequency domain as



Apply KCL at the top node of the impedance of the capacitor to get

$$\frac{\mathbf{V}_s - \mathbf{V}}{10^4} = \frac{\mathbf{V}}{\dfrac{1}{j100C}} + \frac{\mathbf{V}}{10^4} \quad \Rightarrow \quad \frac{1}{2}\mathbf{V}_s = \left(1 + j\left(5\times10^5\right)C\right)\mathbf{V}$$

Apply KCL at the inverting node of the op amp to get

$$\frac{\mathbf{V}}{10^4} + \frac{\mathbf{V}_o}{R} = 0 \quad \Rightarrow \quad \mathbf{V}_o = -\frac{R}{10^4}\mathbf{V}$$

so

$$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{-\dfrac{R}{2\times10^4}}{1 + j\left(5\times10^5\right)C}$$

Converting the input and output sinusoids to phasors gives

$$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{8\angle135°}{4\angle0°} = 2\angle135°$$

so

$$2\angle135° = \frac{-\dfrac{R}{2\times10^4}}{1 + j\left(5\times10^5\right)C} = \frac{\dfrac{R}{2\times10^4}}{\sqrt{1 + \left[\left(5\times10^5\right)C\right]^2}}\angle180° - \tan^{-1}\left(\left(5\times10^5\right)C\right)$$

Equating angles gives

$$135° = 180° - \tan^{-1}\left(\left(5\times10^5\right)C\right) \quad \Rightarrow \quad C = \frac{\tan\left(45°\right)}{5\times10^5} = 2\times10^{-6} = 2\ \mu\text{F}$$

Next, equating magnitudes gives

$$2 = \frac{\dfrac{R}{2\times10^4}}{\sqrt{1+\left(5\times10^5\right)\left(2\times10^{-6}\right)}} = \frac{\dfrac{R}{2\times10^4}}{\sqrt{2}} \quad \Rightarrow \quad R = 10^4 = 10\ \text{k}\Omega$$

P10.10-11 The input to the circuit shown in Figure P10.10-11 is the voltage source voltage, $v_s(t)$. The output is the voltage $v_o(t)$. The input

$$v_s(t) = 2.5\cos(1000t)\text{ V}$$

causes the output to be

$$v_o(t) = 8\cos(1000t + 104°)\text{ V}.$$

Determine the values of the resistances R_1 and R_2.

Answers: R_1 = 1515 Ω and R_2 = 20 kΩ.

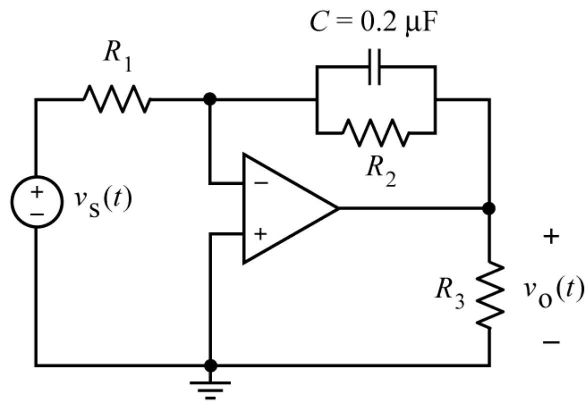
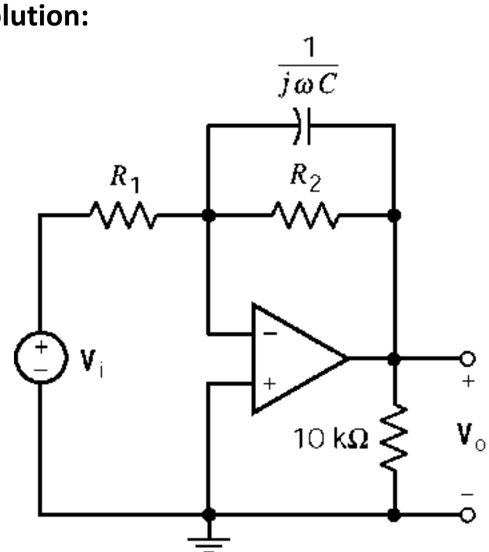


Figure P10.10-11

Solution:



$$R_2 \left\| \frac{1}{j\omega C} = \frac{R_2}{1 + j\omega CR_2}\right.$$

$$\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = -\frac{\dfrac{R_2}{1 + j\omega CR_2}}{R_1} = -\frac{\dfrac{R_2}{R_1}}{1 + j\omega CR_2}$$

$$\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{\dfrac{R_2}{R_1}}{\sqrt{1 + (\omega CR_2)^2}} e^{j(180 - \tan^{-1}\omega CR_2)}$$

In this case the angle of $\dfrac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$ is specified to be 104° so $CR_2 = \dfrac{\tan(180° - 104°)}{1000} = 0.004$ and the magnitude of $\dfrac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$ is specified to be $\dfrac{8}{2.5}$ so $\dfrac{\dfrac{R_2}{R_1}}{\sqrt{1+16}} = \dfrac{8}{2.5} \Rightarrow \dfrac{R_2}{R_1} = 13.2$. One set of values that satisfies these two equations is $C = 0.2\ \mu\text{F}$, $R_1 = 1515\ \Omega$, $R_2 = 20\ \text{k}\Omega$.

Section 10.11 The Complete Response

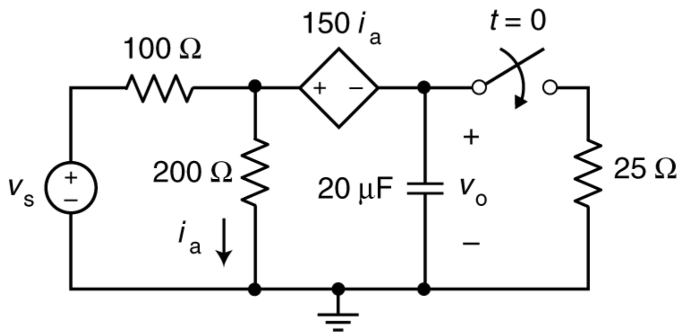
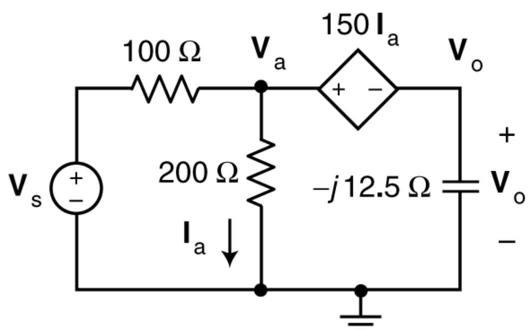


Figure P10.11-1

P10.11-1 The input to the circuit shown in Figure P10.11-1 is the voltage $v_s = 12\cos(4000t)$ V. The output is the capacitor voltage, v_o. Determine v_o.

Solution:

Before the switch closes the circuit is represented in the frequency domain as



Express the dependent source voltage in terms of the node voltages:

$$150\,\mathrm{I}_a = \mathbf{V}_a - \mathbf{V}_o$$

Using Ohm's law

$$150\frac{\mathbf{V}_a}{200} = \mathbf{V}_a - \mathbf{V}_o$$

so $$\mathbf{V}_o = \frac{1}{4}\mathbf{V}_a$$

Apply KCL to the supernode corresponding to the dependent source to get

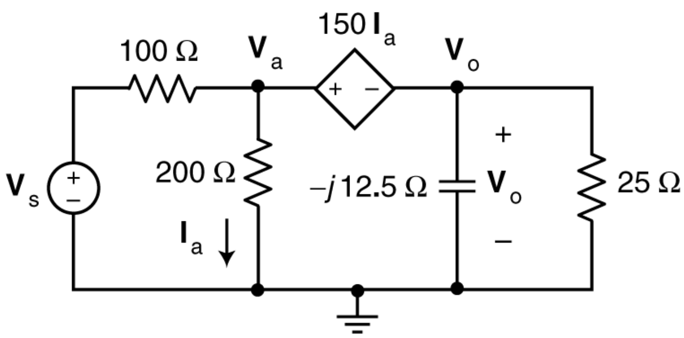
$$\frac{\mathbf{V}_s - \mathbf{V}_a}{100} = \frac{\mathbf{V}_a}{200} + \frac{\mathbf{V}_o}{-j12.5} \quad\Rightarrow\quad \mathbf{V}_o = \frac{-j12.5}{100}\mathbf{V}_s - \left(\frac{-j12.5}{100} + \frac{-j12.5}{200}\right)\mathbf{V}_a$$

$$\mathbf{V}_o = -j\,0.125(12\angle 0°) + j\,0.1875(4\mathbf{V}_o)$$

$$\mathbf{V}_o(1 - j0.75) = -j\,0.125(12\angle 0°) \quad\Rightarrow\quad \mathbf{V}_o = \frac{-j\,0.125(12\angle 0°)}{1 - j0.75} = 1.2\angle -53.1°\ \mathrm{V}$$

The corresponding sinusoid is $1.2\cos(4000t - 53.1°)$ V. The initial capacitor voltage is $v_o(0) = 1.2\cos(-53.1°) = 0.7205$ V.

The steady state response ***after the switch closes*** is the forced response. The circuit is represented in the frequency domain as



Express the dependent source voltage in terms of the node voltages:

$$150\,\mathrm{I}_a = \mathbf{V}_a - \mathbf{V}_o$$

Using Ohm's law

$$150\frac{\mathbf{V}_a}{200} = \mathbf{V}_a - \mathbf{V}_o$$

so $$\mathbf{V}_o = \frac{1}{4}\mathbf{V}_a$$

Apply KCL to the supernode corresponding to the dependent source to get

$$\frac{\mathbf{V}_s - \mathbf{V}_a}{100} = \frac{\mathbf{V}_a}{200} + \frac{\mathbf{V}_o}{-j12.5} + \frac{\mathbf{V}_o}{25} \Rightarrow \mathbf{V}_o\left(\frac{1}{-j12.5} + \frac{1}{25}\right) = \frac{1}{100}\mathbf{V}_s - \left(\frac{1}{100} + \frac{1}{200}\right)\mathbf{V}_a$$

Multiply by 200 to get

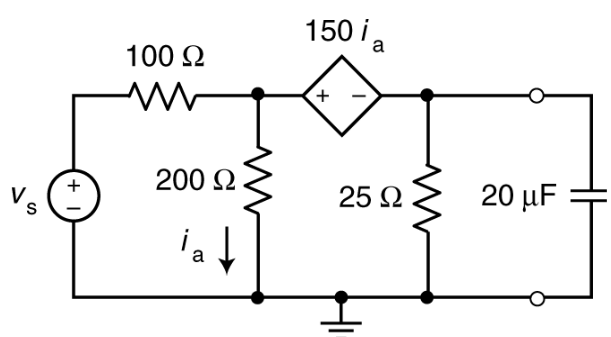
$$\mathbf{V}_o(8 + j16) = 2(12\angle 0°) - 3(4\mathbf{V}_o) \Rightarrow \mathbf{V}_o = \frac{24}{20 + j16} = 0.937\angle -38.7°\ \mathrm{V}$$

The corresponding sinusoid is the forced response:

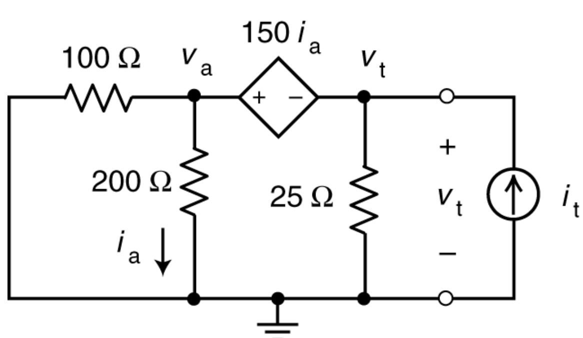
$$v_f(t) = 0.937\cos(4000t - 38.7°)\ \mathrm{V}$$

The natural response is $$v_n(t) = k\,e^{-t/\tau}\ \mathrm{V}$$

To determine the time constant τ we need find to find the Thevenin resistance of the part of the circuit connected to the capacitor after the switch closes. Here's the circuit:



The terminals separate the capacitor from the part of the circuit connected to the capacitor. Now (1) remove the capacitor, (2) replace the voltage source by a short circuit to set the input to zero and (3) connect a current source to the terminals to get



The Thevenin resistance is given by

$$R_t = \frac{v_t}{i_t}$$

Express the dependent source voltage in terms of the node voltages to get

$$v_a - v_t = 150\, i_a = 150 \frac{v_a}{200} \quad \Rightarrow \quad v_a = 4\, v_t$$

Apply KCL to the supernode corresponding to the dependent source to get

$$i_t = \frac{v_a}{100} + \frac{v_a}{200} + \frac{v_t}{25} = \frac{4\, v_t}{100} + \frac{4\, v_t}{200} + \frac{v_t}{25} = \frac{v_t}{10} \quad \Rightarrow \quad R_t = \frac{v_t}{i_t} = 10\ \Omega$$

The time constant is $\tau = R_t\, C = 10\left(20\times 10^{-6}\right) = 0.2\times 10^{-3} = 0.2 \text{ ms}$

The natural response is $v_n\left(t\right) = k\, e^{-t/\tau} = k\, e^{-5000t} \text{ V}$

The complete response is

$$v_o\left(t\right) = 0.937\cos\left(4000\, t - 38.7°\right) + k\, e^{-5000t} \text{ V for } t \ge 0$$

Using the initial condition we calculate

$$0.7205 = v_o\left(0\right) = 0.937\cos\left(-38.7°\right) + k \quad \Rightarrow \quad k = -0.0108$$

Finally $v_o\left(t\right) = 0.937\cos\left(4000\, t - 38.8°\right) - 0.0108\, e^{-5000t} \text{ V for } t \ge 0$

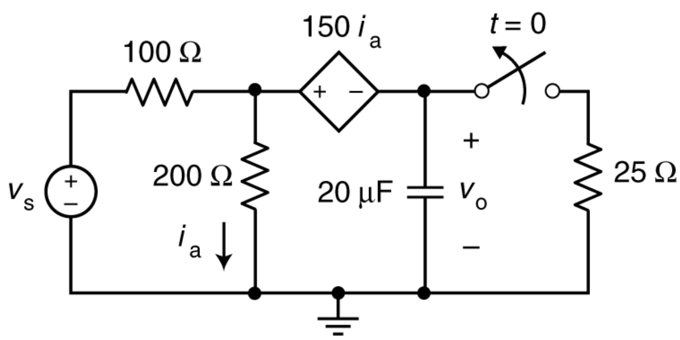
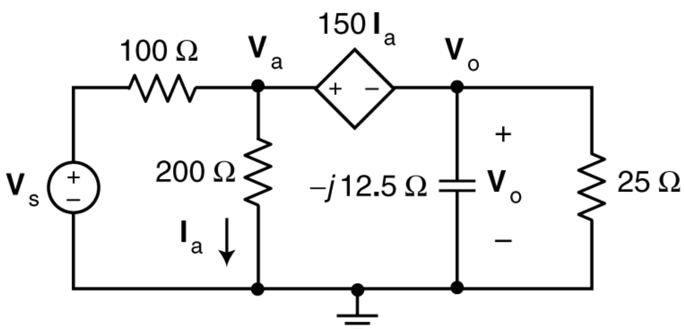


Figure P10.11-2

P10.11-2 The input to the circuit shown in Figure P10.11-2 is the voltage $v_s = 12\cos(4000t)$ V. The output is the capacitor voltage, v_o. Determine v_o.

Solution:

Before the switch opens the circuit is represented in the frequency domain as



Express the dependent source voltage in terms of the node voltages:

$$150\,\mathrm{I_a} = \mathbf{V}_a - \mathbf{V}_o$$

Using Ohm's law

$$150\frac{\mathbf{V}_a}{200} = \mathbf{V}_a - \mathbf{V}_o$$

so $$\mathbf{V}_o = \frac{1}{4}\mathbf{V}_a$$

Apply KCL to the supernode corresponding to the dependent source to get

$$\frac{\mathbf{V}_s - \mathbf{V}_a}{100} = \frac{\mathbf{V}_a}{200} + \frac{\mathbf{V}_o}{-j12.5} + \frac{\mathbf{V}_o}{25} \Rightarrow \quad \mathbf{V}_o\left(\frac{1}{-j12.5} + \frac{1}{25}\right) = \frac{1}{100}\mathbf{V}_s - \left(\frac{1}{100} + \frac{1}{200}\right)\mathbf{V}_a$$

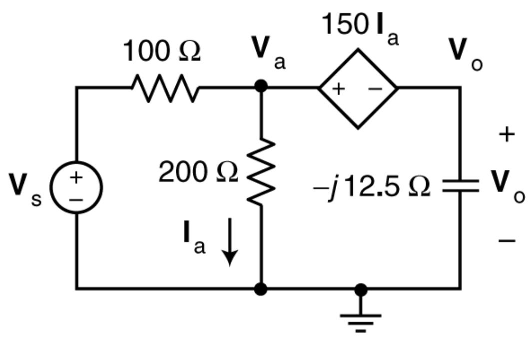
Multiply by 200 to get

$$\mathbf{V}_o(8 + j16) = 2(12\angle 0°) - 3(4\mathbf{V}_o) \quad \Rightarrow \quad \mathbf{V}_o = \frac{24}{20 + j16} = 0.937\angle -38.7° \text{ V}$$

The corresponding sinusoid is $\quad 0.937\cos(4000t - 38.7°)$ V

The initial condition is $\quad v_o(0) = 0.937\cos(-38.7°) = 0.7313$ V

The steady state response ***after the switch closes*** is the forced response. The circuit is represented in the frequency domain as



Express the dependent source voltage in terms of the node voltages:

$$150\,\mathrm{I}_a = \mathbf{V}_a - \mathbf{V}_o$$

Using Ohm's law

$$150\frac{\mathbf{V}_a}{200} = \mathbf{V}_a - \mathbf{V}_o$$

so
$$\mathbf{V}_o = \frac{1}{4}\mathbf{V}_a$$

Apply KCL to the supernode corresponding to the dependent source to get

$$\frac{\mathbf{V}_s - \mathbf{V}_a}{100} = \frac{\mathbf{V}_a}{200} + \frac{\mathbf{V}_o}{-j12.5} \quad \Rightarrow \quad \mathbf{V}_o = \frac{-j12.5}{100}\mathbf{V}_s - \left(\frac{-j12.5}{100} + \frac{-j12.5}{200}\right)\mathbf{V}_a$$

$$\mathbf{V}_o = -j\,0.125(12\angle 0°) + j\,0.1875(4\mathbf{V}_o)$$

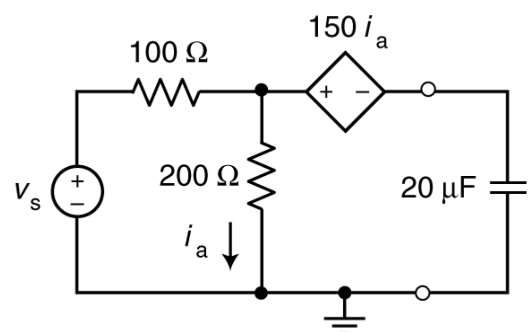
$$\mathbf{V}_o(1 - j0.75) = -j\,0.125(12\angle 0°) \quad \Rightarrow \quad \mathbf{V}_o = \frac{-j\,0.125(12\angle 0°)}{1 - j0.75} = 1.2\angle -53.1° \text{ V}$$

The corresponding sinusoid is the forced response:

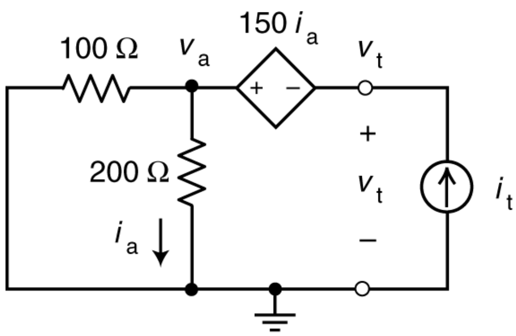
$$v_f(t) = 1.2\cos(4000\,t - 53.1°) \text{ V}$$

The natural response is
$$v_n(t) = k\,e^{-t/\tau} \text{ V}$$

To determine the time constant τ we need find to find the Thevenin resistance of the part of the circuit connected to the capacitor after the switch closes. Here's the circuit:



The terminals separate the capacitor from the part of the circuit connected to the capacitor. Now (1) remove the capacitor, (2) replace the voltage source by a short circuit to set the input to zero and (3) connect a current source to the terminals to get



The Thevenin resistance is given by

$$R_t = \frac{v_t}{i_t}$$

Express the dependent source voltage in terms of the node voltages to get

$$v_a - v_t = 150\, i_a = 150 \frac{v_a}{200} \quad \Rightarrow \quad v_a = 4\, v_t$$

Apply KCL to the supernode corresponding to the dependent source to get

$$i_t = \frac{v_a}{100} + \frac{v_a}{200} = \frac{4\, v_t}{100} + \frac{4\, v_t}{200} = 0.06\, v_t \quad \Rightarrow \quad R_t = \frac{v_t}{i_t} = 16.67\ \Omega$$

The time constant is $\tau = R_t\, C = 16.67\left(20\times 10^{-6}\right) = 0.333\times 10^{-3} = 0.333\ \text{ms}$

The natural response is $v_n(t) = k\, e^{-t/\tau} = k\, e^{-3000t}\ \text{V}$

The complete response is

$$v_o(t) = 1.2\cos\left(4000\, t - 53.1°\right) + k\, e^{-3000t}\ \text{V for } t \ge 0$$

Using the initial condition we calculate

$$0.7313 = v_o(0) = 1.2\cos\left(-53.1°\right) + k \quad \Rightarrow \quad k = 0.0108$$

Finally $v_o(t) = 1.2\cos\left(4000\, t - 53.1°\right) + 0.0108\, e^{-3000t}\ \text{V for } t \ge 0$

Section 10.12 Using MATLAB to Analyze Electric Circuits

10.12-1 Determine the mesh currents for the circuit shown in Figure P10.12-1 when

$v_s(t) = 12\cos(2500t + 60°)$ V and $i_s(t) = 2\cos(2500t - 15°)$ mA

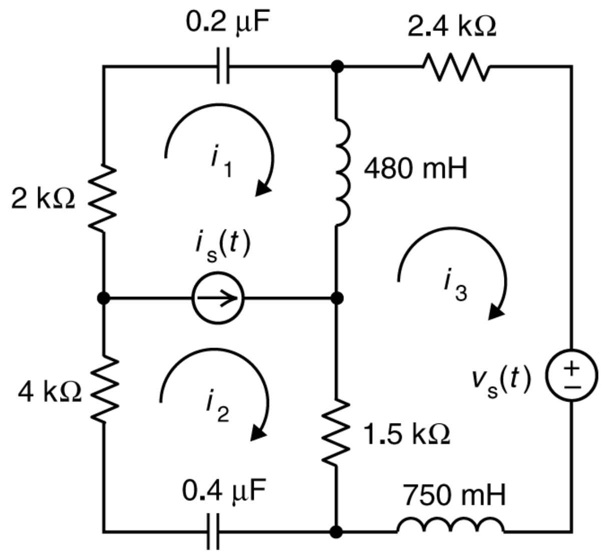
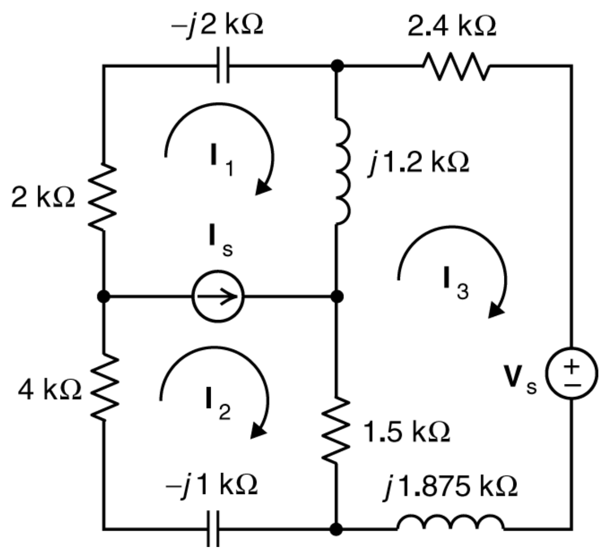


Figure P10.12-1

Solution:

Represent the circuit in the frequency domain:



Represent the source current in terms of the mesh currents: $\mathbf{I}_2 - \mathbf{I}_1 = \mathbf{I}_s = 0.002\angle -15°$ A

Apply KVL to the supermesh corresponding to the current source:

$$(2000 - j\,2000)\mathbf{I}_1 + j1200(\mathbf{I}_1 - \mathbf{I}_3) + 1500(\mathbf{I}_2 - \mathbf{I}_3) + (4000 - j1000)\mathbf{I}_2 = 0$$

Apply KVL to mesh 3:

$$(2400 + j1875)\mathbf{I}_3 + 1500(\mathbf{I}_3 - \mathbf{I}_2) + j1200(\mathbf{I}_3 - \mathbf{I}_1) = -\mathbf{V}_s = -12\angle 60°$$

In matrix form: $$\begin{bmatrix} -1 & 1 & 0 \\ 2000-j800 & 5500-j1000 & -1500-j1200 \\ -j1200 & -1500 & 3900+j3075 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix}=\begin{bmatrix} 0.002\angle -15° \\ 0 \\ -12\angle 60° \end{bmatrix}$$

Solving, using MATLAB, gives $$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix}=\begin{bmatrix} 1.549\angle -164° \\ 1.039\angle -65° \\ 2.904\angle -148° \end{bmatrix} \text{ mA}$$

In the time domain: $$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}=\begin{bmatrix} 1.549\cos(2500t-164°) \\ 1.039\cos(2500t-65°) \\ 2.904\cos(2500t-148°) \end{bmatrix} \text{ mA}$$

10.12-2 Determine the node voltages for the circuit shown in Figure P10.12-2 when

$v_s(t) = 12\cos(400t + 45°)$ V.

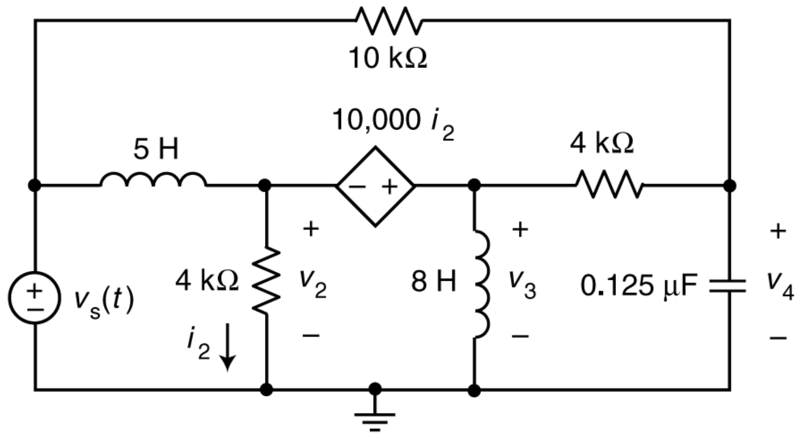
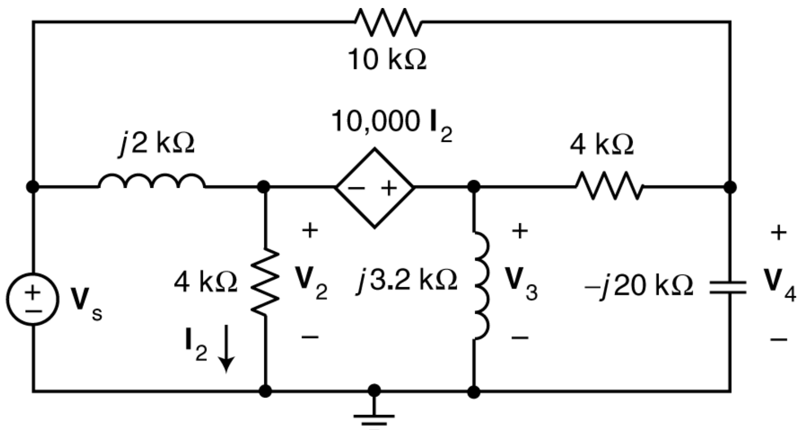


Figure P10.12-2

Solution:

Represent the circuit in the frequency domain:



Represent the dependent source voltage in terms of the node voltages currents:

$$\mathbf{V}_3 - \mathbf{V}_2 = 10000\frac{\mathbf{V}_2}{4000} \quad \Rightarrow \quad \mathbf{V}_3 = 3.5\mathbf{V}_2$$

Apply KCL to the supernode corresponding to the dependent voltage source:

$$\frac{\mathbf{V}_s - \mathbf{V}_2}{j2000} = \frac{\mathbf{V}_2}{4000} + \frac{\mathbf{V}_3}{j3200} + \frac{\mathbf{V}_3 - \mathbf{V}_4}{4000}$$

Rearranging: $$\frac{\mathbf{V}_s}{j2000} = \left(\frac{1}{4000} + \frac{1}{j2000}\right)\mathbf{V}_2 + \left(\frac{1}{4000} + \frac{1}{j3200}\right)\mathbf{V}_3 - \left(\frac{1}{4000}\right)\mathbf{V}_4$$

Apply KCL at node 3: $$\frac{\mathbf{V}_s - \mathbf{V}_4}{10,000} + \frac{\mathbf{V}_3 - \mathbf{V}_4}{4000} = \frac{\mathbf{V}_4}{-j20,000}$$

Rearranging: $$\frac{\mathbf{V}_s}{10,000} = -\frac{\mathbf{V}_3}{4000} + \left(\frac{1}{4000} + \frac{1}{10,000} + \frac{1}{-j\,20,000}\right)\mathbf{V}_4$$

In matrix form: $$\begin{bmatrix} 3.5 & -1 & 0 \\ \frac{1}{4000} + \frac{1}{j\,2000} & \frac{1}{4000} + \frac{1}{j\,3200} & -\frac{1}{4000} \\ 0 & -\frac{1}{4000} & \frac{1}{4000} + \frac{1}{10,000} + \frac{1}{-j\,20,000} \end{bmatrix} \begin{bmatrix} \mathbf{V}_2 \\ \mathbf{V}_3 \\ \mathbf{V}_4 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{\mathbf{V}_s}{j\,2000} \\ \frac{\mathbf{V}_s}{10,000} \end{bmatrix}$$

Solving, using MATLAB, gives $$\begin{bmatrix} \mathbf{V}_2 \\ \mathbf{V}_3 \\ \mathbf{V}_4 \end{bmatrix} = \begin{bmatrix} 3.236\angle 34^\circ \\ 11.324\angle 34^\circ \\ 10.798\angle 29^\circ \end{bmatrix} \text{ V}$$

In the time domain: $$\begin{bmatrix} v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 3.236\cos(400t + 34^\circ) \\ 11.324\cos(400t + 34^\circ) \\ 10.798\cos(400t + 29^\circ) \end{bmatrix} \text{ V}$$

10.12-3 Determine the mesh currents for the circuit shown in Figure P10.12-3

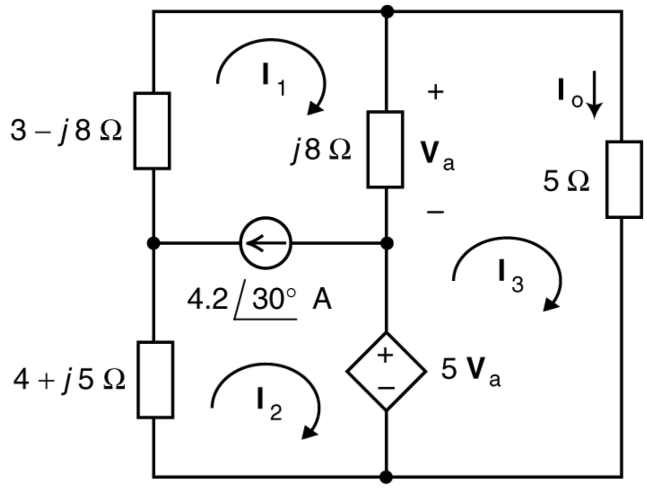


Figure P10.12-3

Solution:

Represent the source current in terms of the mesh currents: $\mathbf{I}_1 - \mathbf{I}_2 = 4.2\angle 30°\ \text{A}$

Apply KVL to the supermesh corresponding to the current source:

$$j8\left(\mathbf{I}_1 - \mathbf{I}_o\right) + 5\left[\,j8\left(\mathbf{I}_1 - \mathbf{I}_o\right)\right] + \left(4 + j5\right)\mathbf{I}_2 + \left(3 - j8\right)\mathbf{I}_1 = 0$$

Apply KVL to mesh 3: $5\mathbf{I}_o + 6\left(j8\right)\left(\mathbf{I}_1 - \mathbf{I}_o\right) = 0$

In matrix form:

$$\begin{bmatrix} 1 & -1 & 0 \\ 3 + j40 & 4 + j5 & -j48 \\ -j48 & 0 & 5 + j48 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_o \end{bmatrix} = \begin{bmatrix} 4.2\angle 30° \\ 0 \\ 0 \end{bmatrix}$$

Solving, using MATLAB, gives

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_o \end{bmatrix} = \begin{bmatrix} 2.204\angle 93.1° \\ 3.758\angle 178° \\ 2.192\angle 99° \end{bmatrix}\ \text{A}$$

10.12-4 Determine the node voltages for the circuit shown in Figure P10.12-4.

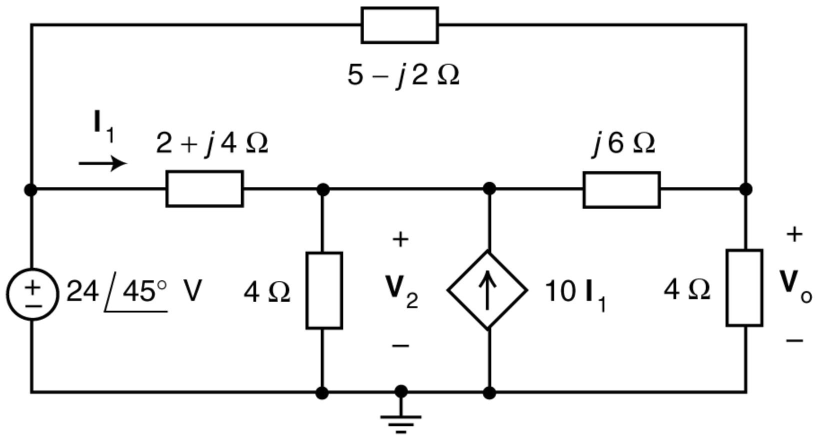


Figure P10.12-4

Solution:

Apply KCL at node 2: $$\frac{24\angle 45° - \mathbf{V}_2}{2+j4} + 10\left(\frac{24\angle 45° - \mathbf{V}_2}{2+j4}\right) = \frac{\mathbf{V}_2}{4} + \frac{\mathbf{V}_2 - \mathbf{V}_o}{j6}$$

Apply KCL at node 3: $$\frac{24\angle 45° - \mathbf{V}_3}{5-j2} + \frac{\mathbf{V}_2 - \mathbf{V}_o}{j6} = \frac{\mathbf{V}_o}{4}$$

In matrix form: $$\begin{bmatrix} \frac{1}{4} + \frac{1}{j6} + \frac{11}{2+j4} & -\frac{1}{j6} \\ -\frac{1}{j6} & \frac{1}{5-j2} + \frac{1}{j6} + \frac{1}{4} \end{bmatrix} \begin{bmatrix} \mathbf{V}_2 \\ \mathbf{V}_o \end{bmatrix} = \begin{bmatrix} \frac{11}{2+j4}(24\angle 45°) \\ \frac{1}{5-j2}(24\angle 45°) \end{bmatrix}$$

Solving, using MATLAB, gives $$\begin{bmatrix} \mathbf{V}_2 \\ \mathbf{V}_o \end{bmatrix} = \begin{bmatrix} 22.13\angle 40.8° \\ 10.07\angle 30.4° \end{bmatrix} \text{ V}$$

10.12-5 The input to the circuit shown in Figure P10.12-5 is the voltage source voltage

$$v_s(t) = 12\cos(20{,}000t + 60°)\text{ V} \quad .$$

and the output is the steady state voltage $v_o(t)$. Use MATLAB to plot the input and output sinusoids.

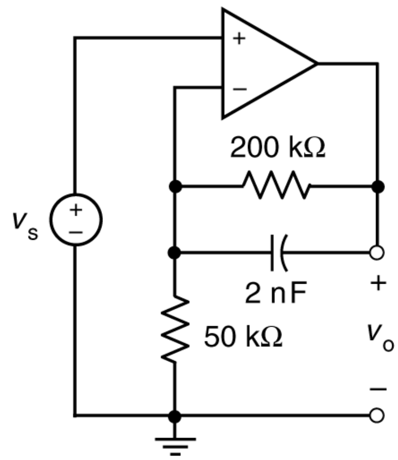
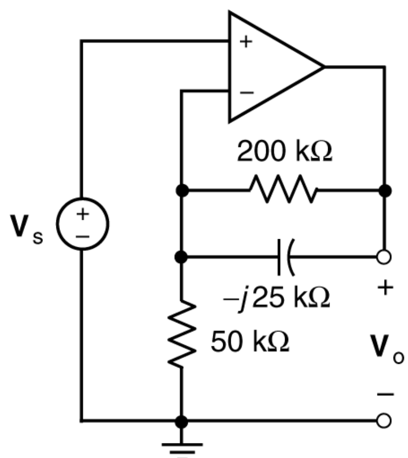


Figure P10.12-5

Solution:

Represent the circuit in the frequency domain:



Write a node equation: $$\frac{12\angle 60° - \mathbf{V}_o}{200{,}000} + \frac{12\angle 60° - \mathbf{V}_o}{-j\,25{,}000} + \frac{12\angle 60°}{20{,}000} = 0$$

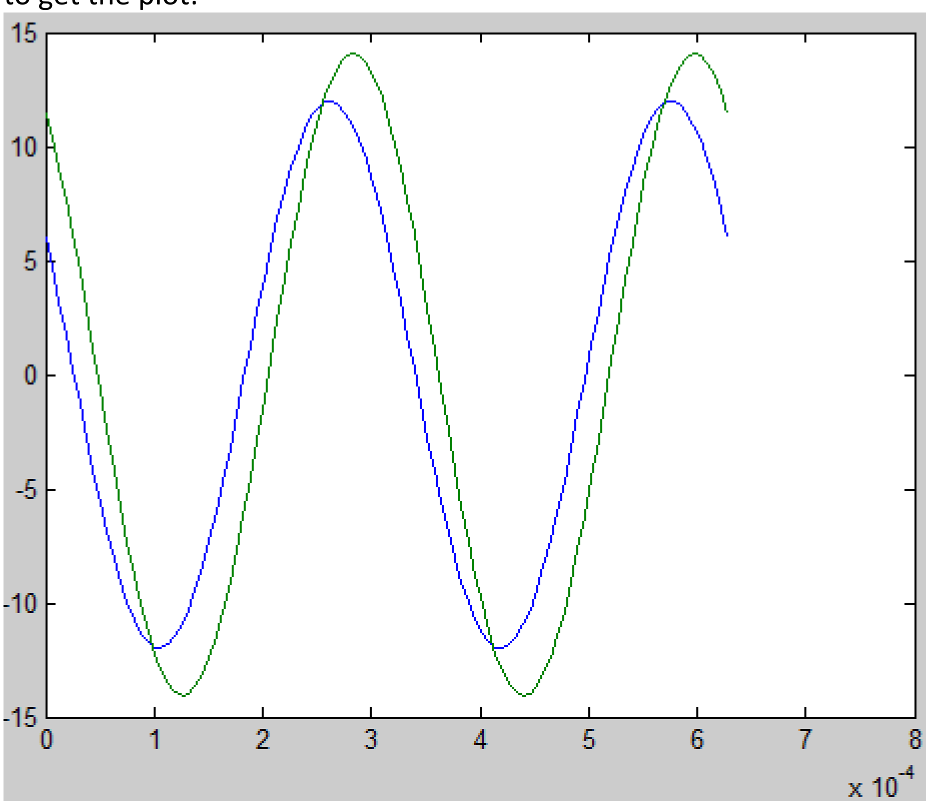
Rearrange: $$\left(\frac{1}{200{,}000} + \frac{1}{-j\,25{,}000} + \frac{1}{20{,}000}\right)12\angle 60° = \left(\frac{1}{200{,}000} + \frac{1}{-j\,25{,}000}\right)\mathbf{V}_o$$

Modify the MATLAB script given in the textbook (and posted on the Student Companion Site for *Introduction to Electric Circuits*):

```
%-----------------------------------------------
%        Describe the input voltage source.
%-----------------------------------------------
w = 20000;
```

```
A = 12;
theta = (pi/180)*60;
Vs = A*exp(j*theta);
%-----------------------------------------------
%              Describe the impedances.
%-----------------------------------------------
R1=50e3; R2=200e3; ZC=-j*25e3;
%-----------------------------------------------
%    Calculate the phasor corresponding to the
%                 output voltage.
%-----------------------------------------------
Vo=(1/R2 + 1/ZC + 1/R1)*Vs/(1/R2 + 1/ZC);
B = abs(Vo)
phi = angle(Vo)
%-----------------------------------------------
%
%-----------------------------------------------
T = 2*pi/w;
tf = 2*T; N = 100; dt = tf/N;
t = 0 : dt : tf;
%-----------------------------------------------
%     Plot the input and output voltages.
%-----------------------------------------------
for k = 1 : 101
   vs(k) = A * cos(w * t(k) + theta);
   vo(k) = B * cos(w * t(k) + phi);
end
plot (t, vs, t, vo)
```

to get the plot:



10.12-6 The input to the circuit shown in Figure P10.12-6 is the voltage source voltage

$$v_s(t) = 3\cos(4000t + 30°)\ \text{V} \quad .$$

and the output is the steady state voltage $v_o(t)$. Use MATLAB to plot the input and output sinusoids.

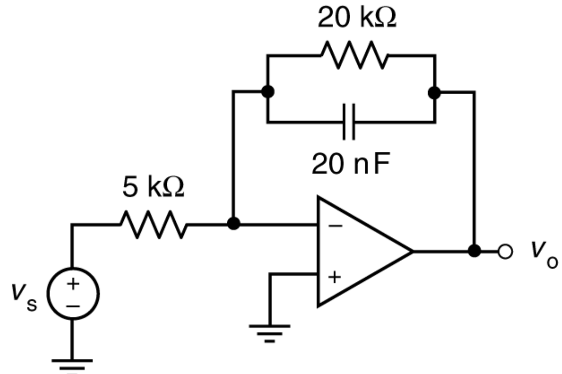
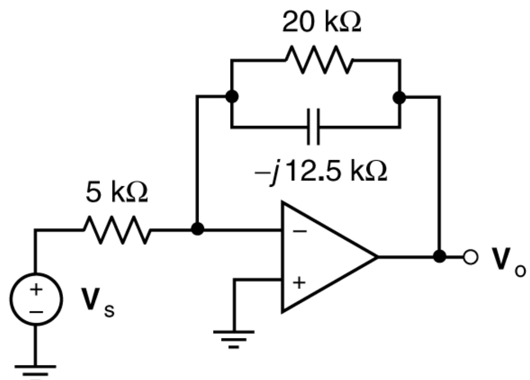


Figure P10.12-6

Solution:

Represent the circuit in the frequency domain:



Recognize this circuit as an inverting amplifier to write:

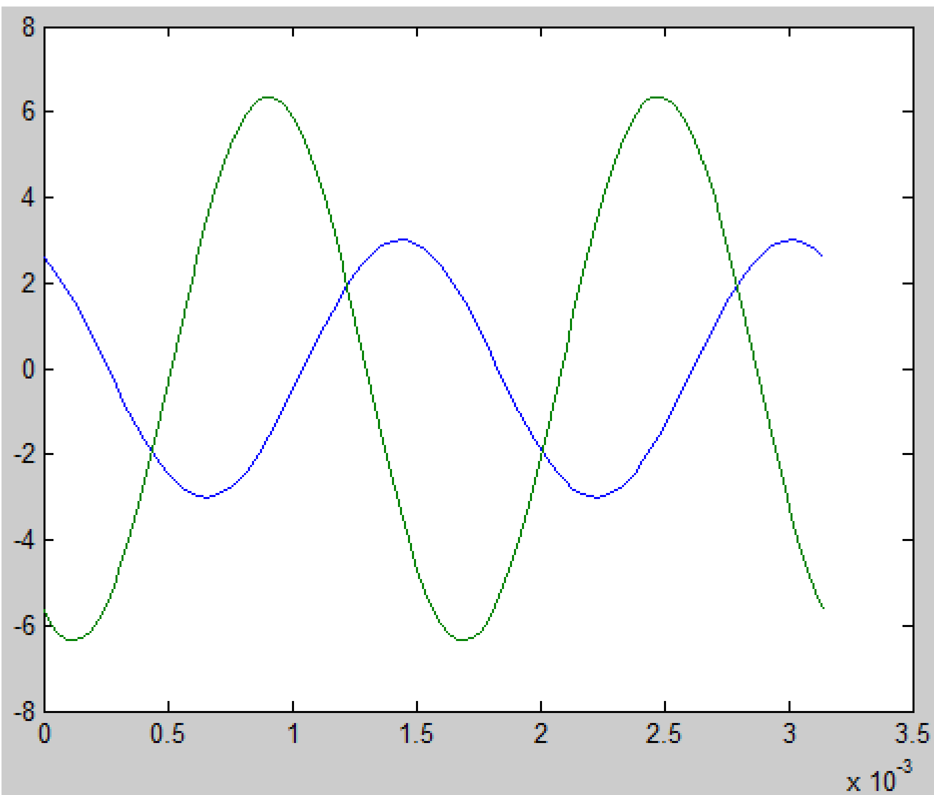
$$\mathbf{V}_o = \left(-\frac{20 \| -j12.5}{5}\right)\mathbf{V}_s = -\frac{20 \| -j12.5}{5}(12\angle 60°)$$

Modify the MATLAB script given in the textbook (and posted on the Student Companion Site for *Introduction to Electric Circuits*):

```
%-----------------------------------------------
%         Describe the input voltage source.
%-----------------------------------------------
w = 4000;
A = 3;
theta = (pi/180)*30;
Vs = A*exp(j*theta);
%------------------------------------------------
%              Describe the impedances.
%------------------------------------------------
R1=5e3; R2=20e3; ZC=-j*12.5e3;
```

```
Zp=R2*ZC/(R2+ZC);
%-----------------------------------------------
%    Calculate the phasor corresponding to the
%                 output voltage.
%-----------------------------------------------
Vo=(-Zp/R1)*Vs;
B = abs(Vo)
phi = angle(Vo)
%-----------------------------------------------
%
%-----------------------------------------------
T = 2*pi/w;
tf = 2*T; N = 100; dt = tf/N;
t = 0 : dt : tf;
%-----------------------------------------------
%     Plot the input and output voltages.
%-----------------------------------------------
for k = 1 : 101
   vs(k) = A * cos(w * t(k) + theta);
   vo(k) = B * cos(w * t(k) + phi);
end
plot (t, vs, t, vo)
```

to get the plot:



Section 10.14 How Can We Check…?

P 10.14-1 Computer analysis of the circuit in Figure P 10.14-1 indicates that the values of the node voltages are $\mathbf{V}_1 = 20 \angle{-90°}$ and $\mathbf{V}_2 = 44.7 \angle{-63.4°}$. Are the values correct?

Hint: Calculate the current in each circuit element using the values of $\mathbf{V}_1$ and $\mathbf{V}_2$. Check to see whether KCL is satisfied at each node of the circuit.

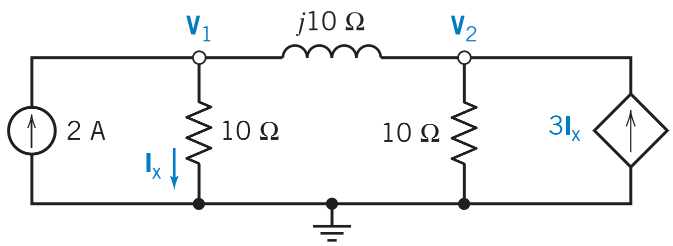


Figure P 10.14-1

Solution:

Generally, it is more convenient to divide complex numbers in polar form. Sometimes, as in this case, it is more convenient to do the division in rectangular form.

Express $\mathbf{V}_1$ and $\mathbf{V}_2$ as: $\mathbf{V}_1 = -j\,20$ and $\mathbf{V}_2 = 20 - j\,40$

KCL at node 1:

$$2 - \frac{\mathbf{V}_1}{10} - \frac{\mathbf{V}_1 - \mathbf{V}_2}{j10} = 2 - \frac{-j\,20}{10} - \frac{-j\,20 - (20 - j\,40)}{j10} = 2 + j\,2 - 2 - j\,2 = 0$$

KCL at node 2:

$$\frac{\mathbf{V}_1 - \mathbf{V}_2}{j10} - \frac{\mathbf{V}_2}{10} + 3\left(\frac{\mathbf{V}_1}{10}\right) = \frac{-j\,20 - (20 - j\,40)}{j10} - \frac{20 - j\,40}{10} + 3\left(\frac{-j\,20}{10}\right) = (2 + j\,2) - (2 - j4) - j\,6 = 0$$

The currents calculated from $\mathbf{V}_1$ and $\mathbf{V}_2$ satisfy KCL at both nodes, so it is very likely that the $\mathbf{V}_1$ and $\mathbf{V}_2$ are correct.

P 10.14-2 Computer analysis of the circuit in Figure P 10.14-2 indicates that the mesh currents are

$i_1(t) = 0.39 \cos(5t + 39°)$ A and $i_2(t) = 0.28 \cos(5t + 180°)$ A.

Is this analysis correct?

Hint: Represent the circuit in the frequency domain using impedances and phasors. Calculate the voltage across each circuit element using the values of $\mathbf{I}_1$ and $\mathbf{I}_2$. Check to see whether KVL is satisfied for each mesh of the circuit.

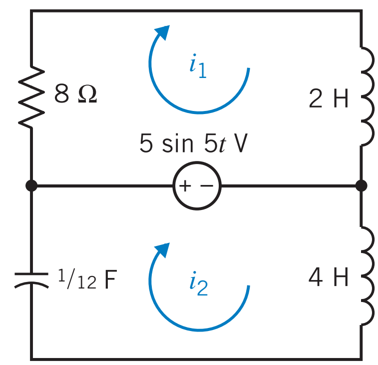
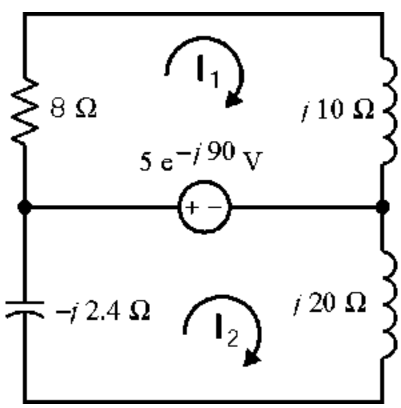


Figure P 10.14-2

Solution:



$\mathbf{I}_1 = 0.390 \angle 39°$ and $\mathbf{I}_2 = 0.284 \angle 180°$

Generally, it is more convenient to multiply complex numbers in polar form. Sometimes, as in this case, it is more convenient to do the multiplication in rectangular form.

Express $\mathbf{I}_1$ and $\mathbf{I}_2$ as: $\mathbf{I}_1 = 0.305 + j\,0.244$ and $\mathbf{I}_2 = -0.284$

KVL for mesh 1:

$$8\left(0.305 + j\,0.244\right) + j10\left(0.305 + j\,0.244\right) - (-j\,5) = j\,10 \neq 0$$

Since KVL is not satisfied for mesh 1, the mesh currents are not correct.

Here is a MATLAB file for this problem:

```
Vs = -j*5;
Z1 = 8;
Z2 = j*10;
Z3 = -j*2.4;
Z4 = j*20;
% Mesh equations in matrix form
Z = [  Z1+Z2     0;
         0     Z3+Z4 ];
V = [ Vs;
     -Vs  ];
I = Z\V
abs(I)
angle(I)*180/3.14159
% Verify solution by obtaining the algebraic sum of voltages for
% each mesh. KVL requires that both M1 and M2 be zero.
M1 = -Vs + Z1*I(1) +Z2*I(1)
M2 = Vs + Z3*I(2) + Z4*I(2)
```

P 10.14-3 Computer analysis of the circuit in Figure P 10.14-3 indicates that the values of the node voltages are

$$v_1(t) = 19.2 \cos(3t + 68°) \text{ V}$$

and

$$v_2(t) = 2.4 \cos(3t + 105°) \text{ V}.$$

Is this analysis correct?

Hint: Represent the circuit in the frequency domain using impedances and phasors. Calculate the current in each circuit element using the values of $\mathbf{V}_1$ and $\mathbf{V}_2$. Check to see whether KCL is satisfied at each node of the circuit.

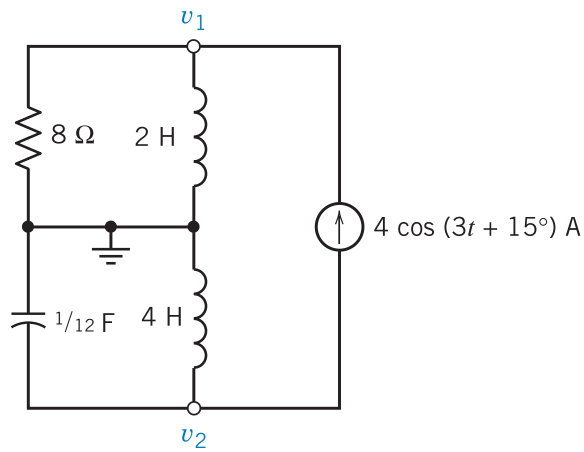
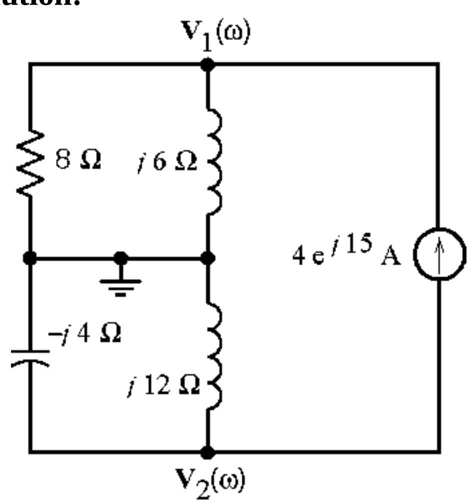


Figure P 10.14-3

Solution:



$\mathbf{V}_1 = 19.2\angle 68°$ and $\mathbf{V}_2 = 24\angle 105°$ V

KCL at node 1 :

$$\frac{19.2\angle 68°}{8} + \frac{19.2\angle 68°}{j6} - 4\angle 15 = 0$$

KCL at node 2:

$$\frac{24\angle 105°}{-j4} + \frac{24\angle 105°}{j12} + 4\angle 15 = 0$$

The currents calculated from $\mathbf{V}_1$ and $\mathbf{V}_2$ satisfy KCL at both nodes, so it is very likely that the $\mathbf{V}_1$ and $\mathbf{V}_2$ are correct.

Here is a MATLAB file for this problem:

```
Is = 4*exp(j*15*3.14159/180);
Z1 = 8;
Z2 = j*6;
Z3 = -j*4;
Z4 = j*12;
Y = [  1/Z1 + 1/Z2       0;
            0       1/Z3 + 1/Z4 ];
I = [ Is;
     -Is  ];
V = Y\I
abs(V)
angle(V)*180/3.14159
% Verify solution by obtaining the algebraic sum of currents for
% each node. KCL requires that both M1 and M2 be zero.
M1 = -Is + V(1)/Z1 + V(1)/Z2
M2 = Is + V(2)/Z3 + V(2)/Z4
```

P 10.14-4 A computer program reports that the currents of the circuit of Figure P 10.14-4 are

$$\mathbf{I} = 0.2\ \angle 53.1°\ \text{A},\ \mathbf{I}_1 = 632\ \angle{-18.4°}\ \text{mA, and } \mathbf{I}_2 = 190\ \angle 71.6°\ \text{mA}.$$

Verify this result.

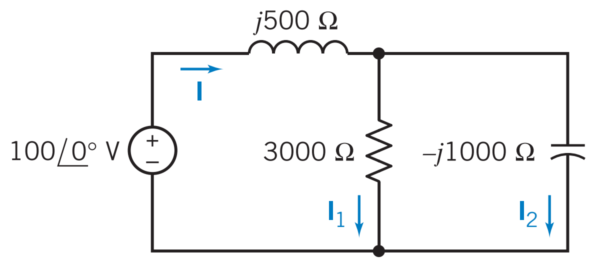
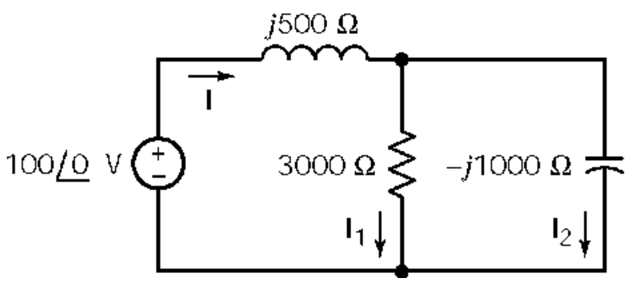
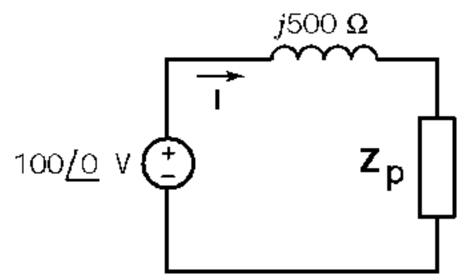


Figure P 10.14-4

Solution: First, replace the parallel resistor and capacitor by an equivalent impedance

$$\mathbf{Z}_P = \frac{(3000)(-j1000)}{3000-j1000} = 949\ \angle -72° = 300 - j900\ \Omega$$





The current is given by

$$\mathbf{I} = \frac{\mathbf{V}_S}{j500+\mathbf{Z}_P} = \frac{100\ \angle 0^{\circ}}{j500+300-j900} = 0.2\angle 53°\ \text{A}$$

Current division yields

$$\mathbf{I}_1 = \left(\frac{-j1000}{3000-j1000}\right)(0.2\ \angle 53°) = \ 63.3\ \angle -18.5°\ \text{mA}$$

$$\mathbf{I}_2 = \left(\frac{3000}{3000-j1000}\right)(0.2\ \angle 53°) = 190\angle 71.4°\ \text{mA}$$

The reported value of $\mathbf{I}_1$ is off by an order of magnitude.

P 10.14-5 The circuit shown in Figure P 10.14-5 was built using a 2 percent resistor having a nominal resistance of 500 Ω and a 10 percent capacitor with a nominal capacitance of 5 μF. The steady-state capacitor voltage was measured to be

$$v(t) = 18.3 \cos(200t - 24°) \text{ V}$$

The voltage source represents a "signal generator." Suppose that the signal generator was adjusted so carefully that errors in the amplitude, frequency, and angle of the voltage source voltage are all negligible. Is the measured response explained by the component tolerances? That is, could the measured $v(t)$ have been produced by this circuit with a resistance R that is within 2 percent of 500 Ω and a capacitance C that is within 5 percent of 5 μF?

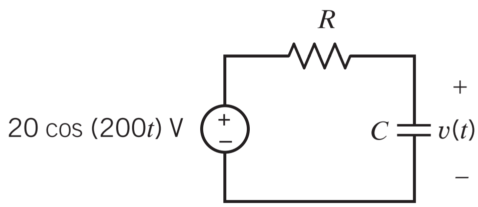


Figure P 10.14-5

Solution:

Represent the circuit in the frequency domain using phasors and impedances. Use voltage division to get

$$18.3\angle -24° = \frac{\dfrac{1}{j200C}}{R + \dfrac{1}{j200C}} \times 20\angle 0°$$

So $$0.915\angle -24° = \frac{1}{1 + j200CR} = \frac{1}{\sqrt{1+(200CR)^2}}\angle -\tan^{-1}(200CR)$$

Equating angles gives

$$-24° = -\tan^{-1}(200CR) \quad \Rightarrow \quad 200CR = \tan(24°) = 0.4452$$

The nominal component values cause $200CR = 0.5$. So we expect that the actual component values are smaller than the nominal values.

Try $$C = 5(1 - 0.10)\times 10^{-6} = 4.5\ \mu\text{F}$$

Then $$R = \frac{0.4452}{200 \times 4.5 \times 10^{-6}} = 494.67\ \Omega$$

Since $\dfrac{500 - 494.67}{500} = 0.01066 = 1.066\%$ this resistance is within 2% of 500 Ω. We conclude that the measured angle could have been caused by a capacitance that is within 10% of 5 μF and the resistance is within 2% of 500 Ω. Let's check the amplitude. We require

$$\frac{1}{\sqrt{1+\left(0.4452\right)^{2}}}=0.9136\simeq 0.915$$

So the measured amplitude could also have been caused by the given circuit with $C = 4.5$ μF and $R = 494.67$ Ω.

We conclude that he measured capacitor voltage could indeed have been produced by the given circuit with a resistance that is within 2 % of 500 Ω and a capacitance that is within 10% of 5 μF.

PSpice Problems

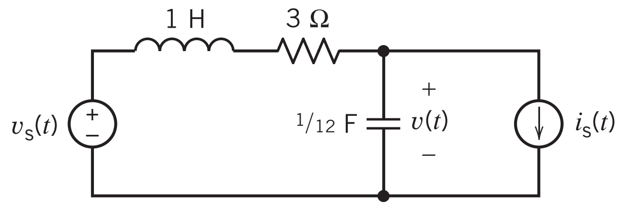


Figure SP 10.1

SP 10-1 The circuit shown in Figure SP 10.1 has two inputs, $v_s(t)$ and $i_s(t)$, and one output, $v(t)$. The inputs are given by

$$v_s(t) = 10 \sin (6t + 45°) \text{ V}$$

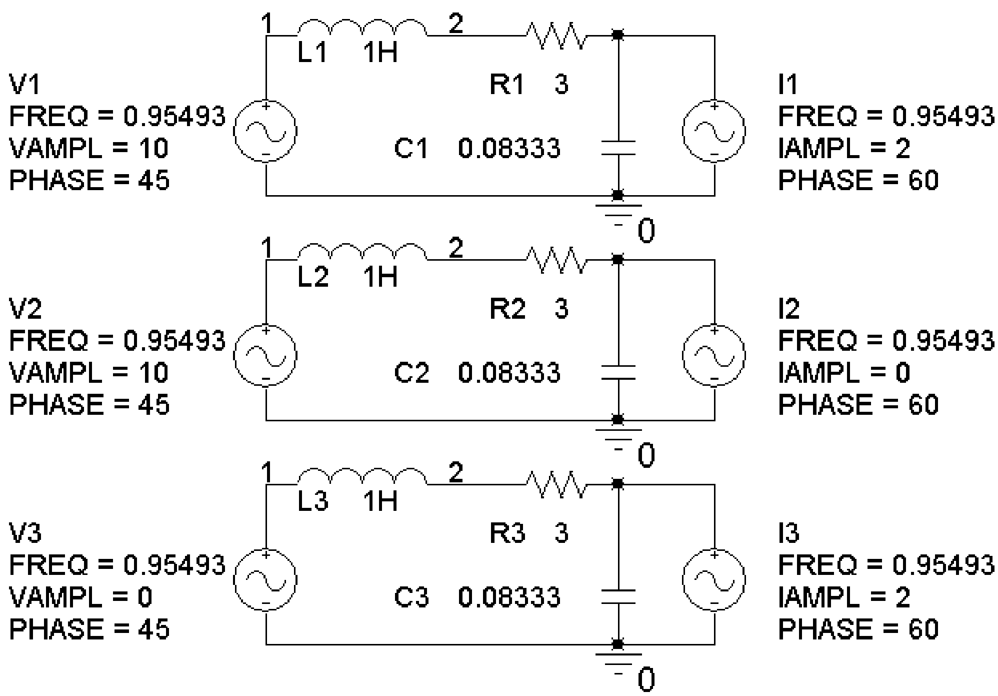
and $$i_s(t) = 2 \sin (6t + 60°) \text{ A}$$

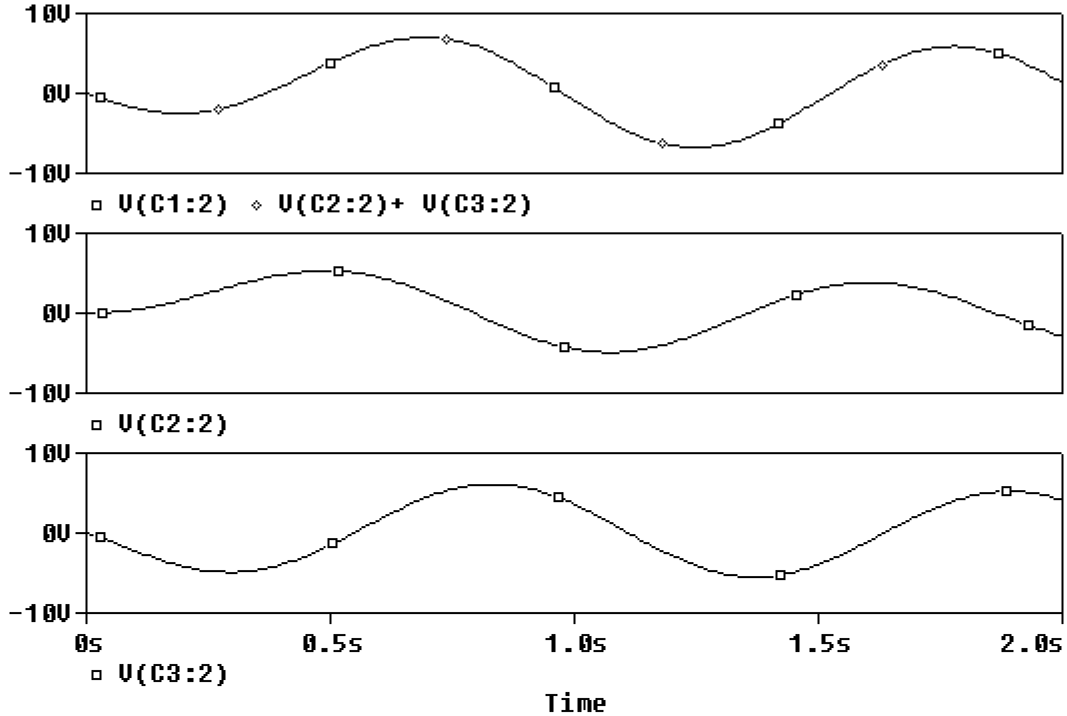
Use PSpice to demonstrate superposition. Simulate three versions of the circuit simultaneously. (Draw the circuit in the PSpice workspace. "Cut and paste" to make two copies. Edit the part names in the copies to avoid duplicate names. For example, the resistor will be R1 in the original circuit. Change R1 to R2 and R3 in the two copies.) Use the given $v_s(t)$ and $i_s(t)$ in the first version. Set $i_s(t) = 0$ in the second version and $v_s(t) = 0$ in the third version. Plot the capacitor voltage, $v(t)$, for all three versions of the circuit. Show that the capacitor voltage in the first version of the circuit is equal to the sum of the capacitor voltages in the second and third versions.

Hint: Use PSpice parts VSIN and ISIN for the voltage and current source. PSpice uses hertz rather than rad/s as the unit for frequency.

Remark: Notice that $v(t)$ is sinusoidal and has the same frequency as $v_s(t)$ and $i_s(t)$.

Solution:




10V
0V
-10V
V(C1:2) V(C2:2)+ V(C3:2)
10V
0V
-10V
V(C2:2)
10V
0V
-10V
0s
0.5s
1.0s
1.5s
2.0s
V(C3:2)
Time

SP 10-2 The circuit shown in Figure SP 10.1 has two inputs, $v_s(t)$ and $i_s(t)$, and one output, $v(t)$. The inputs are given by

$$v_s(t) = 10 \sin(6t + 45°) \text{ V}$$

and $$i_s(t) = 2 \sin(18t + 60°) \text{ A}$$

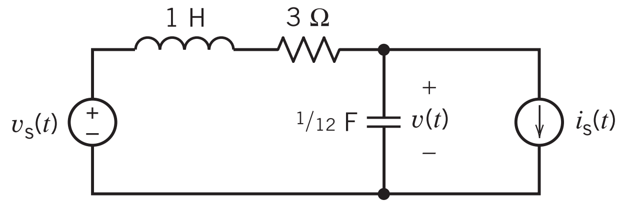


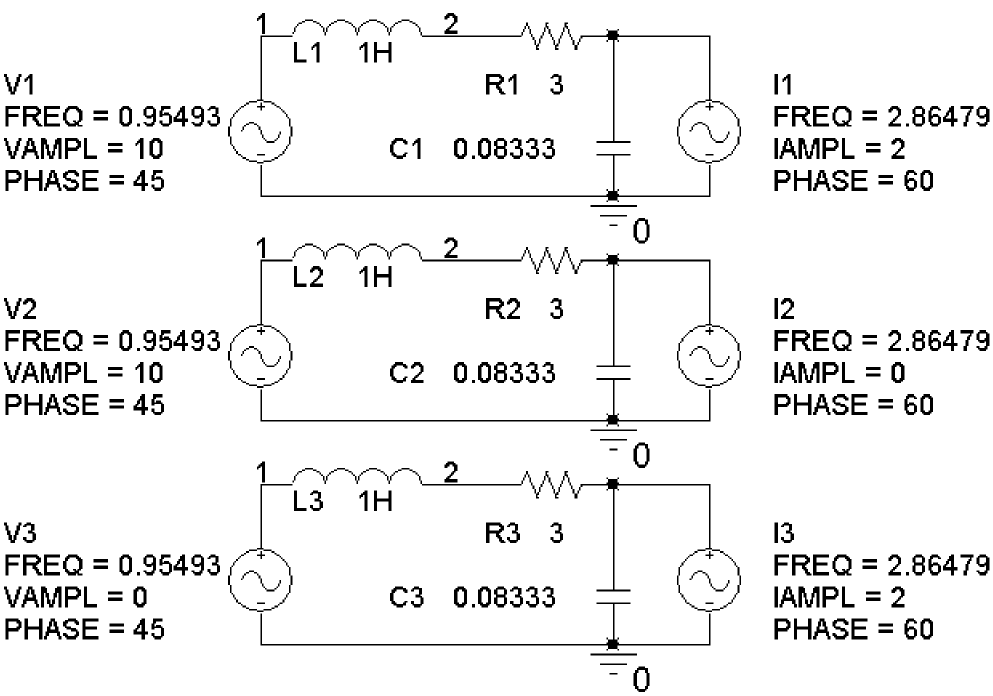
Figure SP 10.1

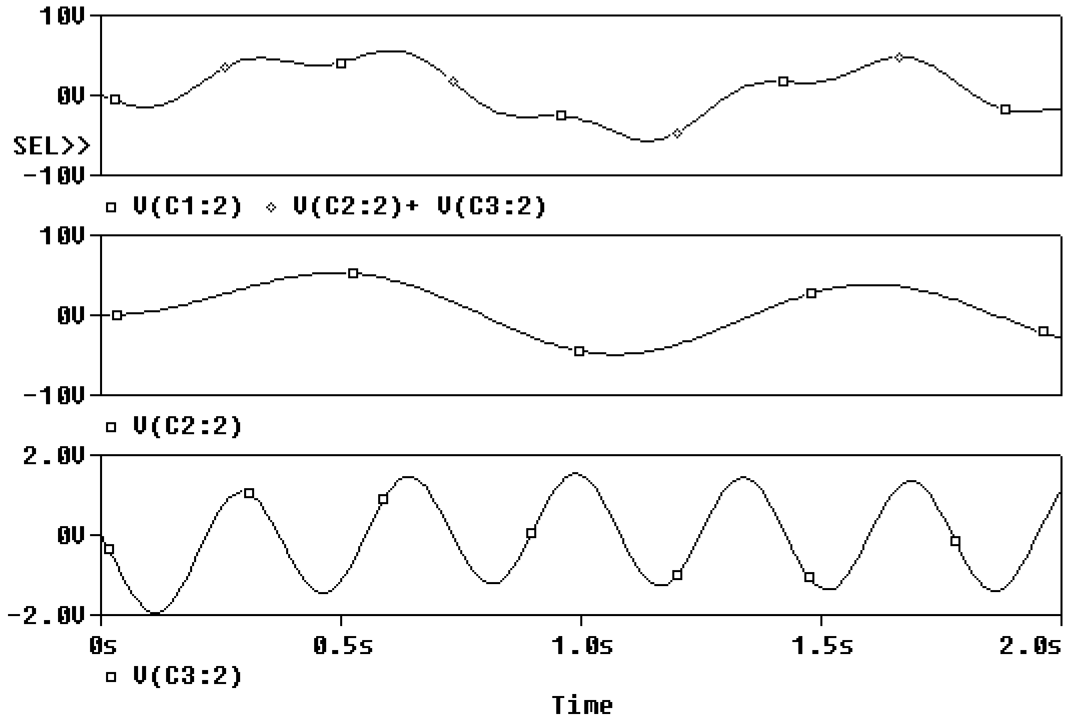
Use PSpice to demonstrate superposition. Simulate three versions of the circuit simultaneously. (Draw the circuit in the PSpice workspace. "Cut and paste" to make two copies. Edit the part names in the copies to avoid duplicate names. For example, the resistor will be R1 in the original circuit. Change R1 to R2 and R3 in the two copies.) Use the given $v_s(t)$ and $i_s(t)$ in the first version. Set $i_s(t) = 0$ in the second version and $v_s(t) = 0$ in the third version. Plot the capacitor voltage, $v(t)$, for all three versions of the circuit. Show that the capacitor voltage in the first version of the circuit is equal to the sum of the capacitor voltages in the second and third versions.

Hint: Use PSpice parts VSIN and ISIN for the voltage and current source. PSpice uses hertz rather than rad/s as the unit for frequency.

Remark: Notice that $v(t)$ is not sinusoidal.

Solution:




10V
0V
SEL>>
-10V
V(C1:2) V(C2:2)+ V(C3:2)
10V
0V
-10V
V(C2:2)
2.0V
0V
-2.0V
0s
0.5s
1.0s
1.5s
2.0s
V(C3:2)
Time

SP 10-3 The circuit shown in Figure SP 10-1 has two inputs, $v_s(t)$ and $i_s(t)$, and one output, $v(t)$. The inputs are given by

$$v_s(t) = 10 \sin(6t + 45°) \text{ V}$$

and $$i_s(t) = 0.8 \text{ A}$$

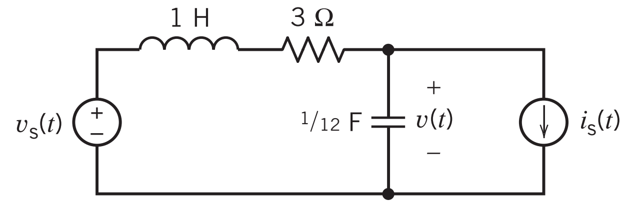


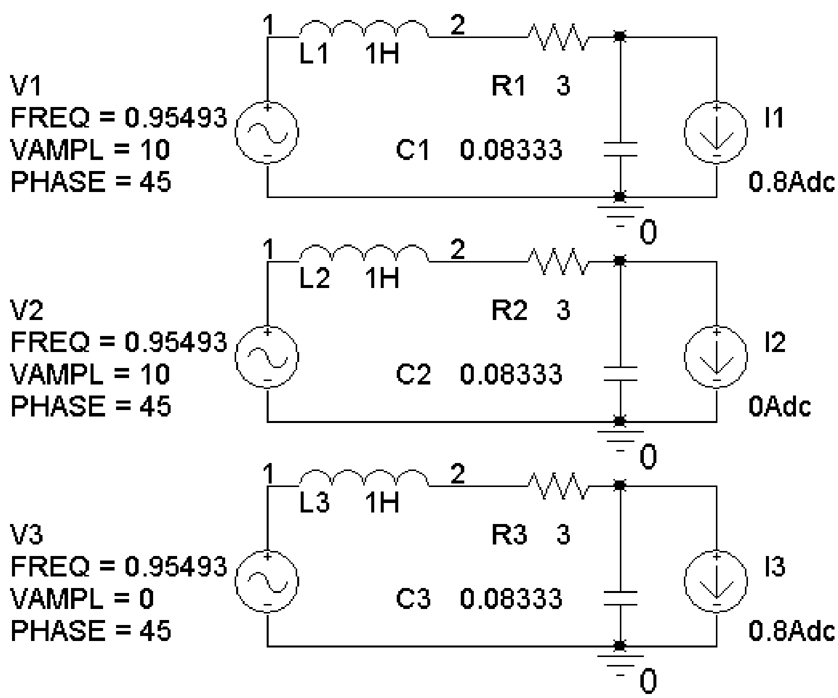
Figure SP 10.1

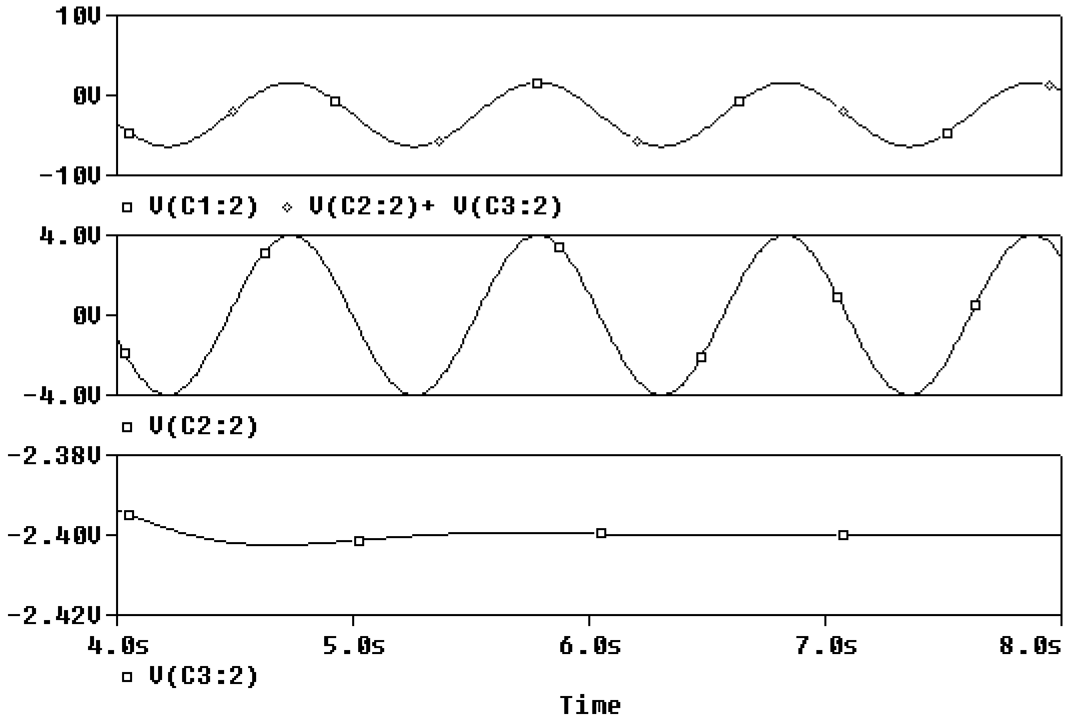
Use PSpice to demonstrate superposition. Simulate three versions of the circuit simultaneously. (Draw the circuit in the PSpice workspace. "Cut and paste" to make two copies. Edit the part names in the copies to avoid duplicate names. For example, the resistor will be R1 in the original circuit. Change R1 to R2 and R3 in the two copies.) Use the given $v_s(t)$ and $i_s(t)$ in the first version. Set $i_s(t) = 0$ in the second version and $v_s(t) = 0$ in the third version. Plot the capacitor voltage, $v(t)$, for all three versions of the circuit. Show that the capacitor voltage in the first version of the circuit is equal to the sum of the capacitor voltages in the second and third versions.

Hint: Use PSpice part VSIN and IDC for the voltage and current source. PSpice uses hertz rather than rad/s as the unit for frequency.

Remark: Notice that $v(t)$ looks sinusoidal, but it's not sinusoidal because of the dc offset.

Solution:





10V
0V
-10V
V(C1:2) V(C2:2)+ V(C3:2)
4.0V
0V
-4.0V
V(C2:2)
-2.38V
-2.40V
-2.42V
4.0s
5.0s
6.0s
7.0s
8.0s
V(C3:2)
Time

SP 10-4 The circuit shown in Figure SP 10-1 has two inputs, $v_s(t)$ and $i_s(t)$, and one output, $v(t)$. When inputs are given by

$$v_s(t) = V_m \sin 6t \text{ V}$$

and $$i_s(t) = I\text{m A}$$

the output will be

$$v_o(t) = A \sin(6t + \theta) + B \text{ V}$$

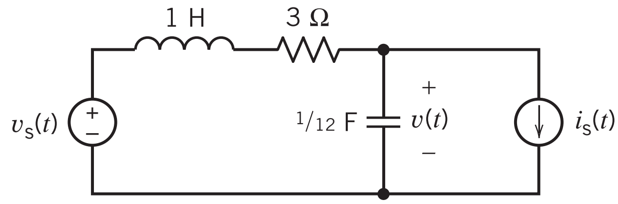


Figure SP 10.1

Linearity requires that A be proportional to V_m and that B be proportional to I_m. Consequently, we can write $A = k_1 V_m$ and $B = k_2 I_m$, where k_1 and k_2 are constants yet to be determined.

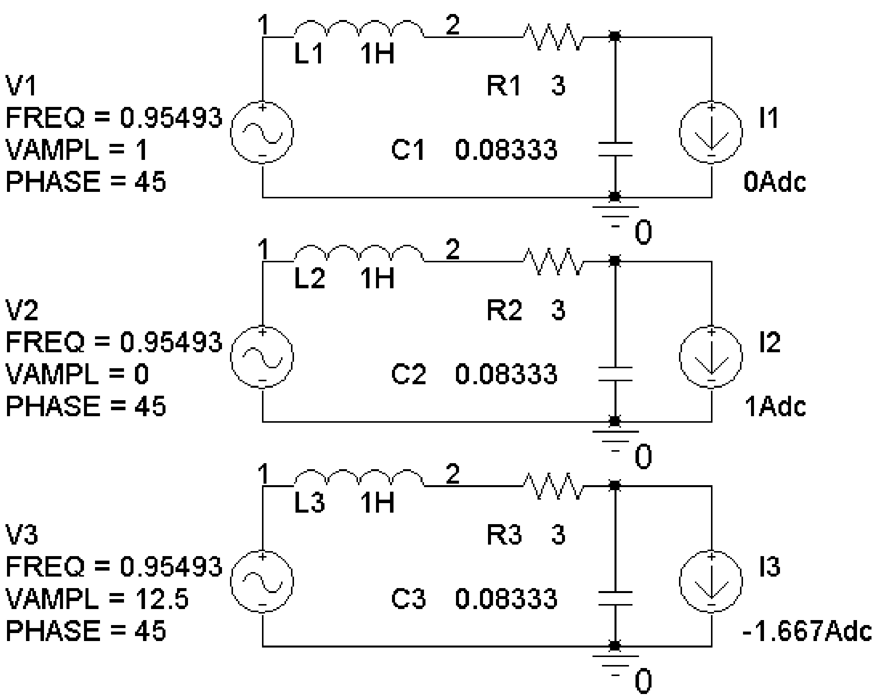
(a) Use PSpice to determine the value of k_1 by simulating the circuit using V_m = 1 V and I_m = 0.
(b) Use PSpice to determine the value of k_2 by simulating the circuit using V_m = 0 V and I_m = 1.
(c) Knowing k_1 and k_2, specify the values of V_m and I_m that are required to cause

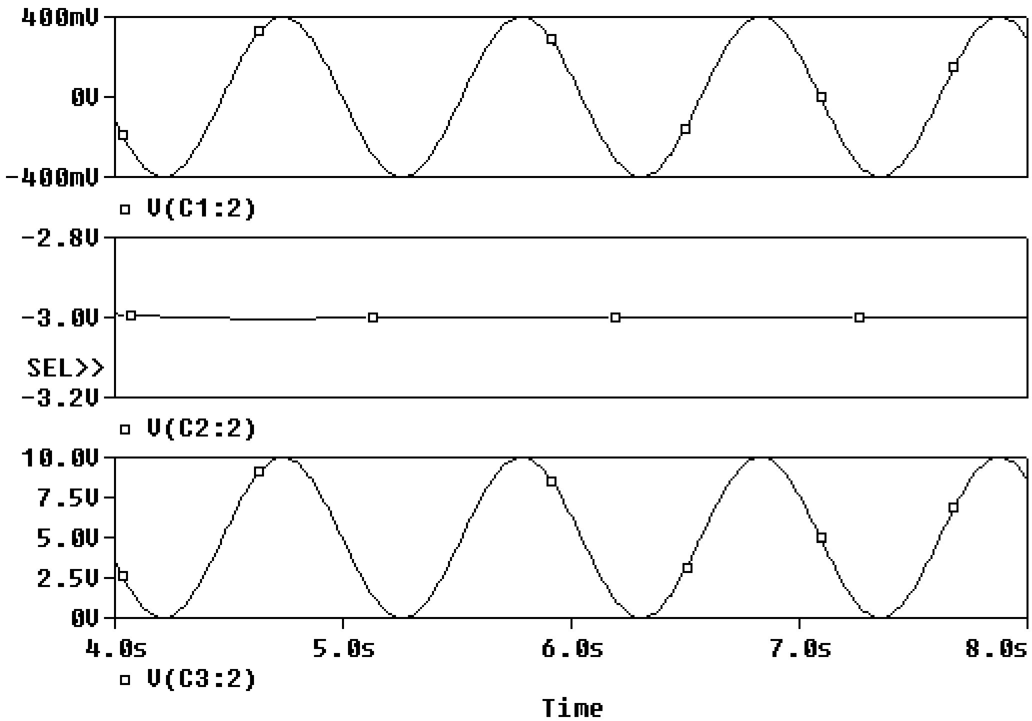
$$v_o(t) = 5 \sin(6t + \theta) + 5 \text{ V}$$

Simulate the circuit using PSpice to verify the specified values of V_m and I_m.

Solution:

The following simulation shows that k_1 = 0.4and k_2 = -3 V/A. The required values of V_m and I_m are V_m = 12.5 V and I_m = -1.667 A.




400mV
0V
-400mV
V(C1:2)
-2.8V
-3.0V
SEL>>
-3.2V
V(C2:2)
10.0V
7.5V
5.0V
2.5V
0V
4.0s
5.0s
6.0s
7.0s
8.0s
V(C3:2)
Time

Design Problems

DP 10-1 Design the circuit shown in Figure DP 10-1 to produce the specified output voltage $v_o(t) = 8\cos(1000t + 104°)$ V when provided with the input voltage $v_i(t) = 2.5\cos(1000t)$ V.

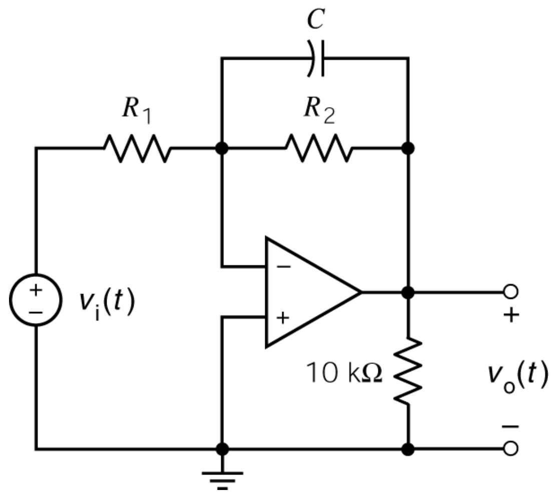
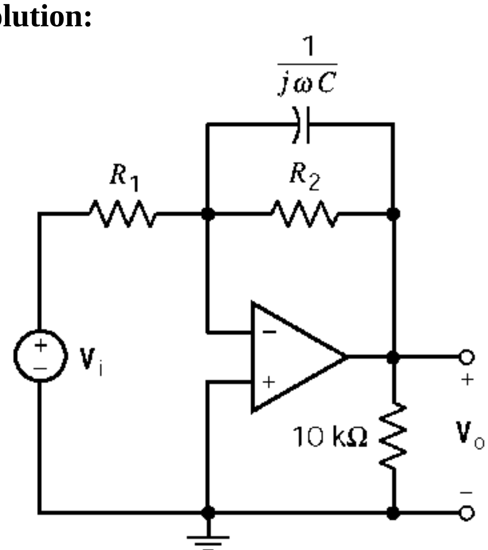


Figure DP 10-1

Solution:



$$R_2 \left\| \frac{1}{j\omega C} = \frac{R_2}{1 + j\omega CR_2}\right.$$

$$\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = -\frac{\dfrac{R_2}{1 + j\omega CR_2}}{R_1} = -\frac{\dfrac{R_2}{R_1}}{1 + j\omega CR_2}$$

$$\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{\dfrac{R_2}{R_1}}{\sqrt{1 + (\omega CR_2)^2}} e^{j\left(180 - \tan^{-1}\omega CR_2\right)}$$

In this case the angle of $\dfrac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$ is specified to be 104° so $CR_2 = \dfrac{\tan(180° - 104°)}{1000} = 0.004$ and

the magnitude of $\dfrac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$ is specified to be $\dfrac{8}{2.5}$ so $\dfrac{\dfrac{R_2}{R_1}}{\sqrt{1+16}} = \dfrac{8}{2.5} \quad \Rightarrow \quad \dfrac{R_2}{R_1} = 13.2$. One set of values that satisfies these two equations is $C = 0.2\ \mu\text{F},\ R_1 = 1515\ \Omega,\ R_2 = 20\ \text{k}\Omega$.

DP 10-2 Design the circuit shown in Figure DP 10-2 to produce the specified output voltage $v_o(t) = 2.5\cos(1000t - 76°)$ V when provided with the input voltage $v_i(t) = 12\cos(1000t)$ V.

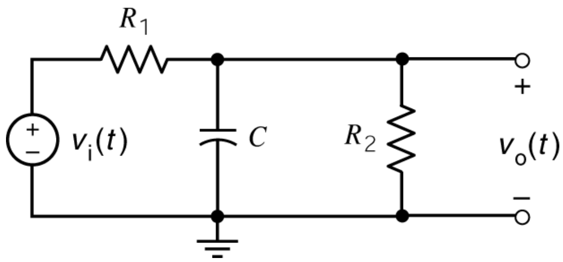
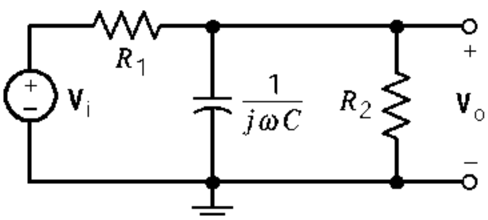


Figure DP 10-2

Solution:



$$R_2 \left\| \frac{1}{j\omega C} = \frac{R_2}{1 + j\omega CR_2}\right.$$

$$\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{\dfrac{R_2}{1 + j\omega CR_2}}{R_1 + \dfrac{R_2}{1 + j\omega CR_2}} = \frac{K}{1 + j\omega CR_p}$$

where $K = \dfrac{R_1}{R_1 + R_2}$ and $R_p = \dfrac{R_1 R_2}{R_1 + R_2}$

$$\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{K}{\sqrt{1 + (\omega CR_p)^2}} e^{-j\tan^{-1}\omega CR_p}$$

In this case the angle of $\dfrac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$ is specified to be -76° so

$C R_p = C\dfrac{R_1 R_2}{R_1 + R_2} = -\dfrac{\tan(-76)}{1000} = 0.004$ and the magnitude of $\dfrac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$ is specified to be $\dfrac{2.5}{12}$ so

$\dfrac{K}{\sqrt{1+16}} = \dfrac{2.5}{12} \quad \Rightarrow \quad 0.859 = K = \dfrac{R_2}{R_1 + R_2}$. One set of values that satisfies these two equations is

$C = 0.2\ \mu\text{F},\ R_1 = 23.3\ \text{k}\Omega,\ R_2 = 142\ \text{k}\Omega$.

DP 10-3 Design the circuit shown in Figure DP 10-3 to produce the specified output voltage $v_o(t) = 2.5\cos(40t + 14°)$ V when provided with the input voltage $v_i(t) = 8\cos(40t)$ V.

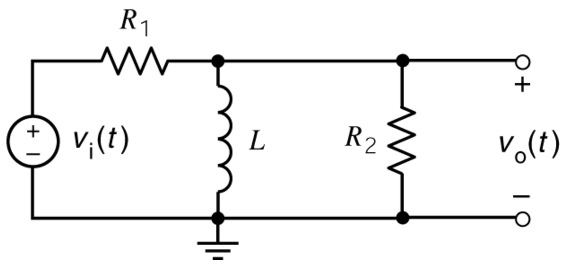
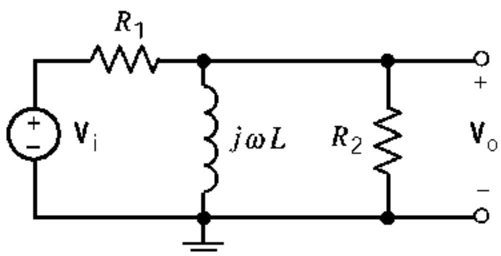


Figure DP 10-3

Solution:



$$\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{\dfrac{j\omega L R_2}{R_2 + j\omega L}}{R_1 + \dfrac{j\omega L R_2}{R_2 + j\omega L}} = \frac{j\omega \dfrac{L}{R_1}}{1 + j\omega \dfrac{L}{R_p}}$$

where $R_p = \dfrac{R_1 R_2}{R_1 + R_2}$

$$\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{\omega \dfrac{L}{R_1}}{\sqrt{1 + \left(\omega \dfrac{L}{R_p}\right)^2}} e^{j\left(90 - \tan^{-1} \omega \frac{L}{R_p}\right)}$$

In this case the angle of $\dfrac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$ is specified to be 14° so

$\dfrac{L}{R_p} = \dfrac{L(R_1 + R_2)}{R_1 R_2} = \dfrac{\tan(90° - 14°)}{40} = 0.1$ and the magnitude of $\dfrac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$ is specified to be $\dfrac{2.5}{8}$ so

$\dfrac{40 \dfrac{L}{R_1}}{\sqrt{1+16}} = \dfrac{2.5}{8} \quad \Rightarrow \quad \dfrac{L}{R_1} = 0.0322$. One set of values that satisfies these two equations is

$L = 1\,\text{H},\ R_1 = 31\,\Omega,\ R_2 = 14.76\,\Omega$.

DP 10-4 Show that it is not possible to design the circuit shown in Figure DP 10-4 to produce the specified output voltage $v_o(t)$ = 2.5cos(40t – 14°) when provided with the input voltage $v_i(t)$ = 8cos(40t) V.

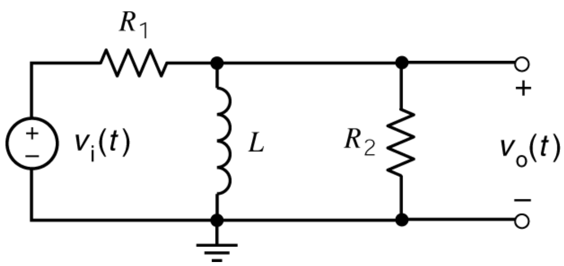
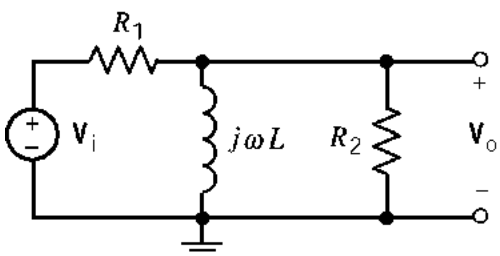


Figure DP 10-4

Solution:



$$\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{\dfrac{j\omega L R_2}{R_2 + j\omega L}}{R_1 + \dfrac{j\omega L R_2}{R_2 + j\omega L}} = \frac{j\omega \dfrac{L}{R_1}}{1 + j\omega \dfrac{L}{R_p}}$$

where $R_p = \dfrac{R_1 R_2}{R_1 + R_2}$

$$\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{\omega \dfrac{L}{R_1}}{\sqrt{1 + \left(\omega \dfrac{L}{R_p}\right)^2}} e^{j\left(90 - \tan^{-1} \omega \frac{L}{R_p}\right)}$$

In this case the angle of $\dfrac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$ is specified to be –14°. This requires

$$\frac{L}{R_p} = \frac{L\left(R_1 + R_2\right)}{R_1 R_2} = \frac{\tan(90+14)}{40} = -0.1$$

This condition cannot be satisfied with positive element waves.

DP 10-5 A circuit with an unspecified R, L, and C is shown in Figure DP 10-5. The input source is

$$i_s = 10 \cos 1000t \text{ A},$$

and the goal is to select the R, L, and C so that the node voltage is

$$v = 80 \cos 1000t \text{ V}.$$

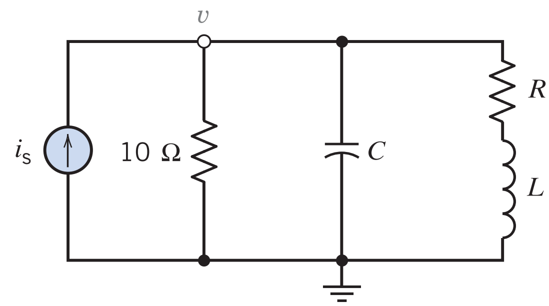
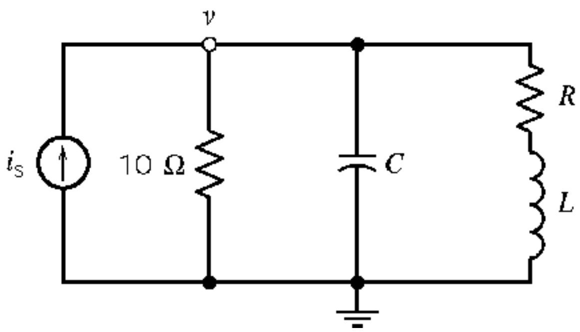


Figure DP 10-5

Solution:



$$\mathbf{Z}_1 = 10\ \Omega \qquad \mathbf{Y}_1 = \frac{1}{10}\ \text{S}$$
$$\mathbf{Z}_2 = \frac{1}{j\omega C} \qquad \mathbf{Y}_2 = j\omega C$$
$$\mathbf{Z}_3 = R + j\omega L \qquad \mathbf{Y}_3 = \frac{1}{R + j\omega L}$$

$$v(t) = 80 \cos (1000t - \theta) \text{ V} \quad \Rightarrow \quad \mathbf{V} = 80\angle -\theta \text{ V}$$
$$i_S(t) = 10 \cos 1000t \text{ A} \quad \Rightarrow \quad \mathbf{I}_s = 10\angle 0° \text{ A}$$

try $\theta = 0°$. Then

$$(80\angle -\infty)\left[\frac{1}{10} + \frac{1}{R + j\omega L} + j\omega C\right] = 10\angle 0° \quad \Rightarrow \quad R + 10 - 10\,\omega^2 LC + j(\omega L + 10\,\omega RC) = 1.25\,R + j1.25\,\omega L$$

Equate real part: $40 - 40\omega^2 LC = R$ where $\omega = 1000$ rad/sec

Equate imaginary part: $40\,RC = L$

Solving yields $R = 40(1 - 4\times10^7\,RC^2)$

Now try $\underline{R = 20\ \Omega} \Rightarrow \ 1 - 2(1 - 4\times10^7(20)C^2)$

which yields $C = 2.5\times10^{-5}\,\text{F} = \underline{25\ \mu\text{F}}$ so $L = 40\ RC = 0.02\ \text{H} = \underline{20\ \text{mH}}$

Now check the angle of the voltage. First

$$\mathbf{Y}_1 = 1/10 = 0.1\ \text{S}$$
$$\mathbf{Y}_2 = j0.25\ \text{S}$$
$$\mathbf{Y}_3 = 1/(20 + j20) = .025 - j.025\ \text{S}$$

then

$$\mathbf{Y} = \mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3 = 0.125\text{, so } \mathbf{V} = \mathbf{Y}\mathbf{I}_s = (0.125\angle 0°)(10\angle 0°) = 1.25\angle 0° \text{ V}$$

So the angle of the voltage is $\theta = 0°$, which satisfies the specifications.

DP 10-6 The input to the circuit shown in Figure DP 10-6 is the voltage source voltage

$$v_s(t) = 10 \cos(1000t) \text{ V}$$

The output is the steady-state capacitor voltage

$$v_o(t) = A \cos(1000t + \theta) \text{ V}$$

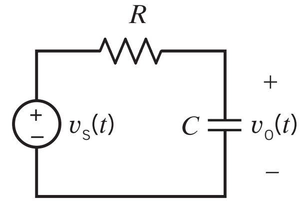


Figure DP 10-6

(a) Specify values for R and C such that $\theta = -30°$. Determine the resulting value of A.

(b) Specify values for R and C such that $A = 5$ V. Determine the resulting values of θ.

(c) Is it possible to specify values for R and C such that $A = 4$ and $\theta = -60°$? (If not, justify your answer. If so, specify R and C.)

(d) Is it possible to specify values of R and C such that $A = 7.07$ V and $\theta = -45°$? (If not, justify your answer. If so, specify R and C.)

Solution:

Represent the circuit in the frequency domain using phasors and impedances. Using voltage division gives

$$A\angle\theta = \frac{\dfrac{1}{j1000C}}{R + \dfrac{1}{j1000C}} \times 10\angle 0° = \frac{10}{1 + j10^3 RC}$$

Equating magnitudes and angles gives

$$A = \frac{10}{\sqrt{1 + 10^6 R^2 C^2}} \quad \Rightarrow \quad RC = \frac{\sqrt{\left(\dfrac{10}{A}\right)^2 - 1}}{1 + j10^3 RC}$$

and

$$\theta = -\tan^{-1}\left(10^3 RC\right) \quad \Rightarrow \quad RC = \frac{\tan(-\theta)}{10^3}$$

(a) $$\theta = -30° \quad \Rightarrow \quad RC = \frac{\tan(30°)}{10^3} = \frac{0.577}{10^3}.$$

Pick $C = 1$ μF, then $R = \dfrac{0.577}{10^6 \times 10^3} = 577\ \Omega$ and $A = 8.66$ V.

(b) $$A = 5 \text{ V} \quad \Rightarrow \quad RC = \frac{\sqrt{\left(\dfrac{10}{5}\right)^2 - 1}}{10^3} = \frac{\sqrt{3}}{10^3}.$$

Pick C = 1 μF, then $R = \dfrac{\sqrt{3}}{10^{-6} \times 10^{3}} = 1732\ \Omega$ and $\theta = -60°$.

(c)

$$A = 4 \quad \Rightarrow \quad RC = \frac{\sqrt{\left(\frac{10}{4}\right)^2 - 1}}{10^3} = \frac{2.29}{10^3}$$

$$\theta = -60° \quad \Rightarrow \quad RC = \frac{\tan(60°)}{10^3} = \frac{1.73}{10^3}$$

Since RC cannot be both 0.00229 and 0.00173 simultaneously, the specifications cannot be satisfied using this circuit.

(d)

$$A = 7.07 \quad \Rightarrow \quad RC = \frac{\sqrt{\left(\frac{10}{7.07}\right)^2 - 1}}{10^3} = 10^{-3}$$

$$\theta = -45° \quad \Rightarrow \quad RC = \frac{\tan(45°)}{10^3} = 10^{-3}$$

Both specifications can be satisfied by taking R = 1000 Ω and C = 1 μF.

Chapter 11: AC Steady State Power

Exercises

Exercise 11.3-1

Determine the instantaneous power delivered to an element and sketch $p(t)$ when the element is (a) a resistance R and (b) an inductor L. The voltage across the element is $v(t) = V_m \cos(\omega t + \theta)$ V.

Answer: **(a)** $P_R = \dfrac{V_m^2}{2R}[1 + \cos(2\omega t + 2\theta)]$ W **(b)** $P_L = \dfrac{V_m^2}{2\omega L}\cos(2\omega t + 2\theta - 90°)$ W

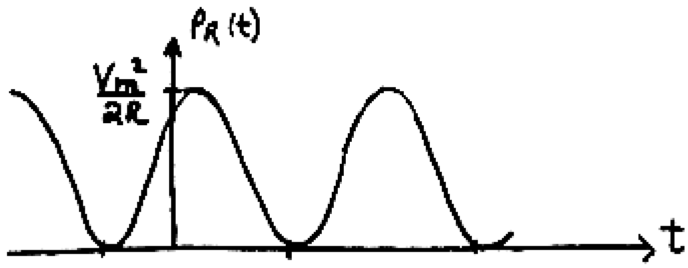
Solution:

(a) When the element is a resistor, the current has the same phase angle as the voltage:

$$i(t) = \frac{v(t)}{R} = \frac{V_m}{R}\cos(\omega t + \theta) \text{ A}$$

The instantaneous power delivered to the resistor is given by

$$p_R(t) = v(t)\cdot i(t) = V_m\cos(\omega t + \theta)\cdot\frac{V_m}{R}\cos(\omega t + \theta) = \frac{V_m^{\,2}}{R}\cos^2(\omega t + \theta) = \frac{V_m^{\,2}}{2R} + \frac{V_m^{\,2}}{2R}\cos(2\omega t + \theta)$$

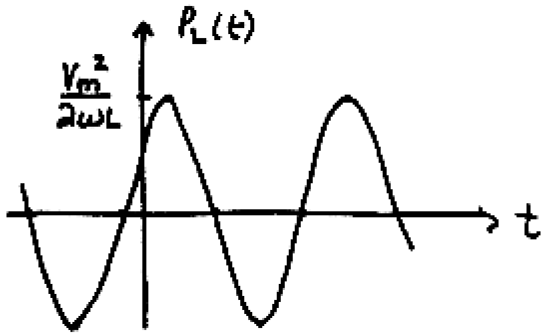


(b) When the element is an inductor, the current will lag the voltage by 90°.

$$\mathbf{Z}_L = j\omega L = \omega L\angle 90°\ \Omega \quad\Rightarrow\quad \mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{V_m\angle\theta}{\omega L\angle 90°} = \frac{V_m}{\omega L}\angle(\theta - 90°)$$

The instantaneous power delivered to the inductor is given by

$$p_L(t) = i(t)\cdot v(t) = \frac{V_m}{\omega L}\cos(\omega t + \theta - 90^\circ)\cdot V_m\cos(\omega t + \theta) = \frac{V_m^{\,2}}{2\omega L}\cos(2\omega t + 2\theta - 90°) \text{ W}$$



Exercise 11.4-1
Find the effective value of the following currents: (a) cos 3t + cos 3t; (b) sin 3t + cos(3t + 60°); (c) 2 cos 3t + 3 cos 5t
Answer: (a) $\sqrt{2}$ (b) 0.366 (c) 2.55

Ex. 11.4-1

(a)
$$i(t) = 2\cos 3t \text{ A} \quad \Rightarrow \quad I_{\text{eff}} \; \frac{I_{\text{max}}}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2} \text{ A}$$

(b)
$$i(t) = \cos(3t - 90^\circ) + \cos(3t + 60^\circ) \text{ A}$$

$$\mathbf{I} = (1\angle{-90^\circ}) + (1\angle 60^\circ) = -j + \frac{1}{2} + j\;\frac{\sqrt{3}}{2} = 0.518\angle{-15^\circ} \text{ A}$$

$$i(t) = 0.518 \;\cos\;(3t - 15^\circ) \text{ A} \Rightarrow I_{\text{eff}} = \frac{0.518}{\sqrt{2}} = 0.366 \text{ A}$$

(c)
$$I_{\text{eff}}{}^2 = \left(\frac{2}{\sqrt{2}}\right)^2 + \left(\frac{3}{\sqrt{2}}\right)^2 \quad \Rightarrow \quad I_{\text{eff}} = 2.55 \text{ A}$$

Exercise 11.5-1 Determine the average power delivered to each element of the circuit shown in Figure E 11.5-1. Verify that average power is conserved.
Answer: 4.39 + 0 = 4.39 W

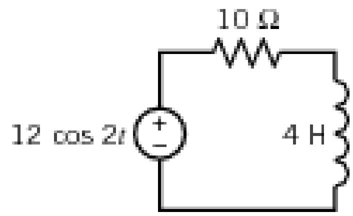
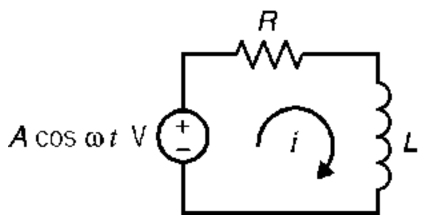


Figure E 11.5-1

Solution:



Analysis using Mathcad (ex11_5_1.mcd):

Enter the parameters of the voltage source: $A := 12$ $\omega := 2$

Enter the values of R and L $R := 10$ $L := 4$

The impedance seen by the voltage source is: $Z := R + j \cdot \omega \cdot L$

The mesh current is: $I := \dfrac{A}{Z}$

The complex power delivered by the source is: $Sv := \dfrac{\overline{I} \cdot (I \cdot Z)}{2}$ $Sv = 4.39 + 3.512i$

The complex power delivered to the resistor is: $Sr := \dfrac{\overline{I} \cdot (I \cdot R)}{2}$ $Sr = 4.39$

The complex power delivered to the inductor is: $Sl := \dfrac{\overline{I} \cdot (I \cdot j \cdot \omega \cdot L)}{2}$ $Sl = 3.512i$

Verify Sv = Sr + Sl : $Sr + Sl = 4.39 + 3.512i$ $Sv = 4.39 + 3.512i$

Exercise 11.5-2 Determine the complex power delivered to each element of the circuit shown in Figure E 11.5-2. Verify that complex power is conserved.
Answer: $6.606 + j5.248 - j3.303 + 6.606 = j1.982$ VA

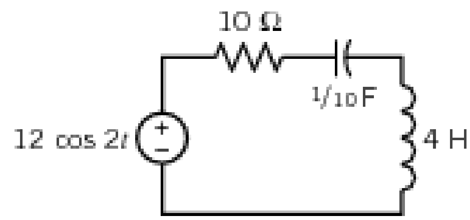
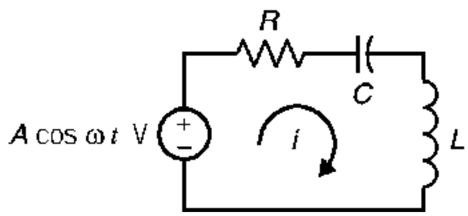


Figure E 11.5-2

Solution:



Analysis using Mathcad (ex11_5_2.mcd):

Enter the parameters of the voltage source: $A := 12$ $\omega := 2$

Enter the values of R, L ad C $R := 10$ $L := 4$ $C := 0.1$

The impedance seen by the voltage source is: $Z := R + j\cdot\omega\cdot L + \dfrac{1}{j\cdot\omega\cdot C}$

The mesh current is: $I := \dfrac{A}{Z}$

The complex power delivered by the source is: $Sv := \dfrac{\bar{I}\cdot(I\cdot Z)}{2}$ $Sv = 6.606 + 1.982i$

The complex power delivered to the resistor is: $Sr := \dfrac{\bar{I}\cdot(I\cdot R)}{2}$ $Sr = 6.606$

The complex power delivered to the inductor is: $Sl := \dfrac{\bar{I}\cdot(I\cdot j\cdot\omega\cdot L)}{2}$ $Sl = 5.284i$

The complex power delivered to the capacitor is: $Sc := \dfrac{\bar{I}\cdot\left(I\cdot\dfrac{1}{j\cdot\omega\cdot C}\right)}{2}$ $Sc = -3.303i$

Verify Sv = Sr + Sl + Sc : $Sr + Sl + Sc = 6.606 + 1.982i$ $Sv = 6.606 + 1.982i$

Exercise 11.6-1 A circuit has a large motor connected to the ac power lines [$\omega = (2\pi)60 = 377$ rad/s]. The model of the motor is a resistor of 100 ω in series with an inductor of 5 H. Find the power factor of the motor.
Answer: $pf = 0.053$ lagging

Solution:

$$pf = \cos(\angle \mathbf{Z}) = \cos\left[\tan^{-1}\left(\frac{\omega L}{R}\right)\right] = \cos\left[\tan^{-1}\frac{(377)(5)}{100}\right] = 0.053 \text{ lagging}$$

Exercise 11.6-2 A circuit has a load impedance $Z = 50 + j80\ \Omega$, as shown in Figure 11.6-5. Determine the power factor of the uncorrected circuit. Determine the impedance $\mathbf{Z}_C$ required to obtain a corrected power factor of 1.0.
Answer : $\mathbf{Z}_C = -j111.25\ \Omega$

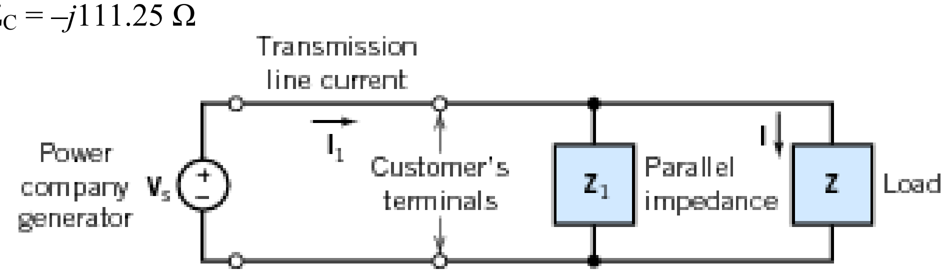


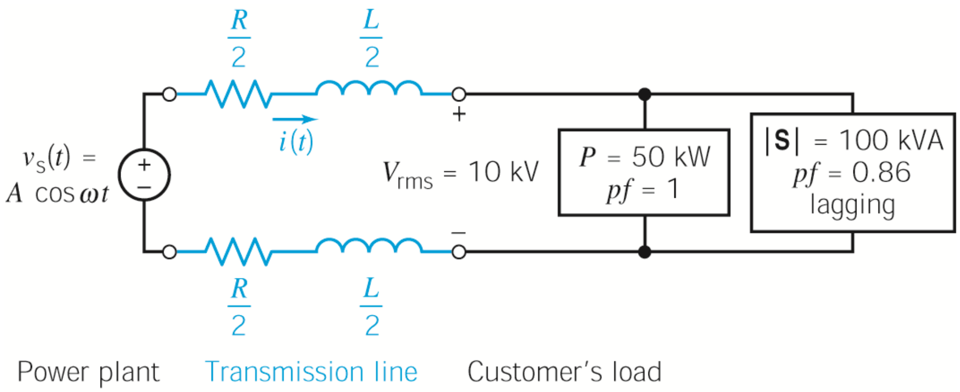
Figure 11.6-5

Solution:

$$pf = \cos(\angle \mathbf{Z}) = \cos\left[\tan^{-1}\left(\frac{X}{R}\right)\right] = \cos\left[\tan^{-1}\left(\frac{80}{50}\right)\right] = 0.53 \text{ lagging}$$

$$X_C = \frac{(50)^2 + (80)^2}{50 \tan(\cos^{-1} 1) - 80} = -111.25\ \Omega \quad \Rightarrow \quad \mathbf{Z}_C = -j\ 111.25\ \Omega$$

Exercise 11.6-3 Determine the power factor for the total plant of Example 11.6-1 when the resistive heating load is decreased to 30 kW. The motor load and the supply voltage remain as described in Example 11.6-1.
Answer: $pf = 0.915$



(a)
Figure from Example 11.6-1

Solution:

$$P_T = 30 + 86 = 116 \text{ kW} \text{ and } Q_T = 51 \text{ kVAR}$$

$$\mathbf{S}_T = P_T + j\,Q_T = 116 + j51 = 126.7\ \angle 23.7° \text{ kVA}$$

$$pf_{\text{plant}} = \cos 23.7° = 0.915$$

Exercise 11.6-4 A 4-kW, 110-V_{rms} load, as shown in Figure 11.6-5, has a power factor of 0.82 lagging. Find the value of the parallel capacitor that will correct the power factor to 0.95 lagging when ω = 377 rad/s.
Answer: C = 0.324 mF

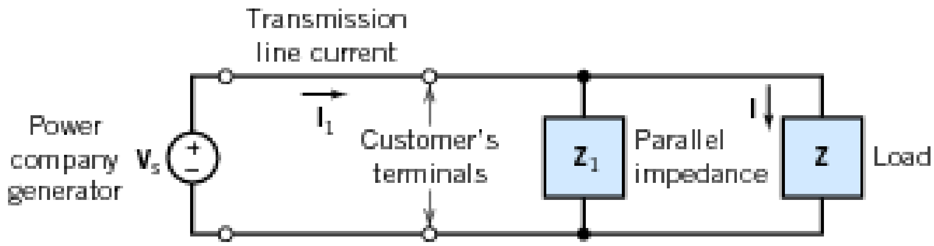


Figure 11.6-5

Solution:

$$P = V_{\text{rms}} I_{\text{rms}} \cos\theta \quad \Rightarrow \quad I_{\text{rms}} = \frac{P}{V_{\text{rms}}\cos\theta} = \frac{4000}{(110)(.82)} = 44.3 \text{ A}$$

$$\mathbf{Z} = \frac{V_{\text{rms}}}{I_{\text{rms}}} \angle\cos^{-1}(0.82) = 2.48\ \angle 34.9^\circ = 2.03 + j1.42 = R + jX$$

To correct power factor to 0.95 requires

$$X_1 = \frac{R^2 + X^2}{R\tan(\cos^{-1} pfc) - X} = \frac{(2.03)^2 + (1.42)^2}{(2.03)\ \tan\ (18.19^\circ) - 1.42} = -\ 8.16\ \Omega$$

$$C = \frac{-1}{\omega X_1} = \underline{325\ \mu\text{F}}$$

Exercise 11.7-1 Determine the average power absorbed by the resistor in Figure 11.7-2*a* for these two cases:

(a) $v_A(t) = 12 \cos 3t$ V and $v_B(t) = 4 \cos 3t$ V;

(b) $v_A(t) = 12 \cos 4t$ V and $v_B(t) = 4 \cos 3t$ V

Answer: (a) 2.66 W (b) 4.99 W

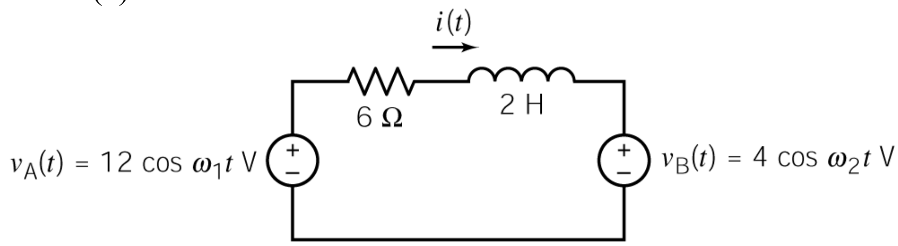


Figure 11.7-2*a*

Solution:

(a) $\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = (1.414\angle -45°) + (0.4714\angle 135°) = 0.9428\angle -45° \text{ A}$

$$\Rightarrow \quad p = \frac{0.9428^2}{2}(6) = 2.66 \text{ W}$$

(b)

$$\mathbf{I}_1 = 1.2\angle 53° \text{ A} \quad \Rightarrow \quad p_1 = \frac{1.2^2}{2}(6) = 4.32 \text{ W}$$

$$\mathbf{I}_2 = 0.4714\angle 135° \text{ A} \quad \Rightarrow \quad p_2 = \frac{0.4714^2}{2}(6) = 0.666 \text{ W}$$

$$\therefore p = p_1 + p_2 = 4.99 \text{ W}$$

Exercise 11.8-1 For the circuit of Figure 11.8-1, find $\mathbf{Z}_L$ to obtain the maximum power transferred when the Thévenin equivalent circuit has $\mathbf{V}_t = 100\angle 0°$V and $\mathbf{Z}_t = 10 + j14\ \Omega$. Also determine the maximum power transferred to the load.

Answer: $\mathbf{Z}_L = 10 - j14\ \Omega$ and $P = 125$ W

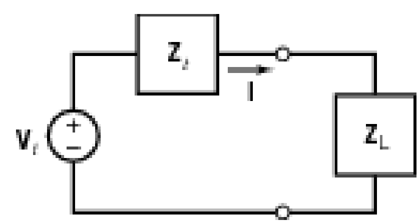
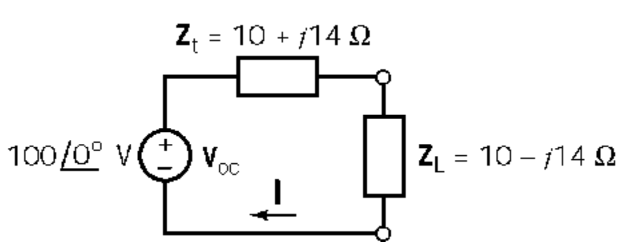


Figure 11.8-1

Solution:



For maximum power transfer

$$\mathbf{Z}_L = \mathbf{Z}_t^* = 10 - j14\ \Omega$$

$$\mathbf{I} = \frac{100}{(10 + j14) + (10 - j14)} = 5 \text{ A}$$

$$P_L = \left(\frac{5}{\sqrt{2}}\right)^2 \text{Re}\{10 - j14\} = 125 \text{ W}$$

Exercise 11.8-2 A television receiver uses a cable to connect the antenna to the TV, as shown in Figure E 11.8-2, with $v_s = 4 \cos \omega t$ mV. The TV station is received at 52 MHz. Determine the average power delivered to each TV set if (a) the load impedance is $\mathbf{Z} = 300\ \Omega$; (b) two identical TV sets are connected in parallel with $\mathbf{Z} = 300\ \Omega$ for each set; (c) two identical sets are connected in parallel and $\mathbf{Z}$ is to be selected so that maximum power is delivered at each set.
Answer: (a) 9.6 nW (b) 4.9 nW (c) 5 nW

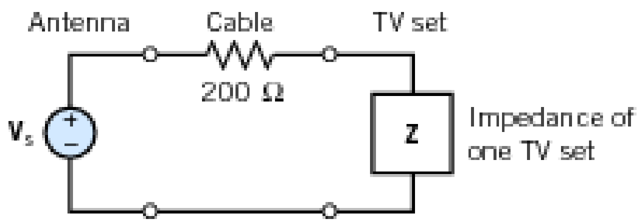


Figure E 11.8-2

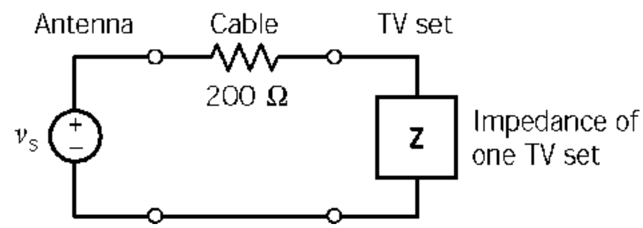
Solution:

If the station transmits a signal at 52 MHz then

$$\omega = 2\pi f = 104\pi\times10^6 \text{ rad/sec}$$

so the received signal is

$$v_s(t) = 4\cos(104\pi\times10^6\, t) \text{ mV}$$



(a) If the receiver has an input impedance of $\mathbf{Z}_{\text{in}} = 300\ \Omega$ then

$$V_{\text{in}} = \frac{\mathbf{Z}_{\text{in}}}{R+\mathbf{Z}_{\text{in}}} V_s = \frac{300}{200+300}\times 4\times10^{-3} = 2.4 \text{ mV} \quad\Rightarrow\quad P = \frac{1}{2}V_{\text{in}}^2\left(\frac{1}{\mathbf{Z}_{\text{in}}}\right) = \frac{\left(2.4\times10^{-3}\right)^2}{2(300)} = \underline{9.6 \text{ nW}}$$

(b) If two receivers are connected in parallel then $\mathbf{Z}_{\text{in}} = 300\|300 = 150\ \Omega$ and

$$V_{\text{in}} = \frac{\mathbf{Z}_{\text{in}}}{R+\mathbf{Z}_{\text{in}}} V_S = \frac{150}{200+150}\ (4\times10^{-3}) = 1.71\times10^{-3} \text{ V}$$

$$\text{total } P = \frac{V_{\text{in}}^{\ 2}}{2}\left(\frac{1}{\mathbf{Z}_{\text{in}}}\right) = \frac{(1.71\times10^{-3})^2}{2(150)} = 9.7 \text{ nW or } 4.85 \text{ nW to each set}$$

(c) In this case, we need $\mathbf{Z}_{\text{in}} = R\|R = 200\ \Omega \quad\Rightarrow\quad R = 400\ \Omega$, where R is the input impedance of each television receiver. Then

$$P_{\text{total}} = \frac{V_{\text{in}}^{\ 2}}{2\,\mathbf{Z}_{\text{in}}} = \frac{(2\times10^{-3})^2}{2(200)} = 10 \text{ nW} \quad\Rightarrow\quad 5 \text{ nW to each set}$$

Exercise 11.9-1 Determine the voltage v_o for the circuit of Figure E 11.9-1.
Hint: Write a single mesh equation. The currents in the two coils are equal to each other and equal to the mesh current.
Answer: $v_o = 14 \cos 4t$ V

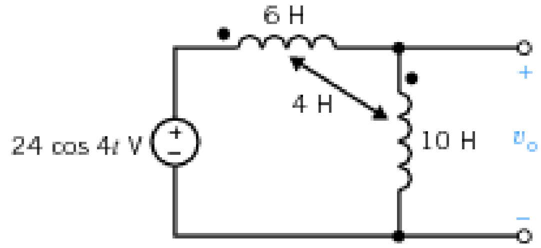
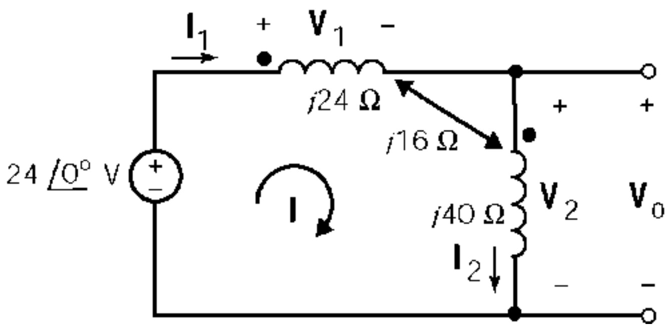


Figure E 11.9-1

Solution:



Coil voltages:

$$\mathbf{V}_1 = j24\,\mathbf{I}_1 + j16\,\mathbf{I}_2 = j40\,\mathbf{I}$$

$$\mathbf{V}_2 = j16\,\mathbf{I}_1 + j40\,\mathbf{I}_2 = j56\,\mathbf{I}$$

Mesh equation:

$$24 = \mathbf{V}_1 + \mathbf{V}_2 = j40\,\mathbf{I} + j56\,\mathbf{I} = j96\,\mathbf{I}$$

$$\mathbf{I} = \frac{24}{j96} = -j\frac{1}{4}$$

$$\mathbf{V}_o = \mathbf{V}_2 = (j56)\left(-j\frac{1}{4}\right) = 14$$

$$v_o = 14 \cos 4t \text{ V}$$

Exercise 11.9-2 Determine the voltage v_o for the circuit of Figure E 11.9-2.
Hint: This exercise is the same as Exercise 11.9-1, except for the position of the dot on the vertical coil.
Answer: $v_o = 18 \cos 4t$ V

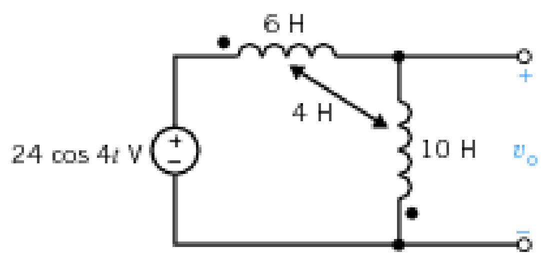
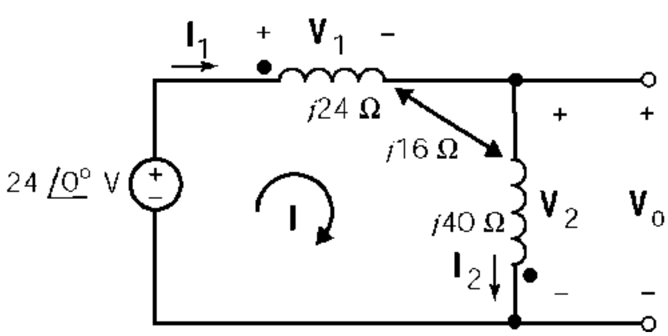


Figure E 11.9-2

Solution:



Coil voltages:

$$\mathbf{V}_1 = j24\,\mathbf{I}_1 - j16\,\mathbf{I}_2 = j8\,\mathbf{I}$$

$$\mathbf{V}_2 = -j16\,\mathbf{I}_1 + j40\,\mathbf{I}_2 = j24\,\mathbf{I}$$

Mesh equation:

$$24 = \mathbf{V}_1 + \mathbf{V}_2 = j8\,\mathbf{I} + j24\,\mathbf{I} = j32\,\mathbf{I}$$

$$\mathbf{I} = \frac{24}{j32} = -j\frac{3}{4}$$

$$\mathbf{V}_o = \mathbf{V}_2 = (j24)\left(-j\frac{3}{4}\right) = 18$$

$$v_o = 18 \cos 4t \text{ V}$$

Exercise 11.9-3 Determine the current i_o for the circuit of Figure E 11.9-3.

Hint: The voltage across the vertical coil is zero because of the short circuit. The voltage across the horizontal coil induces a current in the vertical coil. Consequently, the current in the vertical coil is not zero.

Answer: $i_o = 1.909 \cos(4t - 90°)$ A

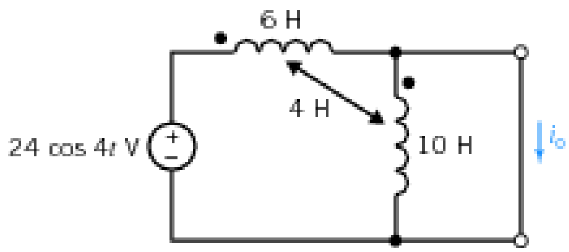
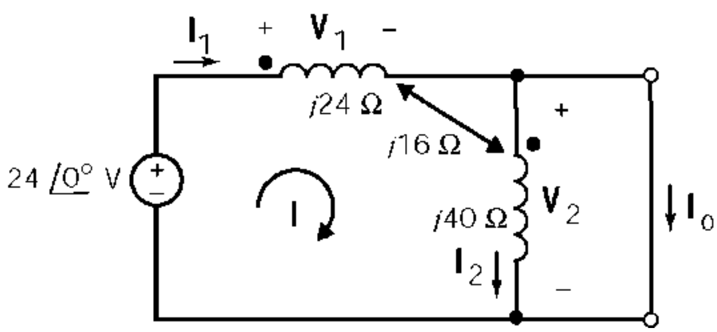


Figure E 11.9-3

Solution:



$$0 = \mathbf{V}_2 = j16\,\mathbf{I}_1 + j40\,\mathbf{I}_2$$

$$\Rightarrow \mathbf{I}_1 = -\frac{40}{16}\mathbf{I}_2 = -2.5\ \mathbf{I}_2$$

$$\mathbf{V}_s = \mathbf{V}_1 = j24\,\mathbf{I}_1 + j16\,\mathbf{I}_2$$

$$= j(24(-2.5)+16)\,\mathbf{I}_2$$

$$= -j44\,\mathbf{I}_2$$

$$\mathbf{I}_2 = \frac{24}{-j44} = j\frac{6}{11}$$

$$\mathbf{I}_o = \mathbf{I}_1 - \mathbf{I}_2 = (-2.5-1)\,\mathbf{I}_2$$

$$= -3.5\,\mathbf{I}_2$$

$$= -3.5\left(j\frac{6}{11}\right) = -j1.909$$

$i_o = 1.909 \cos(4t - 90°)$ A

Exercise 11.9-4 Determine the current i_o for the circuit of Figure E 11.9-4.
Hint: This exercise is the same as Exercise 11.9-3, except for the position of the dot on the vertical coil.
Answer: $i_o = 0.818 \cos(4t - 90°)$ A

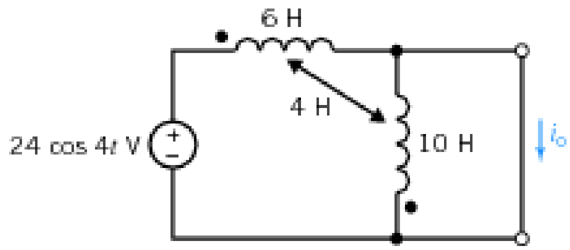
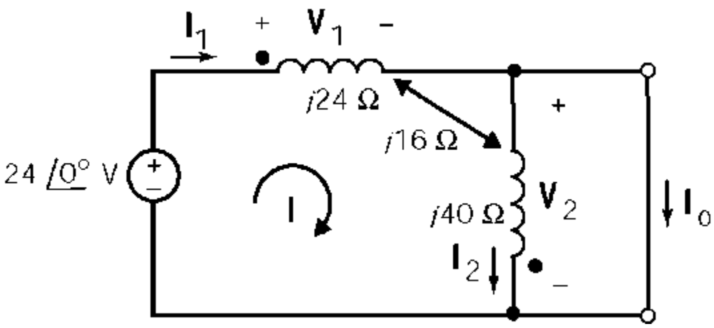


Figure E 11.9-4

Solution:



$$0 = \mathbf{V}_2 = -j16\,\mathbf{I}_1 + j40\,\mathbf{I}_2$$

$$\Rightarrow \mathbf{I}_1 = \frac{40}{16}\mathbf{I}_2 = 2.5\ \mathbf{I}_2$$

$$\mathbf{V}_s = \mathbf{V}_1 = j24\,\mathbf{I}_1 - j16\,\mathbf{I}_2$$
$$= j(24(2.5) - 16)\,\mathbf{I}_2$$
$$= j44\,\mathbf{I}_2$$

$$\mathbf{I}_2 = \frac{24}{j44} = -j\frac{6}{11}$$

$$\mathbf{I}_o = \mathbf{I}_1 - \mathbf{I}_2 = (2.5-1)\,\mathbf{I}_2$$
$$= 1.5\,\mathbf{I}_2$$
$$= 1.5\left(-j\frac{6}{11}\right) = -j0.818$$

$i_o = 0.818 \cos(4t - 90°)$ A

Exercise 11.10-1 Determine the impedance $\mathbf{Z}_{ab}$ for the circuit of Figure E 11.10-1. All the transformers are ideal.

Answer: $\mathbf{Z}_{ab} = 4.063\mathbf{Z}$

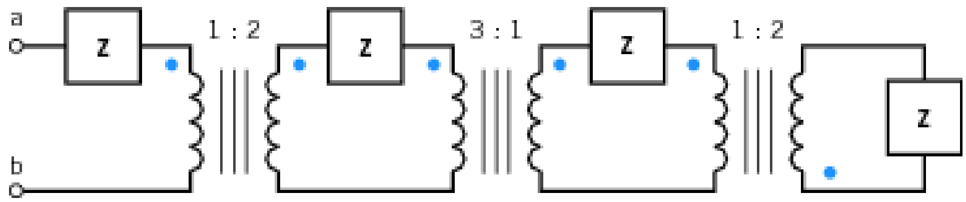
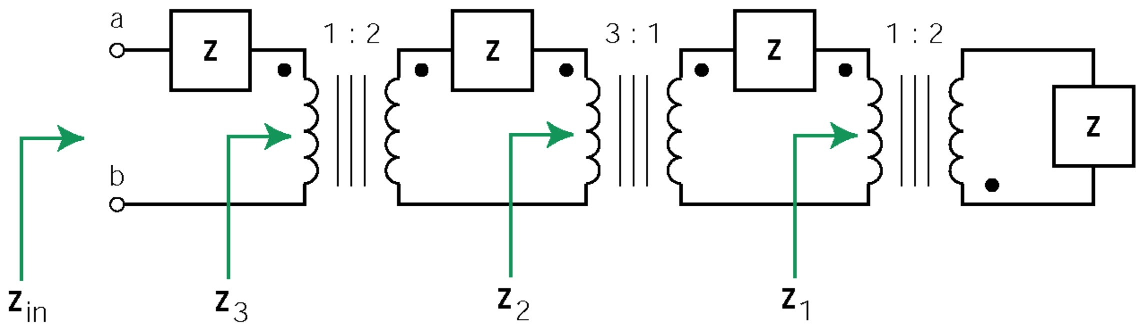


Figure E 11.10-1

Ex. 11.10-1



$$\mathbf{Z}_1 = \frac{\mathbf{Z}}{n_3^2} = \frac{\mathbf{Z}}{4}, \quad \mathbf{Z}_2 = \frac{1}{n_2^2}\left(\mathbf{Z} + \frac{\mathbf{Z}}{n_3^2}\right) = 9\left(\mathbf{Z} + \frac{\mathbf{Z}}{4}\right) \quad \text{and} \quad \mathbf{Z}_3 = \frac{1}{n_1^2}(\mathbf{Z} + \mathbf{Z}_2)$$

then

$$\mathbf{Z}_{ab} = \mathbf{Z}_{in} = \mathbf{Z} + \mathbf{Z}_3 = \mathbf{Z} + \frac{1}{4}\left(\mathbf{Z} + 9\left(\mathbf{Z} + \frac{\mathbf{Z}}{4}\right)\right) = 4.0625\ \mathbf{Z}$$

PROBLEMS

Section 11.3 Instantaneous Power and Average Power

P 11.3-1 An *RLC* circuit is shown in Figure P 11.3-1. Find the instantaneous power delivered to the inductor when $i_s = 1 \cos \omega t$ A and $\omega = 6283$ rad/s.

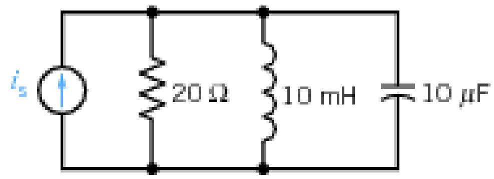
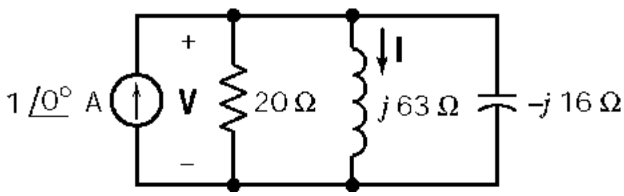


Figure P 11.3-1

Solution:



$$1\angle 0° = \frac{\mathbf{V}}{20} + \frac{\mathbf{V}}{j63} + \frac{\mathbf{V}}{-j16} \Rightarrow \mathbf{V} = 14.6\angle -43°\text{ V}$$

$$\mathbf{I} = \frac{\mathbf{V}}{j63} = 0.23\angle -133°\text{ A}$$

$$\begin{aligned} p(t) = i(t)v(t) &= 0.23\cos\ (2\pi\cdot 10^3 t - 133^\circ)\times 14.6\ \cos\ (2\pi\cdot 10^3 t - 43^\circ) \\ &= 3.36\ \cos\ (2\pi\cdot 10^3 t - 133^\circ)\ \cos\ (2\pi\cdot 10^3 t - 43^\circ) \\ &= 1.68\ (\cos\ (90^\circ) + \cos\ (4\pi\cdot 10^3 t - 176^\circ)) \\ &= 1.68\ \cos\ (4\pi\cdot 10^3 t - 176^\circ) \end{aligned}$$

P 11.3-2 Find the average power absorbed by the 0.6-kΩ resistor and the average power supplied by the current source for the circuit of Figure P 11.3-2.

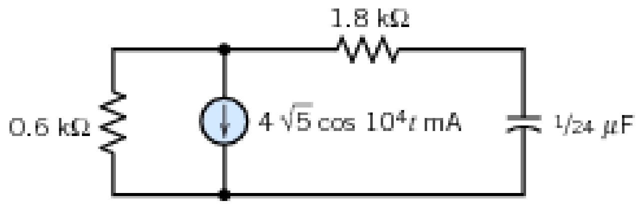
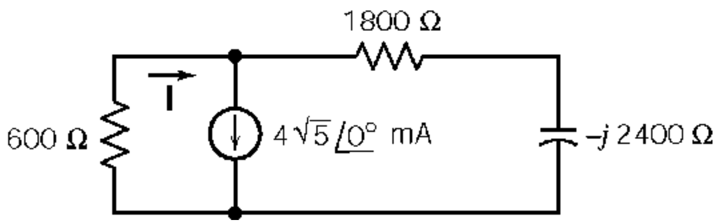


Figure P 11.3-2

Solution:



Current division:

$$\mathbf{I} = 4\sqrt{5}\left[\frac{1800-j2400}{1800-j2400+600}\right] = 5\sqrt{\frac{5}{2}}\angle -8.1° \text{ mA}$$

$$P_{600\Omega} = \frac{|\mathbf{I}|^2\,600}{2} = 300(25)\left(\frac{5}{2}\right) = 1.875\times10^4 \ \mu\text{W} = 18.75 \text{ mW}$$

$$P_{\text{source}} = \frac{|\mathbf{V}||\mathbf{I}|\cos\theta}{2} = \frac{1}{2}(600)\left(5\sqrt{\frac{5}{2}}\right)\left(4\sqrt{5}\right)\cos(-8.1^\circ) = 2.1\times10^4 \ \mu\text{W} = 21 \text{ mW}$$

P 11.3-3 Use nodal analysis to find the average power absorbed by the 20-Ω resistor in the circuit of Figure P 11.3-3.
Answer: $P = 200$ W

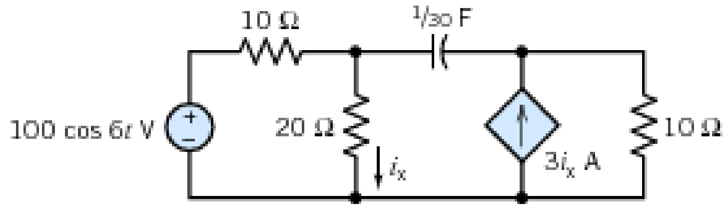
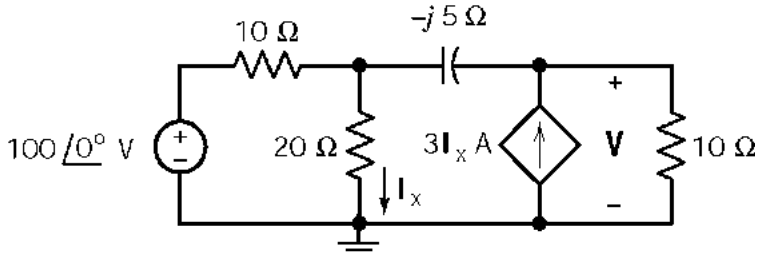


Figure P 11.3-3

Solution:



Node equations:

$$\frac{20\mathbf{I}_X - 100}{10} + \mathbf{I}_X + \frac{20\mathbf{I}_X - \mathbf{V}}{-j5} = 0 \quad \Rightarrow \quad \mathbf{I}_X(20 - j15) - \mathbf{V} = -j50$$

$$\frac{\mathbf{V} - 20\mathbf{I}_X}{-j5} - 3\mathbf{I}_X + \frac{\mathbf{V}}{10} = 0 \quad \Rightarrow \quad \mathbf{I}_X(-40 + j30) + \mathbf{V}(2 - j) = 0$$

Solving the node equations using Cramer's rule yields

$$\mathbf{I}_X = \frac{j50(2-j)}{(40-j30)-(20-j15)(2-j)} = \frac{50\sqrt{5}\angle 63.4^\circ}{25\angle 53.1^\circ} = 2\sqrt{5}\angle 10.3^\circ \text{ A}$$

Then

$$P_{AVE} = \frac{|\mathbf{I}_X|^2}{2}(20) = 10\left(2\sqrt{5}\right)^2 = 200 \text{ W}$$

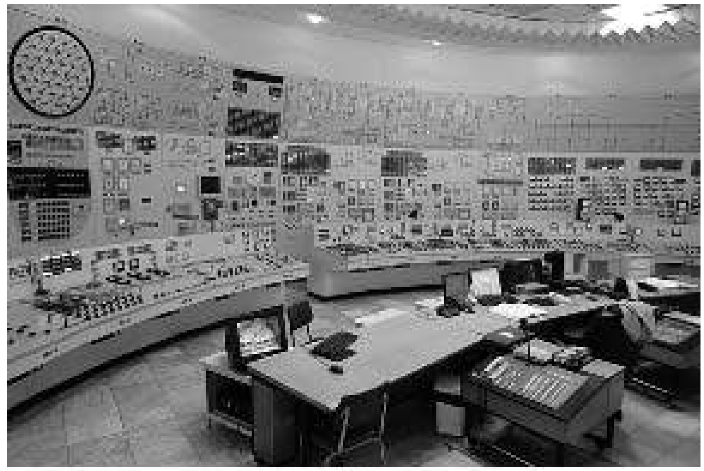
P 11.3-4 Nuclear power stations have become very complex to operate, as illustrated by the training simulator for the operating room of the Pilgrim Power Station shown in Figure P 11.3-4*a*. One control circuit has the model shown in Figure P 11.3-4*b*. Find the average power delivered to each element.

Answer: $P_{\text{source current}} = -12.8\,\text{W}$

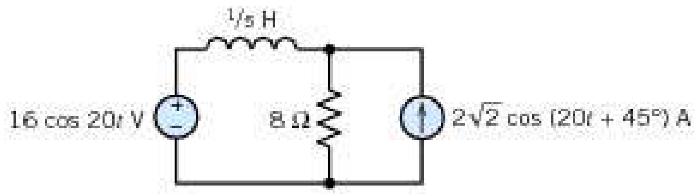
$P_{8\Omega} = 6.4\,\text{W}$

$P_{\text{L}} = 0\,\text{W}$

$P_{\text{voltage source}} = 6.4\,\text{W}$



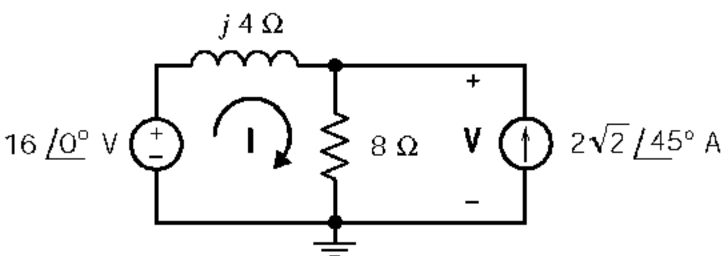
(*a*)



(*b*)

P 11.3-4

Solution:



A node equation:

$$\frac{(\mathbf{V}-16)}{j4}+\frac{\mathbf{V}}{8}-(2\sqrt{2}\angle 45°)=0$$

$$\Rightarrow \mathbf{V}=\left(16\sqrt{\frac{2}{5}}\right)\angle 18.4°\text{ V}$$

Then

$$\mathbf{I}=\frac{16-\mathbf{V}}{j4}=\sqrt{3.2}\ \angle -116.6°\text{ A}$$

$$P_{\text{AVE }8\Omega}=\frac{1}{2}\times\frac{|\mathbf{V}|^2}{8}=\frac{1}{2}\times\frac{\left(16\sqrt{\frac{2}{5}}\right)^2}{8}=6.4\text{ W absorbed}$$

$$P_{\text{AVE current source}}=-\frac{1}{2}|\mathbf{V}|\left(2\sqrt{2}\right)\cos\,\theta=-\frac{1}{2}\left(16\sqrt{\frac{2}{5}}\right)\left(2\sqrt{2}\right)\ \cos\ \left(26.6°\right)=-12.8\text{ W absorbed}$$

$$P_{\text{AVE inductor}}=0$$

$$P_{\text{AVE voltage source}}=-\frac{1}{2}(16)|\mathbf{I}|\ \cos\theta=-\frac{1}{2}(16)(\sqrt{3.2})\cos(-116.6°)=6.4\text{ W absorbed}$$

P 11.3-5 Find the average power delivered to each element for the circuit of Figure P 11.3-5.

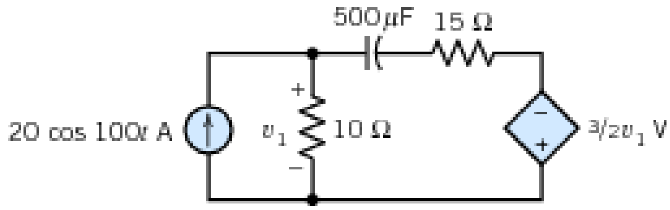
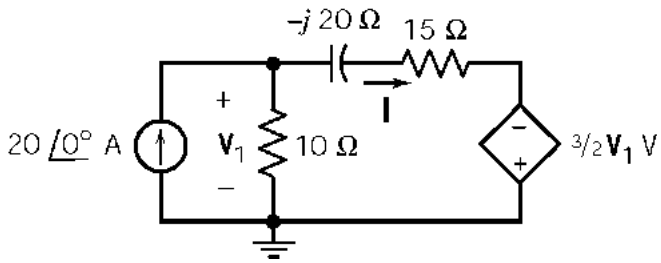


Figure P 11.3-5

Solution:



A node equation:

$$-20+\frac{\mathbf{V}_1}{10}+\frac{\mathbf{V}_1+(3/2)\mathbf{V}_1}{15-j20}=0 \quad\Rightarrow\quad \mathbf{V}_1=50\sqrt{5}\ \angle -26.6^\circ \text{ V}$$

Then

$$\mathbf{I}=\frac{\mathbf{V}_1+(3/2)\ \mathbf{V}_1}{15-j20}=\frac{(5/2)\mathbf{V}_1}{25\angle -53.1^\circ}=5\sqrt{5}\ \angle 26.6^\circ \text{ A}$$

Now the various powers can be calculated:

$$P_{\text{AVE }10\Omega}=\frac{1}{2}\frac{|\mathbf{V}_1|^2}{10}=\frac{1}{2}\frac{(50\sqrt{5})^2}{10}=625 \text{ W absorbed}$$

$$P_{\text{AVE current source}}=-\frac{1}{2}|\mathbf{V}|(20)\cos\theta=-\frac{1}{2}(50\sqrt{5})(20)\cos\ (-26.6^\circ)=-1000 \text{ W absorbed}$$

$$P_{\text{AVE }15\Omega}\ =\ \frac{|\mathbf{I}|^2}{2}(15)=-\frac{(5\sqrt{5})^2}{2}(15)=937.5 \text{ W absorbed}$$

$$P_{\text{AVE voltage source}}\ =\ -\frac{1}{2}|\mathbf{I}|\left|\frac{3}{2}\mathbf{V}_1\right|\cos\ \theta=-\frac{1}{2}(5\sqrt{5})(75\sqrt{5})\cos(-53.1^\circ)=-562.5 \text{ W absorbed}$$

$$P_{\text{AVE capacitor}}\ =\ 0 \text{ W}$$

P 11.3-6 A student experimenter in the laboratory encounters all types of electrical equipment. Some pieces of test equipment are battery-operated or operate at low voltage so that any hazard is minimal. Other types of equipment are isolated from electrical ground so that there is no problem if a grounded object makes contact with the circuit. Some types of test equipment, however, are supplied by voltages that can be hazardous or have dangerous voltage outputs. The standard power supply used in the United States for power and lighting in laboratories is the 120, grounded, 60-Hz sinusoidal supply. This supply provides power for much of the laboratory equipment, so an understanding of its operation is essential in its safe use (Bernstein, 1991).

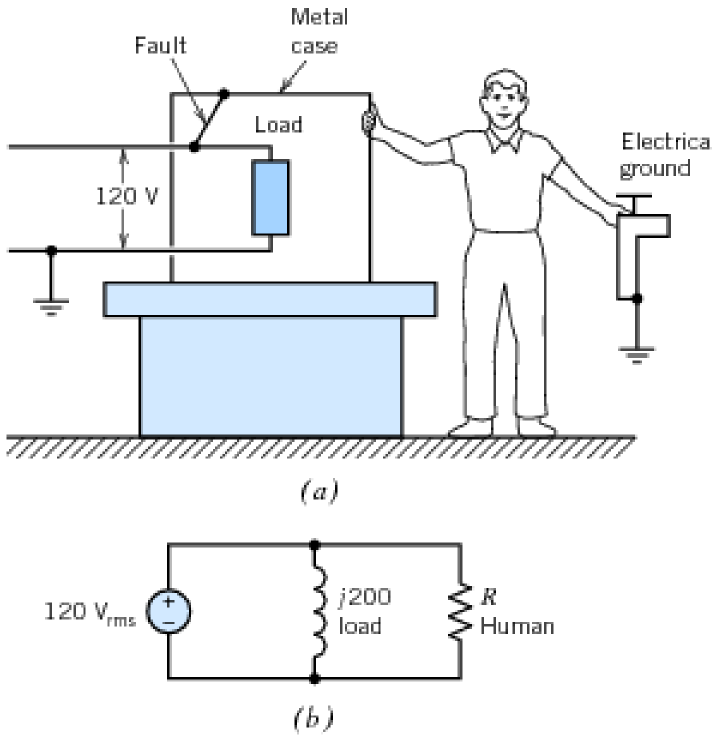


Figure P 11.3-6

Consider the case where the experimenter has one hand on a piece of electrical equipment and the other hand on a ground connection, as shown in the circuit diagram of Figure P 11.3-6*a*.

The hand-to-hand resistance is 200 **Ω**. Shocks with an energy of 30 J are hazardous to humans. Consider the model shown in Figure P 11.3-6*b*, which represents the human with *R*. Determine the energy delivered to the human in 1 s.

Solution:

$$\mathbf{Z} = \frac{200\ (j200)}{200\ (1+j)} = \frac{200\ \angle 90°}{\sqrt{2}\ \angle 45°} = \frac{200}{\sqrt{2}} \angle 45°\ \Omega$$

$$\mathbf{I} = \frac{120\angle 0^\circ}{\frac{200}{\sqrt{2}}\angle 45^\circ} = 0.85\ \angle -45°\ \text{A},\quad \mathbf{I}_\text{R} = \left(\frac{j200}{200 + j200}\right)\mathbf{I} = 0.6\angle 0°\ \text{A}$$

$$P = |\mathbf{I}|^2 R = (0.6)^2 (200) = 72\ \text{W}\ \text{ and }\ w = (72)(1) = 72\ \text{J}$$

P 11.3-7 An *RLC* circuit is shown in Figure P 11.3-7 with a voltage source $v_s = 7 \cos 10t$ V.

(a) Determine the instantaneous power delivered to the circuit by the voltage source.

(b) Find the instantaneous power delivered to the inductor.

Answer: (a) $p = 7.54 + 15.2 \cos(20t - 60.3°)$ W

(b) $p = 28.3 \cos(20t - 30.6°)$ W

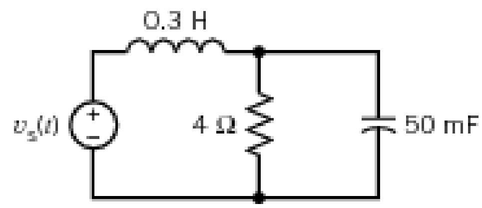
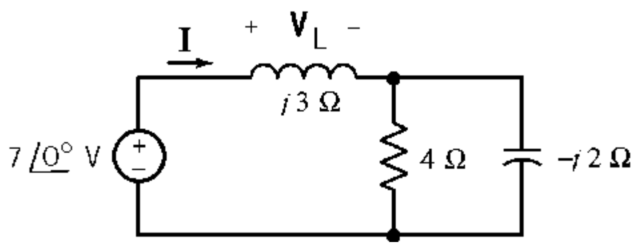


Figure P 11.3-7

Solution:



$$\mathbf{Z} = j3 + \frac{4(-j2)}{4-j2} = 0.8 + j1.4$$

$$= 1.6\ \angle 60.3^\circ\ \Omega$$

$$\therefore \mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{7\angle 0^\circ}{1.6\angle 60.3^\circ} = 4.38\ \angle -60.3^\circ\ \text{A}$$

$$i(t) = 4.38\cos(10t - 60.3^\circ)\ \text{A}$$

The instantaneous power delivered by the source is given by

$$p(t) = v(t)\cdot i(t) = (7\cos 10t)(4.38\cos(10t - 60.3°)) = \frac{(7)(4.38)}{2}\left[\cos(60.3°) + \cos(20t - 60.3°)\right]$$

$$= 7.6 + 15.3\cos(20t - 60.3°)\ \text{W}$$

The inductor voltage is calculated as

$$\mathbf{V}_L = \mathbf{I}\cdot\mathbf{Z}_L = (4.38\ \angle -60.3°)(j3) = 13.12\ \angle 29.69°\ \text{V}$$

$$v_L(t) = 13.12\cos(10t + 29.69°)\ \text{V}$$

The instantaneous power delivered to the inductor is given by

$$p_L(t) = v_L(t)\cdot i(t) = \left[(13.12\ \cos\ (10t + 29.69°)(4.38\cos(10t - 60.3^\circ)\right]$$

$$= \frac{57.47}{2}\left[\cos(29.69° + 60.3°) + \cos(20t + 29.69° - 60.3°)\right]$$

$$= 28.7\ \cos\ (20t - 30.6°)\ \text{W}$$

P 11.3-8

(a) Find the average power delivered by the source to the circuit shown in Figure P 11.3-8.

(b) Find the power absorbed by resistor R_1.

Answer: (a) 30 W

(b) 20 W

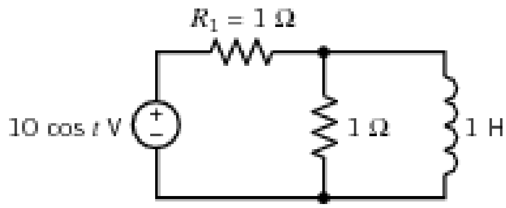
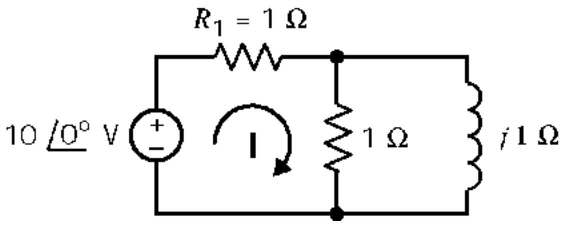


Figure P 11.3-8

Solution:



The equivalent impedance of the parallel resistor and inductor is $\mathbf{Z} = \frac{(1)(j)}{1+j} = \frac{1}{2}(1+j)\ \Omega$. Then

$$\mathbf{I} = \frac{10\angle 0^{\circ}}{1+\frac{1}{2}(1+j)} = \frac{20}{3+j} = \frac{20}{\sqrt{10}}\angle -18.4^{\circ}\text{ A}$$

(a) $P_{\text{source}} = \frac{|\mathbf{I}||\mathbf{V}|}{2}\cos\theta = \frac{(10)\left(\frac{20}{\sqrt{10}}\right)}{2}\cos(-18.4^{\circ}) = 30.0\ \text{W}$

(b) $P_{R_1} = \frac{|\mathbf{I}|^2 R_1}{2} = \frac{\left(\frac{20}{\sqrt{10}}\right)^2 (1)}{2} = 20\ \text{W}$

Section 11.4 Effective Value of a Periodic Waveform

P 11.4-1 Find the rms value of the current i for (a) $i = 2 - 4 \cos 2t$ A, (b) $i = 3 \sin \pi t + \sqrt{2} \cos \pi t$ A, and (c) $i = 2 \cos 2t + 4\sqrt{2} \cos (2t + 45°) + 12 \sin 2t$ A.
Answer: (a) $2\sqrt{3}$ (b) 2.35 A (c) $5\sqrt{2}$ A

Solution:

(a) $i = 2 - 4\cos 2t = i_1 + i_2$ (Treat i as two sources of different frequencies.)

$$\text{2A source:} \quad I_{\text{eff}} = \lim_{T\to\infty} \sqrt{\frac{1}{T}\int_o^T (2)^2 \, dt} = 2 \text{ A}$$

and

$$4\cos 2t \text{ source:} \quad \text{I}_{\text{eff}} = \frac{4}{\sqrt{2}} \text{ A}$$

The total is calculated as

$$I_{\text{eff}}{}^2 = (2)^2 + \left(\frac{4}{\sqrt{2}}\right)^2 = 12 \text{ A} \quad \Rightarrow \quad I_{\text{rms}} = I_{\text{eff}} = \sqrt{12} = 2\sqrt{3} \text{ A}$$

(b)
$$i(t) = 3\cos(\pi t - 90°) + \sqrt{2} \cos \pi t \quad \Rightarrow \quad \mathbf{I} = (3\angle -90°) + (\sqrt{2}\angle 0°)$$
$$= \sqrt{2} - j3 = 3.32\angle -64.8° \text{ A}$$

$$I_{\text{rms}} = \frac{3.32}{\sqrt{2}} = 2.35 \text{ A}$$

(c)
$$i(t) = 2\cos 2t + 4\sqrt{2}\cos(2t + 45°) + 12\cos(2t - 90°)$$

$$\mathbf{I} = (2\angle 0°) + (4\sqrt{2}\angle 45°) + (12\angle -90°) = (2+4) + (j4 - j12) = 10\angle -53.1° \text{ A}$$

$$I_{\text{rms}} = \frac{10}{\sqrt{2}} = 5\sqrt{2} \text{ A}$$

P 11.4-2 Determine the rms value for each of the waveforms shown in Figure P 11.4-2.
Answer: (a) 4.10 V (b) 4.81 V (c) 4.10

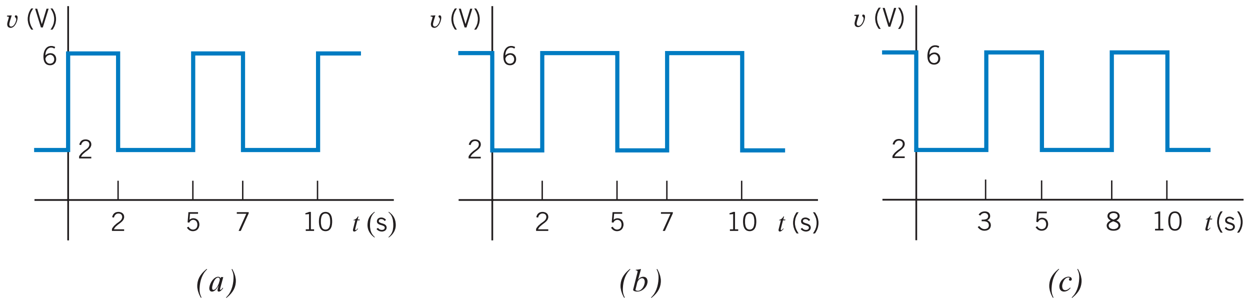


Figure P 11.4-2

Solution:

(a)
$$V_{\text{rms}} = \sqrt{\frac{1}{5}\left(\int_0^2 6^2\,dt + \int_2^5 2^2\,dt\right)} = \sqrt{\frac{1}{5}\left(\int_0^2 36\,dt + \int_2^5 4\,dt\right)} = \sqrt{\frac{1}{5}(72+12)} = \sqrt{\frac{84}{5}} = 4.10 \text{ V}$$

(b)
$$V_{\text{rms}} = \sqrt{\frac{1}{5}\left(\int_0^2 2^2\,dt + \int_2^5 6^2\,dt\right)} = \sqrt{\frac{1}{5}\left(\int_0^2 4\,dt + \int_2^5 36\,dt\right)} = \sqrt{\frac{1}{5}(8+108)} = \sqrt{\frac{116}{5}} = 4.81 \text{ V}$$

(c)
$$V_{\text{rms}} = \sqrt{\frac{1}{5}\left(\int_0^3 2^2\,dt + \int_3^5 6^2\,dt\right)} = \sqrt{\frac{1}{5}\left(\int_0^3 4\,dt + \int_3^5 36\,dt\right)} = \sqrt{\frac{1}{5}(12+72)} = \sqrt{\frac{84}{5}} = 4.10 \text{ V}$$

P 11.4-3 Determine the rms value for each of the waveforms shown in Figure P 11.4-3.

Answer: (a) 4.16 V (b) 4.16 V (c) 4.16

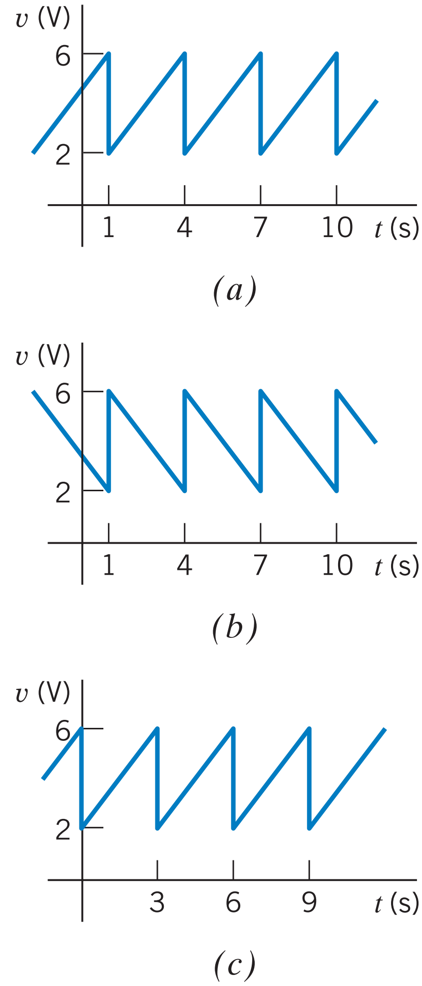


Figure P 11.4-3

Solution:

(a)

$$V_{\text{rms}} = \sqrt{\frac{1}{3}\int_1^4\left(\frac{4}{3}t+\frac{2}{3}\right)^2 dt}$$

$$= \sqrt{\frac{4}{27}\int_1^4(2t+1)^2\,dt}$$

$$= \sqrt{\frac{4}{27}\int_1^4\left(4t^2+4t+1\right)dt}$$

$$= \sqrt{\frac{4}{27}\left(\left.\frac{4t^3}{3}\right|_1^4+\left.\frac{4t^2}{2}\right|_1^4+\left.t\right|_1^4\right)}$$

$$= \sqrt{\frac{4}{27}\left((85.33-1.33)+(2)(16-1)+3\right)}$$

$$= \sqrt{\frac{4}{27}(117)} = 4.16 \text{ V}$$

(b)

$$V_{\text{rms}} = \sqrt{\frac{1}{3}\int_1^4\left(-\frac{4}{3}t+\frac{22}{3}\right)^2 dt}$$

$$= \sqrt{\frac{4}{27}\int_1^4(-2t+11)^2\,dt}$$

$$= \sqrt{\frac{4}{27}\int_1^4\left(4t^2-44t+121\right)dt}$$

$$= \sqrt{\frac{4}{27}\left(\left.\frac{4t^3}{3}\right|_1^4-\left.\frac{44t^2}{2}\right|_1^4+\left.121\,t\right|_1^4\right)}$$

$$= \sqrt{\frac{4}{27}\left(84+(-22)15+(121)3\right)}$$

$$= \sqrt{\frac{4}{27}(117)} = 4.16 \text{ V}$$

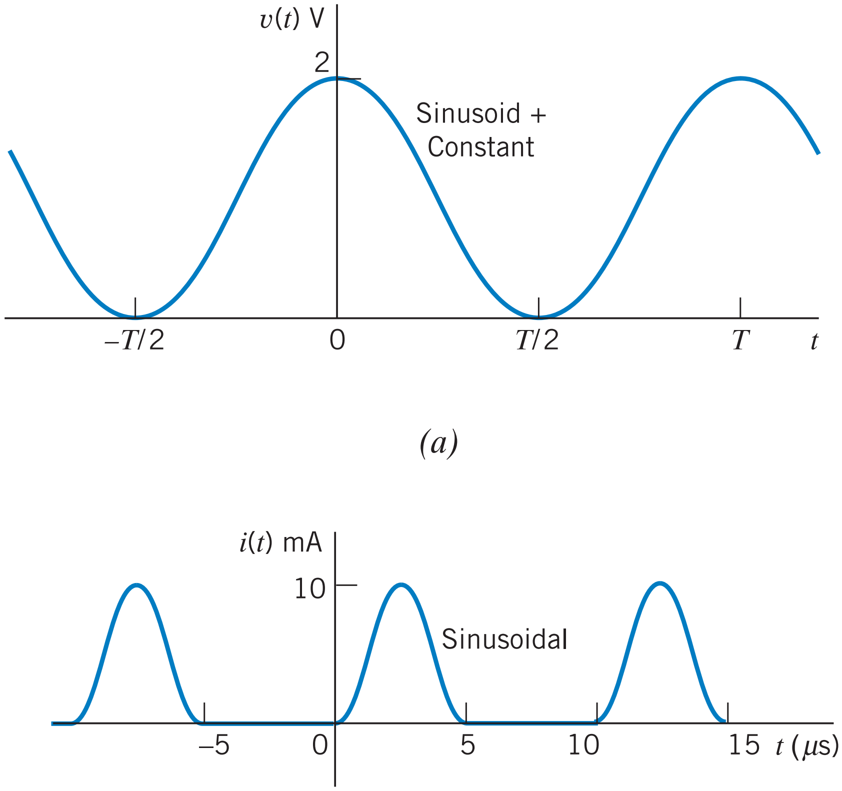
(c) $$V_{\text{rms}} = \sqrt{\frac{1}{3}\int_0^3 \left(\frac{4}{3}t+2\right)^2\,dt} = \sqrt{\frac{4}{27}\int_0^3 (2t+3)^2\,dt} = \sqrt{\frac{4}{27}\int_0^3 \left(4t^2+12t+9\right)dt}$$

$$= \sqrt{\frac{4}{27}\left(\left.\frac{4t^3}{3}\right|_0^3 + \left.\frac{12t^2}{2}\right|_0^3 + 9t\Big|_0^3\right)}$$

$$= \sqrt{\frac{4}{27}(36+54+27)}$$

$$= \sqrt{\frac{4}{27}(117)} = 4.16 \text{ V}$$

P 11.4-4 Find the rms value for each of the waveforms of Figure P 11.4-4.
Answer: $V_{\text{rms}} = 1.225$ V, $I_{\text{rms}} = 5$ mA



(b)

Figure P 11.4-4

Solution:
(a)

$$v(t) = 1 + \cos\left(\frac{2\pi}{T}t\right) = v_{\text{dc}} + v_{\text{ac}}$$

$$v_{\text{dc eff}}{}^2 = \left(\frac{1}{T}\int_0^T 1dt\right) = \left.\frac{t}{T}\right|_0^T = \left(\frac{T}{T} - 0\right) = 1 \text{ V} \quad \text{and} \quad v_{\text{ac eff}}{}^2 = \frac{1}{\sqrt{2}} \text{ V}$$

$$v_{\text{eff}}{}^2 = v_{\text{dc eff}}{}^2 + v_{\text{ac eff}}{}^2 = \sqrt{1^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = 1.225 \text{ V}$$

(b) The period of the sinusoid is half the period of $i(t)$. Let T be the period of $i(t)$ and $T/2$ be the period of the sinusoid. Then

$$i(t) = \begin{cases} 10\sin\dfrac{4\pi}{T}t & 0 < t < \dfrac{T}{4} \\ -10\sin\dfrac{4\pi}{T}t & \dfrac{T}{2} < t < \dfrac{3T}{4} \\ 0 & otherwise \end{cases}$$

$$I_{RMS} = \frac{100}{T}\left[\int_0^{T/4} \sin^2\frac{4\pi}{T}t\,dt + \int_{T/4}^{3T/4} \sin^2\frac{4\pi}{T}t\,dt\right]$$

$$= \frac{100}{T}\left[\left(\frac{t}{2} - \frac{\sin\frac{8\pi}{T}t}{\frac{18\pi}{T}}\right)_0^{T/4} + \left(\frac{t}{2} - \frac{\sin\frac{8\pi}{T}t}{\frac{18\pi}{T}}\right)_{T/2}^{2T/4}\right] = 25$$

$$I_{\text{rms}} = \sqrt{25} \quad \Rightarrow \quad I_{\text{rms}} = 5 \text{ mA}$$

P 11.4-5 Find the rms value of the voltage $v(t)$ shown in Figure P 11.4-5.

Answer: $V_{rms} = 4.24$ V

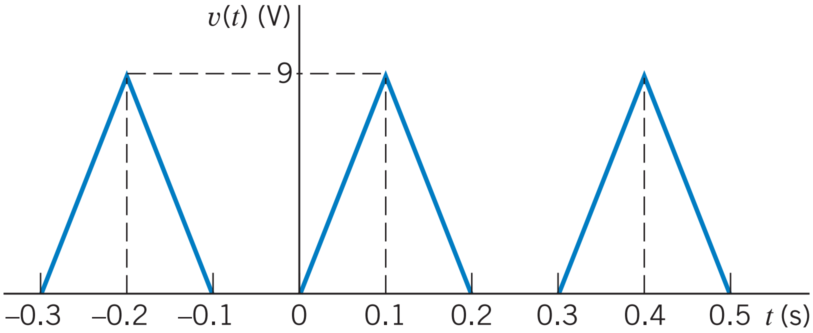


Figure P 11.4-5

Solution:

$$v(t) = \begin{cases} 90\,t & 0 \le t \le 0.1 \\ 90(0.2 - t) & 0.1 \le t \le 0.2 \\ 0 & 0.2 \le t \le 0.3 \end{cases}$$

$$V_{rms}^2 = \frac{1}{.3}\left[\int_0^{0.1} (90t)^2\,dt + \int_{0.1}^{0.2} \left[90(0.2-t)\right]^2 dt\right] = \frac{90^2}{.3}\left[\int_0^{0.1} t^2 dt + \int_{0.1}^{0.2} (0.2-t)^2\,dt\right]$$

$$= \frac{90^2}{.3}\left[\frac{.001}{3} + \frac{.001}{3}\right] = 18 \text{ V}$$

$$V_{rms} = \sqrt{18} = 4.24 \text{ V}$$

P 11.4-6 Find the effective value of the current waveform shown in Figure P 11.4-6.

Answer: $I_{eff} = 8.66$

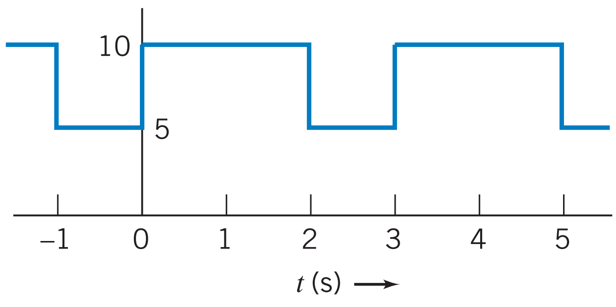


Figure P 11.4-6

Solution:

$$I_{eff} = \sqrt{\frac{1}{T}\int_0^t i^2(t)\,dt} = \sqrt{\frac{1}{3}\left[\int_0^2 (10)^2\,dt + \int_2^3 (5)^2\,dt\right]} = 8.66$$

P 11.4-7 Calculate the effective value of the voltage across the resistance R of the circuit shown in Figure P 11.4-7 when ω = 100 rad/s.
Hint: Use superposition.
Answer: V_{eff} = 4.82 V

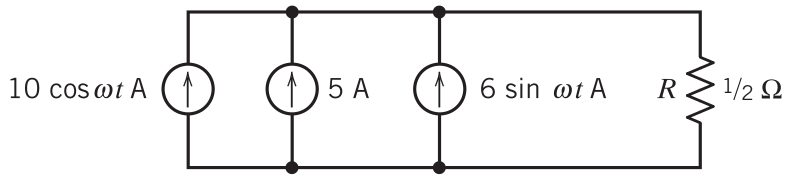
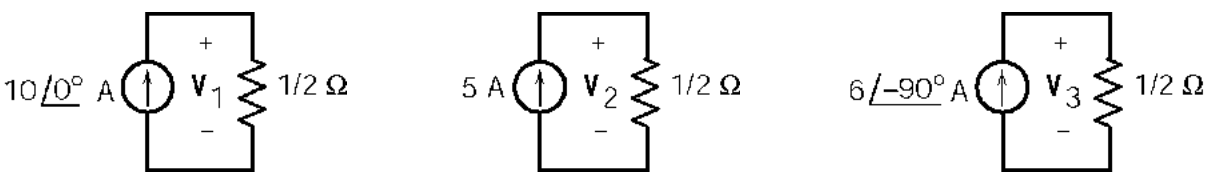


Figure P 11.4-7

Solution:
Use superposition:



$\mathbf{V}_1 = 5\angle 0°$ V $\mathbf{V}_2 = 2.5$ V (dc) $\mathbf{V}_3 = 3\angle -90°$ V

$\mathbf{V}_1$ and $\mathbf{V}_3$ are phasors having the same frequency, so we can add them:

$$\mathbf{V}_1 + \mathbf{V}_3 = (5\angle 0°) + (3\angle -90°) = 5 - j3 = 5.83\angle -31.0° \text{ V}$$

Then

$$v_{\text{R}}(t) = v_1(t) + (v_2(t) + v_3(t)) = 2.5 + 5.83\cos(100t - 31.0°) \text{ V}$$

Finally

$$V_{\text{R}_{\text{eff}}}^2 = (2.5)^2 + \left(\frac{5.83}{\sqrt{2}}\right)^2 = 23.24 \text{ V} \quad \Rightarrow \quad V_{\text{R}_{\text{eff}}} = 4.82 \text{ V}$$

Section 11.5 Complex Power

P 11.5-1 The complex power delivered by the voltage source in Figure P 11.5-1 is **S** = 3.6 + *j*7.2 V A. Determine the values of the resistance, *R*, and inductance, *L*.

Answer: *R* = 4 Ω and *L* = 2 H

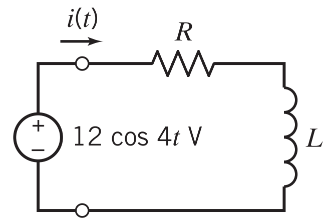


Figure P 11.5-1

Solution:

$$\mathbf{I}^* = \frac{2\,\mathbf{S}}{12\angle 0°} = \frac{2\left(3.6 + j\,7.2\right)}{12\angle 0°} = 0.6 + j1.2 = 1.342\angle 63.43° \text{ A}$$

$$R + j\,4\,L = \frac{12\angle 0°}{1.342\angle -63.43} = 8.94\angle 63.43 = 4 + j\,8 \quad \Rightarrow \quad R = 4 \;\; \Omega \text{ and } L = 2 \;\; \text{H}$$

P 11.5-2 The complex power delivered by the voltage source in Figure P 11.5-2 is **S** = 18 + *j*9 VA. Determine the values of the resistance, *R*, and inductance, *L*.

Answer: *R* = 4 Ω and *L* = 2 H

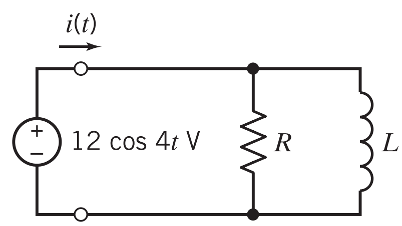


Figure P 11.5-2

Solution:

$$\mathbf{I}^* = \frac{2\,\mathbf{S}}{12\angle 0°} = \frac{2\left(18 + j\,9\right)}{12\angle 0°} = 3 + j1.5 = 3.35\angle 26.56° \text{ A}$$

$$\frac{1}{R} + \frac{1}{j\,4\,L} = \frac{1}{R} - j\frac{1}{4\,L} = \frac{3.35\angle -26.56°}{12\angle 0°} = 0.2791\angle -26.56 = 0.250 - j\,0.125$$

$$\Rightarrow \quad R = 4 \;\; \Omega \text{ and } L = 2 \;\; \text{H}$$

P 11.5-3 Determine the complex power delivered by the voltage source in the circuit shown in Figure P 11.5-3.
Answer: **S** = 7.2 + *j*3.6 VA

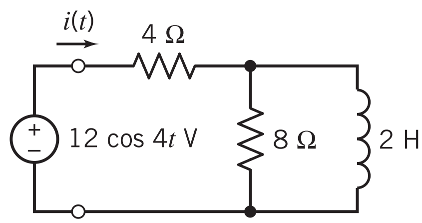
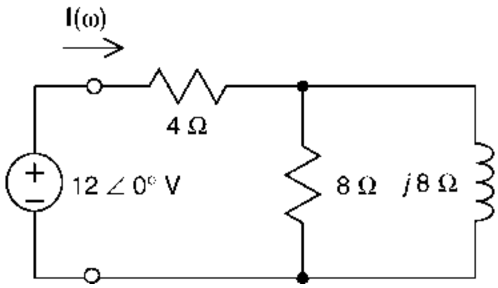


Figure P 11.5-3

Solution:



Let

$$\mathbf{Z}_p = \frac{8(j8)}{8+j8} = \frac{j8}{1+j} \times \frac{1-j}{1-j} = \frac{8+j8}{2} = 4+j4 \ \text{V}$$

Next

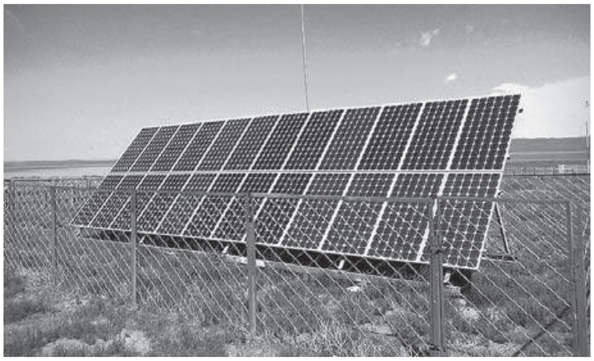
$$\mathbf{I} = \frac{12\angle 0°}{4+\mathbf{Z}_p} = \frac{12\angle 0°}{4+(8+j8)} = 1.342\angle -26.6° \ \text{A}$$

Finally

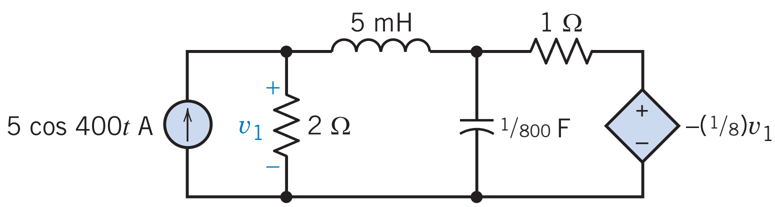
$$\mathbf{S} = \frac{(12\angle 0°)(1.342\angle -26.6°)^*}{2} = 7.2+j3.6 \ \text{VA}$$

P 11.5-4 Many engineers are working to develop photovoltaic power plants that provide ac power. An example of an experimental photovoltaic system is shown in Figure P 11.5-4*a*. A model of one portion of the energy conversion circuit is shown in Figure P 11.5-4*b*. Find the average, reactive, and complex power delivered by the dependent source.

Answer: **S** = + *j*8/9 VA



(a)

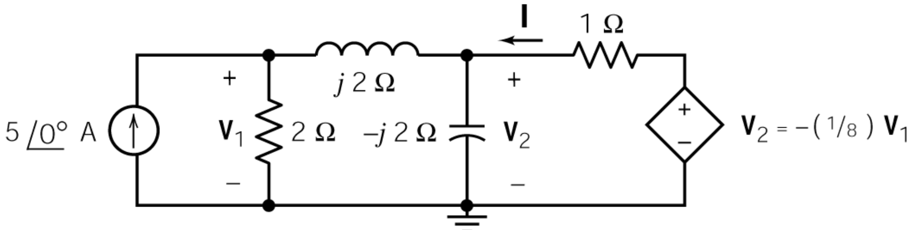


(b)

Figure P 11.5-4

Soution:

Represent the circuit in the frequency domain and label the node voltages:



The node equations are:

$$5\angle 0 = \frac{\mathbf{V}_1}{2} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j2} = 0 \quad \Rightarrow \quad \mathbf{V}_1(1-j) + j\,\mathbf{V}_2 = 10$$

$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{j2} = \frac{\mathbf{V}_2}{-j2} + \frac{\mathbf{V}_2 - \left(\frac{1}{8}\mathbf{V}_1\right)}{1} = 0 \quad \Rightarrow \quad \mathbf{V}_1(0.25 + j) + \mathbf{V}_2(2) = 0$$

Using MATLAB:

$$\begin{bmatrix} 1-j & j \\ 0.25+j & 2 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \end{bmatrix} \quad \Rightarrow \quad \begin{aligned} \mathbf{V}_1 &= 5.33\angle 36.9^\circ \ \text{V} \\ \mathbf{V}_2 &= 2.75\angle -67.2^\circ \ \text{V} \end{aligned}$$

then

$$\mathbf{I} \ = \ \frac{-\frac{1}{8}\mathbf{V}_1 - \mathbf{V}_2}{1} = \ 2.6\bar{6}\angle 126.9^\circ \ \text{A}$$

Now the complex power can be calculated as

$$\mathbf{S} = \frac{\mathbf{I}^*\left(-\left(\frac{1}{8}\right)\mathbf{V}_1\right)}{2} = \frac{\left(2.667\angle -126.9^\circ\right)^*\left(-\frac{5.33}{8}\angle 36.9^\circ\right)}{2} = j\,0.8889 = j\frac{8}{9} \ \text{VA}$$

Finally

$$\mathbf{S} = P + jQ = j\frac{8}{9} \quad \Rightarrow \quad P = 0, \ \ Q = \frac{8}{9} \ \text{VAR}$$

(Checked using MATLAB and LNAP)

P 11.5-5 For the circuit shown in Figure P 11.5-5, determine **I** and the complex power **S** delivered by the source when $\mathbf{V} = 50\ \angle 120°\ \mathbf{V}$ rms.
Answer: $\mathbf{S} = 100 + j75$ VA

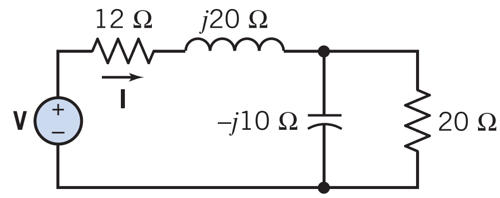
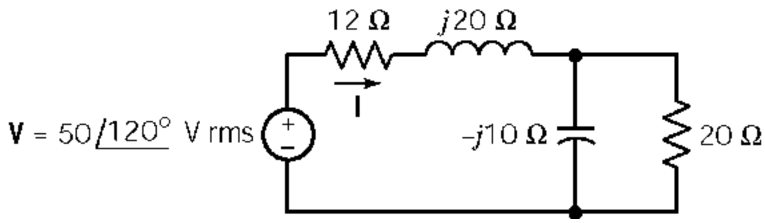


Figure P 11.5-5

Solution:



$$\mathbf{I} = \frac{50\angle 120°}{16+j12} = \frac{50\angle 120°}{20\angle 36.87°} = 2.5\angle 83.13°\ \text{A}$$

$$\mathbf{S} = \mathbf{VI}^* = (50\angle 120°)(2.5\angle -83.13°) = 125\ \angle 36.87° = 100 + j75\ \ \text{VA}$$

P 11.5-6 For the circuit of Figure P 11.5-6, determine the complex power of the *R*, *L*, and *C* elements and show that the complex power delivered by the sources is equal to the complex power absorbed by the *R*, *L*, and *C* elements.

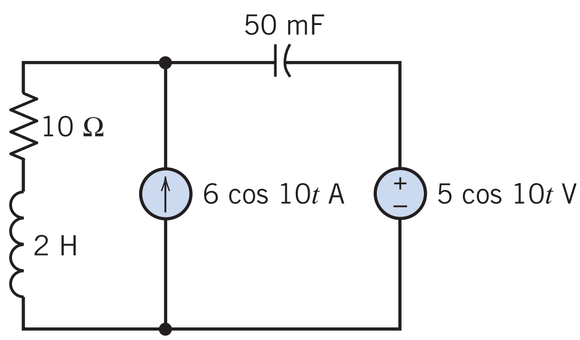
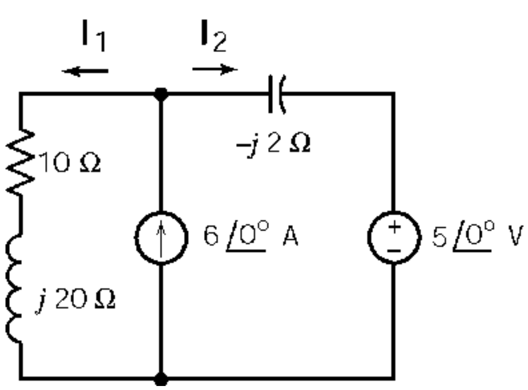


Figure P 11.5-6

Solution:



KVL:

$$(10+j20)\mathbf{I}_1 = 5\angle 0° - j2\mathbf{I}_2$$
$$\Rightarrow\ (10+j20)\mathbf{I}_1 + j2\mathbf{I}_2 = 5\angle 0°$$

KCL:

$$\mathbf{I}_1 + \mathbf{I}_2 = 6\angle 0°$$

Solving these equations using Cramer's rule:

$$\Delta = \begin{vmatrix} 10+j20 & j2 \\ 1 & 1 \end{vmatrix} = 10+j18$$

$$\mathbf{I}_1 = \frac{1}{\Delta}\begin{vmatrix} 5 & j2 \\ 6 & 1 \end{vmatrix} = \frac{5-j12}{10+j18} = 0.63\angle 232° \text{ A } = -0.39 - j0.5 \text{ A}$$

$$\mathbf{I}_2 = 6 - \mathbf{I}_1 = 6 + 0.39 + j.5 = 6.39 + j.5 = 6.41\angle 4.47° \text{ A}$$

Now we are ready to calculate the powers. First, the powers delivered:

$$\mathbf{S}_{5\angle 0°} = \frac{1}{2}\left(5\angle 0^{\circ}\right)\left(-\mathbf{I}_2^{*}\right) = 2.5\left(6.41\angle(180-4.47)\right) = -16.0 + j1.25 \text{ VA}$$

$$\mathbf{S}_{6\angle 0°} = \frac{1}{2}\left[5 - j2\mathbf{I}_2\right]\left(6\angle 0^{\circ}\right) = \left[5 - j2\left(6.39 + j.5\right)\right]3 = 18.0 - j38.3 \text{ VA}$$

$$\mathbf{S}_{\substack{\text{Total}\\ \text{delivered}}} = \mathbf{S}_{5\angle 0°} + \mathbf{S}_{6\angle 0°} = \underline{2.0 - j37.2} \text{ VA}$$

Next, the powers absorbed:

$$\mathbf{S}_{10\Omega} = \frac{1}{2}10\left|\mathbf{I}_1\right|^2 = \frac{10}{2}\left(.63\right)^2 = 2.0 \text{ VA}$$

$$\mathbf{S}_{j20\Omega} = \frac{j20}{2}\left|\mathbf{I}_1\right|^2 = j4.0 \text{ VA}$$

$$\mathbf{S}_{-j2\Omega} = \frac{1}{2}\left(-j2\right)\left|\mathbf{I}_2\right|^2 = -j\ \left(6.41\right)^2 = -j41.1 \text{ VA}$$

$$\mathbf{S}_{\substack{\text{Total}\\ \text{absorbed}}} = \underline{2.0 - j37.1 \text{ VA}}$$

To our numerical accuracy, the total complex power delivered is equal to the total complex power absorbed.

P 11.5-7 A circuit is shown in Figure P 11.5-7 with an unknown impedance **Z**. However, it is known that $v(t) = 100 \cos(100t + 20°)$ V and $i(t) = 25 \cos(100t - 10°)$ A. (a) Find **Z**. (b) Find the power absorbed by the impedance. (c) Determine the type of element and its magnitude that should be placed across the impedance **Z** (connected to terminals a–b) so that the voltage $v(t)$ and the current entering the parallel elements are in phase.

Answer: (a) $4 \angle 30° \ \Omega$ (b) 1082.5 W (c) 1.25 mF

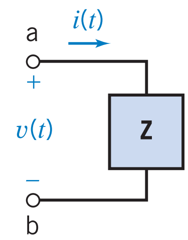
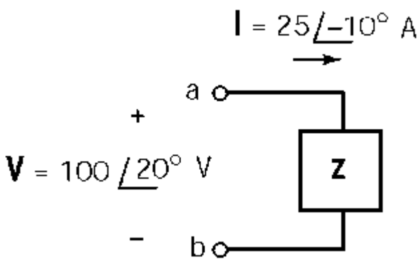


Figure P 11.5-7

Solution:



(a) $$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{100\angle 20°}{25\angle -10°} = \underline{4\angle 30° \ \Omega}$$

(b) $$P = \frac{|\mathbf{I}||\mathbf{V}|\cos\theta}{2} = \frac{(100)(25)\cos 30°}{2} = 1082.5 \text{ W}$$

(c) $\mathbf{Y} = \dfrac{\mathbf{1}}{\mathbf{Z}} = 0.25\angle -30° = 0.2165 - j0.125$ S. To cancel the phase angle we add a capacitor having an admittance of $\mathbf{Y}_C = j0.125$ S. That requires $\omega C = 0.125 \Rightarrow C = 1.25$ mF.

P 11.5-8 Find the complex power delivered by the voltage source and the power factor seen by the voltage source for the circuit of Figure P 11.5-8.

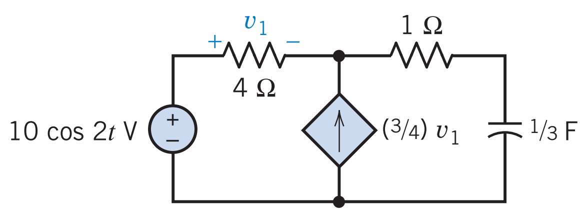
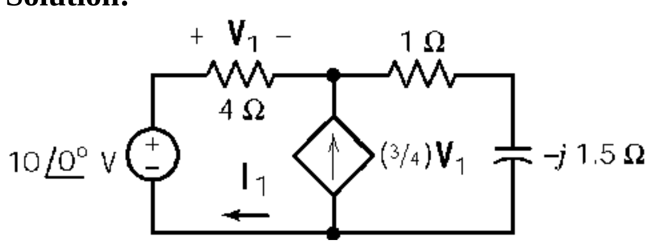


Figure P 11.5-8.

Solution:



Apply KCL at the top node to get

$$-\frac{\mathbf{V}_1}{4} - \frac{3}{4}\mathbf{V}_1 + \frac{10 - \mathbf{V}_1}{1 - j\dfrac{3}{2}} = 0 \quad \Rightarrow \quad \mathbf{V}_1 = 4\angle 36.9° \text{ V}$$

Then $$\mathbf{I}_1 = \frac{\mathbf{V}_1}{4} = 1\angle 36.9° \text{ A}$$

The complex power delivered by the source is calculated as

$$\mathbf{S} = \frac{(1\angle 36.9°)^* (10\angle 0°)}{2} = 5\angle -36.9° \text{ VA}$$

Finally $$pf = \cos(-36.9°) = .8 \text{ leading}$$

P 11.5-9 The circuit in Figure P 11.5-9 consists of a source connected to a load.

(a) Suppose R = 9 Ω and L = 5 H. Determine the average, complex, and reactive powers delivered by the source to the load.

(b) Suppose R = 15 Ω and L = 3 H. Determine the average, complex, and reactive powers delivered by the source to the load.

(c) Suppose the source delivers 8.47 + j14.12 VA to the load. Determine the values of the resistance, R, and the inductance, L.

(d) Suppose the source delivers 14.12 + j8.47 VA to the load. Determine the values of the resistance, R, and the inductance, L.

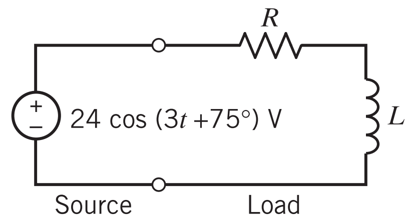
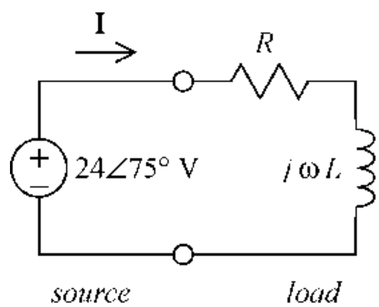


Figure P 11.5-9

Solution:

Represent the circuit in the frequency domain as



(a)
$$\mathbf{I} = \frac{24\angle 75°}{9 + j15} = \frac{24\angle 75°}{17.5\angle 59°} = 1.37\angle 16° \text{ A}$$

$$\mathbf{S} = \frac{1}{2}(24\angle 75°)(1.37\angle 16°)^* = \frac{24(1.37)}{2}\angle(75-16)° = 16.44\angle 59° = 8.47 + j14.1 \text{ VA}$$

so $P = 8.47$ W and $Q = 14.1$ VAR

(b)
$$\mathbf{I} = \frac{24\angle 75°}{15 + j9} = \frac{24\angle 75°}{17.5\angle 31°} = 1.37\angle 44° \text{ A}$$

$$\mathbf{S} = \frac{1}{2}(24\angle 75°)(1.37\angle 44°)^* = 16.44\angle 31° = 14.1 + j8.47 \text{ VA}$$

(c)
$$\mathbf{I} = \left(\frac{2(8.47 + j14.12)}{24\angle 75°}\right)^* = \left(\frac{2(16.44\angle 59°)}{24\angle 75°}\right)^* = (1.37\angle -16°)^* = 1.37\angle 16° \text{ A}$$

$$R + j3L = \frac{24\angle 75°}{1.37\angle 16°} = 17.5\angle 59° = 9 + j15 \ \Omega$$

$$R = 9\ \Omega \quad \text{and} \quad L = \frac{15}{3} = 5\ \text{H}$$

(d)
$$\mathbf{I} = \left(\frac{2(14.12 + j8.47)}{24\angle 75°} \right)^* = (1.37\angle -44°)^* = 1.37\angle 44°\ \text{A}$$

$$R + j3L = \frac{24\angle 75°}{1.37\angle 44°} = 17.5\angle 31° = 15 + j9\ \Omega$$

$$R = 15\ \Omega \quad \text{and} \quad L = \frac{9}{3} = 3\ \text{H}$$

P 11.5-10 The circuit in Figure P 11.5-10 consists of a source connected to a load. Suppose the amplitude of the source voltage is doubled so that

$$v_i(t) = 48 \cos(3t + 75°)\text{V}.$$

How will each of the following change?

(a) The impedance of the load
(b) The complex power delivered to the load
(c) The load current

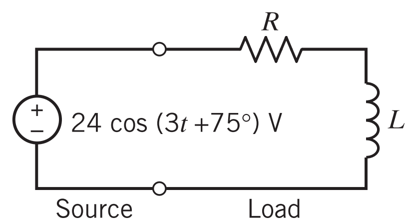
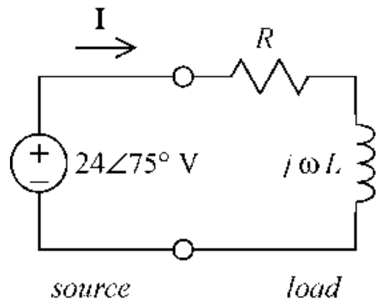


Figure P 11.5-10

Solution:
Represent the circuit in the frequency domain as



Doubling the amplitude of $v_i(t)$:

(c) doubles the amplitude of the load current.
(a) does not change the impedance.
(b) multiplies the complex power by $2^2 = 4$.

P 11.5-11 The circuit in Figure P 11.5-11 consists of a source connected to a load. Suppose the phase angle of the source voltage is doubled so that $v_i(t) = 24 \cos(3t + 150°)$ V. How will the following change?

(a) The impedance of the load

(b) The complex power delivered to the load

(c) The load current

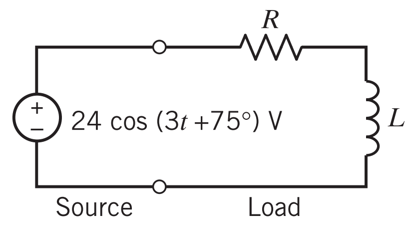
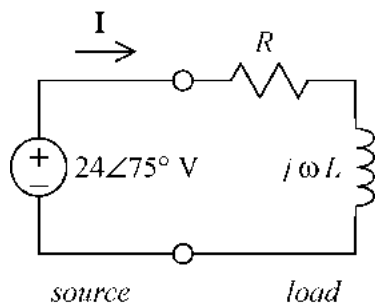


Figure P 11.5-11

Solution:

Represent the circuit in the frequency domain as



Doubling the angles of $v_i(t)$ increases the angle of $v_i(t)$ by 75°

(c) increases the angle of the load current by 75°.

(a) does not change the impedance.

(b) does not change the complex power.

P 11.5-12 The circuit in Figure P 11.5-12 consists of a source connected to a load. The complex power delivered by the source to the load is **S** = 6.61 + *j*1.98 VA. Determine the values of *R* and *C*.

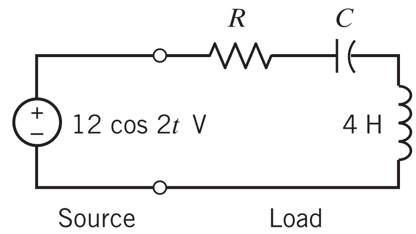
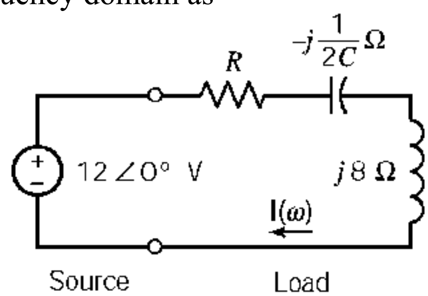


Figure P 11.5-12

Solution:
Represent the circuit in the frequency domain as



$$\mathbf{S}=\frac{1}{2}\mathbf{V}\mathbf{I}^{*} \quad \Rightarrow \quad \mathbf{I}=\left(\frac{2\mathbf{S}}{\mathbf{V}}\right)^{*}$$

so

$$\mathbf{I}(\omega)=\left(\frac{2(6.61+j1.98)}{12e^{j0}}\right)^{*}=(1.10+j0.33)^{*}=(1.10-j0.33)=1.15e^{-j1.67}\text{ A}$$

Using KVL gives

$$\left(R-j\frac{1}{2C}+j8\right)\left(1.15e^{-j16.7}\right)=12e^{j0}$$

$$R+j\left(8-\frac{1}{2C}\right)=\frac{12e^{j0}}{1.15e^{-j16.7}}=10.4e^{j16.7}=10+j3\ \Omega$$

So $$R=10\ \Omega$$

And $$8-\frac{1}{2C}=3 \quad \Rightarrow \quad \frac{1}{2C}=5 \quad \Rightarrow \quad C=\frac{1}{10}\text{ F}$$

P 11.5-13 Design the circuit shown in Figure P 11.5-13, that is, specify values for *R* and *L* so that the complex power delivered to the *RL* circuit is 8 + *j*6 VA.
Answer: *R* = 5.76 Ω and *L* = 2.16 H

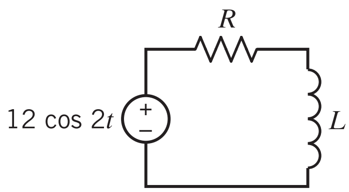
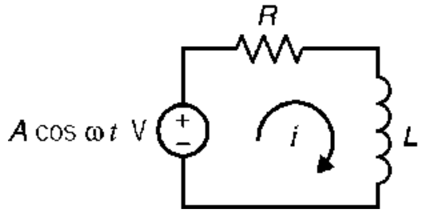


Figure P 11.5-13

Solution:



Analysis using Mathcad (ex11_5_3.mcd):

Enter the parameters of the voltage source: $A := 12$ $\omega := 2$

Enter the Average and Reactive Power delivered to the RL circuit: $P := 8$ $Q := 6$

The complex power delivered to the RL circuit is: $S := P + j \cdot Q$

The impedance seen by the voltage source is: $Z := \dfrac{A^2}{2 \cdot \overline{S}}$

Calculate the required values of R and L $R := \mathrm{Re}(Z)$ $L := \dfrac{\mathrm{Im}(Z)}{\omega}$ $R = 5.76$ $L = 2.16$

The mesh current is: $I := \dfrac{A}{Z}$

The complex power delivered by the source is: $Sv := \dfrac{\overline{I} \cdot (I \cdot Z)}{2}$ $Sv = 8 + 6i$

The complex power delivered to the resistor is: $Sr := \dfrac{\overline{I} \cdot (I \cdot R)}{2}$ $Sr = 8$

The complex power delivered to the inductor is: $Sl := \dfrac{\overline{I} \cdot (I \cdot j \cdot \omega \cdot L)}{2}$ $Sl = 6i$

Verify Sv = Sr + Sl : $Sr + Sl = 8 + 6i$ $Sv = 8 + 6i$

P11.5-14

The source voltage in the circuit shown in Figure P11.5-14 is $\mathbf{V}_s = 24 \angle 30°$ V. Consequently

$$\mathbf{I}_1 = 3.13 \angle 25.4° \text{ A}, \ \mathbf{I}_2 = 1.99 \angle 52.9° \text{ A and } \mathbf{V}_4 = 8.88 \angle -10.6° \text{ V}$$

Determine (a) the average power absorbed by $\mathbf{Z}_4$, (b) the average power absorbed by $\mathbf{Z}_1$, and (c) the complex power delivered by the voltage source. (All phasors are given using peak, not RMS, values.)

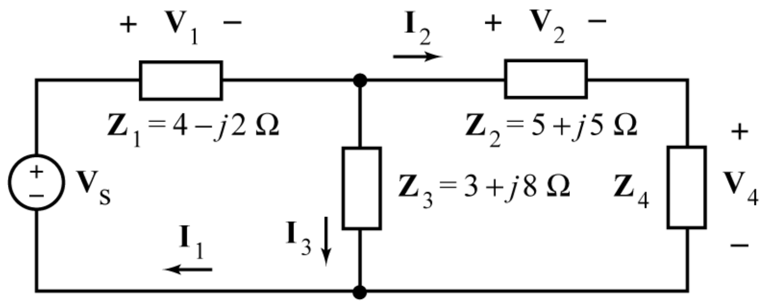


Figure P11.5-14

Solution:

a. The average power absorbed by $\mathbf{Z}_4$ is $P_4 = \frac{(8.88)(1.99)}{2}\cos(-10.6° - 52.9°) = 3.942$ W.

b. The average power absorbed by $\mathbf{Z}_1$ is $P_1 = \left(\frac{3.13^2}{2}\right)4 = 19.59$ W.

c. The complex power delivered by the voltage source is

$$\mathbf{S} = \frac{(24\angle 30°)(3.13\angle 25.4°)^*}{2} = 37.56\angle 4.6° = 37.44 + j3.01 \text{ VA}$$

Section 11.6 Power Factor

P 11.6-1 An industrial firm has two electrical loads connected in parallel across the power source. Power is supplied to the firm at 4000 V rms. One load is 30 kW of heating use, and the other load is a set of motors that together operate as a load at 0.6 lagging power factor and at 150 kVA. Determine the total current and the plant power factor.

