Electric Circuits
Lecture 8 Energy and Power

EE2210, Fall 2022

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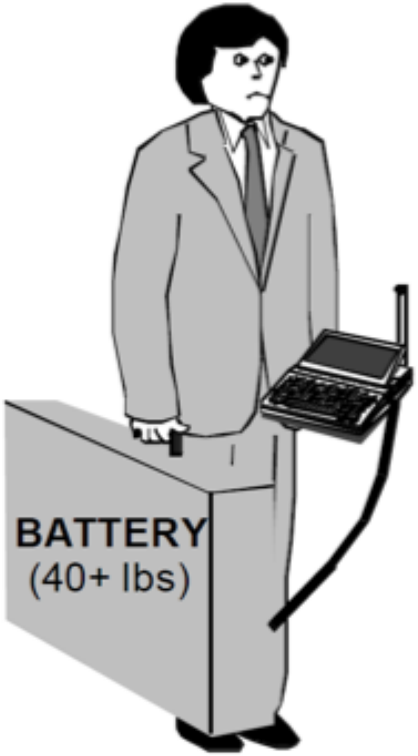
Why Worry about Energy?

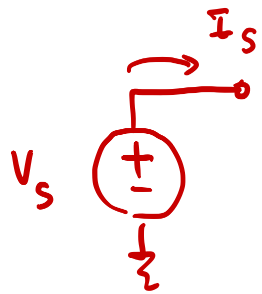
- Portable devices

- How long will the battery last?
  - In standby mode
  - In active mode
- Will the chip overheat and self-destruct?



Static and Dynamic Power

- Static power: power loss due to static current drawn from the power supply.

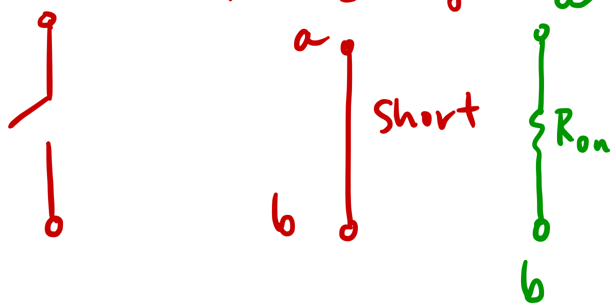


$$P_s = V_s \cdot I_s$$

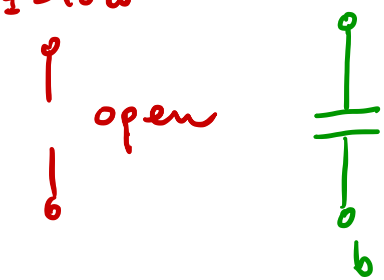
- Dynamic power: power loss due to the switching current required to charge and discharge capacitors.

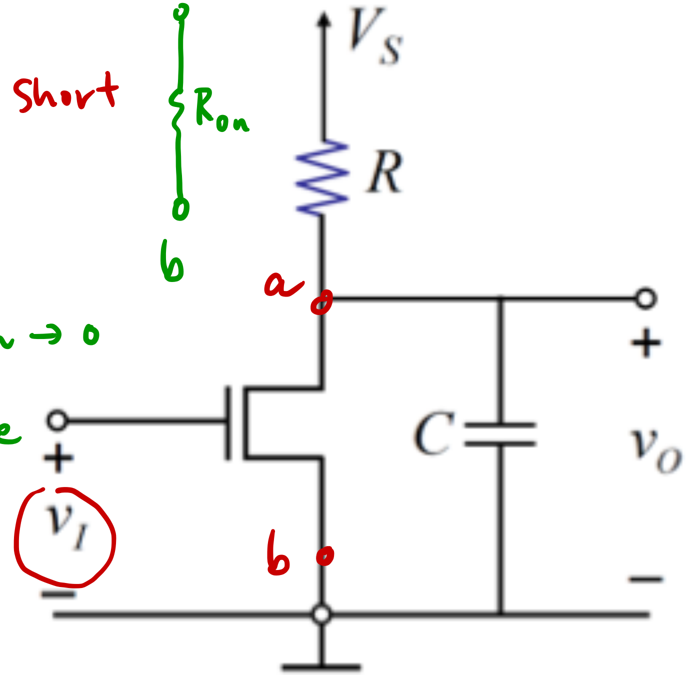
Energy Dissipation in MOSFET Gates

switch v_I = high



v_I = low





ideally, $R_{on} \to 0$

R_{on}: on resistance

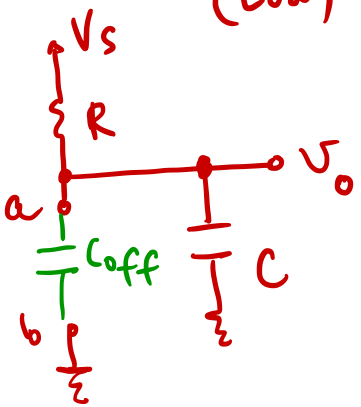
ideally $C_{off} \to 0$

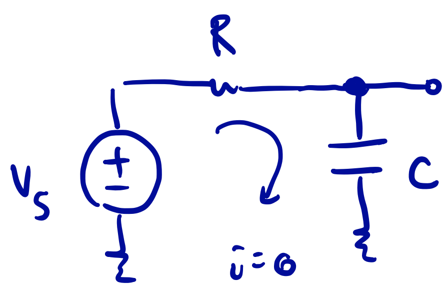
2) when $v_I = V_S$ (High)

($v_I \geq V_{TH}$ 0.3v ~ 1v

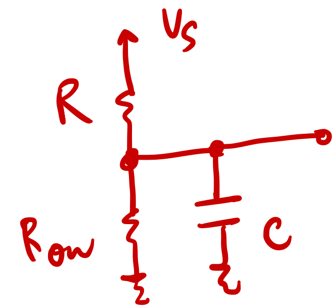
V_{TH}: threshold voltage)

1) when $v_I = 0$ (Low)

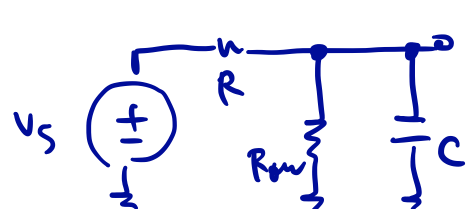




$v_o = V_s$ (High)



$v_o = 0$ (Low)



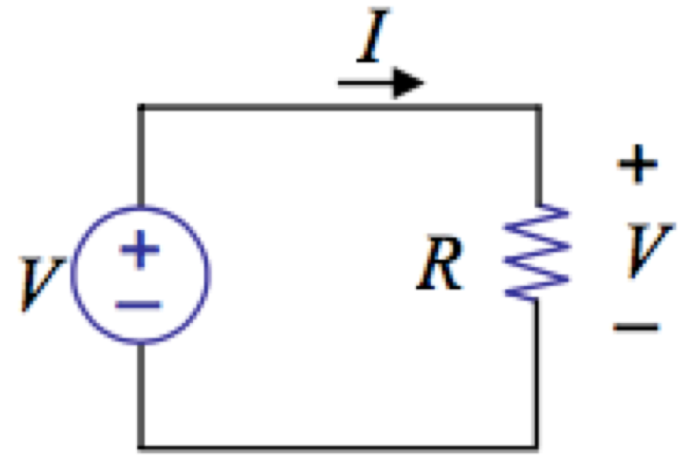
$v_o = V_s \cdot \dfrac{R_{on}}{R+R_{on}}$

