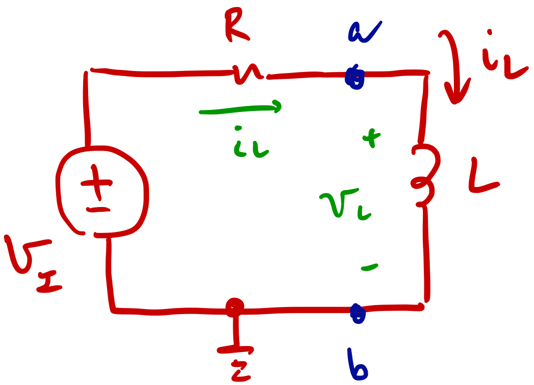
RL Circuit in Series with a Voltage Source



• Given $v_I(t) = V_I\, u(t) = \begin{cases} V_I, & t \ge 0 \\ 0, & t < 0 \end{cases}$

$i_L(t=0) = 0\,A$

Find $i_L(t)$ for $t \ge 0$.

KVL: $V_I - i_L \cdot R - L \cdot \dfrac{di_L}{dt} = 0$

1) Particular: assume $i_{L,p} = K$, $V_I - K \cdot R - L \cdot 0 = 0$

$$\Rightarrow K = \frac{V_I}{R}$$

2) Homogeneous: assume $i_{L,H} = A \cdot e^{st}$

$$Ae^{st} \cdot R + L \cdot A \cdot s \cdot e^{st} = 0 \quad \Rightarrow R + L \cdot s = 0 \Rightarrow s = -\frac{R}{L}$$

$$i_{L,H} = Ae^{-R/L \cdot t}$$

3) Total solution $i_L = \dfrac{V_I}{R} + Ae^{-R/L \cdot t}$

$$i_L(t=0) = 0 = \frac{V_I}{R} + A \Rightarrow A = \frac{-V_I}{R}$$

$$i_L(t) = \frac{V_I}{R} - \frac{V_I}{R} e^{-R/L \cdot t} \quad A.$$

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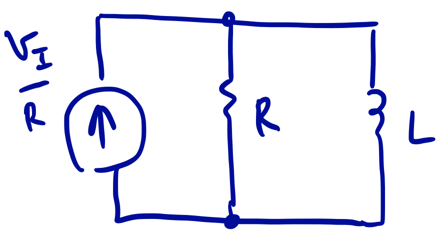


$i_L(t=0) = 0$ A

$i_L(t \to \infty) = \dfrac{V_I}{R}$

L → short

$i_L(t) = \dfrac{V_I}{R} + \left[0 - \dfrac{V_I}{R}\right] \cdot e^{-t/\tau}$

$\tau = L/R$

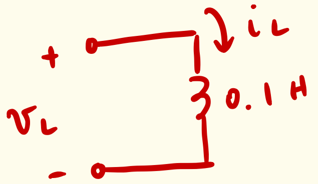
- RC , RL

✓ $x(t) = \underline{x(\infty)} + \left[\underline{x(0) - x(\infty)}\right] \cdot e^{-t/\tau}$

τ : time constant

$\tau = R \cdot C \quad , \quad \frac{L}{R}$

Example 1.

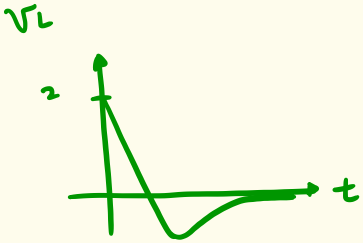


Given $i_L = 20te^{-2t}$ A for $t > 0$

$i_L(0) = 0$

Find v_L, power, energy.

$$v_L = 0.1 \frac{di_L}{dt} = 0.1 \cdot 20 \left(-2te^{-2t} + e^{-2t}\right) = 2e^{-2t}(1-2t)$$



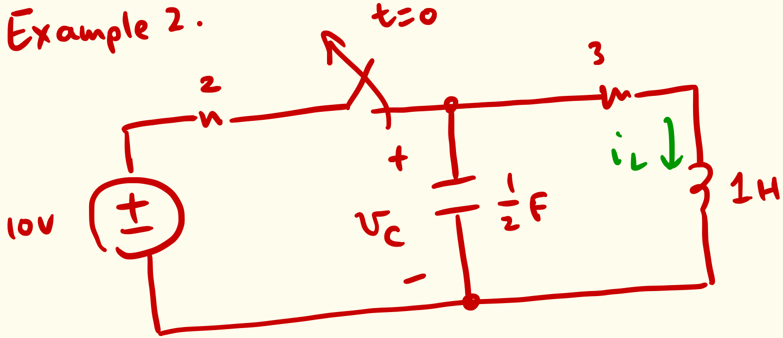
$$p = i_L \cdot v_L = 20te^{-2t} \cdot 2e^{-2t}(1-2t)$$