Answer: I = 42.5 A rms and $pf = 1/\sqrt{2}$

Solution:

Heating: $P = 30$ kW

Motor:
$$\left.\begin{array}{r}\theta = \cos^{-1}(0.6) = 53.1^\circ \\ |\mathbf{S}| = 150 \text{ kVA}\end{array}\right\} \Rightarrow \left\{\begin{array}{l}P = 150\cos 53.1^\circ = 90 \text{ kW} \\ Q = 150\sin 53.1^\circ = 120 \text{ kVAR}\end{array}\right.$$

Total (plant):
$$\left.\begin{array}{r}P = 30 + 90 = 120 \text{ kW} \\ Q = 0 + 120 = 120 \text{ kVAR}\end{array}\right\} \Rightarrow \mathbf{S} = 120 + j120 = 170\angle 45^\circ \text{ VA}$$

The power factor is $pf = \cos\ 45^\circ = 0.707$ lagging.

The rms current required by the plant is $|\mathbf{I}| = \dfrac{|\mathbf{S}|}{|\mathbf{V}|} = \dfrac{170 \text{ kVA}}{4 \text{ kV}} = 42.5 \text{ A rms}$.

P 11.6-2 Two electrical loads are connected in parallel to a 400-V rms, 60-Hz supply. The first load is 12 kVA at 0.7 lagging power factor; the second load is 10 kVA at 0.8 lagging power factor. Find the average power, the apparent power, and the power factor of the two combined loads.

Answer: Total power factor = 0.75 lagging

Solution:

Load 1:
$$P_1 = |\mathbf{S}|\cos\theta = (12 \text{ kVA})(0.7) = 8.4 \text{ kW}$$
$$Q_1 = |\mathbf{S}|\sin\left(\cos^{-1}(.7)\right) = (12 \text{ kVA})\sin(45.6^\circ) = 8.57 \text{ kVAR}$$

Load 2:
$$P_2 = (10 \text{ kVA})(0.8) = 8 \text{ kW}$$
$$Q_2 = 10\sin\left(\cos^{-1}(0.8)\right) = 10\sin(36.9^\circ) = 6.0 \text{ kVAR}$$

Total:
$$\mathbf{S} = P + jQ = 8.4 + 8 + j(8.57 + 6.0) = 16.4 + j14.57 = 21.9\angle 41.6^\circ \text{ kVA}$$

The power factor is $pf = \cos(41.6^\circ) = 0.75$. The average power is P = 16.4 kW. The apparent power is |**S**| = 21.9 kVA.

P 11.6-3 The source of Figure P 11.6-3 delivers 50 VA with a power factor of 0.8 lagging. Find the unknown impedance **Z**.

Answer: $\mathbf{Z} = 6.39\ \angle 26.6°\ \Omega$

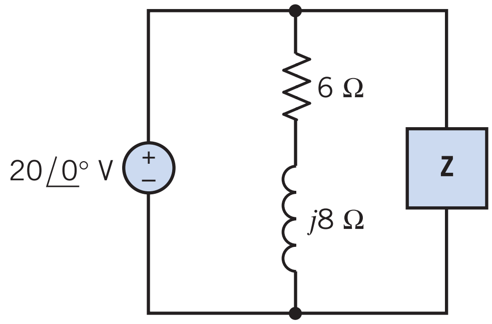
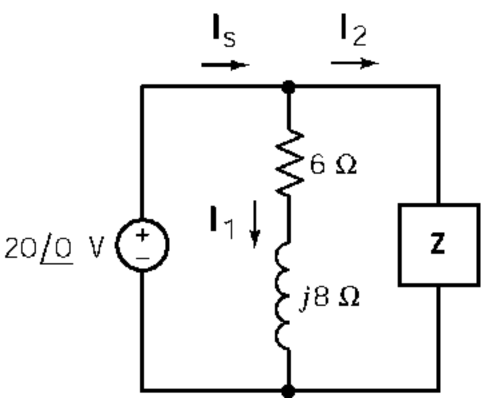


Figure P 11.6-3

Solution:



The source current can be calculated from the apparent power:

$$\mathbf{S} = \frac{\mathbf{V}_s\mathbf{I}_s{}^*}{2} \Rightarrow \mathbf{I}_s{}^* = \frac{2\,\mathbf{S}}{\mathbf{V}_s} = \frac{2\left(50\angle\cos^{-1}0.8\right)}{20\angle 0°} = 5\angle 36.9°\ \text{A}$$

$$\mathbf{I}_s = 5\angle -36.9° = 4 - \text{j}3\ \text{A}$$

Next

$$\mathbf{I}_1 = \frac{\mathbf{V}_s}{6 + j8} = \frac{20\angle 0°}{10\angle 53.1°} = 2\angle -53.1° = 1.2 - j1.6\ \ \text{A}$$

$$\mathbf{I}_2 = \mathbf{I}_s - \mathbf{I}_1 = 4 - j3 - 1.2 + j1.6 = 2.8 - j1.4$$
$$= 3.13\angle -26.6°\ \text{A}$$

Finally,

$$\mathbf{Z} = \frac{\mathbf{V}_s}{\mathbf{I}_2} = \frac{20\angle 0°}{3.13\angle -26.6°} = 6.39\angle 26.6°\ \Omega$$

P 11.6-4 Manned space stations require several continuously available ac power sources. Also, it is desired to keep the power factor close to 1. Consider the model of one communication circuit shown in Figure P 11.6-4. If an average power of 500 W is dissipated in the 20Ω resistor, find (a) V_{rms}, (b) $I_{s\ rms}$, (c) the power factor seen by the source, and (d) $|\mathbf{V}_s|$.

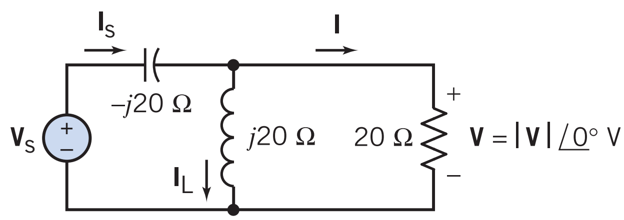


Figure P 11.6-4

Solution:

(Using all rms values.)

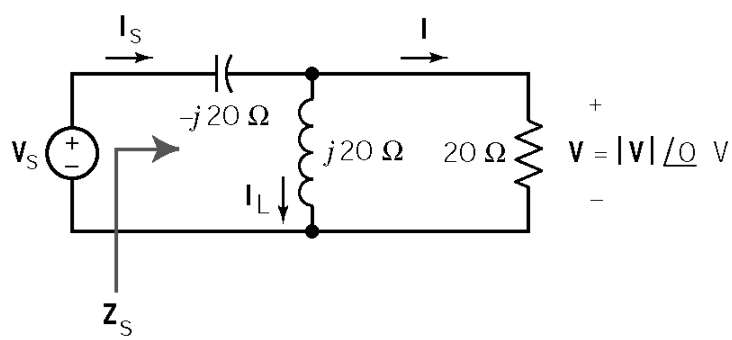
(a)

$$P=|\mathbf{I}|^2 R=\frac{|\mathbf{V}|^2}{R}\Rightarrow|\mathbf{V}|^2=P\cdot R=(500)(20)\quad\Rightarrow\quad|\mathbf{V}|=100\text{ Vrms}$$

(b)

$$\mathbf{I}_s=\mathbf{I}+\mathbf{I}_L=\frac{\mathbf{V}}{20}+\frac{\mathbf{V}}{j20}=\frac{100\angle 0^\circ}{20}+\frac{100\angle 0^\circ}{j20}=5-j5=5\sqrt{2}\angle-45^\circ\text{ A}$$

(c)



$$\mathbf{Z}_s=-j20+\frac{(20)(j20)}{20+j20}=10\sqrt{2}\angle-45^\circ\ \Omega$$

$$pf=\cos\left(-45^\circ\right)=\frac{1}{\sqrt{2}}\text{ leading}$$

(d) No average power is dissipated in the capacitor or inductor. Therefore,

$$\underset{\text{source}}{\text{P}_{\text{AVE}}}=\underset{20\Omega}{\text{P}_{\text{AVE}}}=500\text{ W}\quad\Rightarrow\quad|\mathbf{V}_s||\mathbf{I}_s|\cos\theta=500\Rightarrow|\mathbf{V}_s|=\frac{500}{|\mathbf{I}_s|\cos\theta}=\frac{500}{\left(5\sqrt{2}\right)\left(\frac{1}{\sqrt{2}}\right)}=100\text{ V}$$

P 11.6-5 Two impedances are supplied by $\mathbf{V} = 100\ \angle 160°\ V_{rms}$, as shown in Figure P 11.6-5, where $\mathbf{I} = 2\ \angle 190°$ A rms. The first load draws P_1 = 23.2 W and Q_1 = 50 VAR. Calculate $\mathbf{I}_1$, $\mathbf{I}_2$, the power factor of each impedance, and the total power factor of the circuit.

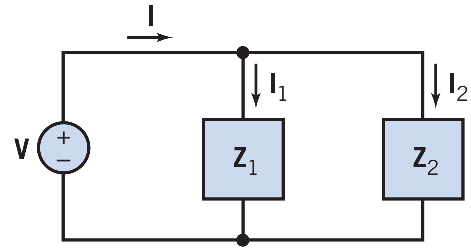


Figure P 11.6-5

Solution:

Load 1:

$$\mathbf{V} = 100\angle 160° \text{ Vrms}$$

$$\mathbf{I} = 2\angle 190° \text{ Arms} = -1.97 - j0.348 \text{ Arms}$$

$$P_1 = 23.2 \text{ W},\ Q_1 = 50 \text{ VAR}$$

$$\mathbf{S}_1 = P_1 + jQ_1 = 23.2 + j50 = 55.12\angle 65.1° \text{ VA}$$

$$pf_1 = \cos 65.1° = 0.422 \text{ lagging}$$

$$\mathbf{I}_1^* = \frac{\mathbf{S}_1}{\mathbf{V}_s} = \frac{55.12\angle 65.1°}{100\angle 160°} = 0.551\angle -94.9°, \text{ so } \mathrm{I}_1 = 0.551\angle 94.9° \text{ Arms}$$

Load 2:

$$\mathbf{I}_2 = \mathbf{I} - \mathbf{I}_1 = -1.97 - j0.348 + 0.047 - j0.549 = 2.12\angle -155° \text{ Arms}$$

$$\mathbf{S}_2 = \mathbf{V}\,\mathbf{I}_2{}^* = (100\angle 160°)(2.12\angle 155°)^* = 212\angle 315° = 212\angle -45° = 150 - j150 \text{ VA}$$

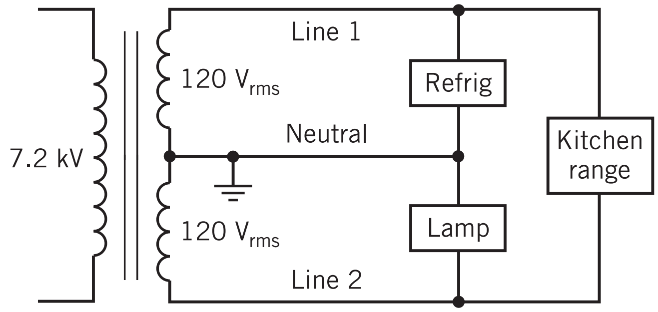
$$pf_2 = \cos(-45°) = 0.707 \text{ leading}$$

Total:

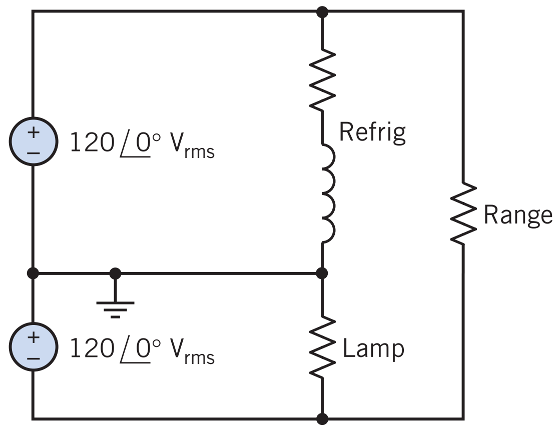
$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 = (23.2 + j50) + (150 - j150) = 173.2 - j100 = 200\angle -30° \text{ VA}$$

$$pf = \cos(-30°) = 0.866 \text{ leading}$$

P 11.6-6 A residential electric supply three-wire circuit from a transformer is shown in Figure P 11.6-6*a*. The circuit model is shown in Figure P 11.6-6*b*. From its nameplate, the refrigerator motor is known to have a rated current of 8.5 A rms. It is reasonable to assume an inductive impedance angle of 45° for a small motor at rated load. Lamp and range loads are 100 W and 12 kW, respectively.



(a)



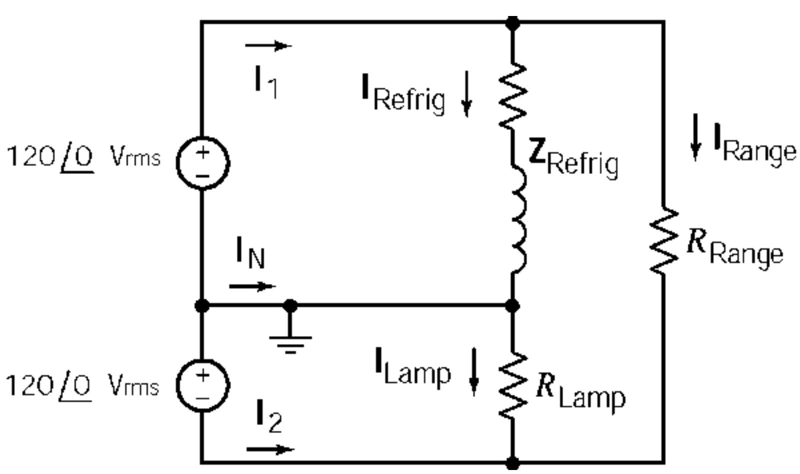
(b)

(a) Calculate the currents in line 1, line 2, and the neutral wire.

(b) Calculate: (i) P_{refrig}, Q_{refrig}, (ii) P_{lamp}, Q_{lamp}, and (iii) P_{total}, Q_{total}, $\mathbf{S}_{total}$, and overall power factor.

(c) The neutral connection resistance increases, because of corrosion and looseness, to 20 Ω. (This must be included as part of the neutral wire.) Use mesh analysis and calculate the voltage across the lamp.

Solution:
(Using rms values)



$$\left|\mathbf{Z}_{refrig}\right| = \frac{120}{8.5} = 14.12\ \Omega$$

$$\mathbf{Z}_{refrig} = 14.12\angle 45° = 10 + j10\ \Omega$$

$$R_{lamp} = \frac{\mathbf{V}^2}{P} = \frac{(120)^2}{100} = 144\ \Omega$$

$$R_{range} = \frac{(240)^2}{12{,}000} = 4.8\ \Omega$$

(a)

$$\mathbf{I}_{refrig} = \frac{120\angle 0°}{10 + j10} = 8.5\angle -45°\ \text{Arms}\ , \quad \mathbf{I}_{lamp} = \frac{120\angle 0°}{144} = 0.83\angle 0°\ \text{Arms}$$

and

$$\mathbf{I}_{\text{range}} = \frac{240\angle 0°}{4.8} = 50\angle 0° \text{ Arms}$$

From KCL:

$$\mathbf{I}_1 = \mathbf{I}_{\text{refrig}} + \mathbf{I}_{\text{range}} = 56 - j6 = \underline{56.3\angle -6.1° \text{ Arms}}$$

$$\mathbf{I}_2 = -\mathbf{I}_{\text{lamp}} - \mathbf{I}_{\text{range}} = \underline{50.83\angle 180°} \text{ Arms}$$

$$\mathbf{I}_{\text{N}} = -\mathbf{I}_1 - \mathbf{I}_2 = \underline{7.92\angle 131° \text{ Arms}}$$

(b)

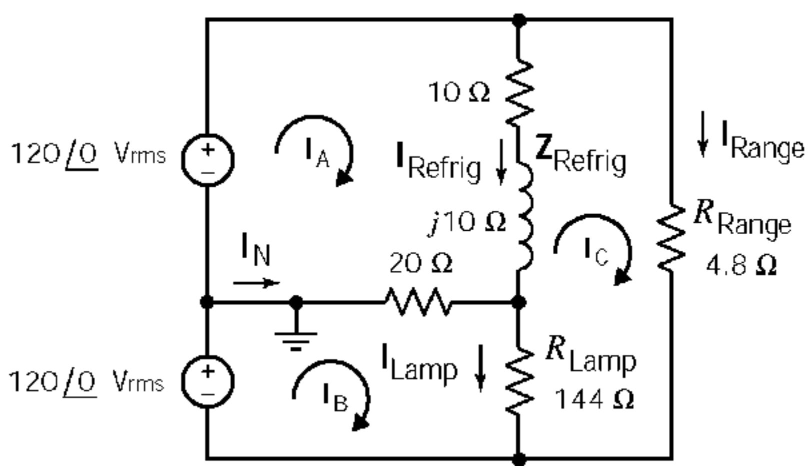
$$P_{\text{refrig}} = \left|\mathbf{I}_{\text{refrig}}\right|^2 R_{\text{refrig}} = \underline{722.5 \text{ W}} \quad \text{and} \quad Q_{\text{refrig}} = \left|\mathbf{I}_{\text{refrig}}\right|^2 X_{\text{refrig}} = \underline{722.5 \text{ VAR}}$$

$$P_{\text{lamp}} = 100 \text{ W} \quad \text{and} \quad Q_{\text{lamp}} = 0$$

$$\left.\begin{array}{r} P_{\text{total}} = 722 + 100 + 12{,}000 = 12.82 \text{ kW} \\ Q_{\text{total}} = 722 + 0 + 0 = 722 \text{ VAR} \end{array}\right\} \Rightarrow \mathbf{S} = 12{,}822 + j722 = 12.84\angle 3.2° \text{ kVA}$$

The overall power factor is $pf = \cos(3.2°) = 0.998$,

(c)



Mesh equations:

$$\begin{bmatrix} 30 + j10 & -20 & -10 - j10 \\ -20 & 164 & -144 \\ -10 - j10 & -144 & 158.8 + j10 \end{bmatrix} \begin{bmatrix} \mathbf{I}_{\text{A}} \\ \mathbf{I}_{\text{B}} \\ \mathbf{I}_{\text{C}} \end{bmatrix} = \begin{bmatrix} 120\angle 0° \\ 120\angle 0° \\ 0 \end{bmatrix}$$

Solve to get:

$$\mathbf{I}_{\text{A}} = 64.3 - j1.57 = 64.3\angle -1.7° \text{ Arms}$$

$$\mathbf{I}_{\text{B}} = 61.3 - j0.19 = 61.3\angle -0.5° \text{Arms}$$

$$\mathbf{I}_{\text{C}} = 60 + j0 = 60\angle 0° \text{ Arms}$$

The voltage across the lamp is

$$\left|\mathbf{V}_{\text{lamp}}\right| = R_{\text{lamp}} \left|\text{I}_{\text{B}} - \text{I}_{\text{C}}\right| = 144\left|1.27\angle -8.6°\right| = 183.2 \text{ Vrms}$$

(checked: MATLAB 7/20/04)

P 11.6-7 A motor connected to a 220-V supply line from the power company has a current of 7.6 A. Both the current and the voltage are rms values. The average power delivered to the motor is 1317 W.

(a) Find the apparent power, the reactive power, and the power factor when $\omega = 377$ rad/s.

(b) Find the capacitance of a parallel capacitor that will result in a unity power factor of the combination.

(c) Find the current in the utility lines after the capacitor is installed.

Answer: (a) $pf = 0.788$ (b) $C = 56.5\ \mu\text{F}$ (c) $I = 6.0$ A rms

Solution:

(Using all rms values)

(a) $VI = 220(7.6) = \underline{1672\text{ VA}}$

$$pf = \frac{P}{VI} = \frac{1317}{1672} = \underline{.788}$$

$$\theta = \cos^{-1}\text{pf} = 38.0^\circ \Rightarrow \underline{Q = VI\sin\theta = 1030\text{ VAR}}$$

(b) To restore the pf to 1.0, a capacitor is required to eliminate Q by introducing $-Q$, then

$$1030 = \frac{V^2}{X_c} = \frac{(220)^2}{X_c} \Rightarrow \text{X}_\text{c} = 47\Omega$$

$$\underline{\therefore\ C = 1/\omega X = 1/(377)(47) = 56.5\mu\text{F}}$$

(c) $P = VI\cos\theta$ where $\theta = 0^\circ$

then $1317 = 220\text{I}$

$\therefore\ \underline{I = 6.0\text{Arms for corrected}}\ pf$

*Note $I =$ 7.6Arms for uncorrected pf.

P 11.6-8 Two loads are connected in parallel across a 1000-V rms, 60-Hz source. One load absorbs 500 kW at 0.6 power factor lagging, and the second load absorbs 400 kW and 600 kVAR. Determine the value of the capacitor that should be added in parallel with the two loads to improve the overall power factor to 0.9 lagging.
Answer: $C = 2.2\ \mu\text{F}$

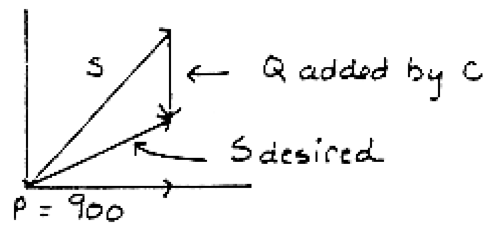
Solution:
First load:

$$\mathbf{S}_1 = P + jQ = P\left(1 + j\tan\left(\cos^{-1}(.6)\right)\right) = 500\left(1 + j\tan 53.1°\right) = 500 + j667 \text{ kVA}$$

Second load: $\mathbf{S}_2 = 400 + j600 \text{ kVA}$
Total:

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 = 900 + j1267 \text{ kVA}$$

$\mathbf{S}_{\text{desired}} = P + jP\ \tan\ (\cos^{-1}(.90)) = 900 + j436 \text{ VA}$



From the vector diagram: $\mathbf{S}_{\text{desired}} = \mathbf{S} + jQ$. Therefore

$900 + j436 = 900 + j1277 + jQ \quad \Rightarrow \quad Q = -841 \text{ VAR}$

$$\frac{|\mathbf{V}|^2}{\mathbf{Z}^*} = -j841 \ \Rightarrow\ \mathbf{Z}^* = \frac{|\mathbf{V}|^2}{-j841} = \frac{(1000)^2}{-j841} = j1189 \ \Rightarrow\ \mathbf{Z} = -j1189 = -\frac{j}{377\,C}$$

Finally,

$$C = \frac{1}{(1189)(377)} = 2.20\ \mu\text{F}$$

P 11.6-9 A voltage source with a complex internal impedance is connected to a load, as shown in Figure P 11.6-9. The load absorbs 1 kW of average power at 100 V rms with a power factor of 0.80 lagging. The source frequency is 200 rad/s.

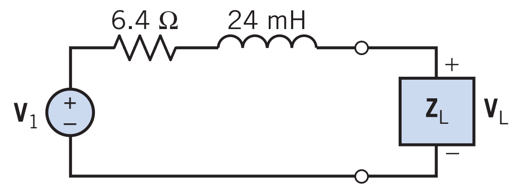


Figure P 11.6-9

(a) Determine the source voltage $\mathbf{V}_1$.

(b) Find the type and value of the element to be placed in parallel with the load so that maximum power is transferred to the load.

Solution:

(a) $\mathbf{S} = P + jQ = P + jP\tan(\cos^{-1} pf) = 1000 + j1000\tan(\cos^{-1} 0.8) = 1000 + j750 \text{ VA}$

Let $\mathbf{V}_L = 100\angle 0° \text{ Vrms}$. Then $\mathbf{I}^* = \dfrac{\mathbf{S}}{\mathbf{V}_L} = \dfrac{1000+j750}{100\angle 0^\circ} = 10 + j7.5 \quad \Rightarrow \quad \mathbf{I} = 10 - j7.5 \text{ Arms}$

$$\mathbf{Z}_L = \frac{\mathbf{V}_L}{\mathbf{I}} = \frac{100\angle 0°}{12.5\angle -36.9°} = 8\angle 36.9° = 6.4 + j4.8 \ \text{ Vrms}$$

$$\mathbf{V}_1 = [6.4 + j(200)(.024) + \mathbf{Z}_L)(\mathbf{I}) = (12.8 + j9.6)(10 - j7.5) = 200\angle 0° \text{ Vrms}$$

(b) For maximum power transfer, we require $\left(6.4 + j4.8\right)^* = \mathbf{Z}_L \parallel \mathbf{Z}_{new} = \dfrac{1}{\mathbf{Y}_L + \mathbf{Y}_{new}}$.

$$\frac{1}{(6.4 - j4.8)} = \mathbf{Y}_L + \mathbf{Y}_{new} \Rightarrow \mathbf{Y}_{new} = \frac{1}{6.4 - j4.8} - \frac{1}{6.4 + j4.8} = j0.15 \text{ S}$$

Then $\mathbf{Z}_{new} = -j6.67\ \Omega$ so we need a capacitor given by

$$\frac{1}{\omega C} = 6.67 \ \Rightarrow \ C = \frac{1}{(6.67)(200)} = 750\ \mu\text{F}$$

P 11.6-10 The circuit shown in Figure P 11.6-10*a* can be represented in the frequency domain as shown in Figure P 11.6-10*b*. In the frequency domain, the value of the mesh current is **I** = 1.076 ∠–38.3° A.

(a) Determine the complex power supplied by the voltage source.

(b) Given that the complex power received by $\mathbf{Z}_1$, is 6.945 + *j* 13.89 VA, determine the values of R_1 and L_1.

(c) Given that the real power received by $\mathbf{Z}_3$ is 4.63 W at a power factor of 0.56 lagging, determine the values of R_3 and L_3.

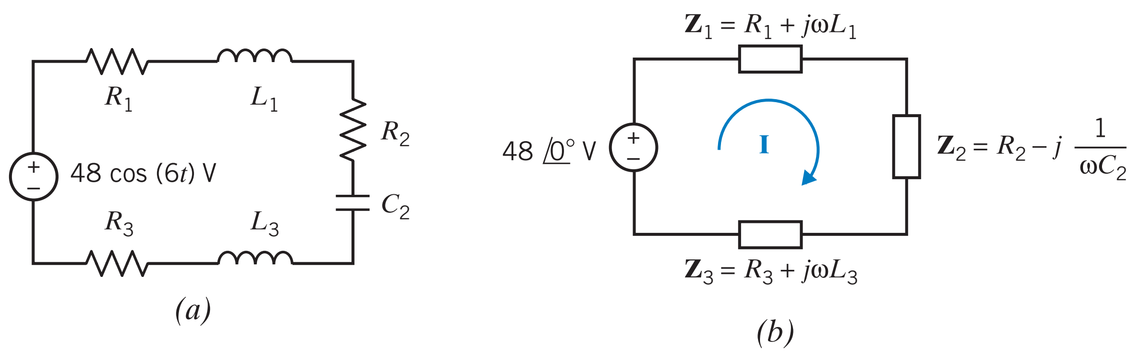


Figure P 11.6-10

Solution:

(a) $\mathbf{S} = \dfrac{(48\angle 0°)(1.076\angle -38.3°)^*}{2} = 25.82\angle 38.3° = 20.27 + j16 \text{ VA}$

(b) $\mathbf{V}_1 = \dfrac{2\,\mathbf{S}_1}{\mathbf{I}^*} = \dfrac{2(6.945 + j\,13.89)}{(1.076\angle -38.3°)^*} = \dfrac{2(15.53\angle 63.4°)}{(1.076\angle 38.3°)} = 28.87\angle 25.1° \text{ V}$

$$\mathbf{Z}_1 = \frac{\mathbf{V}_1}{\mathbf{I}} = \frac{28.87\angle 25.1°}{1.076\angle -38.3°} = 26.83\angle 63.4° = 12 + j24 = 12 + j6(4)\ \Omega$$

$R_1 = 12\ \Omega$ and $L_1 = 4$ H.

(c) $\theta = \cos^{-1}(0.56) = 56°$, $|\mathbf{S}| = \dfrac{P}{pf} = \dfrac{4.63}{0.56} = 8.268 \text{ VA}$

$$\mathbf{V}_3 = \frac{2(8.268\angle 56°)}{(1.076\angle -38.3°)^*} = 15.37\angle 17.7° \text{ V}$$

$$\mathbf{Z}_3 = \frac{\mathbf{V}_3}{\mathbf{I}} = \frac{15.37\angle 17.7°}{1.076\angle -38.3°} = 14.28\angle 56° = 8 + j11.83 = 8 + j6(1.97)\ \Omega$$

$R_3 = 8\ \Omega$ and $L_3 = 2$ H.

P 11.6-11 The circuit in Figure P 11.6-11 consists of a source connected to a load. The source delivers 14.12 W to the load at a power factor of 0.857 lagging. What are the values of the resistance, R, and the inductance, L?

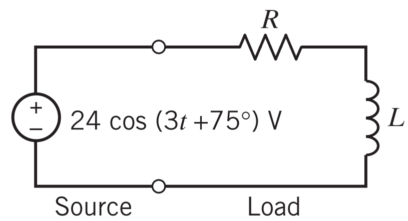
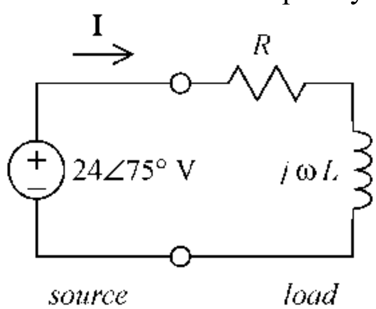


Figure P 11.6-11

Solution:

Represent the circuit in the frequency domain as



$$pf = 0.857 \text{ lagging} \Rightarrow \begin{cases} 0.857 = \cos(\theta) \\ \text{and} \\ \theta > 0 \end{cases}$$

so

$$\theta = 31°.$$

Next

$$14.12 = P = |\mathbf{S}|\cos\theta = |\mathbf{S}|(0.857)$$

so

$$|\mathbf{S}| = \frac{14.12}{0.857} = 16.48 \text{ VA}$$

Then

$$\mathbf{S} = 16.48\angle 31° = 14.12 + j8.49$$

and

$$\mathbf{I} = \left(\frac{2\mathbf{S}}{\mathbf{V}}\right)^* = \left[\frac{2(16.48\angle 31°)}{24\angle 75°}\right]^* = 1.37\angle 44°$$

$$R + j3L = \frac{24\angle 75°}{1.37\angle 44°} = 17.5\angle 31° = 15 + j9\ \Omega$$

so

$$R = 15\ \Omega \qquad L = 3 \text{ H}$$

P 11.6-12 The circuit in Figure P 11.6-12 consists of a source connected to a load. Determine the impedance of the load and the complex power delivered by the source to the load under each of the following conditions:

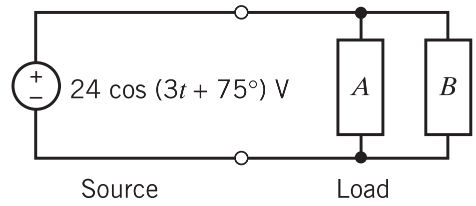


Figure P 11.6-12

(a) The source delivers $14.12 + j8.47$ VA to load *A* and $8.47 + j14.12$ VA to load *B*.

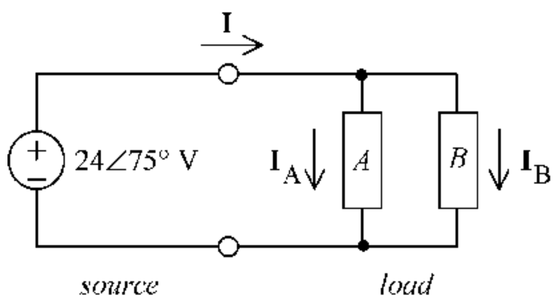
(b) The source delivers $8.47 + j14.12$ VA to load *A*, and the impedance of load *B* is $15 + j9$ Ω.

(c) The source delivers 14.12 W to load *A* at a power factor of 0.857 lagging, and the impedance of load *B* is $9 + j15$ Ω.

(d) The impedance of load *A* is $15 + j9$ Ω, and the impedance of load *B* is $9 + j15$ Ω.

Solution:

Represent the circuit in the frequency domain as



(a)

$$\mathbf{I}_A = \left(\frac{2(14.12 + j8.47)}{24\angle 75°}\right)^* = 1.37\angle 44° \text{ A}$$

$$\mathbf{I}_B = \left(\frac{2(8.47 + j14.12)}{24\angle 75°}\right)^* = 1.37\angle 16° \text{ A}$$

$$\mathbf{I} = \mathbf{I}_A + \mathbf{I}_B = (1.37\angle 44°) + (1.37\angle 16°) = (0.986 + j0.954) + (1.319 + j0.377)$$
$$= 2.305 + j1.331 = 2.662\angle 30° \text{ A}$$

$$\mathbf{Z} = \frac{24\angle 75°}{2.662\angle 30°} = 9.016\angle 45°$$

$$\mathbf{S} = \frac{1}{2}(24\angle 75°)(2.662\angle 30°)^* = 31.9\angle 45° = 22.59 + j22.59 \text{ VA}$$

(b)

$$\mathbf{I}_A = \left(\frac{2(8.47 + j14.12)}{24\angle 75°} \right)^* = 1.37\angle 16° \text{ A}$$

$$\mathbf{I}_B = \frac{24\angle 75°}{15 + j9} = 1.37\angle 44° \text{ A}$$

$$\mathbf{I} = \mathbf{I}_A + \mathbf{I}_B = 2.662\angle 30° \text{ A}$$

$$\mathbf{Z} = \frac{24\angle 75°}{2.662\angle 30°} = 9.016\angle 45° \ \Omega$$

$$\mathbf{S} = 22.59 + j22.59 \text{ VA}$$

(c)

$$\mathbf{P} = 14.12 \text{ W} = \frac{24|\mathbf{I}_A|}{2}\cos(75 - \theta_A)$$

$$\left.\begin{matrix} 0.857 = \cos(75 - \theta_A) \\ 75 - \theta_A > 0 \end{matrix}\right\} \quad \Rightarrow \quad \theta_A = 75° - 31° = 44°$$

Then

$$|\mathbf{I}_A| = \frac{2(14.12)}{24\cos(31°)} = 1.37$$

so

$$\mathbf{I}_A = 1.37\angle 44° \text{ A}$$

Also

$$\mathbf{I}_B = \frac{24\angle 75°}{9 + j15} = 137\angle 16° \text{ A}$$

$$\mathbf{I} = \mathbf{I}_A + \mathbf{I}_B = 2.662\angle 30° \text{ A}$$

(d)

$$\mathbf{I}_A = \frac{24\angle 75°}{15 + j9} = 1.37\angle 44°$$

$$\mathbf{I}_B = \frac{24\angle 75°}{9 + j15} = 1.37\angle 16°$$

$$\mathbf{I} = \mathbf{I}_A + \mathbf{I}_B = 2.662\angle 30° \text{ A}$$

$$\mathbf{Z} = \frac{24\angle 75°}{2.662\angle 30°} = 9.016\angle 45° \ \Omega$$

$$\mathbf{S} = 22.59 + j22.59 \text{ VA}$$

P 11.6-13 Figure P 11.6-13 shows two possible representations of an electrical load. One of these representations is used when the power factor of the load is lagging, and the other is used when the power factor is leading. Consider two cases:

(a) At the frequency $\omega = 4$ rad/s, the load has the power factor $pf = 0.8$ lagging.

(b) At the frequency $\omega = 4$ rad/s, the load has the power factor $pf = 0.8$ leading.

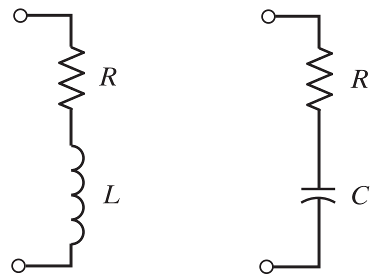


Figure P 11.6-13

In each case, choose one of the two representations of the load. Let $R = 6\ \Omega$ and determine the value of the capacitance, C, or the inductance, L.

Solution:

Let $\mathbf{Z} = R + jX = \sqrt{R^2+X^2}\angle\tan^{-1}\left(\frac{X}{R}\right)$ be the impedance of the load. Further, let $\mathbf{V} = A\angle\theta$ and $\mathbf{I} = B\angle\phi$ be the voltage across and current through the load. Then

$$\sqrt{R^2+X^2}\angle\tan^{-1}\left(\frac{X}{R}\right) = R + jX = \mathbf{Z} = \frac{A\angle\theta}{B\angle\phi} = \frac{A}{B}\angle(\theta-\phi)$$

Equating angles gives $\tan^{-1}\left(\frac{X}{R}\right) = \theta - \phi$

Also, the complex power delivered to the load is

$$\mathbf{S} = (A\angle\theta)(B\angle\phi)^* = \frac{AB}{2}\cos(\theta-\phi) + j\frac{AB}{2}\sin(\theta-\phi)$$

So $pf = \cos(\theta-\phi) \quad\Rightarrow\quad \theta-\phi = \cos^{-1}(pf)$

(Remark: $\cos^{-1}(pf)$ is positive when the power factor is lagging and negative when the power factor is leading.)

a) $pf = 0.8 \text{ lagging} \quad\Rightarrow\quad \cos^{-1}(0.8) = 36.9°$

We require $\frac{X}{R} = \tan(36.9) = 0.75 \quad\Rightarrow\quad X = 6(0.75) = 4.5\ \Omega$

Choose the inductor to implement the positive reactance. Then

$$4.5 = \omega L = 4L \quad\Rightarrow\quad L = 1.125 \text{ H}$$

b) $pf = 0.8 \text{ leading} \quad\Rightarrow\quad \cos^{-1}(0.8) = -36.9°$

We require $\frac{X}{R} = \tan(-36.9) = -0.75 \quad\Rightarrow\quad X = 6(-0.75) = -4.5\ \Omega$

Choose the capacitor to implement the negative reactance. Then

$$-4.5 = -\frac{1}{\omega C} = -\frac{1}{4C} \quad\Rightarrow\quad C = \frac{8}{9} = 0.889 \text{ F}$$

P 11.6-14 Figure P 11.6-14 shows two possible representations of an electrical load. One of these representations is used when the power factor of the load is lagging, and the other is used when the power factor is leading. Consider two cases:

(a) At the frequency $\omega = 4$ rad/s, the load has the power factor $pf = 0.8$ lagging.

(b) At the frequency $\omega = 4$ rad/s, the load has the power factor $pf = 0.8$ leading.

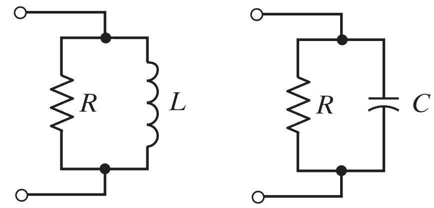


Figure P 11.6-14

In each case, choose one of the two representations of the load. Let $R = 6\ \Omega$ and determine the value of the capacitance, C, or the inductance, L.

Solution:

Let $\mathbf{Y} = \dfrac{1}{R} + \dfrac{1}{jX} = \dfrac{1}{R} - j\dfrac{1}{X} = \sqrt{\dfrac{1}{R^2} + \dfrac{1}{X^2}}\ \angle \tan^{-1}\left(-\dfrac{R}{X}\right)$ be the admittance of the load. Further, let $\mathbf{V} = A\angle\theta$ and $\mathbf{I} = B\angle\phi$ be the voltage across and current through the load. Then

$$\sqrt{\frac{1}{R^2} + \frac{1}{X^2}}\ \angle \tan^{-1}\left(-\frac{R}{X}\right) = \mathbf{Y} = \frac{B\angle\phi}{A\angle\theta} = \frac{B}{A}\angle(\phi - \theta)$$

Equating angles gives $$\tan^{-1}\left(-\frac{R}{\omega L}\right) = (\phi - \theta)$$

Also, the complex power delivered to the load is

$$\mathbf{S} = (A\angle\theta)(B\angle\phi)^* = \frac{AB}{2}\cos(\theta - \phi) + j\frac{AB}{2}\sin(\theta - \phi)$$

So $$pf = \cos(\theta - \phi) \quad\Rightarrow\quad \theta - \phi = \cos^{-1}(pf)$$

(Remark: $\cos^{-1}(pf)$ is positive when the power factor is lagging and negative when the power factor is leading.)

Therefore

$$\tan^{-1}\left(-\frac{R}{X}\right) = (\phi - \theta) = -\cos^{-1}(pf) \quad\Rightarrow\quad -\frac{R}{X} = \tan\left(-\cos^{-1}(pf)\right) = -\tan\left(\cos^{-1}(pf)\right)$$

$$\Rightarrow \frac{R}{X} = \tan\left(\cos^{-1}(pf)\right)$$

a) $$pf = 0.8 \text{ lagging} \quad\Rightarrow\quad \cos^{-1}(0.8) = 36.9°$$

We require $$\frac{R}{X} = \tan(36.9) = 0.75 \quad\Rightarrow\quad X = \frac{6}{0.75} = 8\ \Omega$$

Choose the inductor to implement the positive reactance. Then

$$8 = \omega L = 4L \quad\Rightarrow\quad L = 2\ \text{H}$$

b) $$pf = 0.8 \text{ leading} \quad \Rightarrow \quad \cos^{-1}(0.8) = -36.9°$$

We require $$\frac{R}{X} = \tan(-36.9) = -0.75 \quad \Rightarrow \quad X = \frac{6}{-0.75} = -8\ \Omega$$

Choose the capacitor to implement the negative reactance. Then

$$-8 = -\frac{1}{\omega C} = -\frac{1}{4C} \quad \Rightarrow \quad C = \frac{1}{32} = 31.25 \text{ mF}$$

P 11.6-15 Figure P 11.6-15 shows two electrical loads. Express the power factor of each load in terms of ω, R, and L.

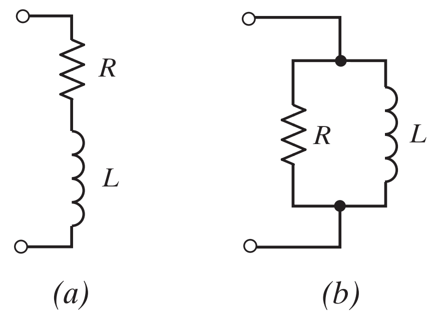


Figure P 11.6-15

Solution:

Let $\mathbf{V} = A\angle\theta$ and $\mathbf{I} = B\angle\phi$ be the voltage across and current through the load. The complex power delivered to the load is

$$\mathbf{S} = (A\angle\theta)(B\angle\phi)^* = \frac{AB}{2}\cos(\theta-\phi) + j\frac{AB}{2}\sin(\theta-\phi)$$

So $$pf = \cos(\theta-\phi) \quad \Rightarrow \quad \theta-\phi = \cos^{-1}(pf)$$

(Remark: $\cos^{-1}(pf)$ is positive when the power factor is lagging and negative when the power factor is leading.)

a.) Let $$\mathbf{Z} = R + j\omega L = \sqrt{R^2+\omega^2 L^2}\angle\tan^{-1}\left(\frac{\omega L}{R}\right)$$

Also $$\mathbf{Z} = \frac{A\angle\theta}{B\angle\phi} = \frac{A}{B}\angle(\theta-\phi)$$

Equating angles gives

$$\tan^{-1}\left(\frac{\omega L}{R}\right) = \theta-\phi$$

so $$\tan^{-1}\left(\frac{\omega L}{R}\right) = \theta-\phi = \cos^{-1}(pf) \quad \Rightarrow \quad pf = \cos\left(\tan^{-1}\left(\frac{\omega L}{R}\right)\right) \text{ lagging}$$

(The power factor is lagging because $\tan^{-1}\left(\frac{\omega L}{R}\right)$ is an angle in the first quadrant.)

b.) Let $$\mathbf{Y}=\frac{1}{R}+\frac{1}{j\omega L}=\frac{1}{R}-j\frac{1}{\omega L}=\sqrt{\frac{1}{R^2}+\frac{1}{\omega^2 L^2}}\angle\tan^{-1}\left(-\frac{R}{\omega L}\right)$$

Also

$$\mathbf{Y}=\frac{B\angle\phi}{A\angle\theta}=\frac{B}{A}\angle(\phi-\theta)$$

Equating angles gives $$\tan^{-1}\left(-\frac{R}{\omega L}\right)=(\phi-\theta)$$

$$\tan^{-1}\left(-\frac{R}{\omega L}\right)=(\phi-\theta)=-\cos^{-1}(pf) \;\Rightarrow\; -\frac{R}{\omega L}=\tan\left(-\cos^{-1}(pf)\right)=-\tan\left(\cos^{-1}(pf)\right)$$

so $$pf=\cos\left(\tan^{-1}\left(\frac{R}{\omega L}\right)\right)\text{ lagging}$$

(The power factor is lagging because $\tan^{-1}\left(\frac{R}{\omega L}\right)$ is an angle in the first quadrant.)

P 11.6-16 Figure P 11.6-16 shows two electrical loads. Express the power factor of each load in terms of ω, R, and C.

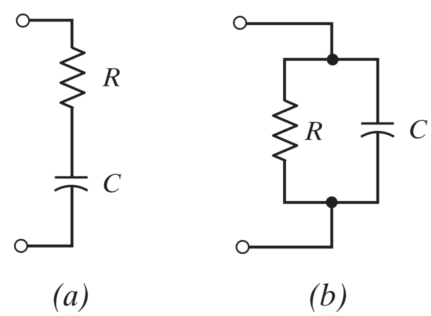


Figure P 11.6-16

Solution:

Let $\mathbf{V}=A\angle\theta$ and $\mathbf{I}=B\angle\phi$ be the voltage across and current through the load. The complex power delivered to the load is

$$\mathbf{S}=(A\angle\theta)(B\angle\phi)^*=\frac{AB}{2}\cos(\theta-\phi)+j\frac{AB}{2}\sin(\theta-\phi)$$

So $$pf=\cos(\theta-\phi) \;\Rightarrow\; \theta-\phi=\cos^{-1}(pf)$$

(Remark: $\cos^{-1}(pf)$ is positive when the power factor is lagging and negative when the power factor is leading.)

a.) Let $$\mathbf{Z}=R+\frac{1}{j\omega C}=R-j\frac{1}{\omega C}=\sqrt{R^2+\frac{1}{\omega^2 C^2}}\angle\tan^{-1}\left(-\frac{1}{\omega C R}\right)$$

Also $$\mathbf{Z}=\frac{A\angle\theta}{B\angle\phi}=\frac{A}{B}\angle(\theta-\phi)$$

Equating angles gives $\tan^{-1}\left(-\dfrac{1}{\omega C R}\right)=\theta-\phi$

So $\tan^{-1}\left(-\dfrac{1}{\omega C R}\right)=\theta-\phi=\cos^{-1}(pf) \quad\Rightarrow\quad pf=\cos\left(\tan^{-1}\left(-\dfrac{1}{\omega C R}\right)\right)$ leading

(The power factor is leading because $\tan^{-1}\left(\dfrac{-1}{\omega C R}\right)$ is an angle in the third quadrant.)

b.) Let $\mathbf{Y}=\dfrac{1}{R}+j\omega C=\sqrt{\dfrac{1}{R^2}+\omega^2 C^2}\ \angle\tan^{-1}(\omega C R)$

Also $\mathbf{Y}=\dfrac{B\angle\phi}{A\angle\theta}=\dfrac{B}{A}\angle(\phi-\theta)$

Equating angles gives $\tan^{-1}(\omega C R)=(\phi-\theta)$

So $\tan^{-1}(\omega C R)=-\cos^{-1}(pf) \quad\Rightarrow\quad pf=\cos\left(-\tan^{-1}(\omega C R)\right)$ leading

(The power factor is leading because $-\tan^{-1}(\omega C R)$ is an angle in the third quadrant.)

P11.6-17

The source voltage in the circuit shown in Figure P11.6-17 is $\mathbf{V}_s = 24\angle 30°$ V. Consequently

$$\mathbf{I}_1 = 3.13\angle 25.4° \text{ A},\ \mathbf{I}_2 = 1.99\angle 52.9° \text{ A and } \mathbf{V}_4 = 8.88\angle -10.6° \text{ V}$$

Determine (a) the average power absorbed by $\mathbf{Z}_4$, (b) the average power absorbed by $\mathbf{Z}_1$, and (c) the complex power delivered by the voltage source. (All phasors are given using peak, not RMS, values.)

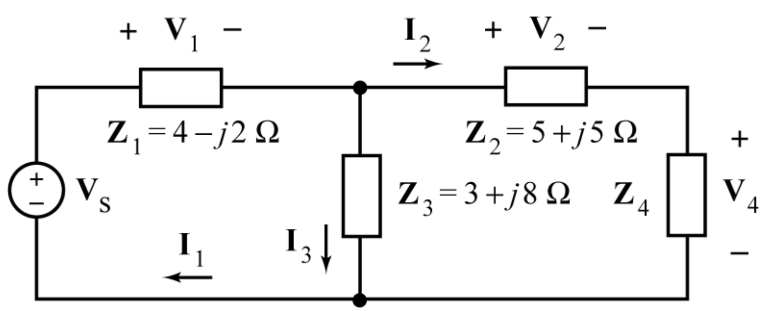


Figure P11.6-17

Solution:

a. The power factor of $\mathbf{Z}_1$ is $pf_1=\cos\left(\tan^{-1}\left(\dfrac{-2}{4}\right)\right)=\cos(-26.6°)=0.894$ leading.

b. The power factor of $\mathbf{Z}_3$ is $pf_3=\cos\left(\tan^{-1}\left(\dfrac{8}{3}\right)\right)=\cos(69.4°)=0.352$ lagging.

c. The power factor of $\mathbf{Z}_4$ is $pf_4=\cos(-10.6°-52.9°)=\cos(-63.5°)=0.45$ leading.

P11.6-18 The current source the circuit shown in Figure P11.6-18 supplies 131.16 – j36.048 VA and the voltage source supplies 64.2275 – 87.8481VA. Determine the values of the impedances $\mathbf{Z}_1$ and $\mathbf{Z}_2$.

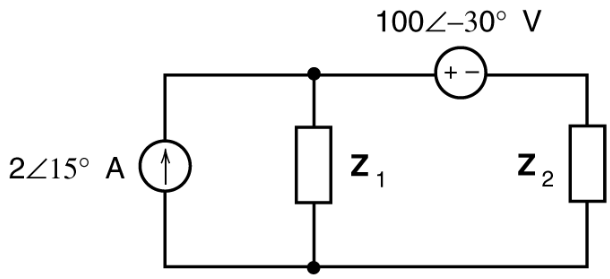
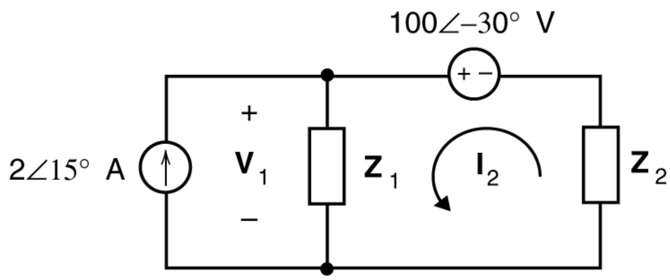


Figure P11.6-18

Solution:

The current source this circuit supplies 131.16 – j36.048 VA and the voltage source supplies 64.2275 – 87.8481VA. Determine the values of the impedances $\mathbf{Z}_1$ and $\mathbf{Z}_2$.



The voltage $\mathbf{V}_1$ is calculated from the current source current and complex power:

$$\mathbf{V}_1 = \frac{2\mathbf{S}}{\mathbf{I}^*} = \frac{2(131.16 - j36.048)}{(2\angle 15°)^*} = 136.02\angle -0.368° \text{ V}$$

The current $\mathbf{I}_2$ is calculated from the voltage source voltage and complex power:

$$\mathbf{I}_2 = \left(\frac{2\mathbf{S}}{\mathbf{V}}\right)^* = \left(\frac{2(64.2275 + j87.8481)}{100\angle -30°}\right)^* = 2.1765\angle -83.829° \text{ A}$$

The impedances are calculated as

$$\mathbf{Z}_1 = \frac{\mathbf{V}_1}{2\angle 15° + \mathbf{I}_2} = \frac{136.02\angle -0.368°}{2\angle 15° + (2.1765\angle -83.829°)} = 40 + j\,30\ \Omega$$

$$\mathbf{Z}_2 = \frac{\mathbf{V}_1 - 100\angle -30°}{-\mathbf{I}_2} = \frac{136.02\angle -0.368° - 100\angle -30°}{-2.1765\angle -83.829°} = 20 - j\,25\ \Omega$$

Section 11.7 The Power Superposition Principle

P 11.7-1 Find the average power absorbed by the 2-Ω resistor in the circuit of Figure P 11.7-1.
Answer: $P = 413$ W

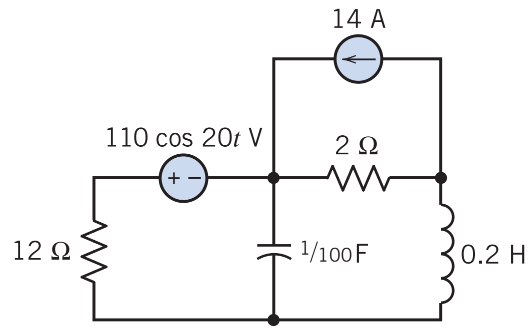
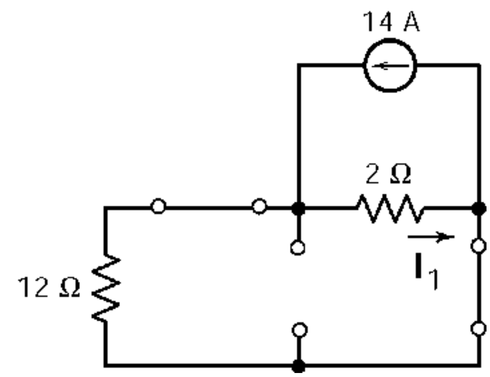


Figure P 11.7-1

Solution:

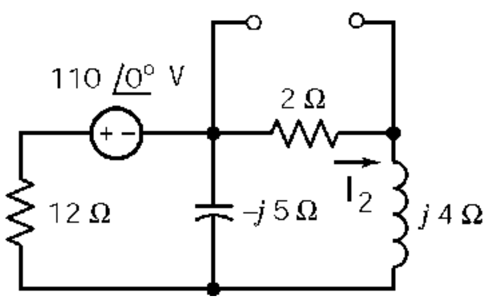
Use superposition since we have two different frequency sources. First consider the dc source (ω = 0):

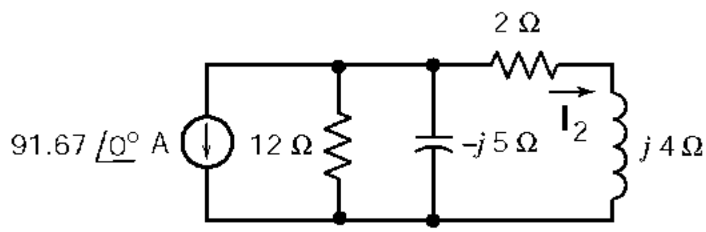


$$\mathbf{I}_1 = 14\left(\frac{12}{12+2}\right) = 12 \text{ A}$$

$$P_1 = \mathbf{I}_1^2\, R = (12)^2(2) = 288 \text{ W}$$

Next, consider the ac source (ω = 20 rad/s):





After a source transformation, current division gives

$$\mathbf{I}_2 = -9.167\left[\frac{\dfrac{-j60}{(12-j5)}}{\dfrac{-j60}{(12-j5)}+2+j4}\right] = \frac{25}{\sqrt{5}}\angle 116.6° \text{ A}$$

Then
$$P_2 = \frac{|\mathbf{I}_2|^2}{2}(2) = \frac{(125)(2)}{2} = 125 \text{ W}$$

Now using power superposition

$$P = P_1 + P_2 = 288 + 125 = 413 \text{ W}$$

P 11.7-2 Find the average power absorbed by the 8-Ω resistor in the circuit of Figure P 11.7-2.
Answer: $P = 22$ W

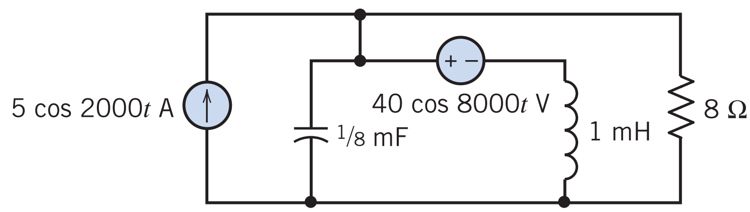
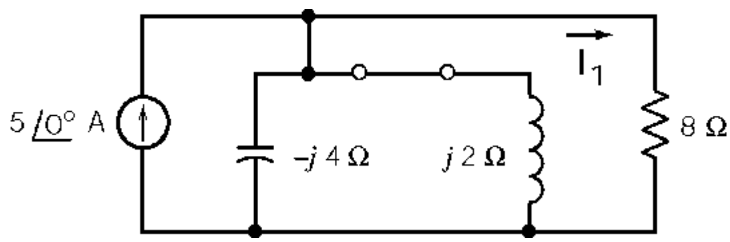


Figure P 11.7-2

Solution:
Use superposition since we have two different frequency sources. First consider ω = 2000 rad/s source:

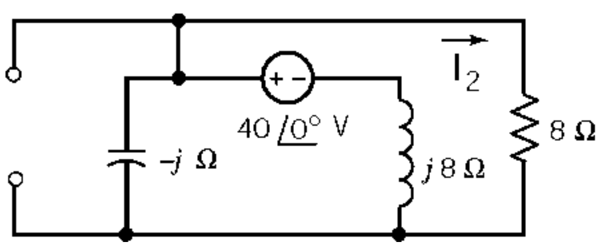
Current division yields

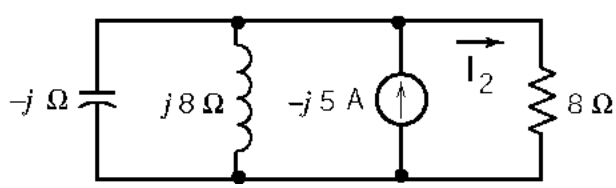
$$\mathbf{I}_1 = 5\left[\frac{\frac{8}{-j2}}{\frac{8}{-j2}+8}\right] = \frac{5}{\sqrt{5}}\angle 63.4° \text{ A}$$



Then $$P_1 = \frac{|\mathbf{I}_1|^2 8}{2} = 20 \text{ W}$$

Next consider ω = 8000 rad/s source.





Current division yields

$$\mathbf{I}_2 = -j5\left[\frac{\frac{8}{j7}}{\frac{8}{j7}+8}\right] = \frac{5}{\sqrt{50}}\angle -171.9° \text{ A}$$

Then $$P_2 = \frac{|\mathbf{I}_2|^2 8}{2} = 2 \text{ W}$$

Now using power superposition

$$P = P_1 + P_2 = 22 \text{ W}$$

P 11.7-3 For the circuit shown in Figure P 11.7-3, determine the average power absorbed by each resistor, R_1 and R_2. The voltage source is v_s = 10 + 10 cos (5t + 40°) V, and the current source is i_s = 4 cos (5t – 30°) A.

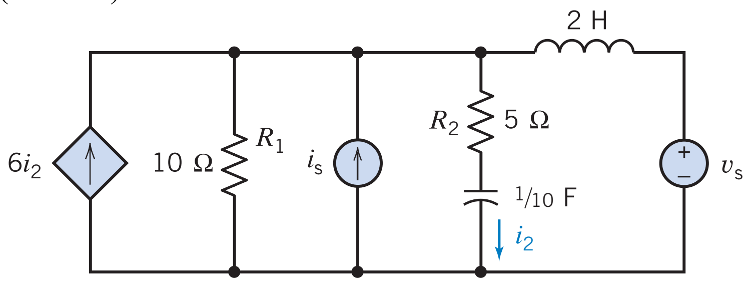


Figure P 11.7-3

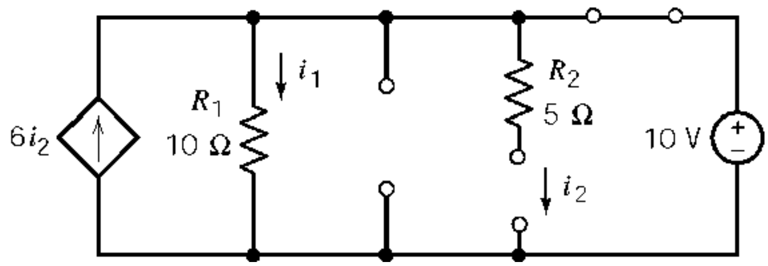
P11.7-3

Use superposition since we have two different frequencies. First consider the dc source (ω = 0):

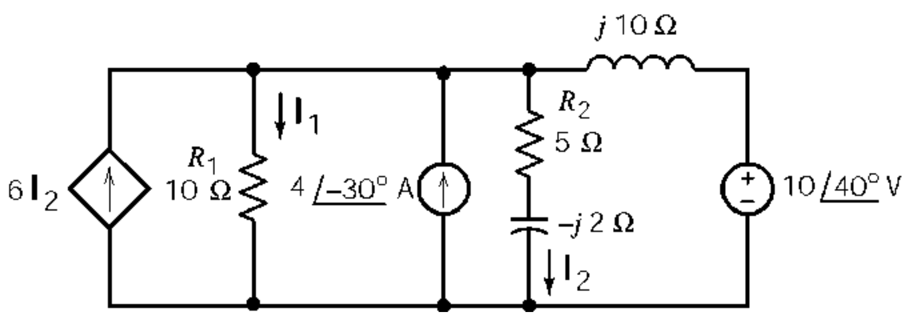
$$i_2(t) = 0 \text{ and } i_1(t) = 10\left(\frac{1}{10}\right) = 1 \text{ A}$$

$$P_{R_1} = i_1^2 R_1 = 1^2(10) = 10 \text{ W}$$

$$P_{R_2} = 0 \text{ W}$$



Next consider ω = 5 rad/s sources.



Apply KCL at the top node to get

$$-6\mathbf{I}_2 + \mathbf{I}_1 + \mathbf{I}_2 - (4\angle -30°) + \frac{(10\mathbf{I}_1 - 10\angle 40°)}{j10} = 0$$

Apply KVL to get $-10\mathbf{I}_1 + (5 - j2)\mathbf{I}_2 = 0$

Solving these equations gives

$$\mathbf{I}_1 = -0.56\angle -64.3° \text{ A and } \mathbf{I}_2 = -1.04\angle -42.5° \text{ A}$$

Then $$P_{R_1} = \frac{|\mathbf{I}_1^2|R_1}{2} = \frac{(0.56)^2(10)}{2} = 1.57 \text{ W and } P_{R_2} = \frac{|\mathbf{I}_2^2|R_2}{2} = \frac{(1.04)^2(5)}{2} = 2.7 \text{ W}$$

Now using power superposition

$$P_{R_1} = 10 + 1.57 = \underline{11.57 \text{ W}} \text{ and } P_{R_2} = 0 + 2.7 = \underline{2.7 \text{ W}}$$

P 11.7-4 For the circuit shown in Figure P 11.7-4, determine the effective value of the resistor voltage v_R and the capacitor voltage v_C.

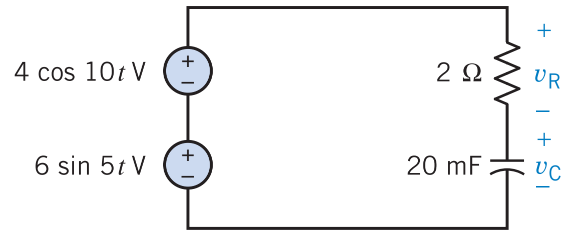


Figure P 11.7-4

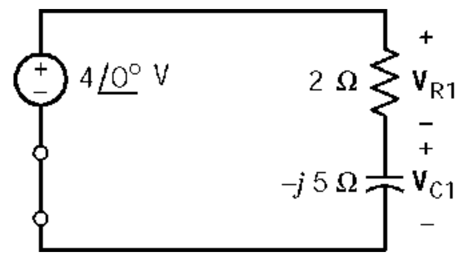
Solution:

Use superposition since we have two different frequencies. First consider the ω = 10 rad/s source:

$$\mathbf{I}_1 = \frac{\mathbf{V_1}}{\mathbf{Z}} = \frac{4\angle 0°}{2-j5} = 0.28 + j0.7 \text{ A}$$

$$\mathbf{V}_{R_1} = 2\mathbf{I}_1 = 2(0.28 + j.7) = 0.56 + j1.4 = 1.51\ \angle 68.2° \text{ V}$$

$$\mathbf{V}_{C_1} = -j5I_1 = 3.77\ \angle -21.8° \text{ V}$$



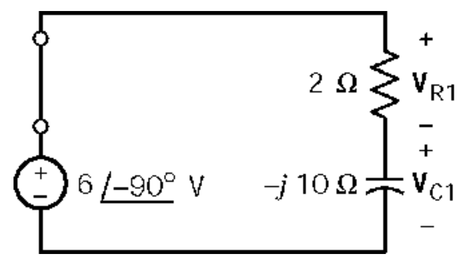
Next consider ω = 5 rad/s source.

$$\mathbf{I}_2 = \frac{\mathbf{V}_2}{\mathbf{Z}} = \frac{6\angle -90°}{2-j10} = 0.577 - j0.12 \text{ A}$$

$$\mathbf{V}_{R_2} = 2\mathbf{I}_2 = 2(.577 - j0.12) = 1.15 - j0.24$$

$$= 1.17\angle -11.8° \text{ V}$$

$$\mathbf{V}_{C_2} = -j10\mathbf{I}_2 = 5.9\angle 258.3° \text{ V}$$



Now using superposition

$$v_R(\text{t}) = 1.51\cos(10t + 68.2°) + 1.17\cos(5t - 11.8°) \text{ V}$$

$$v_C(\text{t}) = 3.77\cos(10t - 21.8°) + 5.9\cos(5t - 258.3°) \text{ V}$$

Then

$$V_{\text{Reff}}^2 = \left(\frac{1.51}{\sqrt{2}}\right)^2 + \left(\frac{1.17}{\sqrt{2}}\right)^2 = 1.82 \Rightarrow \underline{V_{\text{Reff}} = 1.35 \text{ V}}$$

$$V_{\text{Ceff}}^2 = \left(\frac{3.77}{\sqrt{2}}\right)^2 + \left(\frac{5.9}{\sqrt{2}}\right)^2 = 24.52 \Rightarrow \underline{V_{\text{Ceff}} = 4.95 \text{ V}}$$

Section 11.8 The Maximum Power Transfer Theorem

P 11.8-1 Determine values of R and L for the circuit shown in Figure P 11.8-1 that cause maximum power transfer to the load.
Answer: R = 800 Ω and L = 1.6 H

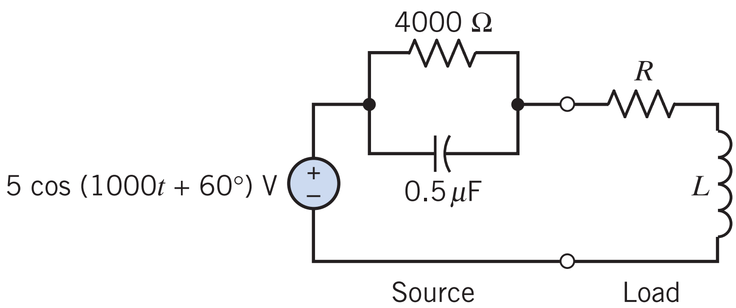


Figure P 11.8-1

Solution:

$$\mathbf{Z}_t = 4000 \,||\, -j2000 = 800 - j1600 \ \ \Omega$$

$$\mathbf{Z}_L = \mathbf{Z}_t^* = 800 + j1600 \ \ \Omega$$

$$R + j1000L = 800 + j1600 \Rightarrow \begin{cases} R = 800 \ \Omega \\ L = 1.6 \text{ H} \end{cases}$$

P 11.8-2 Is it possible to choose R and L for the circuit shown in Figure P 11.8-2 so that the average power delivered to the load is 12 mW?
Answer: Yes

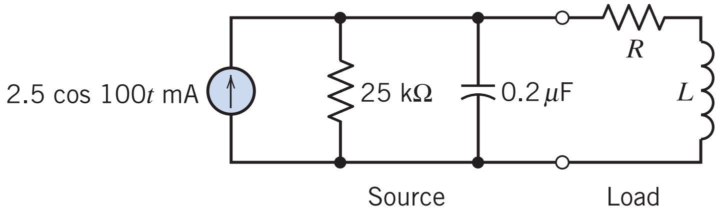


Figure P 11.8-2

Solution:

$$\mathbf{Z}_t = 25{,}000 \,||\, -j50{,}000 \ = \ 20{,}000 - j10{,}000 \ \ \Omega$$

$$\mathbf{Z}_L = \mathbf{Z}_t^* = 20{,}000 + j10{,}000 \ \ \Omega$$

$$R + j\omega L = 20{,}000 + j10{,}000 \Rightarrow \begin{cases} R = 20 \text{ k}\Omega \\ 100L = 10{,}000 \\ L = 100 \text{ H} \end{cases}$$

After selecting these values of R and L,

$$|\mathbf{I}| = 1.4 \text{ mA} \ \text{ and } \ P_{\max} = \left(\frac{0.14\times10^{-2}}{\sqrt{2}}\right)^2 \left(20\times10^3\right) = 19.5 \text{ mW}$$

Since $P_{\max} > 12$ mW, yes, we can deliver 12 mW to the load.

P 11.8-3 The capacitor has been added to the load in the circuit shown in Figure P 11.8-3 in order to maximize the power absorbed by the 4000-Ω resistor. What value of capacitance should be used to accomplish that objective?
Answer: 0.1 μF

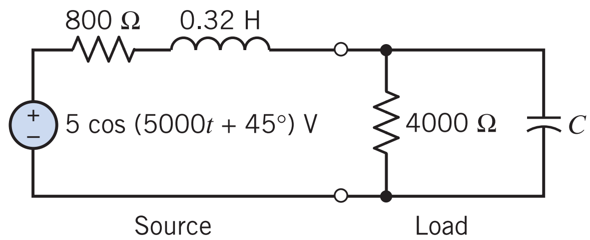


Figure P 11.8-3

Solution:

$$\mathbf{Z}_t = 800 + j1600\ \Omega \quad \text{and} \quad \mathbf{Z}_L = \frac{R\left(\dfrac{-j}{\omega C}\right)}{R - \dfrac{j}{\omega C}} = \frac{R - j\omega R^2 C}{1 + (\omega RC)^2}$$

$$\mathbf{Z}_L = \mathbf{Z}_t^* \quad \Rightarrow \quad \frac{R\left(\dfrac{-j}{\omega C}\right)}{R - \dfrac{j}{\omega C}} = \frac{R - j\omega R^2 C}{1 + (\omega RC)^2} = 800 - j1600\ \Omega$$

Equating the real parts gives

$$800 = \frac{R}{1 + (\omega RC)^2} = \frac{4000}{1 + [(5000)(4000)C]^2} \quad \Rightarrow \quad C = 0.1\ \mu\text{F}$$

P 11.8-4 What is the value of the average power delivered to the 2000-Ω resistor in the circuit shown in Figure P 11.8-4? Can the average power delivered to the 2000-Ω resistor be increased by adjusting the value of the capacitance?
Answer: 8 mW. No.

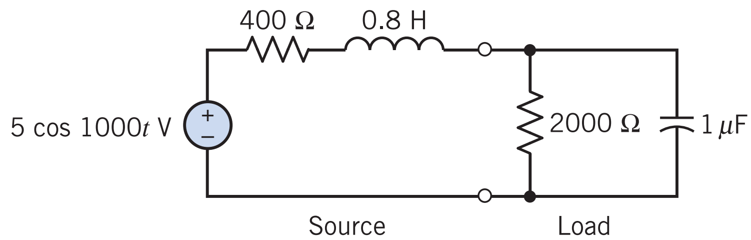


Figure P 11.8-4

Solution:

$$\mathbf{Z}_t = 400 + j800 \ \Omega \ \text{ and } \ \mathbf{Z}_L = 2000 \ || \ -j1000 = 400 - j800 \ \Omega$$

Since $\mathbf{Z}_L = \mathbf{Z}_t^*$ the average power delivered to the load is maximum and cannot be increased by adjusting the value of the capacitance. The voltage across the 2000 Ω resistor is

$$\mathbf{V}_R = 5\frac{\mathbf{Z}_L}{\mathbf{Z}_t + \mathbf{Z}_L} = 2.5 - j5 = 5.59e^{-j63.4} \ \text{V}$$

So

$$P = \left(\frac{5.59}{\sqrt{2}}\right)^2 \frac{1}{2000} = 7.8 \ \text{mW}$$

is the average power delivered to the 2000 Ω resistor.

P 11.8-5 What is the value of the resistance *R* in Figure P 11.8-5 that maximizes the average power delivered to the load?

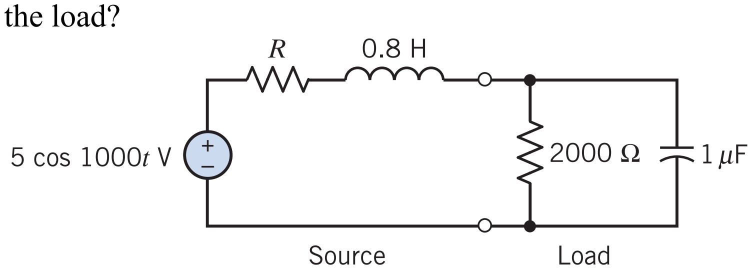


Figure P 11.8-5

Solution:
Notice that $\mathbf{Z}_t$,not $\mathbf{Z}_L$, is being adjusted .When $\mathbf{Z}_t$ is fixed, then the average power delivered to the load is maximized by choosing $\mathbf{Z}_L = \mathbf{Z}_t$*. In contrast, when $\mathbf{Z}_L$ is fixed, then the average power delivered to the load is maximized by minimizing the real part of $\mathbf{Z}_t$. In this case, choose $R = 0$. Since no average power is dissipated by capacitors or inductors, all of the average power provided by source is delivered to the load.

Section 11.9 Coupled Inductors

P 11.9-1 Two magnetically coupled coils are connected as shown in Figure P 11.9-1. Show that an equivalent inductance at terminals a–b is $L_{ab} = L_1 + L_2 - 2M$.

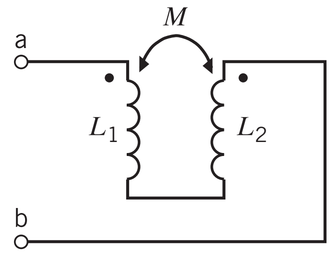
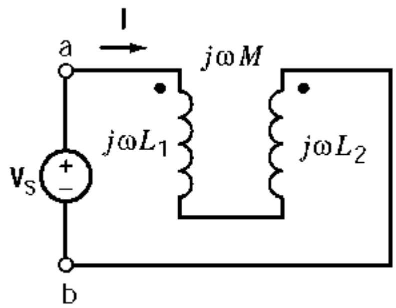


Figure P 11.9-1

Solution:



$$\mathbf{V}_s = \mathbf{I}\, j\omega L_1 - \mathbf{I}\, j\omega M + \mathbf{I}\, j\omega L_2 - \mathbf{I}\, j\omega M$$

$$\Rightarrow j\omega(L_1 + L_2 - 2M) = \frac{\mathbf{V}_s}{\mathbf{I}}$$

Therefore

$$L_{ab} = L_1 + L_2 - 2M$$

P 11.9-2 Two magnetically coupled coils are shown connected in Figure P 11.9-2. Find the equivalent inductance L_{ab}.

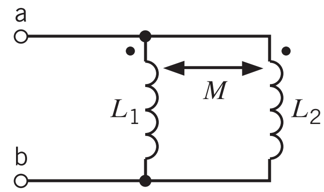
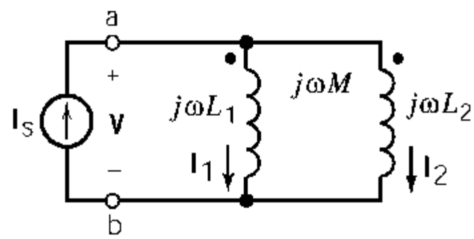


Figure P 11.9-2

Solution:



The coil voltages are given by:

$$\mathbf{V}_1 = \mathbf{I}_1 j\omega L_1 + \mathbf{I}_2 j\omega M$$

$$\mathbf{V}_2 = \mathbf{I}_2 j\omega L_2 + \mathbf{I}_1 j\omega M$$

From KVL the coil voltages are equal so

$$\mathbf{I}_1 j\omega L_1 + \mathbf{I}_2 j\omega M = \mathbf{I}_2 j\omega L_2 + \mathbf{I}_1 j\omega M \quad \Rightarrow \quad \mathbf{I}_1 j\omega\left(L_1 - M\right) = \mathbf{I}_2 j\omega\left(L_2 - M\right)$$

$$\Rightarrow \quad \mathbf{I}_1 = \left(\frac{L_2 - M}{L_1 - M}\right)\mathbf{I}_2$$

From KCL

$$\mathbf{I}_s = \mathbf{I}_1 + \mathbf{I}_2 = \left(\frac{L_2 - M}{L_1 - M} + 1\right)\mathbf{I}_2 = \left(\frac{L_2 + L_1 - 2M}{L_1 - M}\right)\mathbf{I}_2$$

Next

$$\mathbf{I}_2 = \left(\frac{L_1 - M}{L_2 + L_1 - 2M}\right)\mathbf{I}_s \quad \text{and} \quad \mathbf{I}_1 = \left(\frac{L_2 - M}{L_1 - M}\right)\mathbf{I}_2 = \left(\frac{L_2 - M}{L_1 - M}\right)\left(\frac{L_1 - M}{L_2 + L_1 - 2M}\right)\mathbf{I}_s$$

Substituting gives

$$\mathbf{V}_1 = j\omega L_1 \mathbf{I}_1 + j\omega M \mathbf{I}_2 = j\omega M\left(\frac{L_2 - M}{L_2 + L_1 - 2M}\right)\mathbf{I}_s + j\omega M\left(\frac{L_1 - M}{L_2 + L_1 - 2M}\right)\mathbf{I}_s$$

$$= j\omega\left(\frac{M\left(L_2 - M\right) + M\left(L_1 - M\right)}{L_2 + L_1 - 2M}\right)\mathbf{I}_s = j\omega\left(\frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}\right)\mathbf{I}_s$$

Finally

$$\mathbf{Z} = \frac{\mathbf{V}_1}{\mathbf{I}_s} = j\omega\left(\frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}\right)$$

P 11.9-3 The source voltage of the circuit shown in Figure P 11.9-3 is $v_s = 141.4 \cos 100t$ V. Determine $i_1(t)$ and $i_2(t)$.

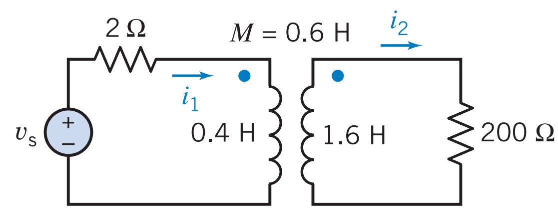
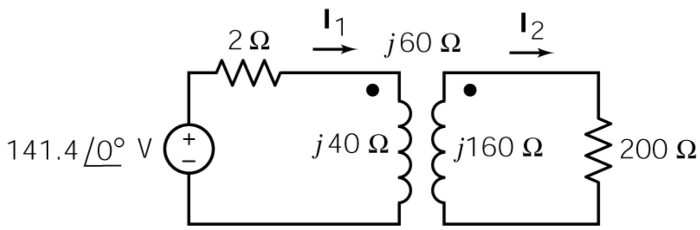


Figure P 11.9-3

Solution:



Mesh equations:

$$-141.4\angle 0° + 2\mathbf{I}_1 + j40\mathbf{I}_1 - j60\mathbf{I}_2 = 0$$

$$200\mathbf{I}_2 + j160\mathbf{I}_2 - j60\mathbf{I}_1 = 0 \Rightarrow \mathrm{I}_2 = (0.23\angle 51°)\mathbf{I}_1$$

Solving yields

$$\mathbf{I}_1 = 4.17\angle -68° \text{ A} \text{ and } \mathbf{I}_2 = 0.96\angle -17° \text{A}$$

Finally

$$\underline{i_1(t) = 4.17 \cos(100t - 68°) \text{ A}} \text{ and } \underline{i_2(t) = 0.96 \cos(100t - 17°) \text{ A}}$$

(checked: LNAPAC 7/21/04)

P 11.9-4 A circuit with a mutual inductance is shown in Figure P 11.9-4. Find the voltage $\mathbf{V}_2$ when $\omega = 5000$.

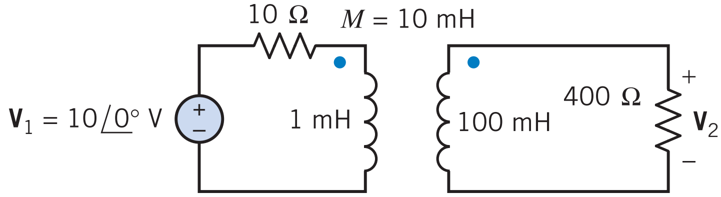
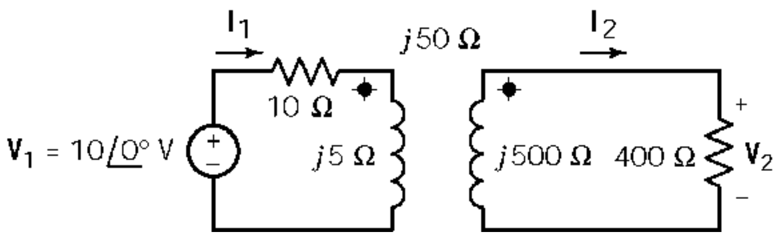


Figure P 11.9-4

Solution:



Mesh equations:

$$(10+j5)\,\mathbf{I}_1 - j50\,\mathbf{I}_2 = 10$$

$$-j50\,\mathbf{I}_1 + (400+j500)\,\mathbf{I}_2 = 0$$

Solving the mesh equations using Cramer's rule:

$$\mathbf{I}_2 = \frac{(10+j5)(0)-(-j50)(10)}{(10+j5)\,(400+j500)\,-(-j50)^2} = \,0.062\,\angle 29.7°\text{ A}$$

Then

$$\mathbf{V}_2 = 400\,\mathbf{I}_2 = 400\,(0.062\,\angle 29.7°)\; = 24.8\,\angle 29.7°$$

P 11.9-5 Determine $v(t)$ for the circuit of Figure P 11.9-5 when $v_s = 10 \cos 30t$ V.

Answer: $v(t) = 23 \cos(30t + 9°)$ V

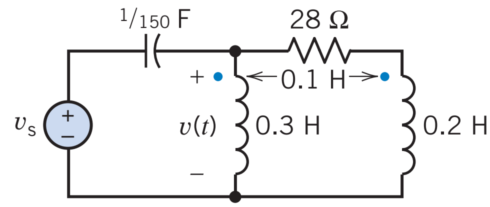
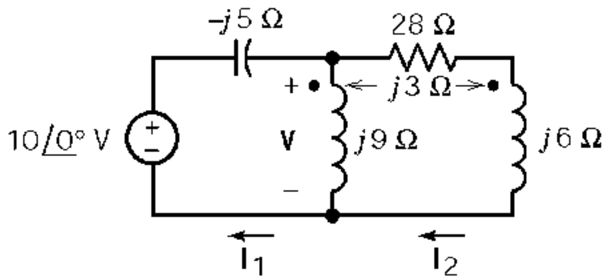


Figure P 11.9-5

Solution:



Mesh equations:

$$10 = -j5\mathbf{I}_1 + \left[j9\left(\mathbf{I}_1 - \mathbf{I}_2\right) + j3\mathbf{I}_2 \right]$$
$$0 = 28\mathbf{I}_2 + \left[j6\mathbf{I}_2 + j3\left(\mathbf{I}_1 - \mathbf{I}_2\right) \right] - \left[j9\left(\mathbf{I}_1 - \mathbf{I}_2\right) + j3\mathbf{I}_2 \right]$$

or

$$\begin{bmatrix} j4 & -j6 \\ -j6 & 28 + j9 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$

Solving the mesh equations, e.g. using MATLAB, yields

$$\mathbf{I}_1 = 2.62\angle -72° \text{ A} \quad \text{and} \quad \mathbf{I}_2 = 0.53\angle 0° \text{ A}$$

then

$$\mathbf{V} = j9(\mathbf{I}_1 - \mathbf{I}_2) + j3\mathbf{I}_2 = j9\mathbf{I}_1 - j6\mathbf{I}_2 = 23\angle 10° \text{ V}$$

Finally

$$v(t) = 23\cos(30t + 10°) \text{ V}$$

(checked: LNAPAC 7/21/04)

P 11.9-6 Find the total energy stored in the circuit shown in Figure P 11.9-6 at $t = 0$ if the secondary winding is (a) open-circuited, (b) short-circuited, (c) connected to the terminals of a 7-Ω resistor.

Answer: (a) 15 J (b) 0 J (c) 5 J

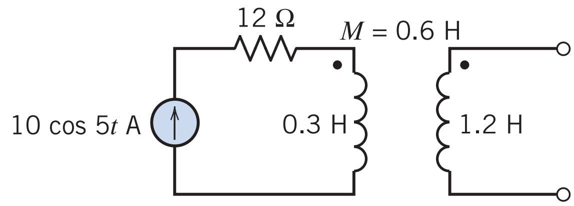
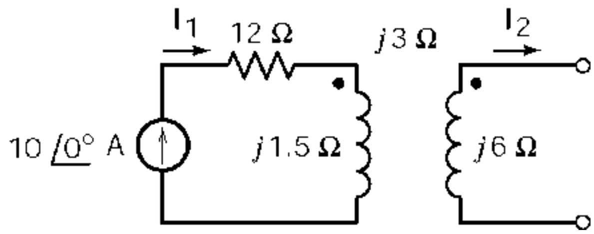


Figure P 11.9-6

Solution:

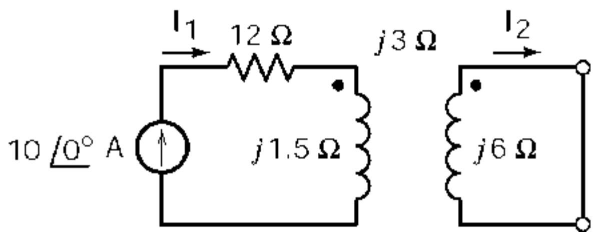
(a)



$$\mathbf{I}_2 = 0 \;\Rightarrow\; \mathrm{I}_1 = 10\ \angle 0°\ \mathrm{A} \;\Rightarrow\; i_1(0) = 10\ \mathrm{A}$$

$$w = \frac{L_1\ i_1^{\,2}(0)}{2} = \frac{(0.3)\ (10)^2}{2} = 15\ \mathrm{J}$$

(b)



Mesh equations:

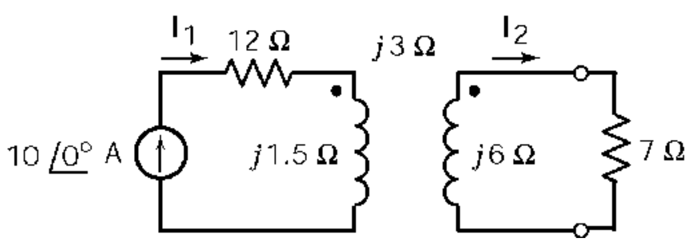
$$j6\ \mathbf{I}_2 \text{ - } j3\ \mathbf{I}_1 = 0 \;\Rightarrow\; \mathbf{I}_1 = 2\ \mathbf{I}_2$$

$$\mathbf{I}_1 = 10\angle 0°\ \mathrm{A} \;\Rightarrow\; \mathrm{I}_2 = 5\angle 0°\ \mathrm{A}$$

Then

$$w = \frac{1}{2}L_1 i_2^{\,2}(0) + \frac{1}{2}L_2 i_1^{\,2}(0) - M\, i_1(0)\, i_2(0) = \frac{1}{2}(0.3)(10)^2 + \frac{1}{2}(1.2)(5)^2 - (0.6)\ (10)(5) = \ 0$$

(c)



$$(7 + j6)\ \mathbf{I}_2 - j3\ \mathbf{I}_1 = 0$$

$$\mathbf{I}_2 = 3.25\ \angle 49.4°\ \mathrm{A}$$

$$i_2(t) = 3.25 \cos(5t + 49.4°)\ \mathrm{A}$$

$$i_2(0) = 2.12\ \mathrm{A}$$

Finally

$$w = \frac{1}{2}(0.3)\ (10)^2 + \frac{1}{2}(1.2)\ (2.12)^2 - (0.6)\ (10)\ (2.12) = 5.0\ \mathrm{J}$$

P 11.9-7 Find the input impedance, **Z**, of the circuit of Figure P 11.9-7 when $\omega = 1000$ rad/s.
Answer: $\mathbf{Z} = 8.4\ \angle 14°\ \Omega$

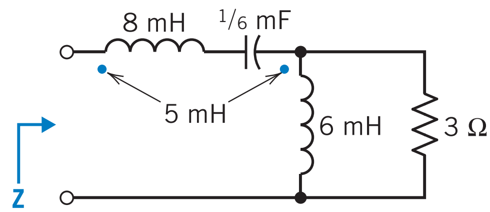
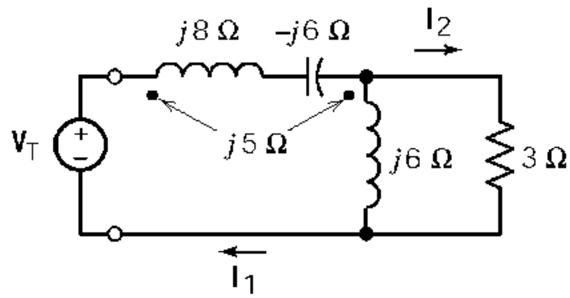


Figure P 11.9-7

Solution:



Mesh equations:

$$-\mathbf{V}_T + j8\mathbf{I}_1 + j5(\mathbf{I}_1 - \mathbf{I}_2) - j6\mathbf{I}_1 + j6(\mathbf{I}_1 - \mathbf{I}_2) + j5\mathbf{I}_1 = 0$$

$$3\mathbf{I}_2 + j6(\mathbf{I}_2 - \mathbf{I}_1) - j5\mathbf{I}_1 = 0$$

Solving yields

$$\mathbf{I}_2 = \frac{j11}{3+j6}\,\mathbf{I}_1$$

$$\mathbf{V}_T = \mathbf{I}_1(j18) + \mathbf{I}_2(-j11) = \left(j18 + \frac{j11}{3+j6}\right)\mathbf{I}_1$$

Then

$$\mathbf{Z} = \frac{\mathbf{V}_T}{\mathbf{I}_1} = j18 + \frac{j11}{3+j6} = 8.28\ \angle 13° = 8.0667 + j1.8667\ \Omega$$

(checked: LNAPAC 7/21/04)

P 11.9-8 A circuit with three mutual inductances is shown in Figure P 11.9-8. When $v_s = 10 \cos 2t$ V, $M_1 = 2$ H, and $M_2 = M_3 = 1$ H, determine the capacitor voltage $v(t)$.

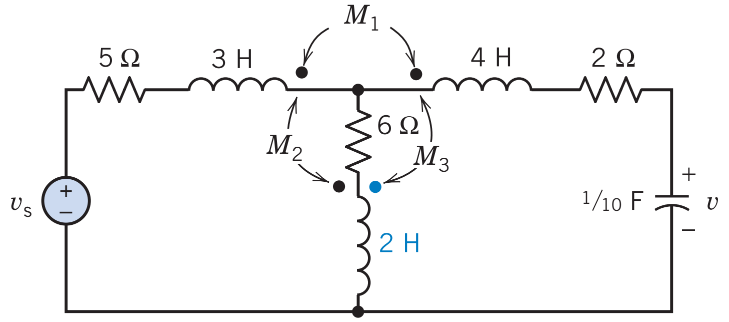
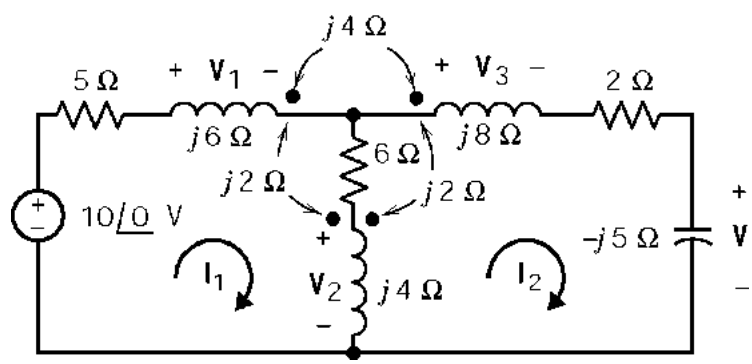


Figure P 11.9-8

Solution:



The coil voltages are given by

$$\mathbf{V}_1 = j6\mathbf{I}_1 - j2(\mathbf{I}_1 - \mathbf{I}_2) - j4\mathbf{I}_2 = j4\mathbf{I}_1 - j2\mathbf{I}_2$$
$$\mathbf{V}_2 = j4(\mathbf{I}_1 - \mathbf{I}_2) - j2\mathbf{I}_1 + j2\mathbf{I}_2 = j2\mathbf{I}_1 - j2\mathbf{I}_2$$
$$\mathbf{V}_3 = j8\mathbf{I}_2 - j4\mathbf{I}_1 + j2(\mathbf{I}_1 - \mathbf{I}_2) = -j2\mathbf{I}_1 + j6\mathbf{I}_2$$

The mesh equations are

$$5\,\mathbf{I}_1 + \mathbf{V}_1 + 6\,(\mathbf{I}_1 - \mathbf{I}_2) + \mathbf{V}_2 = 10\angle 0^\circ$$
$$-\mathbf{V}_2 + 6\,(\mathbf{I}_2 - \mathbf{I}_1) + 2\,\mathbf{I}_2 + \mathbf{V}_3 - j5\,\mathbf{I}_2 = 0$$

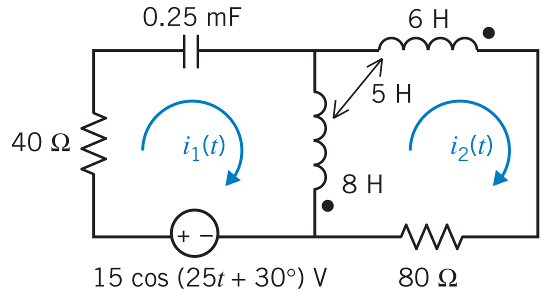
Combining and solving yields

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 11+j6 & 10 \\ -6-j4 & 0 \end{vmatrix}}{\begin{vmatrix} 11+j6 & -6-j4 \\ -6-j4 & 8+j3 \end{vmatrix}} = \frac{60 + j40}{50 + j33} = 1.2\angle 0.28^\circ \text{ A}$$

Finally

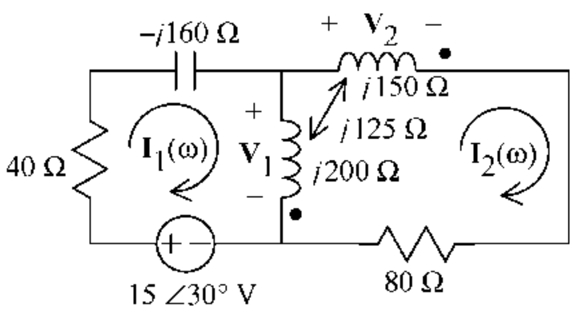
$$\mathbf{V} = -j5\,\mathbf{I}_2 = 6.0\angle -89.72^\circ \text{ A} \quad \Rightarrow \quad v(t) = 6\cos(2t - 89.7^\circ) \text{ V}$$

P 11.9-9 The currents $i_1(t)$ and $i_2(t)$ in Figure, P 11.9-9 are mesh currents. Represent the circuit in the frequency domain and write the mesh equations.



P 11.9-9

Solution:



The equations describing the coupled coils give:

$$\mathbf{V}_1 = j200(\mathbf{I}_1 - \mathbf{I}_2) + j125\,\mathbf{I}_2 = j200\mathbf{I}_1 - j75\,\mathbf{I}_2$$
$$\mathbf{V}_2 = j150\,\mathbf{I}_2 + j125(\mathbf{I}_1 - \mathbf{I}_2) = j125\,\mathbf{I}_1 + j25\,\mathbf{I}_2$$

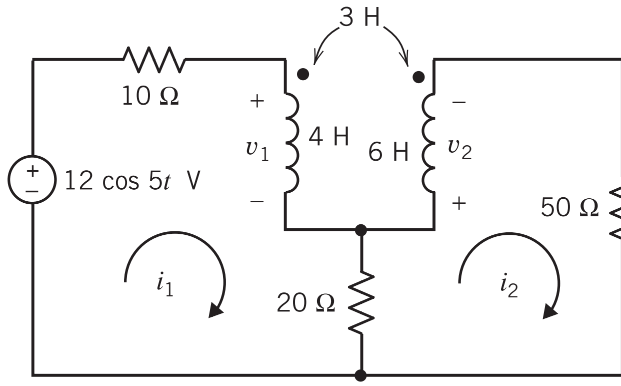
The mesh equation for the left mesh is

$$-j\,160\,\mathbf{I}_1 + \mathbf{V}_1 - 15\angle 30° + 40\,\mathbf{I}_1 = 0$$
$$-j\,160\,\mathbf{I}_1 + j200\mathbf{I}_1 - j75\,\mathbf{I}_2 - 15\angle 30° + 40\,\mathbf{I}_1 = 0$$
$$(40 + j40)\,\mathbf{I}_1 - j75\,\mathbf{I}_2 = 15\angle 30°$$

The mesh equation for the right mesh is

$$\mathbf{V}_2 + 80\,\mathbf{I}_2 - \mathbf{V}_1 = 0$$
$$j125\,\mathbf{I}_1 + j25\,\mathbf{I}_2 + 80\,\mathbf{I}_2 - (j200\mathbf{I}_1 - j75\,\mathbf{I}_2) = 0$$
$$-j75\,\mathbf{I}_1 + (80 + j100)\,\mathbf{I}_2 = 0$$

P 11.9-10 Determine the mesh currents for the circuit shown in Figure P 11.9-10.

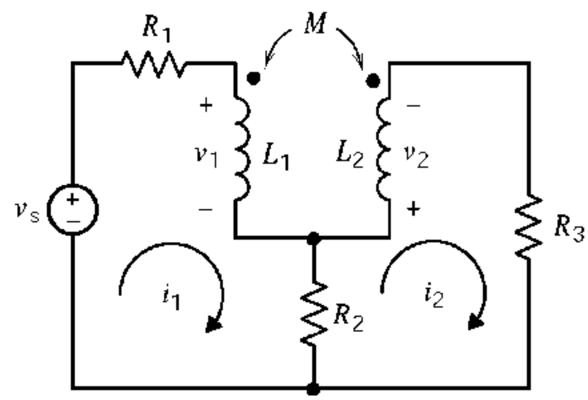


P 11.9-10

Solution:

In the frequency domain, the coil voltages are given by

$$\mathbf{V}_1 = j\omega L_1\,\mathbf{I}_1 - j\omega M\,\mathbf{I}_2$$
$$\mathbf{V}_2 = j\omega L_2\,\mathbf{I}_2 - j\omega M\,\mathbf{I}_1$$



The mesh equations are

$$R_1\,\mathbf{I}_1 + \mathbf{V}_1 + R_2\left(\mathbf{I}_1 - \mathbf{I}_2\right) = \mathbf{V}_s$$
$$\mathbf{V}_2 + R_3\,\mathbf{I}_2 - R_2\left(\mathbf{I}_1 - \mathbf{I}_2\right) = 0$$

Substituting for the coil voltages gives

$$\begin{bmatrix} R_1 + R_2 + j\omega L_1 & -\left(R_2 + j\omega M\right) \\ -\left(R_2 + j\omega M\right) & R_2 + R_3 + j\omega L_2 \end{bmatrix}\begin{bmatrix}\mathbf{I}_1 \\ \mathbf{I}_2\end{bmatrix} = \begin{bmatrix}\mathbf{V}_s \\ 0\end{bmatrix}$$

Using the given values

$$\begin{bmatrix} 30 + j20 & -\left(20 + j15\right) \\ -\left(20 + j15\right) & 70 + j30 \end{bmatrix}\begin{bmatrix}\mathbf{I}_1 \\ \mathbf{I}_2\end{bmatrix} = \begin{bmatrix}12\angle 0 \\ 0\end{bmatrix}$$

Solving, for example using MATLAB, gives

$$\begin{bmatrix}\mathbf{I}_1 \\ \mathbf{I}_2\end{bmatrix} = \begin{bmatrix}0.4240\angle -28.9^\circ \\ 0.1392\angle -15.2^\circ\end{bmatrix}$$

Back in the time domain, the mesh currents are

$$\begin{bmatrix}i_1 \\ i_2\end{bmatrix} = \begin{bmatrix}0.4240\,\cos\left(5t - 28.9^\circ\right) \\ 0.1392\,\cos\left(5t - 15.2^\circ\right)\end{bmatrix}\text{ A}$$

(checked using LNAP, 9/9/04)

P 11.9-11 Determine the coil voltages, v_1, v_2, v_3, and v_4, for the circuit shown in Figure P 11.9-11.

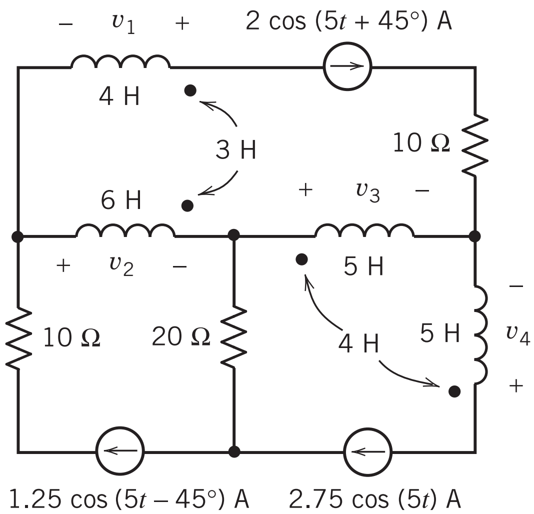
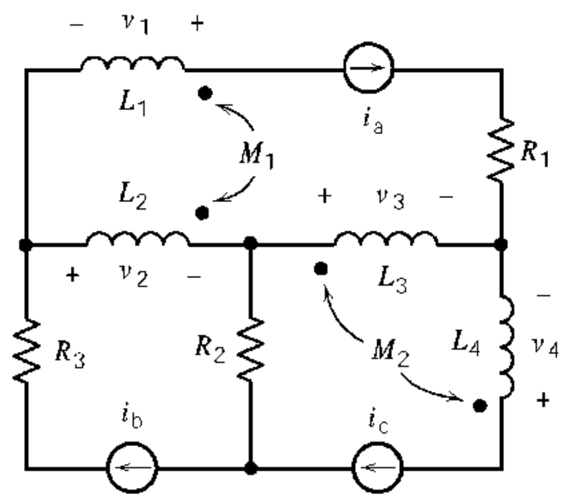


Figure P 11.9-11

Solution:

In the frequency domain, the coil voltages are given by



$$\begin{aligned}\mathbf{V}_1 &= -j\omega L_1\mathbf{I}_a + j\omega M_1\left(\mathbf{I}_a - \mathbf{I}_b\right)\\ &= j\omega\left(M_1 - L_1\right)\mathbf{I}_a - j\omega M_1\mathbf{I}_b\\ &= (-j5)(2\angle 45°) + (-j15)(1.25\angle -45°)\\ &= 21.25\angle -106.9° \text{ V}\end{aligned}$$

$$\begin{aligned}\mathbf{V}_2 &= j\omega L_2\left(\mathbf{I}_b - \mathbf{I}_a\right) + j\omega M_1\mathbf{I}_a\\ &= j\omega\left(M_1 - L_2\right)\mathbf{I}_a + j\omega L_2\mathbf{I}_b\\ &= (-j15)(2\angle 45°) + (j30)(1.25\angle -45°)\\ &= 48.02\angle 6.4° \text{ V}\end{aligned}$$

$$\begin{aligned}\mathbf{V}_3 = j\omega L_3\left(\mathbf{I}_c - \mathbf{I}_a\right) - j\omega M_2\mathbf{I}_c &= -j\omega L_2\mathbf{I}_a + j\omega\left(L_3 - M_2\right)\mathbf{I}_c\\ &= (-j25)(2\angle 45°) + (j5)(2.75\angle 0°) = 41.43\angle -31.4° \text{ V}\end{aligned}$$

$$\begin{aligned}\mathbf{V}_4 = -j\omega L_4\mathbf{I}_c + j\omega M_2\left(\mathbf{I}_c - \mathbf{I}_a\right) &= -j\omega M_2\mathbf{I}_a + j\omega\left(M_2 - L_4\right)\mathbf{I}_c\\ &= (-j20)(2\angle 45°) + (-j5)(2.75\angle 0°) = 50.66\angle -56.1° \text{ V}\end{aligned}$$

Back in the time domain, the coil voltages are

$$v_1 = 21.25\cos(5t - 106.9°) \text{ V},\; v_2 = 48.02\cos(5t + 6.3°) \text{ V},$$
$$v_3 = 41.43\cos(5t - 31.4°) \text{ V and } v_4 = 50.66\cos(5t - 56.1°) \text{ V}$$

(checked using LNAP, 9/9/04)

P 11.9-12 Figure P 11.9-12 shows three similar circuits. In each, the input to the circuit is the voltage of the voltage source, $v_s(t)$. The output is the voltage across the right-hand coil, $v_o(t)$. Determine the steady-state output voltage, $v_o(t)$, for each of the three circuits.

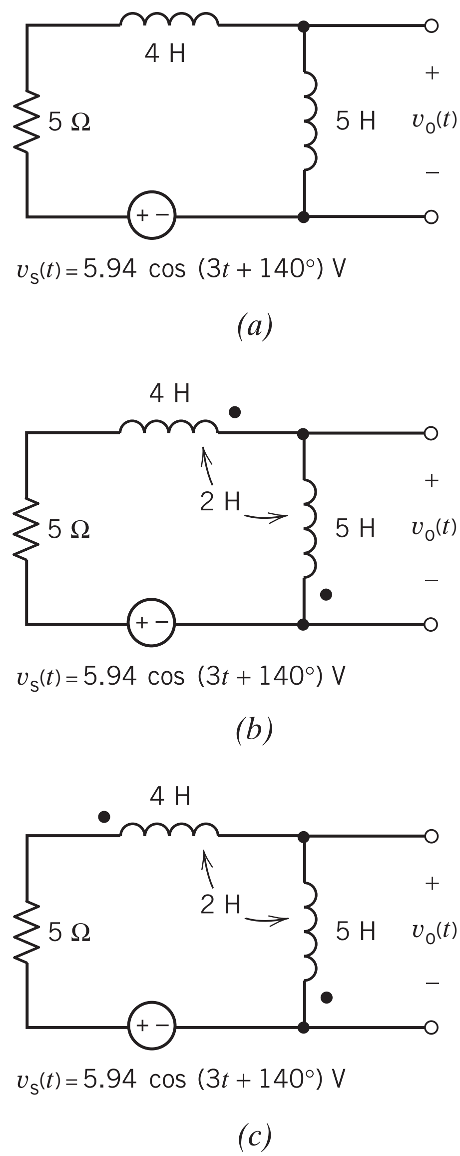
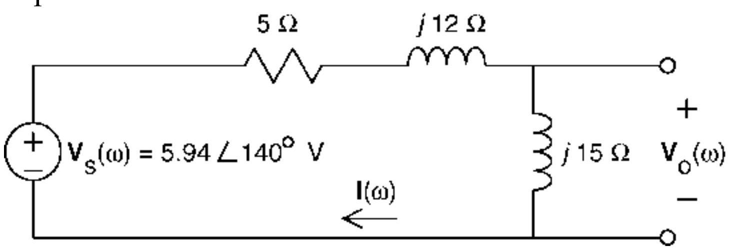


Figure P 11.9-12

Solution:

(*a*) In (*b*) and (*c*) the coils are coupled by mutual inductance, but not in (*a*). This circuit can be represented in the frequency domain, using phasors and impedances.



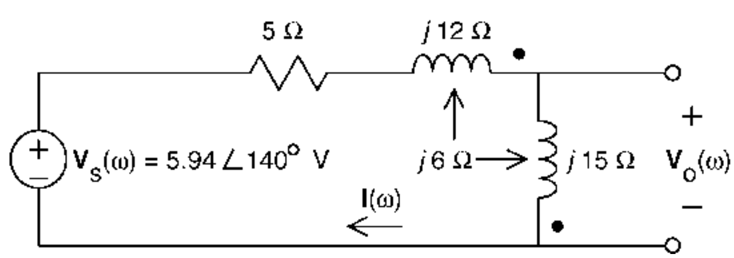
Voltage division gives

$$\mathbf{V}_o = \frac{j15}{5 + j12 + j15} 5.94\angle 140°$$
$$= (0.5463\angle 10.5°)(5.94\angle 140°) = 3.245\angle 150.5° \text{ V}$$

In the time domain, the output voltage is given by

$$v_o(t) = 3.245 \cos(3t + 150.5°) \text{ V}$$

(*b*) The input voltage is a sinusoid and the circuit is at steady state. The output voltage is also a sinusoid and has the same frequency as the input voltage. Consequently, the circuit can be represented in the frequency domain, using phasors and impedances.



This circuit of a single mesh. Notice that the mesh current, **I**(ω), enters the undotted ends of both coils. Apply KVL to the mesh to get

$$5\,\mathbf{I}(\omega)+\left(j12\,\mathbf{I}(\omega)+j6\,\mathbf{I}(\omega)\right)+\left(j6\,\mathbf{I}(\omega)+j15\,\mathbf{I}(\omega)\right)-5.94\angle 140^\circ=0$$

$$5\,\mathbf{I}(\omega)+\left(j12+j6+j6+j15\right)\mathbf{I}(\omega)-5.94\angle 140^\circ=0$$

$$\mathbf{I}(\omega)=\frac{5.94\angle 140^\circ}{5+j\left(12+6+6+15\right)}=\frac{5.94\angle 140^\circ}{5+j39}=\frac{5.94\angle 140^\circ}{39.3\angle 83}=0.151\angle 57^\circ\ \text{ A}$$

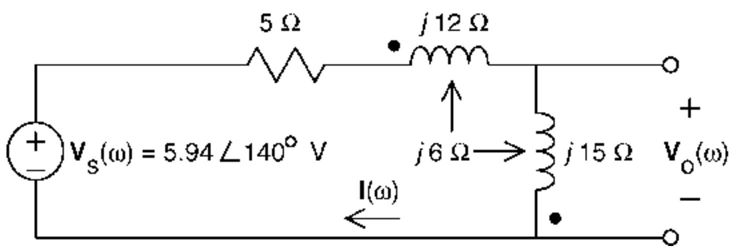
Notice that the voltage, $\mathbf{V_o}(\omega)$, across the right-hand coil and the mesh current, $\mathbf{I}(\omega)$, adhere to the passive convention. The voltage across the right-hand coil is given by

$$\begin{aligned}\mathbf{V_o}(\omega)=j\,15\,\mathbf{I}(\omega)+j\,6\,\mathbf{I}(\omega)=j\,21\,\mathbf{I}(\omega)&=j\,21\left(0.151\angle 57^\circ\right)\\&=(21\angle 90^\circ)\left(0.151\angle 57^\circ\right)\\&=3.17\angle 147^\circ\ \text{ V}\end{aligned}$$

In the time domain, the output voltage is given by

$$v_o(t)=3.17\cos\left(3t+147^\circ\right)\ \text{ V}$$

(*c*) Circuit (*c*) is very similar to the circuit (*b*). There is only one difference: the dot of the left-hand coil is located at the right of the coil in (*c*) and at the left of the coil in (*b*). As before, our first step is to represent the circuit in the frequency domain, using phasors and impedances.



This circuit consists of a single mesh. Notice that the mesh current, $\mathbf{I}(\omega)$, enters the dotted end of the left-hand coil and the undotted end of the right-hand coil. Apply KVL to the mesh to get

$$5\,\mathbf{I}(\omega)+\left(j12\,\mathbf{I}(\omega)-j6\,\mathbf{I}(\omega)\right)+\left(-j6\,\mathbf{I}(\omega)+j15\,\mathbf{I}(\omega)\right)-5.94\angle 140^\circ=0$$

$$5\,\mathbf{I}(\omega)+\left(j12-j6-j6+j15\right)\mathbf{I}(\omega)-5.94\angle 140^\circ=0$$

$$\mathbf{I}(\omega)=\frac{5.94\angle 140^\circ}{5+j\left(12-6-6+15\right)}=\frac{5.94\angle 140^\circ}{5+j15}=\frac{5.94\angle 140^\circ}{15.8\angle 71.6}=0.376\angle 68.4^\circ\ \text{ A}$$

Notice that the voltage, $\mathbf{V_o}(\omega)$, across the right-hand coil and the mesh current, $\mathbf{I}(\omega)$, adhere to the passive convention. The voltage across the right-hand coil is given by

$$\begin{aligned}\mathbf{V_o}(\omega)=j\,15\,\mathbf{I}(\omega)-j\,6\,\mathbf{I}(\omega)=j\,9\,\mathbf{I}(\omega)&=j\,9\left(0.376\angle 68.4^\circ\right)\\&=(9\angle 90^\circ)\left(0.376\angle 68.4^\circ\right)=3.38\angle 158.4^\circ\ \text{ V}\end{aligned}$$

In the time domain, the output voltage is given by

$$v_o(t)=3.38\cos\left(3t+158.4^\circ\right)\ \text{ V}$$

P 11.9-13 Figure P 11.9-13 shows three similar circuits. In each, the input to the circuit is the voltage of the voltage source,

$$v_s(t) = 5.7 \cos(4t + 158°) \text{ V}$$

The output in each circuit is the voltage across the right-hand coil, $v_o(t)$. Determine the steady-state output voltage, $v_o(t)$, for each of the three circuits.

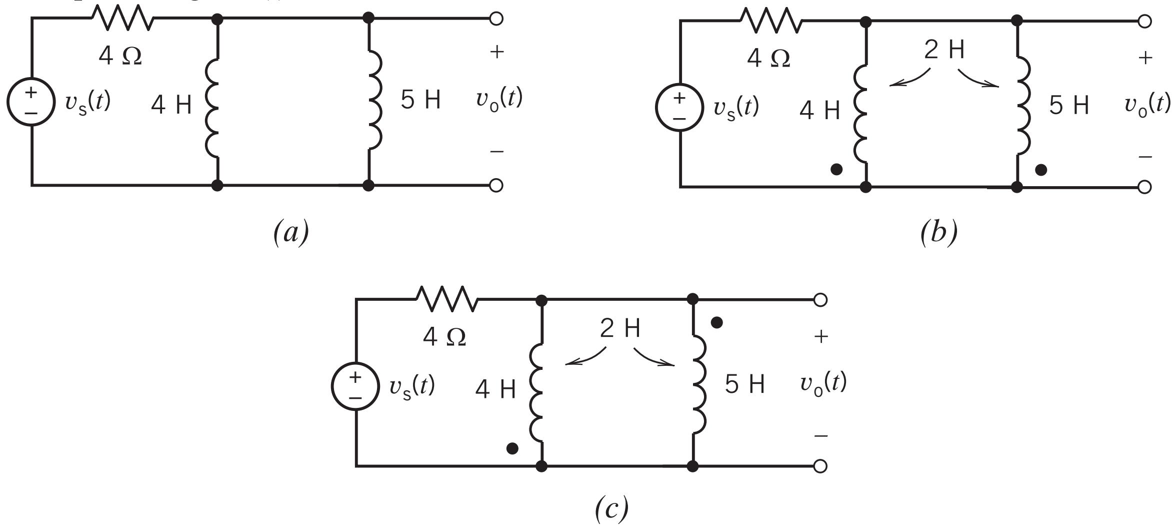
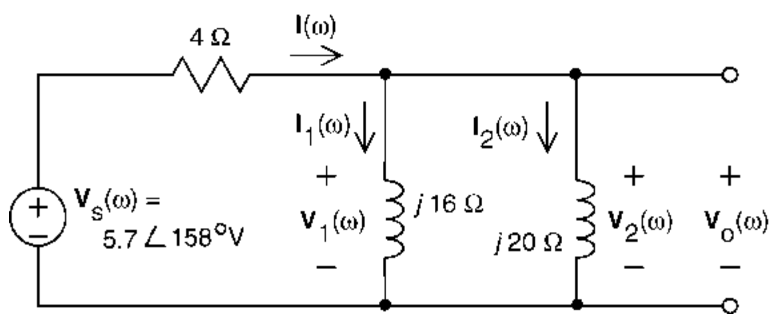


Figure P 11.9-13

Solution:

(*a*) In (*b*) and (*c*) the coils are coupled by mutual inductance, but not in (*a*). This circuit can be represented in the frequency domain, using phasors and impedances.



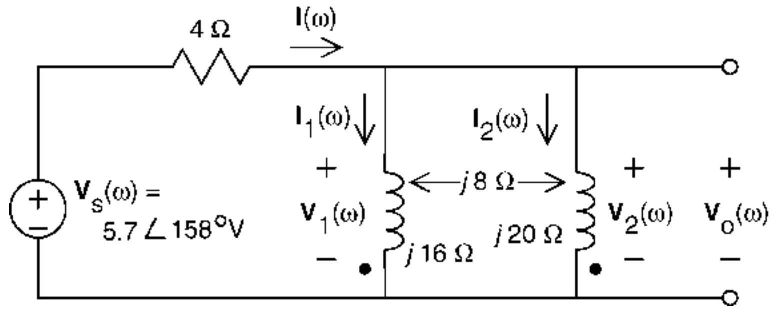
Voltage division gives

$$\mathbf{V}_o = \frac{j16 \,||\, j20}{4 + (j16 \,||\, j20)} 5.7\angle 158° = (0.9119\angle 24°)(5.7\angle 158°) = 5.198\angle 182° \text{ V}$$

In the time domain, the output voltage is given by

$$v_o(t) = 5.2 \cos(4t + 182°) \text{ V}$$

(*b*) The input voltage is a sinusoid and the circuit is at steady state. The output voltage is also a sinusoid and has the same frequency as the input voltage. Consequently, the circuit can be represented in the frequency domain, using phasors and impedances.



The coil currents, $\mathbf{I_1}(\omega)$ and $\mathbf{I_2}(\omega)$, and the coil voltages, $\mathbf{V_1}(\omega)$ and $\mathbf{V_2}(\omega)$, are labeled. Reference directions for these currents and voltages have been selected so that the current and voltage of each coil adhere to the passive convention. Notice that both coil currents, $\mathbf{I_1}(\omega)$ and $\mathbf{I_2}(\omega)$, enter the undotted ends of their respective coils. The device equations for coupled coils are:

$$\mathbf{V}_1(\omega) = j16\,\mathbf{I}_1(\omega) + j8\,\mathbf{I}_2(\omega) \qquad (1)$$

and

$$\mathbf{V}_2(\omega) = j8\,\mathbf{I}_1(\omega) + j20\,\mathbf{I}_2(\omega) \qquad (2)$$

The coils are connected in parallel, consequently $\mathbf{V}_1(\omega) = \mathbf{V}_2(\omega)$. Equating the expressions for $\mathbf{V_1}(\omega)$ and $\mathbf{V_2}(\omega)$ gives

$$j16\,\mathbf{I}_1(\omega) + j8\,\mathbf{I}_2(\omega) = j8\,\mathbf{I}_1(\omega) + j20\,\mathbf{I}_2(\omega)$$

$$j8\,\mathbf{I}_1(\omega) = j12\,\mathbf{I}_2(\omega)$$

$$\mathbf{I}_1(\omega) = \frac{3}{2}\mathbf{I}_2(\omega)$$

Apply Kirchhoff's Current Law (KCL) to the top node of the coils to get

$$\mathbf{I}(\omega) = \mathbf{I}_1(\omega) + \mathbf{I}_2(\omega) = \frac{3}{2}\mathbf{I}_2(\omega) + \mathbf{I}_2(\omega) = \frac{5}{2}\mathbf{I}_2(\omega)$$

Therefore

$$\mathbf{I}_1(\omega) = \frac{3}{5}\mathbf{I}(\omega) \quad \text{and} \quad \mathbf{I}_2(\omega) = \frac{2}{5}\mathbf{I}(\omega) \qquad (3)$$

Substituting the expressions for $\mathbf{I_1}(\omega)$ and $\mathbf{I_2}(\omega)$ from Equation 3 into Equation 1 gives

$$\mathbf{V}_1(\omega) = j16\left(\frac{3}{5}\mathbf{I}(\omega)\right) + j8\left(\frac{2}{5}\mathbf{I}(\omega)\right) = j\,\frac{16(3)+8(2)}{5}\mathbf{I}(\omega) = j\,12.8\,\mathbf{I}(\omega) \qquad (4)$$

Apply KVL to the mesh consisting of the voltage source, resistor and left-hand coil to get

$$4\,\mathbf{I}(\omega) + \mathbf{V}_1(\omega) - 5.7\angle 158° = 0$$

Using Equation 4 gives $\quad 4\,\mathbf{I}(\omega) + j\,12.8\,\mathbf{I}(\omega) - 5.7\angle 158° = 0$

Solving for **I** (ω) gives $\quad \mathbf{I}(\omega) = \dfrac{5.7\angle 158°}{4 + j\,12.8} = \dfrac{5.7\angle 158°}{13.41\angle 73°} = 0.425\angle 85° \text{ A}$

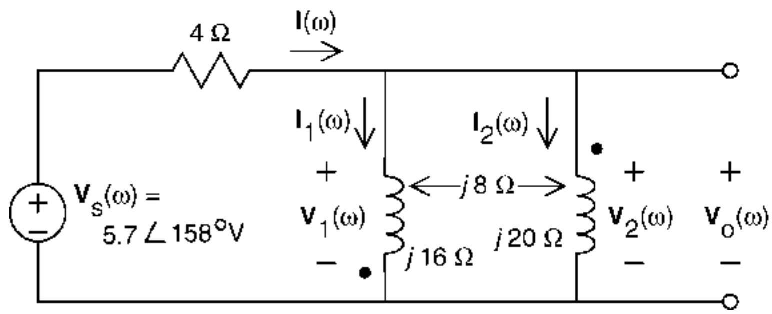
Now the output voltage can be calculated using Equation 4:

$$\mathbf{V_o}(\omega) = \mathbf{V_1}(\omega) = j12.8\,\mathbf{I}(\omega) = j12.8\,(0.425\angle 85°)$$
$$= (12.8\angle 90°)(0.425\angle 85°) = 5.44\angle 175° \text{ V}$$

In the time domain, the output voltage is given by

$$v_o(t) = 5.44\cos(4t + 175°) \text{ V}$$

(*c*) Circuit (*c*) is very similar to the circuit (*b*). There is only one difference: the dot of the right-hand coil is located at the bottom of the coil in (*b*) and at the top of the coil in (*c*). Here is the circuit from (*c*) represented in the frequency domain, using impedances and phasors.



The coil currents, $\mathbf{I_1}(\omega)$ and $\mathbf{I_2}(\omega)$, and the coil voltages, $\mathbf{V_1}(\omega)$ and $\mathbf{V_2}(\omega)$, are labeled. Reference directions for these currents and voltages have been selected so that the current and voltage of each coil adhere to the passive convention. Notice that one of coil currents, $\mathbf{I_1}(\omega)$, enters the undotted end of the coil while the other coil current, $\mathbf{I_2}(\omega)$, enters the dotted end of the coil.

The device equations for coupled coils are:

$$\mathbf{V_1}(\omega) = j16\,\mathbf{I_1}(\omega) - j8\,\mathbf{I_2}(\omega) \tag{5}$$

and

$$\mathbf{V_2}(\omega) = -j8\,\mathbf{I_1}(\omega) + j20\,\mathbf{I_2}(\omega) \tag{6}$$

The coils are connected in parallel, consequently $\mathbf{V_1}(\omega) = \mathbf{V_2}(\omega)$. Equating the expressions for $\mathbf{V_1}(\omega)$ and $\mathbf{V_2}(\omega)$gives

$$j16\,\mathbf{I_1}(\omega) - j8\,\mathbf{I_2}(\omega) = -j8\,\mathbf{I_1}(\omega) + j20\,\mathbf{I_2}(\omega)$$

$$j24\,\mathbf{I_1}(\omega) = j28\,\mathbf{I_2}(\omega)$$

$$\mathbf{I_1}(\omega) = \frac{28}{24}\mathbf{I_2}(\omega) = \frac{7}{6}\mathbf{I_2}(\omega)$$

Apply Kirchhoff's Current Law (KCL) to the top node of the coils to get

$$\mathbf{I}(\omega) = \mathbf{I_1}(\omega) + \mathbf{I_2}(\omega) = \frac{7}{6}\mathbf{I_2}(\omega) + \mathbf{I_2}(\omega) = \frac{13}{6}\mathbf{I_2}(\omega)$$

Therefore

$$\mathbf{I_1}(\omega) = \frac{7}{13}\mathbf{I}(\omega) \quad \text{and} \quad \mathbf{I_2}(\omega) = \frac{6}{13}\mathbf{I}(\omega) \tag{7}$$

Substituting the expressions for $\mathbf{I_1}(\omega)$ and $\mathbf{I_2}(\omega)$ from Equation 7 into Equation 5 gives

$$\mathbf{V_1}(\omega) = j16\left(\frac{7}{13}\mathbf{I}(\omega)\right) - j8\left(\frac{6}{13}\mathbf{I}(\omega)\right) = j\frac{16(7)-8(6)}{13}\mathbf{I}(\omega) = j\,4.9\,\mathbf{I}(\omega) \qquad (8)$$

Apply KVL to the mesh consisting of the voltage source, resistor and left-hand coil to get

$$4\,\mathbf{I}(\omega) + \mathbf{V_1}(\omega) - 5.7\angle 158° = 0$$

Using Equation 8 gives $\quad 4\,\mathbf{I}(\omega) + j\,4.9\,\mathbf{I}(\omega) - 5.7\angle 158° = 0$

Solving for **I** (ω) gives

$$\mathbf{I}(\omega) = \frac{5.7\angle 158°}{4 + j\,4.9} = \frac{5.7\angle 158°}{6.325\angle 51°} = 0.901\angle 107° \text{ A}$$

Now the output voltage can be calculated using Equation 8:

$$\mathbf{V_o}(\omega) = \mathbf{V_1}(\omega) = j\,4.9\,\mathbf{I}(\omega) = j\,4.9\,(0.901\angle 107°)$$
$$= (4.9\angle 90°)(0.901\angle 107°) = 4.41\angle 197° \text{ V}$$

In the time domain, the output voltage is given by

$$v_o(t) = 4.41\cos(4t + 197°) \text{ V}$$

P11.9-14
The circuit shown in Figure 11.9-14 is represented in the time domain. Determine coil voltages v_1 and v_2.

Answer: $v_1 = 104.0 \cos(6t + 46.17°)$ V
and $v_2 = 100.6 \cos(6t + 63.43°)$ V.

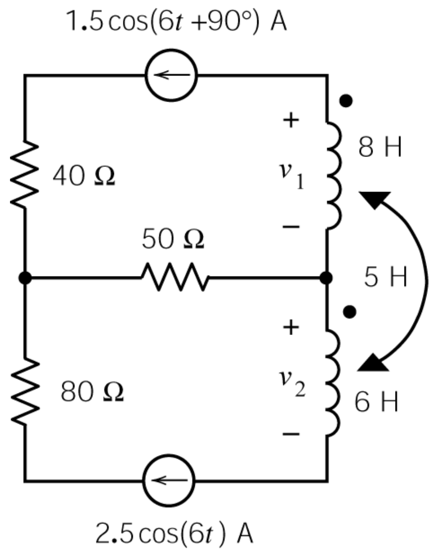
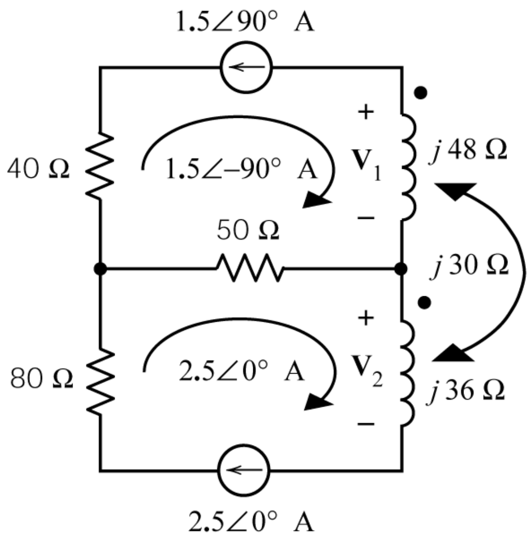


Figure 11.9-14

Solution:



Represent the circuit in the frequency domain as shown. Then

$$\mathbf{V}_1 = j\,48(1.5\angle-90°) + j30(2.5\angle 0°)$$
$$= j\,48(-j1.5) + j30(2.5) = 72 + j75$$
$$= 103.97\angle 46.17° \text{ V}$$

and

$$\mathbf{V}_2 = j\,36(2.5\angle 0°) + j30(1.5\angle-90°)$$
$$= j\,36(2.5) + j30(-j1.5) = 45 + j\,90$$
$$= 100.62\angle 63.43° \text{ V}$$

P11.9-15

The circuit shown in Figure P11.9-15 is represented in the frequency domain (e.g. $-j30\ \Omega$ is the impedance due to the mutual inductance of the coupled coils). Suppose $\mathbf{V}(\omega) = 70\angle 0°$ V. Then $\mathbf{I}_1(\omega) = B\angle\theta$ A and $\mathbf{I}_2(\omega) = 0.875\angle -90°$ A. Determine the values of B and θ.

Answer: B = 1.75 A and θ = –90 °

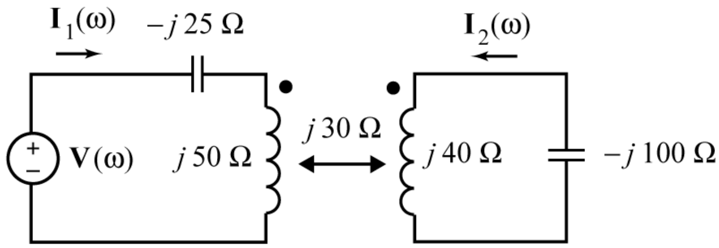
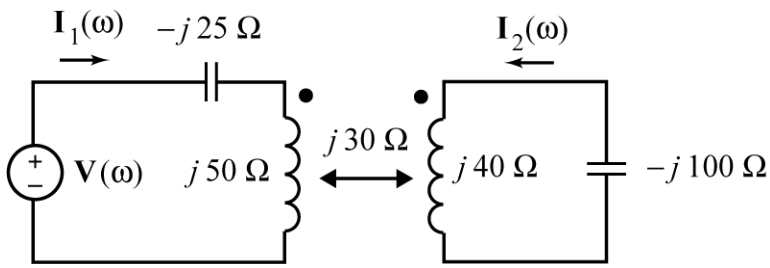


Figure 11.9-15

Solution:



Suppose $\mathbf{V}(\omega) = 70\angle 0°$ V. Then $\mathbf{I}_1(\omega) = B\angle\theta$ A and $\mathbf{I}_2(\omega) = 0.875\angle -90°$ A.

$$\mathbf{V} = -j\,25\,\mathbf{I}_1 + j\,50\,\mathbf{I}_1 + j\,30\,\mathbf{I}_2 = j\,25\,\mathbf{I}_1 + j\,30\,\mathbf{I}_2 = j\,25\,\mathbf{I}_1 + j\,30(-j\,0.875)$$

$$\mathbf{I}_1 = \frac{\mathbf{V} - j\,30(-j\,0.875)}{j\,25} = \frac{\mathbf{V} - 26.25}{j\,25} = \frac{70 - 26.25}{j\,25} = \frac{43.75}{j\,25} = -j1.75$$

P11.9-16

Determine the values of the inductances L_1 and L_2 in the circuit shown in Figure P11.9-16, given that

$$i(t) = 0.319\cos(4t - 82.23°)\ \text{A}$$

and

$$v(t) = 0.9285\cos(4t - 62.20°)\ \text{V}.$$

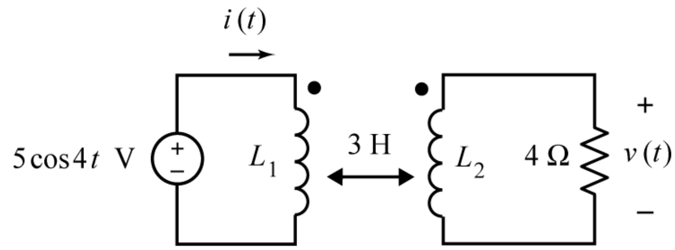
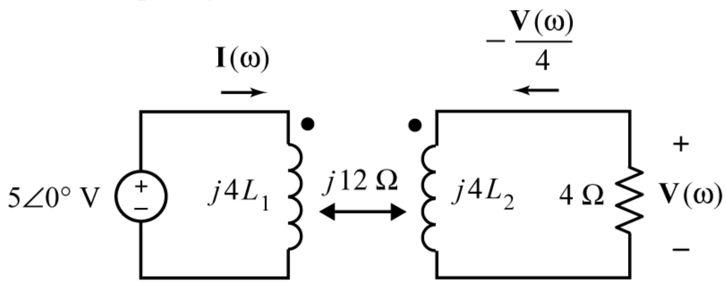


Figure P11.9-16

Solution:

Represent the circuit in the frequency domain as



Apply KVL to the left mesh to get

$$5\angle 0° = j4L_1(0.319\angle -82.23°) + j12\left(-\frac{0.9285\angle -62.20°}{4}\right) \Rightarrow L_1 = \frac{5\angle 0° + j3(0.9285\angle -62.20°)}{j4(0.319\angle -82.23°)}$$

$$= 6.0004 - j0.0005$$

$$\cong 6\ \text{H}$$

Apply KVL to the left mesh to get

$$0.9285\angle -62.20° = j4L_2\left(-\frac{0.9285\angle -62.20°}{4}\right) + j12(0.319\angle -82.23°)$$

$$L_2 = \frac{(0.9285\angle -62.20°) - j12(0.319\angle -82.23°)}{j4\left(-\frac{0.9285\angle -62.20°}{4}\right)}$$

$$= 3.9998 + j0.0005$$

$$\cong 4\ \text{H}$$

P11.9-17 Determine the complex power supplied by the source in the circuit shown in Figure P11.9-17.

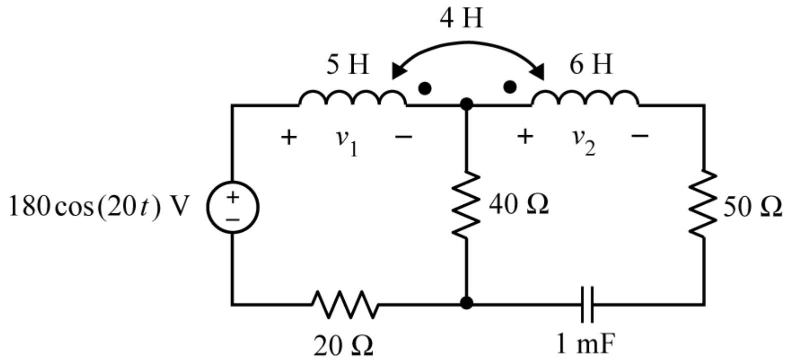
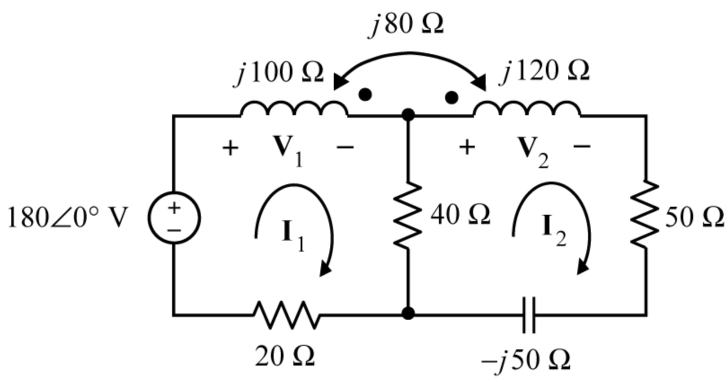


Figure P11.9-17

Solution: Represent the circuit in the frequency domain as



The coil voltages are given by

$$\mathbf{V}_1 = j100\,\mathbf{I}_1 - j80\,\mathbf{I}_2 \quad \text{and} \quad \mathbf{V}_2 = j120\,\mathbf{I}_2 - j80\,\mathbf{I}_1$$

Using KVL

$$\mathbf{V}_1 + 40(\mathbf{I}_1 - \mathbf{I}_2) + 20\,\mathbf{I}_1 - 180\angle 0° = 0$$

$$\mathbf{V}_2 + 50\,\mathbf{I}_2 - j50\,\mathbf{I}_2 - 40(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

Substituting the coil voltages:

$$(j100\,\mathbf{I}_1 - j80\,\mathbf{I}_2) + 40(\mathbf{I}_1 - \mathbf{I}_2) + 20\,\mathbf{I}_1 = 180\angle 0°$$

$$(j120\,\mathbf{I}_2 - j80\,\mathbf{I}_1) + 50\,\mathbf{I}_2 - j50\,\mathbf{I}_2 - 40(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

Simplifying

$$(60 + j100)\,\mathbf{I}_1 - (40 + j80)\,\mathbf{I}_2 = 180\angle 0°$$

$$-(40 + j80)\,\mathbf{I}_1 + (90 + j70\,\mathbf{I}_1)\,\mathbf{I}_2 = 0$$

Solving gives

$$\mathbf{I}_1 = 2.731\angle -26.9° \text{ A} \quad \text{and} \quad \mathbf{I}_2 = 2.142\angle -1.36° \text{ A}$$

The complex power delivered by the source is

$$\mathbf{S} = \frac{\mathbf{VI}^*}{2} = \frac{(180\angle 0°)(2.731\angle -26.9°)^*}{2} = 219.19 + j111.20 \text{ VA}$$

P11.9-18 The input to the circuit shown in Figure P11.9-18 is

$$v_s(t) = 12\cos(5t) \text{ V}$$

The impedance of the load is 20 + j 15 Ω. Determine the complex power (*a*) supplied by the source, (*b*) received by the 20 Ω resistor, (*c*) received by the coupled inductors and (d) received by the load.

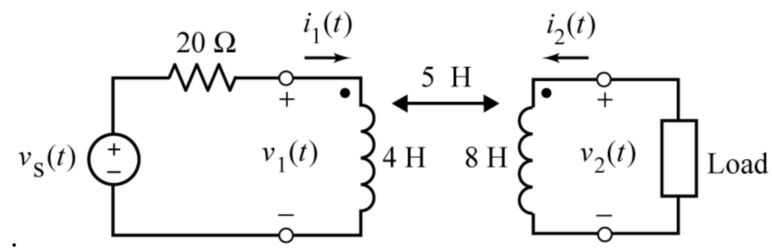
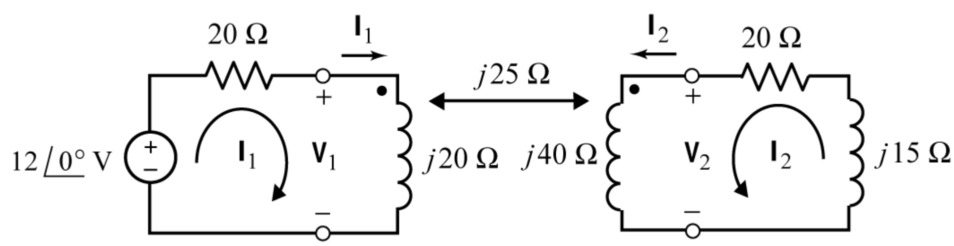


Figure P11.9-18

Solution:

Represent the circuit in the frequency domain as



The coil voltages are given by

$$\mathbf{V}_1 = j20\mathbf{I}_1 - j25\mathbf{I}_2 \quad \text{and} \quad \mathbf{V}_2 = j40\mathbf{I}_2 - j25\mathbf{I}_1$$

Using KVL

$$20\mathbf{I}_1 + \mathbf{V}_1 - 12\angle 45° = 0 \quad \text{and} \quad 20\mathbf{I}_2 + j15\mathbf{I}_2 + \mathbf{V}_2 = 0$$

Substituting the coil voltages:

$$20\mathbf{I}_1 + j20\mathbf{I}_1 - j25\mathbf{I}_2 = 12\angle 0°$$

$$20\mathbf{I}_2 + j15\mathbf{I}_2 + j40\mathbf{I}_2 - j25\mathbf{I}_1 = 0$$

Writhing these equations in matrix form

$$\begin{bmatrix} 20+j20 & -j25 \\ -j25 & 20+j55 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 12\angle 0° \\ 0 \end{bmatrix}$$

Solving gives $\mathbf{I}_1 = 0.4676\angle -22.8° \text{ A} \quad \text{and} \quad \mathbf{I}_2 = 0.1998\angle -2.86° \text{ A}$

(*a*) The complex power delivered by the source is

$$\mathbf{S} = \frac{(12\angle 0°)\mathbf{I}_1^*}{2} = \frac{(12\angle 0°)(0.4676\angle -22.8°)^*}{2} = 2.5855 + j1.0893 \text{ VA}$$

(*b*) The complex power received by the 20 Ω resistor is

$$\mathbf{S} = \frac{|\mathbf{I}_1|^2}{2}(20) = \frac{(0.4676)^2}{2}(20) = 2.1865 + j0 \text{ VA}$$

(*c*) The complex power received by the coupled inductors is

$$\mathbf{S} = \frac{\mathbf{V}_1\mathbf{I}_1{}^*}{2} + \frac{\mathbf{V}_2\mathbf{I}_2{}^*}{2} = 0 + j0.79 \text{ VA}$$

(*d*) The complex power received by the load is

$$\mathbf{S} = \frac{|\mathbf{I}_2|^2}{2}(20 + j15) = \frac{(0.1998)^2}{2}(20 + j15) = 0.399 + j0.299 \text{ VA}$$

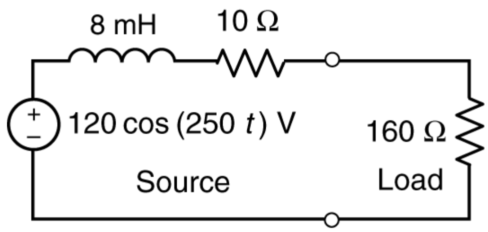
(checked using LNAP 2/6/09)

P11.9-19 Figure P11.9-18*a* shows a source connected to a 160 Ω load. In Figure P11.9-18*a* an ideal transformer and capacitor have been inserted between the source and the load.

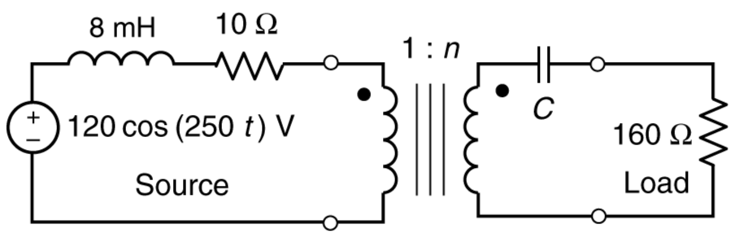
(a) Determine the value of the average power delivered to the 160 Ω load in Figure P11.9-18*a*.

(b) Determine the values of *n* and *C* Figure P11.9-18*b* that maximize the average power in the load.

(c) Using the values of *n* and *C* from part (b), determine the value of the average power delivered to the 160 Ω load in Figure P11.9-18*b*.



(*a*)



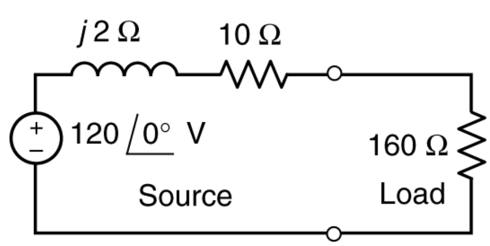
(*b*)

Figure P11.9-19

Solution:

(a)

Represent the circuit in the frequency domain. Using KVL

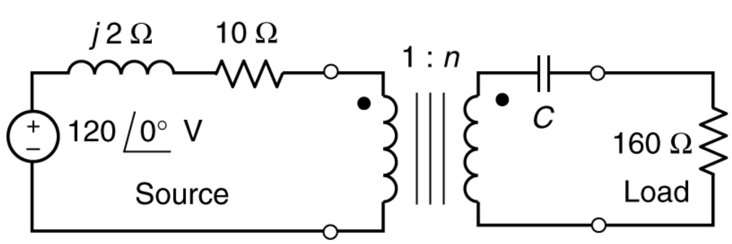


$$\mathbf{I} = \frac{120\angle 0°}{170 + j2} = 0.7058\angle -0.674° \text{ A}$$

and

$$P = \frac{0.7058^2}{2}(160) = 39.85 \text{ W}$$

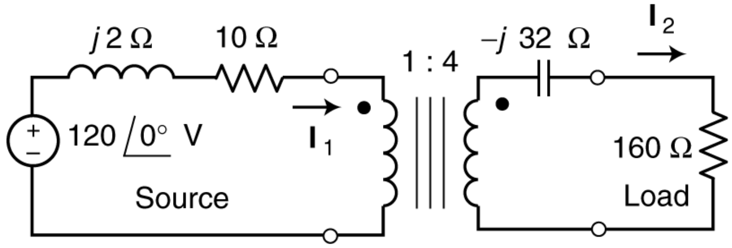
(b)



For maximum power transfer to the 160 Ω load

$$\frac{1}{n^2}(160)=10 \quad \Rightarrow n=4 \text{ and } -(j2)=\left(\frac{1}{4^2}\right)-j\frac{1}{250C} \quad \Rightarrow C=0.125 \text{ mF}$$

(c)
Represent the circuit in the frequency domain



$$\mathbf{I}_1=\frac{120\angle 0°}{10+j2+\left(\frac{1}{4}\right)^2(-j32+160)}=\frac{120\angle 0°}{20}=6\angle 0° \text{ A} \text{ and } \mathbf{I}_2=\frac{1}{4}\mathbf{I}_1=1.5\angle 0° \text{ A}$$

and
$$P=\frac{1.5^2}{2}(160)=180 \text{ W}$$

Section 11.10 The Ideal Transformer

P 11.10-1 Find $\mathbf{V}_1$, $\mathbf{V}_2$, $\mathbf{I}_1$, and $\mathbf{I}_2$ for the circuit of Figure P 11.10-1, when $n = 5$.

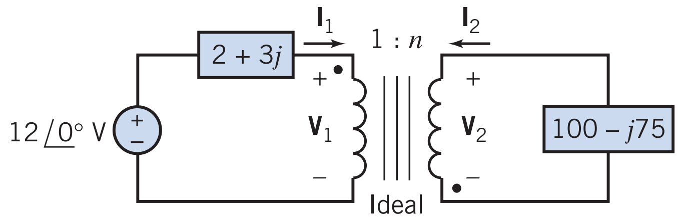
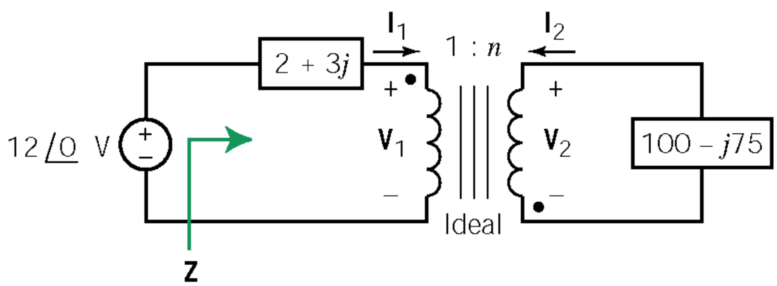


Figure P 11.10-1

Solution:



$$\mathbf{Z} = (2 + j3) + \frac{(100 - j75)}{5^2} = 6\ \Omega$$

$$\mathbf{I}_1 = \frac{12\angle 0°}{\mathbf{Z}} = \frac{12\angle 0°}{6} = 2\text{ A}$$

$$\mathbf{V}_1 = \mathbf{I}_1\left(\frac{100 - j75}{n^2}\right) = (2)\left(\frac{100 - j75}{25}\right) = 10\angle -36.9°\text{ V}$$

$$\mathbf{V}_2 = n\mathbf{V}_1 = 5\ (10\angle -36.9°) = 50\ \angle -36.9°\text{ V}$$

$$\mathbf{I}_2 = \frac{\mathbf{I}_1}{n} = \frac{2}{5}\ \text{ A}$$

P 11.10-2 A circuit with a transformer is shown in Figure P 11.10-2.

(a) Determine the turns ratio.

(b) Determine the value of R_{ab}.

(c) Determine the current supplied by the voltage source.

Answer: (a) n = 5

(b) $R_{ab} = 400\ \Omega$

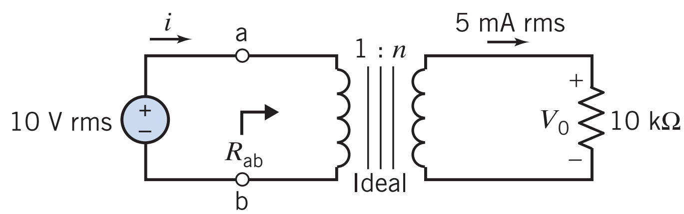


Figure P 11.10-2

Solution:

(a)

$$\mathbf{V}_0 = (5\times10^{-3})(10{,}000) = 50\ \text{V}$$

$$n = \frac{N_2}{N_1} = \frac{\mathbf{V}_0}{\mathbf{V}_1} = \frac{50}{10} = 5$$

(b)

$$R_{ab} = \frac{1}{n^2}R_2 = \frac{1}{25}(10\times10^3) = 400\ \Omega$$

(c)

$$\mathbf{I}_s = \frac{10}{R_{ab}} = \frac{10}{400} = 0.025\ \text{A} = 25\ \text{mA}$$

P 11.10-3 Find the voltage $\mathbf{V}_c$ in the circuit shown in Figure P 11.10-3. Assume an ideal transformer. The turns ratio is $n = 1/3$.

Answer: $\mathbf{V}_c = 21.0\angle -105.3°$

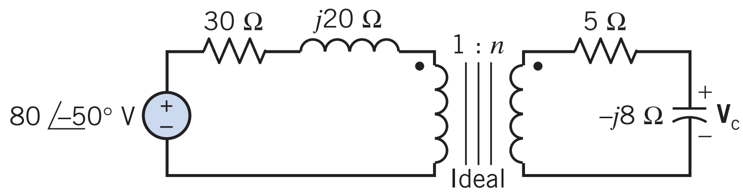
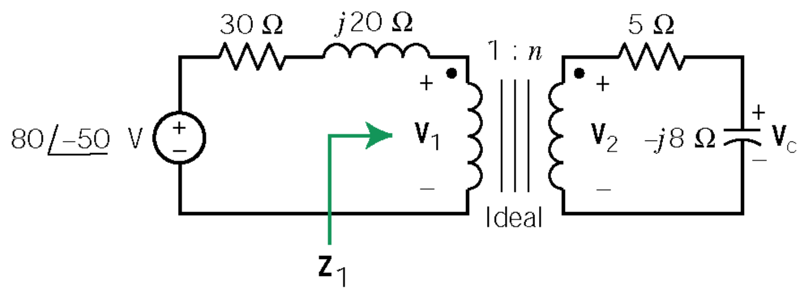


Figure P 11.10-3

Solution:



$$\mathbf{Z}_1 = \frac{1}{n^2}\mathbf{Z}_2 = 9\,\mathbf{Z}_2 = 9(5 - j8) = 45 - j72\ \Omega$$

Using voltage division, the voltage across $\mathbf{Z}_1$ is

$$\mathbf{V}_1 = (80\angle -50°)\left(\frac{45 - j72}{45 - j72 + 30 + j20}\right) = 74.4\angle -73.3°\ \text{V}$$

Then
$$\mathbf{V}_2 = n\mathbf{V}_1 = \frac{74.4\ \angle -73.3°}{3} = 24.8\ \angle -73.3°\ \text{V}$$

Using voltage division again yields

$$\mathbf{V}_c = \mathbf{V}_2\left(\frac{-j8}{5 - j8}\right) = (24.8\angle -73.3°)\left(\frac{8\angle -90°}{\sqrt{89}\angle -58°}\right) = 21.0\angle -105.3°\ \text{V}$$

P 11.10-4 An ideal transformer is connected in the circuit shown in Figure P 11.10-4, where $v_s = 50 \cos 1000t$ V and $n = N_2/N_1 = 5$. Calculate $\mathbf{V}_1$ and $\mathbf{V}_2$.

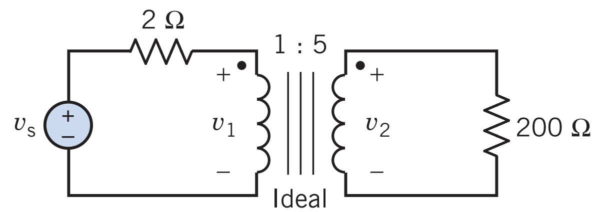


Figure P 11.10-4

Solution:

$$n = 5,\ \ \mathbf{Z}_1 = \frac{200}{(5)^2} = 8\ \Omega \quad \Rightarrow \quad \mathbf{V}_1 = \frac{8}{8+2}(50\angle 0°) = 40\angle 0°\ \text{V} \quad \Rightarrow \quad \mathbf{V}_2 = n\mathbf{V}_1 = 200\angle 0°\ \text{V}$$

P11.10-5 Figure P11.10-5 shows a load connected to a source through an ideal transformer. The input to the circuit is

$$v_s(t) = 12\cos(5t) \text{ V}$$

Determine

(a) The values of the turns ration, n, and load inductance, L, required for maximum power transfer to the load.
(b) The complex power delivered to the transformer by the source.
(c) The complex power delivered to the load by the transformer.

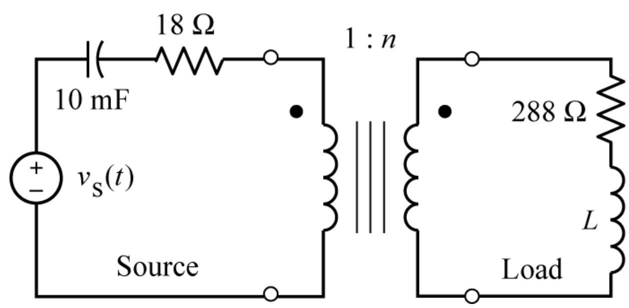


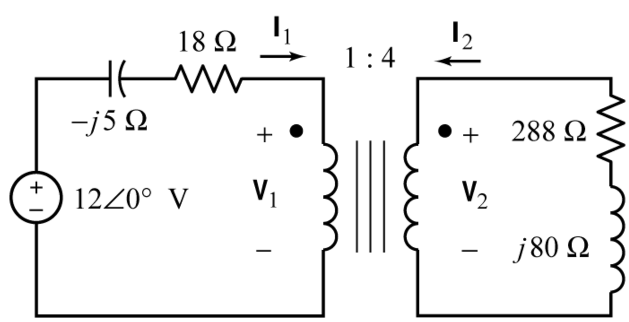
Figure P11.10-5

Solution:

(a) For maximum power transfer: $\frac{1}{n^2}(288 + j\,20\,L) = \left(18 - j\frac{1}{20 \times 0.01}\right)^* = 18 + j\,5$

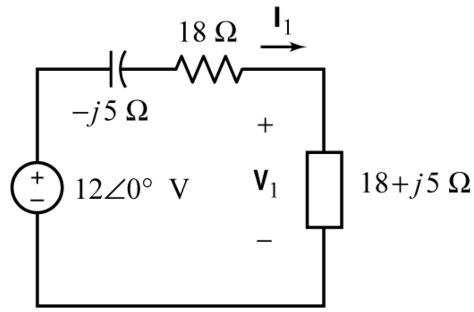
Equating real parts gives $n = \sqrt{\frac{288}{18}} = 4$. Equating imaginary parts gives $L = \frac{5(4^2)}{20} = 4$ H.

(*b*) Represent the circuit in the frequency domain as



Replace the transformer and load by an equivalent impedance

$$\mathbf{Z}_{\text{equiv}} = \frac{1}{4^2}(288 + j\,80) = 18 + j\,5 \ \ \Omega$$



$$\mathbf{I}_1 = \frac{12\angle 0^\circ}{(18 - j5) + (18 + j5)} = \frac{12\angle 0^\circ}{36} = \frac{1}{3}\angle 0^\circ \text{ A}$$

and

$$\mathbf{V}_1 = (18 + j5)\mathbf{I}_1 = (18 + j5)\left(\frac{1}{3}\angle 0^\circ\right) = 6.227\angle 15.5^\circ \text{ V}$$

The secondary coil current and voltages

$$\mathbf{I}_2 = -\frac{1}{4}\mathbf{I}_1 = -\frac{1}{4}\left(\frac{1}{3}\angle 0^\circ\right) = -\frac{1}{12}\angle 0^\circ = -0.0833\angle 0^\circ \text{ A}$$

and

$$\mathbf{V}_2 = \frac{4}{1}\mathbf{V}_1 = 24.91\angle 15.5^\circ \text{ V}$$

The complex power delivered to the transformer by the source.

$$\frac{\mathbf{V}_1\mathbf{I}_1^*}{2} = 1 + j0.277 \text{ VA} = \frac{|\mathbf{I}_1|^2}{2}(18 + j5)$$

(c) The complex power delivered to the load by the transformer is

$$-\frac{\mathbf{V}_2\mathbf{I}_2^*}{2} = 1 + j0.277 \text{ VA} = \frac{|\mathbf{I}_2|^2}{2}(288 + j80)$$

(checked using LNAP and MATLAB 2/6/09)

P 11.10-6 Find the Thévenin equivalent at terminals a–b for the circuit of Figure P 11.10-6 when $v = 16 \cos 3t$ V.
Answer: $V_{oc} = 12$ and $Z_t = 3.75\ \Omega$

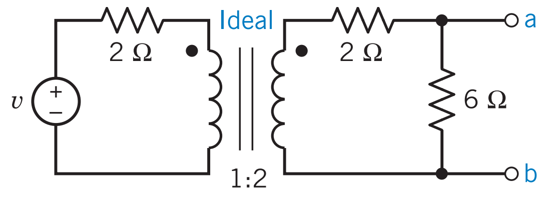
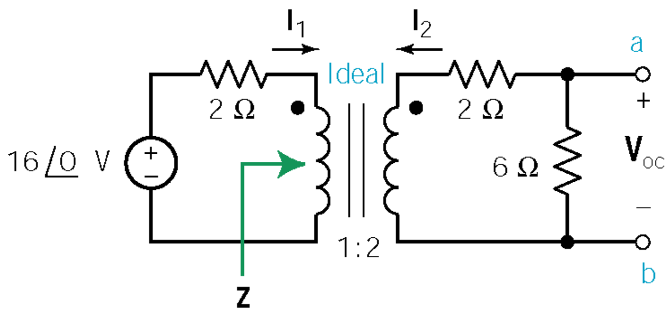


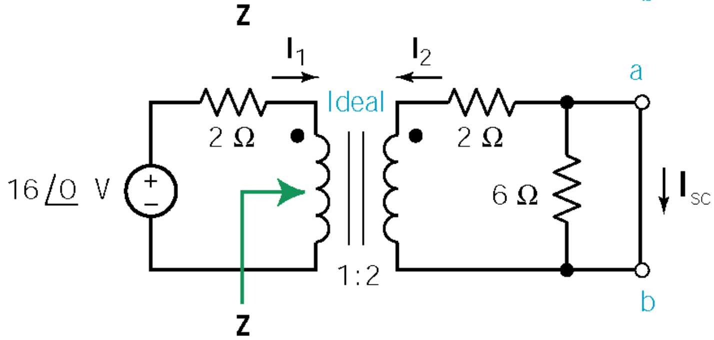
Figure P 11.10-6

Solution:



$$\mathbf{Z} = \frac{1}{2^2}(2+6) = 2\ \Omega$$

$$\mathbf{V}_{oc} = \left(\frac{6}{6+2}\right)(2)\left(\left(\frac{2}{2+2}\right)16\angle 0°\right) = 12\angle 0°\ \text{V}$$



$$\mathbf{Z} = \frac{1}{2^2}(2) = \frac{1}{2}\ \Omega$$

$$\mathbf{I}_{sc} = -\mathbf{I}_2 = \frac{1}{2}\mathbf{I}_1 = \frac{1}{2}\left(\frac{16\angle 0°}{2+\frac{1}{2}}\right) = 3.25\angle 0°\ \text{A}$$

Then

$$\mathbf{Z}_t = \frac{12\angle 0°}{3.2\angle 0°} = 3.75\angle 0°\ \Omega$$

P 11.10-7 Find the input impedance **Z** for the circuit of Figure P 11.10-7.
Answer: $\mathbf{Z} = 6\ \Omega$

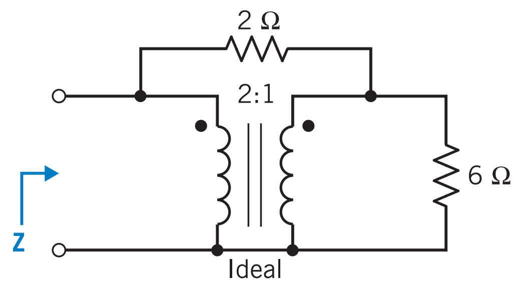
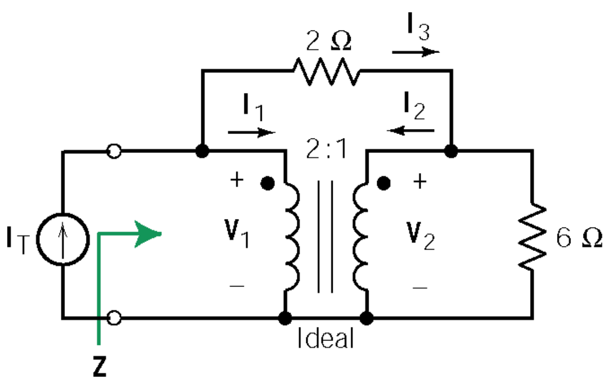


Figure P 11.10-7

Solution:



$$\mathbf{V}_2 = \frac{1}{2}\mathbf{V}_1$$

$$\mathbf{I}_3 = \frac{\mathbf{V}_1 - \mathbf{V}_2}{2} = \frac{\mathbf{V}_1}{4}$$

$$\mathbf{I}_2 = \mathbf{I}_3 - \frac{\mathbf{V}_2}{6} = \frac{\mathbf{V}_1}{6}$$

$$\mathbf{I}_1 = -\frac{1}{2}\mathbf{I}_2 = -\frac{\mathbf{V}_1}{12}$$

$$\mathbf{I}_T = \mathbf{I}_3 + \mathbf{I}_1 = \frac{\mathbf{V}_1}{6}$$

$$\mathbf{Z} = \frac{\mathbf{V}_1}{\mathbf{I}_T} = 6$$

P 11.10-8 In less developed regions in mountainous areas, small hydroelectric generators are used to serve several residences (Mackay, 1990). Assume each house uses an electric range and an electric refrigerator, as shown in Figure P 11.10-8. The generator is represented as $\mathbf{V}_s$ operating at 60 Hz and $\mathbf{V}_2 = 230\ \angle 0°$ V. Calculate the power consumed by each home connected to the hydroelectric generator when $n = 5$.

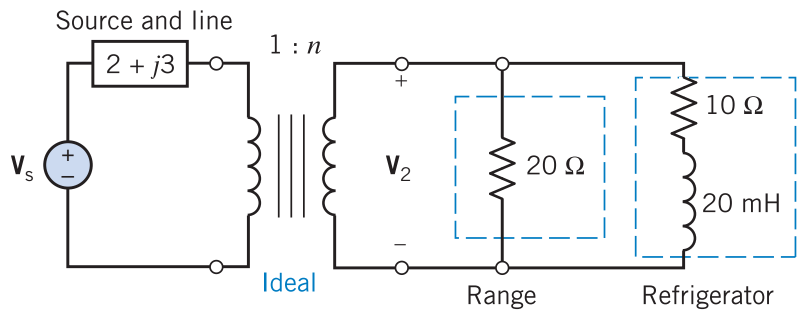
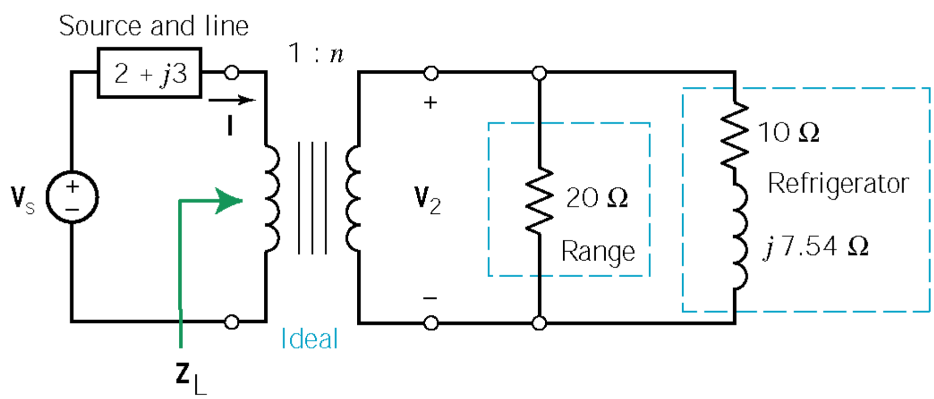


Figure P 11.10-8

Solution:



$$\mathbf{Z}_L = \frac{1}{5^2}\left(\frac{20(10+j\,7.54)}{20+10+j7.54}\right) = \frac{8.1\angle 23°}{25} = 0.3 + j0.13\ \Omega$$

$$P_L = \frac{|\mathbf{V}_L|^2}{2R_2} = \frac{|\mathbf{V}_2|^2}{2R_L} = \frac{(230)^2}{2(0.3)} = \ 88\ \text{kW/home}$$

Therefore, 529 kW are required for six homes.

P 11.10-9 Three similar circuits are shown in Figure P 11.10-9. In each of these circuits

$$v_s(t) = 5\cos(4t + 45°)\ \text{V}.$$

Determine $v_2(t)$ for each of the three circuits.

Answer:

(a) $v_2(t) = 0$ V

(b) $v_2(t) = 1.656\cos(4t + 39°)$ V

(c) $v_2(t) = 2.88\cos(4t + 45°)$ V

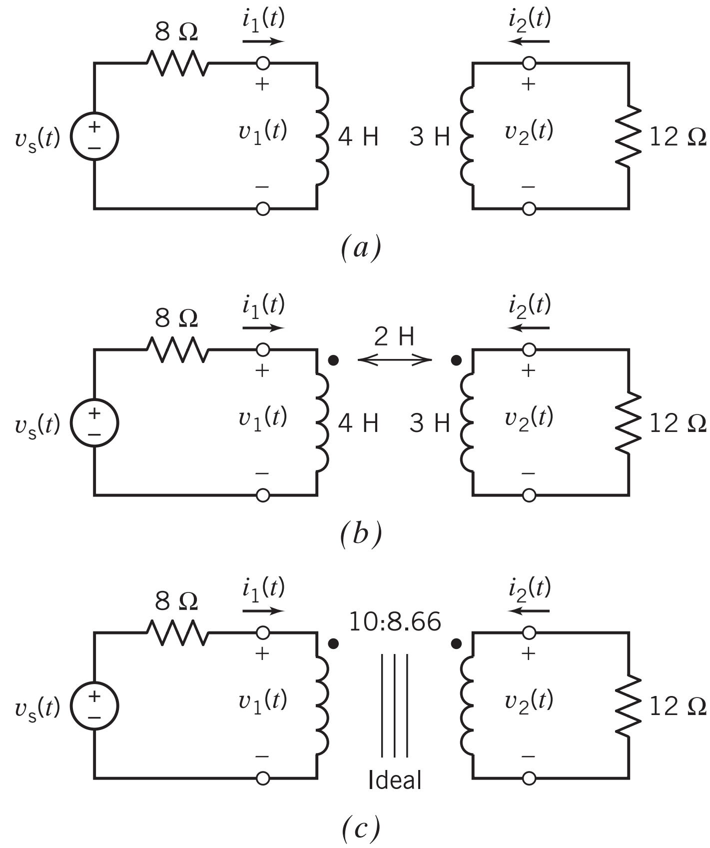
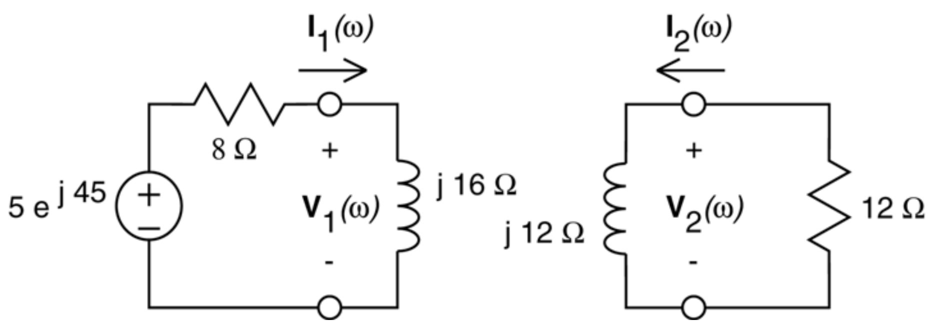


Figure P 11.10-9

Solution:

(a)



Coil voltages:

$$\mathbf{V_1} = j16\,\mathbf{I_1}$$
$$\mathbf{V_2} = j12\,\mathbf{I_2}$$

Mesh equations:

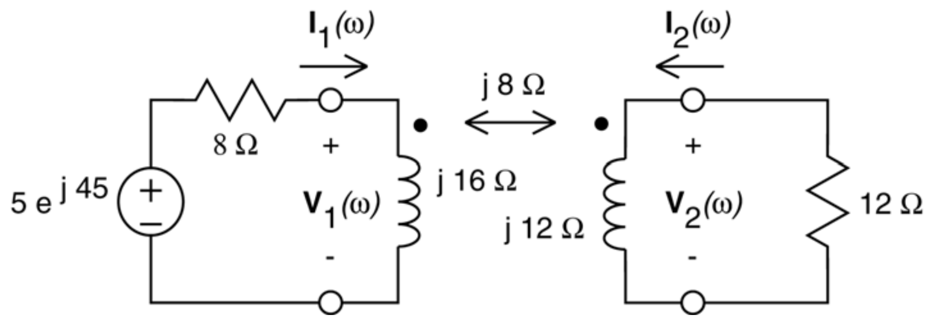
$$8\,\mathbf{I_1} + \mathbf{V_1} - 5\angle 45° = 0$$
$$-12\,\mathbf{I_2} - \mathbf{V_2} = 0$$

Substitute the coil voltages into the mesh equations and do some algebra:

$$8\,\mathbf{I}_1 + j16\,\mathbf{I}_1 = 5\angle 45^\circ \quad \Rightarrow \quad \mathbf{I}_1 = 0.28\angle -18.4^\circ$$
$$12\,\mathbf{I}_2 + j12\,\mathbf{I}_2 = 0 \quad \Rightarrow \quad \mathbf{I}_2 = 0$$

$$\mathbf{V}_2 = -12\,\mathbf{I}_2 = 0$$

(b)



Coil voltages:

$$\mathbf{V}_1 = j16\,\mathbf{I}_1 + j8\,\mathbf{I}_2$$
$$\mathbf{V}_2 = j12\,\mathbf{I}_2 + j8\,\mathbf{I}_1$$

Mesh equations:

$$8\,\mathbf{I}_1 + \mathbf{V}_1 - 5\angle 45^\circ = 0$$
$$-12\,\mathbf{I}_2 - \mathbf{V}_2 = 0$$

Substitute the coil voltages into the mesh equations and do some algebra:

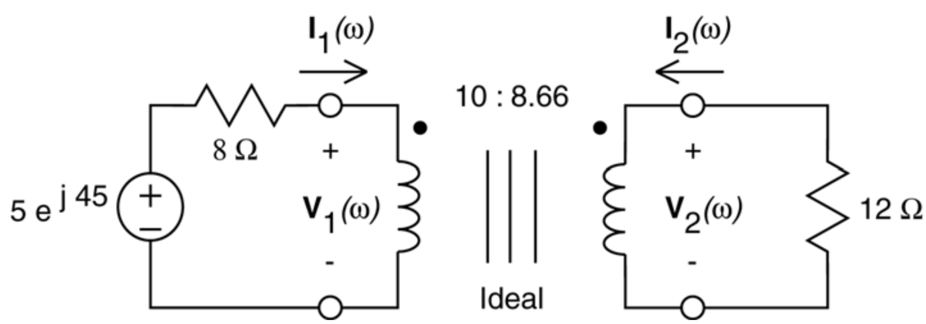
$$8\,\mathbf{I}_1 + \left(j16\,\mathbf{I}_1 + j8\,\mathbf{I}_2\right) = 5\angle 45^\circ$$
$$12\,\mathbf{I}_2 + (j12\,\mathbf{I}_2 + j8\,\mathbf{I}_1) = 0$$

$$\mathbf{I}_1 = -\frac{12 + j12}{j8}\,\mathbf{I}_2 = \frac{3}{2}\left(j-1\right)\mathbf{I}_2$$

$$\left[(8 + j16)\left(\frac{3}{2}\right)(j-1) + j8\right]\mathbf{I}_2 = 5\angle 45^\circ \quad \Rightarrow \quad \mathbf{I}_2 = 0.138\angle -141^\circ$$

$$\mathbf{V}_2 = -12\,\mathbf{I}_2 = 1.656\angle 39^\circ$$

(c)



Coil voltages and currents:

$$\mathbf{V_1} = \frac{10}{8.66}\mathbf{V}_2$$

$$\mathbf{I_1} = -\frac{8.66}{10}\mathbf{I}_2$$

Mesh equations:

$$8\,\mathbf{I_1} + \mathbf{V_1} - 5\angle 45° = 0$$

$$-12\,\mathbf{I_2} - \mathbf{V_2} = 0$$

Substitute into the second mesh equation and do some algebra:

$$-12\left(-\frac{10}{8.66}\mathbf{I_1}\right) = \frac{8.66}{10}\mathbf{V_1} \quad \Rightarrow \quad \mathbf{V_1} = 12\left(\frac{10}{8.66}\right)^2 \mathbf{I_1}$$

$$8\,\mathbf{I_1} + 12\left(\frac{10}{8.66}\right)^2 \mathbf{I_1} = 5\angle 45° \quad \Rightarrow \quad \mathbf{I_1} = 0.208\angle 45°$$

$$\mathbf{V_2} = -12\,\mathbf{I_2} = -12\left(-\frac{10}{8.66}\mathbf{I_1}\right) = \frac{12(10)}{8.66}\,0.208\angle 45° = 2.88\angle 45°$$

P 11.10-10 Find $\mathbf{V}_1$ and $\mathbf{I}_1$ for the circuit of Figure P 11.10-10 when $n = 5$.

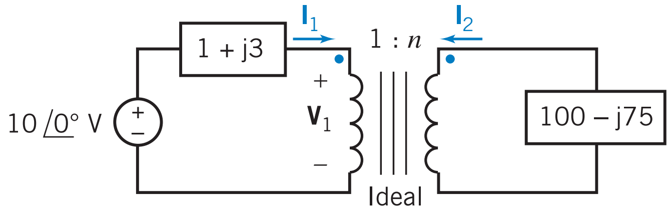
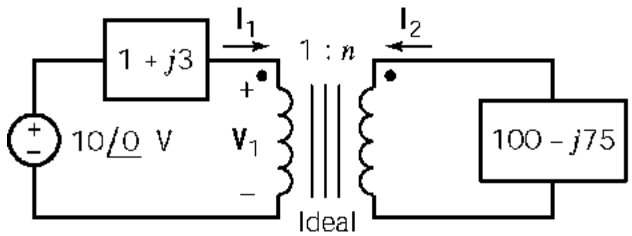
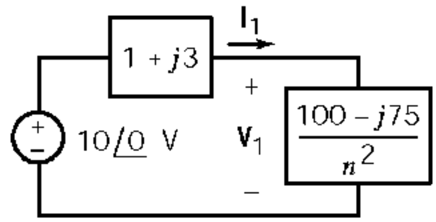


Figure P 11.10-10

Solution:





$$\mathbf{I}_1 = \frac{10\angle 0°}{(1+j3)+\dfrac{100-j75}{5^2}} = \frac{10\angle 0°}{(1+j3)+(4-j3)} = 2\angle 0° \text{ A}$$

$$\mathbf{V}_1 = (4-j3)\,2\angle 0° = 10\angle -36.9° \text{ V}$$

P 11.10-11 Determine v_2 and i_2 for the circuit shown in Figure P 11.10-11 when $n = 2$. Note that i_2 does not enter the dotted terminal.

Answer: $v_2 = 0.68 \cos (10t + 47.7°)$ V
$i_2 = 0.34 \cos (10t + 42°)$ A

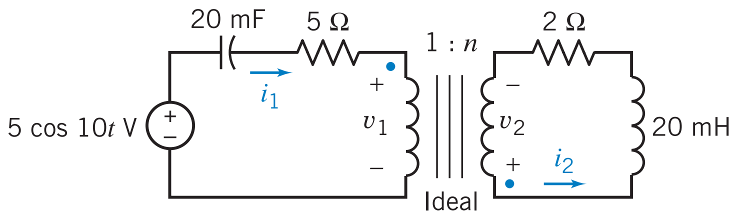
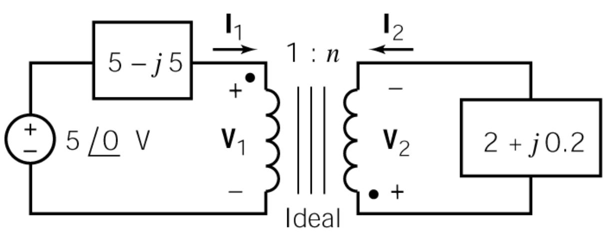
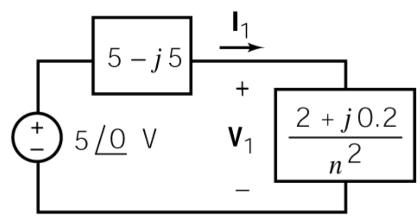


Figure P 11.10-11

Solution:





$$\mathbf{I}_1 = \frac{5\angle 0°}{(5-j5)+\dfrac{2+j0.2}{2^2}} = \frac{5\angle 0°}{5.5+j4.95} = \frac{5\angle 0°}{7.4\angle -42°} = 0.68\angle 42° \text{ A}$$

$$\mathbf{I}_2 = \frac{1}{2}\mathbf{I}_1 = 0.34\angle 42° \text{ A}$$

$$\mathbf{V}_2 = (2+j0.2)\mathbf{I}_2 = (2.01\angle 5.7°)(0.34\angle 42°) = 0.68\angle 47.7° \text{ V}$$

so

$$v_2(t) = 0.68\cos(10t+47.7°) \text{ V and } i_2(t) = 0.34\cos(10t+42°) \text{ A}$$

P11.10-12

The circuit shown in Figure P11.10-12 is represented in the frequency domain. Given the line current is $\mathbf{I}_{\text{Line}} = 0.5761\angle -75.88°\ \text{A}$, determine P_{Source}, the average power supplied by the source, P_{Line}, the average power delivered to the line and P_{Load}, the average power delivered to the load.

Hint: Use conservation of (average) power to check your answers.

Answer: $P_{\text{Source}} = 42.15$ W, $P_{\text{Line}} = 0.6638$ W and $P_{\text{Load}} = 41.49$ W.

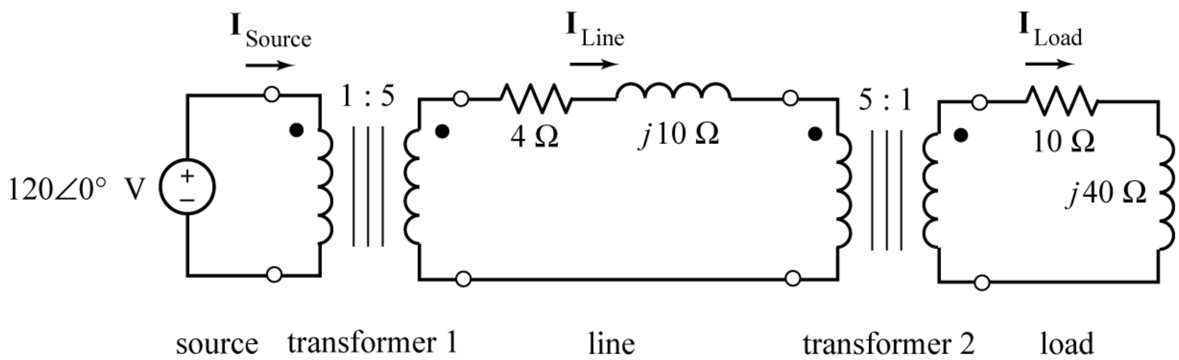


Figure P11.10-12

Solution:

$$\mathbf{I}_{\text{Line}} = \frac{\left(\frac{5}{1}\right)120\angle 0°}{4 + j10 + \left(\frac{5}{1}\right)^2 (10 + j40)} = 0.5761\angle -75.88°\ \text{A}$$

$$\mathbf{I}_{\text{Source}} = \left(\frac{5}{1}\right)\mathbf{I}_{\text{Line}} = 2.8805\angle -75.88°\ \text{A}$$

$$\mathbf{I}_{\text{Load}} = \left(\frac{5}{1}\right)\mathbf{I}_{\text{Line}} = 2.8805\angle -75.88°\ \text{A}$$

$$P_{\text{Source}} = \frac{(120)(2.8805)}{2}\cos\left(0 - (-75.88°)\right) = 42.16\ \text{W}$$

$$P_{\text{Line}} = \frac{0.5761^2}{2}(4) = 0.6638\ \text{W}$$

$$P_{\text{Load}} = \frac{2.8805^2}{2}(10) = 41.49\ \text{W}$$

P11.10-13

The circuit shown in Figure P11.10-13 is represented in the frequency domain. Determine R and X, the real and imaginary parts of the equivalent impedance, $\mathbf{Z}_{eq}$.

Answer: $R = 180\ \Omega$ and $X = 110\ \Omega$

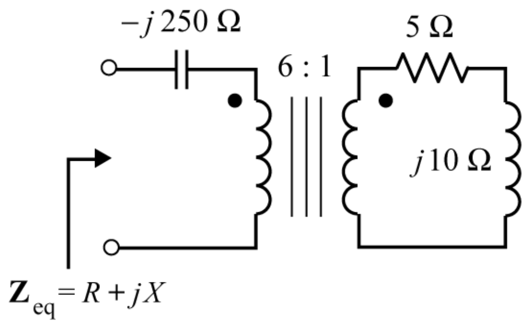
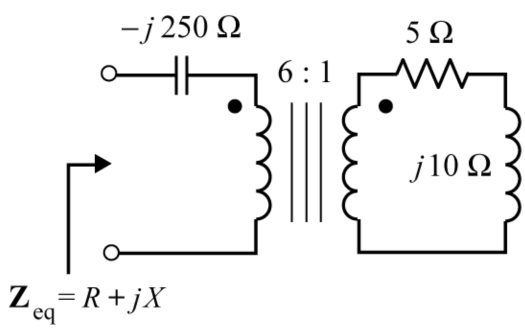


Figure P11.10-13

Solution:



$$\mathbf{Z}_{eq} = -j\,250 + \left(\frac{6}{1}\right)^2 (5 + j10) = 180 + j110\ \Omega$$

P11.10-14 Figure P11.10-14 shows a load connected to a source through an ideal transformer. Determine the complex power delivered to the transformer by the source.

Answer: $\mathbf{S} = 698.3 + j1745.7$ VA

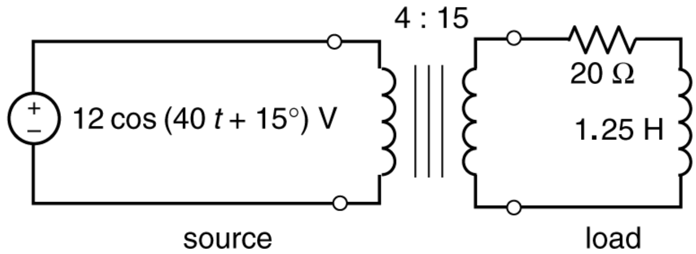
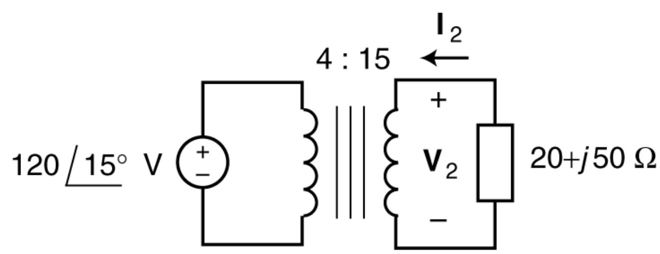


Figure P11.10-14

Solution:

Represent the circuit in the frequency domain. Using the element equation of the ideal transformer:



$$\mathbf{V}_2 = \frac{15}{4}\left(120\angle 15°\right) = 450\angle 15° \text{ V}$$

Using Ohm's law:

$$\mathbf{I}_2 = \frac{\mathbf{V}_2}{20 + j50} = \frac{450\angle 15°}{53.852\angle 68.2°} = 8.3563\angle -53.2°$$

The power supplied by the source is equal to the power received by the load. The power received by the load is calculated as

$$\mathbf{S} = \frac{\left|\mathbf{I}_2\right|^2}{2}\left(20 + j50\right) = \frac{8.3563^2}{2}\left(20 + j50\right) = 698.3 + j1745.7 \text{ VA}$$

(checked using LNAP and MATLAB 2/17/12)

Section 11.11 How Can We Check …?

P 11.11-1 Computer analysis of the circuit shown in Figure P 11.11-1 indicates that when

$$v_s(t) = 12 \cos(4t + 30°) \text{ V}$$

the mesh currents are given by

$$i_1(t) = 2.327 \cos(4t - 25.22°) \text{ A}$$

and

$$i_2(t) = 1.229 \cos(4t - 11.19°) \text{ A}$$

Check the results of this analysis by checking that the average power supplied by the voltage source is equal to the sum of the average powers received by the other circuit elements.

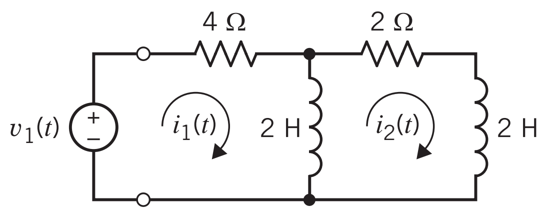


Figure P 11.11-1

P11.11-1

The average power supplied by the source is

$$P_s = \frac{(12)(2.327)}{2}\cos(30° - (-25.22°)) = 7.96 \text{ W}$$

Capacitors and inductors receive zero average power, so the average power supplied by the voltage source should be equal to the sum of the average powers received by the resistors:

$$P_R = \frac{2.327^2}{2}(4) + \frac{1.129^2}{2}(2) = 10.83 + 1.27 = 12.10 \text{ W}$$

The average power supplied by the voltage source is not equal to the sum of the average powers received by the other circuit elements. **The mesh currents cannot be correct.**

(What went wrong? It appears that the resistances of the two resistors were interchanged when the data was entered for the computer analysis. Notice that

$$P_R = \frac{2.327^2}{2}(2) + \frac{1.129^2}{2}(4) = 5.41 + 2.55 = 7.96 \text{ W}$$

The mesh currents would be correct if the resistances of the two resistors were interchanged. The computer was used to analyze the wrong circuit.)

P 11.11-2 Computer analysis of the circuit shown in Figure P 11.11-2 indicates that when

$$v_s(t) = 12 \cos(4t + 30°) \text{ V}$$

the mesh currents are given by

$$i_1(t) = 1.647 \cos(4t - 17.92°) \text{ A}$$

and $$i_2(t) = 1.094 \cos(4t - 13.15°) \text{ A}$$

Check the results of this analysis by checking that the complex power supplied by the voltage source is equal to the sum of the complex powers received by the other circuit elements.

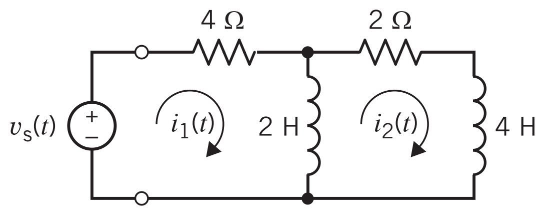


Figure P 11.11-2

Solution:

The average complex supplied by the source is

$$\mathbf{S}_s = \frac{(12\angle 30°)(1.647\angle -17.92°)^*}{2} = \frac{(12\angle 30°)(1.647\angle 17.92°)}{2} = 9.88\angle 47.92° = 6.62 + j\,7.33 \text{ W}$$

The complex power received by the 4 Ω resistor is

$$\mathbf{S}_{4\Omega} = \frac{(4\times 1.647\angle -17.92°)(1.647\angle -17.92°)^*}{2} = 5.43 + j\,0 \text{ VA}$$

The complex power received by the 2 Ω resistor is

$$\mathbf{S}_{2\Omega} = \frac{(2\times 1.094\angle -13.15°)(1.094\angle -13.15°)^*}{2} = 1.20 + j\,0 \text{ VA}$$

The current in the 2 H inductor is

$$(1.647\angle -17.92°) - (1.094\angle -13.15°) = 0.5640\angle -27.19°$$

The complex power received by the 2 H inductor is

$$\mathbf{S}_{2\text{H}} = \frac{(j\,8\times 0.5640\angle -27.19°)(0.5640\angle -27.19°)^*}{2} = 0 + j1.27 \text{ VA}$$

The complex power received by the 4 H inductor is

$$\mathbf{S}_{4\text{H}} = \frac{(j16\times 1.094\angle -13.15°)(1.094\angle -13.15°)^*}{2} = 0 + j\,9.57 \text{ VA}$$

$$\mathbf{S}_{4\Omega}+\mathbf{S}_{2\Omega}+\mathbf{S}_{2\text{H}}+\mathbf{S}_{4\text{H}}=\left(5.43+j\,0\right)+\left(1.20+j\,0\right)+\left(0+j1.27\right)+\left(0+j\,9.57\right)=6.63+j10.84\neq\mathbf{S}_{\text{s}}$$

The complex power supplied by the voltage source is not equal to the sum of the complex powers received by the other circuit elements. **The mesh currents cannot be correct.**

(Suppose the inductances of the inductors were interchanged. Then the complex power received by the 4 H inductor would be

$$\mathbf{S}_{4\text{H}}=\frac{\left(j16\times0.5640\angle-27.19^\circ\right)\left(0.5640\angle-27.19^\circ\right)*}{2}=0+j\,2.54\ \text{ VA}$$

The complex power received by the 2 H inductor would be

$$\mathbf{S}_{2\text{H}}=\frac{\left(j\,8\times1.094\angle-13.15^\circ\right)\left(1.094\angle-13.15^\circ\right)*}{2}=0+j\,4.79\ \text{ VA}$$

$$\mathbf{S}_{4\Omega}+\mathbf{S}_{2\Omega}+\mathbf{S}_{2\text{H}}+\mathbf{S}_{4\text{H}}=\left(5.43+j\,0\right)+\left(1.20+j\,0\right)+\left(0+j\,2.54\right)+\left(0+j\,4.79\right)=6.63+j\,7.33\approx\mathbf{S}_{\text{s}}$$

The mesh currents would be correct if the inductances of the two inductors were interchanged. The computer was used to analyze the wrong circuit.)

P 11.11-3 Computer analysis of the circuit shown in Figure P 11.11-3 indicates that when

$$v_s(t) = 12 \cos(4t + 30°) \text{ V}$$

the mesh currents are given by

$$i_1(t) = 1.001 \cos(4t - 47.01°) \text{ A}$$

and

$$i_2(t) = 0.4243 \cos(4t - 15.00°) \text{ A}$$

Check the results of this analysis by checking that the equations describing currents and voltages of coupled coils are satisfied.

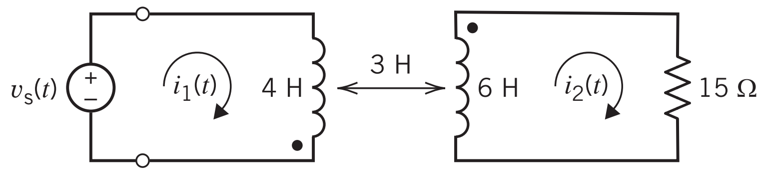


Figure P 11.11-3

Solution:

The voltage across the left coil must be equal to the voltage source voltage. Notice that the mesh currents both enter the undotted ends of the coils. In the frequency domain, the voltage across the left coil is

$$\begin{aligned}(j16)(1.001\angle -47.01°)+(j12)(0.4243\angle -15°) &= 16.016\angle 42.99° + 5.092\angle 75° \\ &= (11.715 + j10.923) + (1.318 + j\,4.918) \\ &= 13.033 + j15.841 = 20.513\angle 50.55°\end{aligned}$$

The voltage across the left coil isn't equal to the voltage source voltage so the computer analysis isn't correct.

What happened? A data entry error was made while doing the computer analysis. Both coils were described as having the dotted end at the top. If both coils had the dot at the top, the equation for the voltage across the right coil would be

$$\begin{aligned}(j16)(1.001\angle -47.01°)-(j12)(0.4243\angle -15°) &= 16.016\angle 42.99° - 5.092\angle 75° \\ &= (11.715 + j10.923) - (1.318 + j\,4.918) \\ &= 10.397 + j\,6.005 = 12.007\angle 30.01°\end{aligned}$$

This is equal to the voltage source voltage. The computer was used to analyze the wrong circuit.

P 11.11-4 Computer analysis of the circuit shown in Figure P 11.11-4 indicates that when

$$v_s(t) = 12\cos(4t + 30°)\text{ V}$$

the mesh currents are given by

$$i_1(t) = 25.6\cos(4t + 30°)\text{ mA}$$

and

$$i_2(t) = 64\cos(4t + 30°)\text{ mA}$$

Check the results of this analysis by checking that the equations describing currents and voltages of ideal transformers are satisfied.

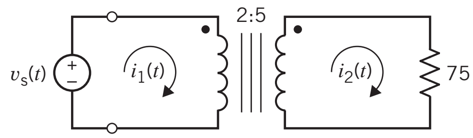


Figure P 11.11-4

Solution:
First check the ratio of the voltages across the coils.

$$\frac{12\angle 30°}{(75)(0.064\angle 30°)} = 2.5 \neq \frac{n_1}{n_2} = \frac{2}{5}$$

The transformer voltages don't satisfy the equations describing the ideal transformer. **The given mesh currents are not correct.**

That' enough but let's also check the ratio of coil currents. (Notice that the reference direction of the $i_2(t)$ is different from the reference direction that we used when discussing transformers.)

$$\frac{0.064\angle 30°}{0.0256\angle 30°} = 2.5 \neq \frac{n_1}{n_2} = \frac{2}{5}$$

The transformer currents don't satisfy the equations describing the ideal transformer.

In both case, we calculated $\frac{n_1}{n_2}$ to be 2.5 instead of $0.4 = \frac{1}{2.5}$. This suggests that a data entry error was made while doing the computer analysis. The numbers of turns for the two coils was interchanged.

PSpice Problems

SP 11-1 The input to the circuit shown in Figure SP 11-1 is the voltage of the voltage source,

$$v_s(t) = 7.5 \sin(5t + 15°) \text{ V}$$

The output is the voltage across the 4-Ω resistor, $v_o(t)$. Use PSpice to plot the input and output voltages.

Hint: Represent the voltage source using the PSpice part called VSIN.

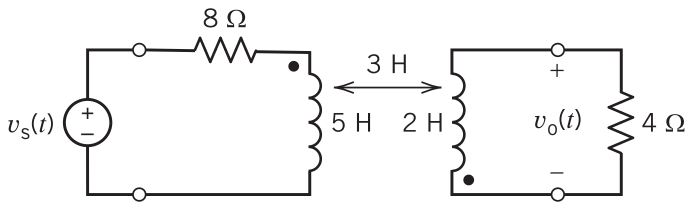
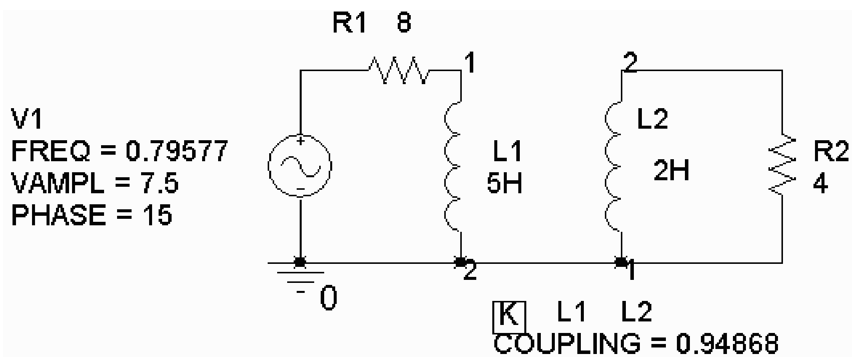
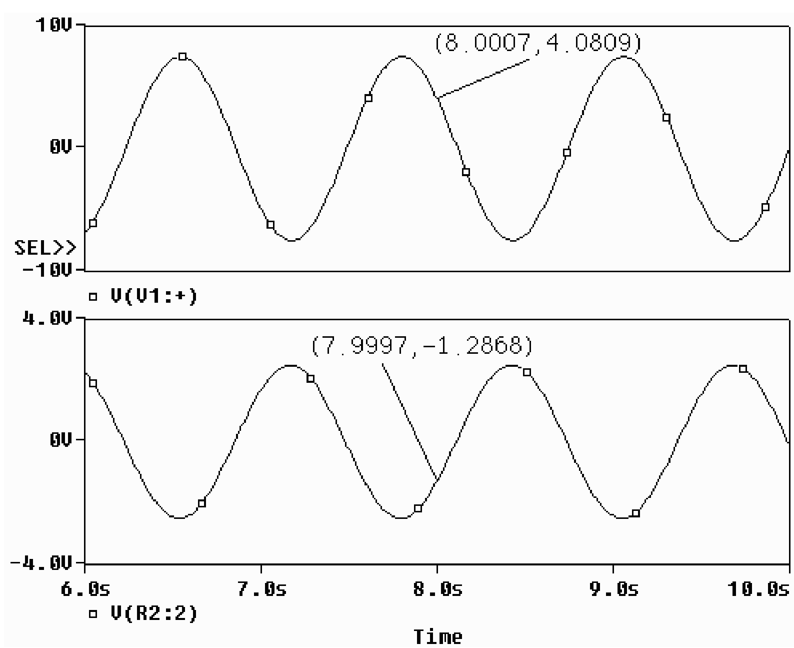


Figure SP 11-1

Solution:

The coupling coefficient is $k = \dfrac{3}{\sqrt{2 \times 5}} = 0.94868$.





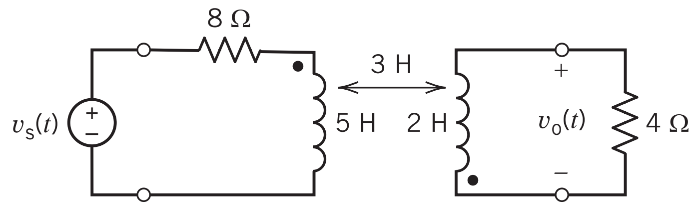


Figure SP 11-1

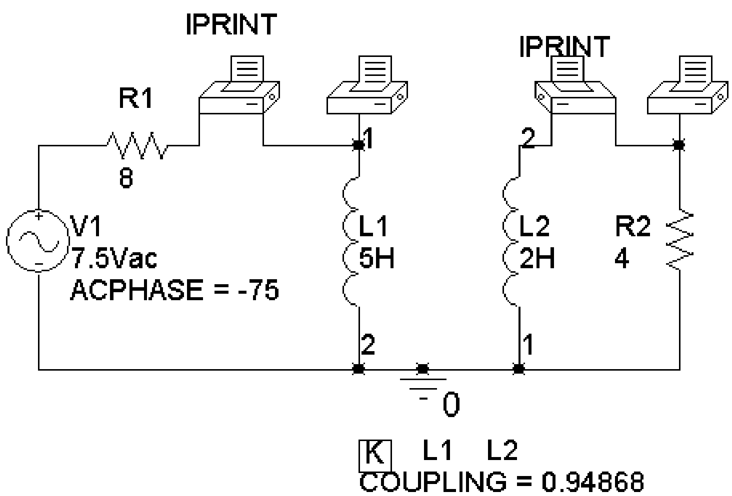
SP 11-2 The input to the circuit shown in Figure SP 11-1 is the voltage of the voltage source,

$$v_s(t) = 7.5 \sin(5t + 15°) = 7.5 \cos(5t - 75°) \text{ V}$$

The output is the voltage across the 4-Ω resistor, $v_o(t)$. Use PSpice to determine the average power delivered to the coupled inductors.

Hint: Represent the voltage source using the PSpice part called VAC. Use printers (PSpice parts called IPRINT and VPRINT) to measure the ac current and voltage of each coil.

Solution: Here is the circuit with printers inserted to measure the coil voltages and currents:



Here is the output from the printers, giving the voltage of coil 2 as 2.498∠107.2°, the current of coil 1 as 0.4484∠-94.57°, the current of coil 2 as 0.6245∠-72.77° and the voltage of coil 1 as 4.292∠-58.74°:

```
FREQ         VM(N00984)  VP(N00984)
7.958E-01    2.498E+00    1.072E+02

FREQ
IM(V_PRINT1)IP(V_PRINT1)
7.958E-01    4.484E-01  -9.457E+01

FREQ
IM(V_PRINT2)IP(V_PRINT2)
7.958E-01    6.245E-01  -7.277E+01

FREQ         VM(N00959)  VP(N00959)
7.958E-01    4.292E+00  -5.874E+01
```

The power received by the coupled inductors is

$$p = \frac{(4.292)(0.4484)}{2}\cos\left(-58.74-(-94.57)\right)+\frac{(2.498)(0.6245)}{2}\cos\left(107.2-(-72.77)\right)$$
$$= 0.78016 - .78000 \approx 0$$

SP 11-3 The input to the circuit shown in Figure SP 11-3 is the voltage of the voltage source,

$$v_s(t) = 48 \cos(4t + 114°) \text{ V}$$

The output is the voltage across the 9-Ω resistor, $v_o(t)$. Use PSpice to determine the average power delivered to the transformer.
Hint: Represent the voltage source using the PSpice part called VAC.

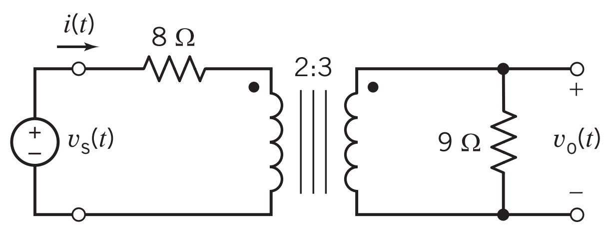
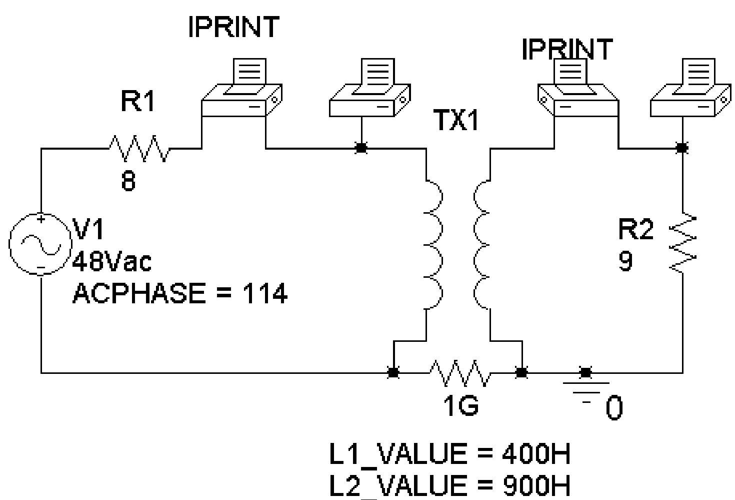


Figure SP 11-3

Solution:



The inductance are selected so that $\sqrt{\frac{L_2}{L_1}} = \frac{N_2}{N_1} = \frac{3}{2}$ and the impedance of these inductors are much larger that other impedance in the circuit. The 1 GΩ resistor simulates an open circuit while providing a connected circuit.

Here is the output from the printers, giving the voltage of coil 2 as 24.00∠114.1°, the current of coil 1 as 4.000∠114.0°, the current of coil 2 as 2.667∠-65.90° and the voltage of coil 1 as 16.00∠114.1°:

```
FREQ         VM(N00984)  VP(N00984)
6.366E-01    2.400E+01   1.141E+02

FREQ
IM(V_PRINT1)IP(V_PRINT1)
6.366E-01    4.000E+00   1.140E+02

FREQ
IM(V_PRINT2)IP(V_PRINT2)
6.366E-01    2.667E+00  -6.590E+01

FREQ         VM(N00959)  VP(N00959)
6.366E-01    1.600E+01   1.141E+02
```

The power received by the transformer is

$$p = \frac{(16)(4)}{2}\cos(114-114) + \frac{(24)(2.667)}{2}\cos(114-(-66))$$
$$= 32 - 32.004 \approx 0$$

SP 11-4 Determine the value of the input impedance, $\mathbf{Z}_t$, of the circuit shown in Figure SP 11-4 at the frequency $\omega = 4$ rad/s.
Hint: Connect a current source across the terminals of the circuit. Measure the voltage across the current source. The value of impedance will be equal to the ratio of the voltage to the current.

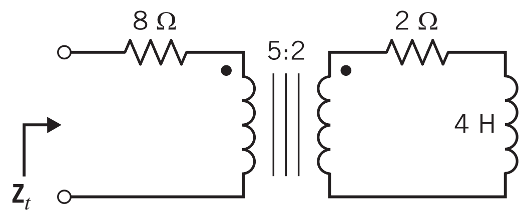
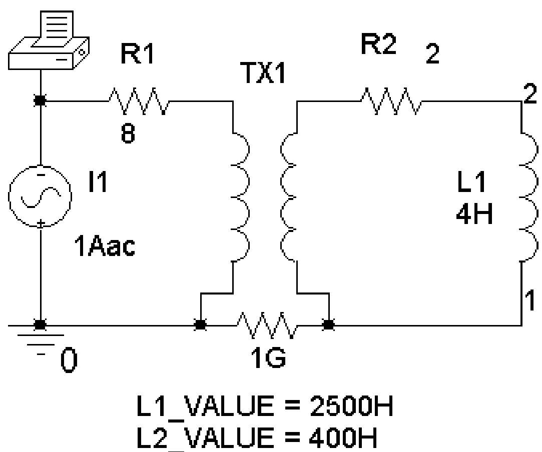


Figure SP 11-4

Solution:



The inductance are selected so that $\sqrt{\dfrac{L_2}{L_1}} = \dfrac{N_2}{N_1} = \dfrac{2}{5}$ and the impedance of these inductors are much larger that other impedance in the circuit. The 1 GΩ resistor simulates an open circuit while providing a connected circuit.

```
FREQ         VM(N00921)  VP(N00921)  VR(N00921)  VI(N00921)
6.366E-01    1.011E+02   7.844E+01   2.025E+01   9.903E+01
```

The printer output gives the voltage across the current source as

$$20.25 + j99.03 = 101.1\angle 78.44° \text{ V}$$

The input impedance is

$$\mathbf{Z}_t = \frac{20.25 + j99.03}{1} = 20.25 + j99.03 \;\Omega = 101.1\angle 78.44° \;\Omega$$

(We expected $\mathbf{Z}_t = 8 + \left(\dfrac{5^2}{2^2}\right)\left(2 + j(4)(4)\right) = 20.5 + j100 \;\Omega$. That's about 1% error.)