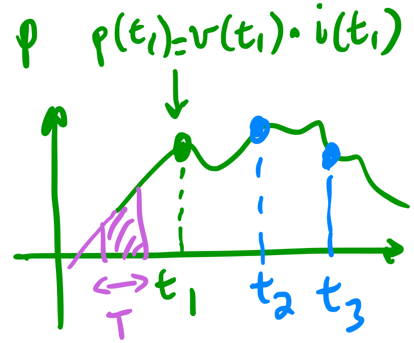
$\simeq 0$ v

First Example

- The power consumed by R

$$p = v \cdot i = V \cdot \frac{V}{R} = I^2 \cdot R \quad (W)$$





Average Power

$$P_{avg} = \frac{1}{t_3 - t_2} \int_{t_2}^{t_3} p(t)\,dt$$

- The energy dissipated during time interval T

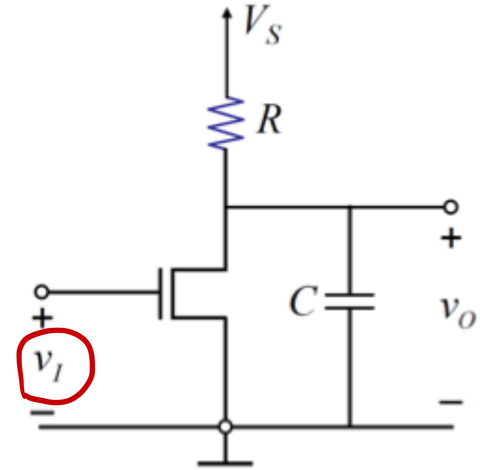
$$E = \int_T p\,dt = P \cdot T \quad (J)$$

$$\text{Energy} = \int_T p(t)\,dt$$

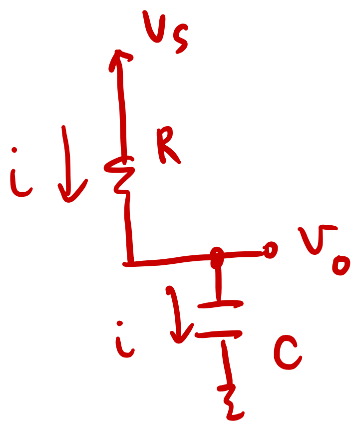
Apply to Our Gate for Static Power

Steady-state

- When the gate is not switching



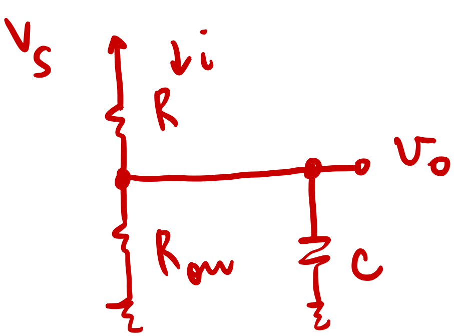
1) when v_I = low



$i = 0$

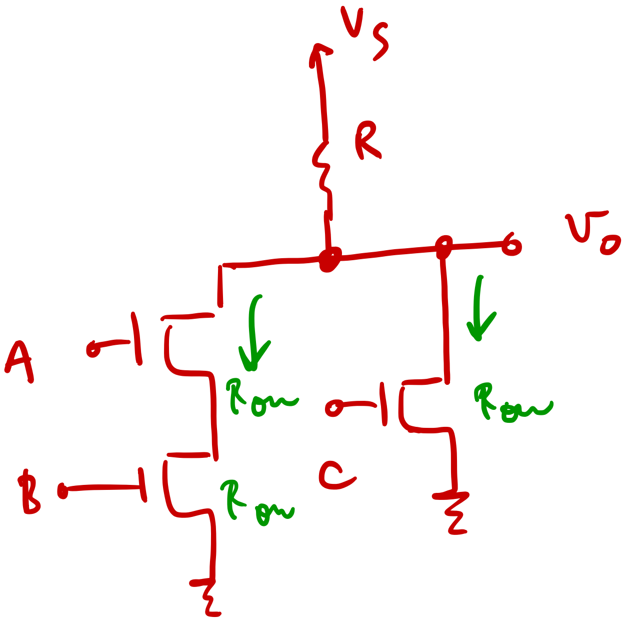
$P = V_S \cdot i = V_S \cdot 0 = 0$ W

2) when v_I = high



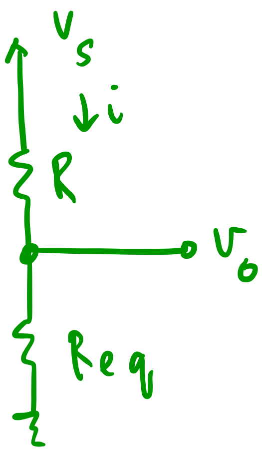
$P = V_S \cdot i = V_S \cdot \frac{V_S}{R + R_{on}} = \frac{V_S^2}{R + R_{on}}$ W

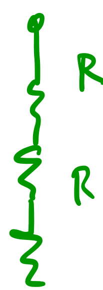
Example   Find the max. static power.



$$P_{static} = V_s \cdot i = V_s \cdot \frac{V_s}{R + R_{eq}} = V_s \cdot \frac{V_s}{R + \frac{2}{3} R_{on}}$$

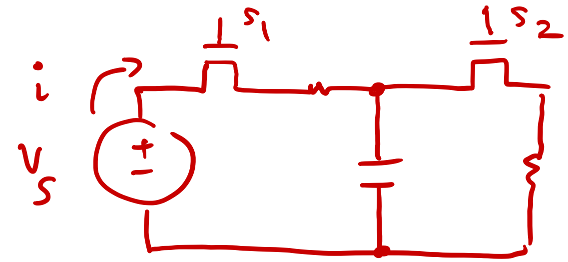
$\Rightarrow$ Min. R_{eq}.   A, B, C = High



