Electric Circuits
Lecture 7 Impulses, Steps, and Ramps

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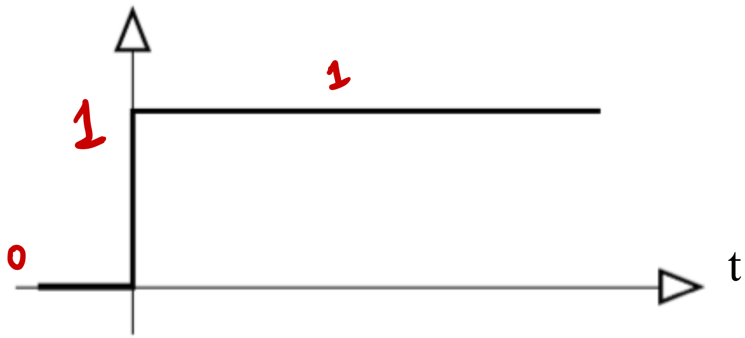
Lecture Outline

- RC and RL (Chapter 7 and 8 in the textbook)
  - Steps
  - Impulses
  - Ramp

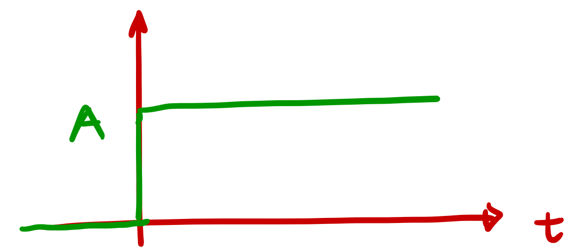
A Step Function

unit step function

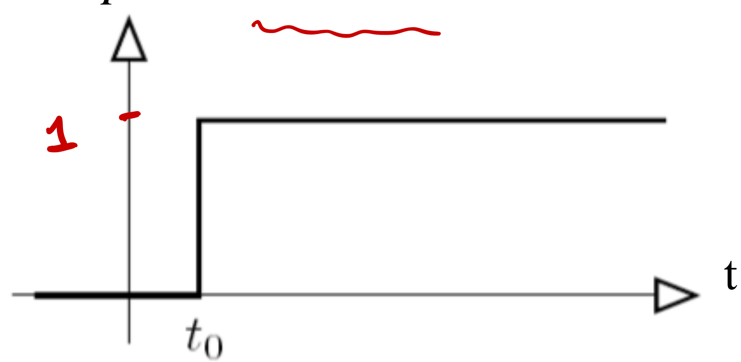
$v_I(t) = u(t)$



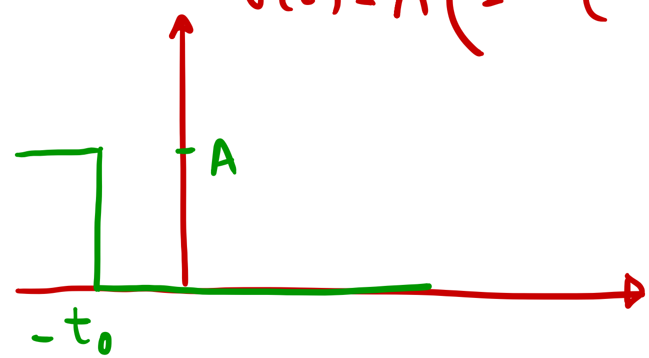
$v(t) = A \cdot u(t)$



$v_I(t) = u(t-t_0)$



$v(t) = A\left(1 - u(t+t_0)\right)$



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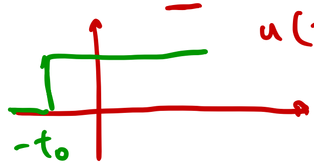
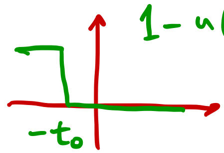