Design Problems

DP 11-1 A 100-kW induction motor, shown in Figure DP 11-1, is receiving 100 kW at 0.8 power factor lagging. Determine the additional apparent power in kVA that is made available by improving the power factor to (a) 0.95 lagging and (b) 1.0. (c) Find the required reactive power in kVAR provided by a set of parallel capacitors for parts (a) and (b). (d) Determine the ratio of kVA released to the kVAR of capacitors required for parts (a) and (b) alone. Set up a table recording the results of this problem for the two values of power factor attained.

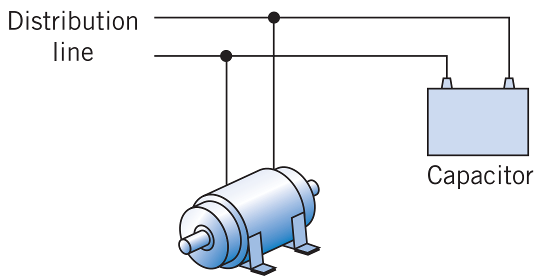


Figure DP 11-1

Solution:

$$\left.\begin{matrix} P = 100 \text{ kW} \\ \text{pf}=0.8 \end{matrix}\right\} \Rightarrow \left\{\begin{matrix} |\mathbf{S}| = \dfrac{P}{pf} = \dfrac{100}{0.8} = 125 \text{ kVA} \\ Q = |\mathbf{S}|\sin\left(\cos^{-1} 0.8\right) = 125\sin\left(36.9°\right) = 75 \text{ kVAR} \end{matrix}\right.$$

(a) Now $pf = 0.95$ so

$$|\mathbf{S}| = \frac{P}{pf} = \frac{100}{0.95} = 105.3 \text{ kVA}$$

$$Q = |\mathbf{S}| \sin(\cos^{-1} 0.95) = 105.3 \sin(18.2°) = 32.9 \text{ kVAR}$$

so an additional 125 − 105.3 = 19.7 kVA is available.

(b) Now $pf = 1$ so

$$|\mathbf{S}| = \frac{P}{pf} = \frac{100}{1} = 100 \text{ kVA}$$

$$Q = |\mathbf{S}| \sin(\cos^{-1} 1) = 0$$

and an additional 125−100= 25 kVA is available.

(c) In part (a), the capacitors are required to reduce Q by 75 – 32.9 = 42.1 kVAR. In part (b), the capacitors are required to reduce Q by 75 – 0 = 75 kVAR.

(d)

Corrected power factor	0.95	1.0
Additional available apparent power	19.7 kVA	25 kVA
Reduction in reactive power	42.1 kVAR	75 kVAR

DP 11-2 Two loads are connected in parallel and supplied from a 7.2-kV rms 60-Hz source. The first load is 50-kVA at 0.9 lagging power factor, and the second load is 45 kW at 0.91 lagging power factor. Determine the kVAR rating and capacitance required to correct the overall power factor to 0.97 lagging.

Answer: $C = 1.01\ \mu$F

Solution:

This example demonstrates that loads can be specified either by kW or kVA. The procedure is as follows:

First load:

$$\left.\begin{matrix} |\mathbf{S}_1| = 50\text{ VA} \\ pf = 0.9 \end{matrix}\right\} \Rightarrow \begin{cases} P_1 = |\mathbf{S}_1|\ pf = (50)(0.9) = 45\text{ W} \\ Q_1 = |\mathbf{S}_1| \sin(\cos^{-1} 0.9) = 50 \sin(25.8°) = 21.8\text{ kVAR} \end{cases}$$

Second load:

$$\left.\begin{matrix} P_2 = 45\text{ W} \\ pf = 0.91 \end{matrix}\right\} \Rightarrow \begin{cases} |\mathbf{S}_2| = \dfrac{P_2}{pf} = \dfrac{45}{0.91} = 49.45\text{ kVA} \\ Q_2 = |\mathbf{S}_2| \sin(\cos^{-1} 0.91) = 49.45 \sin(24.5°) = 20.5\text{ kVAR} \end{cases}$$

Total load:

$$\mathbf{S_L} = \mathbf{S_1} + \mathbf{S_2} = (45 + 45) + \text{j}(21.8+20.5) = 90 + j42.3\text{ kVA}$$

Specified load:

$$\left.\begin{matrix} P_s = 90\text{ W} \\ pf = 0.97 \end{matrix}\right\} \Rightarrow \begin{cases} |\mathbf{S}_s| = \dfrac{P_s}{pf} = \dfrac{90}{0.97} = 92.8\text{ kVA} \\ Q_s = |\mathbf{S}_s| \sin(\cos^{-1} 0.97) = 92.8 \sin(14.1°) = 22.6\text{ kVAR} \end{cases}$$

The compensating capacitive load is $Q_c = 42.3 - 22.6 = 19.7\text{ kVAR}$.

The required capacitor is calculated as

$$X_c = \frac{|\mathbf{V}_c|^2}{Q_c} = \frac{(7.2\times10^3)^2}{19.7\times10^3} = 2626\ \Omega \ \Rightarrow\ C = \frac{1}{377\ (2626)} = 1.01\ \mu\text{F}$$

DP 11-3

(a) Determine the load impedance $\mathbf{Z}_{ab}$ that will absorb maximum power if it is connected to terminals a–b of the circuit shown in Figure DP 11-3.

(b) Determine the maximum power absorbed by this load.

(c) Determine a model of the load and indicate the element values.

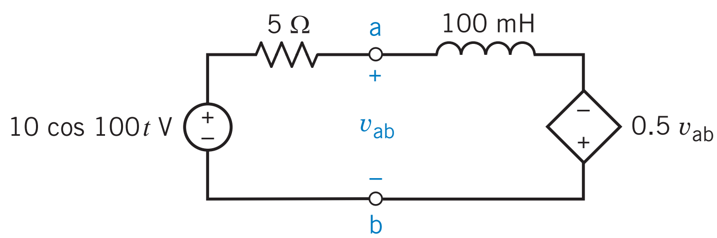
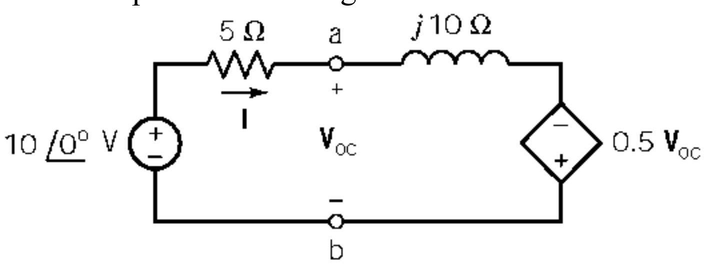


Figure DP 11-3

Solution:

Find the open circuit voltage:



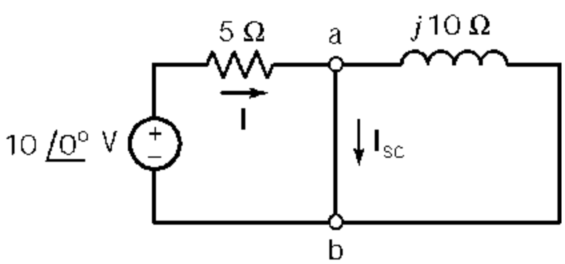
$$-10+5\mathbf{I}+j10\mathbf{I}-0.5\mathbf{V}_{oc} = 0$$

and

$$\mathbf{I} = \frac{10-\mathbf{V}_{oc}}{5}$$

so $\quad \mathbf{V}_{oc} = 8\angle 36.9° = 6.4 + j4.8 \ \text{V}$

Find the short circuit current:



$$\mathbf{I}_{sc} = \frac{10\angle 0°}{5} = 2\angle 0° \ \text{A}$$

The the Thevenin impedance is:

$$\mathbf{Z}_t = \frac{\mathbf{V}_{oc}}{\mathbf{I}_{sc}} = 3.2 + j2.4 \ \Omega$$

The short circuit forces the controlling voltage to be zero. Then the controlled voltage is also zero. Consequently the dependent source has been replaced by a short circuit.

(a) Maximum power transfer requires $\mathbf{Z}_L = \mathbf{Z}_t{}^* = 3.2 - j2.4 \ \Omega$.

(c) $\mathbf{Z}_L$ can be implemented as the series combination of a resistor and a capacitor with

$$R = 3.2 \ \Omega \ \text{ and } \ C = \frac{1}{(100)(2.4)} = 4.17 \ \text{mF}.$$

(b)

$$P_{max} = \frac{|\mathbf{V}_{oc}|^2}{8R} = \frac{64}{8(3.2)} = 2.5 \ \text{W}$$

DP 11-4 Select the turns ratio n necessary to provide maximum power to the resistor R of the circuit shown in Figure DP 11-4. Assume an ideal transformer. Select n when $R = 4$ and 8 Ω.

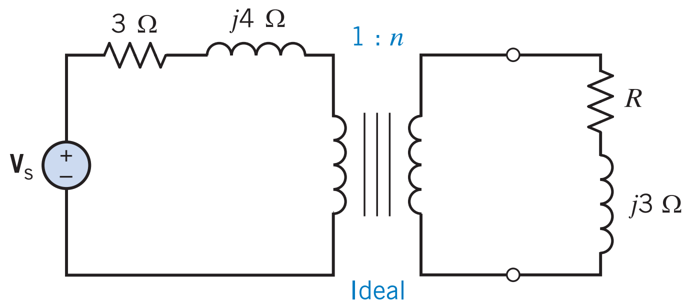
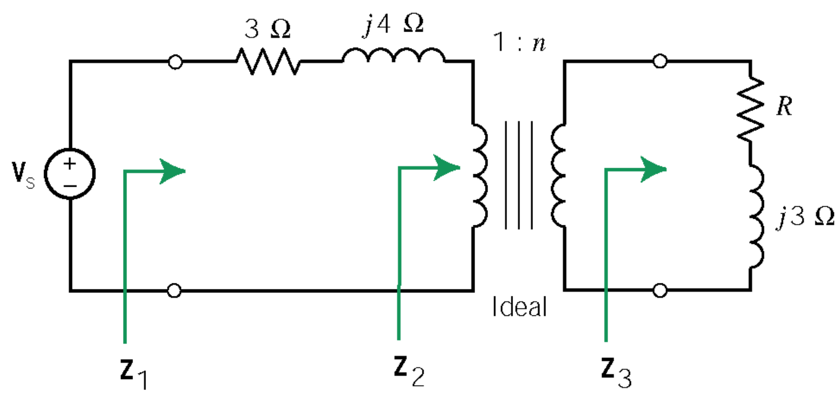


Figure DP 11-4

Solution:



When n is selected to deliver maximum power to $\mathbf{Z}_3$, the value of the maximum power is given as

$$P = \frac{\left(\frac{R}{n^2}\right)\frac{|\mathbf{V}_s|^2}{2}}{\left(3 + \frac{R}{n^2}\right)^2 + \left(\frac{3}{n^2} + 4\right)^2}$$

When $R = 4\ \Omega$,

$$P = \frac{n^2 R |\mathbf{V}_s|^2}{25n^4 + 48n^2 + 25}$$

$$0 = \frac{dP}{dn} = \mathrm{R}|\mathbf{V}_s|^2 \left[\frac{2n(25n^4 + 48n^2 + 25) - n^2(100n^3 + 96n)}{(25n^4 + 48n^2 + 25)^2}\right]$$

$$\Rightarrow \quad -50n^5 + 50n = 0 \quad \Rightarrow \quad n^4 = 1 \quad \Rightarrow \quad n = 1$$

When $R = 8\ \Omega$, a similar calculation gives $n = 1.31$.

DP 11-5 An amplifier in a short-wave radio operates at 100 kHz. The load $\mathbf{Z}_2$ is connected to a source through an ideal transformer, as shown in Figure DP 11-5. The load is a series connection of a 10-Ω resistance and 10-μH inductance. The $\mathbf{Z}_s$ consists of a 1-Ω resistance and a 1-μH inductance.

(a) Select an integer n in order to maximize the energy delivered to the load. Calculate $\mathbf{I}_2$ and the energy to the load.

(b) Add a capacitance C in series with $\mathbf{Z}_2$ in order to improve the energy delivered to the load.

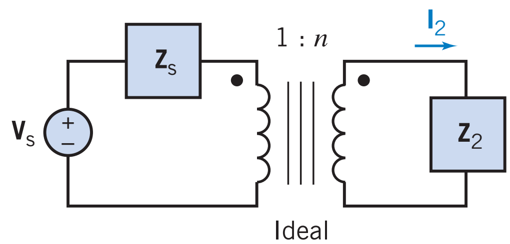
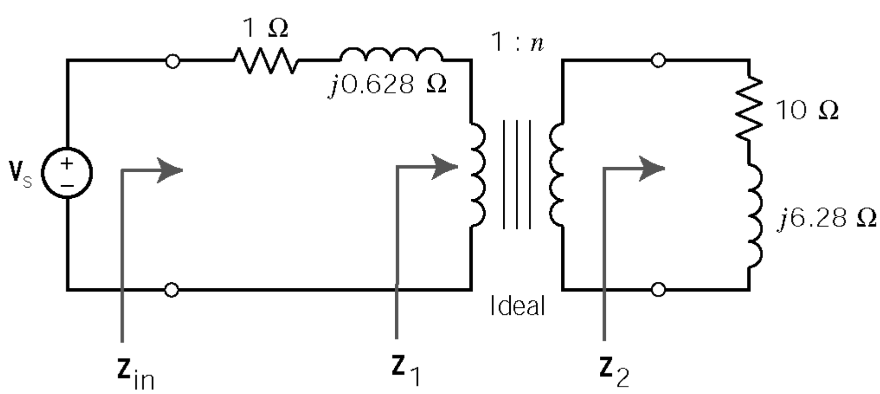


Figure DP 11-5

Solution:



Maximum power transfer requires

$$\frac{10+j6.28}{n^2}=\mathbf{Z}_1=(1+j0.628)^*$$

Equating real parts gives $\quad \frac{10}{n^2}=1 \quad \Rightarrow \quad n=3.16$

Equating imaginary parts requires

$$\frac{jX}{3.16^2}=-j0.628 \quad \Rightarrow \quad X=-6.28$$

This reactance can be realized by adding a capacitance C in series with the resistor and inductor that comprise Z_2. Then

$$-6.28=X=-\frac{1}{(2\pi\times10^5)C}+6.28 \quad \Rightarrow \quad C=\frac{1}{(2\pi\times10^5)(12.56)}=0.1267\mu F$$

DP 11-6 A new electronic lamp (e-lamp) has been developed that uses a radio-frequency sinusoidal oscillator and a coil to transmit energy to a surrounding cloud of mercury gas as shown in Figure DP 11-6*a*. The mercury gas emits ultraviolet light that is transmitted to the phosphor coating, which, in turn, emits visible light. A circuit model of the e-lamp is shown in Figure DP 11.6*b*. The capacitance C and the resistance R are dependent on the lamp's spacing design and the type of phosphor. Select R and C so that maximum power is delivered to R, which relates to the phosphor coating (Adler, 1992). The circuit operates at $\omega_0 = 10^7$ rad/s.

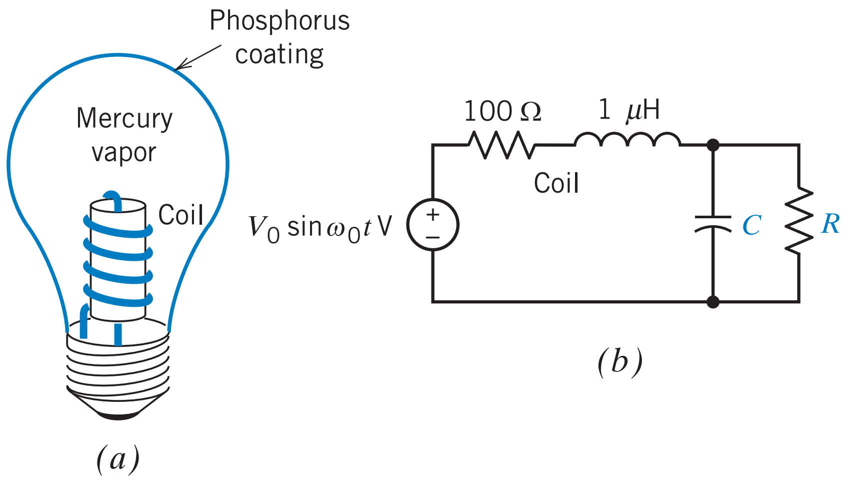
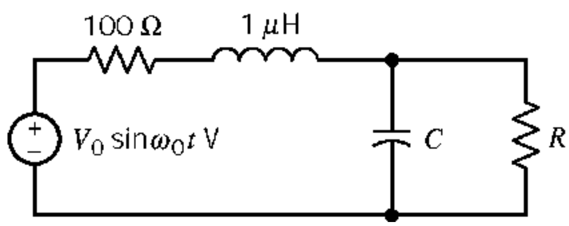


Figure DP 11-6

Solution:



Maximum power transfer requires

$$\frac{1}{j10^7 C} \parallel R = \left(100 + j10^7 \times 10^{-6}\right)^*$$

$$\frac{R}{1 + j10^7 RC} = 100 - j10$$

$$R = (100 - j10)\left(1 + j10^7 RC\right) = 100 + 10^8 RC + j\left(10^9 RC - 10\right)$$

Equating real and imaginary parts yields

$$R = 100 + 10^8 RC \quad \text{and} \quad 10^9 RC - 10 = 0$$

then

$$RC = 10^{-8} \quad \Rightarrow \quad R = 100 + 10^8 R\left(\frac{10^{-8}}{R}\right) = 99\ \Omega \quad \Rightarrow \quad C = \frac{10^{-8}}{99} = 0.101\ \text{nF}$$

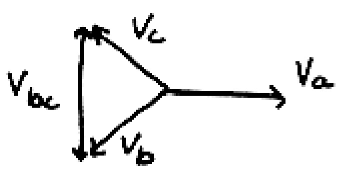
Chapter 12: Three-Phase Circuits

Exercises

Exercise 12.2-1 The Y-connected three-phase voltage source has $\mathbf{V}_c = 120\angle -240° \; V$ rms. Find the line-to-line voltage $\mathbf{V}_{bc}$.
Answer: 207.8 ∠90° V rms

Solution:

$$\mathbf{V}_C = 120\angle -240^\circ \quad \text{so} \quad \mathbf{V}_A = 120\angle 0^\circ \text{ and } \mathbf{V}_B = 120\angle -120^\circ$$



$$\mathbf{V}_{bc} = \sqrt{3}\left(120\right)\angle -90^\circ$$

Exercise 12.3-1 Determine complex power delivered to the three-phase load of a four-wire Y-to-Y circuit such as the one shown in Figure 12.3-1. The phase voltages of the Y-connected source are $\mathbf{V}_a = 120\angle 0°$ V rms, $\mathbf{V}_b = 120\angle -120°$ V rms, and $\mathbf{V}_c = 120\angle 120°$ V rms. The load impedances are $\mathbf{Z}_A = 80 + j50\ \Omega$, $\mathbf{Z}_B = 80 + j80\ \Omega$, and $\mathbf{Z}_C = 100 - j25\ \Omega$.

Answer: $\mathbf{S}_A = 129 + j81$ VA, $\mathbf{S}_B = 90 + j90$ VA, $\mathbf{S}_C = 136 - j34$ VA, and $\mathbf{S} = 355 + j137$ VA

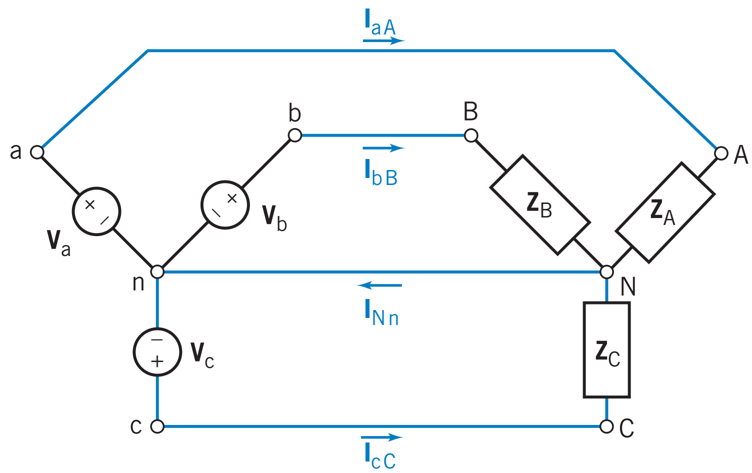


Figure 12.3-1

Solution:

Mathcad analysis (12x4_1.mcd)**:**

Describe the three-phase source: $Vp := 120$

$$Va := Vp \cdot e^{j\cdot\frac{\pi}{180}\cdot 0} \qquad Vb := Va \cdot e^{j\cdot\frac{\pi}{180}\cdot -120} \qquad Vc := Va \cdot e^{j\cdot\frac{\pi}{180}\cdot 120}$$

Describe the three-phase load: $ZA := 80 + j\cdot 50$ $\quad ZB := 80 + j\cdot 80$ $\quad ZC := 100 - j\cdot 25$

Calculate the line currents: $IaA := \frac{Va}{ZA}$ $\quad IbB := \frac{Vb}{ZB}$ $\quad IcC := \frac{Vc}{ZC}$

$IaA = 1.079 - 0.674i$ $\quad IbB = -1.025 - 0.275i$ $\quad IcC = -0.809 + 0.837i$

$|IaA| = 1.272$ $\quad |IbB| = 1.061$ $\quad |IcC| = 1.164$

$\frac{180}{\pi}\cdot \arg(IaA) = -32.005$ $\quad \frac{180}{\pi}\cdot \arg(IbB) = -165$ $\quad \frac{180}{\pi}\cdot \arg(IcC) = 134.036$

Calculate the current in the neutral wire: $INn := IaA + IbB + IcC$ $\quad INn = -0.755 - 0.112i$

Calculate the power delivered to the load:

$SA := \overline{IaA}\cdot IaA \cdot ZA$ $\quad SB := \overline{IbB}\cdot IbB \cdot ZB$ $\quad SC := \overline{IcC}\cdot IcC \cdot ZC$

$SA = 129.438 + 80.899i$ $\quad SB = 90 + 90i$ $\quad SC = 135.529 - 33.882i$

Total power delivered to the load: $SA + SB + SC = 354.968 + 137.017i$

Calculate the power supplied by the source:

$Sa := \overline{IaA} \cdot Va$

$Sa = 129.438 + 80.899i$

$Sb := \overline{IbB} \cdot Vb$

$Sb = 90 + 90i$

$Sc := \overline{IcC} \cdot Vc$

$Sc = 135.529 - 33.882i$

Total power delivered by the source: $Sa + Sb + Sc = 354.968 + 137.017i$

Exercise 12.3-2 Determine complex power delivered to the three-phase load of a four-wire Y-to-Y circuit such as the one shown in Figure 12.3-1. The phase voltages of the Y-connected source are $\mathbf{V}_a = 120\ \angle 0°$ V rms, $\mathbf{V}_b = 120\ \angle -120°$ V rms, and $\mathbf{V}_c = 120\ \angle 120°$ V rms. The load impedances are $\mathbf{Z}_A = \mathbf{Z}_B = \mathbf{Z}_C = 40 + j30\ \Omega$.

Answer: $\mathbf{S}_A = \mathbf{S}_B = \mathbf{S}_C = 230 + j173$ VA and $\mathbf{S} = 691 + j518$ VA

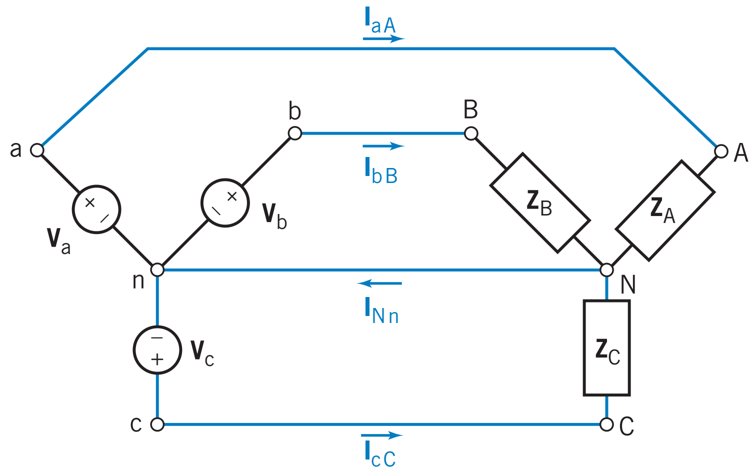


Figure 12.3-1

Solution:

Mathcad analysis (12x4_2.mcd)**:**

Describe the three-phase source: $Vp := 120$

$$Va := Vp \cdot e^{j \cdot \frac{\pi}{180} \cdot 0} \qquad Vb := Va \cdot e^{j \cdot \frac{\pi}{180} \cdot -120} \qquad Vc := Va \cdot e^{j \cdot \frac{\pi}{180} \cdot 120}$$

Describe the three-phase load: $ZA := 40 + j \cdot 30$ $\qquad ZB := ZA$ $\qquad ZC := ZA$

Calculate the line currents: $IaA := \frac{Va}{ZA}$ $\qquad IbB := \frac{Vb}{ZB}$ $\qquad IcC := \frac{Vc}{ZC}$

$IaA = 1.92 - 1.44i$ $\qquad IbB = -2.207 - 0.943i$ $\qquad IcC = 0.287 + 2.383i$

$|IaA| = 2.4$ $\qquad |IbB| = 2.4$ $\qquad |IcC| = 2.4$

$\frac{180}{\pi} \cdot \arg(IaA) = -36.87$ $\qquad \frac{180}{\pi} \cdot \arg(IbB) = -156.87$ $\qquad \frac{180}{\pi} \cdot \arg(IcC) = 83.13$

Calculate the current in the neutral wire: $INn := IaA + IbB + IcC$ $\qquad INn = 0$

Calculate the power delivered to the load:

$SA := \overline{IaA} \cdot IaA \cdot ZA$ $\qquad SB := \overline{IbB} \cdot IbB \cdot ZB$ $\qquad SC := \overline{IcC} \cdot IcC \cdot ZC$

$SA = 230.4 + 172.8i$ $\qquad SB = 230.4 + 172.8i$ $\qquad SC = 230.4 + 172.8i$

Total power delivered to the load: $SA + SB + SC = 691.2 + 518.4i$

Calculate the power supplied by the source:

$Sa := \overline{IaA} \cdot Va$

$Sa = 230.4 + 172.8i$

$Sb := \overline{IbB} \cdot Vb$

$Sb = 230.4 + 172.8i$

$Sc := \overline{IcC} \cdot Vc$

$Sc = 230.4 + 172.8i$

Total power delivered by the source: $Sa + Sb + Sc = 691.2 + 518.4i$

Exercise 12.3-3 Determine complex power delivered to the three-phase load of a three-wire Y-to-Y circuit such as the one shown in Figure 12.3-2. The phase voltages of the Y-connected source are $\mathbf{V}_a = 120\ \angle 0°$ V rms, $\mathbf{V}_b = 120\ \angle{-120°}$ V rms, and $\mathbf{V}_c = 120\ \angle 120°$ V rms. The load impedances are $\mathbf{Z}_A = 80 + j50\ \Omega$, $\mathbf{Z}_B = 80 + j80\ \Omega$, and $\mathbf{Z}_C = 100 - j25\ \Omega$.

Intermediate Answer: $\mathbf{V}_{nN} = 28.89\ \angle{-150.5}$ V rms

Answer: $\mathbf{S} = 392 + j142$ VA

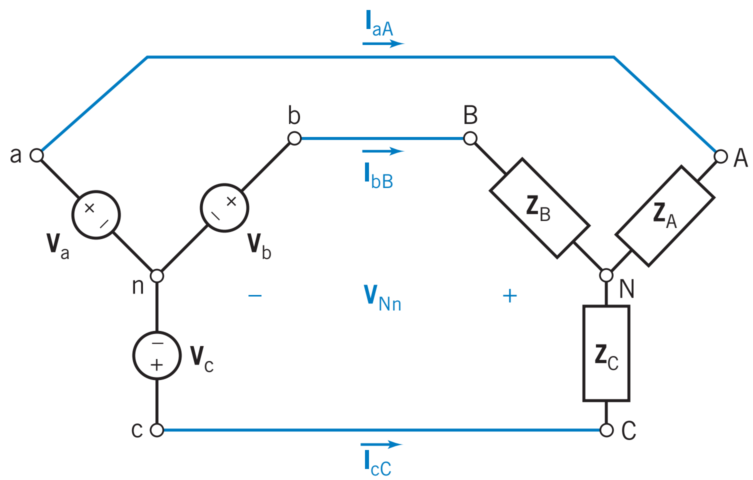


Figure 12.3-2

Solution:

Mathcad analysis (12x4_3.mcd):

Describe the three-phase source: $Vp := 120$

$$Va := Vp \cdot e^{j \cdot \frac{\pi}{180} \cdot 0} \qquad Vb := Va \cdot e^{j \cdot \frac{\pi}{180} \cdot -120} \qquad Vc := Va \cdot e^{j \cdot \frac{\pi}{180} \cdot 120}$$

Describe the three-phase load: $ZA := 80 + j \cdot 50$ $\quad ZB := 80 + j \cdot 80$ $\quad ZC := 100 - j \cdot 25$

Calculate the voltage at the neutral of the load with respect to the neutral of the source:

$$VnN := \frac{ZA \cdot ZC \cdot e^{j \cdot \frac{4}{3} \cdot \pi} + ZA \cdot ZB \cdot e^{j \cdot \frac{2}{3} \cdot \pi} + ZB \cdot ZC}{ZA \cdot ZC + ZA \cdot ZB + ZB \cdot ZC} \cdot Vp$$

$$VnN = -25.137 - 14.236i \qquad |VnN| = 28.888 \qquad \frac{180}{\pi} \cdot \arg(VnN) = -150.475$$

Calculate the line currents: $IaA := \frac{Va - VnN}{ZA}$ $IbB := \frac{Vb - VnN}{ZB}$ $IcC := \frac{Vc - VnN}{ZC}$

$IaA = 1.385 - 0.687i$ $IbB = -0.778 - 0.343i$ $IcC = -0.606 + 1.03i$

$|IaA| = 1.546$ $|IbB| = 0.851$ $|IcC| = 1.195$

$\frac{180}{\pi}\cdot arg(IaA) = -26.403$ $\frac{180}{\pi}\cdot arg(IbB) = -156.242$ $\frac{180}{\pi}\cdot arg(IcC) = 120.475$

Check: $IaA + IbB + IcC = 0$

Calculate the power delivered to the load:

$SA := \overline{IaA}\cdot IaA\cdot ZA$ $SB := \overline{IbB}\cdot IbB\cdot ZB$ $SC := \overline{IcC}\cdot IcC\cdot ZC$

$SA = 191.168 + 119.48i$ $SB = 57.87 + 57.87i$ $SC = 142.843 - 35.711i$

Total power delivered to the load: $SA + SB + SC = 391.88 + 141.639i$

Exercise 12.3-4 Determine complex power delivered to the three-phase load of a three-wire Y-to-Y circuit such as the one shown in Figure 12.3-2. The phase voltages of the Y-connected source are $\mathbf{V}_a = 120\ \angle 0°$ V rms, $\mathbf{V}_b = 120\ \angle{-120°}$ V rms, and $\mathbf{V}_c = 120\ \angle 120°$ V rms. The load impedances are $\mathbf{Z}_A = \mathbf{Z}_B = \mathbf{Z}_C = 40 + j30\ \Omega$.

Answer: $\mathbf{S}_A = \mathbf{S}_B = \mathbf{S}_C = 230 + j173$ VA and $\mathbf{S} = 691 + j518$ VA

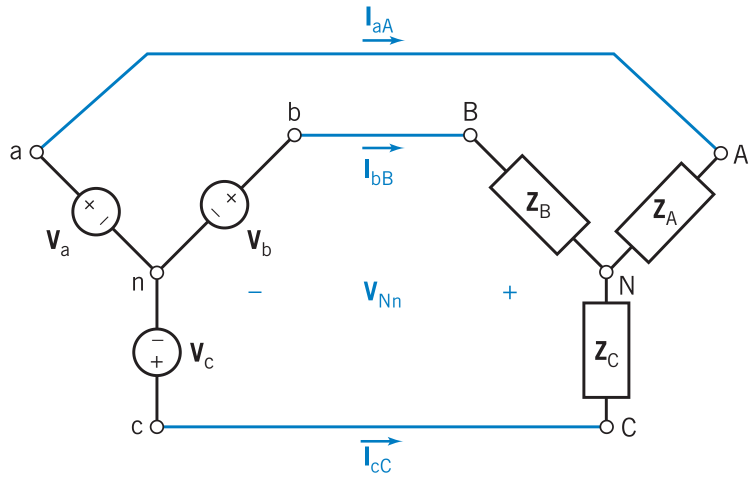


Figure 12.3-2

Solution:

Mathcad analysis (12x4_4.mcd):

Describe the three-phase source: $Vp := 120$

$$Va := Vp \cdot e^{j \cdot \frac{\pi}{180} \cdot 0} \qquad Vb := Va \cdot e^{j \cdot \frac{\pi}{180} \cdot -120} \qquad Vc := Va \cdot e^{j \cdot \frac{\pi}{180} \cdot 120}$$

Describe the three-phase load: $ZA := 40 + j \cdot 30$ $\qquad ZB := ZA$ $\qquad ZC := ZA$

Calculate the voltage at the neutral of the load with respect to the neutral of the source:

$$VnN := \frac{ZA \cdot ZC \cdot e^{j \cdot \frac{4}{3} \cdot \pi} + ZA \cdot ZB \cdot e^{j \cdot \frac{2}{3} \cdot \pi} + ZB \cdot ZC}{ZA \cdot ZC + ZA \cdot ZB + ZB \cdot ZC} \cdot Vp$$

$$VnN = -1.31 \times 10^{-14} + 1.892i \times 10^{-14} \qquad |VnN| = 2.301 \times 10^{-14} \qquad \frac{180}{\pi} \cdot \arg(VnN) = 124.695$$

Calculate the line currents: $IaA := \frac{Va - VnN}{ZA}$ $IbB := \frac{Vb - VnN}{ZB}$ $IcC := \frac{Vc - VnN}{ZC}$

$IaA = 1.92 - 1.44i$ $IbB = -2.207 - 0.943i$ $IcC = 0.287 + 2.383i$

$|IaA| = 2.4$ $|IbB| = 2.4$ $|IcC| = 2.4$

$\frac{180}{\pi} \cdot arg(IaA) = -36.87$ $\frac{180}{\pi} \cdot arg(IbB) = -156.87$ $\frac{180}{\pi} \cdot arg(IcC) = 83.13$

Check: $IaA + IbB + IcC = 1.055 \times 10^{-15} - 2.22i \times 10^{-15}$

Calculate the power delivered to the load:

$SA := \overline{IaA} \cdot IaA \cdot ZA$ $SB := \overline{IbB} \cdot IbB \cdot ZB$ $SC := \overline{IcC} \cdot IcC \cdot ZC$

$SA = 230.4 + 172.8i$ $SB = 230.4 + 172.8i$ $SC = 230.4 + 172.8i$

Total power delivered to the load: $SA + SB + SC = 691.2 + 518.4i$

Exercise 12.5-1

Consider the three-phase circuit shown in Figure 12.5-1. The voltages of the Y-connected source are

$$\mathbf{V}_a = \frac{360}{\sqrt{3}}\angle{-30^\circ}\text{ V rms},\ \mathbf{V}_b = \frac{360}{\sqrt{3}}\angle{-150^\circ}\text{ V rms, and }\mathbf{V}_c = \frac{360}{\sqrt{3}}\angle 90^\circ\text{ V rms}$$

The Δ-connected load is balanced. The impedance of each phase is $\mathbf{Z}_\Delta = 180\angle 45^\circ\ \Omega$. Determine the phase and line currents when the line-to-line voltage is 360 V rms.

Partial Answer: $\mathbf{I}_{AB} = 2\angle 45^\circ$ A rms and $\mathbf{I}_{aA} = 3.46\angle 15^\circ$ A rms

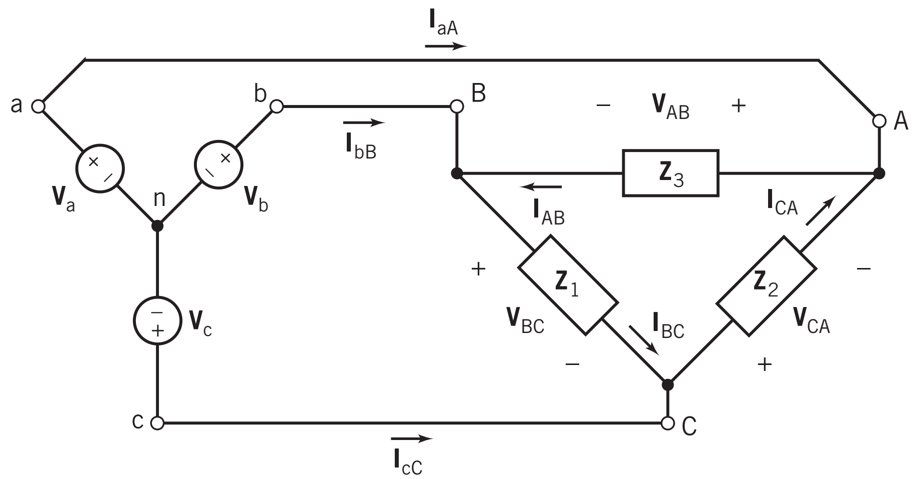
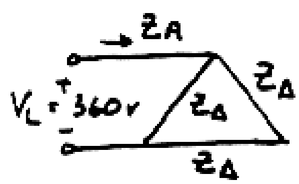


Figure 12.5-1

Solution:

Balanced delta load:



(See Table 12.5-1)

$\mathbf{Z}_\Delta = 180\angle{-45^\circ}$

phase currents:

$$\mathbf{I_{AB}} = \frac{\mathbf{V_{AB}}}{\mathbf{Z}_\Delta} = \frac{360\angle 0^\circ}{180\angle{-45^\circ}} = 2\angle 45^\circ\text{ A}$$

$$\mathbf{I_{BC}} = \frac{\mathbf{V_{BC}}}{\mathbf{Z}_\Delta} = \frac{360\angle{-120^\circ}}{18\angle{-45^\circ}} = 2\angle{-75^\circ}\text{ A}$$

$$\mathbf{I_{CA}} = \frac{\mathbf{V_{CA}}}{\mathbf{Z}_\Delta} = \frac{360\angle 120^\circ}{180\angle{-45^\circ}} = 2\angle 165^\circ\text{ A}$$

line currents:

$$\mathbf{I_A} = \mathbf{I_{AB}} - \mathbf{I_{CA}} = 2\angle 45^\circ - 2\angle 165^\circ = 2\sqrt{3}\angle 15^\circ\text{ A}$$

$$\mathbf{I_B} = 2\sqrt{3}\angle{-105^\circ}\text{ A}$$

$$\mathbf{I_C} = 2\sqrt{3}\angle 135^\circ\text{ A}$$

Exercise 12.6-1 Figure 12.6-1*a* shows a balanced Y-to-Δ three-phase circuit. The phase voltages of the Y-connected source are $\mathbf{V}_a = 110\ \angle 0°$ V rms, $\mathbf{V}_b = 110\ \angle{-120°}$ V rms, and $\mathbf{V}_c = 110\ \angle 120°$ V rms. The line impedances are each $\mathbf{Z}_L = 10 + j25\ \Omega$. The impedances of the Δ-connected load are each $\mathbf{Z}_\Delta = 150 + j270\ \Omega$. Determine the phase currents in the Δ-connected load.

Answer: $\mathbf{I}_{AB} = 0.49\ \angle{-32.5°}$ A rms, $\mathbf{I}_{BC} = 0.49\ \angle{-152.5°}$ A rms, $\mathbf{I}_{CA} = 0.49\ \angle 87.5°$ A rms

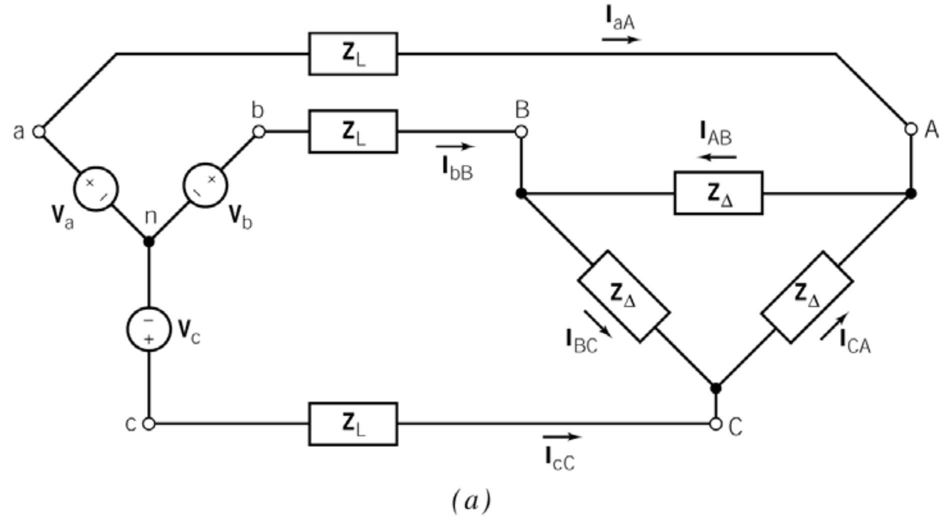


Figure 12.6-1a

Solution:

Three-wire Y-to-Delta Circuit with line impedances

Mathcad analysis (12x4_4.mcd):

Describe the three-phase source: $Vp := 110$

$$Va := Vp \cdot e^{j \cdot \frac{\pi}{180} \cdot 0} \qquad Vb := Va \cdot e^{j \cdot \frac{\pi}{180} \cdot -120} \qquad Vc := Va \cdot e^{j \cdot \frac{\pi}{180} \cdot 120}$$

Describe the delta connected load: $Z1 := 150 + j \cdot 270$ $\quad Z2 := Z1$ $\quad Z3 := Z1$

Convert the delta connected load to the equivalent Y connected load:

$$ZA := \frac{Z1 \cdot Z3}{Z1 + Z2 + Z3} \qquad ZB := \frac{Z2 \cdot Z3}{Z1 + Z2 + Z3} \qquad ZC := \frac{Z1 \cdot Z2}{Z1 + Z2 + Z3}$$

$$ZA = 50 + 90i \qquad ZB = 50 + 90i \qquad ZC = 50 + 90i$$

Describe the three-phase line: $ZaA := 10 + j \cdot 25$ $\quad ZbB := ZaA$ $\quad ZcC := ZaA$

Calculate the voltage at the neutral of the load with respect to the neutral of the source:

$$VnN := \frac{(ZaA + ZA)\cdot(ZcC + ZC)\cdot e^{j \cdot \frac{4}{3} \cdot \pi} + (ZaA + ZA)\cdot(ZbB + ZB)\cdot e^{j \cdot \frac{2}{3} \cdot \pi} + (ZbB + ZB)\cdot(ZcC + ZC)}{(ZaA + ZA)\cdot(ZcC + ZC) + (ZaA + ZA)\cdot(ZbB + ZB) + (ZbB + ZB)\cdot(ZcC + ZC)} \cdot Vp$$

$$VnN = -1.172 \times 10^{-14} + 1.784i \times 10^{-14} \qquad |VnN| = 2.135 \times 10^{-14} \qquad \frac{180}{\pi} \cdot \arg(VnN) = 123.304$$

Calculate the line currents: $IaA := \frac{Va - VnN}{ZA + ZaA}$ $IbB := \frac{Vb - VnN}{ZB + ZbB}$ $IcC := \frac{Vc - VnN}{ZC + ZcC}$

$IaA = 0.392 - 0.752i$ $IbB = -0.847 + 0.036i$ $IcC = 0.455 + 0.716i$

$|IaA| = 0.848$ $|IbB| = 0.848$ $|IcC| = 0.848$

$\frac{180}{\pi}\cdot arg(IaA) = -62.447$ $\frac{180}{\pi}\cdot arg(IbB) = 177.553$ $\frac{180}{\pi}\cdot arg(IcC) = 57.553$

Check: $IaA + IbB + IcC = 0$

Calculate the phase voltages of the Y-connected load:

$VAN := IaA\cdot ZA$ $VBN := IbB\cdot ZB$ $VCN := IcC\cdot ZC$

$|VAN| = 87.311$ $|VBN| = 87.311$ $|VCN| = 87.311$

$\frac{180}{\pi}\cdot arg(VAN) = -1.502$ $\frac{180}{\pi}\cdot arg(VBN) = -121.502$ $\frac{180}{\pi}\cdot arg(VCN) = 118.498$

Calculate the line-to-line voltages at the load:

$VAB := VAN - VBN$ $VBC := VBN - VCN$ $VCA := VCN - VAN$

$|VAB| = 151.227$ $|VBC| = 151.227$ $|VCA| = 151.227$

$\frac{180}{\pi}\cdot arg(VAB) = 28.498$ $\frac{180}{\pi}\cdot arg(VBC) = -91.502$ $\frac{180}{\pi}\cdot arg(VCA) = 148.498$

Calculate the phase currents of the Δ-connected load:

$IAB := \frac{VAB}{Z3}$ $IBC := \frac{VBC}{Z1}$ $ICA := \frac{VCA}{Z2}$

$|IAB| = 0.49$ $|IBC| = 0.49$ $|ICA| = 0.49$

$\frac{180}{\pi}\cdot arg(IAB) = -32.447$ $\frac{180}{\pi}\cdot arg(IBC) = -152.447$ $\frac{180}{\pi}\cdot arg(ICA) = 87.553$

Exercise 12.7-1 Figure 12.6-1*a* shows a balanced Y-to-Δ three-phase circuit. The phase voltages of the Y-connected source are $\mathbf{V}_a = 110\ \angle 0°$ V rms, $\mathbf{V}_b = 110\ \angle{-120°}$ V rms, and $\mathbf{V}_c = 110\ \angle 120°$ V rms. The line impedances are each $\mathbf{Z}_L = 10 + j25\ \Omega$. The impedances of the Δ-connected load are each $\mathbf{Z}_\Delta = 150 + j\,270\ \Omega$. Determine the average power delivered to the Δ-connected load.
Intermediate Answer: $\mathbf{I}_{aA} = 0.848\ \angle{-62.5°}$ A rms and $\mathbf{V}_{AN} = 87.3\ \angle{-1.5°}$ V rms
Answer: $P = 107.9$ W

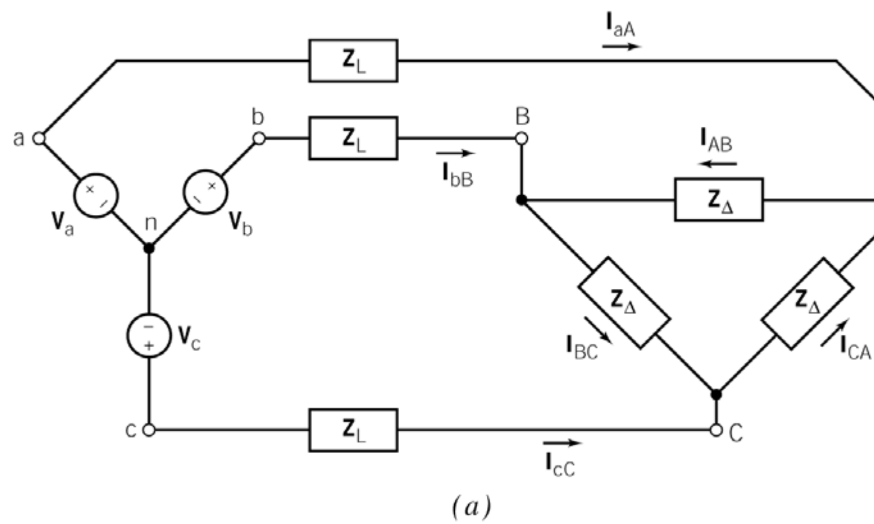


Figure 12.6-1a

Solution:

Continuing Ex. 12.6-1:

Calculate the power delivered to the load:

$SA := \overline{IaA}\cdot IaA\cdot ZA$ $\quad SB := \overline{IbB}\cdot IbB\cdot ZB$ $\quad SC := \overline{IcC}\cdot IcC\cdot ZC$

$SA = 35.958 + 64.725i$ $\quad SB = 35.958 + 64.725i$ $\quad SC = 35.958 + 64.725i$

Total power delivered to the load: $SA + SB + SC = 107.875 + 194.175i$

Exercise 12.8-1 The line current to a balanced three-phase load is 24 A rms. The line-to-line voltage is 450 V rms, and the power factor of the load is 0.47 lagging. If two wattmeters are connected as shown in Figure 12.8-2, determine the reading of each meter and the total power to the load.

Answer: $P_1 = -371$ W, $P_2 = 9162$ W, and $P = 8791$ W

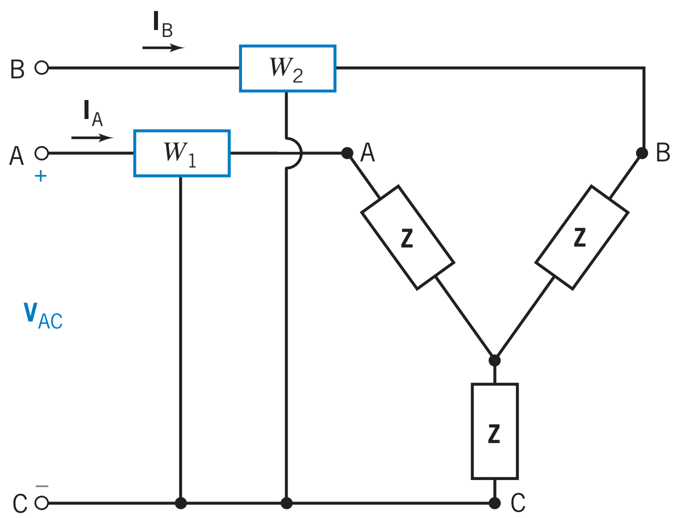


FIGURE 12.8-2

Solution:

$$P_1 = \mathbf{V}_{AB}\,\mathbf{I}_A\,\cos(\theta+30^\circ) + \mathbf{V}_{CB}\,\mathbf{I}_C\,\cos(\theta-30^\circ) = P_1 + P_2$$

$$pf = .4\ lagging \Rightarrow \theta = 61.97^\circ$$

$$So\ P_T = 450(24)\left[\cos 91.97^\circ + \cos 31.97^\circ\right] = 8791\text{ W}$$

$$\therefore\ P_1 = -371\text{ W} \quad P_2 = 9162\text{ W}$$

Exercise 12.8-2 The two wattmeters are connected as shown in Figure 12.8-2 with P_1 = 60 kW and P_2 = 40 kW, respectively. Determine (a) the total power and (b) the power factor.
Answers: (a) 100 kW (b) 0.945 leading

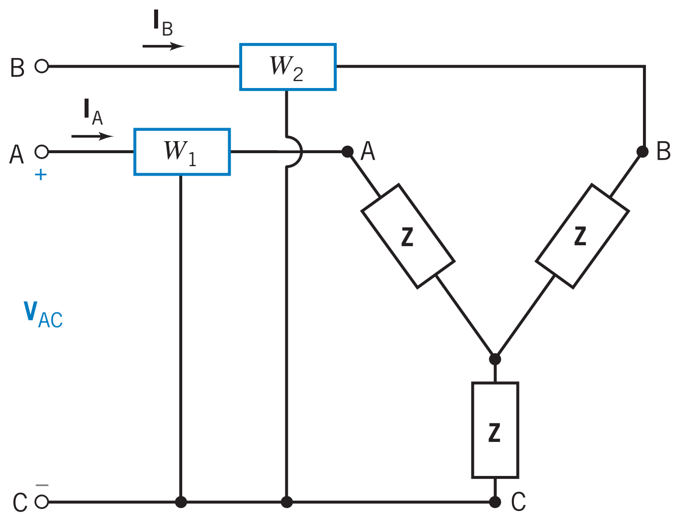


Figure 12.8-2

Ex. 12.8-2

Consider Fig. 12.8-1 with $P_1 = 60\ \text{kW}\quad P_2 = 40\ \text{kW}$.

(a.) $P = P_1 + P_2 = 100\ \text{kW}$

(b.) use equation 12.9-7 to get

$$\tan\theta = \sqrt{3}\,\frac{P_2 - P_1}{\mathrm{P_L} + \mathrm{P_2}} = \sqrt{3}\,\frac{40-60}{100} = -.346 \quad\Rightarrow\quad \theta = -19.11°$$

then

$$pf = \cos\left(-19.110°\right) = \underline{0.945}\quad \text{leading}$$

Problems

Section 12-2: Three Phase Voltages

P 12.2-1 A balanced three-phase Y-connected load has one phase voltage:

$$\mathbf{V}_c = 277\ \angle 45°\ \text{V rms}$$

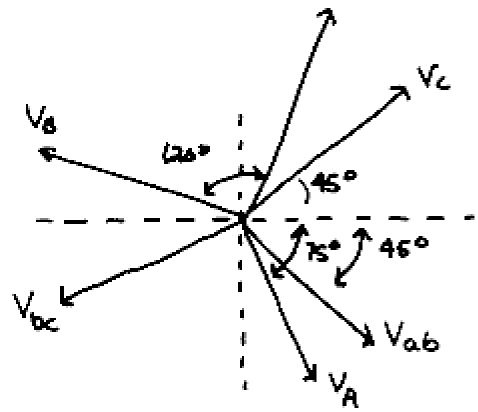
The phase sequence is *abc*. Find the line-to-line voltages $\mathbf{V}_{AB}$, $\mathbf{V}_{BC}$, and $\mathbf{V}_{CA}$. Draw a phasor diagram showing the phase and line voltages.

Solution:

Given $\mathbf{V}_C = 277\ \angle 45°$ and an *abc* phase sequence:

$$\mathbf{V}_A = 277\ \angle\left(45–120\right)° = 277\ \angle -75°$$

$$\mathbf{V}_B = 277\ \angle\left(45°+120\right)° = 277\ \angle 165°$$



$$\begin{aligned}\mathbf{V}_{AB} &= \mathbf{V}_A - \mathbf{V}_B = \left(277\ \angle -75°\right) - \left(277\ \angle 165°\right)\\ &= \left(71.69 - j267.56\right) - \left(-267.56 + j71.69\right)\\ &= 339.25 - j339.25 = 479.77\ \angle -45° \simeq 480\ \angle -45°\end{aligned}$$

Similarly:

$$\mathbf{V}_{BC} = 480\ \angle -165° \text{ and } \mathbf{V}_{CA} = 480\ \angle 75°$$

P 12.2-2 A three-phase system has a line-to-line voltage

$$\mathbf{V}_{BA} = 12{,}470\ \angle -35°\ \text{V rms}$$

with a Y load. Find the phase voltages when the phase sequence is *abc*.

Solution:

$$\mathbf{V}_{AB} = \mathbf{V}_A \times \sqrt{3}\angle 30° \quad \Rightarrow \quad \mathbf{V}_A = \frac{\mathbf{V}_{AB}}{\sqrt{3}\angle 30°}$$

In our case: $\mathbf{V}_{AB} = -\mathbf{V}_{BA} = -\left(12470\ \angle -35°\right) = 12470\ \angle 145°\ \text{V}$

So $\mathbf{V}_A = \dfrac{12470\ \angle 145°}{\sqrt{3}\angle 30°} = 7200\angle 115°$

Then, for an *abc* phase sequence:

$$\mathbf{V}_C = 7200\ \angle\left(115+120\right)° = 7200\ \angle 235° = 7200\ \angle -125°$$

$$\mathbf{V}_B = 7200\ \angle\left(115-120\right)° = 7200\ \angle -5°\ \text{V}$$

P 12.2-3 A three-phase system has a line-to-line voltage

$$\mathbf{V}_{ab} = 1500 \angle 30° \text{ V rms}$$

with a Y load. Determine the phase voltage.

Solution:

$$\mathbf{V}_{ab} = \mathbf{V}_a \times \sqrt{3}\angle 30° \quad \Rightarrow \quad \mathbf{V}_a = \frac{\mathbf{V}_{ab}}{\sqrt{3}\angle 30°}$$

In our case, the line-to-line voltage is

$$\mathbf{V}_{ab} = 1500 \angle 30° \text{ V}$$

So the phase voltage is $\mathbf{V}_a = \dfrac{1500 \angle 30°}{\sqrt{3}\angle 30°} = 866\angle 0° \text{ V}$

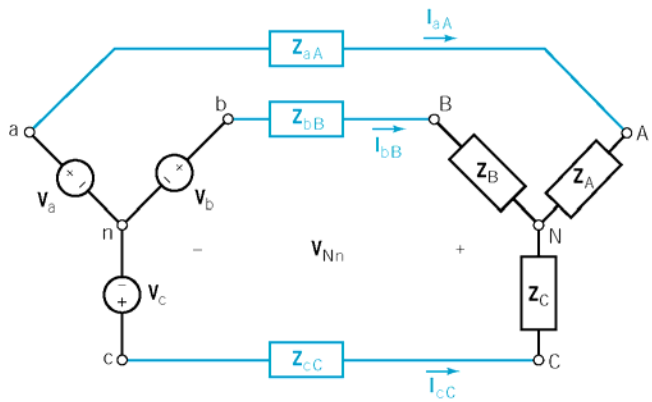
Section 12.3 The Y-to-Y Circuit

P 12.3-1 Consider a three-wire Y-to-Y circuit. The voltages of the Y-connected source are $\mathbf{V}_a = (208/\sqrt{3})\ \angle 0°$ V rms, $\mathbf{V}_b = (208/\sqrt{3})\ \angle{-120°}$ V rms, and $\mathbf{V}_c = (208/\sqrt{3}) \angle 120°$ V rms. The Y-connected load is balanced. The impedance of each phase is $\mathbf{Z} = 12\ \angle 30°\ \Omega$.

(a) Find the phase voltages.
(b) Find the line currents and phase currents.
(c) Show the line currents and phase currents on a phasor diagram.
(d) Determine the power dissipated in the load.

Solution:
Balanced, three-wire, Y-Y circuit:

where $\mathbf{Z}_A = \mathbf{Z}_B = \mathbf{Z}_C = 12\angle 30 = 10.4 + j6$



MathCAD analysis (12p4_1.mcd):

Describe the three-phase source: $Vp := \frac{208}{\sqrt{3}}$

$$Va := Vp \cdot e^{j \cdot \frac{\pi}{180} \cdot 0} \qquad Vb := Va \cdot e^{j \cdot \frac{\pi}{180} \cdot -120} \qquad Vc := Va \cdot e^{j \cdot \frac{\pi}{180} \cdot 120}$$

Describe the balanced three-phase load: $ZA := 10.4 + j \cdot 6 \qquad ZB := ZA \qquad ZC := ZB$

Check: The voltage at the neutral of the load with respect to the neutral of the source should be zero:

$$VnN := \frac{ZA \cdot ZC \cdot e^{j \cdot \frac{4}{3} \cdot \pi} + ZA \cdot ZB \cdot e^{j \cdot \frac{2}{3} \cdot \pi} + ZB \cdot ZC}{ZA \cdot ZC + ZA \cdot ZB + ZB \cdot ZC} \cdot Vp \qquad |VnN| = 2.762 \times 10^{-14}$$

Calculate the line currents: $IaA := \frac{Va - VnN}{ZA} \qquad IbB := \frac{Vb - VnN}{ZB} \qquad IcC := \frac{Vc - VnN}{ZC}$

$IaA = 8.663 - 4.998i \qquad IbB = -8.66 - 5.004i \qquad IcC = -3.205 \times 10^{-3} + 10.002i$

$|IaA| = 10.002 \qquad |IbB| = 10.002 \qquad |IcC| = 10.002$

$\frac{180}{\pi} \cdot arg(IaA) = -29.982 \qquad \frac{180}{\pi} \cdot arg(IbB) = -149.982 \qquad \frac{180}{\pi} \cdot arg(IcC) = 90.018$

Check: $IaA + IbB + IcC = 4.696 \times 10^{-15} - 1.066i \times 10^{-14}$

Calculate the power delivered to the load:

$SA := \overline{IaA} \cdot IaA \cdot ZA$ $\quad$ $SB := \overline{IbB} \cdot IbB \cdot ZB$ $\quad$ $SC := \overline{IcC} \cdot IcC \cdot ZC$

$SA = 1.04 \times 10^3 + 600.222i$ $\quad$ $SB = 1.04 \times 10^3 + 600.222i$ $\quad$ $SC = 1.04 \times 10^3 + 600.222i$

Total power delivered to the load: $\quad SA + SB + SC = 3.121 \times 10^3 + 1.801i \times 10^3$

Consequently:

(a) The phase voltages are

$$\mathbf{V}_a = \frac{208}{\sqrt{3}}\angle 0° = 120\angle 0° \text{ V rms},\ \mathbf{V}_b = 120\angle -120° \text{ V rms and } \mathbf{V}_c = 120\angle 120° \text{ V rms}$$

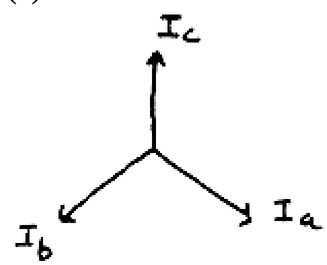
(b) The currents are equal the line currents

$$\mathbf{I}_a = \mathbf{I}_{aA} = 10\angle -30° \text{ A rms},\ \mathbf{I}_b = \mathbf{I}_{bB} = 10\angle -150° \text{ A rms}$$

and

$$\mathbf{I}_c = \mathbf{I}_{cC} = 10\angle 90° \text{ A rms}$$

(c)



(d) The power delivered to the load is $\mathbf{S} = 3.121 + j1.801$ kVA .

P 12.3-2 A balanced three-phase Y-connected supply delivers power through a three-wire plus neutral-wire circuit in a large office building to a three-phase Y-connected load. The circuit operates at 60 Hz. The phase voltages of the Y-connected source are

$$\mathbf{V}_a = 120\angle 0° \text{ V rms},\ \mathbf{V}_b = 120\angle{-120°} \text{ V rms, and } \mathbf{V}_c = 120\angle 120° \text{ V rms.}$$

Each transmission wire, including the neutral wire, has a 2-Ω resistance, and the balanced Y load has a 10-Ω resistance in series with 100 mH. Find the line voltage and the phase current at the load.

Solution:

Balanced, three-wire, Y-Y circuit:

where

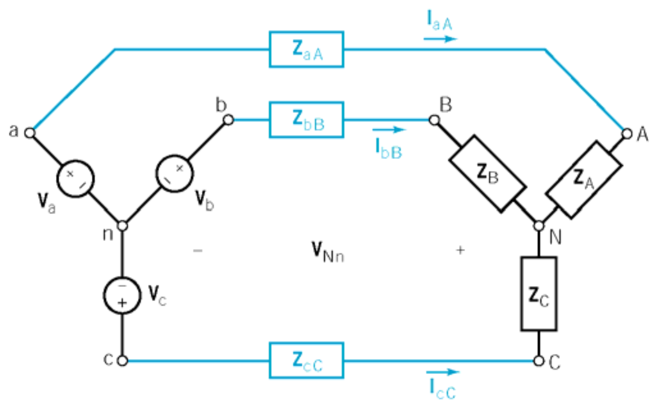
$$\mathbf{V}_a = 120\angle 0° \text{ Vrms},\ \mathbf{V}_b = 120\angle -120° \text{ Vrms}$$

$$\text{and } \mathbf{V}_c = 120\angle 120° \text{ Vrms}$$

$$\mathbf{Z}_A = \mathbf{Z}_B = \mathbf{Z}_C = 10 + j(2\times\pi\times 60)(100\times 10^{-3})$$

$$= 10 + j\,37.7\ \ \Omega$$

and $\mathbf{Z}_{aA} = \mathbf{Z}_{bB} = \mathbf{Z}_{cC} = 2\ \ \Omega$



Mathcad Analysis (12p4_2.mcd):

Describe the three-phase source: $Vp := 120$

$$Va := Vp\cdot e^{j\cdot\frac{\pi}{180}\cdot 0} \qquad Vb := Va\cdot e^{j\cdot\frac{\pi}{180}\cdot -120} \qquad Vc := Va\cdot e^{j\cdot\frac{\pi}{180}\cdot 120}$$

Describe the three-phase load: $ZA := 10 + j\cdot 37.7$ $\quad ZB := ZA$ $\quad ZC := ZB$

Describe the three-phase line: $ZaA := 2$ $\quad ZbB := ZaA$ $\quad ZcC := ZaA$

Calculate the voltage at the neutral of the load with respect to the neutral of the source:

$$VnN := \frac{(ZaA + ZA)\cdot(ZcC + ZC)\cdot e^{j\cdot\frac{4}{3}\cdot\pi} + (ZaA + ZA)\cdot(ZbB + ZB)\cdot e^{j\cdot\frac{2}{3}\cdot\pi} + (ZbB + ZB)\cdot(ZcC + ZC)}{(ZaA + ZA)\cdot(ZcC + ZC) + (ZaA + ZA)\cdot(ZbB + ZB) + (ZbB + ZB)\cdot(ZcC + ZC)}\cdot Vp$$

$$VnN = -8.693\times 10^{-15} + 2.232i\times 10^{-14} \qquad |VnN| = 2.396\times 10^{-14} \qquad \frac{180}{\pi}\cdot\arg(VnN) = 111.277$$

Calculate the line currents: $IaA := \dfrac{Va - VnN}{ZA + ZaA}$ $\quad IbB := \dfrac{Vb - VnN}{ZB + ZbB}$ $\quad IcC := \dfrac{Vc - VnN}{ZC + ZcC}$

$IaA = 0.92 - 2.89i$	$IbB = -2.963 + 0.648i$	$IcC = 2.043 + 2.242i$
$\|IaA\| = 3.033$	$\|IbB\| = 3.033$	$\|IcC\| = 3.033$
$\frac{180}{\pi}\cdot\arg(IaA) = -72.344$	$\frac{180}{\pi}\cdot\arg(IbB) = 167.656$	$\frac{180}{\pi}\cdot\arg(IcC) = 47.656$

Check: $IaA + IbB + IcC = -1.332\times 10^{-15} - 3.109i\times 10^{-15}$

Calculate the phase voltages at the load: $VA := ZA \cdot IaA$ $\quad VB := ZB \cdot IbB$ $\quad VC := ZC \cdot IcC$

$|VA| = 118.301$ $\quad |VB| = 118.301$ $\quad |VC| = 118.301$

$\frac{180}{\pi} \cdot \arg(VA) = 2.801$ $\quad \frac{180}{\pi} \cdot \arg(VB) = -117.199$ $\quad \frac{180}{\pi} \cdot \arg(VC) = 122.801$

Calculate the power delivered to the load:

$SA := \overline{IaA} \cdot IaA \cdot ZA$ $\quad SB := \overline{IbB} \cdot IbB \cdot ZB$ $\quad SC := \overline{IcC} \cdot IcC \cdot ZC$

$SA = 91.996 + 346.824i$ $\quad SB = 91.996 + 346.824i$ $\quad SC = 91.996 + 346.824i$

Consequently, the line-to-line voltages at the source are:

$$\mathbf{V}_{ab} = \mathbf{V}_a \times \sqrt{3}\angle 30° = 120\angle 0° \times \sqrt{3}\angle 30° = 208\angle 30° \text{ Vrms},$$
$$\mathbf{V}_{bc} = 208\angle -120° \text{ Vrms and } \mathbf{V}_{ca} = 208\angle 120° \text{ Vrms}$$

The line-to-line voltages at the load are:

$$\mathbf{V}_{AB} = \mathbf{V}_A \times \sqrt{3}\angle 30° = 118.3\angle 3° \times \sqrt{3}\angle 30° = 205\angle 33° \text{ Vrms},$$
$$\mathbf{V}_{bc} = 205\angle -117° \text{ Vrms and } \mathbf{V}_{ca} = 205\angle 123° \text{ Vrms}$$

and the phase currents are

$$\mathbf{I}_a = \mathbf{I}_{aA} = 10\angle -72° \text{ A rms, } \mathbf{I}_b = \mathbf{I}_{bB} = 3\angle 168° \text{ A rms and } \mathbf{I}_c = \mathbf{I}_{cC} = 3\angle 48° \text{ A rms}$$

P 12.3-3 A Y-connected source and load are shown in Figure P 12.3-3. (a) Determine the rms value of the current $i_A(t)$. (b) Determine the average power delivered to the load.

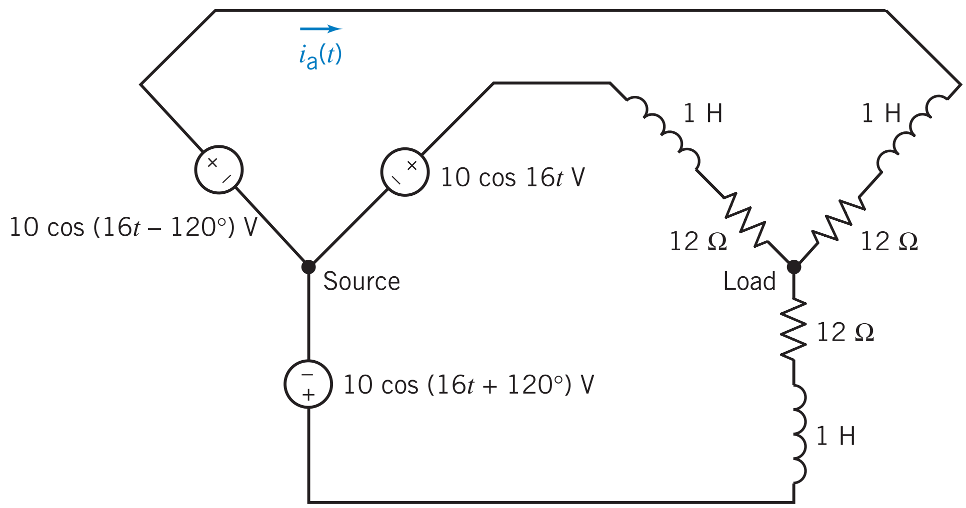
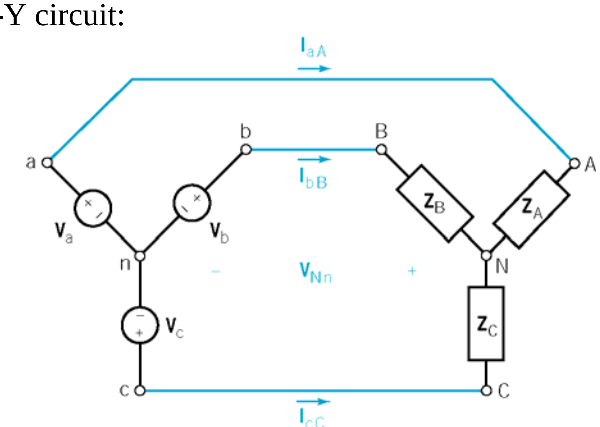


Figure P 12.3-3

Solution:
Balanced, three-wire, Y-Y circuit:



where

$$\mathbf{V}_a = 10\angle 0° \text{ V} = 7.07\angle 0° \text{ V rms}, \ \mathbf{V}_b = 7.07\angle -120° \text{ V rms and } \mathbf{V}_c = 7.07\angle 120° \text{ V rms}$$

and

$$\mathbf{Z}_A = \mathbf{Z}_B = \mathbf{Z}_C = 12 + j(16)(1) = 12 + j16 \ \Omega$$

MathCAD analysis (12p4_3.mcd):

Describe the three-phase source: $Vp := \frac{10}{\sqrt{2}}$

$$Va := Vp \cdot e^{j \cdot \frac{\pi}{180} \cdot 0} \qquad Vb := Va \cdot e^{j \cdot \frac{\pi}{180} \cdot -120} \qquad Vc := Va \cdot e^{j \cdot \frac{\pi}{180} \cdot 120}$$

Describe the balanced three-phase load: $ZA := 12 + j \cdot 16 \qquad ZB := ZA \qquad ZC := ZB$

Check: The voltage at the neutral of the load with respect to the neutral of the source should be zero:

$$VnN := \frac{ZA \cdot ZC \cdot e^{j \cdot \frac{4}{3} \cdot \pi} + ZA \cdot ZB \cdot e^{j \cdot \frac{2}{3} \cdot \pi} + ZB \cdot ZC}{ZA \cdot ZC + ZA \cdot ZB + ZB \cdot ZC} \cdot Vp \qquad |VnN| = 1.675 \times 10^{-15}$$

Calculate the line currents: $IaA := \frac{Va - VnN}{ZA} \qquad IbB := \frac{Vb - VnN}{ZB} \qquad IcC := \frac{Vc - VnN}{ZC}$

$$IaA = 0.212 - 0.283i \qquad IbB = -0.351 - 0.042i \qquad IcC = 0.139 + 0.325i$$

$$|IaA| = 0.354 \qquad |IbB| = 0.354 \qquad |IcC| = 0.354$$

$$\frac{180}{\pi} \cdot \arg(IaA) = -53.13 \qquad \frac{180}{\pi} \cdot \arg(IbB) = -173.13 \qquad \frac{180}{\pi} \cdot \arg(IcC) = 66.87$$

Calculate the power delivered to the load:

$$SA := \overline{IaA} \cdot IaA \cdot ZA \qquad SB := \overline{IbB} \cdot IbB \cdot ZB \qquad SC := \overline{IcC} \cdot IcC \cdot ZC$$

$$SA = 1.5 + 2i \qquad SB = 1.5 + 2i \qquad SC = 1.5 + 2i$$

Total power delivered to the load: $SA + SB + SC = 4.5 + 6i$

Consequently

(a) The rms value of $i_a(t)$ is 0.354 A rms.

(b) The average power delivered to the load is $P = \mathrm{Re}\{\mathbf{S}\} = \mathrm{Re}\{4.5 + j6\} = 4.5$ W

P 12.3-4 An unbalanced Y-Y circuit is shown in Figure P 12.3-4. Find the average power delivered to the load.

Hint: $\mathbf{V}_{\mathrm{Nn}}(\omega) = 27.4 \angle{-63.6}$ V

Answer: 436.4 W

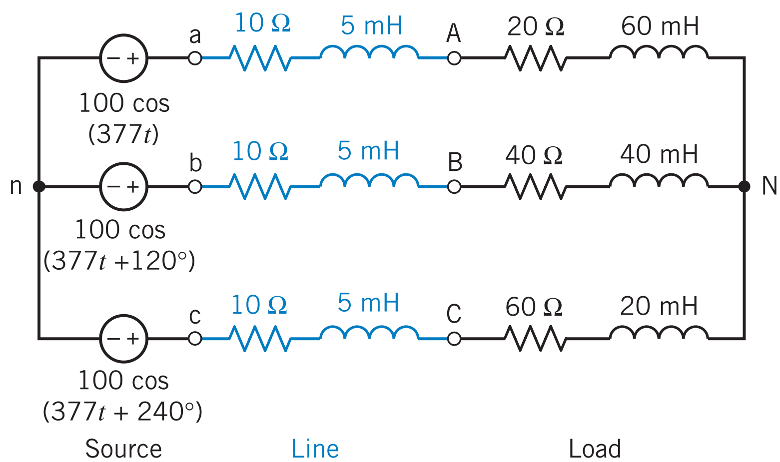
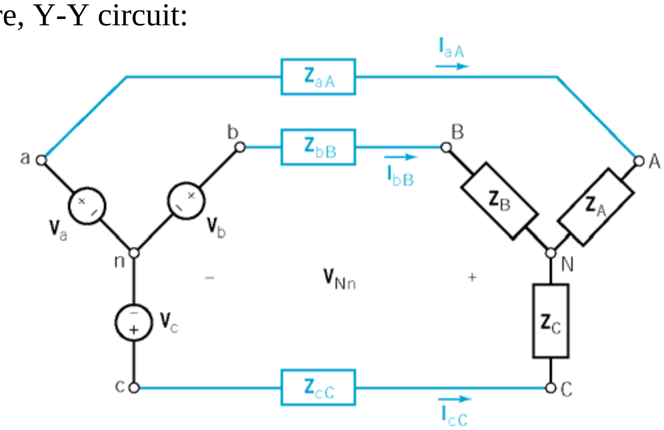


Figure P 12.3-4

Solution:
Unbalanced, three-wire, Y-Y circuit:



where

$$\mathbf{V}_{\mathrm{a}} = 100\angle 0°\ \mathrm{V} = 70.7\angle 0°\ \mathrm{V\ rms},\ \ \mathbf{V}_{\mathrm{b}} = 70.7\angle{-120°}\ \mathrm{V\ rms\ and\ } \mathbf{V}_{\mathrm{c}} = 7.07\angle 120°\ \mathrm{V\ rms}$$

$$\mathbf{Z}_{\mathrm{A}} = 20 + j(377)\left(60\times 10^{-3}\right) = 20 + j\,22.6\ \ \Omega,\ \ \mathbf{Z}_{\mathrm{B}} = 40 + j(377)\left(40\times 10^{-3}\right) = 40 + j\,15.1\ \ \Omega$$

$$\mathbf{Z}_{\mathrm{C}} = 60 + j(377)\left(20\times 10^{-3}\right) = 60 + j\,7.54\ \ \Omega$$

and

$$\mathbf{Z}_{\mathrm{aA}} = \mathbf{Z}_{\mathrm{bB}} = \mathbf{Z}_{\mathrm{cC}} = 10 + j(377)\left(5\times 10^{-3}\right) = 10 + j\,1.89\ \ \Omega$$

Mathcad Analysis (12p4_4.mcd):

Describe the three-phase source: $Vp := 100$

$$Va := Vp \cdot e^{j \cdot \frac{\pi}{180} \cdot 0} \qquad Vb := Va \cdot e^{j \cdot \frac{\pi}{180} \cdot 120} \qquad Vc := Va \cdot e^{j \cdot \frac{\pi}{180} \cdot -120}$$

Enter the frequency of the 3-phase source: $\omega := 377$

Describe the three-phase load: $ZA := 20 + j \cdot \omega \cdot 0.06$ $\quad ZB := 40 + j \cdot \omega \cdot 0.04$ $\quad ZC := 60 + j \cdot \omega \cdot 0.02$

Describe the three-phase line: $ZaA := 10 + j \cdot \omega \cdot 0.005$ $\quad ZbB := ZaA$ $\quad ZcC := ZaA$

Calculate the voltage at the neutral of the load with respect to the neutral of the source:

$$VnN := \frac{(ZaA + ZA) \cdot (ZcC + ZC) \cdot e^{j \cdot \frac{4}{3} \cdot \pi} + (ZaA + ZA) \cdot (ZbB + ZB) \cdot e^{j \cdot \frac{2}{3} \cdot \pi} + (ZbB + ZB) \cdot (ZcC + ZC)}{(ZaA + ZA) \cdot (ZcC + ZC) + (ZaA + ZA) \cdot (ZbB + ZB) + (ZbB + ZB) \cdot (ZcC + ZC)} \cdot Vp$$

$VnN = 12.209 - 24.552i$ $\qquad |VnN| = 27.42$ $\qquad \frac{180}{\pi} \cdot \arg(VnN) = -63.561$

Calculate the line currents: $IaA := \frac{Va - VnN}{ZA + ZaA}$ $\qquad IbB := \frac{Vb - VnN}{ZB + ZbB}$ $\qquad IcC := \frac{Vc - VnN}{ZC + ZcC}$

$IaA = 2.156 - 0.943i$ $\qquad IbB = -0.439 + 2.372i$ $\qquad IcC = -0.99 - 0.753i$

$|IaA| = 2.353$ $\qquad |IbB| = 2.412$ $\qquad |IcC| = 1.244$

$\frac{180}{\pi} \cdot \arg(IaA) = -23.619$ $\qquad \frac{180}{\pi} \cdot \arg(IbB) = 100.492$ $\qquad \frac{180}{\pi} \cdot \arg(IcC) = -142.741$

Calculate the power delivered to the load:

$SA := \frac{\left(\overline{IaA} \cdot IaA\right)}{2} \cdot ZA$ $\qquad SB := \frac{\left(\overline{IbB} \cdot IbB\right)}{2} \cdot ZB$ $\qquad SC := \frac{\left(\overline{IcC} \cdot IcC\right)}{2} \cdot ZC$

$SA = 55.382 + 62.637i$ $\qquad SB = 116.402 + 43.884i$ $\qquad SC = 46.425 + 5.834i$

Total power delivered to the load: $SA + SB + SC = 218.209 + 112.355i$

The average power delivered to the load is $P = \text{Re}\{\mathbf{S}\} = \text{Re}\{218.2 + j112.4\} = 218.2\ \text{W}$

P 12.3-5 A balanced Y-Y circuit is shown in Figure P 12.3-5. Find the average power delivered to the load.

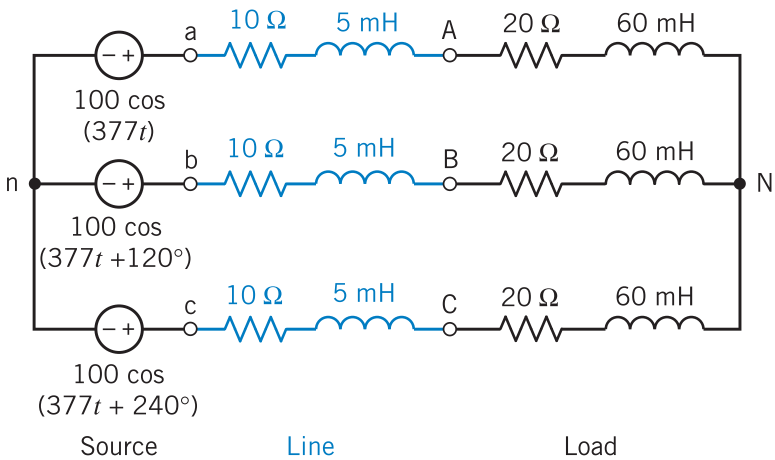
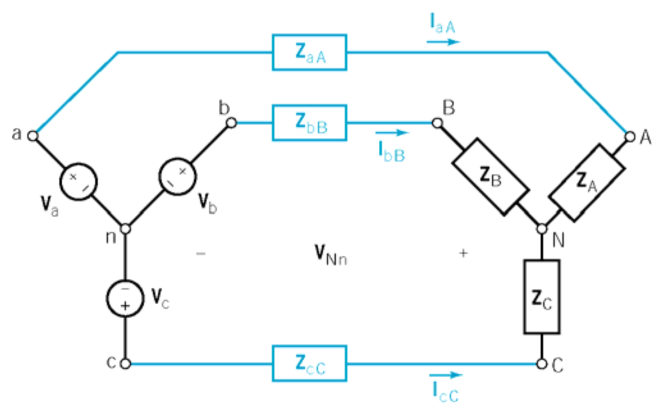


Figure P 12.3-5

Solution:
Balanced, three-wire, Y-Y circuit:



where

$$\mathbf{V}_a = 100\angle 0° \text{ V} = 70.7\angle 0° \text{ V rms}, \ \mathbf{V}_b = 70.7\angle -120° \text{ V rms and } \mathbf{V}_c = 7.07\angle 120° \text{ V rms}$$

$$\mathbf{Z}_A = \mathbf{Z}_B = \mathbf{Z}_C = 20 + j(377)\left(60\times 10^{-3}\right) = 20 + j\,22.6 \ \Omega$$

and

$$\mathbf{Z}_{aA} = \mathbf{Z}_{bB} = \mathbf{Z}_{cC} = 10 + j(377)\left(5\times 10^{-3}\right) = 10 + j\,1.89 \ \Omega$$

Mathcad Analysis (12p4_5.mcd):

Describe the three-phase source: $Vp := 100$

$Va := Vp \cdot e^{j \cdot \frac{\pi}{180} \cdot 0}$ $Vb := Va \cdot e^{j \cdot \frac{\pi}{180} \cdot 120}$ $Vc := Va \cdot e^{j \cdot \frac{\pi}{180} \cdot -120}$

Enter the frequency of the 3-phase source: $\omega := 377$

Describe the three-phase load: $ZA := 20 + j \cdot \omega \cdot 0.06$ $ZB := ZA$ $ZC := ZA$

Describe the three-phase line: $ZaA := 10 + j \cdot \omega \cdot 0.005$ $ZbB := ZaA$ $ZcC := ZaA$

Calculate the voltage at the neutral of the load with respect to the neutral of the source:

$$VnN := \frac{(ZaA + ZA)\cdot(ZcC + ZC)\cdot e^{j \cdot \frac{4}{3} \cdot \pi} + (ZaA + ZA)\cdot(ZbB + ZB)\cdot e^{j \cdot \frac{2}{3} \cdot \pi} + (ZbB + ZB)\cdot(ZcC + ZC)}{(ZaA + ZA)\cdot(ZcC + ZC) + (ZaA + ZA)\cdot(ZbB + ZB) + (ZbB + ZB)\cdot(ZcC + ZC)} \cdot Vp$$

$VnN = -8.982 \times 10^{-15} + 1.879i \times 10^{-14}$ $|VnN| = 2.083 \times 10^{-14}$ $\frac{180}{\pi} \cdot \arg(VnN) = 115.55$

Calculate the line currents: $IaA := \frac{Va - VnN}{ZA + ZaA}$ $IbB := \frac{Vb - VnN}{ZB + ZbB}$ $IcC := \frac{Vc - VnN}{ZC + ZcC}$

$IaA = 1.999 - 1.633i$ $IbB = 0.415 + 2.548i$ $IcC = -2.414 - 0.915i$

$|IaA| = 2.582$ $|IbB| = 2.582$ $|IcC| = 2.582$

$\frac{180}{\pi} \cdot \arg(IaA) = -39.243$ $\frac{180}{\pi} \cdot \arg(IbB) = 80.757$ $\frac{180}{\pi} \cdot \arg(IcC) = -159.243$

Calculate the power delivered to the load:

$SA := \frac{\left(\overline{IaA} \cdot IaA\right)}{2} \cdot ZA$ $SB := \frac{\left(\overline{IbB} \cdot IbB\right)}{2} \cdot ZB$ $SC := \frac{\left(\overline{IcC} \cdot IcC\right)}{2} \cdot ZC$

$SA = 66.645 + 75.375i$ $SB = 66.645 + 75.375i$ $SC = 66.645 + 75.375i$

Total power delivered to the load: $SA + SB + SC = 199.934 + 226.125i$

The average power delivered to the load is $P = \mathrm{Re}\{\mathbf{S}\} = \mathrm{Re}\{200 + j226\} = 200$ W

P 12.3-6 An unbalanced Y-Y circuit is shown in Figure P 12.3-6 Find the average power delivered to the load.

Hint: $\mathbf{V}_{\text{Nn}}(\omega) = 1.755\angle{-29.5}$ V

Answer: 436.4 W

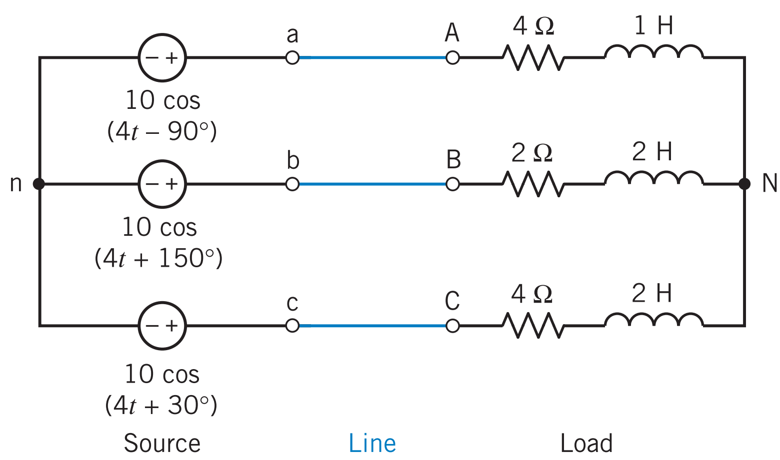
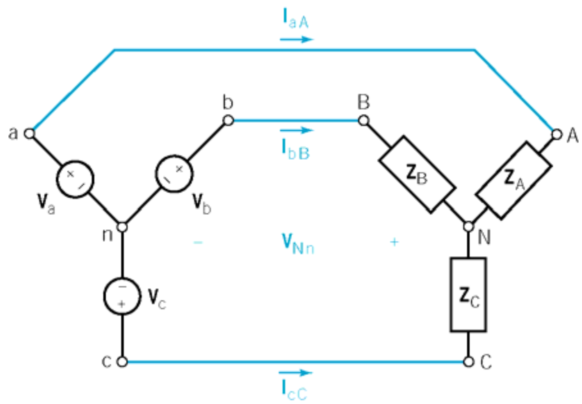


Figure P 12.3-6

Solution:

Unbalanced, three-wire, Y-Y circuit:



where

$$\mathbf{V}_{\text{a}} = 10\angle -90° \text{ V} = 7.07\angle -90° \text{ V rms, } \mathbf{V}_{\text{b}} = 7.07\angle 150° \text{ V rms and } \mathbf{V}_{\text{c}} = 7.07\angle 30° \text{ V rms}$$

and

$$\mathbf{Z}_{\text{A}} = 4 + j(4)(1) = 4 + j\,4 \;\Omega,\; \mathbf{Z}_{\text{B}} = 2 + j(4)(2) = 2 + j\,8 \;\Omega \text{ and } \mathbf{Z}_{\text{C}} = 4 + j(4)(2) = 4 + j\,8 \;\Omega$$

Mathcad Analysis (12p4_6.mcd):

Describe the three-phase source: $Vp := 10$

$Va := Vp \cdot e^{j \cdot \frac{\pi}{180} \cdot -90}$ $Vb := Vp \cdot e^{j \cdot \frac{\pi}{180} \cdot 150}$ $Vc := Vp \cdot e^{j \cdot \frac{\pi}{180} \cdot 30}$

Enter the frequency of the 3-phase source: $\omega := 4$

Describe the three-phase load: $ZA := 4 + j \cdot \omega \cdot 1$ $ZB := 2 + j \cdot \omega \cdot 2$ $ZC := 4 + j \cdot \omega \cdot 2$

Calculate the voltage at the neutral of the load with respect to the neutral of the source:

$$VnN := \frac{ZA \cdot ZC \cdot Vb + ZA \cdot ZB \cdot Vc + ZB \cdot ZC \cdot Va}{ZA \cdot ZC + ZA \cdot ZB + ZB \cdot ZC}$$

$VnN = 1.528 - 0.863i$ $|VnN| = 1.755$ $\frac{180}{\pi} \cdot \arg(VnN) = -29.466$

Calculate the line currents: $IaA := \frac{Va - VnN}{ZA}$ $IbB := \frac{Vb - VnN}{ZB}$ $IcC := \frac{Vc - VnN}{ZC}$

$IaA = -1.333 - 0.951i$ $IbB = 0.39 + 1.371i$ $IcC = 0.943 - 0.42i$

$|IaA| = 1.638$ $|IbB| = 1.426$ $|IcC| = 1.032$

$\frac{180}{\pi} \cdot \arg(IaA) = -144.495$ $\frac{180}{\pi} \cdot \arg(IbB) = 74.116$ $\frac{180}{\pi} \cdot \arg(IcC) = -24.011$

Calculate the power delivered to the load:

$SA := \frac{\left(\overline{IaA} \cdot IaA\right)}{2} \cdot ZA$ $SB := \frac{\left(\overline{IbB} \cdot IbB\right)}{2} \cdot ZB$ $SC := \frac{\left(\overline{IcC} \cdot IcC\right)}{2} \cdot ZC$

$SA = 5.363 + 5.363i$ $SB = 2.032 + 8.128i$ $SC = 2.131 + 4.262i$

Total power delivered to the load: $SA + SB + SC = 9.527 + 17.754i$

The average power delivered to the load is $P = \text{Re}\{\mathbf{S}\} = \text{Re}\{9.527 + j17.754\} = 9.527 \text{ W}$

P 12.3-7 A balanced Y-Y circuit is shown in Figure P 12.3-7. Find the average power delivered to the load.

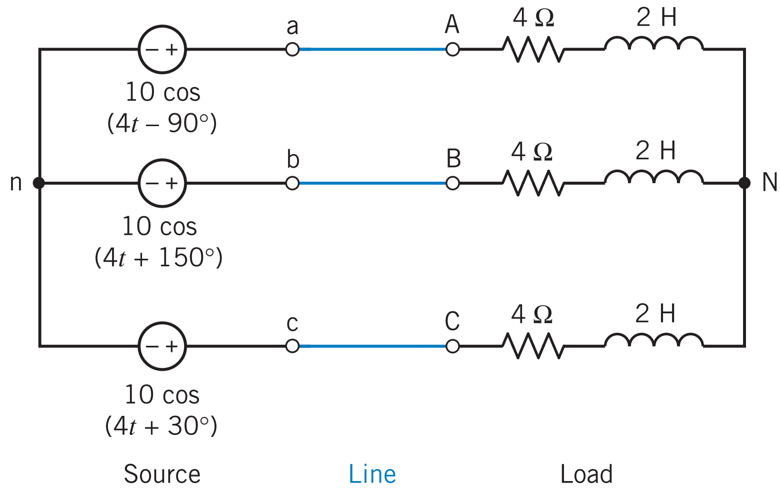
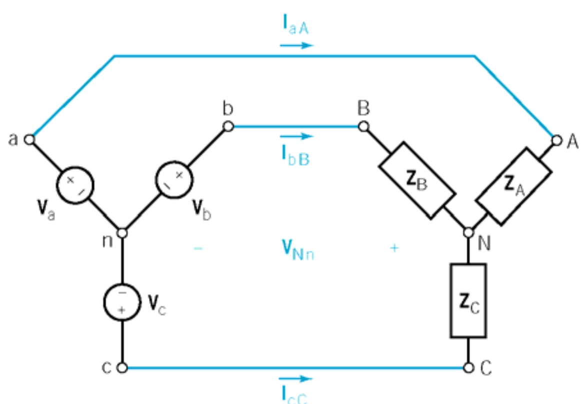


Figure P 12.3-7

Solution:
Balanced, three-wire, Y-Y circuit:



where

$$\mathbf{V}_a = 10\angle -90° \text{ V} = 7.07\angle -90° \text{ V rms}, \ \mathbf{V}_b = 7.07\angle 150° \text{ V rms and } \mathbf{V}_c = 7.07\angle 30° \text{ V rms}$$

and

$$\mathbf{Z}_A = \mathbf{Z}_B = \mathbf{Z}_C = 4 + j(4)(2) = 4 + j\,8 \ \Omega$$

Mathcad Analysis (12p4_7.mcd):

Describe the three-phase source: $Vp := 10$

$$Va := Vp \cdot e^{j \cdot \frac{\pi}{180} \cdot -90} \qquad Vb := Vp \cdot e^{j \cdot \frac{\pi}{180} \cdot 150} \qquad Vc := Vp \cdot e^{j \cdot \frac{\pi}{180} \cdot 30}$$

Enter the frequency of the 3-phase source: $\omega := 4$

Describe the three-phase load: $ZA := 4 + j \cdot \omega \cdot 2$ $\qquad ZB := ZA$ $\qquad ZC := ZA$

The voltage at the neutral of the load with respect to the neutral of the source should be zero:

$$VnN := \frac{ZA \cdot ZC \cdot Vb + ZA \cdot ZB \cdot Vc + ZB \cdot ZC \cdot Va}{ZA \cdot ZC + ZA \cdot ZB + ZB \cdot ZC} \qquad |VnN| = 1.517 \times 10^{-15}$$

Calculate the line currents: $IaA := \frac{Va - VnN}{ZA}$ $\qquad IbB := \frac{Vb - VnN}{ZB}$ $\qquad IcC := \frac{Vc - VnN}{ZC}$

$IaA = -1 - 0.5i$ $\qquad IbB = 0.067 + 1.116i$ $\qquad IcC = 0.933 - 0.616i$

$|IaA| = 1.118$ $\qquad |IbB| = 1.118$ $\qquad |IcC| = 1.118$

$\frac{180}{\pi} \cdot \arg(IaA) = -153.435$ $\qquad \frac{180}{\pi} \cdot \arg(IbB) = 86.565$ $\qquad \frac{180}{\pi} \cdot \arg(IcC) = -33.435$

Calculate the power delivered to the load:

$SA := \frac{\left(\overline{IaA} \cdot IaA\right)}{2} \cdot ZA$ $\qquad SB := \frac{\left(\overline{IbB} \cdot IbB\right)}{2} \cdot ZB$ $\qquad SC := \frac{\left(\overline{IcC} \cdot IcC\right)}{2} \cdot ZC$

$SA = 2.5 + 5i$ $\qquad SB = 2.5 + 5i$ $\qquad SC = 2.5 + 5i$

Total power delivered to the load: $SA + SB + SC = 7.5 + 15i$

The average power delivered to the load is $P = \text{Re}\{\mathbf{S}\} = \text{Re}\{7.5 + j15\} = 7.5 \text{ W}$

Section 12-4: The Δ- Connected Source and Load

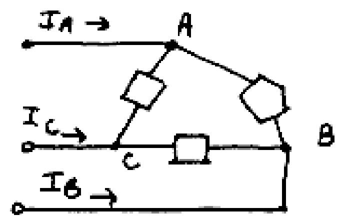
P 12.4-1 A balanced three-phase Δ-connected load has one line current:

$$\mathbf{I}_B = 50\ \angle{-40°}\ \text{A rms}$$

Find the phase currents $\mathbf{I}_{BC}$, $\mathbf{I}_{AB}$, and $\mathbf{I}_{CA}$. Draw the phasor diagram showing the line and phase currents. The source uses the *abc* phase sequence.

Solution:

Given $\mathbf{I}_B = 50\angle -40°$ A rms and assuming the *abc* phase sequence we have



$$\mathbf{I}_A = 50\angle 80°\ \text{A rms} \quad \text{and} \quad \mathbf{I}_C = 50\angle 200°\ \text{A rms}$$

From Eqn 12.6-4

$$\mathbf{I}_A = \mathbf{I}_{AB} \times \sqrt{3}\angle -30° \quad \Rightarrow \quad \mathbf{I}_{AB} = \frac{\mathbf{I}_A}{\sqrt{3}\angle -30°}$$

so

$$I_{AB} = \frac{50\angle 80°}{\sqrt{3}\angle -30°} = 28.9\angle 110°\ \text{A rms}$$

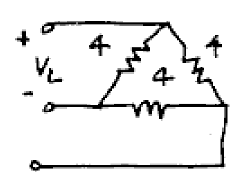
$$I_{BC} = 28.9\angle -10°\ \text{A rms} \quad \text{and} \quad I_{CA} = 28.9\angle -130°\ \text{A rms}$$

P 12.4-2 A three-phase circuit has two parallel balanced Δ loads, one of 5-*Ω* resistors and one of 20-*Ω* resistors. Find the magnitude of the total line current when the line-to-line voltage is 480 V rms.

Solution:

The two delta loads connected in parallel are equivalent to a single delta load with

$$\mathbf{Z}_\Delta = 5 \,||\, 20 = 4\ \ \Omega$$



The magnitude of phase current is

$$I_p = \frac{480}{4} = 120\ \text{A rms}$$

The magnitude of line current is

$$I_L = \sqrt{3}\, I_p = 208\ \text{A rms}$$

Section 12-5: The Y- to Δ- Circuit

P 12.5-1 Consider a three-wire Y-to-Δ circuit. The voltages of the Y-connected source are $\mathbf{V}_a = (208/\sqrt{3})\angle{-30°}$V rms, $\mathbf{V}_b = (208/\sqrt{3})\angle{-150°}$ V rms, and $\mathbf{V}_c = (208/\sqrt{3})\angle 90°$V rms.

The Δ-connected load is balanced. The impedance of each phase is $\mathbf{Z} = 12\angle 30°\ \Omega$. Determine the line currents and calculate the power dissipated in the load.
Answer: $P = 9360$ W

Solution:
We have a delta load with $\mathbf{Z} = 12\angle 30°$. One phase current is

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}} = \frac{\mathbf{V}_A - \mathbf{V}_B}{\mathbf{Z}} = \frac{\left(\frac{208}{\sqrt{3}}\angle{-30°}\right) - \left(\frac{208}{\sqrt{3}}\angle{-150°}\right)}{12\angle 30°} = \frac{208\angle 0°}{12\angle 30°} = 17.31\angle -30° \text{ A rms}$$

The other phase currents are

$$\mathbf{I}_{BC} = 17.31\angle -150° \text{ A rms and } \mathbf{I}_{CA} = 17.31\angle 90° \text{ A rms}$$

One line currents is

$$\mathbf{I}_A = \mathbf{I}_{AB} \times \sqrt{3}\angle -30° = (17.31\angle -30°) \times \left(\sqrt{3}\angle -30°\right) = 30\angle -60° \text{ A rms}$$

The other line currents are

$$\mathbf{I}_B = 30\angle -180° \text{ A rms and } \mathbf{I}_C = 30\angle 60° \text{ A rms}$$

The power delivered to the load is

$$P = 3\left(\frac{208}{\sqrt{3}}\right)(30)\cos\left(60° - 30°\right) = 9360 \text{ W}$$

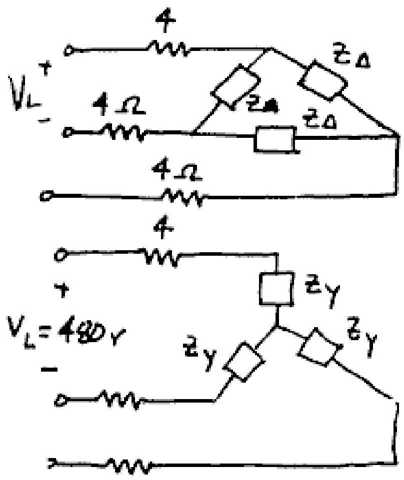
P 12.5-2 A balanced Δ-connected load is connected by three wires, each with a 4-*Ω* resistance, to a Y source with

$\mathbf{V}_a = (480/\sqrt{3})\angle{-30°}$ V rms, $\mathbf{V}_b = (480/\sqrt{3})\angle{-150°}$ V rms, and $\mathbf{V}_c = (480/\sqrt{3})\angle{-90°}$ V rms.

Find the line current $\mathbf{I}_A$ when $\mathbf{Z}_\Delta = 39\angle{-40°}\ \Omega$.

Answer: $\mathbf{I}_A = 17\angle 0.9$ A

Solution:



The balanced delta load with $\mathbf{Z}_\Delta = 39\angle{-40°}\ \ \Omega$ is equivalent to a balanced Y load with

$$\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3} = 13\angle -40^\circ = 9.96 - j8.36\ \ \Omega$$

$$\mathbf{Z}_T = \mathbf{Z}_Y + 4 = 13.96 - j8.36 = 16.3\angle -30.9\ \ \Omega$$

then $$\mathbf{I}_A = \frac{\frac{480}{\sqrt{3}}\angle{-30^\circ}}{16.3\angle{-30.9^\circ}} = 17\angle 0.9^\circ\ \text{ A rms}$$

P 12.5-3 The balanced circuit shown in Figure P 12.5-3 has $\mathbf{V}_{ab} = 380\ \angle 30°$V rms. Determine the phase currents in the load when $\mathbf{Z} = 3 + j4\ \Omega$. Sketch a phasor diagram.

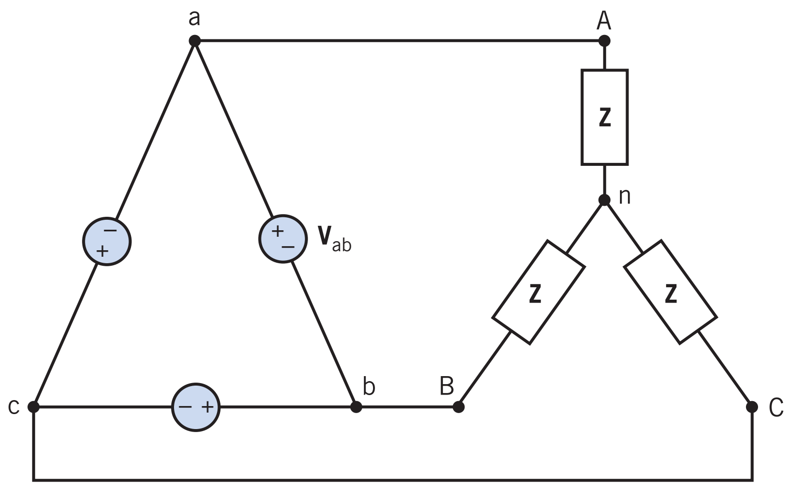
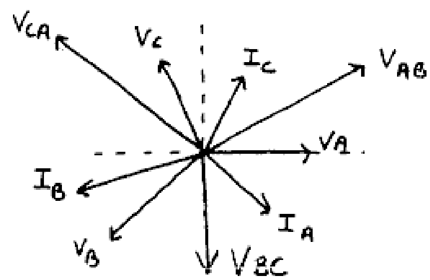


Figure P 12.5-3

Solution:



$$\mathbf{V}_{ab} = \mathbf{V}_a \times \sqrt{3}\angle 30° \quad \Rightarrow \quad \mathbf{V}_a = \frac{\mathbf{V}_{ab}}{\sqrt{3}\angle 30°}$$

In our case, the given line-to-line voltage is

$$\mathbf{V}_{ab} = 380\ \angle 30°\ \text{V rms}$$

So one phase voltage is $\mathbf{V}_a = \dfrac{380\ \angle 30°}{\sqrt{3}\angle 30°} = 200\angle 0°\ \ \text{V rms}$

So

$$V_{AB} = 380\angle 30°\ \text{V rms} \qquad V_A = 220\angle 0°\ \text{V rms}$$

$$V_{BC} = 380\angle\text{-}90°\ \text{V rms} \qquad V_B = 220\angle -120°\ \text{V rms}$$

$$V_{CA} = 380\angle 150°\ \text{V rms} \qquad V_C = 220\angle 120°\ \text{V rms}$$

One phase current is

$$\mathbf{I}_A = \frac{\mathbf{V}_A}{\mathbf{Z}}\ \frac{220\angle 0°}{3+j4} = 44\angle -53.1°\ \ \text{A rms}$$

The other phase currents are

$$\mathbf{I}_B = 44\angle -173.1°\ \ \text{A rms amd } \mathbf{I}_C = 44\angle 66.9°\ \ \text{A rms}$$

P 12.5-4 The balanced circuit shown in Figure P 12.5-3 has $\mathbf{V}_{ab}$ = 380 ∠0° V rms. Determine the line and phase currents in the load when **Z** = 9 + *j*12 *Ω*.

Solution:

$$\mathbf{V}_{ab} = \mathbf{V}_a \times \sqrt{3}\angle 30° \quad \Rightarrow \quad \mathbf{V}_a = \frac{\mathbf{V}_{ab}}{\sqrt{3}\angle 30°}$$

In our case, the given line-to-line voltage is

$$\mathbf{V}_{ab} = 380\ \angle 0°\ \text{V rms}$$

So one phase voltage is $\mathbf{V}_a = \dfrac{380\ \angle 0°}{\sqrt{3}\angle 30°} = 200\angle -30°\ \ \text{V rms}$

So

$$V_{ab} = 380\angle 0°\ \text{V rms} \qquad V_a = 220\angle -30°\ \text{V rms}$$

$$V_{bc} = 380\angle\text{-}120°\ \text{V rms} \qquad V_b = 220\angle -150°\ \text{V rms}$$

$$V_{ca} = 380\angle 120°\ \text{V rms} \qquad V_c = 220\angle 90°\ \text{V rms}$$

One phase current is

$$\mathbf{I}_A = \frac{\mathbf{V}_a}{\mathbf{Z}} = \frac{220\angle -30°}{9+j12} = 14.67\angle -83.1°\ \ \text{A rms}$$

The other phase currents are

$$\mathbf{I}_B = 14.67\angle -203.1°\ \ \text{A rms and } \mathbf{I}_C = 14.67\angle 36.9°\ \ \text{A rms}$$

Section 12-6: Balanced Three-Phase Circuits

P 12.6-1 The English Channel Tunnel rail link is supplied at 25 kV rms from the United Kingdom and French grid systems. When there is a grid supply failure, each end is capable of supplying the whole tunnel but in a reduced operational mode.

The tunnel traction system is a conventional catenary (overhead wire) system similar to the surface main line electric railway system of the United Kingdom and France. What makes the tunnel traction system different and unique is the high density of traction load and the end-fed supply arrangement. The tunnel traction load is considerable. For each half tunnel, the load is 180 MVA (Barnes and Wong, 1991).

Assume that each line-to-line voltage of the Y-connected source is 25 kV rms and the three-phase system is connected to the traction motor of an electric locomotive. The motor is a Y-connected load with $\mathbf{Z} = 150\ \angle 25°\ \Omega$. Find the line currents and the power delivered to the traction motor.

Solution:

$$\mathbf{V}_a = \frac{25}{\sqrt{3}} \times 10^3 \angle 0° \text{ Vrms}$$

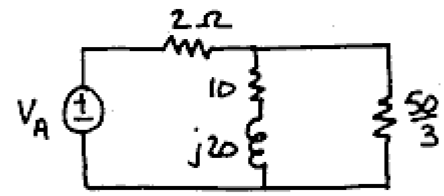
$$\mathbf{I}_A = \frac{\mathbf{V}_a}{\mathbf{Z}} = \frac{\frac{25}{\sqrt{3}} \times 10^3 \angle 0°}{150 \angle 25°} = 96 \angle -25° \text{ A rms}$$

$$P = 3\left|\mathbf{V}_a\right|\left|\mathbf{I}_A\right| \cos\left(\theta_V - \theta_I\right) = 3\left(\frac{25}{\sqrt{3}} \times 10^3\right) 96 \cos(0-25°) = \underline{3.77 \text{ MW}}$$

P 12.6-2 A three-phase source with a line voltage of 45 kV rms is connected to two balanced loads. The Y-connected load has **Z** = 10 + *j*20 *Ω*, and the Δ load has a branch impedance of 50 *Ω*. The connecting lines have an impedance of 2 *Ω*. Determine the power delivered to the loads and the power lost in the wires. What percentage of power is lost in the wires?

Solution:

Convert the delta load to an equivalent Y connected load:



$$\mathbf{Z}_\Delta = 50\ \Omega \quad \mathbf{Z}_\Delta \Rightarrow \hat{\mathbf{Z}}_Y = {}^{50}\!/_{3}\ \Omega$$

To get the per-phase equivalent circuit shown to the right:

The phase voltage of the source is

$$\mathbf{V}_a = \frac{45\times10^3}{\sqrt{3}}\angle 0° = 26\angle 0°\text{ kV rms}$$

The equivalent impedance of the load together with the line is

$$\mathbf{Z}_{eq} = \frac{(10+j20)\dfrac{50}{3}}{10+j20+\dfrac{50}{3}} + 2 = 12 + j5 = 13\angle 22.6°\ \ \Omega$$

The line current is

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_a}{\mathbf{Z}_{eq}} = \frac{26\times10^3\angle 0°}{13\angle 22.6°} = 2000\angle -22.6°\text{ A rms}$$

The power delivered to the parallel loads (per phase) is

$$P_{\text{Loads}} = |\mathbf{I}_{aA}|^2 \times \text{Re}\left\{\frac{(10+j20)\dfrac{50}{3}}{10+j20+\dfrac{50}{3}}\right\} = 4\times10^6\times10 = 40\ \text{ MW}$$

The power lost in the line (per phase) is

$$P_{\text{Line}} = |\mathbf{I}_{aA}|^2 \times \text{Re}\{\mathbf{Z}_{\text{Line}}\} = 4\times10^6\times 2 = 8\ \text{ MW}$$

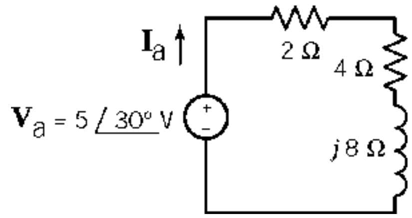
The percentage of the total power lost in the line is

$$\frac{P_{\text{Line}}}{P_{\text{Load}}+P_{\text{Line}}}\times100\% = \frac{8}{40+8}\times100\% = \underline{16.7\%}$$

P 12.6-3 A balanced three-phase source has a Y-connected source with $v_a = 5\cos(2t + 30°)$ connected to a three-phase Y load. Each phase of the Y-connected load consists of a 4-Ω resistor and a 4-H inductor. Each connecting line has a resistance of 2 Ω. Determine the total average power delivered to the load.

Solution:

$$\mathbf{I}_a = \frac{\mathbf{V}_a}{\mathbf{Z}_T} = \frac{5\angle 30°}{6+j8} = 0.5\angle -23° \text{ A} \quad \therefore \left|\mathbf{I}_a\right| = 0.5 \text{ A}$$



$$P_{\text{Load}} = 3\left|\frac{\mathbf{I}_a}{\sqrt{2}}\right|^2 \text{Re}\{\mathbf{Z}_{\text{Load}}\} = 3\times 0.125\times 4 = 1.5 \text{ W}$$

also (but not required) :

$$P_{\text{Source}} = 3\frac{(5)(0.5)}{2}\cos(-30-23) = 2.25 \text{ W}$$

$$P_{\text{line}} = 3\left|\frac{\mathbf{I}_a}{\sqrt{2}}\right|^2 \text{Re}\{\mathbf{Z}_{\text{Line}}\} = 3\times 0.125\times 2 = 0.75 \text{ W}$$

Section 12.7 Instantaneous and Average Power in a Balanced Three-Phase Load

P 12.7-1 Find the power absorbed by a balanced three-phase Y-connected load when

$$\mathbf{V}_{CB} = 208 \angle 15° \text{ V rms} \quad \text{and} \quad \mathbf{I}_B = 3 \angle 110° \text{ A rms}$$

The source uses the *abc* phase sequence.
Answer: $P = 620$ W

Solution:
Assuming the *abc* phase sequence:

$$\mathbf{V}_{CB} = 208\angle 15° \text{ V rms} \Rightarrow \mathbf{V}_{BC} = 208\angle 195° \text{ V rms} \Rightarrow \mathbf{V}_{AB} = 208\angle 315° \text{ V rms}$$

Then

$$\mathbf{V}_A = \frac{\mathbf{V}_{AB}}{\sqrt{3}\angle 30°} = \frac{208\angle 315°}{\sqrt{3}\angle 30°} = \frac{208}{\sqrt{3}}\angle 285° \text{ V rms}$$

also

$$\mathbf{I}_B = 3\angle 110° \text{ A rms} \Rightarrow \mathbf{I}_A = 3\angle 230° \text{ A rms}$$

Finally

$$P = 3|\mathbf{V}_A||\mathbf{I}_A|\cos(\theta_V - \theta_I) = 3(\frac{208}{\sqrt{3}})(3)\cos(285° - 230°) = \underline{620 \text{ W}}$$

P 12.7-2 A three-phase motor delivers 20 hp operating from a 480-V rms line voltage. The motor operates at 85 percent efficiency with a power factor equal to 0.8 lagging. Find the magnitude and angle of the line current for phase A.
Hint: 1 hp = 745.7 W

Solution:
Assuming a lagging power factor:

$$\cos\theta = pf = 0.8 \Rightarrow \theta = 36.9°$$

The power supplied by the three-phase source is given by

$$P_{in} = \frac{P_{out}}{\eta} = \frac{20(745.7)}{0.85} = 17.55 \text{ kW where } 1 \text{ hp} = 745.7 \text{ W}$$

$$P_{in} = 3|\mathbf{I}_A||\mathbf{V}_A|pf \Rightarrow |\mathbf{I}_A| = \frac{P_{in}}{3|\mathbf{V}_A|pf} = \frac{17.55\times 10^3}{3\left(\frac{480}{\sqrt{3}}\right)(0.8)} = 26.4 \text{ A rms}$$

$$\mathbf{I}_A = 26.4\angle -36.9° \text{ A rms when } \mathbf{V}_A = \frac{480}{\sqrt{3}}\angle 0° \text{ V rms}$$

P 12.7-3 A three-phase balanced load is fed by a balanced Y-connected source with a line-to-line voltage of 220 V rms. It absorbs 1500 W at 0.8 power factor lagging. Calculate the phase impedance if it is (a) Δ-connected and (b) Y-connected.

Solution:

(a) For a Δ-connected load, Eqn 12.7-5 gives

$$P_T = 3|\mathbf{V}_P||\mathbf{I}_L|\,pf \qquad \Rightarrow |\mathbf{I}_L| = \frac{P_T}{3|\mathbf{V}_P|pf} = \frac{1500}{3(\frac{220}{\sqrt{3}})(.8)} = 4.92 \text{ A rms}$$

The phase current in the Δ-connected load is given by

$$\mathbf{I}_P = \frac{\mathbf{I}_L}{\sqrt{3}} \quad \Rightarrow \quad |\mathbf{I}_P| = \frac{|\mathbf{I}_L|}{\sqrt{3}} = \frac{4.92}{\sqrt{3}} = 2.84 \text{ A rms}$$

The phase impedance is determined as:

$$\mathbf{Z} = \frac{\mathbf{V}_L}{\mathbf{I}_P} = \frac{|\mathbf{V}_L|}{|\mathbf{I}_P|}\angle(\theta_V - \theta_I) = \frac{|\mathbf{V}_L|}{|\mathbf{I}_P|}\angle\cos^{-1} pf = \frac{220}{2.84}\angle\cos^{-1} 0.8 = 77.44\angle 36.9^\circ \;\; \Omega$$

(b) For a Y-connected load, Eqn 12.7-4 gives

$$P_T = 3|\mathbf{V}_P||\mathbf{I}_L|\,pf \Rightarrow |\mathbf{I}_L| = \frac{P_T}{3|\mathbf{V}_P||\mathbf{I}_L|pf} = \frac{1500}{3(\frac{220}{\sqrt{3}})(.8)} = 4.92 \text{ A rms}$$

The phase impedance is determined as:

$$\mathbf{Z} = \frac{\mathbf{V}_P}{\mathbf{I}_P} = \frac{|\mathbf{V}_P|}{|\mathbf{I}_P|}\angle(\theta_V - \theta_I) = \frac{|\mathbf{V}_P|}{|\mathbf{I}_P|}\angle\cos^{-1} pf = \frac{\frac{220}{\sqrt{3}}}{4.92}\angle\cos^{-1} 0.8 = 25.8\angle 36.9^\circ \;\; \Omega$$

P 12.7-4 A 600-V rms three-phase Y-connected source has two balanced Δ loads connected to the lines. The load impedances are 40 ∠30° *Ω* and 50 ∠–60° *Ω*, respectively. Determine the line current and the total average power.

Solution:

Parallel Δ loads

$$\mathbf{Z}_\Delta = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1+\mathbf{Z}_2} = \frac{(40\angle 30^\circ)\,(50\angle -60^\circ)}{40\angle 30^\circ + 50\angle -60^\circ} = 31.2\ \angle -8.7^\circ\ \Omega$$

$$\mathbf{V}_\text{L} = \mathbf{V}_\text{P},\ \ |\mathbf{I}_\text{P}| = \frac{|\mathbf{V}_\text{P}|}{|Z_\Delta|} = \frac{600}{31.2} = 19.2\text{ A rms},\quad |\mathbf{I}_\text{L}| = \sqrt{3}\ |\mathbf{I}_\text{P}| = 33.3\text{ A rms}$$

So $P = \sqrt{3}\ |\mathbf{V}_\text{L}||\mathbf{I}_\text{L}|\,pf = \sqrt{3}\ (600)\ (33.3)\cos(-8.7^\circ) = \underline{34.2\ \text{kW}}$

P 12.7-5 A three-phase Y-connected source simultaneously supplies power to two separate balanced three-phase loads. The first total load is Δ connected and requires 39 kVA at 0.7 lagging. The second total load is Y connected and requires 15 kW at 0.21 leading. Each line has an impedance 0.038 + *j*0.072 *Ω*/phase. Calculate the line-to-line source voltage magnitude required so that the loads are supplied with 208-V rms line-to-line.

Solution:

We will use

$$\mathbf{S} = |\mathbf{S}|\angle\theta = |\mathbf{S}|\cos\theta + j|\mathbf{S}|\sin\theta = |\mathbf{S}|\,pf + j|\mathbf{S}|\sin\left(\cos^{-1} pf\right)$$

In our case:

$$\tilde{\mathbf{S}}_1 = 39\,(0.7) + j39\sin\left(\cos^{-1}(0.7)\right) = 27.3 + j\ 27.85\ \text{kVA}$$

$$\tilde{\mathbf{S}}_2 = 15 + \frac{15}{0.21}\sin\left(\cos^{-1}(0.21)\right) = 15 - j\ 69.84\ \text{kVA}$$

$$\tilde{\mathbf{S}}_{3\phi} = \tilde{\mathbf{S}}_1 + \tilde{\mathbf{S}}_2 = 42.3 - j\ 42.0\ \text{kVA} \Rightarrow \tilde{\mathbf{S}}_\phi = \frac{\tilde{\mathbf{S}}_{3\phi}}{3} = 14.1 - j14.0\ \text{kVA}$$

The line current is

$$\mathbf{S} = \mathbf{V}_\text{p}\mathbf{I}_\text{L}^* \quad\Rightarrow\quad \tilde{\mathbf{I}}_\text{L} = \left(\frac{\mathbf{S}}{\mathbf{V}_\text{p}}\right)^* = \frac{(14100 + j\ 14000)}{\frac{208}{\sqrt{3}}} = 117.5 + j\ 116.7\text{ A rms} = 167\ \angle 45^\circ\text{ A rms}$$

The phase voltage at the load is required to be $\frac{208}{\sqrt{3}}\angle 0^\circ = 120\angle 0^\circ$ V rms. The source must provide this voltage plus the voltage dropped across the line, therefore

$$\tilde{\mathbf{V}}_{S\phi} = 120\angle 0^\circ + (0.038 + j\ 0.072)(117.5 + j116.7) = 115.9 + j\ 12.9 = 116.6\ \angle 6.4^\circ\text{ V rms}$$

Finally

$$\left|\tilde{\mathbf{V}}_{S\phi}\right| = 116.6\ \text{ V rms}$$

P 12.7-6 A building is supplied by a public utility at 4.16 kV rms. The building contains three balanced loads connected to the three-phase lines:

(a) Δ-connected, 500 kVA at 0.85 lagging
(b) Y-connected, 75 kVA at 0.0 leading
(c) Y connected; each phase with a 150-*Ω* resistor parallel to a 225-*Ω* inductive reactance

The utility feeder is five miles long with an impedance per phase of $1.69 + j0.78$ *Ω*/mile. At what voltage must the utility supply its feeder so that the building is operating at 4.16 kV rms?

Hint: 41.6 kV is the line-to-line voltage of the balanced Y-connected source.

Solution:

The required phase voltage at the load is $\mathbf{V}_P = \dfrac{4.16}{\sqrt{3}}\angle 0° = 2.402\angle 0° \text{ kVrms}$.

Let $\mathbf{I}_1$ be the line current required by the Δ-connected load. The apparent power per phase required by the Δ-connected load is $|\mathbf{S}_1| = \dfrac{500 \text{ kVA}}{3} = 167 \text{ kVA}$. Then

$$\mathbf{S}_1 = |\mathbf{S}_1|\angle\theta = |\mathbf{S}_1|\angle\cos^{-1}(pf) = 167\angle\cos^{-1}(0.85) = 167\angle 31.8° \text{ kVA}$$

and

$$\mathbf{S}_1 = \mathbf{V}_P\mathbf{I}_1^* \quad\Rightarrow\quad \mathbf{I}_1 = \left(\frac{\mathbf{S}_1}{\mathbf{V}_P}\right)^* = \left(\frac{(167\times10^3)\angle 31.8°}{(2.402\times10^3)\angle 0°}\right)^* = 69.6\angle -31.8° = 59 - j36.56 \text{ A rms}$$

Let $\mathbf{I}_2$ be the line current required by the first Y-connected load. The apparent power per phase required by this load is $|\mathbf{S}_2| = \dfrac{75 \text{ kVA}}{3} = 25 \text{ kVA}$. Then, noticing the leading power factor,

$$\mathbf{S}_2 = |\mathbf{S}_2|\angle\theta = |\mathbf{S}_2|\angle\cos^{-1}(pf) = 25\angle\cos^{-1}(0) = 25\angle -90° \text{ kVA}$$

and

$$\mathbf{S}_2 = \mathbf{V}_P\mathbf{I}_2^* \quad\Rightarrow\quad \mathbf{I}_2 = \left(\frac{\mathbf{S}_2}{\mathbf{V}_P}\right)^* = \left(\frac{(25\times10^3)\angle -90°}{(2.402\times10^3)\angle 0°}\right)^* = 10.4\angle 90° = j10.4 \text{ A rms}$$

Let $\mathbf{I}_3$ be the line current required by the other Y-connected load. Use Ohm's law to determine $\mathbf{I}_3$ to be

$$\mathbf{I}_3 = \frac{2402\angle 0°}{150} + \frac{2402\angle 0°}{j\,225} = 16 - j\,10.7 \text{ A rms}$$

The line current is

$$\mathbf{I}_L = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 = 75 - j\,36.8 \text{ A rms}$$

The phase voltage at the load is required to be $\mathbf{V}_P = \frac{4.16}{\sqrt{3}}\angle 0° = 2.402\angle 0°$ kVrms .The source must provide this voltage plus the voltage dropped across the line, therefore

$$\mathbf{V}_{S\phi} = 2402\angle 0^{\circ} + (8.45 + j\ 3.9)\ (75 - j\ 36.8) = 3179\ \angle{-0.3°}\ \text{Vrms}$$

Finally

$$\left|\mathbf{V}_{SL}\right| = \sqrt{3}\ (3179) = \underline{5506\ \ \text{Vrms}}$$

P 12.7-7 The diagram shown in Figure P 12.7-7 has two three-phase loads that form part of a manufacturing plant. They are connected in parallel and require 4.16 kV rms. Load 1 is 1.5 MVA, 0.75 lag *pf*, Δ-connected. Load 2 is 2 MW, 0.8 lagging *pf*, Y-connected. The feeder from the power utility's substation transformer has an impedance of 0.4 + *j*0.8 *Ω*/phase. Determine the following:

(a) The required magnitude of the line voltage at the supply.
(b) The real power drawn from the supply.
(c) The percentage of the real power drawn from the supply that is consumed by the loads.

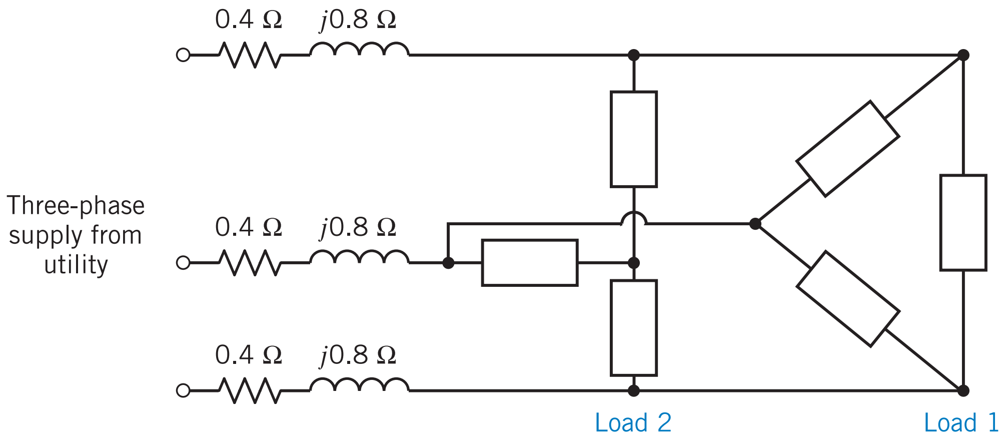


Figure P 12.7-7

Solution:

The required phase voltage at the load is $\mathbf{V}_\mathrm{P} = \dfrac{4.16}{\sqrt{3}}\angle 0° = 2.402\angle 0°$ kVrms .

Let $\mathbf{I}_1$ be the line current required by the Δ-connected load. The apparent power per phase required by the Δ-connected load is $\left|\mathbf{S}_1\right| = \dfrac{1.5 \text{ MVA}}{3} = 0.5$ MVA . Then

$$\mathbf{S}_1 = \left|\mathbf{S}_1\right|\angle\theta = \left|\mathbf{S}_1\right|\angle\cos^{-1}\left(pf\right) = 0.5\angle\cos^{-1}\left(0.75\right) = 0.5\angle 41.4° \text{ MVA}$$

and

$$\mathbf{S}_1 = \mathbf{V}_\mathrm{P}\mathbf{I}_1^* \quad\Rightarrow\quad \mathbf{I}_1 = \left(\frac{\mathbf{S}_1}{\mathbf{V}_\mathrm{P}}\right)^* = \left(\frac{\left(0.5\times 10^6\right)\angle 41.4°}{\left(2.402\times 10^3\right)\angle 0°}\right)^* = 2081.6\angle -41.4° = 1561.4 - j1376.6 \text{ A rms}$$

Let $\mathbf{I}_2$ be the line current required by the first Y-connected load. The complex power, **per phase**, is

$$\mathbf{S}_2 = 0.67 + \frac{0.67}{0.8}\sin\left(\cos^{-1}\left(0.8\right)\right) = 0.67 + j\ 0.5 \text{ MVA}$$

$$\mathbf{I}_2 = \left(\frac{\mathbf{S}_2}{\mathbf{V}_P}\right)^* = \left(\frac{(0.67 + j0.5)\times 10^6}{(2.402\times 10^3)\angle 0°}\right)^* = \left(\frac{(0.833\times 10^6)\angle -36.9°}{(2.402\times 10^3)\angle 0°}\right)^*$$

$$= 346.9\angle -36.9° = 277.4 - j208.3 \text{ A rms}$$

The line current is

$$\mathbf{I}_L = \mathbf{I}_1 + \mathbf{I}_2 = 433.7 - j\,345.9 = 554.7\angle -38.6 \text{ A rms}$$

The phase voltage at the load is required to be $\mathbf{V}_P = \frac{4.16}{\sqrt{3}}\angle 0° = 2.402\angle 0°$ kVrms .The source must provide this voltage plus the voltage dropped across the line, therefore

$$\mathbf{V}_{S\phi} = 2402\angle 0^{\circ} + (0.4 + j\,0.8)\,(433.7 - j\,345.9) = 2859.6\,\angle -38.6° \text{ Vrms}$$

Finally

$$|\mathbf{V}_{SL}| = \sqrt{3}\,(2859.6) = \underline{4953\ \text{Vrms}}$$

The power supplied by the source is

$$P_S = \sqrt{3}\,(4953)\,(554.7)\,\cos\,(4.2^{\circ} + 38.6^{\circ}) = \underline{3.49\ \text{MW}}$$

The power lost in the line is

$$P_{Line} = 3\times\left(554.7^2\right)\times \text{Re}\{0.4 + j\,0.8\} = \underline{0.369\ \text{MW}}$$

The percentage of the power consumed by the loads is

$$\frac{3.49 - 0.369}{3.49}\times 100\% = 89.4\%$$

P 12.7-8 The balanced three-phase load of a large commercial building requires 480 kW at a lagging power factor of 0.8. The load is supplied by a connecting line with an impedance of $5 + j25$ mΩ for each phase. Each phase of the load has a line-to-line voltage of 600 V rms. Determine the line current and the line voltage at the source. Also, determine the power factor at the source. Use the line-to-neutral voltage as the reference with an angle of 0°.

Solution:

The required phase voltage at the load is $\mathbf{V}_{\mathrm{P}} = \frac{600}{\sqrt{3}}\angle 0° = 346.4\angle 0° \text{ Vrms}$.

Let **I** be the line current required by the load. The complex power, **per phase**, is

$$\mathbf{S} = 160 + j\frac{160}{0.8}\sin\left(\cos^{-1}(0.8)\right) = 160 + j\ 120 \text{ kVA}$$

The line current is

$$\mathbf{I} = \left(\frac{\mathbf{S}}{\mathbf{V}_{\mathrm{P}}}\right)^* = \left(\frac{(160 + j120)\times 10^3}{346.4\angle 0°}\right)^* = 461.9 - j346.4 \text{ A rms}$$

The phase voltage at the load is required to be $\mathbf{V}_{\mathrm{P}} = \frac{600}{\sqrt{3}}\angle 0° = 346.4\angle 0° \text{ Vrms}$ .The source must provide this voltage plus the voltage dropped across the line, therefore

$$\mathbf{V}_{\mathrm{S}\phi} = 346.4\angle 0^{\circ} + (0.005 + j\ 0.025)\ (461.9 - j\ 346.4) = 357.5\ \angle 1.6° \text{ Vrms}$$

Finally

$$\left|\mathbf{V}_{\mathrm{SL}}\right| = \sqrt{3}\ (357.5) = \underline{619.2 \text{ Vrms}}$$

The power factor of the source is

$$pf = \cos(\theta_{\mathrm{V}} - \theta_{\mathrm{I}}) = \cos(1.6° - (-37°)) = \underline{0.78}$$

Section 12-8: Two-Wattmeter Power Measurement

P 12.8-1 The two-wattmeter method is used to determine the power drawn by a three-phase 440-V rms motor that is a Y-connected balanced load. The motor operates at 20 hp at 74.6 percent efficiency. The magnitude of the line current is 52.5 A rms. The wattmeters are connected in the A and C lines. Find the reading of each wattmeter. The motor has a lagging power factor.
Hint: 1 hp = 745.7 W

Solution:

$$P_{\text{out}} = 20 \text{ hp} \times 746 \frac{\text{W}}{\text{hp}} = 14920 \text{ W}$$

$$P_{\text{in}} = \frac{P_{\text{out}}}{\eta} = \frac{14920}{0.746} = 20 \text{ kW}$$

$$P_{\text{in}} = \sqrt{3}\, |\mathbf{V}_{\text{L}}||\mathbf{I}_{\text{L}}| \cos\theta \quad \Rightarrow \quad \cos\theta = \frac{P_{\text{in}}}{\sqrt{3}\, |\mathbf{V}_{\text{L}}||\mathbf{I}_{\text{L}}|} = \frac{20 \times 10^3}{\sqrt{3}\,(440)\,(52.5)} = 0.50$$

$$\Rightarrow \quad \theta \cos^{-1}(0.5) = 60°$$

The powers read by the two wattmeters are

$$P_1 = |\mathbf{V}_{\text{L}}||\mathbf{I}_{\text{L}}| \cos(\theta + 30°) = (440)\,(52.5)\cos(60° + 30°) = 0$$

and

$$P_2 = |\mathbf{V}_{\text{L}}||\mathbf{I}_{\text{L}}| \cos(\theta - 30°) = (440)\,(52.5)\cos(60° - 30°) = 20 \text{ kW}$$

P 12.8-2 A three-phase system has a line-to-line voltage of 4000 V rms and a balanced Δ-connected load with **Z** = 40 + *j*30 Ω. The phase sequence is *abc*. Use the two wattmeters connected to lines *A* and *C*, with line *B* as the common line for the voltage measurement. Determine the total power measurement recorded by the wattmeters.
Answer: *P* = 768 kW

Solution:

$$|\mathbf{V}_P| = |\mathbf{V}_L| = 4000 \text{ V rms} \qquad \mathbf{Z}_\Delta = 40 + j\,30 = 50\angle 36.9^\circ$$

$$|\mathbf{I}_P| = \frac{|\mathbf{V}_P|}{|\mathbf{Z}_\Delta|} = \frac{4000}{50} = 80 \text{ A rms} \qquad |\mathbf{I}_L| = \sqrt{3}\,|\mathbf{I}_P| = 138.6 \text{ A rms}$$

$$pf = \cos\,\theta = \cos(36.9^\circ) = 0.80$$

$$P_1 = V_L I_L \cos(\theta + 30^\circ) = 4000\,(138.6)\cos 66.9^\circ = 217.5 \text{ kW}$$

$$P_2 = V_L I_L \cos(\theta - 30^\circ) = 4000\,(138.6)\cos 6.9^\circ = 550.4 \text{ kW}$$

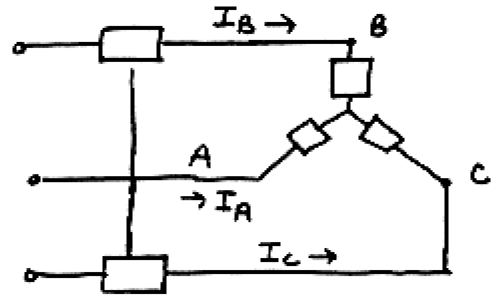
$$P_T = P_1 + P_2 = 767.9 \text{ kW}$$

$$\text{Check}: P_T = \sqrt{3}\,|\mathbf{I}_L||\mathbf{V}_L|\cos\theta = \sqrt{3}\,(4000)\,(138.6)\cos 36.9^\circ$$

$$= 768 \text{ kW} \quad \text{which checks}$$

P 12.8-3 A three-phase system with a sequence *abc* and a line-to-line voltage of 200 V rms feeds a Y-connected load with $\mathbf{Z} = 70.7\ \angle 45°\ \Omega$. Find the line currents. Find the total power by using two wattmeters connected to lines *B* and *C*.
Answer: $P = 400$ W

Solution:



$$V_p = \left|\mathbf{V}_p\right| = \frac{200}{\sqrt{3}} = 115.47 \text{ Vrms}$$

$$\mathbf{V}_A = 115.47\angle 0° \text{ V rms},\ \mathbf{V}_B = 115.47\angle{-120°} \text{ V rms}$$
$$\text{and } \mathbf{V}_C = 115.47\angle 120° \text{ V rms}$$

$$I_A = \frac{V_A}{Z} = \frac{115.47\angle 0°}{70.7\angle 45°} = 1.633\angle{-45°} \text{ A rms}$$

$\mathbf{I}_B = 1.633\ \angle{-165°}$ A rms and $\mathbf{I}_C = 1.633\ \angle 75°$ A rms

$$P_T = \sqrt{3}\ \left|\mathbf{V}_L\right|\left|\mathbf{I}_L\right|\ \cos\theta = \sqrt{3}\ (200)\ (1.633)\ \cos 45^{\circ} = 400 \text{ W}$$

$$P_B = \left|\mathbf{V}_{AC}\right|\ \left|\mathbf{I}_A\right|\ \cos\theta_1 = 200\ (1.633)\ \cos\left(45^{\circ} - 30^{\circ}\right) = 315.47 \text{ W}$$

$$P_C = \left|\mathbf{V}_{BC}\right|\ \left|\mathbf{I}_B\right|\ \cos\theta_2 = 200\ (1.633)\ \cos\left(45^{\circ} + 30^{\circ}\right) = 84.53 \text{ W}$$

P 12.8-4 A three-phase system with a line-to-line voltage of 208 V rms and phase sequence *abc* is connected to a Y-balanced load with impedance 10 ∠–30° Ω and a balanced Δ load with impedance 15 ∠30° Ω. Find the line currents and the total power using two wattmeters.

Solution:

$$\mathbf{Z}_Y = 10\angle -30° \;\; \Omega \;\; \text{and} \;\; \mathbf{Z}_\Delta = 15\angle 30° \;\; \Omega$$

$$\text{Convert } \mathbf{Z}_\Delta \text{ to } \mathbf{Z}_{\hat{Y}} \rightarrow \mathbf{Z}_{\hat{Y}} = \frac{\mathbf{Z}_\Delta}{3} = 5\angle 30° \;\; \Omega$$

$$\text{then } Z_{eq} = \frac{(10\angle -30°)\,(5\angle 30°)}{10\angle -30° + 5\angle 30°} = \frac{50\angle 0°}{13.228\;\angle -10.9°} = 3.78\angle 10.9° \;\; \Omega$$

$$V_p = |\mathbf{V}_p| = \frac{208}{\sqrt{3}} = 120 \;\; \text{V rms}$$

$$\mathbf{V}_A = 120\angle 0° \text{ V rms} \Rightarrow \mathbf{I}_A = \frac{120\angle 0°}{3.78\;\angle 10.9°} = 31.75\;\angle -10.9°$$

$$\mathbf{I}_B = 31.75\angle -130.9°$$

$$\mathbf{I}_C = 31.75\angle 109.1°$$

$$P_T = \sqrt{3} V_L I_L \;\; \cos\theta = \sqrt{3}(208)(31.75)\cos(10.9) = 11.23 \;\; kW$$

$$W_1 = V_L \; I_L \; \cos(\theta - 30°) = 6.24 \;\; \text{kW}$$

$$W_2 = V_L \; I_L \; \cos(\theta + 30°) = 4.99 \;\; \text{kW}$$

P 12.8-5 The two-wattmeter method is used. The wattmeter in line *A* reads 920 W, and the wattmeter in line *C* reads 460 W. Find the impedance of the balanced Δ-connected load. The circuit is a three-phase 120-V rms system with an *abc* sequence.

Answer: **Z**$_\Delta$ = 27.1 ∠–30° Ω

P12.8-5

$$P_T = P_A + P_C = 920 + 460 = 1380 \text{ W}$$

$$\tan\theta = \sqrt{3}\,\frac{P_C - P_A}{P_C + P_A} = \sqrt{3}\,\frac{(-460)}{1380} = -0.577 \quad \Rightarrow \quad \theta = -30^\circ$$

$$P_T = \sqrt{3}\, V_L\, I_L\, \cos\theta \;\; so \; I_L = \frac{P_T}{\sqrt{3}\, V_L \cos\theta} = \frac{1380}{\sqrt{2} \times 120 \times \cos(-30)} = 7.67 \text{ A rms}$$

$$I_P = \frac{I_L}{\sqrt{3}} = 4.43 \;\; \text{A rms} \qquad \therefore |\mathbf{Z}_\Delta| = \frac{120}{4.43} = 27.1\,\Omega \;\; or \; \underline{\mathbf{Z}_\Delta = 27.1\angle -30^\circ}$$

P 12.8-6 Using the two-wattmeter method, determine the power reading of each wattmeter and the total power for Problem P 12.5-1 when **Z** = 0.868 + *j*4.924 Ω. Place the current coils in the *A*-to-*a* and *C*-to-*c* lines.

Solution:

$$\mathbf{Z} = 0.868 + j4.924 = 5\angle 80° \quad \Rightarrow \quad \theta = 80°$$

$$V_L = 380 \text{ V rms}, V_P = \frac{380}{\sqrt{3}} = 219.4 \text{ V rms}$$

$$I_L = I_P \textit{ and } I_P = \frac{V_P}{|\mathbf{Z}|} = 43.9 \text{ A rms}$$

$$P_1 = (380)(43.9)\cos\left(\theta - 30^\circ\right) = 10{,}723 \text{ W}$$

$$P_2 = (380)(43.9)\cos\left(\theta + 30^\circ\right) = -5706 \text{ W}$$

$$P_T = P_1 + P_2 = 5017 \text{ W}$$

Section 12.9 How Can We Check …?

P 12.9-1 A Y-connected source is connected to a Y-connected load (Figure 12.3-1) with **Z** = 10 + *j*4 *Ω*. The line voltage is V_L = 416 V rms. A student report states that the line current $\mathbf{I}_A$ = 38.63 A rms and that the power delivered to the load is 16.1 kW. Verify these results.

Solution:

$$|\mathbf{V}_\mathbf{A}| = \frac{416}{\sqrt{3}} = 240 \text{ V} = |\text{V}_\text{A}|$$

$$\mathbf{Z} = 10 + \text{j}4 = 10.77 \angle 21.8^\circ \ \Omega$$

$$|\mathbf{I}_\text{A}| = \frac{|\mathbf{V}_\text{A}|}{|\mathbf{Z}|} = \frac{240}{10.77} = 22.28 \text{ A rms} \neq 38.63 \text{ A rms}$$

The report is not correct. (Notice that $\frac{38.63}{\sqrt{3}} = 22.3$. It appears that the line-to-line voltage was mistakenly used in place of the phase voltage.)

P 12.9-2 A Δ load with **Z** = 40 + *j*30 *Ω* has a three-phase source with V_L = 240 V rms (Figure 12.3-2). A computer analysis program states that one phase current is 4.8 ∠–36.9° A. Verify this result.

Solution:

$$\mathbf{V}_\text{L} = \mathbf{V}_\text{P} = 240\angle 0^\circ \text{ Vrms}$$

$$\mathbf{Z} = 40 + j30 = 50 \angle 36.9^\circ \ \Omega$$

$$\mathbf{I}_\text{P} = \frac{\mathbf{V}_\text{P}}{\mathbf{Z}} = \frac{240\angle 0^\circ}{50\angle 36.9^\circ} = \underline{4.8 \angle -36.9^\circ \text{ A rms}}$$

The result is correct.

PSpice Problems

SP 12-1 Use PSpice to determine the power delivered to the load in the circuit shown in Figure SP 12-1.

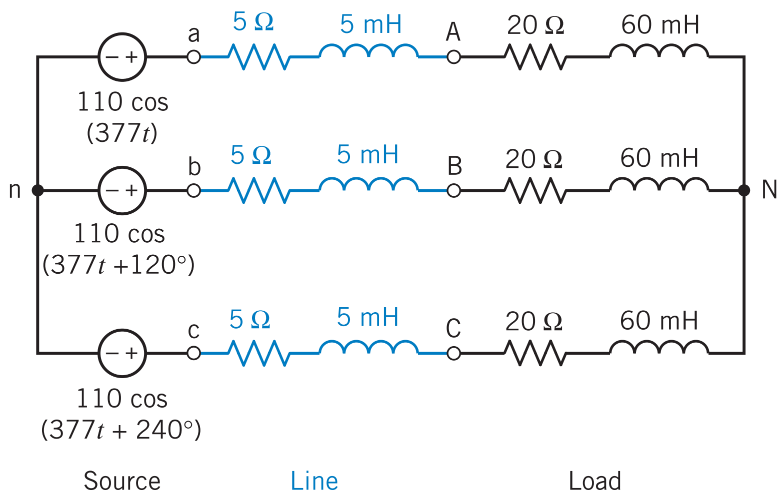
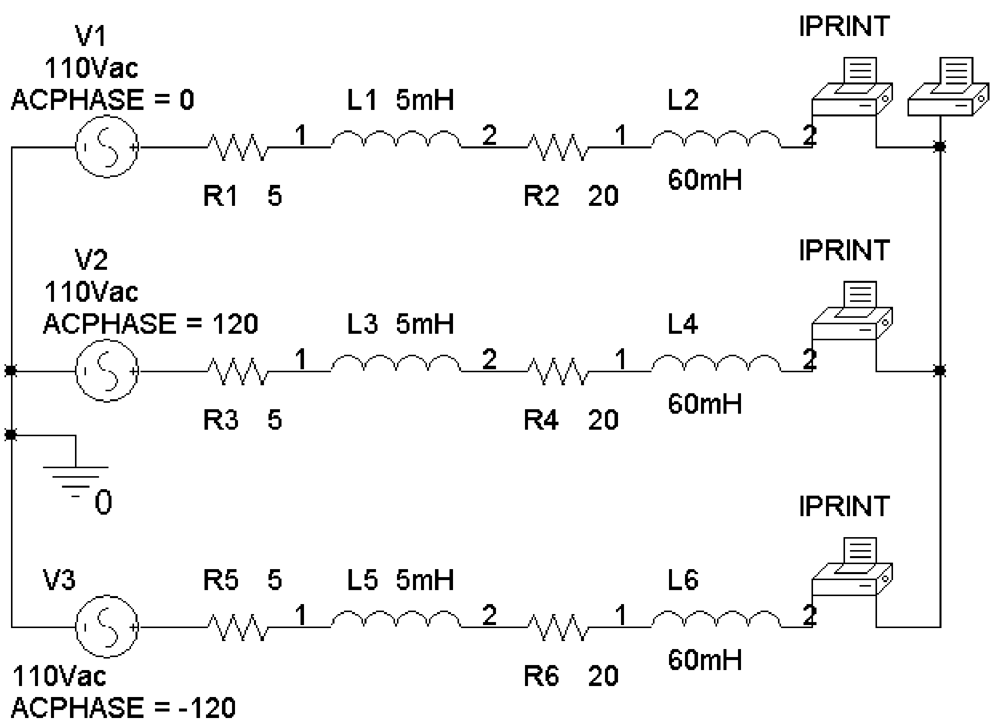


Figure SP 12-1

Solution:




```
FREQ         IM(V_PRINT3)IP(V_PRINT3)IR(V_PRINT3)II(V_PRINT3)
6.000E+01    3.142E+00   -1.644E+02  -3.027E+00  -8.436E-01

FREQ         IM(V_PRINT1)IP(V_PRINT1)IR(V_PRINT1)II(V_PRINT1)
```

```
6.000E+01   3.142E+00  -4.443E+01   2.244E+00  -2.200E+00

FREQ        VM(N01496)  VP(N01496)  VR(N01496)  VI(N01496)
6.000E+01   2.045E-14   2.211E+01   1.895E-14   7.698E-15

FREQ        IM(V_PRINT2)IP(V_PRINT2)IR(V_PRINT2)II(V_PRINT2)
6.000E+01   3.142E+00   7.557E+01   7.829E-01   3.043E+00
```

$$\mathbf{I}_A = 3.142\angle -43.43° \text{ A} \quad \text{and} \quad R_A = 20 \ \Omega \quad \Rightarrow \quad P_A = \frac{3.142^2}{2} 20 = 98.7 \text{ W}$$

$$\mathbf{I}_B = 3.142\angle 75.57° \text{ A} \quad \text{and} \quad R_B = 20 \ \Omega \quad \Rightarrow \quad P_B = \frac{3.142^2}{2} 20 = 98.7 \text{ W}$$

$$\mathbf{I}_C = 3.142\angle -164.4° \text{ A} \quad \text{and} \quad R_C = 20 \ \Omega \quad \Rightarrow \quad P_C = \frac{3.142^2}{2} 20 = 98.7 \text{ W}$$

$$P = 3\left(98.7\right) = 696.1 \text{ W}$$

SP 12-2 Use PSpice to determine the power delivered to the load in the circuit shown in SP 12-2.

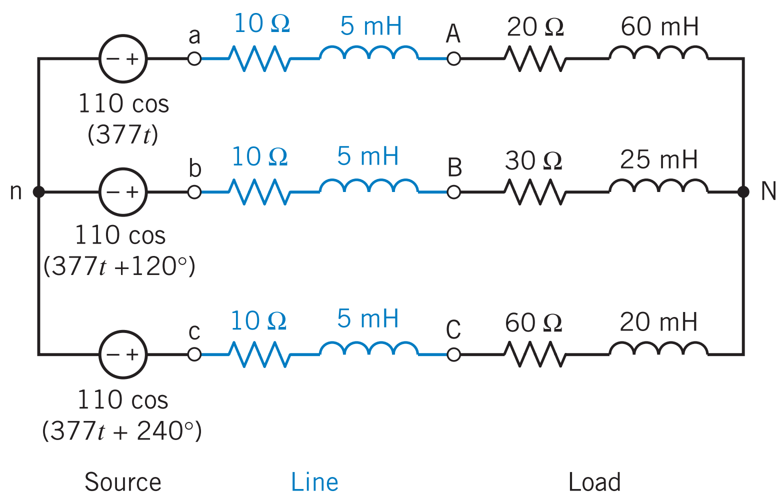
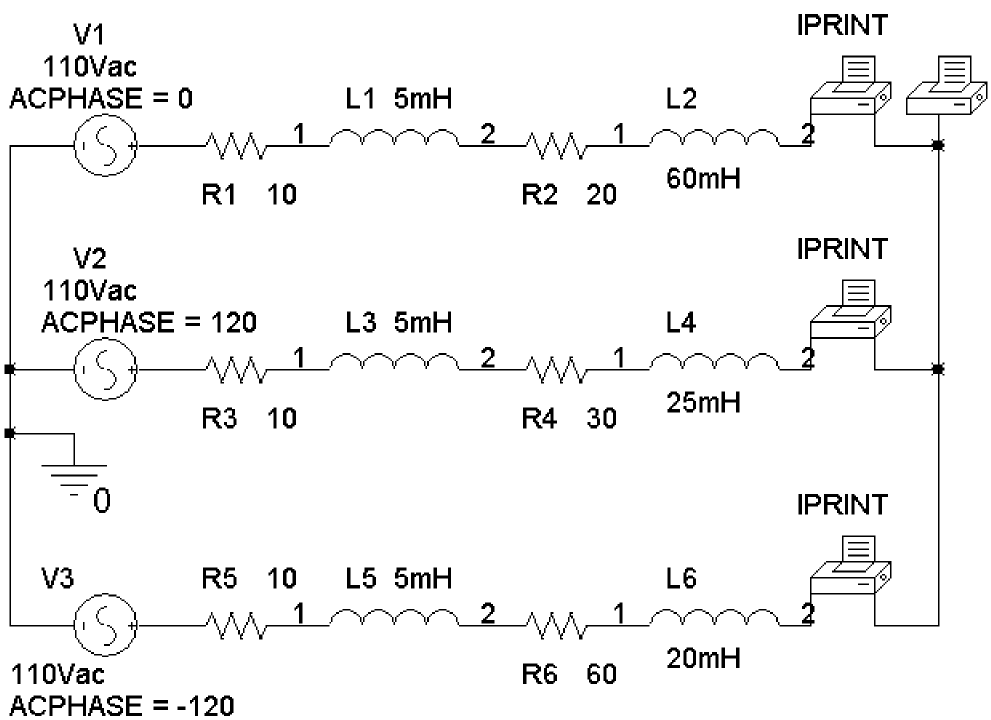


Figure SP 12-2

Solution:




```
FREQ          IM(V_PRINT3)IP(V_PRINT3)IR(V_PRINT3)II(V_PRINT3)
6.000E+01     1.612E+00  -1.336E+02  -1.111E+00  -1.168E+00

FREQ          IM(V_PRINT1)IP(V_PRINT1)IR(V_PRINT1)II(V_PRINT1)
6.000E+01     2.537E+00  -3.748E+01   2.013E+00  -1.544E+00
```

```
FREQ         VM(N01496)  VP(N01496)  VR(N01496)  VI(N01496)
6.000E+01   1.215E+01  -1.439E+01   1.177E+01  -3.018E+00

FREQ         IM(V_PRINT2)IP(V_PRINT2)IR(V_PRINT2)II(V_PRINT2)
6.000E+01   2.858E+00   1.084E+02  -9.023E-01   2.712E+00
```

$$\mathbf{I}_A = 2.537\angle -37.48° \text{ A} \quad \text{and} \quad R_A = 20\ \Omega \quad \Rightarrow \quad P_A = \frac{2.537^2}{2} 20 = 64.4 \text{ W}$$

$$\mathbf{I}_B = 2.858\angle 108.4° \text{ A} \quad \text{and} \quad R_B = 30\ \Omega \quad \Rightarrow \quad P_B = \frac{2.858^2}{2} 30 = 122.5 \text{ W}$$

$$\mathbf{I}_C = 1.612\angle -133.6° \text{ A} \quad \text{and} \quad R_C = 600\ \Omega \quad \Rightarrow \quad P_C = \frac{1.612^2}{2} 60 = 78 \text{ W}$$

$$P = 64.4 + 122.5 + 78 = 264.7 \text{ V}$$

Design Problems

DP 12-1 A balanced three-phase Y source has a line voltage of 208 V rms. The total power delivered to the balanced Δ load is 1200 W with a power factor of 0.94 lagging. Determine the required load impedance for each phase of the Δ load. Calculate the resulting line current. The source is a 208-V rms *ABC* sequence.

Solution:

$P = 400$ W per phase,

$$0.94 = pf = \cos\theta \quad \Rightarrow \quad \theta = \cos^{-1}(0.94) = 20°$$

$$400 = \frac{208}{\sqrt{3}} |\mathbf{I}_L|\, 0.94 \;\Rightarrow |\mathbf{I}_L| = 3.5 \text{ A rms}$$

$$|\mathbf{I}_\Delta| = \frac{|\mathbf{I}_L|}{\sqrt{3}} = 2.04 \text{ A rms}$$

$$|\mathbf{Z}| = \frac{|\mathbf{V}_L|}{|\mathbf{I}_\Delta|} = \frac{208}{2.04} = 101.8\ \Omega$$

$$\underline{\mathbf{Z} = 101.8\ \angle 20^\circ\ \Omega}$$

DP 12-2 A three-phase 240-V rms circuit has a balanced Y-load impedance **Z**. Two wattmeters are connected with current coils in lines *A* and *C*. The wattmeter in line *A* reads 1440 W, and the wattmeter in line *C* reads zero. Determine the value of the impedance.

Solution:

$$|\mathbf{V}_L| = 240 \text{ V rms}$$

$$P_A = |\mathbf{V}_L||\mathbf{I}_L| \cos\ (30^\circ + \theta) = 1440 \text{ W}$$

$$P_C = |\mathbf{V}_L||\mathbf{I}_L| \cos\ (30^\circ - \theta) = 0 \text{ W} \quad \Rightarrow \quad 30 - \theta = 90^\circ \text{ or } \theta = -60^\circ$$

$$\text{then } 1440 = 240\ |\mathbf{I}_L| \cos(-30^\circ) \quad \Rightarrow \quad |\mathbf{I}_L| = 6.93 \text{ A rms}$$

$$|I_L| = |I_P| = \frac{|\mathbf{V}_P|}{|\mathbf{Z}|} \quad \Rightarrow \quad |\mathbf{Z}| = \frac{|\mathbf{V}_P|}{|\mathbf{I}_P|} = \frac{\frac{240}{\sqrt{3}}}{6.93} = 20\ \Omega$$

Finally, $\underline{\mathbf{Z} = 20\ \angle -60^\circ\ \Omega}$

DP 12-3 A three-phase motor delivers 100 hp and operates at 80 percent efficiency with a 0.75 lagging power factor. Determine the required Δ-connected balanced set of three capacitors that will improve the power factor to 0.90 lagging. The motor operates from 480-V rms lines.

Solution:

$$P_{in} = \frac{P_{out}}{\eta} = \frac{100\text{ hp} \times (746\ \frac{\text{W}}{\text{hp}})}{0.8} = 93.2\ \text{kW},\ P_{\phi} = \frac{P_{in}}{3} = 31.07\text{ kW}$$

$V_L = 480$ V rms, $pfc = 0.9$ and $pf = 0.75$. We need the impedance of the load so that we can use Eqn 11.6-7 to calculate the value of capacitance needed to correct the power factor.

$$0.75 = pf = \cos\theta \quad \Rightarrow \quad \theta = \cos^{-1}(0.75) = 41.4°$$

$$31070 = \frac{480}{\sqrt{3}}|\mathbf{I}_p|(0.75) \quad \Rightarrow \quad |\mathbf{I}_p| = 149.5\text{ Arms}$$

$$|\mathbf{Z}| = \frac{|\mathbf{V}_P|}{|\mathbf{I}_P|} = \frac{\frac{480}{\sqrt{3}}}{149.5} = 1.85\ \Omega$$

$$\mathbf{Z} = 1.85\ \angle 41.4°\ \Omega = 1.388 + j1.223\ \Omega$$

The capacitance required to correct the power factor is given by

$$C = \frac{1.365}{1.365^2 + 1.204^2} \times \frac{\left[\tan\ (\cos^{-1}0.75) - \tan\ (\cos^{-1}0.9)\right]}{377} = \underline{434\ \mu\text{F}}$$

(Checked using LNAPAC 6/12/03)

DP 12-4 A three-phase system has balanced conditions so that the per-phase circuit representation can be utilized as shown in Figure DP 12-4. Select the turns ratio of the step-up and step-down transformers so that the system operates with an efficiency greater than 99 percent. The load voltage is specified as 4 kV rms, and the load impedance is 4/3 Ω.

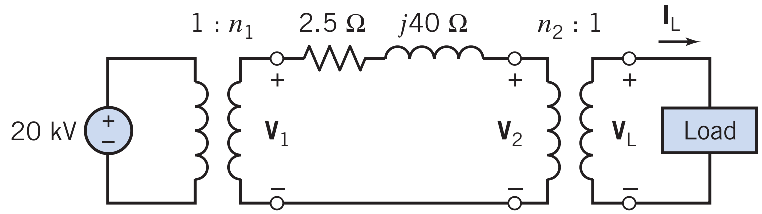


Figure DP 12-4

Solution:

$$\mathbf{V}_L = 4\angle 0° \text{ kV rms}$$

Try $n_2 = 25$ then $\mathbf{V}_2 = \dfrac{n_2}{n_1}\mathbf{V}_L = \dfrac{25}{1} 4000\angle 0° = 100\angle 0° \text{ kVrms}$

$$\mathbf{I}_L = \frac{\mathbf{V}_L}{\mathbf{Z}_L} = \frac{4\times 10^3\angle 0°}{\frac{4}{3}} = 3\angle 0° \text{ kA rms}$$

The line current in 2.5 Ω is $\mathbf{I} = \dfrac{3000\angle 0°}{25} = 120\angle 0° \text{ A rms}$

Thus $\mathbf{V}_1 = (R + jX)\,\mathbf{I} + \mathbf{V}_2$

$$= (2.5 + j40)\,(120\angle 0°) + 100\times 10^3 = 100.4\ \angle 2.7° \text{ kV}$$

Step need : $n_1 = \dfrac{100.4 \text{ kV}}{20 \text{ kV}} = 5.02 \cong 5$

$P_{loss} = |\mathbf{I}|^2 R = |120|^2 (2.5) = 36 \text{ kW},\ P = (4\times 10^3)\,(3\times 10^3) = 12 \text{ MW}$

$\therefore \eta = \dfrac{12 - .036}{12}\times 100\% = 99.7\ \%$ of the power supplied by the source is delivered to the load.

Chapter 13: Frequency Response

Exercises

Exercise 13.2-1 The input to the circuit shown in Figure E 13.2-1 is the source voltage, v_s, and the response is the capacitor voltage, v_o. Suppose $R = 10$ kΩ and $C = 1$ μF. What are the values of the gain and phase shift when the input frequency is $\omega = 100$ rad/s?

Answer: 0.707 and –45°

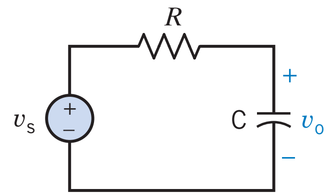
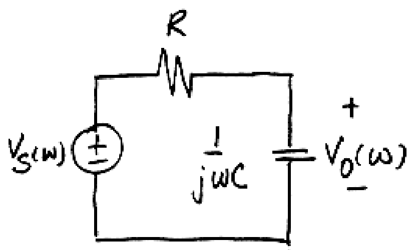


Figure E 13.2-1

Solution:



$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)} = \frac{1}{1 + j\omega C R}$$

$$gain = \frac{1}{\sqrt{1 + (\omega C R)^2}}$$

$$phase\ shift = -\tan^{-1}(\omega C R)$$

When $R = 10^4\ \Omega$, $\omega = 100$ rad/s, and $C = 10^{-6}$ F, then $gain = \frac{1}{\sqrt{2}} = 0.707$ and $phase\ shift = -45^\circ$

Exercise 13.2-2 The input to the circuit shown in Figure E 13.2-2 is the source voltage, v_s, and the response is the resistor voltage, v_o. R = 30 Ω and L = 2 H. Suppose the input frequency is adjusted until the gain is equal to 0.6. What is the value of the frequency?

Answer: 20 rad/s

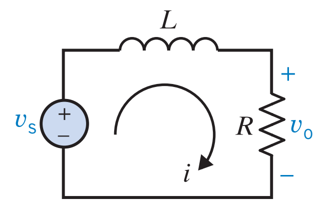
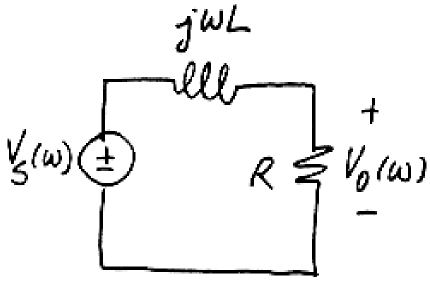


Figure E 13.2-2

Solution:



$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)} = \frac{R}{R+j\omega L}$$

$$gain = \frac{R}{\sqrt{R^2+(\omega L)^2}}$$

$$0.6 = \frac{30}{\sqrt{30^2+(2\omega)^2}} \Rightarrow \omega = \frac{\sqrt{\left(\frac{30}{.6}\right)^2 - 30^2}}{2} = 20 \text{ rad/s}$$

Exercise 13.2-3 The input to the circuit shown in Figure E 13.2-2 is the source voltage, v_s, and the response is the mesh current, i. R = 30 Ω and L = 2 H. What are the values of the gain and phase shift when the input frequency is ω = 20 rad/s?

Answer: 0.02 A/V and –53.1°

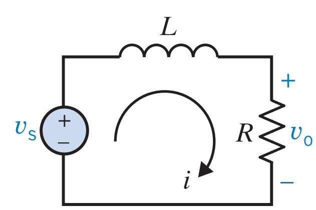


Figure E 13.2-2

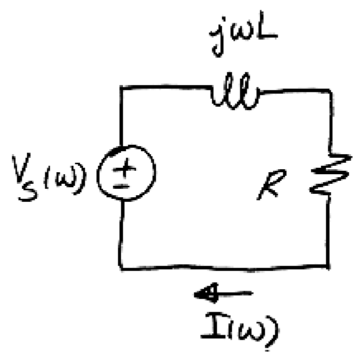
Solution:

$$\mathbf{H}(\omega) = \frac{\mathbf{I}(\omega)}{\mathbf{V}_s(\omega)} = \frac{1}{R+j\omega L}$$

$$gain = \frac{1}{\sqrt{R^2+(\omega L)^2}}$$

$$phase\ shift = -\tan^{-1}\frac{\omega L}{R}$$

When $R = 30\ \Omega,\ L = 2$ H, and $\omega = 20$ rad/s, then



$$gain = \frac{1}{\sqrt{30^2+40^2}} = 0.02\ \frac{\text{A}}{\text{V}} \text{ and } phase\ shift = -\tan^{-1}\left(\frac{40}{30}\right) = -53.1°$$

Exercise 13.2-4 The input to the circuit shown in Figure E 13.2-1 is the source voltage, v_s, and the response is the capacitor voltage, v_o. Suppose $C = 1\ \mu$F. What value of R is required to cause a phase shift equal to –45° when the input frequency is $\omega = 20$ rad/s?

Answer: $R = 50\ k\Omega$

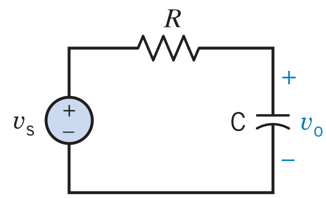
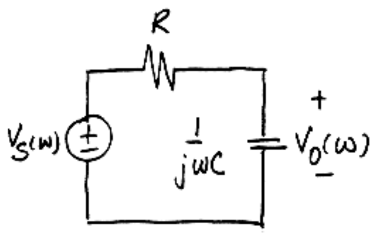


Figure E 13.2-1

Solution:



$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)} = \frac{1}{1 + j\omega C R}$$

$$gain = \frac{1}{\sqrt{1 + (\omega C R)^2}}$$

$$phase\ shift = -\tan^{-1} \omega C R$$

$$-45° = -\tan^{-1}(20 \cdot 10^{-6} \cdot R) \quad \Rightarrow \quad R = \frac{\tan(45^\circ)}{20 \cdot 10^{-6}} = 50 \cdot 10^3 = 50\ \text{k}\Omega$$

Exercise 13.2-5 The input to the circuit shown in Figure E 13.2-1 is the source voltage, v_s, and the response is the capacitor voltage, v_o. Suppose $C = 1\ \mu$F. What value of R is required to cause a gain equal to 1.5 when the input frequency is $\omega = 20$ rad/s?

Answer: No such value of R exists. The gain of this circuit will never be greater than 1.

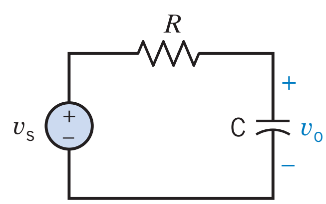
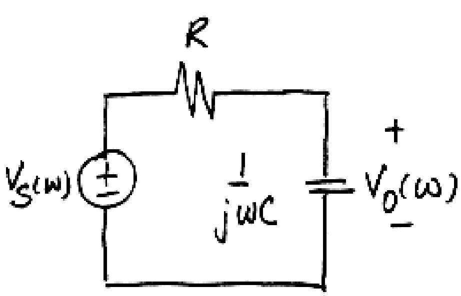


Figure E 13.2-1

Solution:



$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)} = \frac{1}{1 + j\omega C R}$$

$$gain = \frac{1}{\sqrt{1 + (\omega C R)^2}}$$

$\omega, C,$ and R are all positive, or at least nonnegative, so gain ≤ 1. These specifications cannot be met.

Exercise 13.3-1 (a) Convert the gain $|\mathbf{V}_o/\mathbf{V}_s| = 2$ to decibels. (b) Suppose $|\mathbf{V}_o/\mathbf{V}_s| = -6.02$ dB. What is the value of this gain "not in dB"?

Answer: (a) + 6.02 dB (b) 0.5

Solution:

(a) $\text{dB} = 20 \log(2) = 6.02 \text{ dB}$

(b) $10^{-6.02/20} = 0.5$

Exercise 13.3-2 In a certain frequency range, the magnitude of the network function can be approximated as $H = 1/\omega^2$. What is the slope of the Bode plot in this range, expressed in decibels per decade?

Answer: –40 dB/decade

Solution:

$$20\log|\mathbf{H}| = 20\log\left(\frac{1}{\omega^2}\right) = 20\log(\omega)^{-2} = -40\log\omega$$

$$slope = 20\log|\mathbf{H}(\omega_2)| - 20\log|\mathbf{H}(\omega_1)| = -40\log\omega_2 + 40\log\omega_1 = -40\log\left(\frac{\omega_2}{\omega_1}\right)$$

let $\omega_2 = 10\,\omega_1$ to consider 1 decade, then $slope = \underline{-40\log 10 = -40 \text{ dB/decade}}$

Exercise 13.3-3 Consider the network function

$$\mathbf{H}(\omega) = \frac{j\omega A}{B + j\omega C}$$

Find (a) the corner frequency, (b) the slope of the asymptotic magnitude Bode plot for ω above the corner frequency in decibels per decade, (c) the slope of the magnitude Bode plot below the corner frequency, and (d) the gain for ω above the corner frequency in decibels.

Answer: (a) $\omega_0 = B/C$ (b) zero (c) 20 dB/decade (d) 20 $\log_{10} = \dfrac{A}{C}$

Solution:

When $\omega C >> B$, $\mathbf{H}(\omega) \simeq \dfrac{j\omega A}{j\omega C} = \dfrac{A}{C}$

(d) $\left|\mathbf{H}(\omega)\right|$ in dB $= 20\log_{10}\left|\mathbf{H}(\omega)\right| = 20\log_{10}\left(\dfrac{A}{C}\right)$

(b) $\left|\mathbf{H}(\omega)\right|$ does not depend on ω so $slope = 0$

When $\omega C << B$, $\mathbf{H}(\omega) \simeq \dfrac{j\omega A}{B} = j\omega\left(\dfrac{A}{B}\right)$

$\left|\mathbf{H}(\omega)\right|$ in dB = 20 $\log_{10}$ $\left|\mathbf{H}(\omega)\right|$ = 20 $\log_{10}\omega$+20 $\log_{10}\left(\dfrac{A}{B}\right)$

(c) The slope is the coefficient of $\log_{10}\omega$, that is, $slope = 20$ dB/decade

(a) The break frequency is the frequency at which $\omega_0 C = B$, that is, $\omega_0 = \dfrac{B}{C}$

Exercise 13.4-1 For the *RLC* parallel resonant circuit when $R = 8$ kΩ, $L = 40$ mH, and $C = 0.25$ μF, find (a) Q and (b) bandwidth.

Answer: (a) $Q = 20$ (b) $BW = 500$ rad/s

Solution:

$$\text{a)}\quad Q = \omega_0 RC = R\sqrt{\frac{C}{L}} = 8000\sqrt{\frac{2.5\times10^{-7}}{40\times10^{-3}}} = \underline{20}$$

$$\text{b)}\quad BW = \frac{\omega_o}{Q} = \frac{1}{Q\sqrt{LC}} = \frac{1}{20\sqrt{(40\times10^{-3})\,(2.5\times10^{-7})}} = \underline{500\ \text{rad/s}}$$

Exercise 13.4-2 A high-frequency *RLC* parallel resonant circuit is required to operate at $\omega_0 =$ 10 Mrad/s with a bandwidth of 200 krad/s. Determine the required Q and L when $C = 10$ pF.

Answer: $Q = 50$ and $L = 1$ mH

Solution:

$$Q = \frac{\omega_0}{BW} = \frac{10^7}{2\times10^5} = \underline{50}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow L = \frac{1}{\omega_0^{\,2}C} = \frac{1}{(10^7)^2\ (10\times10^{-12})} = \underline{1\ \text{mH}}$$

Exercise 13.4-3 A series resonant circuit has $L = 1$ mH and $C = 10$ μF. Find the required Q and R when it is desired that the bandwidth be 15.9 Hz.

Answer: $Q = 100$ and $R = 0.1$ Ω

Solution:

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\left[(10^{-3})(10^{-5})\right]^{1/2}} = 10^4\ \text{rad/s}$$

$$Q = \frac{\omega_0}{BW} = \frac{10^4}{2\pi\ (15.9)} = \underline{100}$$

$$R = \frac{\omega_0 L}{Q} = \frac{(10^4)(10^{-3})}{100} = \underline{0.1\ \Omega}$$

Exercise 13.4-4 A series resonant circuit has an inductor L = 10 mH. (a) Select C and R so that $\omega_0 = 10^6$ rad/s and the bandwidth is $BW = 10^3$ rad/s. (b) Find the admittance **Y** of this circuit for a signal at $\omega = 1.05\times10^6$ rad/s.

Answer: **(a)** $C = 100\text{pF}, R = 10\,\Omega$

(b) $\mathbf{Y} = \dfrac{10}{1+j97.6}$

Solution:

a)

$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow C = \frac{1}{(10^6)^2\ (0.01)} = \underline{100\ \text{pF}}$$

$$Q = \frac{\omega_0}{BW} = \frac{1}{\omega_0 RC} \Rightarrow R = \frac{BW}{\omega_0^{\ 2}\ C} = \frac{10^3}{(10^6)^2(10^{-10})} = \underline{10\ \Omega}$$

b)

$$Q = \frac{\omega_0}{BW} = \frac{10^6}{10^3} = 1000$$

$$\mathbf{Y} = \frac{1}{1+j\,Q\left(\dfrac{\omega}{\omega_0}-\dfrac{\omega_0}{\omega}\right)} = \frac{1}{1+j1000\left[\dfrac{1.05\times10^6}{10^6}-\dfrac{10^6}{1.05\times10^6}\right]} = \frac{1}{1+j\ 97.6}$$

Section 13-2: Gain, Phase Shift, and the Network Function

P 13.2-1 The input to the circuit shown in Figure P 13.2-1 is the voltage of the voltage source, $v_i(t)$. The output is the voltage, $v_o(t)$, across the parallel connection of the capacitor and 10-**Ω** resistor. Determine the network function, $\mathbf{H}(\omega) = \mathbf{V}_o(\omega)/\mathbf{V}_i(\omega)$, of this circuit.

Answer: $\mathbf{H}(\omega) = \dfrac{0.2}{1 + j4\omega}$

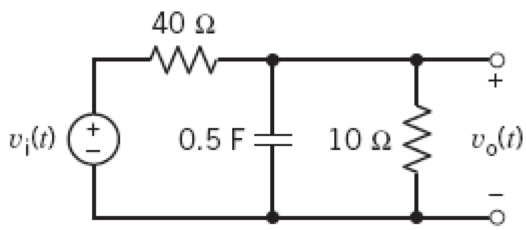
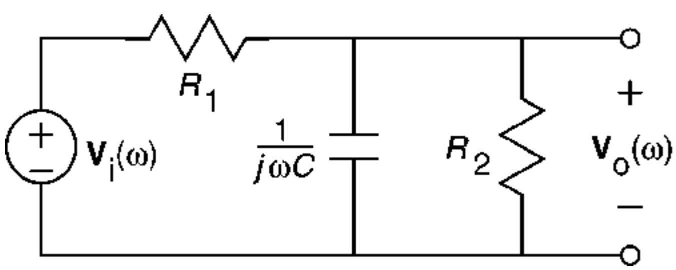


Figure P 13.2-1

Solution:



$$R_2 \,\|\, \frac{1}{j\omega C} = \frac{R_2}{1 + j\omega C\, R_2}$$

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{\dfrac{R_2}{1 + j\omega C R_2}}{R_1 + \dfrac{R_2}{1 + j\omega C R_2}} = \frac{\dfrac{R_2}{R_1 + R_2}}{1 + j\omega C R_p}$$

where $R_p = R_1 \,\|\, R_2$.

When $R_1 = 40\ \Omega$, $R_2 = 10\ \Omega$ and $C = 0.5$ F

$$\mathbf{H}(\omega) = \frac{0.2}{1 + j4\omega}$$

(checked using ELab on 8/6/02)

P 13.2-2 The input to the circuit shown in Figure P 13.2-2 is the voltage of the voltage source, $v_i(t)$. The output is the voltage, $v_o(t)$, across the series connection of the capacitor and 160-kΩ resistor. Determine the network function, $\mathbf{H}(\omega) = \mathbf{V}_o(\omega)/\mathbf{V}_i(\omega)$, of this circuit.

Answer: $\mathbf{H}(\omega)=\dfrac{1+j(0.004)\omega}{1+j(0.005)\omega}$

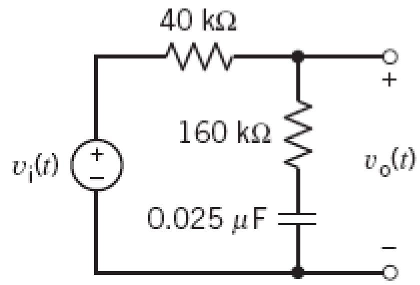
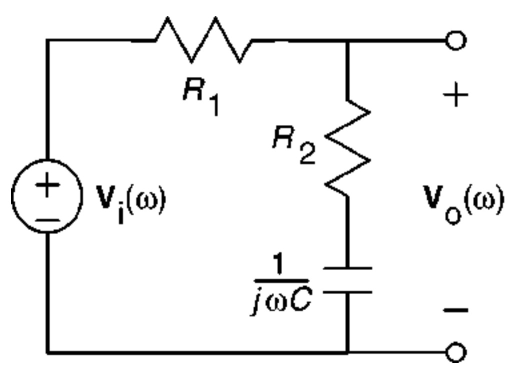


Figure P 13.2-2

Solution:

$$\mathbf{H}(\omega)=\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}=\frac{R_2+\dfrac{1}{j\omega C}}{R_1+R_2+\dfrac{1}{j\omega C}}$$

$$=\frac{1+j\omega C R_2}{1+j\omega C\left(R_1+R_2\right)}$$



When R_1 = 40 kΩ, R_2 = 160 kΩ and C = 0.025 μF

$$\mathbf{H}(\omega)=\frac{1+j\left(0.004\right)\omega}{1+j\left(0.005\right)\omega}$$

(checked using ELab on 8/6/02)

P 13.2-3 The input to the circuit shown in Figure P 13.2-3 is the voltage of the voltage source, $v_i(t)$. The output is the voltage, $v_o(t)$, across the 6-Ω resistor. Determine the network function, $\mathbf{H}(\omega) = \mathbf{V}_o(\omega)/\mathbf{V}_i(\omega)$, of this circuit.

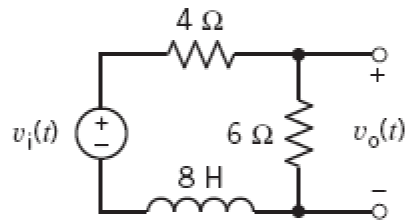
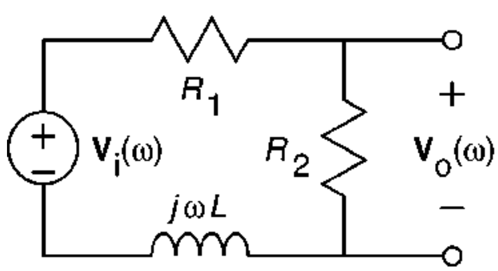


Figure P 13.2-3

Solution:



$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{R_2}{R_1 + R_2 + j\omega L}$$

$$= \frac{\dfrac{R_2}{R_1 + R_2}}{1 + j\omega \dfrac{L}{R_1 + R_2}}$$

When $R_1 = 4\ \Omega$, $R_2 = 6\ \Omega$ and $L = 8$ H

$$\mathbf{H}(\omega) = \frac{0.6}{1 + j(0.8)\omega}$$

(checked using ELab on 8/6/02)

P 13.2-4 The input to the circuit shown in Figure P 13.2-4 is the voltage of the voltage source, $v_i(t)$. The output is the voltage, $v_o(t)$, across the series connection of the inductor and 60-Ω resistor. The network function that represents this circuit is

$$\mathbf{H}(\omega)=\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}=(0.6)\frac{1+j\dfrac{\omega}{12}}{1+j\dfrac{\omega}{20}}$$

Determine the values of the inductance, L, and of the resistance, R.

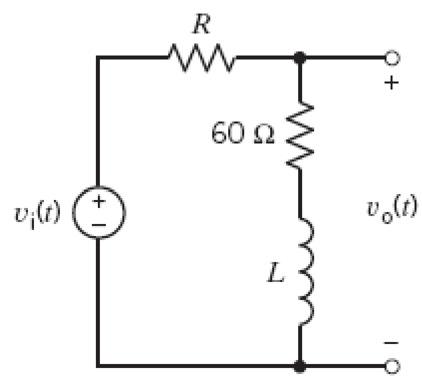
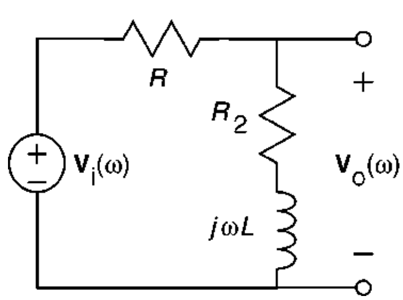


Figure P 13.2-4

Solution:



$$\mathbf{H}(\omega)=\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}=\frac{R_2+j\omega L}{R+R_2+j\omega L}$$

$$=\left(\frac{R_2}{R+R_2}\right)\left(\frac{1+j\omega\dfrac{L}{R_2}}{1+j\omega\dfrac{L}{R+R_2}}\right)$$

Comparing the given and derived network functions, we require

$$\left(\frac{R_2}{R+R_2}\right)\left(\frac{1+j\omega\dfrac{L}{R_2}}{1+j\omega\dfrac{L}{R+R_2}}\right)=(0.6)\frac{1+j\dfrac{\omega}{12}}{1+j\dfrac{\omega}{20}} \Rightarrow \begin{cases}\dfrac{R_2}{R+R_2}=0.6\\ \dfrac{R_2}{L}=12\\ \dfrac{R+R_2}{L}=20\end{cases}$$

Since R_2 = 60 Ω, we have $L=\dfrac{60}{12}=5\ \text{H}$, then $R=(20)(5)-60=40\ \Omega$.

(checked using ELab on 8/6/02)

P 13.2-5 The input to the circuit shown in Figure P 13.2-5 is the voltage of the voltage source, $v_i(t)$. The output is the voltage, $v_o(t)$, across the parallel connection of the capacitor and 2-Ω resistor. The network function that represents this circuit is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{0.2}{1 + j4\omega}$$

Determine the values of the capacitance, *C*, and of the resistance, *R*.

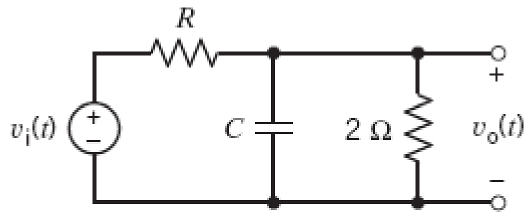
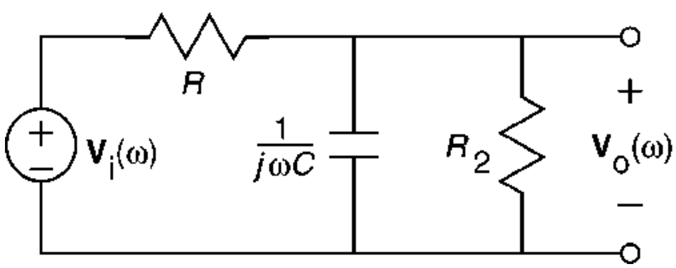


Figure P 13.2-5

Solution:



$$R_2 \parallel \frac{1}{j\omega C} = \frac{R_2}{1 + j\omega C R_2}$$

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{\dfrac{R_2}{1 + j\omega C R_2}}{R + \dfrac{R_2}{1 + j\omega C R_2}} = \frac{\dfrac{R_2}{R + R_2}}{1 + j\omega C R_p}$$

where $R_p = R \parallel R_2$.

Comparing the given and derived network functions, we require

$$\frac{\dfrac{R_2}{R + R_2}}{1 + j\omega C R_p} = \frac{0.2}{1 + j4\,\omega} \quad \Rightarrow \quad \begin{cases} \dfrac{R_2}{R + R_2} = 0.2 \\ C R_p = 4 \end{cases}$$

Since $R_2 = 2\ \Omega$, we have $\frac{2}{R + 2} = 0.2 \quad \Rightarrow \quad R = 8\ \Omega$. Then $R_p = \frac{(2)(8)}{2 + 8} = 1.6\ \Omega$. Finally, $C = \frac{4}{1.6} = 2.5\ \text{F}$.

(checked using ELab on 8/6/02)

P 13.2-6 The input to the circuit shown in Figure P 13.2-6 is the voltage of the voltage source, $v_i(t)$. The output is the voltage, $v_o(t)$, across the capacitor. Determine the network function, $\mathbf{H}(\omega) = \mathbf{V}_o(\omega)/\mathbf{V}_i(\omega)$, of this circuit.

Answer: $\mathbf{H}(\omega) = \dfrac{0.6}{(j\omega)(1+j(0.2)\omega)}$

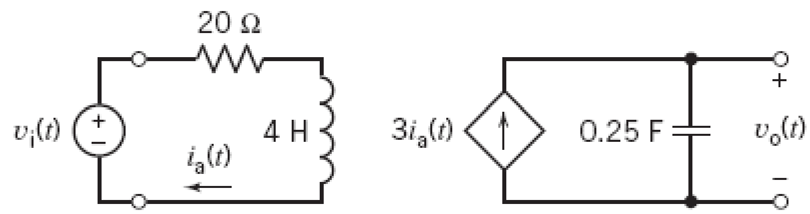
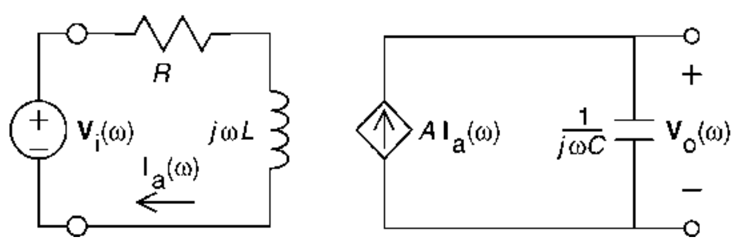


Figure P 13.2-6

Solution:



$$\left.\begin{array}{l} \mathbf{I}_a(\omega) = \dfrac{\mathbf{V}_i(\omega)}{R + j\omega L} \\ \mathbf{V}_o(\omega) = \dfrac{1}{j\omega C}\left(A\,\mathbf{I}_a(\omega)\right) \end{array}\right\} \Rightarrow \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{\dfrac{A}{CR}}{(j\omega)\left(1 + j\omega\dfrac{L}{R}\right)}$$

When $R = 20\ \Omega$, $L = 4$ H, $A = 3$ A/A and $C = 0.25$ F

$$\mathbf{H}(\omega) = \frac{0.6}{(j\omega)(1+j(0.2)\omega)}$$

(checked using LNAP on 12/29/02)

P 13.2-7 The input to the circuit shown in Figure P 13.2-7 is the voltage of the voltage source, $v_i(t)$. The output is the voltage, $v_o(t)$, across the 30-kΩ resistor. The network function of this circuit is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{4}{1 + j\dfrac{\omega}{100}}$$

Determine the value of the capacitance, C, and the value of the gain, A, of the VCVS.

Answer: $C = 5\ \mu$F and $A = 6$ V/V

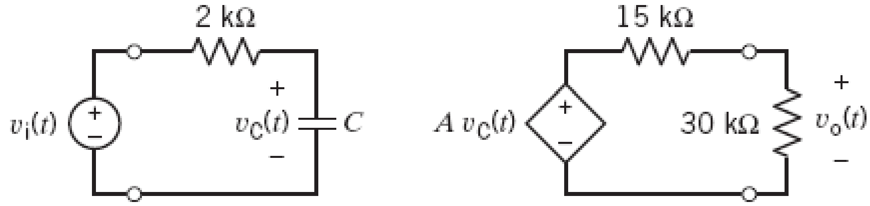
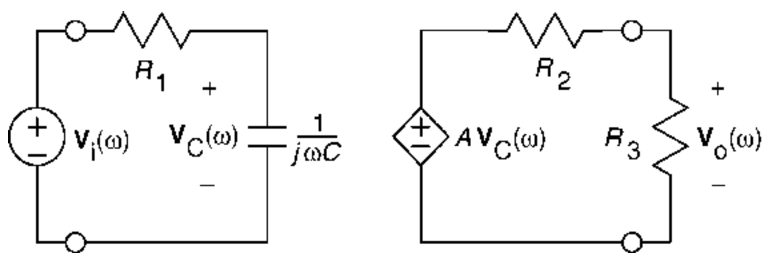


Figure P 13.2-7

Solution:



In the frequency domain, use voltage division on the left side of the circuit to get:

$$\mathbf{V}_C(\omega) = \frac{\dfrac{1}{j\omega C}}{R_1 + \dfrac{1}{j\omega C}} \mathbf{V}_i(\omega) = \frac{1}{1 + j\omega C R_1} \mathbf{V}_i(\omega)$$

Next, use voltage division on the right side of the circuit to get:

$$\mathbf{V}_o(\omega) = \frac{R_3}{R_2 + R_3} A\,\mathbf{V}_C(\omega) = \frac{2}{3} A\,\mathbf{V}_C(\omega) = \frac{\dfrac{2}{3}A}{1 + j\omega C R_1} \mathbf{V}_i(\omega)$$

Compare the specified network function to the calculated network function:

$$\frac{4}{1 + j\dfrac{\omega}{100}} = \frac{\dfrac{2}{3}A}{1 + j\omega C R_1} = \frac{\dfrac{2}{3}A}{1 + j\omega C\, 2000} \quad \Rightarrow \quad 4 = \frac{2}{3}A \ \text{ and } \ \frac{1}{100} = 2000\, C$$

Thus, $C = 5\ \mu$F and $A = 6$ V/V.

(checked using ELab on 8/6/02)

P 13.2-8 The input to the circuit shown in Figure P 13.2-8 is the source voltage, $v_i(t)$, and the response is the voltage across R_L, $v_o(t)$. Find the network function.

Answer: $\mathbf{H}(\omega) = -5/(1 + j\omega/10)$

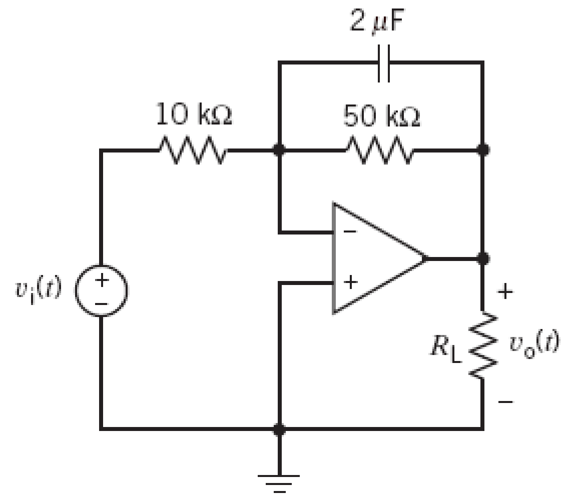
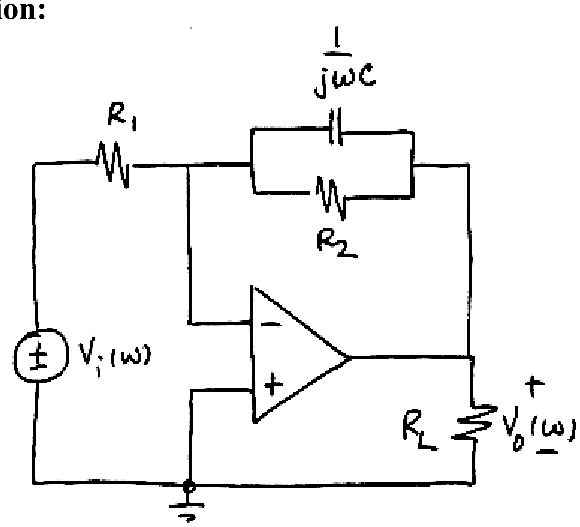


Figure P 13.2-8

Solution:



$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = -\frac{R_2 \Big\| \frac{1}{j\omega C}}{R_1}$$

$$= \frac{-\left(\frac{R_2}{R_1}\right)}{1 + j\omega C R_2}$$

When $R_1 = 10\ k\Omega$, $R_2 = 50\ k\Omega$, and $C = 2\ \mu F$, then $\frac{R_2}{R_1} = 5$ and $R_2 C = \frac{1}{10}$ so $\mathbf{H}(\omega) = \frac{-5}{1 + j\frac{\omega}{10}}$

P 13.2-9 The input to the circuit shown in Figure P 13.2-9 is the source voltage, $v_i(t)$, and the response is the voltage across R_L, $v_o(t)$. Express the gain and phase shift as functions of the radian frequency, ω.

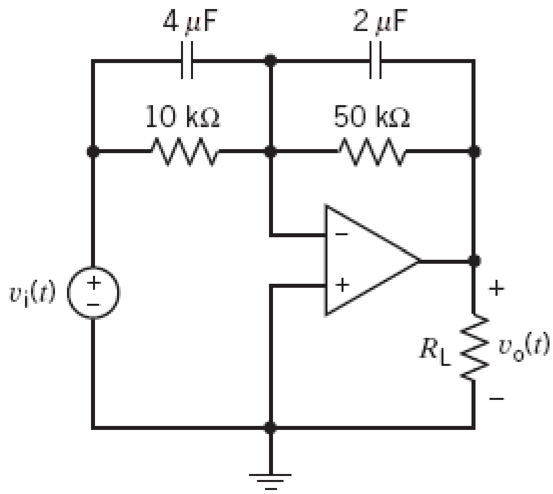
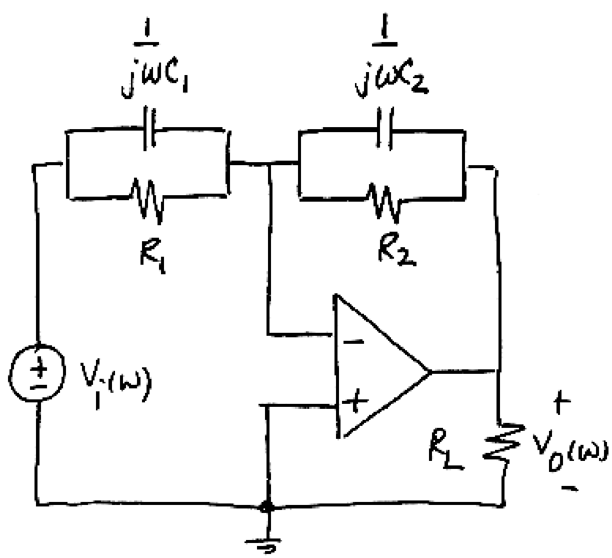


Figure P 13.2-9

Solution:



$$\mathbf{H}(\omega)=\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}=-\frac{R_2\left\|\frac{1}{j\omega C_2}\right.}{R_1\left\|\frac{1}{j\omega C_1}\right.}$$

$$=-\frac{\frac{R_2}{1+j\omega\, C_2R_2}}{\frac{R_1}{1+j\omega C_1R_1}}$$

$$\mathbf{H}(\omega)=-\left(\frac{R_2}{R_1}\right)\left(\frac{1+j\omega\, C_1R_1}{1+j\omega\, C_2R_2}\right)$$

When $R_1=10\text{ k}\Omega$, $R_2=50\text{ k}\Omega$, $C_1=4\ \mu\text{F}$ and $C_2=2\mu\text{F}$,

then $\frac{R_2}{R_1}=5$, $C_1R_1=\frac{1}{25}$ and $C_2R_2=\frac{1}{10}$ so

$$\mathbf{H}(\omega)=-5\left(\frac{1+j\dfrac{\omega}{25}}{1+j\dfrac{\omega}{10}}\right)$$

$$gain = \left|\mathbf{H}(\omega)\right| = (5)\frac{\sqrt{1+\dfrac{\omega^2}{625}}}{\sqrt{1+\dfrac{\omega^2}{100}}}$$

$$phase\ shift = \angle\mathbf{H}(\omega) = 180 + \tan^{-1}\left(\frac{\omega}{25}\right) - \tan^{-1}\left(\frac{\omega}{10}\right)$$

P 13.2-10 The input to the circuit shown in Figure P 13.2-10 is the source voltage, $v_i(t)$, and the response is the voltage across R_L, $v_o(t)$. The resistance, R_1, is 10 kΩ. Design this circuit to satisfy the following two specifications:

(a) The gain at low frequencies is 5.

(b) The gain at high frequencies is 2.

Answer: R_2 = 20 kΩ and R_3 = 30 kΩ

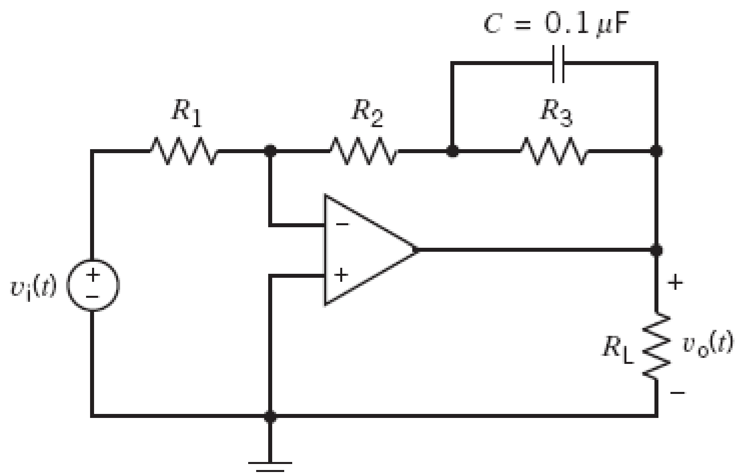


Figure P 13.2-10

Solution

$$R_3 \left\| \frac{1}{j\omega C} \right. = \frac{R_3 \dfrac{1}{j\omega C}}{R_3 + \dfrac{1}{j\omega C}} = \frac{R_3}{1 + j\omega C\, R_3}$$

$$\mathbf{H}(\omega) = -\frac{R_2 + \dfrac{R_3}{1 + j\omega C\, R_3}}{R_1} = -\frac{R_2 + R_3 + j\omega\, R_2 R_3 C}{R_1 + j\omega\, R_1 R_3 C}$$

$$5 = \lim_{\omega \to 0} \left|\mathbf{H}\,(\omega)\right| = \frac{R_2 + R_3}{R_1}$$

$$2 = \lim_{\omega \to \infty} \left|\mathbf{H}\,(\omega)\right| = \frac{R_2}{R_1} \Rightarrow R_2 = 2\,R_1 = 20 \text{ k}\Omega$$

then $\quad R_3 = 5R_1 - R_2 = 30 \text{ k}\Omega$

P 13.2-11 The input to the circuit shown in Figure P 13.2-11 is the source voltage, $v_i(t)$, and the response is the voltage across R_L, $v_o(t)$. Design this circuit to satisfy the following two specifications:

(a) The phase shift at ω = 1000 rad/s is 135°.

(b) The gain at high frequencies is 10.

Answer: R_1 = 1 kΩ and R_2 = 10 kΩ

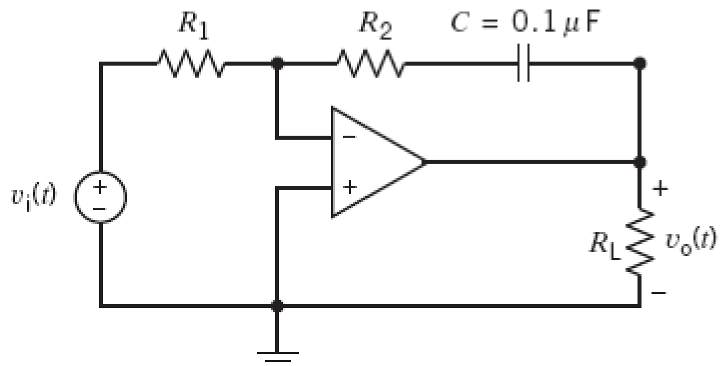


Figure P 13.2-11

Solution:

$$\mathbf{H}(\omega) = -\frac{R_2 + \dfrac{1}{j\omega C}}{R_1} = -\frac{1 + j\omega\, C R_2}{j\omega C R_1}$$

$$\angle\mathbf{H}(\omega) = 180° + \tan^{-1}\left(\omega C R_2\right) - 90°$$

$$\angle\mathbf{H}(\omega) = 135° \Rightarrow \tan^{-1}\left(\omega C R_2\right) = 45° \Rightarrow \omega C R_2 = 1$$

$$\Rightarrow R_2 = \frac{1}{10^3 \times 10^{-7}} = 10 \text{ k}\Omega$$

$$10 = \lim_{\omega\to\infty} \left|\mathbf{H}(\omega)\right| = \frac{R_2}{R_1} \Rightarrow R_1 = \frac{R_2}{10} = 1 \text{ k}\Omega$$

P 13.2-12 The input to the circuit shown in Figure P 13.2-12 is the source voltage, $v_i(t)$, and the response is the voltage across R_L, $v_o(t)$. Design this circuit to satisfy the following two specifications:

(a) The phase shift at ω = 1000 rad/s is 225°.

(b) The gain at high frequencies is 10.

Answer: R_1 = 10 kΩ and R_2 = 100 kΩ

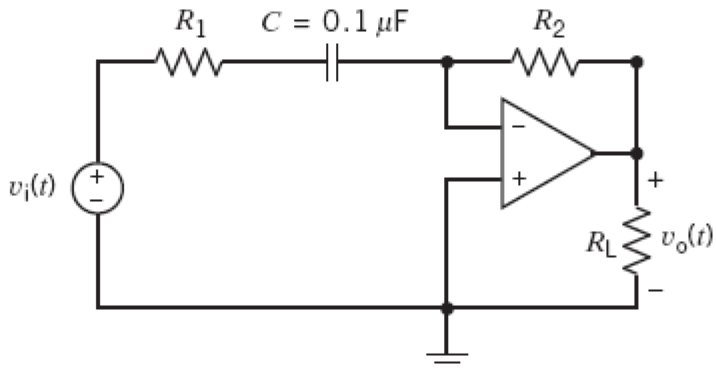


Figure P 13.2-12

Solution:

$$\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)} = \mathbf{H}(\omega) = \frac{-R_2}{R_1 + \dfrac{1}{j\omega C}} = -\frac{j\omega C R_2}{1 + j\omega C R_1}$$

$$10 = \lim_{\omega \to \infty} |\mathbf{H}(\omega)| = \frac{R_2}{R_1} \Rightarrow R_2 = 10 R_1$$

$$\angle\, \mathbf{H}(1000) = 180° + 90° - \tan^{-1} 1000\, C\, R_1 \quad \Rightarrow \quad R_1 = \frac{\tan(270° - 225°)}{1000\, C} = 10 \text{ k}\Omega$$

$$R_2 = 100 \text{ k}\Omega$$

P 13.2-13 The input to the circuit of Figure P 13.2-13 is

$$v_s = 50 + 30\cos(500t + 115°) - 20\cos(2500t + 30°)\ \text{mV}$$

Find the steady-state output voltage, v_o, for (a) $C = 0.1\mu$F and (b) $C = 0.01\mu$F. Assume an ideal op amp.