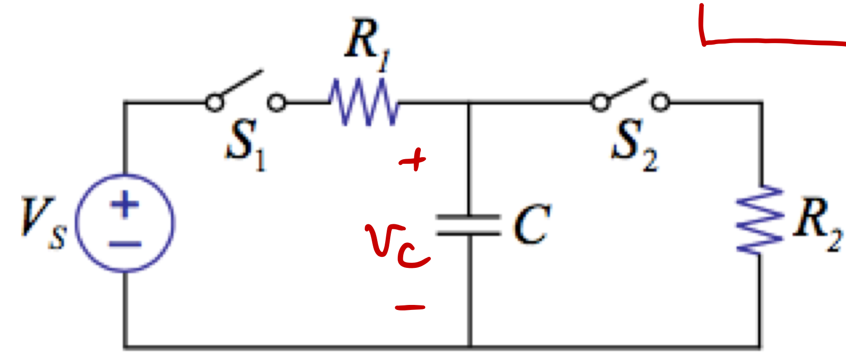


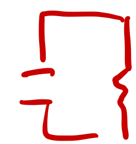
$R_{eq} = R_{on} \parallel 2R_{on}$

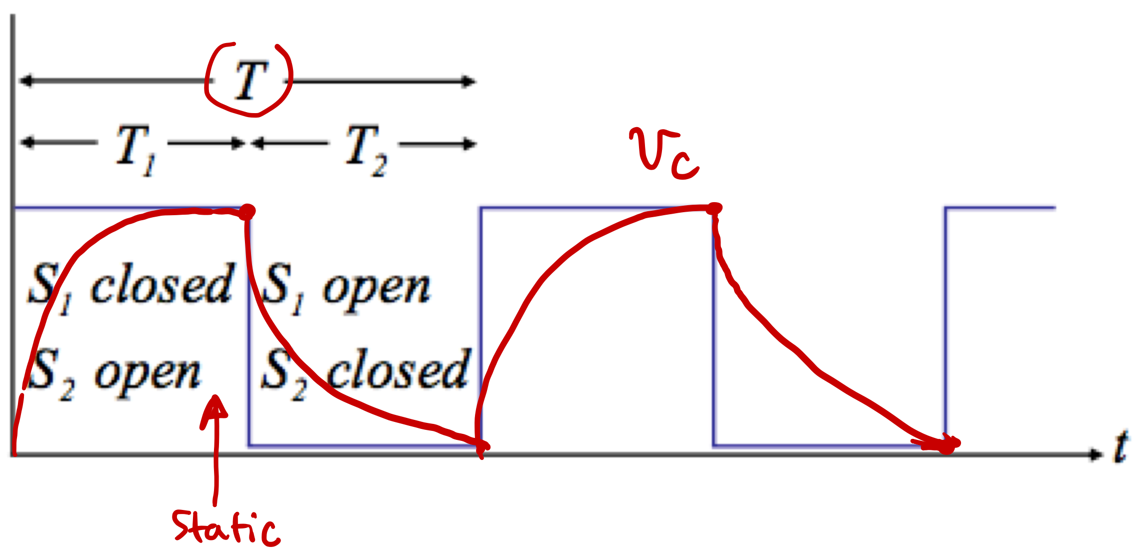
Second Example



- Consider



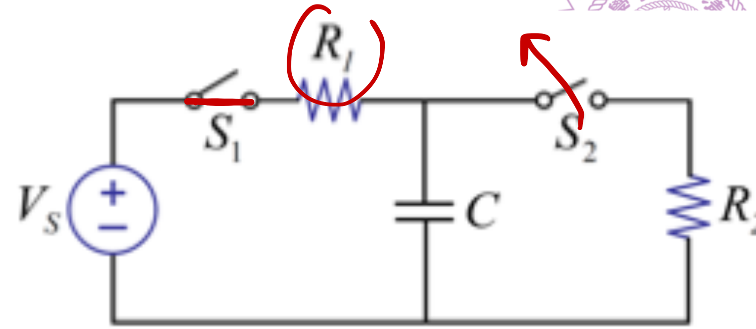


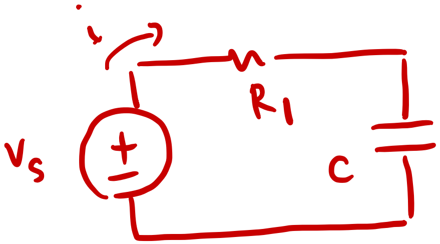


- Find energy dissipated in each cycle.
- Find average power.

Second Example

- During T_1 :
  S_1 is closed and S_2 is open.



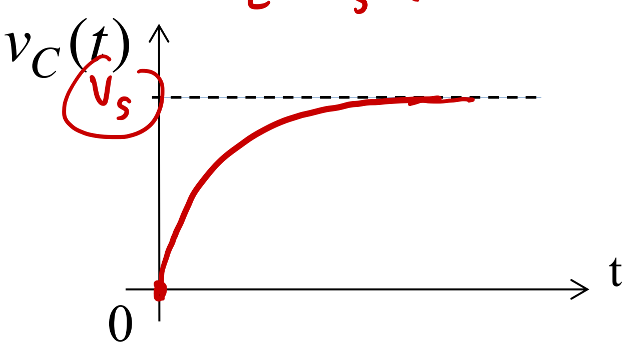


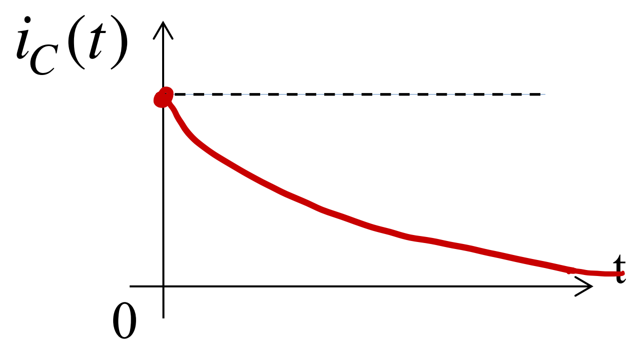
$P_{static} = 0$

$$i_c = C \cdot \frac{dv}{dt} = \frac{v_R}{R} = \frac{V_s - v_c}{R}$$

$$v_c = V_s \cdot \left(1 - e^{-t/R_1C}\right)$$

$$i_c = \frac{V_s}{R_1} \cdot e^{-t/R_1C}$$



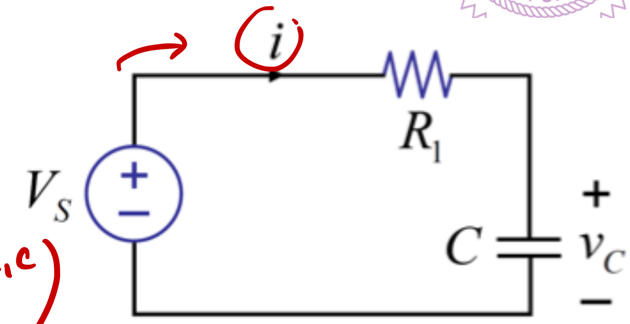


Total Energy Provided by Source During T_1

Total energy provided by V_s (to R_1, C)

$$E = \int_0^{T_1} v \cdot i \, dt = \int_0^{T_1} \underline{\underline{V_s}} \cdot i \, dt$$

$$= \int_0^{T_1} V_s \cdot \frac{V_s}{R_1} e^{-t/R_1C} dt = V_s^2 \cdot C \cdot \left(1 - e^{-t/R_1C}\right)$$



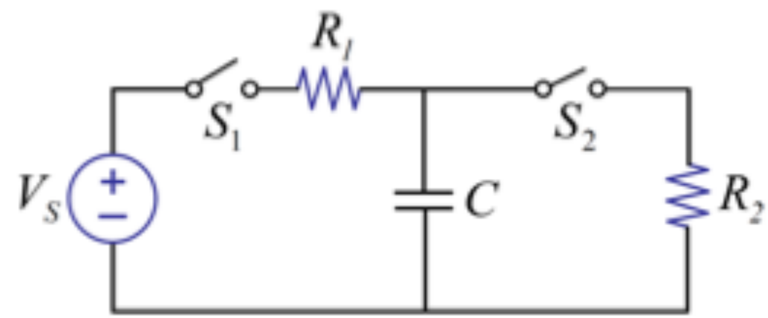
If $T_1 \gg R_1C$, e^{-t/R_1C} can be ignored

$$E_1 \approx V_s^2 \cdot C$$

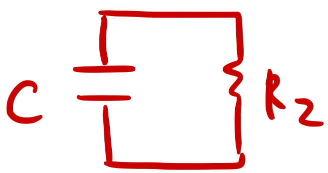
Practice:

- ✓ Energy stored on capacitor C = ? $\frac{1}{2} V_s^2 \cdot C$
- ✓ Energy dissipated in R_1 = ? $\frac{1}{2} V_s^2 \cdot C = \int_0^{T_1} (V_s - v_c) \cdot i_c \, dt$

Second Example



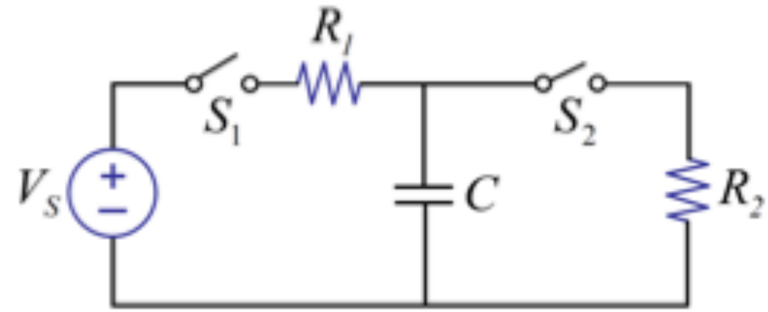
- During T_2 :
  S_2 is closed and S_1 is open.



$v_c(t=T_1) \simeq V_s$

- Initially, $v_c = V_s$
  - Energy stored on capacitor is $\frac{1}{2}CV_S^2$
- Assume $T_2 \gg R_2 \cdot C$
- Capacitor discharges more or less fully during T_2.
- Energy dissipated in R_2 during T_2 is $\frac{1}{2} C \cdot V_s^2 = E_2$ Practice
- E_2 is also independent of R_2.

Putting the Two Together



- Total energy dissipated in each cycle

$$E_{total} = E_{R_1} + E_{R_2} = \underbrace{\frac{1}{2}CV_s^2}_{R_1} + \underbrace{\frac{1}{2}CV_s^2}_{R_2} = \underbrace{CV_s^2}_{\text{provided by } V_s.}$$

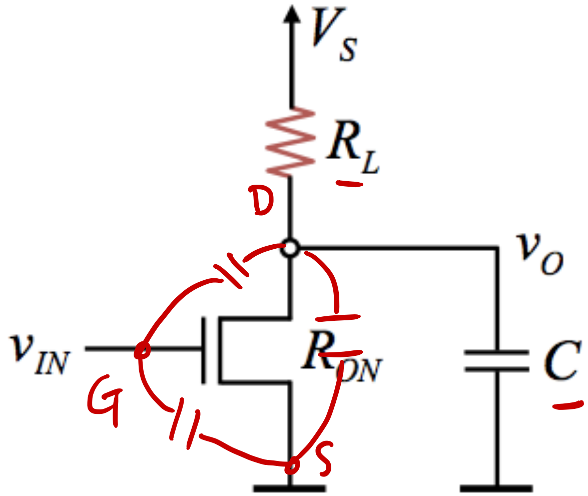
  - The energy dissipated in charging and discharging the capacitor C.
  - Assume C charges and discharges fully. $(T_1 \gg R_1 C,\ T_2 \gg R_2 C)$

- The average power is

$$\bar{P} = \frac{E_{total}}{T_1+T_2} = \frac{CV_s^2}{T_1+T_2} = \frac{CV_s^2}{T} = CV_s^2 \cdot f$$

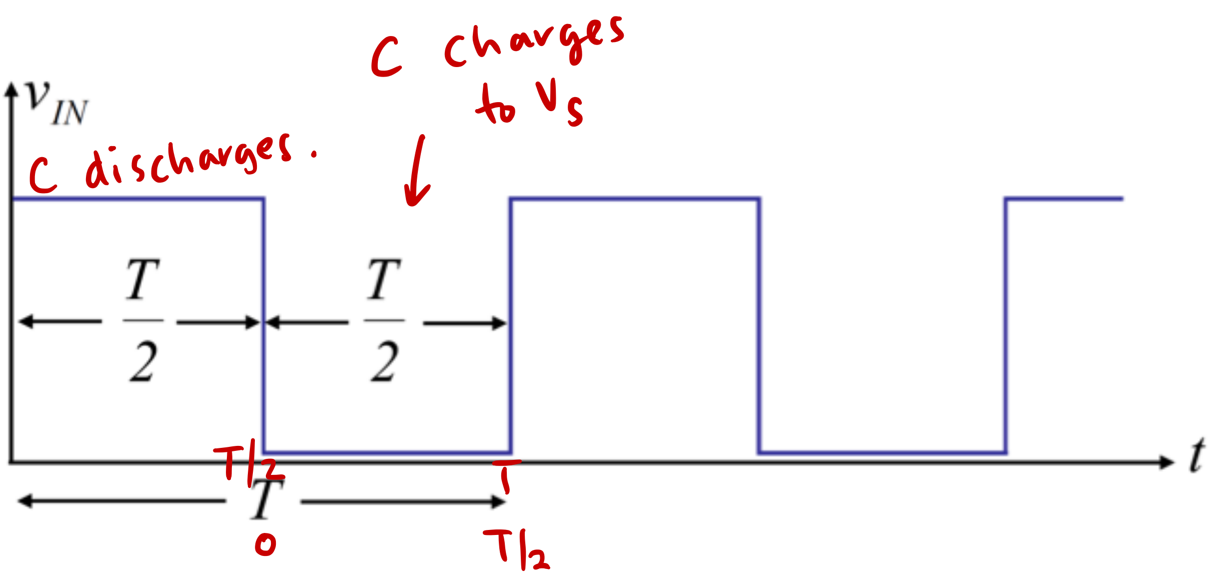
f: frequency (1/s, Hz)

Back to Our Inverter

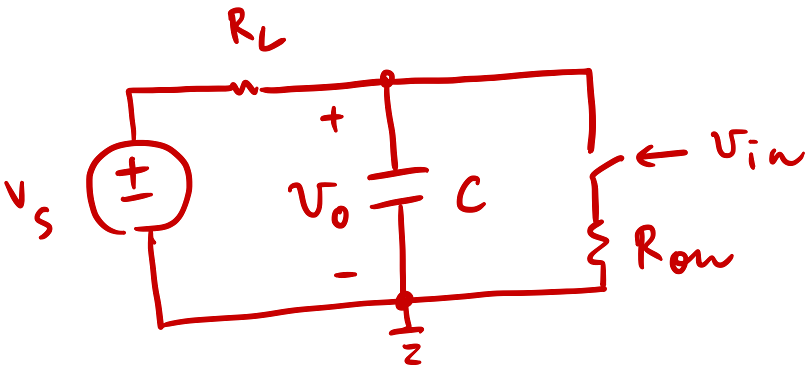


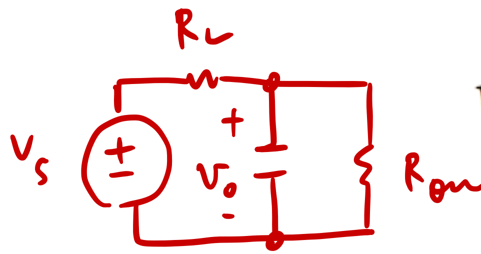
C includes
wire capacitance C_W
and gate capacitance C_{GS} of the
following gates.