$$= 40t \cdot e^{-4t} \cdot (1-2t) \quad W, \quad t > 0$$

$$\text{Energy} = \frac{1}{2} \cdot L \cdot i^2 = \frac{1}{2} \cdot 0.1 \cdot 400t^2 \cdot e^{-4t}$$

$$= 20t^2 \cdot e^{-4t} \quad J, \quad t > 0$$

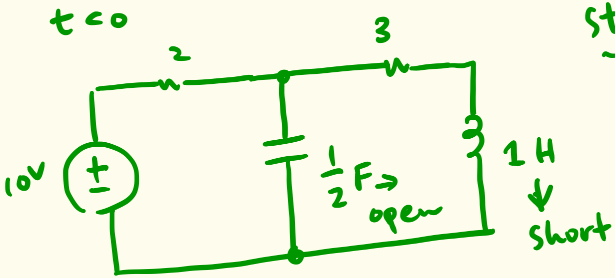
Example 2.



Find $i_L(0^+)$, $v_C(0^+)$

$i_L(0^+) = i_L(0^-)$ ✓

$v_C(0^+) = v_C(0^-)$ ✓

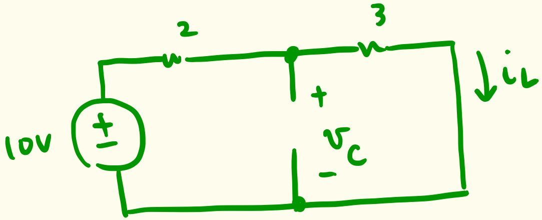


Steady-state for $t<0$

$$\frac{dv_C}{dt} = 0, \quad \frac{di_L}{dt} = 0$$

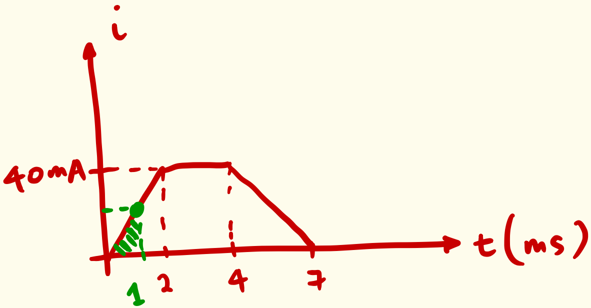
$$i_L(0^-) = \frac{10}{2+3} = 2\ \text{A} = i_L(0^+)$$

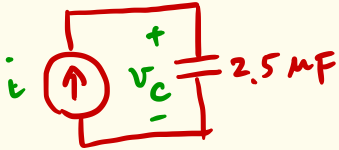
$$v_C(0^-) = \frac{10}{2+3} \times 3 = 6\ \text{V} = v_C(0^+)$$



Example. $v_c(t=0) = -20V$, Find $v_c(t)$ at $t = 1ms, 2ms, 5ms$. ✓

Power, energy





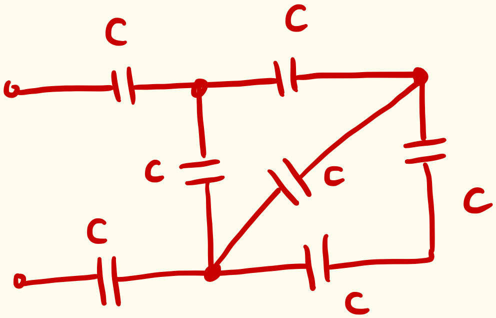
$$v_c(t) = v_c(0) + \frac{1}{c} \cdot \int_0^t i(\tau)\,d\tau = -20 + \frac{1}{2.5 \times 10^{-6}} \cdot \underbrace{\int_0^t i(\tau)\,d\tau}_{\text{area}}$$

$$v_c(t=1ms) = -20 + \frac{1}{2.5 \times 10^{-6}} \times \frac{1}{2} \times 1m \times 20m$$

$$= -16\,V$$

Example 4. Practice

* Series / parallel of R, L, C



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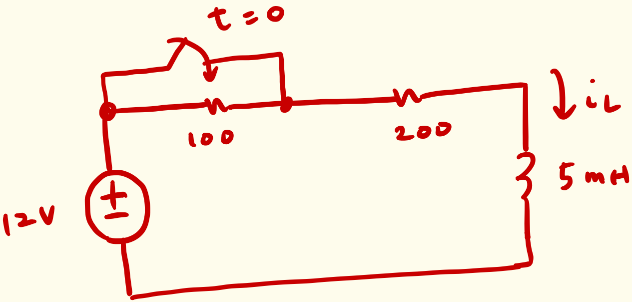
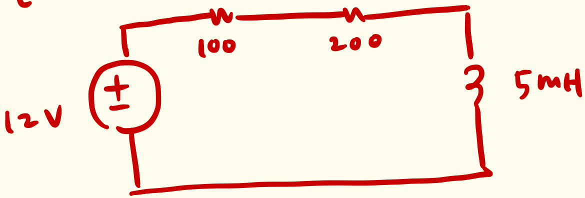
$C_{eq} = \frac{8}{21} C$

Example 5.