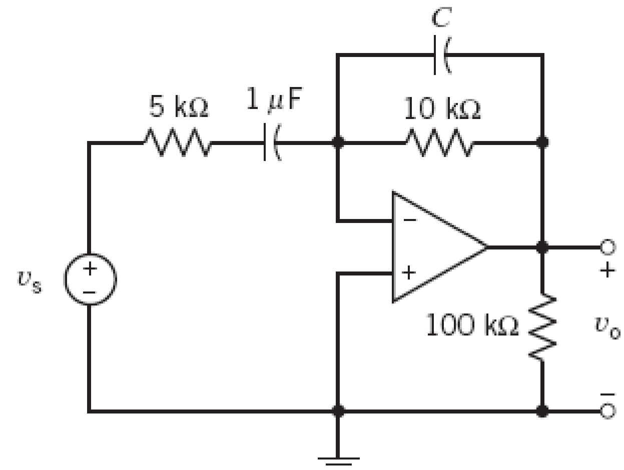
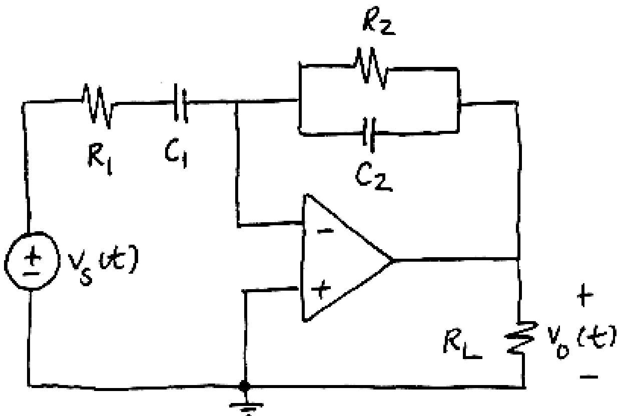


Figure P 13.2-13

Solution:



$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)} = -\frac{R_2 \| \dfrac{1}{j\omega C_2}}{R_1 + \dfrac{1}{j\omega C_1}} = \frac{(-C_1 R_2)\, j\omega}{(1 + j\omega R_1 C_1)\,(1 + j\omega R_2 C_2)}$$

When $R_1 = 5\ \text{k}\Omega$, $C_1 = 1\ \mu\text{F}$, $R_2 = 10\ \text{k}\Omega$ and $C_2 = 0.1\ \mu\text{F}$, then

$$\mathbf{H}(\omega) = \frac{(-0.01)\, j\omega}{\left(1 + j\dfrac{\omega}{200}\right)\left(1 + j\dfrac{\omega}{1000}\right)}$$

so

ω	$\lvert\mathbf{H}(\omega)\rvert$	$\angle\mathbf{H}(\omega)$
0	0	–90°
500	1.66	175°
2500	0.74	116°

Then

$$v_o(t) = (0)\ 50 + (1.66)(30)\cos(500t + 115° + 175°) - (0.74)(20)\cos(2500t + 30° + 116°)$$
$$= 49.8\cos(500t - 70°) - 14.8\cos(2500t + 146°)\ \text{mV}$$

When $R_1 = 5\ \text{k}\Omega$, $C_1 = 1\ \mu\text{F}$, $R_2 = 10\ \text{k}\Omega$ and $C_2 = 0.01\ \mu\text{F}$, then

$$\mathbf{H}(\omega) = -0.01\ \frac{j\omega}{\left(1 + j\dfrac{\omega}{200}\right)\left(1 + j\dfrac{\omega}{10{,}000}\right)}$$

So

ω	$\lvert\mathbf{H}(\omega)\rvert$	$\angle\mathbf{H}(\omega)$
0	0	–90°
500	1.855	–161°
2500	1.934	170°

Then

$$v_o(t) = (0)(50) + (1.855)(30)\cos(500t + 115° - 161°) - (1.934)(20)\cos(2500t + 30° + 170°)$$
$$= 55.65\cos(500t - 46°) - 38.68\cos(2500t + 190°)\ \text{mV}$$

P 13.2-14 The source voltage, v_s, shown in the circuit of Figure P 13.2-14*a* is a sinusoid having a frequency of 500 Hz and an amplitude of 8 V. The circuit is in steady state. The oscilloscope traces show the input and output waveforms as shown in Figure P 13.2-14*b*.

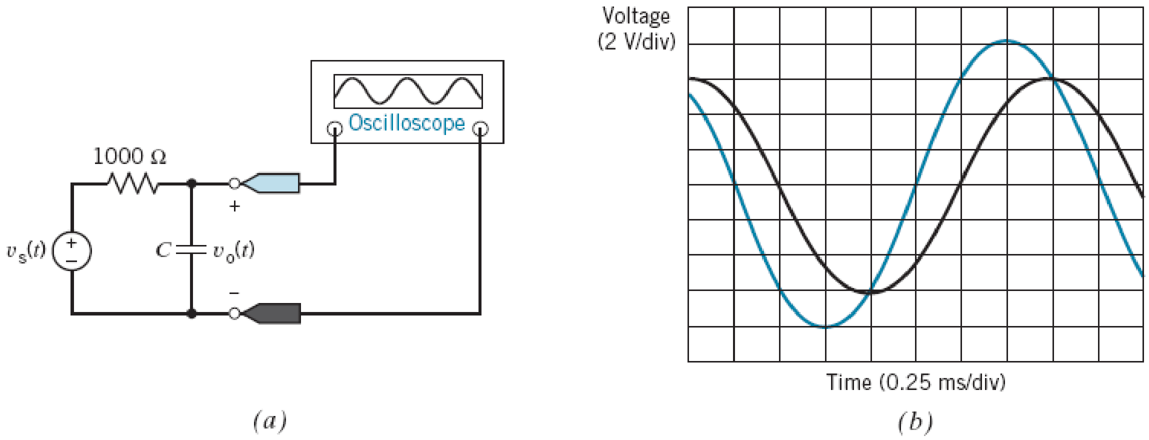


Figure P 13.2-14

(a) Determine the gain and phase shift of the circuit at 500 Hz.

(b) Determine the value of the capacitor.

(c) If the frequency of the input is changed, then the gain and phase shift of the circuit will change. What are the values of the gain and phase shift at the frequency 200 Hz? At 2000 Hz? At what frequency will the phase shift be –45°? At what frequency will the phase shift be –135°?

(d) What value of capacitance would be required to make the phase shift at 500 Hz be –60°? What value of capacitance would be required to make the phase shift at 500 Hz be –300°?

(e) Suppose the phase shift had been –120°at 500 Hz. What would be the value of the capacitor?

Answer: (b) $C = 0.26\ \mu\text{F}$ (e) this circuit can't be designed to produce a phase shift = –120°

Solution:

$$\textit{(a)}\quad |\mathbf{V}_s| = \frac{(8\text{ div})\left(\dfrac{2\text{ V}}{\text{div}}\right)}{2} = 8\text{ V} \quad \text{and} \quad |\mathbf{V}_o| = \frac{(6.2\text{ div})\left(\dfrac{2\text{ V}}{\text{div}}\right)}{2} = 6.2\text{ V} \quad \text{so} \quad \text{gain} = \frac{|\mathbf{V}_o|}{|\mathbf{V}_s|} = \frac{6.2}{8} = 0.775$$

(b) $\mathbf{H}(\omega) = \dfrac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)} = \dfrac{\dfrac{1}{j\omega C}}{R + \dfrac{1}{j\omega C}} = \dfrac{1}{1 + j\omega C R}$.

Let $g = |\mathbf{H}(\omega)| = \dfrac{1}{\sqrt{1+\omega^2 C^2 R^2}}$ then $C = \dfrac{1}{\omega R}\sqrt{\left(\dfrac{1}{g}\right)^2 - 1}$

In this case $\omega = 2\pi \cdot 500 = 3142$ rad/s, $|\mathbf{H}(\omega)| = 0.775$ and $R = 1000\ \Omega$ so $C = 0.26\ \mu$F.

(c) $\angle\,\mathbf{H}(\omega) = -\tan^{-1}\left(\omega R C\right)$ so $\omega = \dfrac{\tan(-\angle\,\mathbf{H}(\omega))}{RC}$

Recalling that R = 1000 Ω and C=0.26μF, we calculate

ω	$\lvert\mathbf{H}(\omega)\rvert$	$\angle\mathbf{H}(\omega)$
$2\pi(200)$	0.95	−18°
$2\pi(2000)$	0.26	−73°

$$\angle\,\mathbf{H}(\omega) = -45° \text{ requires } \omega = \frac{\tan\left(-\left(-45^\circ\right)\right)}{(1000)\left(.26\times10^{-6}\right)} = 3846 \text{ rad}/s$$

$$\angle\,\mathbf{H}(\omega) = -135° \text{ requires } \omega = \frac{\tan\,(-(-135°))}{(1000)(0.26\times10^{-6})} = -3846 \text{ rad/s}$$

A negative frequency is not acceptable. We conclude that this circuit cannot produce a phase shift equal to −135°.

(d)

$$C = \frac{\tan\ (-\angle\mathbf{H}(\omega))}{\omega R} \quad \Rightarrow \quad \begin{cases} C = \dfrac{\tan\ (-60°)}{(2\pi\cdot500)\ (1000)} = \ 0.55\mu\text{F} \\ C = \dfrac{\tan\ (-(-300°))}{(2\pi\cdot500\ )\ (1000)} = -0.55\mu\text{F} \end{cases}$$

A negative value of capacitance is not acceptable and indicates that this circuit cannot be designed to produce a phase shift at −300° at a frequency of 500 Hz.

(e)

$$C \ = \ \frac{\tan(-(-120^\circ))}{(2\pi\cdot500)(1000)} = -0.55\ \mu\text{F}$$

This circuit cannot be designed to produce a phase shift of −120° at 500 Hz.

P 13.2-15 The input to the circuit in Figure P 13.2-15 is the voltage of the voltage source, $v_i(t)$. The output is the voltage $v_o(t)$. The network function of this circuit is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{(-0.1)\, j\omega}{\left(1 + j\dfrac{\omega}{p}\right)\left(1 + j\dfrac{\omega}{125}\right)}$$

Determine the values of the capacitance, C, and the pole, p.

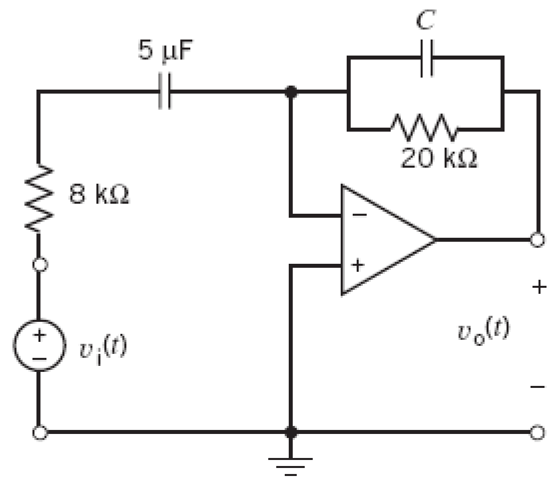
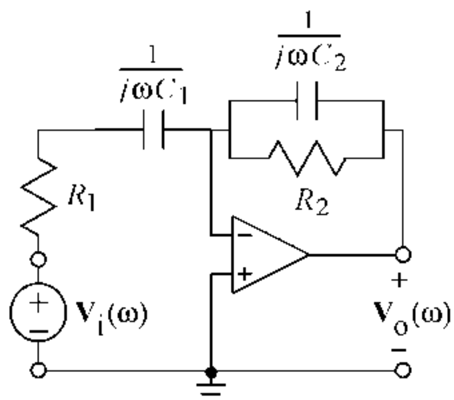


Figure P 13.2-15

Solution:



$$\mathbf{H}(\omega) = -\frac{R_2 \parallel \dfrac{1}{j\omega C_2}}{R_1 + \dfrac{1}{j\omega C_1}} = -\frac{\dfrac{R_2}{1 + j\omega C_2 R_2}}{\dfrac{j\omega C_1 R_1 + 1}{j\omega C_1}}$$

$$= \frac{\left(-C_1 R_2\right) j\omega}{\left(1 + j\omega C_1 R_1\right)\left(1 + j\omega C_2 R_2\right)}$$

$$\frac{\left(-C_1 R_2\right) j\omega}{\left(1 + j\omega C_1 R_1\right)\left(1 + j\omega C_2 R_2\right)} = \frac{(-0.1)\, j\omega}{\left(1 + j\dfrac{\omega}{p}\right)\left(1 + j\dfrac{\omega}{125}\right)} \Rightarrow \begin{cases} -C_1 R_2 = -0.1 \\ C_1 R_1 = \dfrac{1}{p} \text{ or } \dfrac{1}{125} \\ C_2 R_2 = \dfrac{1}{125} \text{ or } \dfrac{1}{p} \end{cases}$$

Since $C_1 = 5\ \mu\text{F}$, $R_1 = 8\ \text{k}\Omega$ and $R_2 = 20\ \text{k}\Omega$

$$C_1 R_1 = \left(5\times10^{-6}\right)\left(8\times10^{3}\right) = \frac{40}{1000} = \frac{1}{25} \neq \frac{1}{125} \Rightarrow p = 25 \text{ rad/s}$$

$$\frac{1}{125} = C_2 R_2 \Rightarrow C_2 = \frac{1}{125 R_2} = \frac{1}{125\left(20\times10^{3}\right)} = 0.4\times10^{-6} = 0.4\ \mu\text{F}$$

P 13.2-16 The input to the circuit in Figure P 13.2-16 is the voltage of the voltage source, $v_s(t)$. The output is the voltage $v_o(t)$. The network function of this circuit is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)} = k\frac{1 + j\dfrac{\omega}{z}}{1 + j\dfrac{\omega}{p}}$$

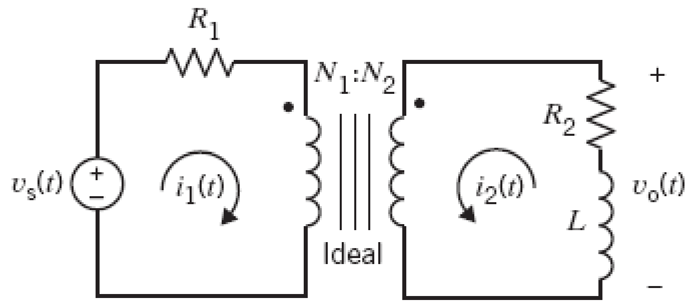


Figure P 13.2-16

Determine expressions that relate the network function parameters *k, z*, and *p* to the circuit parameters R_1, R_2, *L*, N_1, and N_2.

Solution:

$$\mathbf{I}_1(\omega) = \frac{\mathbf{V}_s(\omega)}{R_1 + \left(\dfrac{N_1}{N_2}\right)^2 (R_2 + j\omega L)}$$

$$\mathbf{V}_o(\omega) = -(R_2 + j\omega L)\mathbf{I}_2(\omega) = -(R_2 + j\omega L)\left(-\frac{N_1}{N_2}\mathbf{I}_1(\omega)\right) = \frac{\left(\dfrac{N_1}{N_2}\right)(R_2 + j\omega L)\mathbf{V}_s(\omega)}{R_1 + \left(\dfrac{N_1}{N_2}\right)^2 (R_2 + j\omega L)}$$

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)} = \frac{\left(\dfrac{N_1}{N_2}\right)(R_2 + j\omega L)}{R_1 + \left(\dfrac{N_1}{N_2}\right)^2 (R_2 + j\omega L)} = \frac{\left(\dfrac{N_2}{N_1}\right)R_2}{\left(\dfrac{N_2}{N_1}\right)^2 R_1 + R_2} \frac{1 + j\omega\dfrac{L}{R_2}}{1 + j\omega\dfrac{L}{\left(\dfrac{N_2}{N_1}\right)^2 R_1 + R_2}}$$

Comparing to the given network function:

$$k = \frac{\left(\dfrac{N_2}{N_1}\right)R_2}{\left(\dfrac{N_2}{N_1}\right)^2 R_1 + R_2}, \quad z = \frac{R_2}{L} \text{ and } p = \frac{\left(\dfrac{N_2}{N_1}\right)^2 R_1 + R_2}{L}.$$

P 13.2-17 The input to the circuit in Figure P 13.2-17 is the voltage of the voltage source, $v_s(t)$. The output is the voltage $v_o(t)$. The network function of this circuit is

$$\mathbf{H}(\omega)=\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)}=k\frac{j\omega}{1+j\dfrac{\omega}{p}}$$

Determine expressions that relate the network function parameters k and p to the circuit parameters R_1, R_2, M, L_1, and L_2.

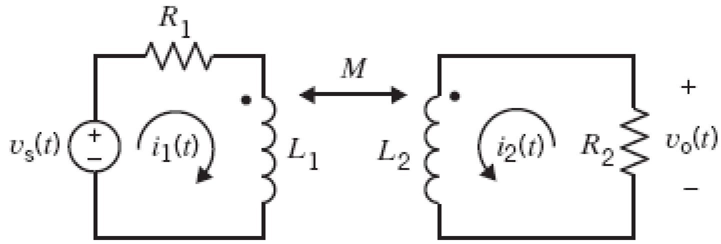


Figure P 13.2-17

Solution: Mesh equations:

$$\mathbf{V}_s(\omega)=\left(R_1+j\omega L_1\right)\mathbf{I}_1(\omega)+j\omega M\,\mathbf{I}_2(\omega)$$

$$0=\left(R_2+j\omega L_2\right)\mathbf{I}_2(\omega)+j\omega M\,\mathbf{I}_1(\omega)$$

Solving the mesh equations

$$\mathbf{I}_1(\omega)=-\frac{R_2+j\omega L_2}{j\omega M}\mathbf{I}_2(\omega)$$

$$\mathbf{V}_s(\omega)=\left[-\left(R_1+j\omega L_1\right)\frac{R_2+j\omega L_2}{j\omega M}+j\omega M\right]\mathbf{I}_2(\omega)$$

$$\mathbf{I}_2(\omega)=\frac{-j\omega M}{\left(R_1+j\omega L_1\right)\left(R_2+j\omega L_2\right)+\omega^2M^2}\mathbf{V}_s(\omega)$$

$$\mathbf{V}_o(\omega)=-R_2\,\mathbf{I}_2(\omega)=\frac{j\omega M\,R_2}{\left(R_1+j\omega L_1\right)\left(R_2+j\omega L_2\right)+\omega^2M^2}\mathbf{V}_s(\omega)$$

$$=\frac{j\omega M\,R_2}{R_1R_2+\omega^2\left(M^2-L_1L_2\right)+j\omega\left(R_1L_2+L_1R_2\right)}\mathbf{V}_s(\omega)$$

$$\mathbf{H}(\omega)=\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)}=\frac{M\,R_2}{R_1R_2+\omega^2\left(M^2-L_1L_2\right)}\frac{j\omega}{1+j\omega\dfrac{R_1L_2+L_1R_2}{R_1R_2+\omega^2\left(M^2-L_1L_2\right)}}$$

Comparing to the given network function:

$$k=\frac{M\,R_2}{R_1R_2+\omega^2\left(M^2-L_1L_2\right)}\quad\text{and}\quad p=\frac{R_1R_2+\omega^2\left(M^2-L_1L_2\right)}{R_1L_2+L_1R_2}$$

P 13.2-18 The input to the circuit in Figure P 13.2-18 is the voltage of the voltage source, $v_i(t)$. The output is the voltage $v_o(t)$. The network function of this circuit is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = k\frac{j\omega}{\left(1 + j\dfrac{\omega}{p_1}\right)\left(1 + j\dfrac{\omega}{p_2}\right)}$$

Determine expressions that relate the network function parameters k, p_1, and p_2 to the circuit parameters R_1, R_2, R_3, R_4, A, C, and L.

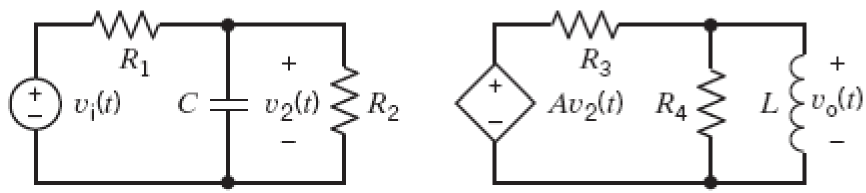


Figure P 13.2-18

P13.2-18

Using voltage division twice gives:

$$\frac{A\mathbf{V}_2(\omega)}{\mathbf{V}_i(\omega)} = \frac{\dfrac{R_2}{1 + j\omega C R_2}}{R_1 + \dfrac{R_2}{1 + j\omega C R_2}} A = \frac{A R_2}{R_1 + R_2 + j\omega C R_1 R_2} = \frac{\dfrac{A R_2}{R_1 + R_2}}{1 + j\omega \dfrac{C R_1 R_2}{R_1 + R_2}}$$

and

$$\frac{\mathbf{V}_o(\omega)}{A\mathbf{V}_2(\omega)} = \frac{\dfrac{j\omega L R_4}{R_4 + j\omega L}}{R_3 + \dfrac{j\omega L R_4}{R_4 + j\omega L}} = \frac{j\omega L R_4}{R_3 R_4 + j\omega L\left(R_3 + R_4\right)} = \frac{L}{R_3}\frac{j\omega}{1 + j\omega\dfrac{L\left(R_3 + R_4\right)}{R_3 R_4}}$$

Combining these equations gives

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{A L R_2}{R_3\left(R_1 + R_2\right)} \frac{j\omega}{\left(1 + j\omega\dfrac{L\left(R_3 + R_4\right)}{R_3 R_4}\right)\left(1 + j\omega\dfrac{C R_1 R_2}{R_1 + R_2}\right)}$$

Comparing to the given network function gives $k = \dfrac{A L R_2}{R_3\left(R_1 + R_2\right)}$ and either $p_1 = \dfrac{R_3 R_4}{L\left(R_3 + R_4\right)}$ and $p_2 = \dfrac{R_1 + R_2}{C R_1 R_2}$ or $p_1 = \dfrac{R_1 + R_2}{C R_1 R_2}$ and $p_2 = \dfrac{R_3 R_4}{L\left(R_3 + R_4\right)}$.

P 13.2-19 The input to the circuit shown in Figure P 13.2-19 is the voltage of the voltage source, v_s. The output of the circuit is the capacitor voltage, v_o. Determine the values of the resistances R_1, R_2, R_3, and R_4 required to cause the network function of the circuit to be

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)} = \frac{21}{\left(1 + j\frac{\omega}{5}\right)\left(1 + j\frac{\omega}{200}\right)}$$

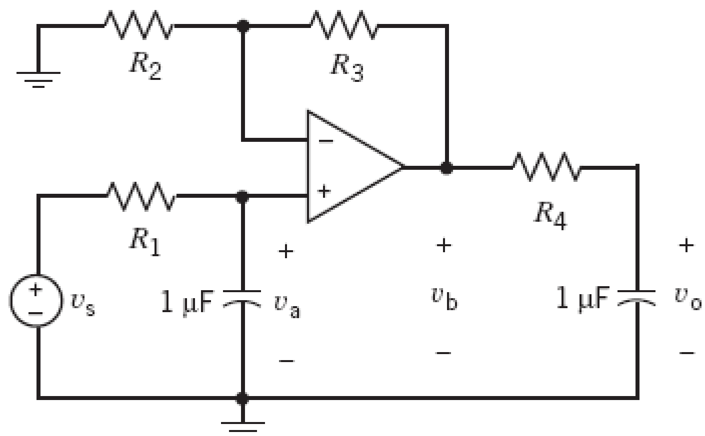
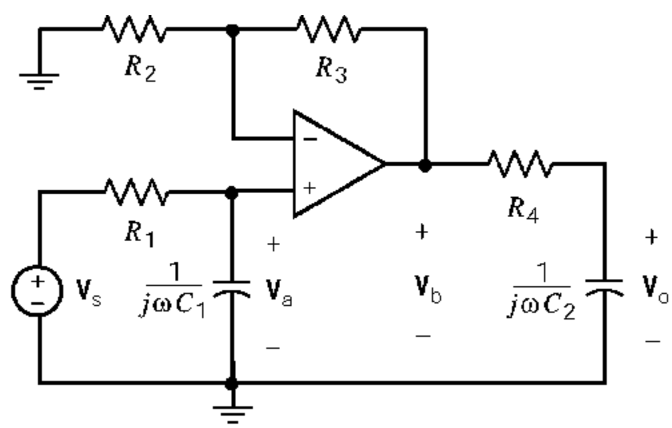


Figure P 13.2-19

Solution: Represent the circuit in the frequency domain.



Apply KCL at the top node of the left capacitor, C_1, to get

$$\frac{\mathbf{V}_a - \mathbf{V}_s}{R_1} + j\omega C_1 \mathbf{V}_a = 0 \quad \Rightarrow \quad \mathbf{V}_a = \frac{1}{1 + j\omega C_1 R_1} \mathbf{V}_s$$

The op amp, together with resistors R_2 and R_3, comprise a noninverting amplifier so

$$\mathbf{V}_b = \left(1 + \frac{R_3}{R_2}\right)\mathbf{V}_a$$

(Alternately, this equation can be obtained by applying KCL at the inverting input node of the op amp.)

Apply KCL at the top node of the right capacitor, C_2, to get

$$\frac{\mathbf{V}_\mathrm{o}-\mathbf{V}_\mathrm{b}}{R_4}+j\omega C_2\,\mathbf{V}_\mathrm{o}=0 \quad\Rightarrow\quad \mathbf{V}_\mathrm{o}=\frac{1}{1+j\omega C_2 R_4}\mathbf{V}_\mathrm{b}$$

Combining these equations gives

$$\mathbf{H}(\omega)=\frac{\mathbf{V}_\mathrm{o}(\omega)}{\mathbf{V}_\mathrm{s}(\omega)}=\frac{1+\dfrac{R_3}{R_2}}{(1+j\omega C_1 R_1)(1+j\omega C_2 R_4)}$$

Comparing to the specified network function gives

$$\frac{1+\dfrac{R_3}{R_2}}{(1+j\omega C_1 R_1)(1+j\omega C_2 R_4)}=\frac{21}{\left(1+j\dfrac{\omega}{5}\right)\left(1+j\dfrac{\omega}{200}\right)}$$

The solution is not unique. For example, we can require

$$1+\frac{R_3}{R_2}=21\,,\; C_1 R_1=\frac{1}{5}=0.2\,,\; C_2 R_4=\frac{1}{200}=0.005$$

With the given values of capacitance, and choosing R_2 = 10 kΩ, we have

R_1 = 200 kΩ, R_2 = 10 kΩ, R_3 = 200 kΩ and R_4 = 5 kΩ

(checked using LNAP 9/14/04)

P 13.2-20 The input to the circuit shown in Figure P 13.2-20 is the voltage of the voltage source, v_s. The output of the circuit is the voltage v_o. Determine the network function

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)}$$

of the circuit.

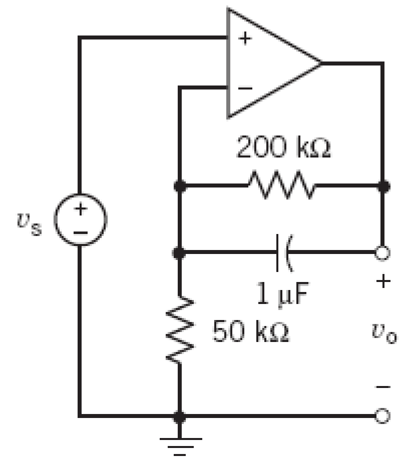
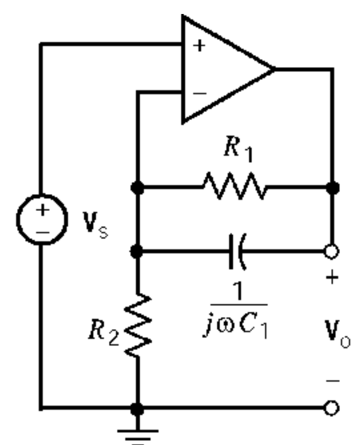


Figure P 13.2-20

Solution:
Represent the circuit in the frequency domain. Apply KCL at the inverting input node of the op amp to get



$$\frac{\mathbf{V}_o - \mathbf{V}_s}{R_1} + j\omega C_1 \left(\mathbf{V}_o - \mathbf{V}_s\right) + \frac{\mathbf{V}_s}{R_2} = 0$$

or

$$\left(R_1 + R_2 + j\omega C_1 R_1 R_2\right)\mathbf{V}_s = \left(R_2 + j\omega C_1 R_1 R_2\right)\mathbf{V}_o$$

so

$$\mathbf{H} = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{R_1 + R_2 + j\omega C_1 R_1 R_2}{R_2 + j\omega C_1 R_1 R_2} = \frac{R_1 + R_2}{R_2} \times \frac{1 + j\omega C_1 \frac{R_1 R_2}{R_1 + R_2}}{1 + j\omega C_1 R_1}$$

With the given values

$$\mathbf{H} = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{25 + j\omega}{5 + j\omega} = 5\,\frac{1 + j\frac{\omega}{25}}{1 + j\frac{\omega}{5}}$$

(checked using LNAP 7/24/05)

P 13.2-21 The input to the circuit shown in Figure P 13.2-21 is the voltage of the voltage source, v_s. The output of the circuit is the capacitor voltage, v_o. Determine the network function

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)}$$

of the circuit.

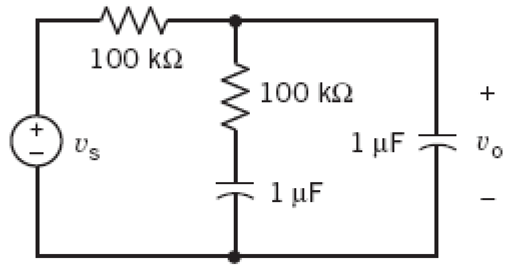
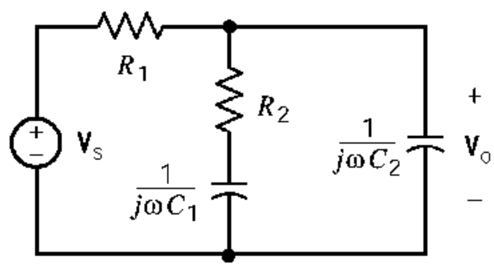


Figure P 13.2-21

Solution:

Represent the circuit in the frequency domain. After determining some equivalent impedances, the network function can be determined using voltage division.



$$\frac{1}{j\omega C_2} \| \left(R_2 + \frac{1}{j\omega C_1}\right) = \frac{\frac{1}{j\omega C_2}\left(R_2 + \frac{1}{j\omega C_1}\right)}{R_2 + \frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}} = \frac{1 + j\omega C_1 R_2}{j\omega\left(C_1 + C_2\right) - \omega^2 R_2 C_1 C_2}$$

Next, using voltage division gives

$$\mathbf{H} = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{\frac{1 + j\omega C_1 R_2}{j\omega\left(C_1 + C_2\right) - \omega^2 R_2 C_1 C_2}}{R_1 + \frac{1 + j\omega C_1 R_2}{j\omega\left(C_1 + C_2\right) - \omega^2 R_2 C_1 C_2}} = \frac{1 + j\omega C_1 R_2}{R_1\left[j\omega\left(C_1 + C_2\right) - \omega^2 R_2 C_1 C_2\right] + 1 + j\omega C_1 R_2}$$

$$= \frac{1 + j\omega C_1 R_2}{1 - \omega^2 C_1 C_2 R_1 R_2 + j\omega\left(C_1 R_1 + C_2 R_1 + R_2 C_1\right)}$$

With the given values

$$\mathbf{H} = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{1 + j\frac{\omega}{10}}{1 - \frac{\omega^2}{100} + j\frac{3\omega}{10}} = \frac{100 + j10\omega}{100 - \omega^2 + j30\omega}$$

P 13.2-22 The input to the circuit shown in Figure P 13.2-22 is the voltage of the voltage source, v_s. The output of the circuit is the capacitor voltage, v_o. The network function of the circuit is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)} = \frac{H_o}{1+j\dfrac{\omega}{p}}$$

Determine the values of H_o and p.

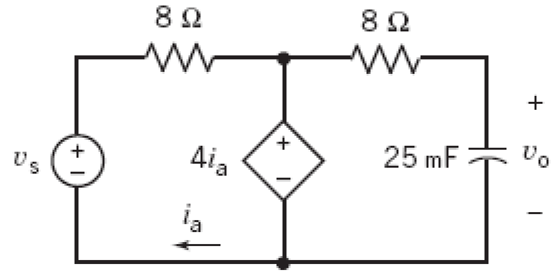
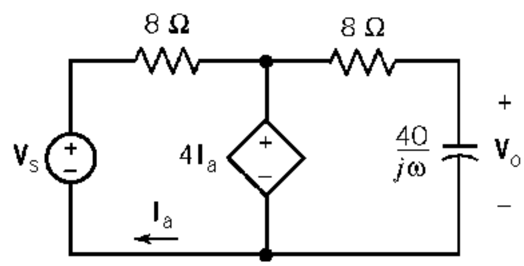


Figure P 13.2-22

Solution:

Represent the circuit in the frequency domain. Apply KVL to the left mesh to get



$$\mathbf{V}_s = 8\mathbf{I}_a + 4\mathbf{I}_a \quad \Rightarrow \quad \mathbf{I}_a = \frac{\mathbf{V}_s}{12}$$

Voltage division gives

$$\mathbf{V}_o = \frac{\dfrac{40}{j\omega}}{8+\dfrac{40}{j\omega}} 4\mathbf{I}_a = \frac{4}{1+\dfrac{j\omega}{5}}\mathbf{I}_a = \frac{4}{1+\dfrac{j\omega}{5}}\left(\frac{\mathbf{V}_s}{12}\right) = \frac{\dfrac{1}{3}}{1+\dfrac{j\omega}{5}}\mathbf{V}_s$$

The network function of the circuit is

$$\mathbf{H} = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{\dfrac{1}{3}}{1+\dfrac{j\omega}{5}}$$

Comparing this network function to the specified network function gives

$$H_o = \frac{1}{3} \text{ and } p = 5$$

(checked using LNAP 9/19/04)

P 13.2-23 The input to the circuit shown in Figure P 13.2-23 is the current of the current source, i_s. The output of the circuit is the resistor current, i_o. The network function of the circuit is

$$\mathbf{H}(\omega)=\frac{\mathbf{I}_o(\omega)}{\mathbf{I}_s(\omega)}=\frac{0.8}{1+j\dfrac{\omega}{40}}$$

Determine the values of the resistances R_1 and R_2.

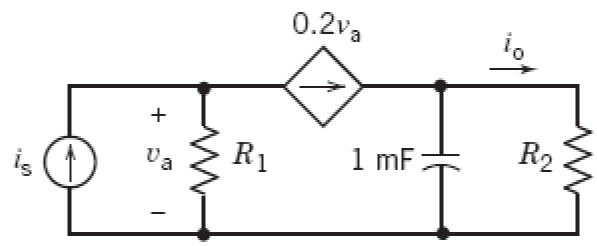
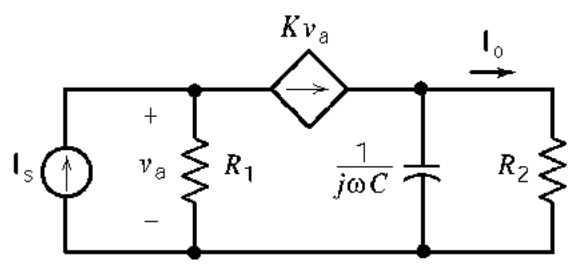


Figure P 13.2-23

Solution:
Represent the circuit in the frequency domain. Apply KCL at the top node of R_1 to get



$$\mathbf{I}_s=\frac{\mathbf{V}_a}{R_1}+K\,\mathbf{V}_a \quad\Rightarrow\quad \mathbf{V}_a=\frac{R_1}{1+K\,R_1}\mathbf{I}_s$$

Current division gives

$$\mathbf{I}_o=\frac{\dfrac{1}{j\omega C}}{\dfrac{1}{j\omega C}+R_2}K\,\mathbf{V}_a=\frac{K}{1+j\omega C\,R_2}\mathbf{V}_a=\frac{K}{1+j\omega C\,R_2}\left(\frac{R_1}{1+K\,R_1}\mathbf{I}_s\right)$$

The network function of the circuit is

$$\mathbf{H}=\frac{\mathbf{I}_o}{\mathbf{I}_s}=\frac{\dfrac{K\,R_1}{1+K\,R_1}}{1+j\omega C\,R_2}$$

Comparing this network function to the specified network function gives

$$\frac{K\,R_1}{1+K\,R_1}=0.8 \text{ and } C\,R_2=\frac{1}{40}$$

With the given values

$$\frac{0.2\,R_1}{1+0.2\,R_1}=0.8 \quad\Rightarrow\quad R_1=20\ \Omega \text{ and } 0.001R_2=\frac{1}{40} \quad\Rightarrow\quad R_2=25\ \Omega$$

(checked using LNAP 9/19/04)

P 13.2-24 The input to the circuit shown in Figure P 13.2-24 is the voltage of the voltage source, v_s. The output of the circuit is the resistor voltage, v_o. Specify values for L_1, L_2, R, and K that cause the network function of the circuit to be

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)} = \frac{1}{\left(1 + j\frac{\omega}{20}\right)\left(1 + j\frac{\omega}{50}\right)}$$

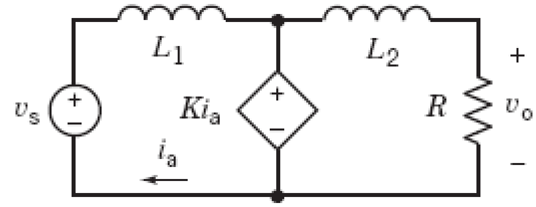
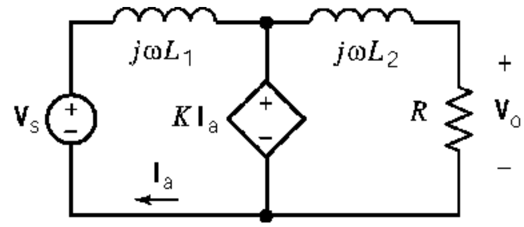


Figure P 13.2-24

Solution:
Represent the circuit in the frequency domain. Apply KVL to the left mesh to get



$$\mathbf{V}_s = j\omega L_1 \mathbf{I}_a + K \mathbf{I}_a \quad \Rightarrow \quad \mathbf{I}_a = \frac{\mathbf{V}_s}{K + j\omega L_1}$$

Voltage division gives

$$\mathbf{V}_o = \frac{R}{R + j\omega L_2} K \mathbf{I}_a = \frac{R}{R + j\omega L_2} K \left(\frac{\mathbf{V}_s}{K + j\omega L_1}\right) = \frac{R K}{\left(R + j\omega L_2\right)\left(K + j\omega L_1\right)} \mathbf{V}_s$$

The network function of the circuit is

$$\mathbf{H} = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{1}{\left(1 + j\omega\frac{L_2}{R}\right)\left(1 + j\omega\frac{L_1}{K}\right)}$$

Comparing this network function to the specified network function gives

$$\frac{L_2}{R} = \frac{1}{20} \text{ and } \frac{L_1}{K} = \frac{1}{50} \quad \text{or} \quad \frac{L_2}{R} = \frac{1}{50} \text{ and } \frac{L_1}{K} = \frac{1}{20}$$

These equations do not have a unique solution. One solution is

$$L_1 = 0.1 \text{ H}, \; L_2 = 0.1 \text{ H}, \; R = 5\ \Omega \text{ and } K = 2 \text{ V/A}$$

(checked using LNAP 9/19/04)

P 13.2-25 The input to the circuit shown in Figure P 13.2-25 is the voltage of the voltage source, v_s. The output of the circuit is the resistor voltage, v_o. Specify values for R and C that cause the network function of the circuit to be

$$\mathbf{H}(\omega)=\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)}=\frac{-8}{1+j\frac{\omega}{250}}$$

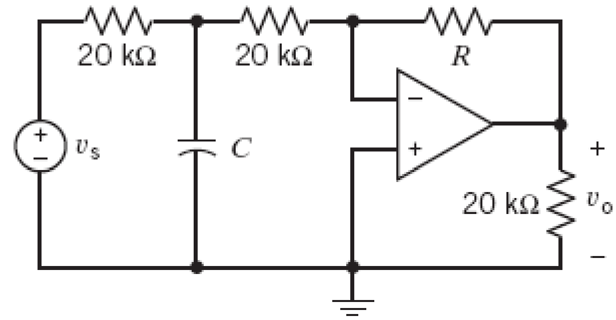
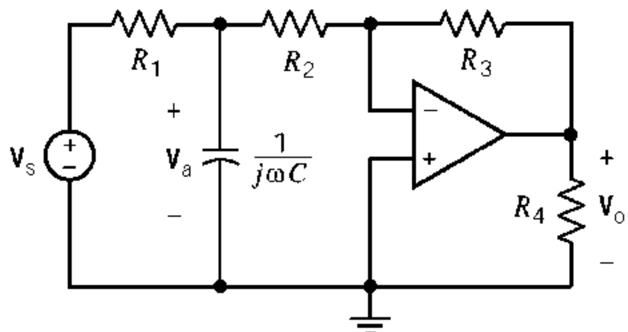


Figure P 13.2-24

Solution: Represent the circuit in the frequency domain.



The node equations are

$$\frac{\mathbf{V}_a-\mathbf{V}_s}{R_1}+\frac{\mathbf{V}_a}{\frac{1}{j\omega C}}+\frac{\mathbf{V}_a}{R_2}=0 \quad\Rightarrow\quad \mathbf{V}_a=\frac{R_2}{R_1+R_2+j\omega C\,R_1\,R_2}\mathbf{V}_s$$

and

$$\frac{\mathbf{V}_a}{R_2}+\frac{\mathbf{V}_o}{R_3}=0 \quad\Rightarrow\quad \mathbf{V}_o=-\frac{R_3}{R_2}\mathbf{V}_a$$

The network function is

$$\mathbf{H}=\frac{\mathbf{V}_o}{\mathbf{V}_s}=\frac{-\frac{R_3}{R_2}R_2}{R_1+R_2+j\omega C\,R_1\,R_2}=\frac{-\frac{R_3}{R_1+R_2}}{1+j\omega C\frac{R_1\,R_2}{R_1+R_2}}$$

Using the given values for R_1 and R_2 and letting $R_3 = R$ gives

$$\mathbf{H}=\frac{\mathbf{V}_o}{\mathbf{V}_s}=\frac{-\frac{R}{4\times10^4}}{1+j\omega C\left(10^4\right)}$$

Comparing this network function to the specified network function gives

$$C\left(10^4\right)=\frac{1}{250} \quad\Rightarrow\quad C=0.4\ \mu\text{F} \ \text{ and } \ \frac{R}{4\times10^4}=8 \quad\Rightarrow\quad R=320\ \text{k}\Omega$$

(checked using LNAP 9/19/04)

P13.2-26

The network function of a circuit is $\mathbf{H}(\omega)=\dfrac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)}=\dfrac{j\,40\,\omega}{120+j\,20\,\omega}$. When the input to this circuit is $v_s(t)=5\cos(5t+15°)$ V, the output is $v_o(t)=A\cos(5t+65.194°)$ V. On the other hand, when the input to this circuit is $v_s(t)=5\cos(8t+15°)$ V, the output is $v_o(t)=8\cos(8t+\theta)$ V. Determine the values of A and θ.

Answers: A = 6.4018 V and θ = 51.87 °

Solution:

When the input to this circuit is $v_s(t)=5\cos(5t+15°)$ V :

$$\mathbf{H}(5)=\frac{j\,40(5)}{120+j\,20(5)}=1.2804\angle 50.194° \quad\Rightarrow\quad v_o(t)=5(1.2804)\cos(5t+15°+50.194°)$$

$$=6.4018\cos(5t+65.194°)\text{ V}$$

When the input to this circuit is $v_s(t)=8\cos(8t-15°)$ V

$$\mathbf{H}(8)=\frac{j\,40(8)}{120+j\,20(8)}=1.6\angle 36.87° \quad\Rightarrow\quad v_o(t)=5(1.6)\cos(8t+15°+36.87°)$$

$$=8\cos(8t+51.87°)\text{ V}$$

P13.2-27

The network function of a circuit is $\mathbf{H}(\omega)=\dfrac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)}=\dfrac{k}{1+j\dfrac{\omega}{p}}$ where $k > 0$ and $p > 0$. When the input to

this circuit is $v_s(t)=12\cos(120t+30°)$ V

the output is $v_o(t)=42.36\cos(120t-48.69°)$ V .

Determine the values of k and p.

Answers: $k = 18$ and $p = 24$ rad/s.

Solution:

$$\frac{k}{1+j\dfrac{120}{p}}=\frac{k}{\sqrt{1+\left(\dfrac{120}{p}\right)^2}}\angle-\tan^{-1}\left(\frac{120}{p}\right) \quad \text{and} \quad \frac{\mathbf{V}_o}{\mathbf{V}_s}=\frac{42.36\angle-48.69°}{12\angle30°}=3.53\angle-78.69°$$

so

$$-\tan^{-1}\left(\frac{120}{p}\right)=-78.69° \quad\Rightarrow\quad \frac{120}{p}=\tan(78.69°)=5 \quad\Rightarrow\quad p=\frac{120}{5}=24 \text{ rad/s}$$

and

$$\frac{k}{\sqrt{1+\left(\dfrac{120}{p}\right)^2}}=\frac{k}{\sqrt{1+(5)^2}}=3.53 \quad\Rightarrow\quad k=3.53\sqrt{26}=18$$

P13.2-28

The network function of a circuit is $\mathbf{H}(\omega)=\dfrac{20}{8+j\omega}$. When the input to this circuit is sinusoidal, the output is also sinusoidal. Let ω_1 be the frequency at which the output sinusoid is twice as large as the input sinusoid and let ω_2 be the frequency at which output sinusoid is delayed by one tenth period with respect to the input sinusoid. Determine the values of ω_1 and ω_2.

Solution:

The gain is 2 at the frequency ω_1 so $2=\dfrac{20}{\sqrt{8^2+\omega_1^{\,2}}}$ and $\omega_1=\sqrt{\left(\dfrac{20}{2}\right)^2-8^2}=6\text{ rad/s}$.

When the frequency is ω_2, the period is $\dfrac{2\pi}{\omega_2}$. Also a delay t_o corresponds to a phase shift $-\omega_2 t_o$. In this case, $t_0=0.1\left(\dfrac{2\pi}{\omega_2}\right)$ so the phase shift is -0.2 π. Then $-0.2\pi=-\tan^{-1}\left(\dfrac{\omega_2}{8}\right)$ so

$\omega_2=8\tan(0.2\pi)=5.8123\text{ rad/s}$.

P13.2-29

The input to the circuit in Figure P13.3-29 is the voltage source voltage, $v_s(t)$. The output is the voltage $v_o(t)$. When the input is $v_s(t) = 8\cos(40t)$ V, the output is $v_o(t) = 2.5\cos(40t + 14°)$ V. Determine the values of the resistances R_1 and R_2.

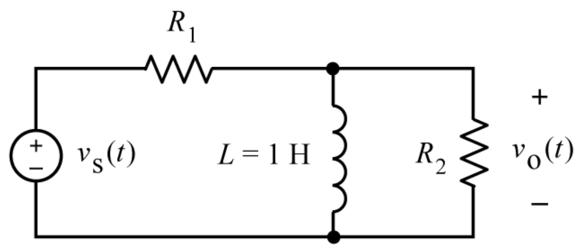
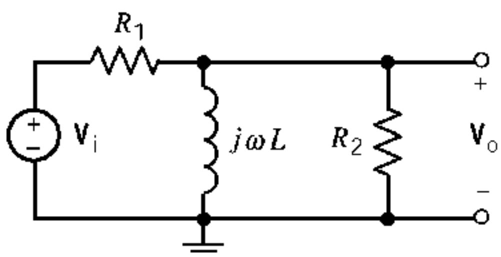


Figure P13.2-29

Solution:



$$\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{\dfrac{j\omega L R_2}{R_2 + j\omega L}}{R_1 + \dfrac{j\omega L R_2}{R_2 + j\omega L}} = \frac{j\omega \dfrac{L}{R_1}}{1 + j\omega \dfrac{L}{R_p}}$$

where $R_p = \dfrac{R_1 R_2}{R_1 + R_2}$

$$\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{\omega \dfrac{L}{R_1}}{\sqrt{1 + \left(\omega \dfrac{L}{R_p}\right)^2}} e^{j\left(90 - \tan^{-1} \omega \frac{L}{R_p}\right)}$$

In this case the angle of $\dfrac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$ is specified to be 14° so $\dfrac{L}{R_p} = \dfrac{L(R_1 + R_2)}{R_1 R_2} = \dfrac{\tan(90° - 14°)}{40} = 0.1$ and

the magnitude of $\dfrac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$ is specified to be $\dfrac{2.5}{8}$ so $\dfrac{40\dfrac{L}{R_1}}{\sqrt{1+16}} = \dfrac{2.5}{8} \Rightarrow \dfrac{L}{R_1} = 0.0322$. One set of values that satisfies these two equations is $L = 1\text{ H},\ R_1 = 31\ \Omega,\ R_2 = 14.76\ \Omega$.

P13.2-30

The input to the circuit shown in Figure P13.2-30 is the voltage source voltage, $v_s(t)$. The output is the voltage $v_o(t)$. The input $v_s(t) = 2.5\cos(1000t)$ V causes the output to be $v_o(t) = 8\cos(1000t + 104°)$ V. Determine the values of the resistances R_1 and R_2.

Answers: $R_1 =$ 1515 Ω and $R_2 =$ 20 kΩ.

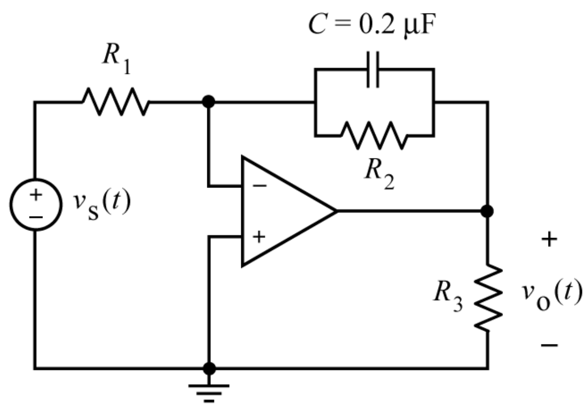
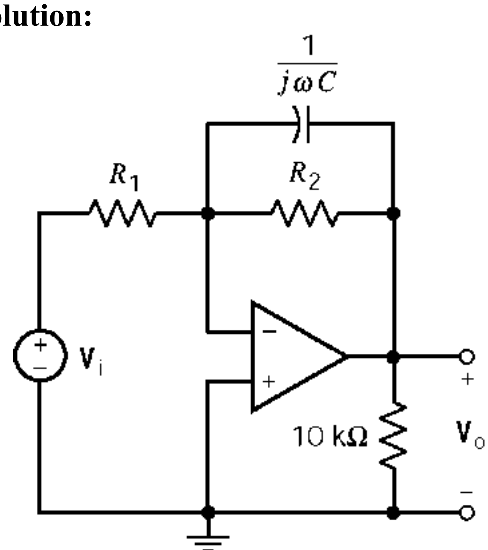


Figure P13.2-30

Solution:



$$R_2 \left\| \frac{1}{j\omega C} = \frac{R_2}{1 + j\omega CR_2} \right.$$

$$\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = -\frac{\dfrac{R_2}{1 + j\omega CR_2}}{R_1} = -\frac{\dfrac{R_2}{R_1}}{1 + j\omega CR_2}$$

$$\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{\dfrac{R_2}{R_1}}{\sqrt{1 + (\omega CR_2)^2}} e^{j\left(180 - \tan^{-1}\omega CR_2\right)}$$

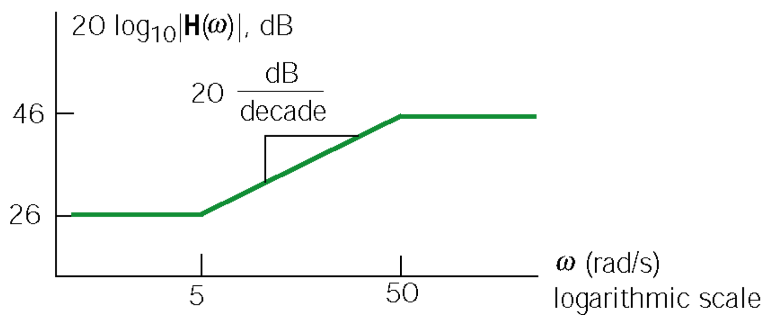
In this case the angle of $\dfrac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$ is specified to be 104° so $CR_2 = \dfrac{\tan(180° - 104°)}{1000} = 0.004$ and the magnitude of $\dfrac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$ is specified to be $\dfrac{8}{2.5}$ so $\dfrac{\dfrac{R_2}{R_1}}{\sqrt{1+16}} = \dfrac{8}{2.5} \quad \Rightarrow \quad \dfrac{R_2}{R_1} = 13.2$. One set of values that satisfies these two equations is $C = 0.2\ \mu\text{F},\ R_1 = 1515\ \Omega,\ R_2 = 20\ \text{k}\Omega$.

Section 13-3: Bode Plots

P 13.3-1 Sketch the magnitude Bode plot of $\mathbf{H}(\omega)=\dfrac{4(5+j\omega)}{\left(1+j\dfrac{\omega}{50}\right)}$

Solution:

$$\mathbf{H}(\omega)=\frac{20\left(1+j\dfrac{\omega}{5}\right)}{\left(1+j\dfrac{\omega}{50}\right)}\approx\begin{cases} 20 & \omega<5 \\ 20\left(j\dfrac{\omega}{5}\right) & 5<\omega<50 \\ \dfrac{20\left(j\dfrac{\omega}{5}\right)}{\left(j\dfrac{\omega}{50}\right)}=200 & 50<\omega \end{cases}$$



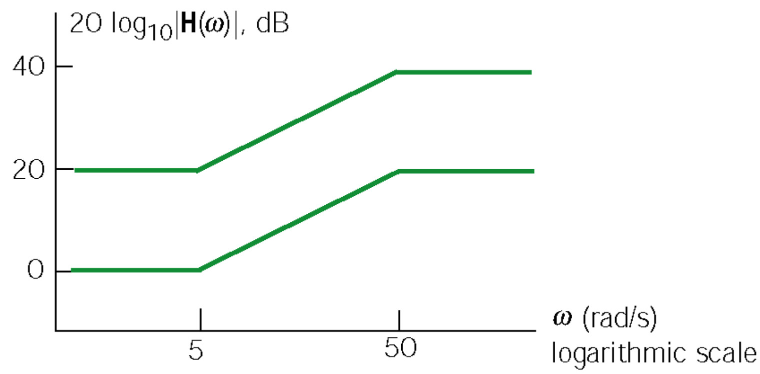
P 13.3-2 Compare the magnitude Bode plots of $\mathbf{H}_1(\omega) = \dfrac{10(5+j\omega)}{50+j\omega}$ and $\mathbf{H}_2(\omega) = \dfrac{100(5+j\omega)}{50+j\omega}$

Solution:

$$\mathbf{H}_1(\omega) = \frac{10\left(5+j\,\omega\right)}{50+j\,\omega} = \frac{1+j\dfrac{\omega}{5}}{1+j\dfrac{\omega}{50}} \qquad \text{and} \qquad \mathbf{H}_2\left(\omega\right) = \frac{100\left(5+j\,\omega\right)}{50+j\,\omega} = 10\,\frac{1+j\dfrac{\omega}{5}}{1+j\dfrac{\omega}{50}}$$

Both $\mathbf{H}_1(\omega)$ and $\mathbf{H}_2(\omega)$ have a pole at $\omega = 50$ rad/s and a zero at $\omega = 5$ rad/s. The slopes of both magnitude Bode plots increase by 20 dB/decade at $\omega = 5$ rad/s and decrease by 20 dB/decade at $\omega =$ 50rad/s. The difference is that for $\omega < 5$rad/s

$$\left|\mathbf{H}_1(\omega)\right| \simeq 1 = 0 \text{ dB} \quad \text{and} \quad \left|\mathbf{H}_2(\omega)\right| \simeq 10 = 20 \text{ dB}$$



P 13.3-3 The input to the circuit shown in Figure P 13.3-3 is the source voltage, $v_{in}(t)$, and the response is the voltage across R_3, $v_{out}(t)$. The component values are R_1 = 5 kΩ, R_2 = 10 kΩ, C_1 = 0.1 μF, and C_2 = 0.1 μF. Sketch the asymptotic magnitude Bode plot for the network function.

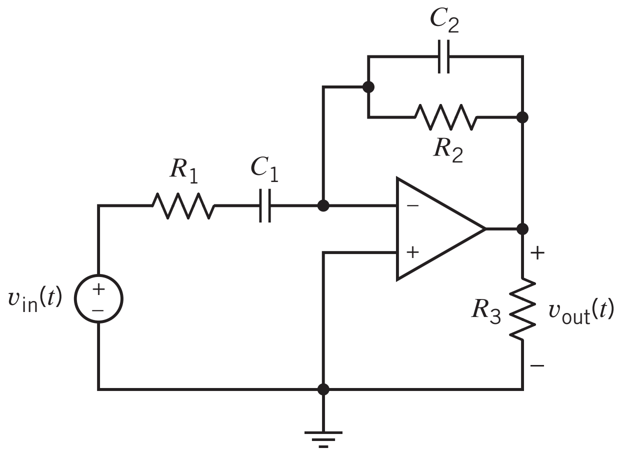


Figure P 13.3-3

Solution:

$$\mathbf{H}(\omega) = -\frac{\dfrac{R_2}{1+j\omega C_2 R_2}}{R_1+\dfrac{1}{j\omega C_1}} = -C_1 R_2 \frac{j\omega}{(1+j\omega R_1 C_1)(1+j\omega R_2 C_2)}$$

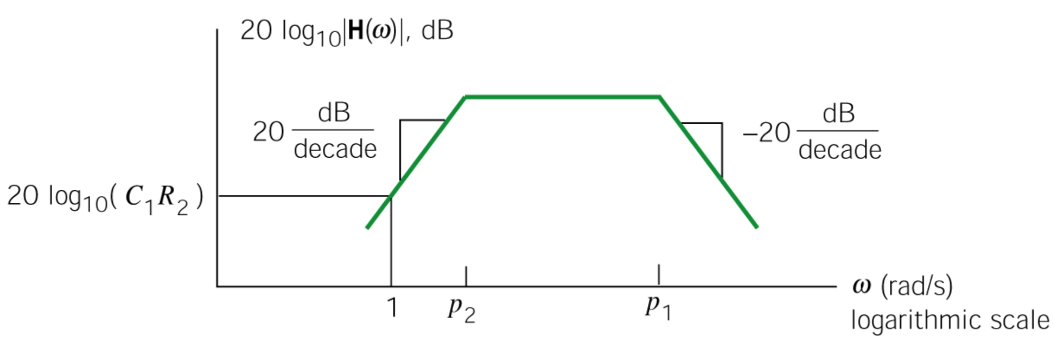
This network function has poles at

$$p_1 = \frac{1}{R_1 C_1} = 2000 \text{ rad/s} \text{ and } p_2 = \frac{1}{R_2 C_2} = 1000 \text{ rad/s}$$

so

$$\mathbf{H}(\omega) \simeq \begin{cases} -(C_1 R_2)\, j\omega & \omega < p_2 \\ -(C_1 R_2)\dfrac{j\omega}{j\omega C_1 R_1} = -\dfrac{R_2}{R_1} = -2 & p_2 < \omega < p_1 \\ -(C_1 R_2)\dfrac{j\omega}{(j\omega C_1 R_1)(j\omega C_2 R_2)} = -\dfrac{1}{j\omega C_2 R_1} & \omega > p_1 \end{cases}$$

Here's the Bode plot:



P 13.3-4 The input to the circuit shown in Figure P 13.3-4 is the source voltage, $v_s(t)$, and the response is the voltage across R_3, $v_o(t)$. Determine $\mathbf{H}(\omega)$ and sketch the Bode diagram.

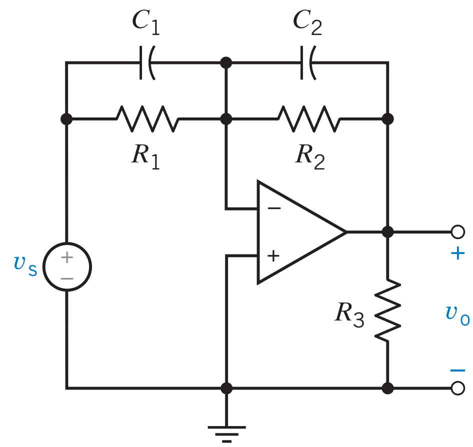
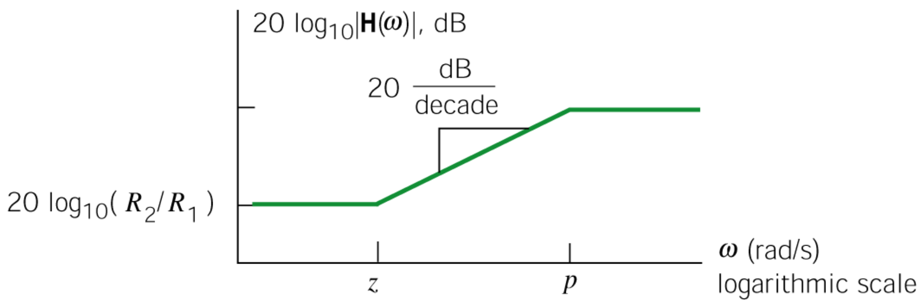


Figure P 13.3-4

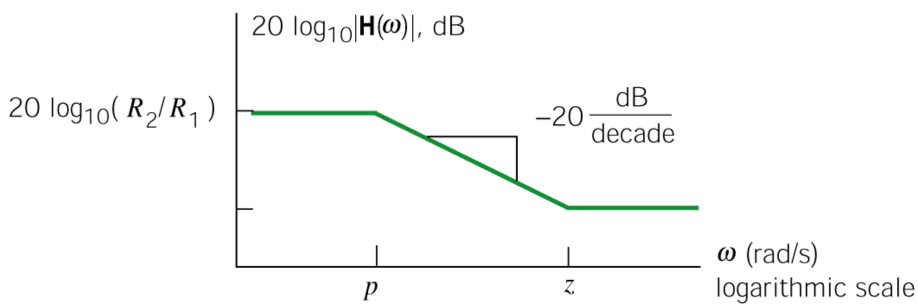
Solution:

$$\mathbf{H}(\omega) = -\frac{\dfrac{R_2}{1+j\omega C_2 R_2}}{\dfrac{R_1}{1+j\omega C_1 R_1}} = -\frac{R_2(1+j\omega C_1 R_1)}{R_1(1+j\omega C_2 R_2)} \quad \text{so} \quad K = -\frac{R_2}{R_1},\ z = \frac{1}{C_1 R_1} \ \text{and}\ p = \frac{1}{C_2 R_2}$$

When $z < p$



When $z > p$



P 13.3-5 The input to the circuit shown in Figure P 13.3-5*a* is the voltage, $v_i(t)$, of the independent voltage source. The output is the voltage, $v_o(t)$, across the capacitor. Design this circuit to have the Bode plot shown in Figure P 13.3-5*b*.

Hint: First show that the network function of the circuit is

$$\mathbf{H}(\omega)=\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}=\frac{j\omega\left(\dfrac{AL\,R_4}{R_1(R_3+R_4)}\right)}{\left(1+j\omega\dfrac{L(R_1+R_2)}{R_1R_2}\right)\left(1+j\omega\dfrac{C\,R_3R_4}{R_3+R_4}\right)}$$

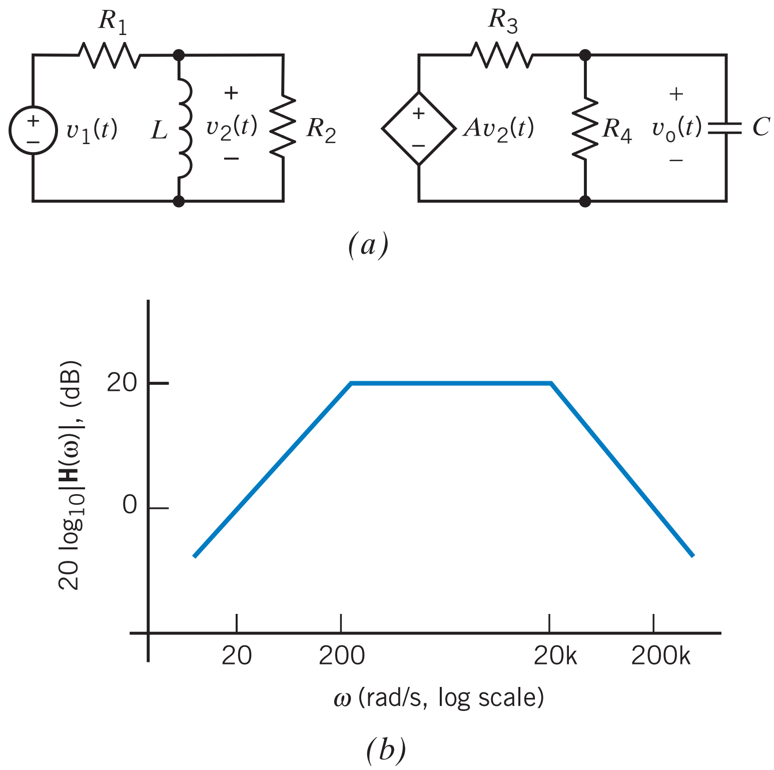


Figure P 13.3-5

Solution: Using voltage division twice gives:

$$\frac{\mathbf{V}_2(\omega)}{\mathbf{V}_i(\omega)}=\frac{\dfrac{j\omega L\,R_2}{R_2+j\omega L}}{R_1+\dfrac{j\omega L\,R_2}{R_2+j\omega L}}=\frac{j\omega L\,R_2}{R_1R_2+j\omega L\left(R_1+R_2\right)}=\frac{L}{R_1}\,\frac{j\omega}{1+j\omega\dfrac{L\left(R_1+R_2\right)}{R_1R_2}}$$

and

$$\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_2(\omega)}=\frac{\dfrac{R_4}{1+j\omega C\,R_4}}{R_3+\dfrac{R_4}{1+j\omega C\,R_4}}A=\frac{A\,R_4}{R_3+R_4+j\omega C\,R_3R_4}=\frac{\dfrac{A\,R_4}{R_3+R_4}}{1+j\omega\dfrac{C\,R_3R_4}{R_3+R_4}}$$

Combining these equations gives

$$\mathbf{H}(\omega)=\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}=\frac{ALR_4}{R_1(R_3+R_4)}\frac{j\omega}{\left(1+j\omega\frac{L(R_1+R_2)}{R_1R_2}\right)\left(1+j\omega\frac{CR_3R_4}{R_3+R_4}\right)}$$

The Bode plot corresponds to the network function:

$$\mathbf{H}(\omega)=\frac{k\,j\omega}{\left(1+j\frac{\omega}{p_1}\right)\left(1+j\frac{\omega}{p_2}\right)}=\frac{k\,j\omega}{\left(1+j\frac{\omega}{200}\right)\left(1+j\frac{\omega}{20000}\right)}$$

$$\mathbf{H}(\omega)\approx\begin{cases}\dfrac{k\,j\omega}{1\cdot 1}=k\,j\omega & \omega\le p_1\\[2ex] \dfrac{k\,j\omega}{\frac{j\omega}{p_1}\cdot 1}=k\,p_1 & p_1\le\omega\le p_2\\[2ex] \dfrac{k\,j\omega}{\frac{j\omega}{p_1}\cdot\frac{j\omega}{p_2}}=\dfrac{k\,p_1\,p_2}{j\omega} & \omega\ge p_2\end{cases}$$

This equation indicates that $|\mathbf{H}(\omega)|=k\,p_1$ when $p_1\le\omega\le p_2$. The Bode plot indicates that $|\mathbf{H}(\omega)|=20$ dB = 10 when $p_1\le\omega\le p_2$. Consequently

$$k=\frac{10}{p_1}=\frac{10}{200}=0.05$$

Finally,

$$\mathbf{H}(\omega)=\frac{0.05\,j\omega}{\left(1+j\frac{\omega}{200}\right)\left(1+j\frac{\omega}{20000}\right)}$$

Comparing the equation for $\mathbf{H}(\omega)$ obtained from the circuit to the equation for $\mathbf{H}(\omega)$obtained from the Bode plot gives:

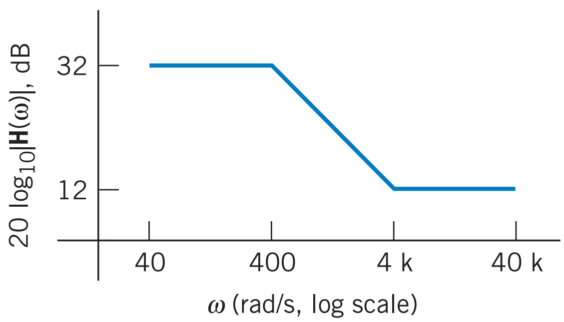
$$0.05=\frac{ALR_4}{R_1(R_3+R_4)},\ 200=\frac{R_1R_2}{L(R_1+R_2)}\text{ and }20000=\frac{R_3+R_4}{C\,R_3R_4}$$

Pick L = 1 H, and $R_1 = R_2$, then $R_1 = R_2$ = 400 Ω. Let C = 0.1 μF and $R_3 = R_4$, then $R_3 = R_4$ = 1000 Ω. Finally, A=40.

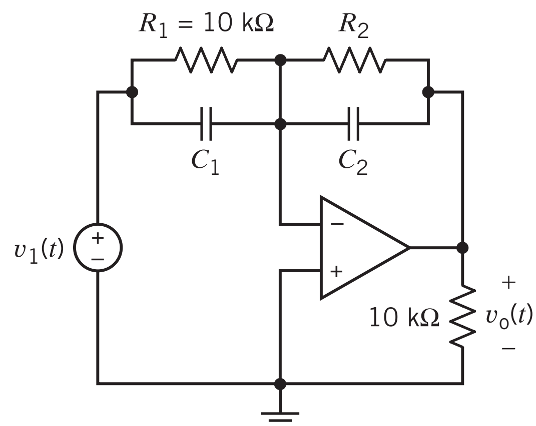
(Checked using ELab 3/5/01)

P 13.3-6 The input to the circuit shown in Figure P 13.3-6*b* is the voltage of the voltage source, $v_i(t)$. The output is the voltage $v_o(t)$. The network function of this circuit is $\mathbf{H}(\omega) = \mathbf{V}_o(\omega)/\mathbf{V}_i(\omega)$. Determine the values of R_2, C_1, and C_2 that are required to make this circuit have the magnitude Bode plot shown in Figure P 13.3-6*a*.

Answer: $R_2 = 400$ kΩ, $C_1 = 25$ nF, and $C_2 = 6.25$ nF



(a)



(b)

Figure P 13.3-6

Solution: From Table 13.3-2:

$$\frac{R_2}{R_1} = k = 32 \text{ dB} = 40 \quad R_2 = 40\left(10\times10^3\right) = 400 \text{ k}\Omega$$

$$\frac{1}{C_2\,R_2} = p = 400 \text{ rad/s} \quad \Rightarrow \quad C_2 = \frac{1}{(400)\left(400\times10^3\right)} = 6.25 \text{ nF}$$

$$\frac{1}{C_1\,R_1} = z = 4000 \text{ rad/s} \quad \Rightarrow \quad C_1 = \frac{1}{(4000)\left(10\times10^3\right)} = 25 \text{ nF}$$

P 13.3-7 The input to the circuit shown in Figure P 13.3-7*b* is the voltage of the voltage source, $v_i(t)$. The output is the voltage $v_o(t)$. The network function of this circuit is $\mathbf{H}(\omega) = \mathbf{V}_o(\omega)/\mathbf{V}_i(\omega)$. The magnitude Bode plot is shown in Figure p 13.3-7*a*. Determine values of the corner frequencies, z and p. Determine value of the low-frequency gain, k.

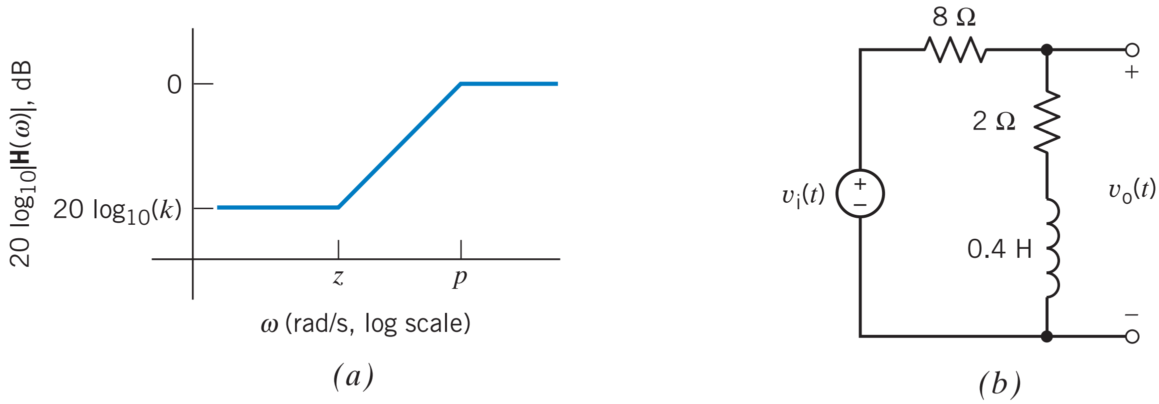
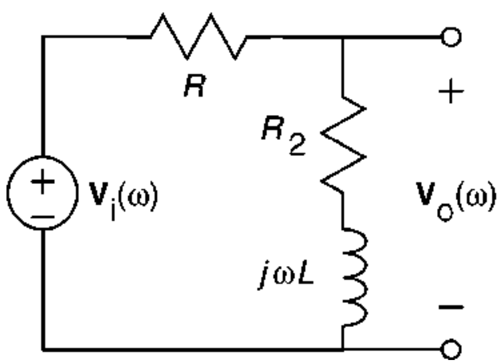


Figure P 13.3-7

Solution:



$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{R_2 + j\omega L}{R + R_2 + j\omega L}$$

$$= \left(\frac{R_2}{R+R_2}\right)\left(\frac{1 + j\omega \frac{L}{R_2}}{1 + j\omega \frac{L}{R+R_2}}\right)$$

$$\mathbf{H}(\omega) = \frac{(0.2)(1 + j(0.2)\omega)}{1 + j(0.04)\omega} \Rightarrow \begin{cases} k = 0.2 \\ z = \dfrac{1}{0.2} = 5 \\ p = \dfrac{1}{0.04} = 25 \end{cases}$$

P 13.3-8 Determine $\mathbf{H}(j\omega)$ from the asymptotic Bode diagram in Figure P 13.3-8.

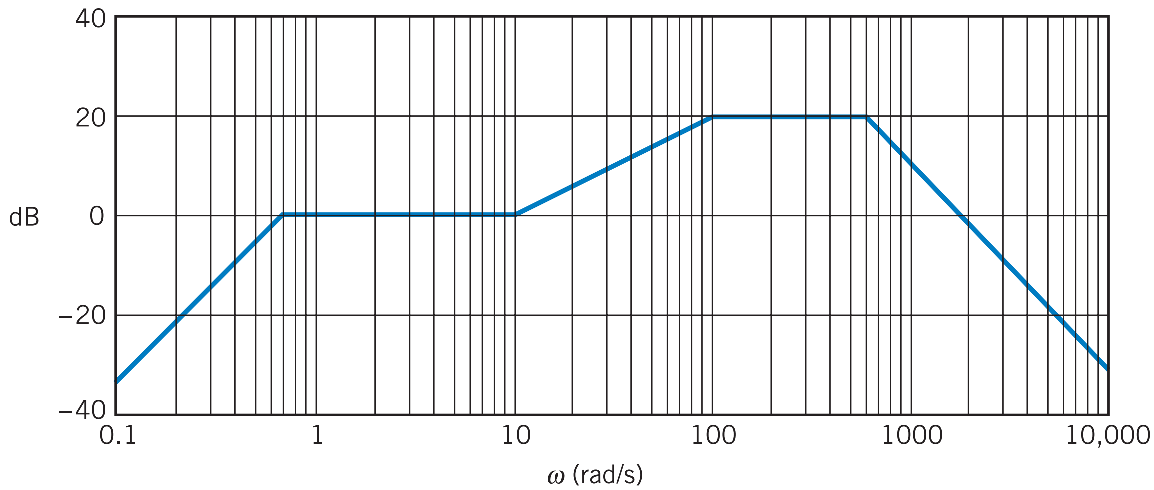


Figure P 13.3-8

Solution

- The slope is 40dB/decade for low frequencies, so the numerator will include the factor $(j\omega)^2$.
- The slope decreases by 40 dB/decade at $\omega = 0.7$rad/sec. So there is a second order pole at $\omega_0 = 0.7$ rad/s. The damping factor of this pole cannot be determined from the asymptotic Bode plot; call it δ_1. The denominator of the network function will contain the factor

$$1 + 2\delta_1 j\frac{\omega}{0.7} - \left(\frac{\omega}{0.7}\right)^2$$

- The slope increases by 20 dB/decade at $\omega = 10$ rad/s, indicating a zero at 10 rad/s.
- The slope decreases by 20 dB/decade at $\omega = 100$ rad/s, indicating a pole at 100 rad/s.
- The slope decreases by 40 dB/decade at $\omega = 600$ rad/s, indicating a second order pole at $\omega_0 =$ 600rad/s. The damping factor of this pole cannot be determined from an asymptotic Bode plot; call it δ_2. The denominator of the network function will contain the factor

$$1 + 2\delta_2 j\frac{\omega}{600} - \left(\frac{\omega}{600}\right)^2$$

$$\mathbf{H}(\omega) = \frac{K(1+j\dfrac{\omega}{10})(j\omega)^2}{\left(1+2\delta_1 j\dfrac{\omega}{0.7}-\left(\dfrac{\omega}{0.7}\right)^2\right)\left(1+2\delta_2 j\dfrac{\omega}{600}-\left(\dfrac{\omega}{600}\right)^2\right)\left(1+j\dfrac{\omega}{100}\right)}$$

To determine K, notice that $|\mathbf{H}(\omega)| = 1$ dB=0 when $0.7 < \omega < 10$. That is

$$1 = \frac{K(1)\omega^2}{-\left(\dfrac{\omega}{0.7}\right)^2(1)(1)} = K(0.7)^2 \;\Rightarrow\; K = 2$$

P 13.3-9 A circuit has a voltage ratio

$$\mathbf{H}(\omega) = \frac{k(1 + j\omega / z)}{j\omega}$$

(a) Find the high- and low-frequency asymptotes of the magnitude Bode plot.

(b) The high- and low-frequency asymptotes comprise the magnitude Bode plot. Over what ranges of frequencies is the asymptotic magnitude Bode plot of **H**(ω) within 1 percent of the actual value of **H**(ω) in decibels?

Solution:

(a)

$$\mathbf{H}(\omega) = \frac{K\left(1 + j\,\frac{\omega}{z}\right)}{j\omega}$$

$$|\mathbf{H}(\omega)| = \frac{K}{\omega}\sqrt{1 + \left(\frac{\omega}{z}\right)^2}$$

$$|\mathbf{H}(\omega)|\ dB = 20\ \log_{10}\ \frac{K}{\omega}\sqrt{1 + \left(\frac{\omega}{z}\right)^2}$$

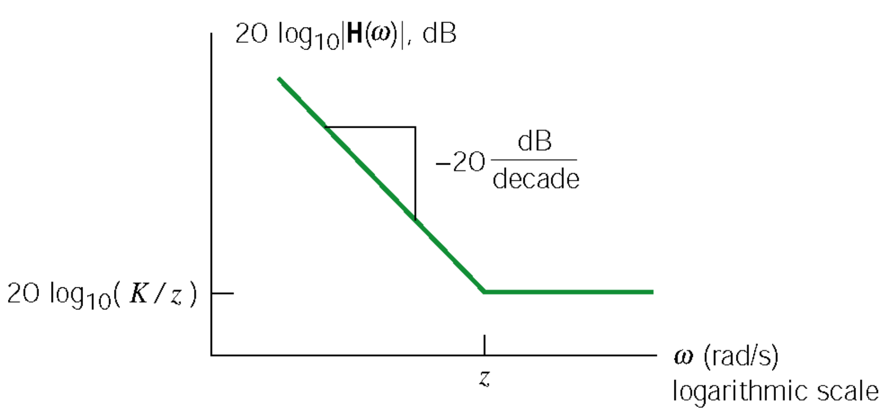
$$= 20\ \log_{10}\ K - 20\log_{10}\ \omega + 20\ \log_{10}\ \sqrt{1 + \left(\frac{\omega}{z}\right)^2}$$

Let $|\mathbf{H}_{\mathrm{L}}(\omega)|\ \mathrm{dB} = 20\ \log_{10} K - 20\log_{10}\omega$

and $|\mathbf{H}_{\mathrm{H}}(\omega)|\ \mathrm{dB} = 20\ \log_{10}\ \frac{K}{z}$

Then $|\mathbf{H}(\omega)|\ \mathrm{dB} \simeq \begin{cases} |\mathbf{H}_{\mathrm{L}}(\omega)|\ \mathrm{dB} & \omega \ll z \\ |\mathbf{H}_{\mathrm{H}}(\omega)|\ \mathrm{dB} & \omega \gg z \end{cases}$

So $|\mathbf{H}_{\mathrm{L}}(\omega)|$ dB and $|\mathbf{H}_{\mathrm{H}}(\omega)|$ dB are the required low and high-frequency asymptotes.



The Bode plot will be within 1% of $|\mathbf{H}(\omega)|$ dB both for $\omega << z$ and for $\omega >> z$. The range when $\omega << z$ is characterized by

$$|\mathbf{H}_L(\omega)| = 0.99|\mathbf{H}(\omega)| \qquad \text{(gains not in dB)}$$

or equivalently

$$20\log_{10}(0.99) = |\mathbf{H}_L(\omega)| \text{ dB} - |\mathbf{H}(\omega)| \text{ dB} \qquad \text{(gains in dB)}$$

$$= 20\log_{10} K - 20\log_{10}\omega - 20\log_{10}\frac{K}{\omega}\sqrt{1+\left(\frac{\omega}{z}\right)^2}$$

$$= -20\log_{10}\sqrt{1+\left(\frac{\omega}{z}\right)^2} = 20\log_{10}\frac{1}{\sqrt{1+\left(\frac{\omega}{z}\right)^2}}$$

Therefore

$$0.99 = \frac{1}{\sqrt{1+\left(\frac{\omega}{z}\right)^2}} \Rightarrow \omega = z\sqrt{\left(\frac{1}{.99}\right)^2-1} = 0.14z \simeq \frac{z}{7}$$

The range when $\omega >> z$ is characterized by

$$|\mathbf{H}_H(\omega)| = .99|\mathbf{H}(\omega)| \qquad \text{(gains not in dB)}$$

or equivalently

$$20\log_{10}0.99 = |\mathbf{H}_H(\omega)| \text{ dB} - |\mathbf{H}(\omega)| \text{ dB} \qquad \text{(gains in dB)}$$

$$= 20\log_{10} K - 20\log_{10} z - 20\log_{10}\frac{K}{\omega}\sqrt{1+\left(\frac{\omega}{z}\right)^2}$$

$$= -20\log_{10}\frac{z}{\omega}\sqrt{1+\left(\frac{\omega}{z}\right)^2} = 20\log_{10}\frac{1}{\sqrt{\left(\frac{z}{\omega}\right)^2+1}}$$

Therefore

$$\frac{z}{\omega} = \sqrt{\left(\frac{1}{.99}\right)^2-1} \Rightarrow \omega = \frac{z}{\sqrt{\left(\frac{1}{.99}\right)^2-1}} = \frac{z}{0.14} \simeq 7z$$

The error is less than 1% when $\omega < \frac{z}{7}$ and when $\omega > 7z$.

P 13.3-10 Physicians use tissue electrodes to form the interface that conducts current to the target tissue of the human body. The electrode in tissue can be modeled by the *RC* circuit shown in Figure P 13.3-10. The value of each element depends on the electrode material and physical construction as well as the character of the tissue being probed. Find the Bode diagram for $\mathbf{V}_o/\mathbf{V}_s = \mathbf{H}(j\omega)$ when R_1 = 1 kΩ, C = 1 μF, and the tissue resistance is R_t = 5 kΩ.

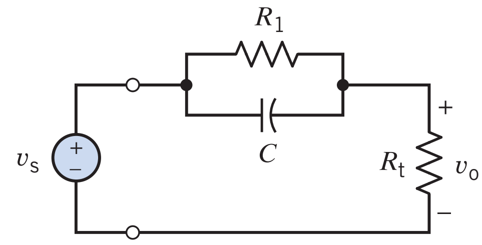


Figure P 13.3-10

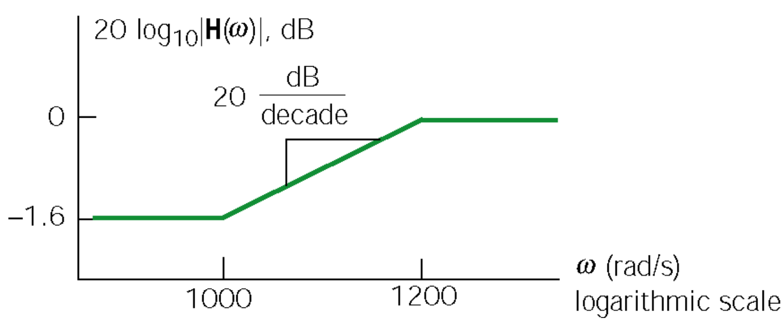
Solution:

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)} = \frac{R_t}{R_t + R_1 \| \frac{1}{j\omega C}} = \frac{R_t}{R_t + \frac{R_1}{1 + j\omega C R_1}}$$

$$= \frac{R_t(1 + j\omega C R_1)}{R_1 + R_t + j\omega C R_1 R_t} = \left(\frac{R_t}{R_1 + R_t}\right) \frac{1 + j\omega C R_1}{1 + j\omega\left(\frac{C R_1 R_t}{R_1 + R_t}\right)}$$

When $R_1 = 1\ \text{k}\Omega$, $C = 1\ \mu\text{F}$ and $R_t = 5\ \text{k}\Omega$

$$\mathbf{H}(\omega) = \frac{5}{6}\left(\frac{1 + j\frac{\omega}{1000}}{1 + j\frac{\omega}{1200}}\right) \quad \Rightarrow \quad \mathbf{H}(\omega) \cong \begin{cases} \frac{5}{6} & \omega < 1000 \\ \left(\frac{5}{6}\right) j\frac{\omega}{1000} & 1000 < \omega < 1200 \\ 1 & \omega < 1200 \end{cases}$$



P 13.3-11 Figure P 13.3-11 shows a circuit and corresponding asymptotic magnitude Bode plot. The input to this circuit shown is the source voltage $v_{in}(t)$, and the response is the voltage $v_o(t)$. The component values are $R_1 = 80\ \Omega$, $R_2 = 20\ \Omega$, $L_1 = 0.03$ H, $L_2 = 0.07$ H, and $M = 0.01$ H. Determine the values of K_1, K_2, p, and z.

Answer: $K_1 = 0.75$, $K_2 = 0.2$, $z = 333$ rad/s, and $p = 1250$ rad/s

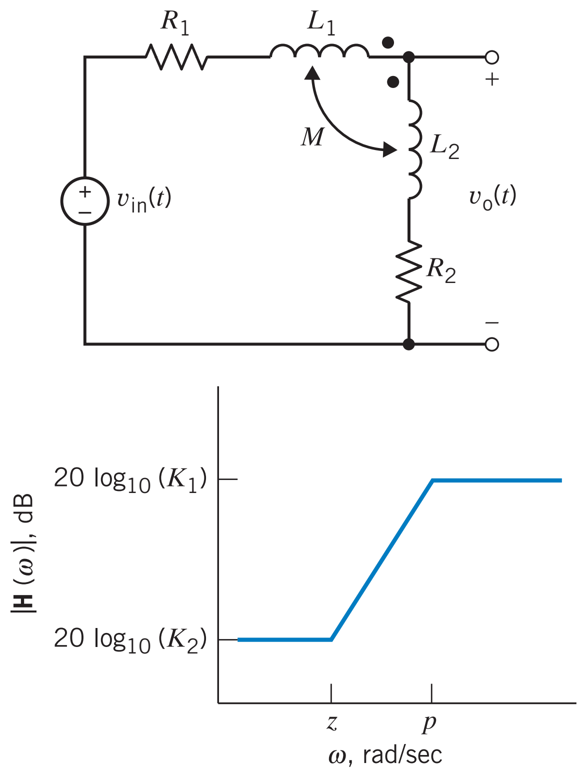
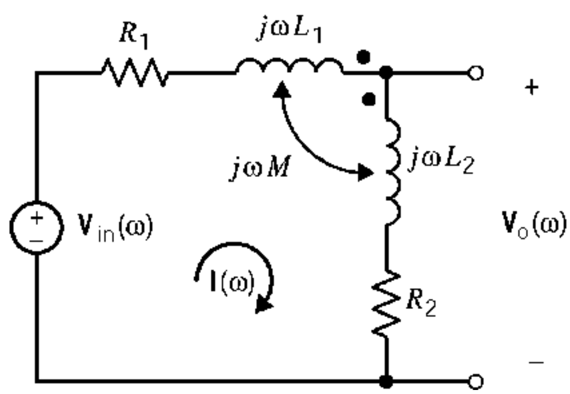


Figure P 13.3-11

Solution:



Mesh equations:

$$\mathbf{V}_{in}(\omega) = \mathbf{I}(\omega)\,[R_1 + (j\omega L_1 - j\omega M) + (-j\omega M + j\omega L_2) + R_2]$$

$$\mathbf{V}_o(\omega) = \mathbf{I}(\omega)\,[(-j\omega M + j\omega L_2) + R_2]$$

Solving yields:

$$\mathbf{H}(\omega) = \frac{V_o(\omega)}{V_{in}(\omega)} = \frac{R_2 + j\omega(L_2 - M)}{R_1 + R_2 + j\omega(L_1 + L_2 - 2M)}$$

Comparing to the given Bode plot yields:

$$K_1 = \lim_{\omega\to\infty} |\mathbf{H}(\omega)| = \frac{L_2 - M}{L_1 + L_2 - 2M} = 0.75 \text{ and } K_2 = \lim_{\omega\to 0} |\mathbf{H}(\omega)| = \frac{R_2}{R_1 + R_2} = 0.2$$

$$z = \frac{R_2}{L_2 - M} = 333 \text{ rad/s and } p = \frac{R_1 + R_2}{L_1 + L_2 - 2M} = 1250 \text{ rad/s}$$

P 13.3-12 The input to the circuit shown in Figure P 13.3-12 is the source voltage $v_{in}(t)$, and the response is the voltage across R_3, $v_{out}(t)$. The component values are R_1 = 10 kΩ, C_1 = 0.025 μF, and C_2 = 0.05 μF. Sketch the asymptotic magnitude Bode plot for the network function.

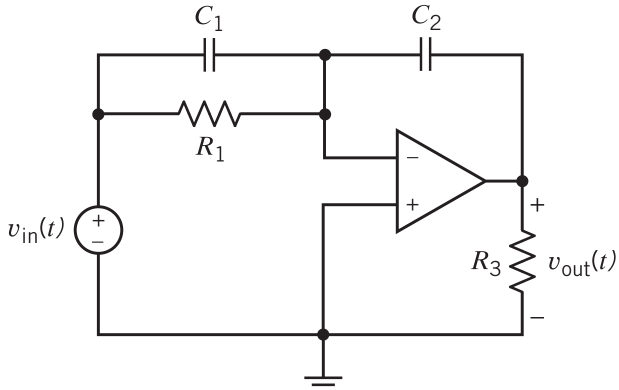


Figure P 13.3-12

Solution:

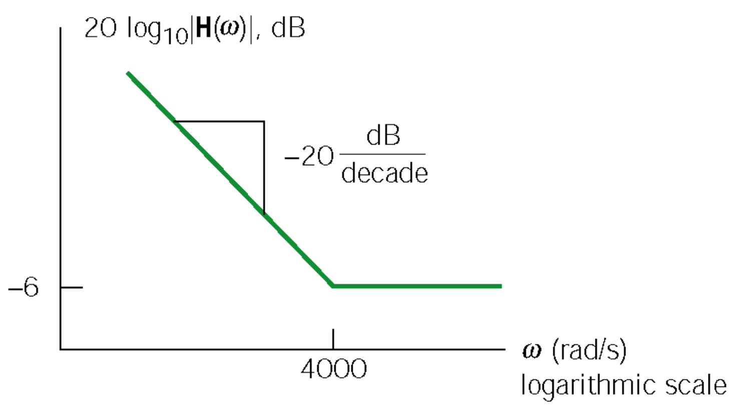
$$\mathbf{H}(\omega) = -\frac{\dfrac{1}{j\omega C_2}}{R_1 \left\| \dfrac{1}{j\omega C_1}\right.} = -\frac{1+j\omega R_1 C_1}{j\omega R_1 C_2} = -\frac{1}{R_1 C_2}\frac{(1+j\omega R_1 C_1)}{j\omega}$$

$$\mathbf{H}(\omega) \simeq \begin{cases} -\dfrac{1}{R_1 C_2}\left(\dfrac{1}{j\omega}\right) & \omega < \dfrac{1}{R_1 C_1} \\ -\dfrac{1}{R_1 C_2}\left(R_1 C_1\right) = -\dfrac{C_1}{C_2} & \omega > \dfrac{1}{R_1 C_1} \end{cases}$$

With the given values:

$$\frac{C_1}{C_2} = \frac{1}{2} = -6 \text{ dB}, \quad \frac{1}{R_1 C_1} = 4000 \text{ rad/s}$$

Here's the Bode plot:



P 13.3-13 Design a circuit that has the asymptotic magnitude Bode plot shown in Figure P 13.3-13.

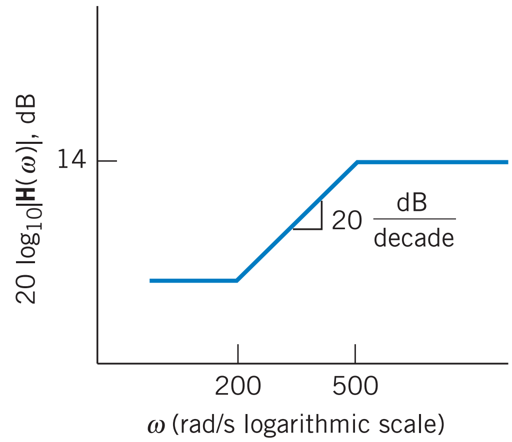
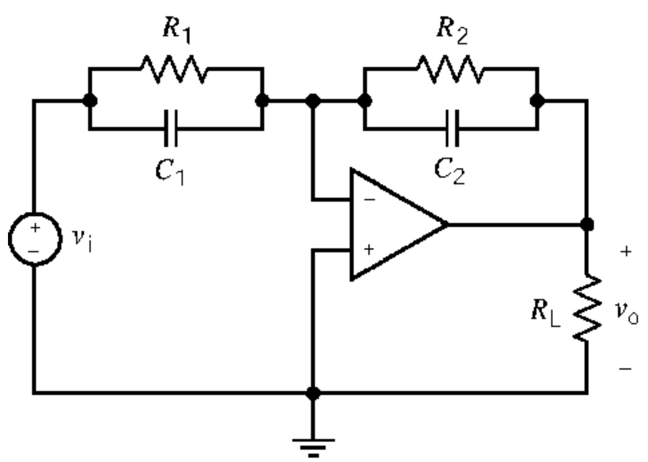


Figure P 13.3-13

Solution: Pick the appropriate circuit from Table 13.3-2.



$$\mathbf{H}(\omega) = -k\,\frac{1 + j\dfrac{\omega}{z}}{1 + j\dfrac{\omega}{p}}$$

where

$$k = \frac{R_2}{R_1}$$

$$z = \frac{1}{C_1 R_1}$$

$$p = \frac{1}{C_2 R_2}$$

We require

$$200 = z = \frac{1}{C_1\,R_1},\quad 500 = p = \frac{1}{C_2 R_2} \quad \text{and} \quad 14\ \text{dB} = 5 = k\frac{p}{z} = \frac{C_1}{C_2}$$

Pick $C_1 = 1\ \mu\text{F}$, then $C_2 = 0.2\ \mu\text{F}$, $R_1 = 5\ \text{k}\Omega$ and $R_2 = 10\ \text{k}\Omega$.

P 13.3-14 Design a circuit that has the asymptotic magnitude Bode plot shown in Figure P 13.3-14.

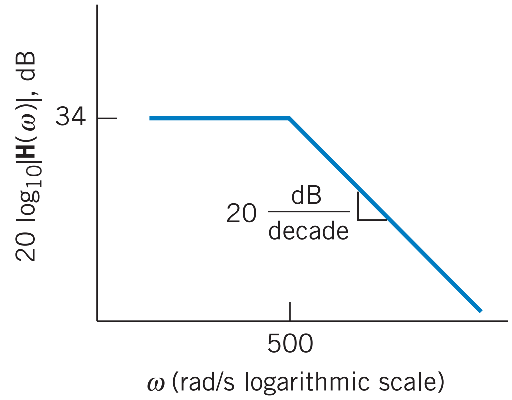
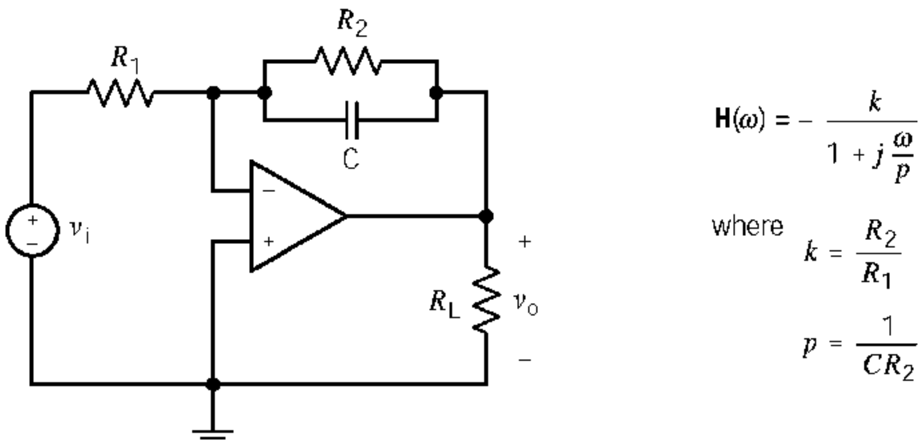


Figure P 13.3-14

Solution: Pick the appropriate circuit from Table 13.3-2.



We require

$$500 = p = \frac{1}{C\,R_2} \quad \text{and} \quad 34\text{ dB} = 50 = \frac{R_2}{R_1}$$

Pick $C = 0.1\ \mu\text{F}$, then $R_2 = 20\ \text{k}\Omega$ and $R_1 = 400\ \Omega$.

P 13.3-15 Design a circuit that has the asymptotic magnitude Bode plot shown in Figure P 13.3-15.

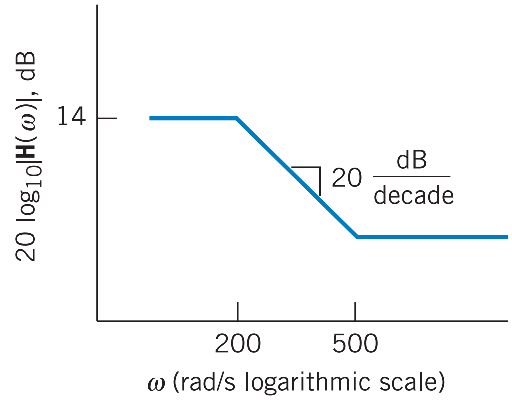
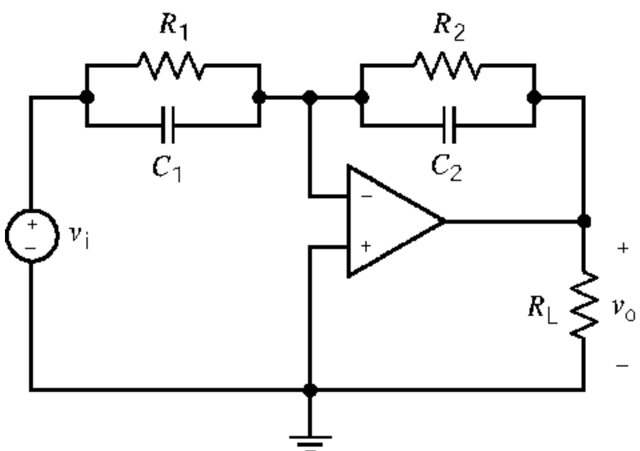


Figure P 13.3-15

Solution: Pick the appropriate circuit from Table 13.3-2.



$$\mathbf{H}(\omega) = -k\,\frac{1 + j\dfrac{\omega}{z}}{1 + j\dfrac{\omega}{p}}$$

where

$$k = \frac{R_2}{R_1}$$

$$z = \frac{1}{C_1 R_1}$$

$$p = \frac{1}{C_2 R_2}$$

We require

$$500 = z = \frac{1}{C_1\,R_1},\ \ 200 = \ p = \frac{1}{C_2 R_2} \ \text{ and }\ 14\text{ dB} = 5 = \ k\,\frac{p}{z} = \frac{C_1}{C_2}$$

Pick $C_1 = 0.1\ \mu$F, then $C_2 = 0.02\ \mu$F, $R_1 = 20$ kΩ and $R_2 = 250$ kΩ.

P 13.3-16 Design a circuit that has the asymptotic magnitude Bode plot shown in Figure P 13.3-16.

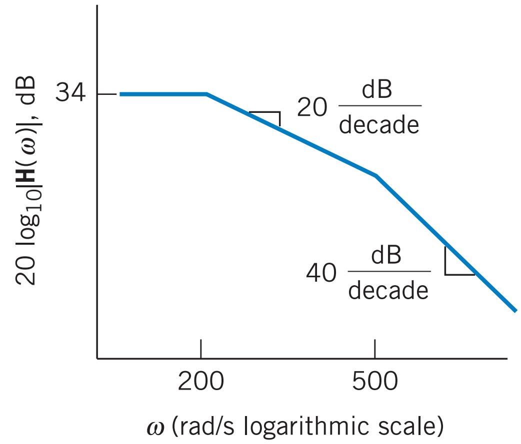
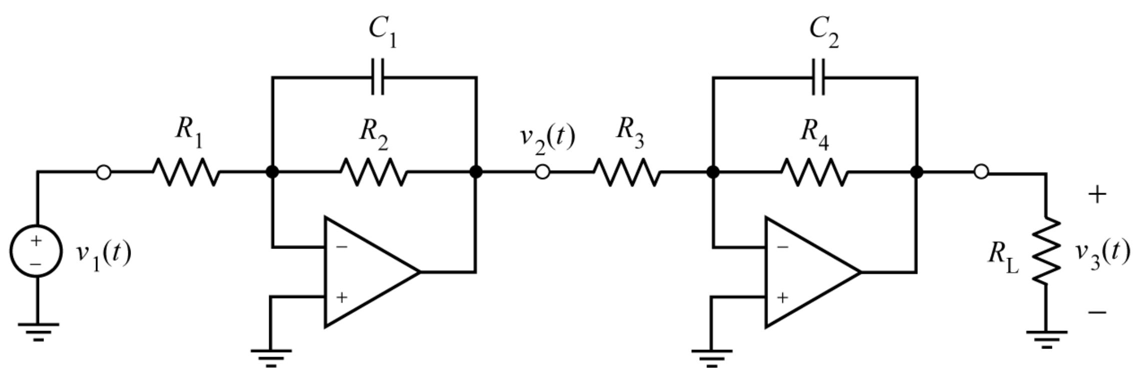


Figure P 13.3-16

Solution: Let's try designing the circuit as a cascade circuit using circuits from Table 13.3-2.



A Cascade Circuit

From Table 13.3-2

$$H_1(\omega) = \frac{\mathbf{V}_2(\omega)}{\mathbf{V}_1(\omega)} = -\frac{k_1}{1 + j\dfrac{\omega}{p_1}} \text{ where } k_1 = \frac{R_2}{R_1} \text{ and } p_1 = \frac{1}{C_1 R_2}$$

and
$$H_2(\omega) = \frac{\mathbf{V}_3(\omega)}{\mathbf{V}_2(\omega)} = -\frac{k_2}{1 + j\dfrac{\omega}{p_2}} \text{ where } k_2 = \frac{R_4}{R_3} \text{ and } p_2 = \frac{1}{C_2 R_4}$$

Consequently
$$H(\omega) = \frac{\mathbf{V}_3(\omega)}{\mathbf{V}_1(\omega)} = \frac{\mathbf{V}_3(\omega)}{\mathbf{V}_2(\omega)} \bullet \frac{\mathbf{V}_2(\omega)}{\mathbf{V}_1(\omega)} = H_1(\omega) \bullet H_2(\omega) = \frac{k_1 k_2}{\left(1 + j\dfrac{\omega}{p_1}\right)\left(1 + j\dfrac{\omega}{p_2}\right)}$$

(Perhaps we should verify that this transfer function is correct before proceeding.. Analysis of the cascade circuit, e.g. using node equations, shows that the transfer function is indeed correct.)

Next, from the Bode plot we see that

$$\mathrm{H}(\omega) = \frac{k}{\left(1 + j\frac{\omega}{p_1}\right)\left(1 + j\frac{\omega}{p_2}\right)}$$

where

$$200 \text{ rad/s} = p_1,\ \ 500 \text{ rad/s} = p_2 \ \text{ and } \ 34 \text{ dB} = 50 = k$$

Pick $C_2 = C_1 = 0.1\ \mu\text{F}$. Then

$$200 = \frac{1}{\left(10^{-7}\right) R_2} \quad \Rightarrow \quad R_2 = 50\,\text{k}\Omega \ \text{ and } \ 500 = \frac{1}{\left(10^{-7}\right) R_4} \quad \Rightarrow \quad R_4 = 20\,\text{k}\Omega$$

Next

$$50 = k = k_1 k_2 = \frac{R_2}{R_1}\frac{R_4}{R_3} = \frac{(20\,\text{k}\Omega)(50\,\text{k}\Omega)}{R_1\, R_3}$$

The solution is not unique. For example, we can choose $R_1 = 4\,\text{k}\Omega$ and $R_3 = 5\,\text{k}\Omega$.

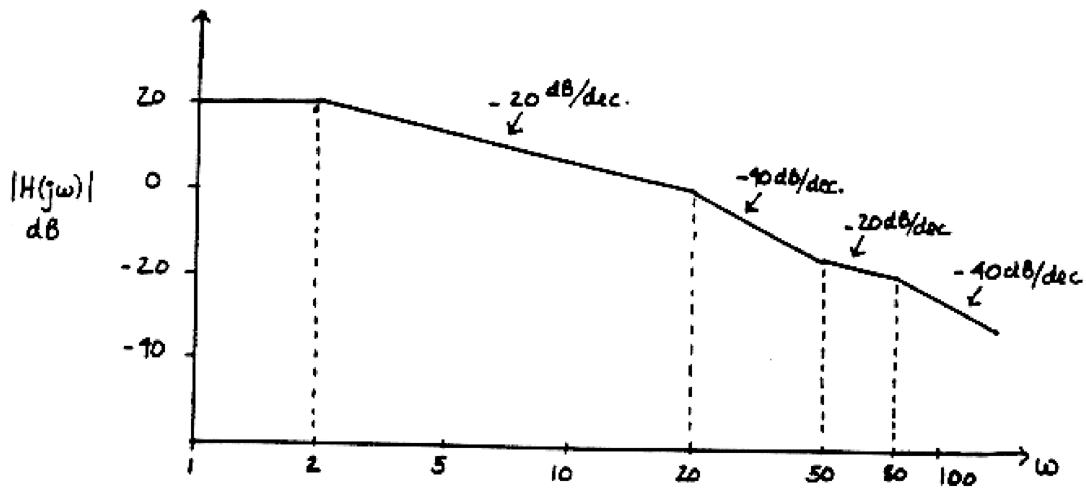
P 13.3-17 The cochlear implant is intended for patients with deafness due to malfunction of the sensory cells of the cochlea in the inner ear (Loeb, 1985). These devices use a microphone for picking up the sound and a processor for converting to electrical signals, and they transmit these signals to the nervous system. A cochlear implant relies on the fact that many of the auditory nerve fibers remain intact in patients with this form of hearing loss. The overall transmission from microphone to nerve cells is represented by the gain function

$$\mathbf{H}(j\omega) = \frac{10(j\omega/50+1)}{(j\omega/2+1)(j\omega/20+1)(j\omega/80+1)}$$

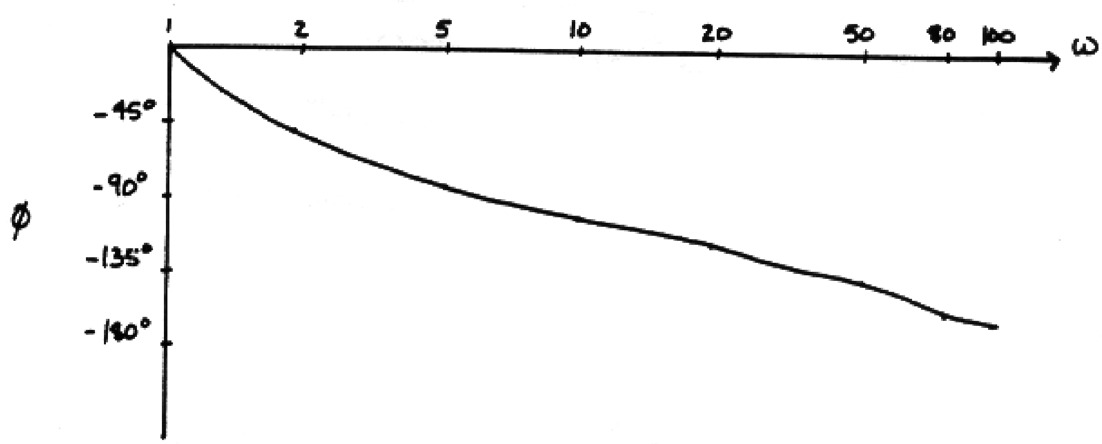
Plot the Bode diagram for $\mathbf{H}(j\omega)$ for $1 \le \omega \le 100$.

Solution:

$$\mathbf{H}(\omega) = \frac{10(1+j\omega/50)}{(1+j\omega/2)(1+j\omega/20)(1+j\omega/80)}$$



$$\varphi = \angle\mathbf{H}(\omega) = \tan^{-1}(\omega/50) - \left(\tan^{-1}(\omega/2) + \tan^{-1}(\omega/20) + \tan^{-1}(\omega/80)\right)$$



P 13.3-18 An operational amplifier circuit is shown in Figure P 13.3-18, where $R_2 = 5$ kΩ and $C = 0.02$ μF.

(a) Find the expression for the network function **H** = $\mathbf{V}_o/\mathbf{V}_s$ and sketch the asymptotic Bode diagram.

(b) What is the gain of the circuit, $|\mathbf{V}_o/\mathbf{V}_s|$, for $\omega = 0$?

(c) At what frequency does $|\mathbf{V}_o/\mathbf{V}_s|$ fall to $1\sqrt{2}$ of its low-frequency value?

Answer: (b) 20 dB and (c) 10,000 rad/s

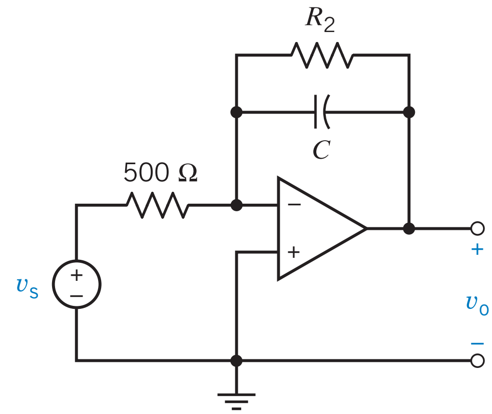
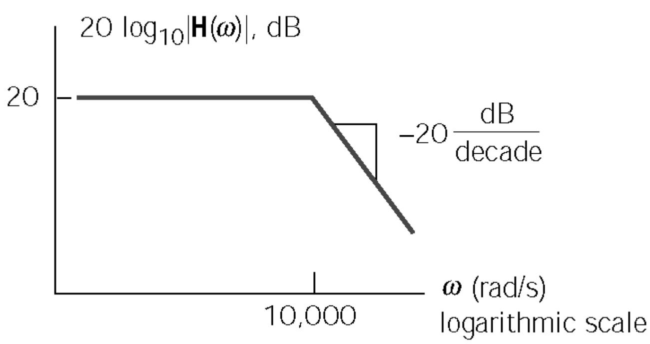


Figure P 13.3-18

Solution:

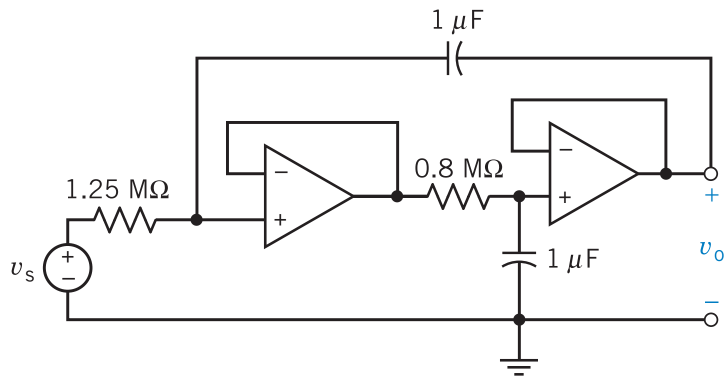


(a)
$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)} = -\frac{R_2/R_1}{1 + j\omega R_2 C}$$
$$= -\frac{10}{1 + j\dfrac{\omega}{10,000}}$$

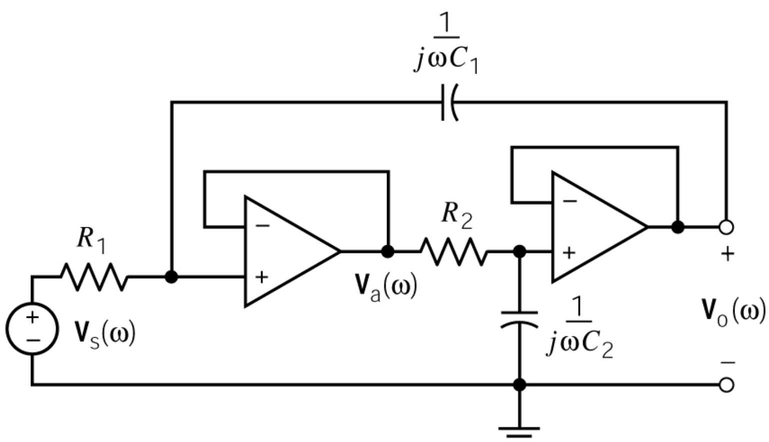
(b) 10 = 20 dB

(c) 10,000 rad/s

P 13.3-19 Determine the network function **H**(ω) for this op amp circuit and plot the Bode diagram.



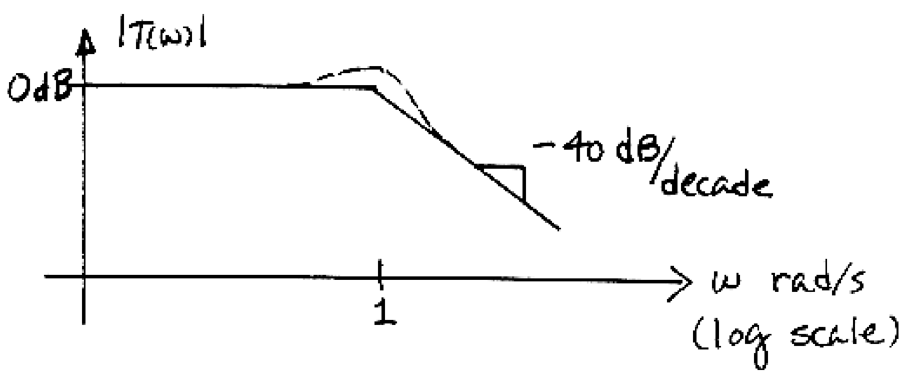
Solution:



$$\left.\begin{aligned} \mathbf{V}_o(\omega) &= \frac{\frac{1}{j\omega C_2}}{R+\frac{1}{j\omega C_2}}\mathbf{V}_a(\omega) \\ 0 &= \frac{\mathbf{V}_a(\omega)-\mathbf{V}_s(\omega)}{R_1}+j\omega C_1(\mathbf{V}_a(\omega)-\mathbf{V}_o(\omega)) \end{aligned}\right\} \Rightarrow \mathbf{V}_o(1+j\omega C_1R_1)(1+j\omega C_2R_2) = j\omega C_1R_1\mathbf{V}_o+\mathbf{V}_s$$

$$\mathbf{T}(\omega)=\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)}=\frac{1}{1+C_2R_2j\omega-\omega^2C_1C_2R_1R_2}=\frac{1}{-\omega^2+0.8j\omega+1}$$

This is a second order transfer function with $\omega_o=1$ and $\delta=0.4$.



P 13.3-20 The network function of a circuit is $\mathbf{H}(\omega) = \dfrac{-3(5+j\omega)}{j\omega(2+j\omega)}$. Sketch the asymptotic magnitude Bode plot corresponding to **H.**

Solution:

$$\mathbf{H}(\omega) = \frac{-3(5+j\omega)}{j\omega(2+j\omega)} = \frac{-\dfrac{15}{2}\left(1+j\dfrac{\omega}{5}\right)}{j\omega\left(1+j\dfrac{\omega}{2}\right)}$$

There is a zero at 5 rad/s and poles at 0 and 2 rad/s. To obtain the asymptotic magnitude Bode plot, use

$$\left|1+j\frac{\omega}{p}\right| = \begin{cases} 1 & \text{for } \omega < p \\ \dfrac{\omega}{p} & \text{for } \omega > p \end{cases}$$

Then

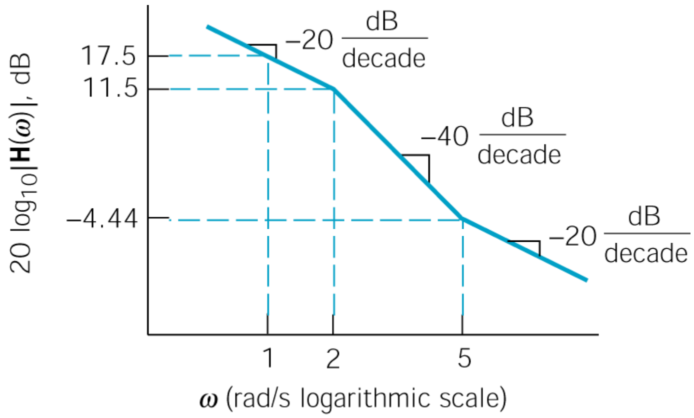
$$H = |\mathbf{H}| = \begin{cases} \dfrac{\dfrac{15}{2}(1)}{\omega(1)} = \dfrac{7.5}{\omega} & \text{for } \omega < 2 \\ \dfrac{\dfrac{15}{2}(1)}{\omega\left(\dfrac{\omega}{2}\right)} = \dfrac{15}{\omega^2} & \text{for } 2 < \omega < 5 \\ \dfrac{\dfrac{15}{2}\left(\dfrac{\omega}{5}\right)}{\omega\left(\dfrac{\omega}{2}\right)} = \dfrac{3}{\omega} & \text{for } \omega > 5 \end{cases}$$

$$20\log_{10} H = \begin{cases} 20\log_{10}(7.5) - 20\log_{10}(\omega) & \text{for } \omega < 2 \\ 20\log_{10}(15) - 2\left[20\log_{10}(\omega)\right] & \text{for } 2 < \omega < 5 \\ 20\log_{10}(3) - 20\log_{10}(\omega) & \text{for } \omega > 5 \end{cases}$$

The slope of the asymptotic magnitude Bode plot is −20 db/decade for $\omega < 2$ and $\omega > 5$ rad/s and is −40 db/decade for $2 < \omega < 5$ rad/s. Also, at $\omega = 1$ rad/s

$$H = \begin{cases} \dfrac{7.5}{1} = 7.5 & \text{at } \omega = 1 \text{ rad/s} \\ \dfrac{7.5}{2} = 3.75 & \text{at } \omega = 2 \text{ rad/s} \\ \dfrac{3}{5} = 0.6 & \text{at } \omega = 5 \text{ rad/s} \end{cases} \Rightarrow 20\log_{10} H = \begin{cases} 20\log_{10}(7.5) = 17.5 \text{ dB} & \text{at } \omega = 1 \text{ rad/s} \\ 20\log_{10}(3.75) = 11.5 \text{ dB} & \text{at } \omega = 2 \text{ rad/s} \\ 20\log_{10}(0.6) = -4.44 \text{ dB} & \text{at } \omega = 5 \text{ rad/s} \end{cases}$$

The asymptotic magnitude Bode plot for **H** is



17.5
11.5
−4.44
20 log10|H(ω)|, dB
−20 dB/decade
−40 dB/decade
−20 dB/decade
1
2
5
ω (rad/s logarithmic scale)

P 13.3-21 The network function of a circuit is $\mathbf{H}(\omega) = \dfrac{(j\omega)^3}{(4+j2\omega)}$. Sketch the asymptotic magnitude Bode plot corresponding to **H.**

Solution:

$$\mathbf{H}(\omega) = \frac{(j\omega)^3}{(4+j2\omega)} = \frac{\frac{1}{4}(j\omega)^3}{1+j\frac{\omega}{2}}$$

There is a pole at 2 rad/s and three zeros at 0 rad/s. To obtain the asymptotic magnitude Bode plot, use

$$\left|1+j\frac{\omega}{p}\right| = \begin{cases} 1 & \text{for } \omega < p \\ \dfrac{\omega}{p} & \text{for } \omega > p \end{cases}$$

Then

$$H = |\mathbf{H}| = \begin{cases} \dfrac{\frac{1}{4}\omega^3}{(1)} = \dfrac{1}{4}\omega^3 & \text{for } \omega < 2 \\ \dfrac{\frac{1}{4}\omega^3}{\left(\frac{\omega}{2}\right)} = \dfrac{1}{2}\omega^2 & \text{for } \omega > 2 \end{cases}$$

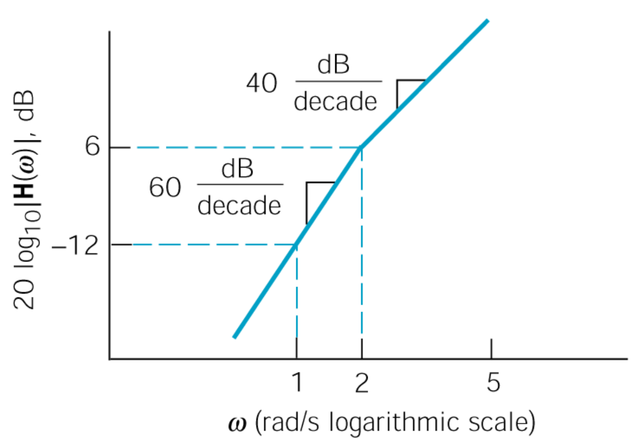
$$20\log_{10} H = \begin{cases} 20\log_{10}(0.25) + 3\left[20\log_{10}(\omega)\right] & \text{for } \omega < 2 \\ 20\log_{10}(0.50) + 2\left[20\log_{10}(\omega)\right] & \text{for } \omega > 2 \end{cases}$$

The slope of the asymptotic magnitude Bode plot is 60 db/decade for $\omega < 2$ rad/s and is 40 db/decade for $\omega > 2$ rad/s. Also,

$$20\log_{10} H = 20\log_{10}(0.25) + 3\left[20\log_{10}(1)\right] = -12 \text{ dB} \quad \text{at } \omega = 1 \text{ rad/s}$$

$$20\log_{10} H = 20\log_{10}(0.25) + 3\left[20\log_{10}(2)\right] = 6 \text{ dB} \quad \text{at } \omega = 2 \text{ rad/s}$$

The asymptotic magnitude Bode plot for **H** is



P 13.3-22 The network function of a circuit is $\mathbf{H}(\omega)=\dfrac{2(j2\omega+5)}{(4+j3\omega)(j\omega+2)}$. Sketch the asymptotic magnitude Bode plot corresponding to **H.**

Solution:

$$\mathbf{H}(\omega)=\frac{2(j2\omega+5)}{(4+j3\omega)(j\omega+2)}=\frac{\frac{5}{4}\left(1+j\frac{\omega}{5/2}\right)}{\left(1+j\frac{\omega}{4/3}\right)\left(1+j\frac{\omega}{2}\right)}$$

There is a zero at 2.5 rad/s and poles at 1.33 and 2 rad/s. To obtain the asymptotic magnitude Bode plot, use

$$\left|1+j\frac{\omega}{p}\right|=\begin{cases}1 & \text{for } \omega<p\\ \dfrac{\omega}{p} & \text{for } \omega>p\end{cases}$$

Then

$$H=|\mathbf{H}|=\begin{cases}\dfrac{\frac{5}{4}(1)}{(1)(1)}=5/4 & \text{for } \omega<4/3 \text{ rad/s}\\[2ex] \dfrac{\frac{5}{4}(1)}{\left(\frac{\omega}{4/3}\right)(1)}=\dfrac{5/3}{\omega} & \text{for } 4/3<\omega<2 \text{ rad/s}\\[2ex] \dfrac{\frac{5}{4}(1)}{\left(\frac{\omega}{4/3}\right)\left(\frac{\omega}{2}\right)}=\dfrac{10/3}{\omega^2} & \text{for } 2<\omega<5/2 \text{ rad/s}\\[2ex] \dfrac{\frac{5}{4}\left(\frac{\omega}{5/2}\right)}{\left(\frac{\omega}{4/3}\right)\left(\frac{\omega}{2}\right)}=\dfrac{4/3}{\omega} & \text{for } \omega>2.5 \text{ rad/s}\end{cases}$$

$$20\log_{10} H = \begin{cases} 20\log_{10}\left(\frac{5}{4}\right) & \text{for } \omega < \frac{4}{3} \\ 20\log_{10}\left(\frac{5}{3}\right) - 20\log_{10}(\omega) & \text{for } \frac{4}{3} < \omega < 2 \\ 20\log_{10}\left(\frac{10}{3}\right) - 40\log_{10}(\omega) & \text{for } 2 < \omega < \frac{5}{2} \\ 20\log_{10}\left(\frac{4}{3}\right) - 20\log_{10}(\omega) & \text{for } \omega > \frac{5}{2} \end{cases}$$

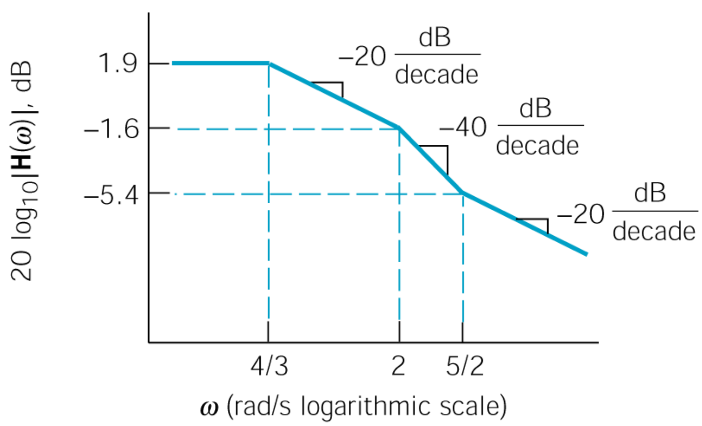
The slope of the asymptotic magnitude Bode plot is −20 db/decade for $4/3 < \omega < 2$ rad/s and $\omega > 5/2$ rad/s and is −40 db/decade for $2 < \omega < 5/2$ rad/s. Also,

$$20\log_{10} H = 20\log_{10}\left(\frac{5}{4}\right) = 1.9 \text{ dB} \quad \text{for } \omega \le \frac{4}{3} \text{ rad/s}$$

$$20\log_{10} H = 20\log_{10}\left(\frac{5}{3}\right) - 20\log_{10}(2) = -1.6 \text{ dB} \quad \text{at } \omega = 2 \text{ rad/s}$$

$$20\log_{10} H = 20\log_{10}\left(\frac{10}{3}\right) - 40\log_{10}\left(\frac{5}{2}\right) = -5.4 \text{ dB} \quad \text{at } \omega = \frac{5}{2} \text{ rad/s}$$

The asymptotic magnitude Bode plot for **H** is



P 13.3-23 The network function of a circuit is $\mathbf{H}(\omega)=\dfrac{4(20+j\omega)(20,000+j\omega)}{(200+j\omega)(2000+j\omega)}$ Sketch the asymptotic magnitude Bode plot corresponding to **H.**

Solution:

$$\mathbf{H}(\omega)=\frac{4\,(20+j\,\omega)\,(20,000+j\,\omega)}{(200+j\,\omega)\,(2000+j\,\omega)}=\frac{4\left(1+j\dfrac{\omega}{20}\right)\left(1+j\dfrac{\omega}{20,000}\right)}{\left(1+j\dfrac{\omega}{200}\right)\left(1+j\dfrac{\omega}{2000}\right)}$$

There are zeros at 20 and 20,000 rad/s and poles at 200 and 2000 rad/s. To obtain the asymptotic magnitude Bode plot, use

$$\left|1+j\frac{\omega}{p}\right|=\begin{cases}1 & \text{for } \omega<p\\ \dfrac{\omega}{p} & \text{for } \omega>p\end{cases}$$

Then

$$H=|\mathbf{H}|=\begin{cases}\dfrac{4(1)(1)}{(1)(1)}=4 & \text{for } \omega<20 \text{ rad/s}\\[2ex] \dfrac{4\left(\dfrac{\omega}{20}\right)(1)}{(1)(1)}=\dfrac{\omega}{5} & \text{for } 20<\omega<200 \text{ rad/s}\\[2ex] \dfrac{4\left(\dfrac{\omega}{20}\right)(1)}{\left(\dfrac{\omega}{200}\right)(1)}=40 & \text{for } 200<\omega<2000 \text{ rad/s}\\[2ex] \dfrac{4\left(\dfrac{\omega}{20}\right)(1)}{\left(\dfrac{\omega}{200}\right)\left(\dfrac{\omega}{2000}\right)}=\dfrac{80000}{\omega} & \text{for } 2000<\omega<20,000 \text{ rad/s}\\[2ex] \dfrac{4\left(\dfrac{\omega}{20}\right)\left(\dfrac{\omega}{20,000}\right)}{\left(\dfrac{\omega}{200}\right)\left(\dfrac{\omega}{2000}\right)}=4 & \text{for } \omega>2000 \text{ rad/s}\end{cases}$$

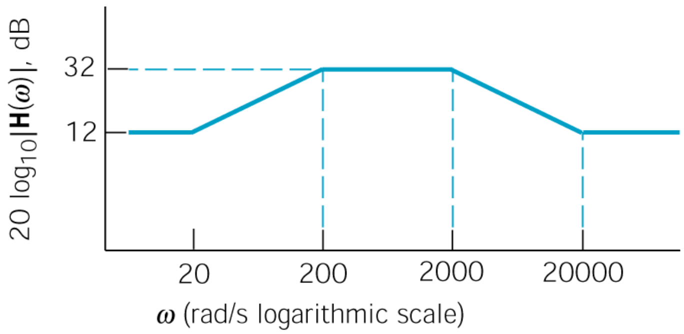
$$20\log_{10}H=\begin{cases}20\log_{10}(4) & \text{for } \omega<20\\ 20\log_{10}(\omega)-20\log_{10}(5) & \text{for } 20<\omega<200\\ 20\log_{10}(40) & \text{for } 200<\omega<2000\\ 20\log_{10}(80000)-20\log_{10}(\omega) & \text{for } 2000<\omega<20,000\\ 20\log_{10}(4) & \text{for } \omega>20,000\end{cases}$$

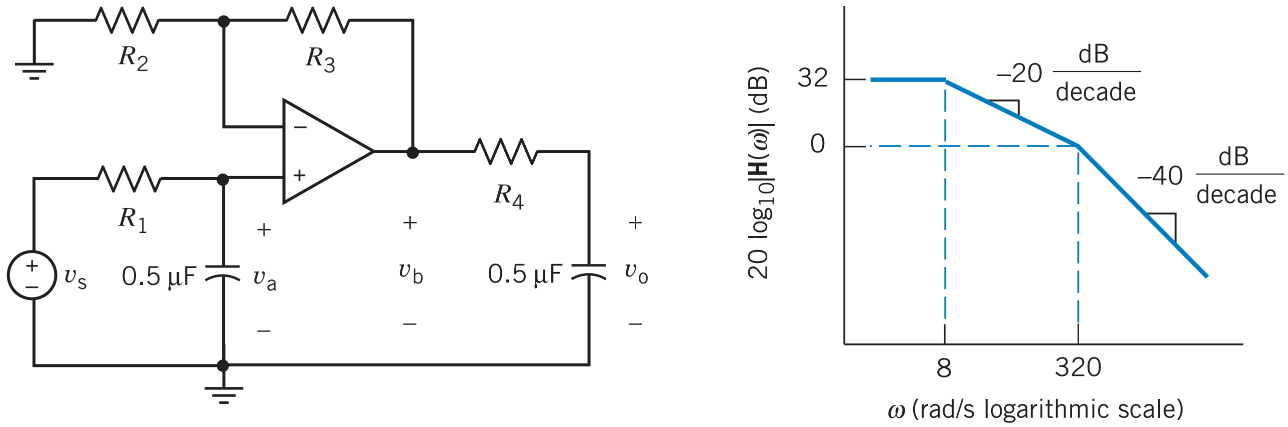
The slope of the asymptotic magnitude Bode plot is 20 db/decade for $20 < \omega < 200$ rad/s and is -20 db/decade for $2000 < \omega < 20{,}000$ rad/s and is 0 db/decade for $\omega < 20$ and $200 < \omega < 2000$ rad/s, and $\omega >$ 20,000 rad/s. Also,

$$20 \log_{10} H = 20 \log_{10} (4) = 12 \text{ dB} \quad \text{for } \omega \le 20 \text{ and } \omega \ge 20{,}000 \text{ rad/s}$$

$$20 \log_{10} H = 20 \log_{10} (40) = 32 \text{ dB} \quad \text{for } 200 \le \omega \le 2000 \text{ rad/s}$$

The asymptotic magnitude Bode plot for **H** is





(a) *(b)*

Figure P 13.3-24

P 13.3-24 The input to the circuit shown in Figure P 13.3-24*a* is the voltage of the voltage source, v_s. The output of the circuit is the capacitor voltage, v_o. The network function of the circuit is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)}$$

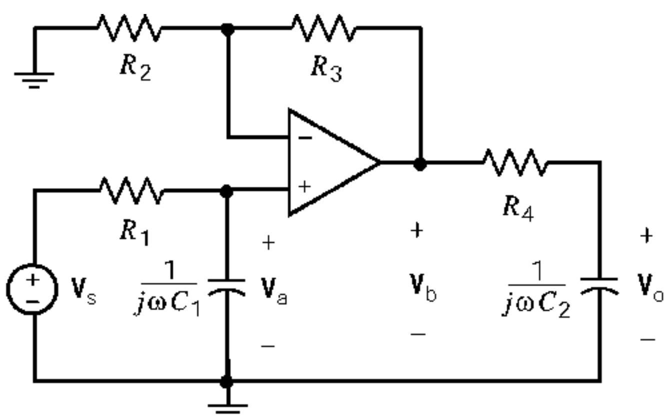
Determine the values of the resistances R_1, R_2, R_3, and R_4 required to cause the network function of the circuit to correspond to the asymptotic Bode plot shown in Figure P 13.3-24b.

Solution:

From Figure P13.3-24*b*, $\mathbf{H}(\omega)$ has poles at 8 and 320 rad/s and has a low frequency gain equal to 32 dB = 40. Consequently, the network function corresponding to the Bode plot is

$$\mathbf{H}(\omega) = \frac{\pm 40}{\left(1 + j\frac{\omega}{8}\right)\left(1 + j\frac{\omega}{320}\right)}$$

Next, we find the network function corresponding to the circuit. Represent the circuit in the frequency domain.



Apply KCL at the top node of the left capacitor, C_1, to get

$$\frac{\mathbf{V}_a - \mathbf{V}_s}{R_1} + j\omega C_1 \mathbf{V}_a = 0 \quad \Rightarrow \quad \mathbf{V}_a = \frac{1}{1 + j\omega C_1 R_1} \mathbf{V}_s$$

The op amp, together with resistors R_2 and R_3, comprise a noninverting amplifier so

$$\mathbf{V}_b = \left(1 + \frac{R_3}{R_2}\right) \mathbf{V}_a$$

(Alternately, this equation can be obtained by applying KCL at the inverting input node of the op amp.) Apply KCL at the top node of the right capacitor, C_2, to get

$$\frac{\mathbf{V}_o - \mathbf{V}_b}{R_4} + j\omega C_2 \mathbf{V}_o = 0 \quad \Rightarrow \quad \mathbf{V}_o = \frac{1}{1 + j\omega C_2 R_4} \mathbf{V}_b$$

Combining these equations gives

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)} = \frac{1 + \frac{R_3}{R_2}}{\left(1 + j\omega C_1 R_1\right)\left(1 + j\omega C_2 R_4\right)}$$

Comparing to the specified network function gives

$$\frac{1 + \frac{R_3}{R_2}}{\left(1 + j\omega C_1 R_1\right)\left(1 + j\omega C_2 R_4\right)} = \frac{\pm 40}{\left(1 + j\frac{\omega}{8}\right)\left(1 + j\frac{\omega}{320}\right)}$$

The solution is not unique. For example, we can require

$$1 + \frac{R_3}{R_2} = 40, \; C_1 R_1 = \frac{1}{8} = 0.125, \; C_2 R_4 = \frac{1}{320} = 0.00758$$

With the given values of capacitance, and choosing R_2 = 10 kΩ, we have

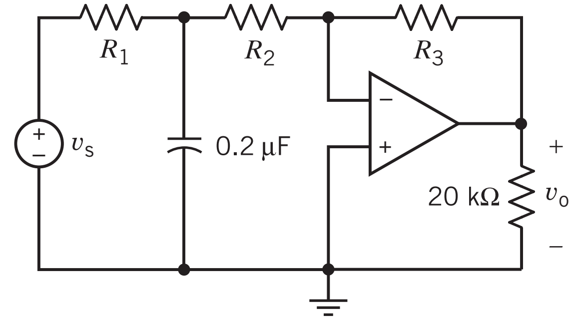
R_1 = 250 kΩ, R_2 = 10 kΩ, R_3 = 390 kΩ and R_4 = 6.25 kΩ

(checked using LNAP 10/1/04)

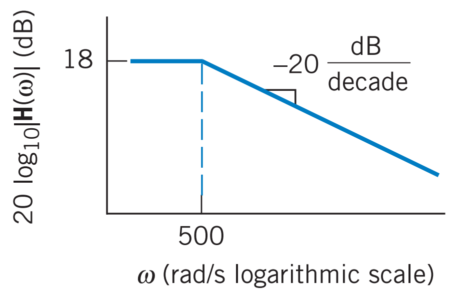
P 13.3-25 The input to the circuit shown in Figure P 13.3-25a is the voltage of the voltage source, v_s. The output of the circuit is the voltage, v_o. The network function of the circuit is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)}$$

Determine the values of the resistances R_1, R_2, and R_3 required to cause the network function of the circuit to correspond to the asymptotic Bode plot shown in Figure P 13.3-25*b*.



(a)



(b)

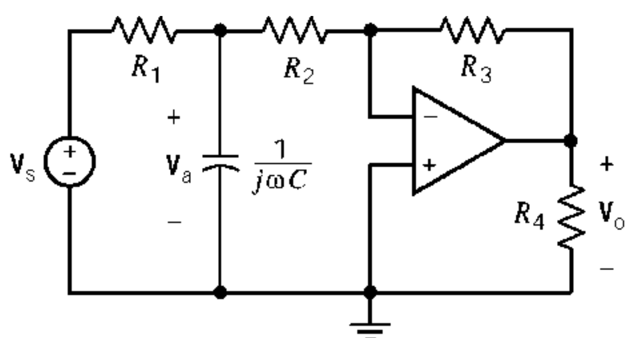
Figure P 13.3-25

Solution:

From Figure P13.3-25*b*, $\mathbf{H}(\omega)$ has a pole at 500 rad/s and a low frequency gain of 18 dB = 8. Consequently, the network function corresponding to the Bode plot is

$$\mathbf{H}(\omega) = \frac{\pm 8}{\left(1 + j\dfrac{\omega}{500}\right)}$$

Next, we find the network function corresponding to the circuit. Represent the circuit in the frequency domain.



The node equations are

$$\frac{\mathbf{V}_a - \mathbf{V}_s}{R_1} + \frac{\mathbf{V}_a}{\dfrac{1}{j\omega C}} + \frac{\mathbf{V}_a}{R_2} = 0 \quad \Rightarrow \quad \mathbf{V}_a = \frac{R_2}{R_1 + R_2 + j\omega C R_1 R_2}\mathbf{V}_s$$

and
$$\frac{\mathbf{V}_a}{R_2}+\frac{\mathbf{V}_o}{R_3}=0 \quad \Rightarrow \quad \mathbf{V}_o=-\frac{R_3}{R_2}\mathbf{V}_a$$

The network function is

$$\mathbf{H}=\frac{\mathbf{V}_o}{\mathbf{V}_s}=\frac{-\frac{R_3}{R_2}R_2}{R_1+R_2+j\omega C R_1 R_2}=\frac{-\frac{R_3}{R_1+R_2}}{1+j\omega C\frac{R_1 R_2}{R_1+R_2}}$$

Comparing to the specified network function gives

$$\frac{-\frac{R_3}{R_1+R_2}}{1+j\omega C\frac{R_1 R_2}{R_1+R_2}}=\frac{\pm 8}{\left(1+j\frac{\omega}{500}\right)}$$

We require

$$\frac{R_3}{R_1+R_2}=8 \quad \text{and} \quad C\frac{R_1 R_2}{R_1+R_2}=\frac{1}{500}=0.002$$

The solution is not unique. With the given values of capacitance, and choosing $R_1 = R_2$, we have

$$R_1 = R_2 = 20\ \text{k}\Omega \text{ and } R_3 = 320\ \text{k}\Omega$$

(checked using LNAP 10/1/04)

P 13.3-26 The input to the circuit shown in Figure P 13.3-26*a* is the voltage of the voltage source, v_s. The output of the circuit is the voltage, v_o. The network function of the circuit is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)}$$

(a) Determine the values of the resistances, R_1 and R_2, required to cause the network function of the circuit to correspond to the asymptotic Bode plot shown in Figure P 13.3-26*b*.

(b) Determine the values of the gains K_1 and K_2 in Figure P 13.3-26*b*.

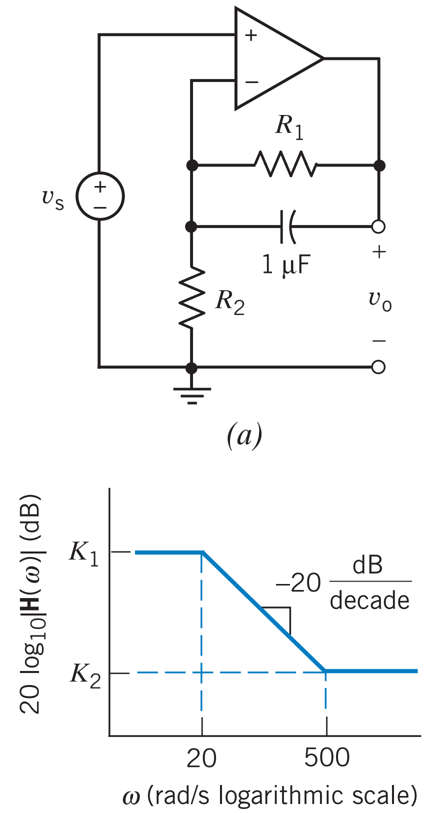


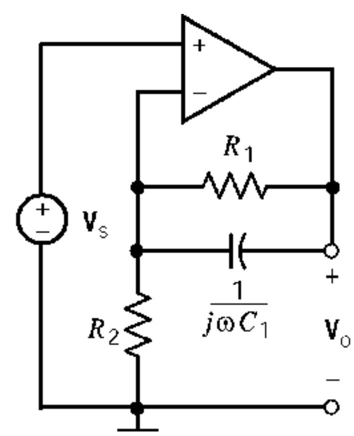
Figure P 13.3-26

Solution:

From Figure P13.3-26*b*, $\mathbf{H}(\omega)$ has a pole at 20 rad/s and a zero at 500 rad/s. Consequently, the network function corresponding to the Bode plot is

$$\mathbf{H}(\omega) = \pm K \frac{\left(1 + j\dfrac{\omega}{500}\right)}{\left(1 + j\dfrac{\omega}{20}\right)}.$$

Next, we find the network function corresponding to the circuit. Represent the circuit in the frequency domain. Apply KCL at the inverting input node of the op amp to get



$$\frac{\mathbf{V}_o - \mathbf{V}_s}{R_1} + j\omega C_1\left(\mathbf{V}_o - \mathbf{V}_s\right) + \frac{\mathbf{V}_s}{R_2} = 0$$

or $$\left(R_1 + R_2 + j\omega C_1 R_1 R_2\right)\mathbf{V}_s = \left(R_2 + j\omega C_1 R_1 R_2\right)\mathbf{V}_o$$

so

$$\mathbf{H} = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{R_1 + R_2 + j\omega C_1 R_1 R_2}{R_2 + j\omega C_1 R_1 R_2} = \frac{R_1 + R_2}{R_2} \times \frac{1 + j\omega C_1 \dfrac{R_1 R_2}{R_1 + R_2}}{1 + j\omega C_1 R_1}$$

a. Comparing to the specified network function gives

$$\frac{R_1+R_2}{R_2}\times\frac{1+j\omega C_1\dfrac{R_1\,R_2}{R_1+R_2}}{1+j\omega C_1\,R_1}=K\,\frac{1+j\dfrac{\omega}{500}}{1+j\dfrac{\omega}{20}}$$

We require $$C_1\,R_1=\frac{1}{20}=.05 \quad\text{and}\quad C\frac{R_1\,R_2}{R_1+R_2}=\frac{1}{500}=0.002$$

Notice that $$K=\frac{R_1+R_2}{R_2}\times\frac{C_1\,R_1}{C_1\dfrac{R_1\,R_2}{R_1+R_2}}=\frac{\dfrac{1}{20}}{\dfrac{1}{500}}=25$$

The solution is not unique. For example, choosing $C = 1$ μF

$R_1 = 50$ kΩ and $R_2 = 2.083$ kΩ

b. The network function is

$$\mathbf{H}(\omega)=25\frac{\left(1+j\dfrac{\omega}{500}\right)}{\left(1+j\dfrac{\omega}{20}\right)}$$

so

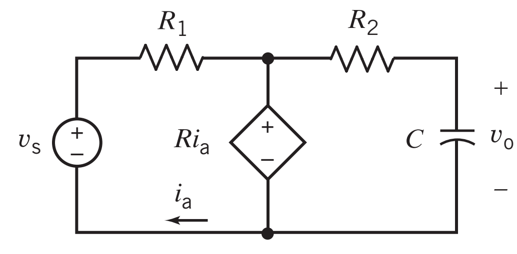
$$K_1=20\log_{10}(25)=28\text{ dB} \quad\text{and}\quad K_2=20\log_{10}\left(25\times\frac{20}{500}\right)=0\text{ dB}$$

(checked using LNAP 10/1/04)

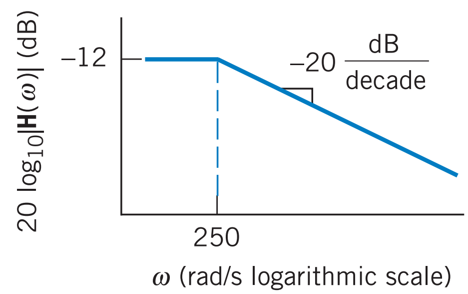
P 13.3-27 The input to the circuit shown in Figure P 13.3-27*a* is the voltage of the voltage source, v_s. The output of the circuit is the voltage, v_o. The network function of the circuit is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)}$$

Determine the values of *R, C,* R_1, and R_2 required to cause the network function of the circuit to correspond to the asymptotic Bode plot shown in Figure P 13.3-27*b*.



(a)



(b)

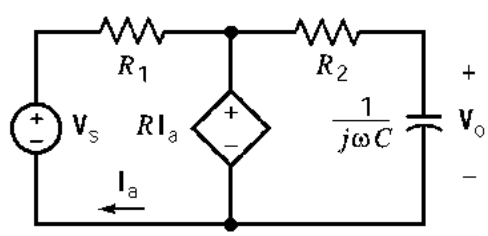
Figure P 13.3-27

Solution:

From Figure P13.3-27*b*, $\mathbf{H}(\omega)$ has a pole at 250 rad/s and a low frequency gain equal to −12 dB = 0.25. Consequently, the network function corresponding to the Bode plot is

$$\mathbf{H}(\omega) = \frac{\pm 0.25}{1 + j\dfrac{\omega}{250}}.$$

Next, we find the network function corresponding to the circuit. Represent the circuit in the frequency domain. Apply KVL to the left mesh to get



$$\mathbf{V}_s = R_1\mathbf{I}_a + R\mathbf{I}_a \quad \Rightarrow \quad \mathbf{I}_a = \frac{\mathbf{V}_s}{R_1 + R}$$

Voltage division gives

$$\mathbf{V}_o = \frac{\dfrac{1}{j\omega C}}{R_2 + \dfrac{1}{j\omega C}} R\mathbf{I}_a = \frac{R}{1 + j\omega C R_2}\mathbf{I}_a = \frac{\dfrac{R}{R_1 + R}}{1 + j\omega C R_2}\mathbf{V}_s$$

The network function of the circuit is

$$\mathbf{H} = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{\dfrac{R}{R_1 + R}}{1 + j\omega C R_2}$$

Comparing to the specified network function gives

$$\frac{\frac{R}{R_1+R}}{1+j\omega C R_2}=\frac{\pm 0.25}{1+j\frac{\omega}{250}}$$

The solution is not unique. We require

$$\frac{R}{R_1+R}=\frac{1}{4} \text{ and } C R_2=\frac{1}{250}=0.004$$

Choosing R = 100 Ω and C = 10 μF we have R_1 = 300 Ω and R_2 = 400 Ω

(checked using LNAP 10/2/04)

P 13.3-28 The input to the circuit shown in Figure P 13.3-28a is the current of the current source, i_s. The output of the circuit is the current i_o. The network function of the circuit is

$$\mathbf{H}(\omega) = \frac{\mathbf{I}_o(\omega)}{\mathbf{I}_s(\omega)}$$

Determine the values of *G, C, R*$_1$, and R_2 required to cause the network function of the circuit to correspond to the asymptotic Bode plot shown in Figure P 13.3-28*b*.

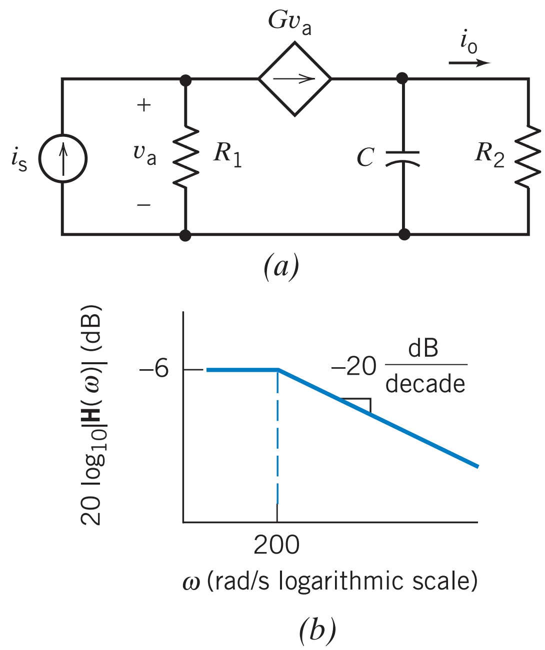


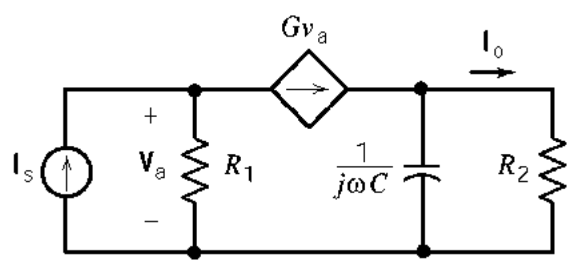
Figure P 13.3-28

Solution:

From Figure P13.3-28*b*, $\mathbf{H}(\omega)$ has a pole at 200 rad/s and a low frequency gain equal to −6 dB = 0. 5. Consequently, the network function corresponding to the Bode plot is

$$\mathbf{H}(\omega) = \frac{\pm 0.5}{1 + j\dfrac{\omega}{200}}.$$

Next, we find the network function corresponding to the circuit. Represent the circuit in the frequency domain. Apply KCL at the top node of R_1 to get



$$\mathbf{I}_s = \frac{\mathbf{V}_a}{R_1} + G\mathbf{V}_a \quad \Rightarrow \quad \mathbf{V}_a = \frac{R_1}{1+GR_1}\mathbf{I}_s$$

Current division gives

$$\mathbf{I}_o = \frac{\dfrac{1}{j\omega C}}{\dfrac{1}{j\omega C} + R_2} G\mathbf{V}_a = \frac{G}{1 + j\omega C R_2}\mathbf{V}_a = \frac{G}{1 + j\omega C R_2}\left(\frac{R_1}{1+GR_1}\mathbf{I}_s\right)$$

The network function of the circuit is

$$\mathbf{H} = \frac{\mathbf{I}_o}{\mathbf{I}_s} = \frac{\dfrac{GR_1}{1+GR_1}}{1 + j\omega C R_2}$$

Comparing this network function to the specified network function gives

$$\frac{G R_1}{1+G R_1}=0.5 \text{ and } C R_2=\frac{1}{200}$$

The solution is not unique. Choosing $G = 0.01$ A/V and $C = 10$ μF gives $R_1 = 100\ \Omega$ and $R_2 = 500\ \Omega$

(checked using LNAP 10/2/04)

P 13.3-29 A first-order circuit is shown in Figure P 13.3-29. Determine the ratio $\mathbf{V}_o/\mathbf{V}_s$ and sketch the Bode diagram when $RC = 0.1$ and $R_1/R_2 = 3$.

Answer: $\mathbf{H} = \left(1 + \frac{R_1}{R_2}\right)\frac{1}{1 + j\omega RC}$

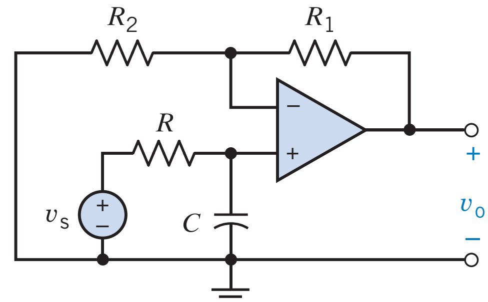


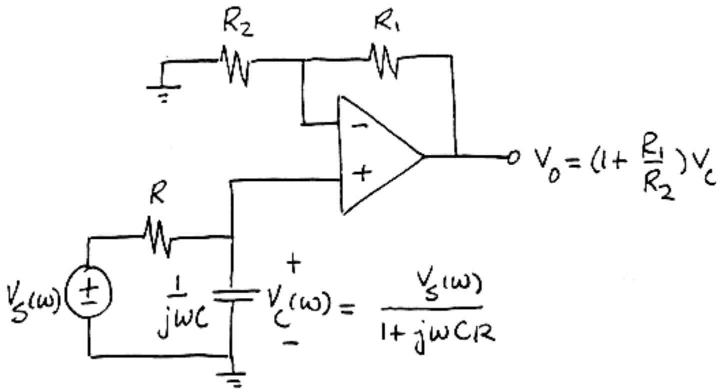
Figure P 13.3-29

Solution:

$$\mathbf{V}_o(\omega) = \left(1 + \frac{R_1}{R_2}\right)\mathbf{V}_c(\omega)$$

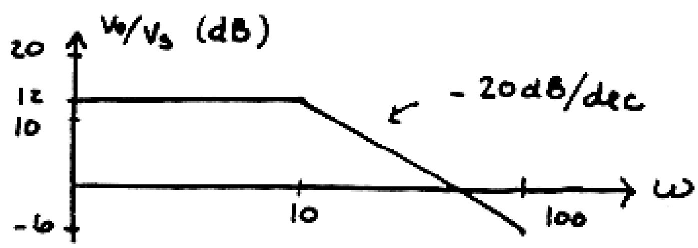
$$= \left(1 + \frac{R_1}{R_2}\right)\left(\frac{1}{1 + j\omega C R}\right)\mathbf{V}_s(\omega)$$

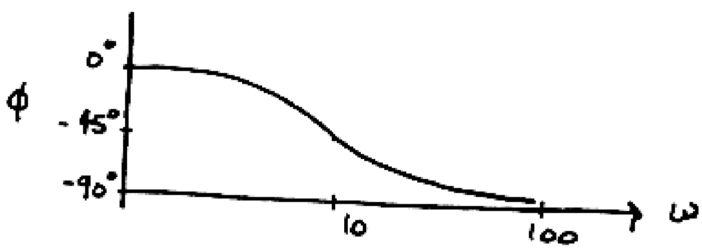
$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)} = \left(1 + \frac{R_1}{R_2}\right)\left(\frac{1}{1 + j\omega C R}\right)$$



When $RC = 0.1$ and $\frac{R_1}{R_2} = 3$,

then $\mathbf{H}(\omega) = \frac{4}{1 + j\frac{\omega}{10}}$





P 13.3-30 *(*a) Draw the Bode diagram of the network function $\mathbf{V}_o/\mathbf{V}_s$ for the circuit of Figure P 13.3-30.

(b) Determine $v_o(t)$ when

$$v_s = 10\cos 20t \text{ V}.$$

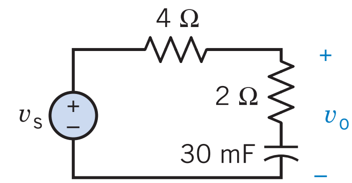
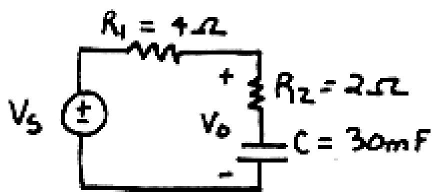


Figure P 13.3-30

Answer: *(b)* $v_o = 4.18\cos(20t - 24.3°)$ V

Solution:

a)



$$\mathbf{Z}_o = R_2 + \frac{1}{j\omega C}$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{\mathbf{Z}_o}{R_1 + \mathbf{Z}_o} = \frac{R_2 + \dfrac{1}{j\omega C}}{R_1 + R_2 + \dfrac{1}{j\omega C}} = \frac{1 + j\dfrac{\omega}{\omega_1}}{1 + j\dfrac{\omega}{\omega_2}}$$

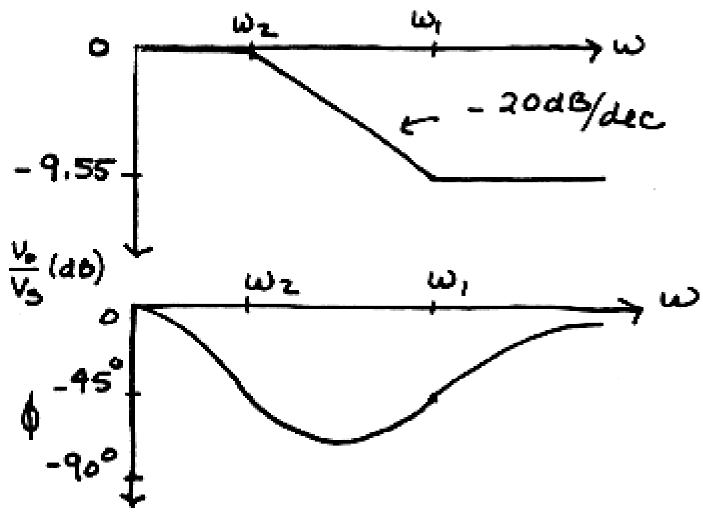
where $\omega_1 = \dfrac{1}{R_2 C} = 16.7$ rad/s

and $\omega_2 = \dfrac{1}{(R_1 + R_2)C} = 5.56$ rad/s

$$v_s(t) = 10\cos 20t \quad \text{or} \quad \mathbf{V}_s = 10\angle 0°$$

$$\therefore \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{1 + j\left(20/16.7\right)}{1 + j\left(20/5.56\right)}$$

$$= \frac{1 + j\,1.20}{1 + j\,3.60} = 0.418\angle -24.3°$$



b) So

$$\mathbf{V}_o = 4.18\angle -24.3°$$

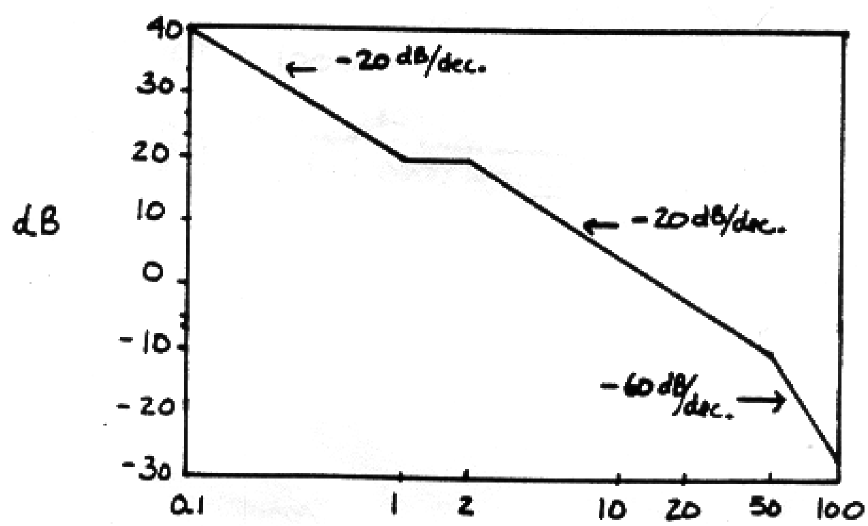
$$\underline{v_o(t) = 4.18\cos(20t - 24.3°) \text{ V}}$$

P 13.3-31 Draw the asymptotic magnitude Bode diagram for

$$\mathbf{H}(\omega) = \frac{10(1+j\omega)}{j\omega(1+j0.5\omega)(1+j0.6(\omega/50)+(j\omega/50)^2)}$$

Hint: At ω = 0.1 rad/s the value of the gain is 40 dB and the slope of the asymptotic Bode plot is –20 dB/decade. There is a zero at 1 rad/s, a pole at 2 rad/s, and a second-order pole at 50 rad/s. The slope of the asymptotic magnitude Bode diagram increases by 20 dB/decade as the frequency increases past the zero, decreases by 20 dB/decade as the frequency increases past the pole, and, finally, decreases by 40 dB/decade as the frequency increases past the second-order pole.

P13.3-31



Section 13-4: Resonant Circuits

P 13.4-1 For a parallel *RLC* circuit with $R = 10\ \text{k}\Omega$, $L = 1/120$ H, and $C = 1/30\ \mu$F, find ω_0, Q, ω_1, ω_2, and the bandwidth BW.

Answer: $\omega_0 = 60$ krad/s, $Q = 20$, $\omega_1 = 58.519$ krad/s, $\omega_2 = 61.519$ krad/s, and $BW = 3$ krad/s

Solution:

For the parallel resonant *RLC* circuit with $R = 10\ \text{k}\Omega$, $L = 1/120$ H, and $C = 1/30\ \mu$F we have

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\left(\frac{1}{120}\right)\left(\frac{1}{30}\ x\ 10^{-6}\right)}} = 60\ \text{k rad/sec}$$

$$Q = R\sqrt{\frac{C}{L}} = 10,000\sqrt{\frac{\frac{1}{30}\times 10^{-6}}{\frac{1}{120}}} = 20$$

$$\omega_1 = -\frac{\omega_0}{2Q} + \sqrt{\left(\frac{\omega_0}{2Q}\right)^2 + \omega_0{}^2} = 58.52\ \text{k rad/s} \quad \text{and} \quad \omega_2 = \frac{\omega_0}{2Q} + \sqrt{\left(\frac{\omega_0}{2Q}\right)^2 + \omega_0{}^2} = 61.52\ \text{k rad/s}$$

$$BW = \frac{1}{RC} = \frac{1}{(10000)\left(\frac{1}{30}\times 10^{-6}\right)} = 3\ \text{krad/s}$$

Notice that $BW = \omega_2 - \omega_1 = \dfrac{\omega_0}{Q}$.

P 13.4-2 A parallel resonant *RLC* circuit is driven by a current source $i_s = 20 \cos \omega t$ mA and shows a maximum response of 8 V at $\omega = 1000$ rad/s and 4 V at 897.6 rad/s. Find *R, L*, and *C*.

Answer: $R = 400\ \Omega$, $L = 50$ mH, and $C = 20\ \mu$F

Solution: For the parallel resonant *RLC* circuit we have

$$|\mathbf{H}(\omega)| = \frac{k}{\sqrt{1+Q^2\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)^2}}$$

so

$$R = k = |\mathbf{H}(\omega_0)| = \frac{8}{20\cdot 10^{-3}} = 400\ \Omega \text{ and } \omega_0 = 1000 \text{ rad/s}$$

At $\omega = 897.6$ rad/s, $|\mathbf{H}(\omega)| = \frac{4}{20.10^{-3}} = 200$, so

$$200 = \frac{400}{\sqrt{1+Q^2\left(\frac{897.6}{1000}-\frac{1000}{897.6}\right)^2}} \quad \Rightarrow \quad Q = 8$$

Then

$$\left.\begin{aligned} \frac{1}{\sqrt{LC}} &= \omega_0 = 1000 \\ 400\sqrt{\frac{C}{L}} &= Q = 8 \end{aligned}\right\} \Rightarrow \begin{aligned} C &= 20\ \mu\text{F} \\ L &= 50 \text{ mH} \end{aligned}$$

P 13.4-3 A series resonant *RLC* circuit has $L = 10$ mH, $C = 0.01\ \mu$F, and $R = 100\ \Omega$. Determine ω_0, Q, and BW.

Answer: $\omega_0 = 10^5$, $Q = 10$, and $BW = 10^4$

Solution:

For the series resonant *RLC* circuit with $R = 100\ \Omega$, $L = 10$ mH, and $C = 0.01\ \mu$F we have

$$\omega_0 = \frac{1}{\sqrt{LC}} = 10^5 \text{ rad/s}, \quad Q = \frac{1}{R}\sqrt{\frac{L}{C}} = 10, \quad BW = \frac{R}{L} = 10^4 \text{ rad/s}$$

P 13.4-4 A quartz crystal exhibits the property that when mechanical stress is applied across its faces, a potential difference develops across opposite faces. When an alternating voltage is applied, mechanical vibrations occur and electromechanical resonance is exhibited. A crystal can be represented by a series *RLC* circuit. A specific crystal has a model with $L = 1$ mH, $C = 10\ \mu$F, and $R = 1\ \Omega$. Find ω_0, Q, and the bandwidth.

Answer: $\omega_0 = 10^4$ rad/s, $Q = 10$, and $BW = 10^3$ rad/s

Solution:

For the series resonant *RLC* circuit with $R = 1\ \Omega$, $L = 1$ mH, and $C = 10\ \mu$F we have

$$\omega_0 = \frac{1}{\sqrt{LC}} = 10^4 \text{ rad/s}, \quad Q = \frac{1}{R}\sqrt{\frac{L}{C}} = 10, \quad BW = \frac{R}{L} = 10^3 \text{ rad/s}$$

P 13.4-5 Design a parallel resonant circuit to have ω_0 = 2500 rad/s, $\mathbf{Z}(\omega_0)$ = 100 Ω, and BW = 500 rad/s.
Answer: R = 100 Ω, L = 8 mH, and C = 20 μF

Solution:
For the parallel resonant *RLC* circuit we have

$$R = \mathbf{Z}(\omega_0) = 100\ \Omega$$

$$\frac{1}{100\,C} = BW = 500 \text{ rad/s} \quad \Rightarrow \quad C = 20\ \mu\text{F}$$

$$\frac{1}{\sqrt{(20 \cdot 10^{-6})L}} = \omega_0 = 2500 \text{ rad/s} \quad \Rightarrow \quad L = 8 \text{ mH}$$

P 13.4-6 Design a series resonant circuit to have ω_0 = 2500 rad/s, $\mathbf{Y}(\omega_0)$ = 1/100 Ω, and BW = 500 rad/s.

Answer: R = 100 Ω, L = 0.2 H, and C = 0.8 μF

P13.4-6
For the series resonant *RLC* circuit we have

$$R = \frac{1}{\mathbf{Y}(\omega_0)} = 100\ \Omega$$

$$\frac{100}{L} = BW = 500 \text{ rad/s} \quad \Rightarrow \quad L = 0.2 \text{ H}$$

$$\frac{1}{\sqrt{(0.2)C}} = \omega_0 = 2500 \text{ rad/s} \quad \Rightarrow \quad C = 0.8\ \mu\text{F}$$

P 13.4-7 The circuit shown in Figure P 13.4-7 represents a capacitor, coil, and resistor in parallel. Calculate the resonant frequency, bandwidth, and Q for the circuit.

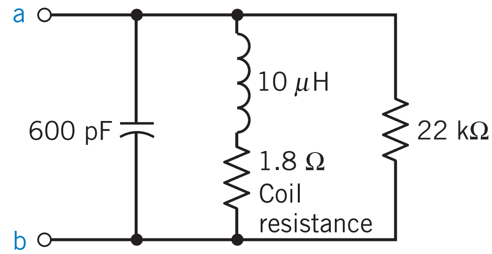
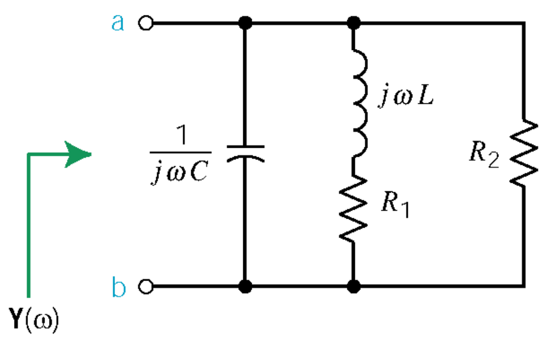


Figure P 13.4-7

Solution:



$C = 600$ pF

$L = 10$ μH

$R_1 = 1.8\ \Omega$

$R_2 = 22$ kΩ

$$\mathbf{Y}(\omega) = j\omega C + \frac{1}{R_1 + j\omega L} + \frac{1}{R_2}$$

$$= \frac{\left(R_1 + R_2 - \omega^2 C L R_2\right) + j\omega\left(L + C R_1 R_2\right)}{R_2\left(R_1 + j\omega L\right)} \times \frac{R_1 - j\omega L}{R_1 - j\omega L}$$

$$= \frac{R_1\left(R_1 + R_2 - \omega^2 C L R_2\right) + \omega^2 L\left(L + C R_1 R_2\right) + j\omega R_1 (L + C R_1 R_2) - j\omega L\left(R_1 + R_2 - \omega^2 C L R_2\right)}{R_2 (R_1 - \omega^2 L^2)}$$

$\omega = \omega_0$ is the frequency at which the imaginary part of $\mathbf{Y}(\omega)$ is zero:

$$R_1\left(L + C R_1 R_2\right) - L\left(R_1 + R_2 - \omega_0^2 C L R_2\right) = 0 \quad \Rightarrow \quad \omega_0 = \sqrt{\frac{L R_2 - C R_1^2 R_2}{C L^2 R_2}} = 12.9 \text{ M rad/sec}$$

P 13.4-8 Consider the simple model of an electric power system as shown in Figure P 13.4-8. The inductance, L = 0.25 H, represents the power line and transformer. The customer's load is R_L = 100 Ω, and the customer adds C = 25 μF to increase the magnitude of $\mathbf{V}_o$.

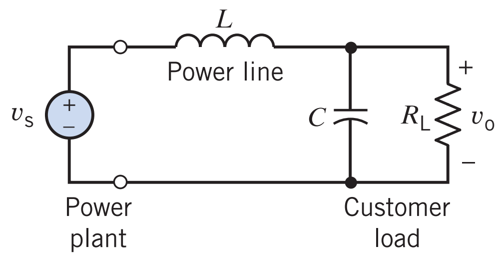


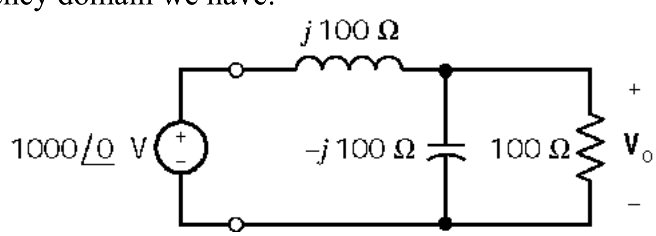
Figure P 13.4-8

The source is v_s = 1000 cos 400t V, and it is desired that $|\mathbf{V}_o|$ also be 1000 V.

(a) Find $|\mathbf{V}_0|$ for R_L = 100 **Ω**.

(b) When the customer leaves for the night, he turns off much of his load, making R_L = 1 k**Ω**, at which point sparks and smoke begin to appear in the equipment still connected to the power line. The customer calls you in as a consultant. Why did the sparks appear when R_L = 1 kΩ?

Solution: In the frequency domain we have:

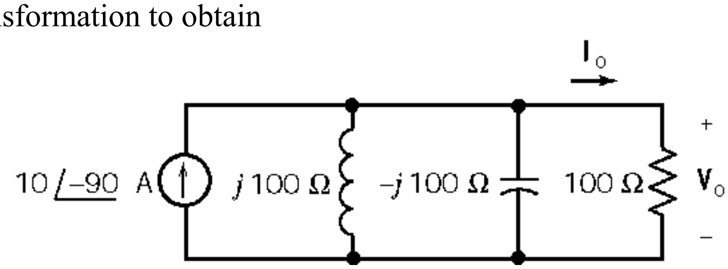


(a) Using voltage division yields

$$\mathbf{V}_o = \left(1000\angle 0^\circ\right) \frac{\dfrac{(100)(-j100)}{100-j100}}{\dfrac{(100)(-j100)}{100-j100}+j100} = \left(1000\angle 0^\circ\right) \frac{\dfrac{100}{\sqrt{2}}\angle -45^\circ}{\dfrac{100}{\sqrt{2}}\angle -45^\circ + j100} = \frac{\dfrac{10^5}{\sqrt{2}}\angle -45^\circ}{50\sqrt{2}\angle 45^\circ} = 1000\angle -90^\circ \text{ V}$$

$$\therefore |\mathbf{V}_O| = 1000 \text{ V}$$

(b) Do a source transformation to obtain



This is a resonant circuit with $\omega_0 = 1/\sqrt{LC}$=400 rad/s. Since this also happens to be the frequency of the input, so this circuit is being operated at resonance. At resonance the admittances of the capacitor and inductor cancel each other, leaving the impedance of the resistor. Increasing the resistance by a factor of 10 will increase the voltage $\mathbf{V}_o$ by a factor of 10. This increased voltage will cause increased currents in both the inductance and the capacitance, causing the sparks and smoke.

P 13.4-9 Consider the circuit in Figure P 13.4-9. $R_1 = R_2 = 1\ \Omega$. Select C and L to obtain a resonant frequency of $\omega_0 = 100$ rad/s.

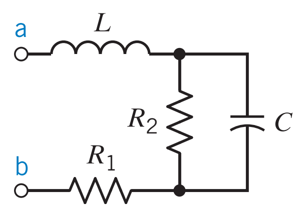
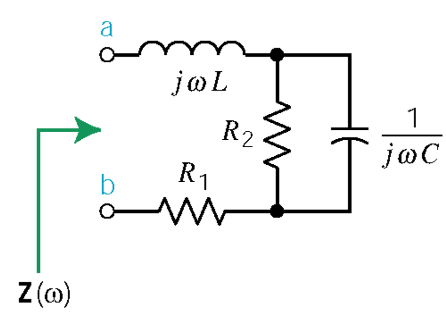


Figure P 13.4-9

Solution:

Let $G_2 = \dfrac{1}{R_2}$. Then



$$\mathbf{Z} = R_1 + j\omega L + \frac{1}{G_2 + j\omega C}$$

$$= \frac{\left(R_1 G_2 + 1 - \omega^2 LC\right) + j\left(\omega L G_2 + \omega C R_1\right)}{G_2 + j\omega C}$$

At resonance, $\angle \mathbf{Z} = 0°$ so

$$\tan^{-1}\frac{\omega L G_2 + \omega C R_1}{\left(R_1 G_2 + 1 - \omega^2 LC\right)} = \tan^{-1}\frac{\omega C}{G_2}$$

so

$$\frac{\omega L G_2 + \omega C R_1}{\left(R_1 G_2 + 1 - \omega^2 LC\right)} = \frac{\omega C}{G_2} \quad \Rightarrow \quad \omega^2 = \frac{C - L G_2^2}{LC^2} \quad \text{and} \quad C > G_2^2 L$$

With $R_1 = R_2 = 1\ \Omega$ and $\omega_0 = 100$ rad/s, $\omega_0{}^2 = 10^4 = \dfrac{C-L}{LC^2}$. Then choose C and calculate L:

$$C = 10 \text{ mF} \Rightarrow L = 5 \text{ mH}$$

Since $C > G_2^2 L$, we are done.

P 13.4-10 For the circuit shown in Figure P 13.4-10,
(a) derive an expression for the magnitude response $|\mathbf{Z}_{in}|$ versus ω,
(b) sketch $|\mathbf{Z}_{in}|$ versus ω, and
(c) find $|\mathbf{Z}_{in}|$ at $\omega = 1/\sqrt{LC}$.

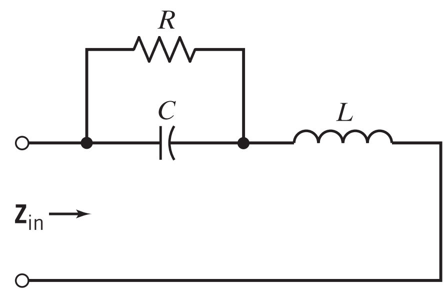
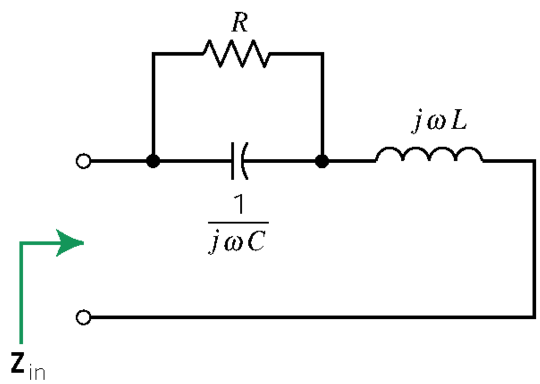


Figure P 13.4-10

Solution:
(a)

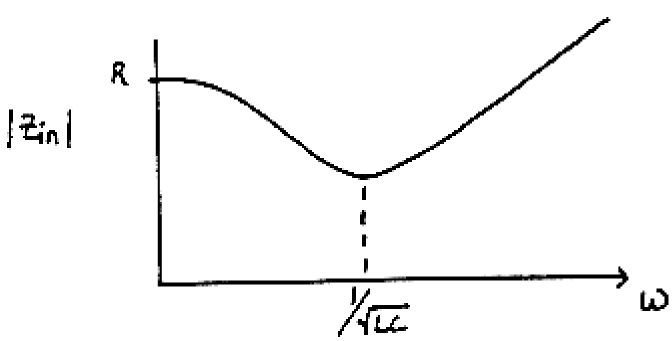


$$\mathbf{Z}_{in} = j\omega L + \frac{\dfrac{R}{j\omega C}}{R + \dfrac{1}{j\omega C}} = \frac{\left(R - \omega^2 R L C\right) + j\omega L}{1 + j\omega R C}$$

Consequently,

$$|\mathbf{Z}_{in}| = \sqrt{\frac{\left(R - \omega^2 R L C\right)^2 + \left(\omega L\right)^2}{1 + \left(\omega R C\right)^2}}$$

(b)



(c)

$$\omega = \frac{1}{\sqrt{LC}} \quad \Rightarrow \quad |\mathbf{Z}_{in}| = \frac{1}{\sqrt{\dfrac{C}{L}\left(1 + \dfrac{R^2 C}{L}\right)}}$$

P 13.4-11 The circuit shown in Figure P 13.4-11 shows an experimental setup that could be used to measure the parameters *k, Q*, and ω_0 of this series resonant circuit. These parameters can be determined from a magnitude frequency response plot for **Y** = **I**/**V**. It is more convenient to measure node voltages than currents, so the node voltages **V** and $\mathbf{V}_2$ have been measured. Express |**Y**| as a function of **V** and $\mathbf{V}_2$.

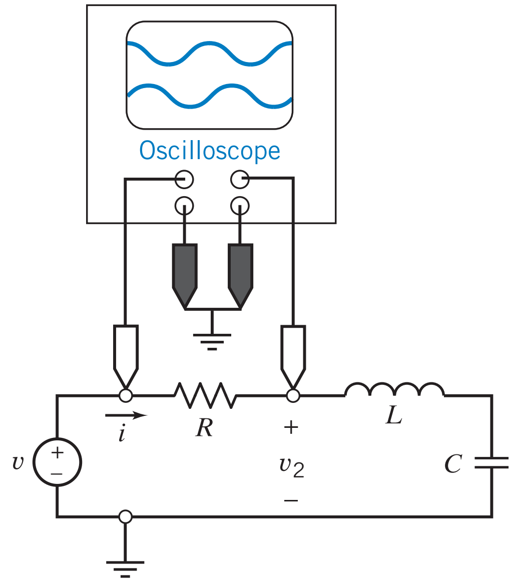


Figure P 13.4-11

Hint: Let $\mathbf{V} = A$ and $\mathbf{V}_2 = B\angle\theta$

Then $\mathbf{I} = \dfrac{(A - B\cos\theta) - jB\sin\theta}{R}$

Answer: $|\mathbf{Y}| = \dfrac{\sqrt{(A - B\cos\theta)^2 + (B\sin\theta)^2}}{AR}$

Solution: Let $\mathbf{V}(\omega) = A\angle 0$ and $\mathbf{V}_2(\omega) = B\angle\theta$. Then

$$\mathbf{Y}(\omega) = \frac{\mathbf{I}(\omega)}{\mathbf{V}(\omega)} = \frac{\dfrac{\mathbf{V}(\omega) - \mathbf{V}_2(\omega)}{R}}{\mathbf{V}(\omega)} = \frac{A - B\angle\theta}{A\,R} = \frac{A - B\cos\theta - j\,B\sin\theta}{A\,R}$$

$$|\mathbf{Y}(\omega)| = \frac{\sqrt{(A - B\cos\theta)^2 + (B\sin\theta)^2}}{A\,R}$$

Section 13-6: Plotting Bode Plots Using MATLAB

P 13.6-1 The input to the circuit shown in Figure P 13.6-1 is the voltage of the voltage source, v_s. The output of the circuit is the voltage, v_o. Use MATLAB to plot the gain and phase shift of this circuit as a function of frequency for frequencies in the range $1 < \omega < 1000$ rad/s.

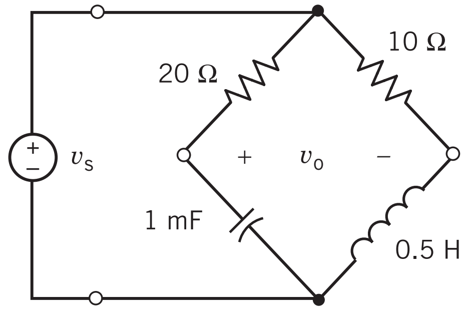
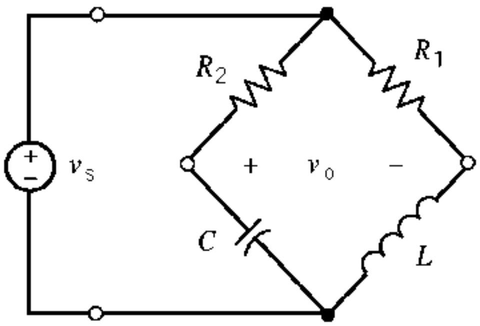


Figure P 13.6-1

Solution:



Using voltage division twice gives

$$\mathbf{V}_o(\omega) = \frac{\dfrac{1}{j\omega C}}{R_2 + \dfrac{1}{j\omega C}} \mathbf{V}_s(\omega) - \frac{j\omega L}{R_1 + j\omega L} \mathbf{V}_s(\omega)$$

so

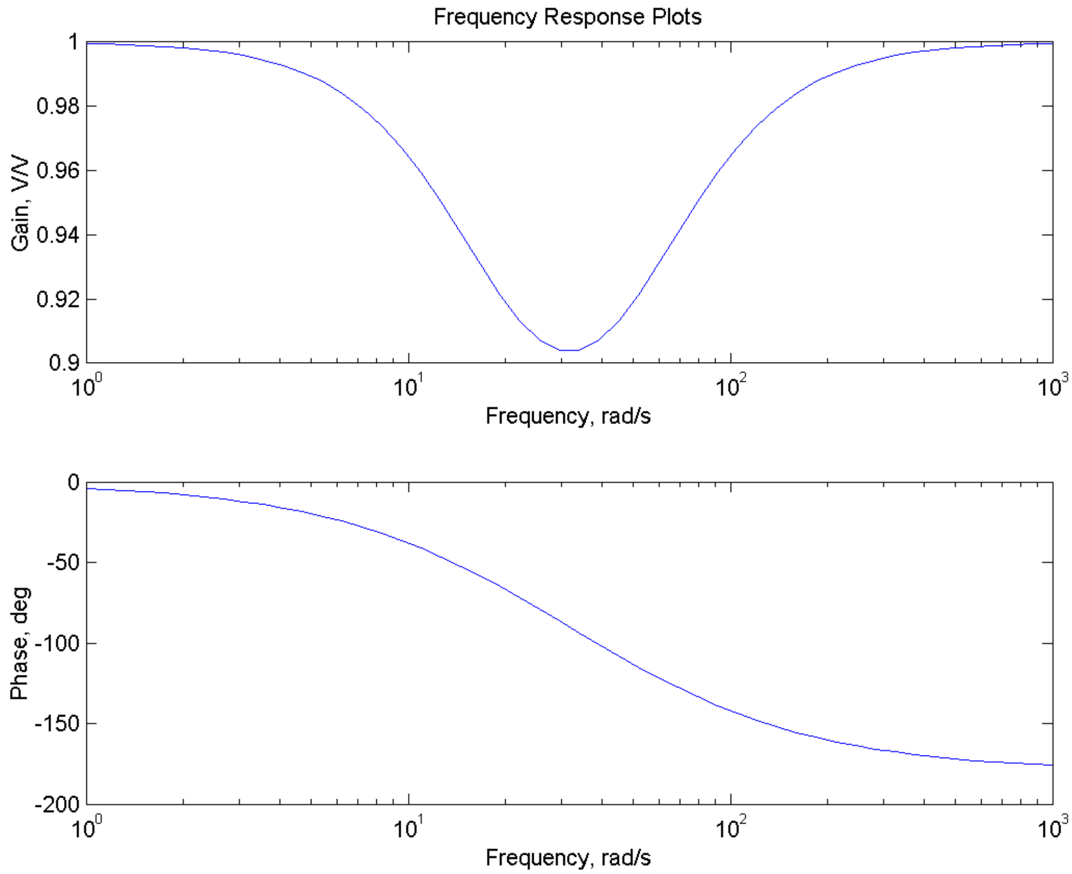
$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)} = \frac{1}{1 + j\omega C R_2} - \frac{j\omega L}{R_1 + j\omega L}$$

Modify the MATLAB script given in Section 13.7 of the text:

```
% P13_7_1.m - plot the gain and phase shift of a circuit

%------------------------------------------------------------
%     Create a list of logarithmically spaced frequencies.
%------------------------------------------------------------

wmin=1;       % starting frequency, rad/s
wmax=1000;    % ending frequency, rad/s

w = logspace(log10(wmin),log10(wmax));

%------------------------------------------------------------
%   Enter values of the parameters that describe the circuit.
%------------------------------------------------------------

R1 = 10;   % Ohms
R2 = 20;   % Ohms
C = 0.001; % Farads
```

```
L = 0.5;   % Henries

%------------------------------------------------------------------
% Calculate the value of the network function at each frequency.
%   Calculate the magnitude and angle of the network function.
%------------------------------------------------------------------

for k=1:length(w)
    H(k) = 1/(1+j*R2*C*w(k)) - j*L*w(k)/(R1+j*L*w(k));
    gain(k) = abs(H(k));
    phase(k) = angle(H(k))*180/pi;
end

%------------------------------------------------------------------
%                    Plot the frequency response.
%------------------------------------------------------------------

subplot(2,1,1), semilogx(w, gain)
xlabel('Frequency, rad/s'), ylabel('Gain, V/V')
title('Frequency Response Plots')
subplot(2,1,2), semilogx(w, phase)
xlabel('Frequency, rad/s'), ylabel('Phase, deg')
```

Here are the plots produced by MATLAB:



P 13.6-2 The input to the circuit shown in Figure P 13.6-2 is the voltage of the voltage source, v_s. The output of the circuit is the voltage, v_o. Use MATLAB to plot the gain and phase shift of this circuit as a function of frequency for frequencies in the range $1 < \omega < 1000$ rad/s.

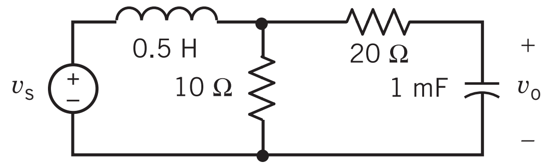
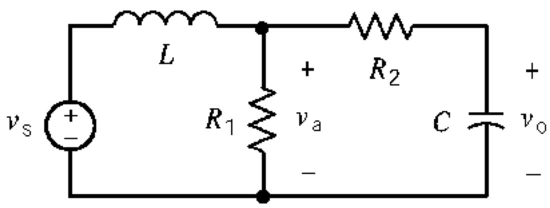


Figure P 13.6-2

Solution:



Let

$$\mathbf{Z}_s = R_2 + \frac{1}{j\omega C} \text{ and } \mathbf{Z}_p = \frac{R_1 \mathbf{Z}_s}{R_1 + \mathbf{Z}_s}$$

Using voltage division twice gives

$$\mathbf{V}_a(\omega) = \frac{\mathbf{Z}_p}{j\omega L + \mathbf{Z}_p} \mathbf{V}_s(\omega) \text{ and } \mathbf{V}_o(\omega) = \frac{\frac{1}{j\omega C}}{R_2 + \frac{1}{j\omega C}} \mathbf{V}_a(\omega)$$

so

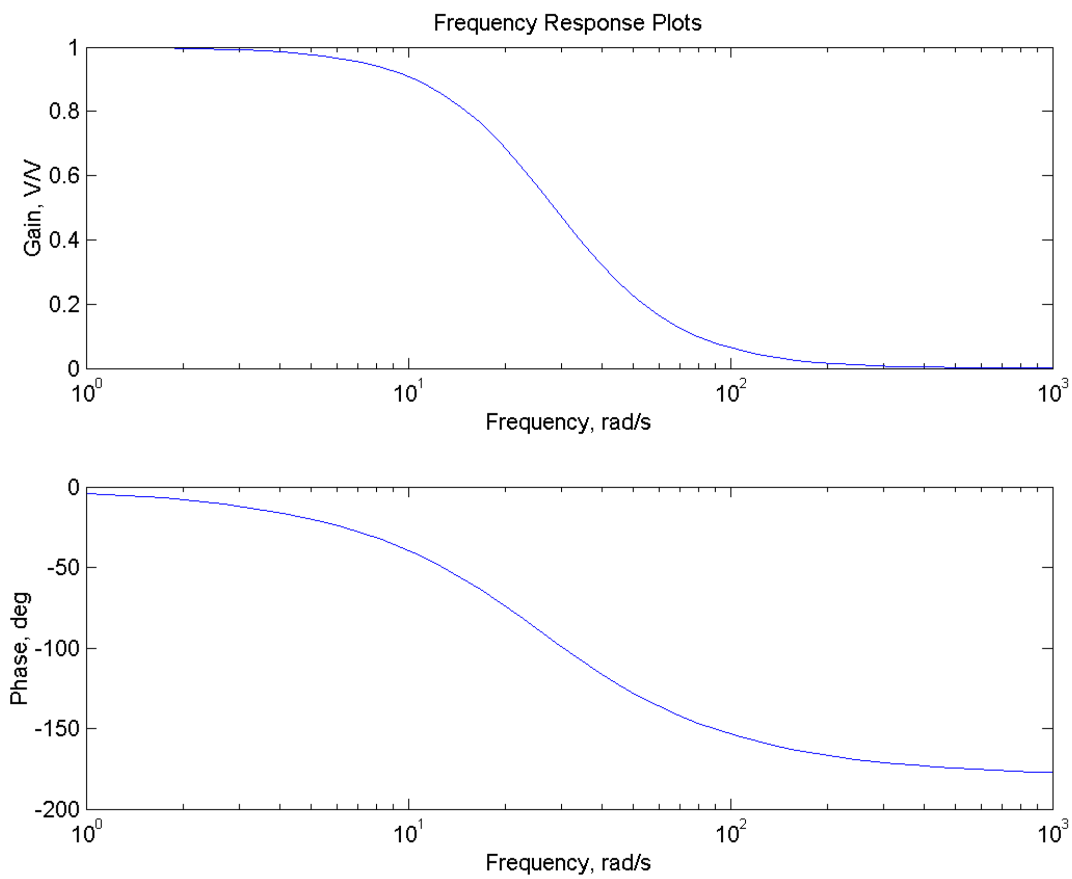
$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)} = \frac{\mathbf{Z}_p}{(j\omega L + \mathbf{Z}_p)(1 + j\omega C R_2)}$$

Modify the MATLAB script given in Section 13.7 of the text:

```
% P13_7_2.m - plot the gain and phase shift of a circuit
pi = 3.14159;
%----------------------------------------------------------------
%     Create a list of logarithmically spaced frequencies.
%----------------------------------------------------------------

wmin=1;       % starting frequency, rad/s
wmax=1000;    % ending frequency, rad/s

w = logspace(log10(wmin),log10(wmax));

%----------------------------------------------------------------
%   Enter values of the parameters that describe the circuit.
%----------------------------------------------------------------

R1 = 10;   % Ohms
R2 = 20;   % Ohms
C = 0.001; % Farads
L = 0.5;   % Henries
```

```
%----------------------------------------------------------------
% Calculate the value of the network function at each frequency.
%   Calculate the magnitude and angle of the network function.
%----------------------------------------------------------------

for k=1:length(w)
    Zs(k) = R2+1/(j*w(k)*C);
    Zp(k) = R1*Zs(k)/(R1+Zs(k));
    H(k) = Zp(k)/((j*w(k)*L+Zp(k))*(1+j*w(k)*C*R2));
    gain(k) = abs(H(k));
    phase(k) = angle(H(k))*180/pi;
end

%----------------------------------------------------------------
%                   Plot the frequency response.
%----------------------------------------------------------------

subplot(2,1,1), semilogx(w, gain)
xlabel('Frequency, rad/s'), ylabel('Gain, V/V')
title('Frequency Response Plots')
subplot(2,1,2), semilogx(w, phase)
xlabel('Frequency, rad/s'), ylabel('Phase, deg')
```

Here are the plots produced by MATLAB:



P 13.6-3 The input to the circuit shown in Figure P 13.6-3 is the voltage of the voltage source, v_s. The output of the circuit is the voltage, v_o. Use MATLAB to plot the gain and phase shift of this circuit as a function of frequency for frequencies in the range $1 < \omega < 1000$ rad/s.

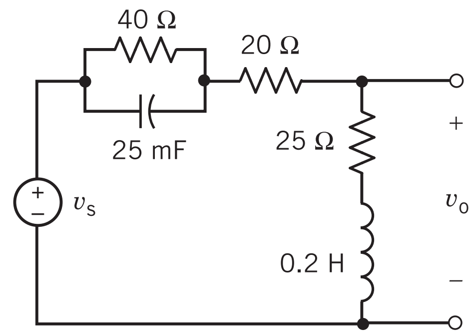
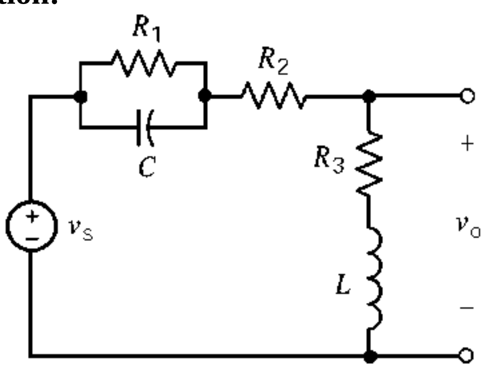


Figure P 13.6-3

Solution:



Let

$$\mathbf{Z}_1 = R_2 + \frac{R_1}{j\omega C R_1} \text{ and } \mathbf{Z}_2 = R_3 + j\omega L$$

Using voltage division gives

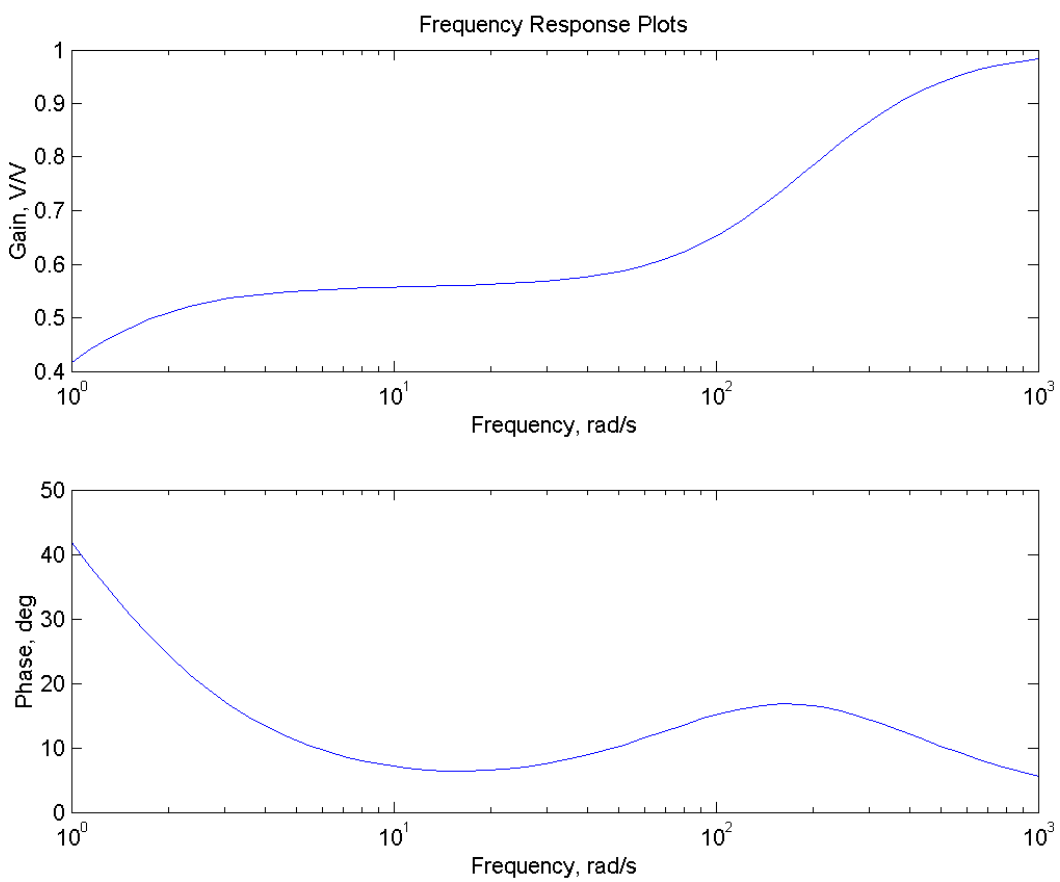
$$\mathbf{V}_a(\omega) = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} \mathbf{V}_s(\omega) \quad \Rightarrow \quad \mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)} = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}$$

Modify the MATLAB script given in Section 13.7 of the text:

```
% P13_7_3.m - plot the gain and phase shift of a circuit
pi = 3.14159;
%----------------------------------------------------------------
%     Create a list of logarithmically spaced frequencies.
%----------------------------------------------------------------

wmin=1;       % starting frequency, rad/s
wmax=1000;    % ending frequency, rad/s

w = logspace(log10(wmin),log10(wmax));

%----------------------------------------------------------------
%   Enter values of the parameters that describe the circuit.
%----------------------------------------------------------------

R1 = 40;   % Ohms
R2 = 20;   % Ohms
R3 = 25;   % Ohms
C = 0.025; % Farads
L = 0.2;   % Henries

%----------------------------------------------------------------
```

```
% Calculate the value of the network function at each frequency.
%   Calculate the magnitude and angle of the network function.
%----------------------------------------------------------------

for k=1:length(w)
    Z1(k) = R2+R1/(j*w(k)*C*R1);
    Z2(k) = R3+j*w(k)*L;
    H(k) = Z2(k)/(Z1(k)+Z2(k));
    gain(k) = abs(H(k));
    phase(k) = angle(H(k))*180/pi;
end

%----------------------------------------------------------------
%                   Plot the frequency response.
%----------------------------------------------------------------

subplot(2,1,1), semilogx(w, gain)
xlabel('Frequency, rad/s'), ylabel('Gain, V/V')
title('Frequency Response Plots')
subplot(2,1,2), semilogx(w, phase)
xlabel('Frequency, rad/s'), ylabel('Phase, deg')
```

Here are the plots produced by MATLAB:



Section 13.8 How Can We Check…?

P 13.8-1 Circuit analysis contained in a lab report indicates that the network function of a circuit is

$$\mathbf{H}(\omega) = \frac{1 + j\dfrac{\omega}{630}}{10\left(1 + j\dfrac{\omega}{6300}\right)}$$

This lab report contains the following frequency response data from measurements made on the circuit. Do these data seem reasonable?

ω, rad/s	200	400	795	1585	3162
$\lvert\mathbf{H}(\omega)\rvert$	0.105	0.12	0.16	0.26	0.460
ω, rad/s	6310	12,600	25,100	50,000	100,000
$\lvert\mathbf{H}(\omega)\rvert$	0.71	1.0	1.0	1.0	1.0

P13.8-1

When $\omega < 630$ rad/s, $\mathbf{H}(\omega) \cong 0.1$, which agrees with the tabulated values of $|\mathbf{H}(\omega)|$ corresponding to $\omega = 200$ and 400 rad/s.

When $\omega > 6300$ rad/s, $\mathbf{H}(\omega) \cong 1.0$, which agrees with the tabulated values of $|\mathbf{H}(\omega)|$ corresponding to $\omega = 12600$, 25000, 50000 and 100000 rad/s.

At $\omega = 6300$ rad/s, we expect $|\mathbf{H}(\omega)| = -3$ dB = 0.707. This agrees with the tabulated value of $|\mathbf{H}(\omega)|$ corresponding to $\omega = 6310$ rad/s.

At $\omega = 630$ rad/s, we expect $|\mathbf{H}(\omega)| = -20$ dB = 0.14. This agrees with the tabulated values of $|\mathbf{H}(\omega)|$ corresponding to $\omega = 400$ and 795 rad/s.

This data does seem reasonable.

P 13.8-2 A parallel resonant circuit (see Figure 13.4-2) has $Q = 70$ and a resonant frequency $\omega_0 = 10{,}000$ rad/s. A report states that the bandwidth of this circuit is 71.43 rad/s. Verify this result.

Solution:

$$BW = \frac{\omega_0}{Q} = \frac{10{,}000}{70} = 143 \neq 71.4 \text{ rad/s}$$. Consequently, this report is not correct.

P 13.8-3 A series resonant circuit (see Figure 13.4-4) has $L = 1$ mH, $C = 10\ \mu$F, and $R = 0.5\ \Omega$. A software program report states that the resonant frequency is $f_0 = 1.59$ kHz and the bandwidth is $BW = 79.6$ Hz. Are these results correct?

Solution:

$$\omega_0 = \frac{1}{\sqrt{LC}} = 10 \text{ k rad/s} = 1.59 \text{ kHz}, \quad Q = \frac{1}{R}\sqrt{\frac{L}{C}} = 20 \text{ and } BW = \frac{R}{L} = 500 \text{ rad/s} = 79.6 \text{ Hz}$$

The reported results are correct.

P 13.8-4 An old lab report contains the approximate Bode plot shown in Figure P 13.8-4 and concludes that the network function is

$$\mathbf{H}(\omega) = \frac{40\left(1 + j\dfrac{\omega}{200}\right)}{\left(1 + j\dfrac{\omega}{800}\right)}$$

Do you agree?

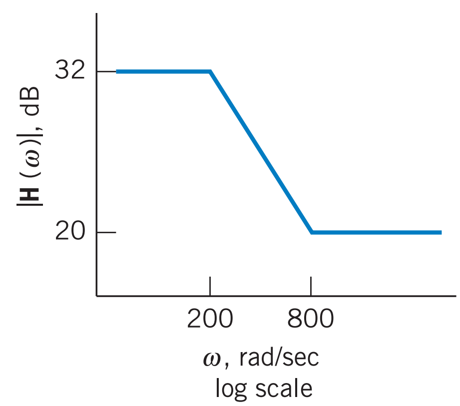


Figure P 13.8-4

Solution:

The network function indicates a zero at 200 rad/s and a pole at 800 rad/s. In contrast, the Bode plot indicates a pole at 200 rad/s and a zero at 800 rad/s. Consequently, the Bode plot and network function don't correspond to each other.

PSpice Problems

SP 13-1 The input to the circuit shown in Figure SP 13-1 is the voltage of the voltage source, $v_i(t)$. The output is the voltage, $v_o(t)$, across the parallel connection of the capacitor and 1-kΩ resistor. The network function that represents this circuit is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{k}{1 + j\dfrac{\omega}{p}}$$

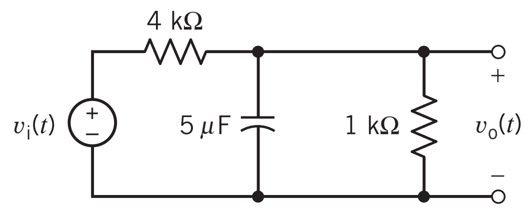
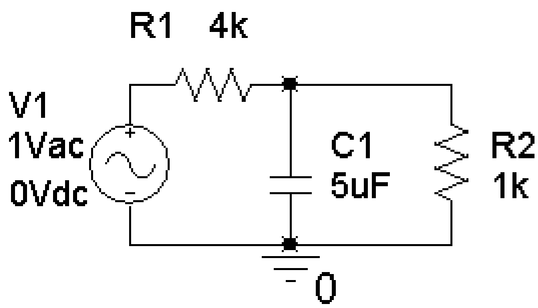


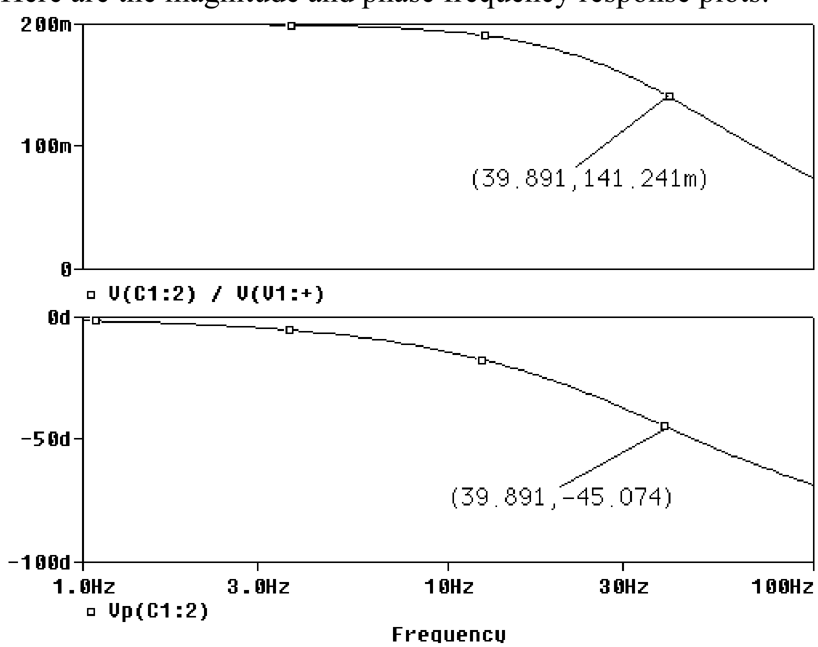
Figure SP 13-1

Use PSpice to plot the frequency response of this circuit. Determine the values of the pole, p, and of the dc gain, k.