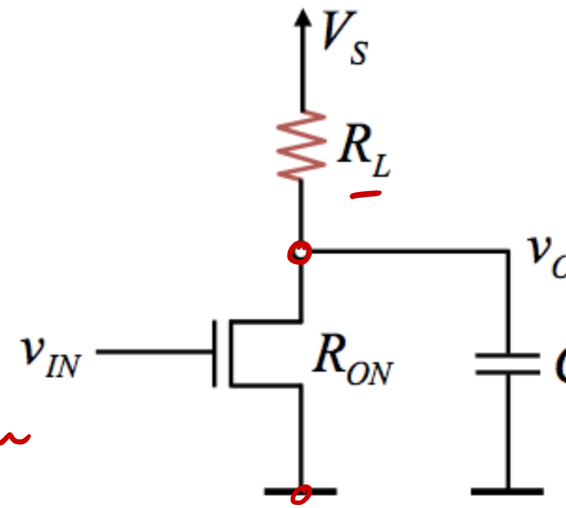
- What is $\overline{P}$ with the following input pattern?



Equivalent Circuit



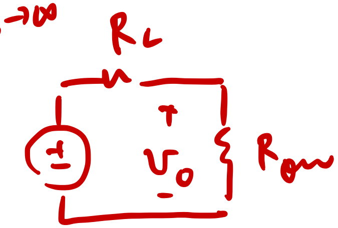




1) When v_{in} is high, discharging C.

$v_{o,initial} = V_s$, $v_{o,final} = \frac{R_{on}}{R_L + R_{on}} \cdot V_s$

$t \to \infty$



$v_{o,final} = V_s \cdot \frac{R_{on}}{R_L + R_{on}}$

$$v_o(t) = V_s \cdot \frac{R_{on}}{R_L + R_{on}} + \left(V_s - \frac{R_{on}}{R_L + R_{on}} V_s\right) \cdot e^{-t/\tau}$$

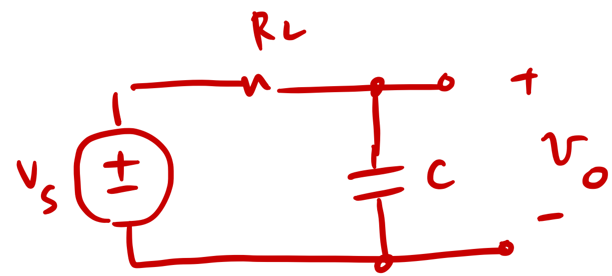
$$\tau = (R_L \| R_{on}) \cdot C$$

$$E_1 = \int_0^{T/2} \left(\frac{(V_s - v_o)^2}{R_L} + \frac{v_o^2}{R_{on}} \right) dt = \frac{V_s^2}{R_L + R_{on}} \cdot \frac{T}{2} + \frac{C \cdot V_s^2 \cdot R_L^2}{2(R_L + R_{on})^2},$$

assuming $\frac{T}{2} \gg \tau$

$$\bar{P}_1 = \frac{E_1}{T/2} = \frac{V_S^2}{R_L + R_{on}} + \frac{C V_S^2 \cdot R_L^2}{(R_L + R_{on})^2} \cdot 2f$$

2) when V_{in} is low, C charging



$$V_{o,\text{initial}} = V_S \cdot \frac{R_{on}}{R_L + R_{on}} \quad \left(\to 0 \text{ when } R_{on} \to 0 \right)$$

$$V_{o,\text{final}} = V_S$$

$$e^{-(t - T/2)/R_L \cdot C}$$

$$V_o(t) = V_S + \left(V_S \frac{R_{on}}{R_L + R_{on}} - V_S \right) \cdot e^{-t/R_L \cdot C}$$

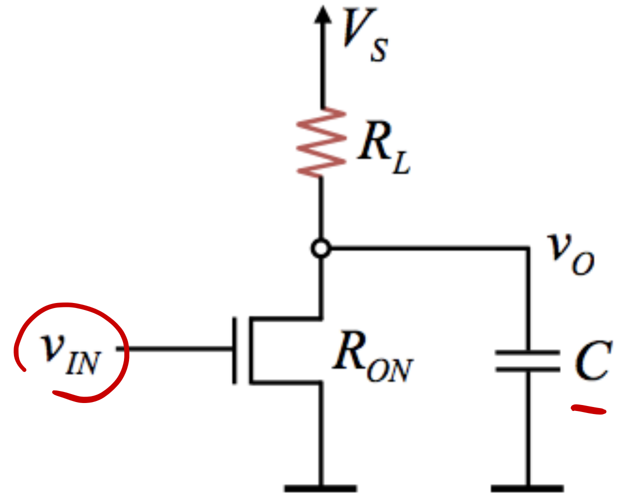
$$\int_{T/2}^{T}$$

$$E_2 = \int_0^{T/2} \frac{(V_S - V_o)^2}{R_L} dt = \frac{C V_S^2 \cdot R_L^2}{2(R_L + R_{on})^2}$$

$$\bar{P}_2 = \frac{E_2}{T/2} = \frac{C V_S^2 \cdot R_L^2}{2(R_L + R_{on})^2} \cdot 2f$$

What is $\overline{P}$ for the Gate?

$$\overline{P} = \frac{V_S^2}{2(R_L + \cancel{R_{ON}})} + CV_S^2 f \frac{R_L^2}{(R_L + \cancel{R_{ON}})^2}$$



- with $\underline{R_L >> R_{ON}}$, $\bar{p} = \underbrace{\frac{V_S^2}{2R_L}}_{\text{Static}} + \underbrace{CV_S^2 \cdot f}_{\text{dynamic}}$

- During active mode, the circuit consumes static power and dynamic power.
- The first term is called the static power dissipation.
  - In both standby mode and active mode.
  - Independent of frequency f.
  - The MOSFET is ON half of the time.
- The second term is called dynamic power dissipation.
  - Proportional to frequency f and capacitance C.

of $\bar{v}_{in}$

Some Numbers

- A chip with 5 million gates running at 3 GHz. frequency

Assume $V_s = 5V$, $R_L = 10k\Omega$, $R_{on} \to 0\Omega$, $C = 1fF$

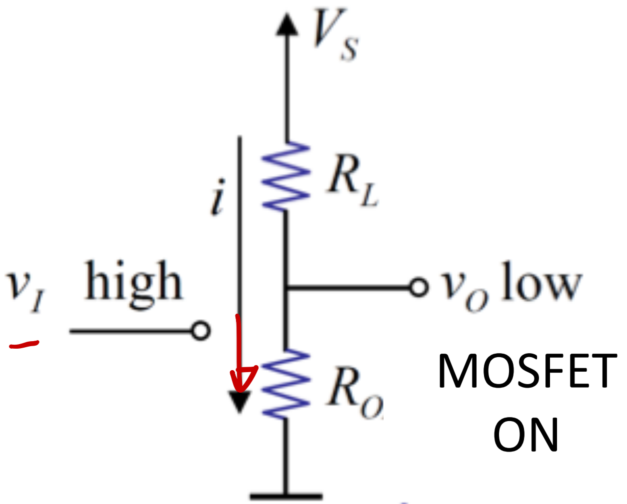
$$\bar{P} = \left(\frac{V_s^2}{2R_L} + CV_s^2 \cdot f\right) \cdot 5 \times 10^6$$

$$= \left(\frac{25}{2 \times 10^4} + 10^{-15} \times 25 \times 3 \times 10^9\right) \times 5 \times 10^6$$