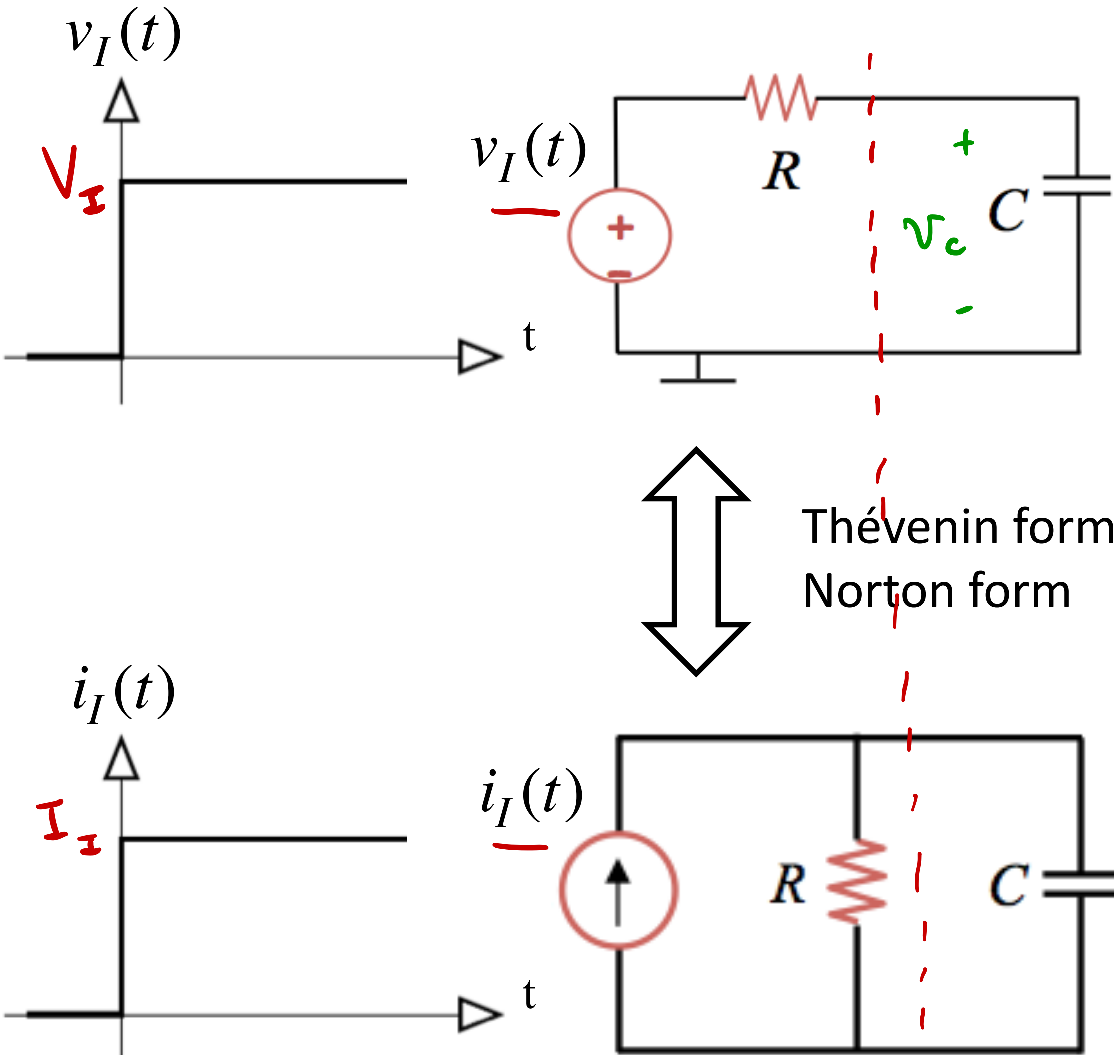
$u(t+t_0)$

$\rightarrow$



$1 - u(t+t_0)$

Rising Step Response of RC Circuit

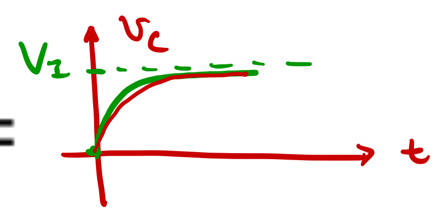


Given $v_I(t)$ or $i_I(t)$ and $v_c(t=0)=0$, then

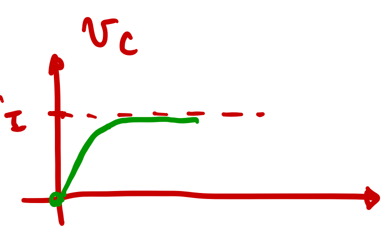
$$v_c(t) = V_I \cdot \left(1 - e^{-t/RC}\right)$$

for $t > 0$.

$$\begin{cases} v_{c,f} = V_I & \tau = RC \\ v_{c,i} = 0 & \\ v_c = V_I + (0 - V_I)\cdot e^{-t/RC} & \end{cases}$$

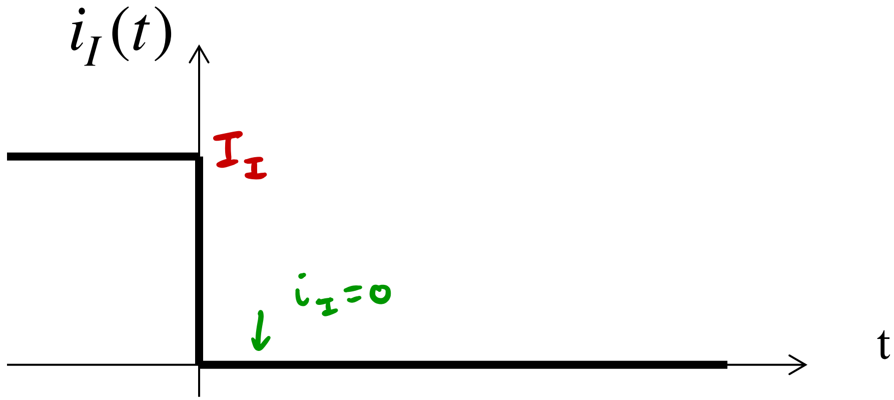


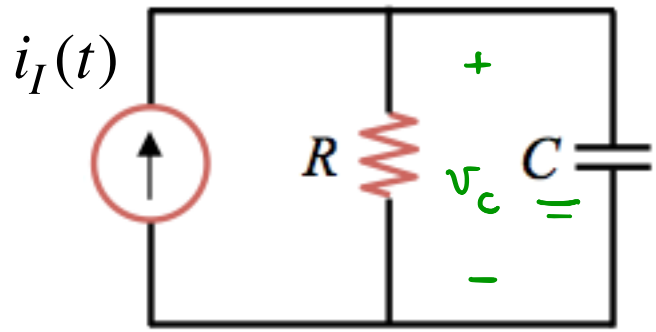
$I_I \cdot R = V_I$

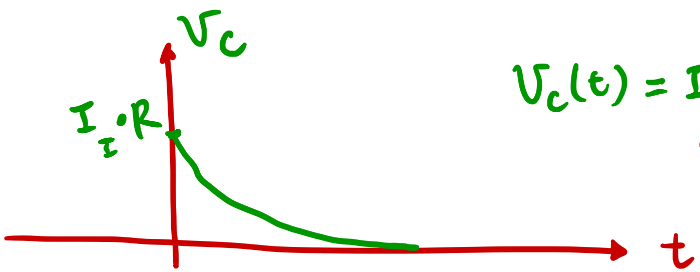


- Both are zero-state response for $t>0$ (initial value, $v_c(0)=0$)

