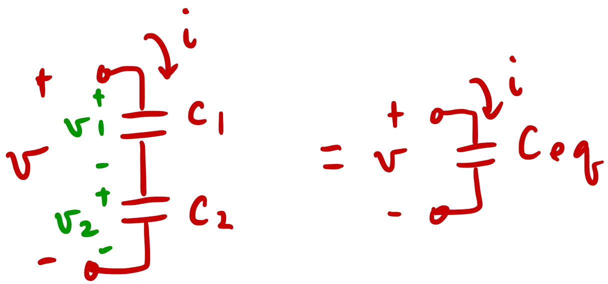
Series and Parallel Connections

$$i = C\frac{dv}{dt}$$



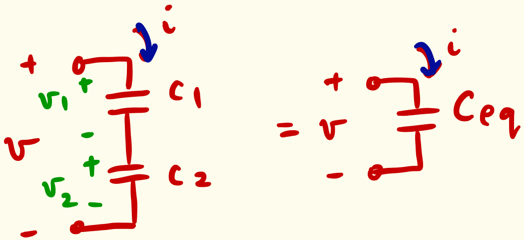
$$v = v_1 + v_2 = \frac{1}{C_1}\cdot\int_{t_0}^{t} i\,d\tau + v_1(t_0) + \frac{1}{C_2}\int_{t_0}^{t} i\,d\tau + v_2(t_0)$$

$$= \left(\frac{1}{C_1} + \frac{1}{C_2}\right)\cdot\int_{t_0}^{t} i\,d\tau + \left[v_1(t_0) + v_2(t_0)\right]$$

$$= \frac{1}{C_{eq}}\cdot\int_{t_0}^{t} i\,d\tau + \Sigma\, v_n(t_0)$$

$$= \frac{1}{C_{eq}}\cdot\int_{t_0}^{t} i\,d\tau + v(t_0)$$

$$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}, \quad v(t_0) = \Sigma\, v_n(t_0) = v_1(t_0) + v_2(t_0)$$

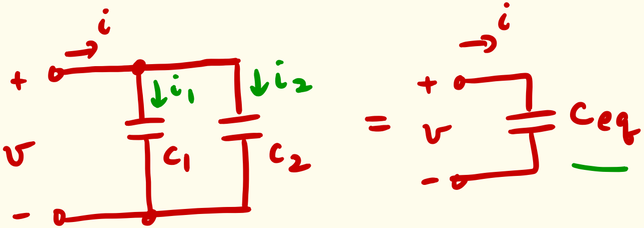


$$Q = \int_{-\infty}^{t} i(\tau)\, d\tau$$

$$Q_1 = Q_2 = Q, \quad Q = C \cdot v$$

$$\Rightarrow \quad C_1 \cdot v_1 = C_2 \cdot v_2$$

$$v = v_1 + v_2 = \frac{Q}{C_1} + \frac{Q}{C_2} = \frac{Q}{C_{eq}} \quad \Rightarrow \quad C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}$$



$$i = i_1 + i_2 = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt}$$

$$= C_{eq} \cdot \frac{dv}{dt}$$

$$\Rightarrow C_1 + C_2 = C_{eq}$$

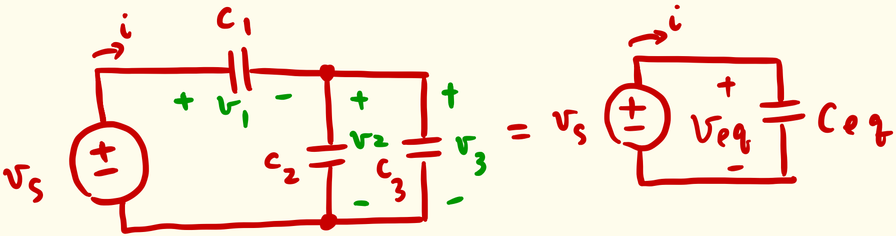
$$i = i_1 + i_2$$

$$\Rightarrow Q = Q_1 + Q_2$$

$$C_{eq} \cdot v = C_1 \cdot v + C_2 \cdot v$$

$$\Rightarrow C_{eq} = C_1 + C_2$$

Example. $C_1 = C_2 = C_3 = 2\ mF$, $v_1(0) = 10V$, $v_2(0) = v_3(0) = 20V$, Find C_{eq}



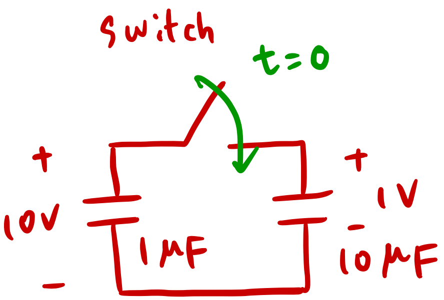
$$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2 + C_3}} = \frac{4}{3}\ mF$$

$$v_{eq}(0) = v_1(0) + v_2(0) = 10 + 20 = 30\ V$$

Charge Sharing in Capacitors

$E = \frac{1}{2} C v^2$

- $C_1 = 1\ \mu\text{F}, C_2 = 10\ \mu\text{F}$ and $v_1 = 10\ \text{V}, v_2 = 1\ \text{V}$
- How much energy is stored before the switch is closed?
- How much energy is lost during the switch closure?