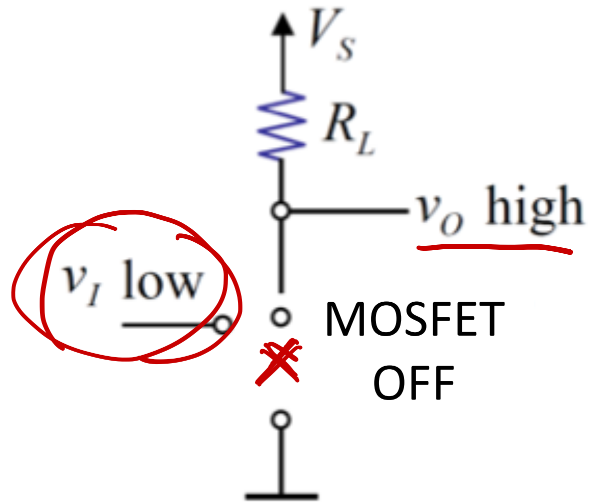
$$= \underbrace{6.25\,kW}_{\text{static}} + \underbrace{375\,kW}_{\text{dynamic}}$$

How to Get Rid of Static Power Dissipation?

- Intuition:

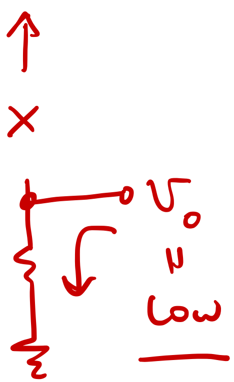


Problem case!!



no problem

V_I high



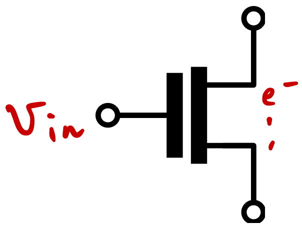
no dc path $\Rightarrow P_{static} = 0$

PMOS

CMOS : NMOS + PMOS

complementary

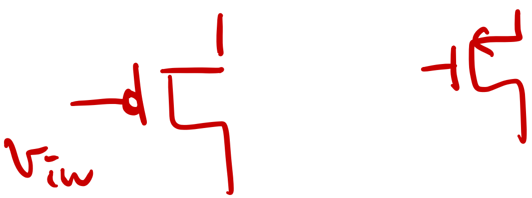
- N-channel MOSFET (NMOS)



v_{in} high, NMOS turns on (R_{on})

v_{in} low, NMOS turns off. (open, C_{off})

- P-channel MOSFET (PMOS)

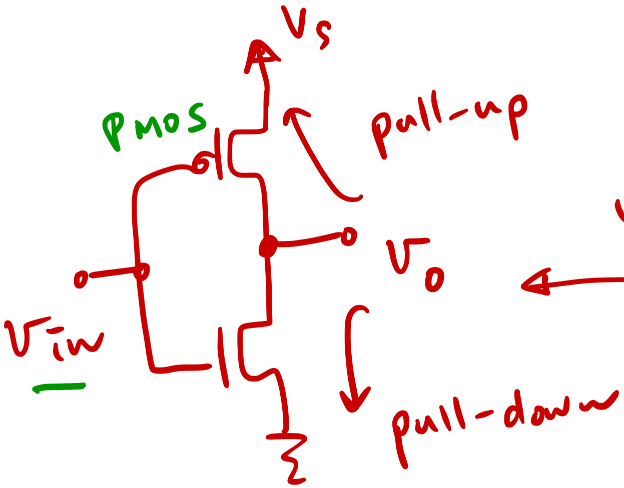


v_{in} high, PMOS turns off (open, C_{off})

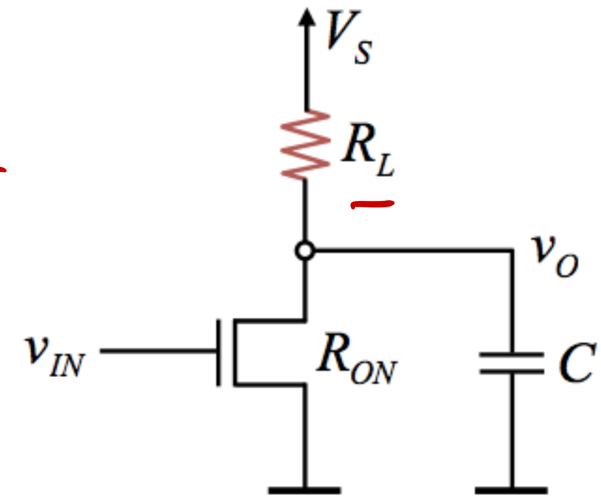
v_{in} low, PMOS turns on (R_{on})

Consider This Circuit

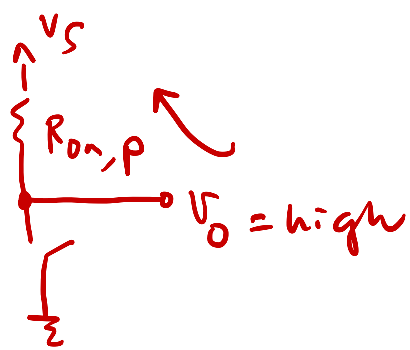
reduce static power

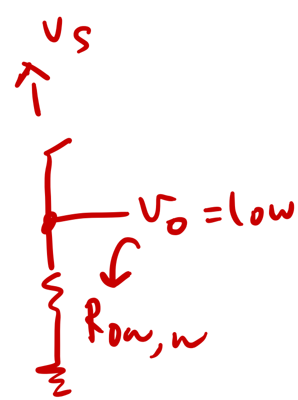


use this instead



Vin=high

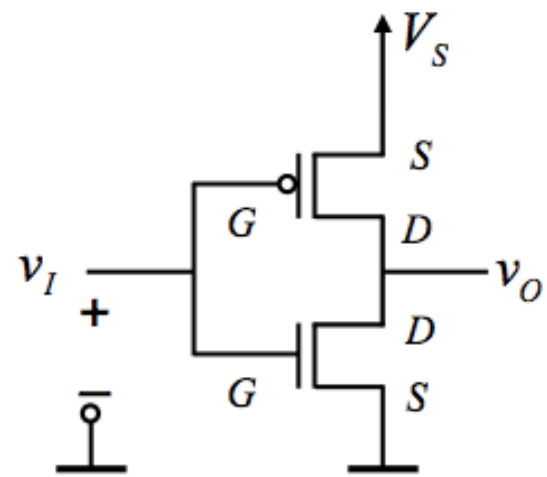




$R_{on,p} \neq R_{on,n}$

- When v_I high, pull-up path should behave like open circuit.