Falling Step Response of RC Circuit





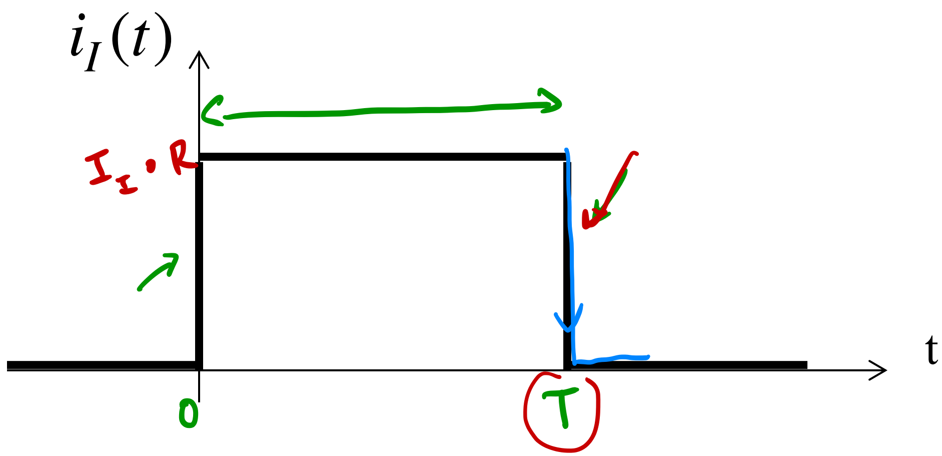


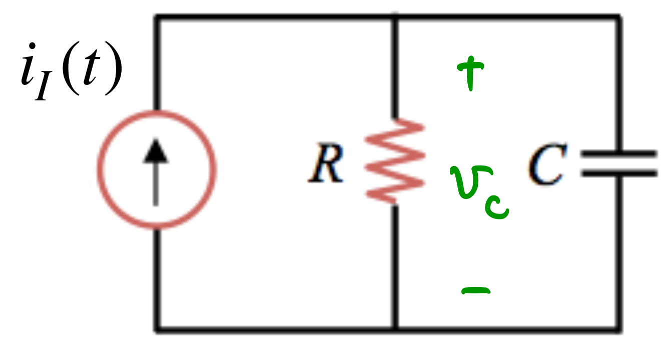
$$v_c(t) = I_I \cdot R \cdot e^{-t/RC}, \quad t > 0$$

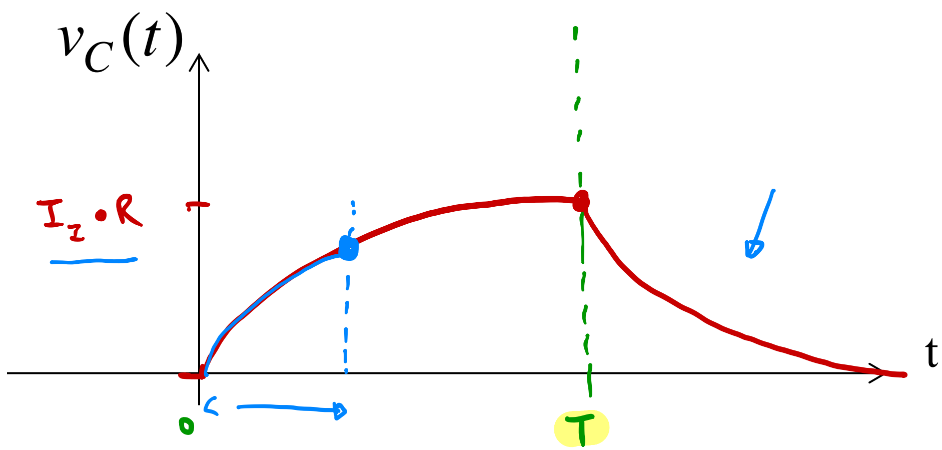
- zero-input response for $t > 0$
  ($i_I(t) = 0$ for $t > 0$)

Pulse Input for RC Circuit

- Rising step followed by falling step (much later)!
  - Pulse width T much larger than time constant RC.