Solution:



Here are the magnitude and phase frequency response plots:



From the magnitude plot, the low frequency gain is $k = 200\text{m} = 0.2$.

From the phase plot, the angle is -45° at $p = 2\pi(39.891) = 251$ rad/s .

SP 13-2 The input to the circuit shown in Figure SP 13-2 is the voltage of the voltage source, $v_i(t)$. The output is the voltage, $v_o(t)$, across the series connection of the inductor and 60-Ω resistor. The network function that represents this circuit is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = k = \frac{1 + j\dfrac{\omega}{z}}{1 + j\dfrac{\omega}{p}}$$

Use PSpice to plot the frequency response of this circuit. Determine the values of the pole, p, of the zero, z, and of the dc gain, k.

Answer: $p = 20$ rad/s, $z = 12$ rad/s, and $k = 0.6$ V/V

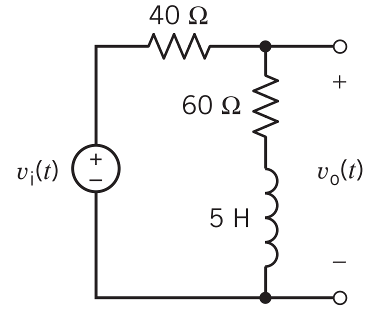
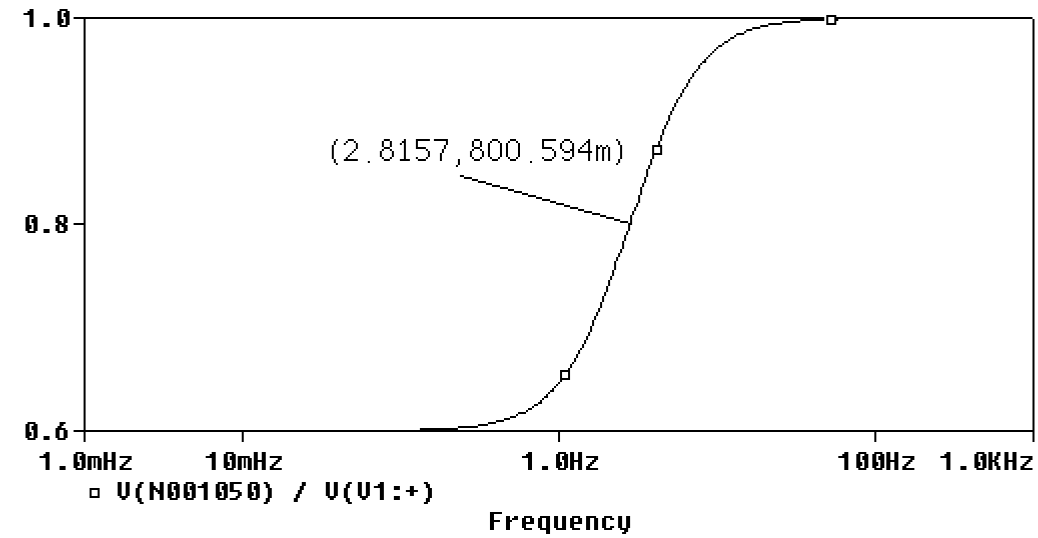


Figure SP 13-2

Solution: Here is the magnitude frequency response plot:



The low frequency gain is $0.6 = \lim_{\omega \to 0} \mathbf{H}(\omega) = k \quad \Rightarrow \quad k = 0.6$.

The high frequency gain is $1 = \lim_{\omega \to \infty} \mathbf{H}(\omega) = k\dfrac{p}{z} \quad \Rightarrow \quad z = (0.6)\,p$

At $\omega = 2\pi(2.8157) = 17.69$ rad/s,

$$0.8 = 0.6\sqrt{\frac{1+\left(\dfrac{17.69}{0.6p}\right)^2}{1+\left(\dfrac{17.69}{p}\right)^2}} \quad \Rightarrow \quad \frac{16}{9} = \frac{p^2+869}{p^2+313} \Rightarrow \quad \frac{16}{9}\left(p^2+313\right) = p^2+869 \Rightarrow \quad (0.77778)\,p^2 = 312.56$$

$$\Rightarrow \quad p = 20 \text{ rad/s and } z = 12 \text{ rad/s}$$

SP 13-3 The input to the circuit shown in Figure SP 13-3 is the voltage of the voltage source, $v_i(t)$. The output is the voltage, $v_o(t)$, across 30-kΩ resistor. The network function that represents this circuit is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{k}{1 + j\dfrac{\omega}{p}}$$

Use PSpice to plot the frequency response of this circuit. Determine the values of the pole, p, and of the dc gain, k.

Answer: p = 100 rad/s and k = 4 V/V

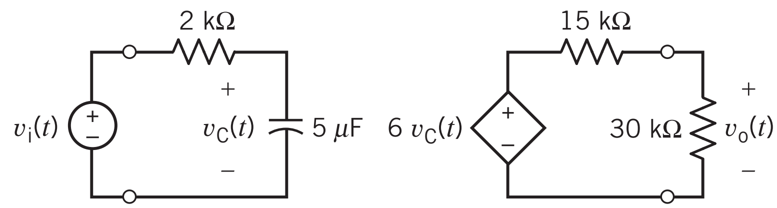
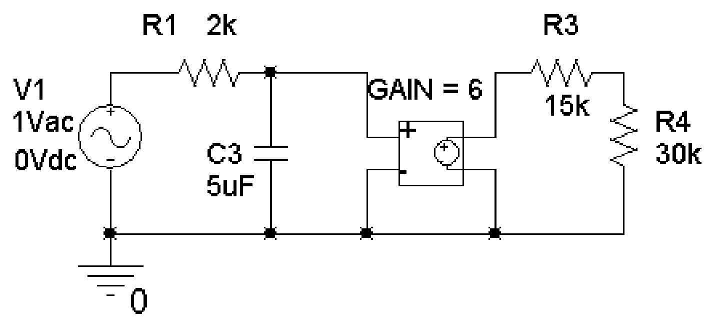
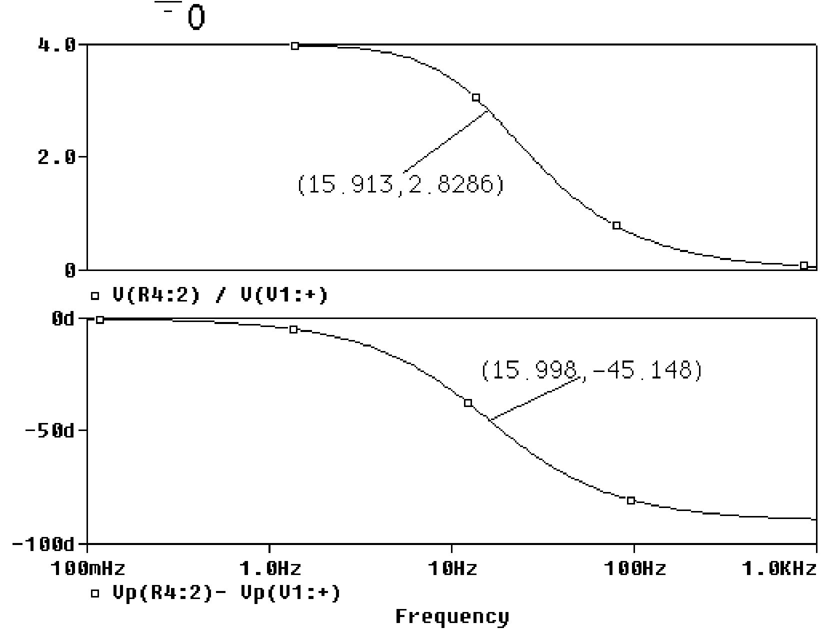


Figure SP 13-3

Solution:





From the magnitude plot, the low frequency gain is k = 4.0. Also, the gain is 4/sqrt(2) = 2.828 at 15.914 hertz.

From the phase plot, the angle is -45° at $p = 2\pi(15.998) = 100.5$ rad/s.

SP 13-4 The input to the circuit shown in Figure SP 13-4 is the voltage of the voltage source, $v_i(t)$. The output is the voltage, $v_o(t)$, across 20-kΩ resistor. The network function that represents this circuit is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{k}{1 + j\dfrac{\omega}{p}}$$

Use PSpice to plot the frequency response of this circuit. Determine the values of the pole, p, and of the dc gain, k.

Answer: p = 10 rad/s and k = 5 V/V

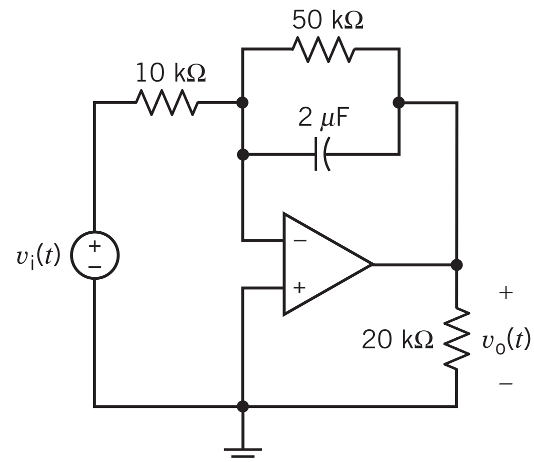
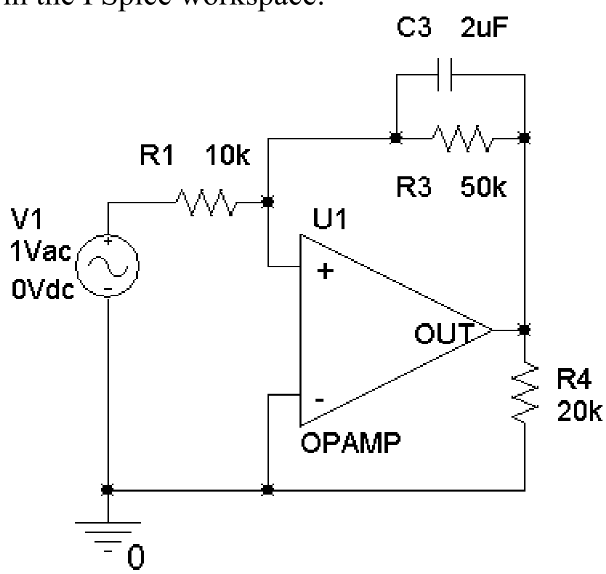


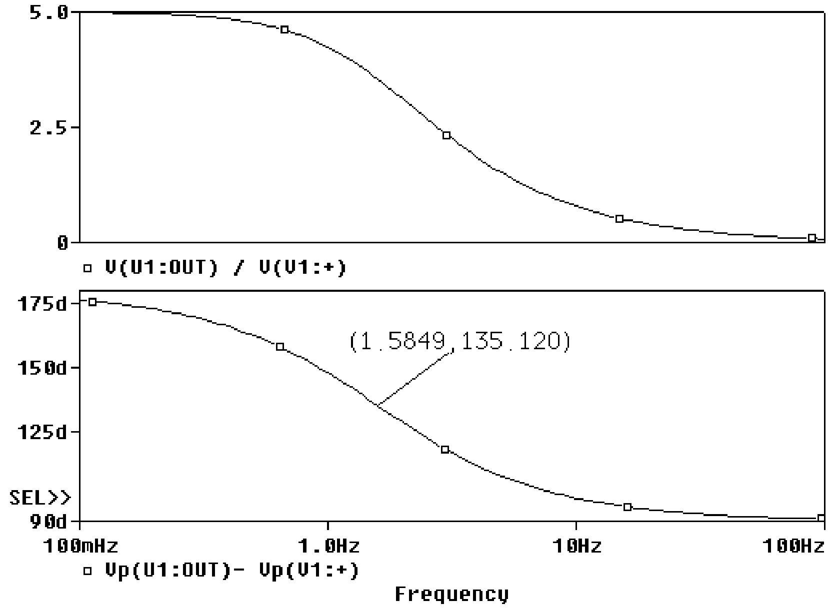
Figure SP 13-4

Solution:

Here's the circuit drawn in the PSpice workspace:



Here are the frequency response plots:



From the magnitude plot, the low frequency gain is $k = 5.0$.
From the phase plot, the angle is 180°-45°=135° at $p = 2\pi\left(1.5849\right) = 9.958$ rad/s.

SP 13-5 Figure SP 13-5 shows a circuit and a frequency response. The frequency response plots were made using PSpice and Probe. V(R3:2) and Vp(R3:2) denote the magnitude and angle of the phasor corresponding to $v_o(t)$. V(V1:+) and Vp(V1:+) denote the magnitude and angle of the phasor corresponding to $v_i(t)$. Hence V(R3:2)/V(V1:+) is the gain of the circuit and Vp(R3:2) – Vp(V1:+) is the phase shift of the circuit.

Determine values for R and C required to cause the circuit correspond to the frequency response.

Hint: PSpice and Probe use m for milli or 10^{-3}. Hence, the label (159.513, 892.827m) indicates that the gain of the circuit is $892.827*10^{-3} = 0.892827$ at a frequency of 159.513 Hz ≈ 1000rad/sec.

Answer: $R = 5\text{k}\Omega$ and $C = 0.2\ \mu\text{F}$

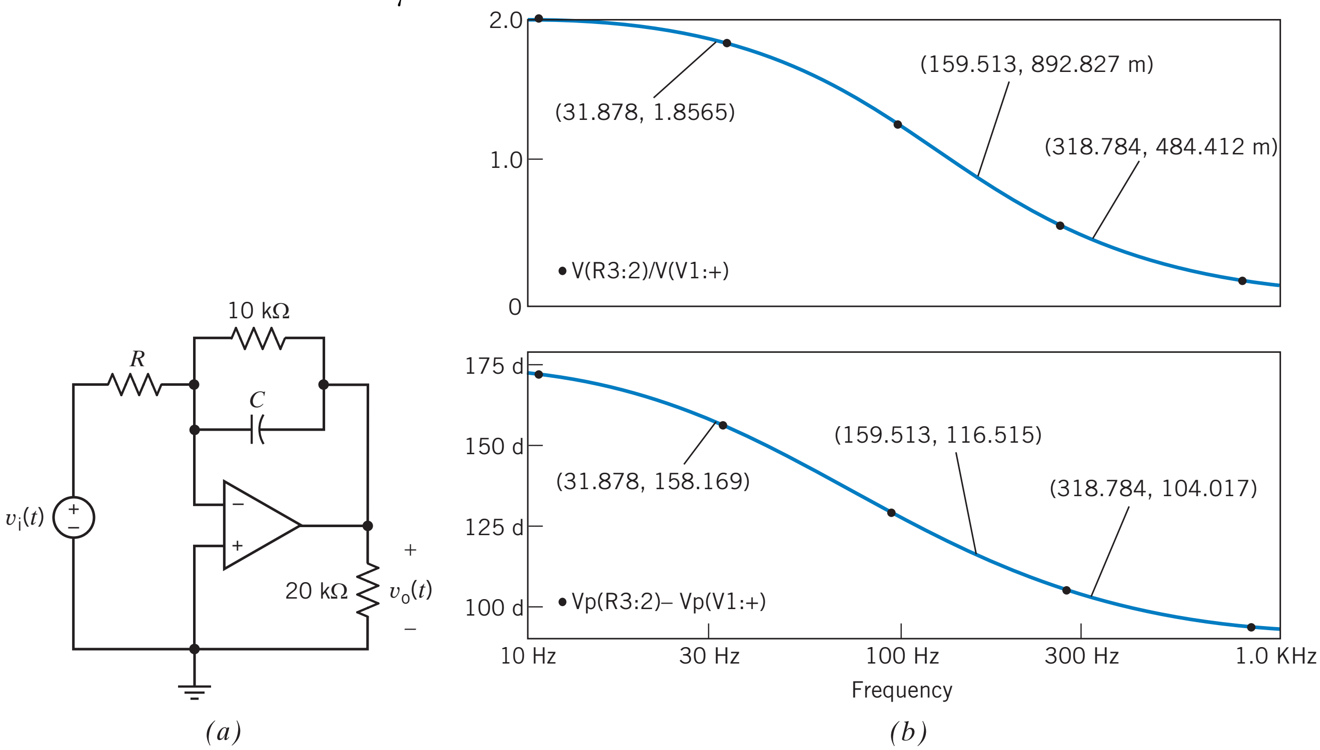


FIGURE SP 13-5 (a) A circuit and (b) the corresponding frequency response.

Solution: From the circuit

$$\mathbf{H}(\omega) = -\frac{\frac{10^4}{R}}{1 + j\omega C\,10^4} = \frac{\frac{10^4}{R}}{\sqrt{1+\left(\omega C\,10^4\right)^2}} \angle -\tan^{-1}\left(\omega C\,10^4\right)$$

From the plot, at ω = 200 rad/sec = 31.83 Hertz **H**(ω) is

$$1.8565\angle 158° = \frac{\frac{10^4}{R}}{\sqrt{1+\left(\omega C\,10^4\right)^2}} \angle -\tan^{-1}\left(\omega C\,10^4\right)$$

Equating phase shifts gives

$$\omega\, C\, 10^4 = 10^3 \frac{C\, R\, 10^4}{R + 10^4} = \tan(22°) = 0.404 \quad \Rightarrow \quad C = 0.2\ \mu\text{F}$$

Equating gains gives

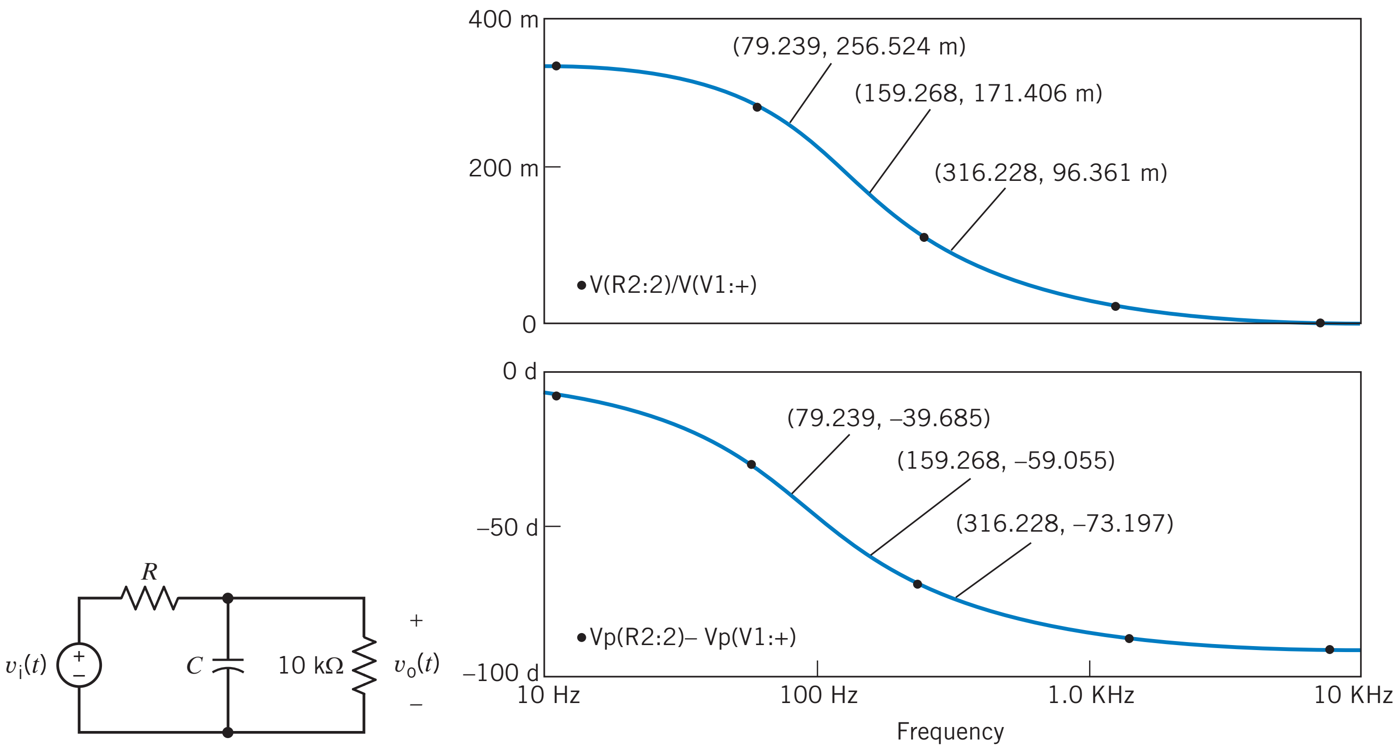
$$1.8565 = \frac{\frac{10^4}{R}}{\sqrt{1 + \left(\omega\, C\, 10^4\right)^2}} = \frac{\frac{10^4}{R}}{\sqrt{1 + \left(0.404\right)^2}} \quad \Rightarrow \quad R = 5\ \text{k}\Omega$$

SP 13-6 Figure SP 13-6 shows a circuit and a frequency response. The frequency response plots were made using PSpice and Probe. V(R2:2) and Vp(R2:2) denote the magnitude and angle of the phasor corresponding to $v_o(t)$. V(V1:+) and Vp(V1:+) denote the magnitude and angle of the phasor corresponding to $v_i(t)$. Hence V(R2:2)/V(V1:+) is the gain of the circuit, and Vp(R2:2) – Vp(V1:+) is the phase shift of the circuit.

Determine values for R and C required to make the circuit correspond to the frequency response.

Hint: PSpice and Probe use m for milli or 10^{-3}. Hence, the label (159.268, 171.408m) indicates that the gain of the circuit is $171.408 * 10^{-3} = 0.171408$ at a frequency of 159.268 Hz ≈ 1000 rad/sec.

Answer: $R = 20$ kΩ and $C = 0.25$ μF



(*a*) (*b*)

FIGURE SP 13-6 (*a*) A circuit and (*b*) the corresponding frequency response.

Solution: From the circuit

$$\mathbf{H}(\omega) = \frac{\dfrac{10^4}{1 + j\omega C R_2}}{\dfrac{10^4}{1 + j\omega C 10^4} + R} = \frac{\dfrac{10^4}{R + 10^4}}{1 + j\omega \dfrac{C R 10^4}{R + 10^4}} = \frac{\dfrac{10^4}{R + 10^4}}{\sqrt{1 + \left(\omega \dfrac{C R 10^4}{R + 10^4}\right)^2}} \angle - \tan^{-1}\left(\omega \frac{C R 10^4}{R + 10^4}\right)$$

From the plot, at ω = 1000 rad/sec = 159.1 Hertz **H**(ω) is

$$0.171408\angle -59^\circ = \frac{\dfrac{10^4}{R+10^4}}{\sqrt{1+\left(\omega \dfrac{C\,R\,10^4}{R+10^4}\right)^2}} \angle -\tan^{-1}\left(\omega \frac{C\,R\,10^4}{R+10^4}\right)$$

Equating phase shifts gives

$$\omega \frac{C\,R\,10^4}{R+10^4} = 10^3 \frac{C\,R\,10^4}{R+10^4} = \tan(59^\circ) = 1.665$$

Equating gains gives

$$0.171408 = \frac{\dfrac{10^4}{R+10^4}}{\sqrt{1+\left(\omega \dfrac{C\,R\,10^4}{R+10^4}\right)^2}} = \frac{\dfrac{10^4}{R+10^4}}{\sqrt{1+(1.665)^2}} \quad \Rightarrow \quad R = 20\text{ k}\Omega$$

Substitute this value of *R* into the equation for phase shift to get:

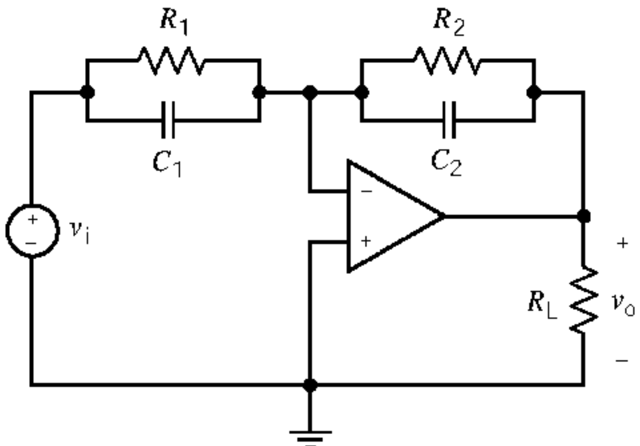
$$1.665 = 10^3 \frac{C\,R\,10^4}{R+10^4} = 10^3 \frac{C\left(20\times 10^3\right)10^4}{\left(20\times 10^3\right)+10^4} \quad \Rightarrow \quad C = 0.25\ \mu\text{F}$$

Design Problems

DP 13-1 Design a circuit that has a low-frequency gain of 2, a high-frequency gain of 5, and makes the transition of $H = 2$ to $H = 5$ between the frequencies of 1 kHz and 10 kHz.

Solution:

Pick the appropriate circuit from Table 13.4-2.



$$\mathbf{H}(\omega) = -k \frac{1 + j\frac{\omega}{z}}{1 + j\frac{\omega}{p}}$$

where

$$k = \frac{R_2}{R_1}$$

$$z = \frac{1}{C_1 R_1}$$

$$p = \frac{1}{C_2 R_2}$$

We require

$$2\pi \times 1000 < z = \frac{1}{C_1 R_1},\ 2\pi \times 10000 > p = \frac{1}{C_2 R_2},\ 2 = k = \frac{R_2}{R_1} \text{ and } 5 = k\frac{p}{z} = \frac{C_1}{C_2}$$

Try $z = 2\pi \times 2000$. Pick $C_1 = 0.05\ \mu\text{F}$. Then

$$R_1 = \frac{1}{C_1 z} = 1.592\ \text{k}\Omega,\ R_2 = 2R_1 = 3.183\ \text{k}\Omega \text{ and } C_2 = \frac{C_1}{k\frac{p}{z}} = \frac{C_1}{2} = 0.01\ \mu\text{F}$$

Check: $p = \dfrac{1}{C_2 R_2} = 31.42\ \text{k rad/s} < 2\pi \cdot 10{,}000\ \text{ rad/s}.$

DP 13-2 Determine L and C for the circuit of Figure DP 13-2 in order to obtain a low-pass filter with a gain of –3 dB at 100 kHz.

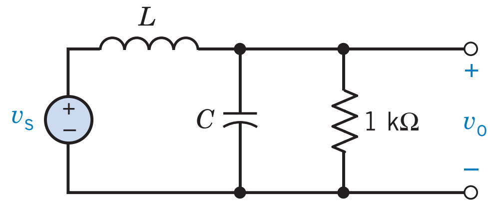
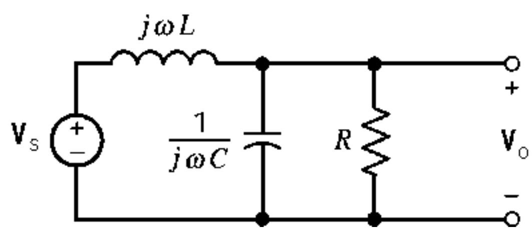


Figure DP 13-2

Solution:



$$\mathbf{H}(\omega)=\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)}=\frac{\frac{1}{j\omega C}\|R}{j\omega L+\left(\frac{1}{j\omega C}\|R\right)}=\frac{\frac{R}{1+j\omega CR}}{j\omega L+\frac{R}{1+j\omega CR}}=\frac{\frac{1}{LC}}{-\omega^2+j\omega\frac{1}{RC}+\frac{1}{LC}}$$

Pick $\frac{1}{\sqrt{LC}}=\omega_0=2\pi(100\cdot10^3)$ rad/s . When $\omega=\omega_0$

$$\mathbf{H}_0(\omega)\;=\;\frac{\frac{1}{LC}}{-\frac{1}{LC}+j\frac{1}{\sqrt{LC}}\frac{1}{RC}+\frac{1}{LC}}$$

So $\left|\mathbf{H}(\omega_0)\right|\;=\;R\sqrt{\frac{C}{L}}$. We require

$$-3\text{ dB}\;=\;0.707\;=\;\left|\mathbf{H}(\omega_0)\right|\;=\;R\sqrt{\frac{C}{L}}\;=\;1000\sqrt{\frac{C}{L}}$$

Finally

$$\left.\begin{aligned}&\frac{1}{\sqrt{LC}}=2\pi(100\cdot10^3)\\&0.707=1000\sqrt{\frac{C}{L}}\end{aligned}\right\}\Rightarrow\begin{aligned}&C=1.13\text{ nF}\\&L=2.26\text{ mH}\end{aligned}$$

DP 13-3 British Rail has constructed an instrumented railcar that can be pulled over its tracks at speeds up to 180 km/hr and will measure the track-grade geometry. Using such a railcar, British Rail can monitor and track gradual degradation of the rail grade, especially the banking of curves, and permit preventive maintenance to be scheduled as needed well in advance of track-grade failure.

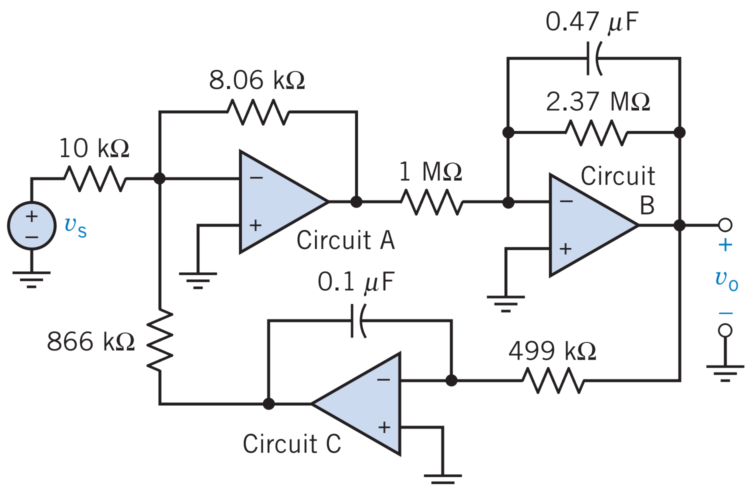


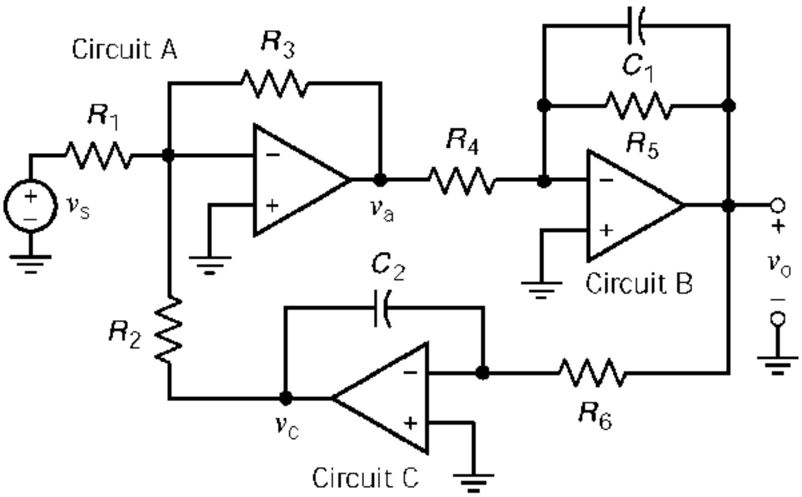
Figure DP 13-3

The instrumented railcar has numerous sensors, such as angular-rate sensors (devices that output a signal proportional to rate of rotation) and accelerometers (devices that output a signal proportional to acceleration), whose signals are filtered and combined in a fashion to create a composite sensor called a compensated accelerometer (Lewis, 1988).

A component of this composite sensor signal is obtained by integrating and high-pass filtering an accelerometer signal. A first-order low-pass filter will approximate an integrator at frequencies well above the break frequency. This can be seen by computing the phase shift of the filter-transfer function at various frequencies. At sufficiently high frequencies, the phase shift will approach 90°, the phase characteristic of an integrator.

A circuit has been proposed to filter the accelerometer signal, as shown in Figure DP 13-3. The circuit is comprised of three sections, labeled A, B, and C. For each section, find an expression for and name the function performed by that section. Then find an expression for the gain function of the entire circuit, $\mathbf{V}_o/\mathbf{V}_s$. For the component values, evaluate the magnitude and phase of the circuit response at 0.01, 0.02, 0.05, 0.1, 0.2, 0.5, 1.0, 2.0, 5.0, and 10.0 Hz. Draw a Bode diagram. At what frequency is the phase response approximately equal to 0°? What is the significance of this frequency?

Solution:



$R_1 = 10\ k\Omega$
$R_2 = 866\ k\Omega$
$R_3 = 8.06\ k\Omega$
$R_4 = 1\ M\Omega$
$R_5 = 2.37\ M\Omega$
$R_6 = 499\ k\Omega$
$C_1 = 0.47\ \mu F$
$C_2 = 0.1\ \mu F$

Circuit A $$\mathbf{V}_a = -\frac{R_3}{R_2}\mathbf{V}_c - \frac{R_3}{R_1}\mathbf{V}_s = -\mathbf{H}_1\mathbf{V}_c - \mathbf{H}_2\mathbf{V}_s$$

Circuit B $$\mathbf{V}_o = -\frac{\frac{R_5}{R_4}}{1 + j\omega C_1 R_5}\mathbf{V}_a = -\mathbf{H}_3\mathbf{V}_a$$

Circuit C $$\mathbf{V}_c = -\frac{1}{j\omega C_2 R_6}\mathbf{V}_o = -\mathbf{H}_4\mathbf{V}_o$$

Then

$$\mathbf{V}_c = \mathbf{H}_3\mathbf{H}_4\mathbf{V}_a$$

$$\mathbf{V}_a = -\mathbf{H}_2\mathbf{V}_s - \mathbf{H}_1\mathbf{H}_3\mathbf{H}_4\mathbf{V}_a \quad \Rightarrow \quad \mathbf{V}_a = \frac{-\mathbf{H}_2}{1 + \mathbf{H}_1\mathbf{H}_3\mathbf{H}_4}\mathbf{V}_s$$

$$\mathbf{V}_o = -\mathbf{H}_3\mathbf{V}_a = \frac{\mathbf{H}_2\mathbf{H}_3}{1 + \mathbf{H}_1\mathbf{H}_3\mathbf{H}_4}\mathbf{V}_s$$

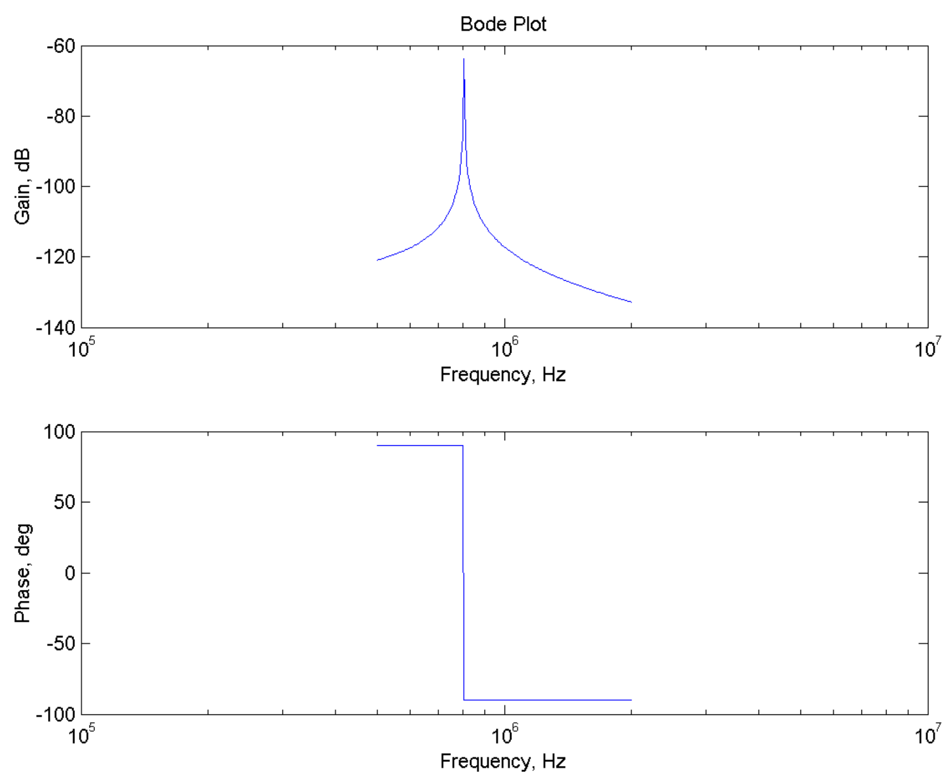
After some algebra

$$\mathbf{V}_o = \frac{j\omega\frac{R_3}{R_1 R_4 C_1}}{\frac{R_3}{R_2 R_4 R_6 C_1 C_2} - \omega^2 + j\frac{\omega}{R_5 C_1}}\mathbf{V}_s$$

This MATLAB program plots the Bode plot:

```
R1=10;        % units: kOhms and mF so RC has units of sec
R2=866;
R3=8.060;
R4=1000;
```

```
R5=2370;
R6=449;
C1=0.00047;
C2=0.0001;

pi=3.14159;
fmin=5*10^5;
fmax=2*10^6;
f=logspace(log10(fmin),log10(fmax),200);
w=2*pi*f;

b1=R3/R1/R4/C1;
a0=R3/R2/R4/R6/C1/C2;
a1=R5/C1;

for k=1:length(w)
    H(k)=(j*w(k)*b1)/(a0-w(k)*w(k)+j+w(k)*a1);
    gain(k)=abs(H(k));
    phase(k)=angle(H(k));
end

subplot(2,1,1), semilogx(f, 20*log10(gain))
xlabel('Frequency, Hz'), ylabel('Gain, dB')
title('Bode Plot')
subplot(2,1,2), semilogx(f, phase*180/pi)
xlabel('Frequency, Hz'), ylabel('Phase, deg')
```



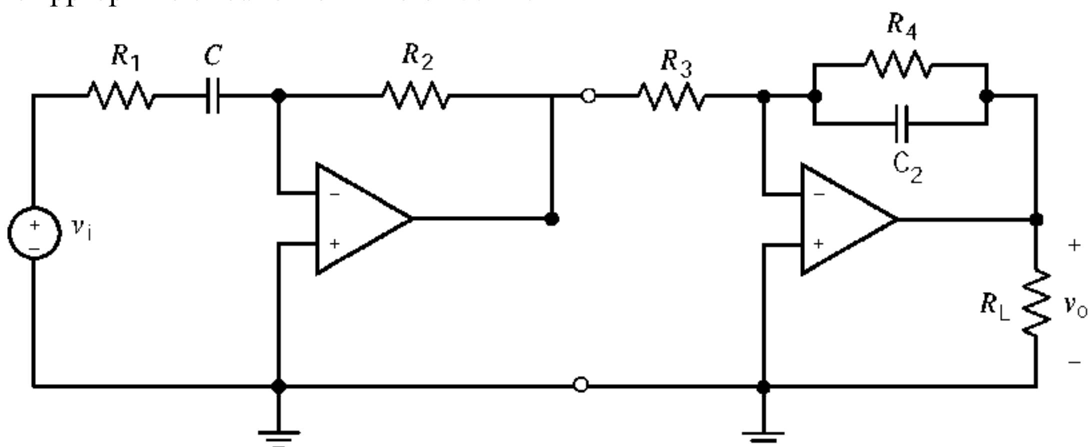
DP 13-4 Design a circuit that has the network function

$$\mathbf{H}(\omega)=\frac{j\omega}{\left(1+j\dfrac{\omega}{200}\right)\left(1+j\dfrac{\omega}{500}\right)}$$

Hint: Use two circuits from Table 13.4-1. Connect the circuits in cascade. That means that the output of one circuit is used as the input to the next circuit. $\mathbf{H}(\omega)$ will be the product of the network functions of the two circuits from Table 13.3-2.

Solution:

Pick the appropriate circuits from Table 13.4-2.



$$\mathbf{H}_1(\omega)=-k_1\frac{j\omega}{1+j\dfrac{\omega}{p_1}}$$

where

$$k_1=R_2C$$

$$p_1=\frac{1}{CR_1}$$

$$\mathbf{H}_2(\omega)=\frac{k_2}{1+j\dfrac{\omega}{p_2}}$$

where

$$k_2=\frac{R_2}{R_1}$$

$$p_2=\frac{1}{CR_2}$$

We require

$$10=-k_1k_2=R_2C_1\frac{R_4}{R_3},\;\; 200\;=\;p_1=\frac{1}{R_1C_1}\;\;\text{and}\;\;500\;=\;p_2=\frac{1}{C_2R_4}$$

Pick $C_1=1\ \mu\text{F}$. Then $R_1=\dfrac{1}{p_1C_1}=\ 5\ \text{k}\Omega$. Pick $C_2=0.1\ \mu\text{F}$. Then $R_4=\dfrac{1}{p_2C_2}=20\ \text{k}\Omega$.

Next $$10=\frac{R_2}{R_3}(10^{-6})(20\cdot 10^{3})\;\Rightarrow\;\frac{R_2}{R_3}=500$$

Let $R_2=500\ \text{k}\Omega$ and $R_3=1\ \text{k}\Omega$.

DP 13-5 Strain-sensing instruments can be used to measure orientation and magnitude of strains running in more than one direction. The search for a way to predict earthquakes focuses on identifying precursors, or changes, in the ground that reliably warn of an impending event. Because so few earthquakes have occurred precisely at instrumented locations, it has been a slow and frustrating quest. Laboratory studies show that before rock actually ruptures—precipitating an earthquake—its rate of internal strain increases. The material starts to fail before it actually breaks. This prelude to outright fracture is called "tertiary creep" (Brown, 1989).

The frequency of strain signals varies from 0.1 to 100 rad/s. A circuit called a band-pass filter is used to pass these frequencies. The network function of the band-pass filter is

$$\mathbf{H}(\omega) = \frac{Kj\omega}{\left(1 + j\dfrac{\omega}{\omega_1}\right)\left(1 + j\dfrac{\omega}{\omega_2}\right)}$$

Specify ω_1, ω_2, and K so that the following are the case:

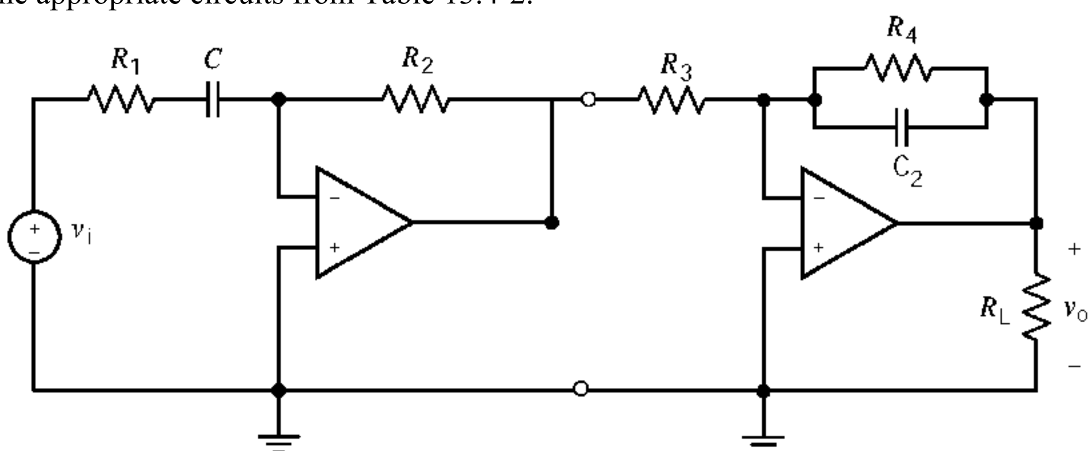
1. The gain is at least 17 dB over the range 0.1 to 100 rad/s.

2. The gain is less than 17 dB outside the range 0.1 to 100 rad/s.

3. The maximum gain is 20 dB.

Solution:

Pick the appropriate circuits from Table 13.4-2.



$$\mathbf{H}_1(\omega) = -k_1 \frac{j\omega}{1 + j\dfrac{\omega}{p_1}} \qquad \mathbf{H}_2(\omega) = \frac{k_2}{1 + j\dfrac{\omega}{p_2}}$$

where $k_1 = R_2 C$, $p_1 = \dfrac{1}{CR_1}$

where $k_2 = \dfrac{R_2}{R_1}$, $p_2 = \dfrac{1}{CR_2}$

We require

$$20\text{ dB} = 10 = -k_1 k_2 = R_2 C_1 \frac{R_4}{R_3},\ \ 0.1\ =\ p_1 = \frac{1}{R_1 C_1}\ \ \text{and}\ \ 100\ =\ p_2 = \frac{1}{C_2 R_4}$$

Pick $C_1 = 20\ \mu\text{F}$. Then $R_1 = \dfrac{1}{p_1 C_1} =$ 500 kΩ. Pick $C_2 = 1\ \mu\text{F}$. Then $R_4 = \dfrac{1}{p_2 C_2} = 10\ \text{k}\Omega$.

Next

$$10 = \frac{R_2}{R_3}(20 \cdot 10^{-6})(10 \cdot 10^{3})\ \Rightarrow\ \frac{R_2}{R_3} = 50$$

Let $R_2 = 200\ \text{k}\Omega$ and $R_3 = 4\ \text{k}\Omega$.

DP 13-6 Is it possible to design the circuit shown in Figure DP 13-6 to have a phase shift of –45°and a gain of 2 V/V both at a frequency of 1000 radians/second using a 0.1 microfarad capacitor and resistors from the range 1 k ohm to 200 k ohm?

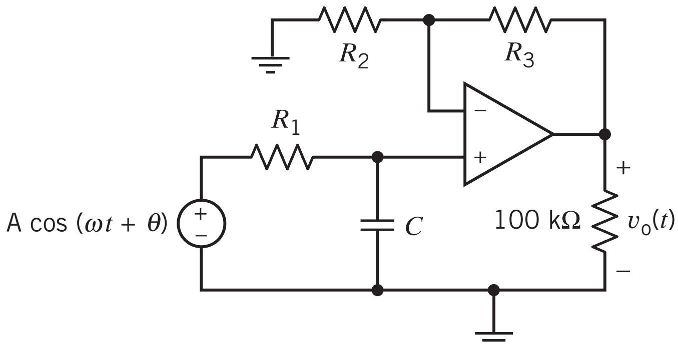


Figure DP 13-6

Solution:

The network function of this circuit is $\mathbf{H}(\omega) = \dfrac{1+\dfrac{R_2}{R_3}}{1+j\omega R_1 C}$

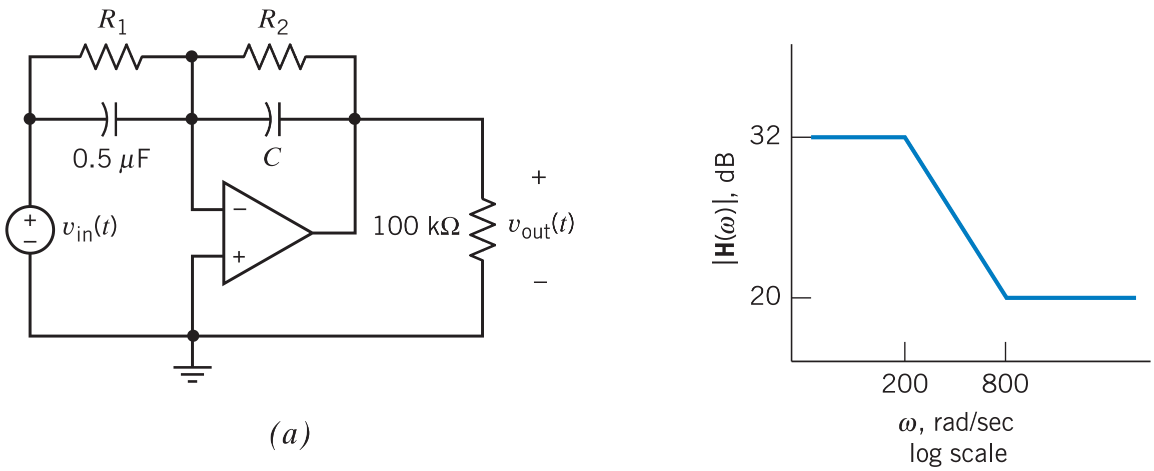
The phase shift of this network function is $\theta = -\tan^{-1}\omega R_1 C$

The gain of this network function is $G = \left|\mathbf{H}(\omega)\right| = \dfrac{1+\dfrac{R_3}{R_2}}{\sqrt{1+(\omega R_1 C)^2}} = \dfrac{1+\dfrac{R_3}{R_2}}{\sqrt{1+(\tan\theta)^2}}$

Design of this circuit proceeds as follows. Since the frequency and capacitance are known, R_1 is calculated from $R_1 = \dfrac{\tan(-\theta)}{\omega C}$. Next pick R_2 = 10kΩ (a convenient value) and calculated R_3 using $R_3 = (G\cdot\sqrt{1+(\tan\theta)^2}-1)\cdot R_2$. Finally

$$\theta = -45 \text{ deg},\ G = 2,\ \omega = 1000 \text{ rad/s} \Rightarrow R_1 = 10 \text{ k}\Omega,\ R_2 = 10 \text{ k}\Omega,\ R_3 = 18.284 \text{ k}\Omega,\ C = 0.1\ \mu\text{F}$$

DP 13-7 Design the circuit shown in Figure DP 13-7*a* to have the asymptotic Bode plot shown in Figure DP 13-7*b*.



(b)

Figure DP 13-7

Solution: From Table 13.4-2 and the Bode plot:

$$800 = z = \frac{1}{R_1(0.5\times10^{-6})} \Rightarrow R_1 = 2.5 \ \text{k}\Omega$$

$$32 \text{ dB} = 40 = \frac{R_2}{R_1} \Rightarrow R_2 = 100 \text{ k}\Omega$$

$$200 = p = \frac{1}{R_2 C} \Rightarrow C = \frac{1}{(200)(100\times10^3)} = 0.05\mu\text{F}$$

$$(\text{Check: } 20 \text{ dB} = 10 = k\frac{p}{z} = \frac{0.5\times10^{-6}}{C} = \frac{0.5\times10^{-6}}{0.05\times10^{-6}})$$

DP 13-8 For the circuit of Figure DP 13.-8, select R_1 and R_2 so that the gain at high frequencies is 10 V/V and the phase shift is 195° at ω = 1000 rad/s. Determine the gain at ω = 10 rad/s.

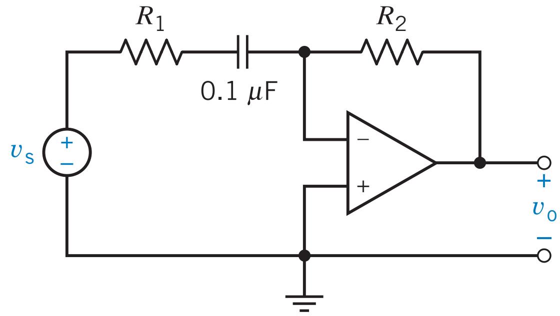


Figure DP 13.-8

Solution:

$$\mathbf{H}(\omega) = \frac{-R_2}{1+\frac{1}{j\omega C}} = -\frac{j\omega C R_2}{1+j\omega C R_1}$$

$$195° = 180 + 90 - \tan^{-1}\omega C R_1 \quad \Rightarrow \quad R_1 = \frac{\tan(270°-195°)}{(1000)(0.1\times10^{-6})} = 37.3\ \text{k}\Omega$$

$$10 = \lim_{\omega\to\infty} |\mathbf{H}(\omega)| = \frac{R_2}{R_1} \Rightarrow R_2 = 10R_1 = 373\ \text{k}\Omega$$

Chapter 14: The Laplace Transform

Exercises

Exercise 14.7-1 Determine the voltage $v_C(t)$ and the current $i_C(t)$ for $t \geq 0$ for the circuit of Figure 14.7-1.

Hint: $v_C(0) = 4$ V

Answer: $v_C(t) = (6 - 2e^{-0.67t})\, u(t)$ V and $i_C(t) = \frac{2}{3} e^{-0.67t} u(t)$ A

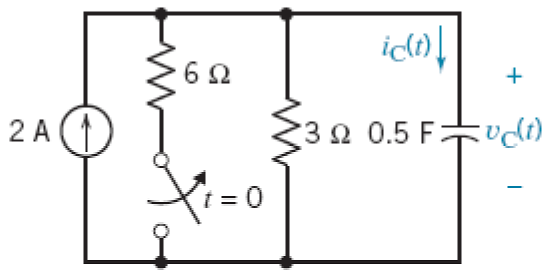
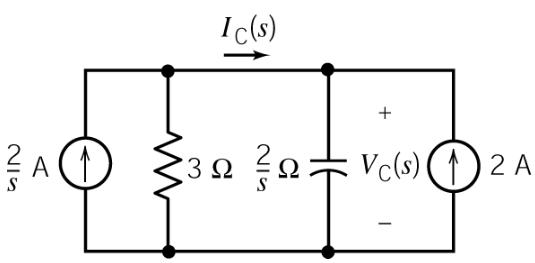


Figure 14.7-1

Solution:



KCL at top node:

$$\frac{V_C(s)}{3} + \frac{s}{2} V_C(s) = \frac{2}{s} + 2$$

$$V_C(s) = \frac{6}{s} - \frac{2}{s + \frac{2}{3}}$$

$$v_C(t) = \left(6 - 2e^{-(2/3)t}\right) u(t) \text{ V}$$

$$I_C(s) = \frac{V_C(s)}{\frac{2}{s}} - 2 = \frac{\frac{2}{3}}{s + \frac{2}{3}} \quad \Rightarrow \quad i_C(t) = \frac{2}{3} e^{-(2/3)t} u(t) \text{ A}$$

Exercise 14.8-1 The transfer function of a circuit is $H(s) = \dfrac{-5s}{s^2+15s+50}$. Determine the impulse response and step response of this circuit.

Answer: (a) impulse response = $\mathscr{L}^{-1}\left[\dfrac{5}{s+5}-\dfrac{10}{s+10}\right] = (5e^{-5t}-10e^{-10t})u(t)$

(b) step response = $\mathscr{L}^{-1}\left[\dfrac{1}{s+10}-\dfrac{1}{s+5}\right] = (e^{-10t}-e^{-5t})u(t)$

Solution:

(a) $\textit{impulse response} = \mathfrak{L}^{-1}\left[\dfrac{5}{s+5}-\dfrac{10}{s+10}\right] = \left(5e^{-5t}-10e^{-10t}\right)u(t)$

(b) $\textit{step response} = \mathfrak{L}^{-1}\left[\dfrac{1}{s+10}-\dfrac{1}{s+5}\right] = \left(e^{-10t}-e^{-5t}\right)u(t)$

Exercise 14.8-2 The impulse response of a circuit is $h(t) = 5e^{-2t}\sin(4t)u(t)$. Determine the step response of this circuit.

Hint: $H(s) = \mathscr{L}[5e^{-2t}\sin(4t)u(t)] = \dfrac{5(4)}{(s+2)^2+4^2} = \dfrac{20}{s^2+4s+20}$

Answer: step response = $\mathscr{L}^{-1}\left[\dfrac{H(s)}{s}\right] = \mathscr{L}^{-1}\left[\dfrac{1}{s}-\dfrac{s+4}{s^2+4s+20}\right]$

$$= \left(1-e^{-2t}\left(\cos 4t+\frac{1}{2}\sin 4t\right)\right)u(t)$$

Solution:

$$H(s) = \mathfrak{L}\left[5e^{-2t}\sin(4t)u(t)\right] = \frac{5(4)}{(s+2)^2+4^2} = \frac{20}{s^2+4s+20}$$

$$\textit{step response} = \mathfrak{L}^{-1}\left[\frac{H(s)}{s}\right] = \mathfrak{L}^{-1}\left[\frac{1}{s}-\frac{s+4}{s^2+4s+20}\right] = (1-e^{-2t}(\cos 4t-\frac{1}{2}\sin 4t))u(t)$$

Exercise 14.10-1 The input to a circuit is the voltage $v_i(t)$. The output is the voltage $v_o(t)$. The transfer function of this circuit is

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{ks}{s^2 + (3-k)s + 2}$$

Determine the following:

(a) The steady-state response when $v_i(t) = 5\cos 2t$ V and the gain of the VCVS is $k = 2$ V/V

(b) The impulse response when $k = 3 - 2\sqrt{2} = 0.17$ V/V

(c) The impulse response when $k = 3 + 2\sqrt{2} = 5.83$ V/V

Answer: (a) $v_o(t) = 7.07\cos(2t - 45°)$ V

(b) $h(t) = 0.17e^{-\sqrt{2}t}\left(1-\sqrt{2}t\right)u(t)$

(c) $h(t) = 5.83e^{\sqrt{2}t}\left(1+\sqrt{2}t\right)u(t)$

Solution:

The poles of the transfer function are $p_{1,2} = \dfrac{-(3-k) \pm \sqrt{(3-k)^2 - 8}}{2}$.

a.) When $k = 2$ V/V, the poles are $p_{1,2} = \dfrac{-1 \pm \sqrt{-7}}{2}$ so the circuit is stable. The transfer function is

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{2s}{s^2 + s + 2}$$

The circuit is stable when k =2 V/V so we can determine the network function from the transfer function by letting s = $j\omega$.

$$\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \mathbf{H}(\omega) = H(s)\Big|_{s=j\omega} = \frac{2s}{s^2+s+2}\bigg|_{s=j\omega} = \frac{2j\omega}{\left(2-\omega^2\right) + j\omega}$$

The input is $v_i(t) = 5\cos 2t$ V. The phasor of the steady state response is determine by multiplying the phasor of the input by the network function evaluated at ω = 2 rad/s.

$$\mathbf{V}_o(\omega) = \mathbf{H}(\omega)\big|_{\omega=2} \times \mathbf{V}_i(\omega) = \left(\frac{2j\omega}{\left(2-\omega^2\right)+j\omega}\bigg|_{\omega=2}\right)(5\angle 0°) = \left(\frac{j4}{-2+j2}\right)(5\angle 0°) = 7.07\angle -45°$$

The steady state response is $v_o(t) = 7.07\cos(2t - 45°)$ V.

b. When $k = 3 - 2\sqrt{2}$, the poles are $p_{1,2} = \dfrac{-2\sqrt{2} \pm \sqrt{0}}{2} = -\sqrt{2}, -\sqrt{2}$ so the circuit is stable. The transfer function is

$$H(s) = \frac{0.17s}{\left(s+\sqrt{2}\right)^2} = \frac{0.17}{\left(s+\sqrt{2}\right)} - \frac{0.17\sqrt{2}}{\left(s+\sqrt{2}\right)^2}$$

The impulse response is

$$h(t) = \mathcal{L}^{-1}\left[H(s)\right] = 0.17\, e^{-\sqrt{2}t}\left(1 - \sqrt{2}\,t\right)u(t)$$

We see that when $k = 3 - 2\sqrt{2}$ the circuit is stable and $\lim_{t\to\infty} |\, h(t)\,| = 0$.

c. When, $k = 3 + 2\sqrt{2}$ the poles are $p_{1,2} = \dfrac{2\sqrt{2} \pm \sqrt{0}}{2} = \sqrt{2}, \sqrt{2}$ so the circuit is not stable. The transfer function is

$$H(s) = \frac{5.83s}{\left(s-\sqrt{2}\right)^2} = \frac{5.83}{\left(s-\sqrt{2}\right)} + \frac{5.83\sqrt{2}}{\left(s-\sqrt{2}\right)^2}$$

The impulse response is

$$h(t) = \mathcal{L}^{-1}\left[H(s)\right] = 5.83\, e^{\sqrt{2}t}\left(1 + \sqrt{2}\,t\right)u(t)$$

We see that when $k = 3 + 2\sqrt{2}$ the circuit is unstable and $\lim_{t\to\infty} |\, h(t)\,| = \infty$.

Exercise 14.12-1 A circuit is specified to have a transfer function of

$$H(s) = \frac{25}{s^2 + 10s + 125}$$

and a unit step response of

$$v_o(t) = 0.1(2 - e^{-5t}(2\cos 10t + 3\sin 10t))u(t)$$

Verify that these specifications are consistent.

Solution:

For the poles to be in the left half of the s-plane, the s-term needs to be positive.

$$V_0(s) = 0.1\left[\frac{2}{s} - \left(2\frac{s+5}{(s+5)^2+10^2} + \frac{10}{(s+5)^2+10^2}\right)\right] = 0.1\left[\frac{2}{s} - \frac{2s+20}{s^2+10s+125}\right]$$

$$= 0.1\frac{2(s^2+10s+125)-(2s+20)s}{s(s^2+10s+125)}$$

$$= 0.1\frac{250}{s(s^2+10s+125)} = \frac{25}{s(s^2+10s+125)}$$

Then

$$H(s) = \frac{V_0(s)}{\mathcal{L}[u(t)]} = \frac{\frac{25}{s(s^2+10s+125)}}{\frac{1}{s}} = \frac{25}{s^2+10s+125}$$

These specifications are consistent.

Section 14.2 Laplace Transforms

P14.2-1 Determine the Laplace Transform of $v(t)=\left(17\,e^{-4t}-147\,e^{-5t}\right)u(t)$ V

Answer: $V(s)=\dfrac{3s+29}{s^2+9s+20}$

Solution:

$$\mathfrak{L}\left[17\,e^{-4t}-14\,e^{-5t}\right]=\frac{17}{s+4}-\frac{14}{s+5}=\frac{17(s+5)-14(s+4)}{(s+4)(s+5)}=\frac{3s+29}{s^2+9s+20}$$

P14.2-2 Determine the Laplace Transform of $v(t)=13\cos(6t-22.62°)$ V.

Answer: $V(s)=\dfrac{12s+30}{s^2+36}$

Solution:

$$\begin{aligned} v(t)&=13\cos(6t-22.62°)=13\cos(-22.62°)\cos(6t)-13\sin(-22.62°)\sin(6t)\\ &=12\cos(6t)+5\sin(6t)\text{ V}\end{aligned}$$

$$\begin{aligned} V(s)&=\mathfrak{L}\left[12\cos(6t)+5\sin(6t)\right]=\mathfrak{L}\left[12\cos(6t)\right]+\mathfrak{L}\left[+5\sin(6t)\right]\\ &=(12)\frac{s}{s^2+36}+(5)\frac{6}{s^2+36}=\frac{12s+30}{s^2+36}\end{aligned}$$

P14.2-3 Determine the Laplace Transform of $v(t)=10\,e^{-5t}\cos(4t+36.86°)u(t)$ V.

Answer: $V(s)=\dfrac{8s+16}{s^2+25s+41}$

Solution:

$$\begin{aligned} 10\cos(4t+36.86°)&=10\cos(36.86°)\cos(4t)-10\sin(36.86°)\sin(4t)\\ &=8\cos(4t)-6\sin(4t)\text{ V}\end{aligned}$$

$$\mathfrak{L}\left[10\cos(4t+36.86°)\right]=(8)\frac{s}{s^2+16}-(6)\frac{4}{s^2+16}=\frac{8s-4}{s^2+16}$$

$$\begin{aligned} V(s)&=\mathfrak{L}\left[10\,e^{-5t}\cos(4t+36.86°)\right]=\mathfrak{L}\left[10\cos(4t+36.86°)\right]\Big|_{s\leftarrow s+5}\\ &=\left.\frac{8s-4}{s^2+16}\right|_{s\leftarrow s+5}=\frac{8(s+5)-4}{(s+5)^2+16}=\frac{8s+16}{s^2+25s+41}\end{aligned}$$

P14.2-4 Determine the Laplace Transform of $v(t)=3t\,e^{-2t}\,u(t)$ V

Answer: $V(s)=\dfrac{3}{s^2+4s+4}$

Solution:

$$\mathfrak{L}\left[3t\,e^{-2t}\right]=\mathfrak{L}[3t]\Big|_{s\leftarrow s+2}=\left.\frac{3}{s^2}\right|_{s\leftarrow s+2}=\frac{3}{(s+2)^2}=\frac{3}{s^2+4s+4}$$

P14.2-5 Determine the Laplace Transform of $v(t)=16(1-2t)e^{-4t}\,u(t)$ V.

Answer: $V(s)=\dfrac{16(s+2)}{s^2+8s+16}$

Solution:

$$\mathfrak{L}\left[16(1-2t)e^{-4t}\right]=\mathfrak{L}[16-32t]\Big|_{s\leftarrow s+4}=\left.\left(\frac{16}{s}-\frac{32}{s^2}\right)\right|_{s\leftarrow s+4}=\frac{16}{s+4}-\frac{32}{(s+4)^2}$$

$$=\frac{16(s+4)-32}{s^2+8s+16}=\frac{16(s+2)}{s^2+8s+16}$$

Section 14-3: Pulse Inputs

P 14.3-1 Determine the Laplace transform of *f*(*t*) shown in Figure P 14.3-1.

Hint: $f(t)=\left(5-\frac{5}{3}t\right)u(t)+\frac{5}{3}\left(t-\frac{21}{5}\right)u\left(t-\frac{21}{5}\right)$

Answer: $F(s)=\dfrac{5e^{-4.2s}+15s-5}{3s^2}$

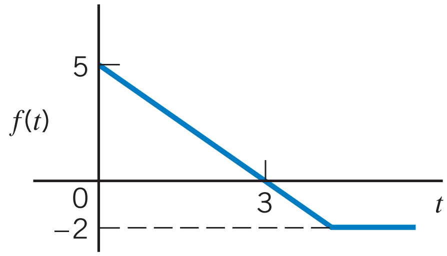
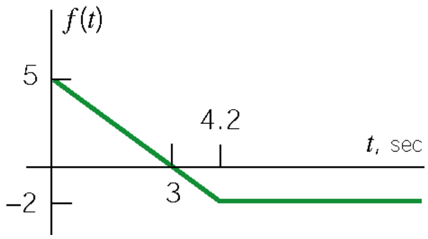


Figure P 14.3-1

Solution:



$$f(t)=\left(-\frac{5}{3}t+5\right)u(t)-\left(-\frac{5}{3}(t-4.2)\right)u(t-4.2)$$

$$F(s)=\left(-\frac{5}{3s^2}+\frac{5}{s}\right)-e^{-4.2s}\left(-\frac{5}{3s^2}\right)=\frac{15s+5\left(e^{-4.2s}-1\right)}{3s^2}$$

P 14.3-2 Use the Laplace transform to obtain the transform of the signal *f*(*t*) shown in Figure P14.3-2.

Answer: $F(s)=\dfrac{3\left(1-e^{-2s}\right)}{s}$

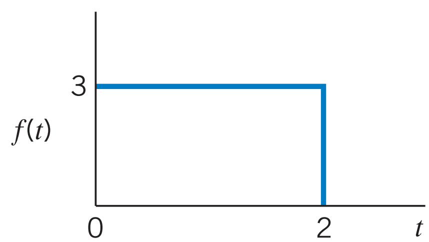
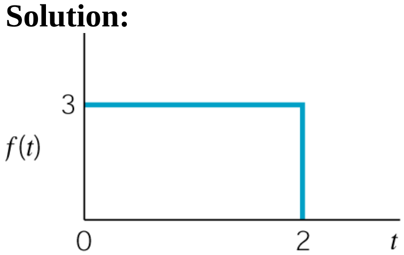


Figure P14.3-2

Solution:



$$F(s)=\int_0^\infty f(t)e^{-st}dt=\int_0^2 3e^{-st}dt=\left.\frac{3e^{-st}}{-s}\right|_0^2=\frac{3(1-e^{-2s})}{s}$$

P 14.3-3 Determine the Laplace transform of *f*(*t*) shown in Figure P 14.3-3.

Answer: $F(s) = \frac{5}{2s^2}\left(1 - e^{-2s} - 2se^{-2s}\right)$

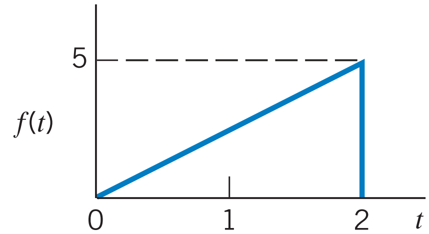
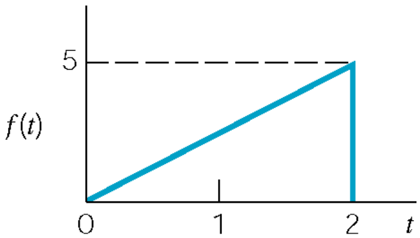


Figure P 14.3-3

Solution:



$$f(t) = \begin{cases} 5/2\ t & 0<t<2 \\ 0 & \text{otherwise} \end{cases}$$

$$f(t) = \frac{5}{2}t\left[u(t)-u(t-2)\right] = \frac{5}{2}t\ u(t) - \frac{5}{2}t\ u(t-2) = \frac{5}{2}\left[t\,u(t)-(t-2)u(t-2)-2u(t-2)\right]$$

$$\therefore F(s) = \mathcal{L}\left[f(t)\right] = \frac{5}{2}\left[\frac{1}{s^2} - \frac{e^{-2s}}{s^2} - \frac{2e^{-2s}}{s}\right] = \underline{\frac{5}{2}\frac{1}{s^2}\left[1-e^{-2s}-2se^{-2s}\right]}$$

P 14.3-4 Consider the pulse shown in Figure P 14.3-4, where the time function follows e^{at} for $0 < t < T$. Find $F(s)$ for the pulse.

Answer: $F(s) = \frac{1-e^{-(s-a)T}}{s-a}$

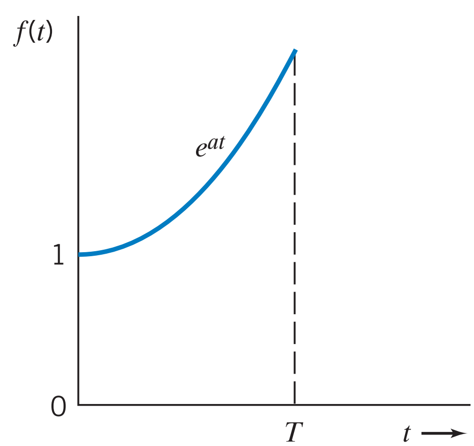


Figure P 14.3-4

Solution:

$$f(t) = \left[u(t)-u(t-T)\right]e^{at} \quad \Rightarrow \quad F(s) = \mathcal{L}\left[e^{at}\left[u(t)-u(t-T)\right]\right]$$

$$\left.\begin{aligned} \mathcal{L}\left[u(t)-u(t-T)\right] &= \frac{1-e^{-sT}}{s} \\ \mathcal{L}\left[e^{at}g(t)\right] &= G(s-a) \end{aligned}\right\} \Rightarrow F(s) = \frac{1-e^{(s-a)T}}{(s-a)}$$

P 14.3-5 Find the Laplace transform for $g(t) = e^{-t}u(t-0.5)$.

Solution:

$$g(t) = e^{-t}u(t-0.5) = e^{-(t+(0.5-0.5))}u(t-0.5) = e^{-0.5}\, e^{-(t-0.5)}u(t-0.5)$$

$$\mathcal{L}\left[e^{-0.5}\, e^{-(t-0.5)}u(t-0.5)\right] = e^{-0.5}\,\mathcal{L}\left[e^{-(t-0.5)}u(t-0.5)\right] = e^{-0.5}\, e^{-0.5s}\,\mathcal{L}\left[e^{-t}u(t)\right] = \frac{e^{0.5}\, e^{-0.5s}}{s+1} = \frac{e^{0.5-0.5s}}{s+1}$$

P 14.3-6 Find the Laplace transform for

$$f(t) = \frac{-(t-T)}{T}u(t-T)$$

Answer: $F(s) = \dfrac{-1e^{-sT}}{Ts^2}$

Solution:

$$\mathcal{L}\left[-\frac{t-T}{T}u(t-T)\right] = e^{-sT}\,\mathcal{L}\left[-\frac{t}{T}u(t)\right] = \frac{e^{-sT}}{T}\,\mathcal{L}\left[-t\,u(t)\right] = -\frac{e^{-sT}}{T\,s^2}$$

Section 14-4: Inverse Laplace Transform

P 14.4-1 Find $f(t)$ when

$$F(s)=\frac{s+3}{s^3+3s^2+6s+4}$$

Answer: $f(s)=\frac{2}{3}e^{-t}-\frac{2}{3}e^{-t}\cos\sqrt{3t}+\frac{1}{\sqrt{3}}e^{-t}\times\sin\sqrt{3t},\ t\geq 0$

Solution

$$F(s)=\frac{s+3}{s^3+3s^2+6s+4}=\frac{s+3}{(s+1)\left[(s+1)^2+3\right]}=\frac{A}{s+1}+\frac{Bs+C}{s^2+2s+4}$$

Where
$$A=\left.\frac{s+3}{(s+1)^2+3}\right|_{s=-1}=\frac{2}{3}$$

Then

$$\frac{(s+3)}{(s+1)(s^2+2s+4)}=\frac{\frac{2}{3}}{s+1}+\frac{Bs+C}{s^2+2s+4}\Rightarrow(s+3)=(\frac{2}{3}+B)\,s^2+\left(\frac{4}{3}+B+C\right)s+\frac{8}{3}+C$$

Equating coefficient yields

$$\text{s}^2:\quad 0=\frac{2}{3}+B\quad\Rightarrow\quad B=-\frac{2}{3}$$
$$\text{s}\ :\quad 1=\frac{4}{3}-\frac{2}{3}+C\quad\Rightarrow\quad C=\frac{1}{3}$$

Then
$$F(s)=\frac{\frac{2}{3}}{s+1}+\frac{-\frac{2}{3}s+\frac{1}{3}}{(s+1)^2+3}=\frac{\frac{2}{3}}{s+1}+\frac{-\frac{2}{3}(s+1)}{(s+1)^2+3}+\frac{\frac{1}{\sqrt{3}}\sqrt{3}}{(s+1)^2+3}$$

Taking the inverse Laplace transform yields

$$f(t)=\frac{2}{3}e^{-t}-\frac{2}{3}e^{-t}\ \cos\sqrt{3}t\ +\frac{1}{\sqrt{3}}e^{-t}\ \sin\sqrt{3}\,t,\quad t\geq 0$$

P 14.4-2 Find $f(t)$ when

$$F(s)=\frac{s^2-2s+1}{s^3+3s^2+4s+2}$$

Solution:

$$F(s)=\frac{s^2-2s+1}{s^3+3s^2+4s+2}=\frac{s^2-2s+1}{(s+1)(s+1-j)(s+1+j)}=\frac{a}{s+1-j}+\frac{a^*}{s+1+j}+\frac{b}{s+1}$$

where

$$b=\left.\frac{s^2-2s+1}{(s+1)^2+1}\right|_{s=-1}=4$$

$$a=\left.\frac{s^2-2s+1}{(s+1)(s+1+j)}\right|_{s=-1+j}=\frac{3-j4}{-2}=-\frac{3}{2}+j2$$

$$a^*=-\frac{3}{2}-j2$$

Then

$$F(s)=\frac{-\frac{3}{2}+j2}{s+1-j}+\frac{-\frac{3}{2}-j2}{s+1+j}+\frac{4}{s+1}$$

Next

$$m=\sqrt{(-3/2)^2+(2)^2}=\frac{5}{2}\quad\text{and}\quad\theta=\tan^{-1}\left(\frac{2}{-\frac{3}{2}}\right)=126.9°$$

From Equation 14.5-8

$$f(t)=\left[5e^{-t}\cos(t+127°)+4e^{-t}\right]u(t)$$

P 14.4-3

Find $f(t)$ when

$$F(s)=\frac{5s-1}{s^3-3s-2}$$

Answer: $f(t) = -e^{-t} + 2te^{-t} + e^{2t},\ t \geq 0$

Solution:

$$F(s)=\frac{5s-1}{(s+1)^2(s-2)}=\frac{A}{s+1}+\frac{B}{(s+1)^2}+\frac{C}{s-2}$$

where $$B=\left.\frac{5s-1}{s-2}\right|_{s=-1}=2 \quad \text{and} \quad C=\left.\frac{5s-1}{(s+1)^2}\right|_{s=2}=1$$

Then $$A=\frac{d}{ds}\left[(s+1)^2 F(s)\right]\Big|_{s=-1}=\left.\frac{-9}{(s-2)^2}\right|_{s=-1}=-1$$

Finally $$F(s)=\frac{-1}{s+1}+\frac{2}{(s+1)^2}+\frac{1}{s-2} \quad \Rightarrow \quad f(t)=\left[-e^{-t}+2te^{-t}+e^{2t}\right]u(t)$$

P 14.4-4 Find the inverse transform of

$$\mathrm{Y}(s)=\frac{1}{s^3+3s^2+4s+2}$$

Answer: $y(t) = e^{-t}(1 - \cos t),\ t \geq 0$

Solution:

$$Y(s)=\frac{1}{(s+1)(s^2+2s+2)}=\frac{1}{(s+1)\left[(s+1)^2+1\right]}=\frac{A}{s+1}+\frac{Bs+C}{(s+1)^2+1}$$

where
$$A=\left.\frac{1}{s^2+2s+2}\right|_{s=-1}=1$$

Next

$$\frac{1}{(s+1)(s^2+2s+2)}=\frac{1}{s+1}+\frac{Bs+C}{s^2+2s+2} \Rightarrow 1=s^2+2s+2+(Bs+C)(s+1)$$

$$\Rightarrow 1=(B+1)s^2+(B+C+2)s+C+2$$

Equating coefficients:

$$s^2: 0=B+1 \Rightarrow B=-1$$
$$s: 0=B+C+2 \Rightarrow C=-1$$

Finally
$$Y(s)=\frac{1}{s+1}-\frac{s+1}{(s+1)^2+1} \Rightarrow y(t)=\left[e^{-t}-e^{-t}\cos t\right]u(t)$$

P 14.4-5 Find the inverse transform of

$$F(s)=\frac{2s+6}{(s+1)(s^2+2s+5)}$$

Solution:

$$F(s)=\frac{2(s+3)}{(s+1)(s^2+2s+5)}=\frac{1}{s+1}+\frac{-(s+1)}{(s+1)^2+4}+\frac{2}{(s+1)^2+4}$$

$$f(t)=\left[e^{-t}-e^{-t}\cos(2t)+e^{-t}\sin(2t)\right]u(t)$$

P 14.4-6 Find the inverse transform of

$$F(s)=\frac{2s+6}{s\left(s^2+3s+2\right)}$$

Answer: $f(t) = [3 - 4e^{-t}+e^{-2t}]\, u(t)$

Solution:

$$F(s)=\frac{2(s+3)}{s(s+1)(s+2)}=\frac{A}{s}+\frac{B}{s+1}+\frac{C}{s+2}$$

where

$$A=sF(s)\Big|_{s=0}=\frac{2(s+3)}{(s+1)(s+2)}\Bigg|_{s=0}=3,\quad B=(s+1)\,F(s)\Big|_{s=-1}=\frac{2(s+3)}{s(s+2)}\Bigg|_{s=-1}=-4$$

and

$$(s+2)\,F(s)\Big|_{s=-2}=\frac{2(s+3)}{s(s+1)}\Bigg|_{s=-2}=C=1$$

Finally

$$F(s)=\frac{3}{s}+\frac{-4}{s+1}+\frac{1}{s+2}\quad\Rightarrow\quad f(t)=\left(3-4e^{-t}+e^{-2t}\right)u(t)$$

P 14.4-7 Find the inverse transform of $F(s)$ expressing $f(t)$ in cosine and angle forms.

(a) $F(s)=\dfrac{8s-3}{s^2+4s+13}$ (b) $F(s)=\dfrac{3e^{-s}}{s^2+2s+17}$

Answer: (a) $f(t) = 10.2e^{-2t}\cos(3t + 38.4°)$, $t \geq 0$

(b) $f(t) = \frac{3}{4}e^{-(t-1)}\sin[4(t-1)]$, $t \geq 1$

Solution:

(a)

$$F(s)=\frac{8s-3}{s^2+4s+13}=\frac{1}{2}\times\frac{2(8s-3)}{(s+2)^2+9}$$

$$\therefore\ a=2,\ c=8,\ \omega=3\ \&\ ca-\omega d=-3\ \Rightarrow\ d=\frac{-3-(8)(2)}{-3}=7.33$$

$$\therefore\ \theta=\tan^{-1}\left(\frac{6.33}{8}\right)=38.4^\circ,\ m=\sqrt{(8)^2+(6.33)^2}=10.85$$

$$\Rightarrow\ \underline{f(t)=10.85\,e^{-2t}\cos(3t+42.5)u(t)}$$

(b)

Given $F(s)=\dfrac{3e^{-s}}{s^2+2s+17}$, first consider $F_1(s)=\dfrac{3}{s^2+2s+17}=\dfrac{1}{2}\times\dfrac{(2(3))}{(s+1)^2+16}$.

Identify $a=1,\ c=0,\ \omega=4\ \textit{and}\ -\omega d=3\Rightarrow d=-3/4$. Then $m=|d|=3/4,\ \theta=\tan^{-1}\left(\dfrac{-3/4}{0}\right)=-90°$

So $f_1(t)=(3/4)e^{-t}\sin 4t\,u(t)$. Next, $F(s)=e^{-s}F_1(s)\Rightarrow f(t)=f_1(t-1)$. Finally

$$\therefore f(t)=(3/4)e^{-(t-1)}\ \sin\left[4(t-1)\right]u(t-1)$$

P 14.4-8 Find the inverse transform of $F(s)$.

(a) $f(s) = \dfrac{s^2 - 5}{s(s+1)^2}$ (b) $f(s) = \dfrac{4s^2}{(s+3)^3}$

Answer: (a) $f(t) = -5 + 6e^{-t} + 4te^{-t}$, $t \geq 0$

(b) $f(t) = 4e^{-3t} - 24te^{-3t} + 18t^2e^{-3t}$, $t \geq 0$

Solution:

(a)

$$F(s) = \frac{s^2-5}{s(s+1)^2} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{(s+1)^2}$$

where

$$A = sF(s)|_{s=0} = \frac{-5}{1} = -5 \text{ and } C = (s+1)^2 F(s)|_{s=-1} = \frac{1-5}{-1} = 4$$

Multiply both sides by $s(s+1)^2$

$$s^2 - 5 = -5(s+1)^2 + Bs(s+1) + 4s \quad \Rightarrow \quad B = 6$$

Then

$$F(s) = \frac{-5}{s} + \frac{6}{s+1} + \frac{4}{(s+1)^2}$$

Finally

$$f(t) = \left(-5 + 6e^{-t} + 4te^{-t}\right), \quad t \geq 0$$

(b)

$$F(s) = \frac{4s^2}{(s+3)^3} = \frac{A}{(s+3)} + \frac{B}{(s+3)^2} + \frac{C}{(s+3)^3}$$

where

$$A = \frac{1}{2}\frac{d^2}{ds^2}\left[(s+3)^3 F(s)\right]_{s=-3} = 4, \quad B = \frac{d}{ds}\left[(s+3)^3 F(s)\right]_{s=-3} = -24$$

and

$$C = (s+3)^3 F(s)_{s=-3} = 36$$

Then

$$F(s) = \frac{4}{(s+3)} + \frac{-24}{(s+3)^2} + \frac{36}{(s+3)^3}$$

Finally

$$f(t) = \left(4 - 24t + 18t^2\right)e^{-3t}, \quad t \geq 0$$

Section 14-5: Initial and Final Value Theorems

P 14.5-1 A function of time is represented by

$$F(s) = \frac{2s^2 - 3s + 4}{s^3 + 3s^2 + 2s}$$

(a) Find the initial value of *f*(*t*) at *t* = 0.

(b) Find the value of *f*(*t*) as *t* approaches infinity.

Solution:

(a)

$$f(0) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{2s^2 - 3s + 4}{s^2 + 3s + 2} = \frac{2s^2}{s^2} = 2$$

(b)

$$f(\infty) = \lim_{s\to 0} sF(s) = \frac{4}{2} = 2$$

P 14.5-2 Find the initial and final values of *v*(*t*) when

$$V(s) = \frac{(s+16)}{s^2 + 4s + 12}$$

Answer: *v*(0) = 1, *v*(∞) = 0 V

Solution:

Initial value:

$$v(0) = \lim_{s\to\infty} sV(s) = \lim_{s\to\infty} \frac{s(s+16)}{s^2 + 4s + 12} = \lim_{s\to\infty} \frac{s^2 + 16s}{s^2 + 4s + 12} = 1$$

Final value:

$$v(\infty) = \lim_{s\to 0} s\left(\frac{s+16}{s^2 + 4s + 12}\right) = \lim_{s\to 0} \frac{s^2 + 16s}{s^2 + 4s + 12} = 0$$

(Check: *V*(*s*) is stable because $\text{Re}\{p_i\} < 0$ since $p_i = -2 \pm 2.828j$. We expect the final value to exist.)

P 14.5-3 Find the initial and final values of $v(t)$ when

$$V(s)=\frac{(s+10)}{(3s^2+2s^2+1s)}$$

Answer: $v(0) = 0$, $v(\infty) = 10$ V

Solution:

Initial value: $$v(0)=\lim_{s\to\infty} sV(s) = \lim_{s\to\infty} \frac{s^2+10s}{3s^3+2s^2+s} = 0$$

Final value: $$v(\infty)=\lim_{s\to 0} sV(s)=\lim_{s\to 0} \frac{s(s+10)}{s(3s^2+2s+1)}=10$$

(Check: $V(s)$ is stable because $p_i = -0.333 \pm 0.471\, j$. We expect the final value to exist.)

P 14.5-4 Find the initial and final values of $f(t)$ when

$$F(s)=\frac{-2(s+7)}{s^2-2s+10}$$

Answer: initial value = –2; final value does not exist

Solution:

Initial value: $$f(0)=\lim_{s\to\infty} s\,F(s)=\lim_{s\to\infty} \frac{-2s^2-14s}{s^2-2s+10}=-2$$

Final value: $F(s)$ is not stable because $\text{Re}\{p_i\}>0$ since $p_i = 1\pm 3\,j$. No final value of $f(t)$ exists.

P14.5-5

Given that $\mathcal{L}\left[v(t)\right]=\dfrac{a\,s+b}{s^2+8\,s}$ where $v(t)$ is the voltage shown in Figure P14.5-5, determine the values of *a* and *b*.

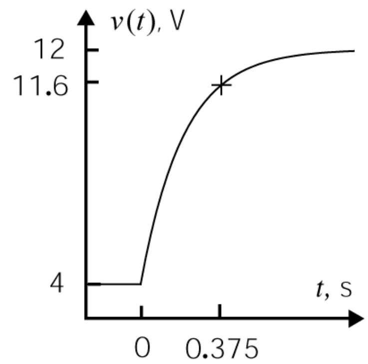


Figure P14.5-5

Solution:

From the plot, *v*(0) = 4 V and $\lim_{t\to\infty} v(t)=12\text{ V}$. From the final value theorem,

$$\lim_{t\to\infty} v(t)=\lim_{s\to 0} s\,V(s)=\lim_{s\to 0} s\,\frac{a\,s+b}{s^2+8\,s}=\lim_{s\to 0}\frac{a\,s+b}{s+8}=\frac{b}{8}\,.$$

Consequently, $12=\dfrac{b}{8} \quad\Rightarrow\quad b=96$. From the initial value theorem

$$\lim_{t\to 0} v(t)=\lim_{s\to\infty} s\,V(s)=\lim_{s\to\infty} s\,\frac{a\,s+b}{s^2+8\,s}=\lim_{s\to\infty}\frac{a\,s+b}{s+8}=a$$

Consequently *a* = 4.

P14.5-6

Given that $\mathcal{L}\left[v(t)\right]=\dfrac{a\,s+b}{2\,s^2+40\,s}$ where $v(t)$ is the voltage shown in Figure P14.5-6, determine the values of *a* and *b*.

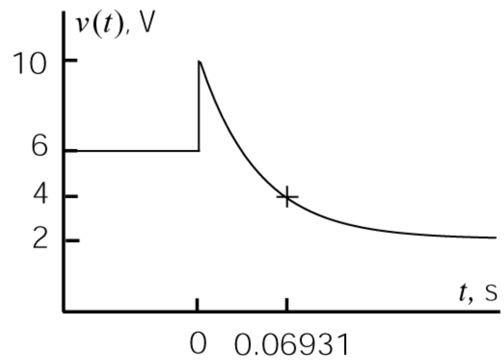


Figure P14.5-6

Solution:

From the plot, $v(0+)$ = 10 V and $\lim_{t\to\infty} v(t)=2\text{ V}$. From the final value theorem,

$$\lim_{t\to\infty} v(t)=\lim_{s\to 0} sV(s)=\lim_{s\to 0} s\frac{a\,s+b}{2\,s^2+40\,s}=\lim_{s\to 0}\frac{a\,s+b}{2\,s+40}=\frac{b}{40}.$$

Consequently, $2=\dfrac{b}{40} \quad\Rightarrow\quad b=80$. From the initial value theorem

$$\lim_{t\to 0+} v(t)=\lim_{s\to\infty} sV(s)=\lim_{s\to\infty} s\frac{a\,s+b}{2\,s^2+40\,s}=\lim_{s\to\infty}\frac{a\,s+b}{2\,s+40}=\frac{a}{2}$$

Consequently $10=\dfrac{a}{2} \quad\Rightarrow\quad a=20$.

Section 14.6 Solution of Differential Equations Describing a Circuit

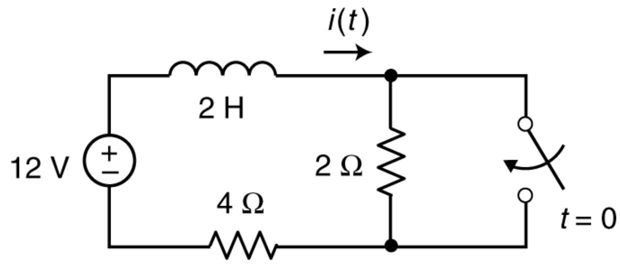


Figure P14.6-1

P14.6-1 The circuit shown in Figure P14.6-1 is at steady state before the switch closes at time $t = 0$. Determine the inductor current, $i(t)$, after the switch closes.

Solution:

The initial inductor current is $i(0) = \frac{12}{6} = 2$ A

Apply KVL after the switch closes $12 = 2\frac{d}{dt}i(t) + 4i(t)$

Take the Laplace Transform of both sides of this equation:

$$\frac{12}{s} = 2\left[s\,I(s) - 2\right] + 4\,I(s)$$

Solve for $I(s)$: $I(s) = \frac{2s+6}{s(s+2)} = \frac{3}{s} - \frac{1}{s+2}$

Taking the Inverse Laplace Transform: $i(t) = 3 - e^{-2t}$ A

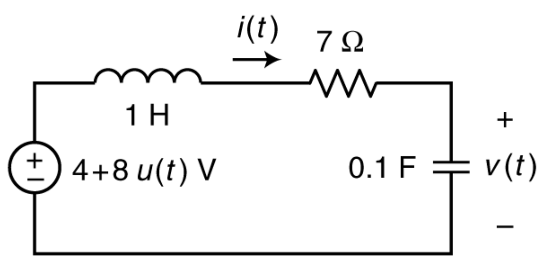


Figure P14.6-2

P14.6-2 The circuit shown in Figure P14.6-1 is represented by the differential equation

$$\frac{d^2v(t)}{dt^2} + 7\frac{d\,v(t)}{dt} + 10\,v(t) = 120$$

after time $t = 0$. The initial conditions are

$$i(0) = 0 \quad \text{and} \quad v(0) = 4 \text{ V}$$

Determine the capacitor, $v(t)$, after time $t = 0$.

Solution:

Let's take the Laplace Transform of both sides of the differential equation.

First, using Table 14.2-2 $$\pounds\left[\frac{d\,v(t)}{dt}\right]=sV(s)-4$$

Next, notice that $$i(t)=0.1\frac{d\,v(t)}{dt}\quad\Rightarrow\quad\frac{d\,v(0)}{dt}=\frac{i(0)}{0.1}=0$$

Using Table 14.2-2 again $$\pounds\left[\frac{d^2v(t)}{dt^2}\right]=s\,\pounds\left[\frac{d\,v(t)}{dt}\right]-\frac{d\,v(0)}{dt}=s^2V(s)-4s$$

Now we have: $$s^2V(s)-4s+7\left(sV(s)-4\right)+10V(s)=\frac{120}{s}$$

or: $$\left(s^2+7s+10\right)V(s)=\frac{120}{s}+28+4s$$

Solve for $V(s)$: $$V(s)=\frac{4s^2+28s+120}{s(s+2)(s+5)}=\frac{3}{s}-\frac{1}{s+2}=\frac{12}{s}+\frac{-\frac{40}{3}}{s+2}+\frac{\frac{16}{3}}{s+5}$$

Taking the inverse Laplace transform: $v(t)=12-\frac{40}{3}e^{-2t}+\frac{16}{3}e^{-5t}$ A

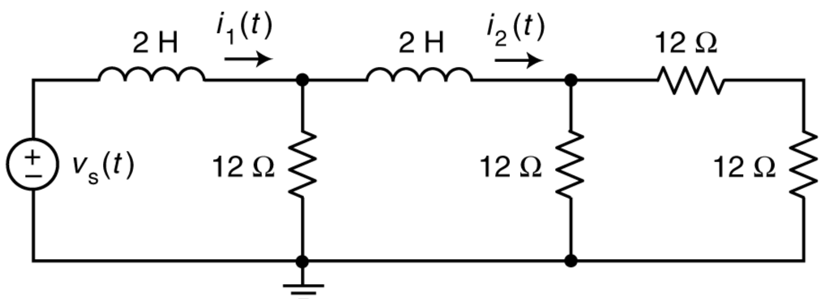


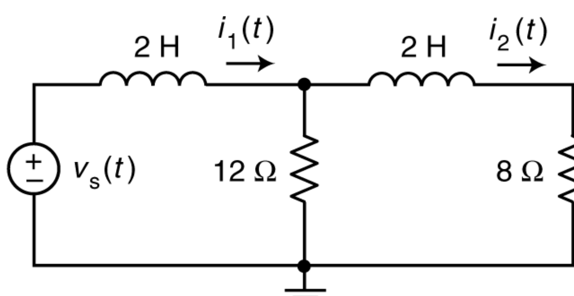
Figure P14.6-3

P14.6-3 The circuit shown in Figure P14.6-3 is at steady state before time $t = 0$. The input to the circuit is

$$v_s(t)=2.4u(t)\ \text{V}$$

Consequently, the initial conditions are $i_1(0)=0$ and $i_2(0)=0$. Determine the inductor current, $i_2(t)$, after time $t = 0$.

Solution:
First, let's simplify the circuit by replacing the 3 12-Ω resistors at the right of the circuit by an equivalent resistor:



Write the mesh equations $$v_s(t) = 2\frac{d\,i_1(t)}{dt} + 12\left(i_1(t) - i_2(t)\right)$$

and $$0 = 2\frac{d\,i_2(t)}{dt} + 8\,i_2(t) - 12\left(i_1(t) - i_2(t)\right) = 2\frac{d\,i_2(t)}{dt} + 20\,i_2(t) - 12\,i_1(t)$$

Taking the Laplace Transforms of these equations:

$$\frac{2.4}{s} = 2\,s\,I_1(s) + 12\left(I_1(s) - I_2(s)\right)$$

and $$0 = 2\,s\,I_2(s) + 20\,I_2(s) - 12\,I_1(s)$$

Next, some algebra: $$I_1(s) = \frac{1}{6}s\,I_2(s) + \frac{5}{3}I_2(s) = \left(\frac{1}{6}s + \frac{5}{3}\right)I_2(s)$$

$$\frac{2.4}{s} = (2\,s + 12)\,I_1(s) - 12 I_2(s) = (2\,s + 12)\left(\frac{1}{6}s + \frac{5}{3}\right)I_2(s) - 12 I_2(s)$$

Solving for $I_2(s)$: $$I_2(s) = \frac{7.2}{2\left(s^2 + 16\,s + 24\right)} = \frac{0.3}{s} + \frac{0.03794}{s + 14.3} - \frac{0.33974}{s + 1.68}$$

Taking the Inverse Laplace Transform gives $i_2(t)$ for $t \geq 0$:

$$i_2(t) = 300 + 39.74\,e^{-14.3t} - 339.74\,e^{-1.68t}\ \text{A}$$

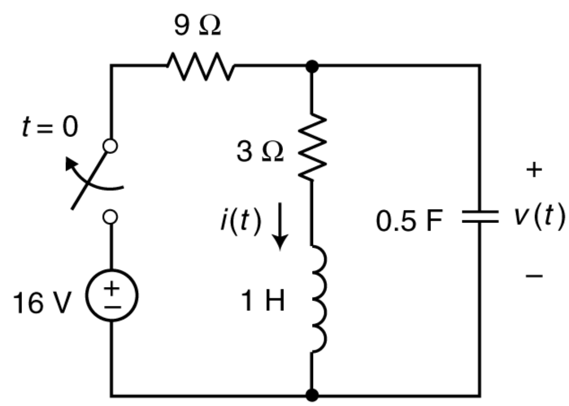


Figure P14.6-4

P14.6-4 The circuit shown in Figure P14.6-4 is at steady state before the switch opens at time $t = 0$. Determine the capacitor voltage, $v(t)$, after the switch opens.

Solution:

The initial conditions are $$v(0) = \frac{3}{3+9}(12) = 4\ \text{V} \quad \text{and} \quad i(0) = -\frac{v(0)}{3} = -\frac{4}{3}\ \text{A}$$

Apply KVL after the switch opens $$0 = \frac{1}{2}\frac{d^2 v(t)}{dt^2} + \frac{3}{2}\frac{d\,v(t)}{dt} + v(t)$$

That is $$0 = \frac{d^2 v(t)}{dt^2} + 3\frac{d\,v(t)}{dt} + 2\,v(t)$$

Let’s take the Laplace Transform of both sides of the differential equation.

First, using Table 14.2-2 $$\pounds\left[\frac{dv(t)}{dt}\right] = sV(s) - 4$$

Next, notice that $$i(t) = -0.5\frac{dv(t)}{dt} \Rightarrow \frac{dv(0)}{dt} = -\frac{i(0)}{0.5} = \frac{8}{3} \text{ A}$$

Using Table 14.2-2 again $$\pounds\left[\frac{d^2v(t)}{dt^2}\right] = s\pounds\left[\frac{dv(t)}{dt}\right] - \frac{dv(0)}{dt} = s^2V(s) - 4s - \frac{2}{3}$$

Now we have: $$\left(s^2V(s) - 4s - \frac{8}{3}\right) + 3\left(sV(s) - 4\right) + 2V(s) = 0$$

Solve for $V(s)$: $$V(s) = \frac{4s + \frac{44}{3}}{(s+1)(s+2)} = \frac{\frac{32}{3}}{s+1} - \frac{\frac{20}{3}}{s+2}$$

Taking the Inverse Laplace Transform: $v(t) = \frac{32}{3}e^{-t} - \frac{20}{3}e^{-2t}$ V

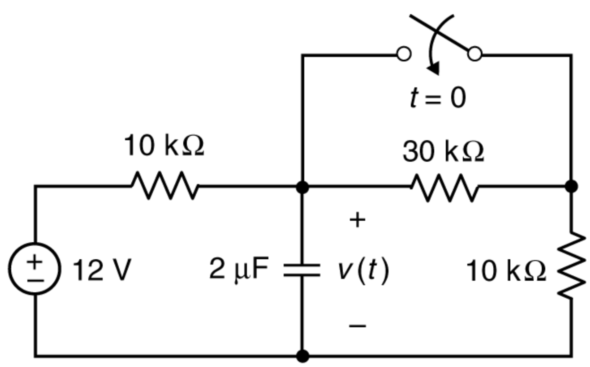


Figure P14.6-5

P14.6-5 The circuit shown in Figure P14.6-5 is at steady state before the switch closes at time $t = 0$. Determine the capacitor voltage, $v(t)$, after the switch closes.

Solution:

The initial capacitor voltage is $$v(0) = \frac{40}{10+40}(12) = 0.98 \text{ V}$$

Write a node equation after the switch closes:

$$\frac{v_s(t) - v(t)}{10\times10^3} = \left(2\times10^{-6}\right)\frac{d}{dt}v(t) + \frac{v(t)}{10\times10^3}$$

$$1200 - 100v(t) = 2\frac{d}{dt}v(t) + 100v(t)$$

$$600 = \frac{d}{dt}v(t) + 100v(t)$$

Take the Laplace Transform of both sides of this equation:

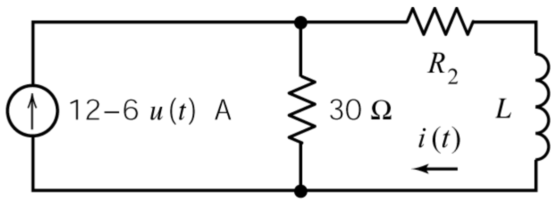
$$\frac{600}{s} = \left[sV(s) - 0.98\right] + 100V(s)$$

Solve for *V*(*s*): $$V(s) = \frac{600\,s + 0.98}{s(s+100)} = \frac{6}{s} - \frac{5.02}{s+100}$$

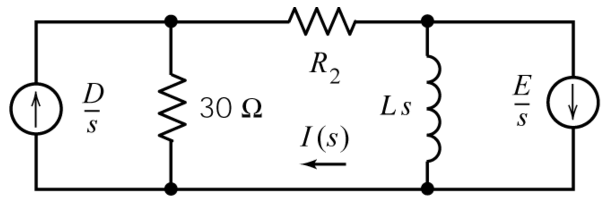
Taking the Inverse Laplace Transform: $v(t) = 6 - 5.02e^{-100t}$ V

Section 14.7 Circuit Analysis Using Impedance and Initial Conditions

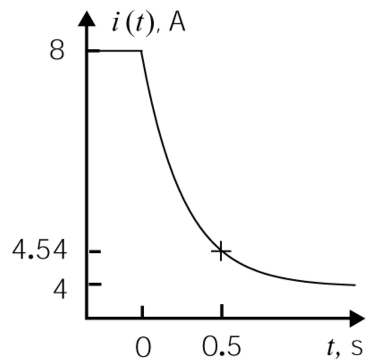
P14.7-1
Figure P14.7-1a shows a circuit represented in the time domain. Figure P14.7-1b shows the same circuit, now represented in the complex frequency domain. Figure P14.7-1c shows a plot of the inductor current.



(a)



(b)

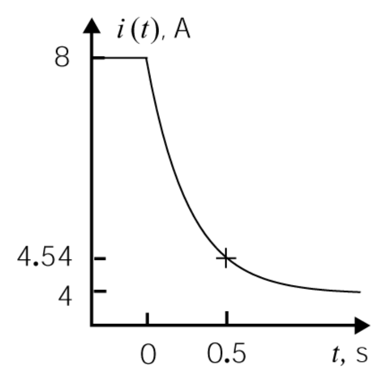


(c)

Figure P14-7-1

Determine the values of *D* and *E* used to represent the circuit in the complex frequency domain. Determine the values of the resistance R_2 and the inductance *L*.

Solution:
First, we find the values of *D* and *E* used to represent the circuit in the complex frequency domain:



$$\mathcal{L}\left[12-6u(t)\right]=\mathcal{L}\left[6u(t)\right]=\frac{6}{s} \quad\Rightarrow\quad D=6 \text{ A.}$$

E is the initial inductor current = 8 A from the plot.

Next, we find the values of the resistance R_2 and the inductance *L*:

The circuit is at steady state before $t = 0$, so the inductor acts like a short circuit. Using current division, $8=\left(\frac{30}{30+R_2}\right)12 \quad\Rightarrow\quad R_2=15\ \Omega$. Similarly, the circuit will at steady state for $t\to\infty$. Again, the inductor acts like a short circuit. Using current division,

$$4 = \left(\frac{30}{30+R_2}\right)6 \quad \Rightarrow \quad R_2 = 15\ \Omega\,.$$

The inductor current can be represented as $v(t) = 4 + 4e^{-at}$ for $t \geq 0$. From the plot,

$$4.54 = 4 + 4e^{-a(0.5)} \quad \text{so} \quad a = \frac{\ln\left(\frac{4.54-4}{4}\right)}{-0.5} = 4.005 \cong 4 \ \ 1/\text{s}\,.$$

then
$$\frac{1}{4} = \tau = \frac{L}{15+30} \quad \Rightarrow \quad L = \frac{45}{4} = 11.25\ \text{H}\,.$$

As a check, apply KVL to the center mesh in the complex frequency domain to get

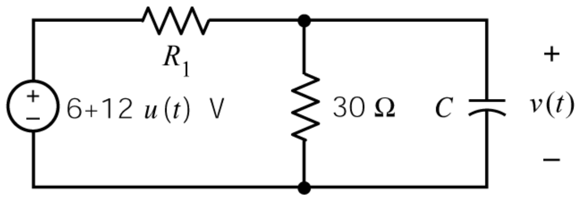
$$R_2\, I(s) + L\,s\left(I(s) - \frac{E}{s}\right) + R_1\left(I(s) - \frac{D}{s}\right) = 0 \quad \Rightarrow \quad I(s) = \frac{L\,s\,\frac{E}{s} + R_1\,\frac{D}{s}}{L\,s + R_1 + R_2} = \frac{E\,s + \frac{R_1\,D}{L}}{s\left(s + \frac{R_1 + R_2}{L}\right)}$$

Substituting values gives
$$I(s) = \frac{8\,s + 16}{s(s+4)} = \frac{4}{s} + \frac{4}{s+4}$$

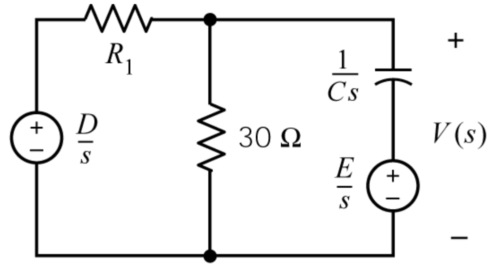
Taking the inverse Laplace transform gives

$$i(t) = \mathcal{L}^{-1}\left[I(s)\right] = \mathcal{L}^{-1}\left[\frac{4}{s} + \frac{4}{s+4}\right] = 4 + 4e^{-4t} \ \ \text{A} \quad \text{for } t \geq 0$$

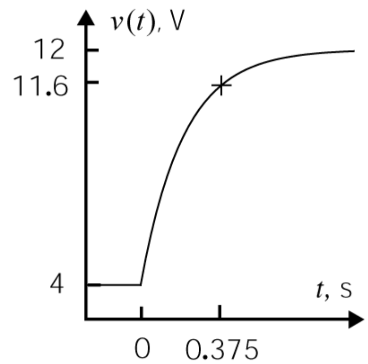
P14.7-2
Figure P14.7-2a shows a circuit represented in the time domain. Figure P14.7-2b shows the same circuit, now represented in the complex frequency domain. Figure P14.7-2c shows a plot of the inductor current.



(a)



(b)



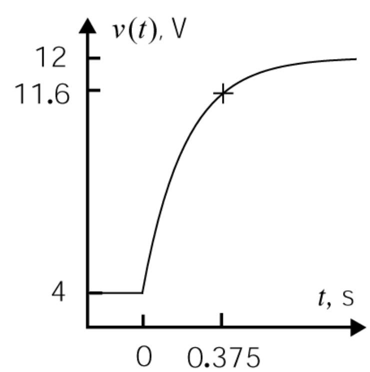
(c)

Figure P14-7-2

Determine the values of D and E used to represent the circuit in the complex frequency domain. Determine the values of the resistance R_1 and the capacitance C.

Solution:

First, we find the values of D and E used to represent the circuit in the complex frequency domain:



$$\mathcal{L}\left[6+12u(t)\right]=\mathcal{L}\left[18u(t)\right]=\frac{18}{s} \quad \Rightarrow \quad D=18 \text{ V.}$$

E is the initial capacitor voltage = 4 V from the plot.

Next, we determine the values of the resistance R_1 and the capacitance C:

The circuit is at steady state before $t = 0$, so the capacitor acts like an open circuit. Using voltage division, $4=\left(\frac{30}{R_1+30}\right)6 \quad \Rightarrow \quad R_1=15\ \Omega$. Similarly, the circuit will at steady state for $t \to \infty$. Again, the capacitor acts like an open circuit. Using voltage division,

$$12 = \left(\frac{30}{R_1 + 30} \right) 18 \quad \Rightarrow \quad R_1 = 15\ \Omega .$$

The capacitor voltage can be represented as $v(t) = 12 - 8e^{-at}$ for $t \ge 0$. From the plot,

$$11.6 = 12 - 8e^{-a(0.375)} \quad \text{so} \quad a = \frac{\ln\left(\frac{11.6 - 12}{-8} \right)}{-0.375} = 7.9886 \cong 8 \;\; 1/\text{s} .$$

then $$\frac{1}{8} = \tau = (15 \,\|\, 30) C = 10C \quad \Rightarrow \quad C = \frac{1}{80} = 12.5 \text{ mF} .$$

As a check, apply KCL at the top node of the 30 Ω resistor, R_2 to get

$$\frac{V(s) - \frac{D}{s}}{R_1} + \frac{V(s)}{R_2} + Cs\left(V(s) - \frac{E}{s} \right) = 0 \quad \Rightarrow \quad V(s) = \frac{Es + \frac{D}{R_1 C}}{s\left(s + \frac{R_1 + R_2}{R_1 R_2 C} \right)}$$

Substituting values and performing a partial fraction expansion gives

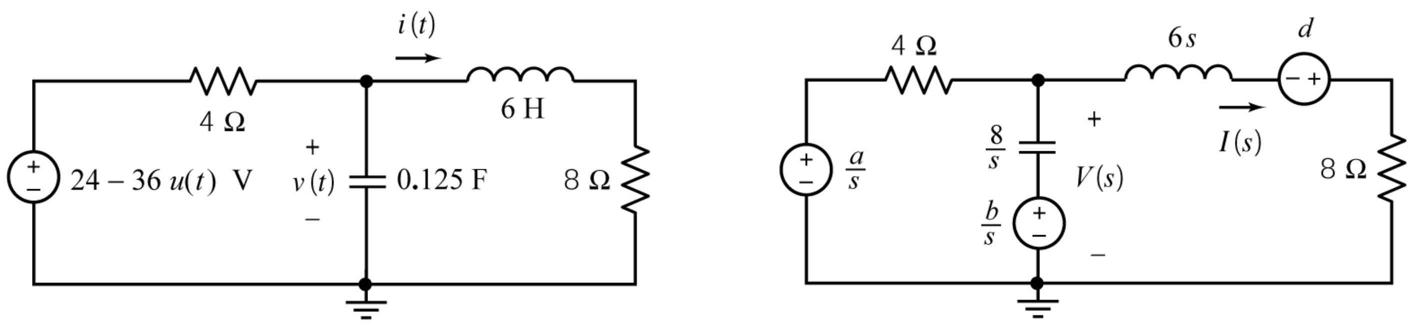
$$V(s) = \frac{4s + 96}{s(s+8)} = \frac{12}{s} - \frac{8}{s+8}$$

Taking the inverse Laplace transform gives

$$v(t) = \mathcal{L}^{-1}\left[V(s) \right] = \mathcal{L}^{-1}\left[\frac{12}{s} - \frac{8}{s+8} \right] = 12 - 8e^{-8t} \text{ V} \quad \text{for } t \ge 0$$

P14.7-3

Figure P14.7-3a shows a circuit represented in the time domain. Figure P14.7-3b shows the same circuit, now represented in the complex frequency domain. Determine the values of *a, b* and *d* used to represent the circuit in the complex frequency domain.



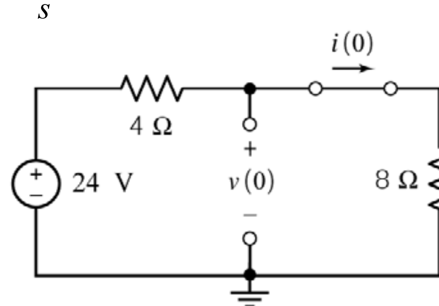
(a) (b)

Figure P14.7-3

Solution:

$$24-36u(t)=-12 \quad \text{for } t>0 \quad \mathcal{L}[-12]=\frac{-12}{s} \quad \Rightarrow \quad a=-12 \text{ V.}$$

The circuit is at steady state before $t = 0$, and the input is constant, so the capacitor acts like an open circuit and the inductor acts like a short circuit.



$$v(0)=\left(\frac{8}{4+8}\right)24=16 \text{ V} \quad \text{and} \quad i(0)=\frac{24}{4+8}=2 \text{ A}$$

Consequently, $b=v(0)=16$ V and $d=L\,i(0)=(6)(2)=12$.

P 14.7-4 The input to the circuit shown in Figure P 14.7-4 is the voltage of the voltage source, 12 V. The output of this circuit is the voltage, $v_o(t)$, across the capacitor. Determine $v_o(t)$ for $t > 0$.

Answer: $v_o(t) = -(4 + 2e^{-t/2})\text{V}$ for $t > 0$

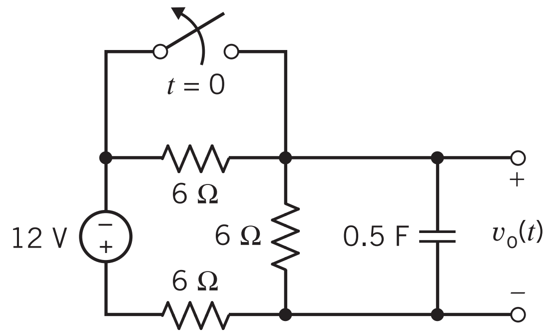
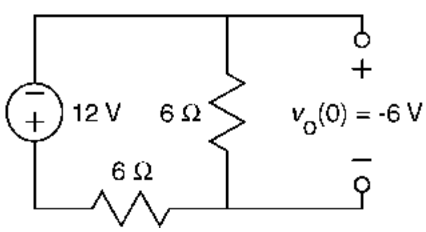


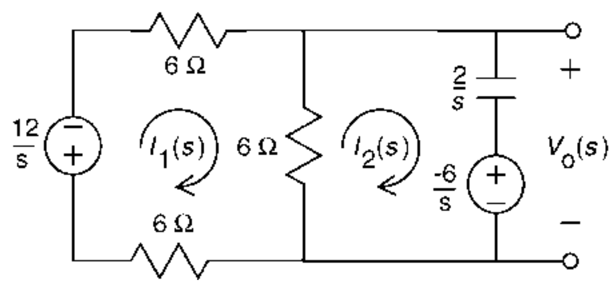
Figure P 14.7-4

Solution:

$t < 0$



time domain



frequency domain

Mesh equations in the frequency domain:

$$6I_1(s)+6\big(I_1(s)-I_2(s)\big)+6I_1(s)+\frac{12}{s}=0 \;\Rightarrow\; I_1(s)=\frac{2}{3}I_2(s)-\frac{2}{3s}$$

$$\frac{2}{s}I_2(s)-\frac{6}{s}-6\big(I_1(s)-I_2(s)\big)=0 \;\Rightarrow\; \left(6+\frac{2}{s}\right)I_2(s)-6I_1(s)=\frac{6}{s}$$

Solving for $I_2(s)$: $$\left(6+\frac{2}{s}\right)I_2(s)-6\left(\frac{2}{3}I_2(s)-\frac{2}{3s}\right)=\frac{6}{s} \;\Rightarrow\; I_2(s)=\frac{\frac{1}{2}}{s+\frac{1}{2}}$$

Calculate for $V_o(s)$: $$V_o(s)=\frac{1}{2}I_2(s)-\frac{6}{s}=\frac{1}{2}\left(\frac{\frac{1}{2}}{s+\frac{1}{2}}\right)-\frac{6}{s}=\frac{-2}{s+\frac{1}{2}}-\frac{4}{s}$$

Take the Inverse Laplace transform:

$$v_o(t)=-\left(4+2e^{-t/2}\right)\ \text{V for } t>0$$

(Checked using LNAP, 12/29/02)

P 14.7-5 The input to the circuit shown in Figure P 14.7-5 is the voltage of the voltage source, 12 V. The output of this circuit is the current, $i(t)$, in the inductor. Determine $i(t)$ for $t > 0$.

Answer: $i(t) = -3(1 + e^{-0.8t})$ A for $t > 0$

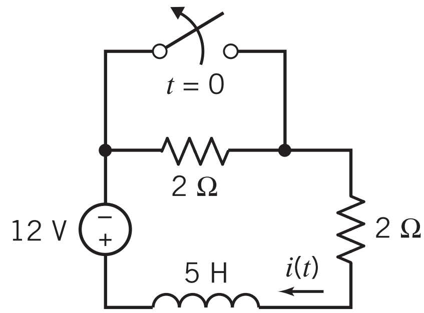
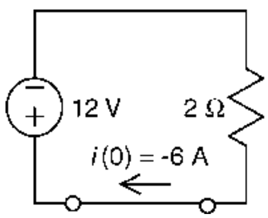


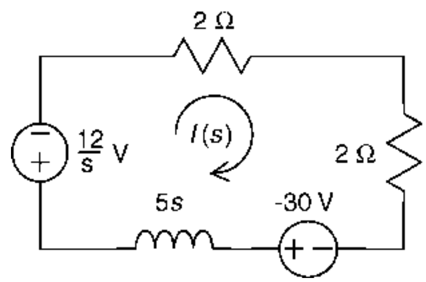
Figure P 14.7-5

Solution:

$t < 0$



time domain



frequency domain

Writing a mesh equation:

$$(4+5s)I(s)+30+\frac{12}{s}=0 \quad \Rightarrow \quad I(s)=\frac{-6\left(s+\frac{2}{5}\right)}{s\left(s+\frac{4}{5}\right)}=-\left(\frac{3}{s}+\frac{3}{s+\frac{4}{5}}\right)$$

Take the Inverse Laplace transform:

$$i(t)=-3\left(1+e^{-0.8t}\right) \text{ A for } t>0$$

(Checked using LNAP, 12/29/02)

P 14.7-6 The input to the circuit shown in Figure P 14.7-6 is the voltage of the voltage source, 18 V. The output of this circuit, the voltage across the capacitor, is given by

$$v_o(t) = 6 + 12e^{-2t} \quad \text{V} \quad \text{when} \quad t > 0$$

Determine the value of the capacitance, C, and the value of the resistance, R.

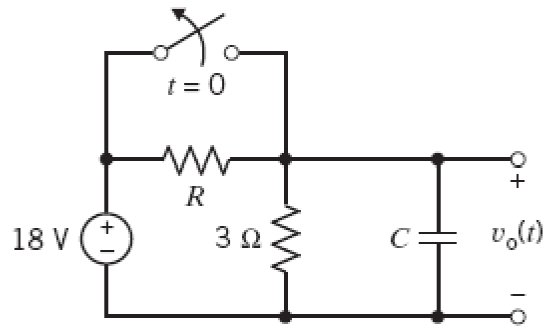
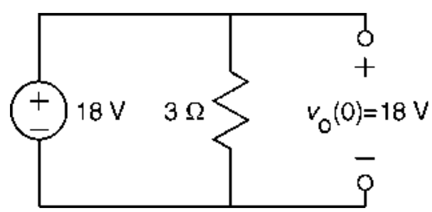


Figure P 14.7-6

Solution:

Steady-state for $t < 0$:



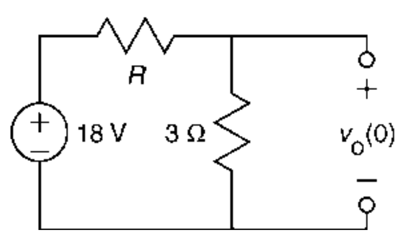
From the equation for $v_o(t)$:

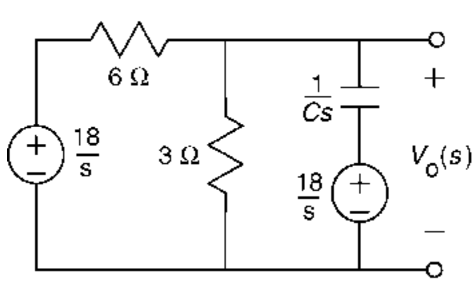
$$v_o(\infty) = 6 + 12\,e^{-2(\infty)} = 6 \text{ V}$$

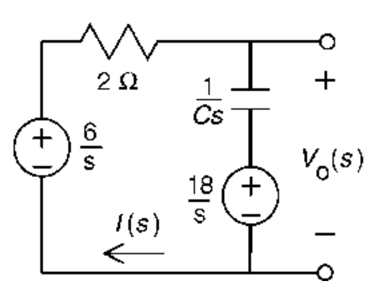
From the circuit: $v_o(\infty) = \dfrac{3}{R+3}(18)$

Therefore: $6 = \dfrac{3}{R+3}(18) \quad \Rightarrow \quad \underline{R = 6\ \Omega}$

Steady-state for $t > 0$:







$$I(s)\left(2 + \frac{1}{Cs}\right) + \frac{18}{s} - \frac{6}{s} = 0 \quad \Rightarrow \quad I(s) = \frac{-6}{s + \dfrac{1}{2C}}$$

$$V_o(s)=\frac{1}{Cs}I(s)+\frac{18}{s}=\frac{1}{Cs}\left(\frac{-6}{s+\frac{1}{2C}}\right)+\frac{18}{s}=\frac{-12}{s}+\frac{12}{s+\frac{1}{2C}}+\frac{18}{s}=\frac{12}{s+\frac{1}{2C}}+\frac{6}{s}$$

Taking the inverse Laplace transform:

$$v_o(t)=6+12\,e^{-t/2C}\ \text{V for } t>0$$

Comparing this to the given equation for $v_o(t)$, we see that $2=\frac{1}{2C} \Rightarrow \underline{C=0.25\ \text{F}}$.

(Checked using LNAP, 12/29/02)

P 14.7-7 The input to the circuit shown in Figure P 14.7-7 is the voltage source voltage

$$v_s(t) = 3 - u(t) \quad \text{V}$$

The output is the voltage

$$v_o(t) = 10 + 5e^{-100t} \quad \text{V} \quad \text{for} \quad t \geq 0$$

Determine the values of R_1 and R_2.

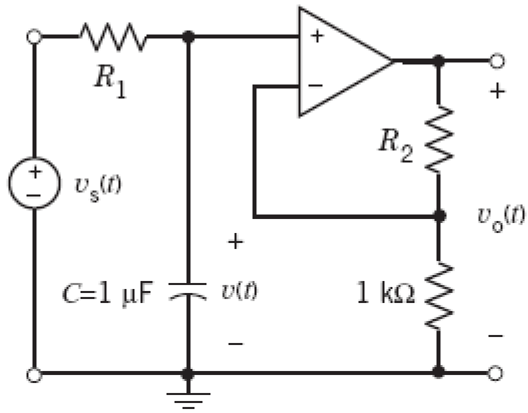


Figure P 14.7-7

Solution:

We will determine $V_o(s)$, the Laplace transform of the output, twice, once from the given equation and once from the circuit. From the given equation for the output, we have

$$V_o(s) = \frac{10}{s} + \frac{5}{s+100}$$

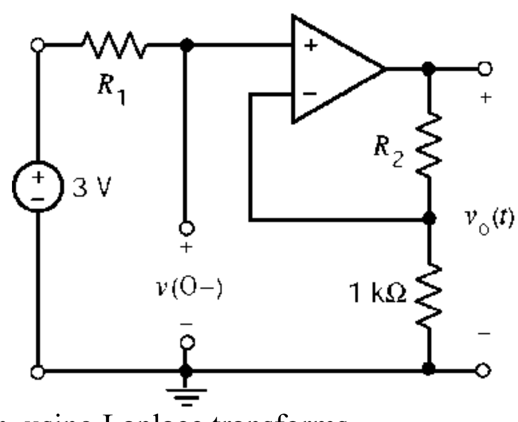
Next, we determine $V_o(s)$ from the circuit. For $t \geq 0$, we represent the circuit in the frequency domain using the Laplace transform. To do so we need to determine the initial condition for the capacitor.

When $t < 0$ and the circuit is at steady state, the capacitor acts like an open circuit. Apply KCL at the noninverting input of the op amp to get

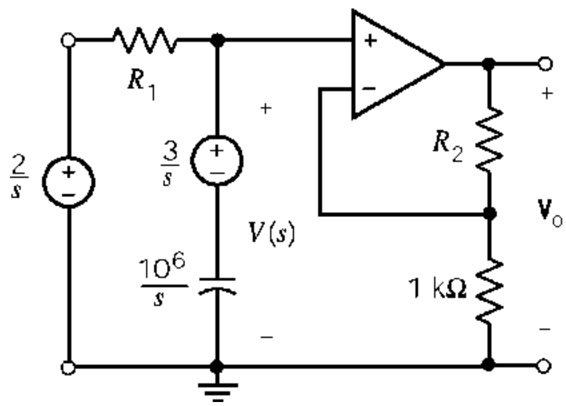
$$\frac{3 - v(0-)}{R_1} = 0 \quad \Rightarrow \quad v(0-) = 3 \text{ V}$$

The initial condition is

$$v(0+) = v(0-) = 3 \text{ V}$$



Now we can represent the circuit in the frequency domain, using Laplace transforms.



Apply KCL at the noninverting input of the op am to get

$$\frac{\frac{2}{s}-V(s)}{R_1}=\frac{V(s)-\frac{3}{s}}{\frac{10^6}{s}}$$

Solving gives

$$V(s)=\frac{3s+2\frac{10^6}{R_1}}{s\left(s+\frac{10^6}{R_1}\right)}=\frac{2}{s}+\frac{1}{\left(s+\frac{10^6}{R_1}\right)}$$

Apply KCL at the inverting input of the op amp to get

$$\frac{V_o(s)-V(s)}{R_2}=\frac{V(s)}{1000} \quad\Rightarrow\quad V_o(s)=\left(1+\frac{R_2}{1000}\right)V(s)=\left(1+\frac{R_2}{1000}\right)\left(\frac{2}{s}+\frac{1}{\left(s+\frac{10^6}{R_1}\right)}\right)$$

The expressions for $V_o(s)$ must be equal, so

$$\frac{10}{s}+\frac{5}{s+100}=\left(1+\frac{R_2}{1000}\right)\left(\frac{2}{s}+\frac{1}{\left(s+\frac{10^6}{R_1}\right)}\right)$$

Equating coefficients gives

$$1+\frac{R_2}{1000}=5 \quad\Rightarrow\quad R_2=4\ \text{k}\Omega \quad\text{and}\quad \frac{10^6}{R_1}=100 \quad\Rightarrow\quad R_1=10\ \text{k}\Omega$$

(checked using LNAPTR 7/31/04)

P 14.7-8 Determine the inductor current, $i_L(t)$, in the circuit shown in Figure P 14.7-8 for each of the following cases:

(a) $R = 2\ \Omega$, $L = 4.5$ H, $C = 1/9$ F, $A = 5$ mA, $B = -2$ mA

(b) $R = 1\ \Omega$, $L = 0.4$ H, $C = 0.1$ F, $A = 1$ mA, $B = -2$ mA

(c) $R = 1\ \Omega$, $L = 0.08$ H, $C = 0.1$ F, $A = 0.2$ mA, $B = -2$ mA

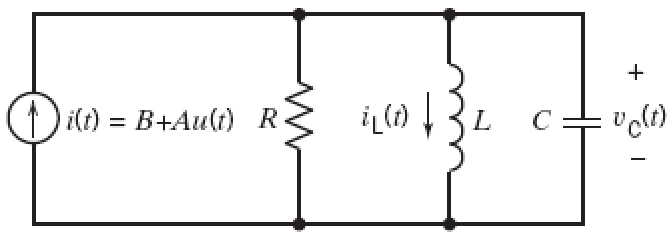
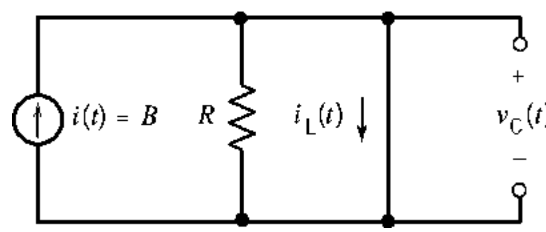


Figure P 14.7-8

Solution:

For $t < 0$, The input is constant. At steady state, the capacitor acts like an open circuit and the inductor acts like a short circuit.

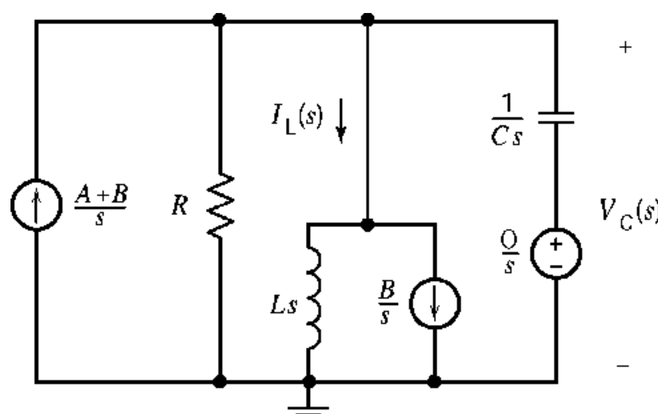


The circuit is at steady state at time $t = 0-$ so

$$v_C(0-) = 0 \quad \text{and} \quad i_L(0-) = B$$

The capacitor voltage and inductor current are continuous so $v_C(0+) = v_C(0-)$ and $i_L(0+) = i_L(0-)$.

For $t < 0$, represent the circuit in the frequency domain using the Laplace transform as shown. $V_C(s)$ is the node voltage at the top node of the circuit. Writing a node equation gives



$$\frac{A+B}{s} = \frac{V_C(s)}{R} + \frac{B}{s} + \frac{V_C(s)}{Ls} + CsV_C(s)$$

so

$$\frac{A}{s} = \frac{Ls + R + RLCs^2}{RLs} V_C(s)$$

Then

$$V_C(s) = \frac{ARL}{RLCs^2 + Ls + R} = \frac{\frac{A}{C}}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}$$

and $$I_L(s)=\frac{V_C(s)}{Ls}+\frac{B}{s}=\frac{\frac{A}{LC}}{s\left(s^2+\frac{1}{RC}s+\frac{1}{LC}\right)}+\frac{B}{s}$$

a.) When $R=2\ \Omega,\ L=4.5\ \text{H},\ C=\frac{1}{9}\ \text{F},\ A=5\ \text{mA}$ and $B=-2\ \text{mA}$, then

$$I_L(s)=\frac{10}{s\left(s^2+4.5s+2\right)}+\frac{-2}{s}=\frac{3}{s}+\frac{\frac{5}{7}}{s+4}-\frac{\frac{40}{7}}{s+\frac{1}{2}}$$

Taking the inverse Laplace transform gives

$$i_L(t)=3+\frac{5}{7}e^{-4t}-\frac{5}{7}e^{-0.5t}\ \text{mA}\quad \text{for } t\ge 0$$

b.) When $R=1\ \Omega,\ L=0.4\ \text{H},\ C=0.1\ \text{F},\ A=1\ \text{mA}$ and $B=-2\ \text{mA}$, then

$$I_L(s)=\frac{25}{s\left(s^2+10s+25\right)}+\frac{-2}{s}=\frac{25}{s(s+5)^2}+\frac{-2}{s}=-\left(\frac{1}{s}+\frac{5}{(s+5)^2}+\frac{1}{s+5}\right)$$

Taking the inverse Laplace transform gives

$$i_L(t)=-\left(1+5te^{-5t}-e^{-5t}\right)\ \text{mA}\quad \text{for } t\ge 0$$

c.) When $R=1\ \Omega,\ L=0.08\ \text{H},\ C=0.1\ \text{F},\ A=0.2\ \text{mA}$ and $B=-2\ \text{mA}$, then

$$I_L(s)=\frac{25}{s\left(s^2+10s+125\right)}+\frac{-2}{s}=\frac{-1.8}{s}+\frac{-0.2s-2}{(s+5)^2+10^2}=\frac{-1.8}{s}-0.2\frac{s+5}{(s+5)^2+10^2}-0.1\frac{10}{(s+5)^2+10^2}$$

Taking the inverse Laplace transform gives

$$i_L(t)=-1.8-e^{-5t}\left(0.2\cos(10t)+0.1\sin(10t)\right)\ \text{mA}\quad \text{for } t\ge 0$$

P 14.7-9 Determine the capacitor current, $i_c(t)$, in the circuit shown in Figure P 14.7-9 for each of the following cases:

(a) $R = 3\ \Omega$, $L = 2$ H, $C = 1/24$ F, $A = 12$ V

(b) $R = 2\ \Omega$, $L = 2$ H, $C = 1/8$ F, $A = 12$ V

(c) $R = 10\ \Omega$, $L = 2$ H, $C = 1/40$ F, $A = 12$ V

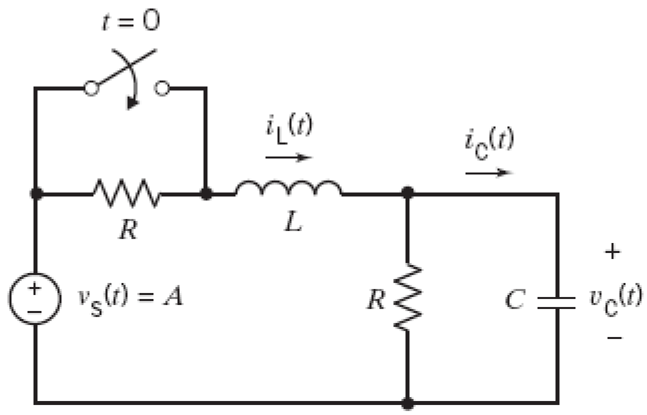
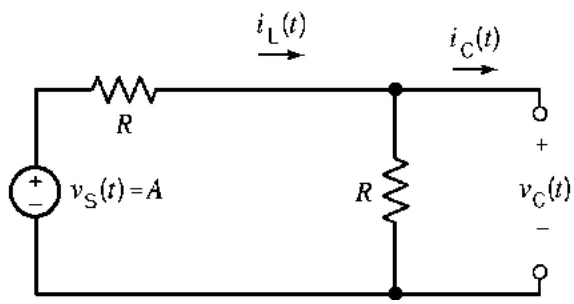


Figure P 14.7-9

Solution:

For $t < 0$, the switch is open and the circuit is at steady state. and the circuit is at steady state. At steady state, the capacitor acts like an open circuit.

$$i(t) = \frac{A}{2R} \quad \text{and} \quad v_C(t) = \frac{A}{2}$$



Consequently,

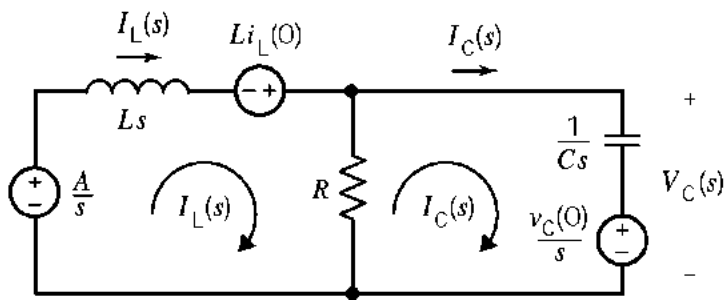
$$i(0-) = \frac{A}{2R} \quad \text{and} \quad v_C(0-) = \frac{A}{2}$$

Also

$$i_C(0-) = 0$$

The capacitor voltage and inductor current are continuous so $v_C(0+) = v_C(0-)$ and $i_L(0+) = i_L(0-)$.

For $t > 0$, the voltage source voltage is 12 V. Represent the circuit in the frequency domain using the Laplace transform as shown.



$I_L(s)$ and $I_C(s)$ are mesh currents. Writing a mesh equations gives

$$L s I_L(s) - \frac{AL}{2R} + R\left(I_L(s) - I_C(s)\right) - \frac{A}{s} = 0$$

$$\frac{1}{Cs} I_C(s) - \frac{A}{2s} + R\left(I_L(s) - I_C(s)\right) = 0$$

Or, in matrix form
$$\begin{pmatrix} Ls + R & -R \\ -R & R + \frac{1}{Cs} \end{pmatrix} \begin{pmatrix} I_L(s) \\ I_C(s) \end{pmatrix} = \begin{pmatrix} \frac{AL}{2R} + \frac{A}{s} \\ -\frac{A}{2s} \end{pmatrix}$$

$$I_C(s) = \frac{(Ls + R)\left(-\frac{A}{2s}\right) + R\left(\frac{AL}{2R} + \frac{A}{s}\right)}{(Ls + R)\left(R + \frac{1}{Cs}\right) - R^2} = \frac{\frac{A}{2L}}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}$$

a.) When $R = 3\ \Omega,\ L = 2\ \text{H},\ C = \frac{1}{24}\ \text{F}$ and $A = 12\ \text{V}$,

$$I_C(s) = \frac{3}{s^2 + 8s + 12} = \frac{2}{(s+2)(s+6)} = \frac{\frac{3}{4}}{s+2} - \frac{\frac{3}{4}}{s+8}.$$

Taking the inverse Laplace transform gives

$$i_C(t) = \left(\frac{3}{4}e^{-2t} - \frac{3}{4}e^{-6t}\right)u(t)\ \text{A}$$

b.) $R = 2\ \Omega,\ L = 2\ \text{H},\ C = \frac{1}{8}\ \text{F}$ and $A = 12\ \text{V}$,

$$I_C(s) = \frac{3}{s^2 + 4s + 4} = \frac{3}{(s+2)^2}$$

Taking the inverse Laplace transform gives

$$i_C(t) = 3t\,e^{-2t}\,u(t)\ \text{A}$$

c.) $R = 10\ \Omega,\ L = 2\ \text{H},\ C = \frac{1}{40}\ \text{F}$ and $A = 12\ \text{V}$

$$I_C(s) = \frac{3}{s^2 + 4s + 20} = \frac{3}{(s+2)^2 + 16} = \frac{3}{4} \times \frac{4}{(s+2)^2 + 16}$$

Taking the inverse Laplace transform gives

$$i_C(t) = \frac{3}{4}e^{-2t}\sin(4t)u(t)\ \text{A}$$

(checked using LNAP 4/11/01)

P 14.7-10 The voltage source voltage in the circuit shown in Figure P 14.7-10 is

$$v_s(t) = 12 - 6u(t) \text{ V}$$

Determine $v(t)$ for $t \geq 0$.

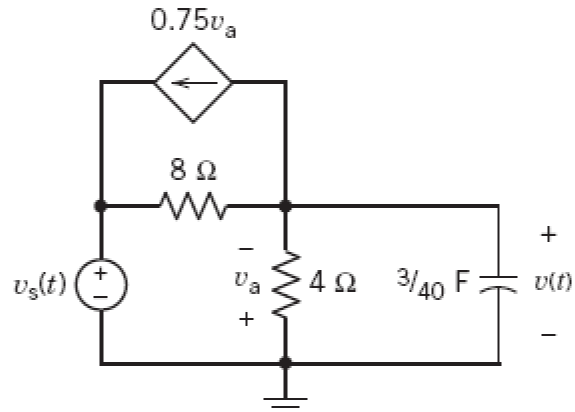
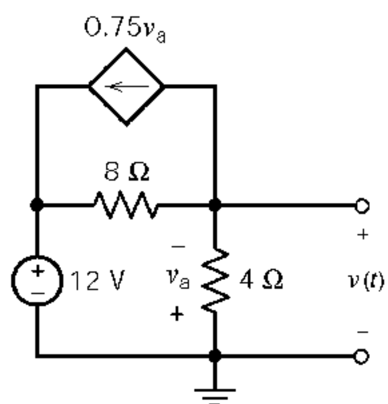


Figure P 14.7-10

Solution:

For $t < 0$, The input is 12 V. At steady state, the capacitor acts like an open circuit.



Notice that $v(t)$ is a node voltage. Express the controlling voltage of the dependent source as a function of the node voltage:

$$v_a = -v(t)$$

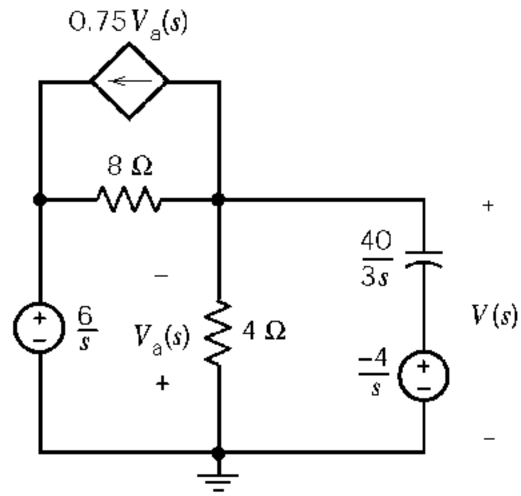
Writing a node equation:

$$-\left(\frac{12 - v(t)}{8}\right) + \frac{v(t)}{4} + \left(-\frac{3}{4}v(t)\right) = 0$$

$$-12 + v(t) + 2\,v(t) - 6\,v(t) = 0 \quad \Rightarrow \quad v(t) = -4 \text{ V}$$

$$v(0+) = v(0-) = -4 \text{ V}$$

For $t < 0$, represent the circuit in the frequency domain using the Laplace transform as shown.



$V(s)$ is a node voltage. Express the controlling voltage of the dependent source in terms of the node voltages

$$V_a(s) = -V(s)$$

Writing a node equation gives

$$\frac{V(s)-\frac{6}{s}}{8} + \frac{V(s)}{4} + \frac{3s}{40}\left(V(s)+\frac{4}{s}\right) = 0.75\,V(s)$$

Solving gives

$$(s-5)V(s) = \frac{10}{s} - 4 \quad \Rightarrow \quad V(s) = \frac{10}{s(s-5)} - \frac{4}{s-5} = \frac{-2}{s} + \frac{2}{s-5} - \frac{4}{s-5} = -2\left(\frac{1}{s} + \frac{1}{s-5}\right)$$

Taking the inverse Laplace transform gives

$$v(t) = -2\left(1+e^{5t}\right) \text{ V} \quad \text{for } t \ge 0$$

This voltage becomes very large as time goes on.

P 14.7-11 Determine the output voltage, $v_o(t)$, in the circuit shown in Figure P 14.7-11.

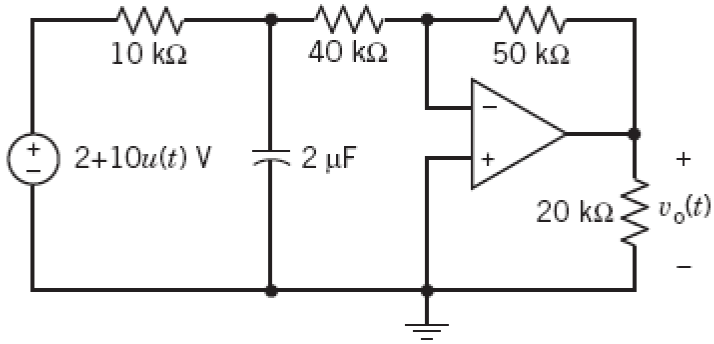
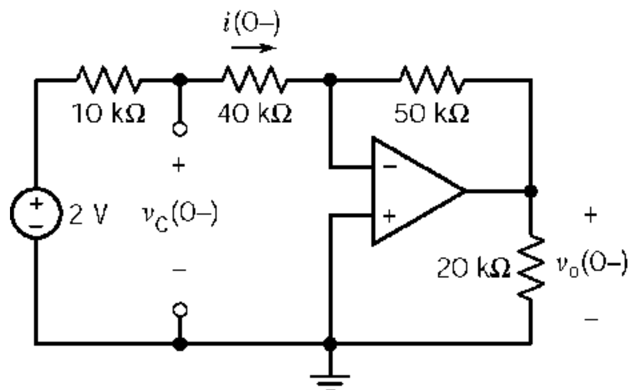


Figure P 14.7-11

Solution:

For $t < 0$, the voltage source voltage is 2 V and the circuit is at steady state. At steady state, the capacitor acts like an open circuit.



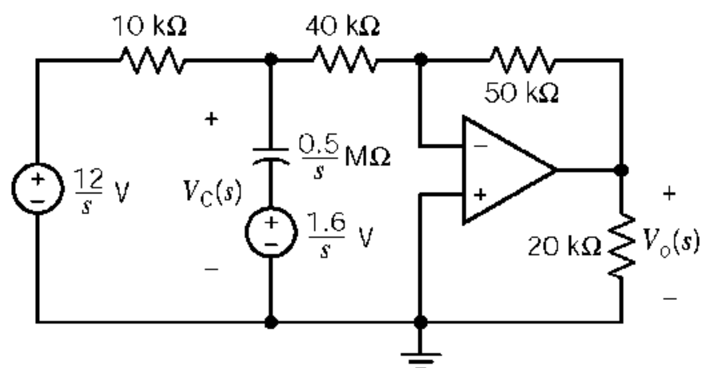
$$i(0-) = \frac{2-0}{10\times10^3 + 40\times10^3} = 0.04 \text{ mA}$$

and

$$v_C(0-) = (40\times10^3)(0.04\times10^{-3}) = 1.6 \text{ V}$$

The capacitor voltage is continuous so $v_C(0+) = v_C(0-)$.

For $t > 0$, the voltage source voltage is 12 V. Represent the circuit in the frequency domain using the Laplace transform as shown.



$V_C(s)$ and $V_o(s)$ are node voltages. Writing a node equation gives

$$\frac{V_C(s)-\frac{12}{s}}{10\times10^3} + \frac{V_C(s)-\frac{1.6}{s}}{\frac{0.5\times10^6}{s}} + \frac{V_C(s)}{40\times10^3} = 0 \Rightarrow 4\left(V_C(s)-\frac{12}{s}\right) + 0.08\,s\left(V_C(s)-\frac{1.6}{s}\right) + V_C(s) = 0$$

$$V_C(s)(0.08s+5) = \frac{48}{s} + 0.128 \Rightarrow V_C(s) = \frac{80s+48}{s(0.08s+5)} = \frac{1.6s+600}{s(s+62.5)} = \frac{9.6}{s} + \frac{-8}{s+62.5}$$

Taking the inverse Laplace transform gives

$$v_C(t) = 9.6 - 8\,e^{-62.5t} \text{ V} \quad \text{for } t \geq 0$$

The 40 kΩ resistor, 50 kΩ resistor and op amp comprise an inverting amplifier so

$$v_o(t) = -\frac{50}{40} v_C(t) = -\frac{50}{40}\left(9.6 - 8\,e^{-62.5t}\right) = -12 + 10\,e^{-62.5t} \text{ V} \quad \text{for } t \geq 0$$

so

$$v_o(t) = \begin{cases} -2 \text{ V} & \text{for } t \leq 0 \\ -12 + 10\,e^{-62.5t} \text{ V} & \text{for } t \geq 0 \end{cases}$$

(checked using LNAP 10/11/04)

P 14.7-12 Determine the capacitor voltage, $v(t)$, in the circuit shown in Figure P 14.7-12.

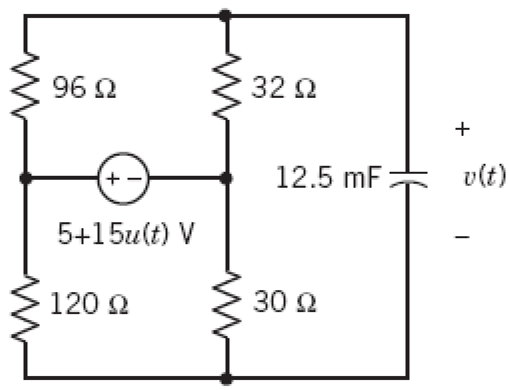
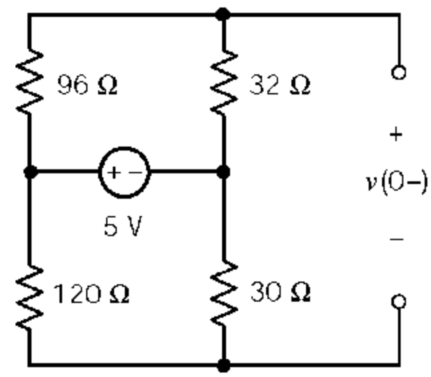


Figure P 14.7-12

Solution:

For $t < 0$, the voltage source voltage is 5 V and the circuit is at steady state. At steady state, the capacitor acts like an open circuit. Using voltage division twice



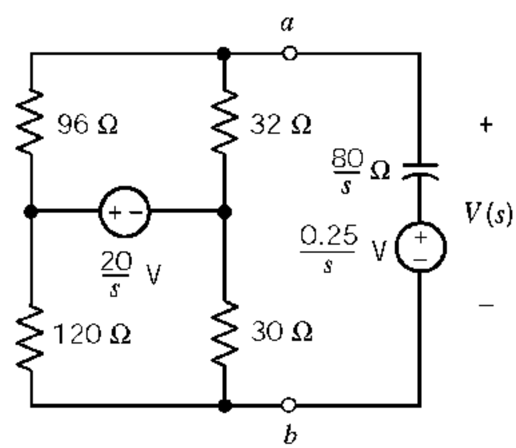
$$v(0-) = \frac{32}{32+96}5 - \frac{30}{120+30}5 = 0.25 \text{ V}$$

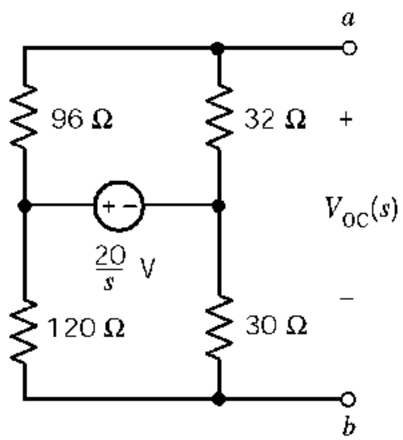
and

$$v(0+) = v(0-) = 0.25 \text{ V}$$

For $t > 0$, the voltage source voltage is 20 V. Represent the circuit in the frequency domain using the Laplace transform as shown.

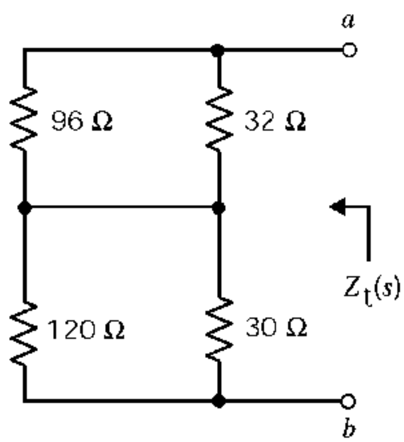
We could write mesh or node equations, but finding a Thevenin equivalent of the part of the circuit to the left of terminals *a*-*b* seems promising.





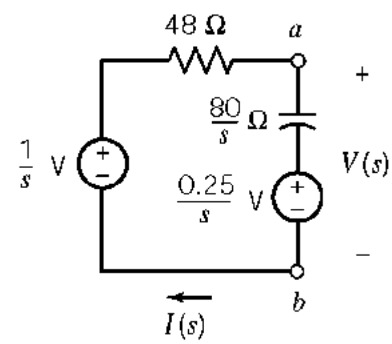
Using voltage division twice

$$V_{oc}(s) = \left(\frac{32}{32+96}\right)\frac{20}{s} - \left(\frac{30}{120+30}\right)\frac{20}{s} = \frac{5-4}{s} = \frac{1}{s} \text{ V}$$



$$Z_t = (96 \parallel 32) + (120 \parallel 30) = 24 + 24 = 48\ \Omega$$

After replacing the part of the circuit to the left of terminals *a*-*b* by its Thevenin equivalent circuit as shown



$$I(s) = \frac{\frac{1}{s} - \frac{0.25}{s}}{48 + \frac{80}{s}} = \frac{0.75}{48s+80}$$

$$V(s) = \frac{80}{s} I(s) + \frac{0.25}{s} = \left(\frac{80}{s}\right)\frac{0.75}{48s+80} + \frac{0.25}{s}$$

$$V(s) = \frac{60}{s(48s+80)} + \frac{0.25}{s} = \frac{1.25}{s(s+1.67)} + \frac{0.25}{s} = \frac{0.75}{s} + \frac{-0.75}{s+1.67} + \frac{0.25}{s} = \frac{1}{s} + \frac{-0.75}{s+1.67}$$

Taking the inverse Laplace transform gives

$$v(t) = 1 - 0.75e^{-1.67t} \text{ V} \quad \text{for } t \ge 0$$

Then

$$v(t) = \begin{cases} 0.25 \text{ V} & \text{for } t \le 0 \\ 1 - 0.75e^{-1.67t} \text{ V} & \text{for } t \ge 0 \end{cases}$$

(checked using LNAP 7/1/04)

P 14.7-13 Determine the voltage $v_o(t)$ for $t \ge 0$ for the circuit of Figure P 14.7-13.

Hint: $v_C(0) = 4$ V

Answer: $v_o(t) = 24e^{0.75t}\, u(t)$ V (This circuit is unstable.)

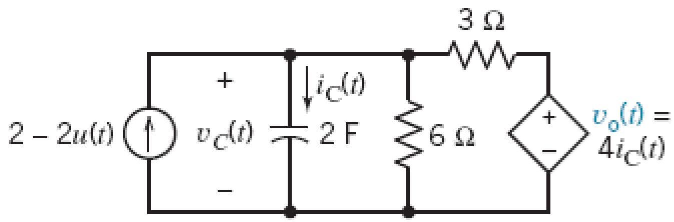
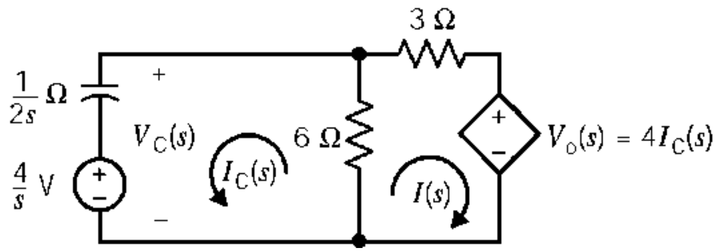


Figure P 14.7-13

Solution:



Mesh Equations:

$$-\frac{4}{s}-\frac{1}{2s}I_C(s)-6\big(I(s)+I_C(s)\big)=0 \;\Rightarrow\; -\frac{4}{s}=\left(6+\frac{1}{2s}\right)I_C(s)+6I(s)$$

$$6\big(I(s)-I_C(s)\big)+3I(s)+4I_C(s)=0 \;\Rightarrow\; I(s)=-\frac{10}{9}I_C(s)$$

Solving for $I_C(s)$:

$$-\frac{4}{s}=\left(-\frac{2}{3}+\frac{1}{2s}\right)I_C(s) \;\Rightarrow\; I_C(s)=\frac{6}{s-\frac{3}{4}}$$

So $V_o(s)$ is

$$V_o(s)=4I_C(s)=\frac{24}{s-\frac{3}{4}}$$

Back in the time domain:

$$v_o(t)=24e^{0.75t}\,u(t)\text{ V}\quad\text{for } t\ge 0$$

P 14.7-14 Determine the current $i_L(t)$ for $t \geq 0$ for the circuit of Figure P 14.7-14.

Hint: $v_C(0) = 8$ V and $i_L(0) = 1$ A

Answer: $i_L(t) = \left(e^{-t}\cos 2t + \frac{1}{2}e^{-t}\sin 2t\right)u(t)\,\text{A}$

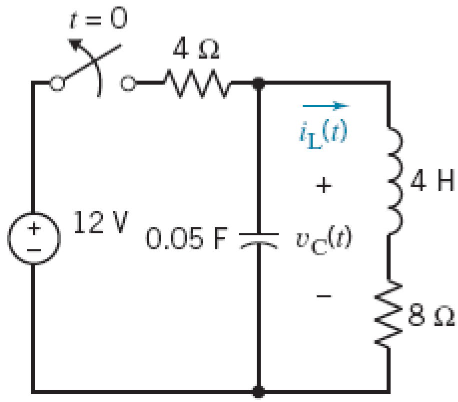
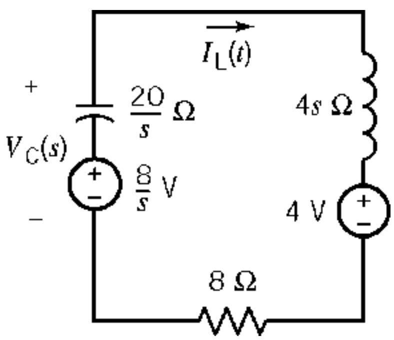


Figure P 14.7-14

Solution:



KVL:

$$\frac{8}{s} + 4 = \left(\frac{20}{s} + 8 + 4s\right) I_L(s)$$

so

$$I_L(s) = \frac{2+s}{s^2+2s+5} = \frac{(s+1)+1}{(s+1)^2+4}$$

Taking the inverse Laplace transform:

$$i_L(t) = \left(e^{-t}\cos 2t + \frac{1}{2}e^{-t}\sin 2t\right)u(t)\ \text{A}$$

P14.7-15

The circuit shown in Figure P14.7-23 is at steady state before the switch opens at time $t = 0$. Determine the inductor voltage $v(t)$ for $t > 0$.

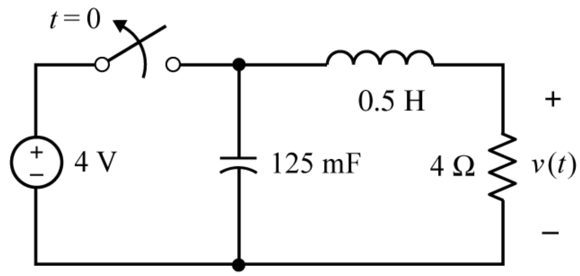


Figure P14.7-15

Solution:

The circuit shown in Figure P14.7-23 is at steady state before the switch opens at time $t = 0$. Determine the inductor voltage $v(t)$ for $t > 0$.

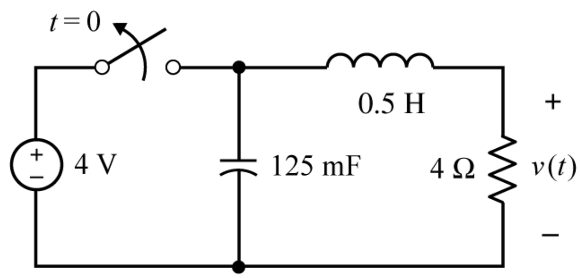
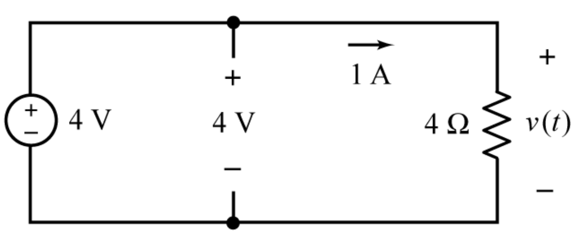
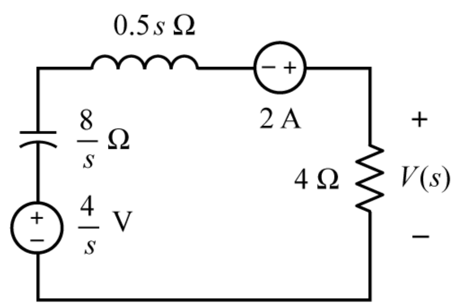


Figure P14.7-15

Determine the initial conditions, i.e. the inductor current and capacitor voltage at $t = 0$, as shown in the circuit on the left below. Use those initial conditions to represent the circuit in the s-domain as s shown in the circuit on the left below.





Analysis of the s-domain circuit shows that

$$V(s) = \left(\frac{4}{0.5s + 4 + \frac{8}{s}} \right) \left(\frac{4}{s} + 2 \right) = \frac{16(s+2)}{s^2 + 8s + 16} = \frac{16(s+2)}{(s+4)^2}$$

Performing the partial fraction expansion, we get

$$\frac{16(s+2)}{(s+4)^2} = \frac{k}{s+4} + \frac{-32}{(s+4)^2} \quad \Rightarrow \quad 16(s+2) = k(s+2) - 32 \quad \Rightarrow \quad k = 16$$

Finally

$$V(s) = \frac{16}{s+4} + \frac{-32}{(s+4)^2}$$

Now use linearity, $e^{-at} f(t) \leftrightarrow F(s+a)$ and $t \leftrightarrow \frac{1}{s^2}$ to find the inverse Laplace transform

$$v(t) = 16 e^{-4t} \mathscr{L}^{-1}\left[\frac{1}{s} - \frac{2}{s^2}\right] = 16(1-2t)e^{-4t} \text{ for } t > 0$$

P14.7-16
The circuit shown in Figure P14.7-16 is at steady state before time $t = 0$. Determine the resistor voltage $v(t)$ for $t > 0$.

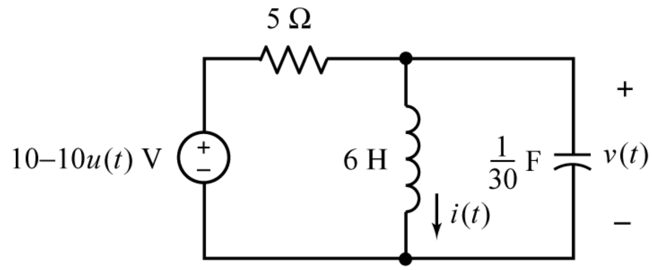


Figure P14.7-16

Solution:
The circuit shown in Figure 14.7-16 is at steady state before time $t = 0$. Determine the resistor voltage $v(t)$ for $t > 0$.

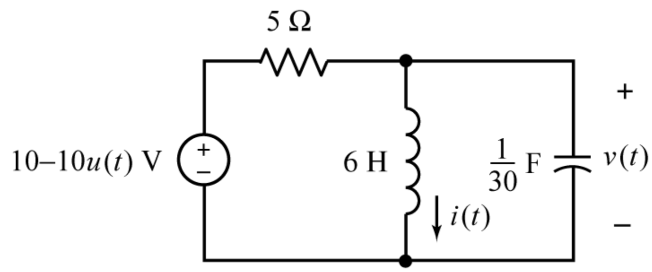
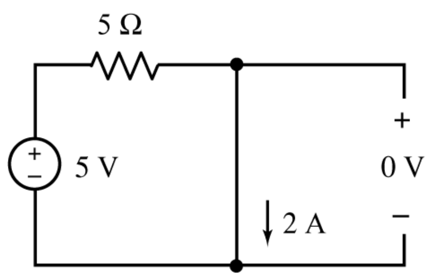
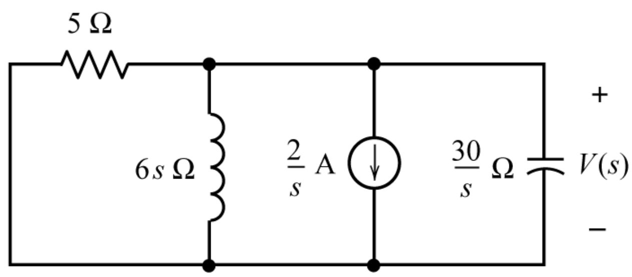


Figure P14.7-16

Determine the initial conditions, i.e. the inductor current and capacitor voltage at $t = 0$, as shown in the circuit on the left below. Use those initial conditions to represent the circuit in the s-domain as s shown in the circuit on the left below.





Analysis of the s-domain circuit shows that

$$\frac{V(s)}{5}+\frac{V(s)}{6s}+\frac{2}{s}+\left(\frac{s}{30}\right)V(s)=0 \quad\Rightarrow\quad \left(6s+5+s^2\right)V(s)=-60 \quad\Rightarrow\quad V(s)=\frac{-60}{s^2+6s+5}$$

Performing the partial fraction expansion, we get

$$V(s)=\frac{-60}{s^2+6s+5}=\frac{-15}{s+1}+\frac{15}{s+5}$$

The inverse Laplace transform is

$$v(t)=15\left(e^{-5t}-e^{-t}\right)\text{ V for } t>0$$

P14.7.17

The input to the circuit shown in Figure P14.7-17 is the voltage source voltage

$$v_i(t) = 10 + 5u(t) \text{ V} = \begin{cases} 10 \text{ V when } t < 0 \\ 15 \text{ V when } t > 0 \end{cases}$$

Determine the response, $v_o(t)$. Assume that the circuit is at steady state when $t < 0$. Sketch $v_o(t)$ as a function of t.

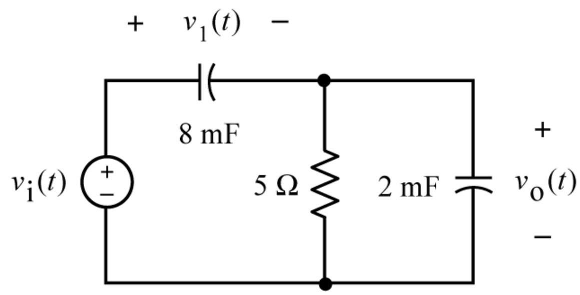
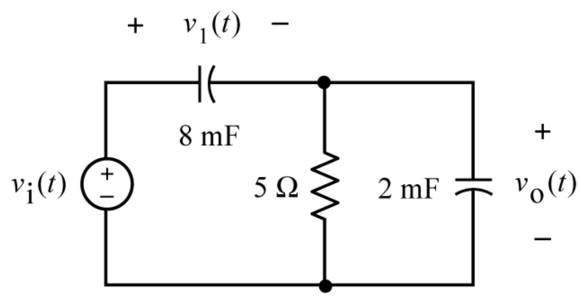


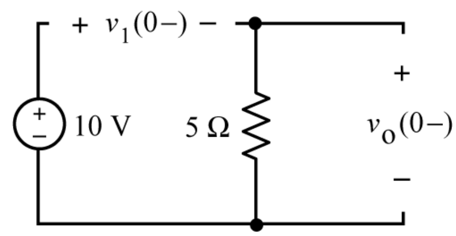
Figure P14.7-17

Solution:

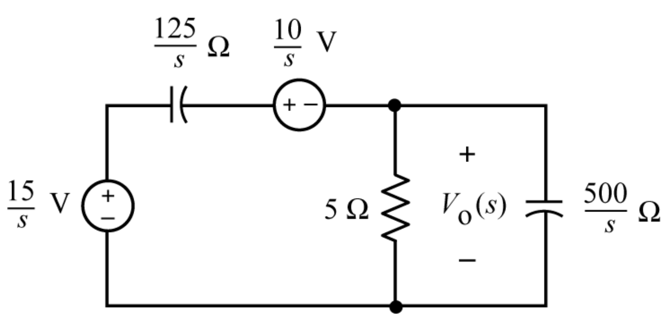
$$v_i(t) = 10 + 5u(t) \text{ V} = \begin{cases} 10 \text{ V when } t < 0 \\ 15 \text{ V when } t > 0 \end{cases}$$



The circuit is at steady state when $t < 0$ so the capacitors act like open circuits. Since the current in the resistor is 0 A, $v_o(0-) = 0$ V. Then KVL gives $v_1(0-) = 10$ V.



Use the initial conditions to represent the circuit for t>0 in the s-domain:



Calculate

$$5 \| \frac{500}{s} = \frac{500}{s+100}$$

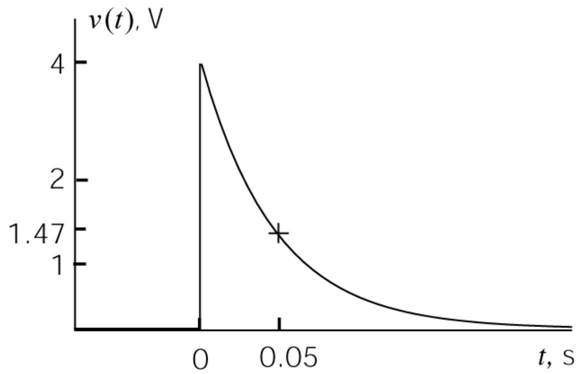
and

$$\frac{5 \| \frac{500}{s}}{\frac{125}{s} + 5 \| \frac{500}{s}} = \frac{0.8s}{s+20}$$

Use voltage division to write

$$V_o(s) = \frac{5 \| \frac{500}{s}}{\frac{125}{s} + 5 \| \frac{500}{s}} \left(\frac{15}{s} - \frac{10}{s} \right) = \frac{0.8s}{s+20} \left(\frac{5}{s} \right) = \frac{4}{s+20}$$

Taking the inverse Laplace transforms gives $v_o(t) = 4e^{-20t}$ V when $t > 0$.



In summary

$$v_o(t) = \begin{cases} 0\ \text{V} & \text{when } t < 0 \\ 4e^{-20t}\ \text{V} & \text{when } t > 0 \end{cases}$$

Let $t = 0.05$ s and calculate

$$v_o(0.05) = 4e^{-20(0.05)} = 1.47\ \text{V}$$

P14.7-18

The input to the circuit shown in Figure P14.7-18 is the current source current

$$i(t) = 25 - 15u(t) \text{ mA} = \begin{cases} 25 \text{ mA when } t < 0 \\ 10 \text{ mA when } t > 0 \end{cases}$$

Determine the response, $i_2(t)$. Assume that the circuit is at steady state when $t < 0$. Sketch $i_2(t)$ as a function of t.

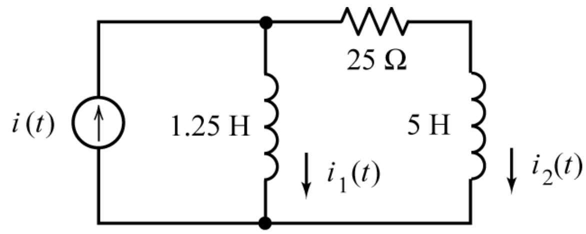
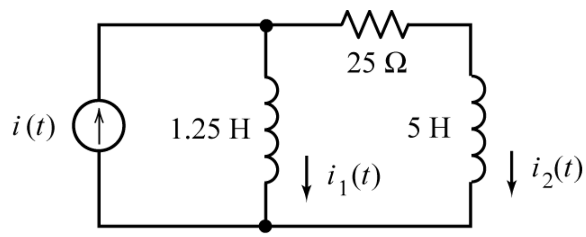


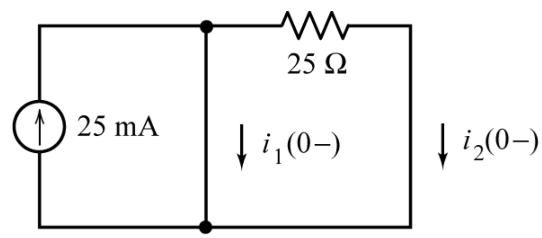
Figure P14.7-18

Solution:

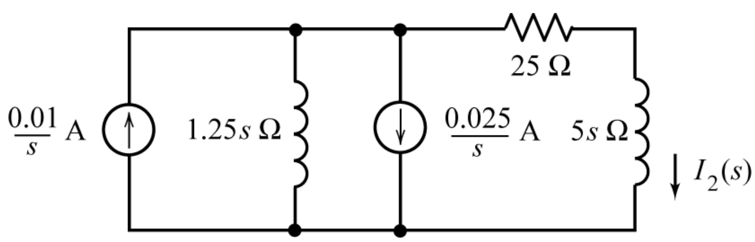
$$i(t) = 25 - 15u(t) \text{ mA} = \begin{cases} 25 \text{ mA when } t < 0 \\ 10 \text{ mA when } t > 0 \end{cases}$$



The circuit is at steady state when $t < 0$ so the inductors act like short circuits. Since the voltage across the resistor is 0 V, $i_2(0-) = 0$ V. Then KCL gives $i_1(0-) = 25$ mA.



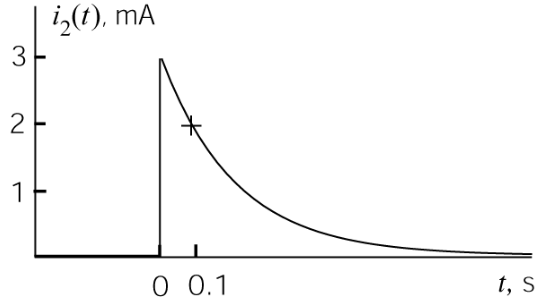
Use the initial conditions to represent the circuit for t>0 in the s-domain:



Use current division to write

$$I_2(s) = \frac{1.25s}{1.25s + (25 + 5s)}\left(\frac{0.01}{s} - \frac{0.025}{s}\right) = \frac{0.2s}{s+4}\left(-\frac{0.015}{s}\right) = \frac{-0.003}{s+20}$$

Taking the inverse Laplace transforms gives $i_2(t) = 3e^{-4t}$ mA when $t > 0$.



In summary

$$i_2(t) = \begin{cases} 0 \text{ mA} & \text{when } t < 0 \\ 3e^{-4t} \text{ mA} & \text{when } t > 0 \end{cases}$$

Let $t = 0.1$ s and calculate

$$i_2(0.1) = 3e^{-4(0.1)} = 2.01 \text{ mA}$$

P 14.7-19 All new homes are required to install a device called a ground fault circuit interrupter (GFCI) that will provide protection from shock. By monitoring the current going to and returning from a receptacle, a GFCI senses when normal flow is interrupted and switches off the power in 1/40 second. This is particularly important if you are holding an appliance shorted through your body to ground. A circuit model of the GFCI acting to interrupt a short is shown in Figure P 14.7-19. Find the current flowing through the person and the appliance, $i(t)$, for $t \geq 0$ when the short is initiated at $t = 0$. Assume $v = 160 \cos 400t$ and the capacitor is intially uncharged.

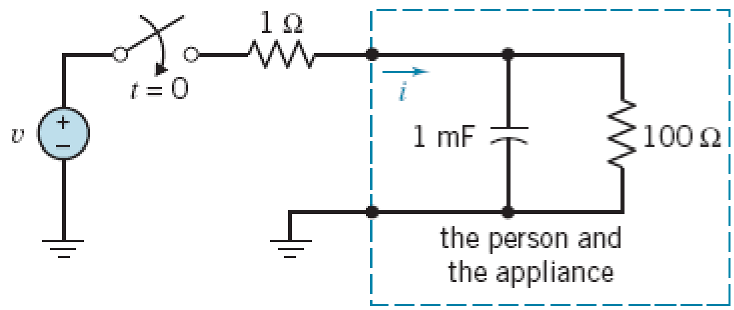
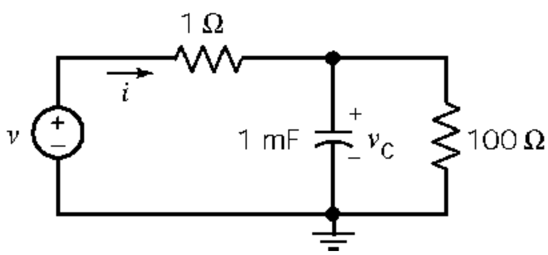


FIGURE P 14.7-19 Circuit model of person and appliance shorted to ground

Solution:



We are given $v(t) = 160\cos 400t$.

The capacitor is initially uncharged, so $v_C(0) = 0$ V. Then

$$i(0) = \frac{160\cos(400\times 0) - 0}{1} = 160 \text{ A}$$

KCL yields
$$10^{-3}\frac{d\,v_C}{dt} + \frac{v_C}{100} = i$$

Apply Ohm's law to the 1 Ω resistor to get

$$i = \frac{v - v_C}{1} \quad \Rightarrow \quad v_C = v - i$$

Solving yields
$$\frac{d\,i}{dt} + 1010\,i = 1600\cos 400t - \left(6.4\times 10^4\right)\sin 400t$$

Taking the Laplace transform yields

$$s\,I(s) - i(0) + (1010)\,I(s) = \frac{1600s}{s^2 + (400)^2} - \frac{\left(6.4\times 10^2\right)(400)}{s^2 + (400)^2}$$

so
$$I(s) = \frac{160}{s+1010} + \frac{1600s - 2.5\times 10^7}{(s+1010)\left[s^2 + (400)^2\right]}$$

Next

$$\frac{1600s - 2.5\times10^7}{(s+1010)\left[s^2+(400)^2\right]} = \frac{A}{s+1010} + \frac{B}{s+j400} + \frac{B^*}{s-j400}$$

where

$$A = \left.\frac{1600s-2.5\times10^7}{s^2+(400)^2}\right|_{s=-1010} = -23.1,$$

$$B = \left.\frac{1600s-2.5\,x\,10^7}{(s+1010)(s-j400)}\right|_{s=-j400} = \frac{2.56\times10^7\angle1.4^\circ}{8.69\times10^5\angle68.4^\circ} = 11.5 - j27.2 \text{ and } B^* = 11.5 + j27.2$$

Then

$$I(s) = \frac{136.9}{s+1010} + \frac{11.5-j27.2}{s+j400} + \frac{11.5+j27.2}{s-j400}$$

Finally

$$\begin{aligned} i(t) &= 136.9e^{-1010t} + 2(11.5)\cos 400t - 2(27.2)\sin 400t \quad \text{for} \quad t>0 \\ &= 136.9e^{-1010t} + 23.0\cos 400t - 54.4\sin 400t \quad \text{for} \quad t>0 \end{aligned}$$

P 14.7-20 Using the Laplace transform, find $v_c(t)$ for $t > 0$ for the circuit shown in Figure P 14.7-20. The initial conditions are zero.

Hint: Use a source transformation to obtain a single mesh circuit.

Answer: $v_c = -5e^{-2t} + 5(\cos 2t + \sin 2t)$ V

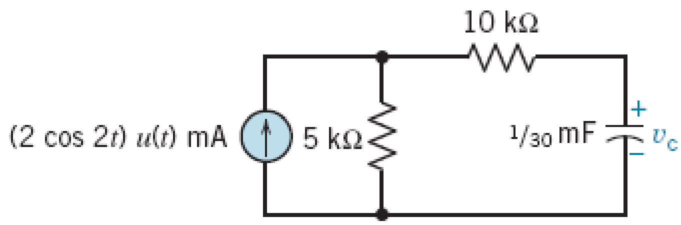
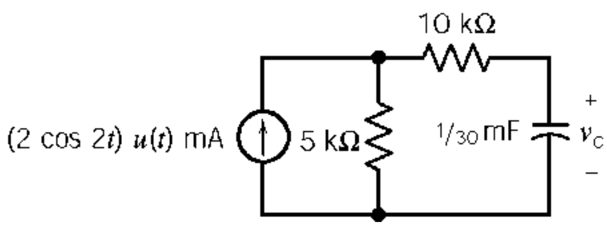
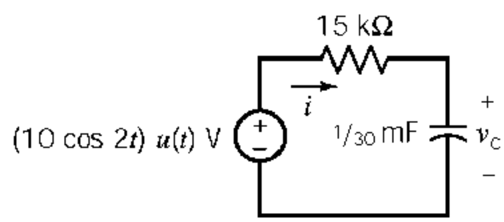


Figure P 14.7-20.

Solution:





$v_C(0) = 0$

$$\left.\begin{aligned} v_c + 15\times10^3\, i &= 10\cos 2t \\ i &= \left(\frac{1}{30}\times10^{-3}\right)\frac{d\,v_c}{dt} \end{aligned}\right\} \Rightarrow \frac{d\,v_c}{dt} + 2\,v_c = 20\cos 2t$$

Taking the Laplace Transform yields:

$$sV_C(s) - v_C(0) + 2V_C(s) = 20\frac{s}{s^2+4} \quad \Rightarrow \quad V_C(s) = \frac{20s}{(s+2)(s^2+4)} = \frac{A}{s+2} + \frac{B}{s+j2} + \frac{B^*}{s-j2}$$

where

$$A = \left.\frac{20s}{s^2+4}\right|_{s=-2} = \frac{-40}{8} = -5, \quad B = \left.\frac{20s}{(s+2)(s-j2)}\right|_{s=-j2} = \frac{5}{1-j} = \frac{5}{2} + j\frac{5}{2} \text{ and } B^* = \frac{5}{2} - j\frac{5}{2}$$

Then

$$V_C(s) = \frac{-5}{s+2} + \frac{\frac{5}{2}+j\frac{5}{2}}{s+j2} + \frac{\frac{5}{2}-j\frac{5}{2}}{s-j2} \quad \Rightarrow \quad v_C(t) = -5e^{-2t} + 5(\cos 2t + \sin 2t) \text{ V}$$

P 14.7-21 Determine the inductor current, *i*(*t*), in the circuit shown in Figure P 14.7-21.

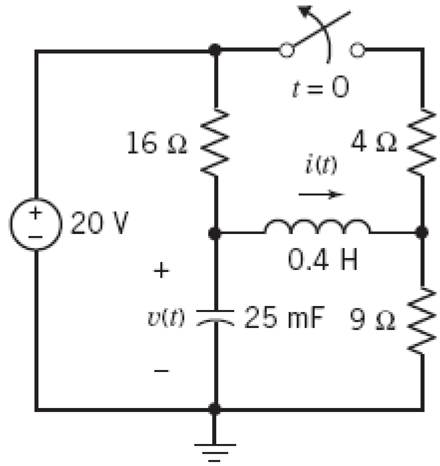
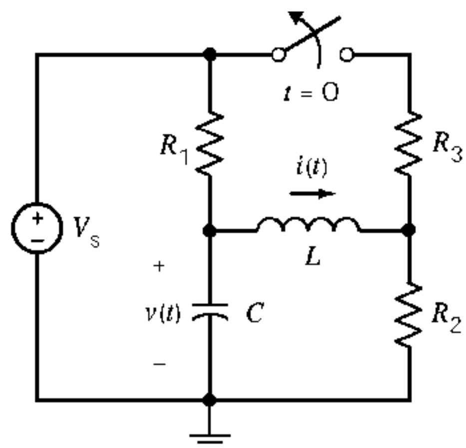


Figure P 14.7-21

Solution:

After the switch opens, apply KCL and KVL to get

$$R_1\left(i(t)+C\frac{d}{dt}v(t)\right)+v(t)=V_s$$



Apply KVL to get

$$v(t)=L\frac{d}{dt}i(t)+R_2\,i(t)$$

Substituting $v(t)$ into the first equation gives

$$R_1\left(i(t)+C\frac{d}{dt}\left(L\frac{d}{dt}i(t)+R_2\,i(t)\right)\right)+L\frac{d}{dt}i(t)+R_2\,i(t)=V_s$$

then

$$R_1\,C\,L\frac{d^2}{dt^2}i(t)+\left(R_1\,C\,R_2+L\right)\frac{d}{dt}i(t)+\left(R_1+R_2\right)i(t)=V_s$$

Dividing by $R_1\,C\,L$:

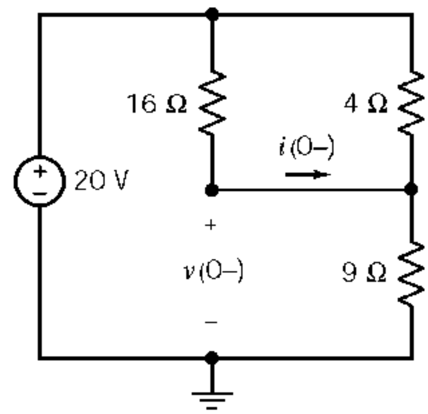
$$\frac{d^2}{dt^2}i(t)+\left(\frac{R_1\,C\,R_2+L}{R_1\,C\,L}\right)\frac{d}{dt}i(t)+\left(\frac{R_1+R_2}{R_1\,C\,L}\right)i(t)=\frac{V_s}{R_1\,C\,L}$$

With the given values: $$\frac{d^2}{dt^2}i(t)+25\frac{d}{dt}i(t)+156.25\,i(t)=125$$

Taking the Laplace transform:

$$\left[s^2 I(s)-\left(\frac{d}{dt}i(0+)+s\,i(0+)\right)\right]+25\left[s\,I(s)-i(0+)\right]+156.25\,I(s)=\frac{125}{s}$$

We need the initial conditions. For $t < 0$, the switch is closed and the circuit is at steady state. At steady state, the capacitor acts like an open circuit and the inductor acts like a short circuit. Using voltage division



$$v(0-)=\frac{9}{9+(16\,\|\,4)}\,20=14.754 \text{ V}$$

Then, using current division

$$i(0-)=\left(\frac{4}{16+4}\right)\frac{v(0-)}{9}=0.328 \text{ A}$$

The capacitor voltage and inductor current are continuous so $v(0+)=v(0-)$ and $i(0+)=i(0-)$. After the switch opens

$$v(t)=L\frac{d}{dt}i(t)+R_2\,i(t) \quad \Rightarrow \quad \frac{d}{dt}i(0+)=\frac{v(0+)}{0.4}+\frac{9\,i(0+)}{0.4}=\frac{14.754}{0.4}+\frac{9\,(0.328)}{0.4}=29.508$$

Substituting these initial conditions into the Laplace transformed differential equation gives

$$\left[s^2 I(s)-(29.508+0.328\,s)\right]+25\left[s\,I(s)-0.328\right]+156.25\,I(s)=\frac{125}{s}$$

$$\left(s^2+25\,s+156.25\right)I(s)=\frac{125}{s}+(29.508+0.328\,s)+25\,(0.328)$$

so

$$I(s)=\frac{0.328\,s^2+\left(29.508+25\,(0.328)\right)+125}{s\left(s^2+25\,s+156.25\right)}$$

$$=\frac{0.328\,s^2+\left(29.508+25\,(0.328)\right)+125}{s\,(s+12.5)^2}=\frac{-0.471}{s+12.5}+\frac{23.6}{(s+12.5)^2}+\frac{0.8}{s}$$

Taking the inverse Laplace transform

$$i(t)=0.8+e^{-12.5t}\,(23.6\,t-0.471) \text{ A} \quad \text{for } t\ge 0$$

So

$$i(t)=\begin{cases}0.328 \text{ A} & \text{for } t\le 0\\ 0.8+e^{-12.5t}\,(23.6\,t-0.471) \text{ A} & \text{for } t\ge 0\end{cases}$$

(checked using LNAP 10/11/04)

P 14.7-22 Find $v_2(t)$ for the circuit of Figure P 14.7-22 for $t \geq 0$.

Hint: Write the node equations at a and b in terms of v_1 and v_2. The initial conditions are $v_1(0) = 10$ V and $v_2(0) = 25$ V. The source is $v_s = 50 \cos 2t\, u(t)$ V.

Answer: $v_2(t) = \frac{23}{3}e^{-t} + \frac{16}{3}e^{-4t} + 12 \cos 2t + 12 \sin 2t \text{ V } t \geq 0$

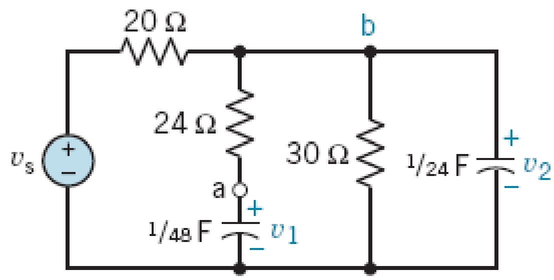
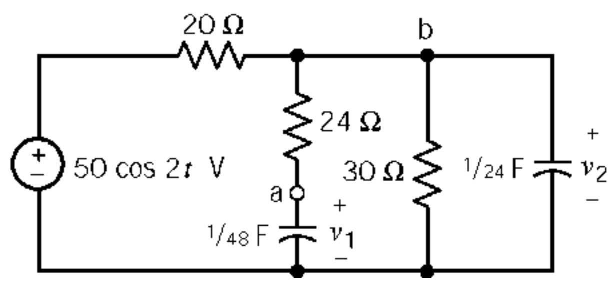


Figure P 14.7-22

Solution:



Apply KCL at node a to get

$$\frac{1}{48}\frac{d\,v_1}{dt} = \frac{v_2 - v_1}{24} \quad \Rightarrow \quad 2\,v_1 + \frac{d\,v_1}{dt} = 2\,v_2$$

Apply KCL at node b to get

$$\frac{v_2 - 50\cos 2t}{20} + \frac{v_2 - v_1}{24} + \frac{v_2}{30} + \frac{1}{24}\frac{d\,v_2}{dt} = 0 \quad \Rightarrow \quad -v_1 + 3\,v_2 + \frac{d\,v_2}{dt} = 60\cos 2t$$

Take the Laplace transforms of these equations, using $v_1(0) = 10$ V and $v_2(0) = 25$ V, to get

$$(2+s)\,V_1(s) - 2\,V_2(s) \;=\; 10 \quad \text{and} \quad -V_1(s) + (3+s)\,V_2(s) = \frac{25s^2 + 60s + 100}{s^2 + 4}$$

Solve these equations using Cramer's rule to get

$$V_2(s) = \frac{(2+s)\left(\dfrac{25s^2+60s+100}{s^2+4}\right)+10}{(2+s)\;(3+s)-2} = \frac{(2+s)\left(25s^2+60s+100\right)+10\left(s^2+4\right)}{\left(s^2+4\right)(s+1)(s+4)}$$

$$= \frac{25s^3+120s^2+220s+240}{\left(s^2+4\right)(s+1)(s+4)}$$

Next, partial fraction expansion gives

$$\mathrm{V}_2(s) = \frac{A}{s+j2} + \frac{A^*}{s-j2} + \frac{B}{s+1} + \frac{C}{s+4}$$

where

$$A = \left.\frac{25s^3+120s^2+220s+240}{(s+1)(s+4)(s-j2)}\right|_{s=-j2} = \frac{-240-j240}{-40} = 6+j6$$

$$A^* = 6-j6$$

$$B = \left.\frac{25s^3+120s^2+220s+240}{(s^2+4)(s+4)}\right|_{s=-1} = \frac{115}{15} = \frac{23}{3}$$

$$C = \left.\frac{25s^3+120s^2+220s+240}{(s^2+4)(s+1)}\right|_{s=-4} = \frac{-320}{-60} = \frac{16}{3}$$

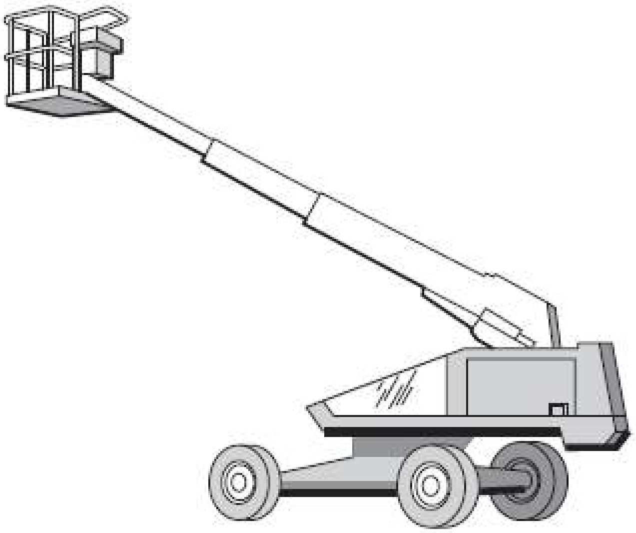
Then

$$V_2(s) = \frac{6+j6}{s+j2} + \frac{6-j6}{s-j2} + \frac{23/3}{s+1} + \frac{16/3}{s+4}$$

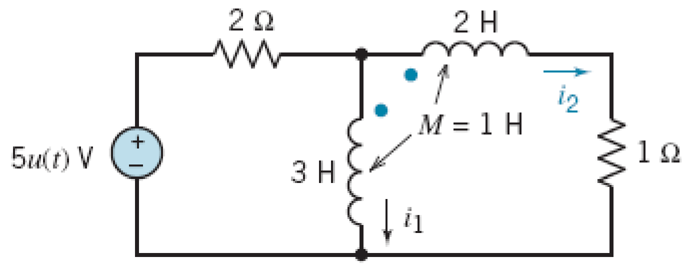
Finally

$$v_2(t) = 12\cos 2t + 12\sin 2t + \frac{23}{3}e^{-t} + \frac{16}{3}e^{-4t} \text{ V} \quad t \geq 0$$

P 14.7-23 The motor circuit for driving the snorkel shown in Figure P 14.7-23*a* is shown in Figure P 14.7-23*b*. Find the motor current $I_2(s)$ when the initial conditions are $i_1(0-) = 2$ A and $i_2(0^-) = 3$ A. Determine $i_2(t)$ and sketch it for 10 s. Does the motor current smoothly drive the snorkel?



(*a*)



(*b*)

Solution:

Here are the equations describing the coupled coils:

$$v_1(t) = L_1 \frac{di_1}{dt} + M\frac{di_2}{dt} \quad \Rightarrow \quad V_1(s) = 3\left(s\,I_1(s)-2\right)+\left(sI_2(s)-3\right) = 3s\,I_1(s)+sI_2(s)-9$$

$$v_2(t) = L_2 \frac{di_2}{dt} + M\frac{di_1}{dt} \quad \Rightarrow \quad V_2(s) = s\left(I_1(s)-2\right)+2\left(sI_2(s)-3\right) = sI_1(s)+2sI_2(s)-8$$

Writing mesh equations:

$$\frac{5}{s} = 2\left(I_1(s)+I_2(s)\right)+V_1 = 2\left(I_1(s)+I_2(s)\right)+3s\,I_1(s)+sI_2(s)-9 \quad \Rightarrow \quad \left(3s+2\right)I_1+\left(s+2\right)I_2 = 9+\frac{5}{s}$$

$$V_1(s) = V_2(s)+1I_2(s) \quad \Rightarrow \quad 3s\,I_1(s)+sI_2(s)-9 = sI_1(s)+2sI_2(s)-8+I_2(s) \quad \Rightarrow \quad 2s\,I_1-\left(s+1\right)I_2 = 1$$

Solving the mesh equations for $I_2(s)$:

$$I_2(s) = \frac{15s+8}{5s^2+9s+2} = \frac{3s+1.6}{\left(s+0.26\right)\left(s+1.54\right)} = \frac{0.64}{s+0.26} + \frac{2.36}{s+1.54}$$

Taking the inverse Laplace transform:

$$i_2(t) = 0.64e^{-0.26t} + 2.36e^{-1.54t} \quad \text{A for } t > 0$$

P14.7-24

Using Laplace transforms, find $v_o(t)$ for t > 0 for the circuit shown in Figure 14.7-24.

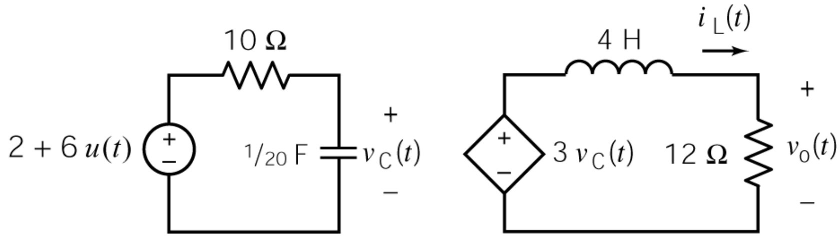
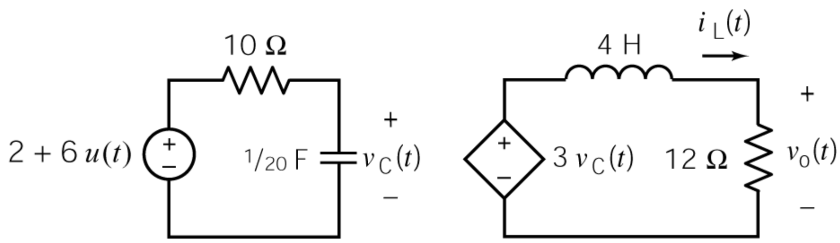


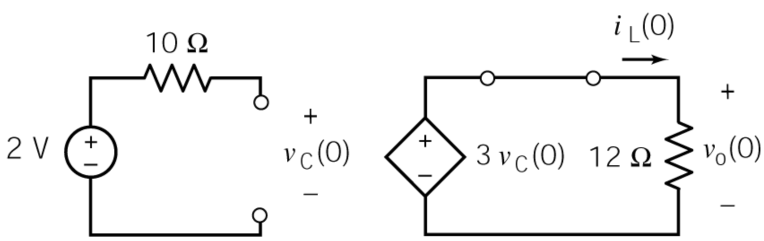
Figure 14.7-24

Solution:

Find $v_o(t)$ for t > 0 for this circuit:

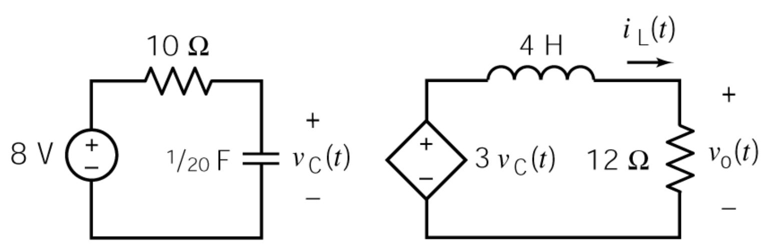


For t < 0, with the circuit at steady state, we have



$$v(0) = 2 \text{ V} \quad \text{and} \quad i(0) = \frac{1}{2} \text{ A}$$

For t > 0 we have



Apply KVL to the left mesh to get

$$10\left(\frac{1}{20}\frac{d}{dt}v_C(t)\right) + v_C(t) - 8 = 0 \quad \Rightarrow \quad \frac{1}{2}\frac{d}{dt}v_C(t) + v_C(t) = 8$$

Apply KVL to the right mesh to get

$$4\frac{d}{dt}i_L(t)+12i_L(t)-3v_C(t)=0 \quad\Rightarrow\quad 4\frac{d}{dt}i_L(t)+12i_L(t)=3v_C(t)$$

Take the Laplace transform of these differential equations to get

$$\frac{1}{2}\left[sV_C(s)-v_C(0)\right]+V_C(s)=\frac{8}{s} \quad\Rightarrow\quad \frac{1}{2}\left[sV_C(s)-2\right]+V_C(s)=\frac{8}{s} \quad\Rightarrow\quad V_C(s)=\frac{2s+16}{s(s+2)}$$

and

$$4\left[sI_L(s)-i_L(0)\right]+12I_L(s)=3V_C(s) \quad\Rightarrow\quad 4\left[sI_L(s)-\frac{1}{2}\right]+12I_L(s)=3V_C(s)$$

$$\Rightarrow\quad (s+3)I_L(s)=\frac{3}{4}V_C(s)+\frac{1}{2}$$

$$\Rightarrow\quad (s+3)I_L(s)=\frac{3}{4}\left(\frac{2s+16}{s(s+2)}\right)+\frac{1}{2}$$

$$\Rightarrow\quad I_L(s)=\frac{\frac{1}{2}s^2+\frac{5}{2}s+12}{s(s+2)(s+3)}$$

$$\Rightarrow\quad I_L(s)=\frac{2}{s}+\frac{-\frac{9}{2}}{s+2}+\frac{3}{s+3}$$

Taking the inverse Laplace transform gives

$$i_L(t)=\left(2-\frac{9}{2}e^{-2t}+3e^{-3t}\right)u(t)\ \text{ A}$$

Finally $$v_o(t)=12i_L(t)=\left(24-54e^{-2t}+36e^{-3t}\right)u(t)\ \text{ V}$$

(checked with LNAP 2/28/05)

P14.7-25

The circuit shown in Figure P14.7-25 is at steady state before the switch opens at time $t = 0$. Determine the inductor voltage $v(t)$ for $t > 0$.

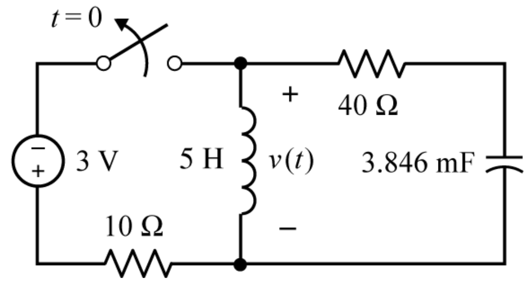
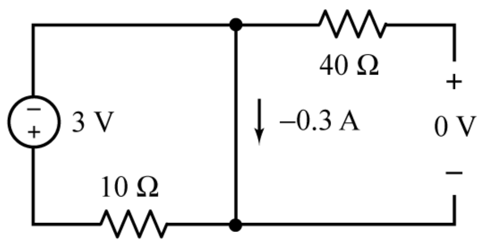
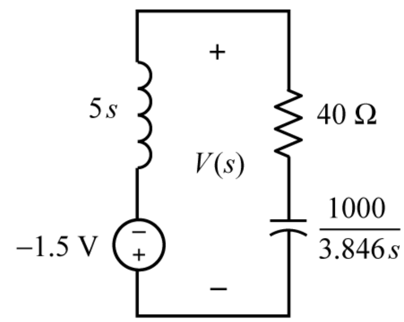


Figure P14.7-25

Solution:

Determine the initial conditions, i.e. the inductor current and capacitor voltage at $t = 0$, as shown in the circuit on the left below. Use those initial conditions to represent the circuit in the s-domain as shown in the circuit on the left below.





Analysis of the s-domain circuit shows that

$$V(s) = -\frac{(-1.5)\left(40+\dfrac{1000}{3.846\,s}\right)}{5\,s+40+\dfrac{1000}{3.846\,s}} = \frac{(1.5)\left(40\,s+\dfrac{1000}{3.846}\right)}{5\,s^2+40\,s+\dfrac{1000}{3.846}} = \frac{12\,s+78}{s^2+8\,s+52}$$

The denominator does not factor any further in the real numbers. Let's complete the square in the denominator

$$V(s) = \frac{12\,s+78}{s^2+8\,s+52} = \frac{12\,s+78}{\left(s^2+8\,s+16\right)+36} = \frac{12\,s+78}{(s+4)^2+36} = \frac{12(s+4)+30}{(s+4)^2+36} = \frac{12(s+4)}{(s+4)^2+6^2} + \frac{5(6)}{(s+4)^2+6^2}$$

Now use the property $e^{-at} f(t) \leftrightarrow F(s+a)$ and the Laplace transform pairs

$$\sin\omega t \;\text{ for } t \ge 0 \leftrightarrow \frac{\omega}{s^2+\omega^2} \quad \text{and} \quad \cos\omega t \;\text{ for } t \ge 0 \leftrightarrow \frac{s}{s^2+\omega^2}$$

to find the inverse Laplace transform

$$v(t) = e^{-4t}\,\mathscr{L}^{-1}\left[\frac{12\,s}{s^2+6^2} + \frac{5(6)}{s^2+6^2}\right] = e^{-4t}\left[\,12\cos(6t) + 5\sin(6t)\,\right] \text{ for } t > 0$$

P14.7-26

The circuit shown in Figure P14.7-26 is at steady state before the switch opens at time $t = 0$. Determine the inductor voltage $v(t)$ for $t > 0$.

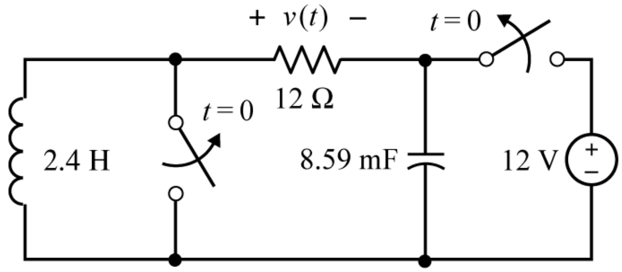


Figure P14.7-26

Solution

The circuit shown in Figure P14.7-22 is at steady state before the switch opens at time $t = 0$. Determine the inductor voltage $v(t)$ for $t > 0$.

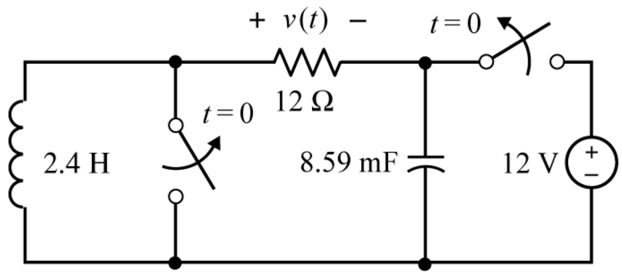
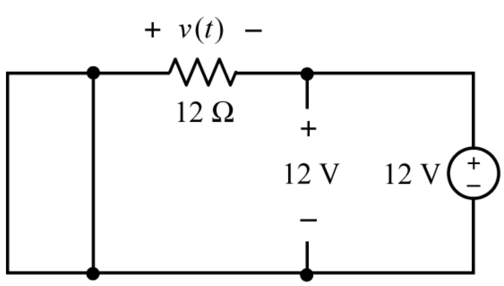
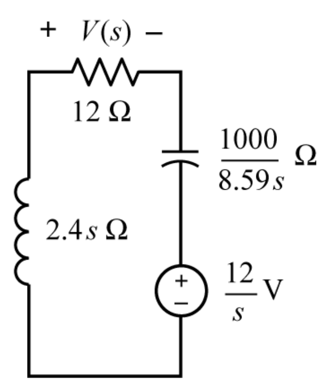


Figure P14.7-26

Determine the initial conditions, i.e. the inductor current and capacitor voltage at $t = 0$, as shown in the circuit on the left below. Use those initial conditions to represent the circuit in the s-domain as s shown in the circuit on the left below.





Analysis of the s-domain circuit shows that

$$V(s) = \left(\frac{-12}{2.4s + 12 + \dfrac{1000}{8.59s}} \right) \left(\frac{12}{s} \right) = \frac{-144}{2.4s^2 + 12s + \dfrac{1000}{8.59}} = \frac{-60}{s^2 + 5s + 48.5}$$

The denominator does not factor any further in the real numbers. Let’s complete the square in the denominator

$$V(s)=\frac{-60}{s^2+5s+48.5}=\frac{-60}{(s+2.5)^2+42.25}=\frac{-60}{(s+2.5)^2+6.5^2}=\frac{-9.23(6.5)}{(s+2.5)^2+6.5^2}$$

Now use $e^{-at}f(t)\leftrightarrow F(s+a)$ and $\sin \omega t$ for $t>0 \leftrightarrow \frac{\omega}{s^2+\omega^2}$ to find the inverse Laplace transform

$$v(t)=e^{-2.5t}\ \mathscr{L}^{-1}\left[\frac{-9.23(6.5)}{(s+2.5)^2+6.5^2}\right]=-9.23\ e^{-2.5t}\ \sin(6.5\ t)\ \text{for } t>0$$

Section 14.8 Transfer Functions

P14.8-1 The input to the circuit shown in Figure P14.8-1 is the voltage $v_{\text{i}}(t)$ and the output is the voltage $v_{\text{o}}(t)$.
Determine the values of *L*, *C*, *k*, R_1 and R_2 that cause the step response of this circuit to be:

$$v_{\text{o}}(t) = \left(5 + 20\,e^{-5000t} - 25\,e^{-4000t}\right) u(t) \ \text{V}$$

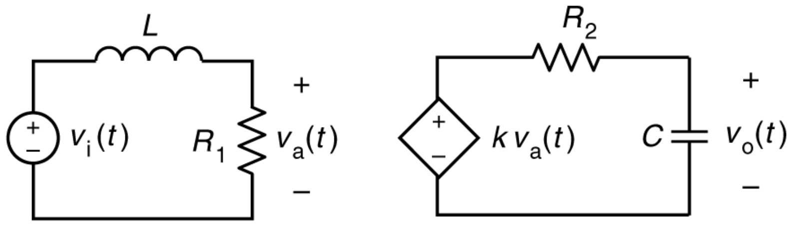
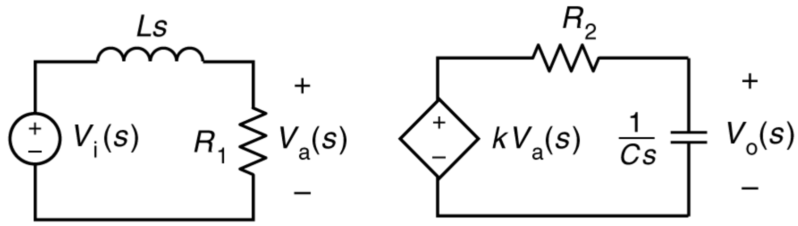


Figure 14.8-1

Answer: One solution is R_1 = 400 Ω, *L* = 0.1 H, *k* = 5 V/V, *C* = 0.1 μF, R_1 = 2 kΩ.