Behavior of the Circuit



- Input HIGH

  $v_I = 5\text{ V}$

  NMOS on
  PMOS off

  $V_o = 0V$

Input LOW

$v_I = 0\text{ V}$

NMOS off
PMOS on

$V_o = 5V$

(optional)

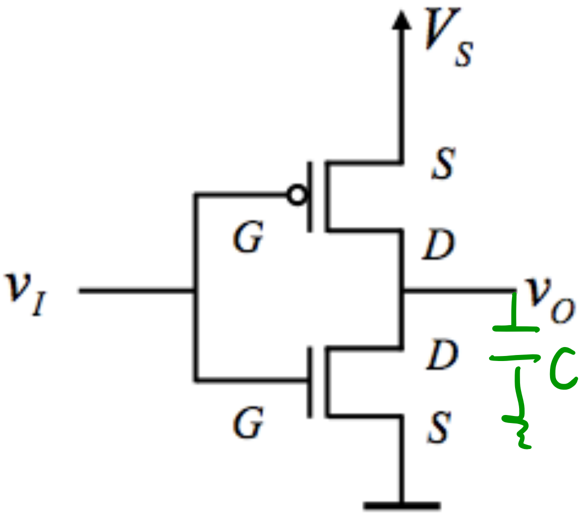
NMOS on
$V_{in} \geq V_{TH,n}$
NMOS off
$V_{in} < V_{TH,n}$

PMOS on
$V_{in} \leq V_{TH,p}$
PMOS off
$V_{in} \geq V_{TH,p}$

  - (Assume $V_{TN} = 1\text{ V}$ and $V_{TP} = -1\text{ V}$)
- Never a path from V_S to ground.
- Called complementary MOS, 'CMOS' logic.

Power

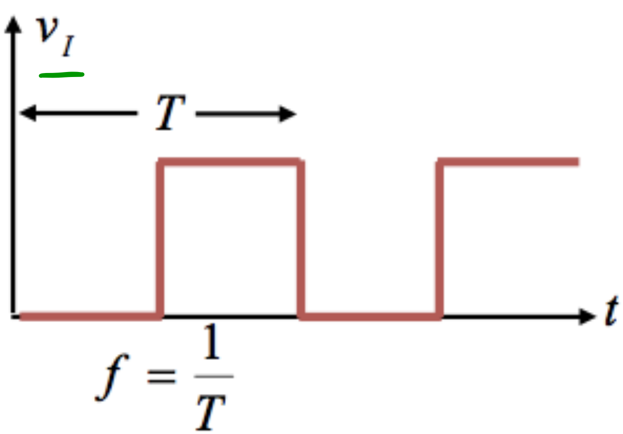
- **Key:** no path from V_S to ground, no static power!



$P_{dynamic} = C \cdot V_S^2 \cdot f$

Practice $v_o = ?$

R_{on}



Our Previous Example

- 5 million gates running at 3 GHz
  - With $V_S = 5\text{ V}, C = 1\, f\text{F}$
- Scaling up
  - Increase frequency and number of gates

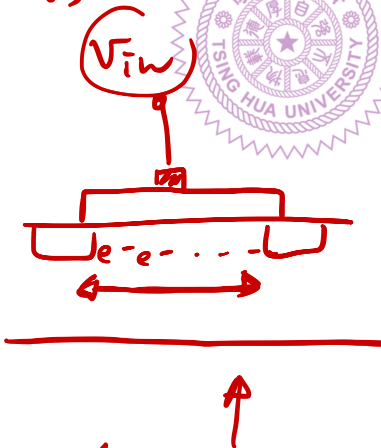
# Gates	f	$\overline{P}$
10^6	100 MHz	~2.5 W
2×10^6	300 MHz	~15 W
2×10^6	600 MHz	~30 W
5×10^6	3 GHz	~375 W
25×10^6	3 GHz	~1875 W

How to Reduce Power $\propto C \cdot V_S^2 \cdot f$

technology
0.35um
0.25um
0.18um
90nm
tsmc

o Reduce C (layout, advanced technology)

- Reduce V_S. 佈局
- Turn OFF circuit and/or clock when not in use.
  - Sleep mode, clock gating.
- Change V_S depending on need.
- Use multicore to reduce f without sacrificing speed of circuits.



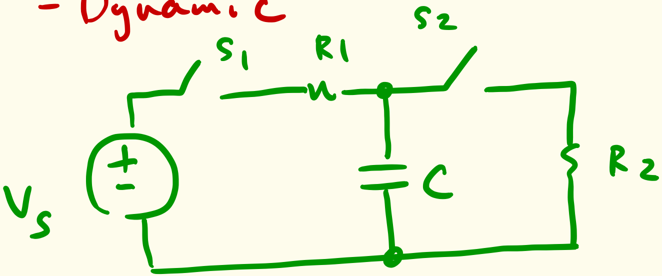
65nm
40nm
28nm
22nm, 16nm/14nm, 10nm,
7nm
5nm
3nm
⋮
(3D) FinFET

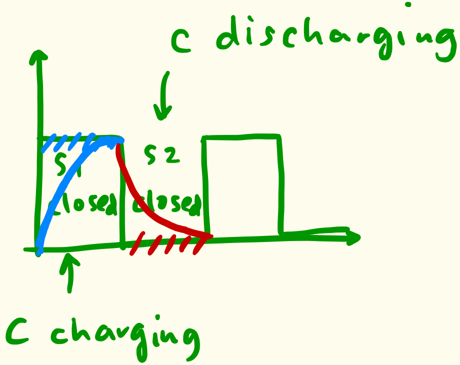
# Gates	f	$\overline{P}$	
2.5×10^6	66 MHz	15 W	Intel Pentium, 0.7 μm, 1993
7.5×10^6	400 MHz	35 W	Intel PII, 0.35 μm, 1997
44×10^6	1 GHz	15 W	Intel PIII, 0.18 μm, 1999
120×10^6	3 GHz	75 W	Intel PIV, 0.13 μm, 2001
615×10^6	0.7 GHz	20 W	Tilera 64-core, 90 nm, 2007
2300×10^6	3.6 GHz	75 W	Intel Nehalem, 8-core, 45 nm, 2011

- Review of lec 8.

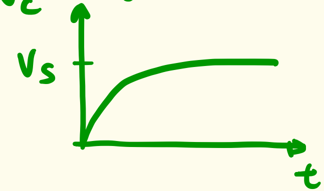
\# - Static

\# - Dynamic

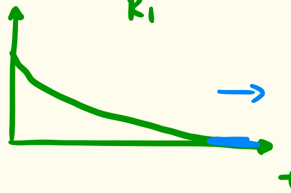




$$v_c = V_s\left(1-e^{-t/R_1C}\right)$$

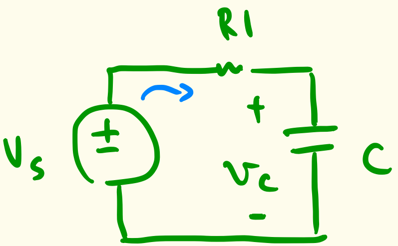


$$i_C = \frac{V_s}{R_1} e^{-t/R_1C}$$



(1) During T_1, C is charging

(p.10)



$$P_{static} = V_s \cdot 0 = 0 \text{ W}$$

$$P_{dynamic} = P_{cap} + P_{R_1} = v_c \cdot i_c + (V_s - v_c) \cdot i_c$$

$$= \frac{V_s^2}{R_1} \cdot e^{-t/R_1C}$$

(P.10)