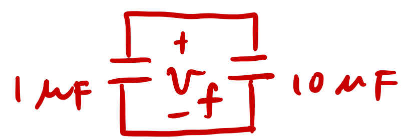


1. $E_1 = \frac{1}{2} \cdot 1\mu \cdot 10^2 = 50\ \mu J$

$E_2 = \frac{1}{2} \cdot 10\mu \cdot 1^2 = 5\ \mu J$

$E_{total} = E_1 + E_2 = 55\ \mu J$

2. Q is the same before t=0 and after t=0.



$Q = 1\mu \cdot 10 + 10\mu \cdot 1 = 1\mu \cdot v_f + 10\mu \cdot v_f$

$\Rightarrow v_f = \frac{20}{11}\ V$

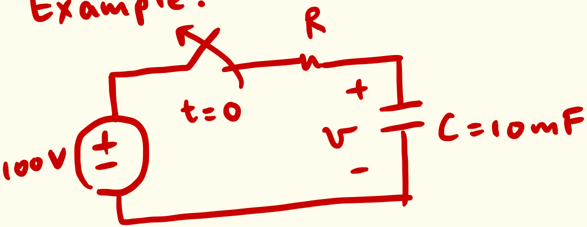
$E_{total} = \frac{1}{2} \cdot 1\mu \cdot v_f^2 + \frac{1}{2} \cdot 10\mu \cdot v_f^2 = 18.2\ \mu J$

3. $Loss = 55\mu - 18.2\mu = 36.8\mu J$

Example.

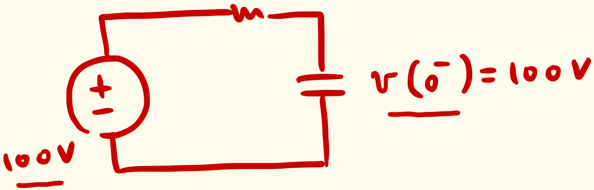
Find v and the energy stored in the capacitor at $\underline{t=0^+}$.

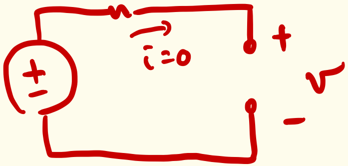
$\underline{t=0^-}$



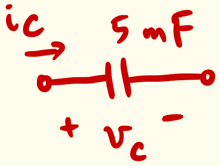
$v(0^+) = v(\underline{0^-}) = 100\ V$

$$E = \frac{1}{2} \cdot 10m \cdot 100^2 = 50\ J$$

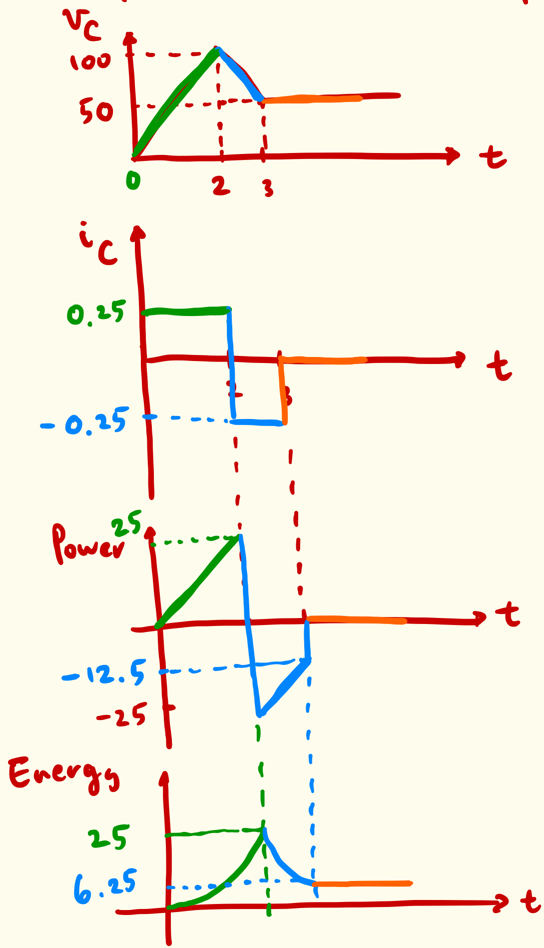




Example.



Given v_c. plot i_c, power, energy.



$$i_c = 5m \cdot \frac{dv_c}{dt}$$

$$= 5m \cdot \frac{100}{2} = 0.25$$

$$5m \cdot \frac{-50}{1} = -0.25$$

$$p = v \cdot i$$

$$E = \int p\, dt$$

$0 < t < 2$

$$E = \int_0^t 12.5t\, dt$$

$$= \frac{12.5}{2} t^2$$