Solution: First, determine the transfer function from the circuit. To do so, represent the circuit in the s-domain as shown below.



Applying voltage division twice

$$V_{\text{a}}(s) = \frac{R_1}{R_1 + L\,s} V_{\text{i}}(s) = \frac{\frac{R_1}{L}}{s + \frac{R_1}{L}} V_{\text{i}}(s) \quad \text{and} \quad V_{\text{o}}(s) = \frac{\frac{1}{C\,s}}{R_2 + \frac{1}{C\,s}} k\,V_{\text{a}}(s) = \frac{\frac{k}{R_2 C}}{s + \frac{1}{R_2 C}} V_{\text{a}}(s)$$

Substituting $V_{\text{a}}(s)$ from the first equation into the second, the transfer function of this circuit is

$$H(s) = \frac{V_{\text{o}}(s)}{V_{\text{i}}(s)} = \frac{\frac{k\,R_1}{R_2 C\,L}}{\left(s + \frac{R_1}{L}\right)\left(s + \frac{1}{R_2 C}\right)}$$

Next, determine the transfer function from the step response.

$$\frac{H(s)}{s} = \mathfrak{L}\left[5 + 20\,e^{-5000t} - 25\,e^{-4000t}\right] = \frac{5}{s} + \frac{20}{s+5000} - \frac{25}{s+4000} = \frac{10^8}{s(s+5000)(s+4000)}.$$

Performing partial fraction expansion:

$$\frac{10^8}{(s+5000)(s+4000)} = H(s) = \frac{\frac{k\,R_1}{R_2 C\,L}}{\left(s + \frac{R_1}{L}\right)\left(s + \frac{1}{R_2 C}\right)}$$

Equality requires

$$\frac{R_1}{L} = 4000 \text{ and } \frac{1}{R_2 C} = 5000 \quad \text{or} \quad \frac{R_1}{L} = 5000 \text{ and } \frac{1}{R_2 C} = 5000$$

and

$$10^8 = k(4000)(5000) \quad \Rightarrow \quad k = 5 \text{ V/V}$$

The solution is not unique. One solution is R_1 = 400 Ω, L = 0.1 H, k = 5 V/V, C = 0.1 μF, R_1 = 2 kΩ.

P14.8-2 The input to the circuit shown in Figure P14.8-2 is the voltage $v_i(t)$ and the output is the voltage $v_o(t)$. Determine the step response of this circuit.

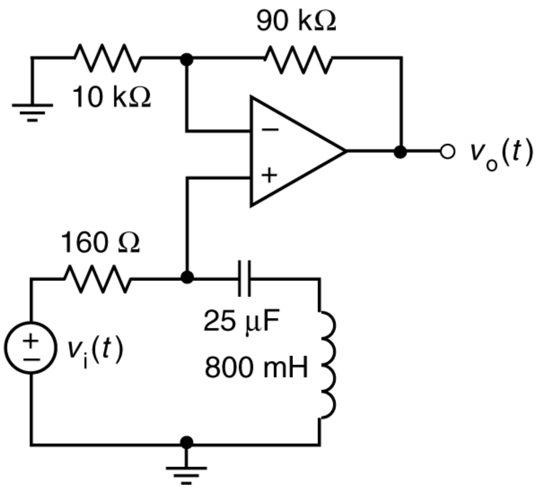


Figure 14.8-2

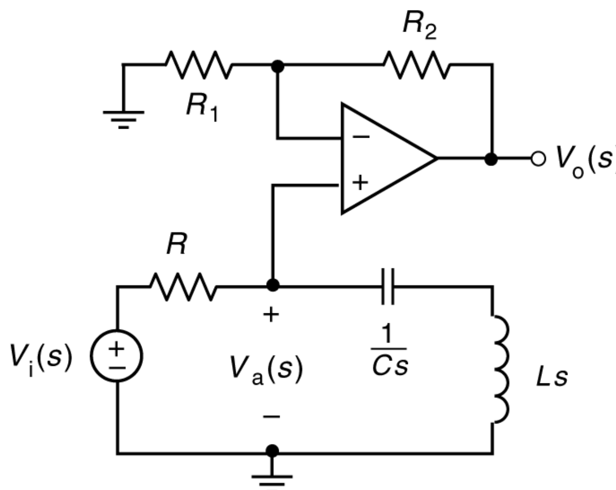
Solution:

The transfer function of this circuit is

$$H(s)=\frac{Ls+\dfrac{1}{Cs}}{R+Ls+\dfrac{1}{Cs}}\left(1+\frac{R_2}{R_1}\right)$$

$$=\frac{10s^2+\dfrac{1}{LC}}{s^2+\dfrac{R}{L}s+\dfrac{1}{LC}}\left(1+\frac{R_2}{R_1}\right)$$

$$=\frac{s^2+500,000}{s^2+200s+50,000}$$



The step response is $v_o(t)=\mathcal{L}^{-1}\left[\dfrac{H(s)}{s}\right]=\mathcal{L}^{-1}\left[\dfrac{10s^2+500,000}{s\left(s^2+200s+50,000\right)}\right]$.

Let's do some algebra:

$$\frac{10s^2+500,000}{s\left(s^2+200s+50,000\right)}=\frac{10}{s}+\frac{-2000}{s^2+200s+50,000}$$

$$=\frac{10}{s}+\frac{-2000}{(s+100)^2+200^2}$$

$$=\frac{10}{s}-10\frac{200}{(s+100)^2+200^2}$$

The step response is

$$v_o(t)=\mathcal{L}^{-1}\left[\frac{10}{s}-10\frac{200}{(s+100)^2+200^2}\right]=10-e^{-100t}\left[10\sin(200t)\right]=\left[10+10e^{-100t}\cos(200t+90°)\right]u(t)\text{ V}$$

P14.8-3 The input to the circuit shown in Figure P14.8-3 is the voltage $v_{\mathrm{i}}(t)$ and the output is the voltage $v_{\mathrm{o}}(t)$. Determine the impulse response of this circuit.

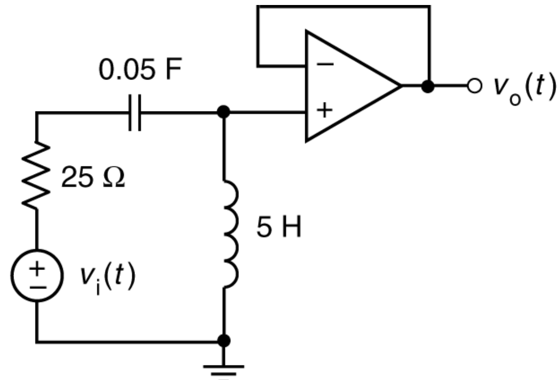
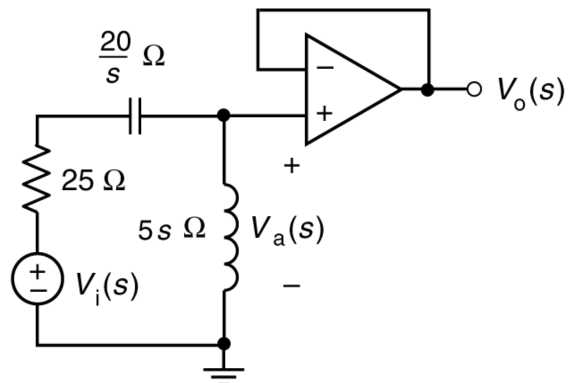


Figure 14.8-3

Solution:

Represent the circuit in the s-domain:



The transfer function of this circuit is

$$H(s)=\frac{V_{\mathrm{o}}(s)}{V_{\mathrm{i}}(s)}=\frac{5s}{25+\dfrac{20}{s}+5s}=\frac{s^2}{s^2+5s+4}$$

The impulse response is $h(t)=\mathcal{L}^{-1}\left[H(s)\right]=\mathcal{L}^{-1}\left[\dfrac{s^2}{s^2+5s+4}\right]$.

Let's do some algebra:

$$\frac{s^2}{s^2+5s+4}=1-\frac{5s+4}{s^2+5s+4}=1-\frac{5s+4}{(s+4)(s+1)}=1-\left(\frac{\dfrac{-16}{-3}}{s+4}+\frac{\dfrac{-1}{3}}{s+1}\right)$$

The impulse response is

$$h(t)=\mathcal{L}^{-1}\left[1-\frac{\dfrac{16}{3}}{s+4}+\frac{\dfrac{1}{3}}{s+1}\right]=\delta(t)+\left(-\frac{16}{3}e^{-4t}+\frac{1}{3}e^{t}\right)u(t)\ \mathrm{V}$$

P14.8-4 The input to the circuit shown in Figure P14.8-4 is the voltage $v_i(t)$ and the output is the voltage $v_o(t)$. Determine the step response of this circuit.

Answer: step response $= \left(5-(5+20t)e^{-4t}\right)u(t)$

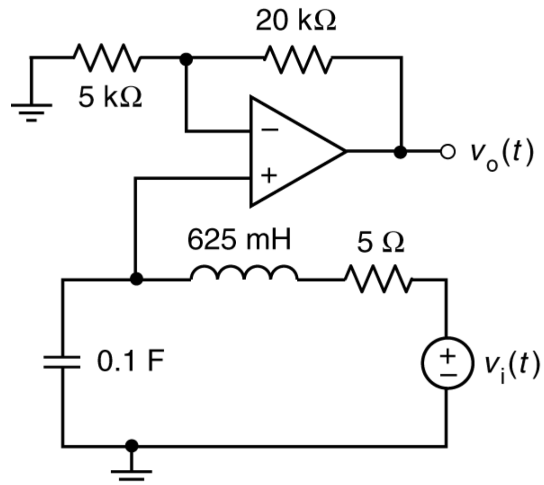
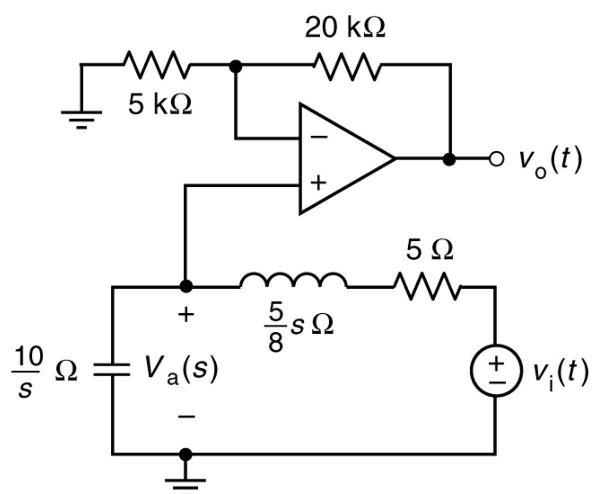


Figure 14.8-4

Solution:

Represent the circuit in the s-domain as shown.



Using voltage division

$$V_a(s) = \frac{\frac{10}{s}}{\frac{20}{s}+\frac{5}{8}s+5}V_i(s) = \frac{16}{s^2+8s+16}V_i(s)$$

Recognizing the noninverting amplifier:

$$V_o(s) = \left(1+\frac{20}{5}\right)V_a(s) = \frac{80}{s^2+8s+16}V_i(s)$$

The transfer function of this circuit is

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{80}{s^2+8s+16} = \frac{80}{(s+4)^2}$$

The step response is

$$\text{step response} = \mathcal{L}^{-1}\left[\frac{H(s)}{s}\right] = \mathcal{L}^{-1}\left[\frac{80}{s(s+4)^2}\right].$$

Performing partial fraction expansion:

$$\frac{80}{s(s+4)^2} = \frac{\frac{80}{16}}{s} + \frac{A}{s+4} + \frac{\frac{80}{-4}}{(s+4)^2}$$

Multiplying both sides by $s(s+4)^2$ and equating coefficients of like powers of *s*:

$$80 = 5(s+4)^2 + As(s+4) + (-20)s \quad \Rightarrow \quad A = -5$$

The step response is

$$\text{step response} = \mathcal{L}^{-1}\left[\frac{5}{s} + \frac{-5}{s+4} + \frac{-20}{(s+4)^2}\right] = \left(5-(5+20t)e^{-4t}\right)u(t)\,\text{V}$$

P14.8-5 The input to the circuit shown in Figure P14.8-5 is the voltage, $v_i(t)$, of the independent voltage source. The output is the voltage, $v_o(t)$, across the 5-kΩ resistor. Specify values of the resistance, R, the capacitance, C, and the inductance, L, such that the transfer function of this circuit is given by

$$H(s)=\frac{V_o(s)}{V_i(s)}=\frac{15\times 10^6}{(s+2000)(s+5000)}$$

Answer: R = 5k Ω, C = 0.5 μF, and L = 1 H (one possible solution)

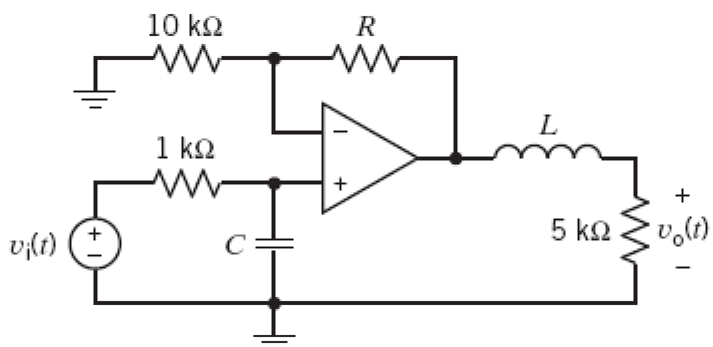
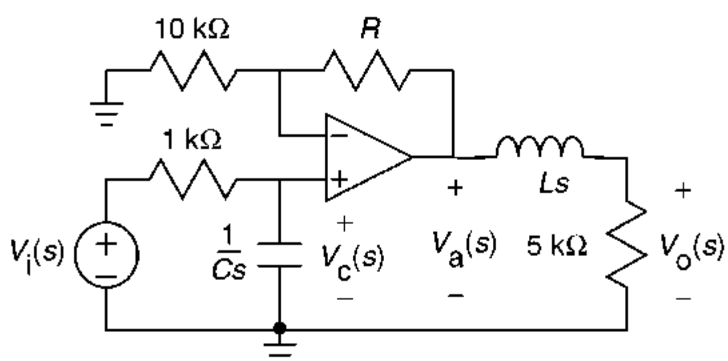


Figure P14.8-5

Solution:

The transfer function can also be calculated from the circuit itself. The circuit can be represented in the frequency domain as



We can save ourselves some work be noticing that the 10000 ohm resistor, the resistor labeled R and the op amp comprise a non-inverting amplifier. Thus

$$V_a(s)=\left(1+\frac{R}{10000}\right)V_c(s)$$

Now, writing node equations,

$$\frac{V_c(s)-V_i(s)}{1000}+CsV_c(s)=0 \quad \text{and} \quad \frac{V_o(s)-V_a(s)}{Ls}+\frac{V_o(s)}{5000}=0$$

Solving these node equations gives

$$H(s)=\frac{\dfrac{1}{1000\,C}\left(1+\dfrac{R}{10000}\right)\dfrac{5000}{L}}{\left(s+\dfrac{1}{1000C}\right)\left(s+\dfrac{5000}{L}\right)}$$

Comparing these two equations for the transfer function gives

$$\left(s+\frac{1}{1000C}\right)=\left(s+2000\right) \quad \text{or} \quad \left(s+\frac{1}{1000C}\right)=\left(s+5000\right)$$

$$\left(s+\frac{5000}{L}\right)=\left(s+2000\right) \quad \text{or} \quad \left(s+\frac{5000}{L}\right)=\left(s+5000\right)$$

$$\frac{1}{1000C}\left(1+\frac{R}{10000}\right)\frac{5000}{L}=15\times10^{6}$$

The solution isn't unique, but there are only two possibilities. One of these possibilities is

$$\left(s+\frac{1}{1000C}\right)=\left(s+2000\right) \quad \Rightarrow \quad C=0.5\ \mu\mathrm{F}$$

$$\left(s+\frac{5000}{L}\right)=\left(s+5000\right) \quad \Rightarrow \quad L=1\ H$$

$$\frac{1}{1000\left(0.5\times10^{6}\right)}\left(1+\frac{R}{10000}\right)\frac{5000}{1}=15\times10^{6} \quad \Rightarrow \quad R=5\ \mathrm{k}\Omega$$

(Checked using LNAP, 12/29/02)

P14.8-6 The input to the circuit shown in Figure P14.8-6 is the voltage, $v_i(t)$, of the independent voltage source. The output is the voltage, $v_o(t)$, across the 10-kΩ resistor. Specify values of the resistances, R_1 and R_2, such that the step response of this circuit is given by

$$v_o(t) = -4(1 - e^{-250t})u(t) \quad \text{V}$$

Answer: R_1 = 10 kΩ and R_2 = 40 kΩ

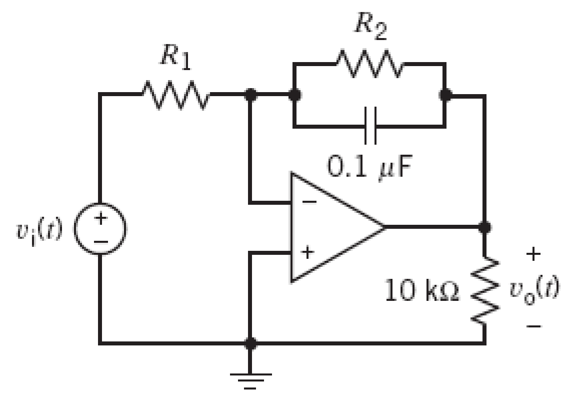


Figure P 14.8-6

Solution:

The transfer function of the circuit is

$$H(s) = -\frac{\dfrac{R_2}{1+R_2 C s}}{R_1} = -\frac{\dfrac{1}{R_1 C}}{s + \dfrac{1}{R_2 C}}$$

The give step response is $v_o(t) = -4\left(1-e^{-250t}\right)u(t)$ V. The correspond transfer function is calculated as

$$\frac{H(s)}{s} = \mathfrak{L}\left\{-4\left(1-e^{-250t}\right)u(t)\right\} = -\left(\frac{4}{s} - \frac{4}{s+250}\right) = \frac{-1000}{s(s+250)} \quad \Rightarrow \quad H(s) = \frac{-1000}{s+250}$$

Comparing these results gives

$$\frac{1}{R_2 C} = 250 \quad \Rightarrow \quad R_2 = \frac{1}{250\,C} = \frac{1}{250\left(0.1\times10^{-6}\right)} = 40\text{ k}\Omega$$

$$\frac{1}{R_1 C} = 1000 \quad \Rightarrow \quad R_1 = \frac{1}{1000\,C} = \frac{1}{1000\left(0.1\times10^{-6}\right)} = 10\text{ k}\Omega$$

(Checked using LNAP, 12/29/02)

P14.8-7 The input to the circuit shown in Figure P14.8-7 is the voltage $v_{\mathrm{i}}(t)$ and the output is the voltage $v_{\mathrm{o}}(t)$. Determine the step response of this circuit.

Answer: $v_{\mathrm{o}}(t)=\left(4\times10^{3}\right)t\,u(t)\ \text{V}$

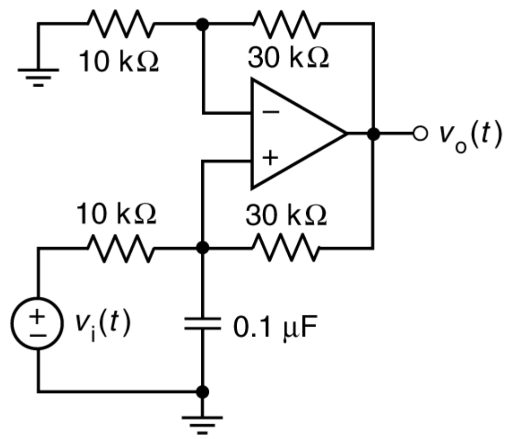
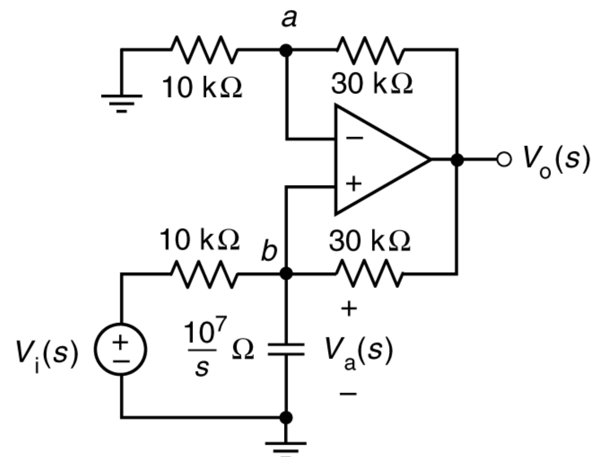


Figure 14.8-7

Solution: First, determine the transfer function from the circuit. To do so, represent the circuit in the s-domain as shown.



Notice that $V_{\mathrm{a}}(s)$ is the node voltage at both node *a* and node *b*. Apply KCL at node *a* to get

$$\frac{V_{\mathrm{a}}(s)}{10,000}+\frac{V_{\mathrm{a}}(s)-V_{\mathrm{o}}(s)}{30,000}=0\ \Rightarrow$$

$$V_{\mathrm{o}}(s)=\left(1+\frac{30,000}{10,000}\right)V_{\mathrm{a}}(s)=4V_{\mathrm{a}}(s)$$

Apply KCL at node *b* to get

$$\frac{V_{\mathrm{i}}(s)-V_{\mathrm{a}}(s)}{10,000}=\frac{V_{\mathrm{a}}(s)-V_{\mathrm{o}}(s)}{30,000}+\frac{V_{\mathrm{a}}(s)}{\frac{10^{7}}{s}}\ \Rightarrow\ \frac{V_{\mathrm{i}}(s)}{10,000}=\frac{V_{\mathrm{a}}(s)}{10,000}+\frac{V_{\mathrm{a}}(s)-4V_{\mathrm{a}}(s)}{30,000}+\frac{sV_{\mathrm{a}}(s)}{10^{7}}$$

$$\Rightarrow\ \frac{V_{\mathrm{i}}(s)}{10,000}=\frac{sV_{\mathrm{a}}(s)}{10^{7}}=\frac{sV_{\mathrm{o}}(s)}{4\times10^{7}}$$

$$\Rightarrow\ V_{\mathrm{o}}(s)=\frac{4\times10^{7}}{10,000\,s}V_{\mathrm{i}}(s)=\frac{4\times10^{3}}{s}V_{\mathrm{i}}(s)$$

The transfer function is

$$H(s)=\frac{V_{\mathrm{o}}(s)}{V_{\mathrm{i}}(s)}=\frac{4\times10^{3}}{s}$$

The step response is

$$\mathfrak{L}^{-1}\left[\frac{H(s)}{s}\right]=\mathfrak{L}^{-1}\left[\frac{4\times10^{3}}{s^{2}}\right]=\left(4\times10^{3}\right)t\,u(t)\ \text{V}.$$

P14.8-8 The input to the circuit shown in Figure P14.8-8 is the voltage $v_i(t)$ and the output is the voltage $v_o(t)$. Determine the step response of this circuit.

Answer: $v_o(t) = \left[2 - \left(\frac{4}{3}e^{-1000t} + \frac{2}{3}e^{-4000t}\right)\right]u(t)$ V

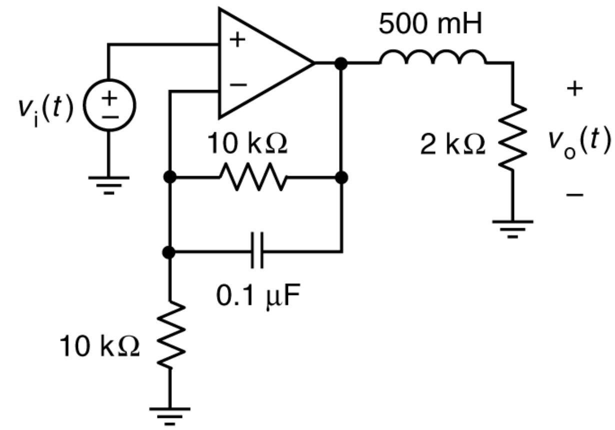
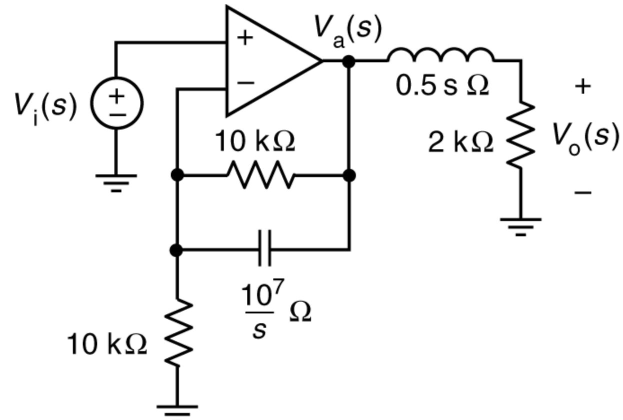


Figure 14.8-8

Solution: First, determine the transfer function from the circuit. To do so, represent the circuit in the s-domain as shown.



Recognizing the noninverting amplifier we

$$V_a(s) = \left(1 + \frac{\frac{10^7}{s} \| 10^4}{10^4}\right) V_i(s) = \left(1 + \frac{\frac{10^7}{s+10^3}}{10^4}\right) V_i(s)$$

$$= \left(\frac{s+2000}{s+1000}\right) V_i(s)$$

(Alternately, this equation can obtained by applying KCL at the inverting input node of the op amp.)

Use voltage division to write

$$V_o(s) = \frac{2000}{2000+0.5s} V_a(s) = \frac{4000}{s+2000} V_a(s)$$

The transfer function is

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{4000(s+2000)}{(s+1000)(s+4000)}$$

The step response is

$$v_o(t) = \mathfrak{L}^{-1}\left[\frac{H(s)}{s}\right] = \mathfrak{L}^{-1}\left[\frac{4000(s+2000)}{s(s+1000)(s+4000)}\right] = \mathfrak{L}^{-1}\left[\frac{2}{s} + \frac{-4/3}{s+1000} + \frac{-2/3}{s+4000}\right].$$

$$v_o(t) = \left[2 - \left(\frac{4}{3}e^{-1000t} + \frac{2}{3}e^{-4000t}\right)\right]u(t) \text{ V}$$

P14.8-9 The input to the circuit shown in Figure P14.8-9 is the voltage, $v_i(t)$, of the independent voltage source. The output is the voltage, $v_o(t)$. The step response of this circuit is

$$v_o(t) = 0.5(1 + e^{-4t})u(t) \quad \text{V}$$

Determine the values of the inductance, L, and the resistance, R.

Answer: $L = 6$ H and $R = 12\ \Omega$

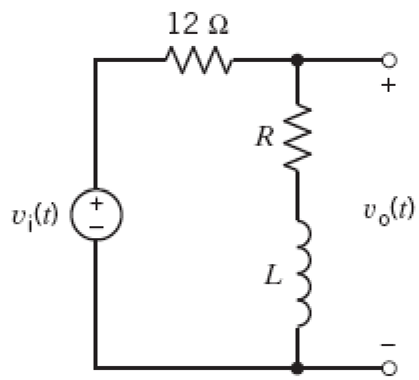


Figure P14.8-9

Solution:

From the circuit:

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{R + Ls}{12 + R + Ls} = \frac{s + \dfrac{R}{L}}{s + \dfrac{12+R}{L}}$$

From the given step response:

$$\frac{H(s)}{s} = \mathcal{L}\left[0.5\left(1 + e^{-4t}\right)u(t)\right] = \frac{0.5}{s} + \frac{0.5}{s+4} = \frac{s+2}{s(s+4)} \quad \Rightarrow \quad H(s) = \frac{s+2}{s+4}$$

Comparing these two forms of the transfer function gives:

$$\left.\begin{array}{l} \dfrac{R}{L} = 2 \\ \dfrac{12+R}{L} = 4 \end{array}\right\} \quad \Rightarrow \quad \frac{12 + 2L}{L} = 4 \quad \Rightarrow \quad L = 6\ \text{H},\ R = 12\ \Omega$$

(Checked using LNAP, 12/29/02)

P 14.8-10 An electric microphone and its associated circuit can be represented by the circuit shown in Figure 14.8-10. Determine the transfer function $H(s) = V_0(s)/V(s)$.

Answer: $\frac{V_o(s)}{V(s)} = \frac{RCs}{(R_1Cs+2)(2RCs+1)-1}$

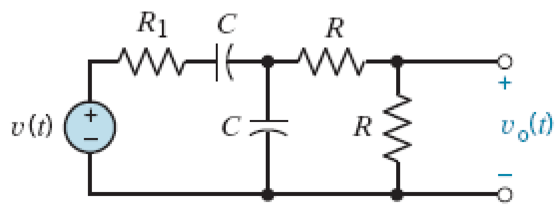
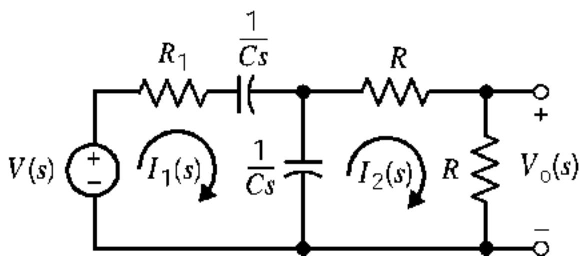


Figure 14.8-10 Microphone circuit

Solution:



Mesh equations:

$$V(s) = \left(R_1+\frac{1}{Cs}+\frac{1}{Cs}\right)I_1(s)-\frac{1}{Cs}I_2(s)$$

$$0 = \left(R+R+\frac{1}{Cs}\right)I_2(s) - \frac{1}{Cs}I_1(s)$$

Solving for $I_2(s)$:

$$I_2(s) = \frac{V(s)\left(\frac{1}{Cs}\right)}{\left(R_1+\frac{2}{Cs}\right)\left(2R+\frac{1}{Cs}\right) - \frac{1}{(Cs)^2}}$$

Then $V_o(s) = R\,I_2(s)$ gives

$$H(s)=\frac{V_0(s)}{V(s)} = \frac{RCs}{[R_1Cs+2][2RCs+1]-1} = \frac{s}{2R_1C\left[s^2 + \frac{4RC+R_1C}{2RR_1C^2}s + \frac{1}{\left(2RR_1C^2\right)^2}\right]}$$

P 14.8-11 Engineers had avoided inductance in long-distance circuits because it slows transmission. Oliver Heaviside proved that the addition of inductance to a circuit could enable it to transmit without distortion. George A. Campbell of the Bell Telephone Company designed the first practical inductance loading coils, in which the induced field of each winding of wire reinforced that of its neighbors so that the coil supplied proportionally more inductance than resistance. Each one of Campbell's 300 test coils added 0.11 H and 12 Ω at regular intervals along 35 miles of telephone wire (Nahin, 1990). The loading coil balanced the effect of the leakage between the telephone wires represented by *R* and *C* in Figure P 14.8-11. Determine the transfer function $V_2(s)/V_1(s)$.

Answer: $\dfrac{V_2(s)}{V_1(s)} = \dfrac{R}{RCLs^2 + (L + R_x RC)s + R_x + R}$

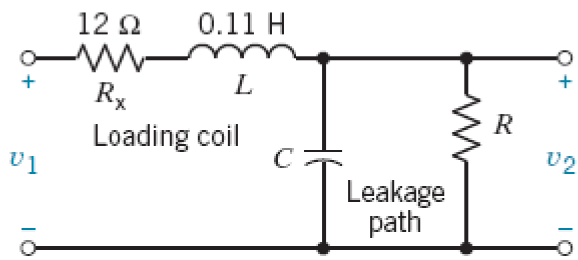
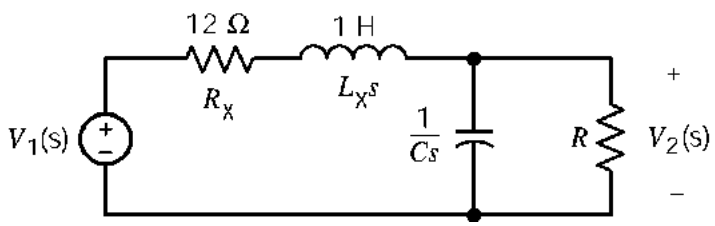


Figure P 14.8-11 Telephone and load coil circuit

Solution:



Let

$$Z_2 = \frac{R\left(\frac{1}{Cs}\right)}{R + \frac{1}{Cs}} = \frac{R}{RCs + 1}$$

$$Z_1 = R_x + L_x s$$

Then

$$\frac{V_2}{V_1} = \frac{Z_2}{Z_1 + Z_2} = \frac{\frac{R}{RCs+1}}{R_x + L_x s + \frac{R}{RCs+1}} = \frac{R}{L_x RCs^2 + (L_x + R_x RC)\, s + R_x + R}$$

$$\frac{V_2}{V_1} = \frac{\frac{1}{L_x C}}{s^2 + \frac{(L_x + R_x RC)}{L_x RC} s + \frac{R_x + R}{L_x RC}}$$

P14.8-12 The input to the circuit shown in Figure P14.8-12 is the current $i(t)$ and the output is the voltage $v(t)$. Determine the impulse response of this circuit.

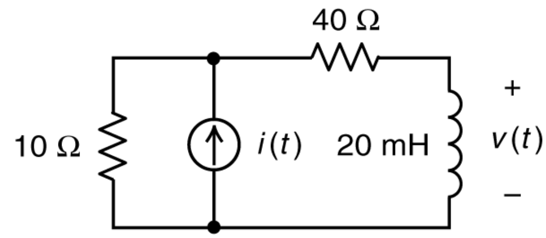
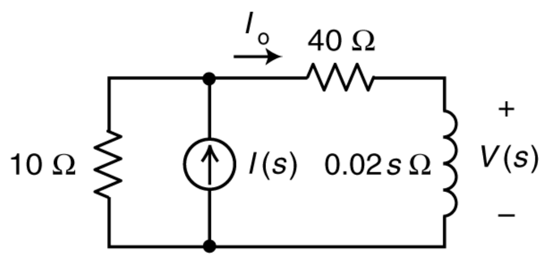


Figure P14.8-12

Solution: First, determine the transfer function from the circuit. To do so, represent the circuit in the s-domain as shown below.



Applying current division $$I_o(s) = \frac{10}{10+40+0.02\,s} I(s) = \frac{500}{s+2500} I(s)$$

Then $$V(s) = 0.02\,s\,I_o(s) = \frac{10\,s}{s+2500} I(s)$$

The transfer function is $$H(s) = \frac{V(s)}{I(s)} = \frac{10\,s}{s+2500} = 10 - \frac{25000}{s+2500}$$

The impulse response is $$\pounds^{-1}\left[H(s)\right] = 10\,\delta(t) - 25000\,e^{-2500t} u(t) \text{ V}$$

P14.8-13 The input to the circuit shown in Figure P14.8-13 is the current $i(t)$ and the output is the voltage $v(t)$. Determine the impulse response of this circuit.

Answer: $v(t)=1.25\times10^7\left(e^{-5000t}-e^{-25000t}\right)u(t)\ \text{V}$

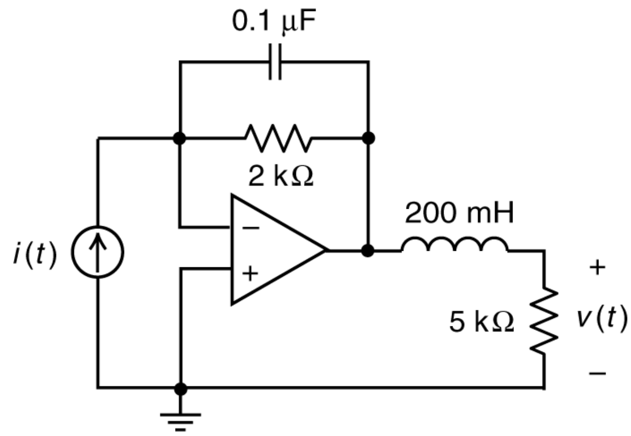
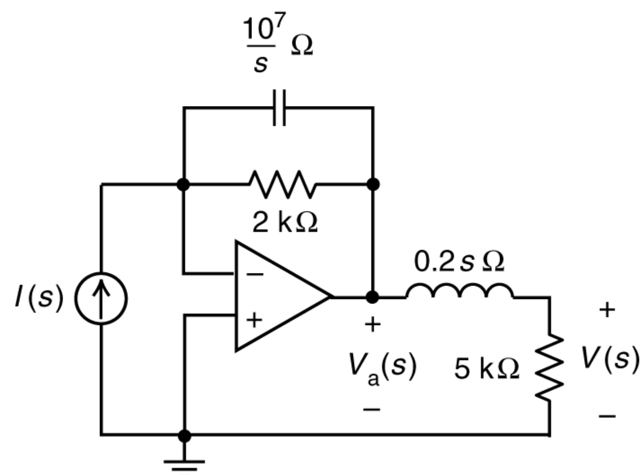


Figure P14.8-13

Solution: First, determine the transfer function from the circuit. To do so, represent the circuit in the s-domain as shown below.



Applying KCL at the inverting input node of the op amp

$$I(s)+\frac{V_a(s)-0}{2000}+\frac{V_a(s)-0}{\frac{10^7}{s}}=0 \Rightarrow I(s)=-\left(\frac{1}{2000}+\frac{s}{10^7}\right)V_a(s)=-\left(\frac{s+5000}{10^7}\right)V_a(s)$$

Using voltage division $$V(s)=\frac{5000}{5000+0.2\,s}V_a(s)=\frac{25000}{s+25000}V_a(s)$$

Combining these equations: $$V(s)=\left(\frac{25000}{s+25000}\right)\left(\frac{10^7}{s+5000}\right)I(s)$$

The transfer function is $$H(s)=\frac{V(s)}{I(s)}=\frac{25\times10^{10}}{(s+5000)(s+25,000)}$$

Partial fraction expansion: $$\frac{25\times10^{10}}{(s+5000)(s+25,000)}=\frac{1.25\times10^7}{s+5000}+\frac{-1.25\times10^7}{s+25,000}$$

The impulse response is $$\pounds^{-1}\left[H(s)\right]=1.25\times10^7\left(e^{-5000t}-e^{-25000t}\right)u(t)\ \text{V}$$

P14.8-14 A series *RLC* circuit is shown in Figure P14.8-14.

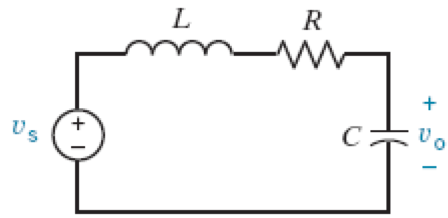
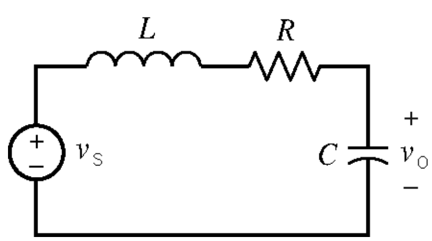


Figure P14.8-14

Determine (a) the transfer function $H(s)$, (b) the impulse response, and (c) the step response for each set of parameter values given in the table below.

	L	*C*	*R*
a	2 H	0.025 F	18 Ω
b	2 H	0.025 F	8 Ω
c	1 H	0.391 F	4 Ω
d	2 H	0.125 F	8 Ω

Solution:



$$H(s)=\frac{V_o(s)}{V_i(s)}=\frac{\frac{1}{Cs}}{Ls+R+\frac{1}{Cs}}=\frac{\frac{1}{LC}}{s^2+\frac{R}{L}s+\frac{1}{LC}}$$

L, *H*	**C, *F***	*R*, Ω	*H*(*s*)
2	0.025	18	$\frac{20}{s^2+9s+20}=\frac{20}{(s+4)(s+5)}$
2	0.025	8	$\frac{20}{s^2+4s+20}=\frac{20}{(s+2)^2+4^2}$
1	0.391	4	$\frac{2.56}{s^2+4s+2.56}=\frac{2.56}{(s+0.8)(s+3.2)}$
2	0.125	8	$\frac{4}{s^2+4s+4}=\frac{4}{(s+2)^2}$

a) $H(s)=\dfrac{20}{(s+4)(s+5)}$

$$\mathcal{L}\{h(t)\}=H(s)=\frac{20}{s+4}-\frac{20}{s+5}\quad\Rightarrow\quad h(t)=\left(20e^{-4t}-20e^{-5t}\right)u(t)$$

$$\mathcal{L}\{\text{step response}\}=\frac{H(s)}{s}=\frac{20}{s(s+4)(s+5)}=\frac{1}{s}+\frac{-5}{s+4}+\frac{4}{s+5}\quad\Rightarrow$$

$$\text{step response }=\left(1+4e^{-5t}-5e^{-4t}\right)u(t)$$

b) $H(s)=\dfrac{20}{(s+2)^2+4^2}$

$$\mathcal{L}\{h(t)\}=H(\text{s})=\frac{5(4)}{(s+2)^2+4^2}\quad\Rightarrow\quad h(t)=5e^{-2t}\sin 4t\,u(t)$$

$$\mathcal{L}\{\text{step response}\}=\frac{H(s)}{s}=\frac{20}{s(s^2+4s+20)}=\frac{1}{s}+\frac{K_1s+K_2}{s^2+4s+20}$$

$$20=s^2+4s+20+s\left(K_1s+K_2\right)=s^2\left(1+K_1\right)+s\left(4+K_2\right)+20$$

$$\Rightarrow K_1=-1,\ K_2=-4$$

$$\mathcal{L}\{\text{step response}\}=\frac{1}{s}+\frac{-(s+2)}{(s+2)^2+4^2}+\frac{-\frac{1}{2}(4)}{(s+2)+4^2}$$

$$\text{step response }=\left(1-e^{-2t}\left(\cos 4t+\frac{1}{2}\sin 4t\right)\right)u(t)$$

c) $H(s)=\dfrac{2.56}{(s+0.8)(s+3.2)}$

$$\mathcal{L}\{h(t)\}=H(s)=\frac{1.07}{s+.8}-\frac{1.07}{s+3.2}\quad\Rightarrow\quad h(t)=1.07\left(\mathrm{e}^{-.8\mathrm{t}}-\mathrm{e}^{-3.2\mathrm{t}}\right)\mathrm{u(t)}$$

$$\mathcal{L}\{\text{step response}\}=\frac{H(s)}{s}=\frac{2.56}{s\,(s+.8)(s+3.2)}=\frac{1}{s}+\frac{\frac{-4}{3}}{s+.8}+\frac{\frac{1}{3}}{s+3.2}$$

$$\text{step response }=\left(1+\frac{1}{3}e^{-3.2t}-\frac{4}{3}e^{-.8t}\right)u(t)$$

d) $H(s)=\dfrac{4}{(s+2)^2}$

$$h(t)=4te^{-2t}u(t)$$

$$\text{step response }=\left(1-(1+2t)e^{-2t}\right)u(t)$$

14.8-15 A circuit is described by the transfer function

$$\frac{V_O}{V_1} = H(S) = \frac{9s+18}{3s^3+18s^2+39s}$$

Find the step response and impulse response of the circuit.

Solution:

For an impulse response, take $V_1(s)=1$. Then

$$V_0(s) = \frac{3(s+2)}{s(s+3-j2)\,(s+3+j2)} = \frac{A}{s} + \frac{B}{s+3-j2} + \frac{B^*}{s+3+j2}$$

where

$$A = sV_0(s)\Big|_{s=0} = 0.462,\;\; B = (s+3-j2)V_0(s)\Big|_{s=-3+j2} = 0.47\angle-119.7^\circ \;\text{ and }\; B^* = 0.47\;\angle 119.7^\circ$$

Then

$$V_0(s) = \frac{0.462}{s} + \frac{0.47\;\angle-119.7^\circ}{s+3-j2} + \frac{0.47\;\angle 119.7^\circ}{s+3+j2}$$

The impulse response is

$$v_0(t) = \left[0.462 + 2(0.47)e^{-3t}\cos\left(2t - 119.7^o\right)\right]u(t)\;\;\text{V}$$

P14.8-16 The input to the circuit shown in Figure P14.8-16 is the voltage of the voltage source, $v_i(t)$, and the output is the voltage, $v_o(t)$, across the 15-kΩ resistor.

(a) Determine the steady-state response, $v_o(t)$, of this circuit when the input is $v_i(t) = 1.5$ V.

(b) Determine the steady-state response, $v_o(t)$, of this circuit when the input is $v_i(t) = 4\cos(100t + 30°)$ V.

(c) Determine the step response, $v_o(t)$, of this circuit.

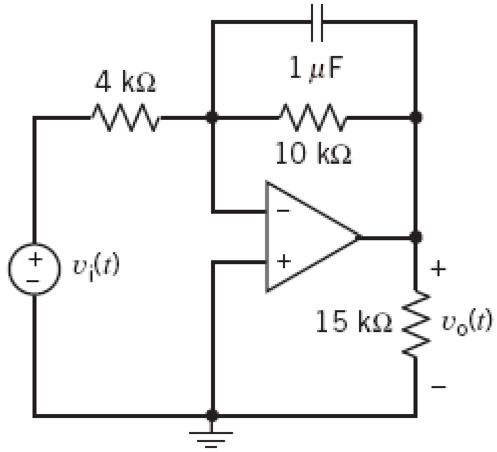


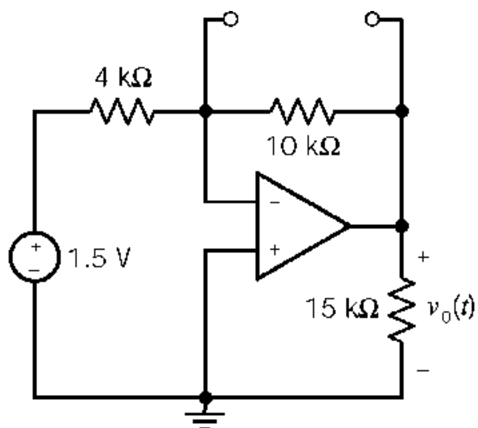
Figure P14.8-16

Solution:

a.

A capacitor in a circuit that is at steady state and has only constant inputs acts like an open circuit. Then

$$v_o(t) = -\frac{10}{4}(1.5) = -3.75 \text{ V}$$



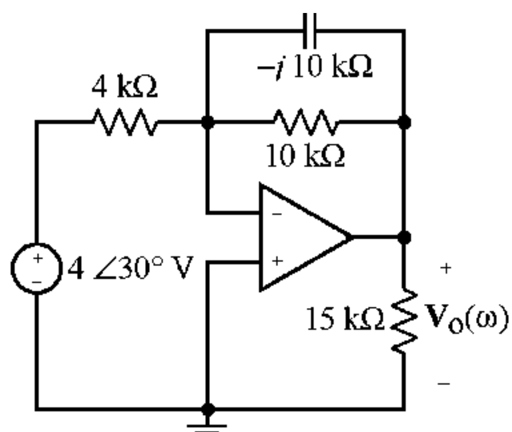
b. Here's the circuit represented in the frequency domain, using phasors and impedances. Writing a node equation at the inverting input node of the op amp gives

$$\frac{4\angle 30°}{4\times 10^3} + \frac{\mathbf{V}_o(\omega)}{-j10\times 10^3} + \frac{\mathbf{V}_o(\omega)}{10\times 10^3} = 0$$

or

$$10\angle 30° + (1+j)\mathbf{V}_o(\omega) = 0$$

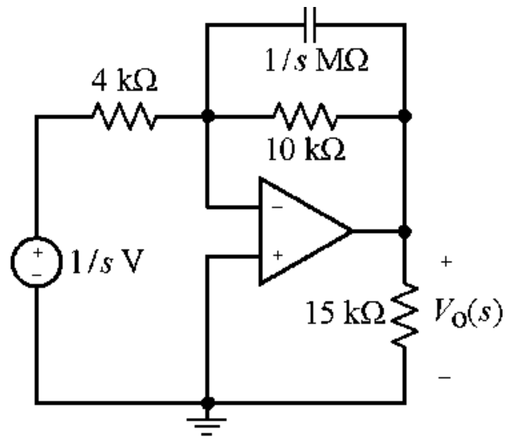
$$\mathbf{V}_o(\omega) = -\frac{10\angle 30°}{1+j} = 7.07\angle 165°$$



Finally,

$v_o(t) = 7.07\cos(100t + 165°)$ V.

c. Here's the circuit represented in the frequency domain, using The Laplace transform (assuming zero initial conditions). Writing a node equation at the inverting input node of the op amp gives



$$\frac{\frac{1}{s}}{4\times10^{3}}+\frac{V_{o}(s)}{\frac{1}{s}\times10^{6}}+\frac{V_{o}(s)}{10\times10^{3}}=0$$

$$\frac{10^{3}}{4s}+(s+100)V_{o}(s)=0$$

$$V_{o}(s)=\frac{250}{s(s+100)}=\frac{-2.5}{s}+\frac{2.5}{s+100}$$

Finally,

$$v_{o}(t)=2.5\left(e^{-100t}-1\right)u(t)\ \ \text{V}$$

P14.8-17 The input to the circuit shown in Figure P14.8-17 is the voltage of the voltage source, $v_i(t)$, and the output is the capacitor voltage, $v_o(t)$. Determine the step response of this circuit.

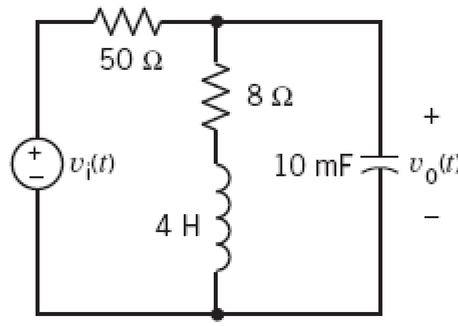
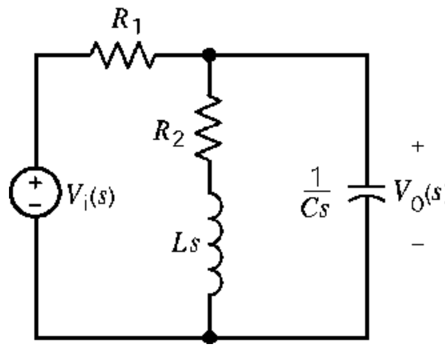


Figure P14.8-17

Solution

Represent the circuit in the frequency domain using the Laplace transform as shown. (Set the initial conditions to zero to calculate the step response.)



First, $$\frac{1}{Cs} \| \left(R_2 + Ls\right) = \frac{\frac{1}{Cs} \times \left(R_2 + Ls\right)}{\frac{1}{Cs} + \left(R_2 + Ls\right)} = \frac{R_2 + Ls}{CLs^2 + CR_2 s + 1}$$

Next, using voltage division,

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{\frac{R_2 + Ls}{CLs^2 + CR_2 s + 1}}{\frac{R_2 + Ls}{CLs^2 + CR_2 s + 1} + R_1} = \frac{R_2 + Ls}{R_2 + Ls + R_1\left(CLs^2 + CR_2 s + 1\right)}$$

$$= \frac{\frac{s}{R_1 C} + \frac{R_2}{R_1 LC}}{s^2 + \frac{L + R_1 R_2 C}{R_1 LC} s + \frac{R_1 + R_2}{R_1 LC}} = \frac{2s + 4}{s^2 + 4s + 29}$$

Using $V_i(s) = \frac{1}{s}$ gives

$$V_o(s) = \frac{H(s)}{s} = \frac{2s+4}{s\left(s^2 + 4s + 29\right)} = \frac{0.1379}{s} + \frac{-0.1379s + 1.4483}{s^2 + 4s + 29}$$

$$= \frac{0.1379}{s} + \frac{-0.1379s + 1.4483}{\left(s+2\right)^2 + 5^2}$$

$$= \frac{0.1379}{s} - 0.1379\frac{s+2}{\left(s+2\right)^2 + 5^2} + 0.3449\frac{5}{\left(s+2\right)^2 + 5^2}$$

Taking the inverse Laplace transform

$$v_o(t) = 0.1379 + e^{-2t}\left(-0.1379\cos(5t) + 0.3448\sin(5t)\right)$$
$$= 0.1379 + 0.3713e^{-2t}\cos\left(5t - 111.8°\right) \text{ V}$$

(checked using LNAP 10/15/04)

P14.8-18 The input to the circuit shown in Figure P14.8-18 is the voltage of the voltage source, $v_i(t)$, and the output is the resistor voltage, $v_o(t)$. Specify values for L_1, L_2, R, and K that cause the step response of the circuit to be

$$v_o(t) = (1 + 0.667e^{-50t} - 1.667e^{-20t})u(t) \quad \text{V}$$

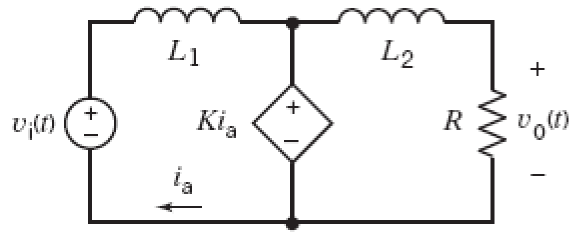


Figure P14.8-18

P14.8-18

First, we determine the transfer function corresponding to the step response. Taking the Laplace transform of the given step response

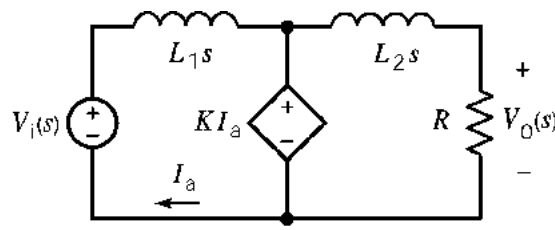
$$\frac{H(s)}{s} = V_o(s) = \frac{1}{s} + \frac{0.667}{s+50} - \frac{1.667}{s+20} = \frac{(s+50)(s+20)+0.667\,s(s+20)-1.667\,s(s+50)}{s(s+50)(s+20)}$$

$$= \frac{1000}{s(s+50)(s+20)}$$

Consequently,

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{1000}{(s+50)(s+20)}$$

Next, we determine the transfer function of the circuit. Represent the circuit in the frequency domain using the Laplace transform as shown. (Set the initial conditions to zero to calculate the transfer function.)



Apply KVL to the left mesh to get

$$V_i(s) = L_1\, s\, I_a(s) + K\, I_a(s) \quad \Rightarrow \quad I_a(s) = \frac{V_i(s)}{K + L_1\, s}$$

Next, using voltage division,

$$V_o(s) = \frac{R}{L_2\, s + R} K\, I_a(s) \quad \Rightarrow \quad V_o(s) = \frac{R\, K}{(L_2\, s + R)(K + L_1\, s)} V_i(s)$$

Then, the transfer function of the circuit is

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{R\, K}{(L_2\, s + R)(L_1\, s + K)} = \frac{\dfrac{R\, K}{L_1\, L_2}}{\left(s + \dfrac{R}{L_2}\right)\left(s + \dfrac{K}{L_1}\right)}$$

Comparing the two transfer functions gives

$$\frac{1000}{(s+50)(s+20)} = H(s) = \frac{\dfrac{R\, K}{L_1\, L_2}}{\left(s + \dfrac{R}{L_2}\right)\left(s + \dfrac{K}{L_1}\right)}$$

We require $1000=\frac{R\,K}{L_1\,L_2}$ and either $50=\frac{R}{L_2}$ and $20=\frac{K}{L_1}$ or $20=\frac{R}{L_2}$ and $50=\frac{K}{L_1}$. These equations do not have a unique solution. One solution is

$$L_1 = 0.1 \text{ H},\ L_2 = 0.1 \text{ H},\ R = 5\ \Omega \text{ and } K = 2 \text{ V/A}$$

(checked using LNAP 10/15/04)

P14.8-19 The input to the circuit shown in Figure P14.8-19 is the voltage of the voltage source, $v_i(t)$, and the output is the capacitor voltage, $v_o(t)$. Determine the step response of this circuit.

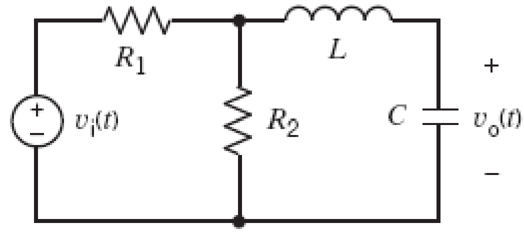
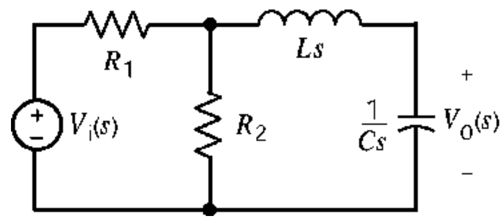


Figure P14.8-19

Solution:

Represent the circuit in the frequency domain using the Laplace transform as shown. (Set the initial conditions to zero to calculate the step response.)



First, $R_2 \,||\left(Ls+\frac{1}{Cs}\right)=\frac{R_2\times\left(Ls+\frac{1}{Cs}\right)}{R_2+\left(Ls+\frac{1}{Cs}\right)}=\frac{R_2\left(CLs^2+1\right)}{CLs^2+CR_2s+1}$

Next, using voltage division twice,

$$H(s)=\frac{V_o(s)}{V_i(s)}=\frac{\frac{R_2\left(CLs^2+1\right)}{CLs^2+CR_2s+1}}{\frac{R_2\left(CLs^2+1\right)}{CLs^2+CR_2s+1}+R_1}\times\frac{\frac{1}{Cs}}{Ls+\frac{1}{Cs}}=\frac{R_2}{\left(R_1+R_2\right)CLs^2+R_1R_2Cs+R_1+R_2}$$

$$=\frac{\frac{R_2}{\left(R_1+R_2\right)LC}}{s^2+\frac{R_1R_2}{\left(R_1+R_2\right)L}s+\frac{1}{LC}}=\frac{8}{s^2+10s+16}$$

Using $V_i(s)=\frac{1}{s}$ gives

$$V_o(s)=\frac{H(s)}{s}=\frac{8}{s\left(s^2+10s+16\right)}=\frac{8}{s(s+2)(s+8)}=\frac{\frac{1}{2}}{s}+\frac{-\frac{2}{3}}{s+2}+\frac{\frac{1}{6}}{s+8}$$

Taking the inverse Laplace transform

$$v_o(t)=\left(\frac{1}{2}-\frac{2}{3}e^{-2t}+\frac{1}{6}e^{-8t}\right)u(t)\ \text{V}$$

(checked using LNAP 10/15/04)

P 14.8-20 The input to the circuit shown in Figure P 14.8-20 is the voltage of the voltage source, $v_i(t)$, and the output is the inductor current, $i_o(t)$. Specify values for *L, C*, and *K* that cause the step response of the circuit to be

$$v_o(t) = (3.2 - (3.2e^{-5t} + 16te^{-5t}))u(t) \quad \text{V}$$

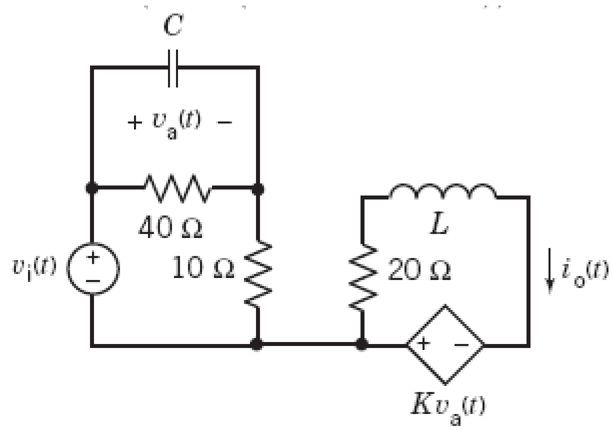


Figure P 14.8-20

Solution:

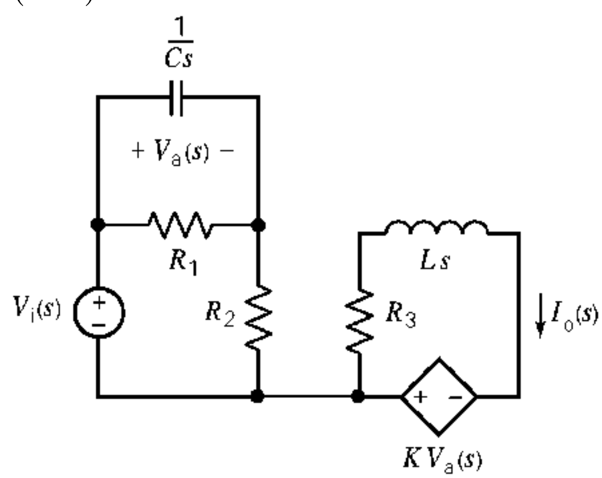
First, we determine the transfer function corresponding to the step response. Taking the Laplace transform of the given step response

$$\frac{H(s)}{s} = I_o(s) = \frac{3.2}{s} - \left(\frac{3.2}{s+5} + \frac{16}{(s+5)^2}\right) = \frac{3.2(s+5)^2 - 3.2s(s+5) + 16s}{s(s+5)^2} = \frac{80}{s(s+5)^2}$$

Consequently,

$$H(s) = \frac{I_o(s)}{V_i(s)} = \frac{80}{(s+5)^2}$$

Next, we determine the transfer function of the circuit. Represent the circuit in the frequency domain using the Laplace transform as shown. (Set the initial conditions to zero to calculate the transfer function.)



First
$$R_1 \parallel \frac{1}{Cs} = \frac{R_1 \times \frac{1}{Cs}}{R_1 + \frac{1}{Cs}} = \frac{R_1}{1 + R_1 Cs}$$

Next, using voltage division,

$$V_a(s) = \frac{\frac{R_1}{1+R_1 Cs}}{\frac{R_1}{1+R_1 Cs} + R_2} V_i(s) = \frac{R_1}{R_1 + R_2 + R_1 R_2 Cs} V_i(s)$$

$$I_o(s) = \frac{KV_a(s)}{Ls + R_3} \Rightarrow I_o(s) = \frac{KR_1}{(Ls + R_3)(R_1 + R_2 + R_1 R_2 Cs)} V_i(s) = \frac{\frac{K}{R_2 CL}}{\left(s + \frac{R_3}{L}\right)\left(s + \frac{R_1 + R_2}{R_1 R_2 C}\right)} V_i(s)$$

Then, the transfer function of the circuit is

$$H(s)=\frac{I_o(s)}{V_i(s)}=\frac{\dfrac{K}{R_2\,C\,L}}{\left(s+\dfrac{R_3}{L}\right)\left(s+\dfrac{R_1+R_2}{R_1\,R_2\,C}\right)}$$

Comparing the two transfer functions gives

$$\frac{80}{(s+5)^2}=H(s)=\frac{\dfrac{K}{R_2\,C\,L}}{\left(s+\dfrac{R_3}{L}\right)\left(s+\dfrac{R_1+R_2}{R_1\,R_2\,C}\right)}$$

We require $$5=\frac{R_1+R_2}{R_1\,R_2\,C}=\frac{40+10}{(40\times 10)C} \quad\Rightarrow\quad C=25\ \text{mF},$$

$$5=\frac{R_3}{L}=\frac{20}{L} \quad\Rightarrow\quad L=4\ \text{H}$$

and $$80=\frac{K}{R_2\,C\,L}=\frac{K}{10(0.025)4} \quad\Rightarrow\quad K=80\ \text{V/V}.$$

(checked using LNAP 10/15/04)

P 14.8-21 The input to a circuit is the voltage $v_i(t)$ and the output is the voltage $v_o(t)$. The impulse response of the circuit is

$$v_o(t) = 6.5e^{-2t}\cos(2t + 22.6°)u(t) \quad \text{V}$$

Determine the step response of this circuit.

Solution:

First,

$$6.5\cos\left(2t+22.6°\right)=6.5\left(\cos 22.6°\right)\cos\left(2t\right)-6.5\left(\sin 22.6°\right)\sin\left(2t\right)=6\cos\left(2t\right)-2.5\sin\left(2t\right)$$

Consequently, the impulse response can be written as

$$v_o\left(t\right)=e^{-2t}\left(6\cos\left(2t\right)-2.5\sin\left(2t\right)\right)u\left(t\right) \;\; \text{V}$$

The transfer function is

$$H\left(s\right)=6\frac{s+3}{\left(s+3\right)^2+2^2}-2.5\frac{2}{\left(s+3\right)^2+2^2}=\frac{6s+13}{\left(s+3\right)^2+2^2}=\frac{6s+13}{s^2+6s+13}$$

The Laplace transform of the step response is

$$\frac{H\left(s\right)}{s}=\frac{6s+13}{s\left(s^2+6s+13\right)}=\frac{1}{s}-\frac{s}{s^2+6s+13}=\frac{1}{s}-\frac{s}{\left(s+3\right)^2+2^2}=\frac{1}{s}-\frac{s+3}{\left(s+3\right)^2+2^2}+\frac{3}{2}\times\frac{2}{\left(s+3\right)^2+2^2}$$

Taking the inverse Laplace transform gives the step response:

$$v_o\left(t\right)=\left(1+e^{-2t}\left(1.5\sin\left(2t\right)-\cos\left(2t\right)\right)\right)u\left(t\right) \;=\; \left(1+1.803e^{-2t}\cos\left(2t-123.7°\right)\right) \;\text{V}$$

P 14.8-22 The input to a circuit is the voltage $v_i(t)$ and the output is the voltage $v_o(t)$. The step response of the circuit is

$$v_o(t) = [1 - e^{-t}(1 + 3t)]u(t) \quad \text{V}$$

Determine the impulse response of this circuit.

Solution:

Taking the Laplace transform of the step response,

$$\frac{H(s)}{s} = \frac{1}{s} - \left[\frac{3}{(s+3)^2} + \frac{1}{s+3}\right] = \frac{1}{s} - \frac{s+6}{(s+3)^2} = \frac{9}{s(s+3)^2}$$

The transfer function is $$H(s) = \frac{9}{(s+3)^2}$$

Taking the inverse Laplace transform gives the impulse response:

$$v_o(t) = 9\,t\,e^{-3t}\,u(t) \text{ V}$$

(checked using LNAP 10/15/04)

P 14.8-23 The input to the circuit shown in Figure P14.8-23 is the voltage of the voltage source, $v_i(t)$, and the output is the voltage, $v_o(t)$. Determine the step response of the circuit.

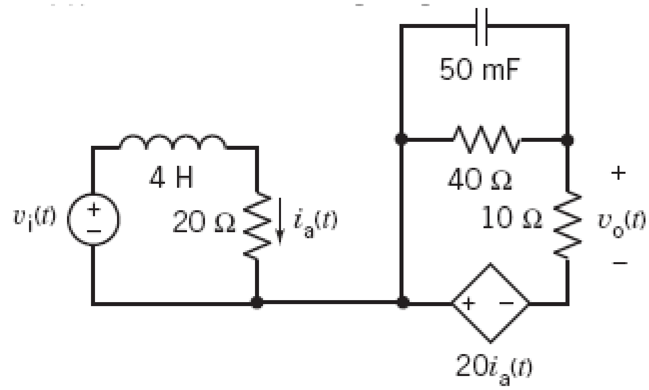
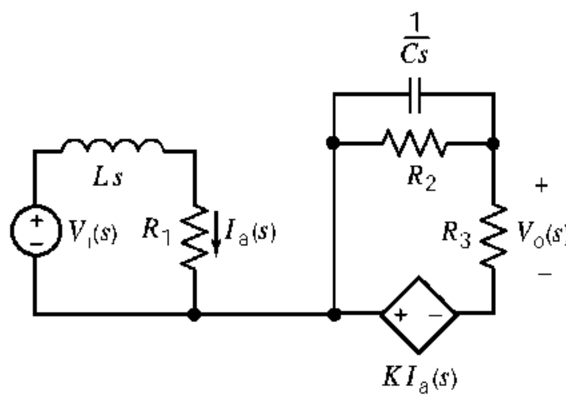


Figure P14.8-23

Solution:

Represent the circuit in the frequency domain using the Laplace transform as shown. (Set the initial conditions to zero to calculate the transfer function.) First,



$$I_a(s) = \frac{V_i(s)}{Ls + R_1}$$

The equivalent impedance of the parallel capacitor and inductor is

$$R_2 \parallel \frac{1}{Cs} = \frac{R_2 \times \frac{1}{Cs}}{R_2 + \frac{1}{Cs}} = \frac{R_2}{1 + R_2 C s}$$

Next, using voltage division,

$$V_o(s) = \frac{R_3}{\frac{R_2}{1 + R_2 C s} + R_3} K I_a(s) = \frac{R_3 + R_2 R_3 C s}{R_2 + R_3 + R_2 R_3 C s} K I_a(s) = \frac{K(R_3 + R_2 R_3 C s)}{(Ls + R_1)(R_2 + R_3 + R_2 R_3 C s)} V_i(s)$$

Then, the transfer function of the circuit is

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{\frac{K}{L}\left(s + \frac{1}{R_2 C}\right)}{\left(s + \frac{R_1}{L}\right)\left(s + \frac{R_2 + R_3}{R_2 R_3 C}\right)} = \frac{5(s + 0.5)}{(s + 5)(s + 2.5)}$$

Using $V_i(s) = \frac{1}{s}$ gives

$$V_o(s) = \frac{H(s)}{s} = \frac{5(s + 0.5)}{s(s + 5)(s + 2.5)} = \frac{0.2}{s} + \frac{-1.8}{s + 5} + \frac{1.6}{s + 2.5}$$

Taking the inverse Laplace transform

$$v_o(t) = \left(0.2 - 1.8e^{-5t} + 1.6e^{-2.5t}\right)u(t) \text{ V}$$

(checked using LNAP 10/15/04)

P14.8-24

The transfer function of a circuit is $H(s)=\dfrac{12}{s^2+8s+16}$. Determine the step response of this circuit.

P14.8-24

The Laplace transform of the step response is:

$$\frac{H(s)}{s}=\frac{12}{s\left(s^2+8s+16\right)}=\frac{12}{s(s+4)^2}=\frac{\frac{3}{4}}{s}+\frac{-3}{(s+4)^2}+\frac{k}{s+4}$$

The constant *k* is evaluated by multiplying both sides of the last equation by $s(s+4)^2$.

$$12=\frac{3}{4}(s+4)^2-3s+ks(s+4)=\left(\frac{3}{4}+k\right)s^2+(3+4k)s+12 \quad\Rightarrow\quad k=-\frac{3}{4}$$

The step response is

$$\mathfrak{L}^{-1}\left[\frac{H(s)}{s}\right]=\left(\frac{3}{4}-e^{-4t}\left(3t+\frac{3}{4}\right)\right)u(t)\ \text{ V}$$

P14.8-25

The transfer function of a circuit is $H(s)=\dfrac{80\,s}{s^2+8\,s+25}$. Determine the step response of this circuit.

P14.8-25

The step response is given by

$$v_o(t)=\mathcal{L}^{-1}\left[\frac{H(s)}{s}\right]=\mathcal{L}^{-1}\left[\frac{80\,s}{s\left(s^2+8\,s+25\right)}\right]=\mathcal{L}^{-1}\left[\frac{80}{s^2+8\,s+25}\right]=\mathcal{L}^{-1}\left[\frac{80}{3}\times\frac{3}{(s+4)^2+3^2}\right]$$

$$=\frac{80}{3}e^{-4t}\sin(3t)u(t)\ \text{ V}$$

P14.8-26 The input to the circuit shown in Figure P14.8-26 is the current $i(t)$ and the output is the current $i_o(t)$. Determine the impulse response of this circuit.

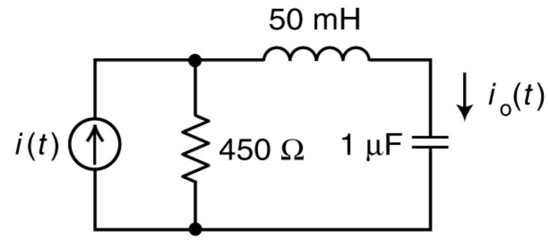
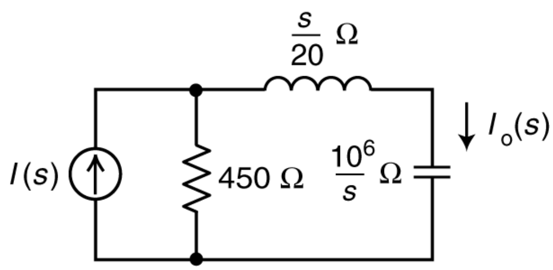


Figure P14.8-26

Solution: First, determine the transfer function from the circuit. To do so, represent the circuit in the s-domain as shown below.



Applying current division $$I_o(s) = \frac{450}{450 + \frac{s}{20} + \frac{10^6}{s}} I(s) = \frac{9000\,s}{s^2 + 9000\,s + 20{,}000{,}000} I(s)$$

The transfer function is $$H(s) = \frac{I_o(s)}{I(s)} = \frac{9000\,s}{s^2 + 9000\,s + 20{,}000{,}000}$$

Performing partial fraction expansion:

$$H(s) = \frac{9000\,s}{s^2 + 9000\,s + 20{,}000{,}000} = \frac{9000\,s}{(s+4000)(s+5000)} = \frac{\frac{9000(-4000)}{1000}}{s+4000} + \frac{\frac{9000(-5000)}{-1000}}{s+5000}$$

$$= \frac{-36000}{s+4000} + \frac{45000}{s+5000}$$

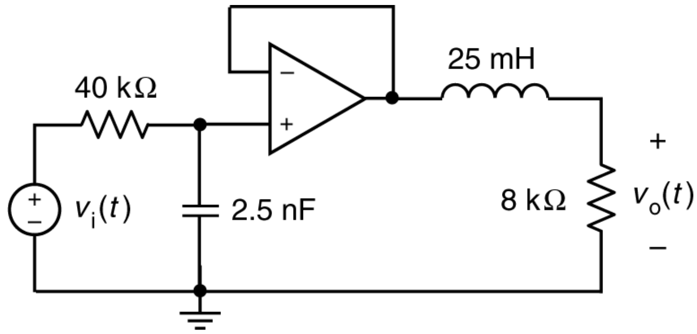
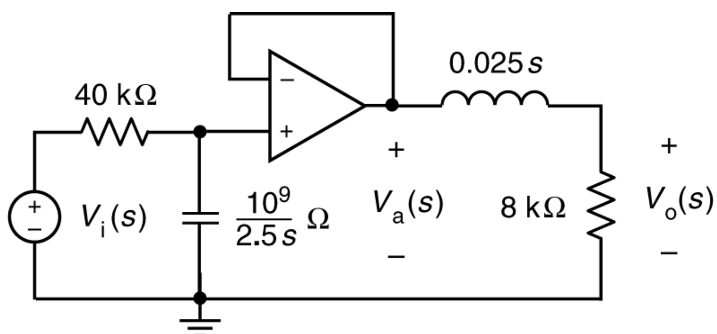


Figure 14.8-27

P14.8-27 The input to the circuit shown in Figure P14.8-29 is the voltage $v_i(t)$ and the output is the voltage $v_o(t)$. Determine the impulse response of this circuit.

Answer: $h(t)=10323\left(e^{-10,000t}-e^{-320,000t}\right)u(t)$ V

Solution: First, determine the transfer function from the circuit. To do so, represent the circuit in the s-domain:



Recognizing the voltage follower, we use voltage division twice:

$$V_a(s)=\frac{\frac{10^9}{2.5s}}{4\times10^4+\frac{10^9}{2.5s}}V_i(s)=\frac{10,000}{s+10,000}V_i(s) \quad \text{and} \quad V_o(s)=\frac{8000}{8000+0.025s}V_a(s)=\frac{320,000}{s+320,000}V_a(s)$$

The transfer function is $H(s)=\dfrac{V_o(s)}{V_i(s)}=\dfrac{3,200,000,000}{(s+10,000)(s+320,000)}=\dfrac{-10323}{s+10,000}+\dfrac{10323}{s+320,000}$

The impulse response is

$$h(t)=\mathcal{L}^{-1}\left[H(s)\right]=\mathcal{L}^{-1}\left[\frac{-10323}{s+10,000}+\frac{10323}{s+320,000}\right]=10323\left(e^{-10,000t}-e^{-320,000t}\right)u(t) \text{ V}.$$

P14.8-28 The input to the circuit shown in Figure P14.8-28 is the voltage $v_i(t)$ and the output is the voltage $v_o(t)$. Determine the impulse response of this circuit.

Answer: $h(t) = \left(10323\,e^{-320{,}000t} - 322.6e^{-10{,}000t}\right)u(t)\ \text{V}$

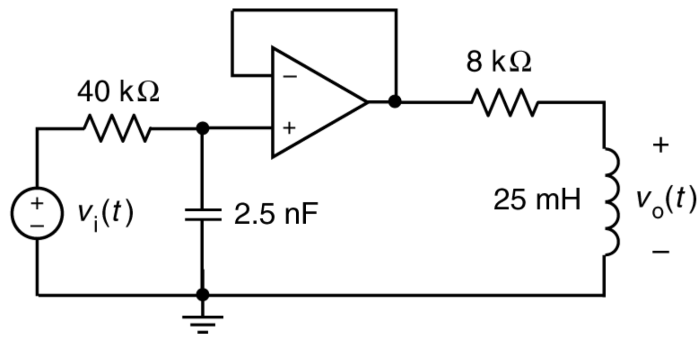
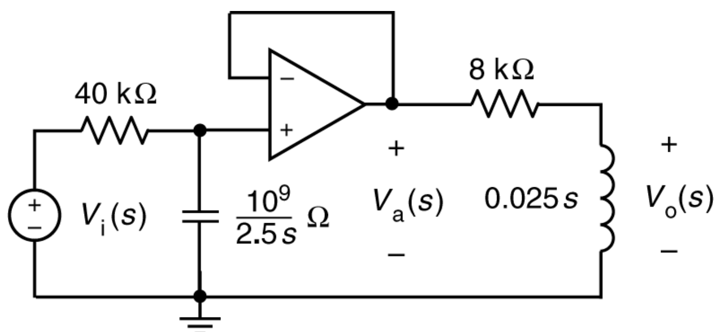


Figure 14.8-28

Solution: First, determine the transfer function from the circuit. To do so, represent the circuit in the s-domain:



Recognizing the voltage follower, we use voltage division twice:

$$V_a(s) = \frac{4\times10^4}{4\times10^4 + \dfrac{10^9}{2.5s}} V_i(s) = \frac{s}{s+10{,}000} V_i(s) \quad \text{and} \quad V_o(s) = \frac{0.025s}{8000+0.025s} V_a(s) = \frac{320{,}000}{s+320{,}000} V_a(s)$$

The transfer function is

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{320{,}000s}{(s+10{,}000)(s+320{,}000)} = \frac{-322.6}{s+10{,}000} + \frac{10323}{s+320{,}000}$$

The impulse response is

$$h(t) = \mathcal{L}^{-1}\left[H(s)\right] = \mathcal{L}^{-1}\left[\frac{-322.6}{s+10{,}000} + \frac{10323}{s+320{,}000}\right] = \left(10323\,e^{-320{,}000t} - 322.6e^{-10{,}000t}\right)u(t)\ \text{V}.$$

P14.8-29 The input to the circuit shown in Figure P14.8-29 is the voltage $v_{\text{i}}(t)$ and the output is the voltage $v_{\text{o}}(t)$. Determine the impulse response of this circuit.

Answer: $h(t)=\delta(t)+\left(322.6e^{-10,000t}-330323\,e^{-320,000t}\right)u(t)\ \text{V}$

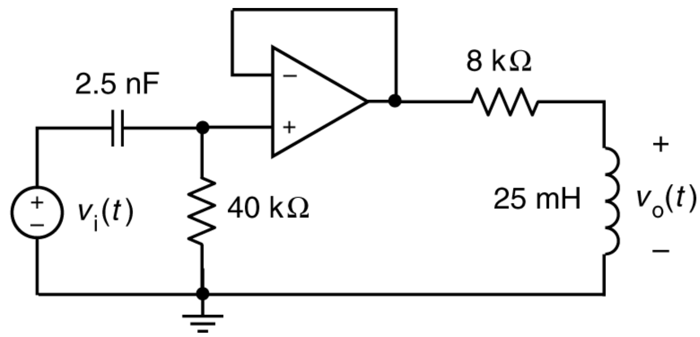
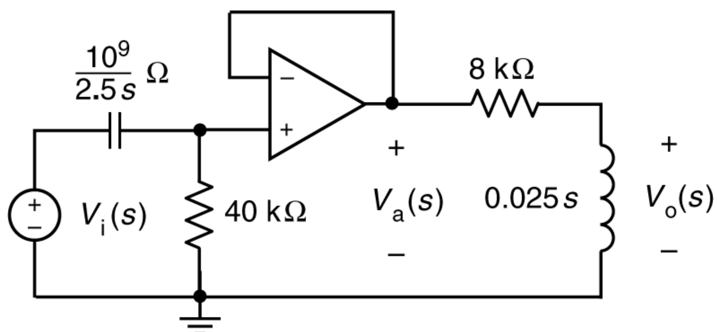


Figure 14.8-29

Solution: First, determine the transfer function from the circuit. To do so, represent the circuit in the s-domain:



Recognizing the voltage follower, we use voltage division twice:

$$V_{\text{a}}(s)=\frac{4\times10^4}{4\times10^4+\dfrac{10^9}{2.5\,s}}V_{\text{i}}(s)=\frac{s}{s+10,000}V_{\text{i}}(s)\quad\text{and}\quad V_{\text{o}}(s)=\frac{0.025\,s}{8000+0.025\,s}V_{\text{a}}(s)=\frac{s}{s+320,000}V_{\text{a}}(s)$$

The transfer function is

$$H(s)=\frac{V_{\text{o}}(s)}{V_{\text{i}}(s)}=\frac{s^2}{(s+10,000)(s+320,000)}$$

$$=1-\frac{330,000\,s+3,200,000,000}{(s+10,000)(s+320,000)}=1+\frac{322.6}{s+10,000}+\frac{-330323}{s+320,000}$$

The impulse response is

$$h(t)=\mathcal{L}^{-1}\left[H(s)\right]=\mathcal{L}^{-1}\left[1+\frac{322.6}{s+10,000}+\frac{-330323}{s+320,000}\right]=\delta(t)+\left(322.6e^{-10,000t}-330323\,e^{-320,000t}\right)u(t)\ \text{V}.$$

Section 14-9: Convolution Theorem

P14.9-1 The input to the circuit shown in Figure P14.6-1a is the voltage $v_i(t)$ shown in Figure P14.9-1. Plot the output, $v_o(t)$, of the circuit.

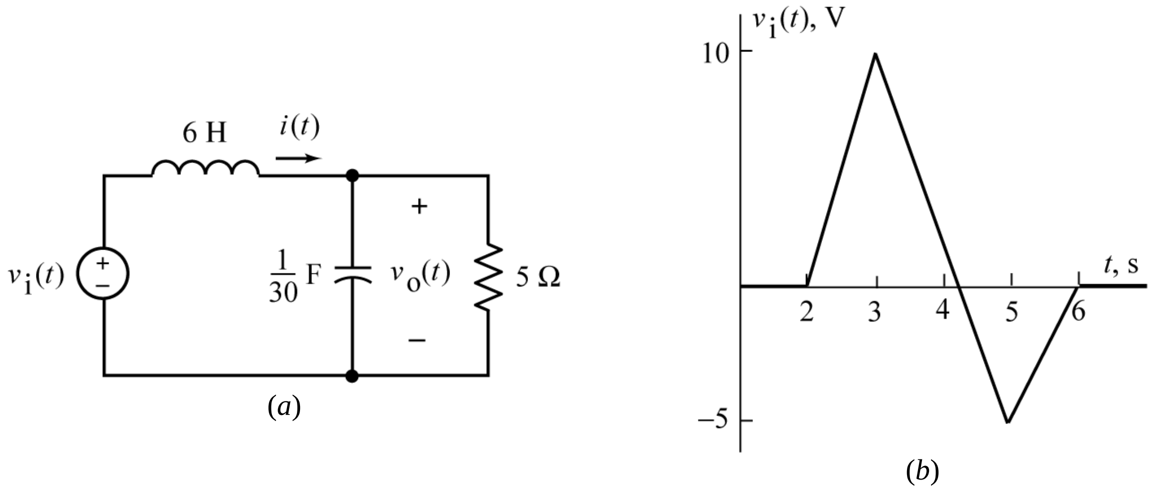
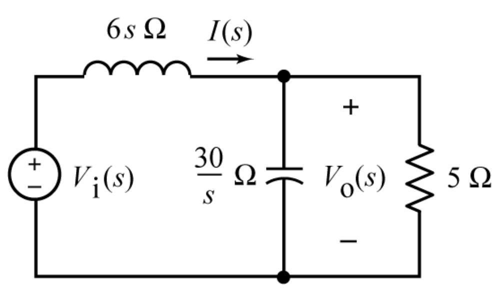


Figure P14.9-1

Solution:
To solve this problem using convolution, we first represent the input is by the function

$$v_i(t) = \begin{cases} 0 & t \le 2 \\ 10t - 20 & 2 \le t \le 3 \\ -7.5t + 32.5 & 3 \le t \le 5 \\ 5t - 30 & 5 \le t \le 6 \\ 0 & 6 \le t \end{cases}$$

Next, we obtain the impulse response. To do so, assume that the initial conditions are zero and represent the circuit in the s-domain as



Using voltage division and equivalent impedance, the transfer function is

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{\frac{30}{s} \| 5}{6s + \frac{30}{s} \| 5} = \frac{\frac{30}{s+6}}{6s + \frac{30}{s+6}} = \frac{5}{s^2 + 6s + 5} = \frac{1.25}{s+1} - \frac{1.25}{s+5}$$

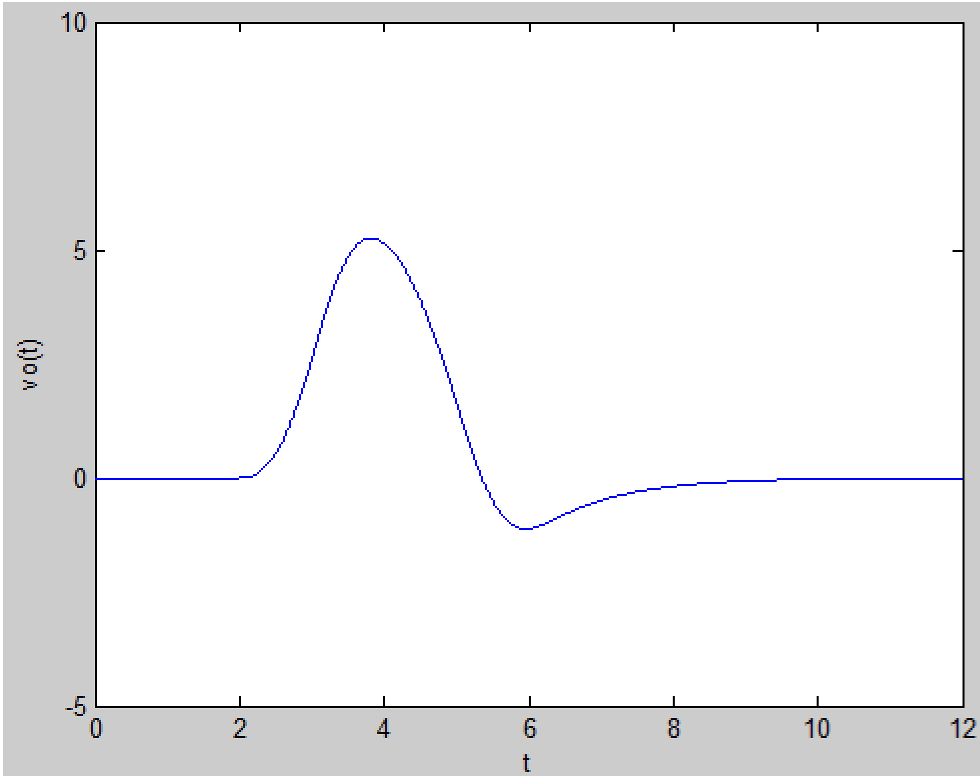
The impulse response is

$$h(t) = \pounds^{-1}\left[H(s)\right] = \pounds^{-1}\left[\frac{1.25}{s+1} - \frac{1.25}{s+5}\right] = \left(1.25e^{-t} - 1.25e^{-5t}\right)u(t)$$

Edit the MATLAB script from Example 14.9.1 to obtain

```
% P14_9_1.m - plots the output for Problem 14.9-1
% --------------------------------------------------
--
%  Obtain a list of equally spaced instants of time
% --------------------------------------------------
--
t0 = 0; % begin
tf = 12;  % end
N = 5000;  % number of points plotted
dt = (tf-t0)/N; % increment
t = t0:dt:tf; % time in seconds

% --------------------------------------------------
--
% Obtain the input x(t) and the impulse response
h(t)
% --------------------------------------------------
--
for k = 1 : length(t)
    if t(k) < 2
        x(k) = 0;
    elseif t(k) < 3
        x(k) = -20 + 10*t(k); %
    elseif t(k) < 5
        x(k) = +32.5 - 7.5*t(k); %
    elseif t(k) < 6
        x(k) = -30 + 5*t(k); %
    else
        x(k) = 0;
    end
end
x=x*dt;
h=1.25*exp(-t)-1.25*exp(-5*t);

% --------------------------------------------------
--
%             Perform the convolution
% --------------------------------------------------
--
y=conv(x,h);

% --------------------------------------------------
--
%             Plot the output y(t)
% --------------------------------------------------
--
plot(t,y(1:length(t)))
axis([t0, tf, -5, 10])
xlabel('t')
ylabel('y(t)')
```

Running this script produces the required plot of the output voltage:



P14.9-2 The input to the circuit shown in Figure P14.9-2a is the voltage $v_i(t)$ shown in Figure P14.9-2. (Perhaps $v_i(t)$ represents the binary sequence 1101 which, in turn, might represent the decimal number 15.) Plot the output, $v_o(t)$, of the circuit.

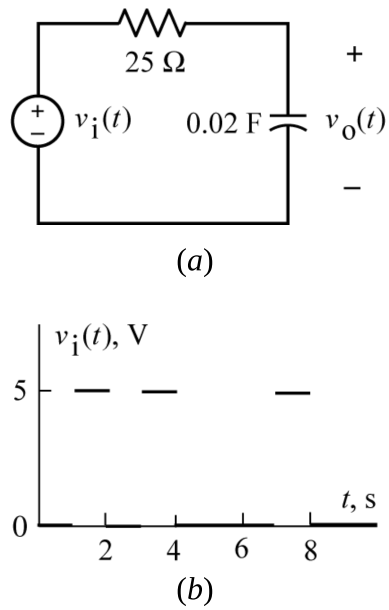


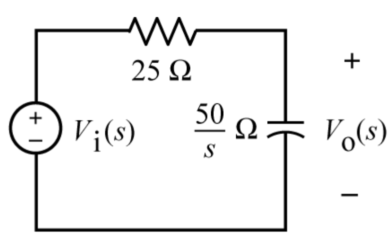
Figure P14.9-2

Solution:

To solve this problem using convolution, we first represent the input is by the function

$$v_i(t) = \begin{cases} 5 & 1 \le t \le 2 \text{ or } 3 \le t \le 5 \text{ or } 7 \le t \le 8 \\ 0 & \text{otherwise} \end{cases}$$

Next, we obtain the impulse response. To do so, assume that the initial conditions are zero and represent the circuit in the s-domain as



Using voltage division, the transfer function is

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{\frac{50}{s}}{\frac{50}{s} + 25} = \frac{2}{s+2}$$

The impulse response is

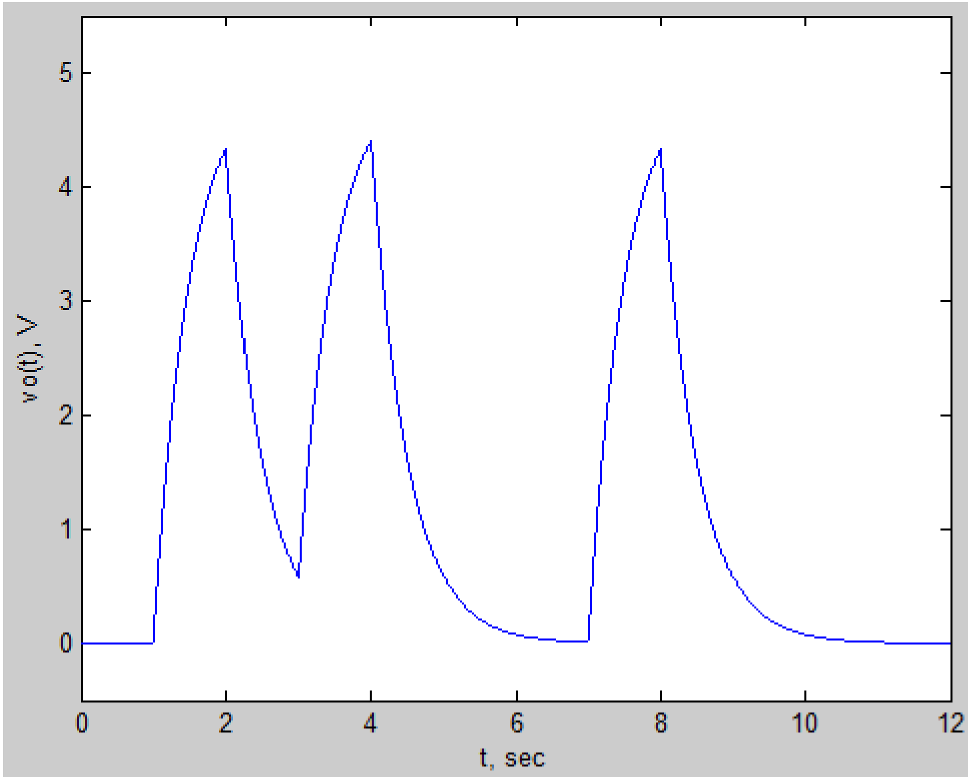
$$h(t) = £^{-1}\left[H(s)\right] = £^{-1}\left[\frac{2}{s+2}\right] = 2e^{-2t}u(t)$$

$$h(t) = 2e^{-2t}$$

Edit the MATLAB script from Example 14.9.1 to obtain

```
% P14_9_2.m - plots the output for Problem 14.9-2
% -------------------------------------------------
--
%  Obtain a list of equally spaced instants of time
% -------------------------------------------------
--
t0 = 0; % begin
tf = 12;  % end
N = 5000;  % number of points plotted
dt = (tf-t0)/N; % increment
t = t0:dt:tf; % time in seconds

% -------------------------------------------------
--
% Obtain the input x(t) and the impulse response
h(t)
% -------------------------------------------------
--
for k = 1 : length(t)
    x(k) = 0;
    if (t(k)>1) & (t(k)<2)
        x(k) = 5;
    end
    if (t(k)>3) & (t(k)<4)
        x(k) = 5;
    end
    if (t(k)>7) & (t(k)<8)
        x(k) = 5;
    end
end
x=x*dt;
h=2*exp(-2*t);

% -------------------------------------------------
--
%             Perform the convolution
% -------------------------------------------------
--
y=conv(x,h);

% -------------------------------------------------
--
%             Plot the output y(t)
% -------------------------------------------------
--
plot(t,y(1:length(t)))
axis([t0, tf, -0.5, 5.5])
xlabel('t, sec')
ylabel('vo(t), V')
```

Running this script produces the required plot of the output voltage:


vo(t), V
t, sec
0
1
2
3
4
5
0
2
4
6
8
10
12

Section 14-10: Stability

P 14.10-1 The input to the circuit shown in Figure P 14.10-1 is the voltage, $v_i(t)$, of the independent voltage source. The output is the voltage, $v_o(t)$, across the resistor labeled R. The step response of this circuit is

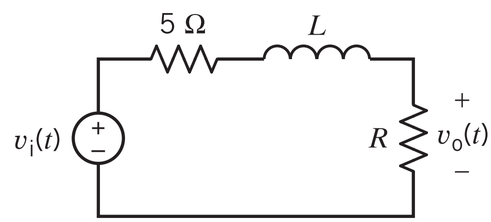


Figure P 14.10-1

$$v_o(t) = (3/4)(1 - e^{-100t})u(t) \quad \text{V}$$

(a) Determine the value of the inductance, L, and the value of the resistance, R.

(b) Determine the impulse response of this circuit.

(c) Determine the steady-state response of the circuit when the input is $v_i(t) = 5 \cos 100\, t$ V.

Solution:

a. From the given step response:

$$\frac{H(s)}{s} = \mathcal{L}\left[\frac{3}{4}\left(1-e^{-100t}\right)u(t)\right] = \frac{75}{s(s+100)}$$

From the circuit: $$H(s) = \frac{R}{R+5+Ls} \quad \Rightarrow \quad \frac{H(s)}{s} = \frac{\frac{R}{L}}{s\left(s+\frac{R+5}{L}\right)}$$

Comparing gives

$$\left.\begin{array}{l} \dfrac{R}{L} = 75 \\ \dfrac{R+5}{L} = 100 \end{array}\right\} \Rightarrow \begin{array}{l} R = 15\ \Omega \\ L = 0.2\ \text{H} \end{array}$$

b. The impulse response is $$h(t) = \mathcal{L}^{-1}\left[\frac{75}{s+100}\right] = 75\, e^{-100t} u(t)$$

c.

$$\mathbf{H}(\omega)\Big|_{\omega=100} = \frac{75}{j100+100} = \frac{3}{4\sqrt{2}} \angle -45°$$

$$\mathbf{V}_o(\omega) = \left(\frac{3}{4\sqrt{2}} \angle 45°\right)(5\angle 0°) = \frac{15}{4\sqrt{2}} \angle -45° \ \text{V}$$

$$v_o(t) = 2.652 \cos(100t - 45°) \ \text{V}$$

(Checked using LNAP, 12/29/02)

P 14.10-2 The input to the circuit shown in Figure P 14.10-2 is the voltage, $v_i(t)$, of the independent voltage source. The output is the voltage, $v_o(t)$, across the capacitor. The step response of this circuit is

$$v_o(t) = [5 - 5e^{-2t}(1 + 2t)]u(t) \text{ V}$$

Determine the steady-state response of this circuit when the input is

$$vi(t) = 5 \cos(2t + 45°) \text{ V}$$

Answer: $v_o(t) = 12.5 \cos(2t - 45°)$ V

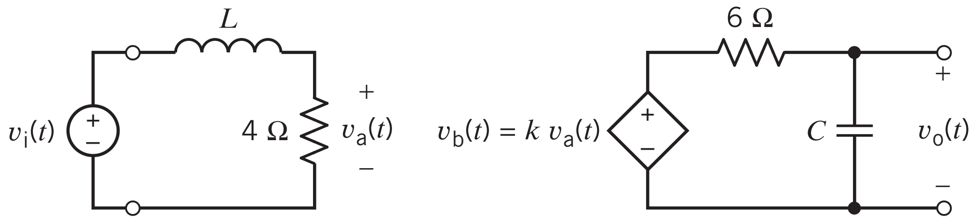


Figure P 14.10-2

Solution:

The transfer function of this circuit is given by

$$\frac{H(s)}{s} = \mathcal{L}\left[\left(5 - 5e^{-2t}(1+2t)\right)u(t)\right] = \frac{5}{s} + \frac{-5}{s+2} + \frac{-10}{(s+2)^2} = \frac{20}{(s+2)^2} \quad \Rightarrow \quad H(s) = \frac{20}{(s+2)^2}$$

This transfer function is stable so we can determine the network function as

$$\mathbf{H}(\omega) = H(s)\big|_{s=j\omega} = \left.\frac{20}{(s+2)^2}\right|_{s=j\omega} = \frac{20}{(2+j\omega)^2}$$

The phasor of the output is

$$\mathbf{V}_o(\omega) = \frac{20}{(2+j2)^2}(5\angle 45°) = \frac{20}{\left(2\sqrt{2}\angle 45°\right)^2}(5\angle 45°) = 12.5\angle -45° \text{ V}$$

The steady-state response is

$$v_o(t) = 12.5\cos(2t - 45°) \text{ V}$$

(Checked using LNAP, 12/29/02)

P 14.10-3 The input to a linear circuit is the voltage $v_i(t)$ and the response is the voltage $v_o(t)$. The impulse response, $h(t)$, of this circuit is

$$h(t) = 30te^{-5t}u(t)\text{V}$$

Determine the steady-state response of this circuit when the input is

$$v_i(t) = 10\cos(3t) \quad \text{V}$$

Answer: $v_o(t) = 8.82\cos(3t - 62°)$ V

Solution:

The transfer function of the circuit is $H(s) = \mathcal{L}^{-1}\left[30\,t\,e^{-5t}u(t)\right] = \dfrac{30}{(s+5)^2}$. The circuit is stable so we can determine the network function as

$$\mathbf{H}(\omega) = H(s)\Big|_{s=j\omega} = \frac{30}{(s+5)^2}\Bigg|_{s=j\omega} = \frac{30}{(5+j\omega)^2}$$

The phasor of the output is

$$\mathbf{V}_o(\omega) = \frac{30}{(5+j3)^2}(10\angle 0°) = \frac{30}{(5.83\angle 31°)^2}(10\angle 0°) = 8.82\angle -62° \;\; \text{V}$$

The steady-state response is

$$v_o(t) = 8.82\cos(3t - 62°) \;\; \text{V}$$

P 14.10-4 The input to a circuit is the voltage v_s. The output is the voltage v_o. The step response of the circuit is

$$v_o(t) = (40 + 1.03e^{-8t} - 41e^{-320t})u(t)$$

Determine the network function

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)}$$

of the circuit and sketch the asymptotic magnitude Bode plot.

Solution:

$$\frac{H(s)}{s} = \mathcal{L}\left[\left(40 - 41.03\, e^{-8t} + 1.03\, e^{-320t}\right)u(t)\right] = \frac{40}{s} - \frac{41.03}{s+8} + \frac{1.03}{s+320} = \frac{102400}{s(s+8)(s+320)}$$

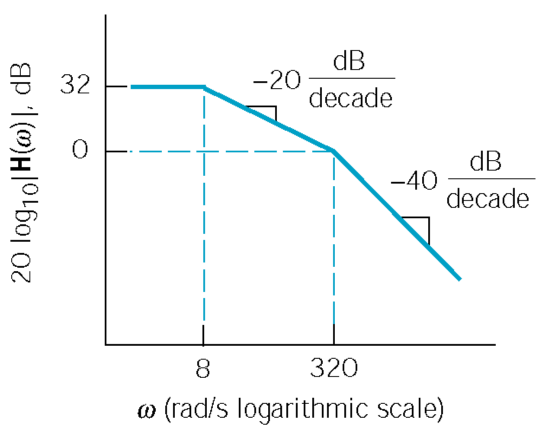
so

$$H(s) = \frac{102400}{(s+8)(s+320)}$$

The poles of the transfer function are $s_1 = -8$ rad/s and $s_2 = -320$ rad/s, so circuit is stable. Consequently,

$$\mathbf{H}(\omega) = H(s)\big|_{s=j\omega} = \frac{102400}{(j\omega+8)(j\omega+320)} = \frac{40}{\left(1+j\dfrac{\omega}{8}\right)\left(1+j\dfrac{\omega}{320}\right)}$$

The network function has poles at 8 and 320 rad/s and has a low frequency gain equal to 32 dB = 40. Consequently, the asymptotic magnitude Bode plot is



P 14.10-5 The input to a circuit is the voltage v_s. The output is the voltage v_o. The step response of the circuit is

$$v_o(t) = 60(e^{-2t} - e^{-6t})u(t)$$

Determine the network function

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)}$$

of the circuit and sketch the asymptotic magnitude Bode plot.

Solution:

$$\frac{H(s)}{s} = \mathcal{L}\left[60\left(e^{-2t} - e^{-6t}\right)u(t)\right] = \frac{60}{s+2} - \frac{60}{s+6} = \frac{240}{(s+2)(s+6)}$$

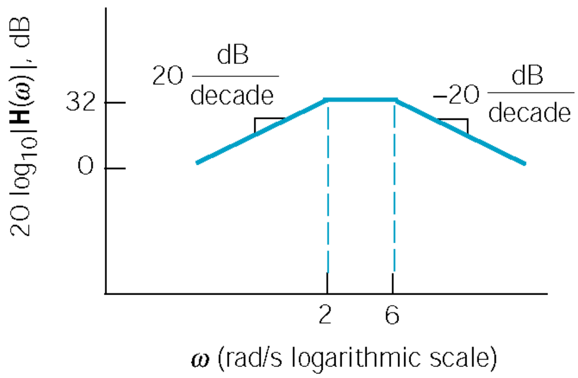
so

$$H(s) = \frac{240\,s}{(s+6)(s+2)}$$

The poles of the transfer function are $s_1 = -2$ rad/s and $s_2 = -6$ rad/s, so circuit is stable. Consequently,

$$\mathbf{H}(\omega) = H(s)\Big|_{s=j\omega} = \frac{240\;j\omega}{(j\omega+2)(j\omega+6)} = \frac{20\;j\omega}{\left(1+j\dfrac{\omega}{2}\right)\left(1+j\dfrac{\omega}{6}\right)}$$

The network function has poles at 2 and 6 rad/s. The asymptotic magnitude Bode plot has a gain equal to 40 = 32 dB between 2 and 6 rad/s. Consequently, the asymptotic magnitude Bode plot is



P 14.10-6 The input to a circuit is the voltage v_s. The output is the voltage v_o. The step response of the circuit is

$$v_o(t) = (4 + 32e^{-90t})u(t)$$

Determine the network function

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)}$$

of the circuit and sketch the asymptotic magnitude Bode plot.

Solution:

$$\frac{H(s)}{s} = \mathcal{L}\left[\left(4+32\,e^{-90t}\right)u(t)\right] = \frac{4}{s} + \frac{32}{s+90} = \frac{36\,s+360}{s(s+90)} = \frac{36(s+10)}{s(s+90)}$$

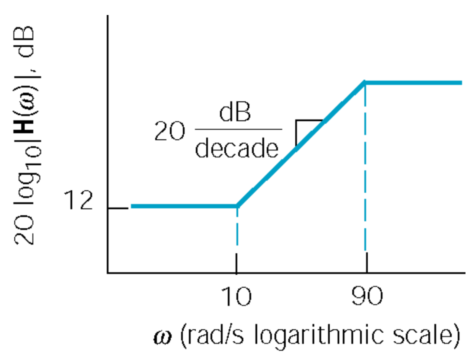
so

$$H(s) = 36\frac{(s+10)}{(s+90)}$$

The pole of the transfer function $s_1 = -90$ rad/s , so circuit is stable. Consequently,

$$\mathbf{H}(\omega) = H(s)\Big|_{s=j\omega} = 36\,\frac{(j\omega+10)}{(j\omega+90)} = 4\frac{\left(1+j\dfrac{\omega}{10}\right)}{\left(1+j\dfrac{\omega}{90}\right)}$$

The network function has a zero at 10 rad/s and a pole at 90 rad/s. The low frequency gain is equal to 4 = 12 dB. Consequently, the asymptotic magnitude Bode plot is



P 14.10-7 The input to a circuit is the voltage v_s. The output is the voltage v_o. The step response of the circuit is

$$v_o(t) = \frac{5}{3}\left(e^{-5t} - e^{-20t}\right)u(t)\,\text{V}$$

Determine the steady-state response of the circuit when the input is

$$v_s(t) = 12 \cos(30t) \text{ V}$$

Solution:

$$\frac{H(s)}{s} = \mathcal{L}\left[\frac{5}{3}\left(e^{-5t} - e^{-20t}\right)u(t)\right] = \frac{1.67}{s+5} - \frac{1.67}{s+20} = \frac{25}{(s+5)(s+20)}$$

so

$$H(s) = \frac{25\,s}{(s+5)(s+20)}$$

The poles of the transfer function are $s_1 = -5$ rad/s and $s_2 = -20$ rad/s, so the circuit is stable. Consequently the network function of the circuit is,

$$\mathbf{H}(\omega) = H(s)\Big|_{s=j\omega} = \frac{25\,j\omega}{(j\omega+5)(j\omega+20)} = \frac{0.25\,j\omega}{\left(1+j\dfrac{\omega}{5}\right)\left(1+j\dfrac{\omega}{20}\right)}$$

Using $\mathbf{V}_o(\omega) = \mathbf{H}(\omega)\mathbf{V}_s(\omega)$ at ω = 30 rad/s gives

$$\mathbf{V}_o(\omega) = \frac{0.25(j30)}{\left(1+j\dfrac{30}{5}\right)\left(1+j\dfrac{30}{20}\right)}(12\angle 0) = \frac{j90}{(1+j6)(1+j1.5)} = 8.2\angle -47° \text{ V}$$

Back in the time domain, the steady state response is

$$v_o(t) = 8.2\cos(30t - 47°) \text{ V}$$

(checked using LNAP 10/12/04)

P 14.10-8 The input to a circuit is the voltage v_s. The output is the voltage v_o. The impluse response of the circuit is

$$v_o(t) = e^{-5t}(10 - 50t)u(t) \text{ V}$$

Determine the steady-state response of the circuit when the input is

$$v_s(t) = 12 \cos(10t) \text{ V}$$

Solution:

$$H(s) = \mathcal{L}\left[e^{-5t}(10-50t)u(t)\right] = \frac{10}{s+5} - \frac{50}{(s+5)^2} = \frac{10(s+5)-50}{(s+5)^2} = \frac{10\,s}{(s+5)^2}$$

The poles of the transfer function are $s_1 = -5$ rad/s and $s_2 = -5$ rad/s, so the circuit is stable. Consequently the network function of the circuit is,

$$\mathbf{H}(\omega) = H(s)\Big|_{s=j\omega} = \frac{10\,j\omega}{(j\omega+5)^2} = \frac{0.4\,j\omega}{\left(1+j\dfrac{\omega}{5}\right)^2}$$

Using $\mathbf{V}_o(\omega) = \mathbf{H}(\omega)\mathbf{V}_s(\omega)$ at $\omega = 10$ rad/s gives

$$\mathbf{V}_o(\omega) = \frac{0.4(j10)}{\left(1+j\dfrac{10}{5}\right)^2}(12\angle 0) = \frac{j48}{(1+j2)^2} = 9.6\angle -37° \text{ V}$$

Back in the time domain, the steady state response is

$$v_o(t) = 9.6\cos(10t - 37°) \text{ V}$$

(checked using LNAP 10/12/04)

P14.10-9 The input to a circuit is the voltage v_s. The output is the voltage v_o. The step response of the circuit is

$$v_o(t) = \left(1 - e^{-20t}\left(\cos(4t) + 0.5\sin(4t)\right)\right)u(t) \text{ V}$$

Determine the steady state response of the circuit when the input is

$$v_s(t) = 12\cos(4t) \text{ V}$$

Solution:

$$\frac{H(s)}{s} = \mathcal{L}\left[\left(1 - e^{-2t}\left(\cos(4t) + 0.5\sin(4t)\right)\right)u(t)\right] = \frac{1}{s} - \left[\frac{(s+2)}{(s+2)^2 + 4^2} + \frac{0.5(4)}{(s+2)^2 + 4^2}\right]$$

$$= \frac{1}{s} - \frac{s+4}{s^2 + 4s + 20}$$

$$= \frac{s^2 + 4s + 20 - s(s+4)}{s(s^2 + 4s + 20)} = \frac{20}{s(s^2 + 4s + 20)}$$

so

$$H(s) = \frac{20}{s^2 + 4s + 20}$$

The poles of the transfer function are $s_{1,2} = \dfrac{-4 \pm \sqrt{16 - 80}}{2} = -2 \pm j4$, so the circuit is stable.

Consequently the network function of the circuit is,

$$\mathbf{H}(\omega) = H(s)\Big|_{s=j\omega} = \frac{20}{20 - \omega^2 + 4j\omega}$$

Using $\mathbf{V}_o(\omega) = \mathbf{H}(\omega)\mathbf{V}_s(\omega)$ at ω = 4 rad/s gives

$$\mathbf{V}_o(\omega) = \frac{20}{20 - 4^2 + 4(j4)}(12\angle 0) = \frac{240}{4 + j16} = 14.6\angle -76° \text{ V}$$

Back in the time domain, the steady state response is

$$v_o(t) = 14.6\cos(4t - 76°) \text{ V}$$

(checked using LNAP 10/12/04)

P14.10_10

The transfer function of a circuit is $H(s)=\dfrac{20}{s+8}$. When the input to this circuit is sinusoidal, the output is also sinusoidal. Let ω_1 be the frequency at which the output sinusoid is twice as large as the input sinusoid and let ω_2 be the frequency at which output sinusoid is delayed by one tenth period with respect to the input sinusoid. Determine the values of ω_1 and ω_2.

Solution:

The circuit is stable so $\mathbf{H}(\omega)=H(s)\big|_{s\leftarrow j\omega}=\dfrac{20}{8+j\omega}$.

The gain is 2 at the frequency ω_1 so $2=\dfrac{20}{\sqrt{8^2+\omega_1{}^2}}$ and $\omega_1=\sqrt{\left(\dfrac{20}{2}\right)^2-8^2}=6 \text{ rad/s}$.

When the frequency is ω_2, the period is $\dfrac{2\pi}{\omega_2}$. Also a delay t_o corresponds to a phase shift $-\omega_2 t_o$.

In this case, $t_0=0.1\left(\dfrac{2\pi}{\omega_2}\right)$ so the phase shift is -0.2 π. Then $-0.2\pi=-\tan^{-1}\left(\dfrac{\omega_2}{8}\right)$ so

$\omega_2=8\tan(0.2\pi)=5.8123 \text{ rad/s}$.

P14.10-11

The input to a linear circuit is the voltage, v_i. The output is the voltage, v_o. The transfer function of the circuit is

$$H(s) = \frac{V_o(s)}{V_i(s)}$$

The poles and zeros of $H(s)$ are shown on the pole-zero diagram in Figure P14.10-11. (There are no zeros.) The dc gain of the circuit is

$$\mathbf{H}(0) = 5$$

Determine the step response of the circuit.

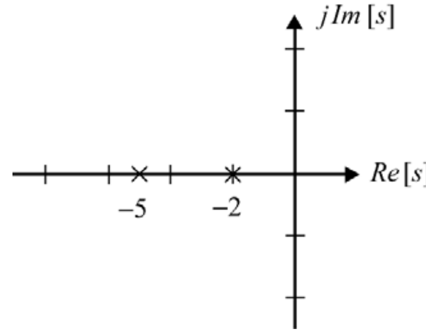


Figure P14.10-11

Solution:

The transfer function of the circuit is

$$H(s) = \frac{a}{(s+2)(s+5)}$$

where *a* is a constant to be determined. The circuit is stable because all its poles lie in the right half of the *s*-plane. Consequently,

$$\mathbf{H}(\omega) = H(s)\Big|_{s=j\omega} = \frac{a}{(2+j\omega)(5+j\omega)} = \frac{\frac{a}{10}}{\left(1+j\frac{\omega}{2}\right)\left(1+j\frac{\omega}{5}\right)}$$

At dc ($\omega = 0$)

$$5 = \mathbf{H}(0) = \frac{a}{10} \quad \Rightarrow \quad a = 50$$

The step response is given by

$$v_o(t) = \mathcal{L}^{-1}\left[\frac{H(s)}{s}\right] = \mathcal{L}^{-1}\left[\frac{50}{s(s+2)(s+5)}\right] = \mathcal{L}^{-1}\left[\frac{5}{s} + \frac{\frac{10}{3}}{s+5} - \frac{\frac{25}{3}}{s+2}\right] = \left(5 + \frac{10}{3}e^{-5t} - \frac{25}{3}e^{-2t}\right)u(t) \text{ V}$$

P14.10-12

The input to a linear circuit is the voltage, v_i. The output is the voltage, v_o. The transfer function of the circuit is

$$H(s) = \frac{V_o(s)}{V_i(s)}$$

The poles and zeros of $H(s)$ are shown on the pole-zero diagram in Figure 14.10-12. At $\omega = 5$ rad/s the gain of the circuit is

$$\mathbf{H}(5) = 10$$

Determine the step response of the circuit.

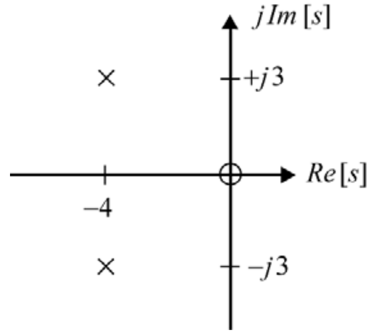


Figure P14.10-12

Solution:

The transfer function of the circuit is

$$H(s) = \frac{a(s-0)}{(s+4-j3)(s+4+j3)} = \frac{a\,s}{s^2+8s+25}$$

where *a* is a constant to be determined. The circuit is stable because all its poles lie in the right half of the *s*-plane. Consequently,

$$\mathbf{H}(\omega) = H(s)\Big|_{s=j\omega} = \frac{j\,a\,\omega}{(j\omega)^2 + j8\omega + 25} = \frac{j\,a\,\omega}{(25-\omega^2) + j8\omega}$$

At $\omega = 5$ rad/s

$$10 = \frac{j\,a\,5}{(25-5^2)+j8(5)} = \frac{a}{8} \quad \Rightarrow \quad a = 80$$

The step response is given by

$$v_o(t) = \mathcal{L}^{-1}\left[\frac{H(s)}{s}\right] = \mathcal{L}^{-1}\left[\frac{80\,s}{s(s^2+8s+25)}\right] = \mathcal{L}^{-1}\left[\frac{80}{s^2+8s+25}\right] = \mathcal{L}^{-1}\left[\frac{80}{3}\times\frac{3}{(s+4)^2+3^2}\right]$$

$$= \frac{80}{3}e^{-4t}\sin(3t)u(t) \text{ V}$$

P14.10-13

The input to a linear circuit is the voltage, v_i. The output is the voltage, v_o. The transfer function of the circuit is

$$H(s)=\frac{V_o(s)}{V_i(s)}$$

The poles and zeros of $H(s)$ are shown on the pole-zero diagram in Figure P14.10-13. (There is a double pole at s = – 4.) The dc gain of the circuit is

$$\mathbf{H}(0)=5$$

Determine the step response of the circuit.

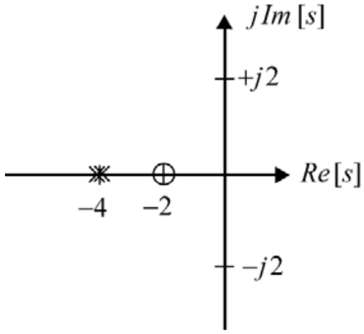


Figure P14.10-13

Solution:

The transfer function of the circuit is

$$H(s)=\frac{a(s+2)}{(s+4)^2}$$

where *a* is a constant to be determined. The circuit is stable because all its poles lie in the right half of the *s*-plane. Consequently,

$$\mathbf{H}(\omega)=H(s)\big|_{s=j\omega}=\frac{a(j\omega+2)}{(j\omega+4)^2}=\frac{\frac{a}{8}\left(1+j\frac{\omega}{2}\right)}{\left(1+j\frac{\omega}{4}\right)^2}$$

At dc ($\omega = 0$)

$$5=\mathbf{H}(0)=\frac{a}{8}\quad\Rightarrow\quad a=40$$

The step response is given by

$$v_o(t)=\mathcal{L}^{-1}\left[\frac{H(s)}{s}\right]=\mathcal{L}^{-1}\left[\frac{40(s+2)}{s(s+4)^2}\right]=\mathcal{L}^{-1}\left[\frac{5}{s}-\frac{5}{s+4}+\frac{20}{(s+4)^2}\right]=\left(5+(20t-5)e^{-4t}\right)u(t)\ \text{ V}$$

P14.10-14

The input to a circuit is the voltage, v_i. The step response of the circuit is

$$v_o = 5e^{-4t}\sin(2t)u(t) \text{ V}$$

Sketch the pole-zero diagram for this circuit.

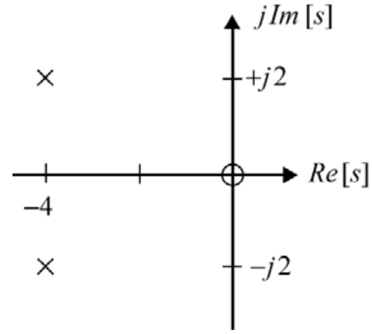
Solution:

$$\frac{H(s)}{s} = \mathcal{L}\left[5e^{-4t}\sin(2t)u(t)\right] = 5\times\frac{2}{s^2+2^2}\bigg|_{s\leftarrow s+4} = \frac{10}{(s+4)^2+2^2}$$

$$= \frac{10}{s^2+8s+20} = \frac{10}{(s+4-j2)(s+4+j2)}$$

Consequently

$$H(s) = \frac{10\,s}{(s+4-j2)(s+4+j2)}$$



P14.10-15

The input to a circuit is the voltage, v_i. The step response of the circuit is

$$v_o = 5t\,e^{-4t}\,u(t) \text{ V}$$

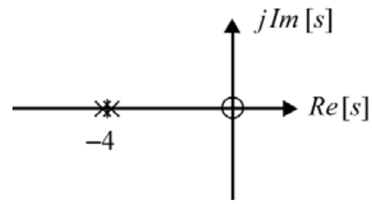
Sketch the pole-zero diagram for this circuit.

Solution:

$$\frac{H(s)}{s} = \mathcal{L}\left[5t\,e^{-4t}\,u(t) \text{ V}\right] = 5\times\frac{1}{s^2}\bigg|_{s\leftarrow s+4} = \frac{5}{(s+4)^2}$$

Consequently

$$H(s) = \frac{5\,s}{(s+4)^2}$$



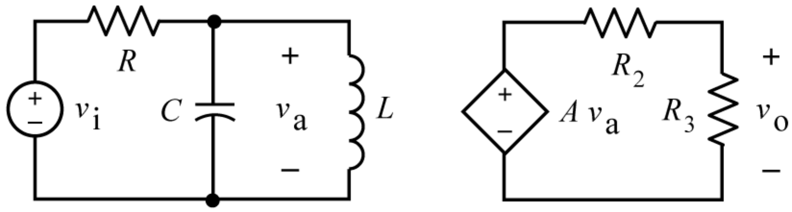


Figure 14.10-16

P14.10-16 The input to the circuit shown in Figure P14.10-16 is the voltage, v_i, of the voltage source. The output is the voltage, v_o, across resistor R_3. The transfer function of this circuit is

$$H(s) = \frac{120\,s}{s^2 + 24\,s + 208}$$

a. Determine values of circuit parameters *A*, *R*, R_2, R_3, *L* and *C* that cause the circuit to have the specified transfer function.

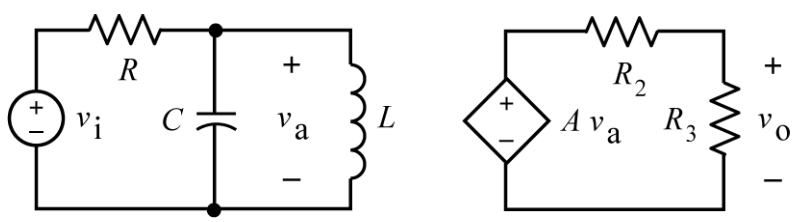
b. Determine the step response of this circuit.

c. Determine the steady state response of the circuit to the input $v_i(t) = 3.2\cos(10\,t + 30°)$ V .

Solution:

The input is v_i, the output is v_o ,and the transfer function is

$$H(s) = \frac{120\,s}{s^2 + 24\,s + 208}$$



a. Using voltage division twice we see that the transfer function is:

$$V_o(s) = \left(\frac{L\,s \,||\, \frac{1}{C\,s}}{R + \left(L\,s \,||\, \frac{1}{C\,s} \right)} \right) \left(\frac{A\,R_3}{R_2 + R_3} \right) V_i(s)$$

where

$$L\,s \parallel \frac{1}{C\,s} = \frac{\frac{L}{C}}{L\,s+\frac{1}{C\,s}} \times \frac{\frac{s}{L}}{\frac{s}{L}} = \frac{\frac{s}{C}}{s^2+\frac{1}{L\,C}} \quad \text{and} \quad \frac{L\,s \parallel \frac{1}{C\,s}}{R+\left(L\,s \parallel \frac{1}{C\,s}\right)} = \frac{\frac{\frac{s}{C}}{s^2+\frac{1}{L\,C}}}{R+\frac{\frac{s}{C}}{s^2+\frac{1}{L\,C}}} = \frac{\frac{1}{R\,C}s}{s^2+\frac{1}{R\,C}s+\frac{1}{L\,C}}$$

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{\left(\frac{A\,R_3}{R_2+R_3}\right)\frac{1}{R\,C}s}{s^2+\frac{1}{R\,C}s+\frac{1}{L\,C}} = \frac{120\,s}{s^2+24\,s+208}$$

Equating coefficients gives:

$$\frac{1}{L\,C} = 208\,,\; \frac{1}{R\,C} = 24 \;\text{ and }\; \frac{A\,R_3}{R_2+R_3} = 5$$

The solution of these three equations is not unique as they involve five unknowns. After some trial and error we see that $C = \frac{1}{64}\ \text{F} = 15.625\ \text{mF}$ is a convenient value of capacitance. The we calculate $L = \frac{64}{208}\ \text{H} = 307.7\ \text{mH}$ and $R = \frac{64}{24}\ \Omega = 2.67\ \Omega$. Suppose we choose $R_2 = R_3 = 5\ \Omega$ then $\frac{A}{2} = 5$ is required so we chose A = 10 V/V.

b. The step response is

$$step\;\; response = L^{-1}\left[\frac{H(s)}{s}\right]$$

Consider

$$\frac{H(s)}{s} = \frac{120\,s}{s\left(s^2+24\,s+208\right)} = \frac{120}{s^2+24\,s+208} = \frac{120}{(s+12)^2+64} = \frac{15(8)}{(s+12)^2+8^2} = 15\left[\frac{8}{s^2+8^2}\right]_{s\leftarrow s+12}$$

Here the notation $\left[F(s)\right]_{s\leftarrow s+a}$ indicates that we replace each s in $F(s)$ by $s + a$. The result is written as $F(s+a)$ so

$$F(s+a) = \left[F(s)\right]_{s\leftarrow s+a}$$

The Laplace transform pair

$$\sin(\omega t) \leftrightarrow \frac{\omega}{s^2+\omega^2}$$

and the Laplace transform property

$$e^{-at}\, f(t) \leftrightarrow F(s+a)$$

are now used to determine the step response

$$step\ \ response = L^{-1}\left[\frac{H(s)}{s}\right] = 15\, e^{-12t}\sin(8t)\ \text{V}\ \ \text{for } t > 0$$

Since we expect the step response to be 0 for $t < 0$, we can write

$$step\ \ response = L^{-1}\left[\frac{H(s)}{s}\right] = 15\, e^{-12t}\sin(8t)\, u(t)\ \text{V}$$

c. The poles of this circuit are the roots of $s^2 + 24s + 208$:

$$s_{1,2} = \frac{-24 \pm \sqrt{24^2 - 4(1)(208)}}{2} = -12 \pm j8$$

The real part of all the poles are negative so the circuit is stable. Consequently

$$\mathbf{H}(\omega) = H(s)\Big|_{s=j\omega} = \frac{120(j\omega)}{(j\omega)^2 + 24(j\omega) + 208} = \frac{j120\,\omega}{208 - \omega^2 + j\,24\,\omega}$$

When ω = 10 rad/s

$$\mathbf{H}(10) = \frac{j1200}{108 + j\,240} = 4.56\angle 24°$$

When the input is $v_{\text{i}}(t) = 3.2\cos(10t + 30°)$ V , the steady state response is

$$v_{\text{o}}(t) = 3.2(4.56)\cos(10t + 30° + 24°) = 14.59\cos(10t + 54°)\ \ \text{V}.$$

Section 14.11 Partial Fraction Expansion Using MATLAB

P14.11-1

Find the inverse Laplace transform of $V(s)=\dfrac{11.6\,s^2+91.83\,s+186.525}{s^3+10.95\,s^2+35.525\,s+29.25}$

Solution:
Using MATLAB:

```
>> num = [11.6  91.83  186.525];
>> den = [1  10.95  35.525  29.25];
>> [r,p]=residue(num,den)
r =
    8.2000
   -3.6000
    7.0000
p =
   -5.2000
   -4.5000
   -1.2500
```

Consequently

$$V(s)=\frac{8.2}{s-(-5.2)}+\frac{-3.6}{s-(-4.5)}+\frac{7}{s-(-1.25)}=\frac{8.2}{s+5.2}+\frac{-3.6}{s+4.5}+\frac{7}{s+1.25}$$

and

$$v(t)=8.2\,e^{-5.2t}-3.6\,e^{-4.5t}+7\,e^{-1.25t} \quad \text{for } t>0$$

P14.11-2

Find the inverse Laplace transform of $V(s)=\dfrac{8s^3+139s^2+774s+1471}{s^4+12s^3+77s^2+296s+464}$

Solution:
Using MATLAB:

```
>> num = [8 139 774 1471];
>> den = [1 12 77 296 464];
>> [r,p]=residue(num,den)
r =
   3.0000 - 6.0000i
   3.0000 + 6.0000i
   2.0000
   3.0000
p =
  -2.0000 + 5.0000i
  -2.0000 - 5.0000i
  -4.0000
  -4.0000
```

Consequently

$$V(s)=\frac{3-j6}{s-(-2+j5)}+\frac{3+j6}{s-(-2-j5)}+\frac{2}{s-(-4)}+\frac{3}{(s-(-4))^2}$$

Using the Laplace transform pair

$$e^{ct}\left[2a\cos(dt)-2b\sin(dt)\right]\leftrightarrow\frac{a+jb}{s-(c+jd)}+\frac{a-jb}{s-(c-jd)}$$

with $a=3$, $b=-6$, $c=-2$ and $d=5$ we have

$$v(t)=e^{-2t}\left(6\cos(5t)+12\sin(5t)\right)+e^{-4t}\left(2+3t\right)\quad \text{for } t>0$$

P14.11-3

Find the inverse Laplace transform of $V(s)=\dfrac{s^2+6s+11}{s^3+12s^2+48s+64}=\dfrac{s^2+6s+11}{(s+4)^3}$

Solution:
Using MATLAB:

```
>> num = [1 6 11];
>> den = [1 12 48 64];
>> [r,p]=residue(num,den)
r =
    1.0000
   -2.0000
    3.0000
p =
   -4.0000
   -4.0000
   -4.0000
```

Consequently

$$V(s)=\frac{1}{s-(-4)}+\frac{-2}{(s-(-4))^2}+\frac{3}{(s-(-4))^3}$$

and

$$v(t)=e^{-4t}\left(1-2t+3t^2\right) \quad \text{for } t>0$$

P14.11-4

Find the inverse Laplace transform of $V(s) = \frac{-60}{s^2 + 5s + 48.5}$

Solution:

The denominator does not factor any further in the real numbers. Let's complete the square in the denominator

$$V(s) = \frac{-60}{s^2 + 5s + 48.5} = \frac{-60}{(s+2.5)^2 + 42.25} = \frac{-60}{(s+2.5)^2 + 6.5^2} = \frac{-9.23(6.5)}{(s+2.5)^2 + 6.5^2}$$

Now use $e^{-at} f(t) \leftrightarrow F(s+a)$ and $\sin \omega t$ for $t > 0 \leftrightarrow \frac{\omega}{s^2 + \omega^2}$ to find the inverse Laplace transform

$$v(t) = e^{-2.5t} \mathscr{L}^{-1}\left[\frac{-9.23(6.5)}{(s+2.5)^2 + 6.5^2}\right] = -9.23 \; e^{-2.5t} \sin(6.5\, t) \text{ for } t > 0$$

Using MATLAB:

```
>> num = [-60];
>> den = [1 5 48.5];
>> [r,p]=residue(num,den)
r =
        0 + 4.6154i
        0 - 4.6154i
p =
  -2.5000 + 6.5000i
  -2.5000 - 6.5000i
```

Using the Laplace transform pair

$$e^{ct}\left[2a\cos(d\,t) - 2b\sin(d\,t)\right] \leftrightarrow \frac{a + jb}{s - (c + jd)} + \frac{a - jb}{s - (c - jd)}$$

with $a = 0$, $b = 4.6154$, $c = -2.5$ and $d = 6.5$ we have

$$v(t) = -9.2308\, e^{-2.5t} \sin(6.5t) \quad \text{for } t > 0$$

P14.11-5

Find the inverse Laplace transform of $V(s)=\dfrac{-30}{s^2-25}$

Solution:
Using MATLAB:

```
>> num = [-30];
>> den = [1  -25];
>> [r,p]=residue(num,den)
r =
    -3
     3
p =
     5
    -5
```

Consequently

$$V(s)=\frac{-3}{s-5}+\frac{3}{s-(-5)}=\frac{-3}{s-5}+\frac{3}{s+5}$$

and

$$v(t)=-3e^{5t}+3e^{-5t} \quad \text{for } t>0$$

P14.11-6 The input to the circuit shown in Figure P14.11-6 is the voltage $v_i(t)$ and the output is the voltage $v_o(t)$. Determine the output when the input

$$v_i(t) = 5\cos(4000t)u(t) \text{ mV}$$

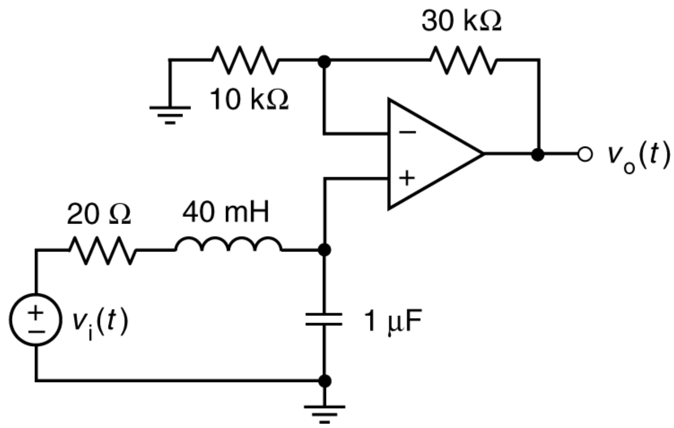
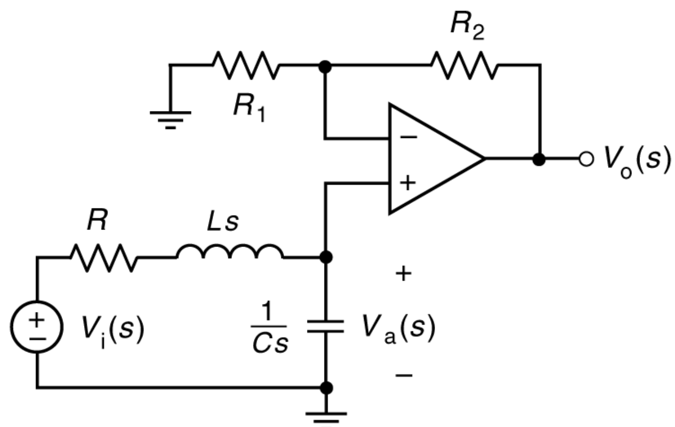


Figure 14.11-6

Solution:
The transfer function of this circuit is



$$H(s) = \frac{\frac{1}{Cs}}{R + Ls + \frac{1}{Cs}}\left(1 + \frac{R_2}{R_1}\right)$$

$$= \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}\left(1 + \frac{R_2}{R_1}\right)$$

$$= \frac{100\times10^6}{s^2 + 500s + 25\times10^6}$$

The Laplace transform of the input is $V_i(s) = \mathcal{L}\left[5\cos(4000t)u(t)\right] = \frac{5s}{s^2 + 4000^2}$

The output is $v_o(t) = \mathcal{L}^{-1}\left[\frac{100\times10^6}{s^2 + 500s + 25\times10^6}\left(\frac{5s}{s^2 + 4000^2}\right)\right]$. We can use MATLAB to calculate this inverse transform:

```
>> den=conv([1 500 25e6],[1 0 16e6]);
>> num=[500e6 0];
>> [r,p]=residue(num,den)

r =
```

```
 -26.4706 + 6.0370i
 -26.4706 - 6.0370i
  26.4706 - 5.8824i
  26.4706 + 5.8824i


p =

  1.0e+003 *

  -0.2500 + 4.9937i
  -0.2500 - 4.9937i
  -0.0000 + 4.0000i
  -0.0000 - 4.0000i
```

Consequently

$$V_o(s) = \frac{-26.4706 + j\,6.0370}{s-(-250+j\,4993.7)} + \frac{-26.4706 - j\,6.0370}{s-(-250+j\,4993.7)} + \frac{-26.4706 + j\,5882.4}{s-(j\,4000)} + \frac{-26.4706 - j\,5882.4}{s-(j\,4000)}$$

Using the Laplace transform pair

$$e^{ct}\left[2a\cos(d\,t) - 2b\sin(d\,t)\right] \leftrightarrow \frac{a+jb}{s-(c+jd)} + \frac{a-jb}{s-(c-jd)}$$

with $a = -26.4706$, $b = 6.0370$, $c = -250$ and $d = 4993.7$ we have

$$\mathfrak{L}^{-1}\left[\frac{-26.4706+j\,6.0370}{s-(-250+j\,4993.7)} + \frac{-26.4706-j\,6.0370}{s-(-250+j\,4993.7)}\right] = e^{-250t}\left[-52.9\cos(4993.7\,t) - 12.073\sin(4993.7\,t)\right]$$

$$= 54.26\,e^{-250t}\cos(4993.7\,t + 167°)$$

Using the Laplace transform pair

$$e^{ct}\left[2a\cos(d\,t) - 2b\sin(d\,t)\right] \leftrightarrow \frac{a+jb}{s-(c+jd)} + \frac{a-jb}{s-(c-jd)}$$

with $a = 26.4706$, $b = 5.8824$, $c = 0$ and $d = 4000$ we have

$$\mathfrak{L}^{-1}\left[\frac{26.4706+j\,5882.4}{s-(j\,4000)} + \frac{26.4706-j\,5882.4}{s-(j\,4000)}\right] = e^{0}\left[52.9\cos(4000\,t) - 11.765\sin(4000\,t)\right]$$

$$= 54.2\,\cos(4000\,t - 12.5°)$$

Finally

$$v_o(t) = \left[54.26\,e^{-250t}\cos(4993.7\,t + 167°) + 54.2\,\cos(4000\,t - 12.5°)\right]u(t)\ \text{mV}$$

Section 14.12 How Can We Check…?

P 14.12-1 Computer analysis of the circuit of Figure P14.12-1 indicates that

$$v_C(t) = 6 + 3.3e^{-2.1t} + 2.7e^{-15.9t} \text{ V}$$

and
$$i_L(t) = 2 + 0.96e^{-2.1t} + 0.04e^{-15.9t} \text{ A}$$

after the switch opens at time $t = 0$. Verify that this analysis is correct by checking that (a) KVL is satisfied for the mesh consisting of the voltage source, inductor, and 12-Ω resistor and (b) KCL is satisfied at node b.

Hint: Use the given expressions for $i_L(t)$ and $v_C(t)$ to determine expressions for $v_L(t)$, $i_C(t)$, $v_{R1}(t)$, $i_{R2}(t)$, and $i_{R3}(t)$.

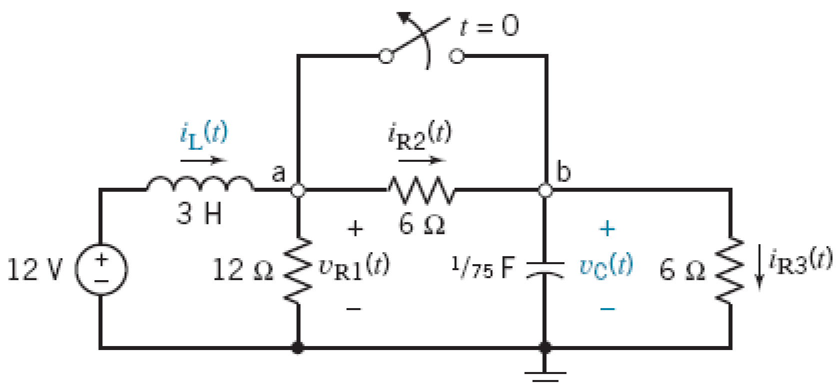


Figure P14.12-1

Solution:

$$v_L(t) = 3\frac{d}{dt}i_L(t) = -6e^{-2.1t} - 2e^{-15.9t}$$

$$i_C(t) = \frac{1}{75}\frac{d}{dt}v_C(t) = -0.092e^{-2.1t} - 0.575e^{-15.9t}$$

$$v_{R1}(t) = 12 - v_L(t) = 12 + 6e^{-2.1t} + 2e^{-15.9t}$$

$$i_{R2}(t) = \frac{12 - \left(v_L(t) + v_C(t)\right)}{6} = 1 + 0.456e^{-2.1t} - 0.123e^{-15.9t}$$

$$i_{R3}(t) = \frac{v_C(t)}{6} = 1 + 0.548e^{-2.1t} + 0.452e^{-15.9t}$$

Thus,
$$-12 + v_L(t) + v_{R1}(t) = 0 \quad \text{and} \quad i_{R2}(t) = i_C(t) + i_{R3}(t)$$

as required. The analysis is correct.

P 14.12-2 Analysis of the circuit of Figure P 14.12-2 when $v_C(0) = -12$ V indicates that

$$i_1(t) = 18e^{0.75t} \text{ A} \quad \text{and} \quad i_2(t) = 20e^{0.75t} \text{ A}$$

after $t = 0$. Verify that this analysis is correct by representing this circuit, including $i_1(t)$ and $i_2(t)$, in the frequency domain using Laplace transforms. Use $I_1(s)$ and $I_2(s)$ to calculate the element voltages and verify that these voltages satisfy KVL for both meshes.

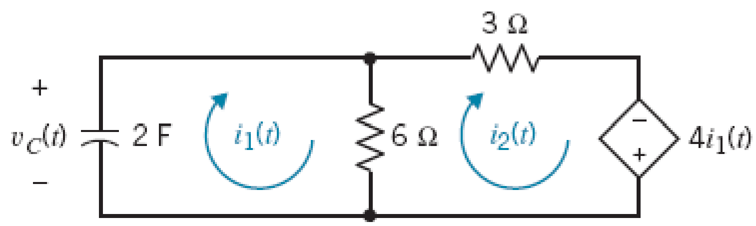


Figure P 14.12-2

Solution:

$$I_1(s) = \frac{18}{s - \frac{3}{4}} \text{ and } I_2(s) = \frac{20}{s - \frac{3}{4}}$$

KVL for left mesh: $$\frac{12}{s} + \frac{1}{2s}\left(\frac{18}{s-\frac{3}{4}}\right) + 6\left(\frac{18}{s-\frac{3}{4}} - \frac{20}{s-\frac{3}{4}}\right) = 0 \quad \text{(ok)}$$

KVL for right mesh: $$-6\left(\frac{18}{s-\frac{3}{4}} - \frac{20}{s-\frac{3}{4}}\right) + 3\left(\frac{20}{s-\frac{3}{4}}\right) - 4\left(\frac{18}{s-\frac{3}{4}}\right) = 0 \quad \text{(ok)}$$

The analysis is correct.

P 14.12-3 Figure P 14.12-3 shows a circuit represented in (a) the time domain and (b) the frequency domain using Laplace transforms. An incorrect analysis of this circuit indicates that

$$I_L(s) = \frac{s+2}{s^2+s+5} \text{ and } V_C(s) = \frac{-20(s+2)}{s(s^2+s+5)}$$

(a) Use the initial and final value theorems to identify the error in the analysis. (b) Correct the error.

Hint: Apparently the error occurred as $V_C(s)$ was calculated from $I_L(s)$.

Answer: $V_C(s) = -\frac{20}{s}\left(\frac{s+2}{s^2+s+5}\right)+\frac{8}{s}$

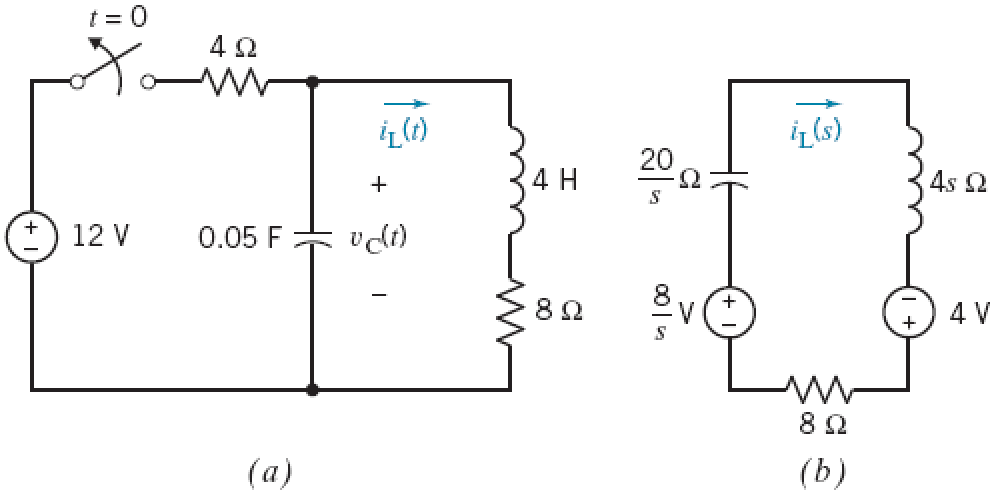


Figure P14.12-3

Solution:

Initial value of $I_L(s)$: $\lim_{s\to\infty} s\frac{s+2}{s^2+s+5} = 1$ (ok)

Final value of $I_L(s)$: $\lim_{s\to 0} s\frac{s+2}{s^2+s+5} = 0$ (ok)

Initial value of $V_C(s)$: $\lim_{s\to\infty} s\frac{-20(s+2)}{s(s^2+s+5)} = 0$ (not ok)

Final value of $V_C(s)$: $$\lim_{s \to 0} s \frac{-20(s+2)}{s(s^2+s+5)} = -8 \quad \text{(not ok)}$$

Apparently the error occurred as $V_C(s)$ was calculated from $I_L(s)$. Indeed, it appears that $V_C(s)$ was calculated as $-\frac{20}{s} I_L(s)$ instead of $-\frac{20}{s} I_L(s) + \frac{8}{s}$. After correcting this error

$$V_C(s) = -\frac{20}{s}\left(\frac{s+2}{s^2+s+5}\right) + \frac{8}{s}.$$

Initial value of $V_C(s)$: $$\lim_{s \to \infty} s\left(\frac{-20(s+2)}{s(s^2+s+5)} + \frac{8}{s}\right) = 8 \quad \text{(ok)}$$

Final value of $V_C(s)$: $$\lim_{s \to 0} s\left(\frac{-20(s+2)}{s(s^2+s+5)} + \frac{8}{s}\right) = 0 \quad \text{(ok)}$$

PSpice Problems

SP 14-1 The input to the circuit shown in Figure SP 14-1 is the voltage of the voltage source, $v_i(t)$. The output is the voltage across the capacitor, $v_o(t)$. The input is the pulse signal specified graphically by the plot. Use PSpice to plot the output, $v_o(t)$, as a function of t.

Hint: Represent the voltage source using the PSpice part named VPULSE.

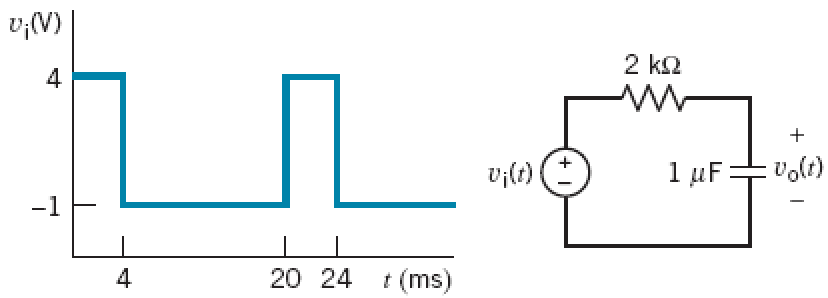
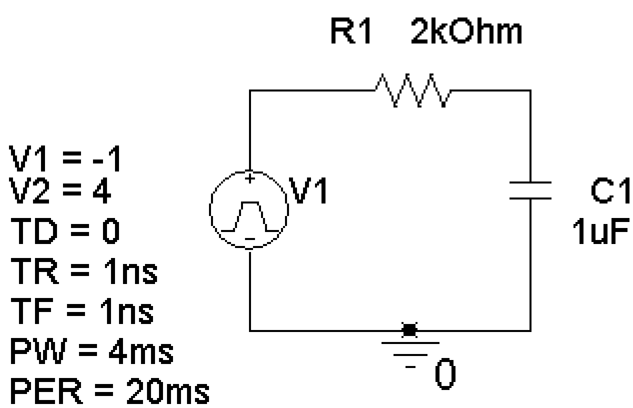
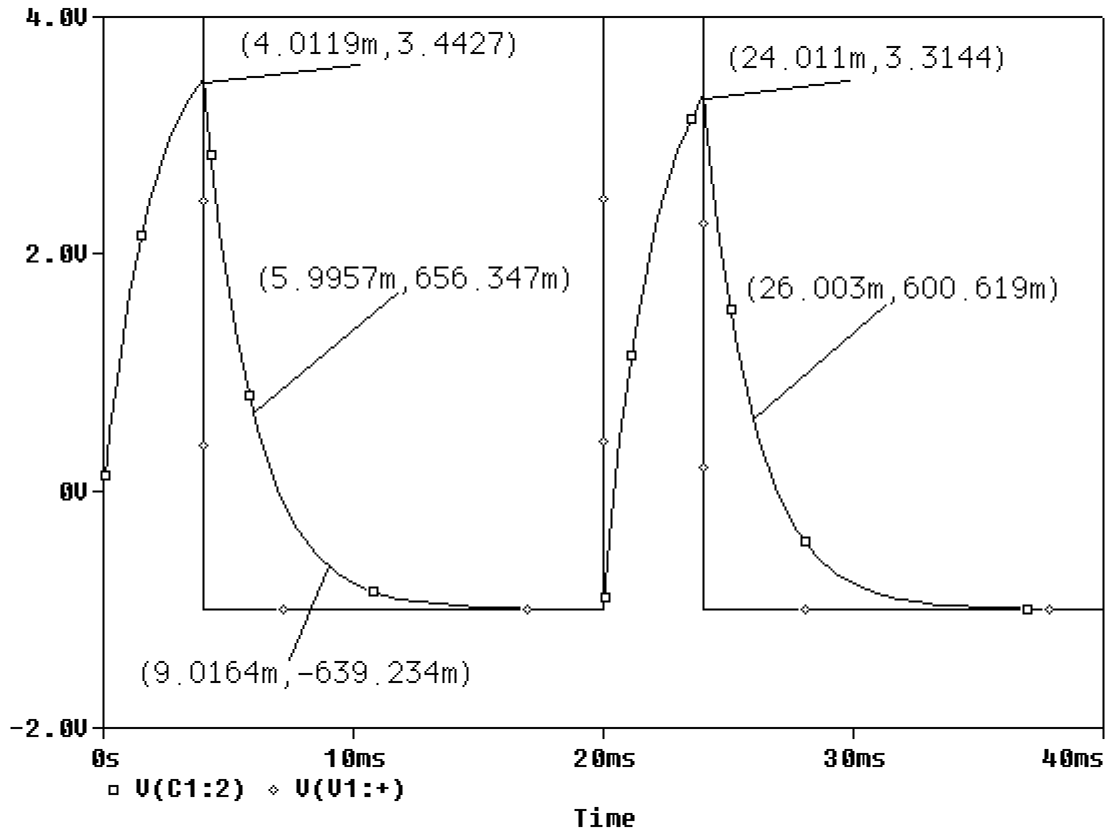


Figure SP 14-1

Solution:





4.0V
2.0V
0V
-2.0V
(4.0119m,3.4427)
(24.011m,3.3144)
(5.9957m,656.347m)
(26.003m,600.619m)
(9.0164m,-639.234m)
0s
10ms
20ms
30ms
40ms
V(C1:2)
V(V1:+)
Time

SP 14-2 The circuit shown in Figure SP 14-2 is at steady state before the switch closes at time $t = 0$. The input to the circuit is the voltage of the voltage source, 12 V. The output of this circuit is the voltage across the capacitor, $v(t)$. Use PSpice to plot the output, $v(t)$, as a function of t. Use the plot to obtain an analytic representation of $v(t)$, for $t > 0$.

Hint: We expect $i(t) = A + \text{B}\, e^{-t/\tau}$ for $t > 0$, where A, B, and τ are constants to be determined.

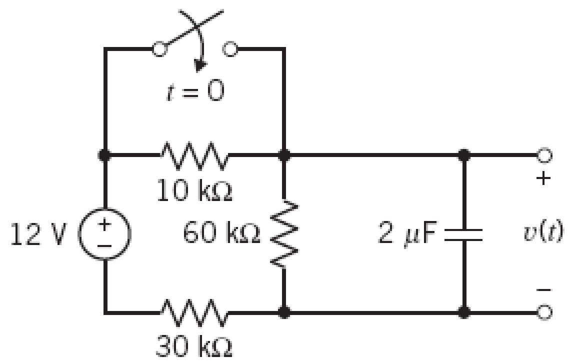
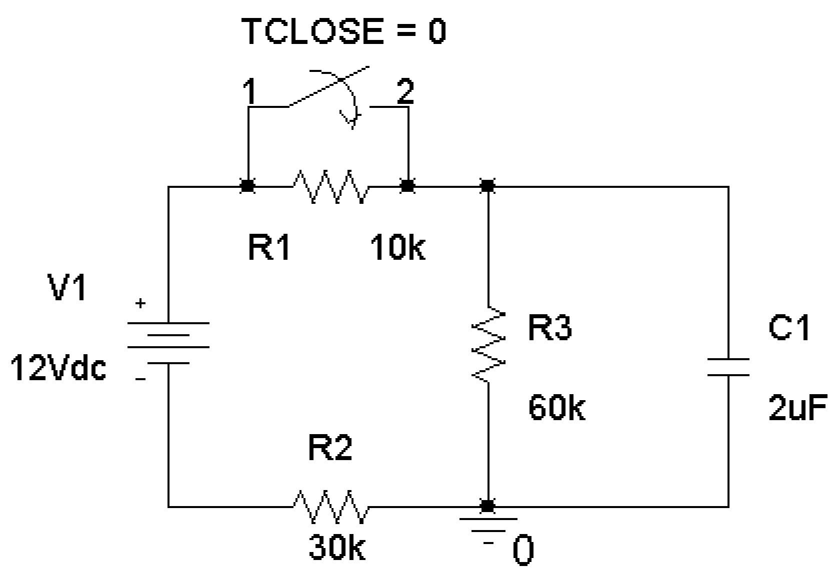
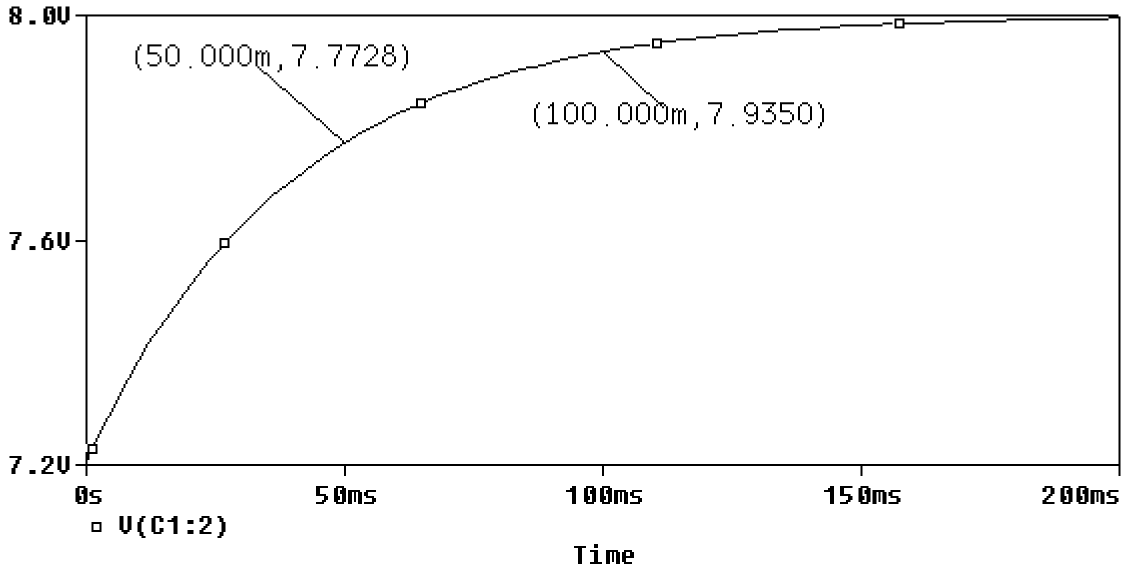


Figure SP 14-2

Solution:





$$v(t) = A + B\,e^{-t/\tau} \quad \text{for } t > 0$$

$$\left.\begin{array}{l} 7.2 = v(0) = A + B\,e^{0} \quad \Rightarrow \quad 7.2 = A + B \\ 8.0 = v(\infty) = A + B\,e^{-\infty} \quad \Rightarrow \quad A = 8.0 \text{ V} \end{array}\right\} \Rightarrow \quad B = -0.8 \text{ V}$$

$$7.7728 = v(0.05) = 8 - 0.8\,e^{-0.05/\tau} \quad \Rightarrow \quad -\frac{0.05}{\tau} = \ln\left(\frac{8 - 7.7728}{0.8}\right) = -1.25878$$

$$\Rightarrow \quad \tau = \frac{0.05}{1.25878} = 39.72 \text{ ms}$$

Therefore

$$v(t) = 8 - 0.8\,e^{-t/0.03972} \text{ V} \quad \text{for } t > 0$$

SP 14-3 The circuit shown in Figure SP 14-3 is at steady state before the switch closes at time $t = 0$. The input to the circuit is the current of the current source, 4 mA. The output of this circuit is the current in the inductor, $i(t)$. Use PSpice to plot the output, $i(t)$, as a function of t. Use the plot to obtain an analytic representation of $i(t)$ for $t > 0$.

Hint: We expect $i(t) = A + \mathrm{B}\, e^{-t/\tau}$ for $t > 0$, where A, B, and τ are constants to be determined.

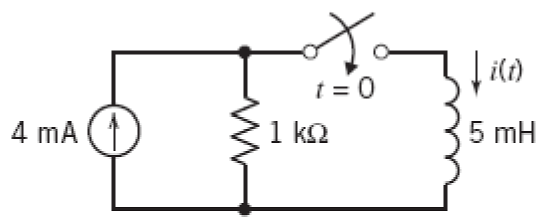
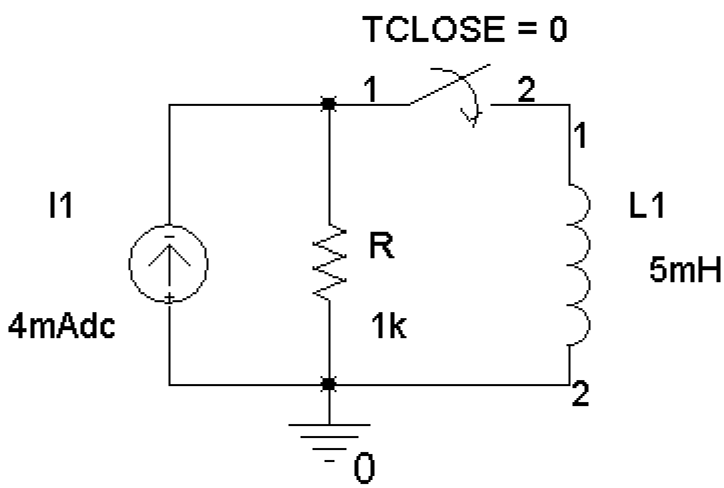
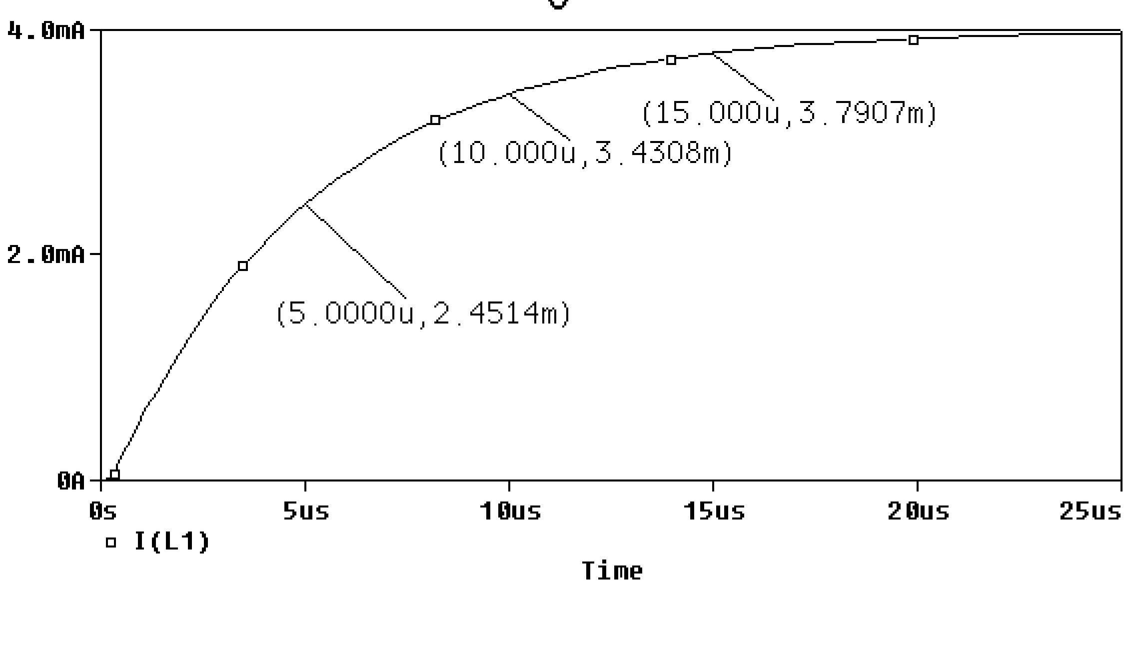


Figure SP 14-3

Solution:





$$i(t) = A + B\,e^{-t/\tau} \quad \text{for } t > 0$$

$$\left.\begin{array}{r} 0 = i(0) = A + B\,e^{0} \quad \Rightarrow \quad 0 = A + B \\ 4\times10^{-3} = i(\infty) = A + B\,e^{-\infty} \quad \Rightarrow \quad A = 4\times10^{-3}\ \text{A} \end{array}\right\} \Rightarrow \quad B = -4\times10^{-3}\ \text{A}$$

$$2.4514\times10^{-3} = v(5\times10^{-6}) = \left(4\times10^{-3}\right) - \left(4\times10^{-3}\right) e^{-\left(5\times10^{-6}\right)/\tau}$$

$$\Rightarrow \quad -\frac{5\times10^{-6}}{\tau} = \ln\left(\frac{\left(4-2.4514\right)\times10^{-3}}{4\times10^{-3}}\right) = -0.94894$$

$$\Rightarrow \quad \tau = \frac{5\times10^{-6}}{0.94894} = 5.269\ \mu\text{s}$$

Therefore

$$i(t) = 4 - 4\,e^{-t/5.269\times10^{-6}}\ \text{A} \quad \text{for } t > 0$$

SP 14-4 The input to the circuit shown in Figure SP 14-4 is the voltage of the voltage source, $v_i(t)$. The output is the voltage across the capacitor, $v_o(t)$. The input is the pulse signal specified graphically by the plot. Use PSpice to plot the output, $v_o(t)$, as a function of t for each of the following cases:

(a) $C = 1$ F, $L = 0.25$ H, $R_1 = R_2 = 1.309\ \Omega$

(b) $C = 1$ F, $L = 1$ H, $R_1 = 3\ \Omega$, $R_2 = 1\ \Omega$

(c) $C = 0.125$ F, $L = 0.5$ H, $R_1 = 1\ \Omega$, $R_2 = 4\ \Omega$

Plot the output for these three cases on the same axis.

Hint: Represent the voltage source using the PSpice part named VPULSE

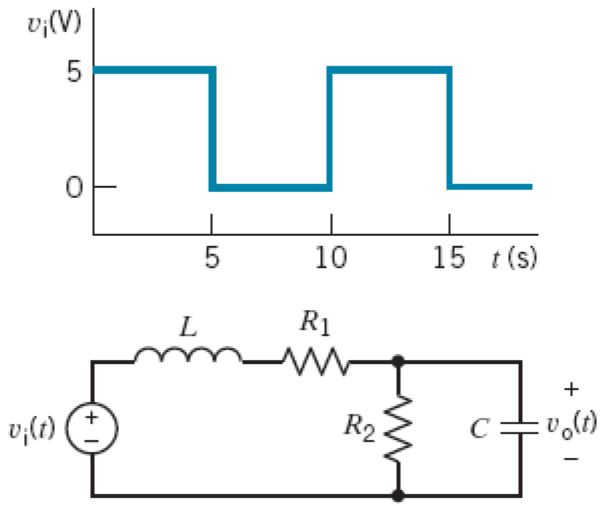
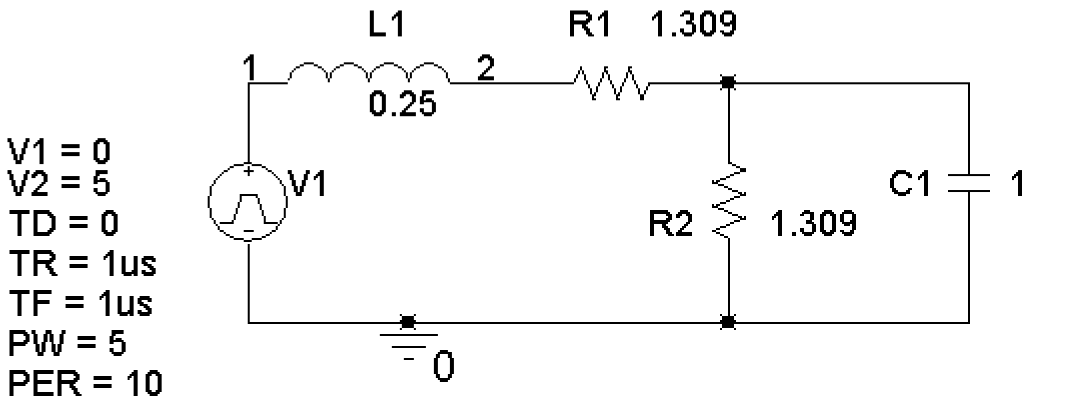
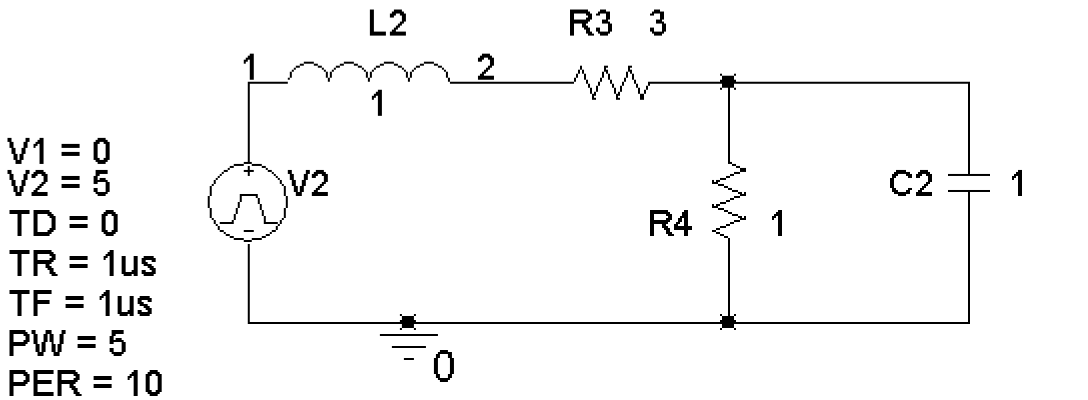


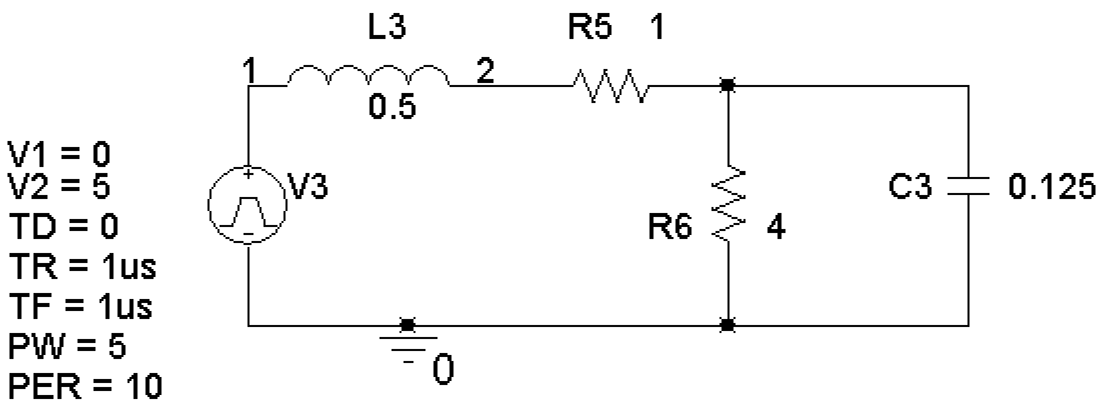
Figure SP 14-4

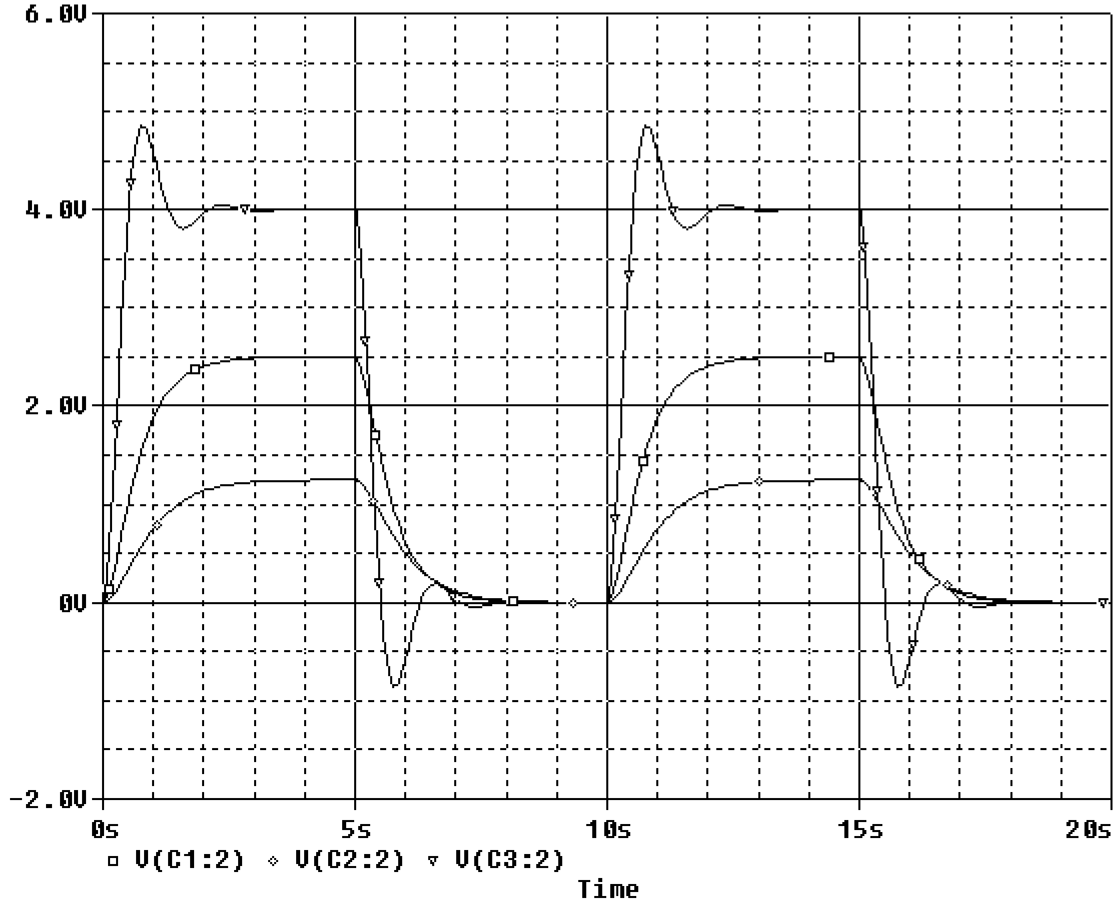
Solution:

Make three copies of the circuit: one for each set of parameter values. (Cut and paste, but be sure to edit the labels of the parts so, for example, there is only one R1.)


L1
R1 1.309
1
2
0.25
V1 = 0
V2 = 5
TD = 0
TR = 1us
TF = 1us
PW = 5
PER = 10
V1
R2
1.309
C1
1
0


L2
R3 3
1
2
1
V1 = 0
V2 = 5
TD = 0
TR = 1us
TF = 1us
PW = 5
PER = 10
V2
R4
1
C2
1
0


L3
R5 1
1
2
0.5
V1 = 0
V2 = 5
TD = 0
TR = 1us
TF = 1us
PW = 5
PER = 10
V3
R6
4
C3
0.125
0



V(C1:2), V(C2:2) and V(C3:2) are the capacitor voltages, listed from top to bottom.

SP 14-5 The input to the circuit shown in Figure SP 14-5 is the voltage of the voltage source, $v_i(t)$. The output is the voltage, $v_o(t)$, across resistor R_2. The input is the pulse signal specified graphically by the plot. Use PSpice to plot the output, $v_o(t)$, as a function of t for each of the following cases:

(a) $C = 1$ F, $L = 0.25$ H, $R_1 = R_2 = 1.309\ \Omega$

(b) $C = 1$ F, $L = 1$ H, $R_1 = 3\ \Omega$, $R_2 = 1\ \Omega$

(c) $C = 0.125$ F, $L = 0.5$ H, $R_1 = 1\ \Omega$, $R_2 = 4\ \Omega$

Plot the output for these three cases on the same axis.

Hint: Represent the voltage source using the PSpice part named VPULSE.

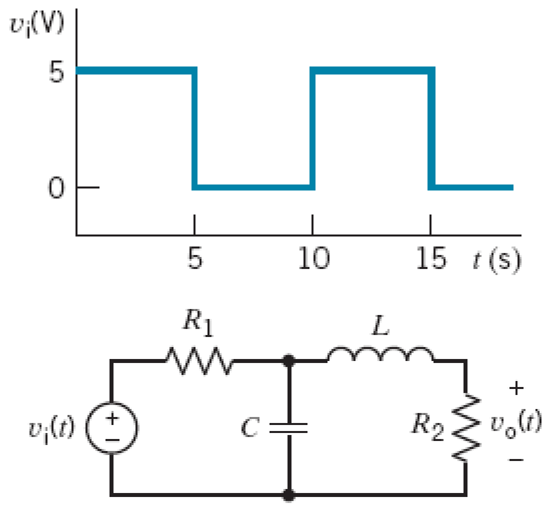
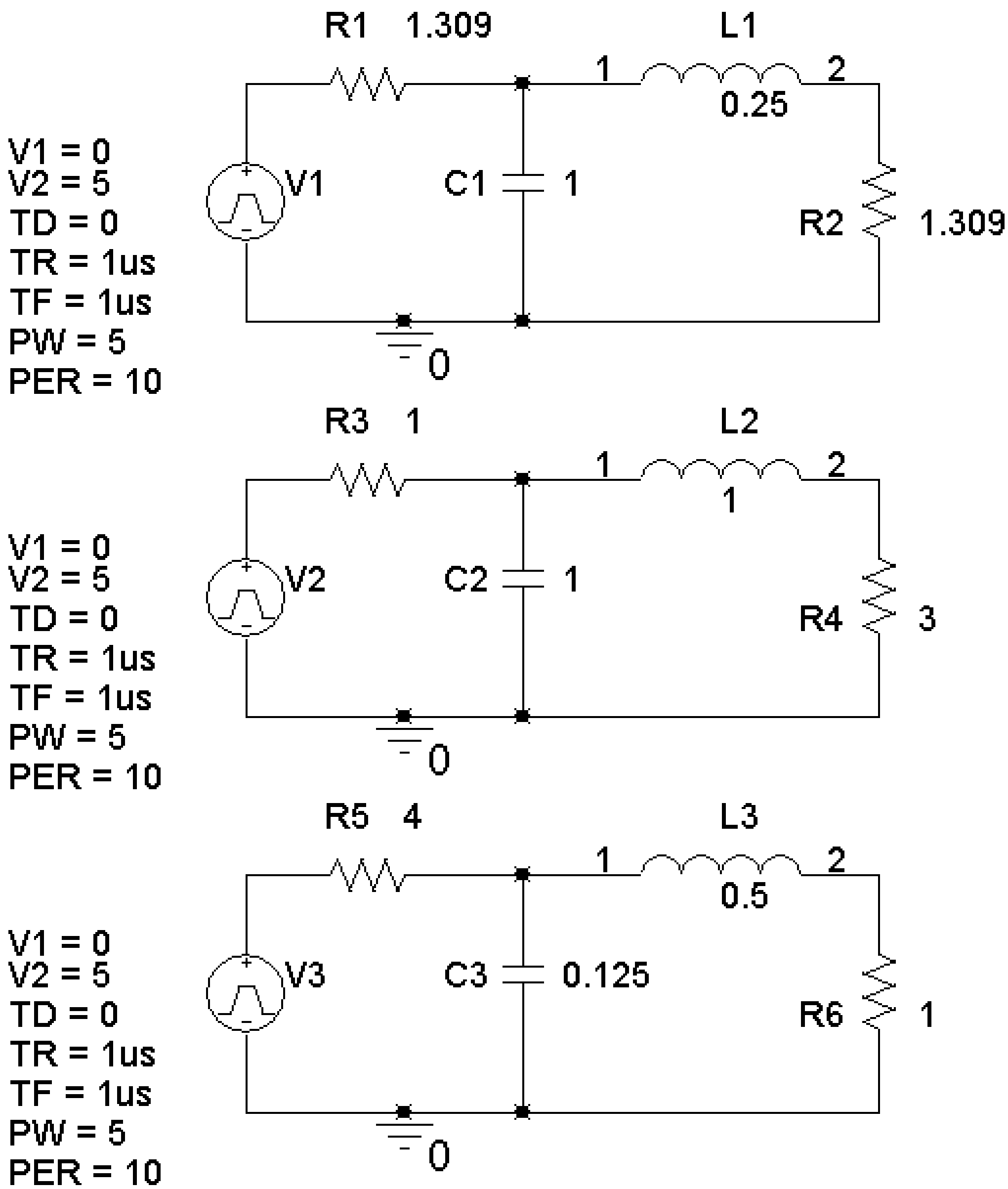
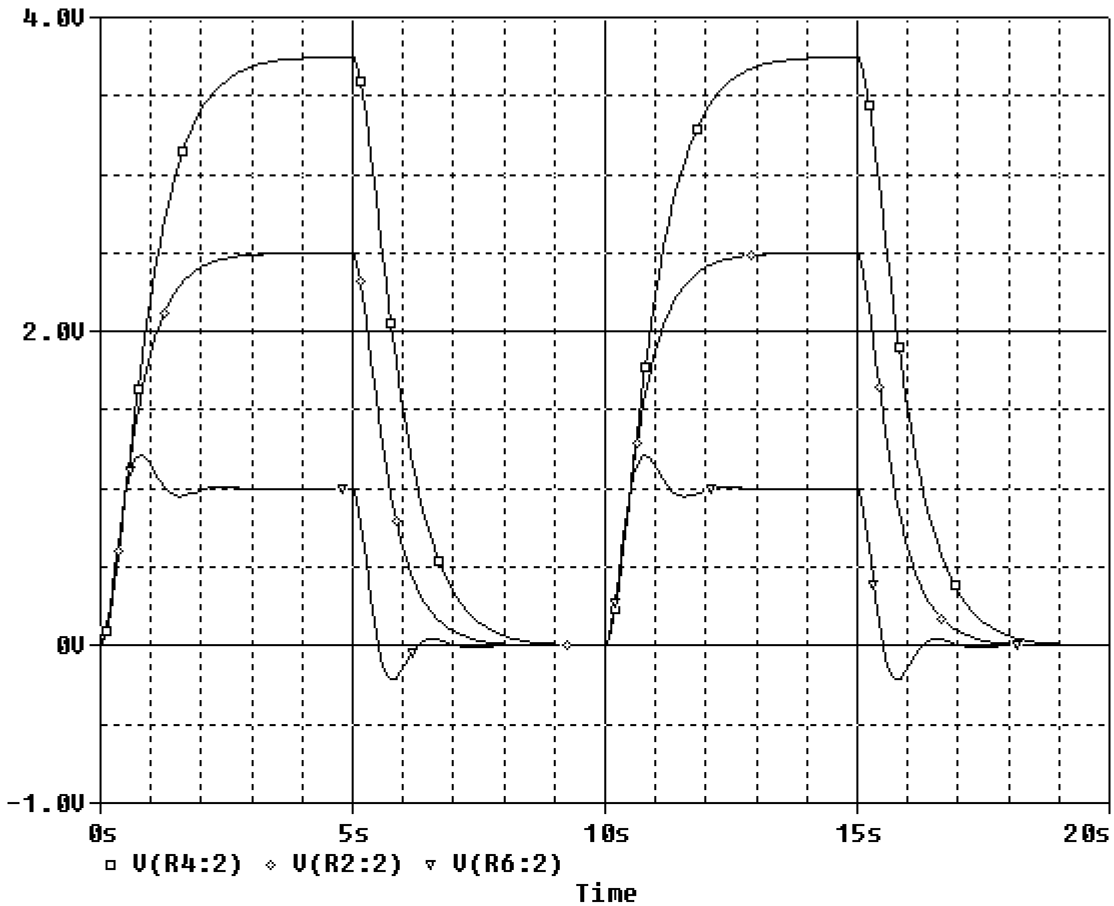


Figure SP 14-5

Solution:

Make three copies of the circuit: one for each set of parameter values. (Cut and paste, but be sure to edit the labels of the parts so, for example, there is only one R1.)


R1 1.309
L1
1
2
0.25
V1 = 0
V2 = 5
TD = 0
TR = 1us
TF = 1us
PW = 5
PER = 10
V1
C1 1
R2 1.309
0
R3 1
L2
1
2
1
V1 = 0
V2 = 5
TD = 0
TR = 1us
TF = 1us
PW = 5
PER = 10
V2
C2 1
R4 3
0
R5 4
L3
1
2
0.5
V1 = 0
V2 = 5
TD = 0
TR = 1us
TF = 1us
PW = 5
PER = 10
V3
C3 0.125
R6 1
0



V(R2:2), V(R4:2) and V(R6:2) are the output voltages, listed from top to bottom.

Design Problems

DP 14-1 Design the circuit in Figure DP 14-1 to have a step response equal to

$$v_o = 5te^{-4t}u(t) \text{ V}$$

Hint: Determine the transfer function of the circuit in Figure DP 14-1 in terms of *k*, *R*, *C*, and *L*. Then determine the Laplace transform of the step response of the circuit in Figure DP 14-1. Next, determine the Laplace transform of the given step response. Finally, determine values of *k*, *R*, *C*, and *L* that cause the two step responses to be equal.

Answer: Pick L = 1 H, then k = 0.625 V/V, R = 8 Ω, and C = 0.0625 F. (This answer is not unique.)

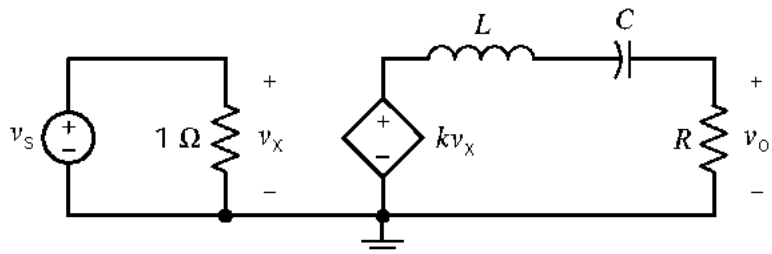


Figure DP 14-1

Solution:

Equating the Laplace transform of the step response of the give circuit to the Laplace transform of the given step response:

$$V_o(s) = \frac{\frac{kR}{L}}{s^2 + \frac{R}{L}s + \frac{1}{LC}} = \frac{5}{(s+4)^2}$$

Equating the poles:

$$s_{1,2} = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2} = -4 \pm j0$$

Summarizing the results of these comparisons:

$$\frac{R}{2L} = 4, \quad R = 2\sqrt{\frac{L}{C}} \quad \text{and} \quad \frac{kR}{L} = 5$$

Pick L = 1 H, then k = 0.625 V/V, R = 8 Ω and C = 0.0625 F.

DP 14-2 Design the circuit in Figure DP 14-1 to have a step response equal to

$$v_o = 5e^{-4t}\sin(2t)u(t)\text{ V}$$

Hint: Determine the transfer function of the circuit in Figure DP 14-1 in terms of *k, R, C*, and *L*. Then determine the Laplace transform of the step response of the circuit in Figure DP 14-1. Next, determine the Laplace transform of the given step response. Finally, determine values of *k, R, C*, and *L* that cause the two step responses to be equal.

Answer: Pick L = 1 H, then k = 1.25 V/V, R = 8 Ω, and C = 0.05 F. (This answer is not unique.)

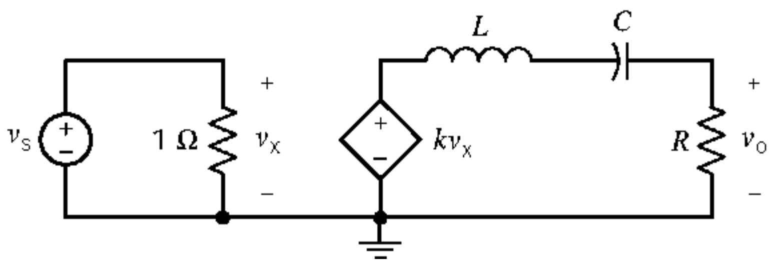


Figure DP 14-1

Solution:

Equating the Laplace transform of the step response of the give circuit to the Laplace transform of the given step response:

$$V_o(s) = \frac{\frac{kR}{L}}{s^2 + \frac{R}{L}s + \frac{1}{LC}} = \frac{10}{(s+4)^2+4} = \frac{10}{s^2+8s+20}$$

Equating coefficients:

$$\frac{R}{L} = 8, \quad \frac{1}{LC} = 20 \quad \text{and} \quad \frac{kR}{L} = 10$$

Pick L = 1 H, then k = 1.25 V/V, R = 8 Ω and C = 0.05 F.

DP 14-3 Design the circuit in Figure DP 14-1 to have a step response equal to

$$v_o = 5(e^{-2t} - e^{-4t})u(t) \text{ V}$$

Hint: Determine the transfer function of the circuit in Figure DP 14-1 in terms of *k*, *R*, *C*, and *L*. Then determine the Laplace transform of the step response of the circuit in Figure DP 14-1. Next, determine the Laplace transform of the given step response. Finally, determine values of *k*, *R*, *C*, and *L* that cause the two step responses to be equal.

Answer: Pick L = 1 H, then k = 1.667 V/V, R = 6 Ω, and C = 0.125 F. (This answer is not unique.)

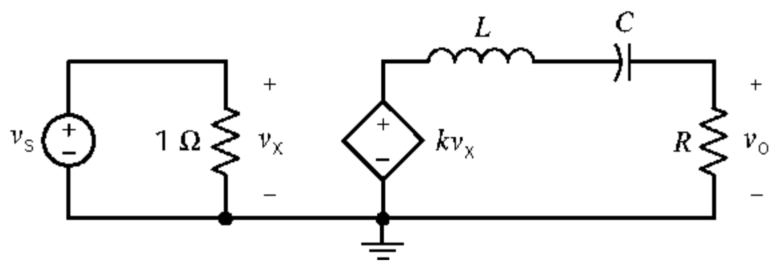


Figure DP 14-1

Solution:

Equating the Laplace transform of the step response of the give circuit to the Laplace transform of the given step response:

$$V_o(s) = \frac{\frac{kR}{L}}{s^2 + \frac{R}{L}s + \frac{1}{LC}} = \frac{5}{s+2} - \frac{5}{s+4} = \frac{10}{s^2 + 6s + 8}$$

Equating coefficients:

$$\frac{R}{L} = 6, \quad \frac{1}{LC} = 8 \quad \text{and} \quad \frac{kR}{L} = 10$$

Pick L = 1 H, then k = 1.667 V/V, R = 6 Ω and C = 0.125 F.

DP 14-4 Show that the circuit in Figure DP 14-1 cannot be designed to have a step response equal to

$$v_o = 5(e^{-2t} + e^{-4t})u(t)\ \text{V}$$

Hint: Determine the transfer function of the circuit in Figure DP 14-1 in terms of *k, R, C*, and *L*. Then determine the Laplace transform of the step response of the circuit in Figure DP 14-1. Next, determine the Laplace transform of the given step response. Notice that these two functions have different forms and so cannot be made equal by any choice of values of *k*, *R*, *C*, and *L*.

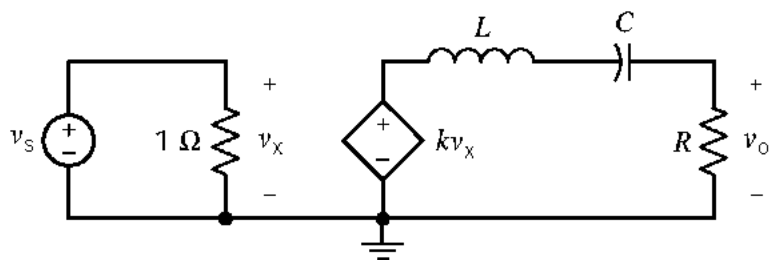


Figure DP 14-1

Solution:

Comparing the Laplace transform of the step response of the give circuit to the Laplace transform of the given step response:

$$V_o(s) = \frac{\frac{kR}{L}}{s^2 + \frac{R}{L}s + \frac{1}{LC}} \neq \frac{5}{(s+2)} + \frac{5}{(s+4)} = \frac{10s+30}{s^2+6s+8}$$

These two functions can not be made equal by any choice of *k*, *R*, *C* and *L* because the numerators have different forms.

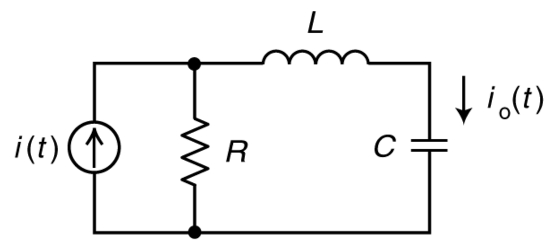


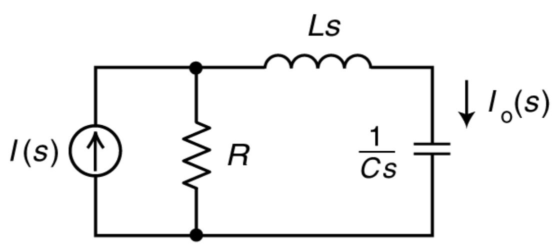
Figure DP14 -5

DP14-5 The input to the circuit shown in Figure DP14-5 is the current $i(t)$ and the output is the current $i_o(t)$. Determine the values of *R*, *L* and *C* that cause the impulse of this circuit to be:

$$i_o(t) = \left(k_1 e^{-2000t} + k_1 e^{-8000t}\right) u(t) \text{ A}$$

where k_1 and k_2 are unspecified constants.
Answer: One solution is *L* = 125 mH, *R* = 1250 Ω and *C* = 0.5 μF..

Solution: First, determine the transfer function from the circuit. To do so, represent the circuit in the s-domain as shown below.



Applying current division $\quad I_o(s) = \dfrac{R}{R + Ls + \dfrac{1}{Cs}} I(s) = \dfrac{\dfrac{R}{L}s}{s^2 + \dfrac{R}{L}s + \dfrac{1}{LC}} I(s)$

The transfer function is $\quad H(s) = \dfrac{I_o(s)}{I(s)} = \dfrac{\dfrac{R}{L}s}{s^2 + \dfrac{R}{L}s + \dfrac{1}{LC}}$

Next, determine the transfer function from the impulse response.

$$H(s) = \mathfrak{L}\left[k_1 e^{-2000t} + k_2 e^{-8000t}\right] = \frac{k_1}{s+2000} + \frac{k_2}{s+8000} = \frac{(k_1+k_2)s + 2000(4k_1+k_2)}{(s+2000)(s+8000)}$$
$$= \frac{(k_1+k_2)s + 2000(4k_1+k_2)}{s^2 + 10{,}000s + 16{,}000{,}000}.$$

Compare the denominators of these transfer functions to get

$$s^2+\frac{R}{L}s+\frac{1}{LC}=s^2+10,000\,s+16,000,000$$

This equation requires $\frac{R}{L}=10,000$ and $\frac{1}{LC}=16,000,000$

These equations don't have a unique solution. We can get one solution by picking a value for *L* and calculating the corresponding values of *R* and *C*. Arbitrarily choosing *L* = 125 mH we calculate
R = 1250 Ω and *C* = 0.5 μF. With these values

$$H(s)=\frac{I_o(s)}{I(s)}=\frac{\frac{R}{L}s}{s^2+\frac{R}{L}s+\frac{1}{LC}}=\frac{10,000\,s}{s^2+10,000\,s+16,000,000}$$

Performing partial fraction expansion:

$$H(s)=\frac{10,000\,s}{s^2+10,000\,s+16,000,000}=\frac{10,000\,s}{(s+2000)(s+8000)}=\frac{-\frac{1}{3}\times 10000}{s+2000}+\frac{\frac{4}{3}\times 10000}{s+8000}$$

Consequently $\pounds^{-1}\left[H(s)\right]=\left[\left(-\frac{1}{3}\times 10000\right)e^{-2000t}+\left(\frac{4}{3}\times 10000\right)e^{-8000t}\right]u(t)$ A

as required.

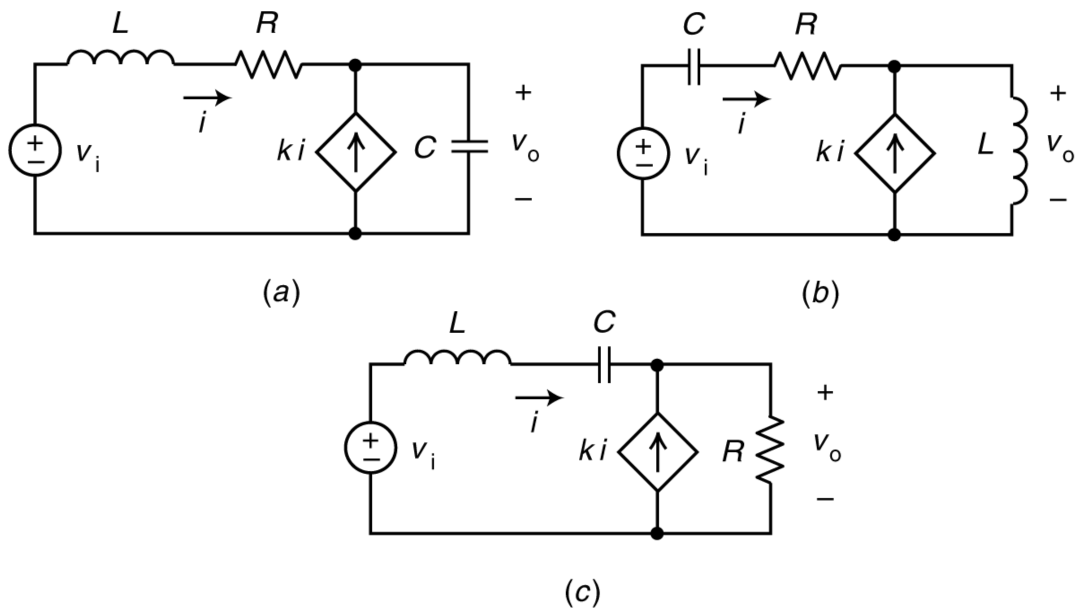


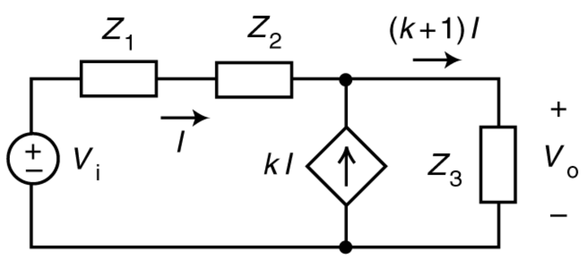
Figure DP14 -6

DP14-6 The input to each of the circuits shown in Figure DP14-6 is the voltage $v_i(t)$ and the output is the voltage $v_o(t)$. Chose one of the circuits shown in Figure DP14-6 and design it to have the step response

$$v_o(t) = \left(\frac{4}{3}e^{-20t} - \frac{1}{3}e^{-5t}\right)u(t) \text{ V}$$

Answer: One solution is to choose Circuit *b* with $L = 1$ H, $R = 125\ \Omega$, $C = 2$ mF and $k = 4$ A/A.

Solution: First, determine the transfer functions of the circuits. To do so, notice that all three circuits in Figure DP14-6 can be represented by the s-domain circuit shown below:



KCL has already been used to determine the current in Z_3. Apply KVL to the outside loop to get

$$Z_1 I + Z_2 I + Z_3(k+1)I = V_i$$

The output is given by $$V_o(s) = Z_3(k+1)I = \frac{Z_3(k+1)}{Z_1 + Z_2 + Z_3(k+1)} V_i$$

The transfer function is $H(s)=\dfrac{V_o(s)}{V_i(s)}=\dfrac{Z_3(k+1)}{Z_1+Z_2+Z_3(k+1)}$

Substituting the appropriate expressions for Z_1, Z_2 and Z_3, we get

$$H_a(s)=\frac{\frac{k+1}{LC}}{s^2+\frac{R}{L}s+\frac{k+1}{LC}},\quad H_b(s)=\frac{s^2}{s^2+\frac{R}{(k+1)L}s+\frac{1}{(k+1)LC}}$$ and

$$H_c(s)=\frac{(k+1)\frac{R}{L}s}{s^2+(k+1)\frac{R}{L}s+\frac{1}{LC}}$$

We require

$$\frac{H(s)}{s}=\mathfrak{L}\left[\frac{4}{3}e^{-20t}-\frac{1}{3}e^{-5t}\right]=\frac{\frac{4}{3}}{s+20}-\frac{\frac{1}{3}}{s+5}=\frac{\frac{4}{3}(s+5)-\frac{1}{3}(s+20)}{(s+20)(s+5)}=\frac{s}{s^2+25s+100}.$$

The required transfer function is $H(s)=\dfrac{s^2}{s^2+25s+100}$

Noticing that $H_b(s)$ has the same form as $H(s)$, we select circuit *b* from Figure DP14-6 and require:

$$\frac{s^2}{s^2+25s+100}=H(s)=H_b(s)=\frac{s^2}{s^2+\frac{R}{(k+1)L}s+\frac{1}{(k+1)LC}}$$

That is: $\dfrac{R}{(k+1)L}=25$ and $\dfrac{1}{(k+1)LC}=100$

These equations don't have a unique solution. One solution is L = 1 H, R = 125 Ω, C = 2 mF and k = 4 A/A.

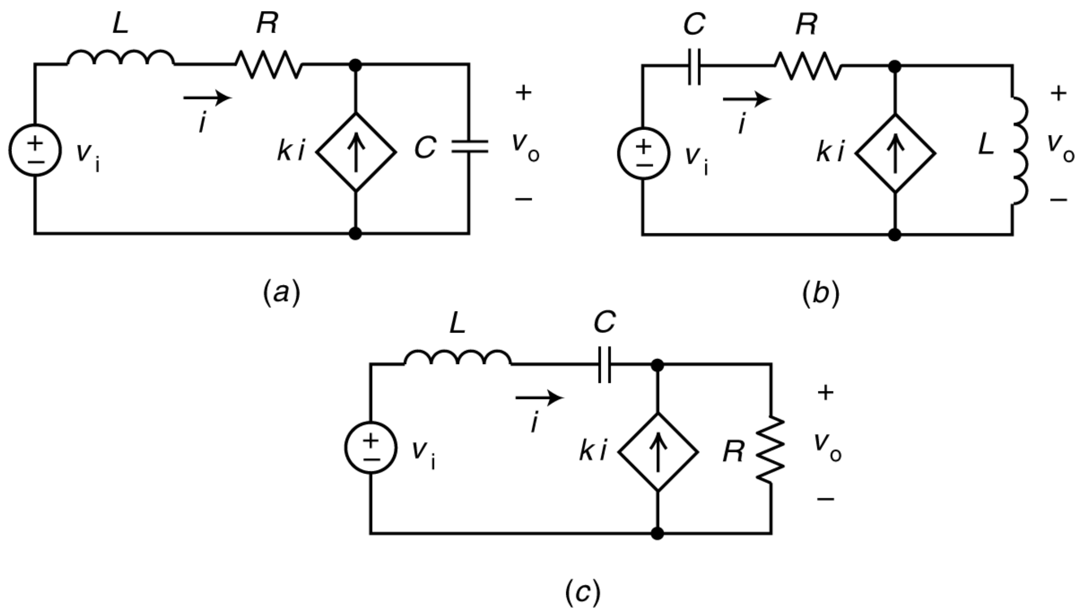


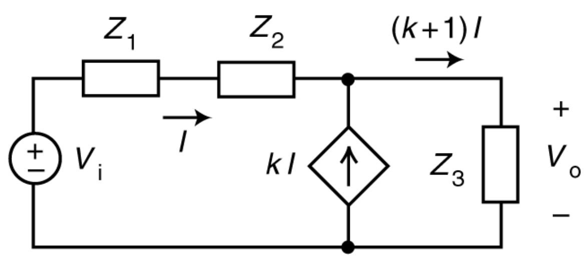
Figure DP14 -6

DP14-7 The input to each of the circuits shown in Figure DP14-6 is the voltage $v_i(t)$ and the output is the voltage $v_o(t)$. Chose one of the circuits shown in Figure DP14-6 and design it to have the step response

$$v_o(t) = 5\left(e^{-10t} - e^{-15t}\right)u(t) \text{ V}$$

Answer: One solution is to choose Circuit *c* with *L* = 1/3 H, *R* = 1.6667 Ω, *C* = 2 mF and *k* = 4 A/A.

Solution: First, determine the transfer functions of the circuits. To do so, notice that all three circuits in Figure DP14-6 can be represented by the s-domain circuit shown below:



KCL has already been used to determine the current in Z_3. Apply KVL to the outside loop to get

$$Z_1 I + Z_2 I + Z_3(k+1)I = V_i$$

The output is given by $$V_o(s) = Z_3(k+1)I = \frac{Z_3(k+1)}{Z_1 + Z_2 + Z_3(k+1)} V_i$$

The transfer function is $H(s)=\dfrac{V_o(s)}{V_i(s)}=\dfrac{Z_3(k+1)}{Z_1+Z_2+Z_3(k+1)}$

Substituting the appropriate expressions for Z_1, Z_2 and Z_3, we get

$$H_a(s)=\frac{\frac{k+1}{LC}}{s^2+\frac{R}{L}s+\frac{k+1}{LC}},\quad H_b(s)=\frac{s^2}{s^2+\frac{R}{(k+1)L}s+\frac{1}{(k+1)LC}}$$ and

$$H_c(s)=\frac{(k+1)\frac{R}{L}s}{s^2+(k+1)\frac{R}{L}s+\frac{1}{LC}}$$

We require

$$\frac{H(s)}{s}=\mathfrak{L}\left[5\left(e^{-10t}-e^{-15t}\right)\right]=\frac{5}{s+10}-\frac{5}{s+15}=\frac{5(s+15)-5(s+10)}{(s+10)(s+15)}=\frac{25}{s^2+25s+150}.$$

The required transfer function is $H(s)=\dfrac{25s}{s^2+25s+150}$

Noticing that $H_c(s)$ has the same form as $H(s)$, we select circuit *c* from Figure DP14-6 and require:

$$\frac{25s}{s^2+25s+150}=H(s)=H_c(s)=\frac{(k+1)\frac{R}{L}s}{s^2+(k+1)\frac{R}{L}s+\frac{1}{LC}}$$

That is: $(k+1)\dfrac{R}{L}=25$ and $\dfrac{1}{LC}=150$

These equations don't have a unique solution. One solution is L = 1/3 H, R = 1.6667 Ω, C = 2 mF and
k = 4 A/A.

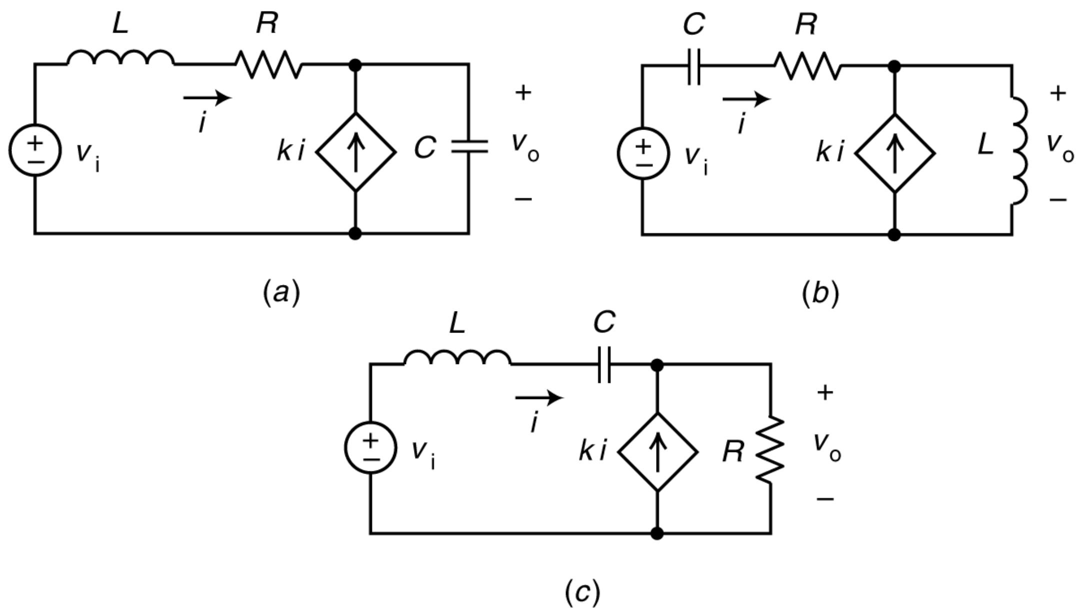


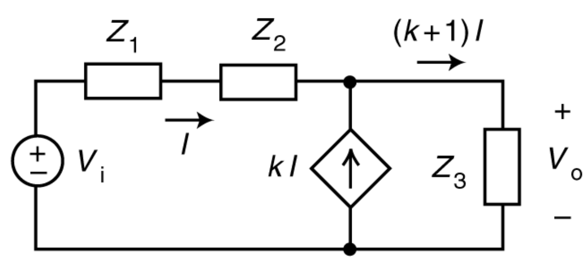
Figure DP14 -6

DP14-10 The input to each of the circuits shown in Figure DP14-6 is the voltage $v_{\text{i}}(t)$ and the output is the voltage $v_{\text{o}}(t)$. Chose one of the circuits shown in Figure DP14-6 and design it to have the step response

$$v_{\text{o}}(t) = e^{-10t}\cos(40t)u(t) \ \text{V}$$

Answer: None of the circuits in Figure DP14-6 can produce the required step response.