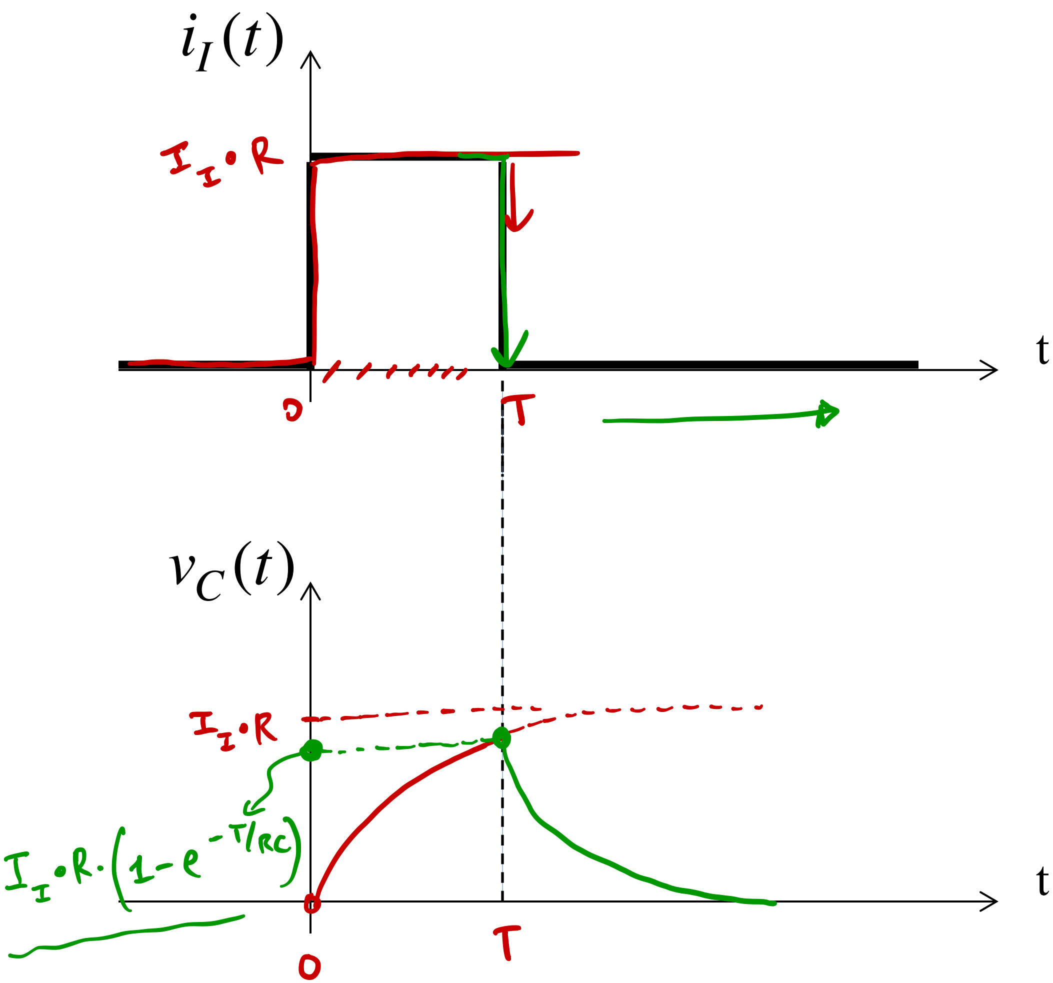
$v_c(t) = 0, \quad t < 0$

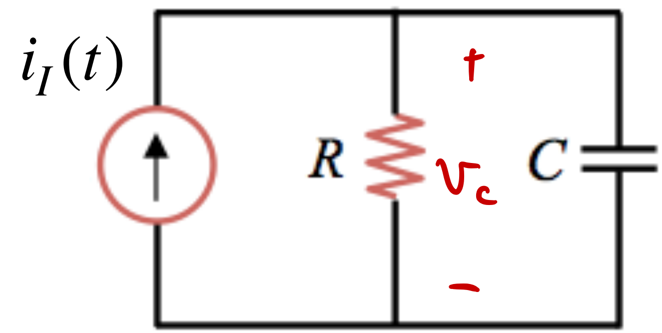
$v_c(t) = I_I \cdot R \cdot \left(1 - e^{-t/RC}\right), \quad 0 \le t < T$

$v_c(t) = I_I \cdot R \cdot e^{-(t-T)/RC}, \quad t \ge T$

Short Pulse Input to RC Circuit

- Rising step followed by falling step (shortly thereafter)!





$v_c(t) = 0 \quad , \quad t < 0$

$v_c(t) = I_I \cdot R \cdot \left(1 - e^{-t/RC}\right)$

$0 \le t < T,$

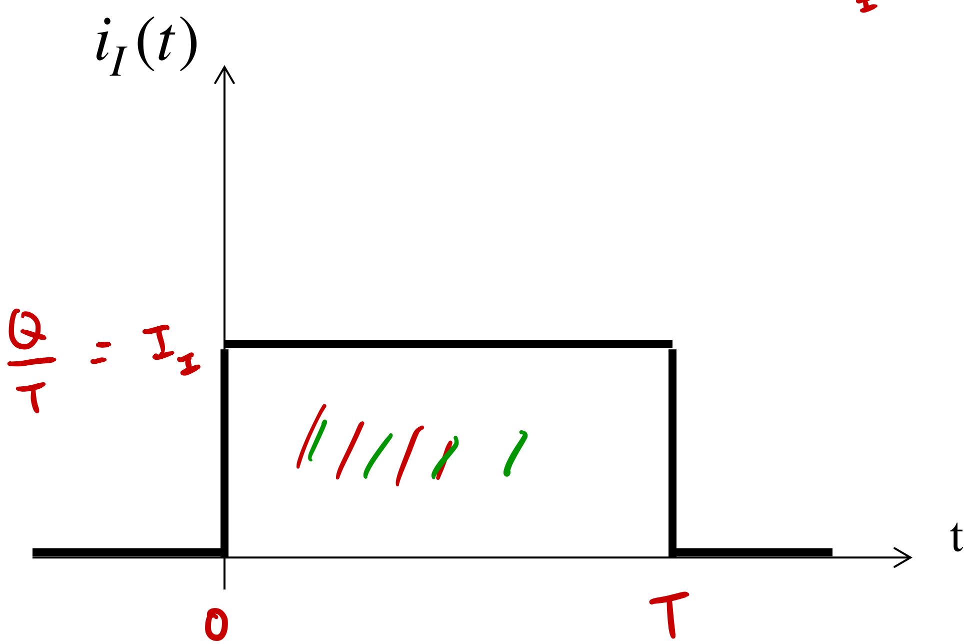
$v_c(t) = \underbrace{I_I \cdot R \cdot \left(1 - e^{-T/RC}\right)} \cdot e^{-(t-T)/RC}$

