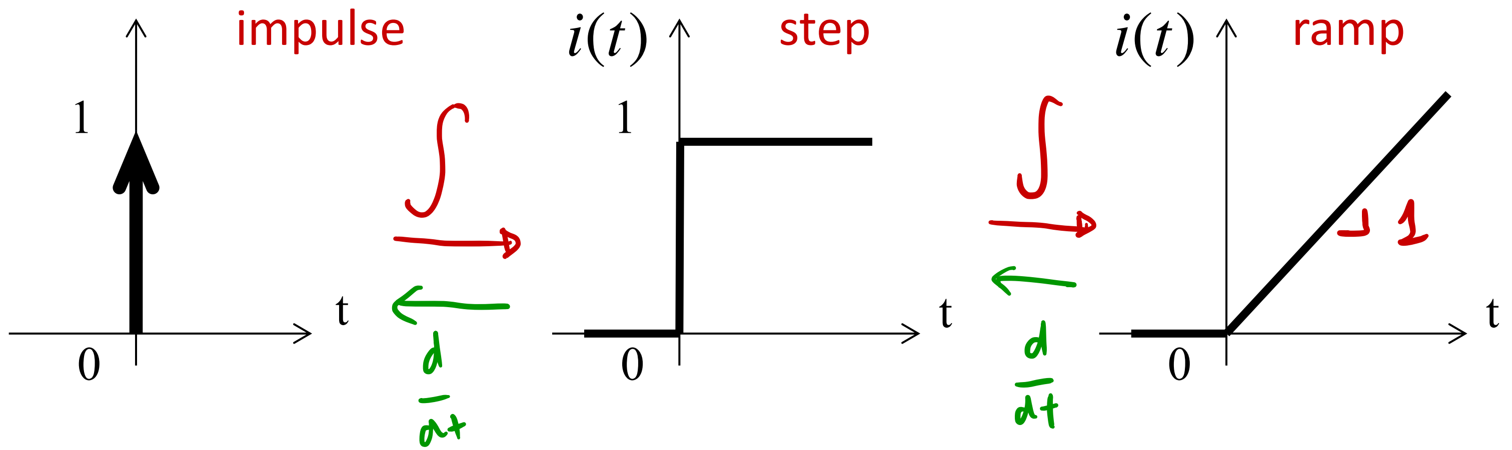
Superposition with Initial Conditions and Sources

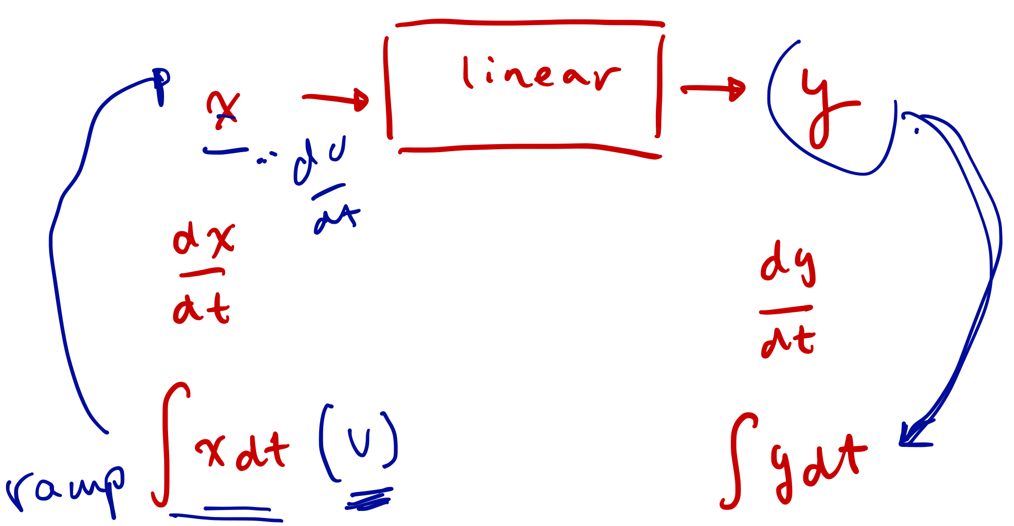
- Note that superposition does NOT apply when initial condition exists.
- Superposition does apply when initial conditions are zero.
- One way of solving circuits with multiple sources and multiple initial conditions
  - Treat initial conditions like sources.
  - Find response to each source or each initial condition acting alone.
  - Sum all the partial responses for total response.

$$v_c(t) = \underbrace{f(v_1) + f(v_2) + f(i_1)}_{\text{zero-state response, } v_c(t=0)=0} + \cdots \quad \underbrace{f(v_{c,0})}$$