Solution: First, determine the transfer functions of the circuits. To do so, notice that all three circuits in Figure DP14-6 can be represented by the s-domain circuit shown below:



KCL has already been used to determine the current in Z_3. Apply KVL to the outside loop to get

$$Z_1 I + Z_2 I + Z_3(k+1)I = V_{\text{i}}$$

The output is given by $$V_{\text{o}}(s) = Z_3(k+1)I = \frac{Z_3(k+1)}{Z_1 + Z_2 + Z_3(k+1)}V_{\text{i}}$$

The transfer function is $H(s)=\dfrac{V_o(s)}{V_i(s)}=\dfrac{Z_3(k+1)}{Z_1+Z_2+Z_3(k+1)}$

Substituting the appropriate expressions for Z_1, Z_2 and Z_3, we get

$$H_a(s)=\frac{\frac{k+1}{LC}}{s^2+\frac{R}{L}s+\frac{k+1}{LC}},\quad H_b(s)=\frac{s^2}{s^2+\frac{R}{(k+1)L}s+\frac{1}{(k+1)LC}} \quad \text{and}$$

$$H_c(s)=\frac{(k+1)\frac{R}{L}s}{s^2+(k+1)\frac{R}{L}s+\frac{1}{LC}}$$

We require

$$\frac{H(s)}{s}=\mathcal{L}\left[e^{-20t}\cos(40t)\right]=\frac{s+20}{(s+20)^2+40^2}=\frac{s+20}{s^2+40s+2000}.$$

The required transfer function is $H(s)=\dfrac{s(s+20)}{s^2+40s+2000}$

Noticing that none of the circuits in Figure DP14-6 has a transfer function of the same form as $H(s)$, we conclude that none of the circuits in Figure DP14-6 can produce the required step response.

Chapter 15: – Fourier Series and Fourier Transform

Exercises

Exercise 15.2-1 Suppose $f_1(t)$ and $f_2(t)$ are periodic functions having the same period, T. Then $f_1(t)$ and $f_2(t)$ can be represented by the Fourier series

$$f_1(t) = a_{10} + \sum_{n=1}^{\infty} (a_{1n} \cos\ (n\omega_0 t) + b_{1n} \sin\ (n\omega_0 t))$$

And
$$f_2(t) = a_{20} + \sum_{n=1}^{\infty} (a_{2n} \cos\ (n\omega_0 t) + b_{2n} \sin\ (n\omega_0 t))$$

Determine the Fourier series of the function

$$f(t) = k_1 f_1(t) + k_2 f_2(t)$$

Answer: $$f(t) = (k_1 a_{10} + k_2 a_{20}) + \sum_{n=1}^{\infty} ((k_1 a_{1n} + k_2 a_{2n}) \cos(n\omega_0 t) + (k_1 b_{1n} + k_2 b_{2n}) \sin(n\omega_0 t))$$

Solution:

Notice that
$$f(t-T) = f_1(t-T) + f_2(t-T) = f_1(t) + f_2(t) = f(t)$$

Therefore, $f(t)$ is a periodic function having the same period, T. Next

$$\begin{aligned} f(t) &= k_1 f_1(t) + k_2 f_2(t) \\ &= k_1 \left[a_{10} + \sum_{n=1}^{\infty} \left(a_{1n} \cos\left(n\,\omega_0\, t\right) + b_{1n} \sin\left(n\,\omega_0\, t\right) \right) \right] \\ &\qquad + k_2 \left[a_{20} + \sum_{n=1}^{\infty} \left(a_{2n} \cos\left(n\,\omega_0\, t\right) + b_{2n} \sin\left(n\,\omega_0\, t\right) \right) \right] \\ &= \left(k_1\, a_{10} + k_2\, a_{20}\right) + \sum_{n=1}^{\infty} \left(\left(k_1\, a_{1n} + k_2\, a_{2n}\right) \cos\left(n\,\omega_0\, t\right) + \left(k_1\, b_{1n} + k_2\, b_{2n}\right) \sin\left(n\,\omega_0\, t\right) \right) \end{aligned}$$

Exercise 15.2-2 Determine the Fourier series when $f(t) = K$, a constant.
Answer: $a_0 = K$ and $a_n = b_n = 0$ for $n \ge 1$

Solution:

$f(t) = K$ is a Fourier Series. The coefficients are $a_0 = K$; $a_\mathrm{n} = b_\mathrm{n} = 0$ for $n \ge 1$.

Exercise 15.2-3 Determine the Fourier series when $f(t) = A \cos \omega_0 t$.
Answer: $a_0 = 0$, $a_1 = A$, $a_n = 0$ for $n > 1$, and $b_n = 0$

Solution:

$f(t) = A\cos(\omega_0 t)$ is a Fourier Series. $a_1 = A$ and all other coefficients are zero.

Exercise 15.3-1 Determine the Fourier series for the waveform *f*(*t*) shown in Figure E 15.3-1. Each increment of time on the horizontal axis is π/8 s, and the maximum and minimum are +1 and –1, respectively.

Answer: $f(t) = \frac{4}{\pi}\sum_{n=1}^{N}\frac{1}{n}\sin n\omega_0 t$ and n odd, $\omega_0 = 4$ rad/s

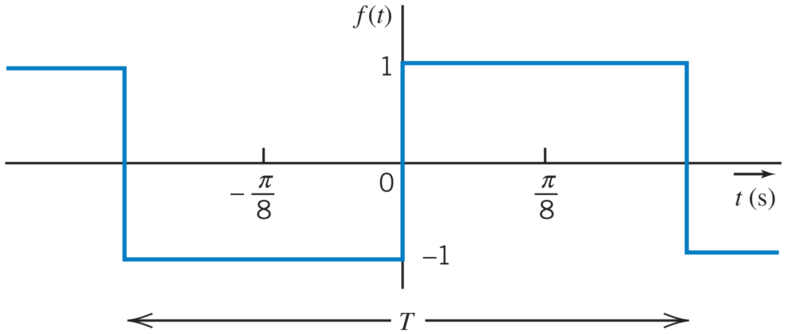
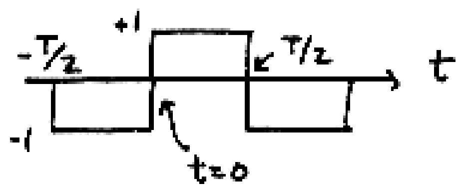


Figure E 15.3-1

Solution



$$T = 4\left(\frac{\pi}{8}\right) = \frac{\pi}{2},\ \omega_0 = \frac{2\pi}{T} = 4\ \text{rad/s}$$

Set origin at t = 0, so have an odd function; then $a_n = 0$ for $n = 0,1,\ldots$ Also, f(t) has half wave symmetry, so $b_n = 0$ for n = even. For odd *n*, we have

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\ \sin(n\omega_0 t)\ dt = -\frac{2}{T}\int_{-T/2}^{0}\sin(n\omega_0 t)\ dt + \frac{2}{T}\int_{0}^{T/2}\sin(n\omega_0 t)\ dt$$

$$= \frac{4}{T}\int_{0}^{T/2}\sin(n\omega_0 t)\ dt$$

$$= \frac{4}{n\omega_0 T}\left(1-\cos\left(n\omega_0\frac{T}{2}\right)\right) = \begin{cases}\dfrac{4}{n\pi} & \text{when } n \text{ is odd}\\ 0 & \text{when } n \text{ is even}\end{cases}$$

Finally,

$$\underline{f(t) = \frac{4}{\pi}\sum_{n}^{\infty}\frac{1}{n}\sin(n\omega_0 t)},\quad n \text{ odd and } \omega_0 = 4\ \text{rad/s}$$

Exercise 15.3-2 Determine the Fourier series for the waveform *f*(*t*) shown in Figure E 15.3-2. Each increment of time on the horizontal grid is π/6 s, and the maximum and minimum values of *f*(*t*) are 2 and –2, respectively.

Answer: $f(t)=\frac{-24}{\pi^2}\sum_{n=1}^{N}\frac{1}{n^2}\sin(n\pi/3)\sin n\omega_0 t$ and n odd, $\omega_0 = 2$ rad/s

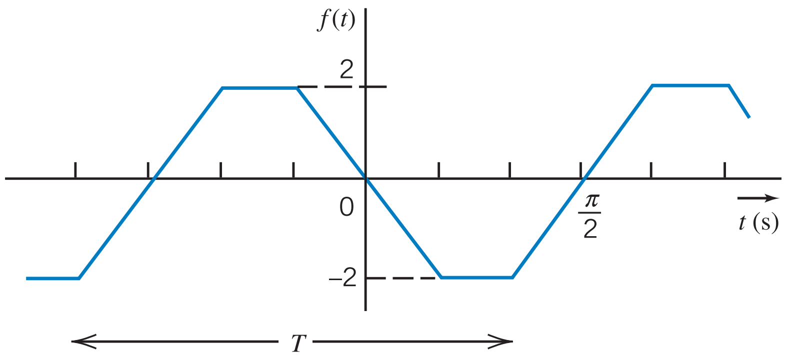


Figure E 15.3-2

Solution:

$$T=\pi\ ,\ \omega_0=\frac{2\pi}{T}=2$$

odd function with quarter wave symmety $\Rightarrow \begin{cases} a_0 = 0\ , & a_n = 0 \quad \text{for all } n \\ b_n = 0 & n = \text{even} \end{cases}$

$$b_n = \frac{8}{\pi}\int_0^{\pi/4} f(t)\ \sin\ n\omega_0 t\ dt \quad \text{where } f(t) = \begin{cases} \frac{-2t}{\pi/6} & 0 < t < \pi/6 \\ -2 & \pi/6 \le t < \pi/4 \end{cases}$$

Thus $b_n = \frac{-24}{\pi^2}\ \frac{1}{n^2}\ \sin\left(\frac{n\pi}{3}\right)$

so $f(t) = \frac{-24}{\pi^2}\ \sum_{\substack{n=1 \\ odd\ n}}^{\infty} \frac{1}{n^2}\ \sin\left(\frac{n\pi}{3}\right)\ \sin\ (2nt)$

Exercise 15.3-3 For the periodic signal $f(t)$ shown in Figure E 15.3-3, determine whether the Fourier series contains (a) sine and cosine terms and (b) even harmonics and (c) calculate the dc value.
Answer: (a) Yes, both sine and cosine terms; (b) no even harmonics; (c) $a_0 = 0$

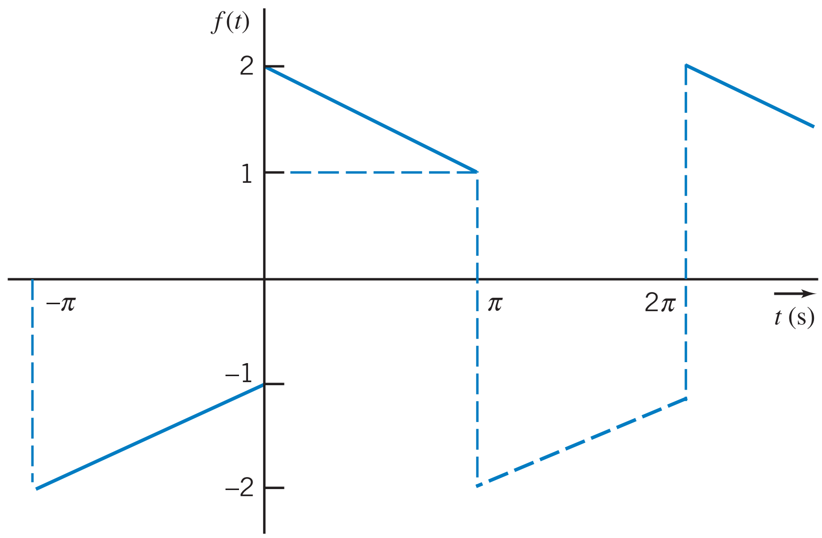


Figure E 15.3-3

Solution:

a) is neither even nor odd. $f(t)$ will contain both sine and cosine terms

b) $\frac{1}{4}$ wave symmetry $\Rightarrow$ no even harmonics

c) average value of $f(t) = 0 \Rightarrow a_0 = 0$

Exercise 15.5-1 Find the exponential Fourier coefficients for the function shown in Figure E 5.5-1.

Answer: $\mathbf{C}_n = 0$ for even n and $\mathbf{C}_n = \dfrac{2}{jn\pi}$ for odd n

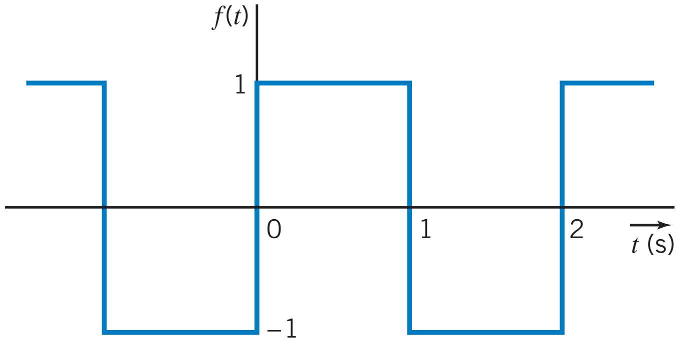


Figure E 15.5-1

Solution:

$$T = 2\text{ s},\ \omega_0 = \frac{2\pi}{T} = \pi \text{ rad/s}$$

$$C_n = \frac{1}{2}\int_0^2 f(t)e^{-jn\pi t}\,dt = \frac{1}{2}\int_0^1 e^{-jn\pi t}\,dt - \frac{1}{2}\int_1^2 e^{-jn\pi t}\,dt$$

$$= \frac{1}{2\,jn\pi}\left[-e^{-jn\pi} + 1 + e^{-j2n\pi} - e^{-jn\pi}\right] = \frac{1}{jn\pi}(1 - e^{-jn\pi})$$

$$C_n = \begin{cases} \dfrac{2}{jn\pi} & n \text{ odd} \\ 0 & n \text{ even} \end{cases}$$

Finally,

$$f(t) = \frac{2}{j\pi}\left[e^{j\pi t} + \frac{1}{3}e^{j3\pi t} + \frac{1}{5}e^{j5\pi t} + \dots - e^{-j\pi t} - \frac{1}{3}e^{-j3\pi t} - \frac{1}{5}e^{-j5\pi t} - \dots\right]$$

Exercise 15.5-2 Determine the complex Fourier coefficients for the waveform shown in Figure E 15.5-2.

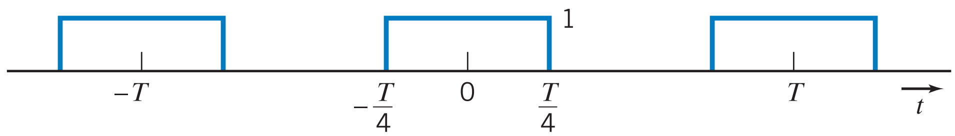


Figure E 15.5-2

Solution:

$$C_n = \frac{1}{T}\int_{-T/4}^{T/4} e^{-j\omega_0 nt}\, dt = \frac{1}{T}\left(\frac{T}{-j2\pi n}\right) e^{-j2\pi nt/T}\Big|_{-T/4}^{T/4} = \frac{1}{-j2\pi n}\left[e^{-j\pi n/2} - e^{j\pi n/2}\right]$$

$$= \frac{1}{-j2\pi n}\left[-j\,2\sin\left(\frac{\pi n}{2}\right)\right] = \frac{\sin\left(\frac{\pi n}{2}\right)}{\pi n}$$

$$C_n = \begin{cases} \frac{(-1)^{(n-1)/2}}{\pi n} & n \text{ odd} \\ 0 & n \text{ even},\ n \neq 0 \\ 1/2 & n = 0 \end{cases}$$

Exercise 15.8-1 Find the response of the circuit of Figure 15.8-1 when $R = 10$ kΩ, $C = 0.4$ mF, and v_s is the triangular wave considered in Example 15.3-1 (Figure 15.3-2). Include all terms that exceed 2 percent of the fundamental term.

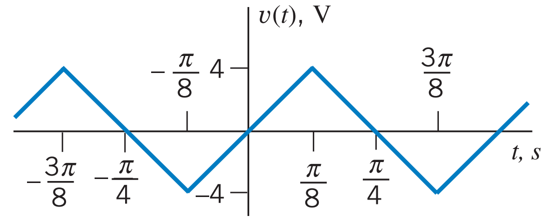
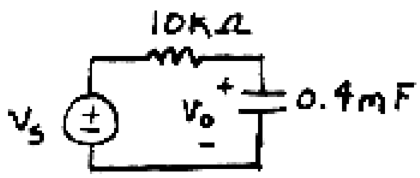


Figure 15.3-2

Answer: $v_o(t) \approx 0.20 \sin(4t - 86°) - 0.008 \sin(12t - 89°)$ V

Solution:



$$\omega_0 = 4 \text{ rad/s}$$

From Example 15.4-1:

$$v_s(t) = 3.24 \sum_{\substack{n=1 \\ \text{odd } n}}^{N} \frac{1}{n^2}\left(\sin\frac{n\pi}{2}\right)\sin n\omega_0 t = 3.24\left(\sin 4t - \frac{1}{9}\sin 12t + \frac{1}{25}\sin 20t - \frac{1}{49}\sin 28t \cdots\right)$$

The network function of the circuit is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega CR} = \frac{1}{1 + j\omega 4}$$

Evaluating the network function at the frequencies of the input series

$$\mathbf{H}(n4) = \frac{1}{1 + j16n} \qquad n = 1,3,5...$$

n	**H**(n4)
1	0.062∠-86°
3	0.021∠-89°
5	0.012∠-89°
7	0.0009∠-89°

Using superposition

$$v_o(t) = 3.24\left((0.062)\sin(4t - 86°) - \frac{0.021}{9}\sin(12t - 89°) + \frac{0.012}{25}\sin(20t - 89°) - \frac{0.0009}{49}\sin(28t - 89°)\cdots\right)$$

$$v_o(t) = (0.2009)\sin(4t - 86°) - (.00756)\sin(12t - 89°)$$
$$+ (0.00156)\sin(20t - 89°) - (5.95\times10^{-5})\sin(28t - 89°)\cdots$$

Discarding the terms that are smaller than 2% of the fundamental term leaves

$$v_o(t) = (0.2009)\sin(4t - 86°) - (.00756)\sin(12t - 89°)$$

Exercise 15.9-1 Determine the Fourier transform of $f(t) = e^{-at}u(t)$, where $u(t)$ is the unit step function.

Answer: $$F(j\omega) = \frac{1}{a + j\omega}$$

Solution:

$$f(t) = e^{-at}u(t)$$

$$F(\omega) = \int_{-\infty}^{+\infty} f(t)\, e^{-j\omega t}\, dt = \int_{0}^{\infty} e^{-at} e^{-j\omega t}\, dt = \left. \frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \right|_{0}^{\infty} = \frac{1}{a + j\omega}$$

Exercise 15.10-1 Find the Fourier transform of $f(at)$ for $a > 0$ when $F(\omega) = \mathcal{F}\,[f(t)]$.

Answer: $\mathcal{F}\,[f(at)] = \frac{1}{a} F\left(\frac{\omega}{a}\right)$

Solution:

$$\mathcal{F}\{f(at)\} = \int_{-\infty}^{\infty} f(at) e^{-j\omega t}\, dt$$

$$\text{Let } \tau = at \quad \Rightarrow \quad t = \frac{\tau}{a}$$

$$\mathcal{F}\{f(at)\} = \int_{-\infty}^{\infty} f(\tau) e^{-j\omega\tau/a}\, d\frac{\tau}{a} = \frac{1}{a}\int_{-\infty}^{\infty} f(\tau) e^{-j(\omega/a)\tau}\, d\tau = \frac{1}{a} F\left(\frac{\omega}{a}\right)$$

Exercise 15.10-2 Show that the Fourier transform of a constant dc waveform $f(t) = A$ for $-\infty \le t \le \infty$ is $F(\omega) = 2\pi A\delta(\omega)$ by obtaining the inverse transform of $F(\omega)$.

Solution:

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \left(2\pi\delta(\omega) A\right) e^{j\omega t}\, dt = \frac{1}{2\pi}\int_{0^-}^{0^+} \left(2\pi\delta(\omega) A\right) dt = A$$

Exercise 15.11-1 Calculate the Fourier transform and draw the Fourier spectrum for $f(t)$ shown in Figure 15.11-1, where

$$f(t) = A \cos \omega_0 t$$

for all t.

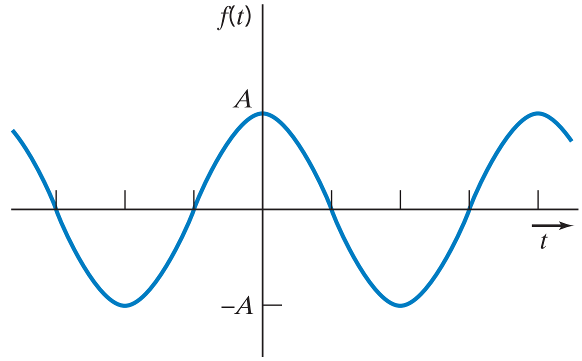


Figure 15.11-1

Answer: $F(\omega) = \pi A\delta(\omega + \omega_0) + \pi A\delta(\omega - \omega_0)$

Solution:

$$\mathcal{F}^{-1}\left\{\delta\left(\omega-\omega_0\right)\right\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \delta\left(\omega-\omega_0\right) e^{j\omega t}\, d\omega = \frac{1}{2\pi}\, e^{j\omega_0 t}$$

Take the Fourier Transform of both sides to get: $\mathcal{F}\left(e^{j\omega_0 t}\right) = 2\pi\delta\left(\omega-\omega_0\right)$

$$\mathcal{F}\left\{A\cos\omega_0 t\right\} = \mathcal{F}\left\{A\left(\frac{e^{j\omega_0 t}+e^{-j\omega_0 t}}{2}\right)\right\} = \frac{A}{2}\left(\mathcal{F}\left(e^{j\omega_0 t}\right)+\mathcal{F}\left(e^{-j\omega_0 t}\right)\right) = \frac{A}{2}\left(2\pi\delta\left(\omega-\omega_0\right)+2\pi\delta\left(\omega+\omega_0\right)\right)$$

$$= A\pi\delta\left(\omega-\omega_0\right)+A\pi\delta\left(\omega+\omega_0\right)$$

Exercise 15.12-1 An ideal band-pass filter passes all frequencies between 24 rad/s and 48 rad/s without attenuation and completely rejects all frequencies outside this passband.

(a) Sketch $|V_0|^2$ for the filter output voltage when the input voltage is

$v(t) = 120e^{-24t}u(t)\,\text{V}$

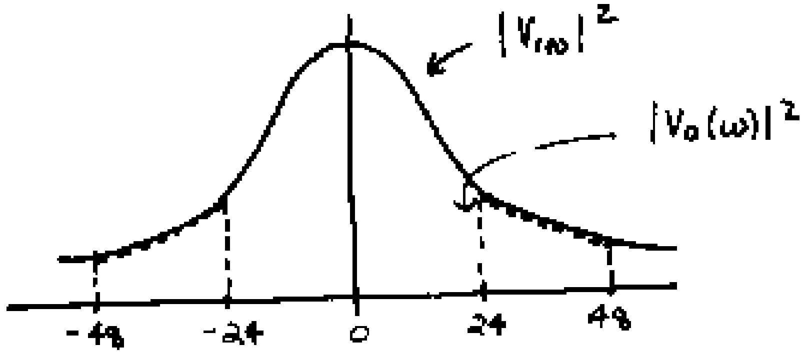
(b) What percentage of the input signal energy is available in the signal at the output of the ideal filter?

Answer: (b) 20.5%

Solution:

a)

$$V_{in}(\omega) = \frac{120}{24 + j\omega} \quad \Rightarrow \quad V_{in}(\omega) = \frac{120^2}{24^2 + \omega^2} = \frac{14400}{576 + \omega^2}$$



b)

$$W_{in} = \frac{1}{\pi}\int_0^{\infty} \frac{14400}{576 + \omega^2}\, d\omega = \frac{14400}{\pi}\left(\frac{1}{24}\tan^{-1}\left(\frac{\omega}{24}\right)\right)_0^{\infty} = 300 \text{ J}$$

$$W_{out} = \frac{1}{\pi}\int_{24}^{48} \frac{14400}{576 + \omega^2}\, d\omega = \frac{14400}{\pi}\left(\frac{1}{24}\tan^{-1}\left(\frac{\omega}{24}\right)\right)_{24}^{48} = 61.3 \text{ J}$$

$$\therefore \eta = \frac{W_{out}}{W_{in}} \times 100\% = \frac{61.3}{300} \times 100\% = 20.5\%$$

Exercise 15.13-1 Derive the Fourier transform for

$$f(t) = te^{-at} \quad t \geq 0$$
$$= te^{at} \quad t \leq 0$$

Answer: $\dfrac{-j4a\omega}{\left(a^2+\omega^2\right)^2}$

Solution:

$$f^+(t) = te^{-at}$$
$$f^-(t) = te^{at} \quad \Rightarrow \quad f^-(-t) = -te^{-at}$$
$$\therefore F^+(s) = \frac{1}{(s+a)^2} \quad \text{and} \quad F^-(s) = \frac{-1}{(s+a)^2}$$

Then $F(\omega) = F^+(s)\Big|_{s=j\omega} + F^-(s)\Big|_{s=-j\omega} = \left.\dfrac{1}{(s+a)^2}\right|_{s=j\omega} + \left.\dfrac{-1}{(s+a)^2}\right|_{s=-j\omega}$

$$= \frac{1}{(a+j\omega)^2} - \frac{1}{(a-j\omega)^2} = \frac{-j4a\omega}{\left(a^2+\omega^2\right)^2}$$

Section 15.2: The Fourier Series

P 15.2-1 Find the trigonometric Fourier series for a periodic function $f(t)$ that is equal to t^2 over the period from $t = 0$ to $t = 2$.

Solution:

$T = 2$ s $\Rightarrow$ $\omega_0 = \dfrac{2\pi}{2} = \pi$ rad/s and $f(t) = t^2$ for $0 \le t \le 2$. The coefficients of the Fourier series are given by:

$$a_0 = \frac{1}{2}\int_0^2 t^2 dt = 4/3$$

$$a_n = \frac{2}{2}\int_0^2 t^2 \cos n\pi t \, dt = \frac{4}{(n\pi)^2}$$

$$b_n = \frac{2}{2}\int_0^2 t^2 \sin n\pi t \, dt = \frac{-4}{n\pi}$$

$$\therefore \quad \underline{f(t) = \frac{4}{3} + \frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\cos n\pi t - \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin n\pi t}$$

P 15.2-2 A "staircase" periodic waveform is described by its first cycle as

$$f(t)=\begin{cases}1 & 0<t<0.25\\ 2 & 0.25<t<0.5\\ 0 & 0.5<t<1\end{cases}$$

Find the Fourier series for this function.

Solution:

$$a_n = \frac{2}{T}\left[\int_0^{\frac{T}{4}}\cos\left(n\frac{2\pi}{T}t\right)dt+\int_{\frac{T}{4}}^{\frac{T}{2}}2\cos\left(n\frac{2\pi}{T}t\right)dt\right]=\frac{1}{n\pi}\left[\sin\left(n\frac{2\pi}{T}t\right)\Bigg|_0^{\frac{T}{4}}+2\sin\left(n\frac{2\pi}{T}t\right)\Bigg|_{\frac{T}{4}}^{\frac{T}{2}}\right]$$

$$=\frac{1}{n\pi}\left[(\sin\left(\frac{n\pi}{2}\right)-0)+2\left((\sin n\pi)-\sin\left(\frac{n\pi}{2}\right)\right)\right]$$

$$=-\frac{1}{n\pi}\sin\left(\frac{n\pi}{2}\right)=\begin{cases}\frac{(-1)^{(n+1)/2}}{n\pi} & \text{odd } n\\ 0 & \text{even } n\end{cases}$$

$$b_n=\frac{2}{T}\left[\int_0^{\frac{T}{4}}\sin\left(n\frac{2\pi}{T}t\right)dt+\int_{\frac{T}{4}}^{\frac{T}{2}}2\sin\left(n\frac{2\pi}{T}t\right)dt\right]=-\frac{1}{n\pi}\left[\cos\left(n\frac{2\pi}{T}t\right)\Bigg|_0^{\frac{T}{4}}+2\cos\left(n\frac{2\pi}{T}t\right)\Bigg|_{\frac{T}{4}}^{\frac{T}{2}}\right]$$

$$=-\frac{1}{n\pi}\left[(2\cos(n\pi)-1)-\cos\frac{n\pi}{2}\right]$$

$$=\begin{cases}\frac{3}{n\pi} & n \text{ is odd}\\ -\frac{2}{n\pi} & n=2,6,10,\ldots\\ 0 & n=4,8,12,\ldots\end{cases}$$

P 15.2-3 Determine the Fourier series for the sawtooth function shown in Figure P 15.2-3.

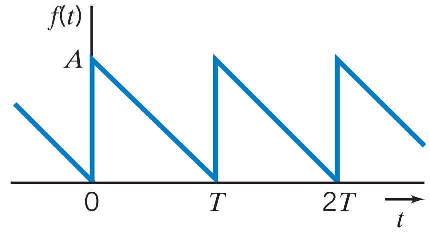


Figure P 15.2-3

Solution:

$$a_0 = \text{average value of } f(t) = \frac{A}{2}$$

$$f(t) = A\left(1 - \frac{t}{T}\right) \qquad \text{for } 0 \le t \le T$$

$$a_n = \frac{2}{T}\int_0^T A\left(1-\frac{t}{T}\right)\cos\left(n\frac{2\pi}{T}t\right)dt = \frac{2A}{T}\left[\int_0^T \cos\left(n\frac{2\pi}{T}t\right)dt - \frac{1}{T}\int_0^T t\cos\left(n\frac{2\pi}{T}t\right)dt\right]$$

$$= \frac{2A}{T}\left[0 - \frac{1}{T}\left.\frac{\cos\left(n\frac{2\pi}{T}t\right) + \left(n\frac{2\pi}{T}t\right)\sin\left(n\frac{2\pi}{T}t\right)}{\left(n\frac{2\pi}{T}\right)^2}\right|_0^T\right]$$

$$= \frac{-A}{2n^2\pi^2}\left[\cos(2n\pi) - \cos(0) + 2n\pi\sin(2n\pi) - 0\right] = 0$$

$$b_n = \frac{2}{T}\int_0^T A\left(1-\frac{t}{T}\right)\sin\left(n\frac{2\pi}{T}t\right)dt = \frac{2A}{T}\left[\int_0^T \sin\left(n\frac{2\pi}{T}t\right)dt - \frac{1}{T}\int_0^T t\sin\left(n\frac{2\pi}{T}t\right)dt\right]$$

$$= \frac{2A}{T}\left[0 - \frac{1}{T}\left.\frac{\sin\left(n\frac{2\pi}{T}t\right) - \left(n\frac{2\pi}{T}t\right)\cos\left(n\frac{2\pi}{T}t\right)}{\left(n\frac{2\pi}{T}\right)^2}\right|_0^T\right]$$

$$= \frac{-A}{2n^2\pi^2}\left[\left(\sin(2n\pi) - \sin(0)\right) - \left(2n\pi\cos(2n\pi) - 0\right)\right] = \frac{A}{n\pi}$$

$$f(t) = \frac{A}{2} + \sum_{n=1}^{\infty} \frac{A}{n\pi}\sin\left(n\frac{2\pi}{T}t\right)$$

P 15.2-4 Find the Fourier series for the periodic function *f*(*t*) that is equal to *t* over the period from *t* = 0 to *t* = 2 s.

Solution:

$$T = 2 \text{ s}, \quad \omega_0 = \frac{2\pi}{2} = \pi \text{ rad/s}, \; a_0 = \text{average value of } f(t) = 1,$$

$$f(t) = t \qquad \text{for } 0 \le t \le 2$$

$$a_n = \frac{2}{2}\int_0^2 t\cos(n\pi t)\,dt = \left.\frac{\cos(n\pi t) + (n\pi t)\sin(n\pi t)}{(n\pi)^2}\right|_0^2$$

$$= \frac{1}{n^2\pi^2}\left[\cos(2n\pi) - \cos(0) + 2n\pi\sin(2n\pi) - 0\right]$$

$$= 0$$

$$b_n = \frac{2}{2}\int_0^2 t\sin(n\pi t)\,dt = \left.\frac{\sin(n\pi t) - (n\pi t)\cos(n\pi t)}{(n\pi)^2}\right|_0^2$$

$$= \frac{1}{n^2\pi^2}\left[\left(\sin(2n\pi) - \sin(0)\right) - \left(2n\pi\cos(2n\pi) - 0\right)\right]$$

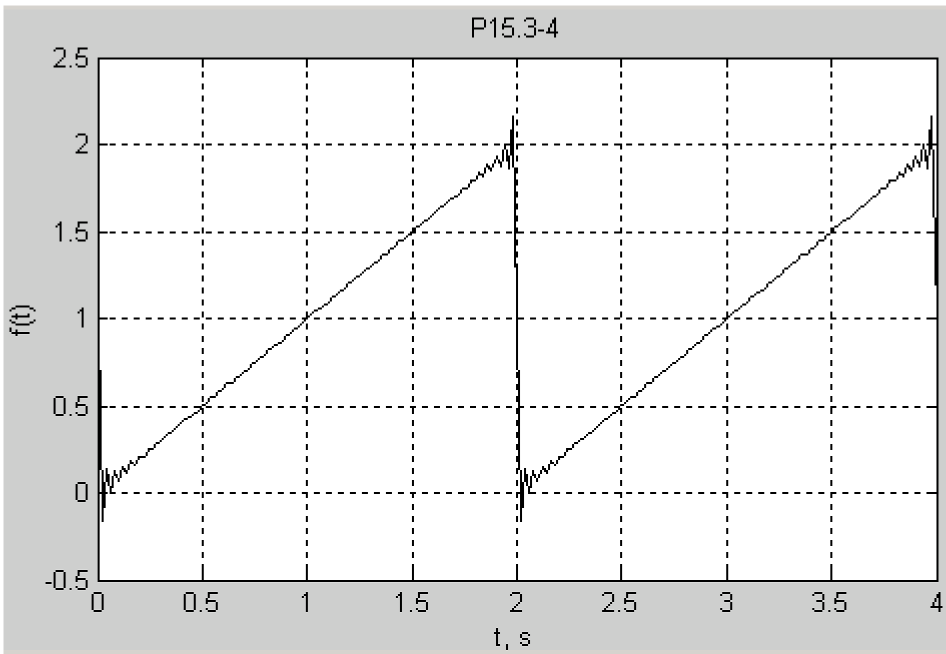
$$= \frac{-2}{n\pi}$$

$$f(t) = 1 - \sum_{n=1}^{\infty}\frac{2}{n\pi}\sin\left(n\frac{2\pi}{T}t\right)$$

Use Matlab to check this answer:

```
% P15.2-4
pi=3.14159;
A=2;                  % input waveform parameters
T=2;                  % period

w0=2*pi/T;            % fundamental frequency, rad/s
tf=2*T;               % final time
dt=tf/200;            % time increment
t=0:dt:tf;            % time, s

a0=A/2;               % avarage value of input
v1=0*t+a0;            % initialize input as vector

for n=1:1:51          % for each term in the Fourier series ...
   an=0;                          % specify coefficients of the input
series
   bn=-A/pi/n;
   cn=sqrt(an*an + bn*bn);      % convert to magnitude and angle form
   thetan=-atan2(bn,an);
```

```
   v1=v1+cn*cos(n*w0*t+thetan); % add the next term of the input
Fourier series
end

plot(t, v1,'black')    % plot the Fourier series

grid
xlabel('t, s')
ylabel('f(t)')
title('P15.3-4')
```



Section 15.3 Symmetry of the Function *f* (*t*)

P 15.3-1 Determine the Fourier series of the voltage waveform shown in Figure P 15.3-1.

Answer: $v_d(t) = \sum_{n=1}^{\infty} \frac{12}{n\pi} \sin\left(n\frac{\pi}{2}t\right)$

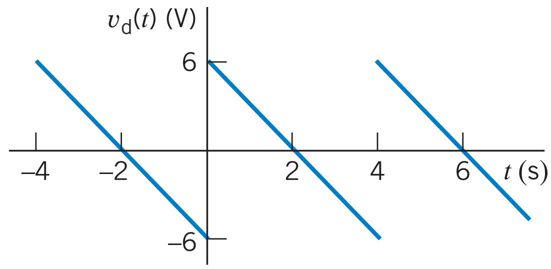


Figure P 15.3-1

Solution:

$$T = 4 \text{ s} \quad \Rightarrow \quad \omega_0 = \frac{2\pi}{4} = \frac{\pi}{2} \text{ rad/s}.$$

The coefficients of the Fourier series are:

$$a_0 = \text{average value of } v_d(t) = 0$$

$a_n = 0$ because $v_d(t)$ is an odd function of t.

$$\begin{aligned}
b_n &= \frac{1}{2}\int_0^4 (6-3t)\sin\left(n\frac{\pi}{2}t\right)dt \\
&= 3\int_0^4 \sin\left(n\frac{\pi}{2}t\right)dt - \frac{3}{2}\int_0^4 t\sin\left(n\frac{\pi}{2}t\right)dt \\
&= 3\left[\frac{-\cos\left(n\frac{\pi}{2}t\right)}{n\frac{\pi}{2}}\right]_0^4 - \frac{3}{2}\left[\frac{1}{\frac{n^2\pi^2}{4}}\left(\sin\left(n\frac{\pi}{2}t\right) - \left(n\frac{\pi}{2}t\right)\cos\left(n\frac{\pi}{2}t\right)\right)\right]_0^4 \\
&= \frac{6}{n\pi}\left(1-\cos(2n\pi)\right) - \frac{6}{n^2\pi^2}\left(\left(\sin(2n\pi)-0\right) - \left(2n\pi\cos(2n\pi)\right)\right) = \frac{12}{n\pi}
\end{aligned}$$

The Fourier series is:

$$v_d(t) = \sum_{n=1}^{\infty} \frac{12}{n\pi}\sin\left(n\frac{\pi}{2}t\right)$$

P 15.3-2 Determine the Fourier series of the voltage waveform shown in Figure P 15.3-2.
Hint: $v_c(t) = v_d(t-1) - 6$, where $v_d(t)$ is the voltage considered in problem P 15.3-1.

Answer: $v_c(t) = -6 + \sum_{n=1}^{\infty} \frac{12}{n\pi} \sin\left(n\frac{\pi}{2}t - n\frac{\pi}{2}\right)$

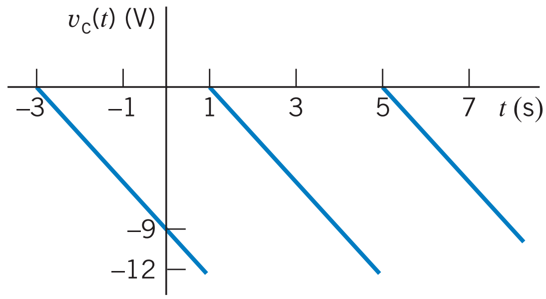


Figure P 15.3-2

Solution:

$$v_c(t) = v_d(t-1) - 6 = -6 + \sum_{n=1}^{\infty} \frac{12}{n\pi} \sin\left(n\frac{\pi}{2}(t-1)\right) = -6 + \sum_{n=1}^{\infty} \frac{12}{n\pi} \sin\left(n\frac{\pi}{2}t - n\frac{\pi}{2}\right)$$

P 15.3-3 Determine the Fourier series of the voltage waveform shown in Figure P 15.3-3.

Answer: $v_a(t) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{18}{n^2\pi^2}\left(1-\cos\left(\frac{n\pi}{3}\right)\right)\cos\left(n\frac{1000\pi}{3}t\right)$

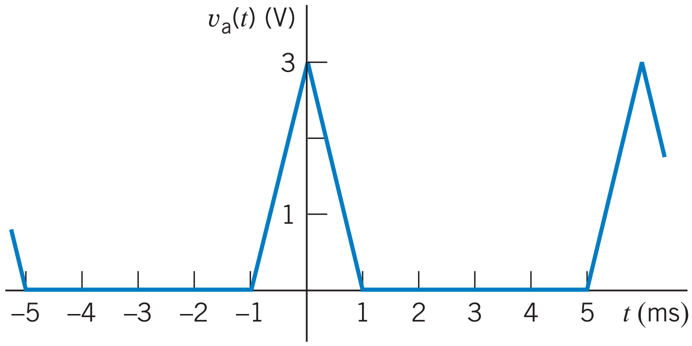


Figure P 15.3-3

Solution: $T = 6 \text{ ms} = 0.006 \text{ s} \Rightarrow \omega_o = \frac{2\pi}{.006} = \frac{1000\pi}{3} \text{ rad/s} = \frac{\pi}{3} \text{ krad/s}$

The coefficients of the Fourier series are:

$$a_0 = \text{average value of } v_a(t) = \frac{\frac{3\times 2}{2}}{6} = \frac{1}{2} \text{ V}$$

$b_n = 0$ because $v_a(t)$ is an even function of t.

$$a_n = 2\left(\frac{2}{0.006}\right)\int_0^{0.001}(3-3000t)\cos\left(n\frac{1000\pi}{3}t\right)dt$$

$$= 2000\int_0^{0.001}\cos\left(n\frac{1000\pi}{3}t\right)dt - \left(2\times 10^6\right)\int_0^{0.001} t\cos\left(n\frac{1000\pi}{3}t\right)dt$$

$$= 2000\left[\frac{\sin\left(n\frac{1000\pi}{3}t\right)}{n\frac{1000\pi}{3}} - \frac{1000}{\frac{n^2 10^6\pi^2}{9}}\left(\cos\left(n\frac{1000\pi}{3}t\right) + \left(n\frac{1000\pi}{3}t\right)\sin\left(n\frac{1000\pi}{3}t\right)\right)\right]_0^{0.001}$$

$$= 2000\left[\frac{3}{n1000\pi}\left(\sin\left(n\frac{\pi}{3}\right)-0\right) - \frac{9}{n^2 10^3\pi^2}\left(\left(\cos\left(n\frac{\pi}{3}\right)-1\right) + \left(\left(n\frac{\pi}{3}\right)\sin\left(n\frac{\pi}{3}\right)-0\right)\right)\right]$$

$$= \frac{6}{n\pi}\sin\left(n\frac{\pi}{3}\right) - \left(\frac{18}{n^2\pi^2}\right)\left(\cos\left(n\frac{\pi}{3}\right)-1\right) - \frac{6}{n\pi}\sin\left(n\frac{\pi}{3}\right)$$

$$= -\left(\frac{18}{n^2\pi^2}\right)\left(\cos\left(n\frac{\pi}{3}\right)-1\right)$$

The Fourier series is $v_a(t) = \frac{1}{2} + \sum_{n=1}^{\infty}\frac{18}{n^2\pi^2}\left(1-\cos\left(\frac{n\pi}{3}\right)\right)\cos\left(n\frac{1000\pi}{3}t\right)$

P 15.3-4 Determine the Fourier series of the voltage waveform shown in Figure P 15.3-4. ***Hint:*** $v_b(t) = v_a(t - 0.002) - 1$, where $v_a(t)$ is the voltage considered in Problem P 15.4-3.

Answer:

$$v_b(t) = -\frac{1}{2} + \sum_{n=1}^{\infty} \frac{18}{n^2\pi^2}\left(1 - \cos\left(\frac{n\pi}{3}\right)\right)$$
$$\cos\left(n\frac{1000\pi}{3}t - n\frac{2\pi}{3}\right)$$

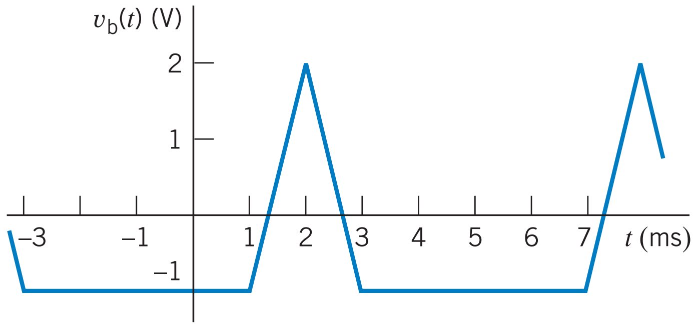


Figure P 15.3-4

Solution:

$$v_b(t) = v_a(t - 0.002) - 1 = -1 + \frac{1}{2} + \sum_{n=1}^{\infty} \frac{18}{n^2\pi^2}\left(1 - \cos\left(\frac{n\pi}{3}\right)\right)\cos\left(n\frac{1000\pi}{3}(t - 0.002)\right)$$

$$= -\frac{1}{2} + \sum_{n=1}^{\infty} \frac{18}{n^2\pi^2}\left(1 - \cos\left(\frac{n\pi}{3}\right)\right)\cos\left(n\frac{1000\pi}{3}t - n\frac{2\pi}{3}\right)$$

P 15.3-5 Find the trigonometric Fourier series of the sawtooth wave, $f(t)$, shown in Figure Figure P 15.3-5.

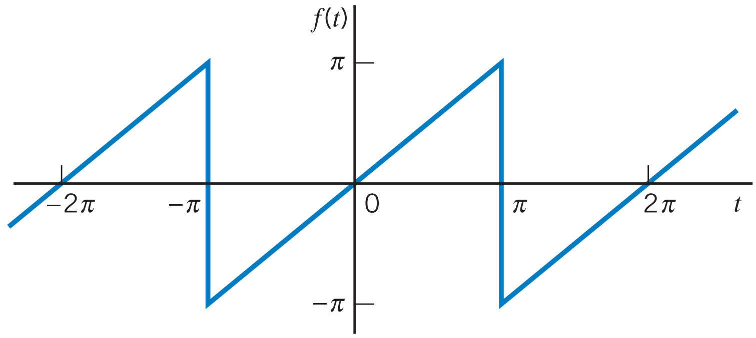
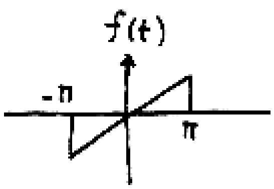


Figure P 15.3-5

Solution:



$T = 2\pi,\ \omega_0 = \dfrac{2\pi}{2\pi} = 1$

average value: $a_0 = 0$

$b_n = \dfrac{2}{T}\int_0^T f(t)\ \sin\ n\omega_0\ t\ dt$

$f(t) = t \qquad -\pi < t < \pi$

$a_n = 0$ since have odd function

$$b_n = \frac{2}{2\pi}\int_{-\pi}^{\pi} t \sin nt\ dt = \frac{1}{\pi}\left[\frac{\sin\ nt}{n^2} - \frac{t\ \cos\ nt}{n}\right]_{-\pi}^{\pi} = \frac{-2(-1)^n}{n}$$

$b_1 = 2, \quad b_2 = -1, \quad$ and $\quad b_3 = 2/3$

P 15.3-6 Determine the Fourier series for the waveform shown in Figure P 15.3-6. Calculate a_0, a_1, a_2, and a_3.

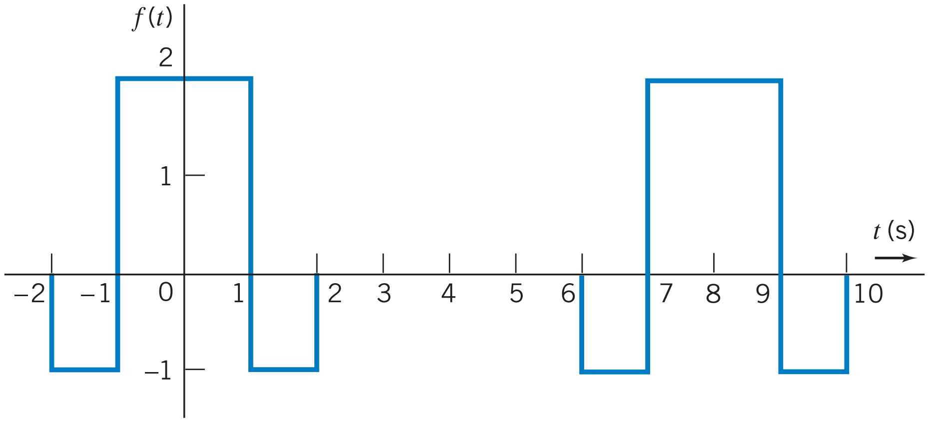


Figure P 15.3-6

Solution:

$T = 8 \text{ s}, \ \omega_0 = \pi/4 \text{ rad/s}$

$b_n = 0$ because $f(t)$ is an even function

$$a_0 = \text{average} = \frac{(2\times2) - 2\times1}{8} = 1/4$$

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos(n\omega_0 t)\,dt$$

$$= \frac{4}{8}\left[\int_0^1 2\cos\left(n\frac{\pi}{4}t\right)dt - \int_1^2 \cos\left(n\frac{\pi}{4}t\right)dt\right]$$

$$= \frac{2}{n\pi}\left[3\sin\frac{n\pi}{4} - \sin\frac{n\pi}{2}\right]$$

$a_1 = .714, \ a_2 = .955, \ a_3 = .662$

P 15.3-7 Determine the Fourier series for $f(t)=\left|A\cos\omega t\right|$.

Solution:

$$\omega_0 = 2\omega,\ T=\frac{\pi}{\omega}$$

$$a_0 = \frac{\omega}{\pi}\int_{\frac{-\pi}{2\omega}}^{\frac{\pi}{2\omega}} A\ \cos\omega t\ dt\ = \frac{2A}{\pi}$$

$$a_n = \frac{2\omega}{\pi}\int_{\frac{-\pi}{2\omega}}^{\frac{\pi}{2\omega}} A\cos(\omega t)\cos(2n\omega t)\ dt = \frac{2\omega A}{2\pi}\int_{\frac{-\pi}{2\omega}}^{\frac{\pi}{2\omega}} \cos((2n-1)\omega t)\cos((2n+1)n\omega t)\ dt$$

$$= \frac{2\omega A}{\pi}\left[\frac{\sin(2n-1)\omega t}{2(2n-1)\omega}+\frac{\sin(2n+1)\omega t}{2(2n+1)\omega}\right]_{-\frac{\pi}{2\omega}}^{\frac{\pi}{2\omega}}$$

$$= \frac{2A}{\pi}\left[\frac{\sin(2n-1)\frac{\pi}{2}}{2n-1}+\frac{\sin(2n+1)\frac{\pi}{2}}{2n+1}\right]$$

$$= \frac{2A}{\pi\left(4n^2-1\right)}\left[(2n+1)\ \sin(2n-1)\ \frac{\pi}{2}-(2n-1)\ \sin(2n-1)\frac{\pi}{2}\right]$$

$$= -\frac{4A}{\pi\left(4n^2-1\right)}\cos(n\pi)\ =\ -\frac{4A(-1)^n}{\pi\left(4n^2-1\right)}$$

$b_n = 0$ since $f(t)$ is an even function.

P 15.3-8 Find the trigonometric Fourier series for the function of Figure P 15.3-8. The function is the positive portion of a cosine wave.

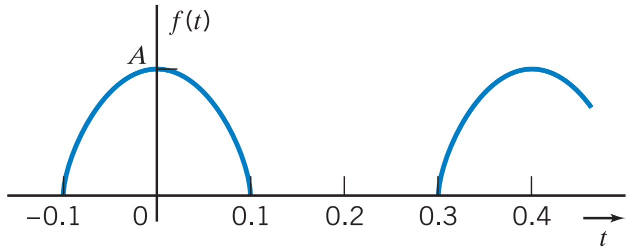


Figure P 15.3-8

Solution:

$$T = 0.4 \text{ s}, \quad \Rightarrow \quad \omega_0 = \frac{2\pi}{T} = 5\pi \ \text{rad/s}$$

$$f(t) = \begin{cases} A\ \cos\ \omega_0 t & 0 \le t \le .1 \\ 0 & .1 \le t < .3 \\ A\ \cos\ \omega_0 t & .3 \le t \le .4 \end{cases}$$

Choose period $-.1 \le t \le .3$ for integral

$$a_0 = \frac{1}{T}\int_{-.1}^{.1} A\cos\ \omega_0 t = A/\pi$$

$$a_n = \frac{2}{T}\int_{-.1}^{.1} A\cos\omega_0 t \ \ \cos\ n\omega_0 t\ dt$$

$$a_1 = 5A\int_{-.1}^{.1}\cos^2\omega_0 t\ dt = \frac{A}{2}$$

$$a_n = 5A\int_{-.1}^{.1}\cos\omega_0 t\ \cos n\omega_0 t\ dt$$

$$= 5A\int_{-.1}^{.1}\frac{1}{2}\left[\cos\ 5\pi(1+n)t + \cos\ 5\pi(1-n)t\right]dt$$

$$= \underline{\frac{2A}{\pi}\ \frac{\cos\ (n\pi/2)}{1-n^2}} \quad n \ne 1$$

$\underline{b_n = 0}$ because the function is even.

P 15.3-9 Determine the Fourier series for $f(t)$ shown in Figure P 15.3-9.
Answer: $a_n = a_0 = 0$; $b_n = 0$ *for even n* $= 8/(n^2\pi^2)$ *for* $n = 1, 5, 9$, and $= -8/(n^2\pi^2)$ *for* $n = 3, 7, 11$

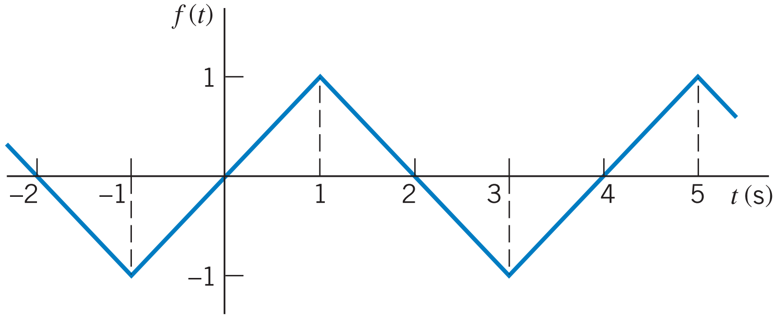


Figure P 15.3-9

Solution:

$$a_0 = 0 \text{ because the average value is zero}$$

$$a_n = 0 \text{ because the function is odd}$$

$$b_n = 0 \text{ for even due to } \frac{1}{4} \text{ wave symmetry}$$

Next:

$$b_n = \int_{-T/4}^{T/4} t \sin\left(n\omega_0 t\right) dt = \frac{8 \sin\left(\frac{n\pi}{2}\right) - 4n\pi \cos\left(\frac{n\pi}{2}\right)}{n^2\pi^2} = \begin{cases} \dfrac{8}{n^2 \pi^2} & \text{for } n = 1,5,9, \ldots \\ -\dfrac{8}{n^2 \pi^2} & \text{for } n = 3,7,11, \ldots \end{cases}$$

P 15.3-10 Determine the Fourier series for the periodic signal shown in Figure P 15.3-10.

Answer: $f(t)=\frac{1}{2}+\frac{2}{\pi}\left(\sin t+\frac{1}{3}\sin\ 3t+\frac{1}{5}\sin\ 5t\ +...\right)$

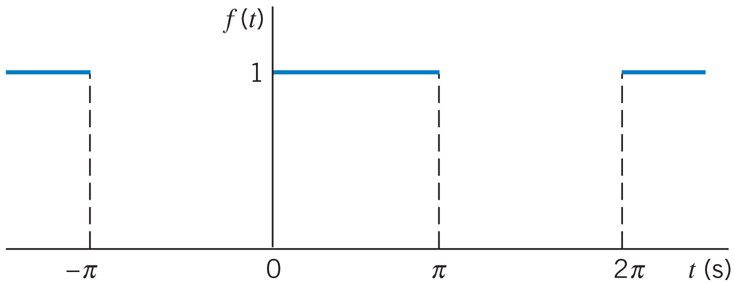


Figure P 15.3-10

Solution:

Refer to Table 15.4-1. Take A = ½ a, a_0= the average value of $f(t)$ = ½, T = 2π so ω_0 = 1 rad/sec.

$$f(t)=\frac{1}{2}+\frac{2}{\pi}\sum_{n=1}^{\infty}\left(\frac{1}{2n-1}\right)\sin\left((2n-1)t\right)=\frac{1}{2}+\frac{2}{\pi}\left(\sin t+\frac{1}{3}\sin 3t+\frac{1}{5}\sin 5t+...\right)$$

Section 15.5 Exponential Form of the Fourier Series

P 15.5-1 Determine the exponential Fourier series of the function

$$f(t) = \left| A \sin(\pi t) \right|$$

shown in Figure P 15.5-1.

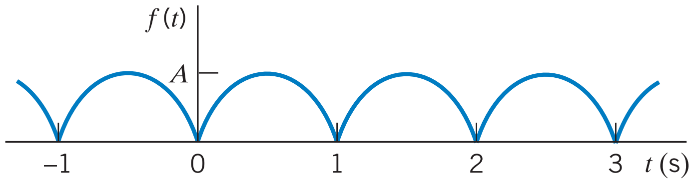


Figure P 15.5-1

Solution:

$T = 1 \quad \Rightarrow \quad \omega_o = \dfrac{2\pi}{1} = 2\pi$, the coefficients of the complex Fourier series are given by:

$$\mathbf{C}_n = \frac{1}{1}\int_0^1 A \sin\left(\pi t\right) e^{-j2\pi n t} dt = \int_0^1 A \left(\frac{e^{j\pi t} - e^{-j\pi t}}{2j}\right) e^{-j2\pi n t} dt$$

$$= \frac{A}{2j}\int_0^1 \left(e^{-j\pi(2n-1)t} - e^{-j\pi(2n+1)t}\right) dt$$

$$= \frac{A}{2j}\left[\frac{e^{-j\pi(2n-1)t}}{-j\pi\left(2n-1\right)} - \frac{e^{-j\pi(2n+1)t}}{-j\pi\left(2n+1\right)}\right]_0^1 = \frac{-2A}{\pi(4n^2-1)}$$

where we have used $e^{\pm j 2\pi n} = 1$ and $e^{j\pi} = e^{-j\pi}$.

P 15.5-2 Determine the exponential Fourier series of the function $f(t)$ shown in Figure P 15.5-2.

Answer: $f(t)=\dfrac{A}{2}+j\dfrac{a}{2\pi}\displaystyle\sum_{\substack{n=-\infty\\n\neq 0}}^{n=\infty}\frac{1}{n}e^{jn2\pi t/T}$

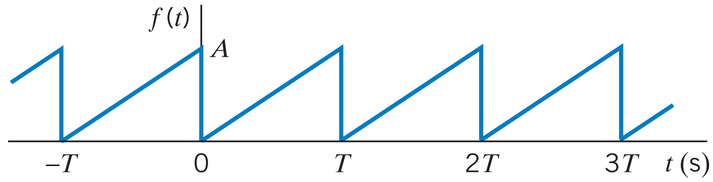


Figure P 15.5-2

Solution:

$$\mathbf{C}_n=\frac{1}{T}\int_0^T\left(\frac{A}{T}t\right)e^{-j\frac{2\pi}{T}nt}\,dt=\frac{A}{T^2}\int_0^T t\,e^{-j\frac{2\pi}{T}nt}\,dt$$

Recall the formula for integrating by parts: $\int_{t_1}^{t_2}u\,dv=u\,v\big|_{t_1}^{t_2}-\int_{t_1}^{t_2}v\,du$. Take $u=t$ and $dv=e^{-j\frac{2\pi}{T}nt}dt$. When $n\neq 0$, we get

$$\mathbf{C}_n=\frac{A}{T^2}\left(\left.\frac{t\,e^{-j\frac{2\pi}{T}nt}}{-j\frac{2\pi}{T}n}\right|_0^T+\frac{1}{j\frac{2\pi}{T}n}\int_0^T e^{-j\frac{2\pi}{T}nt}dt\right)=\frac{A}{T}\left(\frac{T\,e^{-j2\pi n}}{-j2\pi n}+\left.\frac{e^{-j\frac{2\pi}{T}nt}}{\left(\frac{2\pi}{T}n\right)^2}\right|_0^T\right)$$

$$=\frac{A}{T}\left(\frac{T\,e^{-j2\pi n}}{-j2\pi n}+\frac{e^{-j2\pi n}-1}{\left(\frac{2\pi}{T}n\right)^2}\right)=j\frac{A}{2\pi n}$$

Now for $n=0$ we have

$$C_0=\frac{1}{T}\int_0^T\frac{A}{T}t\,dt=\frac{A}{2}$$

Finally,

$$f(t)=\frac{A}{2}+j\frac{A}{2\pi}\sum_{\substack{n=-\infty\\n\neq 0}}^{n=\infty}\frac{1}{n}e^{jn\frac{2\pi}{T}t}$$

P 15.5-3 Determine the exponential Fourier series of the function $f(t)$ shown in Figure P 15.5-3.

Answer: $C_n = \left(\frac{Ad}{T}\right)\frac{\sin\left(\frac{n\pi d}{T}\right)}{\frac{n\pi d}{T}}$

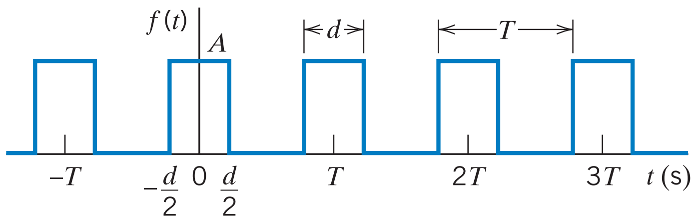


Figure P 15.5-3

Solution:

$$\mathbf{C}_n = \frac{A}{T}\int_{-d/2}^{d/2} e^{-jn\frac{2\pi}{T}t}\,dt = \frac{A}{T}\left[\frac{e^{-jn\frac{2\pi}{T}t}}{-jn\frac{2\pi}{T}}\right]_{-d/2}^{d/2} = \frac{A}{T}\left(\frac{e^{jn\frac{\pi}{T}d}}{jn\frac{2\pi}{T}} - \frac{e^{-jn\frac{\pi}{T}d}}{jn\frac{2\pi}{T}}\right)$$

$$= \frac{A}{n\pi}\left(\frac{e^{jn\frac{\pi}{T}d} - e^{-jn\frac{\pi}{T}d}}{2j}\right)$$

$$= \frac{A}{n\pi}\sin\left(\frac{n\pi d}{T}\right)$$

$$= \left(\frac{Ad}{T}\right)\frac{\sin\left(\frac{n\pi d}{T}\right)}{\frac{n\pi d}{T}}$$

P 15.5-4 Consider two periodic functions, $\hat{f}(t)$ and $f(t)$, that have the same period and are related by

$$\hat{f}(t) = af(t - t_{\mathrm{d}}) + b$$

where a, b, and t_{d} are real constants. Let $\hat{C}_{\mathrm{n}}$ denote the coefficients of the exponential Fourier series of $\hat{f}(t)$ and let C_n denote the coefficients of the exponential Fourier series of $f(t)$. Determine the relationship between $\hat{C}_n$ and C_n.

Answer: $\hat{C}_0 = aC_0 + b$ *and* $\hat{C}_n = ae^{-jn\omega 0td} C_n \quad n \neq 0$

Solution:

$$\widehat{\mathbf{C}_n} = \frac{1}{T}\int_{t_0}^{t_0+T} \left(a\, f\left(t - t_d\right) + b\right) e^{-j\,n\,\omega_o t}\, dt$$

Let $\tau = t - t_d$, then $t = \tau + t_d$.

$$\begin{aligned}
\widehat{\mathbf{C}_n} &= \frac{1}{T}\int_{t_0-t_d}^{t_0+T-t_d} \left(a\, f\left(\tau\right) + b\right) e^{-j\,n\,\omega_o\left(\tau + t_d\right)}\, d\tau \\
&= \frac{1}{T}\int_{t_0-t_d}^{t_0+T-t_d} \left(a\, f\left(\tau\right) + b\right) e^{-j\,n\,\omega_o \tau} e^{-j\,n\,\omega_o t_d}\, d\tau \\
&= \frac{e^{-j\,n\,\omega_o t_d}}{T}\int_{t_0-t_d}^{t_0+T-t_d} \left(a\, f\left(\tau\right) + b\right) e^{-j\,n\,\omega_o \tau}\, d\tau \\
&= \left(a\, e^{-j\,n\,\omega_o t_d}\right)\frac{1}{T}\int_{t_0-t_d}^{t_0+T-t_d} f\left(\tau\right) e^{-j\,n\,\omega_o \tau}\, d\tau + \left(e^{-j\,n\,\omega_o t_d}\right)\frac{1}{T}\int_{t_0-t_d}^{t_0+T-t_d} b\, e^{-j\,n\,\omega_o \tau}\, d\tau
\end{aligned}$$

But $\int_{t_0-t_d}^{t_0+T-t_d} b\, e^{-j\,n\,\omega_o \tau}\, d\tau = b\left[\dfrac{e^{-j\,n\,\omega_o \tau}}{-j\,n\,\omega_o}\right]_{t_0-t_d}^{t_0+T-t_d} = \begin{cases} 0 & n \neq 0 \\ b & = 0 \end{cases}$ so

$$\widehat{C_0} = a\, C_0 + b$$

and

$$\widehat{\mathbf{C}_n} = a\, e^{-j\,n\,\omega_o\, t_d} \mathbf{C}_n \quad n \neq 0$$

***P 15.5-5** Determine the exponential form of the Fourier series for the waveform of Figure P 15.3-6.

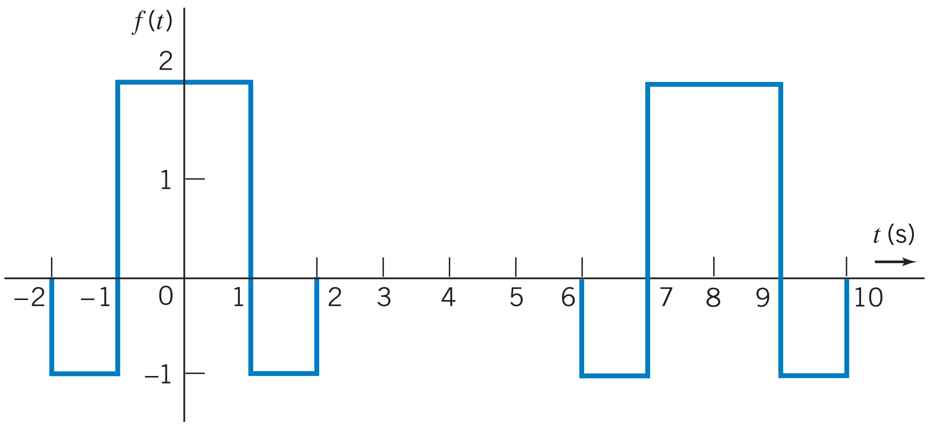


Figure P 15.3-6

Solution:

$$T=8\text{ s},\ \ \omega_0=\frac{2\pi}{T}=\frac{\pi}{4}\text{ rad/s},\ \ C_0=\text{ average value }=\frac{2\times 2-2(1\times 1)}{8}=\frac{1}{4}$$

The coefficients of the exponential Fourier series are calculated as

$$\mathbf{C}_{\text{n}}=\frac{1}{8}\left[\int_{-2}^{-1}-e^{-j\frac{n\pi}{4}t}\,dt+\int_{-1}^{1}2e^{-j\frac{n\pi}{4}t}\,dt+\int_{1}^{2}-e^{-j\frac{n\pi}{4}t}\,dt\right]$$

$$=\frac{1}{8}\left[-\frac{e^{-j\frac{n\pi}{4}t}}{-j\frac{n\pi}{4}}\Bigg|_{-2}^{-1}+2\frac{e^{-j\frac{n\pi}{4}t}}{-j\frac{n\pi}{4}}\Bigg|_{-1}^{1}+-\frac{e^{-j\frac{n\pi}{4}t}}{-j\frac{n\pi}{4}}\Bigg|_{1}^{2}\right]$$

$$=\frac{-j}{2n\pi}\left[\left(e^{j\frac{n\pi}{4}}-e^{j\frac{n\pi}{2}}\right)-2\left(e^{-j\frac{n\pi}{4}}-e^{j\frac{n\pi}{4}}\right)+\left(e^{-j\frac{n\pi}{2}}-e^{-j\frac{n\pi}{4}}\right)\right]$$

$$=\frac{-j}{2n\pi}\left[3\left(e^{j\frac{n\pi}{4}}-e^{-j\frac{n\pi}{4}}\right)-\left(e^{j\frac{n\pi}{2}}-e^{-j\frac{n\pi}{2}}\right)\right]$$

$$=\frac{-j}{2n\pi}\left[3\left(2j\sin\left(\frac{n\pi}{4}\right)\right)-\left(2j\sin\left(\frac{n\pi}{2}\right)\right)\right]=\frac{1}{n\pi}\left[3\sin\left(\frac{n\pi}{4}\right)-\sin\left(\frac{n\pi}{2}\right)\right]$$

and

$$\mathbf{C}_{-\mathrm{n}} = \frac{1}{8}\left[\int_{-2}^{-1} -1\times e^{-j\frac{-n\pi}{4}t}\, dt + \int_{-1}^{1} 2\times e^{-j\frac{-n\pi}{4}t}\, dt + \int_{1}^{2} -1\times e^{-j\frac{-n\pi}{4}t}\, dt\right]$$

$$= \frac{1}{8}\left[-1\times\left.\frac{e^{j\frac{n\pi}{4}t}}{j\frac{n\pi}{4}}\right|_{-2}^{-1} + 2\times\left.\frac{e^{j\frac{n\pi}{4}t}}{j\frac{n\pi}{4}}\right|_{-1}^{1} + (-1)\times\left.\frac{e^{j\frac{n\pi}{4}t}}{j\frac{n\pi}{4}}\right|_{1}^{2}\right]$$

$$= \frac{j}{2n\pi}\left[\left(e^{-j\frac{n\pi}{4}} - e^{-j\frac{n\pi}{2}}\right) - 2\left(e^{j\frac{n\pi}{4}} - e^{-j\frac{n\pi}{4}}\right) + \left(e^{j\frac{n\pi}{2}} - e^{j\frac{n\pi}{4}}\right)\right]$$

$$= \frac{1}{n\pi}\left[\sin\left(\frac{n\pi}{2}\right) - 3\sin\left(\frac{n\pi}{4}\right)\right] = -\mathbf{C}_{\mathrm{n}}$$

The function is represented as

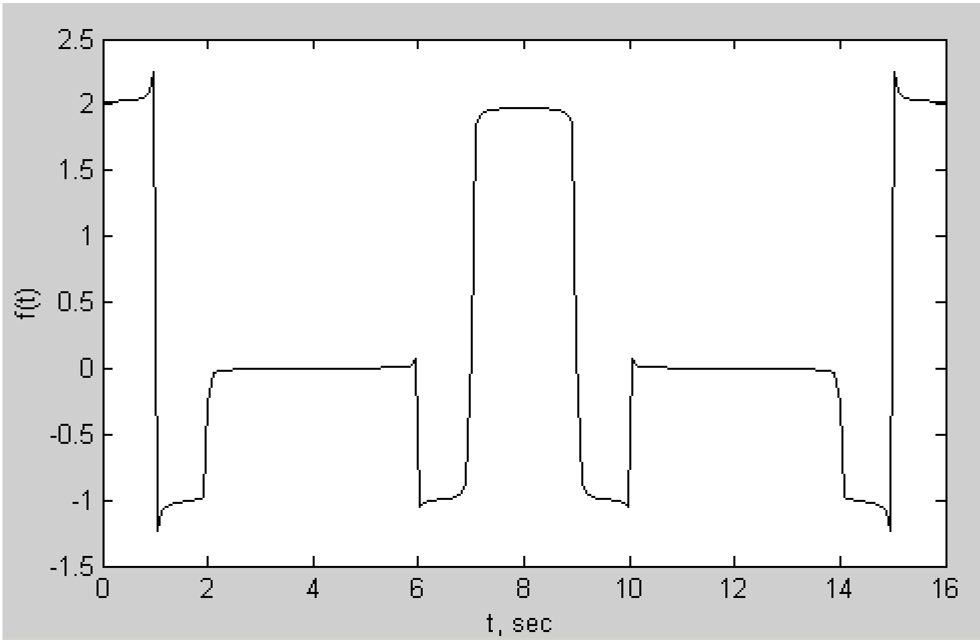
$$f(t) = C_0 + \sum_{n=1}^{\infty}\mathbf{C}_{\mathrm{n}}\; e^{jn\frac{\pi}{4}t} + \sum_{n=-1}^{-\infty}\mathbf{C}_{-\mathrm{n}}\; e^{-jn\frac{\pi}{4}t}$$

$$= \frac{1}{4} + \sum_{n=1}^{\infty}\frac{1}{n\pi}\left[3\sin\left(\frac{n\pi}{4}\right) - \sin\left(\frac{n\pi}{2}\right)\right] e^{jn\frac{\pi}{4}t} + \sum_{n=-1}^{-\infty}\frac{1}{n\pi}\left[\sin\left(\frac{n\pi}{2}\right) - 3\sin\left(\frac{n\pi}{4}\right)\right] e^{-jn\frac{\pi}{4}t}$$

$$= \frac{1}{4} + \sum_{n=1}^{\infty}\frac{2}{n\pi}\left[3\sin\left(\frac{n\pi}{4}\right) - \sin\left(\frac{n\pi}{2}\right)\right] e^{jn\frac{\pi}{4}t}$$

This result can be checked using MATLAB:

```
pi = 3.14159;
N=100;

T = 8;                    % period
t = linspace(0,2*T,200);  % time
c0 = 1/4;                 % average value
w0 = 2*pi/T;              % fundamental frequency

for n = 1: N
  C(n) = -j*((exp(+j*n*pi/4)-exp(+j*n*pi/2))-2*(exp(-j*n*pi/4)-
exp(+j*n*pi/4))+(exp(-j*n*pi/2)-exp(-j*n*pi/4)))/(2*pi*n);
end

for i=1:length(t)
    f(i)=c0;
    for n=1:length(C)
        f(i)=f(i)+C(n)*exp(j*n*w0*t(i))+C(n)*exp(-j*n*w0*t(i));
    end
end

plot(t,f,'black');
xlabel('t, sec');
ylabel('f(t)');
```



2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
f(t)
0
2
4
6
8
10
12
14
16
t, sec

P 15.5-6 Determine the exponential Fourier series for the waveform of Figure P 15.3-6.

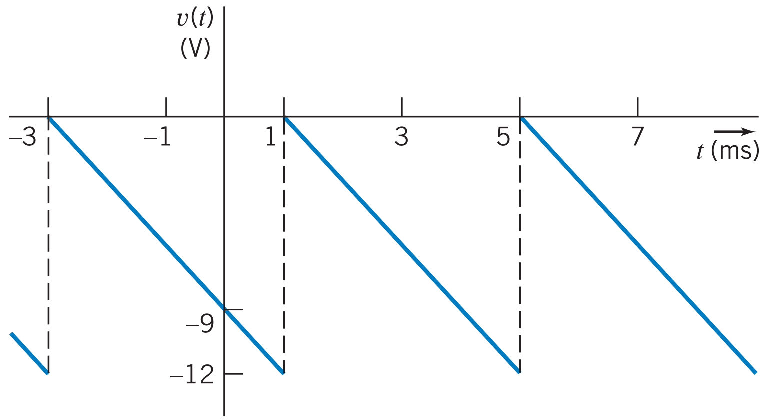


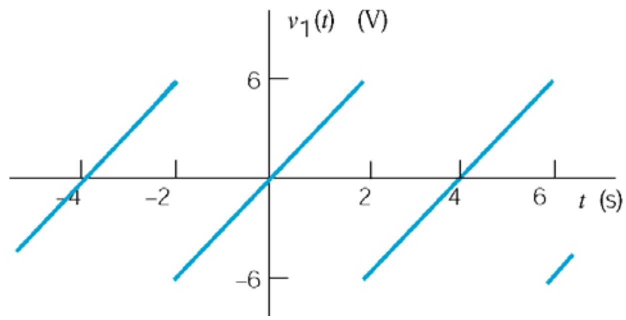
Figure P 15.3-6

Solution:

The function shown at right is related to the given function by

$$v(t) = -v_1(t+1) - 6$$

(Multiply by –1 to flip v_1 upside-down; subtract 6 to fix the average value; replace t by t+1 to shift to the left by 1 s.)



From Table 15.5-1

$$v_1(t) = \sum_{n=-\infty}^{\infty} \frac{j\,A(-1)^n}{n\pi} e^{j n \omega_0 t} = \sum_{n=-\infty}^{\infty} \frac{j\,6(-1)^n}{n\pi} e^{j n \frac{\pi}{2} t}$$

Therefore

$$v(t) = -6 - \sum_{n=-\infty}^{\infty} \frac{j\,6(-1)^n}{n\pi} e^{j n \frac{\pi}{2}(t+1)} = -6 - \sum_{n=-\infty}^{\infty} \left(\frac{j\,6(-1)^n}{n\pi} e^{j n \frac{\pi}{2}} \right) e^{j n \frac{\pi}{2} t}$$

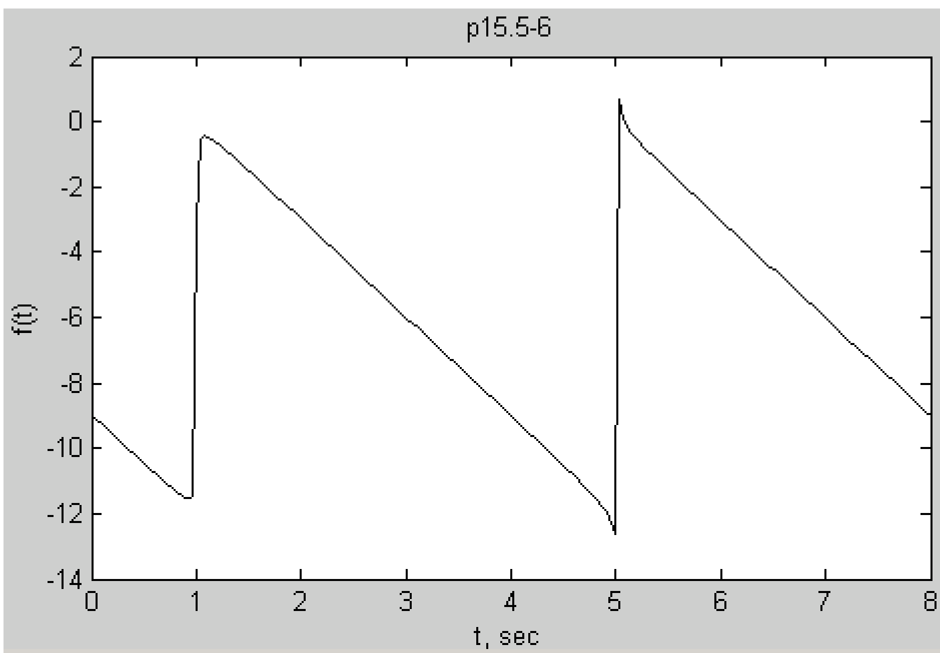
The coefficients of this series are:

$$C_0 = -6 \text{ and } \mathbf{C}_n = -\frac{j\,6(-1)^n}{n\pi} e^{j n \frac{\pi}{2}}$$

This result can be checked using Matlab:

```
pi = 3.14159;
N=100;
A = 6;                      % amplitude
T = 4;                      % period
t = linspace(0,2*T,200);    % time
c0 = -6;                     % average value
```

```
w0 = 2*pi/T;              % fundamental frequency


for n = 1: N
  C(n) = (-j*A*(-1)^n/n/pi)*exp(+j*n*pi/2);
  D(n) = (+j*A*(-1)^n/n/pi)*exp(-j*n*pi/2);
end

for i=1:length(t)
    f(i)=c0;
    for n=1:length(C)
        f(i)=f(i)+C(n)*exp(j*n*w0*t(i))+D(n)*exp(-j*n*w0*t(i));
    end
end

plot(t,f,'black');
xlabel('t, sec');
ylabel('f(t)');
title('p15.5-6')
```



***P 15.5-7** A periodic function consists of rising and decaying exponentials of time constants of 0.2 s each and durations of 1 s each as shown in Figure P 15.5-7. Determine the exponential Fourier series for this function.

Answer: $C_n = \dfrac{5}{(j\pi n)(5+j\pi n)}, n = 1, 3, 5$

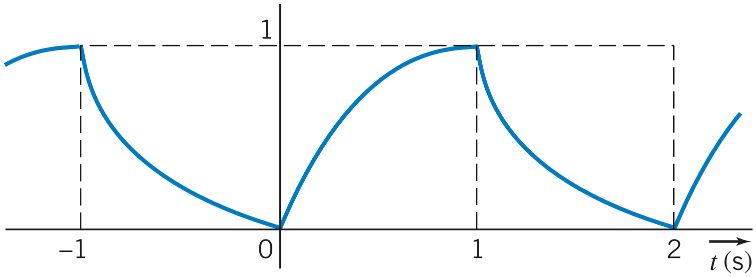


Figure P 15.5-7

Solution:

Represent the function as

$$f(t) = \begin{cases} 1-e^{-5t} & 0 \le t \le 1 \\ e^{-5(t-1)} - e^{-5} & 1 \le t \le 2 \end{cases}$$

(Check: $f(0)=0, \quad f(1)=1-e^{-5} \simeq 1, \quad f(2)=e^{-5}-e^{-5}=0$)

$$T = 2 \text{ s}, \quad \omega_0 = \frac{2\pi}{2} = \pi, \quad \text{also } C_0 = \text{ average value } = \frac{1}{2}$$

The coefficients of the exponential Fourier series are calculated as

$$\mathbf{C}_\text{n} = \frac{1}{2}\left[\int_0^1 \left(1-e^{-5t}\right) e^{-jn\pi t}\, dt + \int_1^2 \left(e^{-5(t-1)} - e^{-5}\right) e^{-jn\pi t}\, dt\right]$$

$$= \frac{1}{2}\left[\left(\int_0^1 e^{-jn\pi t}\, dt - \int_0^1 e^{-5t}\, e^{-jn\pi t}\, dt\right) + \left(e^5 \int_1^2 e^{-(5+jn\pi)t}\, dt - e^{-5}\int_1^2 e^{-jn\pi t}\, dt\right)\right]$$

$$= \frac{1}{2}\left[\left(\left.\frac{e^{-jn\pi t}}{-jn\pi}\right|_0^1 - \left.\frac{e^{-(5+jn\pi)t}}{-(5+jn\pi)}\right|_0^1\right) + \left(e^5 \left.\frac{e^{-(5+jn\pi)t}}{-(5+jn\pi)}\right|_1^2 - e^{-5}\left.\frac{e^{-jn\pi t}}{-jn\pi}\right|_1^2\right)\right]$$

$$= \frac{1}{2}\left[\left(\frac{e^{-jn\pi}-1}{-jn\pi} - \frac{e^{-5}\, e^{-jn\pi}-1}{-(5+jn\pi)}\right) + \left(e^5\, \frac{e^{-(5+jn\pi)2} - e^{-(5+jn\pi)}}{-(5+jn\pi)} - e^{-5}\, \frac{e^{-jn\pi 2} - e^{-jn\pi}}{-jn\pi}\right)\right]$$

$$= \frac{1}{2}\left[\left(\frac{e^{-jn\pi}-1}{-jn\pi} - \frac{e^{-5}\, e^{-jn\pi}-1}{-(5+jn\pi)}\right) + \left(\frac{e^{-5}\, e^{-j2n\pi} - e^{-jn\pi}}{-(5+jn\pi)} - e^{-5}\, \frac{e^{-jn\pi 2} - e^{-jn\pi}}{-jn\pi}\right)\right]$$

$$= \frac{1}{2}\left[\left(\frac{(-1)^n-1}{-jn\pi} - \frac{e^{-5}(-1)^n-1}{-(5+jn\pi)}\right) + \left(\frac{e^{-5}-(-1)^n}{-(5+jn\pi)} - e^{-5}\, \frac{1-(-1)^n}{-jn\pi}\right)\right]$$

The terms that include the factor $e^{-5} = 0.00674$ are small and can be ignored.

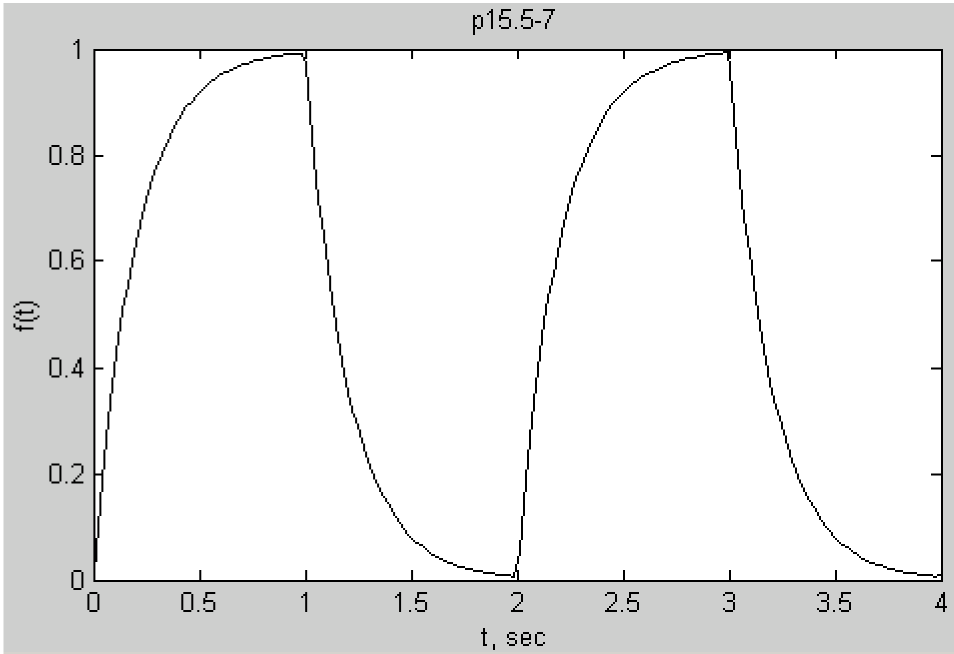
$$\mathbf{C}_\mathrm{n} = \frac{1}{2}\left[\left(\frac{(-1)^n - 1}{-j\,n\,\pi} - \frac{-1}{-(5 + j\,n\,\pi)}\right) + \left(\frac{-(-1)^n}{-(5 + j\,n\,\pi)}\right)\right]$$

$$= \begin{cases} \dfrac{1}{j\,n\,\pi} - \dfrac{1}{5 + j\,n\,\pi} & \text{odd } n \\ 0 & \text{even } n \end{cases}$$

$$= \begin{cases} \dfrac{5}{(j\,n\,\pi)(5 + j\,n\,\pi)} & \text{odd } n \\ 0 & \text{even } n \end{cases}$$

This result can be checked using Matlab:

```
pi = 3.14159;
N=101;
T = 2;                      % period
t = linspace(0,2*T,200);    % time
c0 = 0.5;                     % average value
w0 = 2*pi/T;                % fundamental frequency


for n = 1:2:N
  if n == 2*(n/2)
     C(n) = 5/((+j*pi*n)*(5+j*pi*n));
     D(n) = 5/((-j*pi*n)*(5-j*pi*n));
  else
     C(n)=0;
     D(n)=0
  end
end

for i=1:length(t)
    f(i)=c0;
    for n=1:length(C)
        f(i)=f(i)+C(n)*exp(j*n*w0*t(i))+D(n)*exp(-j*n*w0*t(i));
    end
end

plot(t,f,'black');
xlabel('t, sec');
ylabel('f(t)');
title('p15.5-7')
```


p15.5-7
f(t)
t, sec
1
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
2
2.5
3
3.5
4

Section 15.6 The Fourier Spectrum

P 15.6-1 Determine the cosine-sine Fourier series for the sawtooth waveform shown in Figure P 15.6-1. Draw the Fourier spectra for the first four terms including magnitude and phase.

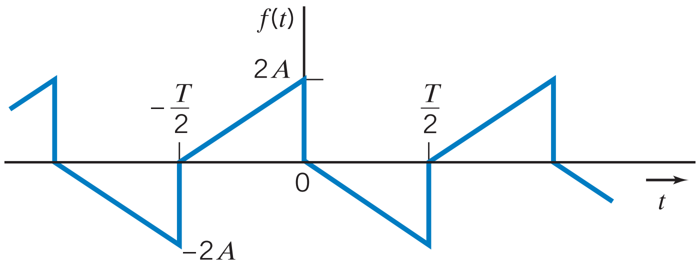


Figure P 15.6-1

Solution:

$$f(t)=\begin{cases}-\dfrac{4A}{T}t & 0\le t<\dfrac{T}{2}\\ -2A+\dfrac{4A}{T}t & \dfrac{T}{2}\le t<T\end{cases},\quad \omega_0=\frac{2\pi}{T},\ \text{Average value}\ =\ 0\ \Rightarrow\ a_0=0$$

The function exhibits half-wave symmetry so the coefficients of the Fourier series are calculated as

$$a_n=b_n=0 \qquad \text{when } n \text{ is even}$$

and

$$a_n=\frac{4}{T}\int_0^{T/2}\underbrace{\left(-\frac{4A}{T}t\right)}_{f(t)}\cos\left(n\frac{2\pi}{T}t\right)dt=-\frac{16A}{T^2}\left[\frac{\cos\left(n\frac{2\pi}{T}t\right)}{\left(n\frac{2\pi}{T}\right)^2}+\frac{t\sin\left(n\frac{2\pi}{T}t\right)}{n\frac{2\pi}{T}}\right]_0^{T/2}$$

$$=-\frac{16A}{T^2}\times\frac{T^2}{(2\pi n)^2}\left(\cos(n\pi)-1\right)$$

$$=\frac{8A}{n^2\pi^2}\qquad \text{when } n \text{ is odd}$$

$$b_n = \frac{4}{T}\int_0^{T/2} \underbrace{\left(-\frac{4A}{T}t\right)}_{f(t)} \sin\left(n\frac{2\pi}{T}t\right)dt = -\frac{16A}{T^2}\left[\frac{\sin\left(n\frac{2\pi}{T}t\right)}{\left(n\frac{2\pi}{T}\right)^2} - \frac{t\cos\left(n\frac{2\pi}{T}t\right)}{n\frac{2\pi}{T}}\right]_{T/2}^{T}$$

$$= \frac{16A}{T^2}\times\frac{T}{2\pi n}\left(T\cos(2n\pi) - \frac{T}{2}\cos(n\pi)\right)$$

$$= \frac{4A}{\pi n} \quad \text{when } n \text{ is odd}$$

so

$$f(t) = \sum_{n=1,3,5...}^{\infty}\left(\frac{8A}{(\pi n)^2}\cos\left(n\frac{2\pi}{T}t\right) + \frac{4A}{\pi n}\sin\left(n\frac{2\pi}{T}t\right)\right)$$

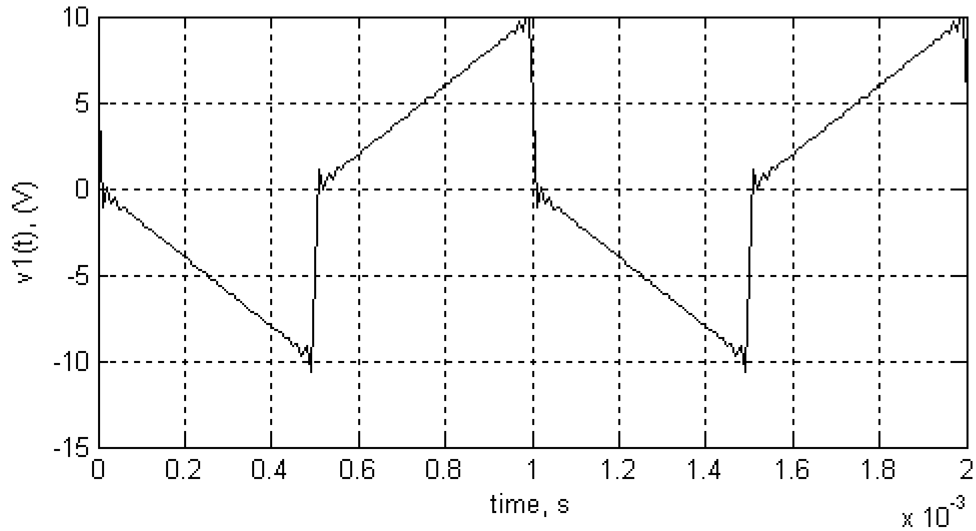
Here are the first 4 nonzero terms of the Fourier spectrum

n	$C_n = \sqrt{a_n^{\ 2} + b_n^{\ 2}}$	$\theta_n = \tan^{-1}\left(\frac{b_n}{a_n}\right)$
1	$1.509 \cdot A$	-57.5°
2	0	0
3	$0.434 \cdot A$	-78.0°
4	0	0
5	$0.257 \cdot A$	-82.7°
6	0	0
7	$0.183 \cdot A$	-84.8°

Check the Fourier Series using MATLAB:

```
% P15_6_1.m Fourier series for problem P15.6-1
pi=3.14159;
A=5;                   % input waveform parameters
T=0.001;               % period

w0=2*pi/T;             % fundamental frequency, rad/s
tf=2*T;                % final time
dt=tf/200;             % time increment
t=0:dt:tf;             % time, s

a0=0;                  % avarage value of input
v1=0*t+a0;             % initialize input as vector
H0=-R2/R1;             % dc gain of the circuit

for n=1:2:51           % for each term in the Fourier series ...
   an=8*A/pi/n/pi/n;             % specify coefficients of the input series
```

```
   bn=4*A/pi/n;
   cn=sqrt(an*an + bn*bn);      % convert to magnitude and angle form
   thetan=atan2(bn,an);
   v1=v1+cn*cos(n*w0*t+thetan); % add the next term of the input Fourier
series
end

plot(t, v1)        % plot the Fourier series

axis([0 tf -15 10])
grid
xlabel('time, s')
ylabel('v1(t), (V)')
```



P 15.6-2 The load current waveform of the variable-speed motor drive depicted in Figure 15.16-1*c* is shown in Figure 15.6-2. The current waveform is a portion of $A \sin \omega_0 t$. Determine the Fourier series of this waveform, and draw the line spectra of $|C_n|$ for the first 10 terms.

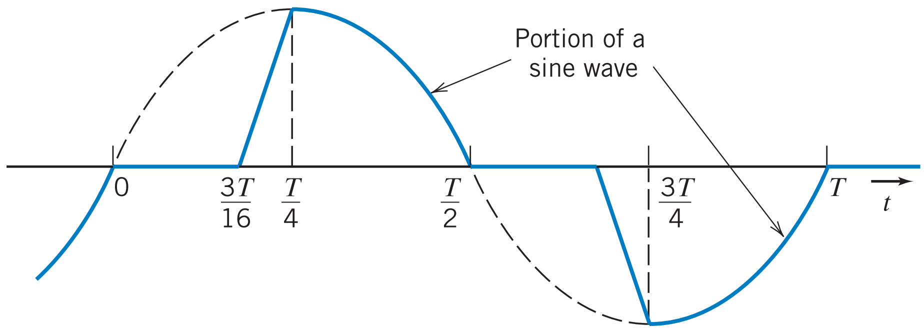


Figure 15.6-2

Solution: Mathcad spreadsheet (p15_6_2.mcd):

$N := 100 \qquad n := 1, 2 .. N \qquad T := 32 \qquad \omega 0 := 2\frac{\pi}{T}$

Calculate the coefficients of the exponential Fourier series:

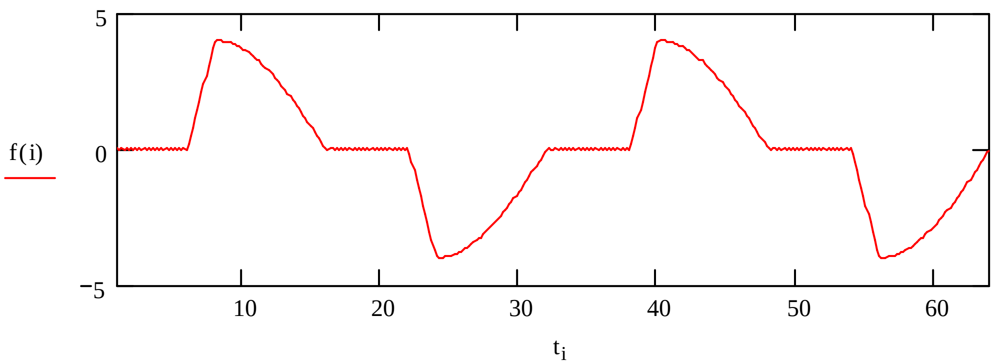
$$C1_n := \frac{4}{T} \cdot \int_{3 \cdot \frac{T}{16}}^{\frac{T}{4}} \left(16 \cdot \frac{t}{T} - 3\right) \exp(-j \cdot n \cdot \omega 0 \cdot t)\, dt \qquad C2_n := \frac{4}{T} \cdot \int_{\frac{T}{4}}^{\frac{T}{2}} \sin\left(\frac{2 \cdot \pi}{T} \cdot t\right) \exp(-j \cdot n \cdot \omega 0 \cdot t)\, dt$$

$$C3_n := \frac{4}{T} \cdot \int_{11 \cdot \frac{T}{16}}^{\frac{3 \cdot T}{4}} \left(11 - \frac{16 \cdot t}{T}\right) \exp(-j \cdot n \cdot \omega 0 \cdot t)\, dt \qquad C4_n := \frac{4}{T} \cdot \int_{3 \cdot \frac{T}{4}}^{T} \sin\left(\frac{2 \cdot \pi}{T} \cdot t\right) \exp(-j \cdot n \cdot \omega 0 \cdot t)\, dt$$

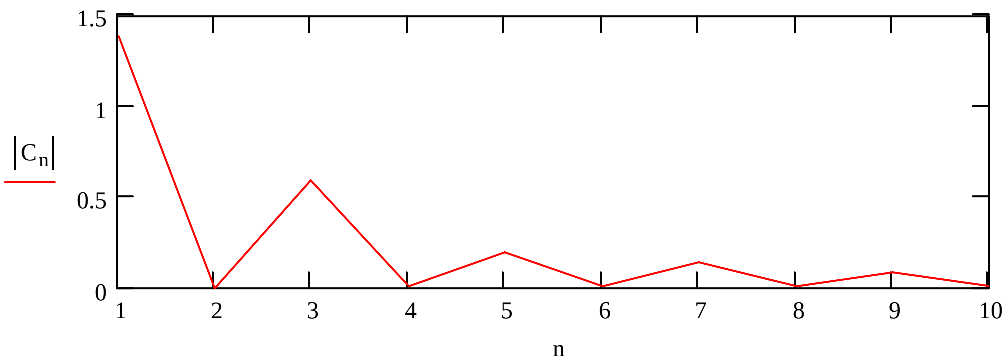
$C_n := C1_n + C2_n + C3_n + C4_n$

Check: Plot the function using it's exponential Fourier series:

$d := \frac{T}{200} \qquad i := 1, 2 .. 400 \qquad t_i := d \cdot i \qquad f(i) := \sum_{n=1}^{N} C_n \cdot \exp(j \cdot n \cdot \omega0 \cdot t_i) + \sum_{n=1}^{N} \overline{C_n} \cdot \exp(-j \cdot n \cdot \omega0 \cdot t_i)$



Plot the magnitude spectrum:



That's not a very nice plot. Here are the values of the coefficients:

$\left|C_n\right| =$

1.385
0
0.589
0
0.195
0
0.139
0
0.082
0
0.039
0
0.027
0
$1.226 \cdot 10^{-3}$
0

$\arg(C_n) \cdot \frac{180}{\pi} =$

-115.853
-90
22.197
-24.775
-113.34
106.837
66.392
-78.232
-69.062
-48.814
109.584
90.415
-25.598
78.14
63.432
163.724

P 15.6-3 The input to a low-pass filter is

$$v_i(t) = 10 \cos t + 10 \cos 10t + 10 \cos 100t \text{ V}$$

The output of the filter is the voltage $v_o(t)$. The network function of the low-pass filter is

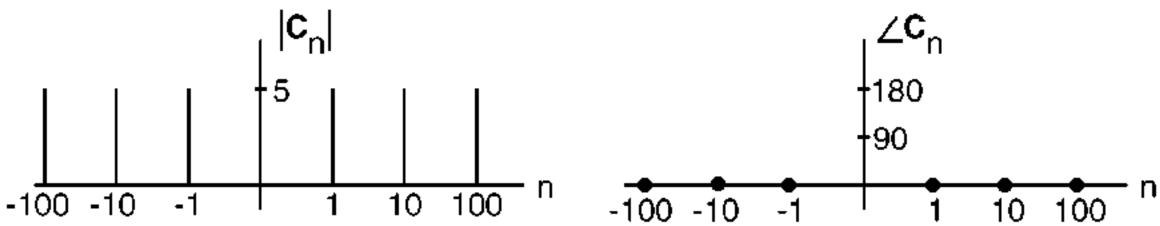
$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{2}{\left(1 + j\frac{\omega}{5}\right)^2}$$

Plot the Fourier spectrum of the input and the output of the low-pass filter.

Solution:
Use Euler's formula to convert the trigonometric series of the input to an exponential series:

$$v_i(t) = 10 \cos t + 10 \cos 10t + 10 \cos 100t \text{ V}$$
$$= 10\frac{e^{t} + e^{-t}}{2} + 10\frac{e^{10t} + e^{-10t}}{2} + 10\frac{e^{100t} + e^{-100t}}{2} = 5e^{-100t} + 5e^{-10t} + 5e^{-t} + 5e^{t} + 5e^{10t} + 5e^{100t}$$

The corresponding Fourier spectrum is:



The network function of the lowpass filter is

$$\mathbf{H}(\omega) = \frac{2}{\left(1 + j\frac{\omega}{5}\right)^2}$$

Evaluating the network function at the frequencies of the input:

ω, rad/s	$\lvert\mathbf{H}(\omega)\rvert$	$\angle\, \mathbf{H}(\omega)$, °
1	1.923	-23
10	0.400	-127
100	0.005	-174

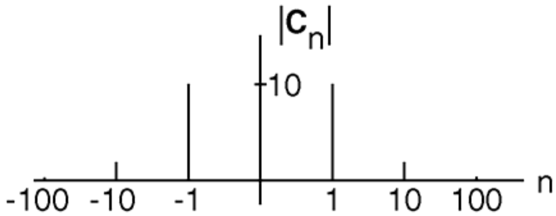
Using superposition:

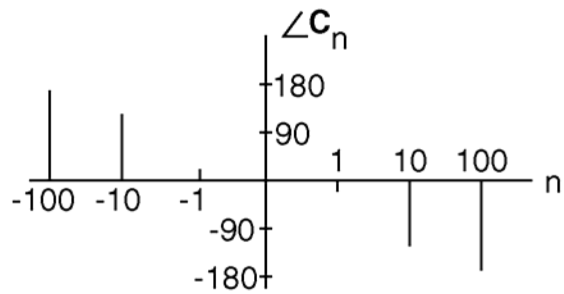
$$v_o(t) = 19.23 \cos(t - 23°) + 4.0 \cos(10t - 127°) + 0.05 \cos(100t - 174°) \text{ V}$$

Use Euler's formula to convert the trigonometric series of the output to an exponential series:

$$v_o(t) = 19.23\,\frac{e^{j(t-23°)} + e^{-j(t-23°)}}{2} + 4.0\,\frac{e^{j(10t-127°)} + e^{-j(10t-127°)}}{2} + 0.05\,\frac{e^{j(100t-174°)} + e^{-j(100t-174°)}}{2} \text{ V}$$

$$= 0.025 e^{j174°} e^{-j100t} + 2.0\, e^{j127°} e^{-j10t} + 9.62 e^{j23°} e^{-jt} + 9.62 e^{-j23°} e^{jt} + 2.0\, e^{-j127°} e^{j10t} + 0.025 e^{-j174°} e^{j100t}$$





P 15.6-4 Draw the Fourier spectra for the waveform shown in Figure P 15.6-4.

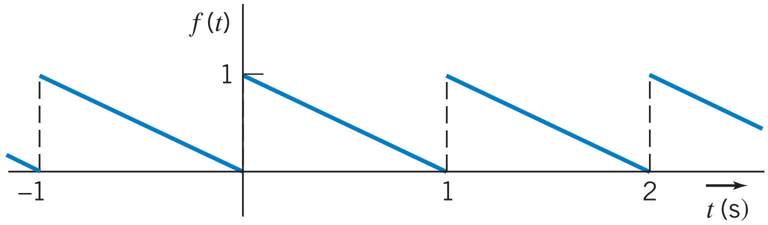


Figure P 15.6-4.

Solution:

$$T = 1\text{ s},\ \ \omega_0 = \frac{2\pi}{T} = 2\pi\ \text{ rad/s},\ \ C_0 = \frac{1}{2}$$
$$f(t) = 1 - t \qquad \text{when } 0 \le t < 1\text{ s}$$

The coefficients of the exponential Fourier series are given by

$$\mathbf{C}_n = \frac{1}{1}\int_0^1 (1-t)\,e^{-j2\pi nt}dt = \int_0^1 e^{-j2\pi nt}\,dt - \int_0^1 t\,e^{-j2\pi nt}\,dt$$

Evaluate the first integral as

$$\int_0^1 e^{-j2\pi nt}\,dt = \left.\frac{e^{-j2\pi nt}}{-j2\pi nt}\right|_0^1 = \frac{e^{-j2\pi n} - 1}{-j2\pi n} = 0$$

To evaluate the second integral, recall the formula for integrating by parts:

$\int_{t_1}^{t_2} u\,dv = u\,v\big|_{t_1}^{t_2} - \int_{t_1}^{t_2} v\,du$. Take $u = t$ and $dv = e^{-j2\pi nt}dt$. Then

$$\int_0^1 t\,e^{-j2\pi nt}\,dt = \left.\frac{t\,e^{-j2\pi nt}}{-j2\pi n}\right|_0^1 - \frac{1}{j2\pi n}\int_0^1 e^{-j2\pi nt}dt$$

$$= \frac{e^{-j2\pi n}}{-j2\pi n} + \left.\frac{e^{-j2\pi nt}}{(j2\pi n)^2}\right|_0^1 = \frac{e^{-j2\pi n}}{-j2\pi n} + \frac{e^{-j2\pi n} - 1}{(j2\pi n)^2} = j\frac{1}{2\pi n}$$

Therefore

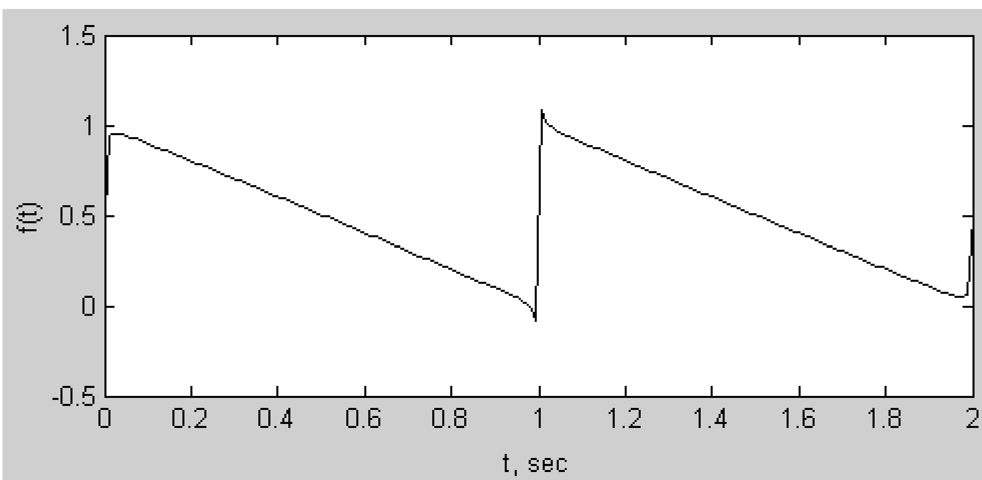
$$C_n = \begin{cases} \dfrac{1}{2} & n = 0 \\ \dfrac{-j}{2\pi n} & n \ne 0 \end{cases}$$

To check these coefficients, represent the function by it's Fourier series:

$$f(t) = \frac{1}{2} + \sum_{n=1}^{n=\infty} \left(\frac{-j}{2\pi n} e^{j2\pi nt} + \frac{j}{2\pi n} e^{-j2\pi nt} \right)$$

Next, use Matlab to plot the function from its Fourier seris (p15_6_4check.m):

```
pi = 3.14159;
N=20;

T = 1;                      % period
t = linspace(0,2*T,200);    % time
c0 = 1/2;                   % average value
w0 = 2*pi/T;                % fundamental frequency


for n = 1: N
  C(n) = -j/(2*pi*n);
end

for i=1:length(t)
    f(i)=c0;
    for n=1:length(C)
        f(i)=f(i)+C(n)*exp(j*n*w0*t(i))-C(n)*exp(-j*n*w0*t(i));
    end
end

plot(t,f,'black');
xlabel('t, sec');
ylabel('f(t)');
```

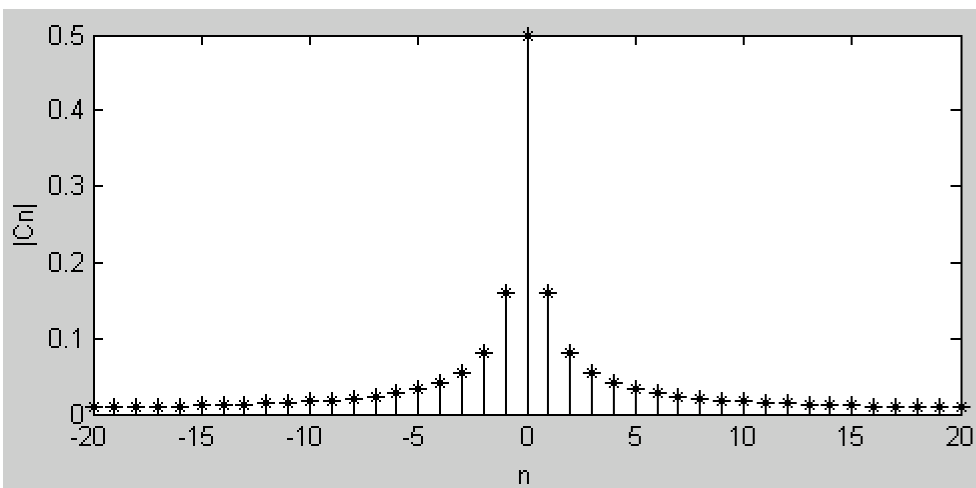


This plot agrees with the given function, so we are confident that the coefficients are correct. The magnitudes of the coefficients of the exponential Fourier series are:

$$|\mathbf{C}_n| = \begin{cases} \dfrac{1}{2} & n = 0 \\ \dfrac{1}{2\pi n} & n \neq 0 \end{cases}$$

Finally, use the “stemplot” in Matlab to plot the Fourier spectrum (p15_6_4spectrum.m):

```
pi = 3.14159;
N=20;
n = linspace(-N,N,2*N+1);

Cn = abs(1/(2*pi)./n); % Division by 0 when n=0 causes Cn(N+1)= NaN.
Cn(N+1)=1/2;       % Fix Cn(N+1); C0=1/2

% Plot the spectrum using a stem plot
stem(n,Cn,'-*k');
xlabel('n');
ylabel('|Cn|');
```



Section 15.7 Circuits and Fourier Series

P 15.7-1 Determine the steady-state response, $v_o(t)$, for the circuit shown in Figure P 15.7-1. The input to this circuit is the voltage $v_c(t)$ shown in Figure P 15.3-2.

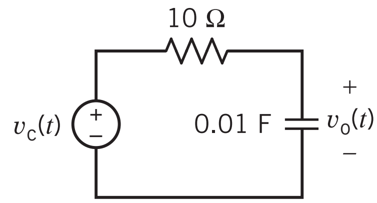


Figure P 15.7-1

Answer: $v_0(t) = -6 + \sum_{n=1}^{\infty} \frac{240}{n\pi\sqrt{400+n^2\pi^2}} \sin\left(n\frac{\pi}{2}t - \left(n\frac{\pi}{2} + \tan^{-1}\left(\frac{n\pi}{20}\right)\right)\right)$

Solution:

The network function of the circuit is:

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{\dfrac{100}{j\omega}}{10+\dfrac{100}{j\omega}} = \frac{1}{1+j\dfrac{\omega}{10}}$$

Evaluating the network function at the harmonic frequencies:

$$\mathbf{H}\left(n\frac{\pi}{2}\right) = \frac{1}{1+j\dfrac{n\pi}{20}} = \frac{20}{20+jn\pi} = \frac{20}{\sqrt{400+n^2\pi^2}} \angle -\tan^{-1}\left(\frac{n\pi}{20}\right)$$

From problem P15.4-2, the Fourier series of the input voltage is

$$v_c(t) = -6 + \sum_{n=1}^{\infty} \frac{12}{n\pi} \sin\left(n\frac{\pi}{2}t - n\frac{\pi}{2}\right)$$

Using superposition, the Fourier series of the output voltage is

$$v_o(t) = -6 + \sum_{n=1}^{\infty} \frac{240}{n\pi\sqrt{400+n^2\pi^2}} \sin\left(n\frac{\pi}{2}t - \left(n\frac{\pi}{2} + \tan^{-1}\left(\frac{n\pi}{20}\right)\right)\right)$$

P 15.7-2 Determine the steady-state response, $v_o(t)$, for the circuit shown in Figure P 15.7-2. The input to this circuit is the voltage $v_b(t)$, shown in Figure P 15.4-4.

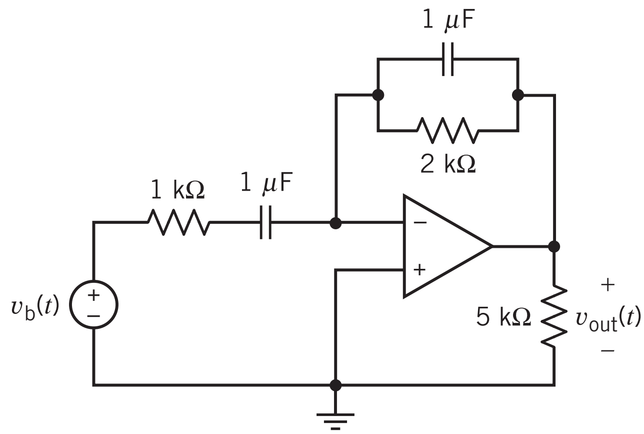


Figure P 15.7-2

Solution: The network function of the circuit is:

$$\mathbf{H}(\omega)=\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}=-\frac{\dfrac{R_2}{1+j\omega C_2R_2}}{R_1+\dfrac{1}{j\omega C_1}}=-\frac{j\omega C_1R_2}{(1+j\omega C_1R_1)(1+j\omega C_2R_2)}$$

$$=-\frac{j\omega(10^{-6})(2000)}{\left(1+j\omega(10^{-6})(1000)\right)\left(1+j\omega(10^{-6})(2000)\right)}$$

$$=-\frac{j\dfrac{\omega}{500}}{\left(1+j\dfrac{\omega}{1000}\right)\left(1+j\dfrac{\omega}{500}\right)}$$

Evaluating the network function at the harmonic frequencies:

$$\mathbf{H}\left(n\frac{1000\pi}{3}\right)=-\frac{jn\dfrac{2\pi}{3}}{\left(1+jn\dfrac{\pi}{3}\right)\left(1+jn\dfrac{2\pi}{3}\right)}$$

From problem 15.3-4, the Fourier series of the input voltage is

$$v_b(t)=-\frac{1}{2}+\sum_{n=1}^{\infty}\frac{18}{n^2\pi^2}\left(1-\cos\left(\frac{n\pi}{3}\right)\right)\cos\left(n\frac{1000\pi}{3}t-n\frac{2\pi}{3}\right)$$

Using superposition, the Fourier series of the output voltage is

$$v_b(t)=\sum_{n=1}^{\infty}\frac{18\times\left|\mathbf{H}\left(n\dfrac{1000\pi}{3}\right)\right|}{n^2\pi^2}\left(1-\cos\left(\frac{n\pi}{3}\right)\right)\cos\left(n\frac{1000\pi}{3}t-n\frac{2\pi}{3}+\angle\mathbf{H}\left(n\frac{1000\pi}{3}\right)\right)$$

P 15.7-3 The input to the circuit shown in Figure P 15.7-3 is the voltage of the voltage source

$$v_{in}(t) = 2 + 4\cos(100t) + 5\cos(400t + 45^\circ)\ \text{V}$$

The output is the voltage across the 5-kω resistor

$$v_{out} = -5 + 7.071\cos(100t + 135^\circ) + c_4\cos(400t + \theta_4)\ \text{V}$$

Determine the values of the resistance, R, the capacitance, C, the coefficient, c_4, and the phase angle, θ_4.

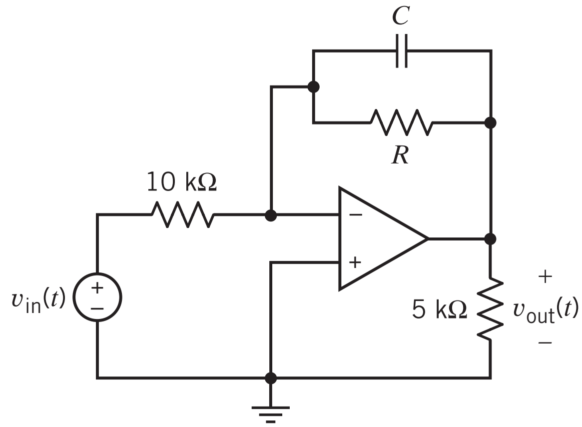


Figure P 15.7-3

Answer: $R = 25$ kΩ; $C = 0.4\ \mu$F, $c_4 = 3.032$ V, and $\theta_4 = 149^\circ$

Solution:

$$\mathbf{H}(\omega) = \frac{\mathbf{V_o}(\omega)}{\mathbf{V_i}(\omega)} = \frac{H_o}{1 + j\dfrac{\omega}{p}}$$

When $\omega = 0$ (dc)

$$-5 = -\frac{R}{10^4}(2) \quad \Rightarrow \quad R = 25\ \text{k}\Omega$$

When $\omega = 100$ rad/s

$$135^\circ = \angle \mathbf{H}(\omega) = 180^\circ - \tan^{-1}(\omega C R) \quad \Rightarrow \quad \tan(45^\circ) = (100)C(25000)$$
$$\Rightarrow \quad C = 0.4\ \ \mu\text{F}$$

$$c_4 = (5)\left|\mathbf{H}(400)\right| = (5)\left|\frac{\dfrac{25000}{10^4}}{1 + j(400)(0.4\times10^{-6})(25000)}\right| = 3.032$$

$$\theta_4 = 45^\circ + \angle \mathbf{H}(400) = 45^\circ + 180 - \tan^{-1}\left(400\times0.4\times10^{-6}\times25000\right) = 149^\circ$$

P 15.7-4 The input to a circuit is the voltage

$$v_i(t) = 2 + 4\cos(25t) + 5\cos(100t + 45^\circ)\ \text{V}$$

The output is the voltage

$$v_o(t) = 5 + 7.071\cos(25t - 45^\circ) + c_4\cos(\omega_4 t + \theta_4)\ \text{V}$$

The network function that represents this circuit is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{H_o}{1 + j\dfrac{\omega}{p}}$$

Determine the values of the dc gain, H_o, the pole, p, the coefficient, c_4, and the phase angle, θ_4.

Answer: $H_o = 2.5$ V/V, $p = 25$ rad/s, $c_4 = 3.032$ V, and $\theta_4 = -31^\circ$

Solution:

When $\omega = 0$ (dc)

$$5 = H_o(2) \quad \Rightarrow \quad H_o = 2.5\ \text{V/V}$$

When $\omega = 25$ rad/s

$$-45^\circ = \angle\mathbf{H}(\omega) = -\tan^{-1}\left(\frac{\omega}{p}\right) \quad \Rightarrow \quad \tan(45^\circ) = \frac{25}{p} \quad \Rightarrow \quad p = 25\ \text{rad/s}$$

$$c_4 = (5)\left|\mathbf{H}(100)\right| = (5)\left|\frac{2.5}{1 + j\dfrac{100}{25}}\right| = 3.03$$

$$\theta_4 = 45^\circ + \angle\mathbf{H}(100) = 45^\circ - \tan^{-1}\left(\frac{100}{25}\right) = -31^\circ$$

P 15.7-5 The input to this circuit in Figure P 15.7-5 is the voltage of the independent voltage source

$v_i(t) = 6 + 4 \cos(1000t) + 5 \cos(3000t + 45°)$ V

The output is the voltage across a 500-Ω resistor

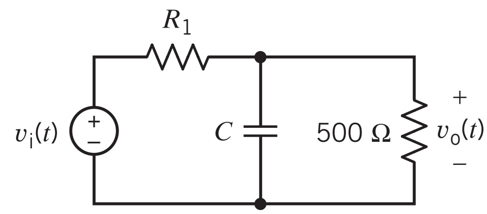


Figure P 15.7-5

$$v_o(t) = 3.75 + 2.34 \cos(1000t - 20.5°) + c_3 \cos(3000t + \theta_3) \text{ V}$$

Determine the values of the resistance, R_1, the capacitance, C, the coefficient, c_3, and the phase angle, θ_3.

Answer: $R_1 = 300\ \Omega$, $C = 2\ \mu$F, $c_3 = 2.076$ V, and $\theta_3 = -3.4°$

Solution:

$$\mathbf{H}(\omega) = \frac{\mathbf{V_o}(\omega)}{\mathbf{V_i}(\omega)} = \frac{R_2}{R_1 + R_2 + j\omega C R_1 R_2}$$

When $\omega = 0$ (dc)

$$\frac{R_2}{R_1 + R_2} = \frac{3.75}{6} \quad \Rightarrow \quad R_1 = \left(\frac{2.25}{3.75}\right) R_2 = \left(\frac{2.25}{3.75}\right)(500) = 300\ \Omega$$

When $\omega = 1000$ rad/s

$$-20.5° = \angle \mathbf{H}(\omega) = -\tan^{-1}\left(\omega C \frac{R_1 R_2}{R_1 + R_2}\right) \quad \Rightarrow \quad \tan(20.5°) = (1000) C \left(\frac{(300)(500)}{800}\right)$$

$$\Rightarrow \quad C = 2\ \mu\text{F}$$

$$c_3 \angle \theta_3 = \left(\frac{500}{800 + j(3000)(2 \times 10^{-6})(500)(300)}\right)(5\angle 45°) = 2.076\angle -3.4$$

P 15.7-6 Find the steady-state response for the output voltage, v_o, for the circuit of Figure P 15.7-6 when $v(t)$ is as described in Figure 15.5-6.

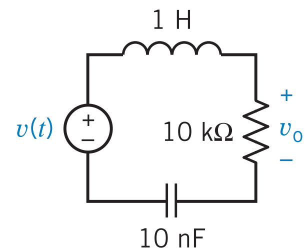
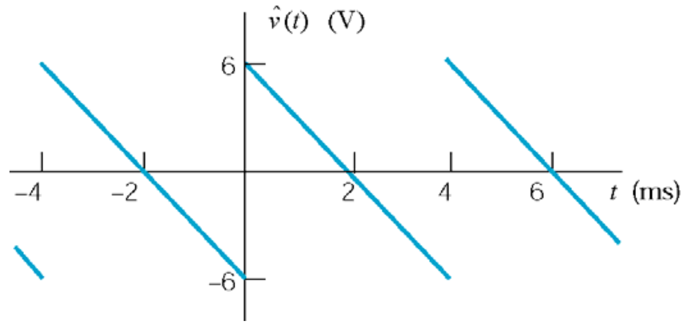


Figure P 15.7-6

Solution:



Rather than find the Fourier Series of $v(t)$ directly, consider the signal $\hat{v}(t)$ shown above. These two signals are related by

$$v(t) = \hat{v}(t-1) - 6$$

since $v(t)$ is <u>delayed</u> by 1 ms and shifted down by 6 V.

The Fourier series of $\hat{v}(t)$ is obtained as follows:

$$T = 4 \text{ ms} \quad \Rightarrow \quad \omega_0 = \frac{2\pi \text{ radians}}{4 \text{ ms}} = \frac{\pi}{2} \text{rad/ms}$$

$$\hat{a}_n = 0 \text{ because the average value of } \hat{v}(t) = 0$$

$$\hat{b}_n = \frac{1}{2}\int_0^4 (6-3t)\sin\left(n\frac{\pi}{2}t\right)dt \quad \text{because } \hat{v}(t) \text{ is an odd function.}$$

$$= 3\int_0^4 \sin\left(n\frac{\pi}{2}t\right)dt - \frac{3}{2}\int_0^4 t\sin\left(n\frac{\pi}{2}t\right)dt$$

$$= 3\frac{-\cos\left(n\frac{\pi}{2}t\right)}{n\frac{\pi}{2}}\Bigg|_0^4 - \frac{3}{2}\left[\left(\frac{1}{\frac{n^2\pi^2}{4}}\right)\sin\left(n\frac{\pi}{2}t\right) - \left(\frac{n\pi}{2}t\right)\cos\left(n\frac{\pi}{2}t\right)\right]_0^4$$

$$= \frac{6}{n\pi}\left(-1+\cos(2n\pi)\right) - \frac{6}{n^2\pi^2}\left(\left(\sin(2n\pi)-0\right) - \left(2n\pi - \cos(2\pi) - 0\right)\right) = \frac{12}{n\pi}$$

Finally, $$\hat{v}(t) = \sum_{n=1}^{\infty} \frac{12}{n\pi} \sin n\frac{\pi}{2} t$$

The Fourier series of $v(t)$ is obtained from the Fourier series of $\hat{v}(t)$ as follows:

$$v(t) = -6 + \sum_{n=1}^{\infty} \frac{12}{n\pi} \sin n\frac{\pi}{2}(t-1) = -6 + \sum_{n=1}^{\infty} \frac{12}{n\pi} \sin\left(n\frac{\pi}{2} t - n\frac{\pi}{2} \right)$$

where t is in ms. Equivalently,

$$v(t) = -6 + \frac{12}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin\left(n\frac{\pi}{2} 10^3 t - n\frac{\pi}{2} \right)$$

where t is in s.

Next, the transfer function of the circuit is $H(s) = \dfrac{R}{\dfrac{1}{Cs} + Ls + R} = \dfrac{\dfrac{R}{L}s}{s^2 + \dfrac{R}{L}s + \dfrac{1}{LC}}$.

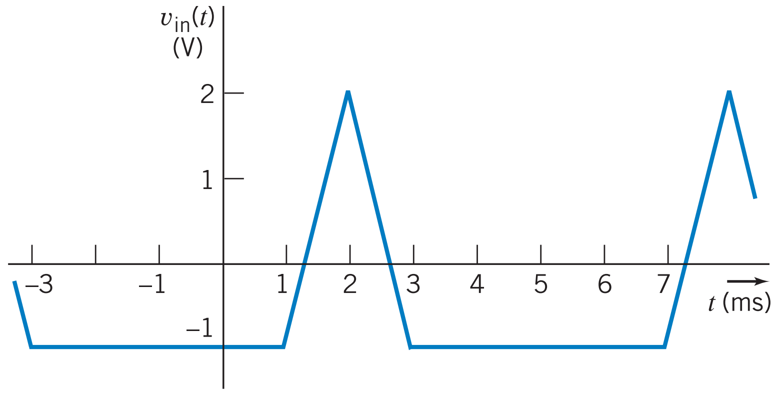
The network function of the circuit is $\mathbf{H}(\omega) = \dfrac{j\omega\dfrac{R}{L}}{\left(\dfrac{1}{LC} - \omega^2\right) + j\omega\dfrac{R}{L}} = \dfrac{10^4\, j\omega}{\left(10^8 - \omega^2\right) + 10^4\, j\omega}$.

We see that $\mathbf{H}(0) = 0$ and

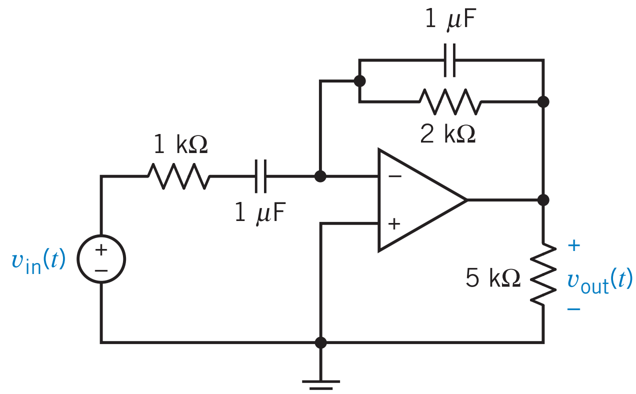
$$\mathbf{H}(n\,\omega_0) = \mathbf{H}\left(n\frac{\pi}{2} 10^3 \right) = \frac{j20n\pi}{\left(400 - n^2\pi^2\right) + j20n\pi} = \frac{1}{\sqrt{\left(400 - n^2\pi^2\right)^2 + 400n^2\pi^2}}\, e^{j\left(90 - \tan^{-1}\frac{20n\pi}{400 - n^2\pi^2}\right)}$$

Finally, $$v_0(t) = \frac{12}{\pi} \sum_{n=1}^{\infty} \frac{\sin\left(n\frac{\pi}{2} 10^3 t - n\frac{\pi}{2} + 90^\circ - \tan^{-1}\left(\frac{20n\pi}{400 - n^2\pi^2} \right) \right)}{n\sqrt{\left(400 - n^2\pi^2\right)^2 + 400n^2\pi^2}}$$

P 15.7-7 Determine the value of the voltage $v_o(t)$ at $t = 4$ ms when v_{in} is shown in Figure 15.7-7*a* and the circuit is shown in Figure P 15.7-7*b*.



(a)



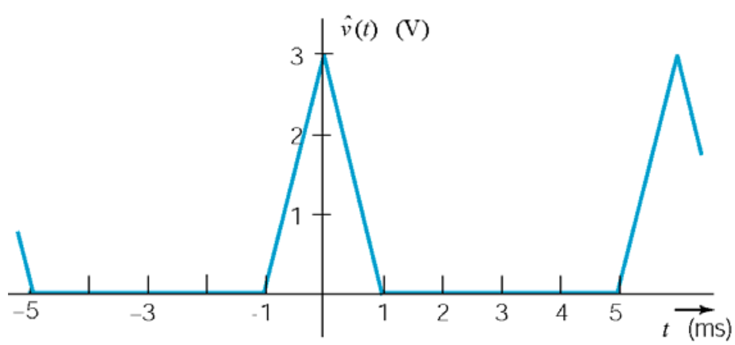
(b)

Figure P 15.7-7

Solution:

Rather than find the Fourier Series of $v(t)$ directly, consider the signal $\hat{v}(t)$ shown below.

These two signals are related by $v(t) = \hat{v}(t-2)-1$



Let's calculate the Fourier Series of $\hat{v}(t)$, taking advantage of its symmetry.

$$T = 6\text{ ms} \quad \Rightarrow \quad \omega_0 = \frac{2\pi\text{ rad}}{6\text{ ms}} = \frac{\pi}{3}\text{ rad/ms}$$

$$a_o = \text{average value of } \hat{v}(t) = \frac{\frac{3.2}{2}}{6} = \frac{1}{2}\text{ V}$$

$b_n = 0$ because $\hat{v}(t)$ is an even function

$$a_n = 2\left(\frac{2}{6}\int_0^1 (3-3t)\cos n\frac{\pi}{3}t\,dt\right)$$

$$a_n = 2\int_0^1 \cos n\frac{\pi}{3}t\,dt - 2\int_0^1 t\cos n\frac{\pi}{3}t\,dt$$

$$= 2\left(\frac{\sin n\frac{\pi}{3}t}{n\frac{\pi}{3}} - \frac{1}{\frac{n^2\pi^2}{9}}\left(\cos n\frac{\pi}{3}t + n\frac{\pi}{3}t\sin n\frac{\pi}{3}t\right)\right)\Bigg|_0^1$$

$$= \frac{6}{n\pi}\sin n\frac{\pi}{3} - \left(\frac{18}{n^2\pi^2}\left(\cos n\frac{\pi}{3}-1\right)+\frac{6}{n\pi}\sin n\frac{\pi}{3}\right) = -\frac{18}{n^2\pi^2}\left(\cos n\frac{\pi}{3}-1\right)$$

so

$$\hat{v}(t) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{18}{n^2\pi^2}\left(1-\cos n\frac{\pi}{3}\right)\cos n\frac{\pi}{3}t$$

$$v(t) = \hat{v}(t-2) - 1 = -\frac{1}{2} + \sum_{n=1}^{\infty} \frac{18}{n^2\pi^2}\left(1-\cos\frac{n\pi}{3}\right)\cos\left(n\frac{\pi}{3}t - n\frac{2\pi}{3}\right)$$

where t is in ms. Equivalently,

$$v(t) = -\frac{1}{2} + \sum_{n=1}^{\infty} \frac{18}{n^2\pi^2}\left(1-\cos\frac{n\pi}{3}\right)\cos\left(n\frac{\pi}{3}10^3 t - n\frac{2\pi}{3}\right)$$

where t is in s.

The network function of the circuit is:

$$\mathbf{H}(\omega) = \frac{-\frac{R_2}{1+j\omega\, C_2R_2}}{R_1+\frac{1}{j\omega C_1}} = -\frac{j\omega\, C_1R_2}{(1+j\omega R_1C_1)(1+j\omega R_2C_2)}$$

Evaluate the network function at the harmonic frequencies of the input to get.

$$\mathbf{H}(n\omega_0) = \mathbf{H}\left(n\frac{\pi}{3}10^3\right) = \frac{-jn\frac{2\pi}{3}}{\left(1+jn\frac{\pi}{3}\right)\left(1+jn\frac{2\pi}{3}\right)}$$

The gain and phase shift are

$$\left|\mathbf{H}(n\,\omega_0)\right| = \frac{n\frac{2\pi}{3}}{\sqrt{\left(1+\frac{n^2\pi^2}{9}\right)\left(1+\frac{4n^2\pi^2}{9}\right)}} = \frac{6n\pi}{\sqrt{\left(9+n^2\pi^2\right)\left(9+4n^2\pi^2\right)}}$$

$$\angle\mathbf{H}(n\,\omega_0) = -90^\circ - \left(\tan^{-1} n\frac{\pi}{3} + \tan^{-1} n\frac{2\pi}{3}\right)$$

The output voltage is

$$v_0(t) = \sum_{n=1}^{\infty} \frac{108\left(1-\cos\frac{n\pi}{3}\right)\cos\left(n\frac{\pi}{3}10^3 t - n\frac{2\pi}{3} - 90^\circ - \tan^{-1} n\frac{\pi}{3} - \tan^{-1} n\frac{2\pi}{3}\right)}{n\pi\sqrt{\left(9+n^2\pi^2\right)\left(9+4n^2\pi^2\right)}}$$

At t = 4 ms =0.004 s

$$v_0(.004) = \sum_{n=1}^{\infty} \frac{108\left(1-\cos\frac{n\pi}{3}\right)\cos\left(n\frac{4\pi}{3} - n\frac{2\pi}{3} - 90^\circ - \tan^{-1} n\frac{\pi}{3} - \tan^{-1} n\frac{2\pi}{3}\right)}{n\pi\sqrt{\left(9+n^2\pi^2\right)\left(9+4n^2\pi^2\right)}}$$

Section 15.9 The Fourier Transform

P 15.9-1 Find the Fourier transform of the function

$$f(t) = -u(-t) + u(t)$$

as shown in Figure P 15.9-1. This is called the signum function.

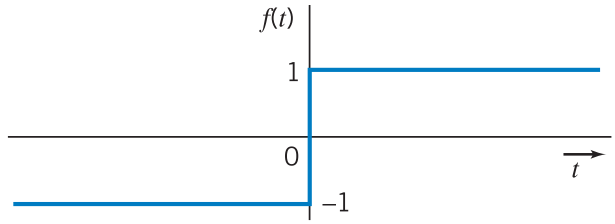


Figure P 15.9-1

Solution:

$$f(t) = -u(-t) + u(t)$$

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t}\, dt = \int_{-\infty}^{0} (-1) e^{-j\omega t}\, dt + \int_{0}^{\infty} (1) e^{-j\omega t}\, dt = -\left.\frac{e^{-j\omega t}}{-j\omega}\right|_{-\infty}^{0} + \left.\frac{e^{-j\omega t}}{-j\omega}\right|_{0}^{\infty} = \frac{2}{j\omega}$$

P 15.9-2 Find the Fourier transform of $f(t) = Ae^{-at}u(t)$ when $a > 0$.

Answer: $F(\omega) = \dfrac{A}{a + j\omega}$

Solution:

$$F(\omega) = \int_{-\infty}^{\infty} Ae^{-at} u(t)\, e^{-j\omega t} dt = \int_{0}^{\infty} Ae^{-at} e^{-j\omega t} dt = \left.\frac{Ae^{-(a+j\omega)t}}{-(a+j\omega)}\right|_{0}^{\infty} = 0 - \frac{A}{-(a+j\omega)} = \frac{A}{a+j\omega}$$

P 15.9-3 Find the Fourier transform of the waveform shown in Figure P 15.9-3.

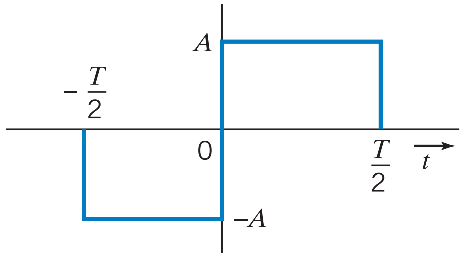
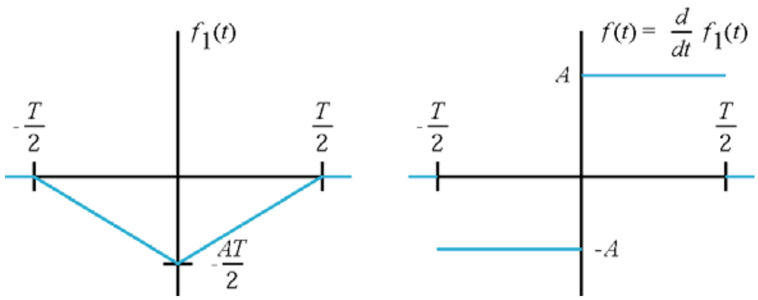


Figure P 15.9-3.

Solution:
First notice that



Then, from line 6 of Table 15.10-2: $\mathcal{F}\left\{f_1(t)\right\} = \left(-\frac{AT}{2}\right)\left(\frac{T}{2}\right) Sa^2\left(\frac{\omega T}{4}\right) = \left(\frac{-AT^2}{4}\right) Sa^2\left(\frac{\omega T}{4}\right)$

From line 7 of Table 15.10-2: $\mathcal{F}\left\{f(t)\right\} = \mathcal{F}\left\{\frac{d}{dt} f_1(t)\right\} = j\omega\, \mathcal{F}\left\{f_1(t)\right\} = -j\omega \frac{AT^2}{4} Sa^2\left(\frac{\omega T}{4}\right)$

This can be written as: $\mathcal{F}\left\{f(t)\right\} = -j\omega \frac{AT^2}{4} \frac{\sin^2\left(\frac{\omega T}{4}\right)}{\left(\frac{\omega T}{4}\right)^2} = \frac{4A}{j\omega} \sin^2\left(\frac{\omega T}{4}\right)$

P 15.9-4 Determine the Fourier transform of $f(t) = 10 \cos 50\, t$.

Answer: $F(\omega) = 10\pi\delta(\omega - 50) + 10\pi\delta(\omega + 50)$

Solution:

First notice that: $\mathcal{F}^{-1}\{\delta(\omega-\omega_0)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\delta(\omega-\omega_0)\, e^{-j\omega t}\, d\omega = \frac{1}{2\pi} e^{-j\omega_0 t}$

Therefore $\mathcal{F}\{e^{-j\omega_0 t}\} = 2\pi\delta(\omega-\omega_0)$. Next, $10 \cos 50t = 5\, e^{j50t} + 5\, e^{-j50t}$.

Therefore $\mathcal{F}\{10 \cos 50t\} = \mathcal{F}\{5\, e^{j50t}\} + \mathcal{F}\{5\, e^{-j50t}\} = 10\pi\delta(\omega-50) + 10\pi\delta(\omega+50)$.

P 15.9-5 Determine the Fourier transform of the pulse shown in Figure P 15.9-5.

Answer:

$$F(j\omega) = \frac{2}{\omega}(\sin\omega - \sin 2\omega) + \frac{j2}{2}(\cos\omega - \cos 2\omega)$$

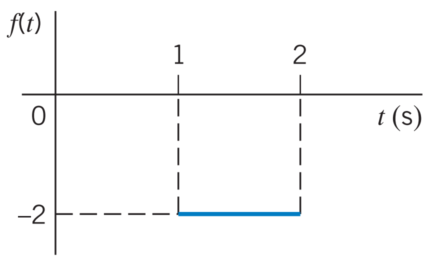


Figure P 15.9-5

Solution:

$$F(\omega) = -2\int_1^2 e^{-j\omega t}\, dt = \left.\frac{-2e^{-j\omega t}}{-j\omega}\right|_1^2 = \frac{2}{j\omega}\left(e^{-j2\omega} - e^{-j\omega}\right) = \frac{2}{j\omega}\left((\cos 2\omega - j\sin 2\omega) - (\cos\omega - j\sin\omega)\right)$$

$$= \frac{2j}{\omega}(\cos\omega - \cos 2\omega) + \frac{2}{\omega}(\sin\omega - \sin 2\omega)$$

P 15.9-6 Determine the Fourier transform of a signal with $f(t) = At/B$ between $t = 0$ and $t = B$ and $f(t) = 0$ elsewhere.

Answer: $F(j\omega) = \frac{A}{B}\left[\frac{-B}{j\omega}e^{-j\omega B} + \frac{1}{\omega^2}e^{-j\omega B} - \frac{1}{\omega^2}\right]$

Solution:

$$F(\omega) = \int_0^B \frac{A}{B} t\, e^{-j\omega t}\, dt = \frac{A}{B}\left[\frac{e^{-j\omega t}}{(-j\omega)^2}(-j\omega t - 1)\right]_0^B = \frac{A}{B}\left[\frac{e^{-j\omega B}}{-\omega^2}(-j\omega B - 1) - \frac{1}{\omega^2}\right]$$

$$= \frac{A}{B}\left[\frac{-Be^{-j\omega B}}{j\omega} + \frac{e^{-j\omega B}}{\omega^2} - \frac{1}{\omega^2}\right]$$

P 15.9-7 Determine the Fourier transform of the waveform $f(t)$ shown in Figure P 15.9-7.

Answer: $F(j\omega) = \dfrac{2}{\omega}(\sin 2\omega - \sin \omega)$

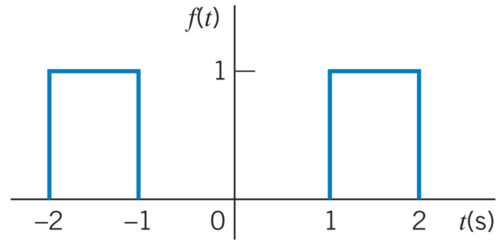


Figure P 15.9-7

Solution:

$$F(\omega) = \int_{-2}^{2} e^{-j\omega t}\,dt - \int_{-1}^{1} e^{-j\omega t}\,dt = \left.\frac{e^{-j\omega t}}{-j\omega}\right|_{-2}^{2} - \left.\frac{e^{-j\omega t}}{-j\omega}\right|_{-1}^{1} = \frac{1}{j\omega}\left(e^{j2\omega} - e^{-j2\omega}\right) - \frac{1}{j\omega}\left(e^{j\omega} - e^{-j\omega}\right)$$

$$= \frac{2}{\omega}\left(\sin 2\omega - \sin \omega\right)$$

Section 15.12 Convolution and Circuit Response

P 15.12-1 Find the current $i(t)$ in the circuit of Figure P 15.12-1 when $i_s(t)$ is the signum function, so that

$$i_s(t) = \begin{cases} +40\text{A} & t > 0 \\ -40\text{A} & t < 0 \end{cases}$$

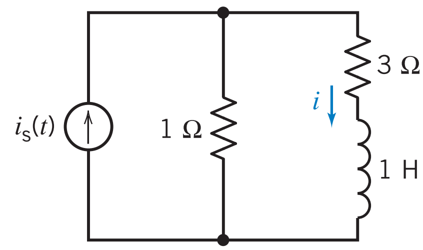


Figure P 15.12-1

Solution:

$$i_s(t) = 40\ \text{signum}(t)$$

$$\mathbf{I}_s(\omega) = 40\left(\frac{2}{j\omega}\right) = \frac{80}{j\omega}$$

$$\mathbf{H}(\omega) = \frac{I(\omega)}{I_s(\omega)} = \frac{1}{4 + j\omega}$$

$$\mathbf{I}(\omega) = \mathbf{H}(\omega)\mathbf{I}_s(\omega) = \frac{1}{4+j\omega} \times \frac{80}{j\omega} = \frac{20}{j\omega} - \frac{20}{4+j\omega}$$

$$\therefore i(t) = 10\ \text{signum}(t) - 20\ e^{-4t}u(t)$$

P 15.12-2 Repeat Problem 15.12-1 when $i_s = 100 \cos 3t$ A.

Solution:

$$i_s(t) = 100 \cos 3t \quad \text{A}$$

$$\mathbf{I}_s(\omega) = 100\pi\left[\delta(\omega - 3) + \delta(\omega + 3)\right]$$

$$\mathbf{H}(\omega) = \frac{I(\omega)}{\mathbf{I}_s(\omega)} = \frac{1}{4 + j\omega}$$

$$\mathbf{I}(\omega) = 100\pi\left[\frac{\delta(\omega-3) + \delta(\omega+3)}{4 + j\omega}\right]$$

$$i(t) = \frac{100\pi}{2\pi}\int_{-\infty}^{\infty}\left[\frac{\delta(\omega-3) + \delta(\omega+3)}{4+j\omega}\right]e^{j\omega t}d\omega$$

$$= 50\left[\frac{e^{-j3t}}{4 - j3} + \frac{e^{j3t}}{4 + j3}\right] = 10\left[e^{-j(3t - 36.9^\circ)} + e^{j(3t-36.9^\circ)}\right] = 20 \cos\left(3t - 36.9^\circ\right)$$

P 15.12-3 The voltage source of Figure P 15.12-3 is

$$v(t) = 10 \cos 2t$$

for all t. Calculate $i(t)$ using the Fourier transform.

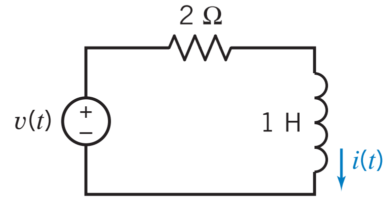


Figure P 15.12-3

Solution:

$$v(t) = 10 \cos 2t$$

$$\mathbf{V}(\omega) = 10\pi\left[\delta(\omega+2) + \delta(\omega-2)\right]$$

$$\mathbf{Y}(\omega) = \frac{1}{2 + j\omega}$$

$$\mathbf{I}(\omega) = \mathbf{Y}(\omega)\mathbf{V}(\omega) = \frac{10\pi\left[\delta(\omega+2) + \delta(\omega-2)\right]}{2 + j\omega}$$

$$i(t) = \frac{10\pi}{2\pi}\int_{-\infty}^{\infty}\left[\frac{\delta(\omega+2) + \delta(\omega-2)}{2 + j\omega}\right]e^{j\omega t}d\omega = 5\left[\frac{e^{-j2t}}{2 - j2} + \frac{e^{j2t}}{2 + j2}\right]$$

$$= \frac{5}{2\sqrt{2}}\left[e^{-j(2t-45)} + e^{j(2t-45)}\right] = \frac{5}{\sqrt{2}}\cos(2t - 45) \quad \text{A}$$

P 15.12-4 Find the output voltage $v_o(t)$ using the Fourier transform for the circuit of Figure P 15.12-4 when

$$v(t) = e^{t}u(-t) + u(t) \text{ V}.$$

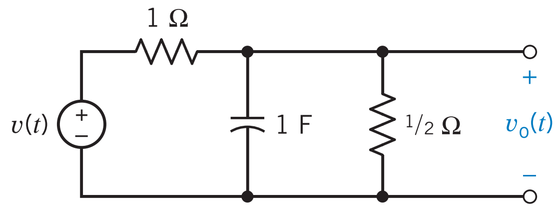


Figure P 15.12-4

Solution:

$$v(t) = e^{t}u(-t) + u(t)$$

$$\mathcal{F}\left\{e^{t}u(-t)\right\} = \int_{-\infty}^{\infty} e^{t}u(-t)e^{-j\omega t}dt = \int_{-\infty}^{0} e^{t}e^{-j\omega t}dt = \left.\frac{e^{(1-j\omega)t}}{1-j\omega}\right|_{-\infty}^{0} = \frac{1}{1-j\omega}$$

$$\mathcal{F}\left\{u(t)\right\} = \pi\delta(\omega) + \frac{1}{j\omega}$$

$$\therefore \mathbf{V}(\omega) = \frac{1}{1-j\omega} + \pi\delta(\omega) + \frac{1}{j\omega}$$

$$\frac{\left(\frac{1}{2}\right)\left(\frac{1}{j\omega}\right)}{\frac{1}{2}+\frac{1}{j\omega}} = \frac{1}{2+j\omega}, \quad \mathbf{H}(\omega) = \frac{\frac{1}{2+j\omega}}{1+\frac{1}{2+j\omega}} = \frac{1}{3+j\omega}$$

$$\mathbf{V}_o(\omega) = \frac{1}{3+j\omega}\left[\frac{1}{1-j\omega} + \pi\delta(\omega) + \frac{1}{j\omega}\right] = \frac{-\frac{1}{12}}{3+j\omega} + \frac{\frac{1}{4}}{1-j\omega} + \frac{\frac{1}{3}}{j\omega} + \frac{\pi\delta(\omega)}{3+j\omega}$$

$$\mathcal{F}^{-1}\left\{\frac{\pi\delta(\omega)}{3+j\omega}\right\} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\pi\delta(\omega)}{3+j\omega}e^{-j\omega t}d\omega = \frac{1}{6}$$

$$\therefore \quad v_o(t) = -\frac{1}{12}e^{-3t}u(t) + \frac{1}{4}e^{t}u(-t) + \frac{1}{6}\,\text{signum}(t) + \frac{1}{6}$$

P 15.12-5 The voltage source of the circuit of Figure P 15.12-5 is $v(t) = 15e^{-5t}$ V. Find the resistance R when it is known that the energy available in the output signal is two-thirds of the energy of the input signal.

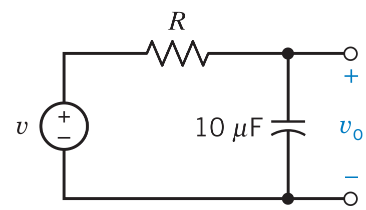


Figure P 15.12-5

Solution:

$$v_s(t) = 15e^{-5t}u(t) \quad \text{V} \quad \Rightarrow \quad \mathbf{V}(\omega) = \frac{15}{5 + j\omega}$$

$$W_s = \int_{-\infty}^{\infty} \left(15e^{-5t}u(t)\right)^2 dt = \int_0^{\infty} \left(15e^{-5t}\right)^2 dt = 22.5 \quad \text{J}$$

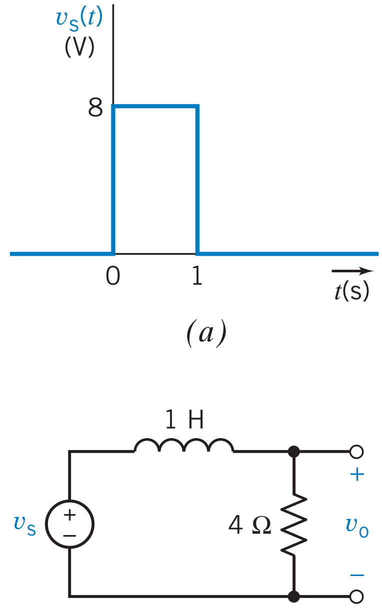
$$\mathbf{H}(\omega) = \frac{\dfrac{1}{j\omega C}}{R + \dfrac{1}{j\omega C}} = \frac{\dfrac{1}{RC}}{\dfrac{1}{RC} + j\omega}$$

C=10 μF. Try R=10 kΩ. Then $\mathbf{V}_o(\omega) = \dfrac{10}{10 + j\omega} \times \dfrac{15}{5 + j\omega}$

$$W_o = \frac{1}{\pi}\int_0^{\infty} \left(\frac{10}{10 + j\omega} \times \frac{15}{5 + j\omega}\right)^2 d\omega = \frac{1}{\pi}\int_0^{\infty} \left(\frac{300}{25 + \omega^2} - \frac{300}{100 + \omega^2}\right) d\omega$$

$$= \frac{1}{\pi}\left[\frac{300}{5}\tan^{-1}\left(\frac{\omega}{5}\right) - \frac{300}{10}\tan^{-1}\left(\frac{\omega}{10}\right)\right]_0^{\infty} = \frac{1}{\pi}\left[60\frac{\pi}{2} - 30\frac{\pi}{2}\right] = 15 \quad \text{J}$$

P 15.12-6 The pulse signal shown in Figure P 15.12-6*a* is the source $v_s(t)$ for the circuit of Figure P 15.12-6*b*. Determine the output voltage, v_o, using the Fourier transform.



(*a*) (*b*)

Figure P 15.12-6

Solution:

$$H(\omega)=\frac{4}{4+j\omega}$$

$$V_s(\omega)=\mathcal{F}\{8u(t)-8u(t-1)\}=\left(8\pi\delta(\omega)+\frac{8}{j\omega}\right)-\left(8\pi\delta(\omega)+\frac{8}{j\omega}\right)e^{-j\omega}$$

$$V_s(\omega)=\frac{8}{j\omega}\left(1-e^{-j\omega}\right)\quad \text{since} \quad \delta(\omega)e^{-j\omega}=\delta(\omega)$$

$$V_o(\omega)=\frac{4}{4+j\omega}\times\frac{8}{j\omega}\left(1-e^{-j\omega}\right)=\left(\frac{8}{j\omega}-\frac{8}{4+j\omega}\right)-\left(\frac{8}{j\omega}-\frac{8}{4+j\omega}\right)e^{-j\omega}$$

Next use $\frac{1}{j\omega}=\frac{1}{j\omega}+\pi\delta(\omega)-\pi\delta(\omega)$ to write

$$V_o(\omega)=\left(8\left(\frac{1}{j\omega}+\pi\delta(\omega)\right)-8\pi\delta(\omega)-\frac{8}{4+j\omega}\right)-\left(8\left(\frac{1}{j\omega}+8\pi\delta(\omega)\right)-\pi\delta(\omega)-\frac{8}{4+j\omega}\right)e^{-j\omega}$$

$$=\left(8\left(\frac{1}{j\omega}+\pi\delta(\omega)\right)-\frac{8}{4+j\omega}\right)-\left(8\left(\frac{1}{j\omega}+\pi\delta(\omega)\right)-\frac{8}{4+j\omega}\right)e^{-j\omega}$$

$$v_o(t)=8u(t)-8e^{-4t}u(t)-\left(8u(t-1)-8e^{-4(t-1)}u(t-1)\right)$$

$$=8(1-e^{-4t})u(t)-8(1-e^{-4(t-1)})u(t-1)\quad \text{V}$$

Section 15.15 How Can We Check…?

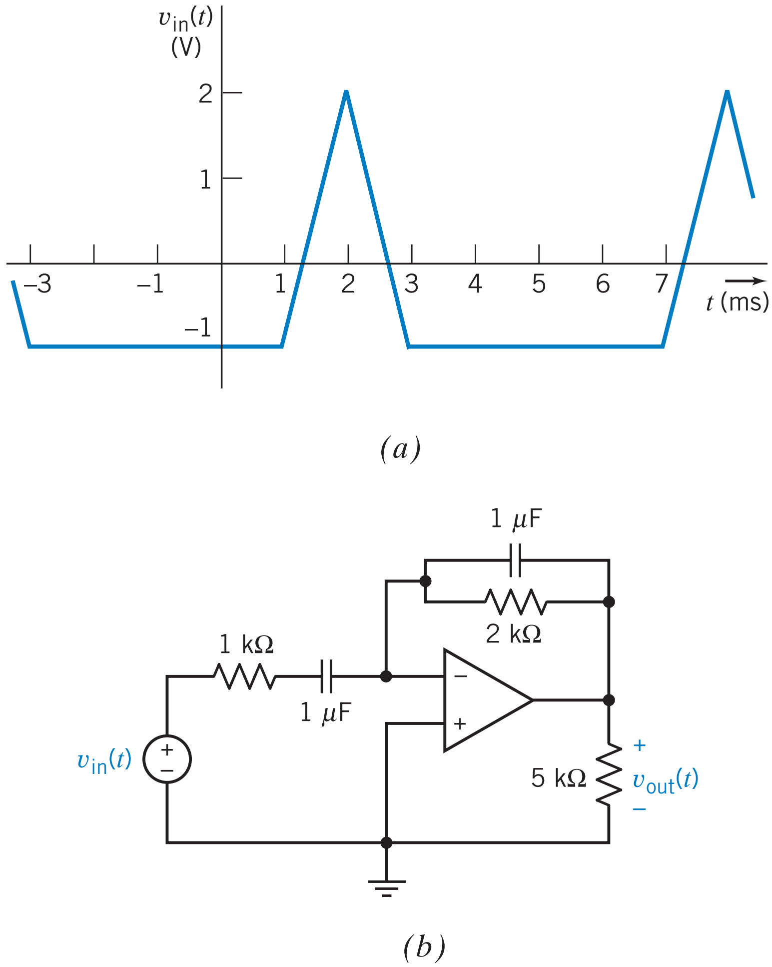


Figure P 15.7-7

P 15.14-1 The Fourier series of $v_{in}(t)$ shown in Figure P 15.8-7 is given as

$$v_{in}(t) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{18}{n^2 \pi}\left(1 - \cos\frac{n\pi}{3}\right)\cos\left(n\frac{\pi}{3}t - n\frac{2\pi}{3}\right) \text{V}$$

Is this the correct Fourier series?

Hint: Check the average value and the fundamental frequency.

Answer: The given Fourier series is not correct.

Solution: The average value of the given series is $a_0 = \frac{1}{2}$ V . But

$$\text{average value of } 1 + v_{in}(t) = \frac{\frac{3\times 2}{2}}{6} = \frac{1}{2} \quad \Rightarrow \quad \text{average value of } v_{in}(t) = -\frac{1}{2} \text{ V}$$

Therefore the given Fourier series is not correct.

P 15.14-2 The Fourier series of $v(t)$ shown in Figure P 15.14-2 is given as

$$v(t) = 9 + \sum_{n=1}^{\infty} \frac{40}{n\pi}\left(\sin\frac{n\pi}{5}\right)\cos\left(n\frac{\pi}{5}t - n\frac{\pi}{5}\right)\text{V}$$

Is this the correct Fourier series?

Hint: Check the average value and the fundamental frequency.

Answer: The given Fourier series is not correct.

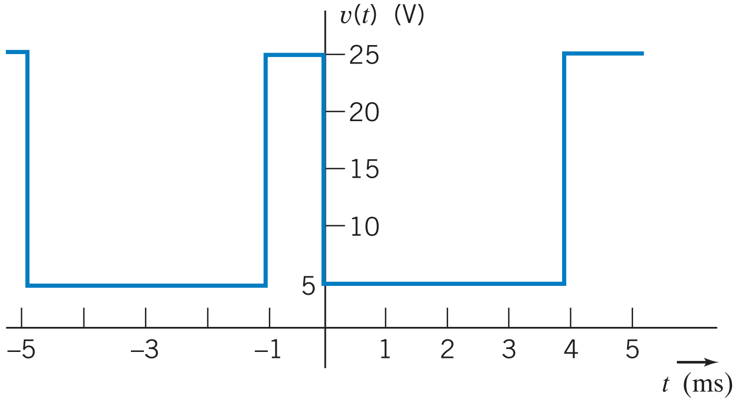


Figure P 15.14-2

Solution:

The fundamental frequency of the given series is $\frac{\pi}{5}$ rad/s. The period of $v(t)$ is $T = 5$ s.

Therefore $\omega_0 = \frac{2\pi}{5}$ rad/s not $\frac{\pi}{5}$ rad/s. Therefore the given Fourier series is not correct.

P 15.14-3 The Fourier series of $v(t)$ shown in Figure SP 15-2 in the next section is given as

$$v(t) = 2\sum_{n=1}^{\infty} \frac{(-1)^n}{n\pi} \cos\ (n2\pi t)\text{V}$$

Is this the correct Fourier series?

Hint: Check the average value and the fundamental frequency. Check for symmetry.

Answer: The given Fourier series is not correct.

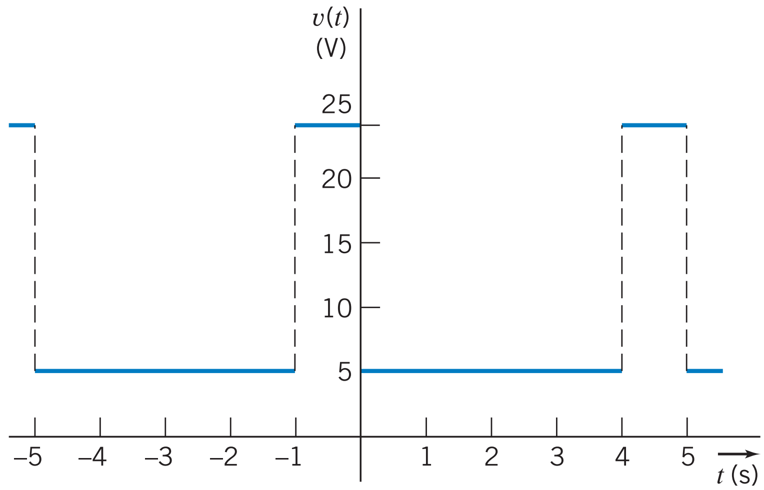


Figure SP 15-2

Solution:
The given series contains only cosine terms, indicating an even function. Instead, $v(t) = -v(-t)$, so the $v(t)$ is an odd function. Therefore the given Fourier series is not correct.

PSpice Problems

SP 15-1 Use PSpice to determine the Fourier coefficients for $v(t)$ shown in Figure SP 15-1.

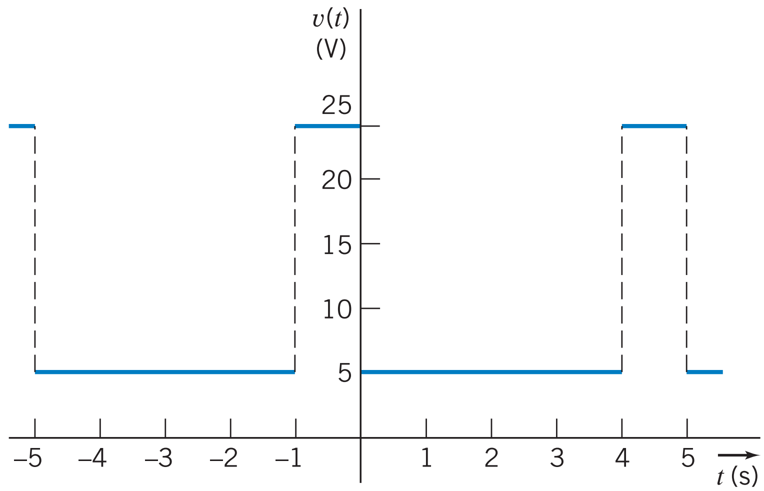


Figure SP 15-1

Solution:

```
Vin    1    0   pulse   (25  5  0  0  0  4  5)
R1     1    0   1

.tran    0.01 5
.four    0.2 v(1)
.probe
.end

FOURIER COMPONENTS OF TRANSIENT RESPONSE V(1)

 DC COMPONENT =     8.960000E+00
```

HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE(DEG)
1	2.000E-01	7.419E+00	1.000E+00	1.253E+02	0.000E+00
2	4.000E-01	6.030E+00	8.127E-01	1.606E+02	3.528E+01
3	6.000E-01	4.061E+00	5.473E-01	-1.642E+02	-2.894E+02
4	8.000E-01	1.935E+00	2.609E-01	-1.289E+02	-2.542E+02
5	1.000E+00	8.000E-02	1.078E-02	-9.360E+01	-2.189E+02
6	1.200E+00	1.182E+00	1.593E-01	1.217E+02	-3.600E+00
7	1.400E+00	1.704E+00	2.297E-01	1.570E+02	3.168E+01
8	1.600E+00	1.537E+00	2.072E-01	-1.678E+02	-2.930E+02
9	1.800E+00	8.954E–01	1.207E-01	-1.325E+02	-2.578E+02

SP 15-2 Use PSpice to determine the Fourier coefficients for $v(t)$ shown in Figure SP 15-2.

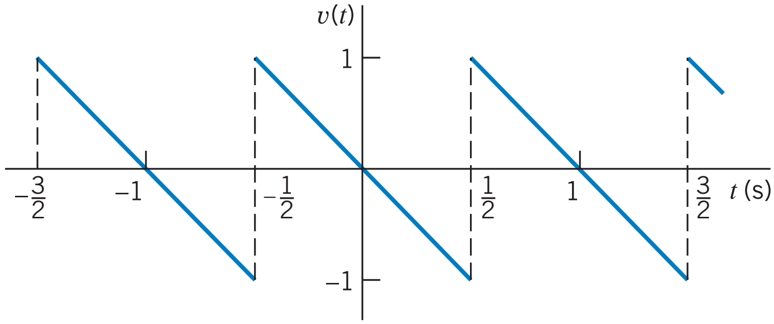


Figure SP 15-2

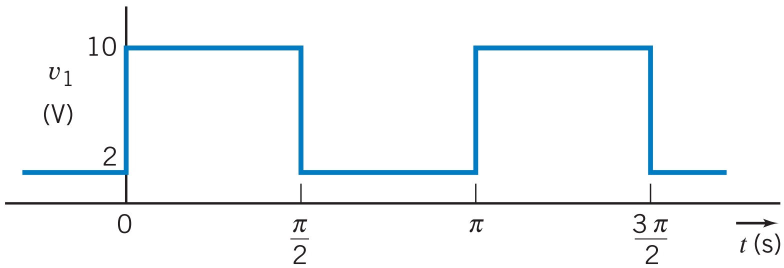
Solution:

```
Vin    1      0      pulse  (1 -1 -0.5 1 0 0 1)
R1     1      0      1

.tran    0.1   1
.four    1   v(1)
.probe
.end

FOURIER COMPONENTS OF TRANSIENT RESPONSE V(1)

 DC COMPONENT =     1.299437E-02
```

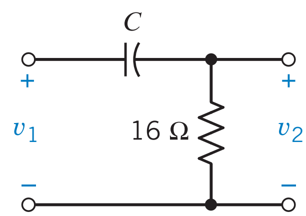
HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	1.000E+00	6.364E-01	1.000E+00	-1.777E+02	0.000E+00
2	2.000E+00	3.180E-01	4.996E-01	4.679E+00	1.823E+02
3	3.000E+00	2.117E-01	3.326E-01	-1.730E+02	4.682E+00
4	4.000E+00	1.585E-01	2.490E-01	9.366E+00	1.870E+02
5	5.000E+00	1.264E-01	1.987E-01	-1.683E+02	9.376E+00
6	6.000E+00	1.051E-01	1.651E-01	1.407E+01	1.917E+02
7	7.000E+00	8.972E-02	1.410E-01	-1.636E+02	1.409E+01
8	8.000E+00	7.817E-02	1.228E-01	1.880E+01	1.965E+02
9	9.000E+00	6.916E-02	1.087E-01	-1.588E+02	1.883E+01

Design Problems

DP 15-1 A periodic waveform shown in Figure DP 15-1*a* is the input signal of the circuit shown in Figure DP 15-1*b*. Select the capacitance *C* so that the magnitude of the third harmonic of $v_2(t)$ is less than 1.4 V and greater than 1.3 V. Write the equation describing the third harmonic of $v_2(t)$ for the value of *C* selected.



(*a*)



(*b*)

Figure DP 15-1

Solution:

$T = \pi \quad \Rightarrow \quad \omega_0 = \dfrac{2\pi}{T} = 2$ rad/s and $a_0 =$ average value $= 6$ V. Using $A = 4$ in row 1 of Table 15.4-2, we get

$$v_1(t) = 6 + \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin(2n-1)(2t) = 6 + \frac{16}{\pi}\sin(2t) + \frac{16}{3\pi}\sin(6t) + \ldots$$

The network function of the circuit is $\mathbf{H}(\omega) = \dfrac{16}{16 + \dfrac{1}{j\omega C}} = \dfrac{j16\omega C}{1 + j16\omega C}$.

The third harmonic is the term at $\omega = 3\omega_0 = 6$ rad/s. The specification requires that

$$1.3 \le \frac{16}{3\pi} \times \frac{16\omega C}{\sqrt{1 + (16\omega C)^2}} \le 1.4$$

This specification is satisfied for by capacitances in the range 812.4 μF to 862.5 μF.

DP 15-2 A dc laboratory power supply uses a nonlinear circuit to convert a sinusoidal voltage obtained from the wall plug to a constant dc voltage. The wall plug voltage is $A \sin \omega_0 t$, where $f_0 = 60$ Hz and $A = 160$ V. The voltage is then rectified so that $v_s = |A \sin \omega_0 t|$. Using the filter circuit of Figure DP 15-2, determine the required inductance L so that the magnitude of each harmonic (ripple) is less than 4 percent of the dc component of the output voltage.

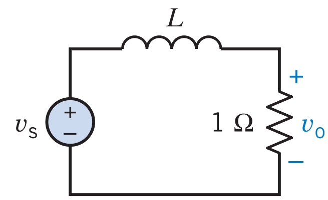


Figure DP 15-2

Solution:

Refer to Table 15.4-2.

$$v_s(t) = \frac{2A}{\pi} - \sum_{n=1}^{N} \frac{4A}{\pi}\left(\frac{1}{4n^2 - 1}\right)\cos(n\omega_0 t)$$

In our case:

$$v_s(t) = \frac{320}{\pi} - \sum_{n=1}^{N} \frac{640}{\pi}\left(\frac{1}{4n^2 - 1}\right)\cos(n377t)$$

Let $v_s(t) = v_{s0} + \sum_{n=1}^{N} v_{sn}(t)$ and $v_0(t) = v_{o0} + \sum_{n=1}^{N} v_{on}(t)$

We require ripple $\leq 0.04 \cdot$ dc output

$$\max\left(\sum_{n=1}^{N} v_{on}(t)\right) \leq 0.04 \cdot v_{o0} \quad \Rightarrow \quad |v_{o1}(t)| \leq 0.04\, v_{o0}$$

but $v_{o0} = v_{s0}$ because the inductor acts like a short at dc.

Next, using the network function of the circuit gives $\mathbf{V}_{on} = \left(\dfrac{R}{R + jn\omega_0 L}\right)\mathbf{V}_{sn}$.

For n=1:

$$\mathbf{V}_{o1} = \frac{R}{R + j\omega_0 L}\,\mathbf{V}_{s1} = \frac{1}{1 + j377L}\,\mathbf{V}_{s1}\text{ , but } |\mathbf{V}_{s1}| = \frac{640}{\pi(3)} \text{ so } \mathbf{V}_{01} = \frac{1}{1 + j377L}\left(\frac{640}{3\pi}\right)$$

We require $|\mathbf{V}_{o1}| \leq 0.04\, v_{o0}$ and $v_{o0} = v_{s0} = \dfrac{320}{\pi}$. Then $\dfrac{1}{\sqrt{1 + (377)^2 L^2}} \cdot \dfrac{640}{3\pi} \leq 0.04\left(\dfrac{320}{\pi}\right)$

Solving for L yields $\underline{L > 44.13 \text{ mH}}$

DP 15-3 A low-pass filter is shown in Figure DP 15-3. The input, v_s, is a half-wave rectified sinusoid with $\omega_0 = 800\pi$ (item 5 of Table 15.5-1). Select L and C so that the peak value of the first harmonic is 1/20 of the dc component for the output, v_o.

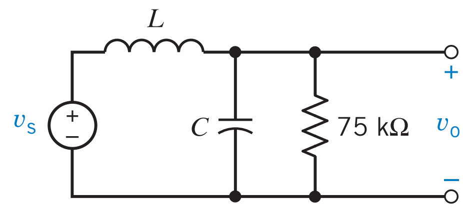


Figure DP 15-3

Solution:

From Table 15.5-1, the Fourier series can represent the input to the circuit as:

$$v_s(t) = \frac{1}{\pi} + \frac{j}{4} e^{j\omega_0 t} + \frac{j}{4} e^{j\omega_0 t} + \sum_{\text{even } n=2}^{\infty} \frac{1}{\pi(1-n^2)} e^{jn\omega_0 t}$$

The transfer function of the circuit is calculated as $\mathbf{V}_{o1} = \dfrac{\mathbf{Z}_p}{\mathbf{Z}_L + \mathbf{Z}_p} \mathbf{V}_{s1}$ where $\mathbf{Z}_p = \dfrac{R}{1 + j\omega RC}$

So

$$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \; = \frac{\dfrac{1}{LC}}{(j\omega)^2 + (j\omega)\dfrac{1}{RC} + \dfrac{1}{LC}}$$

The gain at dc, $\omega = 0$, is 1 so

$$v_{o0} = v_{s0} = \frac{1}{\pi}$$

For $n = 1$

$$|\mathbf{V}_{o1}| = \frac{1}{20}|v_{o0}| = \frac{1}{20}|v_{s0}| = \frac{1}{20\pi} \quad \Rightarrow \quad \frac{\dfrac{1}{LC}}{\sqrt{\left(\dfrac{1}{LC} - \omega^2\right)^2 + \left(\dfrac{\omega}{RC}\right)^2}} \times \frac{1}{4} = \frac{1}{20\pi}$$

We are given $\omega = 800\pi$ and $R = 75$ kΩ. Choosing <u>$L = 0.1$ mH</u> yields <u>$C = 26.5$ mF</u>

Section 16.3: Filters

Exercises

Exercise 16.3-1 Find the transfer function of a first-order Butterworth low-pass filter having a cutoff frequency equal to 1250 rad/s.

Answer: $H(s)=\dfrac{1}{\dfrac{s}{1250}+1}=\dfrac{1250}{s+1250}$

Solution:

$$H_n(s)=\frac{1}{s+1}$$

$$H(s)=H_n\left(\frac{s}{1250}\right)=\frac{1}{\dfrac{s}{1250}+1}=\frac{1250}{s+1250}$$

Problems

P 16.3-1 Obtain the transfer function of a third-order Butterworth low-pass filter having a cutoff frequency equal to 100 hertz.

Answer: $H_{\mathrm{L}}(s)=\dfrac{628^3}{(s+628)(s^2+628s+628^2)}$

Solution:

Equation 16-3.2 and Table 16-3.2 provide a third-order Butterworth low-pass filter having a cutoff frequency equal to 1 rad/s.

$$H_n(s)=\frac{1}{(s+1)(s^2+s+1)}$$

Frequency scaling so that $\omega_c = 2\pi 100 = 628$ rad/s:

$$H_{\mathrm{L}}(s)=\frac{1}{\left(\dfrac{s}{628}+1\right)\left(\left(\dfrac{s}{628}\right)^2+\dfrac{s}{628}+1\right)}=\frac{628^3}{(s+628)(s^2+628s+628^2)}=\frac{247673152}{(s+628)(s^2+628s+394384)}$$

P 16.3-2 A dc gain can be incorporated into Butterworth low-pass filters by defining the transfer function to be

$$H_L(s) = \frac{\pm k}{D(s)}$$

where $D(s)$ denotes the polynomials tabulated in Table 16.3-2 and k is the dc gain. The dc gain k is also called the pass-band gain. Obtain the transfer function of a third-order Butterworth low-pass filter having a cutoff frequency equal to 100 rad/s and a pass-band gain equal to 5.

Solution:

Equation 16-3.2 and Table 16-3.2 provide a third-order Butterworth low-pass filter having a cutoff frequency equal to 1 rad/s and a dc gain equal to 1.

$$H_n(s) = \frac{1}{(s+1)(s^2+s+1)}$$

Multiplying by 5 to change the dc gain to 5 and frequency scaling to change the cutoff frequency to ω_c = 100 rad/s:

$$H_L(s) = \frac{5}{\left(\frac{s}{100}+1\right)\left(\left(\frac{s}{100}\right)^2+\frac{s}{100}+1\right)} = \frac{5 \cdot 100^3}{(s+100)(s^2+100s+100^2)} = \frac{5000000}{(s+100)(s^2+100s+10000)}$$

P 16.3-3 High-pass Butterworth filters have transfer functions of the form

$$H_{\mathrm{H}}(s) = \frac{\pm k s^n}{D_n(s)}$$

where n is the order of the filter, $D_n(s)$ denotes the nth order polynomial in Table 16.3-2, and k is the pass-band gain. Obtain the transfer function of a third-order Butterworth high-pass filter having a cutoff frequency equal to 100 rad/s and a pass-band gain equal to 5.

Answer: $$H_{\mathrm{H}}(s) = \frac{5 \cdot s^3}{(s+100)(s^2+100s+10000)}$$

Solution:

Use Table 16-3.2 to obtain the transfer function of a third-order Butterworth high-pass filter having a cutoff frequency equal to 1 rad/s and a dc gain equal to 5.

$$H_n(s) = \frac{5s^3}{(s+1)(s^2+s+1)}$$

Frequency scaling to change the cutoff frequency to $\omega_c = 100$ rad/s:

$$H_{\mathrm{H}}(s) = \frac{5\left(\frac{s}{100}\right)^3}{\left(\frac{s}{100}+1\right)\left(\left(\frac{s}{100}\right)^2+\frac{s}{100}+1\right)} = \frac{5s^3}{(s+100)(s^2+100s+100^2)} = \frac{5s^3}{(s+100)(s^2+100s+10000)}$$

P 16.3-4 High-pass Butterworth filters have transfer functions of the form

$$H_{\mathrm{H}}(s) = \frac{\pm k s^n}{D_n(s)}$$

where n is the order of the filter, $D_n(s)$ denotes the nth order polynomial in Table 16.3-2, and k is the pass-band gain. Obtain the transfer function of a fourth-order Butterworth high-pass filter having a cutoff frequency equal to 500 hertz and a pass-band gain equal to 5.

Solution:

Use Table 16-3.2 to obtain the transfer function of a fourth-order Butterworth high-pass filter having a cutoff frequency equal to 1 rad/s and a dc gain equal to 5.

$$H_n(s) = \frac{5s^4}{\left(s^2 + 0.765s + 1\right)\left(s^2 + 1.848s + 1\right)}$$

Frequency scaling can be used to adjust the cutoff frequency 500 hertz = 3142 rad/s:

$$H_{\mathrm{H}}(s) = \frac{5 \cdot \left(\dfrac{s}{3142}\right)^4}{\left(\left(\dfrac{s}{3142}\right)^2 + 0.765\left(\dfrac{s}{3142}\right) + 1\right)\left(\left(\dfrac{s}{3142}\right)^2 + 1.848\left(\dfrac{s}{3142}\right) + 1\right)} = \frac{5 \cdot s^4}{\left(s^2 + 2403.6s + 3142^2\right)\left(s^2 + 5806.4s + 3142^2\right)}$$

P 16.3-5 A band-pass filter has two cutoff frequencies, ω_a and ω_b. Suppose that ω_a is quite a bit smaller than ω_b, say $\omega_a < \omega_b/10$. Let $H_L(s)$ be a low-pass transfer function having a cutoff frequency equal to ω_b and $H_H(s)$ be a high-pass transfer function having a cutoff frequency equal to ω_a. A band-pass transfer function can be obtained as a product of low-pass and high-pass transfer functions, $H_B(s) = H_L(s) \cdot H_H(s)$. The order of the band-pass filter is equal to the sum of the orders of the low-pass and high-pass filters. We usually make the orders of the low-pass and high-pass filter equal, in which case the order of the band-pass is even. The pass-band gain of the band-pass filter is the product of pass-band gains of the low-pass and high-pass transfer functions. Obtain the transfer function of a fourth-order band-pass filter having cutoff frequencies equal to 100 rad/s and 2000 rad/s and a pass-band gain equal to 4.

Answer:

$$H_B(s) = \frac{16,000,000 \cdot s^2}{(s^2 + 141.4s + 10.000)(s^2 + 2828s + 4,000,000)}$$

Solution:

First, obtain the transfer function of a second-order Butterworth low-pass filter having a dc gain equal to 2 and a cutoff frequency equal to 2000 rad/s:

$$H_L(s) = \frac{2}{\left(\frac{s}{2000}\right)^2 + 1.414\left(\frac{s}{2000}\right) + 1} = \frac{8000000}{s^2 + 2828s + 4000000}$$

Next, obtain the transfer function of a second-order Butterworth high-pass filter having a passband gain equal to 2 and a cutoff frequency equal to 100 rad/s:

$$H_H(s) = \frac{2 \cdot \left(\frac{s}{100}\right)^2}{\left(\frac{s}{100}\right)^2 + 1.414\left(\frac{s}{100}\right) + 1} = \frac{2 \cdot s^2}{s^2 + 141.4s + 10000}$$

Finally, the transfer function of the bandpass filter is

$$H_B(s) = H_L(s) \times H_H(s) = \frac{16000000 \times s^2}{\left(s^2 + 141.4s + 10000\right)\ \left(s^2 + 2828s + 4000000\right)}$$

P 16.3-6 In some applications band-pass filters are used to pass only those signals having a specified frequency ω_0. The cutoff frequencies of the band-pass filter are specified to satisfy $\sqrt{\omega_a \omega_b} = \omega_0$. The transfer function of the band-pass filter is given by

$$H_B(s) = k\left(\frac{\frac{\omega_0}{Q}s}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}\right)^m$$

The order of this band-pass transfer function is $n = 2m$. The pass-band gain is k. Transfer functions of the type are readily implemented as the cascade connection of identical second-order filter stages. Q is the quality factor of the second-order filter stage. The frequency ω_0 is called the center frequency of the band-pass filter. Obtain the transfer function of a fourth-order band-pass filter having a center frequency equal to 250 rad/s and a pass-band gain equal to 4. Use $Q = 1$.

Answer: $H_B(s) = \dfrac{250,000s^2}{(s^2 + 250s + 62,500)^2}$

Solution:

$$H_B(s) = 4\left(\frac{\frac{250}{1}s}{s^2 + \frac{250}{1}s + 250^2}\right)^2 = \frac{250000s^2}{\left(s^2 + 250s + 62500\right)^2}$$

P 16.3-7 A band-stop filter has two cutoff frequencies, ω_a and ω_b. Suppose that ω_a is quite a bit smaller than ω_b, say $\omega_a < \omega_b/10$. Let $H_L(s)$ be a low-pass transfer function having a cutoff frequency equal to ω_a and $H_H(s)$ be a high-pass transfer function having a cutoff frequency equal to ω_b. A band-stop transfer function can be obtained as a sum of low-pass and high-pass transfer functions, $H_N(s) = H_L(s) + H_H(s)$. The order of the band-pass filter is equal to the sum of the orders of the low-pass and high-pass filters. We usually make the orders of the low-pass and high-pass filter equal, in which case the order of the band-stop is even. The pass-band gains of both the low-pass and high-pass transfer functions are set equal to the pass-band gain of the band-stop filter. Obtain the transfer function of a fourth-order band-stop filter having cutoff frequencies equal to 100 rad/s and 2000 rad/s and a pass-band gain equal to 2.

Answer: $H_N(s) = \dfrac{2s^4 + 282.8s^3 + 40{,}000s^2 + 56{,}560{,}000s + 8\cdot 10^{10}}{(s^2 + 141.4s + 10{,}000)(s^2 + 2828s + 4{,}000{,}000)}$

Solution:
First, obtain the transfer function of a second-order Butterworth high-pass filter having a dc gain equal to 2 and a cutoff frequency equal to 2000 rad/s:

$$H_L(s) = \frac{2\left(\dfrac{s}{2000}\right)^2}{\left(\dfrac{s}{2000}\right)^2 + 1.414\left(\dfrac{s}{2000}\right) + 1} = \frac{2s^2}{s^2 + 2828s + 4000000}$$

Next, obtain the transfer function of a second-order Butterworth low-pass filter having a pass-band gain equal to 2 and a cutoff frequency equal to 100 rad/s:

$$H_H(s) = \frac{2}{\left(\dfrac{s}{100}\right)^2 + 1.414\left(\dfrac{s}{100}\right) + 1} = \frac{20000}{s^2 + 141.4s + 10000}$$

Finally, the transfer function of the band-stop filter is

$$H_N(s) = H_L(s) + H_H(s) = \frac{2s^2\left(s^2 + 141.4s + 10000\right) + 20000\left(s^2 + 2828s + 4000000\right)}{\left(s^2 + 141.4s + 10000\right)\left(s^2 + 2828s + 4000000\right)}$$

$$= \frac{2s^4 + 282.8s^3 + 40000s^2 + 56560000s + 8\cdot 10^{10}}{\left(s^2 + 141.4s + 10000\right)\left(s^2 + 2828s + 4000000\right)}$$

P 16.3-8 In some applications band-stop filters are used to reject only those signals having a specified frequency ω_0. The cutoff frequencies of the band-stop filter are specified to satisfy $\sqrt{\omega_a \omega_b} = \omega_0$. The transfer function of the band-pass filter is given by

$$H_N(s) = k - H_B(s) = k - k\left(\frac{\frac{\omega_0}{Q}s}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}\right)^m$$

The order of this band-stop transfer function is $n = 2m$. The pass-band gain is k. Transfer functions of the type are readily implemented using a cascade connection of identical second-order filter stages. Q is the quality factor of the second-order filter stage. The frequency ω_0 is called the center frequency of the band-stop filter. Obtain the transfer function of a fourth-order band-stop filter having a center frequency equal to 250 rad/s and a pass-band gain equal to 4. Use $Q = 1$.

Answer: $H_N(s) = \dfrac{4(s^2 + 62{,}500)^2}{(s^2 + 250s + 62{,}500)^2}$

Solution:

$$H_N(s) = 4 - 4\left(\frac{\frac{250}{1}s}{s^2 + \frac{250}{1}s + 250^2}\right)^2 = \frac{4\left(s^2 + 62500\right)^2}{\left(s^2 + 250s + 62500\right)^2}$$

P 16.3-9 Transfer functions of the form

$$H_L(s) = k\left(\frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}\right)^m$$

are low-pass transfer functions. (This is not a Butterworth transfer function.) The order of this low-pass transfer function is $n = 2m$. The pass-band gain is k. Transfer functions of this type are readily implemented using a cascade connection of identical second-order filter stages. Q is the quality factor of the second-order filter stage. The frequency ω_0 is the cutoff frequency, ω_c, of the low-pass filter. Obtain the transfer function of a fourth-order low-pass filter having a cutoff frequency equal to 250 rad/s and a pass-band gain equal to 4. Use $Q = 1$.

Solution:

$$H_L(s) = 4\left(\frac{250^2}{s^2 + \frac{250}{1}s + 250^2}\right)^2 = \frac{4 \cdot 250^4}{\left(s^2 + 250s + 62500\right)^2}$$

P 16.3-10 Transfer functions of the form

$$H_{\mathrm{H}}(s) = k\left(\frac{s^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}\right)^m$$

are high-pass transfer functions. (This is not a Butterworth transfer function.) The order of this high-pass transfer function is $n = 2m$. The pass-band gain is k. Transfer functions of the type are readily implemented using a cascade connection of identical second-order filter stages. Q is the quality factor of the second-order filter stage. The frequency ω_0 is the cutoff frequency, ω_c, of the high-pass filter. Obtain the transfer function of a fourth-order high-pass filter having a cutoff frequency equal to 250 rad/s and a pass-band gain equal to 4. Use $Q = 1$.

Solution:

$$H_{\mathrm{H}}(s) = 4\left(\frac{s^2}{s^2 + \frac{250}{1}s + 250^2}\right)^2 = \frac{4 \cdot s^4}{\left(s^2 + 250s + 62500\right)^2}$$

Section 16.4: Second-Order Filters

P 16.4-1 The circuit shown in Figure P 16.4-1 is a second-order band-pass filter. Design this filter to have $k = 1$, $\omega_0 = 1000$ rad/s, and $Q = 1$.

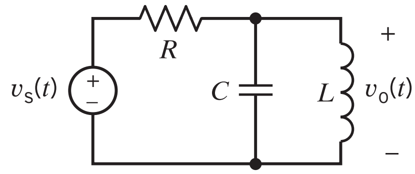


Figure P 16.4-1

Solution

The transfer function is

$$H(s) = \frac{V_o(s)}{V_s(s)} = \frac{\dfrac{sL \times \dfrac{1}{Cs}}{sL + \dfrac{1}{Cs}}}{\dfrac{sL \times \dfrac{1}{Cs}}{sL + \dfrac{1}{Cs}} + R} = \frac{\dfrac{sL}{s^2 LC + 1}}{\dfrac{sL}{s^2 LC + 1} + R} = \frac{sL}{s^2 LCR + sL + R} = \frac{\dfrac{s}{RC}}{s^2 + \dfrac{s}{RC} + \dfrac{1}{LC}}$$

so

$$K = 1, \ \omega_0^2 = \frac{1}{LC} \ \text{ and } \ \frac{1}{RC} = \frac{\omega_0}{Q} \Rightarrow Q = RC\omega_0 = R\sqrt{\frac{C}{L}}$$

Pick $C = 1\ \mu$F. Then $L = \dfrac{1}{C\omega_0^2} = 1$ H and $R = Q\sqrt{\dfrac{L}{C}} = 1000\ \Omega$

P 16.4-2 The circuit shown in Figure P 16.4-2 is a second-order low-pass filter. Design this filter to have k = 1, ω_0 = 200 rad/s, and Q = 0.707.

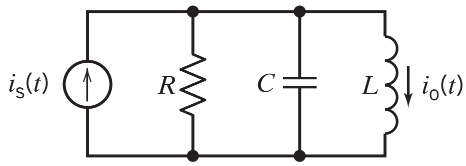


Figure P 16.4-2

Solution:
The transfer function is

$$T(s) = \frac{I_0(s)}{I_s(s)} = \frac{\frac{1}{LC}}{s^2 + \frac{s}{RC} + \frac{1}{LC}}$$

so

$$k = 1 \;,\; \omega_0^2 = \frac{1}{LC} \text{ and } \frac{1}{RC} = \frac{\omega_0}{Q} \Rightarrow Q = RC\omega_0 = R\sqrt{\frac{C}{L}}$$

Pick $C = 1\mu\text{F}$ then $L = \frac{1}{C\omega_0^2} = 25 \text{ H}$ and $R = Q\sqrt{\frac{L}{C}} = 3535 \;\Omega$

P 16.4-3 The circuit shown in Figure P 16.4-3 is a second-order low-pass filter. This filter circuit is called a multiple-loop feedback filter (MFF). The output impedance of this filter is zero, so the MFF low-pass filter is suitable for use as a filter stage in a cascade filter. The transfer function of the low-pass MFF filter is

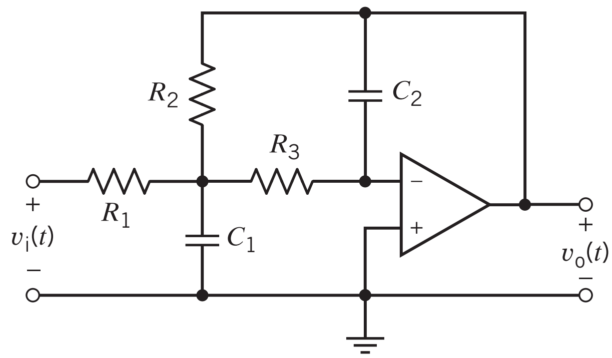


Figure P 16.4-3

$$H_L(s) = \frac{-\dfrac{1}{R_1 R_3 C_1 C_2}}{s^2 + \left(\dfrac{1}{R_1 C_1} + \dfrac{1}{R_2 C_1} + \dfrac{1}{R_3 C_1}\right)s + \dfrac{1}{R_2 R_3 C_1 C_2}}$$

Design this filter to have ω_0 = 2000 rad/s and Q = 8. What is the value of the dc gain?

Hint: Let $R_2 = R_3 = R$ and $C_1 = C_2 = C$. Pick a convenient value of C and calculate R to obtain ω_0 = 2000 rad/s. Calculate R_1 to obtain Q = 8.

Solution:

The transfer function is

$$T(s) = \frac{-\dfrac{1}{R_1 R C^2}}{s^2 + \dfrac{1}{RC}\left(2 + \dfrac{R}{R_1}\right)s + \dfrac{1}{R^2 C^2}}$$

Pick $C = 0.01\ \mu\text{F}$, then

$$\frac{1}{RC} = \omega_0 = 2000 \Rightarrow R = 50000 = 50\ k\Omega$$

$$\frac{\omega_0}{Q} = \frac{1}{RC}\left(2 + \frac{R}{R_1}\right) \Rightarrow R_1 = \frac{R}{Q-2} = 8333 = 8.33\ k\Omega$$

P 16.4-4 The circuit shown in Figure P 16.4-4 is a second-order band-pass filter. This filter circuit is called a multiple-loop feedback filter (MFF). The output impedance of this filter is zero, so the MFF band-pass filter is suitable for use as a filter stage in a cascade filter. The transfer function of the band-pass MFF filter is

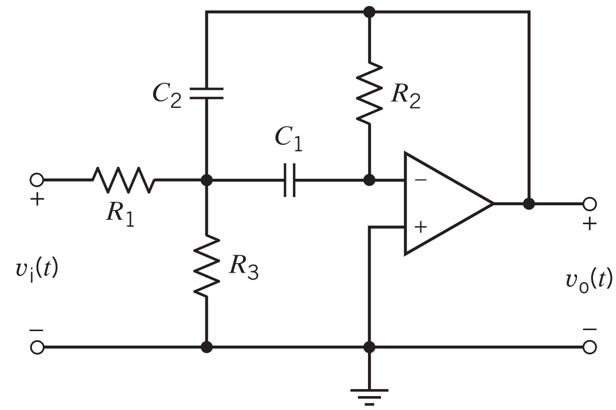


Figure P 16.4-4

$$H_B(s) = \frac{-\dfrac{s}{R_1 C_2}}{s^2 + \left(\dfrac{1}{R_2 C_1} + \dfrac{1}{R_2 C_2}\right)s + \dfrac{R_1 + R_3}{R_1 R_2 R_3 C_1 C_2}}$$

To design this filter, pick a convenient value of C and then use

$$R_1 = \frac{Q}{k\omega_0 C}, \quad R_2 = \frac{2Q}{\omega_0 C}, \quad \text{and} \quad R_3 = \frac{2Q}{\omega_0 C(2Q^2 - k)}$$

Design this filter to have $k = 5$, $\omega_0 = 2000$ rad/s, and $Q = 8$.

Solution: Pick $C = 0.02\ \mu$F. Then $R_1 = 40$ kΩ, $R_2 = 400$ kΩ and $R_3 = 3.252$ kΩ.

P 16.4-5 The circuit shown in Figure P 16.4-5 is a low-pass filter.

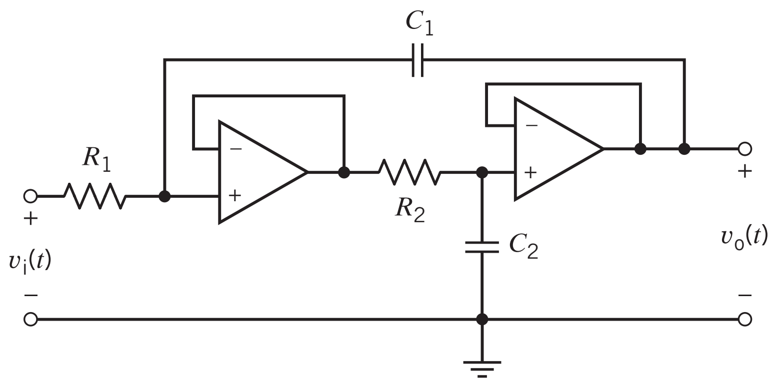


Figure P 16.4-5

The transfer function of this filter is

$$H_L(s) = \frac{\dfrac{1}{R_1 R_2 C_1 C_2}}{s^2 + \dfrac{1}{R_1 C_1} s + \dfrac{1}{R_1 R_2 C_1 C_2}}$$

Design this filter to have $k = 1$, $\omega_0 = 1000$ rad/s, and $Q = 1$.

Solution: Pick $C_1 = C_2 = C = 1\ \mu\text{F}$**. Then**

$$\frac{10^6}{\sqrt{R_1 R_2}} = \omega_0$$

and

$$\frac{1}{R_1 C} = \frac{\omega_0}{Q} \Rightarrow Q = \sqrt{\frac{R_1}{R_2}} \Rightarrow R_2 = \frac{R_1}{Q^2}$$

In this case, since $Q = 1$, we have $R_2 = R_1$ and $R_1 = \dfrac{10^6}{1000} = 1000 = 1\ \text{k}\Omega$

P 16.4-6 The *CR:RC* transformation is used to transform low-pass filter circuits into high-pass filter circuits and vice versa. This transformation is applied to *RC* op amp filter circuits. Each capacitor is replaced by a resistor, while each resistor is replaced by a capacitor. Apply the *CR:RC* transformation to the low-pass filter circuit in Figure P 16.4-5 to obtain the high-pass filter circuit shown in Figure P 16.4-6. Design a high-pass filter to have $k = 1$, $\omega_0 = 1000$ rad/s, and $Q = 1$.

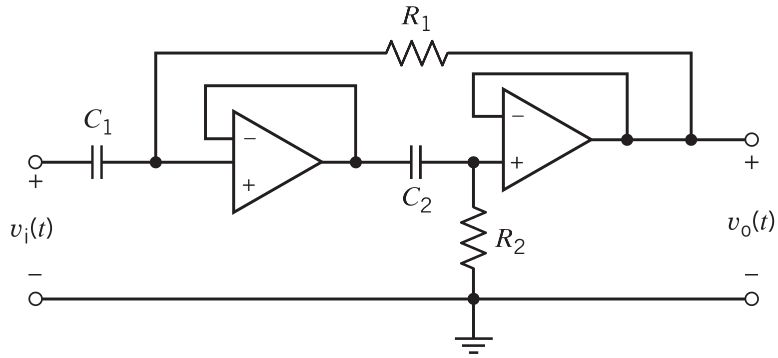
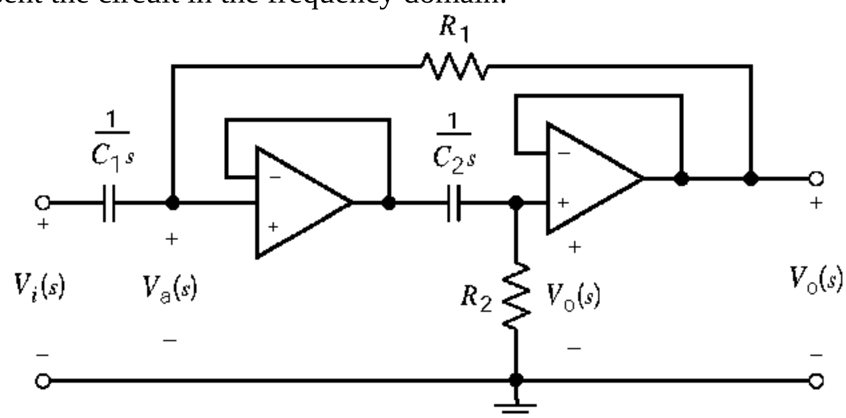


Figure P 16.4-6

P16.4-6 Represent the circuit in the frequency domain:



The node equations are

$$V_0(s) = \frac{R_2}{R_2 + \dfrac{1}{C_2 s}} V_a(s)$$

$$\frac{V_0(s) - V_a(s)}{R_1} - C_1 s\left(V_a(s) - V_i(s)\right) = 0$$

Doing a little algebra

$$\frac{V_o(s)}{R_1} - V_a(s)\left(\frac{1}{R_1} + s\,C_1\right) = -s\,C_1\,V_i(s)$$

Substituting for $V_a(s)$

$$-\frac{V_o(s)}{R_1} - V_o(s)\left(\frac{R_2 C_2 s + 1}{R_2 C_2 s} + \frac{R_1 C_1 s + 1}{R_1}\right) = s C_1 V_i(s)$$

$$\frac{V_o(s)}{V_i(s)} = \frac{(C_1 s) R_1 R_2 C_2 s}{-R_2 C_2 s + (R_2 C_2 s + 1)(R_1 C_1 s + 1)} = \frac{R_1 R_2 C_1 C_2 s^2}{R_1 R_2 C_1 C_2 s^2 + R_1 C_1 s + 1}$$

The transfer function is:

$$H(s) = \frac{V_0(s)}{V_i(s)} = \frac{s^2}{s^2 + \frac{s}{R_2 C_2} + \frac{1}{R_1 R_2 C_1 C_2}}$$

Pick $C_1 = C_2 = C = 1\ \mu$F. Then $\frac{1}{C\sqrt{R_1 R_2}} = \omega_0$ and $\frac{1}{R_2 C} = \frac{\omega_0}{Q} \Rightarrow Q = \sqrt{\frac{R_2}{R_1}} \Rightarrow R_1 Q^2 = R_2$.

In this case $R_1 = R_2 = R$ and $\frac{1}{C\,R} = \omega_0 \Rightarrow R = 1000\ \Omega$.

P 16.4-7 We have seen that transfer functions can be frequency scaled by replacing s by s/k_f each time that it occurs. Alternately, circuits can also be frequency scaled by dividing each capacitance and each inductance by the frequency scaling factor k_f. Either way, the effect is the same. The frequency response is shifted to the right by k_f. In particular, all cutoff, corner, and resonant frequencies are multiplied by k_f. Suppose that we want to change the cutoff frequency of a filter circuit from ω_{old} to ω_{new}. We set the frequency scale factor to

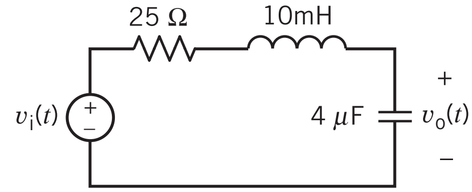


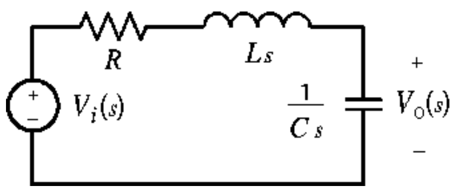
Figure P 16.4-7

$$k_f = \frac{\omega_{new}}{\omega_{old}}$$

and then divide each capacitance and each inductance by k_f. Use frequency scaling to change the cutoff frequency of the circuit in Figure P 16.4-7 to 250 rad/s.

Answer: $k_f = 0.05$.

Solution:



$$T(s) = \frac{V_0(s)}{V_i(s)} = \frac{\frac{1}{Cs}}{Ls + R + \frac{1}{Cs}} = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

When $R = 25\ \Omega$, $L = 10^{-2}$ H and $C = 4\times10^{-6}$ F, then the transfer function is

$$H(s) = \frac{25\times10^6}{s^2 + 2500\,s + 25\times10^6}$$

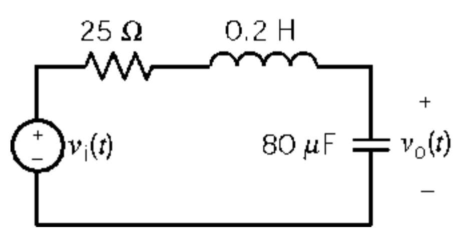
so

$$\omega_{old} = \sqrt{25\times10^6} = 5000$$

and

$$k_f = \frac{\omega_{new}}{\omega_{old}} = \frac{250}{5000} = 0.05$$

The scaled circuit is



P 16.4-8 Impedance scaling is used to adjust the impedances of a circuit. Let k_m denote the impedance scaling factor. Impedance scaling is accomplished by multiplying each impedance by k_m. That means that each resistance and each inductance is multiplied by k_m, but each capacitance is divided by k_m. Transfer functions of the form $H(s) = \frac{V_0(s)}{V_i(s)}$ or $H(s) = \frac{I_0(s)}{I_i(s)}$ are not changed at all by impedance scaling. Transfer functions of the form $H(s) = \frac{V_0(s)}{I_i(s)}$ are multiplied by k_m, whiletransfer functions of the form $H(s) = \frac{I_0(s)}{I_i(s)}$ are divided by k_m. Use impedance scaling to change the values of the capacitances in the filter shown in Figure P 16.4-8 so that the capacitances are in the range 0.01 μF to 1.0 μF. Calculate the transfer function before and after impedance scaling.