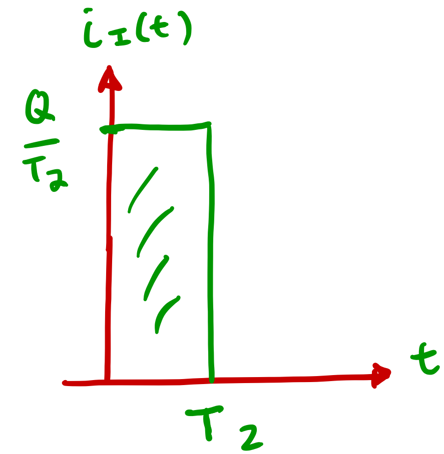
initial condition at t=T

Let's Stare at Current Pulse

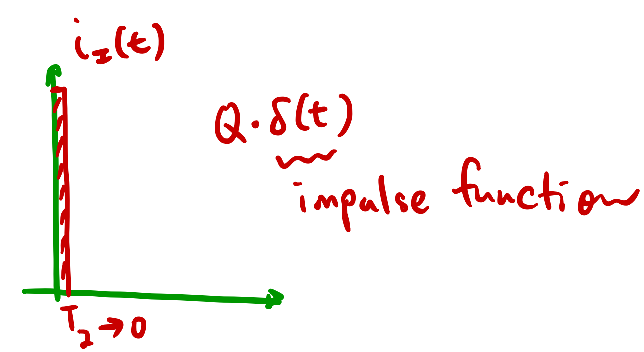
- Current pulse delivers charges.

$Q = I_I \cdot T$



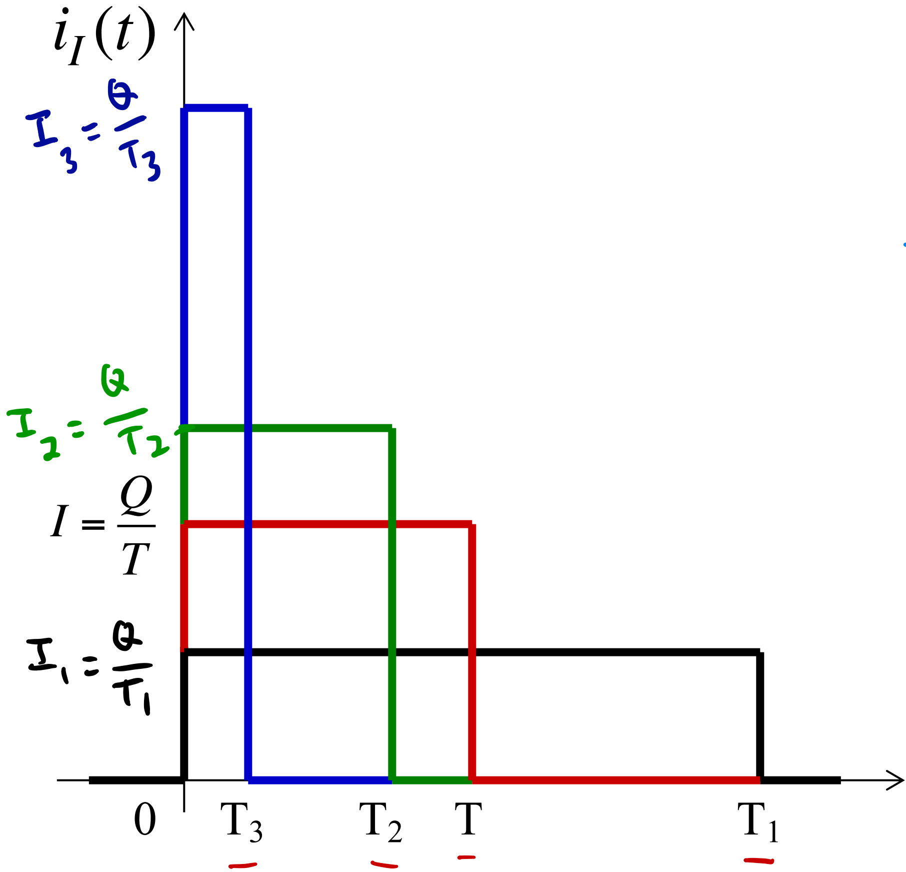


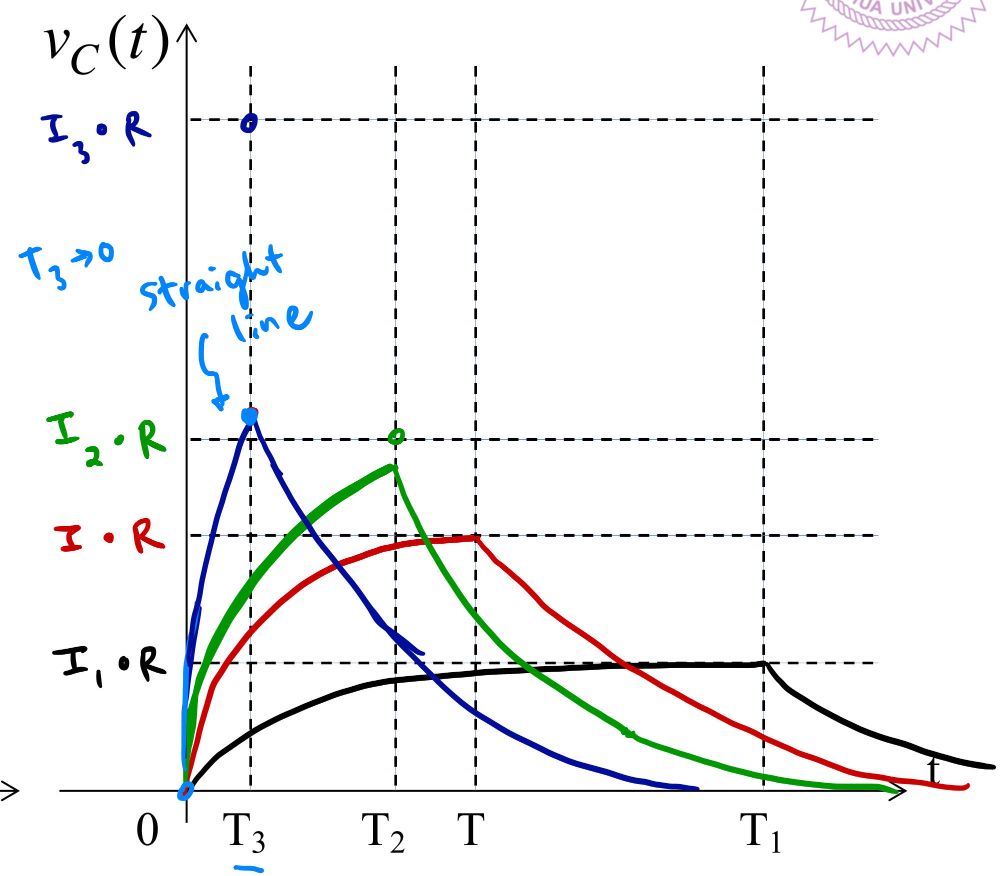
$T_2 \to 0$ $\qquad \frac{Q}{T_2} \to \infty$