Find $i_L(t)$ for $t > 0$.



$t < 0$

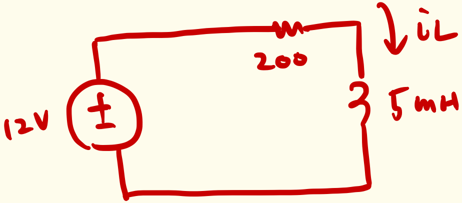


$v_L = 0$

$$i_L = \frac{12}{100 + 200} = 40\,mA$$

$$i_L(0^-) = i_L(0^+) = \underline{40\,mA}$$

$t > 0$



KVL: $v_L + i_L \cdot 200 = 12$, $v_L = 5m \cdot \frac{di_L}{dt}$

$$\Rightarrow \quad 5m \cdot \frac{di_L}{dt} + 200 \cdot i_L = 12$$

$$i_L(t \to \infty) = \frac{12}{200} = 60\,mA$$

$$5m \cdot \frac{di_L}{dt} + 200\, i_L = 12$$ ✓

① Particular: assume $i_{L,p} = K$, $5m \cdot 0 + 200 \cdot K = 12$, $\Rightarrow i_{L,p} = \frac{12}{200}$

$= 0.06$

② Homogeneous, assume $i_{L,H} = Ae^{st}$, $5m \cdot A \cdot s \cdot e^{st} + 200 \cdot Ae^{st} = 0$

$5m \cdot s + 200 = 0 \Rightarrow s = -40000$

③ Total solution $i_L = 0.06 + Ae^{-40000t}$

$i_L(t=0^+) = 40m = 0.06 + A \Rightarrow A = -0.02$

$i_L(t) = 0.06 - 0.02 \cdot e^{-40000t}$ A

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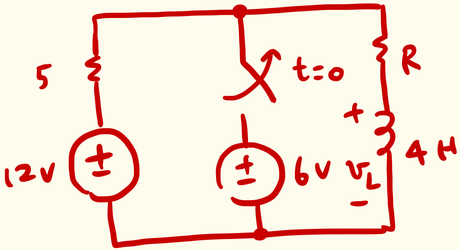
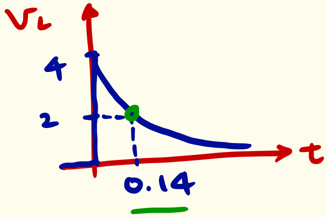
$$i_L(t) = 0.06 + (0.04 - 0.06) \cdot e^{-t/\tau}$$

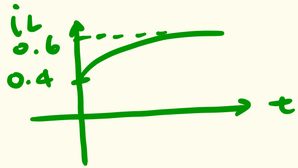
$$\tau = \frac{L}{R} = \frac{5m}{200}$$

Example 6.

Given v_L, Find v_L, R, i_L





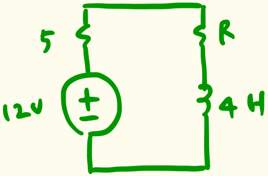


①

$$v_L = \begin{cases} 0 & t<0 \\ \underline{4e^{-at}}, & t \geq 0 \end{cases} \quad \checkmark$$

$$2 = 4e^{-a \cdot 0.14} \Rightarrow a = 5$$

② $t>0$



$$\underline{v_L = 0 + (4-0) \cdot e^{-t/\tau}}, \quad \tau = \frac{4}{5+R} = \frac{1}{5} \Rightarrow R = 15$$

③

$$i_L = \frac{1}{L}\int_0^t v_L \, dt + i(0)$$

$$= \frac{1}{4}\int_0^t 4e^{-5t} \, dt + 0.4$$

$$= \underline{0.6 - 0.2e^{-5t}}$$

Practice $i_L(0)$
$i_L(\infty)$

$$i_L(t) = i_L(\infty) + [i_L(0) - i_L(\infty)] \cdot e^{-t/\tau}$$

$$\tau = \frac{L}{R}$$