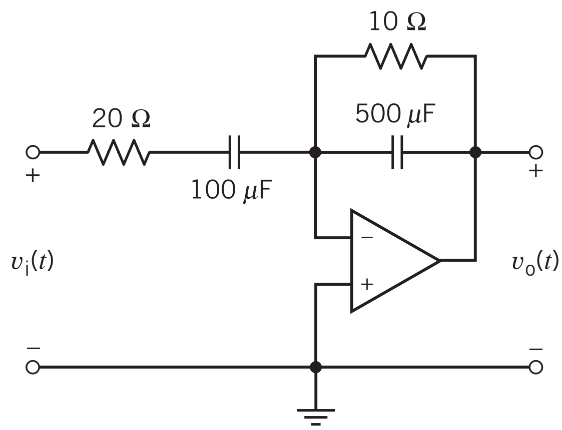
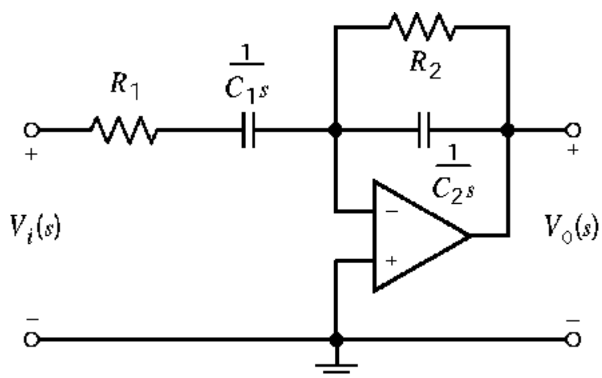


Figure P 16.4-8

Solution:



The transfer function of this circuit is

$$H(s)=\frac{V_0(s)}{V_i(s)}=-\frac{\dfrac{R_2}{1+R_2C_2s}}{R_1+\dfrac{1}{C_1s}}=-\frac{R_2C_1s}{(1+R_1C_1s)(1+R_2C_2s)}=-\frac{\dfrac{1}{R_1C_2}s}{s^2+\left(\dfrac{1}{R_1C_1}+\dfrac{1}{R_2C_2}\right)s+\dfrac{1}{R_1R_2C_1C_2}}$$

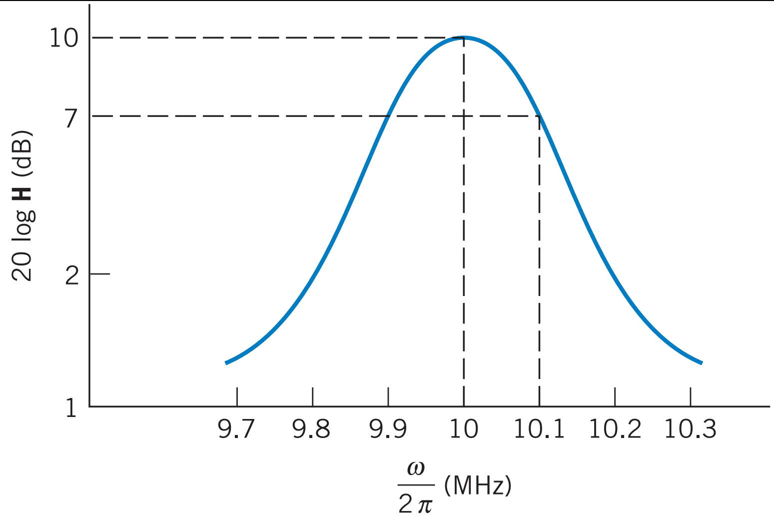
Pick $k_m = 1000$ so that the scaled capacitances will be $\dfrac{100\ \mu\text{F}}{1000}=0.1\ \mu\text{F}$ and $\dfrac{500\ \mu\text{F}}{1000}=0.5\ \mu\text{F}$.

Before scaling $\left(R_1=20\ \Omega,\ C_1=100\ \mu\text{F},\ R_2=10\ \Omega \text{ and } C_2=500\ \mu\text{F}\right)$

$$H(s)=\frac{-100\,s}{s^2+700\,s+10^5}$$

After scaling $\left(R_1=20000\ \Omega=20\ \text{k}\Omega,\ C_1=0.1\ \mu\text{F},\ R_2=10000\ \Omega=10\ \text{k}\Omega,\ C_2=0.5\ \mu\text{F}\right)$

$$H(s)=\frac{-100s}{s^2+700\,s+10^5}$$

P 16.4-9 A band-pass amplifier has the frequency response shown in Figure P 16.4-9. Find the transfer function, $H(s)$. ***Hint:*** $\omega_0 = 2\pi(10 \text{ MHz})$, $k = 10$ dB = 3.16, $BW = 0.2$ MHz, $Q = 50$	 **Figure P 16.4-9**

P16.4-9

This is the frequency response of a bandpass filter, so

$$H(s) = \frac{k\frac{\omega_0}{Q}s}{s^2 + \frac{\omega_0}{Q}s + {\omega_0}^2}$$

From peak of the frequency response

$$\omega_0 = 2\pi \times 10 \times 10^6 = 62.8 \times 10^6 \text{ rad/s and } k = 10 \text{ dB} = 3.16$$

Next

$$\frac{\omega_0}{Q} = \text{BW} = (10.1\times10^6 - 9.9\times10^6)\ 2\pi = (0.2\times10^6)2\pi = 1.26\times10^6 \text{ rad/s}$$

So the transfer function is

$$H(s) = \frac{3.16(1.26)10^6 s}{s^2 + (1.26)10^6 s + 62.8^2 \times 10^{12}} = \frac{(3.98)10^6 s}{s^2 + (1.26)10^6 s + 3.944\times10^{15}}$$

P 16.4-10 A band-pass filter can be achieved using the circuit of Figure P 16.4-10. Find
(a) the magnitude of $\mathbf{H} = \mathbf{V}_o/\mathbf{V}_s$,
(b) the low- and high-frequency cutoff frequencies ω_1 and ω_2, and
(c) the pass-band gain when $\omega_1 \ll \omega \ll \omega_2$,

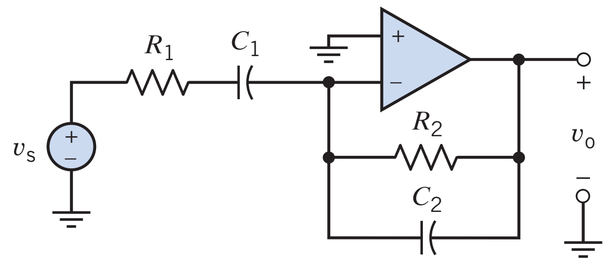


Figure P 16.4-10

Answer:

(b) $\omega_1 = \dfrac{1}{R_1 C_1}$ and $\omega_2 = \dfrac{1}{R_2 C_2}$, (c) pass-band gain $= \dfrac{R_2}{R_1}$

Solution:

(a)

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_0(\omega)}{\mathbf{V}_s(\omega)} = -\frac{\mathbf{Z}_2}{\mathbf{Z}_1} \quad \text{where } \mathbf{Z}_1 = R_1 - \frac{j}{\omega C_1} \text{ and } \mathbf{Z}_2 = \frac{R_2 \times \dfrac{1}{j\omega C_2}}{R_2 + \dfrac{1}{j\omega C_2}}$$

$$\therefore \mathbf{H}(\omega) = -\frac{j\omega R_2 C_1}{\left(1+\dfrac{j\omega}{\omega_1}\right)\left(1+\dfrac{j\omega}{\omega_2}\right)} \quad \text{where } \omega_1 = \frac{1}{R_1 C_1},\ \omega_2 = \frac{1}{R_2 C_2}$$

(b) $$\omega_1 = \frac{1}{R_1 C_1},\ \omega_2 = \frac{1}{R_2 C_2}$$

(c)

$$|\mathbf{H}(\omega)| = \left| -\frac{j\omega R_2 C_1}{\left(0+\dfrac{j\omega}{\omega_1}\right)(1+0)} \right| = \frac{\omega R_2 C_1}{\dfrac{\omega}{\omega_1}} = \omega_1 R_2 C_1 = \frac{R_2}{R_1}$$

P 16.4-11 A unity gain, low-pass filter is obtained from the operational amplifier circuit shown in Figure P 16.4-11. Determine the network function $\mathbf{H}(\omega) = \mathbf{V}_o/\mathbf{V}_s$.

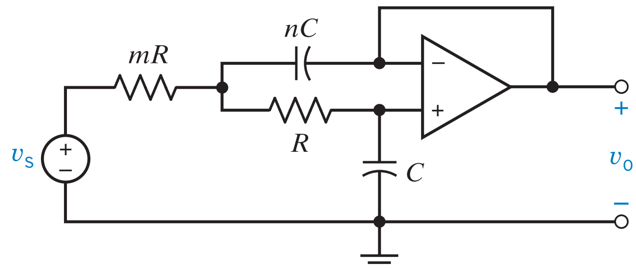
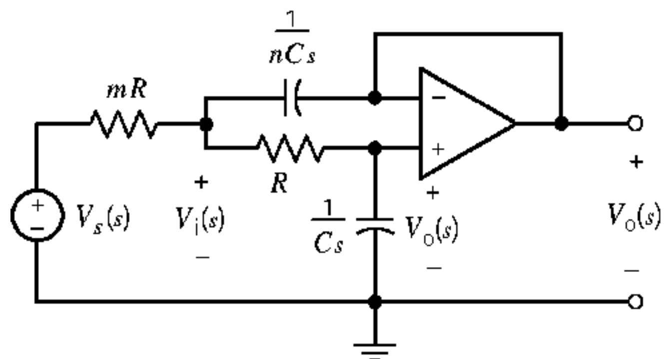


Figure P 16.4-11

Solution:



Voltage division:

$$V_0(s) = \frac{\frac{1}{Cs}}{R + \frac{1}{Cs}} V_1(\mathrm{s}), \ \Rightarrow \ V_1(s) = (1 + sRC)V_0(s)$$

KCL:

$$\frac{V_1 - V_s}{mR} + \frac{V_1 - V_0}{R} + (V_1 - V_0) nCs = 0$$

Combining these equations gives:

$$V_0 \left[\frac{1}{mR} + sC + \frac{sC}{m} + s^2 nRC^2 \right] = \frac{V_s}{mR}$$

Therefore

$$\mathbf{H}(\omega) = \frac{\mathrm{V}_0(\omega)}{\mathrm{V}_\mathrm{S}(\omega)} = \frac{1}{1 + s(m+1)RC + nm R^2 C^2 s^2} = \frac{1}{1 - \left(\frac{\omega}{\omega_0}\right)^2 + j\frac{\omega}{Q\omega_0}}$$

where $\omega_0 = \frac{1}{\sqrt{mn}\, RC}$ and $Q = \frac{\sqrt{mn}}{m+1}$

P 16.4-12 A particular acoustic sensor produces a sinusoidal output having a frequency equal to 5 kHz. The signal from the sensor has been corrupted with noise. Figure P 16.4-12. shows a band-pass filter that was designed to recover the sensor signal from the noise. The voltage v_S represents the noisy signal from the sensor. The filter output, v_o, should be a less noisy signal. Determine the center frequency and bandwidth of this band-pass filter. Assume that the op amp is ideal.

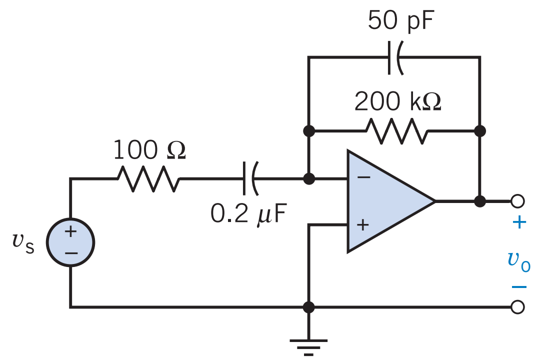
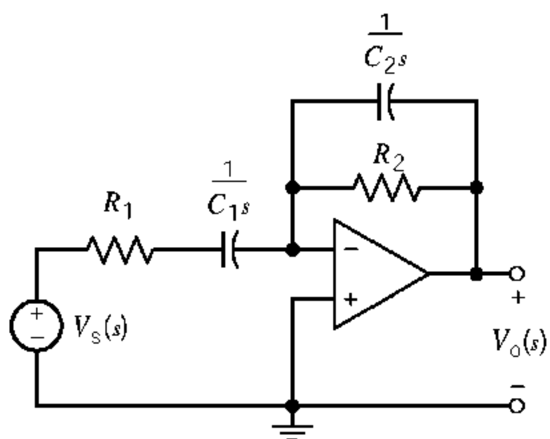


Figure P 16.4-12

Solution:



$$H(s)=\frac{V_0(s)}{V_S(s)}=-\frac{R_2\left\|\frac{1}{C_2 s}\right.}{R_1+\frac{1}{C_1 s}}=-\frac{\frac{R_2}{R_2 C_2 s+1}}{\frac{R_1 C_1 s+1}{C_1 s}}=\frac{-\frac{1}{R_1 C_2}s}{s^2+\left(\frac{1}{R_1 C_1}+\frac{1}{R_2 C_2}\right)s+\frac{1}{R_1 R_2 C_1 C_2}}$$

Substituting the element values $R_1=100\ \Omega$, $R_2=200\ \Omega$, $C_1=0.02\ \mu\text{F}$ and $C_2=50\ \text{pF}$ we determine the center frequency and bandwidth of the filter to be

$$\omega_0=\frac{1}{\sqrt{R_1 R_2 C_1 C_2}}=70.7\ \text{k rad/sec}=2\pi(11.25\ \text{kHz})$$

$$BW=\frac{\omega_0}{Q}=\frac{1}{R_1 C_1}+\frac{1}{R_2 C_2}=150\ \text{k rad/s}=2\pi(23.9\ \text{kHz})$$

Section 16.5: High-Order Filters

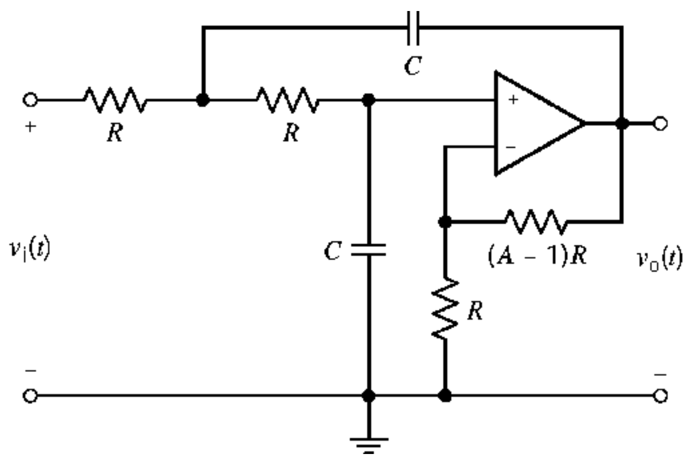
P 16.5-1 Design a low-pass filter circuit that has the transfer function

$$H_L(s) = \frac{628^3}{(s+628)(s^2+628s+628^2)}$$

Answer: See Figure SP 16-1

Solution:
This filter is designed as a cascade connection of a Sallen-Key low-pass filter designed as described in Table 16.4-2 and a first-order low-pass filter designed as described in Table 16.5-2.

Sallen-Key Low-Pass Filter:



MathCad Spreadsheet (p16_5_1_sklp.mcd)

The transfer function is of the form $H(s) = \frac{c}{s^2 + bs + a}$.

Enter the transfer function coefficitents: $a := 628^2$ $b := 628$

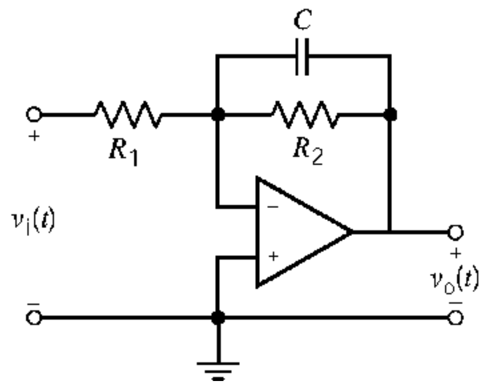
Determine the Filter Specifications: $\omega_0 := \sqrt{a}$ $Q := \frac{\omega_0}{b}$ $\omega_0 = 628$ $Q = 1$

Pick a convenient value for the capacitance: $C := 0.1 \cdot 10^{-6}$

Calculate resistance values: $R := \frac{1}{C \cdot \omega_0}$ $A := 3 - \frac{1}{Q}$ $R = 1.592 \times 10^4$ $R \cdot (A-1) = 1.592 \times 10^4$

Calculate the dc gain. $A = 2$

First-Order Low-Pass Filter:



MathCad Spreadsheet (p16_5_1_1stlp.mcd)

The transfer function is of the form $H(s) = \frac{-k}{s + p}$.

Enter the transfer function coefficitents: $p := 628$ $k := 0.5p$

Pick a convenient value for the capacitance: $C := 0.1 \cdot 10^{-6}$

Calculate resistance values: $R2 := \frac{1}{C \cdot p}$ $R1 := \frac{1}{C \cdot k}$ $R1 = 3.185 \times 10^{4}$ $R2 = 1.592 \times 10^{4}$

P 16.5-2 Design a filter that has the transfer function

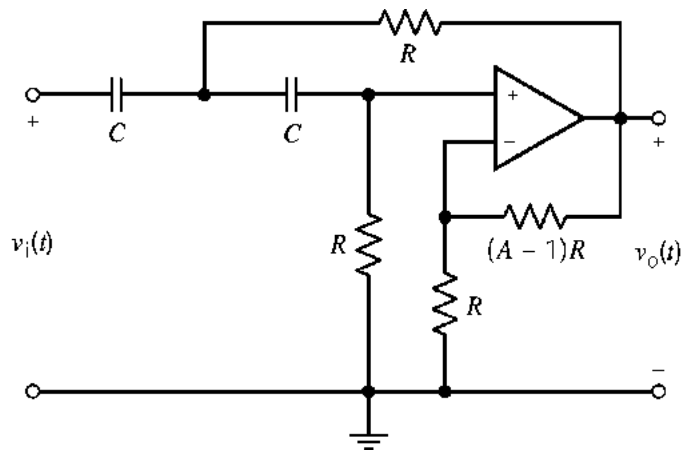
$$H_H(s) = \frac{5 \cdot s^3}{(s + 100)(s^2 + 100s + 10{,}000)}$$

Answer: See Figure SP 16-2.

Solution: This filter is designed as a cascade connection of a Sallen-key high-pass filter, designed as described in Table 16.4-2, and a first-order high-pass filter, designed as described in Table 16.5-2.

The passband gain of the Sallen key stage is 2 and the passband gain of the first-order stage is 2.5 So the overall passband gain is $2 \times 2.5 = 5$

Sallen-Key High-Pass Filter:



MathCad Spreadsheet (p16_5_2_skhp.mcd)

The transfer function is of the form H(s) = $\frac{A s^2}{s^2 + bs + a}$.

Enter the transfer function coefficitents: $a := 10000$ $b := 100$

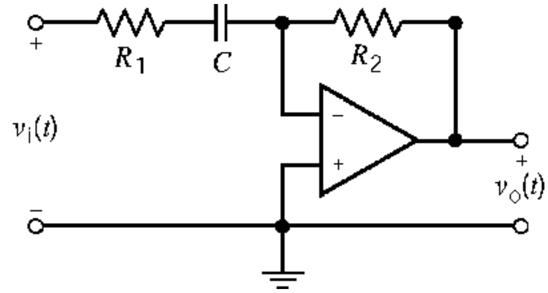
Determine the Filter Specifications: $\omega_0 := \sqrt{a}$ $Q := \frac{\omega_0}{b}$ $\omega_0 = 100$ $Q = 1$

Pick a convenient value for the capacitance: $C := 0.1 \cdot 10^{-6}$

Calculate resistance values: $R := \frac{1}{C \cdot \omega_0}$ $A := 3 - \frac{1}{Q}$ $R = 1 \times 10^5$ $R \cdot (A - 1) = 1 \times 10^5$

Calculate the passband gain. $A = 2$

First-Order High-Pass Filter:



MathCad Spreadsheet (p16_5_2_1sthp.mcd)

The transfer function is of the form H(s) = $\dfrac{-ks}{s + p}$.

Enter the transfer function coefficitents: $p := 100$ $k := 2.5$

Pick a convenient value for the capacitance: $C := 0.1 \cdot 10^{-6}$

Calculate resistance values: $R1 := \dfrac{1}{C \cdot p}$ $R2 := k \cdot R1$ $R1 = 1 \times 10^{5}$ $R2 = 2.5 \times 10^{5}$

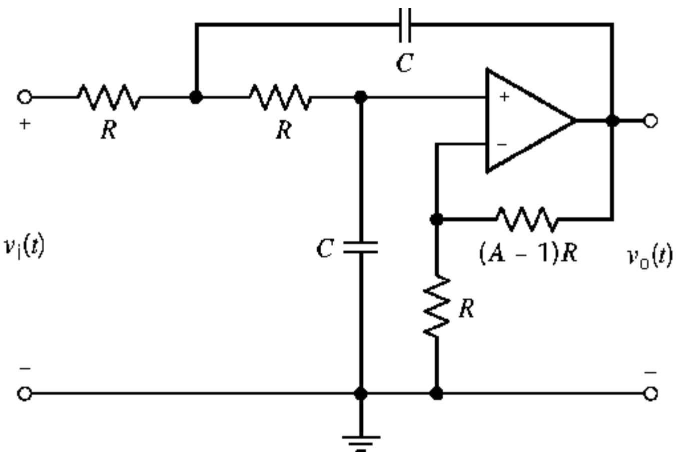
P 16.5-3 Design a filter that has the transfer function

$$H_B(s) = \frac{16{,}000{,}000 \cdot s^2}{(s^2 + 141.4s + 10{,}000)(s^2 + 2828s + 4{,}000{,}000)}$$

Answer: See Figure SP 16-3.

Solution: This filter is designed as a cascade connection of a Sallen-key low-pass filter, a Sallen-key high-pass filter and an inverting amplifier.

Sallen-Key Low-Pass Filter:



MathCad Spreadsheet (p16_5_3_sklp.mcd)

The transfer function is of the form $H(s) = \frac{c}{s^2 + bs + a}$.

Enter the transfer function coefficitents: $a := 4000000$ $b := 2828$

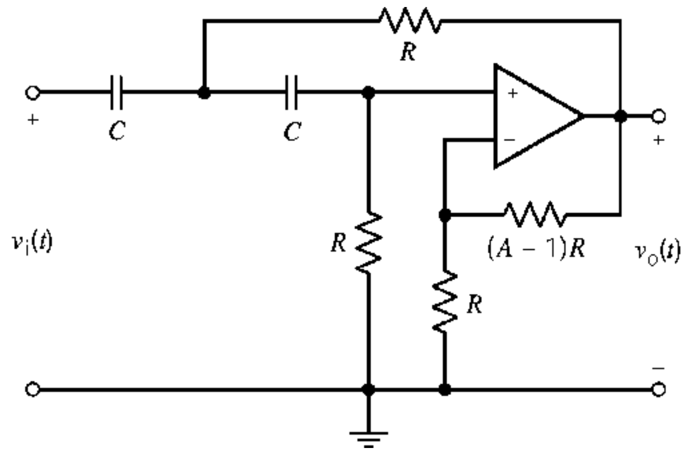
Determine the Filter Specifications: $\omega_0 := \sqrt{a}$ $Q := \frac{\omega_0}{b}$ $\omega_0 = 2 \times 10^3$ $Q = 0.707$

Pick a convenient value for the capacitance: $C := 0.1 \cdot 10^{-6}$

Calculate resistance values: $R := \frac{1}{C \cdot \omega_0}$ $A := 3 - \frac{1}{Q}$ $R = 5 \times 10^3$ $R \cdot (A - 1) = 2.93 \times 10^3$

Calculate the dc gain. $A = 1.586$

Sallen-Key High-Pass Filter:



MathCad Spreadsheet (p16_5_3_skhp.mcd)

The transfer function is of the form H(s) = $\dfrac{c\ s\hat{}2}{s\hat{}2 + bs + a}$.

Enter the transfer function coefficitents: $a := 10000$ $b := 141.4$

Determine the Filter Specifications: $\omega_0 := \sqrt{a}$ $Q := \dfrac{\omega_0}{b}$ $\omega_0 = 100$ $Q = 0.707$

Pick a convenient value for the capacitance: $C := 0.1 \cdot 10^{-6}$

Calculate resistance values: $R := \dfrac{1}{C \cdot \omega_0}$ $A := 3 - \dfrac{1}{Q}$ $R = 1 \times 10^5$ $R \cdot (A-1) = 5.86 \times 10^4$

Calculate the passband gain. $A = 1.586$

Amplifier: The required passband gain is $\dfrac{1.6\times10^6}{141.4\times2828} = 4.00$. An amplifier with a gain equal to $\dfrac{4.0}{2.515} = 1.59$ is needed to achieve the specified gain.

P 16.5-4 Design a filter that has the transfer function

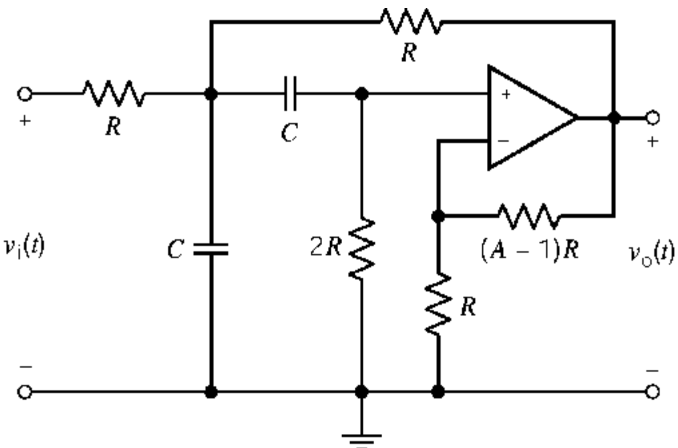
$$H_B(s) = \frac{250{,}000s^2}{(s^2 + 250s + 62{,}500)^2}$$

Answer: See Figure SP 16-4.

Solution:

This filter is designed as the cascade connection of two identical Sallen-key bandpass filters:

Sallen-Key BandPass Filter:



MathCad Spreadsheet (p16_5_4_skbp.mcd)

The transfer function is of the form $H(s) = \dfrac{cs}{s^2 + bs + a}$.

Enter the transfer function coefficitents: $a := 62500$ $b := 250$

Determine the Filter Specifications: $\omega_0 := \sqrt{a}$ $Q := \dfrac{\omega_0}{b}$ $\omega_0 = 250$ $Q = 1$

Pick a convenient value for the capacitance: $C := 0.1 \cdot 10^{-6}$

Calculate resistance values: $R := \dfrac{1}{C \cdot \omega_0}$ $A := 3 - \dfrac{1}{Q}$

$R = 4 \times 10^4$ $2 \cdot R = 8 \times 10^4$ $R \cdot (A - 1) = 4 \times 10^4$

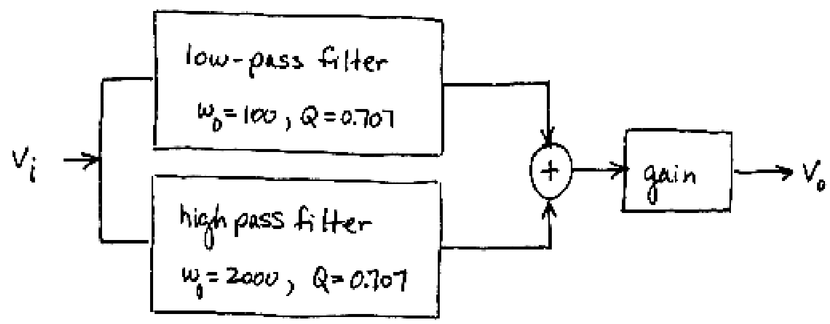
Calculate the pass-band gain. $A \cdot Q = 2$

P 16.5-5 Design a filter that has the transfer function

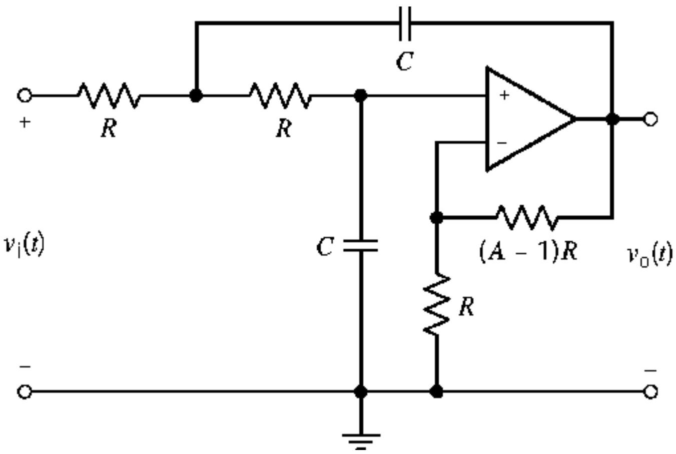
$$H_N(s) = \frac{2s^2}{(s^2 + 2828s + 4{,}000{,}000)} + \frac{20{,}000}{(s^2 + 141.4s + 10{,}000)}$$

Answer: See Figure SP 16-5.

Solution: This filter is designed using this structure:



Sallen-Key Low-Pass Filter:



MathCad Spreadsheet (p16_5_5_sklp.mcd)

The transfer function is of the form H(s) = $\frac{c}{s^2 + bs + a}$.

Enter the transfer function coefficitents: $a := 10000$ $b := 141.4$

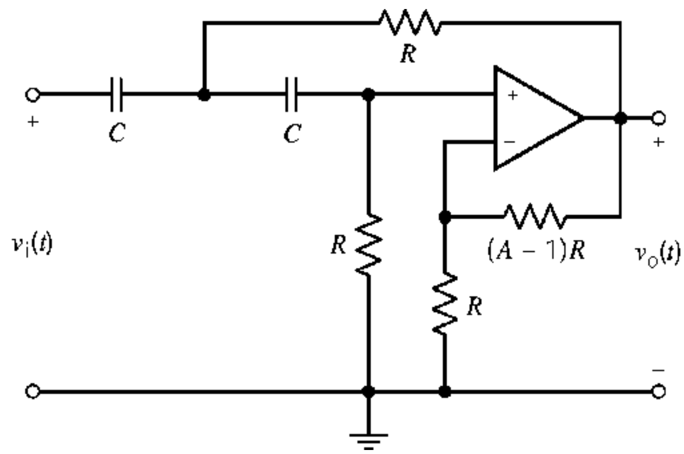
Determine the Filter Specifications: $\omega_0 := \sqrt{a}$ $Q := \frac{\omega_0}{b}$ $\omega_0 = 100$ $Q = 0.707$

Pick a convenient value for the capacitance: $C := 0.1 \cdot 10^{-6}$

Calculate resistance values: $R := \frac{1}{C \cdot \omega_0}$ $A := 3 - \frac{1}{Q}$ $R = 1 \times 10^5$ $R \cdot (A - 1) = 5.86 \times 10^4$

Calculate the dc gain. $A = 1.586$

Sallen-Key High-Pass Filter:



MathCad Spreadsheet (p16_5_5_skhp.mcd)

The transfer function is of the form H(s) = $\dfrac{c\,s^2}{s^2 + bs + a}$.

Enter the transfer function coefficitents: $a := 4000000$ $b := 2828$

Determine the Filter Specifications: $\omega_0 := \sqrt{a}$ $Q := \dfrac{\omega_0}{b}$ $\omega_0 = 2\times 10^3$ $Q = 0.707$

Pick a convenient value for the capacitance: $C := 0.1\cdot 10^{-6}$

Calculate resistance values: $R := \dfrac{1}{C\cdot\omega_0}$ $A := 3 - \dfrac{1}{Q}$ $R = 5\times 10^3$ $R\cdot(A-1) = 2.93\times 10^3$

Calculate the passband gain. $A = 1.586$

Amplifier: The required gain is 2, but both Sallen-Key filters have passband gains equal to 1.586. The amplifier has a gain of $\dfrac{2}{1.586} = 1.26$ to make the passband gain of the entire filter equal to 2.

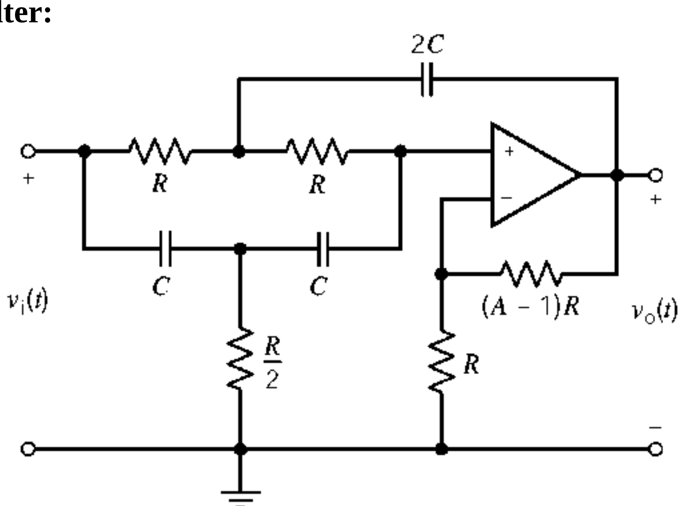
P 16.5-6 Design a filter that has the transfer function

$$H_N(s) = \frac{4(s^2 + 62{,}500)^2}{(s^2 + 250s + 62{,}500)^2}$$

Answer: See Figure SP 16-6.

Solution:
This filter is designed as the cascade connection of two identical Sallen-key notch filters.

Sallen-Key Notch Filter:



MathCad Spreadsheet (p16_5_6_skn.mcd)

The transfer function is of the form H(s) = $\frac{c(s^2 + a)}{s^2 + bs + a}$.

Enter the transfer function coefficitents: $a := 62500$ $b := 250$

Determine the Filter Specifications: $\omega_0 := \sqrt{a}$ $Q := \frac{\omega_0}{b}$ $\omega_0 = 250$ $Q = 1$

Pick a convenient value for the capacitance: $C := 0.1 \cdot 10^{-6}$ $2 \cdot C = 2 \times 10^{-7}$

Calculate resistance values: $R := \frac{1}{C \cdot \omega_0}$ $A := 2 - \frac{1}{2 \cdot Q}$

$R = 4 \times 10^4$ $\frac{R}{2} = 2 \times 10^4$ $R \cdot (A - 1) = 2 \times 10^4$

Calculate the pass-band gain. $A = 1.5$

Amplifier: The required passband gain is 4. An amplifier having gain equal to $\frac{4}{(1.5)(1.5)} = 1.78$ is needed to achieve the required gain.

P 16-5-7

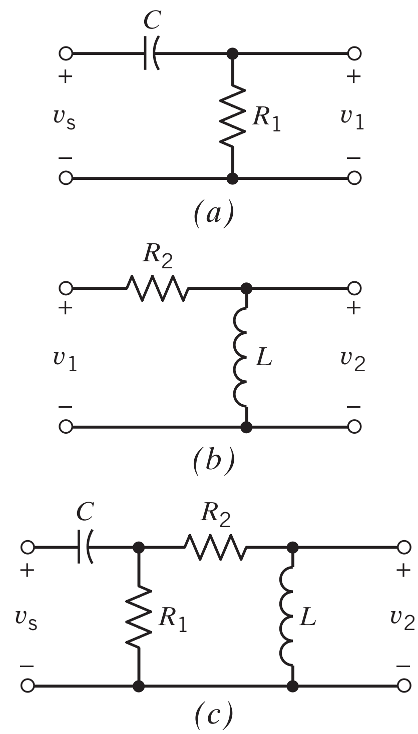
(a) For the circuit of Figure P 16.5-7*a*, derive an expression for the transfer function $H_a(s) = V_1/V_s$.

(b) For the circuit of Figure P 16.5-7*b*, derive an expression for the transfer function $H_b(s) = V_2/V_1$.

(c) Each of the above filters is a first-order filter. The circuit of Figure P 16.5-7*c* is the cascade connection of the circuits of Figure P 16.5-7*a* and Figure P 16.5-7*b*. Derive an expression for the transfer function $H_c(s) = V_2/V_s$ of the second-order circuit in Figure P 16.5-7*c*.

(d) Why doesn't $H_c(s) = H_a(s)\,H_b(s)$?

Hint: Consider loading.



P 16.5-7

Solution:

(a) Using voltage division: $$H_a(s) = \frac{V_1(s)}{V_s(s)} = \frac{R_1}{R_1 + \frac{1}{Cs}} = \frac{R_1 C s}{1 + R_1 C s}$$

(b) Using voltage division again: $$H_b(s) = \frac{V_2(s)}{V_1(s)} = \frac{Ls}{R_2 + Ls}$$

(c) Voltage division again: $$H_c(s) = \frac{V_2(s)}{V_s(s)} = \frac{R_1 \,\|\, (R_2 + Ls)}{\frac{1}{Cs} + R_1 \,\|\, (R_2 + Ls)} \times \frac{Ls}{R_2 + Ls}$$

Doing some algebra:

$$H_c(s) = \frac{V_2(s)}{V_s(s)} = \frac{\frac{R_1 \times (R_2 + Ls)}{R_1 + (R_2 + Ls)}}{\frac{1}{Cs} + \frac{R_1 \times (R_2 + Ls)}{R_1 + (R_2 + Ls)}} \times \frac{Ls}{R_2 + Ls}$$

$$= \frac{R_1 R_2 C s + R_1 L C s^2}{R_1 R_2 C s + R_1 L C s^2 + R_1 + R_2 + Ls} \times \frac{Ls}{R_2 + Ls}$$

$$= \frac{R_1 C s \left(R_2 + L s\right)}{R_1 L C s^2 + \left(R_1 R_2 C + L\right) s + R_1 + R_2} \times \frac{L s}{R_2 + L s}$$

$$= \frac{R_1 L C s^2}{R_1 L C s^2 + \left(R_1 R_2 C + L\right) s + R_1 + R_2}$$

(d) $H_c(s) \neq H_a(s) \times H_b(s)$ because the $R_2, L s$ voltage divider loads the $\frac{1}{C s}$, R_1 voltage divider.

P 16.5-8 Two filter stages are connected in cascade as shown in Figure P 16.5-8. The transfer function of each filter stage is of the form

$$H(s) = \frac{As}{(1 + s/\omega_L)(1 + s/\omega_H)}$$

Determine the transfer function of the fourth-order filter. (Assume that there is no loading.)

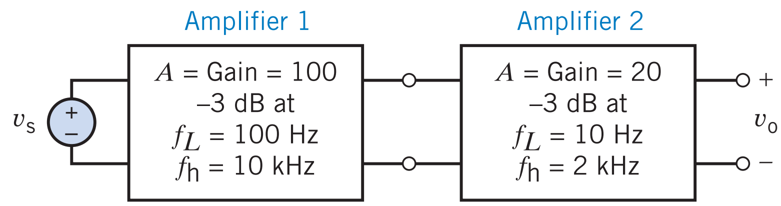
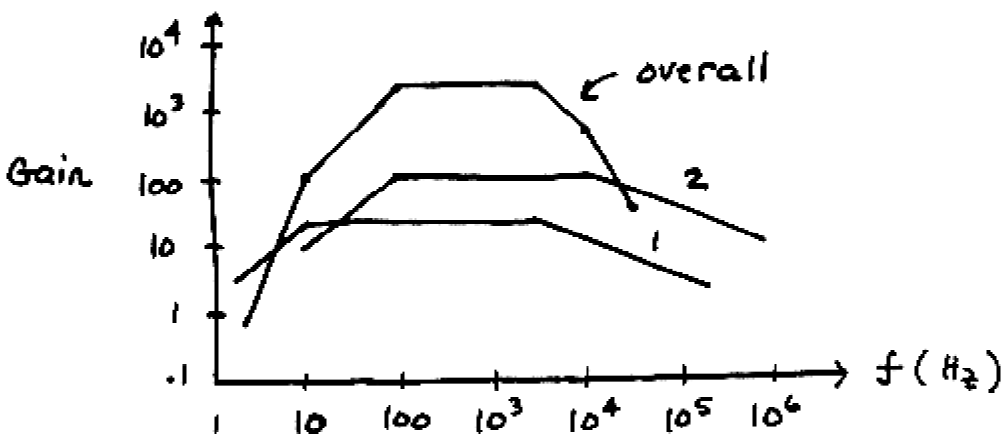


Figure P 16.5-8

Solution:

$$H(s) = \frac{100}{\left(1+\frac{s}{200\pi}\right)\left(1+\frac{s}{20,000\pi}\right)} \times \frac{20}{\left(1+\frac{s}{20\pi}\right)\left(1+\frac{s}{4000\pi}\right)}$$

$$= \frac{2000}{\left(1+\frac{s}{20\pi}\right)\left(1+\frac{s}{200\pi}\right)\left(1+\frac{s}{4000\pi}\right)\left(1+\frac{s}{20,000\pi}\right)}$$



P 16.5-9 A second-order filter uses two identical first-order filter stages as shown in Figure P 16.5-9. Each filter stage is specified to have a cutoff or break frequency at ω_c = 1000 and a pass-band gain of 0 dB. (a) Find the required R_1, R_2, and C. (b) Find the gain of the second-order filter at ω = 10,000 in decibels.

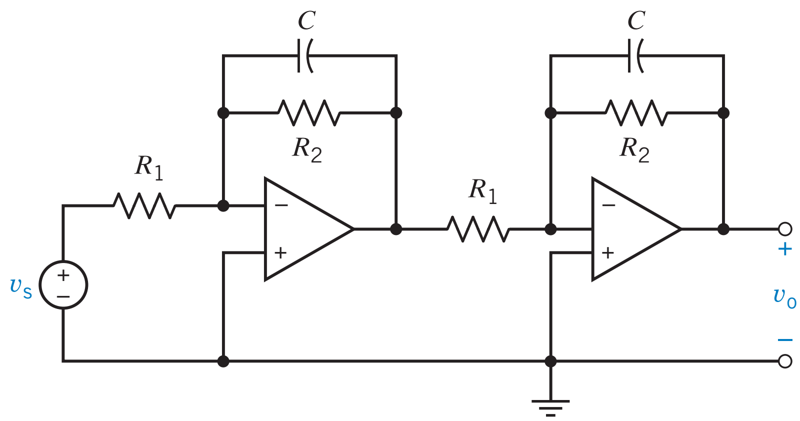


Figure P 16.5-9

Solution:

(a) The transfer function of each stage is

$$H_i(s) = -\frac{R_2 \| \dfrac{1}{C\,s}}{R_1} = -\frac{\dfrac{R_2 \times \dfrac{1}{C\,s}}{R_2 + \dfrac{1}{C\,s}}}{R_1} = -\frac{\dfrac{R_2}{1+R_2\,C\,s}}{R_1} = -\frac{\dfrac{R_2}{R_1}}{1+R_2\,C\,s}$$

The specification that the dc gain is 0 dB = 1 requires $R_2 = R_1$.

The specification of a break frequency of 1000 rad/s requires $\dfrac{1}{R_2\,C} = 1000$.

Pick $C = 0.1\ \mu\text{F}$. Then $R_2 = 10\ \text{k}\Omega$ so $R_1 = 10\ \text{k}\Omega$.

(b)

$$\mathbf{H}(\omega) = \frac{-1}{1+j\dfrac{\omega}{1000}} \times \frac{-1}{1+j\dfrac{\omega}{1000}} \quad \Rightarrow \quad \left|\mathbf{H}(10,000)\right| = \left(\frac{1}{\sqrt{1+10^2}}\right)^2 = \frac{1}{101} = -40.1\ \text{dB}$$

Section 16.7 How Can We Check…?

P 16.7-1 The specifications for a band-pass filter require that ω_0 = 100 rad/s, Q = 5, and k = 3. The transfer function of a filter designed to satisfy these specifications is

$$H(s) = \frac{75s}{s^2 + 25s + 10{,}000}$$

Does this filter satisfy the specifications?

Solution:

$$\omega_0 = \sqrt{10000} = 100 \text{ rad/}s \quad \text{and} \quad \frac{\omega_0}{Q} = 25 \quad \Rightarrow \ Q = \frac{100}{25} = 4 \neq 5$$

This filter does <u>not</u> satisfy the specifications.

P 16.7-2 The specifications for a band-pass filter require that ω_0 = 100 rad/s, Q = 4, and k = 3. The transfer function of a filter designed to satisfy these specifications is

$$H(s) = \frac{75s}{s^2 + 25s + 10{,}000}$$

Does this filter satisfy the specifications?

Solution:

$$\omega_0 = \sqrt{10000} = 100 \text{ rad/}s, \quad \frac{\omega_0}{Q} = 25 \Rightarrow Q = \frac{100}{25} = 4 \text{ and } \quad k = \frac{75}{25} = 3$$

This filter does satisfy the specifications.

P 16.7-3 The specifications for a low-pass filter require that ω_0 = 20 rad/s, Q = .8, and k = 1.5. The transfer function of a filter designed to satisfy these specifications is

$$H(s) = \frac{600}{s^2 + 25s + 400}$$

Does this filter satisfy the specifications?

Solution:

$$\omega_0 = \sqrt{400} = 20 \text{ rad/s}, \quad \frac{\omega_0}{Q} = 25 \Rightarrow Q = \frac{20}{25} = 0.8 \text{ and } \quad k = \frac{600}{400} = 1.5$$

This filter does satisfy the specifications.

P 16.7-4 The specifications for a low-pass filter require that ω_0 = 25 rad/s, Q = .4, and k = 1.2. The transfer function of a filter designed to satisfy these specifications is

$$H(s) = \frac{750}{s^2 + 62.5s + 625}$$

Does this filter satisfy the specifications?

Solution:

$$\omega_0 = \sqrt{625} = 25 \text{ rad/s}, \ \frac{\omega_0}{Q} = 62.5 \Rightarrow Q = \frac{25}{62.5} = 0.4 \text{ and } k = \frac{750}{625} = 1.2$$

This filter does satisfy the specifications.

P 16.7-5 The specifications for a high-pass filter require that ω_0 = 12 rad/s, Q = 4, and k = 5. The transfer function of a filter designed to satisfy these specifications is

$$H(s) = \frac{5s^2}{s^2 + 30s + 144}$$

Does this filter satisfy the specifications?

Solution:

$$\omega_0 = \sqrt{144} = 12 \text{ rad/s and } \frac{\omega_0}{Q} = 30 \Rightarrow Q = \frac{12}{30} = 0.4$$

This filter does <u>not</u> satisfy the specifications.

PSpice Problems

SP 16-1 The filter circuit shown in Figure SP 16-1 was designed to have the transfer function

$$H_L(s) = \frac{628^3}{(s+628)(s^2+628s+628^2)}$$

Use PSpice to verify that the filter circuit does indeed implement this transfer function.

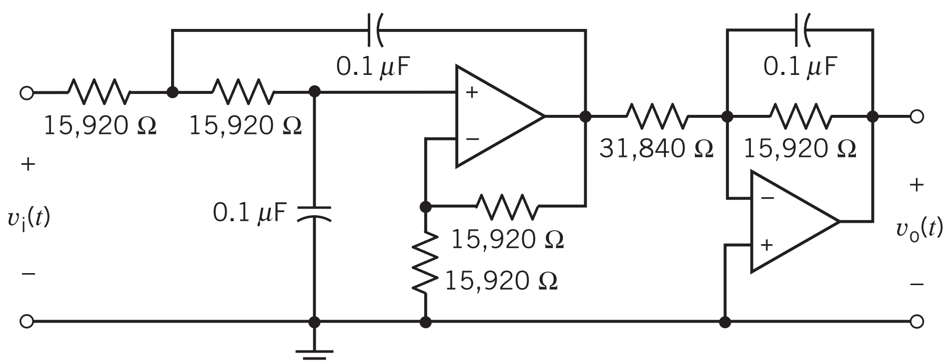
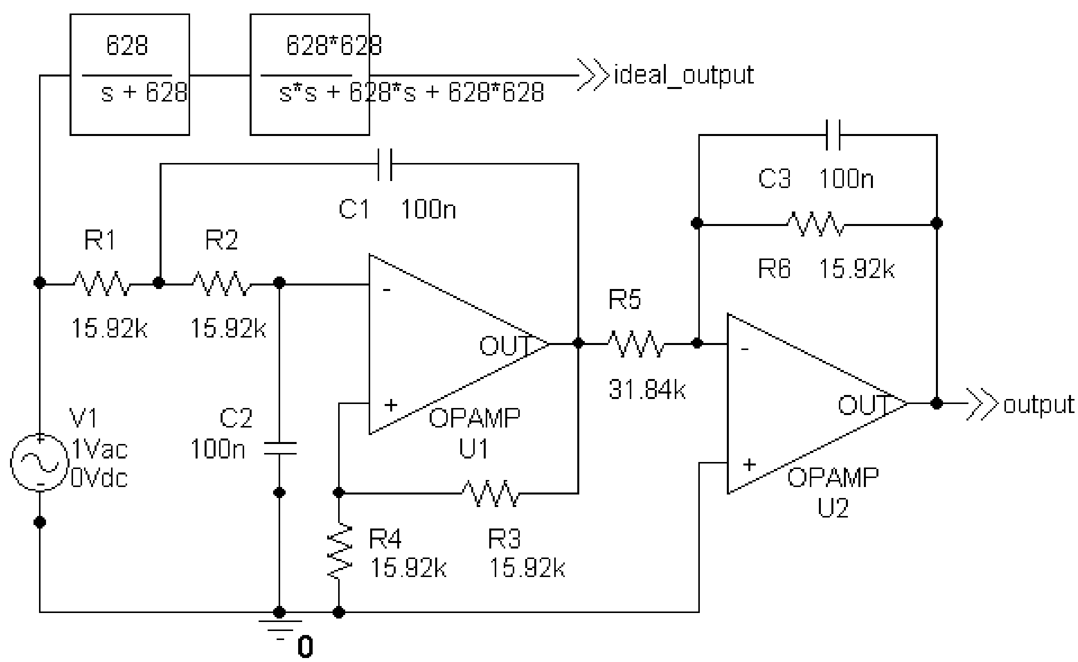
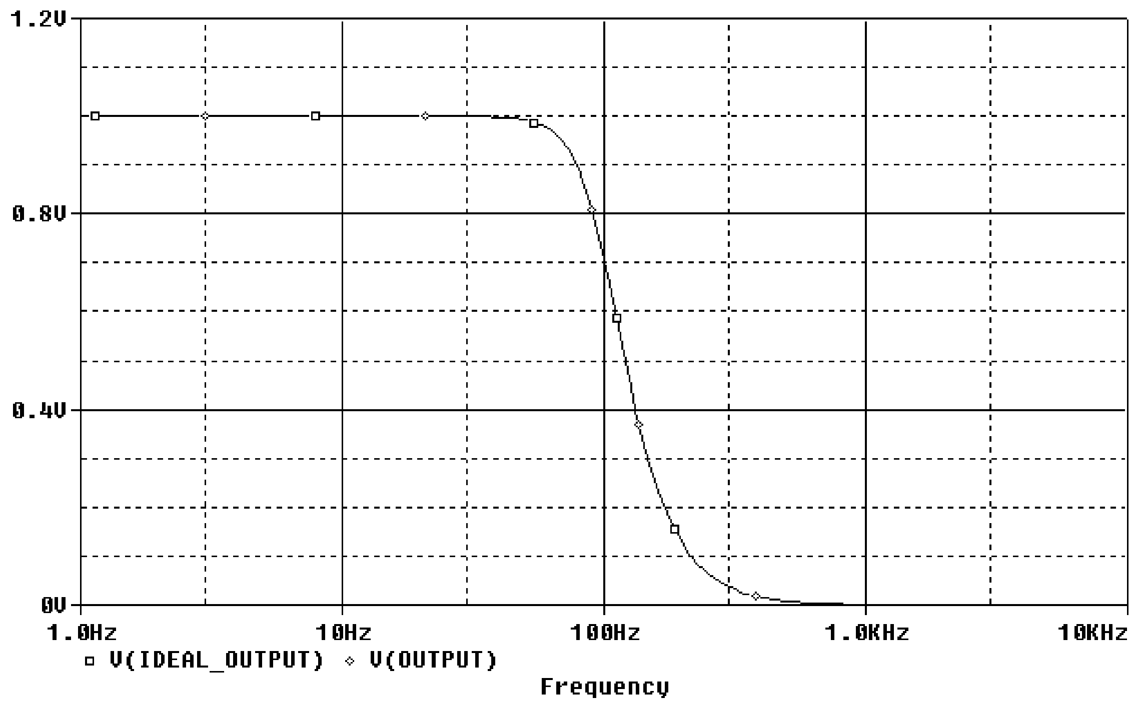


Figure SP 16-1

Solution:





1.2V
0.8V
0.4V
0V
1.0Hz
10Hz
100Hz
1.0KHz
10KHz
V(IDEAL_OUTPUT)
V(OUTPUT)
Frequency

SP 16-2 The filter circuit shown in Figure SP 16-2 was designed to have the transfer function

$$H_H(s) = \frac{5 \cdot s^3}{(s + 100)(s^2 + 100s + 10{,}000)}$$

Use PSpice to verify that the filter circuit does indeed implement this transfer function.

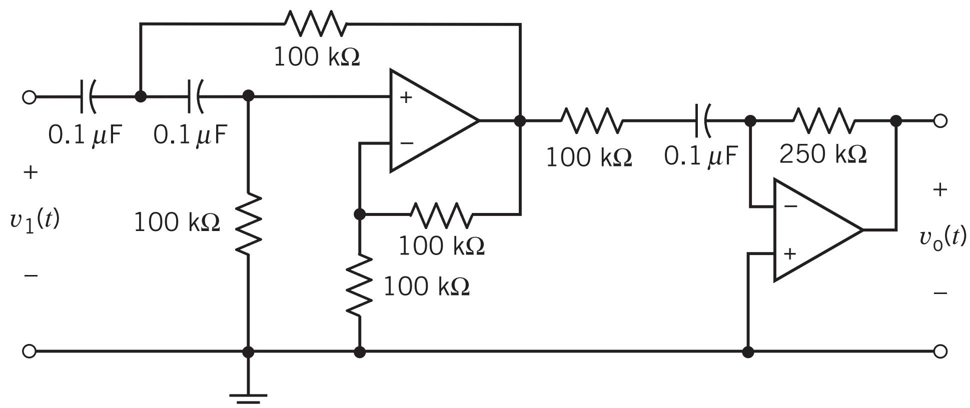
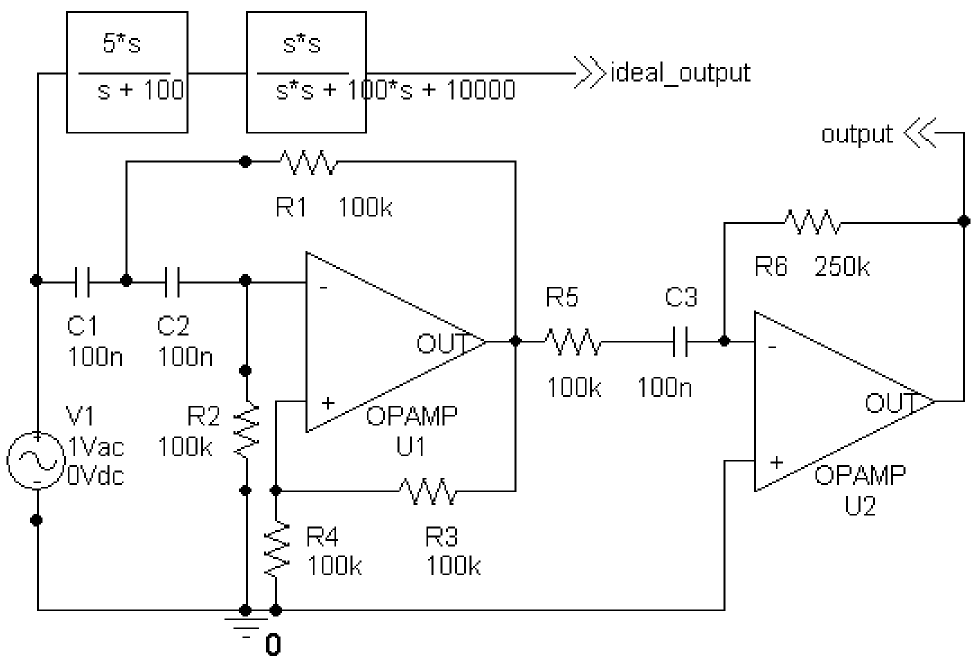
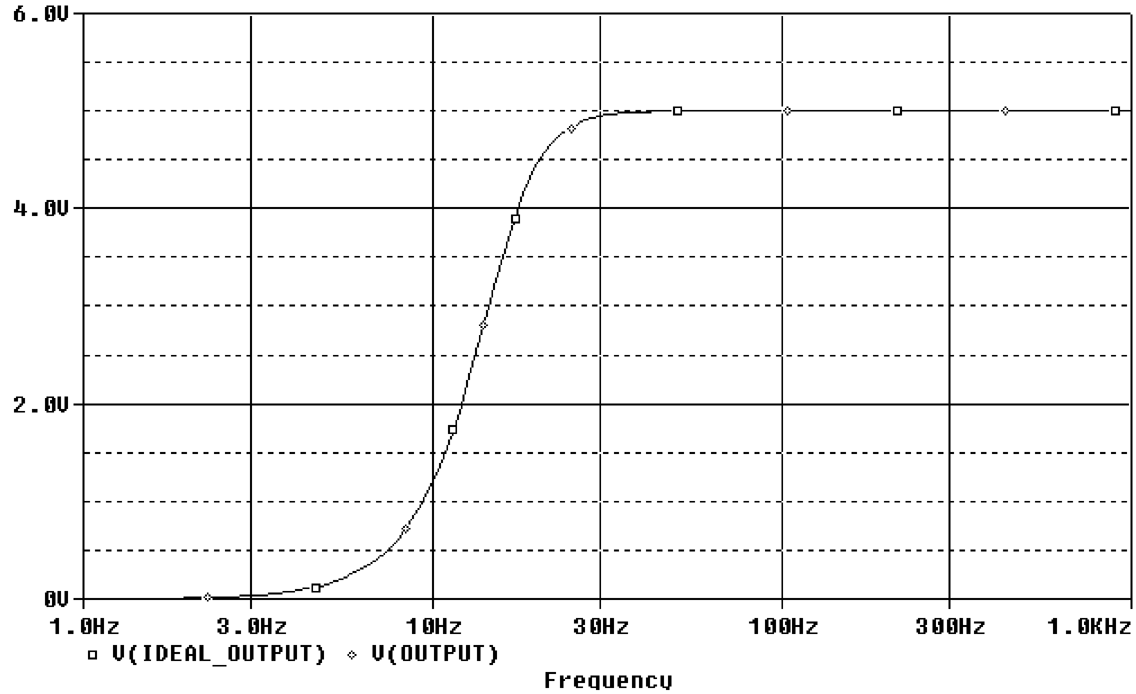


Figure SP 16-2

Solution:




6.0V
4.0V
2.0V
0V
1.0Hz
3.0Hz
10Hz
30Hz
100Hz
300Hz
1.0KHz
V(IDEAL_OUTPUT)
V(OUTPUT)
Frequency

SP 16-3 The filter circuit shown in Figure SP 16-3 was designed to have the transfer function

$$H_B(s) = \frac{16{,}000{,}000 \cdot s^2}{(s^2 + 141.4s + 10{,}000)(s^2 + 2828s + 4{,}000{,}000)}$$

Use PSpice to verify that the filter circuit does indeed implement this transfer function.

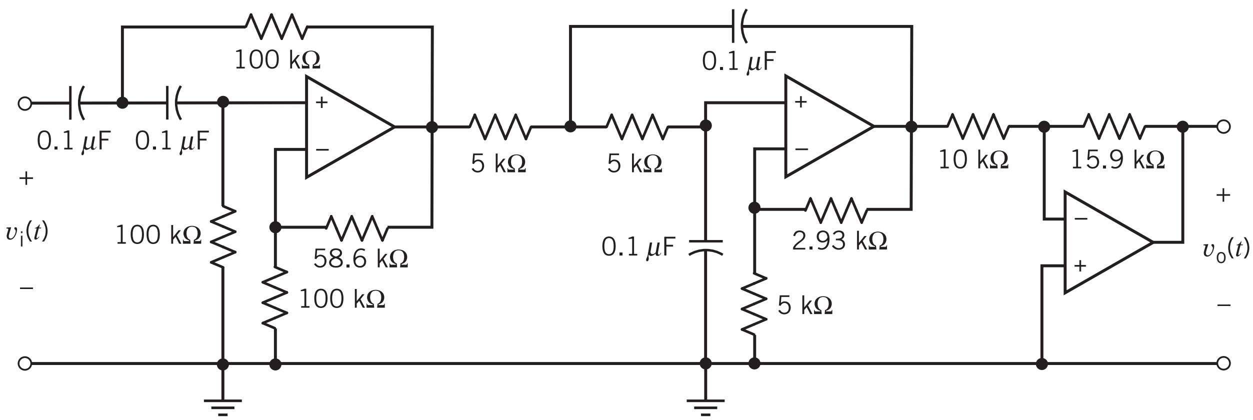
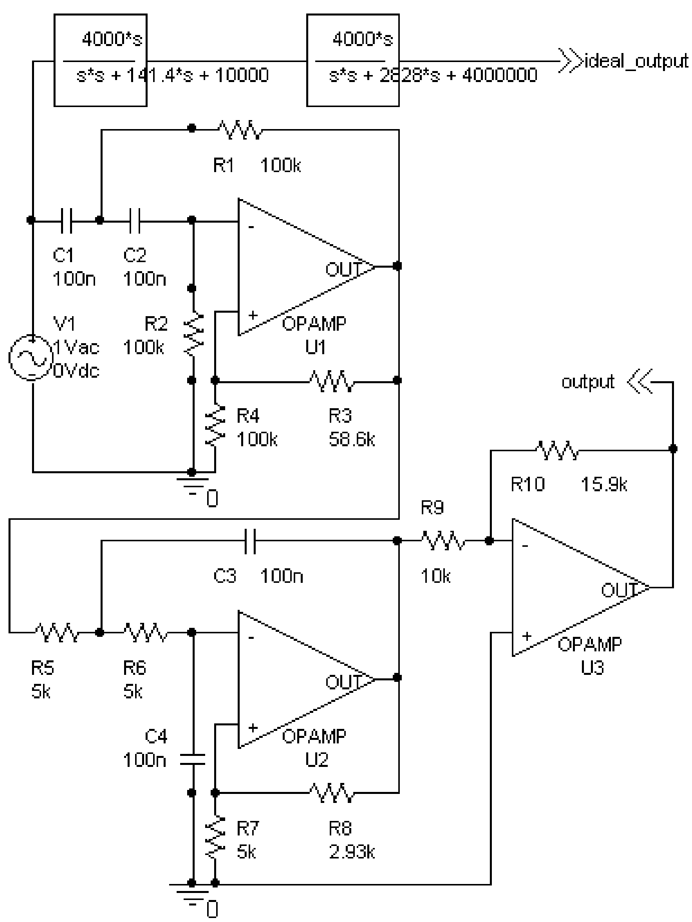
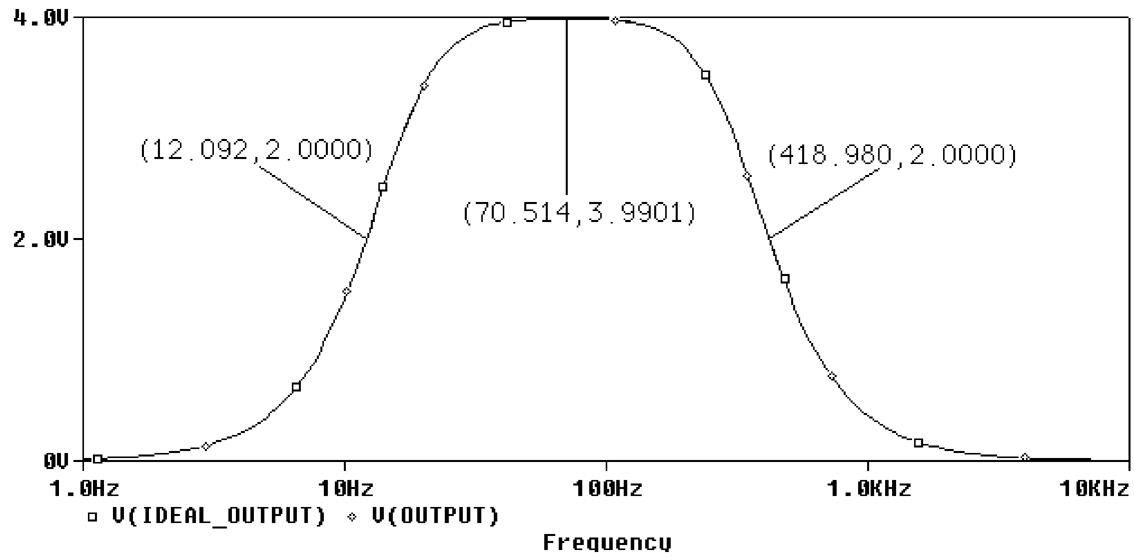


Figure SP 16-3

Solution:





4.0V
2.0V
0V
(12.092,2.0000)
(70.514,3.9901)
(418.980,2.0000)
1.0Hz
10Hz
100Hz
1.0KHz
10KHz
V(IDEAL_OUTPUT) V(OUTPUT)
Frequency

SP 16-4 The filter circuit shown in Figure SP 16-4 was designed to have the transfer function

$$H_B(s) = \frac{250{,}000s^2}{(s^2 + 250s + 62{,}500)^2}$$

Use PSpice to verify that the filter circuit does indeed implement this transfer function.

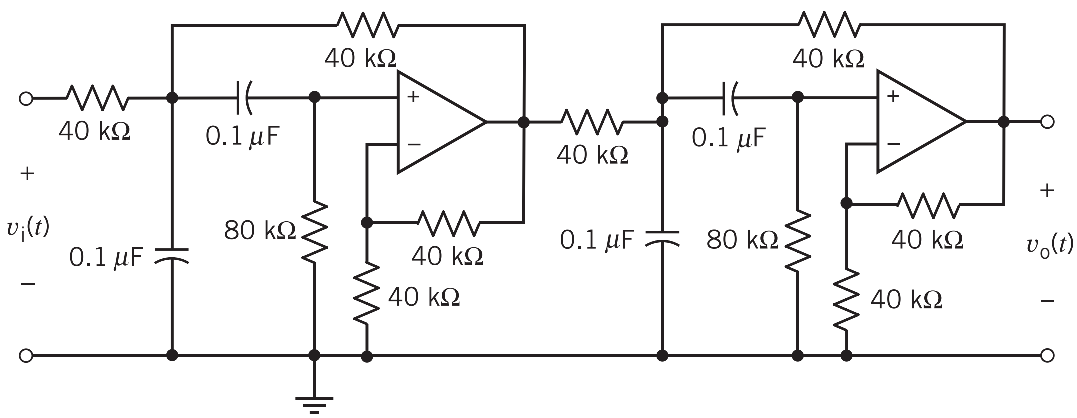
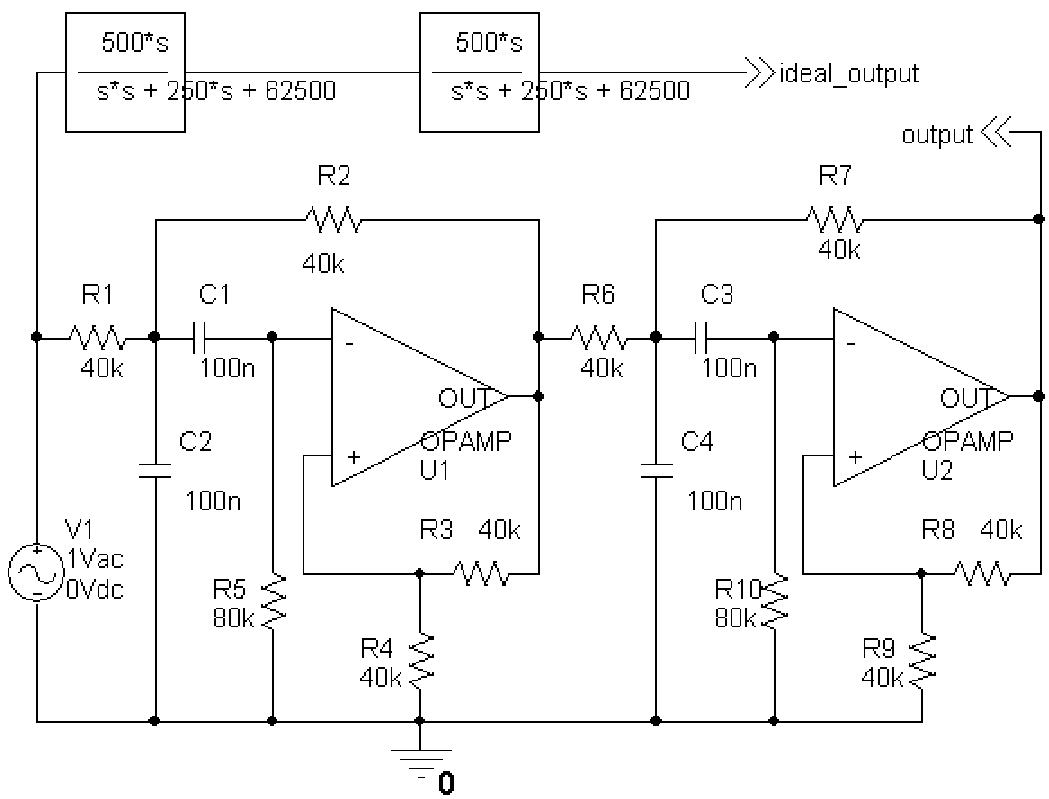
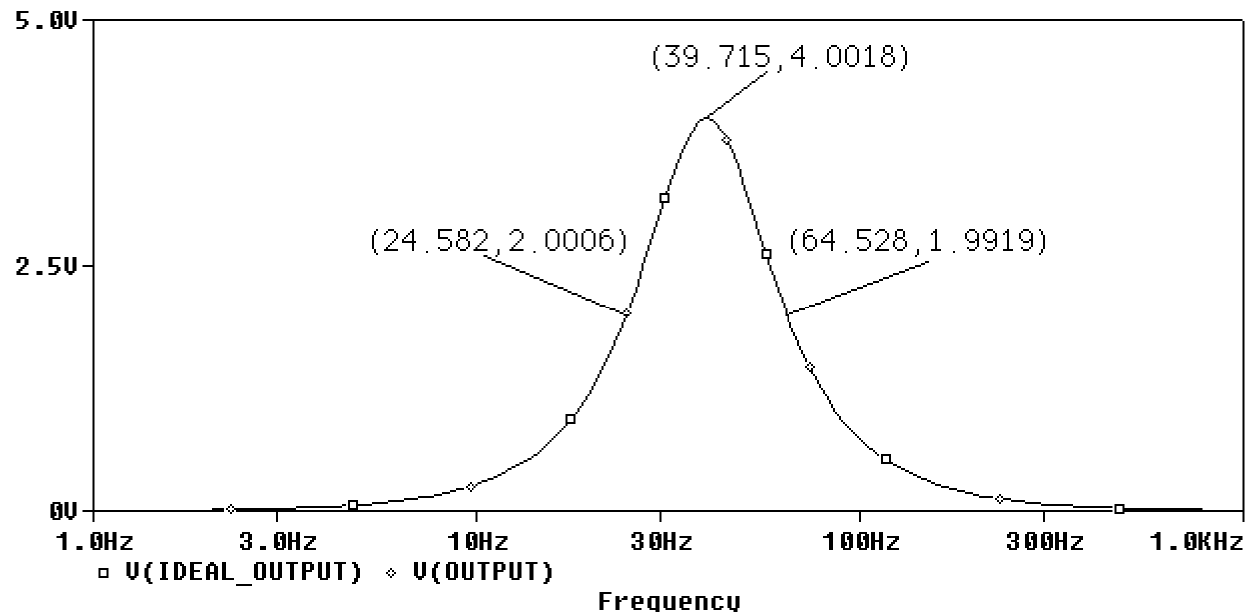


Figure SP 16-4

Solution:




5.0V
2.5V
0V
(39.715,4.0018)
(24.582,2.0006)
(64.528,1.9919)
1.0Hz
3.0Hz
10Hz
30Hz
100Hz
300Hz
1.0KHz
V(IDEAL_OUTPUT)
V(OUTPUT)
Frequency

SP 16-5 The filter circuit shown in Figure SP 16-5 was designed to have the transfer function

$$H_N(s) = \frac{2s^2}{(s^2 + 2828s + 4{,}000{,}000)} + \frac{20{,}000}{(s^2 + 141.4s + 10{,}000)}$$

Use PSpice to verify that the filter circuit does indeed implement this transfer function.

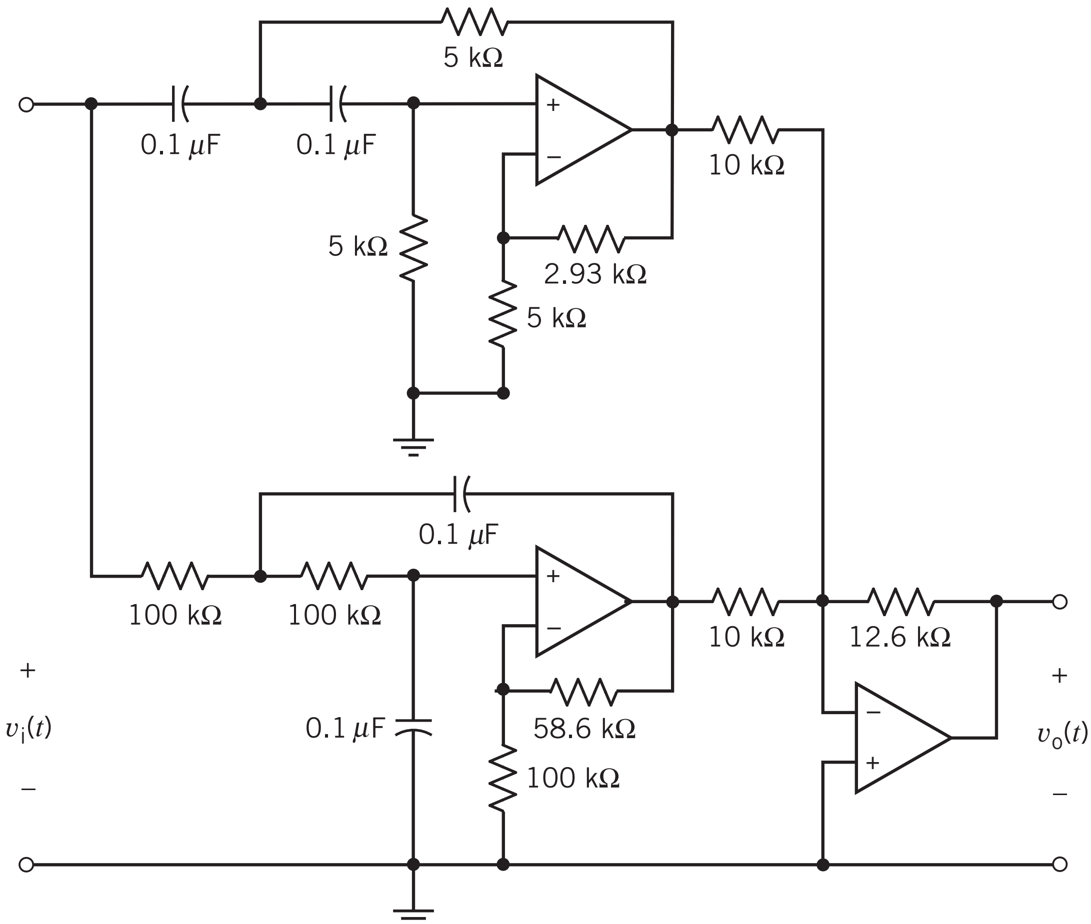
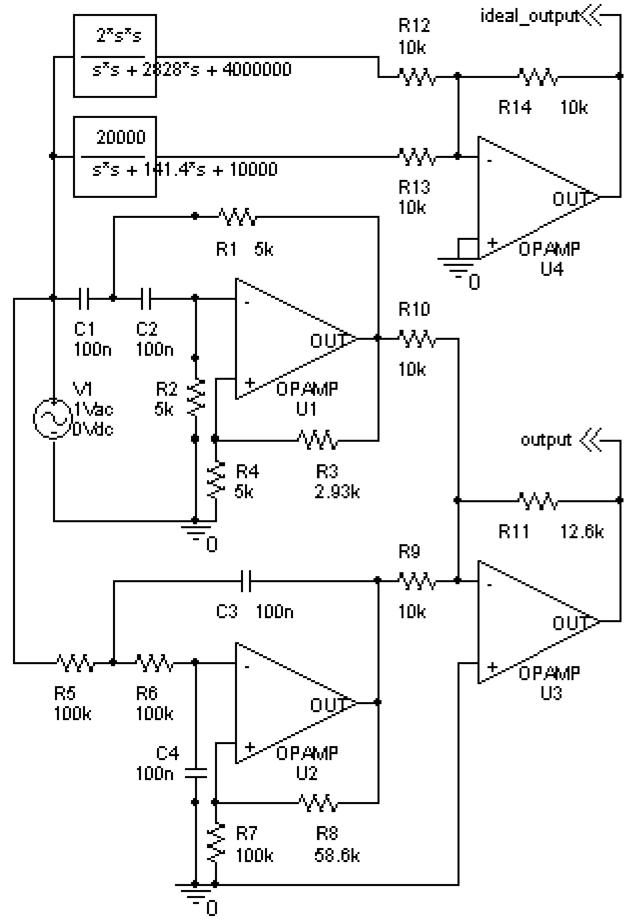
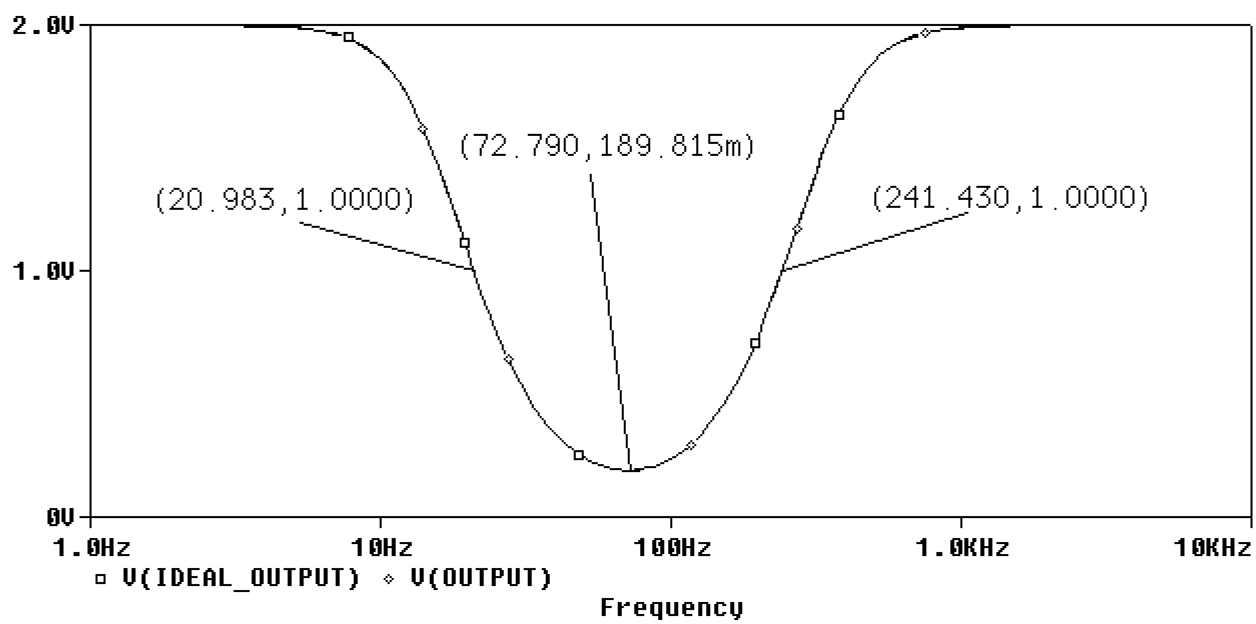


Figure SP 16-5

Solution:


2*s*s
s*s + 2828*s + 4000000
20000
s*s + 141.4*s + 10000
ideal_output
R12
10k
R14 10k
R13
10k
OUT
OPAMP
U4
0
R1 5k
C1
100n
C2
100n
OUT
OPAMP
U1
R10
10k
V1
1Vac
0Vdc
R2
5k
R4
5k
R3
2.93k
output
R11 12.6k
R9
10k
C3 100n
R5
100k
R6
100k
OPAMP
U2
OPAMP
U3
C4
100n
R7
100k
R8
58.6k


2.0V
1.0V
0V
(72.790,189.815m)
(20.983,1.0000)
(241.430,1.0000)
1.0Hz
10Hz
100Hz
1.0KHz
10KHz
V(IDEAL_OUTPUT)
V(OUTPUT)
Frequency

SP 16-6 The filter circuit shown in Figure SP 16-6 was designed to have the transfer function

$$H_N(s) = \frac{4(s^2 + 62{,}500)^2}{(s^2 + 250s + 62{,}500)^2}$$

Use PSpice to verify that the filter circuit does indeed implement this transfer function.

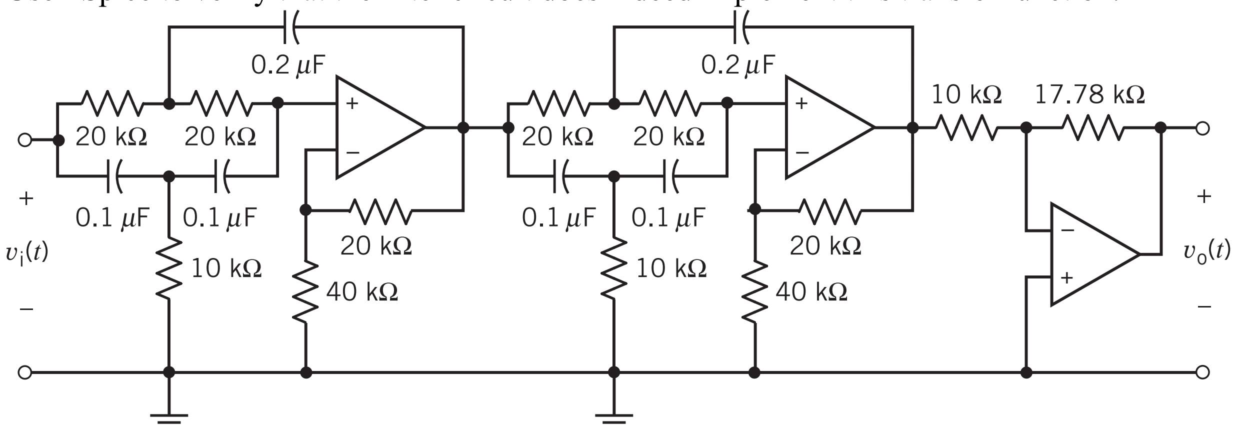
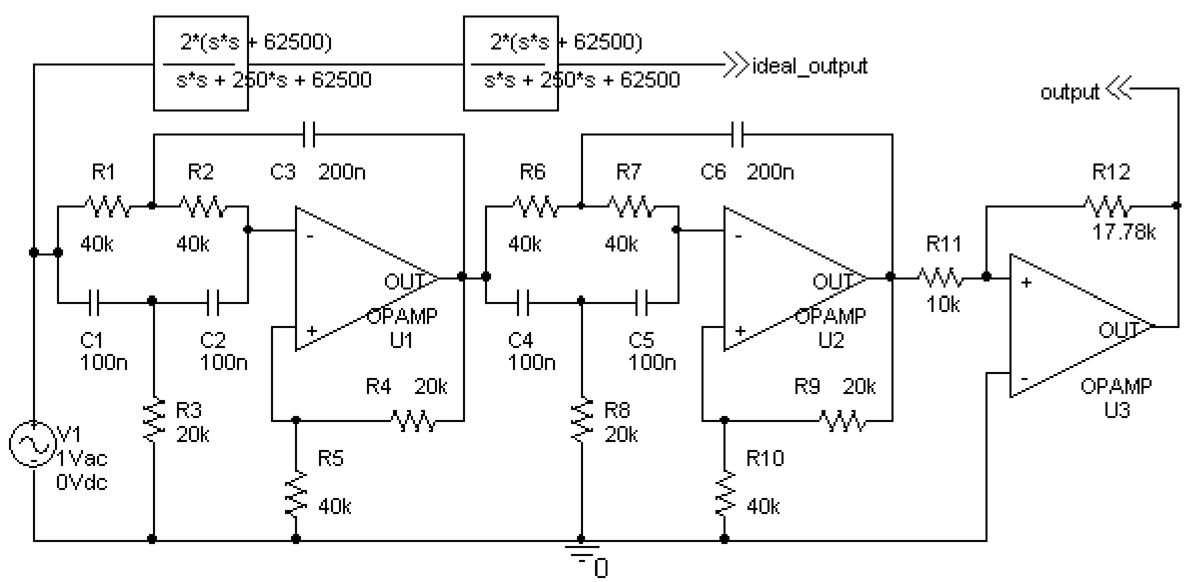
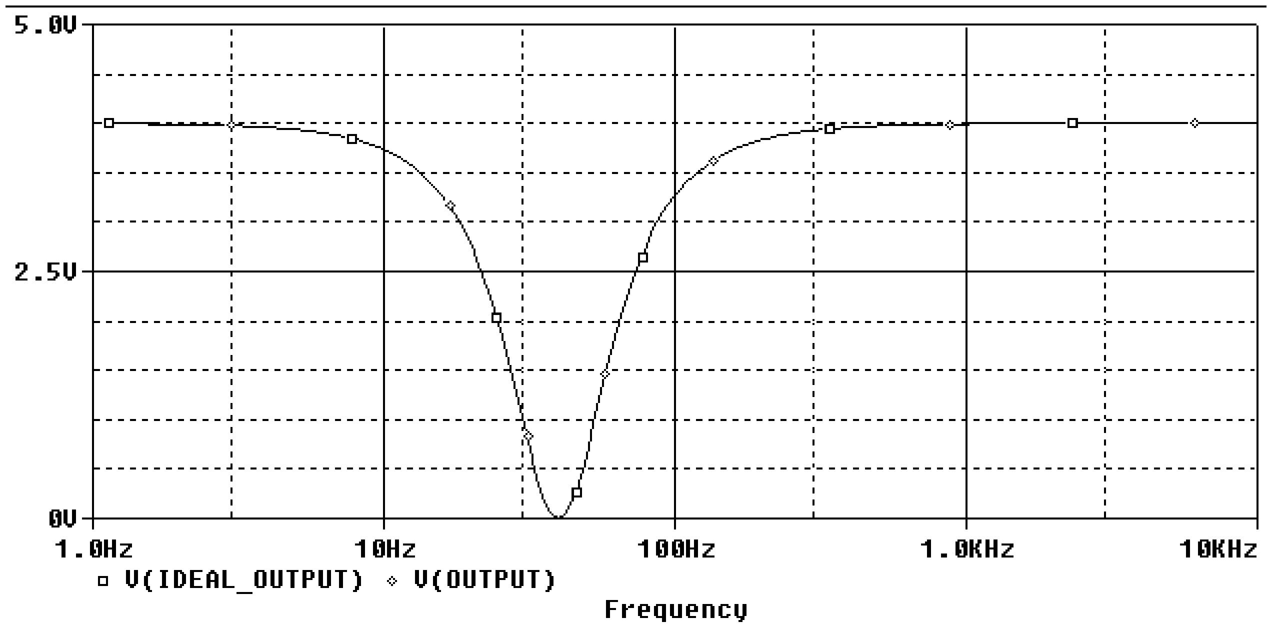


Figure SP 16-6

Solution:




5.0V
2.5V
0V
1.0Hz
10Hz
100Hz
1.0KHz
10KHz
V(IDEAL_OUTPUT)
V(OUTPUT)
Frequency

SP 16-7 A notch filter is shown in Figure SP 16-7. The output of a two-stage filter is v_1, and the output of a three-stage filter is v_2. Plot the Bode diagram of $\mathbf{V}_1/\mathbf{V}_s$ and $\mathbf{V}_2/\mathbf{V}_s$ and compare the results when L = 10 mH and C = 1 μF.

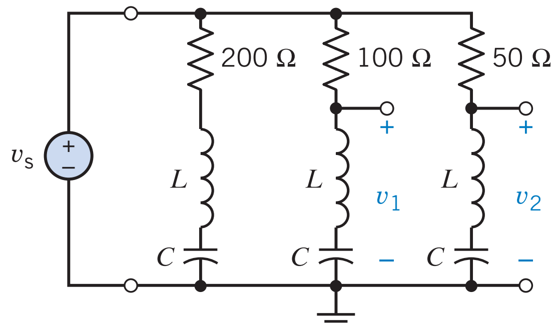
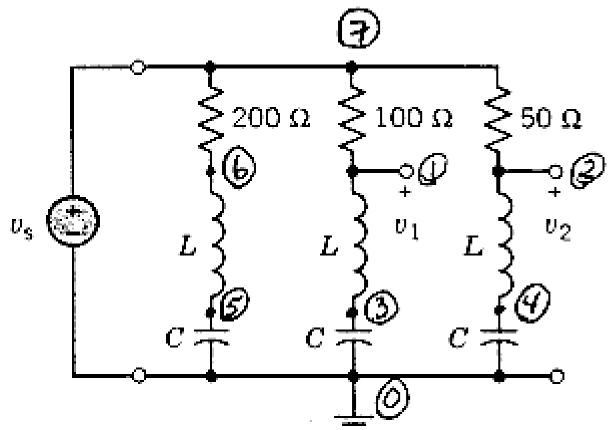
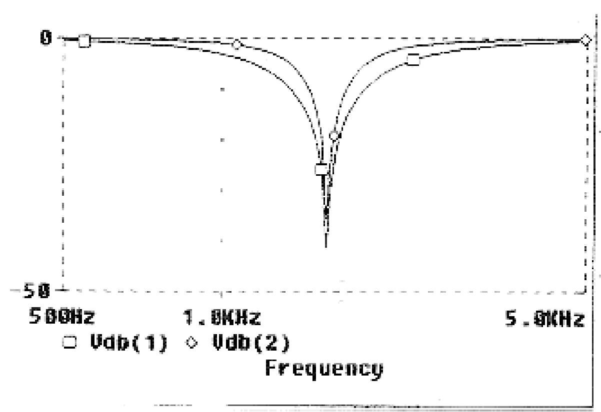


Figure SP 16-7

Solution:




```
Vs   7  0   ac  1
R1   7  6   200
R2   7  1   100
R3   7  2   50
L1   6  5   10m
L2   1  3   10m
L3   2  4   10m
C1   5  0   1u
C2   3  0   1u
C3   4  0   1u

.ac dec 100 100 10k
.probe
.end
```



SP 16-8 An acoustic sensor operates in the range of 5 kHz to 25 kHz and is represented in Figure SP 16-8. by v_s. It is specified that the band-pass filter shown in the figure passes the signal in the frequency range within 3 dB of the center frequency gain. Determine the bandwidth and center frequency of the circuit when the op amp has R_i = 500 kΩ, R_o = 1 kΩ, and $A = 10^6$.

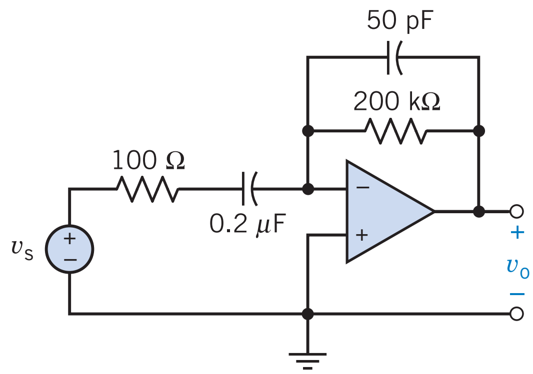
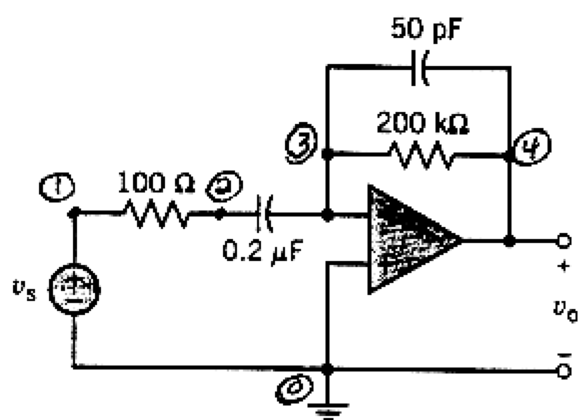
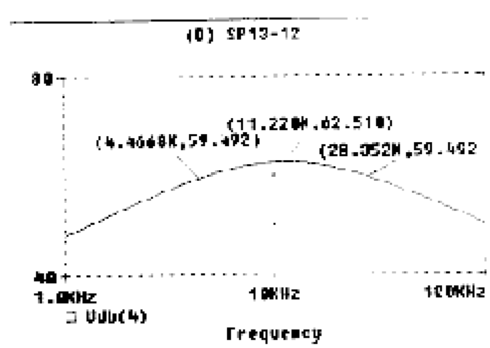


Figure SP 16-8

Solution:




```
Vs    1   0   ac    1
R1    1   2   100
C1    2   3   0.2u
R2    3   4   200k
C2    3   4   50p

Xoa5 3     0    4     FGOA

.subckt FGOA 1 2 4
*nodes listed in order - + o
Ri    1    2   500k
E     3   0   1    2   100k
Ro    4    3   1k
.ends FGOA

.ac dec 100 1k 100k
.probe
.end
```



SP 16-9 Frequently, audio systems contain two or more loudspeakers that are intended to handle different parts of the audio-frequency spectrum. In a three-way setup, one speaker, called a woofer, handles low frequencies. A second, the tweeter, handles high frequencies, while a third, the midrange, handles the middle range of the audio spectrum.

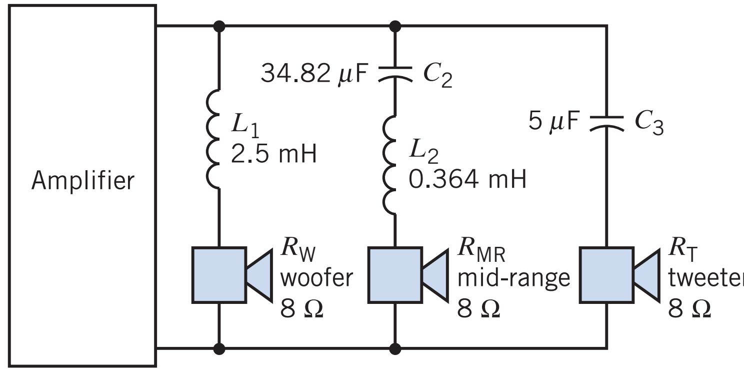
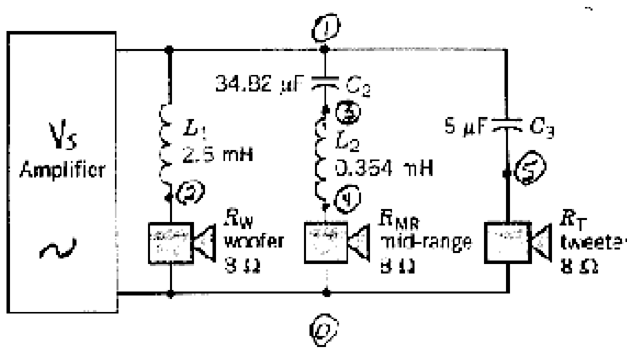


Figure SP 16-9

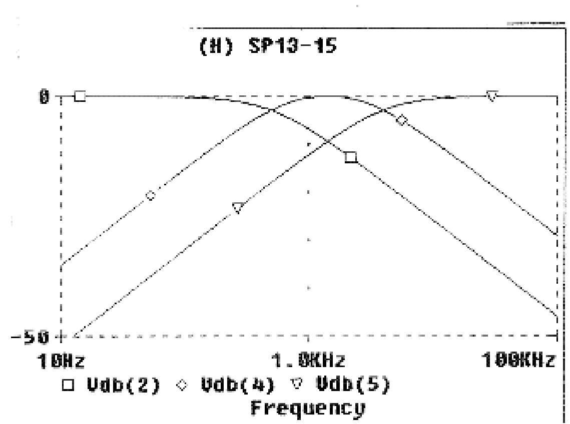
A three-way filter, called a crossover network, is used to split the audio signal into the three bands of frequencies suitable for each speaker. There are many and varied designs. A simple one is based on series *LR*, *CR*, and resonant *RLC* circuits as shown in Figure SP 16-9. All speaker impedances are assumed resistive. The conditions are (1) woofer, at the crossover frequency: $X_{L1} = R_{W;}$ (2) tweeter, at the crossover frequency $X_{C3} = R_{T;}$ and (3) midrange, with components C_2, L_2, and R_{MR} forming a series resonant circuit with upper and lower cutoff frequencies f_u and f_L, respectively. The resonant frequency = $(f_u f_L)^{1/2}$.

When all the speaker resistances are 8 Ω, determine the frequency response and the cutoff frequencies. Plot the Bode diagram for the three speakers. Determine the bandwidth of the midrange speaker section.

Solution:




```
Vs  1   0   ac  1
L1  1   2   2.5m
Rw  2   0   8
C2  1   3   34.82u
L2  3   4   0.364m
Rmr 4   0   8
C3  1   5   5u
Rt  5   0   8

.ac dec 100 10 100k
.probe
.end
```



Bw=4.07k - 493 Hz $\simeq$ 3600Hz

Design Problems

DP 16-1 Design a band-pass filter with a center frequency of 100 kHz and a bandwidth of 10 kHz using the circuit shown in Figure DP 16-1. Assume that C = 100 pF and find R and R_3. Use PSpice to verify the design.

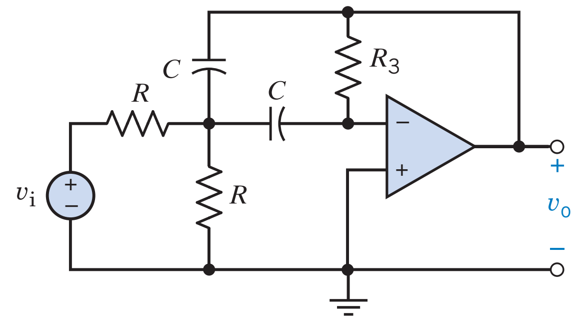


Figure DP 16-1

Solution:

$$\frac{V_0(s)}{V_1(s)} = \frac{-\dfrac{s}{R\,C}}{s+\dfrac{2}{R_3\,C}\,s+\dfrac{2}{R\,R_3\,C^2}}$$

$$2\pi(100.10^3) = \omega_0 = \sqrt{\frac{2}{R\,R_3\,C^2}} \quad \text{and} \quad 2\pi(10.10^3) = \text{BW} = \frac{\omega_0}{\text{Q}} = \frac{2}{R_3\,C}$$

$C = 100$ pF is specified so $R_3 = \dfrac{2}{(100\times10^{-12})\,(2\pi\times10\times10^3)} = 318\text{ k}\Omega$ and $R = \dfrac{2}{R_3 C^2 \omega_0^2} = 1.6\text{ k}\Omega$

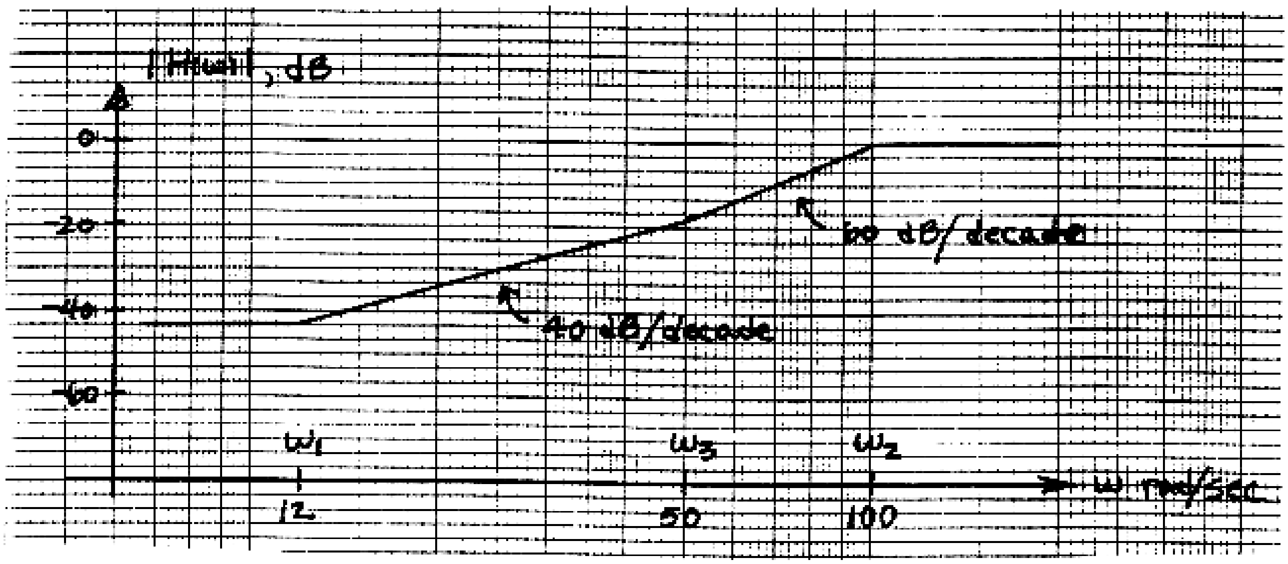
DP 16-2 A communication transmitter requires a band-pass filter to eliminate low-frequency noise from nearby traffic. Measurements indicate that the range of traffic rumble is $2 < \omega < 12$ rad/s. A designer proposes a filter as

$$H(s) = \frac{(1 + s/\omega_1)^2(1 + s/\omega_3)}{(1 + s/\omega_2)^3}$$

where $s = j\omega$.

It is desired that signals with $\omega > 100$ rad/s pass with less than 3-dB loss, while the traffic rumble be reduced by 46 dB or more. Select ω_1, ω_2, and ω_3 and plot the Bode diagram.

Solution:



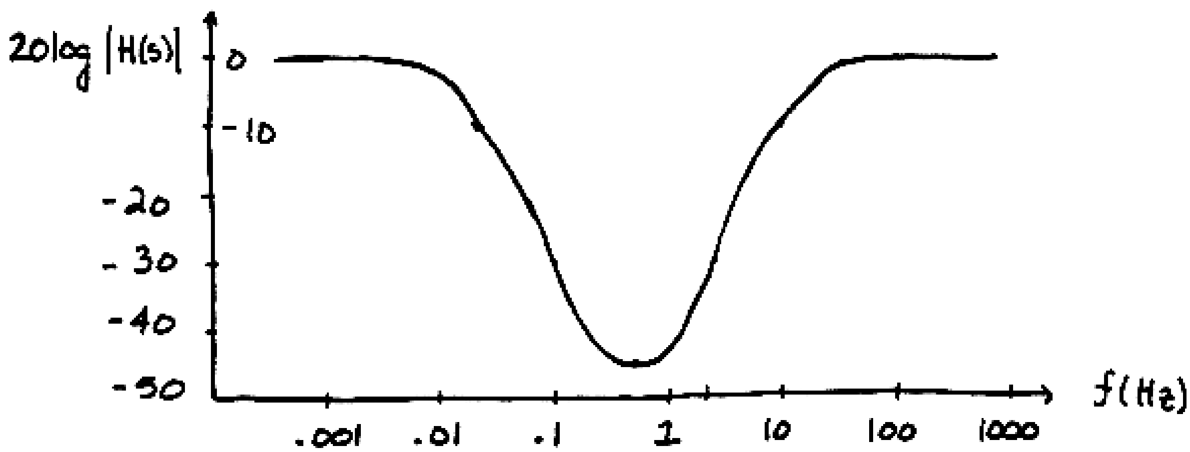
DP 16-3 A communication transmitter requires a band-stop filter to eliminate low-frequency noise from nearby auto traffic. Measurements indicate that the range of traffic rumble is 2 rad/s < ω < 12 rad/s. A designer proposes a filter as

$$H(s) = \frac{(1 + s/\omega_1)^2 (1 + s/\omega_3)^2}{(1 + s/\omega_2)^2 (1 + s/\omega_4)^2}$$

where $s = j\omega$. It is desired that signals above 130 rad/s pass with less than 4-dB loss, while the traffic rumble be reduced by 35 dB or more. Select ω_1, ω_2, ω_3, and ω_4 and plot the Bode diagram.

Solution:

Choose $\omega_1 = 0.1$, $\omega_2 = 2$, $\omega_3 = 5$, $\omega_4 = 100$ rad/s . The corresponding Bode magnitude plot is:



$$H(s) = \frac{\left(1+ s/\omega_1\right)^2 \left(1+ s/\omega_3\right)^2}{\left(1+ s/\omega_2\right)^2 \left(1+ s/\omega_4\right)^2}$$

Minimum gain is -46.2 dB at $f_{\min} = 0.505$ Hz

Chapter 17- Two-Port and Three Port Networks

Exercises

Exercise 17.2-1 Find the T circuit equivalent to the Π circuit shown in Figure E 17.2-1.

Answers: $R_1 = 10\ \Omega$ $R_2 = 12.5\ \Omega$, and $R_3 = 50\ \Omega$

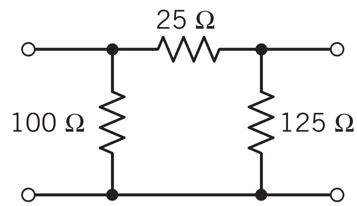
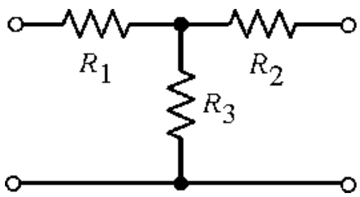


Figure E 17.2-1

Solution:



$$R_1 = \frac{R_a\ R_c}{R_a + R_b + R_c} = \frac{100(25)}{250} = 10\ \Omega$$

$$R_2 = \frac{R_b\ R_c}{R_a + R_b + R_c} = \frac{125(25)}{250} = 12.5\ \Omega$$

$$R_3 = \frac{R_a\ R_b}{R_a + R_b + R_c} = \frac{100(125)}{250} = 50\ \Omega$$

Exercise 17.3-1 Find the Z and Y parameters of the circuit of Figure E 17.3-1.

Answers: $\mathbf{Z} = \begin{bmatrix} 18 & 6 \\ 6 & 9 \end{bmatrix}, \quad \mathbf{Y} = \begin{bmatrix} \frac{1}{14} & -\frac{1}{21} \\ -\frac{1}{21} & \frac{1}{7} \end{bmatrix}$

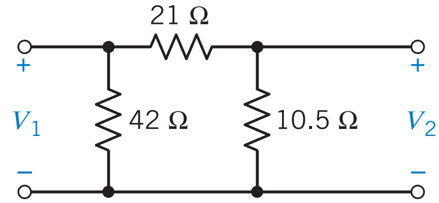
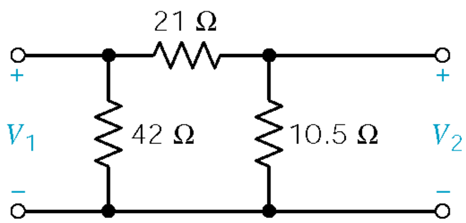


Figure E 17.3-1

Solution:



$$-Y_{12} = -Y_{21} = \frac{1}{21}\text{ S}$$

$$Y_{11} + Y_{12} = \frac{1}{42} \quad \Rightarrow \quad Y_{11} = \frac{1}{42} - \left(-\frac{1}{21}\right) = \frac{3}{42} = \frac{1}{14}\text{ S}$$

$$Y_{22} + Y_{21} = 10.5 \quad \Rightarrow \quad Y_{22} = \frac{1}{10.5} - \left(-\frac{1}{21}\right) = \frac{1}{7}\text{ S}$$

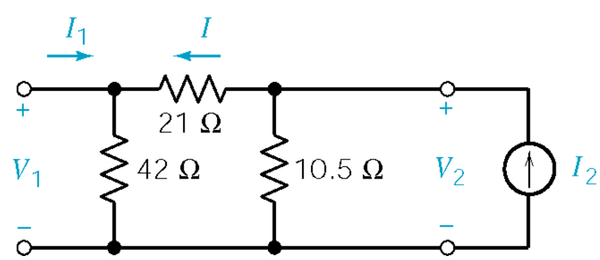
$$\mathbf{Y} = \begin{bmatrix} \frac{1}{14} & -\frac{1}{21} \\ -\frac{1}{21} & \frac{1}{7} \end{bmatrix}\text{ S}$$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{42\ (21+10.5)}{42+31.5} = 18\ \Omega$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{10.5(63)}{73.5} = 9\ \Omega$$

Since $I = \frac{10.5}{73.5} I_2$, then $V_1 = \frac{42(10.5)}{73.5} I_2 = 6 I_2$

$$Z_{12} = Z_{21} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = 6\ \Omega$$



$$\mathbf{Z} = \begin{bmatrix} 18 & 6 \\ 6 & 9 \end{bmatrix}\ \Omega$$

Exercise 17.4-1 Determine the Y parameters of the circuit of Figure 17.4-1.

Answer: $Y = \begin{bmatrix} \frac{1}{6} & \frac{1}{18} \\ -\frac{1}{6} & \frac{17}{18} \end{bmatrix}$

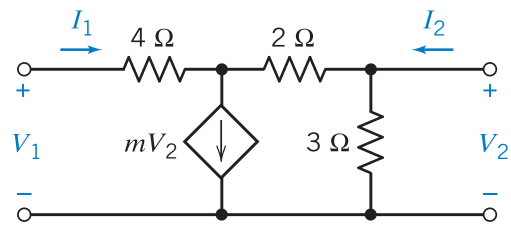
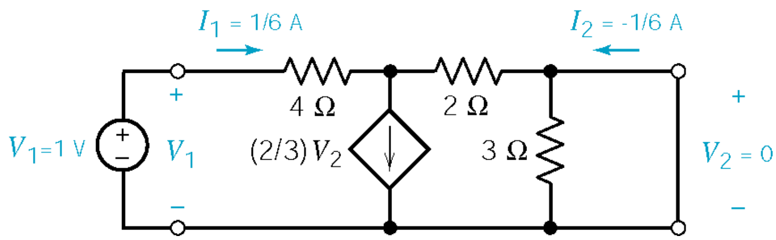


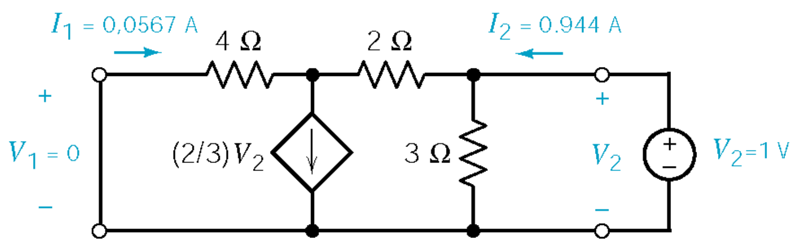
Figure 17.4-1

Solution:



$$Y_{11} = \frac{I_1}{V_1} = \frac{1}{6} \ \text{S}$$

$$Y_{21} = \frac{I_2}{V_1} = -\frac{1}{6} = -0.167 \ \text{S}$$



$$Y_{12} = \frac{I_1}{V_2} = 0.0567 \ \text{S}$$

$$Y_{22} = \frac{I_2}{V_2} = 0.944 \ \text{S}$$

Exercise 17.5-1 Find the hybrid parameter model of the circuit shown in Figure E 17.5-1.

Answers: $h_{11} = 0.9\ \Omega$, $h_{12} = 0.1\ \Omega$, $h_{21} = 4.4\ \Omega$, and $h_{22} = 0.6\ \Omega\text{S}$

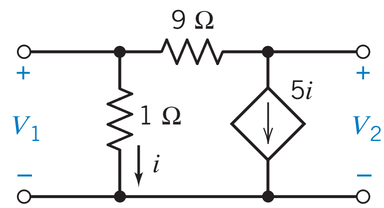
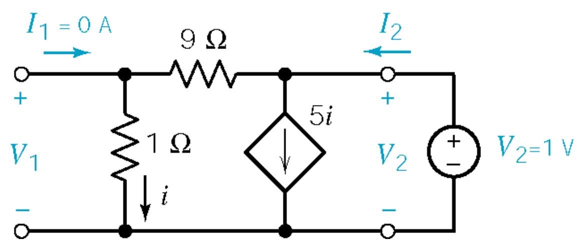


Figure E 17.5-1

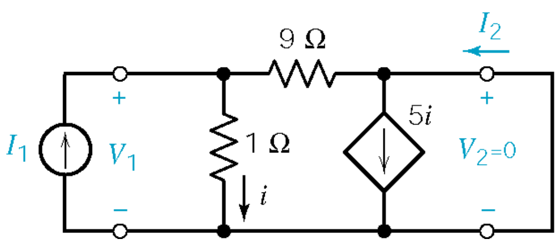
Solution:



$$I_2 = 6i,\ V_2 = (9+1)\,i = 10\,i,\ V_1 = 1i$$

$$h_{22} = \frac{I_2}{V_2} = \frac{6i}{10i} = 0.6\ \text{S}$$

$$h_{12} = \frac{V_1}{V_2} = \frac{i}{10i} = 0.1$$



$$V_1 = 1\,i$$

$$I_1 = i + \frac{V_1}{9} = \frac{10}{9}i$$

$$I_2 = 5i - \frac{V_1}{9} = \frac{44}{9}i$$

Therefore

$$h_{11} = \frac{V_1}{I_1} = \frac{i}{\left(10/9\right)i} = 0.9\ \Omega$$

$$h_{21} = \frac{I_2}{I_1} = \frac{\left(44/9\right)i}{\left(10/9\right)i} = 4.4$$

Exercise 17.6-1 Determine the Z parameters if the Y parameters are

$$Y = \begin{bmatrix} \dfrac{2}{15} & \dfrac{-1}{5} \\ \dfrac{-1}{10} & \dfrac{2}{5} \end{bmatrix}$$

The units are siemens.

Answers: $Z_{11} = 12\ \Omega$, $Z_{12} = 6\ \Omega$, $Z_{21} = 3\ \Omega$, and $Z_{22} = 4\ \Omega$

Solution:

$$\mathbf{Y} = \begin{bmatrix} 2/15 & -1/5 \\ -1/10 & 2/5 \end{bmatrix} \quad \text{and} \quad \Delta Y = \frac{4}{75} - \frac{1}{50} = \frac{1}{30} \quad \Rightarrow \quad \mathbf{Z} = \mathbf{Y}^{-1} = 30\begin{bmatrix} 2/5 & 1/5 \\ 1/10 & 2/15 \end{bmatrix} = \begin{bmatrix} 12 & 6 \\ 3 & 4 \end{bmatrix}$$

Exercise 17.6-2 Determine the T parameters from the Y parameters of Exercise 17.6-1.

Answers: $A = 4$, $B = 10\ \Omega$, $C = 1/3\ S$, and $D = 4/3$

Ex. 17.6-2

$$\mathbf{T} = \begin{bmatrix} -\dfrac{2/5}{(-1/10)} & -\dfrac{1}{(-1/10)} \\ -\dfrac{1/30}{(-1/10)} & -\dfrac{2/15}{(-1/10)} \end{bmatrix} = \begin{bmatrix} 4 & 10 \\ 1/3 & 4/3 \end{bmatrix}$$

Solution:

The transmission parameters of the two-port networks are:

$$\mathbf{T}_a = \begin{bmatrix} 1 & 12 \\ 0 & 1 \end{bmatrix}, \ \mathbf{T}_b = \begin{bmatrix} 1 & 0 \\ 1/6 & 1 \end{bmatrix} \text{ and } \mathbf{T}_c = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

The transmission parameters of the cascade circuit are:

$$\mathbf{T}_a\mathbf{T}_b\mathbf{T}_c = \begin{bmatrix} 1 & 12 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1/6 & 1 \end{bmatrix}\mathbf{T}_c = \begin{bmatrix} 3 & 12 \\ 1/6 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 21 \\ 1/6 & 3/2 \end{bmatrix}$$

Exercise 17.7-1 Determine the total transmission parameters of the cascade connection of three two-port networks shown in Figure E 17.7-1.

Answer: $A = 3$, $B = 21\ \Omega$, $C = 1/6$ S, and $D = 3/2$

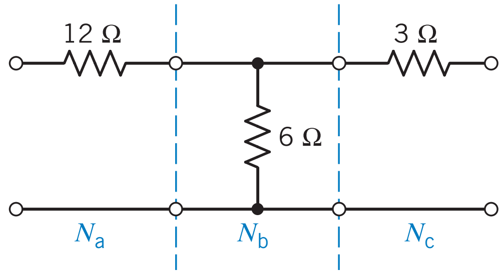


Figure E 17.7-1

Solution:

The transmission parameters of the two-port networks are:

$$\mathbf{T}_a = \begin{bmatrix} 1 & 12 \\ 0 & 1 \end{bmatrix},\ \mathbf{T}_b = \begin{bmatrix} 1 & 0 \\ 1/6 & 1 \end{bmatrix} \text{ and } \mathbf{T}_c = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

The transmission parameters of the cascade circuit are:

$$\mathbf{T}_a\mathbf{T}_b\mathbf{T}_c = \begin{bmatrix} 1 & 12 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1/6 & 1 \end{bmatrix}\mathbf{T}_c = \begin{bmatrix} 3 & 12 \\ 1/6 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 21 \\ 1/6 & 3/2 \end{bmatrix}$$

Exercise 17.8-1 Verify that the circuit shown in Figure E 17.8-1 does indeed have the transfer function

$$\frac{V_o(s)}{V_{in}(s)} = \frac{2s-10}{s^2+27s+2}$$

(The circuits in Figures 17.8-1*a* and E 17.8-1 differ only in the sign of h_{21}.)

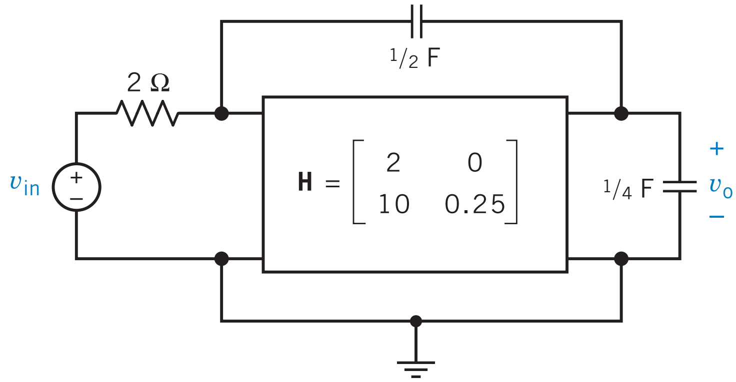
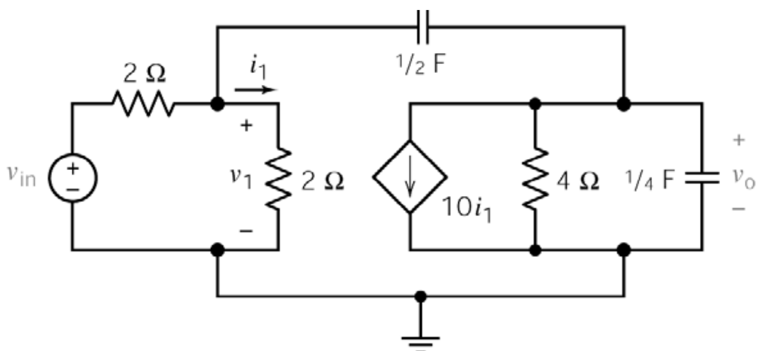


Figure E 17.8-1

Solution:



Node equations:
$$\begin{bmatrix} 1+\frac{s}{2} & -\frac{s}{2} \\ 5-\frac{s}{2} & \frac{3s}{4}+\frac{1}{4} \end{bmatrix} \begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} = \begin{bmatrix} \frac{V_{in}(s)}{2} \\ 0 \end{bmatrix}$$

Using Cramer's Rule:
$$\frac{V_o(s)}{V_{in}(s)} = \frac{-\frac{1}{2}\left(5-\frac{s}{2}\right)}{\left(1+\frac{s}{2}\right)\left(\frac{3s}{4}+\frac{1}{4}\right)+\frac{s}{2}\left(5-\frac{s}{2}\right)} = \frac{2s-10}{s^2+27s+2}$$

Section 17.2 T-to-Π Transformation and Two-Port Three-Terminal Networks

P 17.2-1 Determine the equivalent resistance R_{ab} of the network of Figure P 17.2-1. Use the Π-to-T transformation as one step of the reduction.

Answer: $R_{ab} = 3.2\ \Omega$

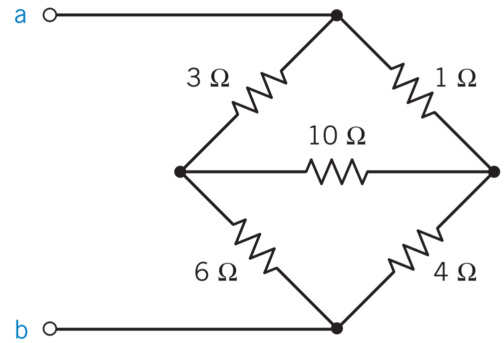
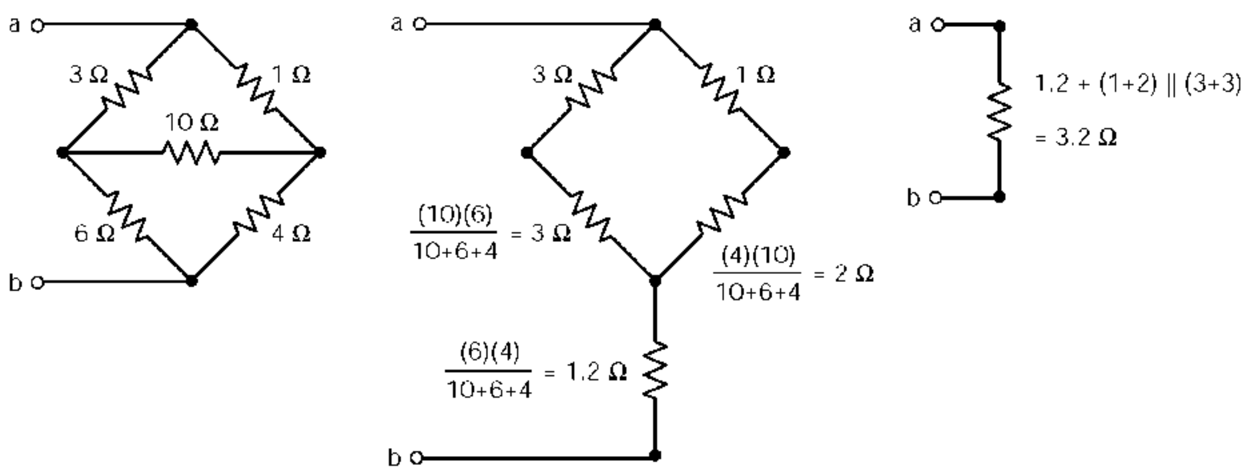


Figure P 17.2-1

Solution:



P 17.2-2 Repeat problem P 17.2-1 when the 6-Ω resistance is changed to 4 Ω and the 10-Ω resistance is changed to 12 Ω.

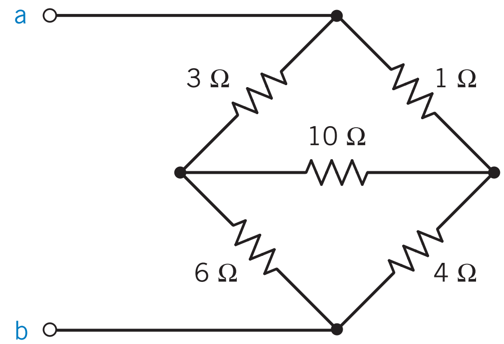
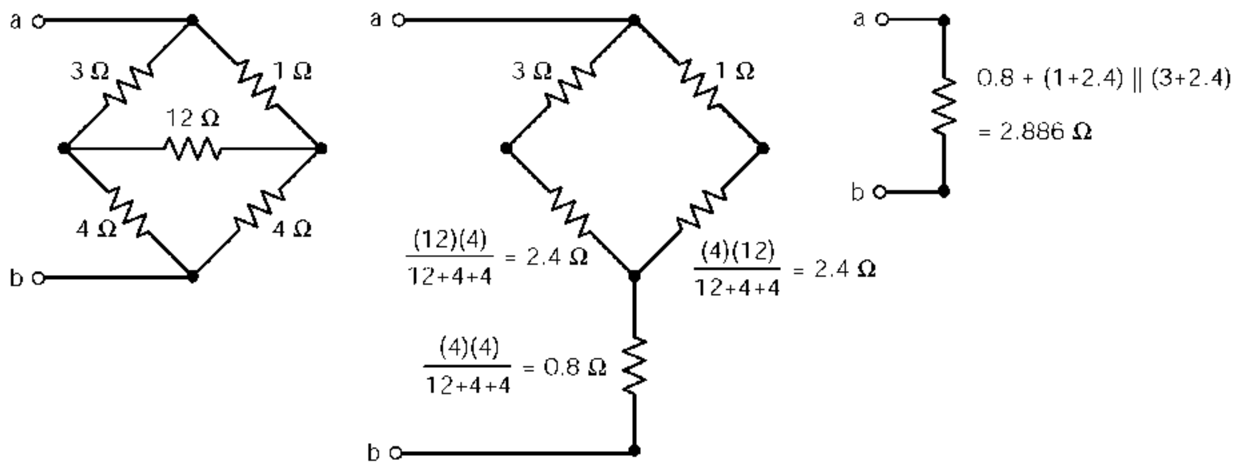


Figure P 17.2-1

Solution:



P 17.2-3 The two-port network of Figure 17.1-1 has an input source V_s with a source resistance R_s connected to the input terminals so that $V_1 = V_s - I_1 R_s$ and a load resistance connected to the output terminals so that $V_2 = -I_2 R_L = I_L R_L$. Find $R_{in} = V_1/I_1$, $A_v = V_2/V_1$, $A_i = -I_2/I_1$, and $A_p = -V_2 I_2/(V_1 I_1)$ by using the Z-parameter model.

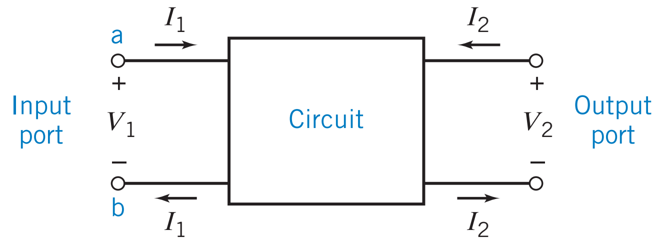
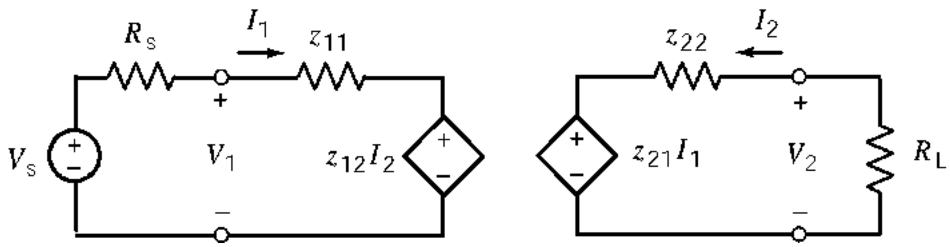


Figure 17.1-1

Solution:



$$I_2 = \frac{-z_{21}\ I_1}{z_{22} + R_L} \Rightarrow A_i = \frac{-I_2}{I_1} = \frac{z_{21}}{z_{22}+R_L} \qquad \text{(forward current gain)}$$

$$R_{in} = \frac{V_1}{I_1} = \frac{z_{11}I_1 + z_{12}I_2}{I_1} = z_{11} - \frac{z_{12}\ A_i\ I_1}{I_1} = z_{11} - \frac{z_{12}\ z_{21}}{\left(z_{22}+R_L\right)} \qquad \text{(input resistance)}$$

$$V_2 = -I_2\ R_L = A_i\ R_L I_1 \ \text{ and } \ V_1 = R_{in} I_1 \Rightarrow A_v = \frac{V_2}{V_1} = \frac{A_i\ R_L}{R_{in}} \qquad \text{(forward voltage gain)}$$

$$\therefore A_p = A_i\ A_v = A_i^2 \frac{R_L}{R_{in}}$$

P 17.2-4 Using the Δ-to-Y transformation, determine the current I when $R_1 = 15\ \Omega$ and $R = 20\ \Omega$ for the circuit shown in Figure P 17.2-4.

Answer: $I = 385$ mA

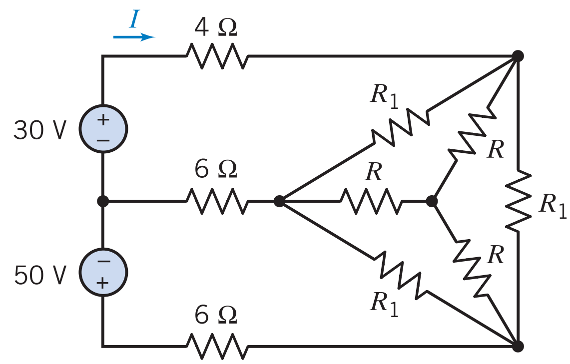
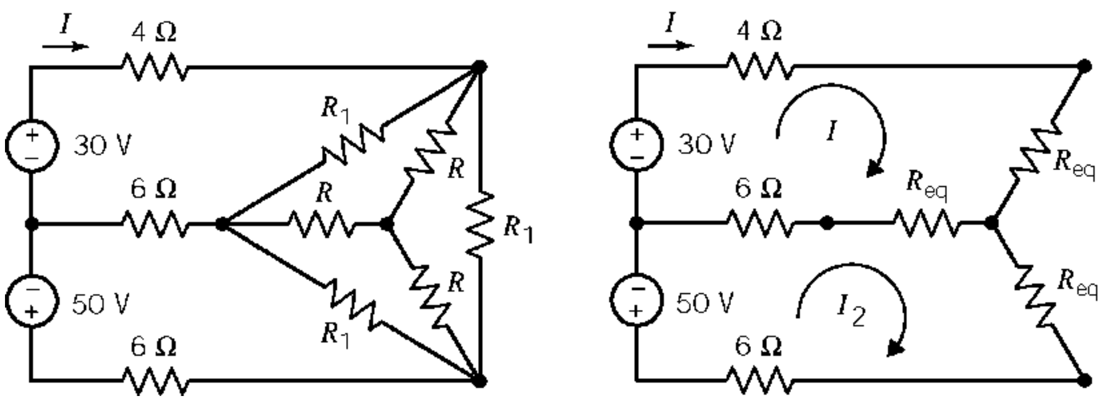


Figure P 17.2-4

P17.2-4

First, simplify the circuit using a Δ-Y transformation:



$$R_{eq} = \frac{R_1}{3} \parallel R = 5 \parallel 20 = 4\ \Omega$$

Mesh equations:

$$30 = 18I - 10I_2$$
$$50 = 10I - 20I_2$$

Solving for the required current:

$$I = \frac{\begin{vmatrix} 30 & -10 \\ 50 & -20 \end{vmatrix}}{18(-20) - (-10)10} = \frac{-100}{-260} = \underline{0.385\ \text{A}}$$

P 17.2-5 Use the Y-to-Δ transformation to determine R_{in} of the circuit shown in Figure P 17.2-5.

Answer: R_{in} = 673.85 Ω

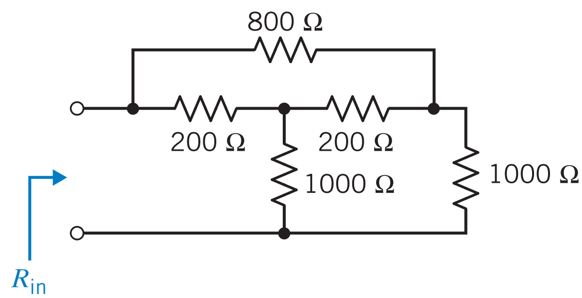
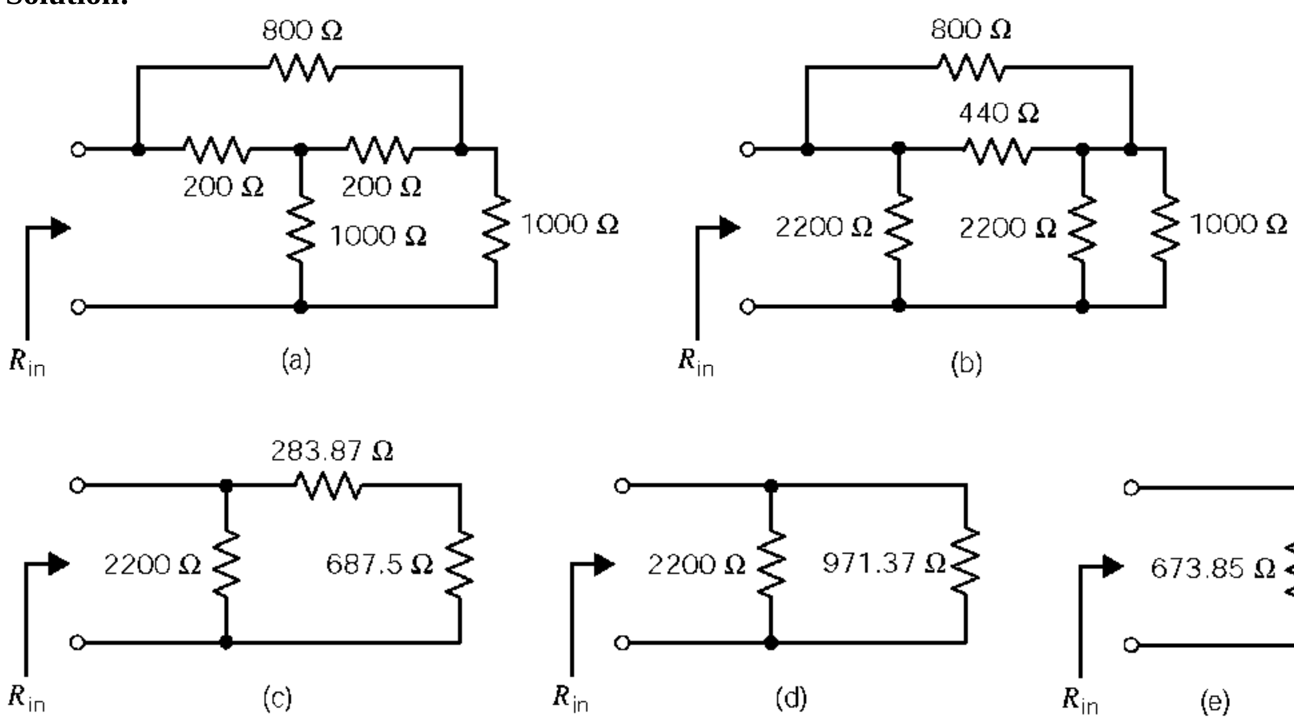


Figure P 17.2-5

Solution:



Section 17-3: Equations of Two-Port Networks

P 17.3-1 Find the *Y* parameters and *Z* parameters for the two-port network of Figure P 17.3-1.

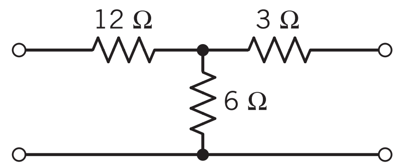
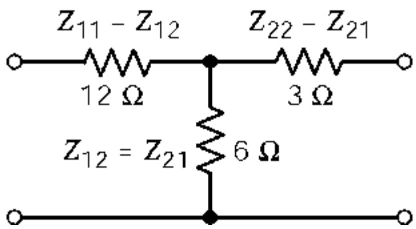


Figure P 17.3-1

Solution:

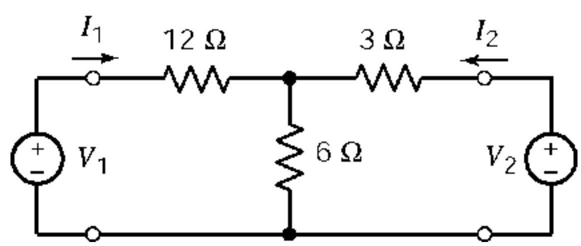


$$Z_{12} = 6\ \Omega$$

$$Z_{11} - Z_{12} = 12\ \Omega \quad \Rightarrow \quad Z_{11} = 18\ \Omega$$

$$Z_{22} - Z_{21} = 3\ \Omega \quad \Rightarrow \quad Z_{22} = 9\ \Omega$$

$$\mathbf{Z} = \begin{bmatrix} 18 & 6 \\ 6 & 9 \end{bmatrix}\ \Omega$$



$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{1}{14}\ \text{S}$$

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = \frac{-6\, I_2}{(6+12)\, V_2} = -\frac{1}{21}\ \text{S} = \text{Y}_{21}$$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{V_2/7}{V_2} = \frac{1}{7}\ \text{S}$$

$$\mathbf{Y} = \begin{bmatrix} 1/14 & -1/21 \\ -1/21 & 1/7 \end{bmatrix}\ \text{S}$$

P 17.3-2 Determine the Z parameters of the ac circuit shown in Figure P 17.3-2.

Answer: $Z_{11} = 2 - j4\ \Omega$, $Z_{12} = Z_{21} = -j4\ \Omega$, $Z_{22} = -j2\ \Omega$

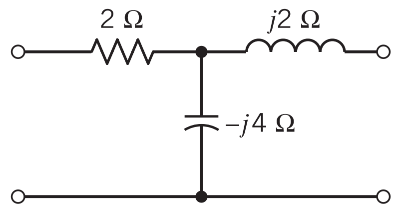
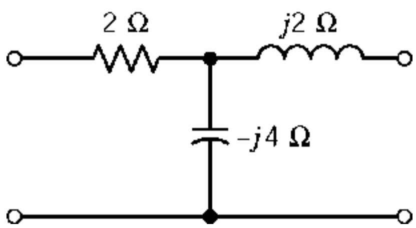


Figure P 17.3-2

Solution:



$$z_{12} = z_{21} = -j\,4\ \Omega$$

$$z_{11} - z_{12} = 2\ \Omega \quad \Rightarrow \quad z_{11} = 2 - j\,4\ \Omega$$

$$z_{22} - z_{21} = j\,2\ \Omega \quad \Rightarrow \quad z_{22} = j\,2 - j\,4 = -j\,2\ \Omega$$

$$\mathbf{Z} = \begin{bmatrix} 2-j4 & -j4 \\ -j4 & -j2 \end{bmatrix}\ \Omega$$

P 17.3-3 Find the *Y* parameters of the circuit of Figure P 17.3-3 when $b = 4$, $G_1 = 2$ S, $G_2 = 1$ S, and $G_3 = 3$ S.

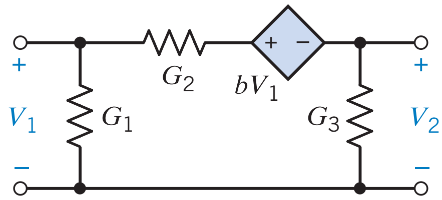
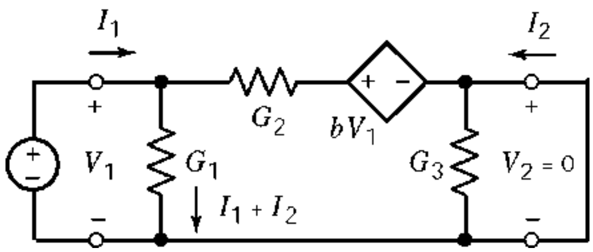


Figure P 17.3-3

Solution:



$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \quad \text{and} \quad Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

$$V_1 = \frac{I_1 + I_2}{G_1} \quad \text{and} \quad \frac{I_1 + I_2}{G_1} + \frac{I_2}{G_2} = bV_1$$

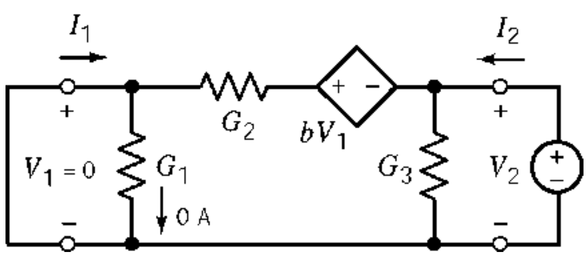
so

$$I_1 = (G_1 - (b-1)\,G_2)\,V_1 = -1\,V_1 \quad \text{and} \quad I_2 = (b-1)\,G_2\,V_1 = 3\,V_1$$

Finally

$$Y_{11} = -1 \text{ S} \quad \text{and} \quad Y_{21} = 3 \text{ S}$$

Next



$$V_2 = \frac{I_1 + I_2}{G_3} \quad \text{and} \quad V_2 = \frac{-I_2}{G_2}$$

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -G_2 = -1 \text{ S}$$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = G_2 + G_3 = 4 \text{ S}$$

P 17.3-4 Find the *Y* parameters for the circuit of Figure P 17.3-4.

Answer: $Y_{11} = 0.3$ S, $Y_{21} = Y_{12} = -0.1$ S, and $Y_{22} = 0.15$ S

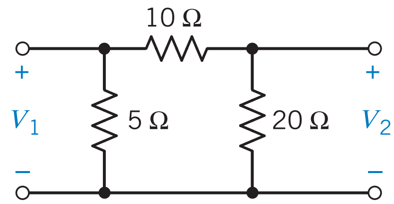
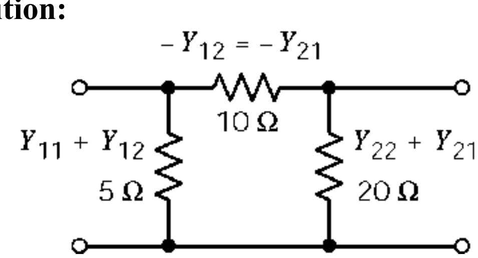


Figure P 17.3-4

Solution:



Using Fig. 17.3-2 as shown:

$$-Y_{12} = -Y_{21} = \ 0.1 \ \text{ S} \ \text{ or } \ Y_{12} = \ Y_{21} = -0.1 \ \text{ S}$$
$$Y_{11} \ = 0.2 - Y_{12} \ = 0.3 \ \ \text{S}$$
$$Y_{22} = \ 0.05 \ - Y_{21} \ = \ 0.15 \ \ \text{S}$$

P 17.3-5 Find the *Y* parameters of the circuit shown in Figure P 17.3-5.

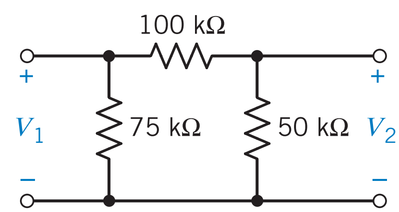
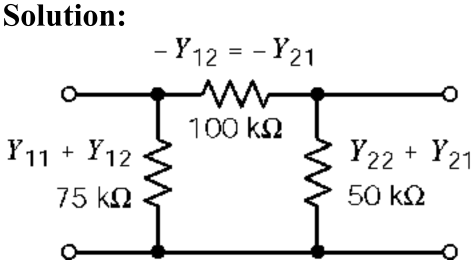


Figure P 17.3-5

Solution:



$$Y_{12} = -10 \ \mu\text{S} = Y_{21}$$
$$Y_{11}+Y_{12}=13.33 \ \mu\text{S}$$
$$Y_{11} = 23.33 \ \ \mu\text{S}$$

$$Y_{22} + Y_{21} = 20 \ \ \mu\text{S} \quad \Rightarrow \quad Y_{22} = \ 30 \ \ \mu\text{S}$$

P 17.3-6 Find the Z parameters for the circuit shown in Figure P 17.3-6 for sinusoidal steady-state response at $\omega = 3$ rad/s.

Answer: $Z_{11} = 3 + j\ \Omega$, $Z_{12} = Z_{21} = -j2\ \Omega$, and $Z_{22} = -j2\ \Omega$

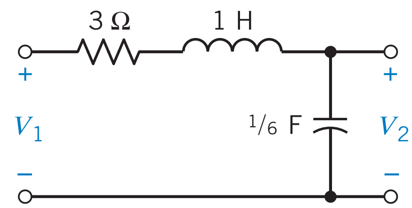
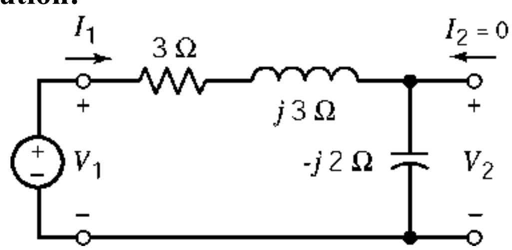


Figure P 17.3-6

Solution:

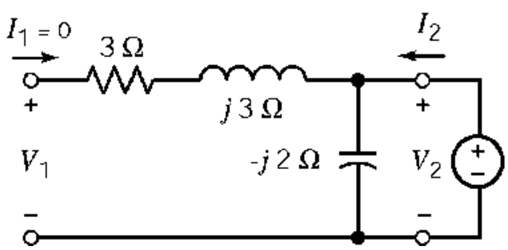


$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = 3 + j3 - j2 = \underline{(3+j)\quad\Omega}$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{-j2\,I_1}{I_1} = \underline{-j2\quad\Omega}$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \underline{-j2\quad\Omega}$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \underline{-j2\quad\Omega}$$



P 17.3-7 Determine the impedance parameters in the s-domain (Laplace domain) for the circuit shown in Figure 17.3-7.

Answer: $Z_{11} = (4s + 1)/s$, $Z_{12} = Z_{21} = 1/s$, and $Z_{22} = (2s^2 + 1)/s$

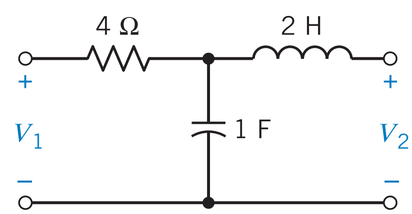
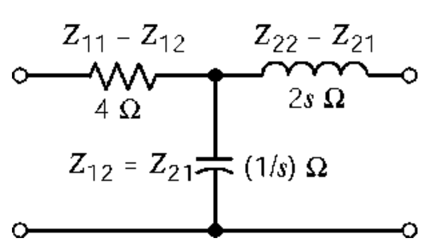


Figure P 17.3-7

Solution:

$$\left.\begin{aligned} Z_{11} - Z_{21} &= 4 \\ Z_{21} - Z_{12} &= \frac{1}{s} \end{aligned}\right\} \Rightarrow \quad Z_{11} = 4 + \frac{1}{s} = \frac{4s+1}{s}$$

$$Z_{22} - Z_{21} = 2s \quad \Rightarrow \quad Z_{22} = 2s + \frac{1}{s} = \frac{2s^2+1}{s}$$

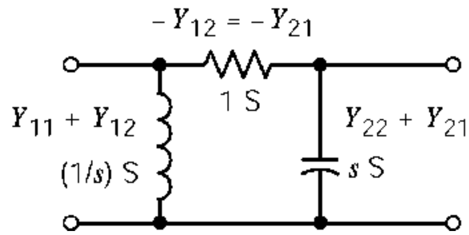


P 17.3-8 Determine a two-port network that is represented by the Y parameters:

$$\mathbf{Y} = \begin{bmatrix} \dfrac{s+1}{s} & -1 \\ -1 & (s+1) \end{bmatrix}$$

Solution:

Try a π circuit as shown at the right.



$$Y_{12} = Y_{21} = -1 \text{ S}$$

$$Y_{11} + Y_{12} = \frac{1}{s} \quad \Rightarrow \quad Y_{11} = \frac{1}{s} - 1 = \frac{s+1}{s}$$

$$Y_{22} + Y_{21} = s \quad \Rightarrow \quad Y_{22} = s - (-1) = s + 1$$

P 17.3-9 Find a two-port network incorporating one inductor, one capacitor, and two resistors that will give the following impedance parameters:

$$\mathbf{Z} = \frac{1}{\Delta}\begin{bmatrix} (s^2+2s+2) & 1 \\ 1 & (s^2+1) \end{bmatrix}$$

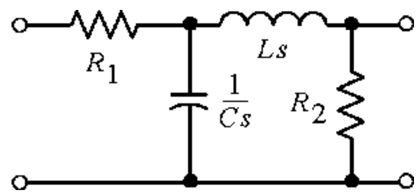
where $\Delta = s^2 + s + 1$.

Solution:

Given:

$$\mathbf{Z} = \begin{bmatrix} \frac{s^2+2s+2}{s^2+s+1} & \frac{1}{s^2+s+1} \\ \frac{1}{s^2+s+1} & \frac{s^2+1}{s^2+s+1} \end{bmatrix}$$

Try :



From the circuit, we calculate:

$$z_{11} = R_1 + \frac{\frac{1}{Cs}\left(R_2 + Ls\right)}{\frac{1}{Cs} + R_2 + Ls} = R_1 + \frac{R_2 + Ls}{1 + R_2 Cs + LCs^2} = \frac{LCR_1 s^2 + \left(R_1 R_2 C + L\right)s + R_1 + R_2}{LCs^2 + R_2 Cs + 1}$$

Comparing to the given z_{11} yields:

$$\left.\begin{array}{c} LC = 1 \\ R_2 C = 1 \\ LCR_1 = 1 \\ R_1 R_2 C + L = 2 \\ R_1 + R_2 = 2 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} R_1 = 1\ \Omega \\ R_2 = 1\ \Omega \\ L = 1\text{ H} \\ C = 1\text{ F} \end{array}\right.$$

Then check z_{12}, z_{21} and z_{22}. The are all okay. If they were not, we would have to try a different circuit structure..

P 17.3-10 An infinite two-port network is shown in Figure P 17.3-10. When the output terminals are connected to the circuit's characteristic resistance R_o, the resistance looking down the line from each section is the same. Calculate the necessary R_o.

Answer: $R_o = (\sqrt{3} - 1)R$

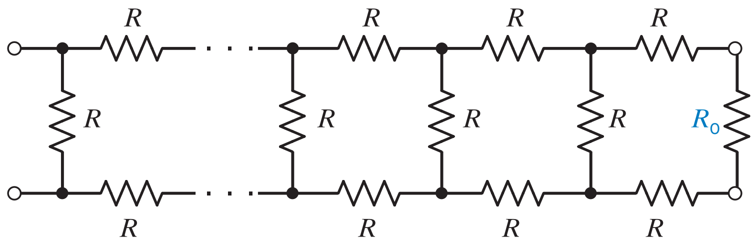
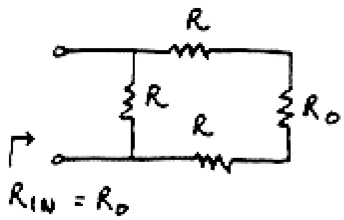


Figure P 17.3-10

Solution:

It is sufficient to require that the input resistance of each section of the circuit is equal to R_o, that is



Then

$$R_o = \frac{R(2R+R_o)}{3R+R_o} \quad \Rightarrow \quad R_o = -R \pm \sqrt{R^2+2R^2} = -R \pm \sqrt{3}\,R \;=\; \underline{(\sqrt{3}-1)\,R}$$

Section 17-4: Z and Y Parameters

P 17.4-1 Determine the *Y* parameters of the circuit shown in Figure P 17.4-1.

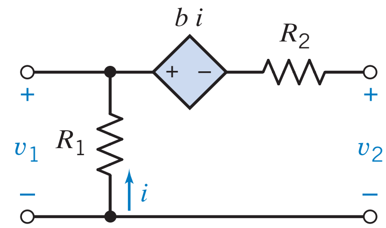
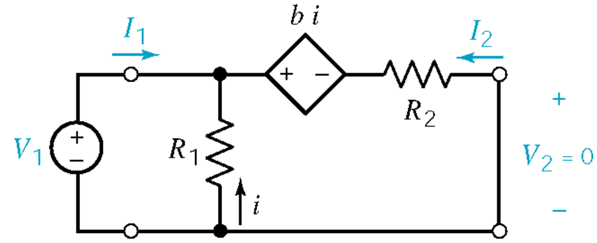


Figure P 17.4-1

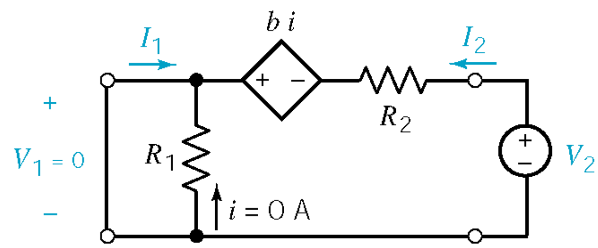
Solution:



$$i = -\frac{V_1}{R_1} \text{ and } I_2 = -\frac{(b + R_1)}{R_1 R_2} V_1$$

$$I_1 = -I_2 - i = \left(\frac{b + R_1 + R_2}{R_1 R_2}\right) V_1$$

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2 = 0} = \frac{b + R_1 + R_2}{R_1 R_2} \text{ and } Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2 = 0} = -\frac{(b + R_1)}{R_1 R_2}$$



$$I_2 = -I_1$$

$$V_2 = R_2 I_2 \quad \Rightarrow \quad I_2 = \frac{V_2}{R_2}$$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1 = 0} = \frac{1}{R_2} \text{ and } Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1 = 0} = -\frac{1}{R_2}$$

Finally

$$\mathbf{Y} = \begin{bmatrix} \frac{b+R_1+R_2}{R_1 R_2} & -\frac{1}{R_2} \\ -\frac{b+R_1}{R_1 R_2} & \frac{1}{R_2} \end{bmatrix}$$

P 17.4-2 An electronic amplifier has the circuit shown in Figure P 17.4-2. Determine the impedance parameters for the circuit.
Answer: $Z_{11} = 4$, $Z_{12} = 3(1 + \alpha)$, $Z_{21} = 3$, and $Z_{22} = 5 + 3\alpha$

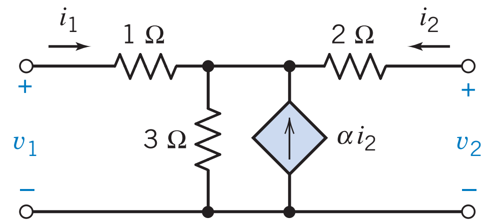
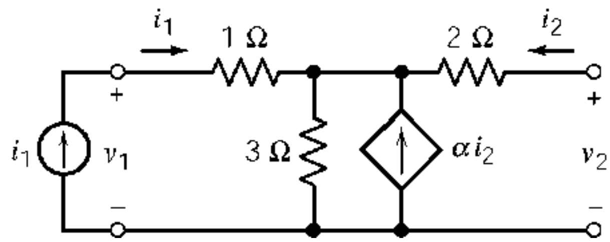


Figure P 17.4-2

Solution:

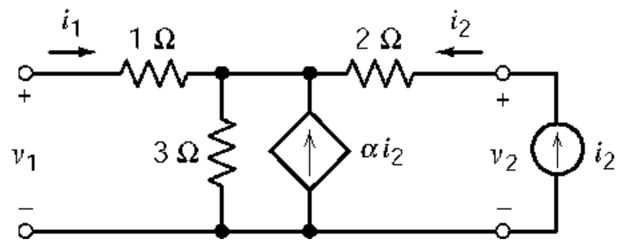
$$i_2 = 0 \quad \Rightarrow \quad \begin{cases} v_1 = (1+3)\,i_1 = 4\,i_1 \\ v_2 = 3\,i_1 \end{cases}$$



therefore

$$Z_{11} = 4\ \Omega \ \text{ and } \ Z_{21} = 3\ \Omega$$

$$i_1 = 0 \quad \Rightarrow \quad \begin{cases} v_1 = 3(\alpha\, i_2 + i_2) \\ v_1 = 3(\alpha\, i_2 + i_2) + 2\,i_2 \end{cases}$$



therefore

$$Z_{12} = 3(1+\alpha) \ \text{ and } \ Z_{21} = 5 + 3\alpha$$

Finally,

$$\mathbf{Z} = \begin{bmatrix} 4 & 3(1+\alpha) \\ 3 & 5+3\alpha \end{bmatrix}$$

P 17.4-3

(a) For the circuit shown in Figure P 17.4-3, determine the two-port Y model using impedances in the *s*-domain.

(b) Determine the response $v_2(t)$ when a current source $i_1 = 1\, u(t)$ A is connected to the input terminals.

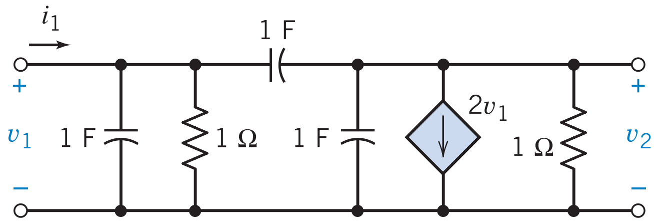
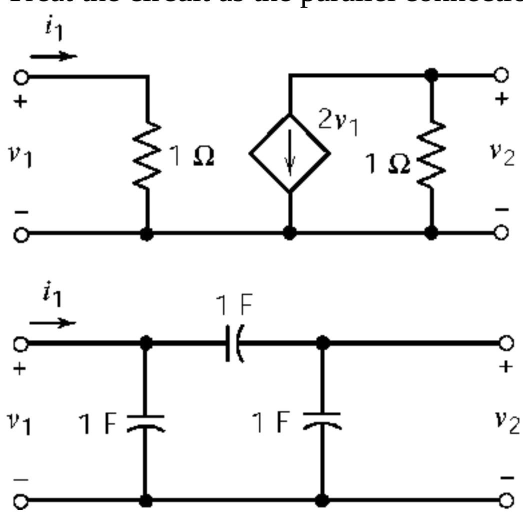


Figure P 17.4-3

Solution:

Treat the circuit as the parallel connection of two 2-port networks:



The admittance matrix of the entire network can be obtained as the sum of the admittance matrices of these two 2-port networks

$$\mathbf{Y} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 2s & -s \\ -s & 2s \end{bmatrix} = \begin{bmatrix} 1+2s & -s \\ 2-s & 1+2s \end{bmatrix}$$

When $i_1(t) = u(t)$:

$$\mathbf{Y}\begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} = \begin{bmatrix} \frac{1}{s} \\ 0 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} = \mathbf{Y}^{-1}\begin{bmatrix} \frac{1}{s} \\ 0 \end{bmatrix} = \frac{\begin{bmatrix} 2s+1 & s \\ s-2 & 2s+1 \end{bmatrix}}{3s^2+6s+1}\begin{bmatrix} \frac{1}{s} \\ 0 \end{bmatrix}$$

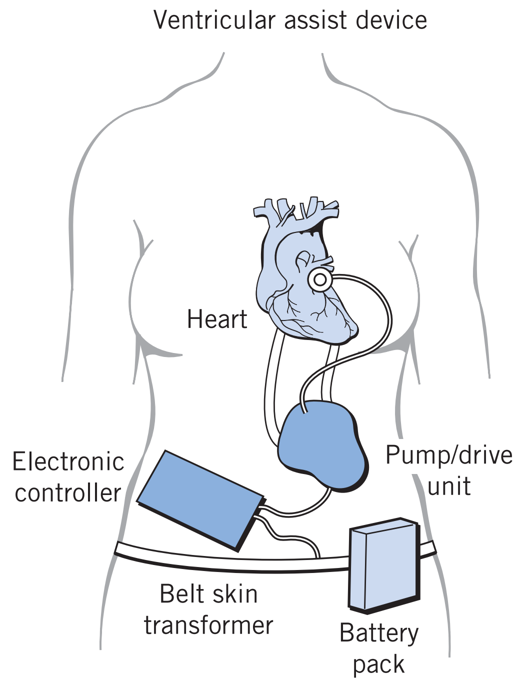
so

$$V_2(s) = \frac{(s-2)}{s(3s^2+6s+1)} = \frac{1}{3}\left[\frac{-6}{s} + \frac{-1.25}{s+1.82} + \frac{7.25}{s+0.184}\right]$$

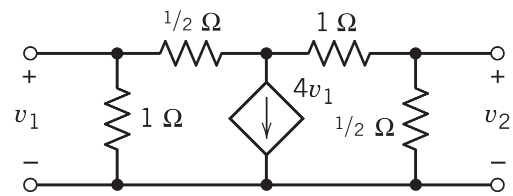
Taking the inverse Laplace transform

$$v_2(t) = \frac{1}{3}\left[-6 - 1.25e^{-1.82t} + 7.25e^{-0.184t}\right] \qquad t \geq 0$$

P 17.4-4 One form of a heart-assist device is shown in Figure P 17.4-4*a*. The model of the electronic controller and pump/drive unit is shown in Figure P 17.4-4*b*. Determine the impedance parameters of the two-port model.



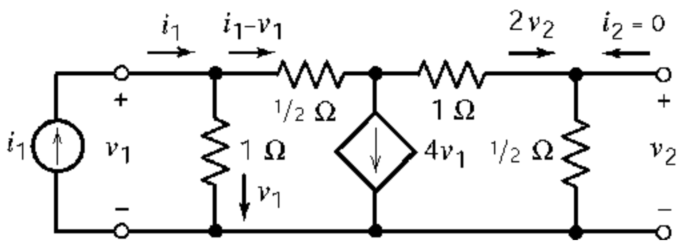
(a)



(b)

Figure P 17.4-4

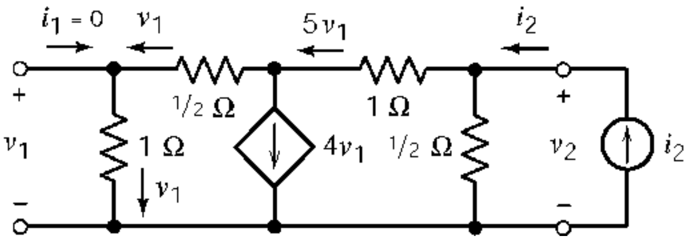
Solution:



KVL: $\frac{1}{2}\left(i_1 - v_1\right) + 2v_2 + v_2 - v_1 = 0$

KCL: $i_1 - v_1 = 4v_1 + 2v_2$

$$\left.\begin{array}{l} i_1 = 3v_1 - 6v_2 \\ i_1 = 5v_1 + 2v_2 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} i_1 = 3v_1 - 6\dfrac{i_1 - 5v_1}{2} \Rightarrow z_{11} = \dfrac{v_1}{i_1} = \dfrac{2}{9}\ \Omega \\ i_1 = 5\dfrac{i_1 + 6v_2}{3} + 2v_2 \Rightarrow z_{21} = \dfrac{v_2}{i_1} = -\dfrac{1}{18}\ \Omega \end{array}\right.$$



KVL: $v_2 = 1\,v_1 + \frac{1}{2}\,v_1 + 5\,v_1 = \frac{13}{2}\,v_1$

KCL: $i_2 = \frac{v_2}{1/2} + 5\,v_1 = 2\,v_2 + 5\,v_1$

$$i_2 = 2\left(\frac{13}{2}\,v_1\right) + 5\,v_1 = 18\,v_1 \quad \Rightarrow \quad z_{12} = \frac{1}{18}\ \Omega$$

and

$$i_2 = 2\,v_2 + 5\left(\frac{2}{13}\,v_2\right) = 2.769\,v_2 \quad \Rightarrow \quad z_{22} = 0.361\,\Omega$$

P 17.4-5 Determine the Y parameters for the circuit shown in Figure P 17.4-5.

Partial Answer: $Y_{12} = -\dfrac{1}{R_2}$ and $Y_{21} = \dfrac{-(1+b)}{R_2}$

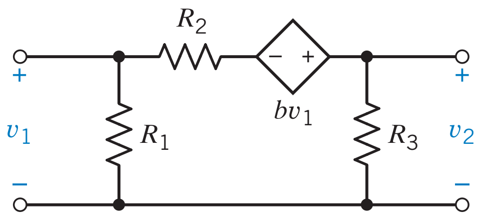
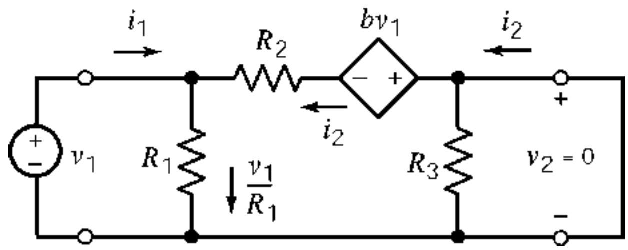


Figure P 17.4-5

Solution:



KCL: $i_1 + i_2 = \dfrac{v_1}{R_1}$

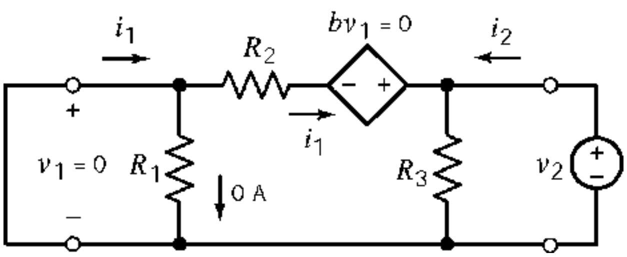
KVL: $-R_2\, i_2 - b\, v_1 + 0 - v_1 = 0$

Then

$$i_2 = -\frac{b+1}{R_2} v_1 \quad \textit{and} \quad i_1 = \left(\frac{1}{R_1} + \frac{b+1}{R_2}\right) v_1 = \frac{R_2 + R_1(b+1)}{R_1 R_2} v_1$$

so

$$y_{21} = \frac{i_2}{v_1} = -\frac{b+1}{R_2} \quad \text{and} \quad y_{11} = \frac{i_1}{v_1} = \frac{R_2 + R_1(b+1)}{R_1 R_2}$$



KVL: $R_2\, i_1 + v_2 = 0 \Rightarrow i_1 = -\dfrac{1}{R_2} v_2$

KCL: $v_2 = R_3 (i_1 + i_2) = R_3\left(-\dfrac{1}{R_2} v_2\right) + R_3\, i_2$

Then

$$y_{12} = \frac{i_1}{v_2} = -\frac{1}{R_2} \quad \text{and} \quad v_2\left(1 + \frac{R_3}{R_2}\right) = R_3\, i_2 \;\Rightarrow\; y_{22} = \frac{i_2}{v_2} = \frac{1}{R_3} + \frac{1}{R_2}$$

Finally

$$\mathbf{Y} = \begin{bmatrix} \dfrac{R_2 + R_1(b+1)}{R_1 R_2} & -\dfrac{1}{R_2} \\ -\dfrac{b+1}{R_2} & \dfrac{R_2 + R_3}{R_2 R_3} \end{bmatrix}$$

Section 17-5: Hybrid Transmission Parameters

P 17.5-1 Find the transmission parameters of the circuit of Figure P 17.5-1.

Answer: $A = 1.2$, $B = 6.8\ \Omega$, $C = 0.1$ S, and $D = 1.4$

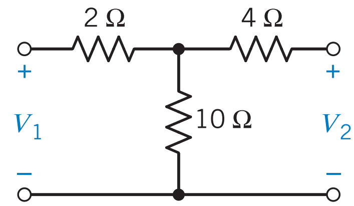
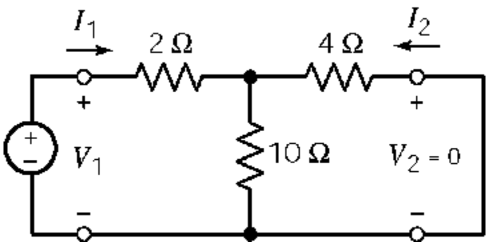


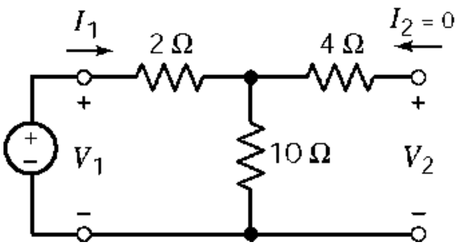
Figure P 17.5-1

Solution:



$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = \frac{34}{5} = 6.8\ \Omega \quad \text{since} \quad -I_2 = \frac{V_1}{2+4\|10} = \frac{5}{34} V_1$$

$$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = \frac{10+4}{10} = 1.4 \quad \text{since} \quad I_2 = -\frac{10}{10+4} I_1$$



$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{12}{10} = 1.2 \quad \text{since} \quad V_2 = \frac{10}{10+2} V_1$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{1}{10} = 0.1\ \text{S}$$

P 17.5-2 An op amp circuit and its model are shown in Figure P 17.5-2. Determine the *h*-parameter model of the circuit and the **H** matrix when R_i = 100 kΩ, $R_1 = R_2$ = 1 MΩ, R_o = 1 kΩ, and $A = 10^4$.

Answer: h_{11} = 600 kΩ, h_{12} = 1/2, $h_{21} = -10^6$, and $h_{22} = 10^{-3}$ S

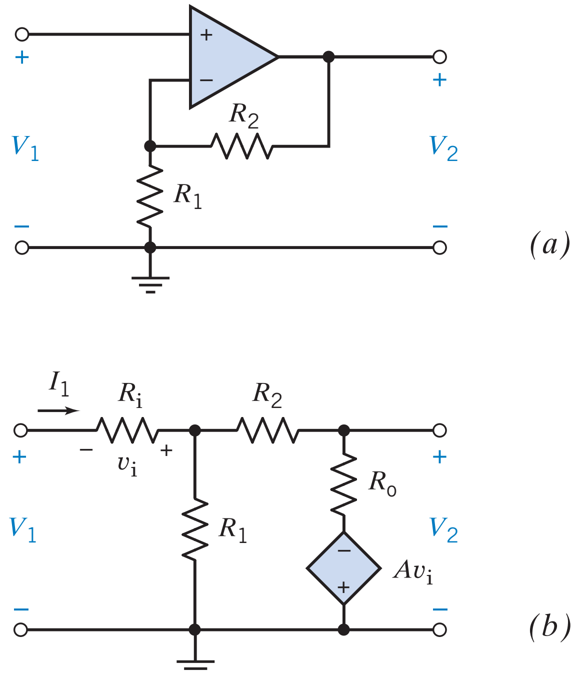


Figure P 17.5-2

Solution:

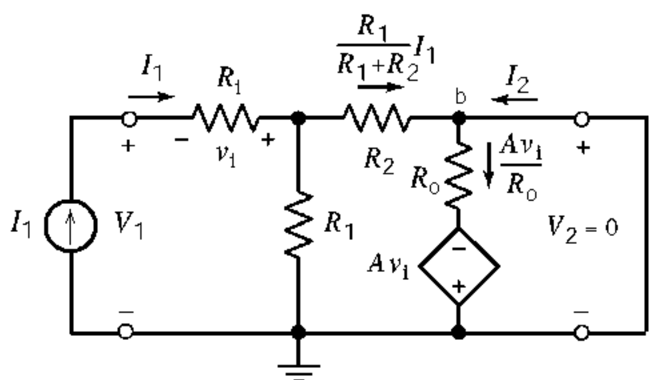
$$V_2 = 0$$

so

$$V_1 = (R_i + R_1 \| R_2)\, I_1$$

therefore

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = R_i + R_1 \| R_2 = 600 \text{ k}\Omega$$



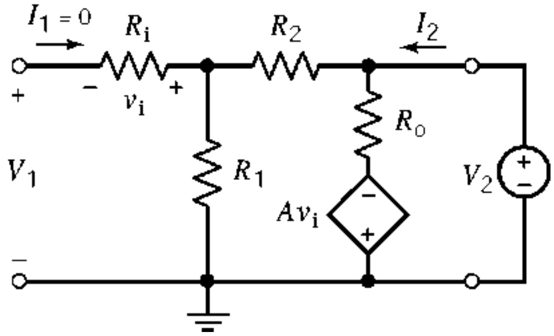
KVL:

$$I_2 + \frac{R_1}{R_1+R_2} I_1 = -A\frac{R_i}{R_o} I_1$$

therefore

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = -\left(A\frac{R_i}{R_o} + \frac{R_1}{R_1+R_2}\right) = -10^6$$

$$I_1 = 0 \quad \Rightarrow \quad v_i = 0 \quad \Rightarrow \quad Av_i = 0$$



so

$$I_2 = \frac{V_2}{R_o \| (R_1+R_2)}$$

therefore

$$h_{22} = \left. \frac{I_2}{V_2} \right|_{I_1=0} = \frac{R_o+R_1+R_2}{R_o(R_1+R_2)} = 10^{-3}$$

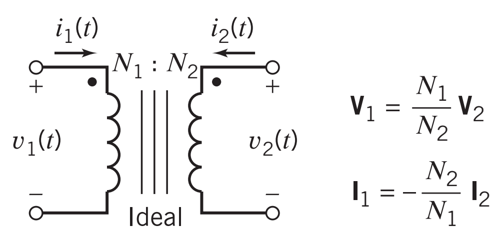
Next,

$$V_1 = \frac{R_1}{R_1+R_2} V_2$$

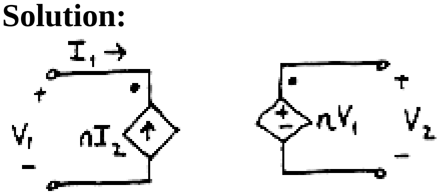
therefore

$$h_{12} = \left. \frac{V_1}{V_2} \right|_{I_1=0} = \frac{R_1}{R_1+R_2} = \frac{1}{2}$$

P 17.5-3 Determine the *h* parameters for the ideal transformer of Section 11.9.



Solution:



Compare : to

$$V_2 = nV_1 \qquad V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_1 = -n\,I_2 \qquad I_2 = h_{21}I_1 + h_{22}V_2$$

Then $h_{11} = 0,\ h_{22} = 0,\ h_{12} = 1/n$ and $h_{21} = 1/-n$

P 17.5-4 Determine the *h* parameters for the T circuit of Figure P 17.5-4.

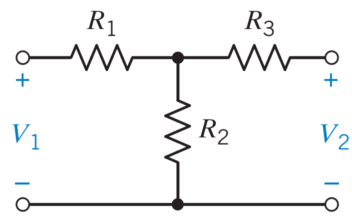
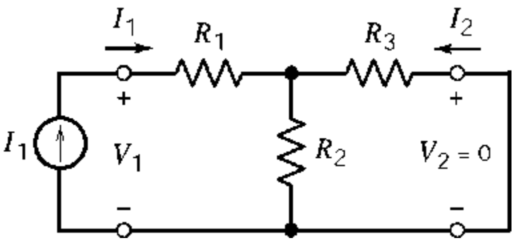


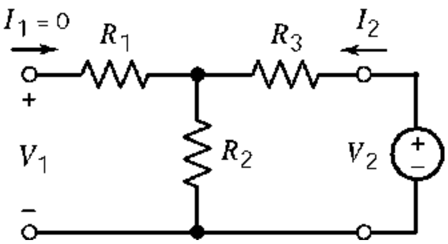
Figure P 17.5-4

Solution:



$$V_1 = \left(R_1 + R_2 \parallel R_3\right) I_1 \Rightarrow h_{11} = R_1 + \frac{R_2 R_3}{R_2 + R_3}$$

$$I_2 = -\frac{R_2}{R_2 + R_3} I_1 \Rightarrow h_{21} = -\frac{R_2}{R_2 + R_3}$$



$$I_2 = \frac{V_2}{R_2 + R_3} \Rightarrow h_{22} = \frac{1}{R_2 + R_3}$$

$$V_1 = \frac{R_2}{R_2 + R_3} V_2 \Rightarrow h_{12} = \frac{R_2}{R_2 + R_3}$$

P 17.5-5 A simplified model of a bipolar junction transistor is shown in Figure P 17.5-5. Determine the *h* parameters of this circuit.

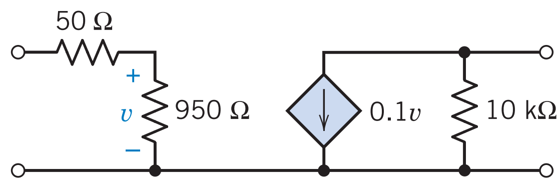
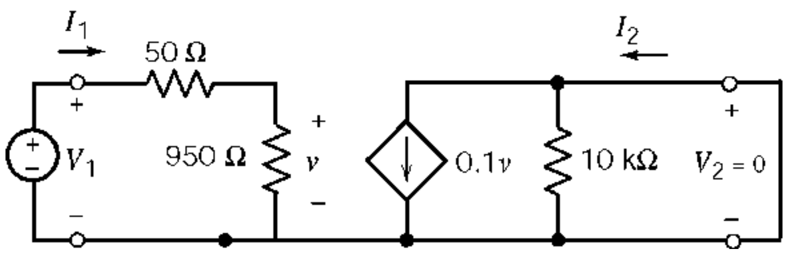


Figure P 17.5-5

Solution:



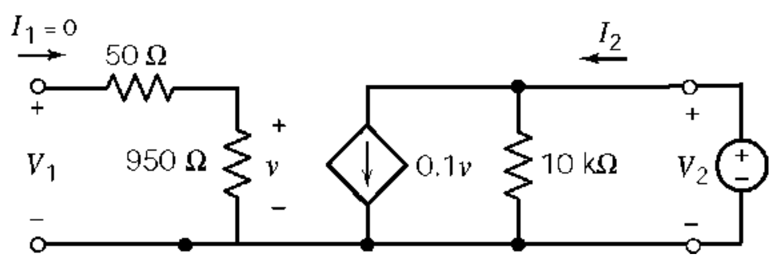
$I_2 = 0.1v$ and $v = 950\, I_1$

so $I_2 = 95\, I_1$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = 50 + 950 = 1000\ \Omega$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = 95$$

$I_1 = 0 \quad \Rightarrow \quad v = 0$



$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = 0$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = 10^{-4}\ \text{S}$$

Section 17.6: Relationships between Two-Port Parameters

P 17.6-1 Derive the relationships between the *Y* parameters and the *h* parameters by utilizing the defining equations for both parameter sets.

Solution:
Start with

$$\text{Y parameters: } \begin{cases} I_1 = Y_{11} V_1 + Y_{12} V_2 \\ I_2 = Y_{21} V_1 + Y_{22} V_2 \end{cases} \quad \text{and} \quad \text{H parameters: } \begin{cases} V_1 = h_{11} I_1 + h_{12} V_2 \\ I_2 = h_{21} I_1 + h_{22} V_2 \end{cases}$$

Solve the Y parameter equations for V_1 and I_2 to put them in the same form as the H parameter equations.

$$\begin{aligned} -Y_{11} V_1 + I_1 &= Y_{12} V_2 \\ -Y_{21} V_1 + I_2 &= Y_{22} V_2 \end{aligned} \quad \Rightarrow \quad \begin{bmatrix} -Y_{11} & 0 \\ -Y_{21} & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -1 & Y_{12} \\ 0 & Y_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -Y_{11} & 0 \\ -Y_{21} & 1 \end{bmatrix}^{-1} \begin{bmatrix} -1 & Y_{12} \\ 0 & Y_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$\therefore\ \mathbf{H} = \begin{bmatrix} -Y_{11} & 0 \\ -Y_{21} & 1 \end{bmatrix}^{-1} \begin{bmatrix} -1 & Y_{12} \\ 0 & Y_{22} \end{bmatrix} = \begin{bmatrix} -\dfrac{1}{Y_{11}} & 0 \\ -\dfrac{Y_{21}}{Y_{11}} & 1 \end{bmatrix} \begin{bmatrix} -1 & Y_{12} \\ 0 & Y_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{1}{Y_{11}} & -\dfrac{Y_{12}}{Y_{11}} \\ \dfrac{Y_{21}}{Y_{11}} & Y_{22} - \dfrac{Y_{12} Y_{21}}{Y_{11}} \end{bmatrix}$$

P 17.6-2 Determine the *Y* parameters if the *Z* parameters are (in ohms):

$$\mathbf{Z} = \begin{bmatrix} 3 & 2 \\ 2 & 6 \end{bmatrix}$$

Solution:

First $\Delta \mathbf{Z} = (3)(6) - (2)(2) = 14$. Then $\mathbf{Y} = \begin{bmatrix} \dfrac{Z_{22}}{\Delta \mathbf{Z}} & -\dfrac{Z_{12}}{\Delta \mathbf{Z}} \\ \dfrac{-Z_{21}}{\Delta \mathbf{Z}} & \dfrac{Z_{11}}{\Delta \mathbf{Z}} \end{bmatrix} = \begin{bmatrix} \dfrac{6}{14} & -\dfrac{2}{14} \\ -\dfrac{2}{14} & \dfrac{3}{14} \end{bmatrix}$.

P 17.6-3 Determine the *h* parameters when the *Y* parameters are (in siemens):

$$\mathbf{Y} = \begin{bmatrix} 0.1 & 0.1 \\ 0.4 & 0.5 \end{bmatrix}$$

Solution:

First $\Delta\mathbf{Y} = (0.1)(0.5) - (0.4)(0.1) = .01\ \text{S}$. Then $\mathbf{H} = \begin{bmatrix} \frac{1}{Y_{11}} & -\frac{Y_{12}}{Y_{11}} \\ \frac{Y_{21}}{Y_{11}} & \frac{\Delta\mathbf{Y}}{Y_{11}} \end{bmatrix} = \begin{bmatrix} 10 & -1 \\ 4 & 0.1 \end{bmatrix}$.

P 17.6-4 A two-port has the following *Y* parameters: $Y_{12} = Y_{21} = -0.4$ S, $Y_{11} = 0.5$ S, and $Y_{22} = 0.6$ S. Determine the *h* parameters.

Answer: $h_{11} = 2\ \Omega$, $h_{21} = -0.8$, $h_{12} = 0.8$, and $h_{22} = 0.28$ S

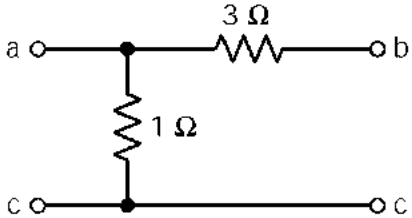
Solution:

First $\Delta\mathbf{Y} = (0.5)(0.6) - (-0.4)(-0.4)\ \text{S}$. Then $\mathbf{H} = \begin{bmatrix} \frac{1}{Y_{11}} & -\frac{Y_{12}}{Y_{11}} \\ \frac{Y_{21}}{Y_{11}} & \frac{\Delta\mathbf{Y}}{Y_{11}} \end{bmatrix} = \begin{bmatrix} 2 & 0.8 \\ -0.8 & 0.28 \end{bmatrix}$

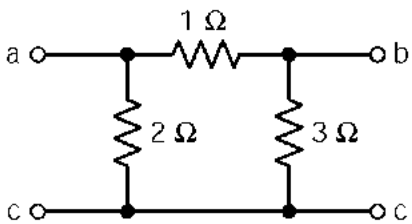
Section 17.7: Interconnection of Two-Port Networks

P 17.7-1 Connect in parallel the two circuits shown in Figure P 17.7-1 and find the *Y* parameters of the parallel combination.
Answer: Y_{11} = 17/6, Y_{12} = Y_{21} = –4/3, and Y_{22} = 5/3

Solution:



$$Y_{12} = Y_{21} = -\tfrac{1}{3}\ \text{S}$$
$$Y_{22} = 0 - Y_{21} = \tfrac{1}{3}\ \text{S}$$
$$Y_{11} + Y_{12} = 1\ \text{S} \quad \Rightarrow \quad Y_{11} = \tfrac{4}{3}\ \text{S}$$
$$\mathbf{Y}_a = \begin{bmatrix} \tfrac{4}{3} & -\tfrac{1}{3} \\ -\tfrac{1}{3} & \tfrac{1}{3} \end{bmatrix}$$

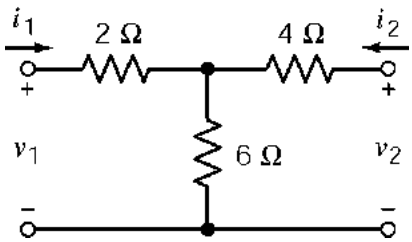


$$Y_{12} = Y_{21} = -1\ \text{S}$$
$$Y_{11} + Y_{12} = \tfrac{1}{2}\ \text{S} \quad \Rightarrow \quad Y_{11} = \tfrac{3}{2}\ \text{S}$$
$$Y_{21} + Y_{22} = \tfrac{1}{3}\ \text{S} \quad \Rightarrow \quad Y_{22} = \tfrac{4}{3}\ \text{S}$$
$$\mathbf{Y}_b = \begin{bmatrix} \tfrac{3}{2} & -1 \\ -1 & \tfrac{4}{3} \end{bmatrix}$$

$$\mathbf{Y} = \mathbf{Y}_a + \mathbf{Y}_b = \begin{bmatrix} (\tfrac{4}{3}+\tfrac{3}{2}) & -\tfrac{4}{3} \\ -\tfrac{4}{3} & \tfrac{5}{3} \end{bmatrix} = \begin{bmatrix} \tfrac{17}{6} & -\tfrac{4}{3} \\ -\tfrac{4}{3} & \tfrac{5}{3} \end{bmatrix}$$

P 17.7-2 For the T network of Figure P 17.7-2, find the *Y* and *T* parameters and determine the resulting parameters after the two two-ports are connected in (a) parallel and (b) cascade. Both two-ports are identical as defined in Figure P 17.7-2

Solution:

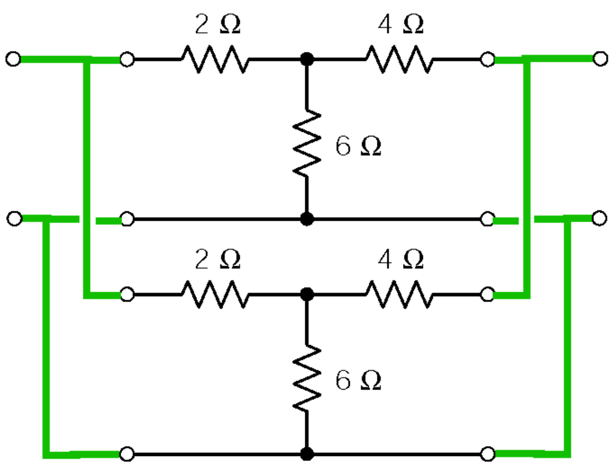


Admittance parameters:

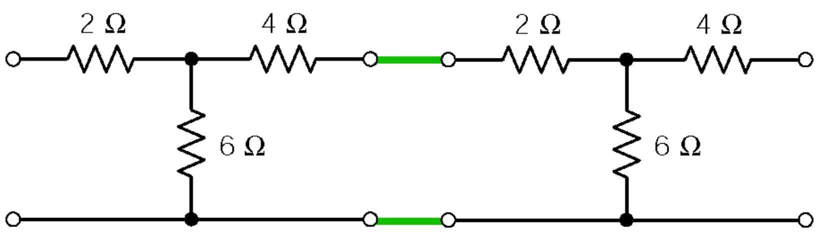
$$\mathbf{Y} = \begin{bmatrix} \frac{10}{44} & \frac{-6}{44} \\ \frac{-6}{44} & \frac{8}{44} \end{bmatrix}$$

Transmission parameters:

$$\mathbf{T} = \begin{bmatrix} \frac{8}{6} & \frac{44}{6} \\ \frac{1}{6} & \frac{10}{6} \end{bmatrix}$$



$$\mathbf{Y}_{\text{p}} = \mathbf{Y} + \mathbf{Y} = \begin{bmatrix} \frac{20}{44} & \frac{-12}{44} \\ \frac{-12}{44} & \frac{16}{44} \end{bmatrix}$$



$$\mathbf{T}_{\text{C}} = \mathbf{T} \cdot \mathbf{T} = \begin{bmatrix} \frac{108}{36} & \frac{792}{36} \\ \frac{18}{36} & \frac{144}{36} \end{bmatrix}$$

P 17.7-3 Determine the *Y* parameters of the parallel combination of the circuits of Figures P 17.7-3*a*, *b*.

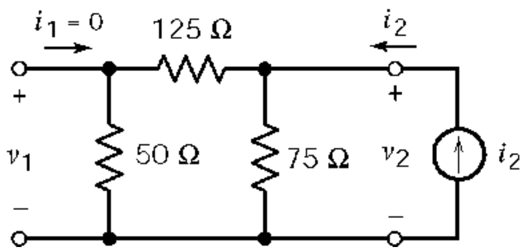
Solution:

$$\mathbf{Y} = \mathbf{Y}_{\text{a}} + \mathbf{Y}_{\text{b}} = \begin{bmatrix} \frac{1}{sL} + sC & -sC \\ -sC & \frac{1}{sL} + sC \end{bmatrix} + \begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{bmatrix}$$

Section 17.8 How Can We Check…?

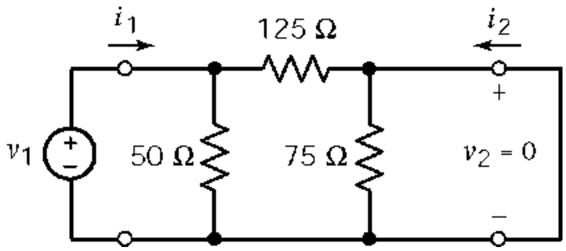
P 17.8-1 A laboratory report concerning the circuit of Figure P 17.8-1 states that $Z_{12} = 15\ \Omega$ and $Y_{11} = 24$ mS. Verify these results.

Solution:



$$V_1 = 50\left(\frac{75}{175+75}\right)I_2 = 15\ I_2$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = 15\ \Omega$$



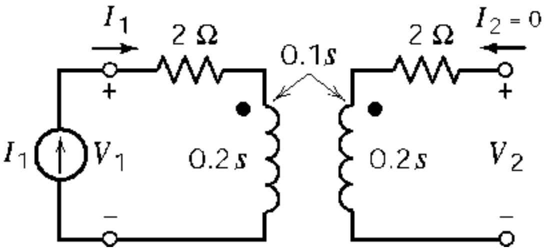
$$I_1 = \left(\frac{1}{50}+\frac{1}{125}\right)V_1 = 0.028\,V_1$$

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = 28\text{ mS}$$

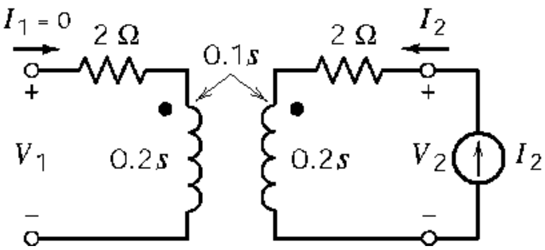
$Y_{11} \neq 24$ mS, so the report is n<u>o</u>t correct.

P 17.8-2 A student report concerning the circuit of Figure P 17.8-2 has determined the transmission parameters as $A = 2(s + 10)/s$, $D = A$, $C = 10/s$, and $B = (3s^2 + 80s + 400)/s^2$. Verify these results when $M = 0.1$ H.

Solution:



$$\left.\begin{array}{l} V_1 = (2+0.2s)\, I_1 \\ V_2 = (0.1s) I_1 \end{array}\right\} \Rightarrow \begin{array}{l} Z_{11} = 2+0.2s = 0.2\,(s+10) \\ Z_{21} = 0.1s \end{array}$$



$$Z_{22} = 2 + 0.2\,s \quad \text{and} \quad Z_{12} = 0.1s$$

$$\Delta\mathbf{Z} = (2+0.2\,s)(2+0.2\,s) - (0.1s)(0.1s) = 0.01(3s^2 + 80\,s + 400)$$

$$\mathbf{T} = \begin{bmatrix} \dfrac{Z_{11}}{Z_{21}} & \dfrac{\Delta\mathbf{Z}}{Z_{21}} \\ \dfrac{1}{Z_{21}} & \dfrac{Z_{22}}{Z_{21}} \end{bmatrix} = \begin{bmatrix} \dfrac{2(s+10)}{s} & \dfrac{0.1(3s^2+80s+400)}{s} \\ \dfrac{10}{s} & \dfrac{2\,(s+10)}{s} \end{bmatrix}$$

This is the transmission matrix given in the report.

Design Problems

DP 17-1 Select R_1 and R so that R_{in} = 16.6 Ω for the circuit of Figure DP 17-1. A design constraint requires that both R_1 and R be less than 10 Ω.

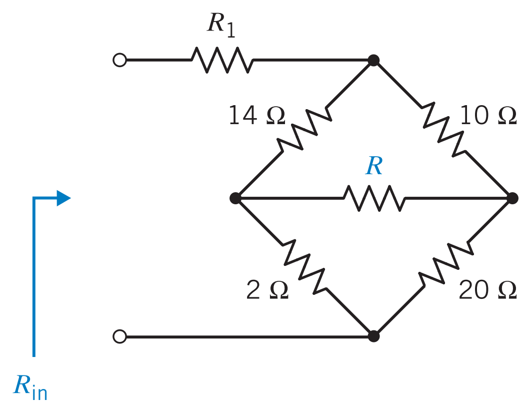
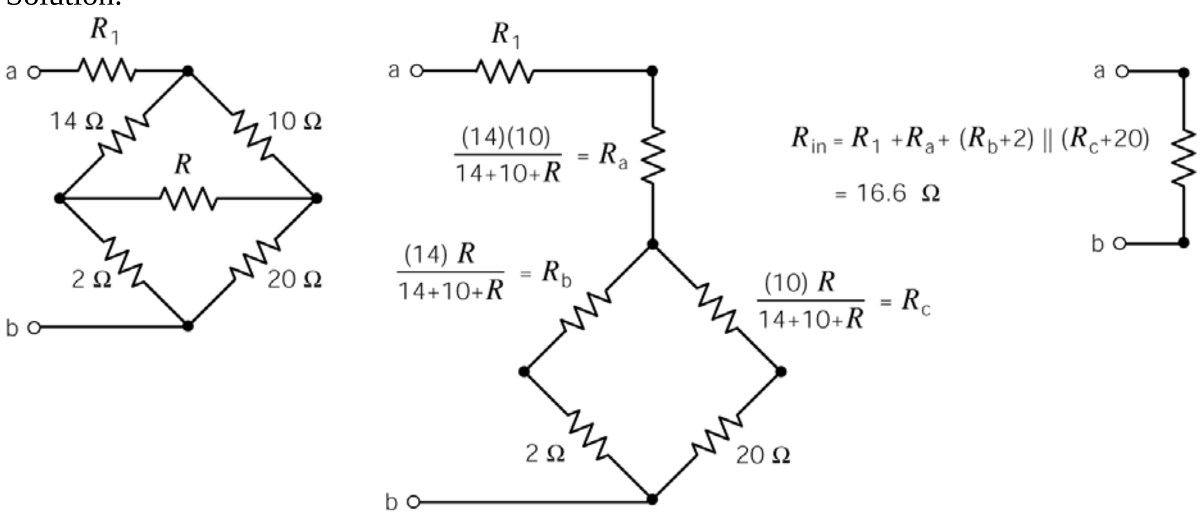


Figure DP 17-1

Solution:



We will need to find R and R_1 by trial and error. A Mathcad spreadsheet will help with the calculations. Given the restrictions $R \le 10\ \Omega$ and $R_1 \le 10\ \Omega$ we will start with $R = 10\ \Omega$ and $R_1 = 10\ \Omega$:

$R1 := 10 \qquad R := 10$

$Ra := \dfrac{14 \cdot 10}{14 + 10 + R} \qquad Rb := \dfrac{14 \cdot R}{14 + 10 + R} \qquad Rc := \dfrac{R \cdot 10}{14 + 10 + R}$

$Rin := R1 + Ra + \dfrac{(Rb + 2) \cdot (Rc + 20)}{Rb + 2 + Rc + 20} \qquad Rin = 18.947$

The specifications can be satisfied by reducing R_1:

$R1 := 7.653 \qquad R := 10$

$Ra := \frac{14 \cdot 10}{14 + 10 + R} \qquad Rb := \frac{14 \cdot R}{14 + 10 + R} \qquad Rc := \frac{R \cdot 10}{14 + 10 + R}$

$Rin := R1 + Ra + \frac{(Rb + 2) \cdot (Rc + 20)}{Rb + 2 + Rc + 20} \qquad Rin = 16.6$

One solution is $R = 7.653\ \Omega$ and $R_1 = 10\ \Omega$.

DP 17-2 The bridge circuit shown in Figure DP 17-2 is said to be balanced when $I = 0$. Determine the required relationship for the bridge resistances when balance is achieved.

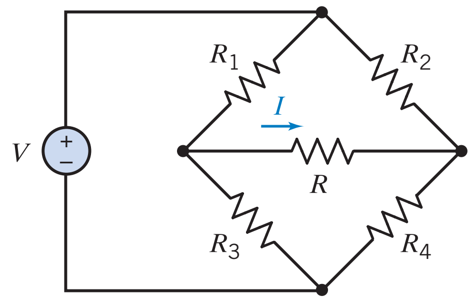
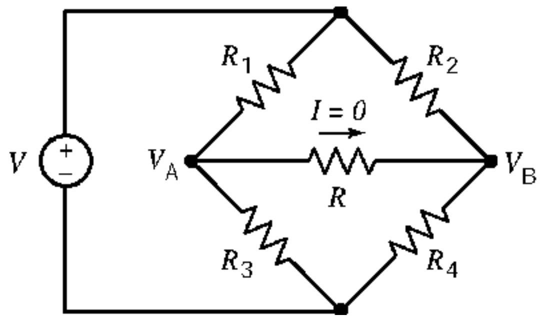


Figure DP 17-2

Solution:



Need $V_A + V_B$ for balance

$$\frac{R_1\,V}{R_1+R_3} = \frac{R_2\,V}{R_2+R_4} \qquad (1)$$

$$\frac{R_3\,V}{R_1+R_3} = \frac{R_4\,V}{R_2+R_4} \qquad (2)$$

Dividing (1) by (2) yields: $\underline{\frac{R_1}{R_3} = \frac{R_2}{R_4}}$.

DP 17-3 A hybrid model of a common-emitter transistor amplifier is shown in Figure DP 17-3. The transistor parameters are h_{21} = 80, h_{11} = 45 Ω, h_{22} = 12.5 μS, and $h_{12} = 5 \times 10^{-4}$. Select R_L so that the current gain i_2/i_1 = 79 and the input resistance of the amplifier is less than 10 Ω.

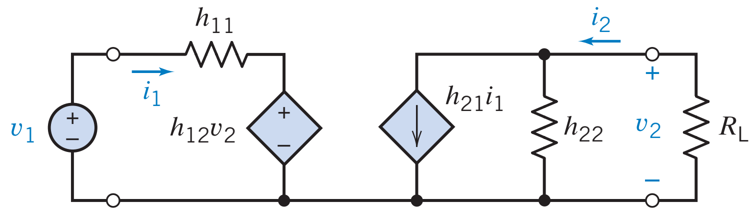


Figure DP 17-3

Solution:

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

and $V_2 = -I_2 R_L \Rightarrow I_2 = h_{21} I_1 - h_{22} R_L I_2$

Next

$$\frac{I_2}{I_1} = h_{21}\left(\frac{1}{1+h_{22} R_L}\right) \Rightarrow A_i = \frac{I_L}{I_1} = -\frac{I_2}{I_1} = -h_{21}\left(\frac{1}{1+h_{22} R_L}\right)$$

We require

$$79 = 80\left(\frac{1}{1+h_{22}R_L}\right) \Rightarrow \frac{79}{80}\left(1+\frac{R_L}{80\times10^3}\right) = 1 \Rightarrow R_L = 1.013\ \text{k}\Omega \cong 1\ \text{k}\Omega$$

Next

$$I_2 = -\frac{V_2}{R_L} = h_{21} I_1 + h_{22} V_2 \Rightarrow V_2 (h_{22} + 1/R_L) = -h_{21} I_1$$

Substituting this expression into the second hybrid equation gives:

$$V_1 = h_{11} I_1 + \frac{h_{12}(-h_{21})}{(h_{22} + 1/R_L)} I_1$$

The input resistance can be approximated as

$$R_{in} \approx h_{11} - h_{12} R_L h_{21} \quad (\text{since } h_{22} << 1/R_L)$$

Finally

$$R_{in} = 45 - (5\times10^{-4})(10^3)(80) = 5\ \Omega < 10\ \Omega$$

DP 17-4 A two-port network connected to a source v_s and a load resistance R_L is shown in
Figure DP 17-4.

(a) Determine the impedance parameters of the two-port network.

(b) Select R_L so that maximum power is delivered to R_L.

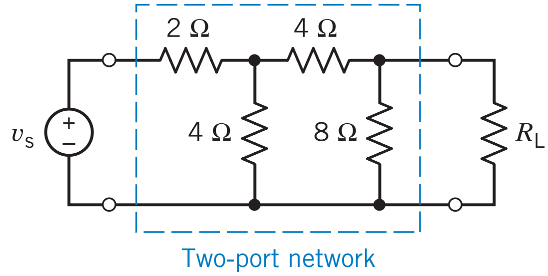
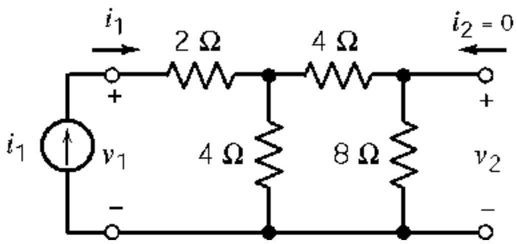


Figure DP 17-4

Solution:

$$Z_{11} = 2 + \frac{4(12)}{4+12} = 5\ \Omega \quad \text{and} \quad Z_{22} = \frac{8(8)}{8+8} = 4\ \Omega$$

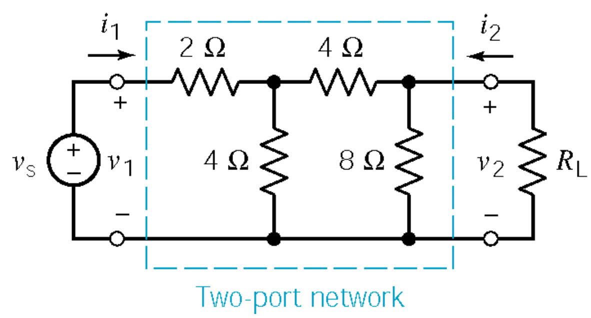


$$V_2 = 8\left[\frac{4}{4+12} I_1\right] = 2 I_1 \quad \Rightarrow \quad Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = 2\ \Omega$$

Similarly $Z_{12} = 2\ \Omega$

Thèvenin: $Z_T = Z_{22} = 4\ \Omega$ so for maximum power transfer, use $R_L = 4\ \Omega$

$$P_{RL} = \frac{\left(\frac{V_s}{2}\right)^2}{4} = 89.3\ \text{W} \Rightarrow V_s = 37.8\ \text{V}$$



DP 17-5

(a) Determine the ABCD (transmission matrix) of the two-port networks shown in Figures DP 17.5*a* and DP 17.5*b*.

(b) Using the results of part (a), find the *s*-domain ABCD matrix of the network shown in Figure DP 17-5*c*.

(c) Given $L_1 = (10/\pi)$ mH, $L_2 = (2.5/\ \pi)$ mH, $C_1 = (0.78/\ \pi)\ \mu$F, $C_2 = C_3 = (1/\ \pi)\ \mu$F, and $R_L = 100\ \Omega$, find the open-circuit voltage gain V_2/V_1 and the short-circuit current gain I_2/I_1 under sinusoidal-state conditions at the following frequencies: 2.5 kHz, 5.0 kHz, 7.5 kHz, 10 kHz, and 12.5 kHz.

Hint: Use the appropriate entries of the ABCD matrix. Also note the resonant frequencies of the circuit.)

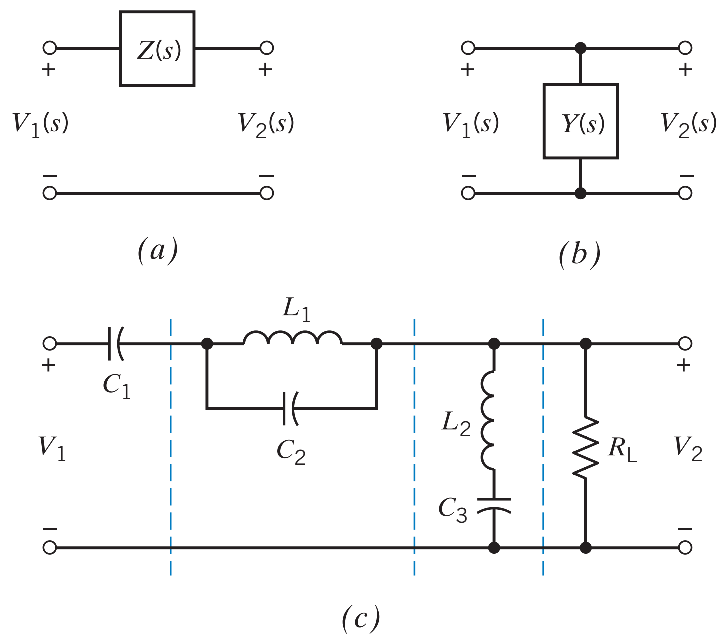
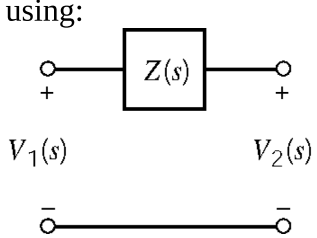


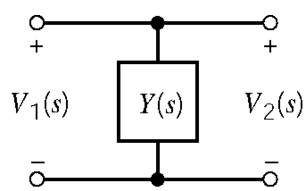
Figure DP 17.5

Solution:

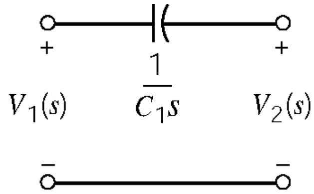
The circuit consists of 4 cascaded stages. Represent each stage by a transmission matrix using:



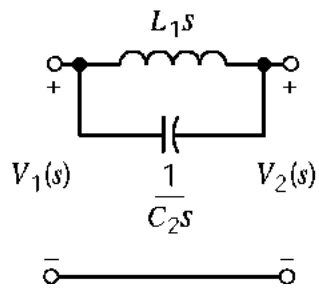
$$\mathbf{T} = \begin{bmatrix} 1 & Z(s) \\ 0 & 1 \end{bmatrix}$$



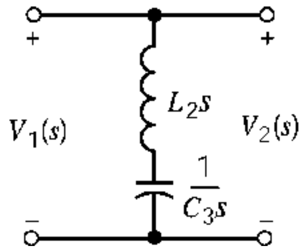
$$\mathbf{T} = \begin{bmatrix} 1 & 0 \\ Y(s) & 1 \end{bmatrix}$$



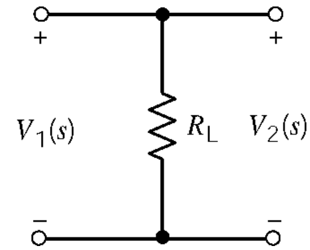
$$\mathbf{T}_{\mathrm{a}} = \begin{bmatrix} 1 & \dfrac{1}{C_1 s} \\ 0 & 1 \end{bmatrix}$$



$$\mathbf{T}_{\mathrm{b}} = \begin{bmatrix} 1 & \dfrac{L_1 s}{L_1 C_2 s^2 + 1} \\ 0 & 1 \end{bmatrix}$$



$$\mathbf{T} = \begin{bmatrix} 1 & 0 \\ \dfrac{C_3 s}{L_2 C_3 s^2 + 1} & 1 \end{bmatrix}$$



$$\mathbf{T}_{\mathrm{d}} = \begin{bmatrix} 1 & 0 \\ \dfrac{1}{R_{\mathrm{L}}} & 1 \end{bmatrix}$$

$$\mathbf{T} = \mathbf{T}_{\mathrm{a}}\,\mathbf{T}_{\mathrm{b}}\,\mathbf{T}_{\mathrm{c}}\,\mathbf{T}_{\mathrm{d}} = \begin{bmatrix} 1 + \dfrac{L_1\left(C_1 + C_2\right)s^2 + 1}{\left(L_1 C_2 s^2 + 1\right)C_1 s} \times \dfrac{L_2 C_3 s^2 + R_{\mathrm{L}} C_3 s + 1}{R_{\mathrm{L}}\left(L_2 C_3 s^2 + 1\right)} & \dfrac{L_1\left(C_1 + C_2\right)s^2 + 1}{\left(L_1 C_2 s^2 + 1\right)C_1 s} \\ \dfrac{L_2 C_3 s^2 + R_{\mathrm{L}} C_3 s + 1}{R_{\mathrm{L}}\left(L_2 C_3 s^2 + 1\right)} & 1 \end{bmatrix}$$