- Pulse becomes impulse as $T \to 0$
  - Denoted as $\underline{Q \cdot \delta(t)}$

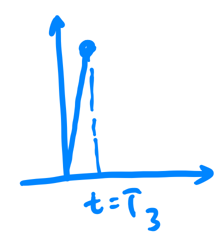
As the Pulse Gets Narrower → Impulse



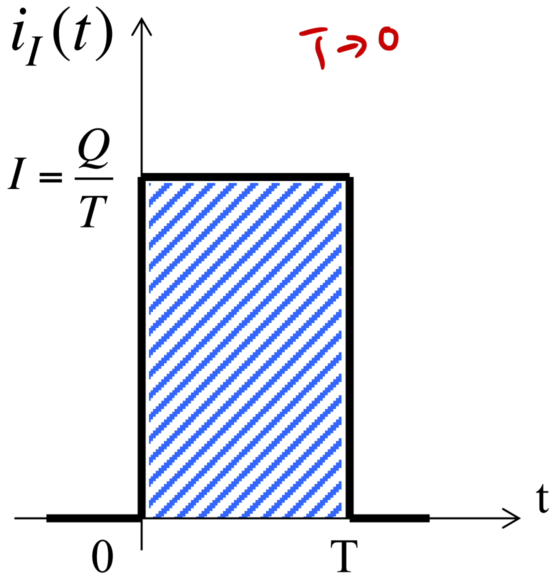


- As $\lim T \to 0$, pulse becomes impulse
  - Denoted by $\delta(t)$
  - Impulse is an important signal type.

$\left.\frac{dv_c}{dt}\right|_{t=0}$



Response to Impulse – Limit Case



$v_c(t) = 0, \quad t < 0$

$v_c(t) = \frac{Q}{T} \cdot R \cdot \left(1 - e^{-t/RC}\right), \quad 0 \le t < T$

$v_c(t) = v_c(T) \cdot e^{-(t-T)/RC}, \quad t > T$

where $v_c(T) = \frac{Q}{T} \cdot R \cdot \left(1 - e^{-T/RC}\right)$

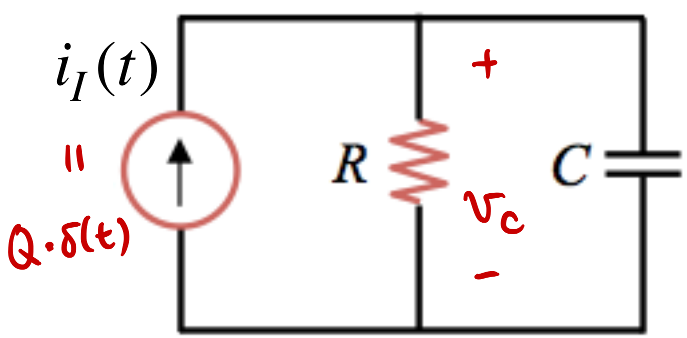
Now consider $T \to 0$, $\quad v_c(T) = ?$

① $\left.\frac{dv_c(t)}{dt}\right|_{t=0} = \frac{Q}{T} \cdot R \cdot \frac{1}{RC} = \frac{Q}{TC}$, then

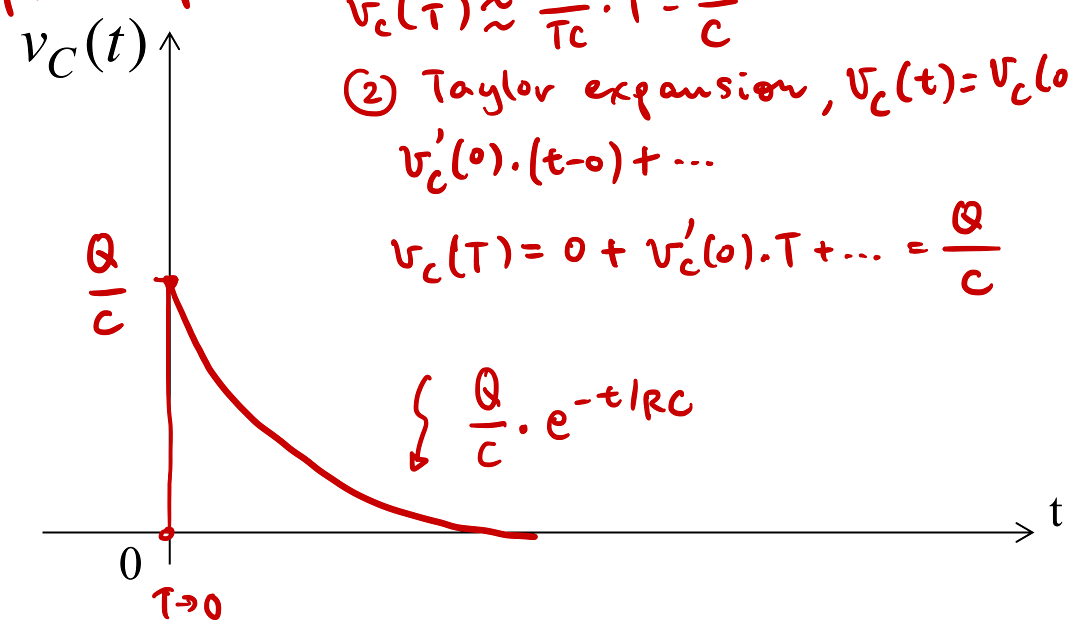
$v_c(T) \approx \frac{Q}{TC} \cdot T = \frac{Q}{C}$