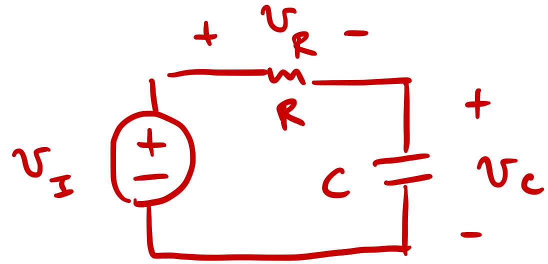
zero-input response
initial condition

Impulses, Steps, and Ramps

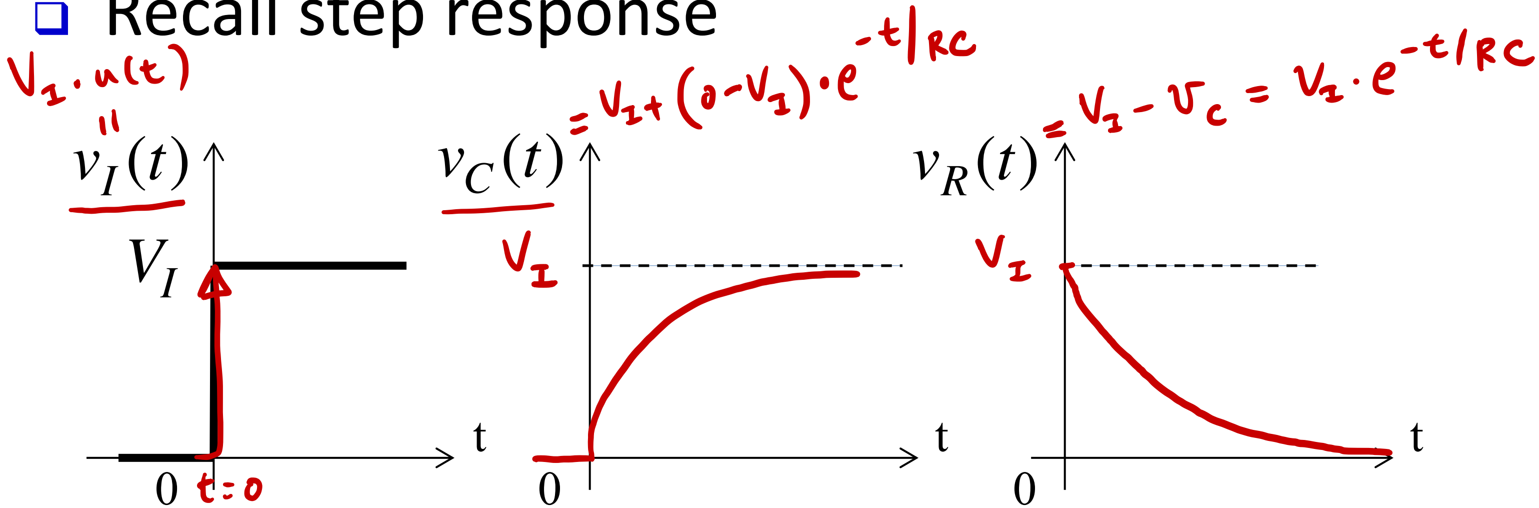




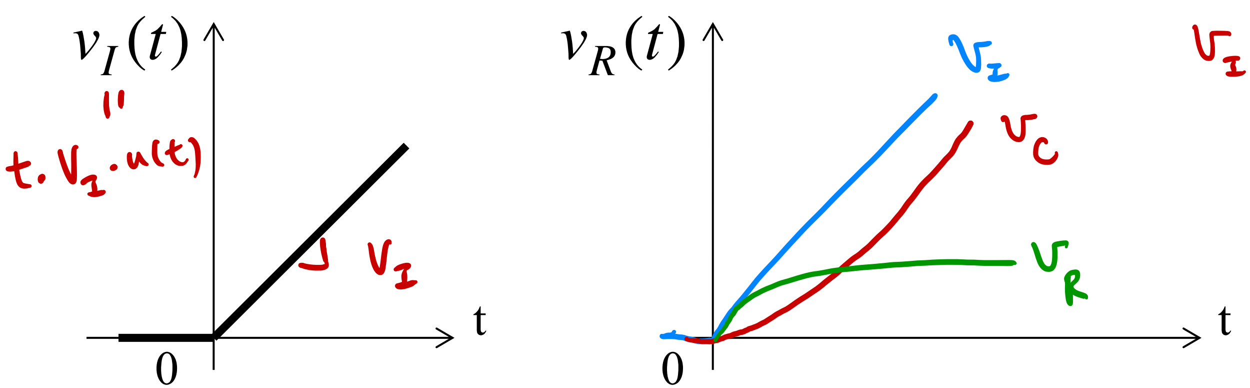
Ramp Input to RC Circuit



- Recall step response



- With ramp input



$v_I = v_C + v_R$

- Ramp response

$$v_c(t) = \int_{-\infty}^{t} V_I\left(1-e^{-t/RC}\right)dt$$

$$= v_c(0^-) + \int_0^t V_I\left(1-e^{-t/RC}\right)dt$$

$$= v_c(0^-) + V_I\left(t + RC\cdot e^{-t/RC}\right)\Big|_0^t$$

$$= 0 + V_I\left(t - 0 + RC\cdot e^{-t/RC} - RC\right)$$

$$= V_I\left(t + RC\cdot e^{-t/RC} - RC\right), \quad t>0$$

$$v_R(t) = \int_{-\infty}^{t} V_I\cdot e^{-t/RC}dt = V_I\cdot(-RC)\cdot e^{-t/RC}\Big|_0^t = V_I\cdot RC\cdot\left(1-e^{-t/RC}\right), \quad t>0$$

Check that $v_c + v_R = v_I$