$t \gg R_1 C$

$$\int_0^{T_1} \frac{V_s^2}{R_1} e^{-t/R_1C}\, dt = V_s^2 \cdot C \cdot \left(1 - \cancel{e^{-t/R_1C}}\right) \simeq V_s^2 \cdot C \quad (J)$$

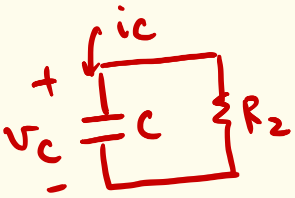
$$E_{cap} = \int_0^{T_1} v_c \cdot i_c\, dt = \int_0^{T_1} V_s\left(1 - e^{-t/R_1C}\right) \cdot \frac{V_s}{R_1} e^{-t/R_1C}\, dt$$

Energy stored in C during 0 to T1. (by Vs)

$$= \frac{V_s^2}{R_1} \int_0^{T_1} \left(e^{-t/R_1C} - e^{-2t/R_1C}\right) dt$$

$$= \frac{V_s^2}{R_1} \left(-R_1 C e^{-t/R_1C} \Big|_0^{T_1} + \frac{R_1 C}{2} e^{-2t/R_1C} \Big|_0^{T_1} \right) = \frac{V_s^2}{R_1} \left(\frac{1}{2} R_1 C\right) = \frac{V_s^2}{2} \cdot C$$

② During T_1 to T (total of T_2)



$P_{static} = 0$

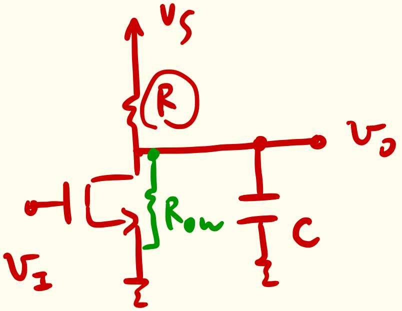
$v_c = V_s e^{-t/R_2 C}$, $\quad i_c = \frac{-v_c}{R_2}$

Energy dissipated by $R_2 = \int_0^{T_2} v_c \cdot (-i_c)\, dt$

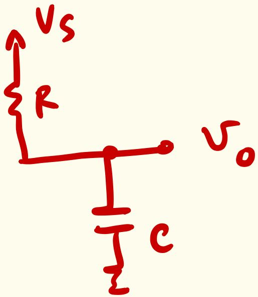
$$= \int_0^{T_2} \frac{V_s^2}{R_2} e^{-2t/R_2 C}\, dt$$

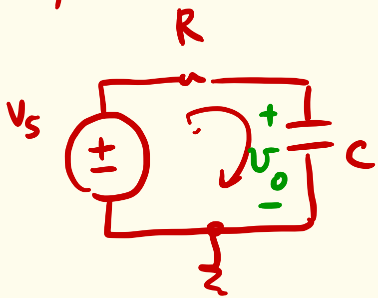
$$= \frac{V_s^2}{R_2} \cdot \frac{1}{2} R_2 C = \frac{V_s^2}{2} C$$

1



v_I = low, v_o = high, C charging





$$P_{static} = 0$$

$$v_o = V_S - V_S \frac{R_L}{R_L + R_{on}} e^{-t/R_L C}$$

$$E_R = \int_0^{T/2} (V_S - v_o) \cdot i_c \, dt = \int_0^{T/2} \frac{V_S^2}{(R_L + R_{on})^2} \cdot R_L \cdot e^{-2t/R_L C} \, dt = \frac{V_S^2 \cdot R_L^2}{(R_L + R_{on})^2} \cdot \frac{C}{2}$$

$$= \frac{V_S^2}{2} \cdot C \cdot \frac{R_L^2}{(R_L + R_{on})^2}$$

(charging)

$$E_{cap} = \int_0^{T/2} v_c \cdot i_c \, dt \qquad i_c = C\frac{dv_o}{dt} = i_R = \frac{v_s - v_o}{R}$$

$$= \int_0^{T/2} \left(V_s - V_s \frac{R_L}{R_{on}+R_L} e^{-t/R_L C} \right) \cdot \left(\frac{V_s}{R_{on}+R_L} \cdot e^{-t/R_L C} \right) dt$$

$$= \underbrace{\frac{1}{2} C V_s^2 \cdot \frac{R_L^2 + 2R_{on}R_L}{(R_{on}+R_L)^2}}_{} = \underbrace{\frac{1}{2} C \cdot v_{c,final}^2}_{} - \underbrace{\frac{1}{2} C \cdot v_{c,initial}^2}_{}$$

$$= \frac{1}{2} C \cdot V_s^2 - \frac{1}{2} \cdot C \cdot V_s^2 \left(\frac{R_{on}}{R_L + R_{on}} \right)^2$$

2

$t=0$

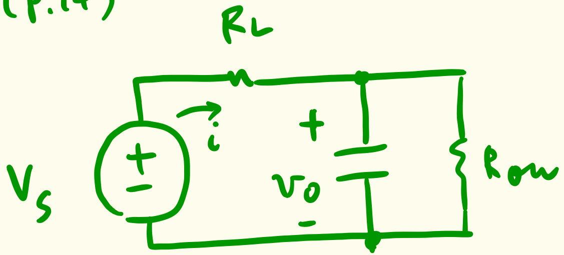
$v_{c,intial}$

$t = T/2$

$v_{c,final}$

(p.14)

C discharging



$$v_o = V_s \frac{R_{on}}{R_L + R_{on}} + \left(V_s - V_s \frac{R_{on}}{R_L + R_{on}}\right) \cdot e^{-t/\tau}$$

$$\tau = (R_L \parallel R_{on}) \cdot C$$

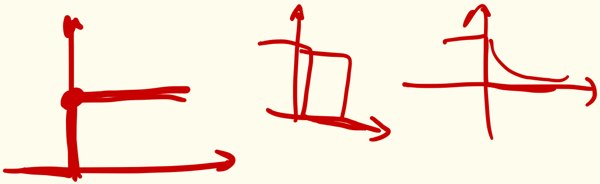
$$\text{Energy provided by } V_s = \int_0^{T/2} V_s \cdot i \, dt = \int_0^{T/2} V_s \cdot \frac{V_s - v_o}{R_L} dt$$

$$= \int_0^{T/2} \frac{V_s^2}{R_L + R_{on}} \left(1 - e^{-t/\tau}\right) dt = \frac{V_s^2}{R_L + R_{on}} \left(t + \tau e^{-t/\tau}\right)\Big|_0^{T/2}$$

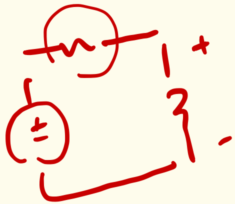
$$= \frac{V_s^2}{R_L + R_{on}} \left(T/2 - 0 + \tau e^{-T/2\tau} - \tau\right)$$

$$= \frac{V_s^2}{R_L + R_{on}} \cdot \frac{T}{2} - \frac{V_s^2 \cdot R_L \cdot R_{on} \cdot C}{(R_L + R_{on})^2} \quad (1)$$

(1) + (2) = p.14



$$\frac{di}{dt} = 0$$



$$V_s + \underbrace{\left(V_s \cdot \frac{R_{on}}{R_L + R_{on}} - V_s\right)}$$