② Taylor expansion, $v_c(t) = v_c(0) + v_c'(0) \cdot (t - 0) + \cdots$

$v_c(T) = 0 + v_c'(0) \cdot T + \cdots = \frac{Q}{C}$



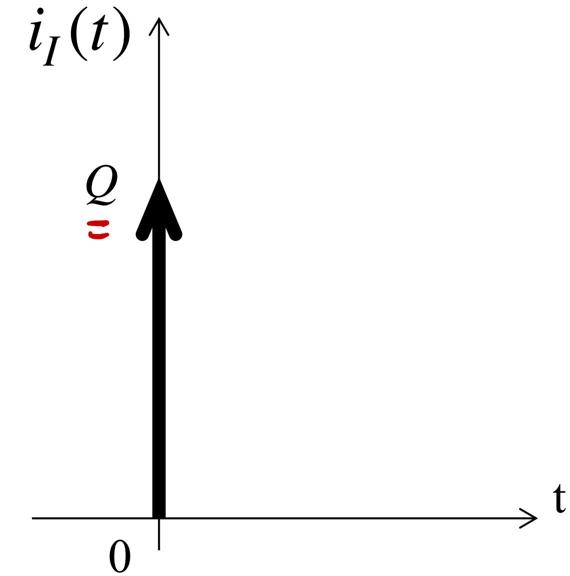
impulse response

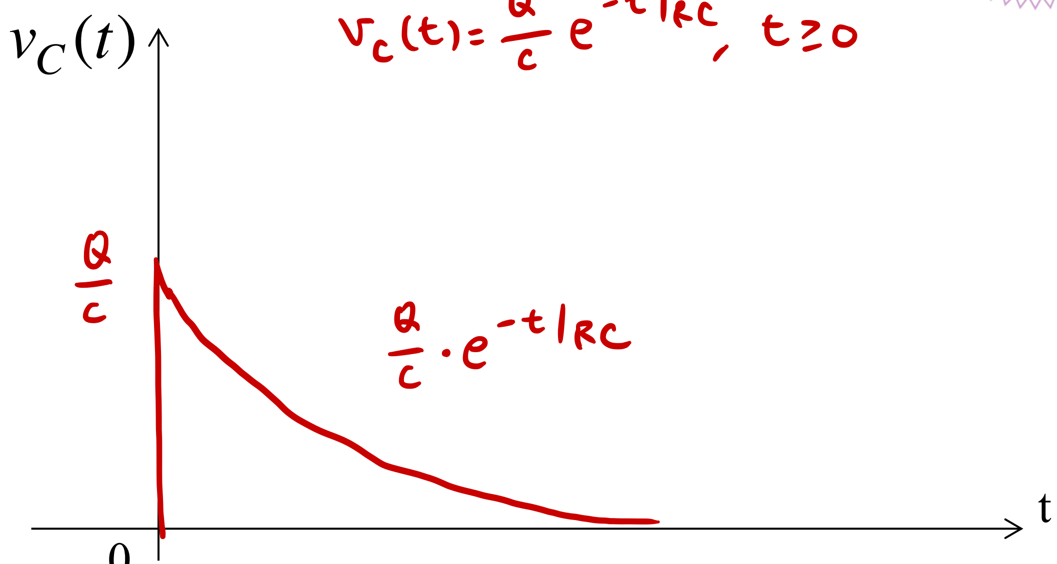


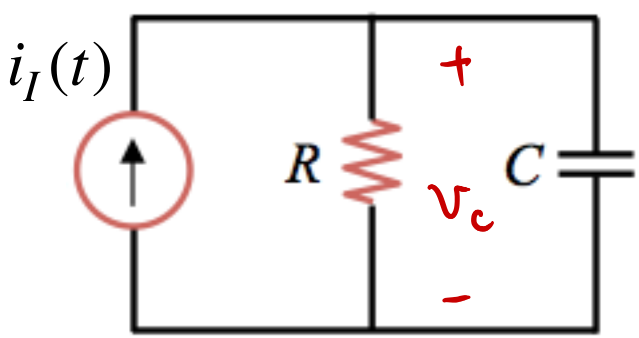
Response to Impulse

Impulse response

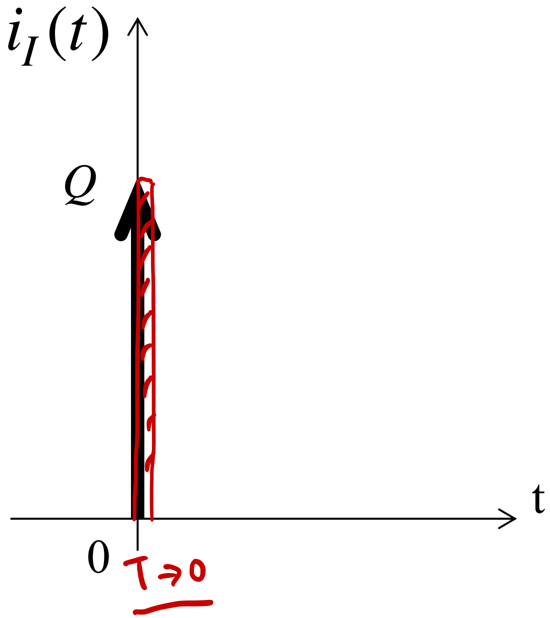
$v_c(t) = \frac{Q}{C} e^{-t/RC}, \quad t \geq 0$

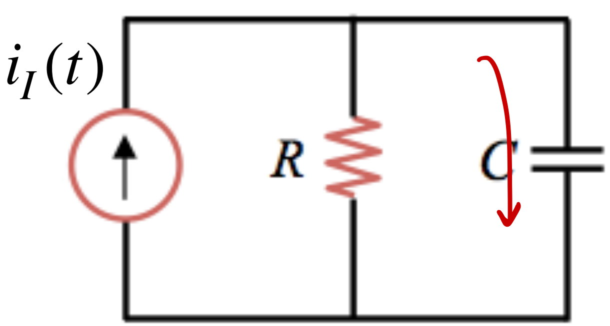






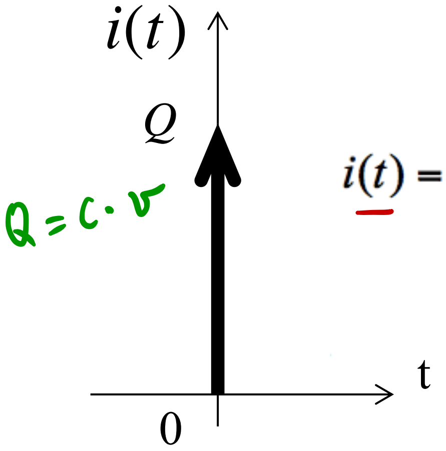
Response to Impulse

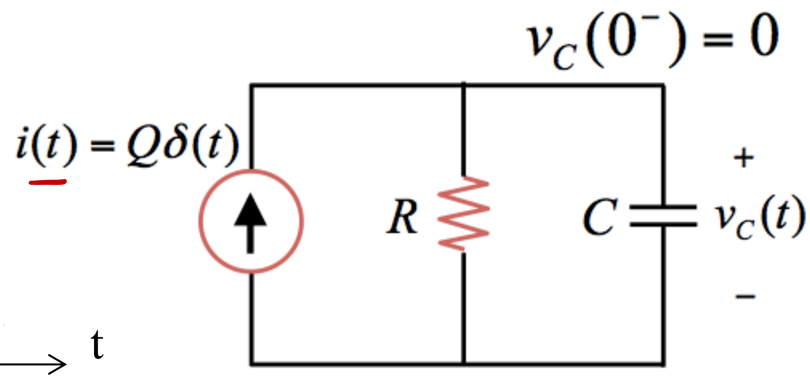


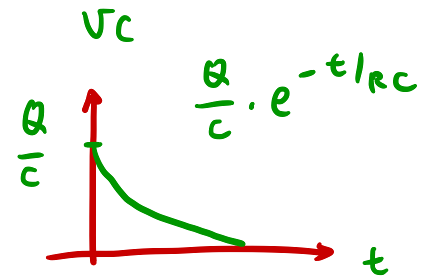


- $v_C(t)$ is largely independent of T or details of the pulse.
  - During delivery, shape of pulse does not matter.
- Capacitor behaves like a short circuit for changes that are much faster than the time constant RC.
- All charge Q gets delivered to capacitor.
  - Recall that, for capacitor, $v = \frac{Q}{C}$
- Impulse sets up initial condition on capacitor.

Current and Voltage Impulses

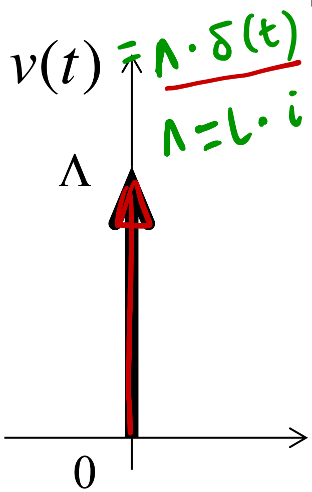


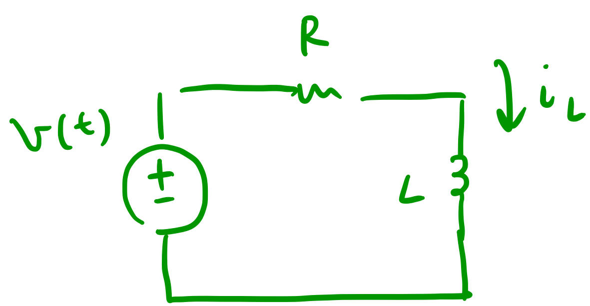


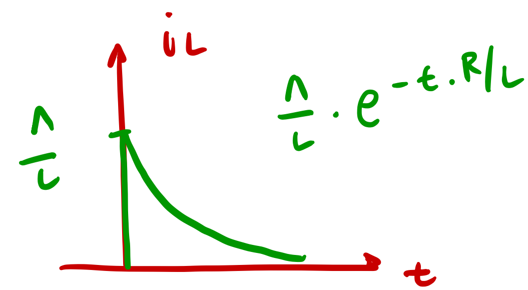


Practice

- Current impulse delivers charge Q.







- Voltage impulse delivers flux linkage.